Math 2210 - Linear Algebra

First prelim - 1 October 2013 - 7:30 to 9:00pm

Name and NetID: _____

Whose discussion section are you enrolled in? Circle one.   Yash Lodha   Wai-kit Yeung
At what time is the discussion section you enrolled in? Circle one.

1:25-2:15pm   2:30-3:20pm   3:35-4:25pm

INSTRUCTIONS

- This test has 6 problems on 6 pages, worth a total of 100 points. Check if you have all 6 pages with questions.

- If you need more space than you are given under a question, write on the back side of the preceding sheet, but be sure to label your work clearly and point out where your final answer to each question is. You also have a 2-sided page for scratchwork at the end.

- No books or electronic devices allowed. You are allowed a one-sided letter size paper of notes.

- Please **show all your work and justify your answers**.

Academic integrity is expected of all Cornell University students at all times, whether in the presence or absence of members of the faculty.

Understanding this, I declare I shall not give, use, or receive unauthorized aid in this examination.

Please sign below to indicate that you have read and agree to these instructions.

_____

Signature of Student

1. *(20 points)* Let

$$A = \begin{pmatrix} 1 & 2 & 2 & 1 \\ 1 & 1 & -1 & -2 \\ 1 & 4 & 8 & 7 \end{pmatrix}, \vec{b}_1 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \vec{b}_2 = \begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix}, \vec{b}_3 = \begin{pmatrix} 2 \\ 2 \\ b \end{pmatrix}.$$

Find all solutions to the systems of equations:

(a) $A\vec{x} = 0$

(b) $A\vec{x} = \vec{b}_1$

(c) $A\vec{x} = \vec{b}_2$

(d) Find all numbers b such that the system $A\vec{x} = \vec{b}_3$ has a solution.

2. *(15 points)* Let P be a 4×4 matrix whose columns are $\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4$. Show that these vectors are linearly independent if and only if P is an invertible matrix.

Hint: Consider the system $P\vec{x} = 0$ as a vector equation. You may use the fact that P is invertible if and only if its row-reduced echelon form is the identity matrix.

3. *(15 points)* Let $A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$. Either find A^{-1} or show that it does not exist.

4. *(15 points)* Let $A = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$

    (a) Find a matrix B such that $AB = I_2$ where I_2 is the 2×2 identity matrix, or show that such a B does not exist.

    (b) Find a matrix B such that $BA = I_3$ where I_3 is the 3×3 identity matrix, or show that such a B does not exist.

5. *(15 points)* For each of the following statements say if it is true or false; give reasons if it is true, a counterexample if it false.

(a) If three vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3$ in $\mathbb{R}^4$ are linearly dependent, then one of them is a sum of the other two.

(b) Three vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3$ in $\mathbb{R}^2$ cannot be linearly independent.

(c) Two vectors $\vec{v}_1, \vec{v}_2$ in $\mathbb{R}^3$ cannot span all of $\mathbb{R}^3$.

6. (a) *(15 points)* Let $R : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation which rotates each vector $\vec{v}$ in $\mathbb{R}^2$ counterclockwise around the origin by an angle of $30°$.

Let $S : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation which projects each vector $\vec{v}$ in $\mathbb{R}^2$ onto the x_1-axis, i.e. $S \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 \\ 0 \end{pmatrix}$.

Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the composition of R and S, i.e., $T(\vec{v}) = S(R(\vec{v}))$ for all v in $\mathbb{R}^2$.

Find the standard matrices A, B and C of the linear transformations R, S and T, respectively. Note $\cos 30° = \frac{\sqrt{3}}{2}$, $\sin 30° = \frac{1}{2}$.

(b) *(5 points)* Which of the linear transformations R, S, T are one-to-one? Which of them are onto $\mathbb{R}^2$? Explain.

This page is for scratch work, it will not be graded unless you point us here from the page where the question was posed.