

Implementation of the Grover search algorithm with Josephson charge qubits

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Abstract

A scheme of implementing the Grover search algorithm based on Josephson charge qubits has been proposed, which would be a key step to scale more complex quantum algorithms and very important for constructing a real quantum computer via Josephson charge qubits. The present scheme is simple but fairly efficient, and easily manipulated because any two-charge-qubit can be selectively and effectively coupled by a common inductance. More manipulations can be carried out before decoherence sets in. Our scheme can be realized within the current technology.

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It is well anticipated that a quantum computer could accomplish a huge task, which a classical computer may never fulfill in an acceptable time. As a result, the realization of the quantum computer has attracted not only many physicists but also many scientific researchers such as computer scientists, electrical engineers and so on. The implementation of quantum algorithms is the basis of inventing quantum computers. Two classes of quantum algorithms have shown great promise of the quantum computer. One is based on Shor's Fourier transform including quantum factoring [1] and Deutsch–Jozsa algorithm [2]. The other is based on the Grover quantum search algorithm [3], which is quadratic speedup compared with the classical one. The Grover search algorithm is very important because many techniques based on the search algorithm are used universally in our lives. The Grover search algorithm is fairly efficient in looking for one item in an

unsorted database of size $N \equiv 2^n$ [3,4]. In order to achieve the task, classical algorithm needs $O(N)$ queries but $O(\sqrt{N})$ queries by the Grover search algorithm. The efficiency of the algorithm has been manipulated experimentally in few-qubit cases via nuclear magnetic resonance (NMR) [5,6] and optics [7,8].

The Grover search algorithm can be used to search one item from 2^n items with n data qubits and one auxiliary working qubit. The process can be concluded as the following four steps: Firstly, prepare the $n + 1$ qubits, in which n data qubits are in $|0\rangle^{\otimes n}$, and one auxiliary working qubit in $|1\rangle_{n+1}$. And perform the $n + 1$ Hadamard transform on the $n + 1$ qubits. Secondly, apply the oracle. The auxiliary working qubit can be omitted after the second step. Thirdly, perform the n Hadamard transform on the n data qubits, and apply a phase shift to the data qubits except $|0\rangle^{\otimes n}$, which can be described by the unitary operator $2|0\rangle^{\otimes n}\langle 0| - I$, where I is identity operation on the data qubits. And perform the n Hadamard transformations on the n data qubits again. Finally, repeat Steps 2 \rightarrow 3 with a finite number of times until obtaining a solution to the

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search problem with high probability. And measure the n data qubits. The number of repetitions [9] for obtaining a finite item is $R = CI \left(\frac{\arccos \sqrt{1/N}}{2 \arccos \sqrt{N-1/N}} \right)$, which is bounded above by $\pi\sqrt{N}/4$.

Because of several advantages over other qubits, e.g., having large-scale integration, relatively high quantum coherence, being manipulated more easily, and being manufactured by modern microfabrication techniques, Josephson charge [10–12] and phase [13,14] qubits, based on the macroscopic quantum effects in low-capacitance Josephson junction circuits [15,16], have recently been used in quantum information processing. Some striking experimental phenomena [17,18] have shown that the Josephson charge and phase qubits are promising candidates of solid-state qubits in quantum information processing. In particular, recent experimental realization of a single charge qubit demonstrates that it is hopeful to construct the quantum computer by means of Josephson charge qubits [19]. Accordingly, implementation of quantum algorithm by Josephson charge qubits is of great importance. Recently, many schemes of quantum algorithms with Josephson charge qubits have been proposed. For example, Vartiainen et al. [20] have implemented a Shor's factorization algorithm on a Josephson charge qubit register. Fazio and Siewert [21] have realized a simple solid-state quantum computer by implementing the Deutsch–Jozsa algorithm in a system of Josephson charge qubits. In this paper, we propose a scheme to implement Grover search algorithm with Josephson charge qubits. The scheme is simple and easily manipulated because any two-charge-qubit can be selectively and effectively coupled by a common inductance. More manipulations can be carried out before decoherence sets in. Our scheme can be realized within the current technology.

Since the earliest Josephson charge qubit scheme [10] was proposed, a series of improved schemes [11,22] have been brought forward. Here, we concern the architecture in [22], which is an efficient scalable quantum computing (QC) architecture via Josephson charge qubits. The Josephson charge qubits structure is shown in Fig. 1. It consists of N cooper-pair boxes (CPBs) coupled by a common superconducting inductance L . For the k th cooper-pair box, a superconducting island with charge $Q_k = 2en_k$

is weakly coupled by two symmetric direct current superconducting quantum interference devices (dc SQUIDs) biased by an applied voltage through a gate capacitance C_k . Assume that the two symmetric dc SQUIDs are identical and all Josephson junctions have Josephson coupling energy E_{Jk}^0 and capacitance C_{Jk} .

