A Novel Quantum-Inspired Evolutionary Algorithm for Solving Combinatorial Optimization Problems

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ABSTRACT
In this paper, we propose a novel quantum-inspired evolutionary algorithm, called NQEA, for solving combinatorial optimization problems. NQEA uses a new Q-bit update operator to increase the balance between the exploration and exploitation of the search space. In the operator, first, the Q-bits of each individual in the population are updated based on the personal best measurement of that individual and the best measurement of current generation. Then, a restriction is applied to each Q-bit to prevent the premature convergence of its values. The results of experiments on the 0-1 knapsack and NK-landscapes problems show that NQEA performs better than a classical genetic algorithm, CGA, and two quantum-inspired evolutionary algorithms, QEA and vQEA, in terms of convergence speed and accuracy.

Categories and Subject Descriptors
I.2.8 [**Artificial Intelligence**]: Problem Solving, Control Methods, and Search

General Terms
Algorithms, Experimentation, Theory

Keywords
Evolutionary Algorithms, Quantum Computing, Combinatorial Optimization, Knapsack Problem, NK-landscapes Problem.

1. INTRODUCTION
Using quantum computation concepts to improve the performance of the evolutionary algorithms on the digital computers led to the development of quantum-inspired evolutionary algorithms. Han and Kim [2] proposed QEA for solving combinatorial optimization problems. QEA is based on the concepts and principles of quantum computing such as quantum bits, linear superposition of states, and quantum gate. Defoin-Platel et al. [1] examined the weaknesses of QEA and figured out the problem of premature convergence in this algorithm, which is originated from hitchhiking phenomenon. They resolved this problem by proposing vQEA. With vQEA the exploration of the search space is driven by the generation best measurement. This measurement is continuously renewed each generation, so it allows the algorithm to explore different regions of the search space. Since vQEA does not employ the global best measurement, it is not able to improve this measurement through searching its neighborhood.

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2. NQEA
We propose a novel quantum-inspired algorithm, called NQEA, for solving combinatorial optimization problems. Figure 1 shows the procedure of NQEA.

procedure NQEA
  $t \leftarrow 0$
1. Initialize population $Q(t)$
  while (**not** termination condition) **do**
    $t \leftarrow t+1$
    for each Q-bit individual $q_i(t)$ **do**
2.   Apply the multi-measurement operator to $q_i(t)$ to form the current measurement $c_i(t)$ and update the personal best measurement $b_i(t)$
    end for
3.   Calculate the generation best measurement $\hat{c}(t)$
4.   Calculate the global best measurement $\hat{b}(t)$
    for each Q-bit individual $q_i(t)$ **do**
5.   Apply the restricted update operator to $q_i(t)$
    end for
  end while
6. Display $\hat{b}(t)$
end procedure

Figure 1. Procedure of NQEA

NQEA maintains a population $Q(t)$ of n Q-bit individuals at generation t. Each Q-bit individual $q_i(t) \in Q(t)$ simultaneously represents multiple binary solutions in the search space with different probabilities and is defined as a string of m Q-bits. The first step in the algorithm is to randomly produce an initial population $Q(0)$. Then, at each generation, quantum operators are applied on each Q-bit individual to construct the values for next generation.

The current measurement $c_i(t)$ and personal best measurement $b_i(t)$ are associated with each Q-bit individual $q_i(t)$. $c_i(t)$ is the binary solution formed by measuring $q_i(t)$ at generation t and $b_i(t)$ is the best binary solution formed by measuring $q_i(t)$ since the first generation of the algorithm.

At each generation t, first, the multi-measurement operator, as defined in [3], is applied to each Q-bit individual $q_i(t)$ to form the current measurement $c_i(t)$ and update the personal best measurement $b_i(t)$. Then, the best among the current measurements and the best among the personal best measurements of all Q-bit individuals are chosen and referred to as the generation best measurement $\hat{c}(t)$ and the global best measurement $\hat{b}(t)$, respectively. Finally, the restricted update operator is applied to each Q-bit individual $q_i(t)$ to update the values of its Q-bits. These steps are repeated until some termination condition is met.

