Math 2210 Prelim 1 Name: ______________________

February 19, 2015

☐ Discussion Section 201 (M 1:25-2:15, Pueschel)
☐ Discussion Section 202 (M 2:30-3:20, Pueschel)
☐ Discussion Section 203 (M 2:30-3:20, Patotski)
☐ Discussion Section 204 (M 3:35-4:25, Patotski)
☐ Discussion Section 205 (M 3:35-4:25, Pueschel)

INSTRUCTIONS — READ THIS NOW

- Relax. Take a deep breath.
- Print your first and last name and check the box indicating which section you are in **right now**.
- This test has **6** problems on **8** pages (including the cover sheet and a page for scrap work). Look over your test package as soon as the exam begins. If you find any missing pages please ask a proctor for another test booklet.
- **SHOW YOUR WORK.** To receive full credit, your answers must be neatly written and logically organized. If you need more space, write on the back side of the preceding sheet, but be sure to label your work clearly.
- This is a 90 minute exam. You may leave early, but if you finish within the last 15 minutes, please stay in your seat. When time is called, put your pencil down immediately and pass your exam booklet to the aisle.
- This is a closed book exam. You are **NOT** allowed to use a calculator. Cell phones may **NOT** be used in the exam rooms, not even as time-keeping devices. All other aids are prohibited.
- Academic integrity is expected of all students of Cornell University at all times, whether in the presence or absence of members of the faculty. Understanding this, I declare I shall not give, use, or receive unauthorized aid in this examination.

Please sign below to indicate that you have read and agree to these instructions.

____
Signature of Student

OFFICIAL USE ONLY	
1. ________	/15
2. ________	/15
3. ________	/15
4. ________	/20
5. ________	/20
6. ________	/15
Total: ________	/100

1. *(15 points)* Let

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & 1 \\ 2 & -1 & 1 \end{bmatrix}, \quad \mathbf{b}_1 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}, \quad \mathbf{b}_2 = \begin{bmatrix} -1 \\ -3 \\ a \end{bmatrix}.$$

(a) Find all solutions to the linear system $A\mathbf{x} = \mathbf{0}$.

(b) Find all solutions to the linear system $A\mathbf{x} = \mathbf{b}_1$.

(c) Find all numbers a, such that the system $A\mathbf{x} = \mathbf{b}_2$ has a solution. For each such number a write down all solutions.

2. *(15 points)* Let $A = \begin{bmatrix} 2 & -1 & -4 \\ 2 & 3 & -1 \\ -2 & 9 & 10 \end{bmatrix}$.

(a) Describe the solution set of $A\mathbf{x} = \mathbf{0}$ in parametric form.

(b) Let $\mathbf{v}_1 = \begin{bmatrix} 2 \\ 2 \\ -2 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} -1 \\ 3 \\ 9 \end{bmatrix}$ and $\mathbf{v}_3 = \begin{bmatrix} -4 \\ -1 \\ 10 \end{bmatrix}$. Are $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

(c) Are $\{\mathbf{v}_1, \mathbf{v}_2\}$ linearly independent? Are $\{\mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answers.

3. *(15 points)*

(a) Suppose A is a 5×3 matrix and $\mathbf{b}$ is a vector in $\mathbb{R}^5$ with the property that $A\mathbf{x} = \mathbf{b}$ has a unique solution. What can you say about the echelon form of A? Justify your answer.

(b) Could a set of three vector in $\mathbb{R}^5$ span all of $\mathbb{R}^5$? Justify your answer.

4. *(20 points)* For each of the following statements say if it is true of false; give reasons if it is true, and a counterexample if it is false.

   (a) Let $S = \{\mathbf{v}_1, \mathbf{v}_1, \mathbf{v}_3, \mathbf{v}_4\}$ be a linearly dependent set of four vectors in $\mathbb{R}^n$. Then each vector in S can be written as a linear combination of the other three vectors.

   (b) Any two vectors $\mathbf{v}_1$, $\mathbf{v}_2$ in $\mathbb{R}^4$ are always linearly independent.

   (c) Let $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ in $\mathbf{R}^4$ be linearly independent and let $\mathbf{w} = 4\mathbf{v}_1 - 3\mathbf{v}_2$ be a linear combination of $\mathbf{v}_1$ and $\mathbf{v}_2$. Then the vectors $\mathbf{w}, \mathbf{v}_3$ are linearly independent.

   (d) Let $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ in $\mathbb{R}^3$ be linearly dependent and

   $$\mathbf{w} = c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + c_3\mathbf{v}_3$$

   for some weights $c_1, c_2, c_3 \in \mathbb{R}$. Then $\mathbf{w}$ can always be written as a linear combination of just two vectors in S.

5. *(20 points)* Let $R : \mathbb{R}^2 \to \mathbb{R}^2$ be the counterclockwise rotation about the origin with angle $\frac{3\pi}{4}$.

(a) What is the matrix A of the linear transformation R?

(b) Let $S : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation with matrix

$$B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

Find all vectors $\mathbf{x}$ fixed by S, that is find all vectors $\mathbf{x}$ such that $B\mathbf{x} = \mathbf{x}$.

(c) Let T be the composition of R and S that is

$$T(\mathbf{x}) = S(R(\mathbf{x})) \quad \text{for all } \mathbf{x} \text{ in } \mathbb{R}^2.$$

Find the matrix C of the linear transformation T.

(d) Is the transformation T invertible? Justify your answer.

6. *(15 points)* Let

$$A = \begin{bmatrix} 0 & 0 & 4 \\ -1 & 0 & 0 \\ 2 & -1 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 4 & a & b \\ 1 & -4 & c \\ 1 & 0 & 4 \end{bmatrix}.$$

Find all numbers a, b and c such that $AB = BA$, or show that such numbers do not exist.

This extra blank page for scratch work is on purpose.