STSCI 5080 Practice Midterm Exam 1

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Name: _____

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Instructions:

- The exam is 60 minutes-long. This exam is closed-books and closed-notes, but you can bring one cheatsheet (one letter size paper). No electronic devices are allowed.
- Page 8 is the formula sheet where you can find some pdf/pmfs used in the exam.
- You can detach the final page and use it as a scratch paper.
- There are five problems. Problem 1 consists of multiple-choice questions. Problems 2–5 are open-ended questions.
- For each of Problems 2–5, i) write your answer after the horizontal line; ii) in case you need more space, you can use the back side of the page, but you have to indicate in the front page that you use the back side. (iii) Writing only the final solution will not be given full credit. Write intermediate steps you used to reach the final solution.
- The total number of points is 80.

Problem	1	2	3	4	5	Total
Points	25	10	15	10	20	80
Score						

Problem 1. Circle the correct choice in each of the following questions.

(1) [5 points] Suppose that you toss a coin three times and all the outcomes occur equally likely. What is the probability that at least two heads appear?

a. $\frac{1}{8}$ b. $\frac{1}{4}$ c. $\frac{3}{8}$ d. $\frac{1}{2}$

(2) [5 points] Suppose that two events A and B are independent and such that $P(A) = 1/3$ and $P(B) = 1/6$. What is the probability of $A \cup B$?

a. $\frac{2}{9}$ b. $\frac{1}{3}$ c. $\frac{4}{9}$ d. $\frac{1}{2}$

(3) [5 points] Suppose that two events A and B are such that $P(B) = 1/3$, and $P(A \cap B) = 1/6$. What is the conditional probability $P(A \mid B)$?

a. $\frac{1}{18}$ b. $\frac{1}{6}$ c. $\frac{1}{3}$ d. $\frac{1}{2}$

(4) [5 points] Suppose that the number of typos on a single page of a certain book follows the Poisson distribution with parameter $\lambda = 1$. What is the probability that there is at least one error on this page?

a. $1 - e^{-1/2}$ b. $e^{-1/2}$ c. $1 - e^{-1}$ d. e^{-1}

(5) [5 points] Find the correct statement about a probability density function (pdf). Only one of them is correct and the other three are incorrect.

a. If f is a pdf, then $-f$ is also a pdf.

b. If f is a pdf, then $2f$ is also a pdf.

c. If f and g are pdfs, then $\frac{1}{2}f + \frac{1}{2}g$ is also a pdf.

b. If f and g are pdfs, then their product fg is also a pdf.

Problem 2. Suppose that the student has to solve a multiple-choice question with four alternatives, and she knows the correct answer with probability 60% and guesses with probability 40%. In case she guesses, she randomly chooses each alternative with probability 25%.

(a) [5 points] What is the probability that she chooses the correct answer?

(b) [5 points] What is the probability that she knew the correct answer given that she chooses the correct answer?

Problem 3. Define a pdf f by

$$f(x) = \begin{cases} \frac{2}{x^2} & \text{if } x \geq 2 \\ 0 & \text{otherwise} \end{cases}.$$

(a) [5 points] Compute the cdf F that corresponds to pdf f.

(b) [5 points] Compute the quantile function F^{-1} of F.

(c) [5 points] Explain how to generate a random variable with cdf F from a uniform random variable U on $[0, 1]$.

Problem 4. Let X and Y be independent exponential random variables with parameters α and β, respectively.

(a) [5 points] Find the joint pdf of (X, Y).

(b) [5 points] Calculate $P(X > Y)$.

Problem 5. Let (X, Y) be a uniform random vector on the set $A = \{(x, y) \in \mathbb{R}^2 \mid 0 \leq y \leq 1 - x^2, 0 \leq x \leq 1\}$.

(a) [5 points] Find the joint pdf of (X, Y). (Hint). Draw the set A and find the area of A.

(b) [10 points] Find the marginal pdfs of X and Y.

(c) [5 points] Find the conditional pdf of X given Y.

Formula sheet

(You can detach this sheet)

- The probability mass function of a Poisson random variable with parameter $\lambda > 0$ is

$$p(x) = \frac{\lambda^x}{x!} e^{-\lambda}, \ x = 0, 1, 2, \ldots,$$

  and $p(x) = 0$ elsewhere.

- The probability density function of an exponential random variable with parameter $\lambda > 0$ is

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}.$$

You can detach this sheet and use it as a scratch paper.