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Quantum technology to expand soft computing

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A B S T R A C T

Soft computing was founded to unify and advance computing methods beyond the limits of binary variables, Boolean logic and Turing machines. David Deutsch invented a way to use massive quantum parallelism to massively improve the power of Turing machines in some applications. Here we give a roadmap to use quantum parallelism to massively improve the power of soft computing, focusing on tasks in optimization ranging from function optimization, finding a needle in a haystack, optimal control, energy technology and reinforcement learning (Quantum artificial general intelligence, AGI) in a path leading up to optimization of physical plants themselves put into quantum superpositions. The path begins with design and test of prototypes built on platforms such as Nuclear Magnetic Resonance, sparse quantum optics, superconductors (sQuID), quantum dots and new high speed electronics.

1. How to combine quantum and soft computing

1.1. Roots of soft computing

Lotfi Zadeh proposed soft computing as a new field of research before 1990. He proposed it as a way to unify and extend the emerging fields of fuzzy logic and neural networks, first, and then to unify these with related work in evolutionary computing and other natural allies. The goal was to build a coalition going beyond the edifice of classical computer science grounded in concepts like Boolean logic, binary variables, digital systems and Turing machines. In his formulation, soft computing aims to develop new general-purpose forms of computing, building beyond the limits of 0/1, black and white thinking, and helping us humans think better about how we can think and reason. The IEEE Computational Intelligence Society was founded by Robert Marks, as another way to unify these same fields, by expanding the IEEE Neural Networks Coucil and Neural networksw Society which he had previously organized [19].

In 1990, the National Science Foundation followed up with a cross-cutting workshop, held at McDonnell-Douglas, working out the details of a new paradigm to unify a soft computing approach, to build true intelligent systems for use in decision and control applications [39]. NSF then followed up with a roadmap for the neural network part of this research, leading up to pathway towards Artificial General Intelligence (AGI) as powerful as that of the mammal brain [31] and for a new way to explain how mammal brains achieve such basic capabilities [21,37]. The core of that new paradigm was the development of Reinforcement Learning and Approximate Dynamic Programming, RLADP, a family of methods and designs which can learn to maximize any user-selected utility function over time, by tuning the parameters of a neural network, a set of rules expressed in words using fuzzy logic, or more classic differentiable models such as econometric models [18,34,39,40]. The concept of cardinal utility function and the development of neural networks for artificial intelligence can be traced back to the work of John Von Neumann [9,27].

1.2. Roots, motives and types of Quantum Information Science and Technology (QuIST)

1.2.1. QuIST based on David Deutsch and Quantum Turing Machines (QTM)

In 1985, the great Oxford physicist David Deutsch proved how his invention, the universal Quantum Turing Machine (QTM), can perform any computation which a standard Turing machine can perform, but can also perform a more general set.

Some of the computations available to the QTM can be simulated by a conventional Turing machine, but only at a cost which rises exponentially with the complexity of the computation. Such algorithms have been found so far only for a few types of computation; however, after it was learned that these included a quantum algorithm capable of breaking the kinds of codes used at the time in almost all advanced cryptography [22], the US government launched major research programs across all agencies in Quantum Information Science and Technology (QuIST). Work on advanced communications and commmunication security related to the QTM technology has also grown in importance [6].

QAGI, RLADP QuATh, tQuA.

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There is now no doubt that QuIST offers future capabilities of ernormous practical importance. For example, Senator MarkWarner, head of the Senate Intelligence Committee, has announced new national strategic priorities, in his support for the Innovation and Competitiveness Act 2021 [26]:

"The United States Innovation and Competition Act meets the threat of an increasingly dominant China by significantly investing in the development and manufacturing of technologies that are critical to national security and economic competitiveness. This includes biotechnology, quantum computing, artificial intelligence, and semiconductors – the tiny chips that power modern technology, from cars, computers, and smartphones, to 'smart' devices …" [29].

However, for many important applications, such as optimization, the best QTM algorithms achieve little more than polynomial speedups for all but a few special cases [4]. The overhead cost associated with standard QTMs (e.g., error correction, decoherence issues) has not yet brought the general methods to market.

Just as neural networks can be implemented directly as a hardware architecture, giving power far beyond the simulation of neural networks on Turing machines, based on optimzation concepts such as error minimization and RLADP, quantum annealing hardware can be used directly on optimization problems such as the quantum version of RLADP.

1.2.2. Beyond the QTM: QuIST for soft computing

By 2015, Howard Brandt, the great thought leader helping coordinate QuIST programs across US agencies up until his death, had recognized the limits of the QTM and, more important, the huge power available by moving to a different paradigm, beyond 0 s and 1 s to quantum soft computing. For his annual conference on QuIST at SPIE in Baltimore, he invited four teams to present new results based on continuous variables, including us [38].

