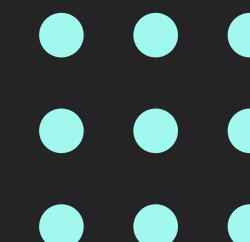




Bienvenidos

Integrantes

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- Luis Huachaca
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Serie de Fourier

- Definicion de serie de Fourier:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right)$$

- Coeficientes:

$$a_0 = \frac{1}{L} \int_{-L}^L f(x) dx$$

$$a_n = \frac{1}{L} \int_{-L}^L f(x) \cos\left(\frac{n\pi x}{L}\right) dx$$

$$b_n = \frac{1}{L} \int_{-L}^L f(x) \sin\left(\frac{n\pi x}{L}\right) dx$$

Sacando : a_0

$$a_0 = \frac{1}{L} \int_{-L}^L f(x) dx$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} x dx$$

$$a_0 = \frac{1}{\pi} \left(\frac{\pi^2}{2} - \frac{-\pi^2}{2} \right)$$

$$a_0 = \frac{1}{\pi} \left| \frac{x^2}{2} \right|_{-\pi}^{\pi}$$

$$a_0 = 0$$

$$a_0 = \frac{1}{\pi} \left(\frac{\pi^2}{2} - \frac{(-\pi)^2}{2} \right)$$

Sacando : a_n

$$a_n = \frac{1}{L} \int_{-L}^L f(x) \cos\left(\frac{n\pi x}{L}\right) dx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} x \cos\left(\frac{n\pi x}{\pi}\right) dx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} x \cos(nx) dx$$

$$a_n = \frac{1}{\pi} \left(\left(\frac{1}{n} x \sin(nx) \Big|_{-\pi}^{\pi} \right) - \left(\frac{1}{n} \int_{-\pi}^{\pi} \sin(nx) dx \right) \right)$$

$$a_n = \frac{1}{\pi} \left(\frac{1}{n} \pi \sin(n\pi) - \frac{1}{n} (-\pi) \sin(-n\pi) + \frac{1}{n^2} \cos(nx) \Big|_{-\pi}^{\pi} \right)$$

$$a_n = \frac{1}{\pi n^2} (\cos(n\pi) - \cos(-n\pi)) = \frac{1}{\pi n^2} (\cos(n\pi) - \cos(n\pi)) = 0$$

Sacando : a_n

$$b_n = \frac{1}{L} \int_{-L}^L f(x) \sin\left(\frac{n\pi x}{L}\right) dx$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} x \sin\left(\frac{n\pi x}{\pi}\right) dx$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} x \sin(nx) dx$$

$$b_n = \frac{1}{\pi} \left(-\frac{1}{n} x \cos(nx) \Big|_{-\pi}^{\pi} + \frac{1}{n} \int_{-\pi}^{\pi} \cos(nx) dx \right)$$

$$b_n = \frac{1}{\pi} \left(-\frac{1}{n} \pi \cos(n\pi) + \frac{1}{n} (-\pi) \cos(-n\pi) + \frac{1}{n^2} \sin(nx) \Big|_{-\pi}^{\pi} \right)$$

$$b_n = \frac{1}{\pi} \left(-\frac{2\pi}{n} \cos(n\pi) + \frac{1}{n^2} \sin(n\pi) - \frac{1}{n^2} \sin(-n\pi) \right)$$



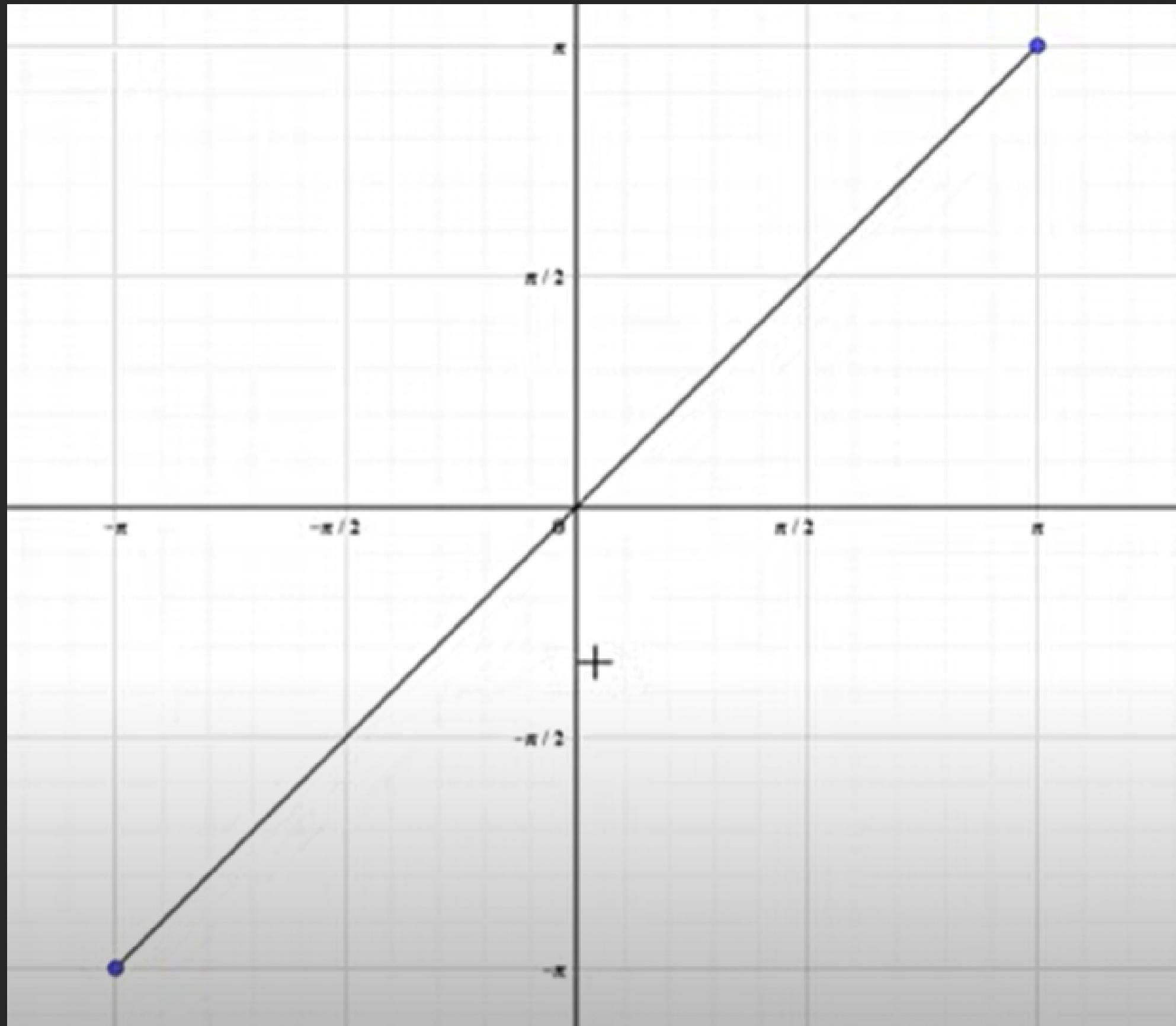
$$b_n = -\frac{2}{n} (-1)^n = \frac{2}{n} (-1)^{n+1}$$