The self-inductance effect of each SQUID loop is usually neglected because of very small size (1 μm) of the loop. Each SQUID pierced by a magnetic flux Φ_{Xk} provides an effective coupling energy $-E_{Jk}(\Phi_{Xk})\cos\phi_{kA(B)}$ with $E_{Jk}(\Phi_{Xk}) = 2E_{Jk}^0 \cos(\pi\Phi_{Xk}/\Phi_0)$, and the flux quantum $\Phi_0 = h/2e$. The effective phase drop $\phi_{kA(B)}$, with subscript $A(B)$ labeling the SQUID above (below) the island, equals the average value, $[\phi_{kA(B)}^L + \phi_{kA(B)}^R]/2$, of the phase drops across two Josephson junctions in the dc SQUID, with superscript L (R) denoting the left (right) Josephson junction.

For any given cooper-pair box, say i , when $\Phi_{Xk} = \frac{1}{2}\Phi_0$ and $V_{Xk} = (2n_k + 1)e/c_k$ for all boxes except $k = i$, the inductance L connects only the i th cooper-pair box to form a superconducting loop, as shown in Fig. 2a.

In the spin-1/2 representation, based on charge states $|0\rangle = |n_i\rangle$ and $|1\rangle = |n_{i+1}\rangle$, the reduced Hamiltonian of the system becomes [22]

$$H = \varepsilon_i(V_{Xi})\sigma_z^{(i)} - \bar{E}_{Ji}(\Phi_{Xi}, \Phi_e, L)\sigma_x^{(i)}, \quad (1)$$

where $\varepsilon_i(V_{Xi})$ is controlled by gate voltage V_{Xi} , while the intrabit coupling $\bar{E}_{Ji}(\Phi_{Xi}, \Phi_e, L)$ depends on inductance L , applied external flux Φ_e through the common inductance, and local flux Φ_{Xi} through the two SQUID loops of the i th cooper-pair box. By controlling Φ_{Xi} and V_{Xi} , the operations of Pauli matrices $\sigma_z^{(i)}$ and $\sigma_x^{(i)}$ are achieved. Thus, any single-qubit operations are realized by Eq. (1).

To manipulate any two-qubit, says i and j , when $\Phi_{Xk} = \frac{1}{2}\Phi_0$ and $V_{Xk} = (2n_k + 1)e/c_k$ for all boxes except $k = i$ and j , the inductance L is only shared by the

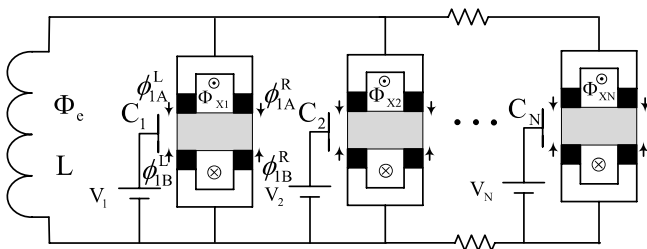


Fig. 1. Josephson charge qubit structure. Each CPB is configured in the charging regime $E_{ck} \gg E_{Jk}^0$ at low temperatures $k_B T \ll E_{ck}$. Furthermore, the superconducting gap Δ is larger than E_{ck} so that quasiparticle tunneling is suppressed in the system.

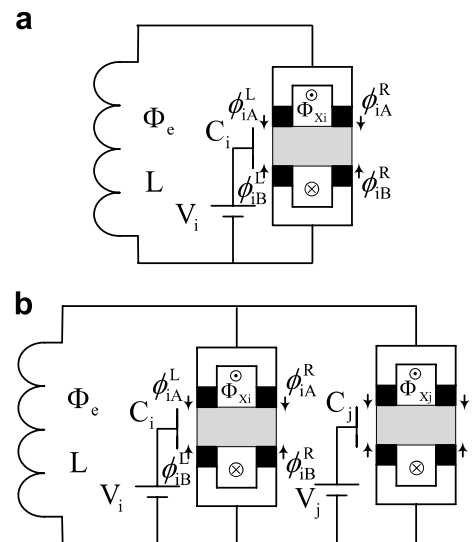


Fig. 2. (a) Single-qubit structure where a CPB is only connected to the inductance. (b) Two-qubit structure where two CPBs are connected to the common inductance.

