1. Find all solutions to the following system of equations:

$$\begin{aligned} 2x_1 + 4x_2 + 2x_3 + 2x_4 &= 6 \\ x_1 + 2x_2 + x_3 + x_4 &= 3 \\ -3x_1 - 6x_2 + x_3 + 5x_4 &= -5 \end{aligned}$$

Write your answer in parametric vector form, that is,

$$\mathbf{x} = \mathbf{v}_0 + c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + \cdots + c_k\mathbf{v}_k,$$

where the vectors $\mathbf{v}_0, \ldots, \mathbf{v}_k$ are specific numerical vectors in $\mathbf{R}^4$ that you must find and any choice of values for the real numbers $c_1, \ldots, c_4$ yields a solution to the system.

2. Let A be the 3×3 matrix

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 4 & -3 & 7 \end{bmatrix}.$$

If A is invertible find A^{-1}. If A is not invertible explain why.

3. Let A be the matrix

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 1 & 2 \end{bmatrix}$$

and $\mathbf{u}$ the vector in $\mathbf{R}^3$

$$\begin{bmatrix} 4 \\ 2 \\ h \end{bmatrix}.$$

For which values of h is $\mathbf{u}$ not in the span of the columns of A?

4. T/F. Determine whether each statement below is true of false. If true give a brief explanation as to why it is true. If false, give an example to show that it it false.

   (a) A homogeneous system of linear equations has a nontrivial solution if and only if there is at least one free variable.

   (b) If A is a 4×3 matrix then there is a b in $\mathbf{R}^4$ such that $Ax = b$ is inconsistent.

   (c) There is a matrix A such that $A\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 7 \\ 8 \\ 1 \end{bmatrix}$.

   (d) If $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is a linearly independent set of vectors, then $\mathbf{v}_3$ is not in the span of $\{\mathbf{v}_1, \mathbf{v}_2\}$.

   (e) If $\mathbf{v}_3$ is not in the span of $\{\mathbf{v}_1, \mathbf{v}_2\}$, then $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ are linearly independent.

(f) If A is a 3×5 matrix, then $A\mathbf{x}$ is in the span of the columns of A.

(g) If $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ are linearly dependent, then $2\mathbf{v}_1, -3\mathbf{v}_2, 8\mathbf{v}_3$ are linearly dependent.

5. A is a 3×2 matrix such that

$$A \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \\ 5 \end{bmatrix} \text{ and } A \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}.$$

(a) Find $A \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

(b) Is $T(\mathbf{x}) = A\mathbf{x}$ a 1-1 linear transformation? Explain.

(c) Is $T(\mathbf{x}) = A\mathbf{x}$ an onto linear transformation? Explain.

6. Prove *using the definition of linear transformation* that if $T : \mathbf{R}^n \to R^n$ and $S : \mathbf{R}^n \to \mathbf{R}^n$ are linear transformations, then the function $\mathbf{x} \to T(S(\mathbf{x}))$ is a linear transformation.