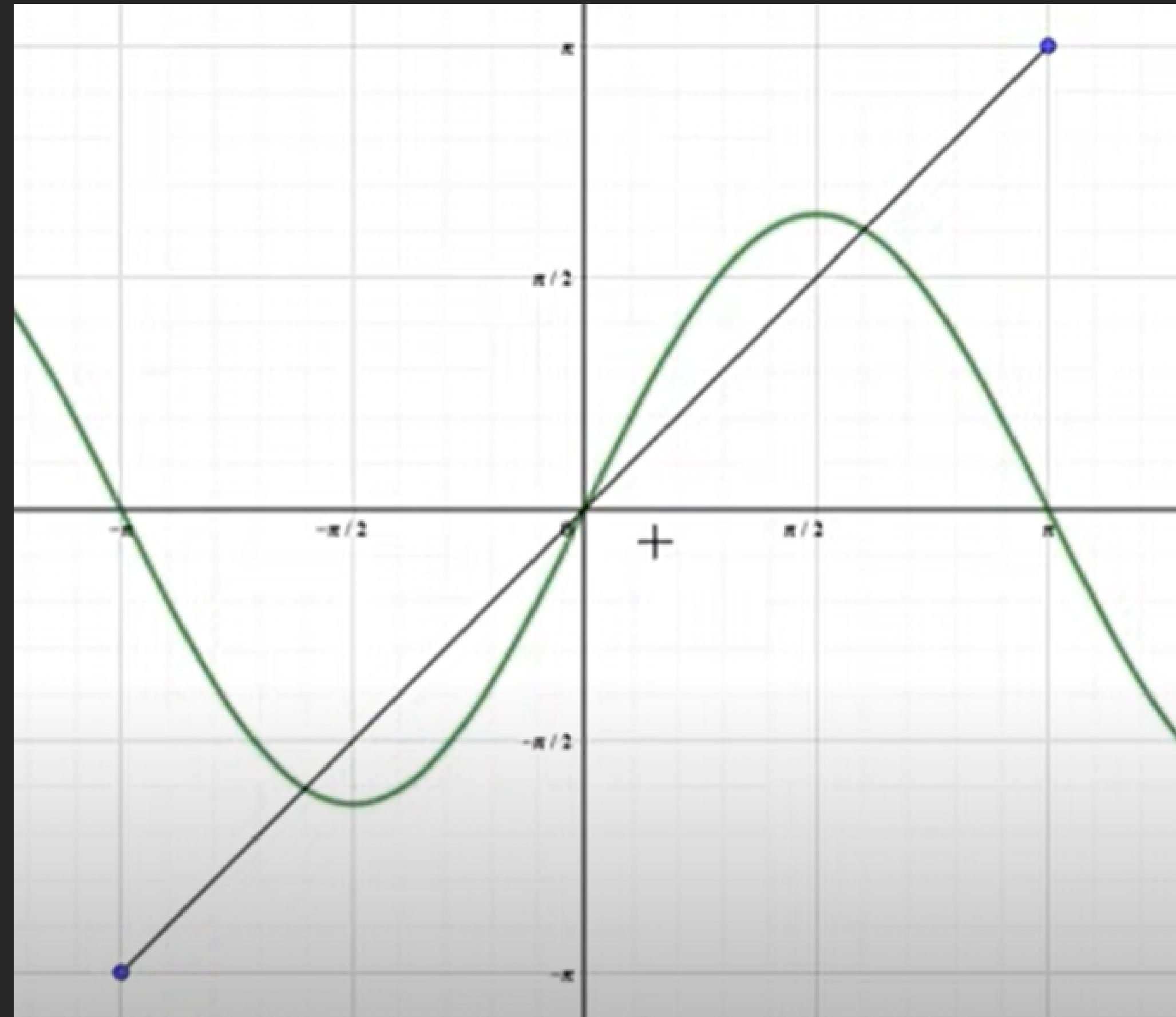
Juntamos...

