Fall 2018 STSCI 5080 Discussion 1 (8/24)

Complements to Lecture 1

Finite, countable, and uncountable sets.

- If a set has finitely many elements, then it is called a finite set. E.g. $A = \{1, 2, 3, 4, 5, 6\}$.
- An infinite set A is said to countable if we can enumerate the elements of A in such a way that $A = \{a_1, a_2, \dots\}$. E.g. $A = \{2, 4, 6, \dots\}$ (the set of even numbers).
- An infinite set is said to be uncountable if it is not countable. E.g. $A = \{t \mid t \geq 0\}$.

Problems

1. (**Rice 1.8.1**) A coin is tossed three times and the sequence of heads and tails is recorded.
   (a) List the sample space.
   (b) List the elements that make up the following events: (1) $A =$ at least two heads, (2) $B =$ the first two tosses are heads, (3) $C =$ the last toss is a tail.
   (c) List the elements of the following events: (1) A^c, (2) $A \cap B$, (3) $A \cup C$.
2. (**Rice 1.8.5**) Let A and B be arbitrary events. Let C be the event that either A occurs or B occurs, but not both. Express C in terms of A and B using any of the basic operations of union, intersection, and complement.

   Hint: Draw Venn's diagram.
3. Generalize de Morgan's laws into three events.
   (a) $(A \cup B \cup C)^c = A^c \cap B^c \cap C^c$.
   (b) $(A \cap B \cap C)^c = A^c \cup B^c \cup C^c$.

Solutions

1. (**Rice 1.8.1**) $h \leftrightarrow$ head and $t \leftrightarrow$ tail. E.g. tht means that the first toss is head, the second toss is tail, and the third toss is head.

   (a) There are 8 possible outcomes and

   $$\Omega = \{hhh, hht, hth, hht, htt, tht, tth, ttt\}.$$

   (b) Just check the conditions. (1) $A = \{hhh, hht, hth, thh\}$. (2) $B = \{hhh, hht\}$. (3) $C = \{hht, htt, tht, ttt\}$.

   (c) (1) The complement of A is the collection of elements that are outside of A, and so $A^c = \{htt, tht, tth, ttt\}$. (2) The intersection of A and B is the collection of common elements of A AND B, and so $A \cap B = \{hhh, hht\}$. (2) The union of A and C is the collection of elements that are elements of A OR C, and so $A \cup C = \{hhh, hht, hth, thh, htt, tht, ttt\}$.

2. (**Rice 1.8.5**) The event such that A occurs or B occurs or both occur is $A \cup B$. We have to exclude from $A \cup B$ the event such that both A and B occur, namely, $A \cap B$. That is

   $$C = (A \cup B) \cap (A \cap B)^c.$$

   Draw Venn's diagram.

3. (a) Introduce a new set $D = B \cup C$; then $A \cup B \cup C = A \cup D$. Apply de Morgan's laws to $A \cup D$ to conclude that

   $$(A \cup B \cup C)^c = (A \cup D)^c = A^c \cap D^c.$$

   Applying de Morgan's laws to $D = B \cup C$ again, we get

   $$D^c = (B \cup C)^c = B^c \cap C^c.$$

   Hence

   $$(A \cup B \cup C)^c = A^c \cap D^c = A^c \cap (B^c \cap C^c) = A^c \cap B^c \cap C^c.$$

   (b) Similar to Case (a). Introduce a new set $D = B \cap C$; then $A \cap B \cap C = A \cap D$. Apply de Morgan's laws and get

   $$(A \cap D)^c = A^c \cup D^c,$$

   and apply de Morgan again to $D = B \cap C$ to get

   $$D^c = B^c \cup C^c.$$

   Hence

   $$(A \cap B \cap C)^c = A^c \cup (B^c \cup C^c) = A^c \cup B^c \cup C^c.$$