ORIE 4630: Spring Term 2019
Homework #6 Solutions

Question 1. [5 points]

Note that August 24, 2011 corresponds to row 1421.

i) Based on output from line 16,

```
> stocks_netRet[1421,2]
[1] 0.03228526
```

or

```
> stocks_netRet$GS[1421]
[1] 0.03228526
```

ii) Based on output from line 10,

```
> mu_f[1421]
[1] 7.936508e-07
```

iii) Based on output from line 18,

```
> stocks_exRet[1421,2]
[1] 0.03228447
```

or

```
> stocks_exRet$GS[1421]
[1] 0.03228447
```

Question 2. [10 points]

Output from lines 20 and 21:

```
> round(fit_netRet$coefficients, 5)
                 AAPL      GS     MCD     NKE     WMT     XOM
(Intercept)   0.00088 0.00009 0.00054 0.00049 0.00019 0.00003
market_netRet 0.97573 1.43971 0.58027 0.89699 0.53363 0.95024
```

i) $\hat{\beta}_{\text{XOM}} = 0.95024$

ii) Goldman Sachs Group Inc (GS) has the largest estimate of β; $\hat{\beta}_{\text{GS}} = 1.43971$

iii) Walmart Inc (WMT) has the smallest estimate of β; $\hat{\beta}_{\text{WMT}} = 0.53363$

iv) The mean can be calculated as follows:

```
> mean(fit_netRet$coefficients[2, ])
[1] 0.8960944
```

v) Goldman Sachs Group Inc (GS) has the largest estimate of β, so Goldman Sachs Group Inc (GS) is the most aggressive.

vi) Walmart Inc (WMT) has the smallest estimate of β; so Walmart Inc (WMT) is the least aggressive.

Question 3. [5 points]

Output from lines 22 and 23:

```
> round(fit_exRet$coefficients, 5)
                AAPL      GS     MCD     NKE     WMT     XOM
market_exRet 0.97709 1.43975 0.58107 0.89781 0.53394 0.95013
```

i) $\hat{\beta}_{\text{XOM}} = 0.95013$; the difference between the estimates is $0.95013 - 0.95024 = -0.00011$

ii) The mean can be calcuated as follows:

```
> mean(fit_exRet$coefficients)
[1] 0.8966328
```

Question 4. [15 points]

The output of the regression for Exxon Mobile Corp (XOM) is

```
Response XOM :

Call:
lm(formula = XOM ~ market_exRet)

Residuals:
      Min        1Q    Median        3Q       Max
-0.071797 -0.004867 -0.000133  0.004839  0.078102

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  2.761e-05  1.802e-04   0.153    0.878
market_exRet 9.501e-01  1.452e-02  65.415   <2e-16 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 0.009746 on 2925 degrees of freedom
Multiple R-squared:  0.594,Adjusted R-squared:  0.5938
F-statistic:  4279 on 1 and 2925 DF,  p-value: < 2.2e-16
```

i) $\hat{\alpha}_{\text{XOM}} = 0.00002761$

ii) $R^2_{\text{XOM}} = 0.594$

iii) The p-value is 0.878.

iv) No, there is no evidence against the null hypothesis $H_0 : \alpha_{\text{XOM}} = 0$, since the p-value 0.878 is very large and exceeds than the specified 5% threshold. If the CAPM holds, then $\alpha_{\text{XOM}} = 0$. Since there is no evidence from the data that $\alpha_{\text{XOM}} \neq 0$, then there is no evidence that Exxon Mobile Corp (XOM) does not follow CAPM.
v) The p-value for testing $H_0 : \alpha_j = 0$ against $H_0 : \alpha_j \neq 0$ for each stock j is given in the following table:

```
> pvals<-rep(0,6); for (i in 1:6)
+   {a=lm(as.matrix(stocks_exRet[,i]) ~ market_exRet);
+   pvals[i]=round(summary(a)$coefficients[1,4],6)}
> rbind(stocknames,pvals)
           [,1]       [,2]       [,3]       [,4]       [,5]       [,6]
stocknames "AAPL"     "GS"       "MCD"      "NKE"      "WMT"      "XOM"
pvals      "0.004599" "0.714938" "0.002148" "0.045762" "0.358192" "0.878222"
```

The stocks for which the p-values are less than 5% are Apple Inc (AAPL), McDonald's Corp (MCD), and Nike Inc (NKE), so these are the stocks for which there is evidence that $\alpha \neq 0$, i.e., for which there is evidence that CAPM does not hold.