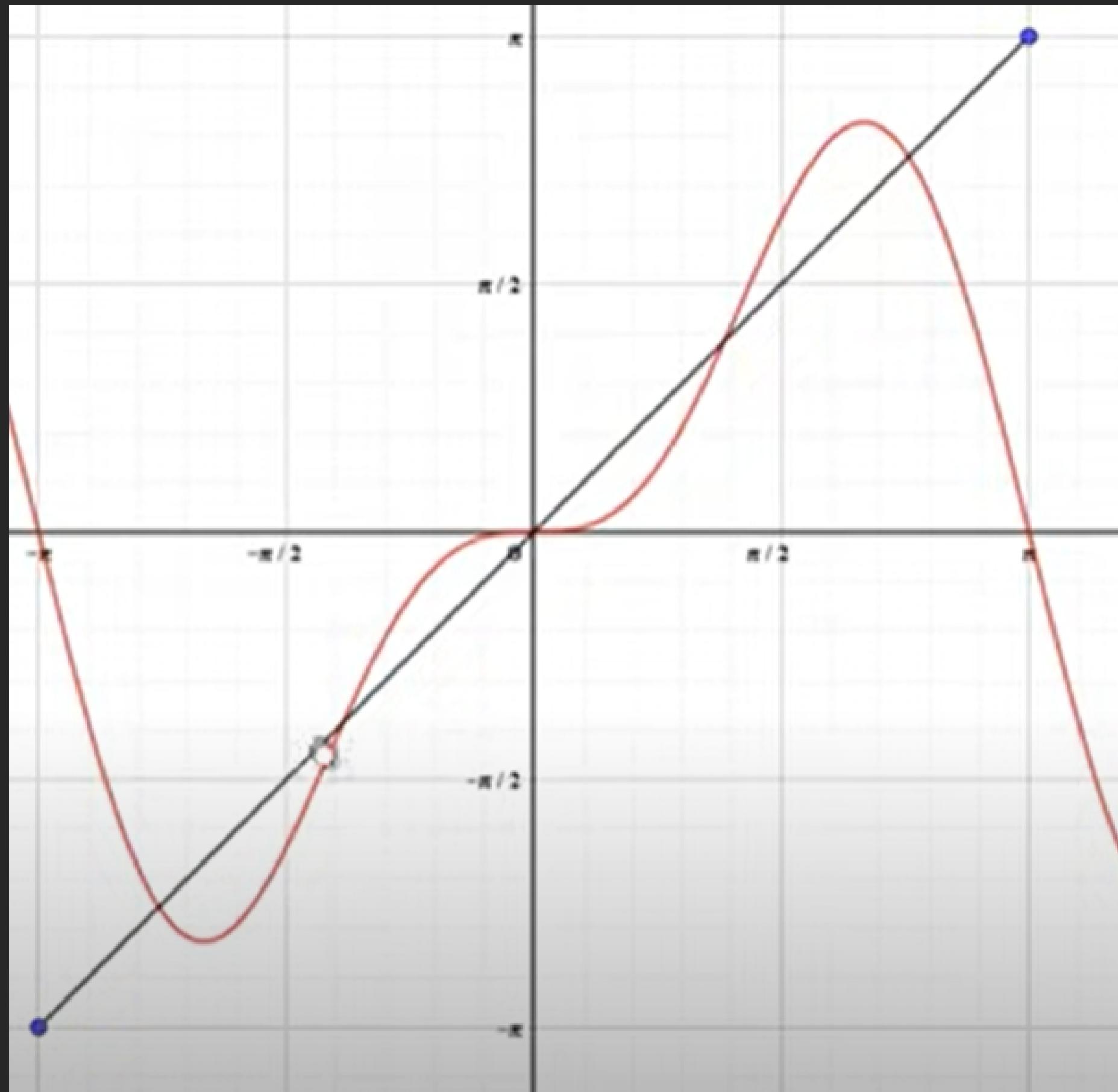
$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right)$$

