Quantum Sort Algorithm Based On Entanglement Qubits {00, 11}

Ammar Odeh
Department of Computer Science and Information System
Al Maarefa Colleges for Science & Technology
Riyadh, KSA
aoudah@mcst.edu.sa

Eman Abdelfattah
Computing Science and Information Technology Department
Sacred Heart University
Fairfield, CT
Abdelfattahe@sacredheart.edu

Abstract- Quantum algorithms have gained a lot of consideration especially for the need of searching unsorted database in a faster way rather than using the classical search mechanisms. This paper introduces a new quantum sort algorithm by employing the quantum features to expedite the searching process in conventional environments. The proposed efficient sorting algorithm emphasizes entanglement qubits property to sort N positive integers.

***Index Terms* – Superposition, Qubit, Entanglement**

I. Introduction

In 1981, Nobel Laureate Richard Feynman introduced quantum computer idea. He observed that the classical computers can't perform the efficiency and the accuracy of the quantum concepts. Thus, we need to build new quantum machines to follow quantum laws[1]. In 1997, Peter Shor introduced a quantum prime factorization algorithm of integer numbers that achieved a polynomial time complexity. In contrast, all known prime factorization classical algorithms need a complexity of exponential time[2]. Later, Lov Grover introduced a significant quantum database search algorithm by using superposition property to search in $\mathrm{N} = 2^{\mathrm{n}}$ unordered items [3]. The complexity of Grover's algorithm is $\mathrm{O}(\sqrt{\mathrm{N}})$ where the complexity of the best classical algorithm is $\mathrm{O}(\mathrm{N})$.

The three fundamental mathematical differences between classical computing and quantum computing are as follows: superposition, entanglement, and probability and uncertainty.

In superposition, Quantum computers assume all possible states can be available at the same time. In other words, quantum registers can provide 0's and 1's at the same moment[4]. This phenomenon causes quantum systems to be more powerful than classical systems. For instance, let us assume system's register can save two bits then classical computing output is two bits of information. But, Quantum computing can simultaneously process 0 and 1 states then the output will be 4 qubits of information. Therefore, when N qubits are added then we can have $2^{\mathrm{N\ qubits}}$ of information generated. Equation 1 represents the superposition of qubit[5]. Data (0) has $|a_0|^2$ probability and data (1) has $|a_1|^2$ probability.

$$|\psi> = a_0|0> + a_1|1> \quad (1)$$

Entanglement is a property of two particles, where if quantum system observes the first one then system can determine the second one. For example, if you have two qubits in superposition and the first qubit value is 0 then instantly the second qubit is going to be 1 without looking at it[6]. Albert Einstein described this characteristic as “Spooky action at distance”[7].

For probability and uncertainty, Quantum system introduces probability to each of the possible states. Quantum system performs computable probability corresponding to the likelihood that any given state can occur[8]. Round by round, this probability is going to increase based on some quantum operations until it gives the system highly reasonable amount of certainty[9]. Figure 1 shows the probability principle.

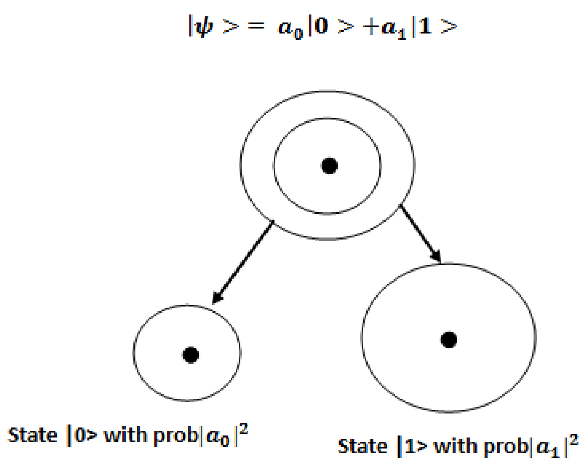

Figure 1. Quantum System Probability Principle

This paper is organized as follows. Section II presents the related work of existing quantum sorting algorithms. In section III, the proposed sorting algorithm is presented. Section IV offers performance evaluation of the proposed sorting algorithm. Finally, section V includes the conclusion.

II. RELATED WORK

Sorting problem can be formulated as follows, given a list of P_n items $P_n = \langle p_0, p_1, p_2 \ldots\ldots p_{n-1} \rangle$ and the output is a sorted list of $P'_n = \langle p'_0, p'_1, p'_2 \ldots\ldots p'_{n-1} \rangle$. However, quantum systems contain special gates to process qubits based on superposition phenomenon. Figures 2 and 3 exemplify the two gates used mainly in quantum sort algorithms[10, 11].

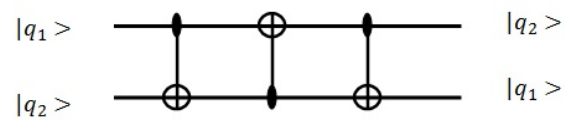

Figure 2. Quantum SWAP Gate

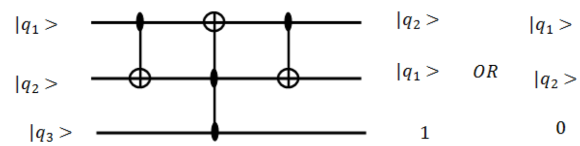

Figure 3. Controlled SWAP Gate

Moreover, to apply comparison between two quantum numbers, sorting system need to use Black box model as shown in Figure 4.

