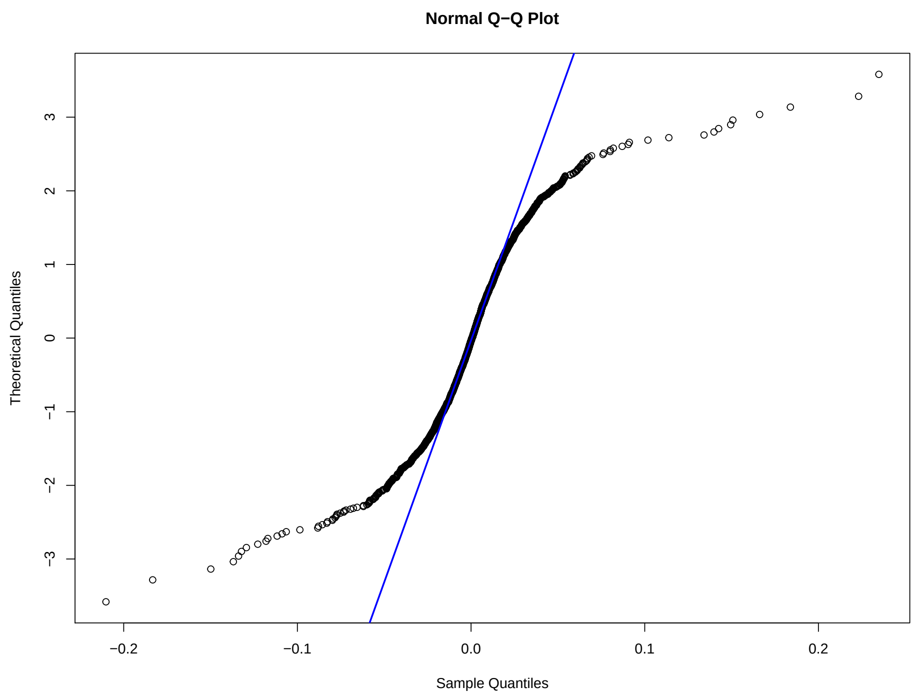
ORIE 4630: Spring Term 2019
Homework #2 Solutions

Question 1. [10 points]

Output from lines 6 and 7:



The points show a "convex-concave" pattern, suggesting that the distribution of the returns has heavier tails than the normal distribution.

Question 2. [10 points]

The output from lines 8 to 11:

```
> skewness(Returns$GS)
[1] 0.2767769
> kurtosis(Returns$GS)
[1] 18.61229
```

Since the normal distribution is symmetric, the skewness is 0. The estimate of the skewness of the returns for Goldman Sachs (GS) is 0.277, which is small, i.e., close to 0. This estimate of skewness suggests that the distribution of the returns is roughly symmetric.

The kurtosis of the normal distribution is 3. The estimate of the kurtosis of the returns for Goldman Sachs (GS) is 18.612, which is much larger than 3. This estimate of kurtosis suggests that the distribution of the returns has much heavier tails than the normal distribution.

These results agree with the normal probability plot in Question 1.

Question 3. [10 points]

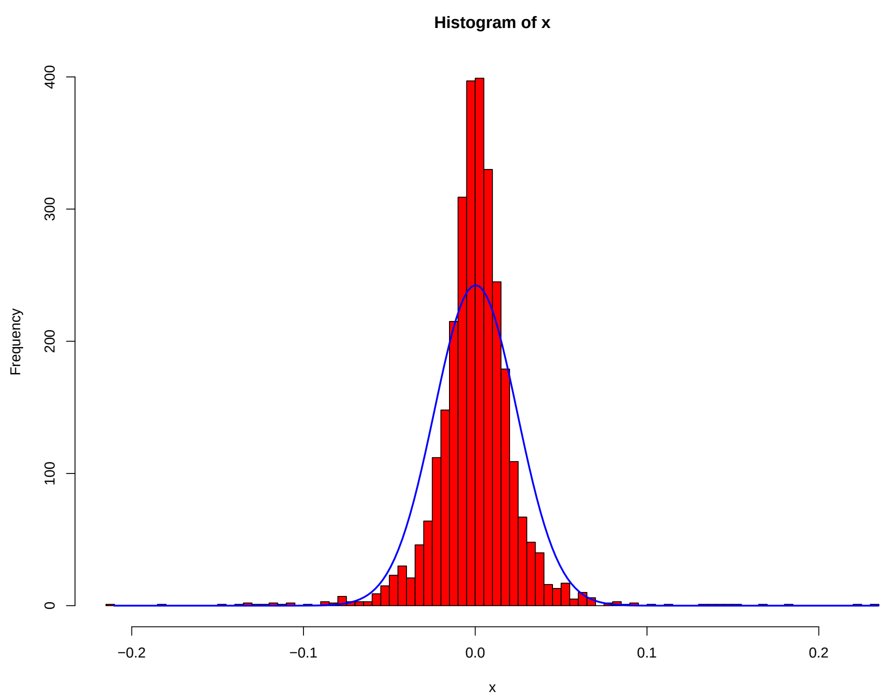
Output from lines 12 and 13:

```
> shapiro.test(Returns$GS)

Shapiro-Wilk normality test

data:  Returns$GS
W = 0.8543, p-value < 2.2e-16

> jarque.test(Returns$GS)

Jarque-Bera Normality Test

data:  Returns$GS
JB = 29764, p-value < 2.2e-16
alternative hypothesis: greater
```

Both tests produce p-values that are essentially 0 (less than 2.2e-16). Consequently, both tests provide strong evidence against the null hypothesis that the returns for Goldman Sachs (GS) are normally distributed. The conclusion is that the returns for Goldman Sachs (GS) are not normally distributed.