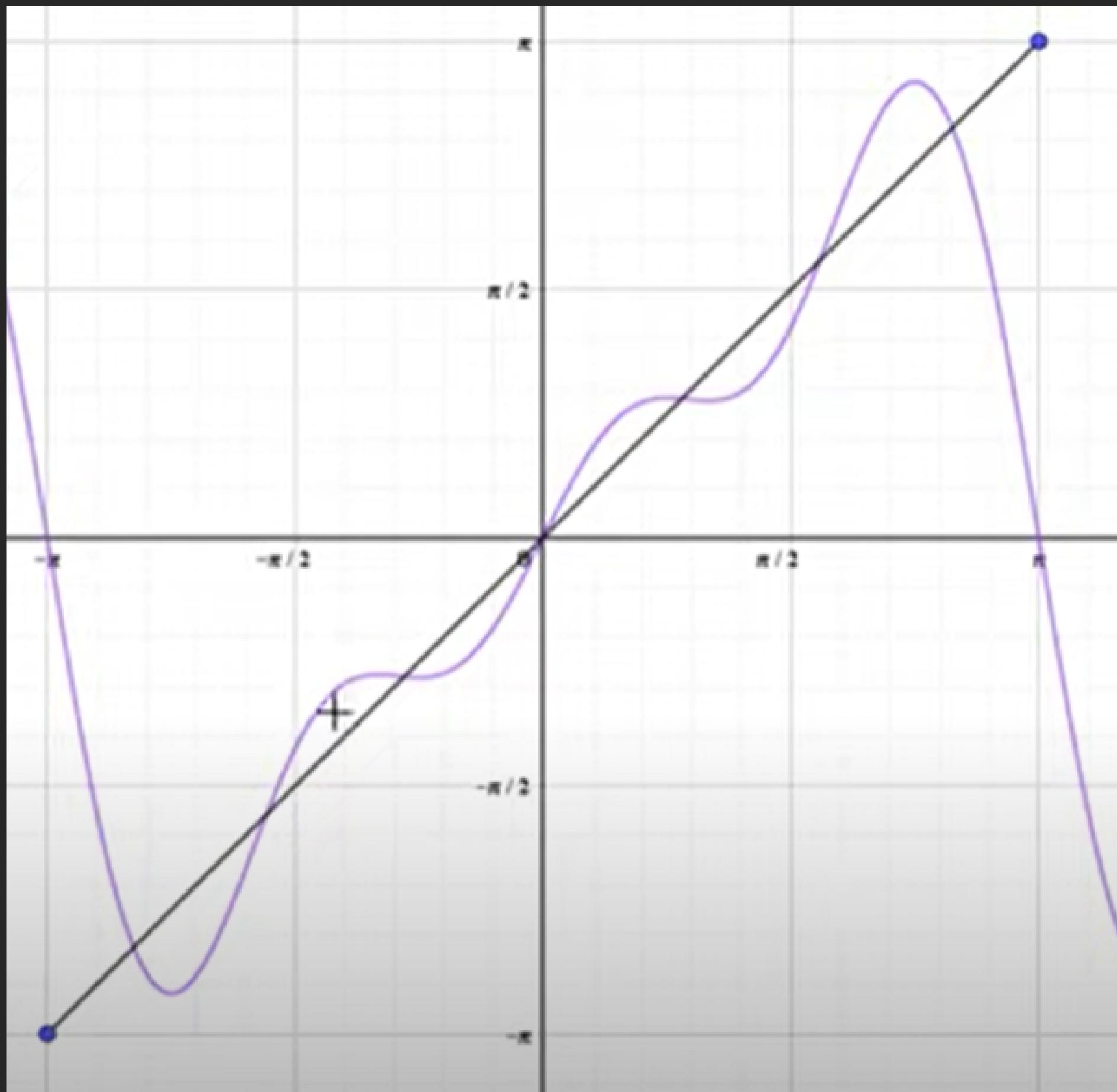
$$f(x) = \sum_{n=1}^{\infty} \frac{2}{n} (-1)^{n+1} \sin(nx)$$

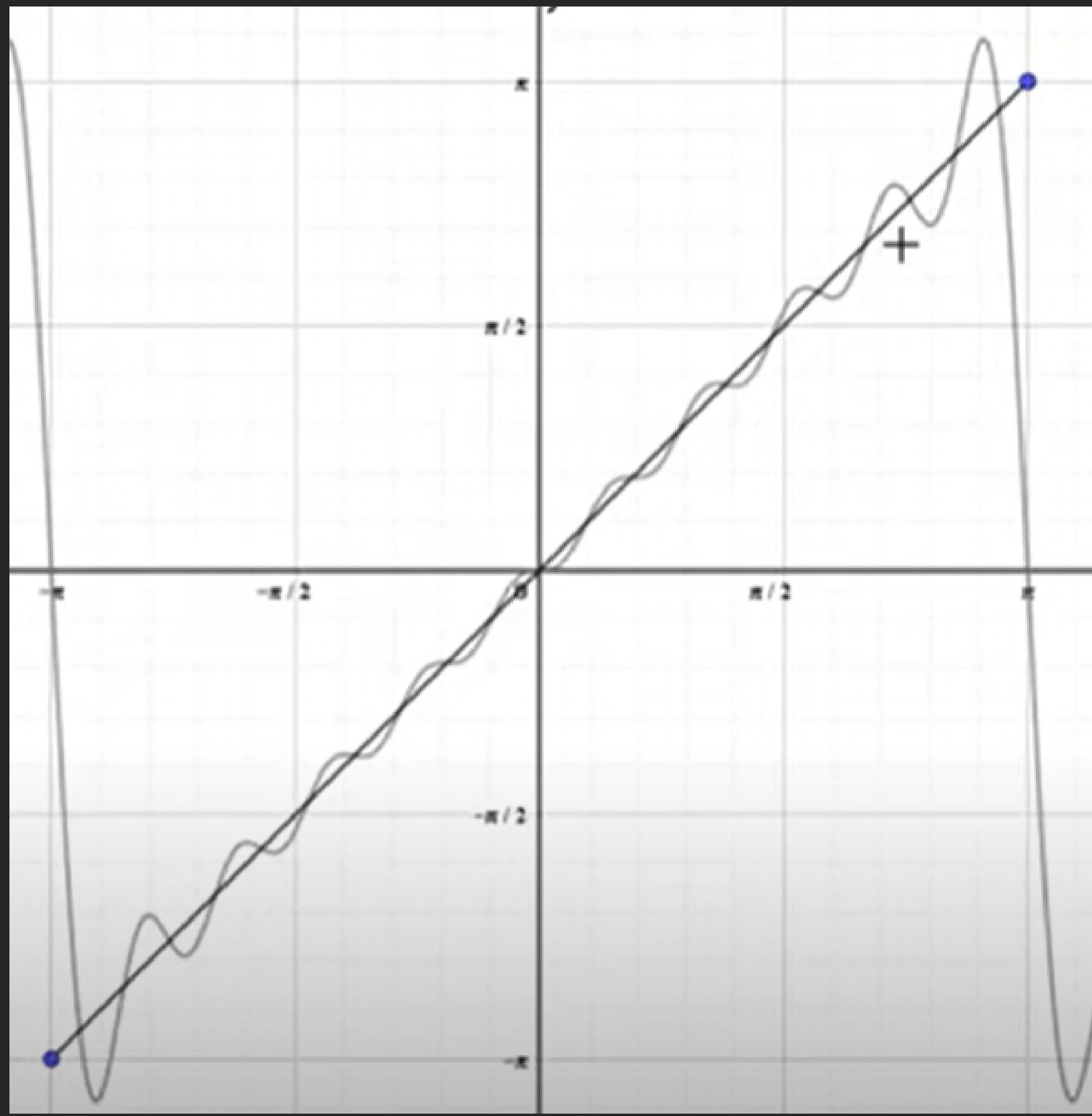
$$x = 2(\sin(x) - \frac{1}{2}\sin(2x) + \frac{1}{3}\sin(3x) - \frac{1}{4}\sin(4x) + \frac{1}{5}\sin(5x) - \dots)$$