In reviewing the prior work, we cited what little relevant prior work we could find in the US. We focused on the challenge of how to combine the universal post-Turing capability of the QTM with another type of post-Turing capability proven for classical neural networks, by Sontag and Siegelmann and others. Intuitively, standard Turing machines offer an aleph-zero countable infinite capability, while Deutsch [11] and Siegelmann and Sontag [23] both offered aleph-one by different pathways. Combining the two offers yet another level of universal capability, alpeh-two learning, which could be extended even further by exploiting new measurement components and models for which preliminary results are encouraging but yet to be fully implemented. We proposed a pathway to build true Quantum Artificial General Intelligence (QAGI), step by step. This paper will describe more concretely what steps come next on that path.

In 2015, the most relevant work in actual operation in quantum soft computing was work by the company D-Wave [1,20]. Instead of trying to build an improved sequential computer designed for bits or qubits, D-Wave built a general purpose working commercial system aimed at solving a very broad class of optimization problems. Almost all higher order problems in AGI or in profit maximization or in prediction [30] map naturally into optimization problems (including optimization over continuous variables, not only binary), which the D-Wave system was constructed to handle. Apolloni et al. [1] showed that D-Wave system can outperform many classial optimization algorithms, such as classical thermal annealing methods.

Unfortunately, the existing D-Wave system relies heavily on a component which we call an orbital quadratic optimizer (OQO), which has limited the range of optimization problems which they can perform well on. Their system includes:

*the OQO, plus
*an important set of attached capabilities which can map any static optimization problem (to minimize F(u̲) for some vector u̲ which may be real or complex) into problems which the OQO can handle, plus
*important interfaces to applications.

This paper will discuss how to design and build a true quantum quadratic optimizer (tQQO), by directly exploiting the physics vision of David Deutsch and the physics of true annealing well-known in condensed matter physics. The development of tQQO, from mathematics to simulation to hardware, is the crucial first step in the roadmap to QAGI. In the next step, the QQO could be inserted directly into D-Wave or any new system like D-Wave, to expand its power. Finally, this paper will also discuss a way to apply this technology to optimize actual physical plants, like radio telescopes and chip diagnostics, through an approach we call the Quantum Annealing of Things. Just as the Quantum Turing Machine can perform some tasks with an exponential advantage over the classical Turing machine, because of the physics it harness, Thermal Quantum Annealing (tQuA) offers similar advantages over classical thermal annealing and other classical optimization methods.

2. Design and path to a true quantum quadratic optimizer

2.1. Key underlying principles

The true QQO is firmly grounded in the same physics that David Deutsch used when he first envisioned the QTM. That same physics has continued to fit reality in very extensive work perfomed on QTMs in recent years.

Deutsch actually began with a theory of how physics actually works [12]. That theory postulates that we actually live in a "multiverse," in which computer chips, cats and other macroscopic objects can be put into a state of quantum superposition. Some of us seriously doubt whether there could be multiple copies of us humans out there, existing in parallel [35], but the empirical evidence is now overwhelming [2] that we can put objects like computer chips and long-distance communication channels into a state of quantum superposition.

Intuitively, then, a QTM with 20 true "qubits" creates 2**20 parallel copies of the same device, performing variations of the same computation – "herding a million Schrodinger cats." QTMs have been built and proven to work with far more than 20 qubits.

Our concept of QQO is to build a new type of u-box, in which millions of Schrodinger cats really are run in parallel to search through millions of possible values for the vector u̲, and settle down on the one which minimizes a figure of merit E(u̲). In our concept, the u box is coupled to a reservoir, an external environment or bath of some kind, which we simply need not model in any detail in the beginning, as we design and test the first true QQO. The figure of merit E is represented by the standard H energy for the system defined by u̲ (accounting for exogenous inputs x̲ if desired). *The natural energy level (temperature) of the reservoir must be less than the minimum value of E.* The idea is that excess energy of E, above its minimum value, can be shed by ordinary dissipation to the reservoir. The reservoir will of course send noise back to the u box, but if the resulting quantum noise is small enough the thermal equlibrium in the u box should be close enough to the true global minimum in many important applications. (Not all applications, but we will give examples of some important ones.)

Note that there is no need for a final stage of selecting which cat we want, or of defuzzification. The choice of a good u̲ is performed by the same thermodynamics operating from the moment when the full system is turned on. The choice of initial state should not matter so much as it would with a QTM. More precisely, to achieve a stochastic search exploiting "a million Schrodinger cats all working in parallel," one may either initialize the system in a rich mixed state over possible values of **u̲**(0), or one may start with a strong coupling (C(t)) between the core and the reservoir, reduced to 0 over timr, as in true annealing as used in solid state physics or materials science.

