# Quantum Advantage in Cryptography

### Renato Renner and Ramona Wolf

Institute for Theoretical Physics, ETH Zürich, 8093 Zürich, Switzerland

Ever since its inception, cryptography has been caught in a vicious circle: Cryptographers keep inventing methods to hide information, and cryptanalysts break them, prompting cryptographers to invent even more sophisticated encryption schemes, and so on. But could it be that quantum information technology breaks this circle? At first sight, it looks as if it just lifts the competition between cryptographers and cryptanalysts to the next level. Indeed, quantum computers will render most of today's public key cryptosystems insecure. Nonetheless, there are good reasons to believe that cryptographers will ultimately prevail over cryptanalysts. Quantum cryptography allows us to build communication schemes whose secrecy relies only on the laws of physics as well as some minimum assumptions about the cryptographic hardware—leaving basically no room for an attack. While we are not yet there, this article provides an overview of the principles and state of the art of quantum cryptography.

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Renato Renner: renner@ethz.ch Ramona Wolf: rawolf@ethz.ch

### 1 Introduction

The art of encryption is as old as the concept of written information. For thousands of years, humans have been using more and more elaborate schemes to hide the content of messages for a variety of purposes, for example, to facilitate secret communication between governments and militaries. But those who wanted to get hold of the secrets did not remain passive, of course: They have come up with increasingly refined methods to get access to encrypted information. Consequently, history is full of examples illustrating how the most sophisticated encryption schemes were rendered useless by the ingenuity of brilliant code-breakers [Kah96]. Is it inevitable that history will always repeat itself? Is it impossible to develop an encryption scheme that cannot be broken?

It is undoubtedly true that currently employed encryption standards such as the RSA cryptosystem [RSA78] and the Diffie-Hellman key exchange method [DH76, Mer78] can, in principle, be broken. They are only computationally secure, which means that breaking them requires the ability to solve a computationally hard problem efficiently. In the case of RSA, for instance, the underlying problem is factoring large numbers into prime factors. For large enough numbers, the expected amount of computational power required to factor them within reasonable time lies beyond the capabilities of state-of-the-art computers. However, as technology advances, what is deemed "large enough" must be constantly adjusted accordingly to ensure the security of RSA encryption. Hence, secrets that are encrypted with the RSA cryptosystem today might be decodable with tomorrow's computers!

Alongside developments in the field of classical computers, the imminent advent of quantum computers poses an additional threat to the security of current encryption standards. Quantum computers exploit the unique features of quantum mechanics and, as such, allow for new kinds of algorithms. Consequently, tasks that are believed to be hard for classical computers can become feasible if one has access to a universal quantum computer. Most famously, quantum computers can factor large numbers efficiently using Shor's algorithm [Sho94] and therefore, once they are realized, render current cryptographic standards such as RSA completely insecure.

Fortunately, we do not have to fear that all ciphers will eventually be broken, not even with the help of a quantum computer. It is well-understood how to design an encryption scheme between two parties remains secure, even if the adversary has all the (classical and quantum) computational power in the universe. We call such a cryptographic scheme information-theoretically secure. The crucial ingredient is a cryptographic key, which is a sequence of shared random bits only known to the two parties that want to exchange private messages. If the key is uniformly random, kept perfectly secret to everyone except the two parties, and no part of it is ever reused, it can be employed in the so-called one-time pad (OTP) scheme [Ver26], which is an example of an information-theoretically secure encryption protocol. To use the one-time pad, the sender adds the cryptographic key to the message they intend to encrypt. The resulting ciphertext is sent to the receiver, who subtracts the key from the ciphertext, thus retrieving the original message. Even though an adversary has access to the ciphertext and knows the general encryption method (namely the one-time pad), they cannot learn anything about the message: If the key fulfills the properties mentioned above, the ciphertext is completely independent from the message, thus it does not reveal any information about it.

As a result, the difficulty in creating a secure encryption method now lies in developing a protocol that generates a cryptographic key with the desired properties. Here, quantum theory comes into play. The unique features of quantum mechanics lend themselves extraordinarily well to the task of distributing a secret key between two parties. Firstly, the intrinsic randomness of quantum states and measurements can be exploited to generate truly random bits. Secondly, the phenomenon of entanglement allows the generation of the same set of random bits in two distant locations. Thirdly, quantum states have the property that they cannot be perfectly copied, in contrast to classical bits. If an adversary attempts to access information encoded in quantum states, they have to somehow interact with the quantum state, which in turn will lead to detectable

<sup>&</sup>lt;sup>1</sup>The term encompasses the fact that this kind of security can be expressed in terms of purely information-theoretic concepts, in contrast to computational security (which requires the notion of computational complexity).

<sup>&</sup>lt;sup>2</sup>Since the key and the message are both bit strings, adding here refers to bitwise addition modulo 2.

changes in the state. Consequently, the communicating parties will be alerted to the attack, before confidential information has been exchanged. The distribution of a secret key via quantum mechanics is known as *quantum key distribution* (QKD) [BB84, Eke91]. One goal of this article is to explain how the features of quantum mechanics guarantee the security of QKD.

So, is quantum cryptography the universal solution? Can it provide information-theoretic security for every aspect of (cyber) security? The answer is: to some extend, yes. Quantum cryptography offers solutions for cryptographic tasks that concern private communication. Here, symmetric encryption schemes are employed for which quantum key distribution provides an information-theoretically secure algorithm (together with the one-time pad). Other cryptographic tasks such as digital signatures, however, require public key encryption (such as RSA); here, quantum cryptography cannot provide any advantageous quantum protocols. The second goal of this article is to explain the role that quantum cryptography plays in the larger context of cyber security and discuss possible applications and limitations.

Finally, while quantum cryptography is a promising research area that has increasingly attracted the industry's attention in recent years, it is still under development. Even though experimental implementations of QKD protocols are reaching ever further distances and higher key rates, the state-of-the-art achievements are still several orders of magnitude away from what is required for practical applications. Therefore, the third goal of this article is to present the landscape of QKD protocols, the progress of their respective experimental realizations and the challenges that still need to be overcome to make QKD market-ready.

The article is structured as follows: In Section 2, we explain what level of security classical cryptosystems can guarantee and where they reach their limitations, in particular with regard to the possible threats posed by quantum computers. In Section 3, we present potential applications of quantum cryptography and explain where it can provide an improvement over classical algorithms and where it is not applicable. We also discuss the goal of post-quantum cryptography and its advantages over current state-of-the-art algorithms. In Section 4, we introduce basic notions of quantum theory and explain how they contribute to the security of quantum cryptography. Finally, in Section 5, we discuss how quantum key distribution works, how to quantify security, and current progress as well as challenges in theory and experiment.

# 2 Limits of classical cryptography

The field of cryptography spans everything concerned with privacy, authentication and confidentiality in the presence of adversarial behavior. An important subfield of cryptography is secure communication, which nowadays mainly involves electronic data such as encrypting emails and other plain-text messages, online banking security, and communication related to the military, governments, and the financial market. Typical tasks include private communication, authentication (i.e., verifying a claim of identity) and digital signatures, to name just a few examples.

Classical cryptography provides the standard algorithms and protocols used to achieve these tasks on a day-to-day basis, such as public-key encryption schemes for digital signatures and key exchange mechanisms for confidentiality. While these protocols have the advantage that they are easy to implement with state-of-the-art technology, they have some disadvantages regarding the level of security they can guarantee.

#### 2.1 Security based on computational complexity

The security of classical protocols is usually based on the assumption that certain problems are hard in terms of their computational complexity. Widely used algorithms that fall into this category are the Diffie-Hellman key exchange method [DH76, Mer78], which relies on the hardness of the discrete logarithm problem, and the RSA cryptosystem [RSA78], whose security depends on the practical difficulty of factoring the product of two large prime numbers. Basing the security of a cryptographic system on the difficulty of specific problems has its issues: Inherently, a necessary condition for the security of these schemes is the assumption that the conjecture  $P \neq NP$  holds, which effectively ensures that hard problems actually exist. While this is widely believed to be true, it has not yet been proven despite decades of exhaustive research in this area [For09]. As long as

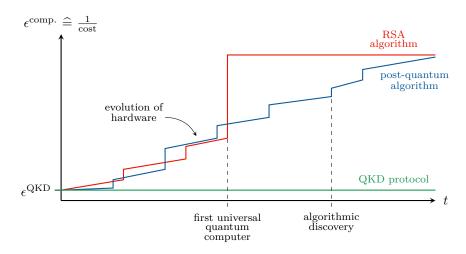


Figure 1: Security of cryptographic protocols over time. The diagram shows schematically the development of the probability  $\epsilon^{\mathrm{comp.}}$  that an encryption scheme is broken over time. We can link this probability to the inverse of the cost it takes to break the encryption scheme (the more computational power is needed to break the scheme, the less likely it is to be broken); keeping one of these two parameters,  $\epsilon^{\mathrm{comp.}}$  or the cost, fixed, the diagram shows the development of the other parameter over time. Classical algorithms (including post-quantum ones) become increasingly insecure over time due to evolution of hardware and algorithmic discoveries. If there exists an efficient quantum algorithm for breaking it (which is the case for RSA), the scheme will immediately become insecure once the first universal quantum computer is built. The failure probability  $\epsilon^{\mathrm{QKD}}$  of quantum key distribution, on the other hand, always remains the same.

the hardness of these problems is only a conjecture, it is always possible that an efficient algorithm is found to solve them, making cryptographic schemes building on them effectively unsafe.

Additionally, even if the conjecture  $P \neq NP$  can be proven to hold, it is still possible that a problem known to be hard can be efficiently solved in many cases in practice. There exist problems that have exponential worst-case complexity but whose empirical average-case complexity is surprisingly low, such as the traveling salesman problem. This shows that the difficulty of a problem in terms of its worst-case computational complexity is not enough to guarantee the security of a cryptosystem based on this problem. It has further to be ensured that in practice, the problem cannot be solved efficiently, i.e., the average-case complexity is high.