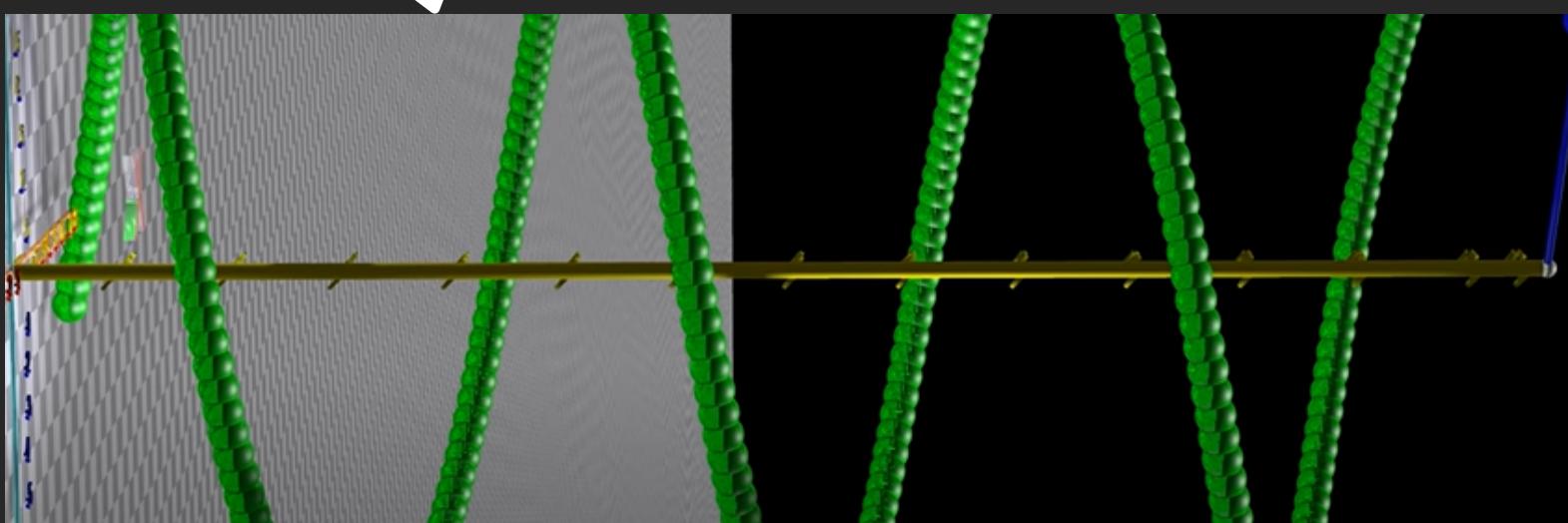
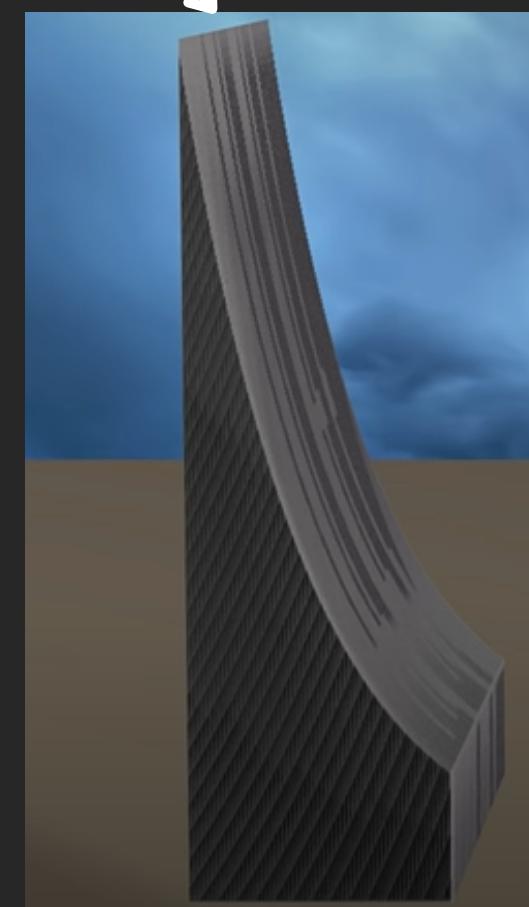