The standard OQO design (Appolloni 1988) aims to minimize E, by varying a different coupling constant in the Hamiltonian, representing the strength of interactions within the core itself, and orbiting around the set of possible states in hope of finding a breakthrough to a better state through quantum tunneling. Dissipation has been observed in that u box in actuality, but is not integrated into the design or the mode of operation.

2.2. Specification of the Hamiltonian for OQO and QQO

The first step in specifying the QQO is to specify the final figure of merit, $E = H$, which it will minimize. The OQO minimizes a slightly simpler target:

$$H = \sum_{j=1}^{n} H_j^0 u_j + \sum_{j,k=1}^{n} H_{jk}^I u_j u_k$$

As we consider the full range of applications, we would specify a slightly more general target:

$$E = H = \sum_{j=1}^{n} H_j^0 u_j + \sum_{j,k=1}^{n} H_{jk}^I u_j u_k + \sum_{j,k=1}^{n,m} H_{jk}^E u_j x_k \tag{1}$$

The vector $\underline{u}$ is composed of n complex numbers u_j (as $j = 1$ to n). The overall task is to find the optimal values for these variables, for the problem details specified by the user. The number n is a measure of the size of the QQO, the maximum number of variables which can be optimized. In OQO, the task is to input matrices H^0, H^I and H^E supplied by a user, and find the value of $\underline{u}$ which minimizes H. The full QQO version of this task also allows the user to insert a vector $\underline{x}$ of exogenous variables, made up of numbers x_k (as $k = 1$ to m).

2.3. Model of the reservoir for initial QQO simulation and design

A rigorous mathematical model of such a system when coupled to a stochastic reservoir would call for a master equation for the dynamics of the density matrix of the system, using the kinds of mathematical tools described in detail by Carmichael [7,8] and Walls and Milburn [28]. However, for purposes of initial simulation and design, using a small number of variables to test the basic principles and performance versus OQO, it should be sufficient to simulate the system on an ordinary, sequential computer, A proper simulation system should combine both ways of representing the uncertainty in $\underline{u}(t)$ over time t: (1) trajectory simulations, starting from a state u but adding a noise factor, a jump stochastic process receiving or sending energy from the u box, under control of parameters to be set by the user running the simulation; (2) an estimated density matrix or simulated wave function, covering the space of possibilities with low resolution, such as a grid with spacing 0.1 or 0.2 (user selected), representing the same.

A natural noise model would be a stochastic choice, to be made at small intervals of time Δt. In each time interval, *a j* between 1 and n is chosen. Two possible jumps are considered: a jump up (noise), where u_j is multiplied by 1+delta, and a jump down (dissipation), where it is divided by 1+delta. The probability of a jump would be calculated based on the change in the total energy E (as given in Eq. (1)), the usual Boltzmann equation, the assumed temperature Hr of the reservoir (a fixed parameter in initial simulations). The user would naturally want to consider values representing the kind of noise levels which might be expected from some of the available types of platforms available for implementing the qubits and the reservoir.

It should not be difficult to translate this kind of model to a full master equation model, in the limit of small time intervals, but for now the first stage is to explore interesting possible values for H and for noise parameters, compared to D-Wave results.

Simulations on a classical digital computer should be enough to demonstrate the advantages of thermal quantum annealing on relatively small but interesting test problems. Before the actual construction of physical prototypes, it might be advantageous to perform a true quantum simulation of the design on larger test problems [17].

For an exact model of the tQQO kind of system, connecting a core and a reservoir across choices of Hamiltonian H and coupling to reservoir, one must analyze the properties of the grand canonical Boltzmann density operator [10] or, if one believes a more neoclassical model of the physics [33], the entropy function for that class of problem [31]. In either case, choices of H and coupling which imply millions of local minima in the error or entropy surface should be tractable with a tQQO design which makes effective use of millions of Schrodinger cats in parallel, which the present D-Wave system does not [20].

