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Letter to the Editor

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Letter to the Editor

Ver Hoef, J. M. (2012), "Who Invented the Delta Method," *The American Statistician*, 66, 124–127: Comment by S. Portnoy and Reply

While the history of the delta method is indeed obscure, I was very surprised not to see reference to Doob (1935) that I knew as the original presentation of the theory of the delta method. This earlier article appeared in a major journal, provided a formal theorem (Theorem 1, p. 166) covering the case of several possibly dependent means, and used the phrase "δ-method." Since the author had a specific application in mind, the Theorem is stated for a studentized statistic that is a (smooth) function of four means and their associated covariance estimates. It does give formal conditions, and it is clear that the results can be extended directly to multivariate means in any (fixed) dimension. In fact, Doob refers to two earlier applications of the method (Kelley 1928, pp. 49–50 and Wright 1934, p. 211), but these do not give a theorem nor do they use the phrase "delta method."

The delta method derives from linearization via differentials followed by simple variance formulas for linear combinations of measurement means. This idea was certainly known in the early 19th century, and scientists like Gauss and Bessel were quite fluent with it. Numerous textbooks from the 19th century (e.g., Airy 1861; Bessel 1876) covered the theory of errors and developed the formulas, often under the name "propagation of error." However, to my knowledge, the first formal mathematical result providing general conditions for asymptotic normality appeared in Doob's article.

It is true that the Doob reference is not well-known, though it is possible to find the reference using an electronic search (without using Doob's name). An "advanced scholar" Google search listed 22 citations. It is possible that my Illinois connection (and interaction with Joe Doob) has made this more familiar to me, but I have mentioned this reference a number of times and often found it familiar to other statisticians as well. In any event, consideration of Doob's article would certainly be expected in a discussion of the history of the delta method.

I do have one personal anecdote about this article. Shortly after coming to Illinois I mentioned to Joe Doob that this statistical result was doubtlessly used far more often (almost always without citation) than any of his other results. He replied that the result was really rather trivial. He deprecated the article as not having the intrinsic interest and excitement of his deeper works in probability theory (with which I had to agree).

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Reply

I wish to sincerely thank Dr. Portnoy for adding to the history of the delta method. I missed Doob (1935); this, and Wright (1934) and Kelley (1928) offer considerably more insight. Note that Doob (1935) listed Columbia University as his institutional affiliation, where he did a post-doc from 1932–1934. Dorfman earned his B.A. in mathematics from Columbia University in 1936, so there is likely a connection. Columbia University is where Doob became interested in probability due to a grant from Hoteling (Snell 1997), and Hoteling may be the common thread, providing the current literature to both Doob and Dorfman.

It seems to me that Doob's result is the limiting distribution version (Section 2.3 of my original article), while Dorfman's was the approximate variance (Section 2.1 of my original article) with the plug-in estimator, but these differences are subtle and may not warrant distinction. Ultimately, though, it is fascinating that Doob also used the "δ-method" name, which would indicate that it was commonly known by that name, at least among the Columbia University crowd, in the early 1930s. Also note that Wright (1934) used a δ notation, while Kelley (1928) used a Δ notation, even though they did not name the method.

Jay M. VER HOEF
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REFERENCE

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Please see Stephen Portnoy's letter for the Doob (1935), Kelley (1928), and Wright (1934) references.