Question 5. [10 points]

Output from line 28:

```
> round(apply(stocks_exRet, 2, mean), 6)
    AAPL       GS      MCD      NKE      WMT      XOM
0.001129 0.000482 0.000678 0.000717 0.000311 0.000273
```

Output from line 29:

```
> round(beta*mean(market_exRet), 6)
                 AAPL       GS     MCD      NKE      WMT      XOM
market_exRet 0.000252 0.000372 0.00015 0.000232 0.000138 0.000245
```

Output from line 30:

```
> round(apply(stocks_exRet, 2, mean)-beta*mean(market_exRet), 6)
                 AAPL       GS      MCD      NKE      WMT     XOM
market_exRet 0.000877 0.000111 0.000528 0.000485 0.000173 2.8e-05
```

i) 0.000273

ii) 0.000245

iii) According to the output from line 30, the stocks having the most extreme difference between the two estimates of mean excess return are Apple Inc (AAPL), McDonald's Corp (MCD), and Nike Inc (NKE). These stocks are exactly the ones for which the tests in Question 4 suggests that CAPM does not hold.

Question 6. [10 points]
Output from line 31:

```
> round(beta*0.009, 6)
                AAPL       GS     MCD     NKE      WMT      XOM
market_exRet 0.008794 0.012958 0.00523 0.00808 0.004805 0.008551
```

i) 0.00523

ii) No, this prediction is not satisfactory. This prediction is based on the assumption that McDonald's Corp (MCD) follows CAPM; however, there is evidence that $\alpha_{\text{NKE}} \neq 0$, so there is evidence against CAPM for McDonald's Corp (MCD).

iii) Another estimate of the excess return for McDonald's Corp (MCD) would be $\hat{\alpha}_{\text{MCD}} + \hat{\beta}_{\text{MCD}} * 0.009$ where the estimates $\hat{\alpha}_{\text{MCD}}$ and $\hat{\beta}_{\text{MCD}}$ are taken from the regression model with intercept and slope from Question 4 . The output of the regression for McDonald's Corp (MCD) is

```
Response MCD :

Call:
lm(formula = MCD ~ market_exRet)

Residuals:
      Min        1Q    Median        3Q       Max
-0.066755 -0.004767 -0.000145  0.004507  0.071061

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.0005280  0.0001719   3.072  0.00215 **
market_exRet 0.5801893  0.0138559  41.873  < 2e-16 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 0.009298 on 2925 degrees of freedom
Multiple R-squared:  0.3748,Adjusted R-squared:  0.3746
F-statistic:  1753 on 1 and 2925 DF,  p-value: < 2.2e-16
```

The estimate is

```
> 0.0005280+0.5801893*0.009
[1] 0.005749704
```

The present estimate 0.00575 would be preferred over the previous estimate 0.00523.

Question 7. [**10 points**]

i) Output from line 34:

```
> round(cor(res), 5)
         AAPL       GS      MCD      NKE      WMT      XOM
AAPL  1.00000 -0.01529  0.00087  0.00724 -0.04209 -0.13686
GS   -0.01529  1.00000 -0.13550 -0.07764 -0.09538 -0.17395
MCD   0.00087 -0.13550  1.00000  0.11933  0.14116 -0.00209
NKE   0.00724 -0.07764  0.11933  1.00000  0.08575 -0.12125
WMT  -0.04209 -0.09538  0.14116  0.08575  1.00000 -0.02906
XOM  -0.13686 -0.17395 -0.00209 -0.12125 -0.02906  1.00000
```

ii) The results of the test are as follows:

```
> cor.test(res[ ,2], res[ ,4])

Pearson's product-moment correlation

data:  res[, 2] and res[, 4]
t = -4.2118, df = 2925, p-value = 2.61e-05
alternative hypothesis: true correlation is not equal to 0
95 percent confidence interval:
 -0.11355149 -0.04152756
sample estimates:
        cor
-0.07764083
```

iii) The p-value 0.0000261 in the correlation test is much smaller than the threshold 5%, which suggests that there is strong evidence against the null hypothesis H_0 : $\rho_{\text{GS,NKE}} = 0$ in favor of the alternative hypothesis that $H_A : \rho_{\text{GS,NKE}} \neq 0$. Since there is evidence that $Corr(\epsilon_{\text{GS},t}, \epsilon_{\text{NKE},t}) \neq 0$, there is evidence that the assumptions of the security characteristic line are violated for Goldman Sachs Group Inc (GS) and Nike Inc (NKE).

Question 8. [**5 points**]

Output from line 37:

```
> round(cov_est1, 6)
         AAPL       GS      MCD      NKE      WMT      XOM
AAPL 0.000426 0.000212 0.000087 0.000136 0.000073 0.000120
GS   0.000212 0.000588 0.000108 0.000182 0.000102 0.000183
MCD  0.000087 0.000108 0.000138 0.000095 0.000061 0.000085
NKE  0.000136 0.000182 0.000095 0.000297 0.000085 0.000116
WMT  0.000073 0.000102 0.000061 0.000085 0.000147 0.000075
XOM  0.000120 0.000183 0.000085 0.000116 0.000075 0.000234
```

