ORIE 4630: Spring Term 2019
Homework #3 Solutions

Question 1. [15 points]

Output from lines 6 to 7:

```
> fit
        m              s              df
 0.0005273569   0.0133452040   2.7340473894
(0.0003051133) (0.0003283650) (0.1609371218)
```

i) 2.734047

ii) 0.000328365

iii) The test statistic is $\dfrac{0.000527357 - 0}{0.000305113} = 1.7284$, which exceeds $z_{0.05} = 1.645$, so the null hypothesis is rejected at the 5% level. This test suggests that the mean is positive.

Question 2. [15 points]

Output from lines10 to 21:

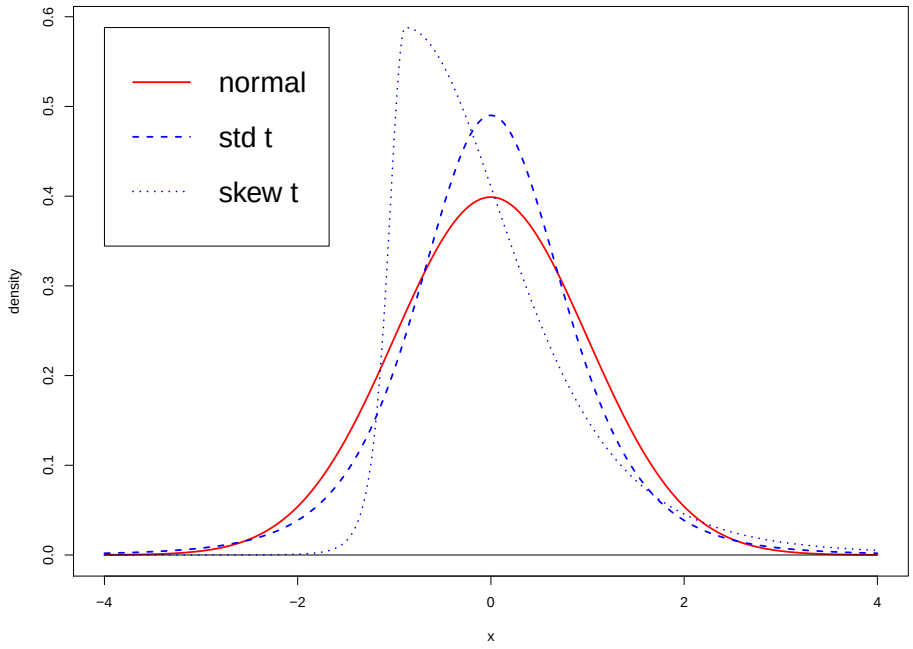
```
> rbind(parameter, round(eststt, 7), round(sestt, 7))
          [,1]        [,2]        [,3]
parameter "mean"      "sd"        "nu"
          "0.0005306" "0.0266574" "2.6507656"
          "0.0003036" "0.001946"  "0.1487207"
```

i) 2.650766

ii) The standard deviation estimate is 0.0266574. These results are not inconsistent; from the fitted standardized Student's t-distribution, the implied estimate of the scale parameter is $0.0266574\sqrt{\dfrac{2.650766 - 2}{2.650766}} = 0.0132082$, which is very close to the estimate of the scale parameter 0.0133452 given in Question 1.

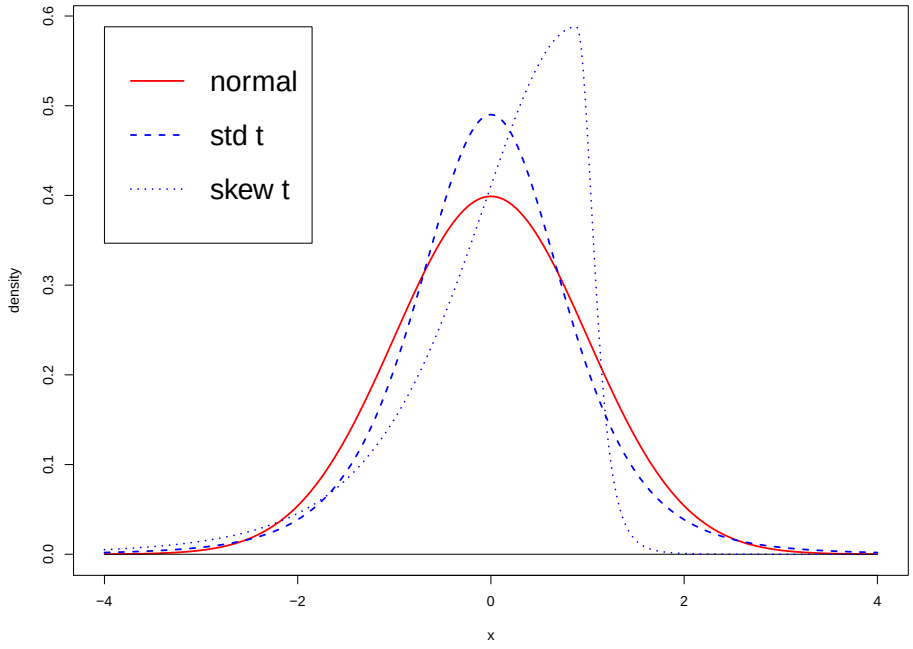
Question 3. [15 points]

i) Output from lines 22 to 33 with $\xi = 2.5$:



ii) The F-S skewed Student's t-density with $\xi = 2.5$ is skewed to the right.