With the imminent advent of quantum computing devices comes another complication that can arise when relying on the hardness of specific tasks: There are types of computations, such as quantum computation, that cannot be sorted into classical complexity classes such as P and NP. While integer factorization and the discrete logarithm problem are believed to be hard for classical computers, there exists a quantum algorithm that can solve them in polynomial runtime, namely Shor's algorithm [Sho94]. Hence, it is insecure to use the Diffie-Hellman key exchange method and RSA encryption in light of the possibility that practical quantum computers will be built. It may sound like a problem that will only impact security in the (more or less) distant future, depending on the development of quantum computers. But, in fact, it is already a serious threat to the confidentiality of data today: An adversary can store information encrypted using the RSA algorithm today in its encrypted form, and decrypt it once they have access to a quantum computer. If the data is supposed to be secret for an extended period of time (which is typical for data related to the military, intelligence agencies, or medical records), that means that quantum computers are already a threat to the security of data today, before they even have been built.

### 2.2 Quantitative measures of security

Another downside of security guarantees that rely on the hardness of specific problems is that they do not provide a quantitative security measure. One cannot directly map the length of the key to the level of security it provides. The decision of how long a key has to be is based on estimates of how long it takes state-of-the-art computers to crack it and how much resources they require for it. This is depicted in Figure 1: The probability that a classical cryptographic scheme can be broken,

denoted  $\epsilon^{\text{comp.}}$ , depends on its computational security and is the inverse of the costs that are required for breaking it. A classical algorithm, for instance the RSA encryption scheme, gradually becomes more and more insecure<sup>3</sup> over time due to evolution of hardware and discoveries regarding algorithms. Once the first universal quantum computer is built, the RSA scheme will immediately become insecure. This is true for any encryption scheme for which there exists an efficient quantum algorithm that breaks it. Post-quantum algorithms, which are classical techniques that cannot be broken efficiently by a quantum computer (see Section 3.3), do not immediately become insecure once a quantum computer is built, but they still suffer from becoming gradually more and more insecure due to hardware and software developments. Consequently, which key length is considered secure in applications based on classical (including post-quantum) algorithms such as RSA has to be continuously updated. The security of quantum key distribution, on the other hand, does not change over time since it is information-theoretically secure and hence does not depend on how powerful state-of-the-art computers are.

### 3 Quantum cryptography within the larger cybersecurity ecosystem

Classical cryptography can only guarantee a limited level of security since it is based on the conjectured hardness of certain problems. Quantum cryptography, on the other hand, can provide information-theoretic security that only relies on the laws of physics. In this section, we explain what this means, which role quantum cryptography can play within a larger cryptographic framework, and its limitations.

### 3.1 Information-theoretic security

As discussed above, classical cryptography is only *computationally* secure. What would be required to guarantee a level of security that is independent of any computational assumptions, i.e., information-theoretic or  $unconditional^4$  security?

It is well understood how one can construct a cryptographic scheme for private communication that is secure independent of any hardness assumptions. The crucial ingredient here is a *cryptographic key*, which is a sequence of uniformly random bits shared between two parties, which we call Alice and Bob. To encrypt a message, Alice computes the bitwise addition modulo 2 of the message and the key and sends the resulting ciphertext to Bob over a public channel. Bob can decrypt the ciphertext by subtracting the key, thus retrieving the original message. This is known as the one-time pad (OTP) encryption method [Ver26], depicted in Figure 2. It is vital, though, that the key fulfills the following requirements:

- 1. The key has to be truly random, which implies that the individual key bits cannot be correlated in any way.
- 2. No part of the key can ever be reused. In particular, this implies that the key has to be at least as large as the message that is supposed to be encrypted.
- 3. It has to be securely delivered to Alice and Bob, who are possibly far apart, such that no one else has any information about the key.

In this case, an eavesdropper keen to learn the message has only access to the ciphertext and the general encryption method cannot learn anything about the encrypted message, which was shown

<sup>&</sup>lt;sup>3</sup>Or, put differently, maintaining the same level of security becomes more and more costly, for example because one has to use longer keys. Therefore, the level of security here is always taken with respect to a fixed amount of resources.

<sup>&</sup>lt;sup>4</sup>The term "unconditional security" is often equated with "information-theoretic security", although strictly speaking, we do not eliminate all assumptions in quantum cryptography. The assumptions we still have to make concern, for example, the underlying theory (we assume that quantum theory is a correct and complete description of nature) or the measurements that are involved in the quantum protocol (this is discussed in detail in Section 5.3). However, in contrast to computational security, we drop all assumptions regarding the computational hardness of certain problems. The term unconditional security is thus to be understood as the statement that no conditions need to be imposed apart from assumptions about the underlying physical theory and the devices, which are usually implicit in classical cryptography. In this work, we use the two terms interchangeably.

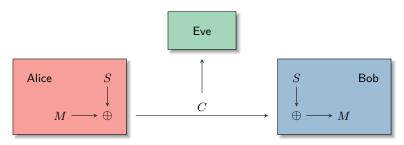


Figure 2: **One-time pad encryption.** Alice and Bob share a cryptographic key S. Alice encrypts the message M by performing bitwise addition modulo 2 on the key and the message,  $M \oplus S = C$ , thus producing the ciphertext C. The ciphertext is then sent over a public channel to Bob. Bob uses the key to retrieve the message from the ciphertext by computing  $C \oplus S = M$ . An eavesdropper (Eve) only has access to the ciphertext C.

by Claude Shannon in 1949 [Sha49]. In addition to showing that the OTP protocol is information-theoretically secure, Shannon proved that any unbreakable encryption scheme must have these characteristics.

Thus, the difficulty in constructing an unconditionally secure encryption system lies in generating a cryptographic key that fulfills the above conditions. This is where quantum mechanics comes into play. Quantum theory gives rise to several phenomena that do not exist in classical physics which can be exploited in cryptography. First, it exhibits an inherent randomness that can be used to generate truly random bits. Second, quantum mechanics gives rise to the phenomenon of entanglement, which is a notion of correlation between two (or more) parties that is stronger than any classical correlation. This can be exploited to generate the same string of bits in two distant locations.

Thirdly, it is impossible to perfectly clone quantum states, which means that (in contrast to classical information) an adversary cannot simply copy information encoded in quantum states. Therefore, to learn anything about the quantum information the adversary has to interact with the quantum system that carries this information, for example via measurements. This, however, leads to detectable changes in the system's state which alert Alice and Bob to the adversary. The process of generating a cryptographic key using quantum theory is called *quantum key distribution* (QKD) and the aspects of quantum theory that guarantee its security are discussed in detail in Section 4.

Building on these properties, the security of quantum cryptography is based on the laws of physics rather than the conjectured hardness of certain problems. This makes it *information-theoretically* secure, i.e., secure against an omnipotent adversary, which means that we not only consider them to have access to quantum computers but to any machine that might be developed in the future, regardless of how powerful it is (as long as it behaves according to the rules of quantum mechanics). This also implies that the security provided by QKD is *future-proof*, i.e., information encoded with it today will be secure arbitrarily long, in contrast to an encoding using classical algorithms! In addition, the security of a quantum key distribution protocol can be quantified in the sense that we can give precise mathematical bounds on the probability that it can be cracked, which is explained in Section 5.2.

### 3.2 Scope of applications for quantum cryptography

With its guaranteed unconditional security, quantum key distribution (QKD) seems to be a promising candidate for replacing classical key generation schemes within larger cryptosystems. Nonetheless, this is only true to some extent. In general, we distinguish between two different kinds of encryption: In symmetric encryption the same key is used to encrypt and decrypt the information, while in asymmetric cryptography two keys are required, a public one and a private one.

### Symmetric encryption

In principle, quantum key distribution is well suited to replace classical symmetric key generation methods. However, it is crucial to compose the quantum key distribution protocol with a suitable

encryption method to preserve its security level. As explained above, combining a QKD protocol with one-time pad encryption provides information-theoretic security. This scheme, however, is somewhat unsuitable when it comes to practical terms since the key has to be at least as long as the message, and no part of it can be reused. Hence, if one wants to encrypt many messages, it is necessary to generate large amounts of secret key. Since the implementation of QKD protocols is considerably more expensive than classical key generation protocols, it is desirable to use the generated key as efficiently as possible.

Unfortunately, there is no real alternative to the one-time pad if one wants to maintain the unconditional security provided by the QKD protocol. One of the standard encryption protocols that are used nowadays is the so-called Advanced Encryption Standard (AES) [Nat01]. The advantage of AES is that it requires a much shorter key to encrypt a given message compared to the one-time pad. The disadvantage, however, is that this encryption method only offers computational security, i.e., it is difficult to break in practice, but it does not provide the information-theoretic security that the one-time pad does. As a result, the composition of a QKD protocol and the AES encryption yields a scheme that is again just computationally secure. Hence, we wouldn't have needed to bother with implementing the QKD protocol in the first place but could have used a classical key generation method to receive the same level of security for the composed protocol.

#### Asymmetric encryption

Asymmetric encryption schemes like RSA are widely used nowadays, for example, to secure communication between web browsers and to create digital signatures. As explained above, the RSA scheme relies on the conjecture that factoring large numbers is difficult on a classical computer. However, since Shor's algorithm provides a way to solve this problem on a quantum computer efficiently, RSA encryption will effectively be unsafe once quantum computers are built. Does quantum cryptography offer a possibility to replace classical asymmetric encryption methods with unconditionally secure quantum ones?

Unfortunately, there are no asymmetric quantum cryptographic protocols; the reason is that quantum key distribution is only suitable for the generation of symmetric keys, which will become apparent when we explain how these protocols work in Section 5. Moreover, the impossibility of information-theoretic security of asymmetric encryption is a much more fundamental phenomenon. The problem with asymmetric encryption is that one of the keys is always public. This prevents any such scheme from being unconditionally secure since the encryption of information with the public key can always, in principle, be reversed (it is necessary to not use irreversible encryption algorithms to ensure that the legitimate recipient can decrypt the message again). The security thus always relies on making the reversion process impractically hard, but if an adversary had unlimited power, they would be able to perform the reversion.