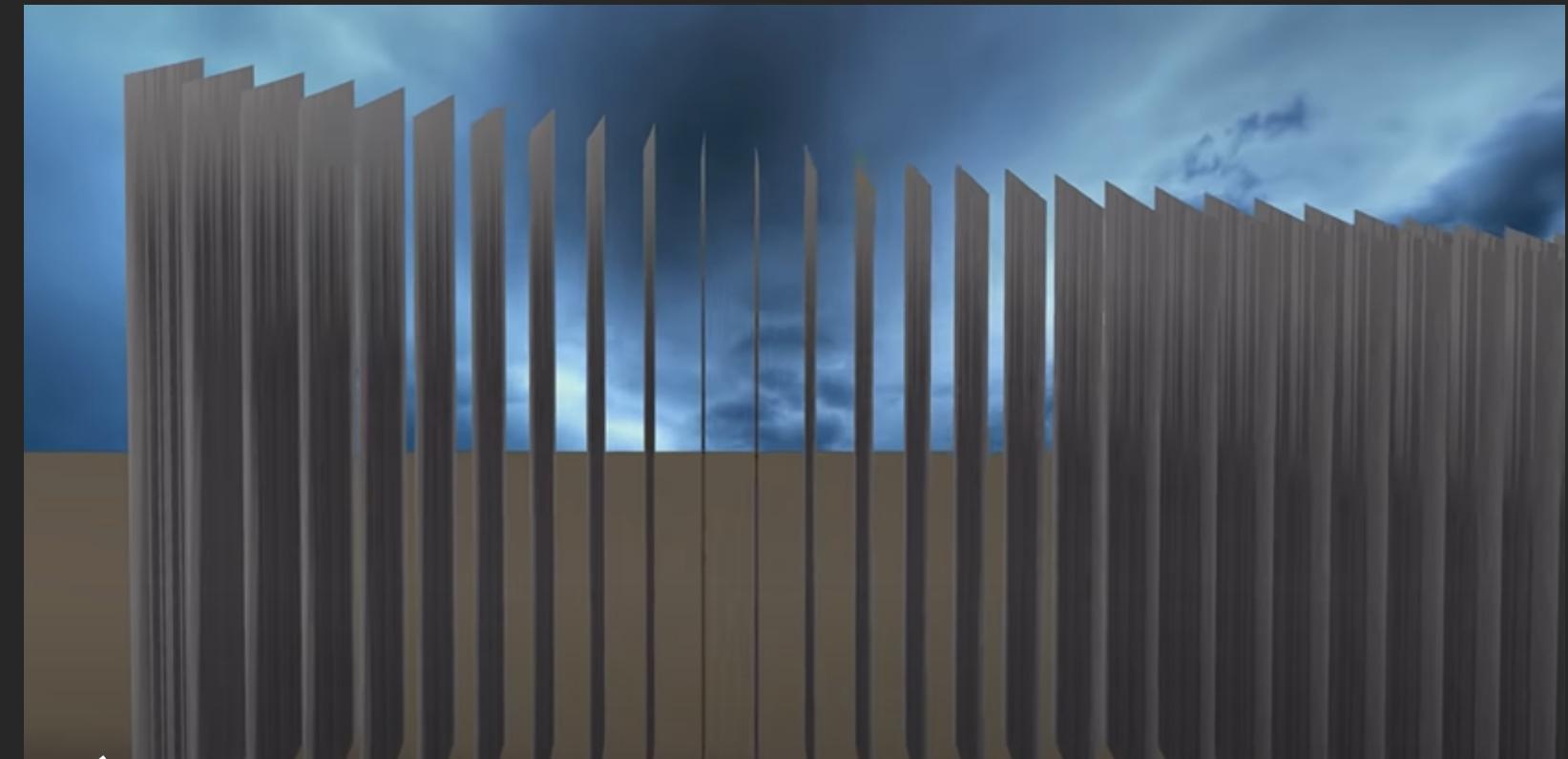
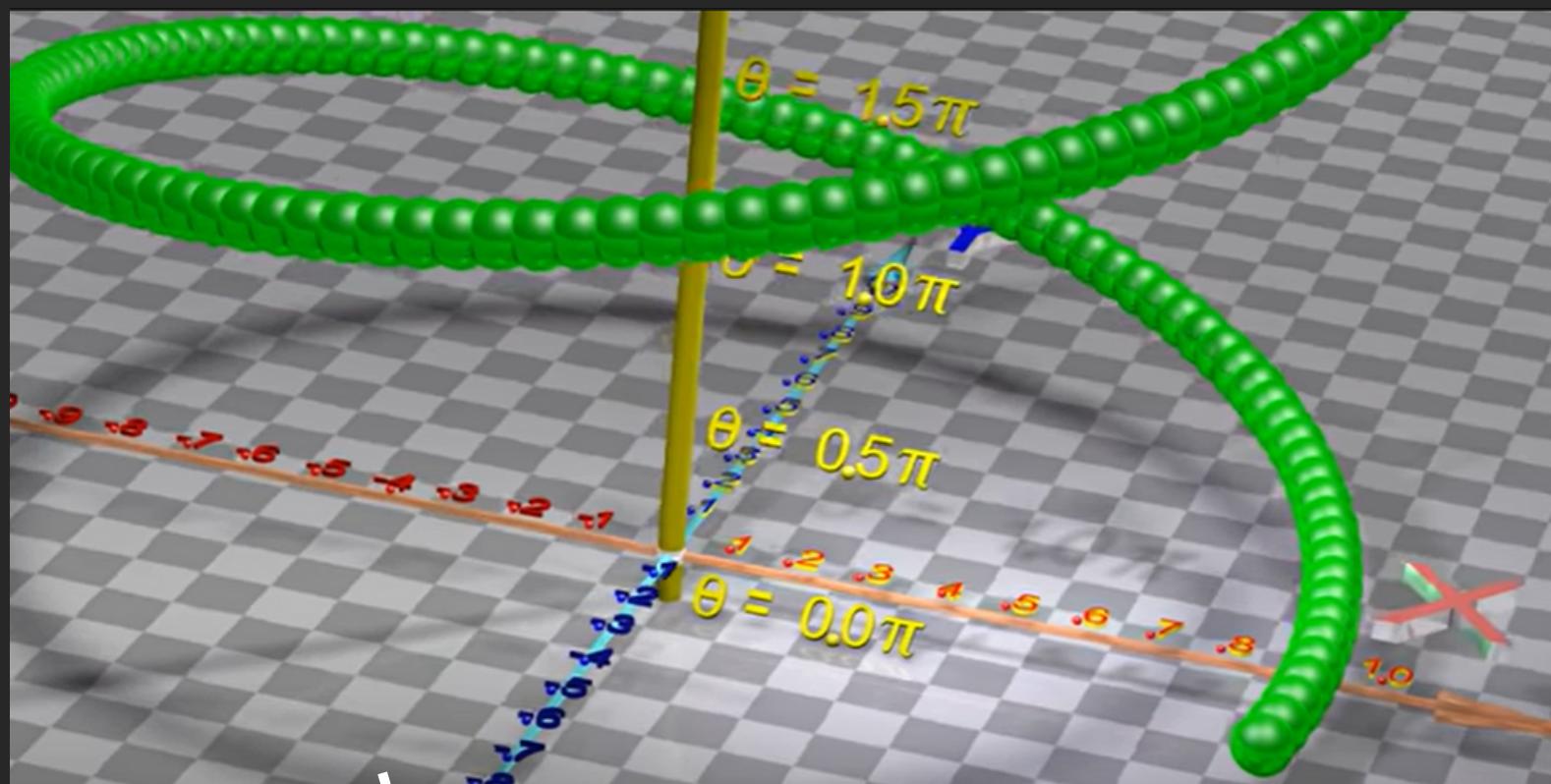




