PRELIM 1
MATH 2210

NAME: ______________________________

TA: ______________ (Yun or Anwesh)

EXAM RULES: Calculators, computers, notes, books, and cheat sheets are not allowed. Cell phones may not be used, not even as time-keeping devices.

PROBLEM	SCORE
1	
2	
3	
4	
5	
6	
TOTAL	

Problem 1. (Homework problems) Let A be an $m \times n$ matrix, and $\mathbf{b}$ be a column vector with m entries. Mark the mathematically CORRECT statements below. You do not need to justify your answers.

- ☐ If the equation $A\mathbf{x} = \mathbf{b}$ is inconsistent, then $\mathbf{b}$ is not in the set spanned by the columns of A.
- ☐ If A has a pivot position in every row, then the equation $A\mathbf{x} = \mathbf{b}$ is inconsistent.
- ☐ If the equation $A\mathbf{x} = \mathbf{0}$ has only the trivial solution and the equation $A\mathbf{x} = \mathbf{b}$ has a solution, then $A\mathbf{x} = \mathbf{b}$ has a unique solution.
- ☐ A system of linear equations with more equations than unknowns is inconsistent.
- ☐ Suppose A is a 3×3 matrix and $\mathbf{y}$ is a vector in $\mathbf{R}^3$ such that the equation $A\mathbf{x} = \mathbf{y}$ does not have a solution. There exists no vector $\mathbf{z}$ in $\mathbf{R}^3$ such that the equation $A\mathbf{x} = \mathbf{z}$ has a unique solution.

Problem 2. (Homework problem) Use the following definition: A **vector space** is a non-empty set V of objects, called **vectors**, with two operations, called **addition** and **scalar multiplication** (multiplication by real numbers), subject to the ten conditions (called axioms in the textbook) listed below. The following properties must hold for all vectors $\mathbf{u}, \mathbf{v}$, and $\mathbf{w}$ in V and for all scalars (numbers) c and d.

1. The sum of $\mathbf{u}$ and $\mathbf{v}$, denoted by $\mathbf{u} + \mathbf{v}$, is in V.
2. The scalar multiple of $\mathbf{u}$ by c, denoted by $c\mathbf{u}$, is in V.
3. $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$.
4. $(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$.
5. There is a **zero vector** $\mathbf{0}$ in V such that $\mathbf{u} + \mathbf{0} = \mathbf{0} + \mathbf{u} = \mathbf{u}$.
6. For each $\mathbf{u}$ in V, there is a vector $-\mathbf{u}$ in V such that $\mathbf{u} + (-\mathbf{u}) = (-\mathbf{u}) + \mathbf{u} = \mathbf{0}$.
7. $1\mathbf{u} = \mathbf{u}$.
8. $c(\mathbf{u} + \mathbf{v}) = c\mathbf{u} + c\mathbf{v}$.
9. $(c + d)\mathbf{u} = c\mathbf{u} + d\mathbf{u}$.
10. $c(d\mathbf{u}) = (cd)\mathbf{u}$.

Fill in the missing axiom numbers in the following proof that $c\,\mathbf{0} = \mathbf{0}$ for every number c.
We get $c\,\mathbf{0} = c(\mathbf{0} + \mathbf{0})$ by Axiom ____.
Hence, $c\,\mathbf{0} = c\,\mathbf{0} + c\,\mathbf{0}$ by Axiom ____.
Adding the negative $-c\,\mathbf{0}$ of $c\,\mathbf{0}$ to both sides we get $c\,\mathbf{0} + (-c\,\mathbf{0}) = [c\,\mathbf{0} + c\,\mathbf{0}] + (-c\,\mathbf{0})$.
Hence, $c\,\mathbf{0} + (-c\,\mathbf{0}) = c\,\mathbf{0} + [c\,\mathbf{0} + (-c\,\mathbf{0})]$ by Axiom ____.
Therefore, $\mathbf{0} = c\,\mathbf{0} + \mathbf{0}$ by Axiom ____.
Thus, $\mathbf{0} = c\,\mathbf{0}$ by Axiom ____.

SOLVE THE FOLLOWING PROBLEMS IN YOUR EXAM BOOKLET.

Problem 3. Find all solutions of the linear system

$$\begin{array}{rrrrl} x_1 & +2x_2 & -3x_3 & -5x_4 & = 6 \\ & x_2 & +8x_3 & -6x_4 & = 1 \\ & x_2 & +10x_3 & -6x_4 & = 11\,. \end{array}$$

Problem 4.
a) Is the set W of all polynomials of the form $at^3 + bt^2 - 3t + d$ a subspace of the vector space of all polynomials? Justify your answer.

b) Is the set W of all matrices of the form $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ with $b \leq 0$ a subspace of the vector space of all 2×2 matrices? Justify your answer.

Problem 5.
a) Is $v = 6t^2 + t - 2$ in the span of $v_1 = t^2 + 3t + 5$, $v_2 = t^2 + 3t + 7$, and $v_3 = 2t^2 + 6t + 3$? If yes, then express v as a linear combination of v_1, v_2, v_3.

b) Is the span of the vectors $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$ a plane (considered as a subspace of $\mathbf{R}^3$)? Justify your answer.

Problem 6. Consider the linear system

$$\begin{array}{rrrl} x_1 & + x_2 & + x_3 & = 2 \\ x_1 & +2x_2 & + x_3 & = 3 \\ x_1 & + x_2 & +(r^2-5)\,x_3 & = r\,, \end{array}$$

where r is a parameter (real number).
a) Reduce the augmented matrix to row echelon form.
b) Determine all values of r for which the linear system has
   1) no solution;
   2) a unique solution;
   3) infinitely many solutions..
c) Solve the system if $r = 2$.