iii) Output from lines 10 to 21 with $\xi = 1/2.5$:



iv) The F-S skewed Student's t-density with $\xi = 1/2.5$ is skewed to the left.

Question 4. [15 points]

Output from lines 34 to 43:

```
> rbind(parameter, round(estskt, 7), round(seskt, 7))
          [,1]        [,2]        [,3]        [,4]
parameter "mean"      "sd"        "nu"        "xi"
          "0.0002299" "0.0266256" "2.6542137" "0.972674"
          "0.0004155" "0.0019335" "0.148922"  "0.0233196"
```

i) The maximum likelihood estimate of ξ is $\hat{\xi} = 0.972674$. This estimate is less than 1, so it suggests the returns are skewed to the left.

ii) The standard error of $\hat{\xi}$ is 0.0233196.

iii) The value of the test statistic is $\dfrac{0.972674 - 1}{0.0233196} = -1.1718$. Since $|-1.1718| < 1.960 = z_{0.975}$, the null hypothesis is not rejected in a test at the 5% level. The conclusion of the test is that there is no evidence the returns are skewed.

Question 5. [20 points]

Output from lines 44 to 48:

```
> fitLSE

Call:
lm(formula = Returns$GS ~ Returns$SP500)

Coefficients:
  (Intercept)  Returns$SP500
   -8.568e-05      1.434e+00

> summary(fitLSE)

Call:
lm(formula = Returns$GS ~ Returns$SP500)

Residuals:
      Min        1Q    Median        3Q       Max
-0.139437 -0.007101 -0.000145  0.006916  0.144971

Coefficients:
                Estimate Std. Error t value Pr(>|t|)
(Intercept)   -8.568e-05  2.997e-04  -0.286    0.775
Returns$SP500  1.434e+00  2.413e-02  59.436   <2e-16 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 0.01621 on 2925 degrees of freedom
Multiple R-squared:  0.547,Adjusted R-squared:  0.5469
```

```
F-statistic:  3533 on 1 and 2925 DF,  p-value: < 2.2e-16

> round(confint(fitLSE, level=0.95), 7)
                   2.5 \%    97.5 \%
(Intercept)   -0.0006732 0.0005019
Returns$SP500  1.3866921 1.4813073
> round(head(fitLSE$residuals), 7)
         1          2          3          4          5          6
-0.0190801 -0.0003300  0.0007390  0.0068111  0.0130959 -0.0053528
```

i) The least squares estimate is $(\hat{\alpha}, \hat{\beta}) = (0.00008568, 1.434)$.

ii) The standard error of $\hat{\beta}$ is 0.02413.

iii) The 95% confidence interval for β is $(1.38669, 1.48131)$.

iv) Since the value 1 is not in the 95% confidence interval for β, the null hypothesis $H_0 : \beta = 1$ would be rejected in favor of the alternative hypothesis $H_A : \beta \neq 1$ in a test at the 5% level.

v) The estimate of σ_ϵ is 0.01621.

Question 6. [20 points]

Output from lines 49 to 63:

```
> rbind(parameter, round(mle, 7), round(se, 7))
          [,1]         [,2]        [,3]        [,4]
parameter "alpha"      "beta"      "sd"        "nu"
          "-0.0003151" "1.3678901" "0.0180398" "2.6336112"
          "0.0002044"  "0.0222406" "0.0012963" "0.1405194"
> cbind(parameter, lowerCL, upperCL)
     parameter lowerCL      upperCL
[1,] "alpha"   "-0.0007156" "8.54e-05"
[2,] "beta"    "1.3242993"  "1.4114809"
[3,] "sd"      "0.0154991"  "0.0205805"
[4,] "nu"      "2.3581982"  "2.9090242"
```

i) The maximum likelihood estimate is $(\hat{\alpha}, \hat{\beta}) = (-0.0003151, 1.367890)$.

ii) The standard error of the maximum likelihood estimate of β is 0.0222406.

iii) The 95% confidence interval for β is $(1.324299, 1.411481)$.

iv) Since the value 1 is not in the 95% confidence interval for β, the null hypothesis $H_0 : \beta = 1$ would be rejected in favor of the alternative hypothesis $H_A : \beta \neq 1$ in a test at the 5% level.

v) The estimate of σ_ϵ is 0.018040.