Math 2210 Prelim 1

Name: ______________________________

September 29, 2015

☐ Discussion 1 (12:20-1:10, Frederik De Keersmaeker)
☐ Discussion 2 (1:25-2:15, Benjamin Hoffman)
☐ Discussion 3 (1:25-2:15, Frederik De Keersmaeker)
☐ Discussion 4 (1:25-2:15, Ian Lizarraga)
☐ Discussion 5 (2:30-3:20, Benjamin Hoffman)
☐ Discussion 6 (2:30-3:20, Frederik De Keersmaeker)
☐ Discussion 7 (3:35-4:25, Benjamin Hoffman)
☐ Discussion 8 (3:35-4:25, Ian Lizarraga)
☐ Discussion 9 (7:30-8:20, Ian Lizarraga)

INSTRUCTIONS — READ THIS NOW

• Relax. Take a deep breath.

• Print your first and last name and check the box indicating which section you are in **right now**.

• This test has **6** problems on **10** pages (including the cover sheet, a page for scratch work, and two extra pages for problems 1 and 4). Look over your test package as soon as the exam begins. If you find any missing pages please ask a proctor for another test booklet.

• **SHOW YOUR WORK.** To receive full credit, your answers must be neatly written and logically organized. If you need more space, write on the back side of the preceding sheet, but be sure to label your work clearly.

• This is a 90 minute exam. You may leave early, but if you finish within the last 15 minutes, please stay in your seat. When time is called, put your pencil down immediately and pass your exam booklet to the aisle.

• This is a closed book exam. You are **NOT** allowed to use a calculator. Cell phones may **NOT** be used in the exam rooms, not even as time-keeping devices. All other aids are prohibited.

• Academic integrity is expected of all students of Cornell University at all times, whether in the presence or absence of members of the faculty. Understanding this, I declare I shall not give, use, or receive unauthorized aid in this examination.

Please sign below to indicate that you have read and agree to these instructions.

____
Signature of Student

OFFICIAL USE ONLY	
1. ________	/20
2. ________	/15
3. ________	/15
4. ________	/20
5. ________	/15
6. ________	/15
Total: ________	/100

1. *(20 points)* Consider the vectors $\mathbf{v_1} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$, $\mathbf{v_2} = \begin{bmatrix} 3 \\ 0 \\ 5 \end{bmatrix}$, $\mathbf{v_3} = \begin{bmatrix} 5 \\ 4 \\ 3 \end{bmatrix}$, and $\mathbf{v_4} = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$ in $\mathbb{R}^3$.

   (a) Solve the system of linear equations $A\mathbf{x} = \mathbf{b}$ for A the 3×4 matrix $[\mathbf{v_1}\ \mathbf{v_2}\ \mathbf{v_3}\ \mathbf{v_4}]$ and $\mathbf{b} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$.

   (b) Interpret part (a) in terms of writing $\mathbf{b}$ as a linear combination.

   (c) Using the answer to part (a), write down a solution to $A\mathbf{y} = \mathbf{0}$ without re-doing the row reduction process.

   (d) The list of vectors $\mathbf{v_1}$, $\mathbf{v_2}$, $\mathbf{v_3}$, $\mathbf{v_4}$ is linearly dependent. For each of the four vectors, if we remove it from the list, will the remaining three vectors be linearly independent? Justify your answer in each case.

This is an extra page for the solution of problem 1, if needed.

2. *(15 points)* Consider the matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 0 & -1 & e \\ 0 & 0 & f \end{bmatrix}$.

(a) Write down a statement of the form "A is invertible if and only if..." giving a condition on the parameters e and f.

(b) In the cases where A is invertible, compute its inverse.

(c) Suppose that $e = -1$ and $f = 0$; your statement in (a) should tell you A is not invertible. This means the linear transformation $T(\mathbf{x}) = A\mathbf{x}$ is not one-to-one, and is not onto. Provide examples of vectors that demonstrate this is true: find $\mathbf{x} \neq \mathbf{y}$ with $T(\mathbf{x}) = T(\mathbf{y})$, and find $\mathbf{z}$ that's not the image of anything by T.

3. *(15 points)*

If $A = \begin{bmatrix} -1 & 2 \\ 3 & 1 \\ 4 & 1 \end{bmatrix}$ and $AB = \begin{bmatrix} -3 & 4 & 3 \\ 2 & 2 & 5 \\ 3 & 2 & 6 \end{bmatrix}$ determine the matrix B.

4. *(20 points)* Let $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ and $B = A - I_2$.

(a) Determine the nullspace $\mathrm{Nul}(B)$, that is find all vectors $\mathbf{x} \in \mathbb{R}^2$ such that $B\mathbf{x} = \mathbf{0}$.

(b) Determine the columnspace $\mathrm{Col}(B)$.

(c) Find a 2×1 column vector $\mathbf{v}$ and a 1×2 row vector $\mathbf{r}$ such that $B = \mathbf{vr}$.

(d) Draw the vectors $A\begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $A\begin{bmatrix} 3 \\ 2 \end{bmatrix}$, $A\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and determine geometrically the transformation $\mathbf{x} \mapsto A\mathbf{x}$ of $\mathbb{R}^2$.

This is an extra page for the solution of problem 4, if needed.

5. *(15 points)*

(a) Define what is LU factorization of a matrix. Compute the LU factorization of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 0 & 2 \end{bmatrix}$.

(b) Give example of an invertible matrix which does not have a LU factorization. Explain carefully why such factorization does not exist.

6. *(15 points)*

(a) Let A be a non-zero 5×3 matrix and B be a non-zero 3×6 matrix. Find all possible values for the rank of the 5×6 matrix AB. Explain why the rank of the product need to be one of these numbers, and construct examples of matrices A, B which show that all these ranks can be achieved.

(b) Same as part (a) for A a non-zero 3×1 matrix and B a non-zero 1×4 matrix.

This extra blank page for scratch work is on purpose.