Output from line 39:

```
> round(cov_est2, 6)
         AAPL       GS      MCD      NKE      WMT      XOM
AAPL 0.000427 0.000216 0.000087 0.000135 0.000080 0.000143
GS   0.000216 0.000588 0.000129 0.000199 0.000118 0.000211
MCD  0.000087 0.000129 0.000138 0.000080 0.000048 0.000085
NKE  0.000135 0.000199 0.000080 0.000297 0.000074 0.000131
WMT  0.000080 0.000118 0.000048 0.000074 0.000147 0.000078
XOM  0.000143 0.000211 0.000085 0.000131 0.000078 0.000234
```

Output from line 40:

```
> round(cov_est1/cov_est2, 4)
       AAPL     GS    MCD    NKE    WMT    XOM
AAPL 0.9990 0.9790 0.9985 1.0094 0.9088 0.8423
GS   0.9790 0.9999 0.8379 0.9149 0.8649 0.8678
MCD  0.9985 0.8379 0.9989 1.1792 1.2774 0.9962
NKE  1.0094 0.9149 1.1792 0.9992 1.1539 0.8807
WMT  0.9088 0.8649 1.2774 1.1539 0.9997 0.9625
XOM  0.8423 0.8678 0.9962 0.8807 0.9625 0.9999
```

i) 0.000182

ii) 0.000199

iii) 0.9149

Question 9. [15 points]

i) Output from line 43:

```
> beta_P
          [,1]
[1,] 0.8966328
```

ii) Output from line 45:

```
> systematic_component_P
             [,1]
[1,] 0.0001237144
```

iii) Output from line 47:

```
> unique_component_P
             [,1]
[1,] 2.794721e-05
```

iv) The estimates of the unique components of risk for the eight stocks are ouptut from line 48:

```
> diag(cov(res))
        AAPL           GS          MCD          NKE          WMT          XOM
2.798226e-04 2.687814e-04 8.641434e-05 1.727510e-04 1.033814e-04 9.494888e-05
```

The estimate of the unique component of risk for the portfolio, 0.00002795, is less than each of the estimates for the stocks. Diversification does appear to reduce the unique component of risk.

v) The estimate of risk for the portfolio is output from line 50:

```
> sigma_P
           [,1]
[1,] 0.01231509
```

The estimates of risk for the stocks are output from line 51:

```
> sqrt(diag(cov_est2))
      AAPL         GS        MCD        NKE        WMT        XOM
0.02065758 0.02424386 0.01176319 0.01722759 0.01213474 0.01529270
```

The estimate of risk for the portfolio is less than the estimates of risk for the stocks, except for McDonald's Corp (MCD) and Walmart Inc (WMT).

Question 10. [15 points]

i) The weights are given are output from line 72:

```
> w
      AAPL         GS        MCD        NKE        WMT        XOM
 2.0000000 -1.0000000  2.0000000 -0.5182826 -0.4817174 -1.0000000
```

In the portfolio, the stocks Goldman Sachs Group Inc (GS), Nike Inc (NKE), Walmart Inc (WMT), and Exxon Mobile Corp (XOM) have negative weights, so they are shorted.

ii) The estimate $\hat{\alpha}_P$ is output from line 74:

```
> alpha_P
            [,1]
[1,] 0.002337173
```

The estimates of the αs for the stocks are output from line 75:

```
> alpha
        AAPL           GS          MCD          NKE          WMT          XOM
0.0008772343 0.0001107307 0.0005279981 0.0004856880 0.0001727728 0.0000276100
```

The estimate of α_P is much larger than the estimates of the αs for the individual stocks.

iii) The estimate $\hat{\beta}_P$ is output from line 77:

```
> beta_P
              [,1]
[1,] -1.742495e-12
```

Thus, $\hat{\beta}_P = 0$

iv) The estimate of risk for the portfolio is output from line 79:

```
> portfolio_risk
           [,1]
[1,] 0.04357832
```

The estimates of the risk for the stocks are output from line 81

```
> rbind(stocknames,round(stocks_risk,5))
           [,1]      [,2]      [,3]      [,4]      [,5]      [,6]
stocknames "AAPL"    "GS"      "MCD"     "NKE"     "WMT"     "XOM"
           "0.02065" "0.02424" "0.01176" "0.01722" "0.01213" "0.01529"
```

The estimate of the risk for the portfolio is much larger than the estimates of the risks for the individual stocks.

v) The estimate of the risk from the market neutral portfolio is $\hat{\sigma}_P = 0.04357832$; the estimate of the risk from the equal-weights portfolio considered in Question 9 is $\hat{\sigma}_P = 0.01231509$. Maximizing α inflates the risk of the portfolio. It seems appealing to maximize α in a market neutral portfolio; the lure is that a consistently profitable portfolio can be obtained regardless of the movement of the market. However, in practice, the market neutral portfolio has considerable risk, so the expected gains of the portfolio are in no way guaranteed.