9/11/18 Kengo Kato

STSCI 5080 Homework 2

- Due is 9/20 (Th) in class.
- Write your name and NetID at the top of the first page, along with the assignment number.
- Use only the one side of the paper. Attach your pages with a staple at the top left corner.
- There are five problems. Each problem is worth 10 points.

Problems

1. Suppose that you have four urns U_1, U_2, U_3, U_4, and for each $k = 1, 2, 3, 4$, urn U_k contains k red balls and $10 - k$ blue balls. Now, you first choose an urn with probability $1/4$ and then draw a ball from the chosen urn.
   (a) Calculate the probability that you draw a red ball. (Hint). The generalized law of total probability.
   (b) Calculate the probability that the chosen urn was U_4 given that you draw a red ball. (Hint). The Bayes rule.
2. Let X be a discrete random variable taking values in $\{0, 1, 2\}$ with $P(X = 0) = p, P(X = 1) = q$, and $P(X = 2) = 1 - p - q$, where p, q satisfy that $0 < p, q < 1$ and $p + q < 1$. Find the cdf of X and draw its graph. (Hint). The cdf of a discrete random variable is a step function.
3. Define a function g by
$$g(x) = \begin{cases} \sqrt{1 - x^2} & \text{if } 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}.$$
   (a) Draw the graph of g.
   (b) Define a function f by $f(x) = cg(x)$ for any real x where $c > 0$ is a constant. If f is a pdf, find the value of c. (Hint). The area of the unit circle is...
4. (**Rice 2.5.45**) Suppose that the lifetime of an electronic component follows an exponential distribution with $\lambda = 0.1$.
   (a) Find the probability that the lifetime is less than 10.
   (b) Find the probability that the lifetime is between 5 and 15.
   (c) Find t such that the probability that the lifetime is greater than t is 0.01.
5. (**Rice 2.5.60**) Find the pdf of $Y = e^X$ where $X \sim N(0, 1)$. This is called the *lognormal density*[1].

[1] You can google the lognormal density and find the answer but I want you to *derive* the lognormal density.