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STSCI 5080 Practice Midterm Exam 2[1]

Problem 1. Circle the correct choice in each of the following questions.

(1) Let X and Y be lifetimes (in year) of two cars, and suppose that X and Y are independent and each follows the exponential distribution with parameter $\lambda = 0.1$. What is the probability that at least one car will be working for more than 10 years?

a. e^{-1} b. e^{-2} c. $1 - e^{-1}$ d. $1 - (1 - e^{-1})^2$

(2) Let X and Y be random variables such that $E(X) = 0, E(X^2) = 1, E(Y) = 1, E(Y^2) = 5$, and $\mathrm{Corr}(X, Y) = 0.5$. What is $\mathrm{Var}(X + 2Y)$?

a. 17 b. 18 c. 21 d. 22

(3) Suppose that we first draw N according to the Poisson distribution with parameter $\lambda = 10$; throw a six-sided die N times and then count the sum of the face values, which is denoted by Y. What is the mean of Y?

a. 3.5 b. 10 c. 30 d. 35

(4) Find the correct statement. Only one of them is correct.

a. If $X_n \xrightarrow{P} X$ and the expectations are defined, then $E(X_n) \to E(X)$.

b. If X and Y are such that $E(X^k) = E(Y^k)$ for all positive intergers k (assuming that thoese moments exist), then X and Y have the same cdf.

c. If X_n and X are continuous with pdfs f_n and f, respectively, and $X_n \xrightarrow{d} X$, then $f_n(x) \to f(x)$ pointwise.

d. None of them are correct.

[1] The actual exam is 1-hour long. The instructions of the first midterm exam apply. In the exam, you will be given a scratch sheet and a formula sheet as in the first midterm exam.

Problem 2. Let $X_1, \ldots, X_n$ be a random sample from the uniform distribution on $[0, 1]$. Let $X_{(1)} = \min_{1 \leq i \leq n} X_i$ and $X_{(n)} = \max_{1 \leq i \leq n} X_i$.

(a) Derive the pdfs of $X_{(1)}$ and $X_{(n)}$.

(b) Find $E(X_{(n)} - X_{(1)})$.

(You can also use this page to answer Problem 2)

Problem 3. Let (X, Y) be a continuous random vector with joint pdf

$$f(x, y) = \begin{cases} 6x & \text{if } x, y \geq 0,\ 0 \leq x + y \leq 1 \\ 0 & \text{otherwise} \end{cases}.$$

(a) Find $\mathrm{Cov}(X, Y)$ and $\mathrm{Corr}(X, Y)$.

(b) Find $E(Y \mid X)$ and $\mathrm{Var}(Y \mid X)$.

(You can also use this page to answer Problem 3)

Problem 4. Let X be a Poisson random variable with parameter λ.

(a) Find the mgf of X.

(b) Find the skewness of X, which is defined by

$$\beta_1 = \frac{E[\{X - E(X)\}^3]}{\{\mathrm{Var}(X)\}^{3/2}}.$$

You may use the following identity: $E[\{X - E(X)\}^3] = E(X^3) - 3E(X)E(X^2) + 2\{E(X)\}^3$.

(c) If Y is a Poisson random variable with parameter κ and Y is independent of X, then show that $X + Y$ follows the Poisson distribution with parameter $\lambda + \kappa$.

(You can also use this page to answer Problem 4)

Problem 5. Let Y_n denote a binomial random variable with parameters n and p where $0 < p < 1$.

(a) Derive the limiting distribution of $\sqrt{n}(Y_n/n - p)$ as $n \to \infty$.

(b) Suppose that we want to estimate $g(p) = p(1 - p)$ which is the variance of the corresponding Bernoulli trial. Find the limiting distribution of $\sqrt{n}\{g(Y_n/n) - g(p)\}$.