### 3.3 Post-quantum cryptography

With the looming advent of quantum computers and the accompanying threat of them being able to crack standard encryption methods, researchers have started to investigate classical techniques that are quantum safe. This term refers to the fact that they cannot be broken efficiently by either a quantum or a conventional computer. Developing quantum-safe algorithms is all the more important since quantum cryptography cannot replace asymmetric classical encryption methods, as explained above. While certain asymmetric encryption schemes such as RSA immediately become insecure with the realization of a universal quantum computer, symmetric encryption schemes such as AES are not as much at risk. They are thought to be most vulnerable to quantum brute force attacks enabled by Grover's algorithm [Gro96]. An efficient countermeasure against these attacks is thus increasing the length of the keys used to ensure security. For example, the AES scheme with 256 bit keys is currently considered quantum-safe since, considering brute force attacks, it is as difficult to break for a quantum computer as the 128 bit key is for a classical computer.

The goal of post-quantum cryptography [BBD09, BL17] is thus to find algorithms that are hard for quantum computers (which, of course, includes classical computers as a special case). In some aspects, it faces similar problems as "traditional" classical cryptography: The security of post-quantum cryptographic schemes relies on the assumption that the underlying problem is hard

	Hardware developments	Software developments	Developments in physical theories
Classical cryptography	not safe	not safe	not safe
Post-quantum cryptography	safe	not safe	not safe
Quantum cryptography	safe	safe	not safe
Non-signaling cryptography	safe	safe	safe

Table 1: **Possible threats for cryptography.** Classical, post-quantum, quantum, and non-signaling cryptography exhibit different levels of security with regard to threats concerning hardware and software developments and new insights into the underlying physical theories.

to crack for quantum computers. Consequently, the question of which algorithms are considered quantum-safe needs to be updated regularly as quantum computers become more powerful and new quantum algorithms are developed. However, when it comes to the question of which algorithms are difficult for a quantum computer, researchers have the disadvantage that we cannot rely on decade-long trials, as is the case with many problems considered difficult to solve on classical computers. In other words, we do not really know yet which problems are hard for quantum computers, for the search for quantum algorithms to solve them only started quite recently.

Despite these difficulties, post-quantum cryptography has a crucial advantage compared to classical cryptography: It does not have to fear the development of a more powerful machine since a universal quantum computer is already the worst-case scenario, at least within the theory of quantum mechanics. The gradual improvement of hardware might make it necessary to adjust the length of the keys used in post-quantum algorithms (as explained in Section 2.2) from time to time, but the development of a universal quantum computer will not make them immediately insecure.

### 3.4 Cryptography beyond quantum theory

It is of course possible that, in addition to the development of hardware and software, new findings regarding the physical theories that describe our universe will also affect cryptography. For instance, quantum theory will likely have to be modified to allow for a theory that unifies quantum mechanics and gravity. This development would then not only have an impact on classical and post-quantum cryptography, but also quantum cryptography, as it relies on quantum theory. However, these changes are expected to be small and will not affect each part of the theory equally. As long as the quantum effects responsible for the security of quantum cryptography (which are explained in Section 4) remain roughly the same, the security of quantum cryptography is not at risk. We can even go one step further and formulate cryptographic protocols that do not depend on the correctness of quantum theory, but only rely on the so-called non-signaling condition, which means it is only required that faster-than-light communication is impossible [BHK05]. This kind of protocol is even secure against developments in physical theories, as we expect any theory that describes our universe to fulfill the non-signaling condition. The vulnerabilities of the different approaches to cryptography to potential threats are summarized in Table 1.

# 4 Concepts of quantum theory for cryptography

In the last section, we have seen that quantum mechanics can guarantee unconditional security due to the fact that it relies on the laws of physics rather than the computational hardness of certain problems. But what is it that makes cryptography based on quantum mechanics so much more powerful than its classical counterpart? In this section, we explain the underlying concepts of quantum physics and show how they play together to guarantee unconditional security.

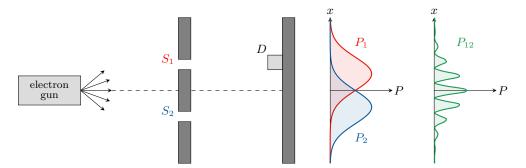


Figure 3: **Double-slit experiment with electrons.** The electron gun sends electrons towards a wall with two slits in it,  $S_1$  and  $S_2$ . Some electrons travel through the slits towards another wall. This wall is equipped with a movable detector D, for example, an electron multiplier that makes a sound whenever it detects an electron. We can use this setup to collect statistics about where the electrons are detected and plot the corresponding probability distributions. Whenever only one of the two slits is open, we see either the probability distribution  $P_1$  or  $P_2$  (depending on which slit is open). When both slits are open we observe the probability distribution  $P_{12}$ . From the diagram it becomes clear that  $P_{12} \neq P_1 + P_2$ , showing that the electron does not simply pass through one of the possible slits, but interferes with itself (which corresponds to being in a superposition of the to possibilities).

### 4.1 The state space of quantum mechanics

When thinking of quantum mechanics, the first thing that comes to mind is usually the Schrödinger equation, which describes the time evolution of a quantum system in terms of its wave function. While this is undoubtedly one of the most famous elements of quantum theory, it is not part of the fundamental concepts that we make use of in quantum cryptography. Instead, we exploit the specific structure and properties of the state space of quantum mechanics, which gives rise to unique quantum phenomena such as superposition, entanglement and uncertainty.

### Superposition

To illustrate these quantum phenomena, let us consider an example. An electron is a subatomic particle that can be in a state where its position is not uniquely determined. If it is part of a hydrogen atom, it orbits a positively charged nucleus at some distance, bound by the Coulomb force. According to quantum mechanics, the exact position of the electron cannot be predicted since this property of the electron does not have a predetermined value. If we nevertheless carry out a position measurement on the electron, we will get a specific position as a result. This result, however, has been produced randomly according to some probability distribution; it didn't exist before the measurement. The electron example shows one of the fundamental properties of quantum theory, called indeterminism. It describes the fact that properties of quantum objects such as position, momentum, and energy do not necessarily have a definite value. We can only assign probabilities to the different values of these properties.

Formally, the phenomenon that the electron does not have a definite position is described by the concept of superposition. Consider the situation in a double-slit experiment as depicted in Figure 3. The electron can either pass through the first slit or the second. Moreover, it can also be in a superposition of passing through the first and the second slit.<sup>5</sup> This is verified in the experiment by the interference pattern detected behind the double-slit that occurs if we only measure where the electron impinges on the screen, but not which path it has taken. If, on the other hand, we measure the position of the electron when passing the double-slit, we will randomly get one of the two positions, slit one or slit two. The measurement forces the electron to decide in which position it is. The concept of quantum superposition has no analog in classical physics; it is essentially different from anything we can observe in any classical theory.

<sup>&</sup>lt;sup>5</sup>This is sometimes described as the electron being in both positions at once, i.e., it takes both possible paths. However, this does not capture the situation correctly: We can only say that the probability of finding the electron in either path upon measuring it is nonzero, but a measurement would always either output the result "electron in path 1" or "electron in path 2", but never the result "electron in path 1 and 2".

The electron in the double-slit experiment is an example of a quantum system that is in a superposition of two states, namely the state "passes through slit  $S_1$ " and the state "passes through slit  $S_2$ ". As such, it is a realization of a so-called quantum bit or qubit, for short. This is a generalization of a classical bit and commonly used in quantum cryptography. The two values, corresponding to two perfectly distinguishable quantum states of the system, are typically denoted  $|0\rangle$  and  $|1\rangle$ . As explained above, a quantum system is not necessarily in one of these possible states but can be in a superposition of them. This means that the general state of a qubit can be written as a linear combination of the two possible states:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle. \tag{1}$$

From the coefficients  $\alpha$  and  $\beta$ , we can calculate the probability of getting the result 0 or 1, respectively, when measuring the qubit:

$$\Pr[0] = |\alpha|^2, \quad \Pr[1] = |\beta|^2. \tag{2}$$

Since the probability of finding the object in any one of the possible states has to be one, it follows that  $|\alpha|^2 + |\beta|^2 = 1$ . If both results, 0 and 1, occur with equal probability upon measuring the qubit, the state is of the form

$$|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle. \tag{3}$$

A priori, a qubit is an abstract notion that does not specify the physical system we are working with. A possible realization of a qubit (apart from the electron in a double-slit experiment) is provided by polarized photons. For example, we can identify the state  $|0\rangle$  with the horizontal polarization and the state  $|1\rangle$  with the vertical polarization. However, these are not the only choices for realizing the two states via directions of polarization. Alternatively, we could choose to identify the state  $|0\rangle$  with a diagonal polarization at angle  $+45^{\circ}$  and the state  $|1\rangle$  with one at angle  $-45^{\circ}$ . To distinguish between these two choices, we can use a different notation for the diagonally polarized states: We denote polarization at angle  $+45^{\circ}$  with the state  $|+\rangle$ , and polarization at angle  $-45^{\circ}$  with  $|-\rangle$ . Analogous to (1), we can hence realize a qubit with the two polarization states  $|+\rangle$  and  $|-\rangle$ , whose general state is

$$|\psi\rangle = \alpha'|+\rangle + \beta'|-\rangle. \tag{4}$$

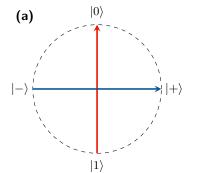
The two pairs of directions of polarization we have introduced above,  $\{|0\rangle, |1\rangle\}$  and  $\{|+\rangle, |-\rangle\}$ , are not independent of each other. They are connected via the relations

$$|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}, \quad |-\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}.$$
 (5)

The two possible ways of using polarization of photons are illustrated in Figure 4(a). Horizontal polarization, denoted  $|0\rangle$ , is visualized as a parallel direction to the red arrow. Vertical polarization  $|1\rangle$  then corresponds to the anti-parallel direction of the red arrow. Analogously,  $|+\rangle$  and  $|-\rangle$  correspond to parallel and anti-parallel directions of the blue arrow, respectively.