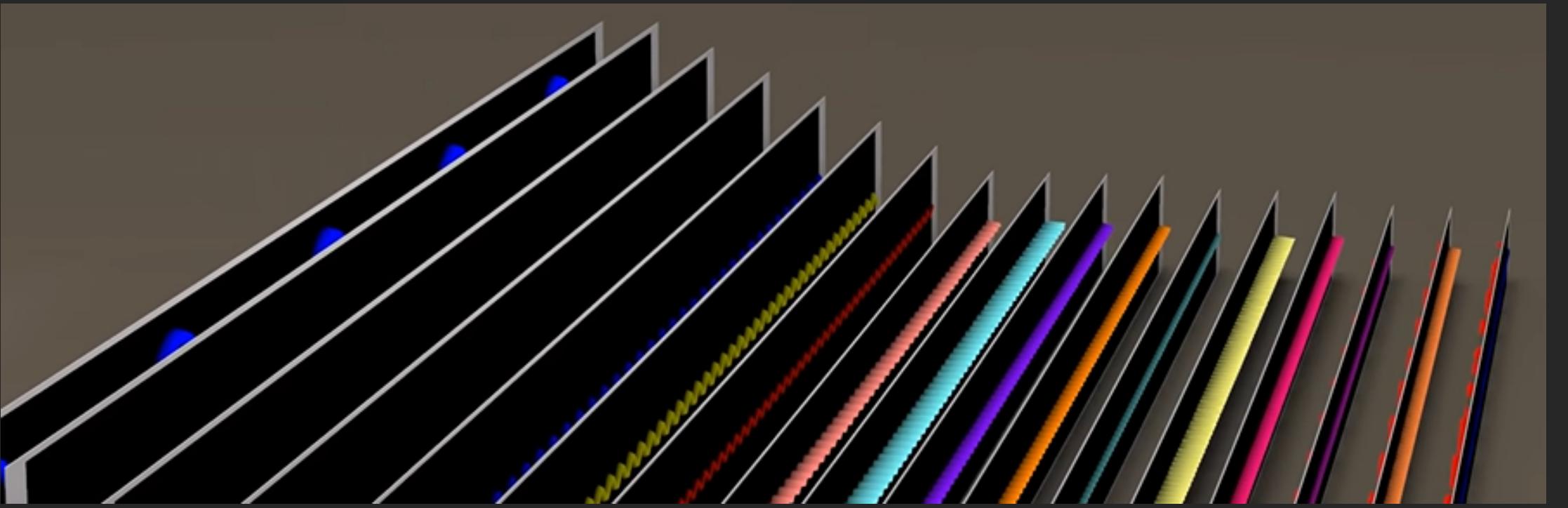






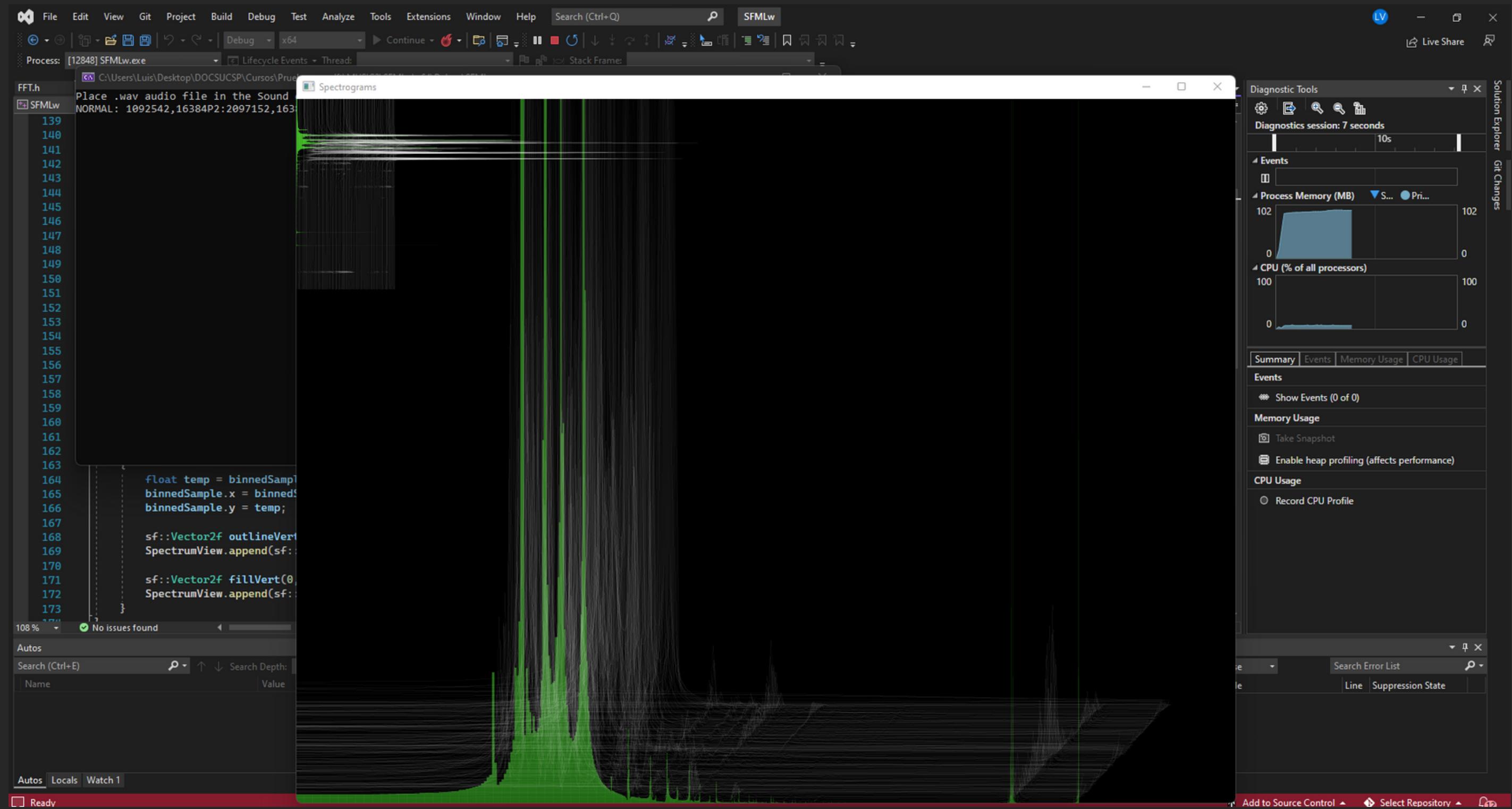


Spectrum con
FFT



Entoonces →

:)



Cooley Tukey

$$X_k = \sum_{m=0}^{N/2-1} x_{2m} e^{-\frac{2\pi i}{N}(2m)k} + \sum_{m=0}^{N/2-1} x_{2m+1} e^{-\frac{2\pi i}{N}(2m+1)k}$$

$$X_k = \sum_{m=0}^{N/2-1} x_{2m} e^{-\frac{2\pi i}{N}(2m)k} + \sum_{m=0}^{N/2-1} x_{2m+1} e^{-\frac{2\pi i}{N}(2m+1)k}$$

$$X_k = \underbrace{\sum_{m=0}^{N/2-1} x_{2m} e^{-\frac{2\pi i}{N/2}mk}}_{\text{DFT of even-indexed part of } x_n} + e^{-\frac{2\pi i}{N}k} \underbrace{\sum_{m=0}^{N/2-1} x_{2m+1} e^{-\frac{2\pi i}{N/2}mk}}_{\text{DFT of odd-indexed part of } x_n} = E_k + e^{-\frac{2\pi i}{N}k} O_k$$

```

void CooleyTukeyFFT::FFT(SampleArray& values)
{
    const size_t N = values.size();
    if (N <= 1) return;

    SampleArray evens = values[std::slice(0, N / 2, 2)];
    SampleArray odds = values[std::slice(1, N / 2, 2)];

    FFT(evens);
    FFT(odds);

    for (size_t i = 0; i < N / 2; i++)
    {
        ComplexVal index = std::polar(1.0, -2 * PI * i / N) * odds[i];
        values[i] = evens[i] + index;
        values[i + N / 2] = evens[i] - index;
    }
}

```