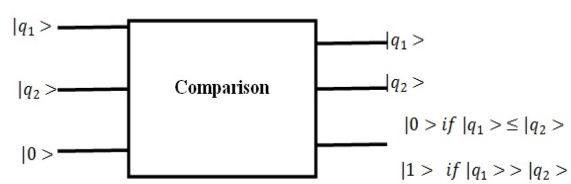

Figure 4. Black Box Model for Qubits Comparison

Farhi *et. al* introduced a quantum searching algorithm by using Insertion sort [12]. The authors proposed algorithm used comparison gates to search in N sorted elements. The complexity for this proposed algorithm is $O(0.526N \log_2 N)$.

A new sorting algorithm introduced by using $\Omega(N \log N)$ queries to compare between N elements [13]. The main drawback of [12] sorting algorithm is the high cost of quantum components, and comparable complexity was achieved by the classical algorithms such as Quick sort and Merge sort.

Quantum existence testing algorithm introduced in [13]. This algorithm is useful to find the minimum and the maximum values among set of elements with time complexity $O(1)$. However, Quantum existence testing algorithm just deals with low size quantum registers.

A novel quantum sort algorithm was introduced by using insertion and decision tree for a set of N elements [15]. The time complexity of this quantum sort algorithm is $O((N-2)\log(n-2))$

III. PROPOSED SORTING ALGORITHM

The proposed algorithm sorts N positive integers based on entanglement qubit property. According to the most significant bit (MSB), 0 or 1 will be inserted. For example, if the number is {001} this means that the most significant bit is 0 then the inserted bit is 0, otherwise 1 will be inserted.

$$\text{Insert }(X) = \begin{cases} 0 & \text{if MSB} == 0 \\ 1 & \text{if MSB} == 1 \end{cases} \qquad (2)$$

According to Equation 2, elements can be split into two groups based on {00, 11} the 11 elements swap to left side and the right side contains the 00 elements. The proposed sorting algorithm consists of the following steps:

Input N positive integers
Output N sorted positive integers
Sort Procedure (N Positive integers)
Begin
Step 1: Read N positive integers.
Step 2: Convert all integers into binary representation.
Step 3: if MSB == 0
Then insert 0 and shift right
Else insert 1 and shift left
Step 4: Discard the two most significant bits
Step 5: Repeat steps 3 and 4 until we have a single qubit.
End

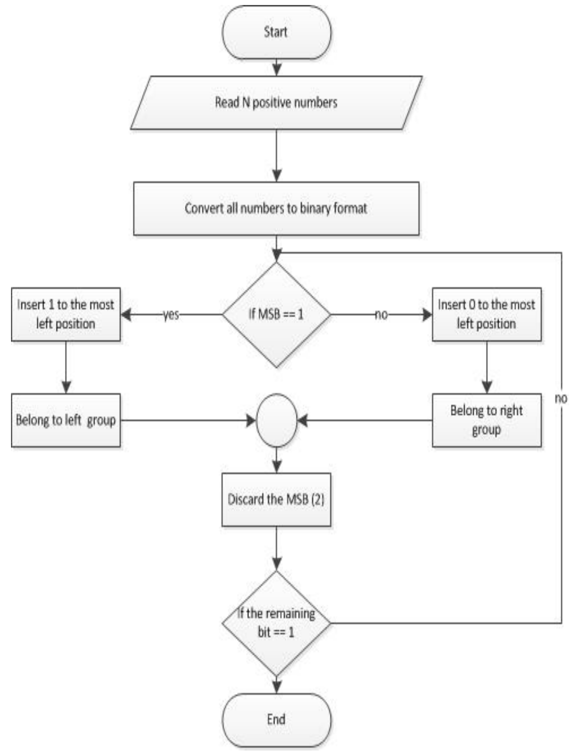

Figure 5. Flow Chart of Proposed Algorithm

IV. PERFORMANCE EVALUATION

The proposed sorting algorithm demonstrates quantum parallelism, due to mutual coherence among qubits. Furthermore, Toffoli quantum gate (Figure 6) enhances system to process more than one qubits simultaneously. Moreover, parallelism also can be applied by using Quantum Phase Registers (QuPhaRegisters) that classify input data into three categories: data state, data phase, and data probability[14]. Figure 7 illustrates an example of the proposed sorting algorithm. The complexity of the proposed efficient sorting algorithm is $O(N) = \frac{N-2}{2}\log\frac{N-2}{2}$ for N positive integers.

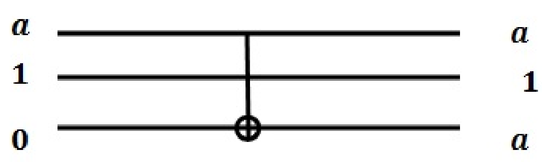

Figure 6. Toffoli Quantum Gate

In Table I, some of the sorting algorithms are presented and compared with the proposed sorting algorithm.