The measurement of the polarization of a photon can be depicted in a similar way, see Figure 4(b). For example, the measurement  $m_1$ , which is illustrated by an arrow that is parallel to the red arrow in Figure 4(a), corresponds to a measurement that can distinguish horizontal and vertical polarization. We say that it is a measurement in the  $\{|0\rangle, |1\rangle\}$ -basis. If the photon is horizontally polarized (which corresponds to the state  $|0\rangle$ ), such a measurement will always yield the result 0. If, instead, the photon is diagonally polarized at angle +45° (i.e., in the state  $|+\rangle$ ), we will get the results 0 and 1 each with a probability of 50% because of the relation (5) and the rule for calculating probabilities given in (2). These probabilities can also be derived from how the two arrows are aligned with each other: The arrow corresponding to the measurement

<sup>&</sup>lt;sup>6</sup>Here, we see a first difference between bits and qubits: While classical bits take either the value 0 or 1, quantum bits can be in a superposition of the two states  $|0\rangle$  and  $|1\rangle$ , and only upon measuring them they have to choose one. This is analogous to the electron in the double-slit experiment, whose path is in a superposition of both slits until we measure which path it takes.



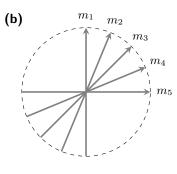


Figure 4: **Realization of a qubit via polarization.** (a) Possible realizations of a qubit via photon polarization. The vertical, red arrow visualizes the states  $|0\rangle$  (parallel to the arrow) and  $|1\rangle$  (anti-parallel to the arrow), and the horizontal, blue arrow visualizes the states  $|+\rangle$  (parallel) and  $|-\rangle$  (anti-parallel). (b) Some possible measurement directions  $m_i$  of photon polarization. The more parallel two arrows are, the more similar are the corresponding probability distributions of their outcomes.

 $m_1$  is orthogonal to the arrow corresponding to the state  $|+\rangle$ . If we interpret the two possible measurement outcomes as the two values a classical bit can take, the above observations show how measuring quantum states in a superposition allows us to generate truly random bits.

We can make a similar observation when instead of changing the state, we change the measurement. What happens when we measure the state  $|0\rangle$  with the measurement direction  $m_5$ , i.e., the one that is parallel to the state  $|+\rangle$ ? In this case, we will also get the two possible results with equal probability (note that the measurement arrow and the state arrow are orthogonal, just as in the previous situation). If we instead choose the measurement  $m_2$ , whose arrow is almost parallel to the red arrow, to measure the state  $|0\rangle$ , we will get the result 0 in a majority of the cases, i.e., we get results which are almost the same as the ones for  $m_1$ .

From these observations, we can derive some general rules: If the measurement arrow points in the same direction as the state arrow, the measurement outcome will always be the same. If the measurement arrow is orthogonal to the state arrow, the result will be completely random. Furthermore, the more parallel two measurement arrows are, the more similar are the respective probability distributions of the results.

### Entanglement

A single quantum particle already shows remarkable features that cannot be observed in classical systems. However, quantum particles do not exist isolated. Several quantum particles can be composed to form a bigger quantum system. These kind of composed quantum systems exhibit an additional feature that is unique to quantum theory, namely entanglement.

Let us first briefly go back to the electron example from above. We have seen that an electron does not have to be in a definite position but can be in a superposition of different positions. We can now form a composite system of two electrons. Similar to single-particle systems, a composite system of two electrons can also be in a superposition. However (and here the full extent of the strangeness of quantum mechanics reveals itself!), it is possible that even though neither electron has a definite position, the distance between them is well-defined. In this case, we say that the two electrons are entangled.

To explain the use of entanglement, we now focus on qubits: Suppose two parties, Alice and Bob, share an entangled state of the form

$$|\Phi\rangle_{AB} = \frac{|0\rangle_A |0\rangle_B + |1\rangle_A |1\rangle_B}{\sqrt{2}} \tag{6}$$

where the qubit with index A belongs to Alice and the one with index B belongs to Bob. This is a so-called *Bell state*. In this case, Alice and Bob do not know whether the individual qubits are in state  $|0\rangle$  or  $|1\rangle$ , but they do know that both qubits are in the same state (since no terms of the form  $|0\rangle_A|1\rangle_B$  or  $|1\rangle_A|0\rangle_B$  appear). The same is true if we consider the diagonal polarization

states  $|+\rangle$  and  $|-\rangle$ , i.e., one can show that the Bell state can be expressed equivalently as

$$|\Phi\rangle_{AB} = \frac{|+\rangle_A|+\rangle_B + |-\rangle_A|-\rangle_B}{\sqrt{2}}.$$
 (7)

That means that whenever Alice and Bob measure their individual qubits in the same basis, they will obtain the same results. As such, shared entanglement can be used as a resource in quantum cryptography, where Alice and Bob use the outcomes of their measurements to produce a secret key. Suppose Alice and Bob share a number of entangled states  $|\Phi\rangle_{AB}$  and measure their respective parts in order to generate a key. As long as their respective measurement bases are always the same, they will obtain perfectly correlated outcomes. If they choose measurement bases that are orthogonal to each other (compare Figure 4), for example Alice measures in the  $\{|0\rangle, |1\rangle\}$ -basis and Bob in the  $\{|+\rangle, |-\rangle\}$ -basis, their outcomes are uncorrelated. Thus, after the measurement step it is necessary that Alice and Bob exchange the information in which basis they have measured each state and discard the uncorrelated outcomes. Note that even though Alice and Bob get correlated results, these results are still perfectly random, and in particular independent of the choice of measurement basis. Hence, publicly revealing the chosen bases does not corrupt the secrecy of the generated bits.

#### Information gain vs. state disturbance

The structure of quantum states described above has a remarkable consequence for the ability to gain information about a system. Consider the scenario described above, where Alice and Bob share a number of entangled states and measure them in either the  $\{|0\rangle, |1\rangle\}$ - or the  $\{|+\rangle, |-\rangle\}$ -basis to generate a shared secret key. Let us now add a third party to the setting, an eavesdropper, called Eve, who wants to gain as much information as possible without being detected. One possible way of gaining information for Eve is to intercept the quantum states and measure them before they reach Alice's and Bob's respective laboratories.

For instance, suppose that Alice produces the entangled states in her lab and sends one half of each state to Bob. Eve can intercept the part that is sent to Bob, measure it, and resend the state to Bob. However, there are two problems here that limit the amount of information Eve can obtain: First, since she does not know in which basis Alice will measure her qubit, she has to guess the measurement basis. If she chooses a different one than Alice, the results are uncorrelated and Eve does not gain any information. Second, upon measuring the qubit that is supposed to be sent to Bob, Eve inevitably disturbs its quantum state if she wants to gain information. This stems from the fact that every informative measurement disturbs the state of the system [Bus09]. In fact, there is a trade-off between the information Eve can learn and the extend to which the state is altered: the more information she gains by measuring, the more she changes the state, which increases the chance that her attack will be detected.

Is it possible for Eve to build a measurement apparatus that measures both quantities at once, polarization in the  $\{|0\rangle, |1\rangle\}$ -basis and in the  $\{|+\rangle, |-\rangle\}$ -basis? This would solve the problem that the outcomes are uncorrelated if Eve chooses a different basis than Alice since she does not have to make a choice anymore. However, in quantum mechanics such apparatuses exist only for a limited set of so-called *jointly measurable* or *compatible* variables, and the two polarization bases do not fall into this set. When generating a key, Alice and Bob will always choose variables that are not jointly measurable, hence it is impossible for Eve to measure both quantities simultaneously.

In classical physics, it is generally not a problem to measure two properties of an object simultaneously. For instance, we can easily measure the position and momentum of a ball rolling down an inclined plane. In contrast, there is no way to measure both the position and the momentum of a quantum particle to arbitrary precision. For example, if we measure the electron's position exactly, our uncertainty of its momentum is very large, and vice versa. This is called *Heisenberg's uncertainty principle* [Hei27], and it is another important difference between classical and quantum physics.

<sup>&</sup>lt;sup>7</sup>If the eavesdropper is detected, Alice and Bob will simply abort the protocol and the eavesdropper does not gain any usable information.

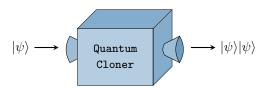


Figure 5: **A universal cloning machine.** This machine is able to copy an arbitrary, unknown quantum state  $|\psi\rangle$  such that we get two perfect copies of the state. Its existence is forbidden by the laws of quantum mechanics.

### 4.2 Quantum cloning is impossible

To gain information about a state in a quantum key generation procedure Eve cannot measure all properties of the quantum state simultaneously, as discussed above. Is it possible that Eve simply copies the state that is sent to Bob? She could then forward the original, undisturbed, state to Bob and perform measurements on her copy of the state. She could even produce several copies of the state in order to perform different measurements.

Fortunately, the structure of quantum mechanics implies that perfectly cloning an arbitrary unknown quantum state is impossible [WZ82]. Why can we not make a copy of a quantum state? Let us see what happens if we assume that it is possible to build a machine that can perfectly clone arbitrary quantum states, as illustrated in Figure 5. In the following, we will show that having access to such a machine would allow faster-than-light communication, which would violate special relativity. Ergo, we can conclude that it must be impossible to clone arbitrary quantum states.