For a later, more complicated phase of simulations and mathematical analysis, success is guaranteed more with stochastic simulations which reflect the grand canonical density matrix or neoclassical entropy function. These would be based on perturbations of the entire vector $\underline{\mathbf{u}}$. We need to fix a procedure which, depending on $\underline{\mathbf{u}}$ and on global simulation parameters to be set by the user, determines $\Pr(\underline{\mathbf{u}^*} \mid \underline{\mathbf{u}}(t))$, the probability that our program chooses the value $\underline{\mathbf{u}^*}$ out of all the values available in the complex surface C^n. In essence, the problem of selecting (programming) this probability distribution for use in these. As Tv goes to zero, the obvious simulation approach here is to assume that $\underline{\mathbf{u}}$ will be perturbed towards a value of minimum energy, simply by choosing a probability of $\underline{\mathbf{u}}$ being changed to $\underline{\mathbf{u}^*}$ with follows a Boltzmann distribution, exp(-k(H-H*)/C(t)), normalized across possible values of $\underline{\mathbf{u}^*}$ such that the integral of $\Pr(\underline{\mathbf{u}^*} \mid \underline{\mathbf{u}}(t))$ over $\underline{\mathbf{u}^*} = 1$, with k a user-selected simulation parameter.

2.4. Test problems of interest

Even when D-Wave is compared with classical or QTM optimizers, we have learned, as expected, that different systems do better on different types of problems. The first major goal here is to identify at least one interesting class of problems for which we expect better performance than with the OQO, and for which genuinely interesting applications seem likely.

For this purpose, we recommend study of "needle in a haystack" kinds of problems. We define these as problems where H is chosen to make the space of possible values of u relatively flat, but lowered by thousand or millions of "haystacks", small regions (like Gaussians) of diverse depth and other parameters. For a true QQO, with well chosen parameters, it would operate like a herd of millions of cats, scattered all over field, exploring different places, with only the best cat(s) dominating the thermodynamic equilibrium. There is no orbiting or quantum tunneling required. Given what has been observed with D-Wave on systems with local minima, we strongly expect a large family of examples to exist which can represent this kind of benefit.

The world of applications contains many such needle in a haystack applications. Two which we consider most interesting are "SETI" and "chip diagnostics".

In SETI [5], we are often confronted with the problem of how to find signals which meet some concept of a needle in a haystack. In a gigantic database, with computer controlled addressing, the challenge is to find those signals which score highest in some quantitative measure of what kind of signal has the best chance of representing extraterrestrial intelligence of some kind. Recent results from this very serious research have been reported in Scientific American, and cry out for much more thorough and complete investigation of the underlying databases. In a similar fashion, chip diagnostics call for more comprehensive, faster searches of computer chips, for which measures of undesired or dysfunctional behavior are often available. (We think of this application as "quantum bromium").

Simple, abstract demonstrations of the advantages would of course be an important step towards physical prototypes and exploration on a larger scale.

2.5. Hardware platforms of interest

The first step in going from proof of principle in simulation to actual design evaluation is to evaluate possible hardware platforms, and extract what ranges of parameters might be implemented in prototype. Current research on QTMs explores dozens of possible hardware platforms, but here it would make sense to limit early exploration to five possibilities, when and as opportunities arise:

(1) Nuclear Magnetic Resonance (NMR), "quantum computing in a coffee cup." NMR has not led to practical QTM quantum computers, but the early demonstration by Gershenfeld and Chuang [13] proved that true quantum entanglement and qubits can be achieved in that system. It could be an excellent starting point here as well, demonstrating the new basic mathematics and principles.
(2) Simple quantum optics platforms, which can be used for experiments probing essential properties of quantum measurement important to the more complete theoretical understanding essential to getting full value not only from QQO but from other quantum technologies [38].
(3) Superconducting qubits (SQuiDS). Squids have been widely used at D-Wave, at NSA and in China [24]. However, it is not clear whether they have the best choice of parameters available for QQO [3,16].
(4) Massive arrays of quantum dots, like what Samsung and LG use in a radical new generation of televisions. New switching needs to be added, but these platforms promise billions of true qubits in systems of only a square meter in size [15].
(5) High frequency electronic systems, like what is used in the most modern cell phones [14]. This may allow the kind of additional flexibility needed for the more advanced technologies beyond even Quantum Annealing of Things (QuATh).

Because temperature parameters are so important to the performance of systems of this type, it is also possible that the new system described in Spivey et al. [25] could allow for better performance in many crucial applications.

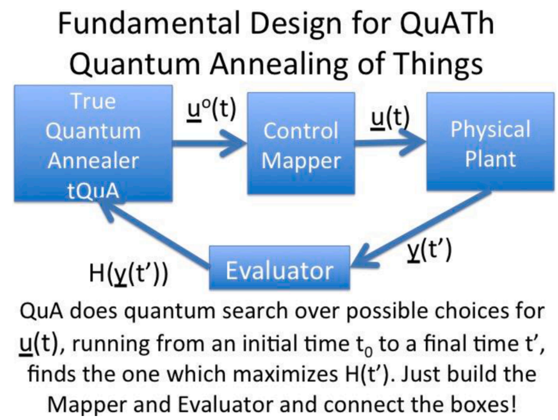
3. Quantum Annealing of Things (QuATh)

Once the benefits of QQO have been demonstrated, we will be ready to explore a few of the many applications possible in a whole new area which we call the Quantum Annealing of Things.