Table I. Big O for Some Sorting Algorithms

Sorting Algorithm	Time Complexity	Operation
Bubble Sort	$O(N^2)$	Exchanging
Merge Sort	$O(N \log N)$	Merging
Quick sort	$O(N \log N)$	Partition
[15]	$\mathrm{O}(0.526\mathrm{N} \log_2 \mathrm{N})$	Partition
[12]	$\Omega(\mathrm{N} \log \mathrm{N})$	Partition
[16]	$O((N-2)\log(N-2))$	Partition
Proposed Algorithm	$O(\frac{\mathrm{N}-2}{2} \log \frac{\mathrm{N}-2}{2})$	Partition

V. Conclusion

In this paper, some quantum computing characteristics were presented such as superposition, entanglement, and probabilistic. The proposed efficient algorithm sorts N positive integers by using one of the interesting quantum computing concepts which is entanglement of qubits. Moreover, Toffoli quantum gate is utilized to accomplish time complexity of $\mathrm{O(N)} = \frac{\mathrm{N}-2}{2} \log \frac{\mathrm{N}-2}{2}$. In addition, the algorithm exploits divide and conquer theory to utilize the parallelism concept.

VI. References

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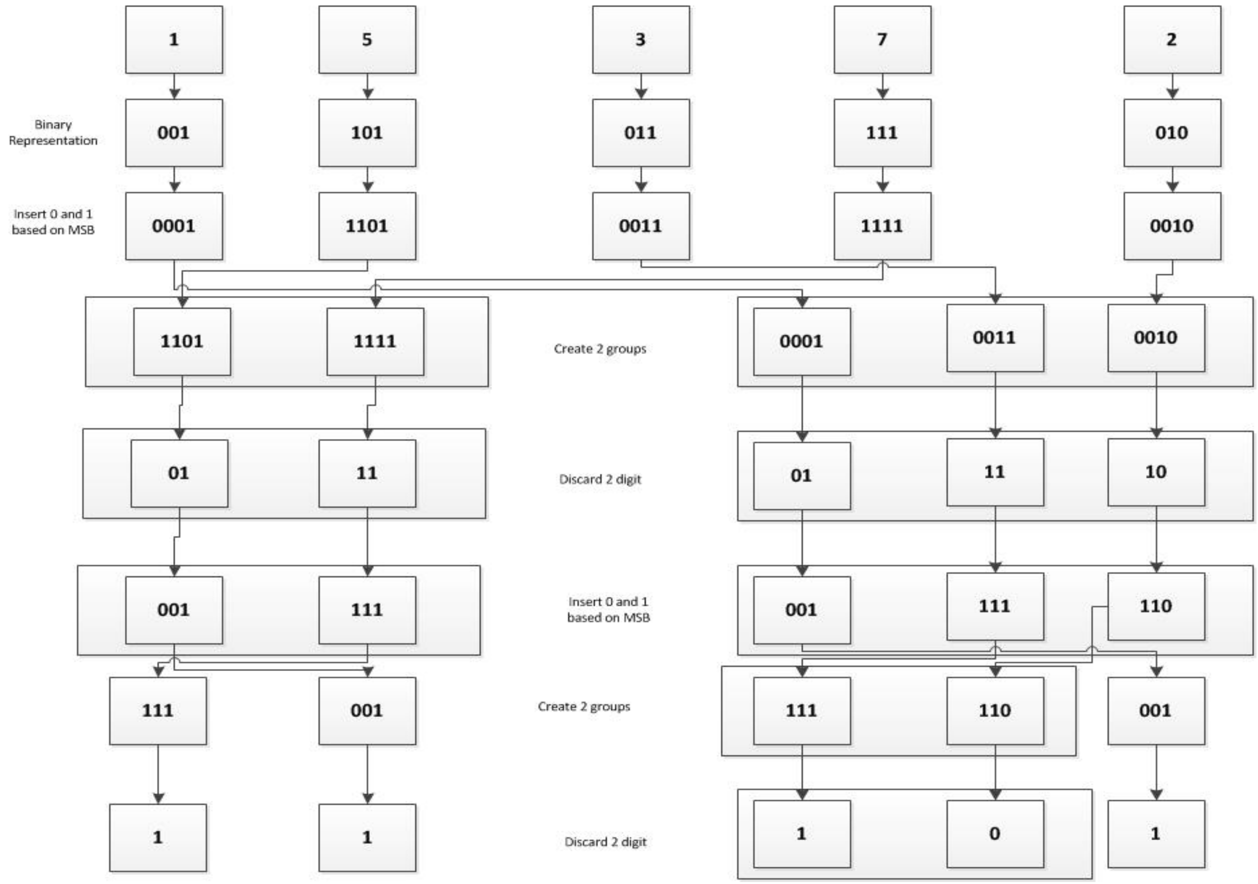

Figure 7. Tree Representation