cooper-pair boxes i and j to form superconducting loops, as shown in Fig. 2b, the Hamiltonian of the system can be reduced to [22,23]

$$H = \sum_{k=i,j} [\varepsilon_k (V_{Xk} \sigma_z^{(k)} - \bar{E}_{Jk} \sigma_x^{(k)})] + \Pi_{ij} \sigma_x^{(i)} \sigma_x^{(j)}, \quad (2)$$

where the interbit coupling Π_{ij} depends on external flux Φ_e through the inductance L , local fluxes Φ_{Xi} and Φ_{Xj} through the SQUID loops. If letting $V_{Xk} = (2n_k + 1)e/c_k$, Eq. (2) can be further reduced to

$$H = -\bar{E}_{Ji} \sigma_x^{(i)} - \bar{E}_{Jj} \sigma_x^{(j)} + \Pi_{ij} \sigma_x^{(i)} \sigma_x^{(j)}. \quad (3)$$

For the simplicity of calculation, we set $\bar{E}_{Ji} = \bar{E}_{Jj} = \Pi_{ij} = \frac{-\pi\hbar}{4\tau}$ (τ is a given period of time) by suitably adjusting parameters. Thus, Eq. (3) becomes

$$H = \frac{-\pi\hbar}{4\tau} (-\sigma_x^{(i)} - \sigma_x^{(j)} + \sigma_x^{(i)} \sigma_x^{(j)}). \quad (4)$$

According to the Hamiltonian H of Eq. (4), on the basis $\{|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), |-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)\}$, we can obtain following evolutions:

$$|+\rangle_{ij} \rightarrow e^{-i\pi\tau/4\tau} |+\rangle_{ij}, \quad (5a)$$

$$|+\rangle_{ij} \rightarrow e^{-i\pi\tau/4\tau} |+\rangle_{ij}, \quad (5b)$$

$$|-\rangle_{ij} \rightarrow e^{-i\pi\tau/4\tau} |-\rangle_{ij}, \quad (5c)$$

$$|-\rangle_{ij} \rightarrow e^{i3\pi\tau/4\tau} |-\rangle_{ij}. \quad (5d)$$

If we select the optimal interaction time $t = \tau$, and perform a single-qubit operation $U = e^{i\pi/4}$, we can obtain

$$|+\rangle_{ij} \rightarrow |+\rangle_{ij}, \quad (6a)$$

$$|+\rangle_{ij} \rightarrow |+\rangle_{ij}, \quad (6b)$$

$$|-\rangle_{ij} \rightarrow |-\rangle_{ij}, \quad (6c)$$

$$|-\rangle_{ij} \rightarrow -|-\rangle_{ij}. \quad (6d)$$

From Eq. (6), we can see that the operation of a controlled phase transform on arbitrary two-qubit has been actually carried out. On the basis of controlled phase gate, the controlled-not gate is also obtained by performing a series of single-qubit operations [22]. A sequence of conditional two-qubits gates and single-qubit operations constitute a universal element for QC [24], so arbitrary multi-qubit operations can be performed in our scheme.

In the following we only discuss the case of two-data-qubits, where we can search a finite item from four items. The circuit diagram of the Grover search with two-data-qubits and one auxiliary working qubit is shown in Fig. 3.

Firstly, in order to implement two-data-qubit Grover search algorithm, we prepare charge qubits 1, 2 and 3 in the state

$$|\phi\rangle_{123} = |++-\rangle_{123}. \quad (7)$$

By choosing appropriate Φ_{Xk} and V_{Xk} , we can perform Hadamard operation on charge qubits 1, 2 and 3

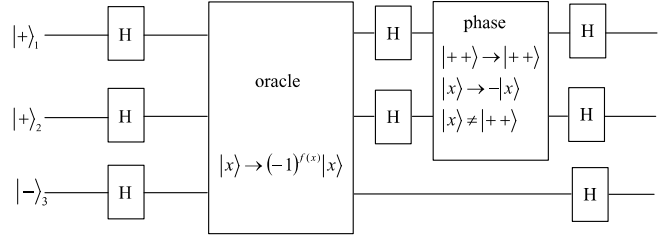


Fig. 3. The circuit diagram for the two-data-qubit Grover search algorithm. H denotes Hadamard transformation. $|+\rangle_1$ and $|+\rangle_2$ are two-data-qubits, $|-\rangle_3$ is an auxiliary working qubit.

$$|+\rangle_1 \rightarrow \frac{1}{\sqrt{2}}(|+\rangle_1 + |-\rangle_1), \quad (8a)$$

$$|+\rangle_2 \rightarrow \frac{1}{\sqrt{2}}(|+\rangle_2 + |-\rangle_2), \quad (8b)$$

$$|-\rangle_3 \rightarrow \frac{1}{\sqrt{2}}(|+\rangle_3 - |-\rangle_3). \quad (8c)$$