In the restricted update operator, first, the values of each Q-bit $q_{i,j}(t)$ are updated by applying the rotation gate used in [2] and then the absolute values are taken. Hence, the range of the rotation angle of $q_{i,j}(t)$ is limited to the first quadrant as shown in Figure 2.

$$q_{i,j}(t+1) = \left| U(\Delta\theta_{i,j}(t)) \cdot q_{i,j}(t) \right| \tag{1}$$

where $\Delta\theta_{i,j}(t)$ is the rotation angle of $q_{i,j}(t)$ towards the “0” or “1” state.

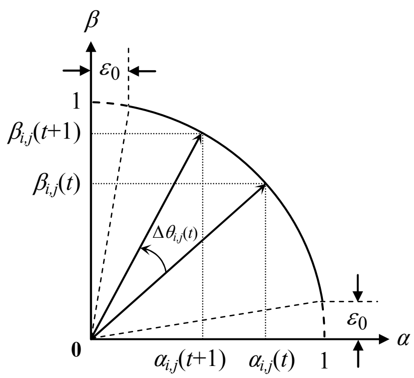

Figure 2. The rotation range of Q-bit $q_{i,j}(t)$

The rotation angle $\Delta\theta_{i,j}(t)$ for each $q_{i,j}(t)$ is calculated as follows:

$$\begin{aligned}\Delta\theta_{i,j}(t) = \theta_1 \cdot r_{i,j}^1(t) \cdot (b_{i,j}(t) - c_{i,j}(t)) + \\ \theta_2 \cdot r_{i,j}^2(t) \cdot (\hat{c}_j(t) - c_{i,j}(t))\end{aligned} \tag{2}$$

where $r_{i,j}^1(t)$ and $r_{i,j}^2(t)$ are two uniformly generated random numbers in the range of [0,1]. θ_1 and θ_2 are two positive coefficients, where θ_1 shows how much confidence a Q-bit individual has in the best measurement of itself, while θ_2 shows how much confidence a Q-bit individual has in the current measurements of other Q-bit individuals.

The rotation gate causes the convergence of each Q-bit towards either “0” or “1” state. However, a Q-bit converged to either “0” or “1” state cannot escape from the state by itself, even though it can be changed by the generation best measurement. Therefore, a restriction is applied to the updated Q-bit $q_{i,j}(t+1)$ to prevent the premature convergence of its values. Applying this restriction causes the convergence of each Q-bit $q_{i,j}(t)$ towards either ε_0 or $\sqrt{1-\varepsilon_0^2}$, instead of 0 or 1.

3. EXPERIMENTS

The performance of NQEA is compared to the performance of a classical genetic algorithm CGA, and two quantum-inspired evolutionary algorithms, QEA [2] and vQEA [1] on the 0-1 knapsack and NK-landscapes benchmark problems.

Tables 1 and 2 report the average fitness of the best solutions found by each algorithm over 10 runs on two instances of the 0-1 knapsack problem with 500 and 1000 items and two instances of NK-landscapes problem with $K = 8$ and $N = 2048, 4096$, respectively. In the experiments of NQEA, the parameters were set to $l_0 = 2$, $\theta_1 = \theta_2 = 0.01\pi$, $n = 10$, and $\varepsilon_0 = 1/m$, where l_0 is the number of measurements. Also, for QEA and vQEA, the settings of the parameters were similar to the setting proposed in [2] and [1], with a population of 10 individuals and $\Delta\theta = 0.01\pi$. In CGA, a tournament selection and a uniform crossover were used. In the experiments of all the above algorithms on the 0-1 knapsack and NK-landscapes problems, the maximum number of generations $t_{\max}$ was set to 1000 and 10000, respectively.

Table 1. Experimental results on the 0-1 knapsack problem

Items	CGA	QEA [2]	vQEA[1]	NQEA
500	2891.4	3008.0	3043.0	**3048.6**
1000	5756.3	5951.0	6062.7	**6116.1**

Table 2. Experimental results on the NK-landscapes problem

K	N	CGA	QEA [2]	vQEA [1]	NQEA
8	2048	0.628	0.645	0.672	**0.679**
8	4096	0.600	0.616	0.661	**0.671**

NQEA has better exploration ability in comparison to QEA because of the use of the generation best measurement in the Q-bit update operation. Moreover, NQEA has better exploitation ability in comparison to vQEA because of the efficient use of the search history.

4. REFERENCES

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