The argumentation is as follows: Consider a scenario where Alice and Bob share a Bell state  $|\Phi\rangle$ , which is an entangled two-qubit state that can be written either in the  $\{|0\rangle, |1\rangle\}$ - or the  $\{|+\rangle, |-\rangle\}$ -basis:

$$|\Phi\rangle_{AB} = \frac{|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B}{\sqrt{2}} = \frac{|+\rangle_A|+\rangle_B + |-\rangle_A|-\rangle_B}{\sqrt{2}},\tag{8}$$

where the first qubit belongs to Alice and the second one to Bob. If Alice measures her part of the state in the  $\{|0\rangle, |1\rangle\}$ -basis and gets either 0 or 1 as the result, Bob's qubit will collapse to the corresponding state. If he then performs a measurement in the  $\{|0\rangle, |1\rangle\}$ -basis on his qubit he will get the same result as Alice with certainty. Similar for the  $\{|+\rangle, |-\rangle\}$ -basis: Whether Alice gets the result + or -, Bob's qubit will collapse to the respective state. However, Bob is unable to learn which basis Alice has measured in just by measuring his qubit. A single measurement outcome does not reveal anything about the probability distribution of the outcomes.

Suppose now that Bob has access to the quantum cloning machine depicted in Figure 5 and he uses it to produce 1.000 perfect copies of his state. After Alice has measured her part of the state Bob measures half of them in the  $\{|0\rangle, |1\rangle\}$ -basis and the other half in the  $\{|+\rangle, |-\rangle\}$ -basis. Say, Alice has measured her qubit in the  $\{|0\rangle, |1\rangle\}$ -basis and has gotten  $|0\rangle$  as a result. The states that Bob measures in the  $\{|0\rangle, |1\rangle\}$ -basis will then produce the output 0 with certainty, while the states that are measures in the  $\{|0\rangle, |1\rangle\}$ -basis will yield random outputs. As a result, Bob will observe (roughly) the following probability distribution:

$$\begin{aligned} \{|0\rangle,|1\rangle\}\text{-basis:} & & \Pr\big[0\big] = 100\% \\ & & \Pr\big[1\big] = 0\% \\ \{|+\rangle,|-\rangle\}\text{-basis:} & & \Pr\big[+\big] = 50\% \\ & & \Pr\big[-\big] = 50\% \end{aligned}$$

From this probability distribution Bob can deduce that Alice has measured in the  $\{|0\rangle, |1\rangle\}$ -basis, which corresponds to the superluminal transmission of one bit (given that they are far apart, but since this argumentation holds for any distance between Alice and Bob we can make this distance sufficiently large). However, since we have assumed that faster-than-light communication is impossible, it follows that perfect cloning of arbitrary quantum states must be impossible.

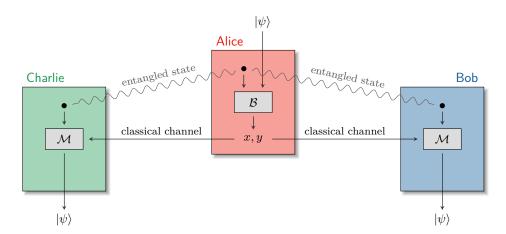


Figure 6: Quantum cloning via teleportation. We can show the monogamy of entanglement via an argument by contradiction: Suppose that Alice possesses a qubit state that is perfectly entangled with both Bob's and Charlie's respective quantum states. She performs a Bell measurement  $\mathcal B$  on this state and the state  $|\psi\rangle$  she wants to clone. This measurement outputs two classical bits x and y, which Alice sends to Bob and Charlie over classical channels. The values of the two bits provide the information which measurement  $\mathcal M$  Bob and Charlie have to perform on their respective states in order to transform those to the state  $|\psi\rangle$ . This scheme effectively copies the qubit state  $|\psi\rangle$  without requiring any knowledge of the state itself. Note that this scheme would only work if it were possible for Alice's state to be entangled with both Bob's and Charlie's state at the same time. Since we know that cloning is impossible, this argument shows that polygamy of entanglement is impossible.

### 4.3 Monogamy of entanglement

So far, we have seen that an eavesdropper can neither gain information by measuring all properties of a quantum system simultaneously nor can they make copies of the states that are sent. Maybe it is possible for Eve instead to entangle her quantum system with Alice's quantum state such that her measurement results are perfectly correlated with Alice's measurement outcomes? In other words, is it possible that Alice's quantum system is perfectly entangled both with Bob's quantum state and with Eve's? In this way, Eve would get the maximum amount of information without being detected since she never has to perform any measurement on Alice's and Bob's respective quantum states.

Fortunately (for the sake of quantum cryptography), this strategy is prohibited by the so-called *monogamy of entanglement* [CKW00]. This term describes the phenomenon that when two quantum systems are perfectly entangled (such as Alice's and Bob's quantum systems are if they are in the state (8)), neither of them can be entangled to a third system. This follows directly from the fact that quantum cloning is impossible, which was shown in the previous section. Or, put differently: If "polygamy" of entanglement were possible, cloning arbitrary unknown quantum states would be possible.

This can be illustrated using a protocol called quantum teleportation [BBC+93], whose goal is to send a quantum state from one party to another (spatially distant) party without using a quantum channel (hence the name "teleportation"). However, the two parties are allowed to exchange classical information. It works as follows: Alice and Bob share a perfectly entangled state (for example, the Bell state in (8)). To teleport an arbitrary qubit state  $|\psi\rangle$  to Bob, Alice measures her part of the entangled state together with the state  $|\psi\rangle$  in a so-called Bell measurement. This measurement effectively outputs two classical bits x and y. Alice then sends these bits to Bob via a classical channel. The bits provide the information which measurement Bob has to perform on his part of the entangled state to transform it into the state  $|\psi\rangle$ . Note that this does not violate the fact that quantum states cannot be cloned: The state  $|\psi\rangle$  is destroyed at Alice's lab by the Bell measurement before it is recreated in Bob's lab.

Suppose it were possible for Alice to be entangled with more than one party. In that case, she could execute the same protocol simultaneously with Bob and a third party, called Charlie (see Figure 6). First, she entangles her quantum state with both Bob's and Charlie's respective quantum

states. She then continues with the protocol steps as described above but sends the results x,y of the Bell measurement to both Bob and Charlie. Both of them can then use this information to perform the measurement  $\mathcal M$  on their quantum systems. Because each of their respective systems was entangled with Alice's state in the beginning, the measurement will produce the initial state  $|\psi\rangle$  in both Bob's and Charlie's lab, effectively cloning it. Since we have shown above that cloning unknown quantum states is impossible, in turn, it must be impossible for Alice to be entangled with two quantum systems at once.

### 5 Quantum key distribution

Quantum key distribution protocols enable two spatially distant parties to establish a shared secret key, which means that both parties have an identical, completely random bit string, such that the knowledge an adversary might have on the key is negligible. Combining such a protocol with the one-time pad scheme (see Figure 2) enables unconditionally secure data encryption. There is a variety of different QKD protocols that focus on different aspects, but their security is always based on the quantum phenomena that were explained in the previous section. Some protocols are designed to account for complications in practical implementations such as imperfect devices and noisy quantum channels, and different kinds of protocols make different kinds of assumptions on the incorporated physical devices, resulting in different levels of security. Here, we explain the general structure of a QKD protocol and present different classes of such protocols. Additionally, we discuss the state of the art in both theory and experiment, and which challenges have yet to be overcome.

### 5.1 The structure of QKD protocols

As a starting point, we describe a general QKD protocol that exploits entanglement between the two honest parties, Alice and Bob. Most protocols are varieties of this one. The general setting as depicted in Figure 7 is the following: Alice and Bob have access to a source (which can be located either inside one of Alice and Bob's laboratories or outside of both) that distributes quantum systems via an insecure quantum channel. This allows Eve to arbitrarily interact with the quantum systems. However, due to the nature of quantum mechanics she will either be detected or not gain any information on the quantum states as explained in the previous section. Furthermore, each party has a quantum device  $\mathcal Q$  to measure the received quantum systems, a classical computing device  $\mathcal C$  that stores and processes all classical information, and a trusted random number generator RNG which provides the randomness required for some steps of the protocol (see below). In addition, Alice and Bob can communicate via an authenticated public classical channel, which means that an eavesdropper can listen to all of their messages, but cannot modify them.

We divide the protocol into two phases: The first one is the *quantum phase*, where quantum systems are distributed and measured, thus acquiring the classical data called the *raw key*. It consists of several rounds, each of them containing the following steps:

- 1. State distribution: The source distributes quantum systems in an entangled state (e.g., the qubit state  $|\Phi\rangle_{AB}$  from (8)) to Alice and Bob via insecure quantum channels. If the source is located in one of the laboratories, say Alice's lab, in this step Alice prepares the system in an entangled state and sends one half of it to Bob via an insecure quantum channel while keeping the other half. During this step, it is possible for an eavesdropper to tamper with the quantum systems.
- 2. **Measurement:** Alice and Bob each randomly and independently of each other select one out of several possible measurements. The necessary randomness for this step is provided by their respective random number generators RNG. For example, they could choose between measuring in the  $\{|0\rangle, |1\rangle\}$ -basis and in the  $\{|+\rangle, |-\rangle\}$ -basis (see Section 4.1). Both parties measure their part of the system using the quantum device  $\mathcal Q$  and store the classical result.

After completing the quantum phase, Alice and Bob share a pair of bit strings, which are partially correlated and partially secret. This stems from the fact that an eavesdropper could have

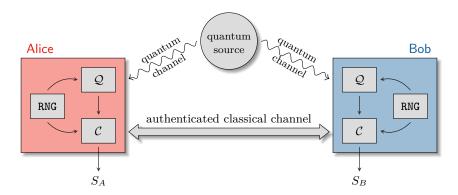


Figure 7: **Setting of a typical QKD protocol.** A source distributes quantum systems in an entangled state to Alice and Bob via insecure quantum channels. The two parties each have access to a quantum device  $\mathcal{Q}$ , a classical computer  $\mathcal{C}$ , and a trusted random number generator RNG. Additionally, Alice and Bob can communicate via an authenticated public classical channel.

performed an attack on the quantum systems during the protocol, thus compromising the entanglement and collecting information on the raw key. In addition, Alice and Bob have to choose their measurements randomly and independently of each other in the second step, which is necessary so that an eavesdropper cannot predict the measurements beforehand and adapting their strategy accordingly. However, that also means that in those rounds where Alice and Bob have chosen incompatible measurements (for example, if Alice has chosen to measure in the  $\{|0\rangle, |1\rangle\}$ -basis and Bob in the  $\{|+\rangle, |-\rangle\}$ -basis), their results are uncorrelated.