So the total state of the charge qubits 1, 2 and 3 becomes

$$|\phi\rangle_{123} = \frac{1}{2\sqrt{2}}(|++\rangle_{12} + |+-\rangle_{12} + |-+\rangle_{12} + |--\rangle_{12})(|+\rangle_3 - |-\rangle_3). \quad (9)$$

Obviously, the four items ($|++\rangle_{12}$, $|+-\rangle_{12}$, $|-+\rangle_{12}$, $|--\rangle_{12}$) have been stored in the data qubits before applying the oracle. Without loss of generality, we search the state $|+-\rangle_{12}$ from the four states. For the two-data-qubit Grover search algorithm, the oracle has only effect on the states to be searched. The auxiliary working qubit can be discarded at this point.

Secondly, in order to achieve the next step in which the function of the oracle is implemented, we can perform the controlled phase operation on the two-charge qubits as in Eq. (6), then perform single-qubit phase shift on the second qubit. These lead the state of charge qubits 1 and 2 to

$$|\phi\rangle_{12} = \frac{1}{2}(|++\rangle_{12} - |+-\rangle_{12} + |-+\rangle_{12} + |--\rangle_{12}). \quad (10)$$

We perform Hadamard operations on charge qubits 1 and 2 as in Eqs. (8a) and (8b). Thus, Eq. (10) can be rewritten as

$$|\phi\rangle_{12} = \frac{1}{2}(|++\rangle_{12} + |+-\rangle_{12} - |-+\rangle_{12} + |--\rangle_{12}). \quad (11)$$

Thirdly, for achieving phase change we can perform single-qubit NOT operations on charge qubits 1, 2, and the controlled phase shift on the two charge qubits as in Eq. (6), which can lead Eq. (11) to

$$|\phi\rangle_{12} = \frac{1}{2}(|++\rangle_{12} - |+-\rangle_{12} + |-+\rangle_{12} - |--\rangle_{12}). \quad (12)$$

Then we perform Hadamard operations on charge qubits 1 and 2 as in Eqs. (8a) and (8b) again. We can obtain the state of qubits 1 and 2

$$|\phi\rangle_{12} = |+-\rangle_{12}. \quad (13)$$

Finally, having been measured by detectors, the state of charge qubits 1 and 2 is the result we want to search. The scheme can generalize to implement a multi-qubit quantum search algorithm by performing arbitrary multi-qubit operations.

Next, we briefly discuss experimental feasibility of the current scheme. For the charge qubits in our scheme, the typically experimental switching time τ_1 during a single-qubit operation is about 0.1 ns [22]. The inductance $L \sim 30$ nH in our proposal is experimentally accessible. In the earlier design [11], the inductance L is about 3.6 μ H, which is difficult to make at nanometer scales. Another improved design [15] greatly reduces the inductance to ~ 120 nH, which is about four times larger than the one used in our scheme. The fluctuations of voltage sources and fluxes result in decoherence for all charge qubits. The gate voltage fluctuations play a dominant role in producing decoherence. The estimated dephasing time is $\tau_4 \sim 10^{-4}$ s [15], which is allowed in principle 10^6 coherent single-qubit manipulations. Owing to using the probe-junction, the phase coherence time is only about 2 ns [19,25]. In this setup, background charge fluctuations and the probe-junction measurement may be two major factors in producing decoherence [22]. The charge fluctuations are principal only in low-frequency region and can be reduced by the echo technique [25] and by controlling the gate voltage to the degeneracy point, but an effective technique for suppressing charge fluctuations is highly desired.

It is necessary to give the following explains. Our scheme is a perfect one in which the fidelity and successful probability of final result are both 1.0. The current technology can realize our scheme. If experimental technology is imperfect, the SQUIDs will be asymmetric and the local flux Φ_e will not completely suppress the Josephson coupling in the considered circuit. These can result in some residual Josephson coupling except the required coupling. In this case, the fidelity and successful probability of the final result will decrease slightly, but the arbitrary single-qubit and two-qubit operations can be still performed.

In a summary, we have proposed a new scheme to implement the Grover search algorithm with Josephson charge qubits. Our scheme is simple but fairly efficient, and easily manipulated because any two-charge-qubit can be selectively and effectively coupled by a common inductance. More manipulations can be carried out before decoherence sets in. Our scheme can be realized within the current technology. The implementation of the algorithm would be a key step to scale more complex quantum algorithms and very important for constructing a real quantum computer with Josephson charge qubits.

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