QuATh aims to demonstrate how ***quantum entanglement applies to macroscopic physical systems in the real world, not just inside of a computer***. Just as the Internet of Things extends global connectivity from programs and data inside computers and internet, to actual controllable physical objects, the Quantum Annealing of Things (QuAth) puts physical objects into states of quantum superposition, in order to optimize physical performance in the physical world.

For example, an effective QQO-based optimizer could be connected to objects like radio telescopes (not just their databases), to improve a million fold the throughput of detection of interesting signals from deep space. Computer chips, biological cells, and even airplanes are also physical objects which can be put into states of quantum superposition.

The basic design for using QuATh in some physical application is shown in the following slide:



To optimize the performance or output of a physical plant, we usually need to allow some time delay between the time when a new control signal $\underline{u}(t)$ is received, and the time when the observed output $\underline{y}$ is ready for evaluation. In this slide, we use the notation "$\underline{u}$" to denote the vector of controls used to control the physical plant, as we normally do in control theory. The user gets to decide what range of possibilities should be considered in the optimization; the "control mapper" is basically a user-defined lookup table to fill in the 2^n table of possible choices offered by $\underline{u}^o$. The evaluator represents the user's goals, the evaluation score to be maximized.

As an example, in the SETI application, $\underline{u}$ would represent settings like promising frequency and direction choices to be used to control an array of radio telescopes. The evaluation would be based on the SETI team's ideas of what kind of signal looks interesting. If the time interval $t'-t_0$ is short, the system could explore millions of settings to find the most interesting signal, for the current exogenous choices, choices which can be changed as the user requests.

A time interval as long as 10 to 20 min is also possible, and may be very useful for many purposes. However, we recommend limiting the time for now, and limiting the spatial range of entanglement, in order to avoid possible issues of safety and confusion if entanglement is extended outside the range of the task at hand.

The underlying mathematics of optimization change substantially when we progress from minimizing a fixed static function f(u) to minimizing the end result of a dynamical system. The obvious way to do this would be use this kind of optimizer to implement the components of existing general RLADP designs [18,36]. Because RLADP is a pathway to build true artificial general intelligence [31,37], this enhancement of RLADP would be a direct pathway to true Quantum Artificial General Intelligence (QAGI).

Nevertheless, when we manipulate information over time in this way, we should remember that there is a possibility that the fundamental principles of quantum electrodynamics may not follow exactly the physics assumed by David Deutsch. New experimental realms may bring out new physics and require new experiments [6,38]. Correct models for quantum dots and for advanced transistors operating beyond 100 GHz, entailing phenomena like collapse or splitting of wave functions, might be different in nature for different components, leading to different options in technology.

4. In conclusion: from quantum soft computing to sustainable intelligent internet

The development of QAGI based on RLADP concepts will be a monumental long-term goal in itself, offering a unified focus to many technical areas which need to be integrated. However, as with classical RLADP, there will be larger societal implications which also need to be considered, especially in the design of larger internet systems using these capabilities. If the future global internet of things (IOT) were organized so as to control everything on earth to maximize one centrally defined utility function U, it would not live up to the highest values and capabilities of the human beings living in that system, unless there are major parallel research efforts to account for those values and capabilities from the start. That is true even in the classical domain.

These parallel research efforts should take advantage of efforts to design not only computer systems but markets, to account more fully for the situation where the utility functions of many players must be considered [18,27]. In complex multiplayer systems, step-by-step progress towards Pareto optimal systems can require a high level of understanding and focus by the humans who build such designs, and the risks of accepting Nash equilibria can be literally fatal. The development of a Sustainable Intelligent Internet (SII) requires the development of techndologies like RLADP and QAGI, but also requires larger scale systems development which can take positive advantage of them.

The three step roadmap presented here assumes, again, the validity of Deutsch (1997) and Chaiken et al. (1995) as models of the underlying physics. As we develop ever more powerful systems, it is possible that new experiments and mathematics [32,33] will point to new technology options which extend this road even further.

Declaration of Competing Interest

I have no financial interests whatsoever concerning this paper. My income this year is from NSF pension, from investment funds, and from the IEEE Frank Rosenblatt Award. I HAVE discussed the possibility of partnering with Prof. James Momoh of Howard University, to work for patents through them, based in part on ideas in this paper.

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