All these aspects are accounted for in the second phase, the classical post-processing. This part of the protocol is purely classical and it has two objectives: First, Alice and Bob estimate the errors in their bit strings. Errors can occur because of noisy channels and imperfect devices, but also because of the attack of an eavesdropper as noted above. To account for the worst-case scenario, all errors are typically attributed to the involvement of an eavesdropper. The amount of errors decides whether Alice and Bob abort the protocol or continue with the classical post-processing. The second objective of this phase is to turn the generated raw key pair that Alice and Bob share into a pair of shorter but identical and secret bit strings, the cryptographic key. The steps are the following:

- 3. Sifting: Alice and Bob publicly compare the bases they have chosen in the second step. Only those rounds where they have chosen the same measurement provide correlated results, hence they discard those rounds where they have chosen different measurements.
- 4. **Parameter estimation:** Alice and Bob randomly choose a number of rounds and announce their respective results. In an ideal scenario with perfect devices and no eavesdropper, the results should always coincide after the sifting step. Hence, the comparison of their results gives them an accurate estimate of the error rate of their strings. If the error rate is higher than a previously determined threshold, Alice and Bob abort the protocol.
- 5. Error correction: After having identified an estimate on the error rate of their strings, Alice and Bob use a classical error correction protocol to turn their correlated strings into identical strings.
- 6. **Privacy amplification:** Alice and Bob employ a privacy amplification procedure to remove Eve's information on the key, which shortens it by a certain amount.

After completing the steps of this protocol, one of two possible scenarios will occur. Either the protocol has aborted, or Alice and Bob hold a pair  $(S_A, S_B)$  of identical, secret bit strings, which they can then use to encrypt messages. The protocol will never (or, to be precise, with negligible probability) output an insecure key.

### 5.2 Quantifying security

In contrast to classical key generation schemes, the structure of a QKD protocol allows us to directly link the length of the generated key to its security. It is important to note that the security is never perfect. For example, in the quantum phase, Eve could guess the measurement bases correctly for each protocol round. In this case, she could measure all the systems without being detected, and her outcomes would be perfectly correlated with Alice's information. Even though this is possible, it is highly unlikely. If the protocol has n rounds, the probability of Eve guessing all bases correctly is vanishingly small, namely  $2^{-n}$ . As mentioned above, it is also possible that the protocol aborts because the eavesdropper has gained too much information on the raw key. In this case, Alice and Bob can simply restart the protocol and make a second attempt at generating a key. The scenario they want to avoid is that the protocol does not abort but outputs an insecure key. An insecure key is a key where either the two bit strings of the key pair are not identical or where Eve has received any information about the key. This results in two quantities that describe the security of a QKD protocol:

- 1.  $\epsilon_{\text{correct}}$  = the probability that the protocol does not abort, but Alice and Bob's bit strings are not identical
- 2.  $\epsilon_{\text{secret}}$  = the probability that the protocol does not abort, but Eve has some information about the generated key pair

The values  $\epsilon_{\text{correct}}$  and  $\epsilon_{\text{secret}}$  are often combined to a single security parameter  $\epsilon := \epsilon_{\text{correct}} + \epsilon_{\text{correct}}$  $\epsilon_{\rm secure}$ . They are directly connected to the secret key rate of the protocol, which is the number of key bits generated per round. In the error correction step, Alice and Bob not only apply an error correction protocol, but also check whether it has been successful. This is typically done by comparing hashes of the respective bit strings, which consumes a small number of bits from the raw key. This procedure has the advantage that Alice and Bob never have to reveal the actual values of their strings. The disadvantage, however, is that there is a certain probability that the bit strings are different even though the corresponding hashes are identical. This probability depends on how many bits have been used to compute the hashes, resulting in a trade-off between the amount of key that is generated and the probability that the bit strings still contain errors. The smaller that probability is made, the more bits from the raw key must be used for error correction, hence shortening the key. A similar situation presents itself in the privacy amplification step, where the error-corrected bit strings are compressed to remove Eve's information. The more they are compressed, the less information Eve has on them; hence it is a trade-off between the length of the key and its secrecy. Therefore, choosing suitable values for these two quantities means finding a trade-off between the amount of key generated and the level of security it guarantees.

How do you find a suitable value of the security parameter  $\epsilon$ ? As it represents the probability that something goes wrong without the protocol aborting, ideally we want it to be as close to zero as possible. However, decreasing the value of  $\epsilon$  is always accompanied by an increase in cost for the implementation of the protocol, since we need more rounds for the same amount of key when  $\epsilon$  decreases. A good guide for determining an appropriate value is to consider two types of costs involved (see Figure 8): One is the cost of decreasing  $\epsilon$ , the other is the cost of insuring possible malfunctions of the device. When choosing the value of  $\epsilon$ , these two costs have to be weighed against each other. Typical values for the two parameters are  $\epsilon_{\text{correct}} = 10^{-10}$  (the probability that Alice and Bob receive non-identical strings), and  $\epsilon_{\text{secret}} = 10^{-5}$  (the probability that an eavesdropper has any information on the key), see for example [AFDF<sup>+</sup>18].

### 5.3 Assumptions and security guarantees

Any security proof we formulate for a QKD protocol is based on assumptions about the systems, states, and measurements and even about the underlying physical theory. Most of these assumptions depend on the kind of protocol that is considered, but there are some fundamental ones independent of this choice:

1. Quantum theory is correct. We assume that quantum theory makes accurate predictions about measurement outcomes. This is supported by the fact that countless experiments

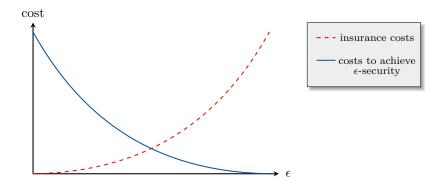


Figure 8: Schematic depiction of the tradeoff between security and costs. The graph shows the costs associated to a given security level  $\epsilon$ . Increasing the security, which corresponds to lowering  $\epsilon$ , requires higher resource costs (depicted in blue, solid). For example,  $\epsilon$  can be made smaller at the cost of increasing the length of the raw key, which in turn requires a higher communication bandwidth. Conversely, the costs for an insurance (depicted in red, dashed) that would pay for the damage incurred by a security breach are proportional to the probability of such a damage event, i.e., they decrease with  $\epsilon$ . An optimal choice of the parameter  $\epsilon$  minimises the total of the resource costs and the insurance costs.

have verified the predictions of quantum mechanics. Nevertheless, the following remark is in order here: While the assumption that quantum mechanics is correct is surely sufficient for quantum cryptography to be secure, it might be too strong for this purpose. The security of quantum cryptography does not rely on all aspects of quantum theory, but only on those we have introduced in Section 4: From the structure of the state space and the requirement that superluminal communication is impossible alone we were able to show that quantum states cannot be cloned and entanglement is monogamous. We did not make use of other elements of quantum theory such as the Schrödinger equation. Consequently, the security of quantum cryptography only requires to assume that those parts of quantum theory discussed in Section 4 are correct. What's more, even if these parts of quantum theory have to be modified, quantum cryptography can still be expected to be secure as long as the changes are minor.<sup>8</sup>

- 2. Free randomness exists. We assume that measurement choices (e.g., basis choices) can be made randomly and independently of the measurement device. This is indicated in Figure 7 by the random number generator RNG and the quantum device  $\mathcal{Q}$  being depicted as two independent devices.
- 3. Quantum theory is complete. We assume that quantum theory is *complete*, which means that there is no extension of the theory that can provide improved predictions. We have stated above that QKD guarantees security against an adversary who can use any attack that is possible within the framework of quantum theory. Completeness guarantees that the adversary cannot obtain any more information on the generated key than what is predicted by quantum theory.<sup>9</sup>

Apart from these fundamental assumptions about the underlying physical theory, there is a variety of additional assumptions one can make about the implementation of the protocol. Generally, the security proof becomes easier the more assumptions we make as each piece of information we have on the devices reduces the possible attacks an eavesdropper can perform. For example, if we assume that the devices do not leak any unauthorized information, we do not have to factor in this possibility in the security proof. On the other hand, the security proof only holds if the

<sup>&</sup>lt;sup>8</sup>This is especially important with regard to inevitable adjustments of quantum theory that have to be made in order to combine it with gravity to a unified theory of quantum gravity. As long as those parts discussed in the previous section are still true, the security of quantum key distribution is still provable in a theory of quantum gravity.

<sup>&</sup>lt;sup>9</sup>It has been shown that the completeness of quantum theory follows from its correctness and the existence of free randomness [CR11]. We nevertheless list this point here independently of the others to emphasize its importance.

assumptions are fulfilled. Consequently, any deviation of the implementation from the assumptions renders the protocol effectively insecure.

In practice, devices used in real-life realizations of QKD protocols rarely conform to the theoretical description of the apparatuses. This opens security loopholes (so-called side-channel attacks) the eavesdropper can exploit to obtain information about the key without being detected. This is known as quantum hacking and has been demonstrated experimentally numerous times. What are typical attacks that exploit such loopholes? If an implementation uses single photons as information carriers, the adversary can exploit that no perfect single-photon sources exist. Sometimes, the source emits a pulse consisting of two or more photons, and these pulses reveal information about Alice's basis choice that the adversary can exploit via the so-called photon number splitting attack [FGSZ01]. As a countermeasure, the decoy-state protocol [Hwa03, Wan05, LMC05] was developed which allows to detect this kind of attack. While this attack exploits vulnerabilities of the sender, the detectors are also susceptible to side-channel attacks. Typical examples are the time-shift attack [MAS06, QFLM07, ZFQ+08], which exploits inaccuracies of the photon detectors, and the detector blinding attack [Mak09, LWW+10, GLLL+11], where the adversary gains control over the detectors by illuminating them with bright laser light.