Decimation in time FFT

```
/*Aplicamos bit_reversal en nuestra data*/
int reversal = 0;
for (int i = 0; i < n; i++) {
    reversal = bit_reversal(i);
    if (i < reversal)
        swap(audio_data[i], audio_data[reversal]);
}
```

$$a = \left\{ \left[(a_0, a_4), (a_2, a_6) \right], \left[(a_1, a_5), (a_3, a_7) \right] \right\}$$

```
if (invert) {
    for (complex<double>& x : audio_data)
        x /= n;
}
```

```
for (int len = 2; len <= n; len <= 1) {
    //Identidad de euler
    double ang = 2 * PI / len;
    if (!invert)
        ang = ang * -1;
    complex<double> euler_w(cos(ang), sin(ang));

    //Diagrama de la mariposa
    for (int i = 0; i < n; i += len) {
        complex<double> temp(1);
        for (int j = 0; j < len / 2; j++) {
            //y0=x0+x1*(raiz_de_unidad); y1=x0-x1*(raiz_de_unidad);
            complex<double> x0 = audio_data[i + j];
            complex<double> x1 = audio_data[i + j + len / 2] * temp;
            audio_data[i + j] = x0 + x1;
            audio_data[i + j + len / 2] = x0 - x1;
            temp *= euler_w;
        }
    }
}
```

$$y_0 = x_0 + x_1 \omega_n^k$$

$$y_1 = x_0 - x_1 \omega_n^k,$$



GRACIAS