Question 4. [15 points]

The output from lines 14 to 19 is:



The mean and standard deviation of matching normal distribution are the same as the mean and standard deviation of the sample. These can be found found from the following output:

```
> mean(Returns$GS)
[1] 0.0002317852
> sd(Returns$GS)
[1] 0.02408091
```

The mean is 0.0002318, and the standard deviation is 0.02408.

Question 5. [20 points]

Output from lines 20 to 23:

```
> fit
              m                            s                            df
   0.0005273569    0.0133452040    2.7340473894
 (0.0003051133) (0.0003283650) (0.1609371218)
```

i) The estimate of the location parameter is 0.0005274.

ii) The estimate of the scale parameter is 0.01335.

iii) The estimate of the degrees of freedom is 2.734.

iv) The standard error for the degrees of freedom is 0.16094.

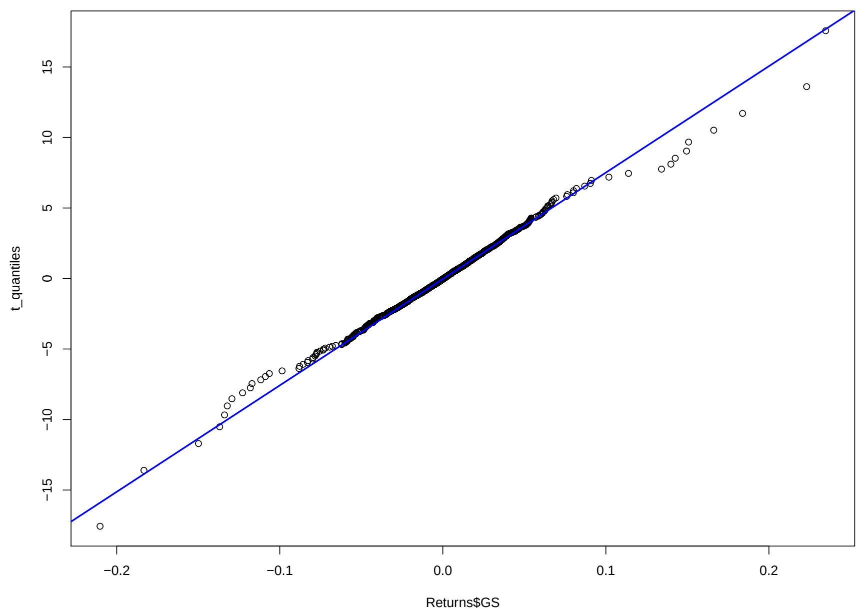
iv) The 95% confidence interval for the degrees of freedom is $2.734 \pm 1.960(0.16094) = (2.419, 3.049)$

v) The mean is the same as the location parameter, 0.0003051.

vi) The standard deviation is $(0.01335)\sqrt{\dfrac{2.734}{2.734-2}} = 0.02577$.

Question 6. [10 points]

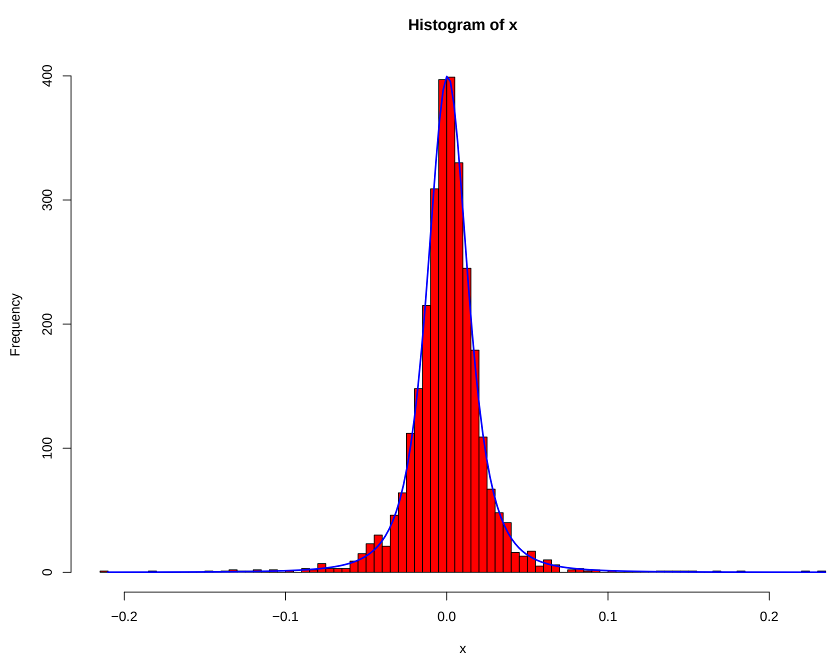
Output from lines 24 to 29:



The points are close to collinear, which indicates that the distribution of the returns for Goldman Sachs (GS) is well-approximated by the Student's t-distribution with 2.734 degrees of freedom.

Question 7. [10 points]

Output from lines 30 to 38:



Question 8. [15 points]

The code is:

```
ind=(Returns$GS<=sort(Returns$GS)[2]
     |Returns$GS>=sort(Returns$GS,decreasing=TRUE)[3])
Returns$Date[ind]
Returns$GS[ind]
```

The output is:

```
> Returns$Date[ind]
[1] "2008-09-19" "2008-10-13" "2008-11-24" "2008-12-01" "2009-01-20"
> Returns$GS[ind]
[1]  0.1838640  0.2231436  0.2348178 -0.1833096 -0.2102226
```

The two smallest returns and their dates are: -0.1833 (December 1, 2008) and -0.2102 (January 20, 2009).

The three largest returns and their dates are: 0.1839 (September 19, 20083), 0.2231 (October 13, 2008), 0.2348 (November 24, 2008).