A way to avoid this vicious circle of design and attacks of quantum cryptographic schemes is device-independent cryptography, which does not make any assumptions on the inner workings of the quantum devices. However, it comes with disadvantages regarding achievable key rates and distances, as explained in Section 5.5. In general, one has to find a trade-off between a reasonable set of assumptions regarding the resulting security and what is practically feasible, which has led to different categories of QKD protocols.

### Device-dependent QKD

The first proposed quantum key distribution protocols, the BB84 protocol [BB84] and the Ekert protocol [Eke91], exactly specified which states have to be prepared and which measurements are carried out on those states. In the subsequent decades, further protocols were presented that also relied on the exact specification of the deployed devices for their security proof, see for example [Ben92, SARG04]. These kinds of protocols fall under the name device-dependent QKD, since the security proof depends on the exact characteristics of the devices. For instance, the QKD protocol described in Section 5.1 can be implemented using Bell states (8) and measurements in the  $\{|0\rangle, |1\rangle\}$ - and the  $\{|+\rangle, |-\rangle\}$ -basis for both Alice and Bob. Based on this choice, it is possible to calculate the key rate depending on the chosen values of  $\epsilon_{\text{correct}}$  and  $\epsilon_{\text{secure}}$ . This is an example of a device-dependent protocol.

While knowing the exact characterization of the incorporated devices simplifies the security proof of the protocol, it has some drawbacks: The resulting security statement only holds if the quantum devices behave exactly as specified, i.e., the quantum source produces only the predefined states and the measurement devices only measure in the two given bases. Consequently, any deviation of the actual devices in the lab compromises the protocol's security.

In practice, the devices never behave exactly as specified. Some examples how implementations deviate from their theoretical description are that quantum states cannot be prepared with arbitrary precision, and measurement devices have dead times and dark counts. A more extensive list of problems regarding practical implementations can be found in [SK14]. This mismatch between theoretical assumptions and realistic implementations has led to the development of QKD protocols that treat the quantum devices (or some of them) as black boxes instead of specifying which measurements they perform. They fall under the name device-independent QKD and semi-device-independent QKD, and their security is based (either partly or solely) on phenomena that do not depend on any assumptions about the inner workings of the devices.<sup>10</sup>

#### Device-independent QKD

In device-independent QKD (DIQKD) protocols [PAB+09, VV14], all quantum devices are treated as black boxes, see Figure 9(a). First, this means that we do not make any assumptions about what

<sup>&</sup>lt;sup>10</sup>It is important to stress that this only concerns the quantum devices we employ in the protocol. The classical devices required for classical post-processing and storage of classical information always have to be trusted.

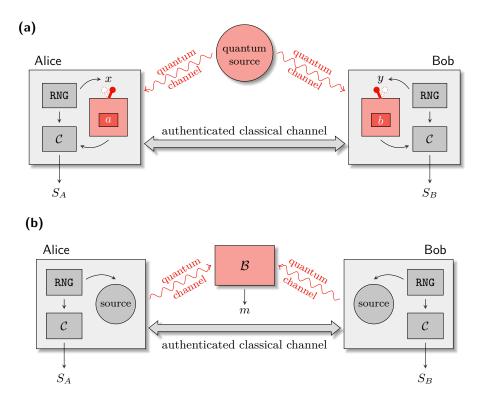


Figure 9: **Untrusted quantum devices.** (a) In device-independent QKD, both the source of quantum states and the measurement devices, along with the quantum channels, are untrusted (indicated in red). The measurements are treated as black boxes with an input x (respectively y for Bob) and a classical output a (respectively b). (b) In measurement-device-independent QKD, the preparation of quantum states takes place in Alice and Bob's respective laboratories. Both of them send a quantum system over an untrusted quantum channel to an untrusted relay where the Bell measurement  $\mathcal B$  takes places, whose outcome m is publicly announced. Here, too, the untrusted parts are marked in red.

quantum systems the source distributes to Alice and Bob. Second, we now view the measurement devices as boxes that take a classical input x (respectively y for Bob) and produce a classical output a (respectively b). While in practice, they are still realized by quantum measurements (where x and y correspond to the choice of measurement), the protocol's security is not derived from the precise states and measurements involved. Instead, the devices itself are being tested throughout the execution of the protocol and the security is derived from these tests. This is typically achieved by evaluating a so-called  $Bell\ inequality\ [Bel64,\ CHSH69]$ , which allows us to certify entanglement between Alice and Bob (and thus how much information an eavesdropper could have gained) simply from the input-output statistics of the boxes.

By removing all assumptions about the quantum devices from the security analysis, a higher level of safety is achieved, as the security proof applies to all realizations of the states and measurements involved in the protocol. As such, we do not have to worry about deviations from the theoretical description of the protocol. This even includes the case of malicious devices, where the manufacturer has manipulated the devices to obtain information about the key. Naturally, the improved security guarantees come at a cost. Not making any assumptions about the inner workings of the quantum devices also implies that we have to consider the worst case, where the eavesdropper, in principle, has absolute control over the devices. Consequently, we obtain lower key rates than in device-dependent settings, resulting in higher requirements on experimental implementations.

#### Semi-device-independent QKD

As an alternative to treating all quantum devices as black boxes, it is possible to formulate protocols that fall somewhere between device-dependent QKD (where all quantum devices are trusted) and

DIQKD (where none of the quantum devices are trusted). A wide range of protocols can be formulated in this area: one can either trust the source that produces the quantum systems or the measurement devices, or trust only one of the involved parties. A variety of protocols has been formulated here, all of which fall into the category of *semi-device-independent QKD*.

While we cannot discuss all of these approaches in detail here, there is one particular approach that stands out: Most attacks that exploit the implementation of a protocol rather than its theoretical concepts aim at the detectors. To circumvent this problem, measurement-device-independent (MDI) QKD [BP12, LCQ12] rules out the possibility of exploiting imperfect detectors by removing them from the trusted part of the setting (i.e., Alice and Bob's respective labs) to an untrusted relay where the measurement takes place, see Figure 9(b). Alice and Bob then independently prepare quantum states instead of receiving and measuring them. The measurement is a so-called Bell measurement which entangles the previously uncorrelated systems sent by Alice and Bob. The result of the measurement, together with the parties' respective local information about the prepared states, enables Alice and Bob to generate correlated key bits.

MDI QKD cannot provide the same level of security as DIQKD since the quantum state preparation has to be trusted. On the other hand, the characterization of the state preparation results in fewer requirements on experimental implementations and easier security proofs. In addition, MDI QKD also solves a problem stemming from an implicit assumption in both device-dependent and device-independent QKD: To ensure the protocol's security, we have to ensure that no unauthorized information leaves Alice's and Bob's respective labs. This can be achieved by shielding the labs accordingly, but there will always remain a weak spot in the shielding where the quantum systems distributed by the source enter the lab. At least for a brief moment, the outside world has access to the laboratory, and it is crucial to ensure that during this time no unauthorized information such as the measurement settings is leaked. This requirement, however, is often difficult to meet in practice. The advantage of MDI QKD is that this assumption is not necessary because the measurement process takes place at an untrusted relay anyway.

### 5.4 Discrete- vs. continuous-variable QKD

All of the protocols discussed above fall in the category of discrete-variable QKD (DV QKD) because the information is encoded in a discrete way. Typically, this is implemented by using single photons as information carriers, exploiting for example different polarization directions to realize the two states of a qubit. The key is then established via detection of individual photons. In practice, however, there are no perfect single-photon sources and detectors, which has consequences for the security analysis of the protocol: If one does not account for this deviation from perfect single photons in the security proof, it opens loopholes that an adversary can exploit to obtain information on the key. An example of such an attack is the so-called photon number splitting attack, which exploits that sometimes more than one photon is emitted [FGSZ01]. Including these deviations, on the other hand, generally makes the theoretical analysis more difficult and, additionally, decreases the key rate of the protocol.

These challenges have led to the idea of encoding information in properties of light that are continuous, such as the quadrature components of the electromagnetic field, yielding continuous values as measurement results. These protocols go by the name *continuous-variable QKD* (CV QKD) [WPGP+12]. The great advantage of this kind of protocols is that their implementation only requires standard telecommunication technology which is also used in classical optical communication. On the theoretical side, CV QKD faces the same challenges as DV QKD: A complete security proof includes proving security against general attacks under reasonable assumptions and for meaningful security parameters. The fundamentally different way of encoding information (namely via continuous instead of discrete quantities) however, leads to new challenges regarding the security analysis of such protocols since most of the techniques are specific to DV QKD and cannot directly be extended to the CV case.

### 5.5 State of the art and challenges in theory and experiments

In recent years, enormous progress has been made in all areas of quantum cryptography, both in theory and experiment, with the aim of realizing a trustworthy QKD system with the best possible security guarantee. However, although QKD is at a completely different level today than it was ten years ago, there are still many challenges to overcome before it can be considered ready for the market, both on the theoretical and experimental side.

In general, there are three parameters that allow us to benchmark and compare different protocols and implementations:

- 1. Security: The most vital aspect of any quantum cryptographic primitive is security, which can be divided into three aspects. First, we need to state the assumptions we make on the incorporated devices, which means that we need to decide between device-dependent, semi-device-independent, and device-independent QKD. The second aspect is the kind of attacks we consider in the security proof: While allowing for the most general attacks by the eavesdropper provides the best security, these attacks more complicated to analyze than those where the eavesdropper only has limited power, and the necessary techniques are developed to different levels for different kinds of protocols. The third aspect of security are the values for the two security parameters discussed in Section 5.2. Since they essentially represent the probability that the protocol fails without Alice and Bob being aware of it, choosing sensible values for them is a central part of the analysis of any protocol.
- 2. Achievable distance: Another important criterion to decide whether the protocol is useful for a particular application is the distance over which a key can be established. Depending on the protocol and the implementation, this is limited by different factors. One main limitation, especially with regard to optical implementations, is the photon loss in optical fibers.
- 3. **Key rate:** The rate at which a key is generated is a significant performance measure for any QKD protocol. A protocol that can generate a key over long distances is only of practical relevance if the key rate is sufficiently high. The long-term goal is to close the gap between typically deployed classical key rates (around 100 Gbit/s) and those achievable with quantum protocols. Limiting factors here generally are the efficiency and dead-time of the operated detectors. For protocols that use a source of entanglement, the rate at which entangled states can be successfully generated is another crucial aspect that limits the key rate.

It is generally impossible to have optimal values for all three factors listed above: Increasing the distance between the two parties in a QKD protocol results in additional losses, which reduces the key rate. Likewise, requiring small  $\epsilon$ -values comes with additional costs for error correction and privacy amplification, also reducing the key rate. Therefore, a crucial part of any practical realization of a QKD protocol is to find a reasonable trade-off between these three aspects, depending on the focus of the implementation. In the remainder of this section, we present the state-of-the-art theoretical end experimental developments for the different kinds of protocols presented in the previous section and the main challenges that still have to be overcome.

### 5.5.1 Security proofs

On the theoretical side, the ultimate goal for any QKD protocol is to prove its security against general attacks where we assume that an adversary is only limited in their attacks by the laws of physics. Since proving the security of the different types of protocols presented in Section 5.3 is increasingly challenging the fewer assumption we make, naturally the current state of the respective proofs is different.

In device-dependent QKD we have the most complete security proofs (see, for example, [TL17]). Given that we accept the assumptions we make in this case (such as the specification of the quantum states and measurements), there exist techniques for proving the security of these protocols against general attacks. A typical strategy for such a proof is to first analyze the protocol for collective attacks, i.e., where the adversary applies the same attack in every round, and the individual rounds are uncorrelated. For this case, powerful numerical tools have been developed [CML16, WLC18]. From this, security against general attacks can then be inferred via techniques based on the quantum de Finetti theorem [Ren08, CKR09].

In device-independent QKD, one can use a similar strategy to prove security. First, one can prove security against collective attacks using numerical methods such as the one presented by Brown et al. [BFF21]. This can then be lifted to a full security proof using the so-called Entropy

Accumulation Theorem [DFR20, AFDF+18]. Hoewever, the biggest issue that is unsolved to this day is composability of DIQKD. A protocol is said to be composably secure if it can be used as a subprotocol in any other routine without compromising security. This is a natural requirement if the generated key is supposed to be used in another cryptographic protocol such as the one-time-pad scheme. While composable security can be shown for device-dependent QKD [PR21], it has not yet been rigorously proven for device-independent protocols. Even worse, there is evidence that DI QKD is not composably secure when the same devices are being reused in the composition. It was pointed out by Barrett et al. [BCK13] that a malicious device could store data that was generated during the first execution of a protocol, and upon a second run of the protocol leak the data from the first run. Since all the techniques used to prove security only deal with a single execution of the protocol, these kind of attacks are not accounted for. Note that this is not a problem in the device-dependent case: Here, we can make the assumption that the devices do not store any information after the execution of the protocol is finished. This kind of assumptions cannot be made in the DI QKD case since they are direct assumptions on the incorporated quantum devices and therefore contradict the idea of device-independence.

If information is encoded in continuous variables instead of discrete ones we face new challenges regarding the theoretical analysis. Many techniques that are used in the discrete-variable case cannot be directly extended to CV QKD, such as the quantum de Finetti theorem and the Entropy Accumulation Theorem. Consequently, there is no general security proof yet, only proofs for a limited number of protocols that use specific states and measurements. An overview of the current state of security proofs in CV QKD can be found in [DL15].

### 5.5.2 Experimental implementations

Before we discuss achievements in the respective categories of protocols, we want to mention one challenge that all of them face: There is a fundamental limit to the distance over which a key can be generated due to losses in the quantum channel that connects the two parties [PLOB17]. One approach to overcome the distance limit is the use of quantum repeaters [BDCZ98, SSdRG11]. Unlike classical signals, the quantum signals used in QKD cannot be noiselessly amplified due to the no-cloning theorem (see Section 4.2). However, using quantum memories makes it possible to establish a pair of entangled states over high distances. The idea is to divide the quantum channel into segments (which can be made sufficiently small) and generate entangled pairs between the intermediate stations. These entangled pairs are then used to carry out the quantum teleportation protocol described in Section 4.3, effectively teleporting entanglement from one party to the other via the intermediate stations. Even though this approach, in principle, allows to overcome the distance limit, it is not yet experimentally realized. Even though much progress has been made on implementing quantum repeaters (see, for example, [LHL+21, LRGR+21]), the main problem is still that state-of-the-art quantum memories cannot store quantum systems for the time span that is needed to establish the entangled state between the two distant parties. It is worth noting that once quantum computers become available, they can be used as quantum memories in repeaters.

### Device-dependent QKD

Device-dependent QKD has the advantage that the devices are trusted; hence we can use their complete characterization in the security proof. Therefore, it is not surprising that the experiments that have achieved the highest key rates and distances fall into this category. In 2018, Yuan et al. [YMK<sup>+</sup>18] have reached a secret key rate of 13.72 Mbit/s over a distance of 10 km with security parameter  $\epsilon = 10^{-10}$ . While this experiment demonstrated that high key rates are achievable in principle, the distance is too short for practical purposes. An implementation that focuses on large distances was carried out by Boaron et al. [BBR<sup>+</sup>18] in 2018, where a record distance of 421 km was achieved with  $\epsilon = 10^{-9}$ . However, at this distance, the key rate was only 0.25 bit/s, which is too small for practical purposes.

These implementations demonstrate how much the distance affects the achievable secret key rate. A possible way to overcome this complication is to use low-Earth orbit satellites as links. Compared to terrestrial channels, satellite-to-ground communication has much smaller losses, which makes it a promising candidate for long-distance quantum communication. With this approach, it

is possible to establish a secure communication channel over a distance of 7,600 km (the distance between Xinglong, China and Graz, Austria) as shown in 2018 by Liao et al. [LCH<sup>+</sup>18].

It is important to emphasize that in these implementations, we always have to trust the devices (which includes the satellites). Hence, we need to be aware of potential attacks that aim at exploiting inevitable deviations from the protocol by any device that is part of the practical implementation and possibly take countermeasures.

#### Device-independent QKD

Device-independent QKD can naturally guarantee the highest level of security as it requires the fewest assumptions on the implementation. While this level of security is desirable for practical purposes, it comes at a cost: The requirements for practical implementations are higher in the device-independent case, in the sense that they can tolerate fewer noise and losses in the devices. It is therefore not surprising that it has taken until 2021 for the first experimental implementations to successfully generate a secret key in the device-independent setting as reported in [NDN+21, ZvLR+21, LZZ+21].

The difficulty of realizing DIQKD experimentally is reflected in the achieved levels of security, distance, and secret key rates. Of the three experiments, only the implementation by Nadlinger et al. [NDN<sup>+</sup>21] was shown to be secure against the most general kind of attacks. Furthermore, the experiments have been carried out over distances between and 3.5 m and 700 m, with secret key rates between  $8.7 \cdot 10^{-4}$  bits/s and 466 bits/s. These numbers are far away from what is needed for practical use, especially because long distances are paired with small key rates.

Depending on the implementation, the challenges that have to be addressed are of different nature: If photons are used as information carriers, it is possible to prepare entangled states at very high rates. However, the downside of photonic implementations is the high losses in experimental equipment such as filters and optical fibers, which increases with the distance. As discussed above, quantum repeaters could help overcome this problem. Other implementations employ trapped ions or atoms as information carriers. The problem here is that the rate at which entangled states are generated is much lower than in the photonic case. On the upside, losses are not such a big problem with these realizations.

#### Semi-device-independent QKD

The fact that semi-device-independent QKD represents a compromise between security and practicality is reflected in the achievable distances and key rates. The current record for measurement-device-independent protocols, demonstrated by Yin et al. [YCY+16] in 2016 is a distance of 404 km with a key rate of  $3.2 \cdot 10^{-4}$ . Additionally, for smaller distances the group was able to achieve much higher key rates; for example, over a distance of 207 km a key rate of  $9.55 \, \text{bit/s}$  was achieved. These numbers illustrate that the additional information we have on the devices in MDI QKD (compared to DIQKD) makes the experimental realization much easier, although not as easy as in the device-dependent case.

### 6 Outlook

Classical cryptographic protocols suffer from the fact that their security is based on the conjectured hardness of specific problems. Consequently, advancements in hardware and software development always constitute potential threats to their security. In contrast, quantum cryptography does not suffer the same fate. Taking advantage of the unique properties of quantum mechanics, security is based solely on the laws of physics. This means that the encryption cannot be broken regardless of how much power an adversary has. Moreover, quantum cryptography is future-proof: Even if classical encryption schemes can guarantee that no computer today has enough power to break them, an adversary who stores the encrypted messages can decrypt them once more powerful devices (for example, quantum computers) have been developed. In contrast, information encoded via quantum cryptography will still be secure in the future independent of potential developments in hardware and software.

One could argue that the great disadvantage of quantum cryptography lies in the fact that it is expensive and difficult to realize. While it is undoubtedly true that quantum cryptography poses many challenges on experimental implementations, it is important to note that it is strictly simpler to realize than quantum computers. One of the most significant missing elements for experimental realizations at the moment are reliable quantum memories. These are also needed for quantum computers, but their requirements are much lower in QKD applications: Both the number of qubits and the time for which quantum states need to be stored are shorter in QKD than what is needed to build a universal quantum computer.

Once the technical issues have been resolved, quantum key distribution can hence offer a real advantage over classical cryptography. In classical cryptography, the development of an encryption algorithm is always followed by the search for an algorithm that breaks it, which itself is followed by the search for a new secure encryption algorithm, forming a vicious circle. Quantum cryptography breaks this circle: It ensures that the encryption can never be broken regardless of any developments in hardware and software, even taking quantum computers into account. Moreover, quantum protocols are not only secure during their execution, but information encrypted today will remain secure forever, independent of developments in (quantum) software and hardware. Accordingly, quantum cryptography can guarantee everlasting security.

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