Math 2210

Prelim Exam 1 (February 23, 2017)

NAME: ______________________ **Section:** _____________

- Time: you have 90 minutes (7:30-9:00PM).
- Show all your work and justifications. The final answers must be "reasonably" simplified. For example, a rational number must be given in the form $\frac{a}{b}$ for some integers a and b.
- The use of calculators (and other electronic devices) or notes is not permitted.
- You may use both sides of the paper.
- Make sure you have **7 pages** and **5 problems** before starting the exam.

Academic integrity is expected of all students of Cornell University at all times. Understanding this, I declare I shall not give, use, or receive unauthorized aid.

SIGNATURE: ______________________

Problem 1: ___ / 20
Problem 2: ___ / 20
Problem 3: ___ / 20
Problem 4: ___ / 20
Problem 5: ___ / 20

Total: ___ / 100

Problem 1: Consider the system of linear equations

$$\begin{cases} x - 2y + z &= 3 \\ 2x + y + 3z &= b \\ 3x + ay - z &= 5 \end{cases}$$

(a) For which values of a and b, if any, does this system have a *unique solution*?

(b) For which values of a and b, if any, does this system have *no solution*?

(c) For which values of a and b, if any, does this system have *infinitely many solutions*?

Problem 2: Let $T\colon \mathbb{R}^4 \to \mathbb{R}^4$ be the transformation defined by

$$T(x_1, x_2, x_3, x_4) = (x_4, x_1, x_2, x_3) \ .$$

(a) Verify that this transformation is *linear*.

(b) Find the *standard matrices* for T and T^2.

(Here $T^2(\mathbf{x}) = T \circ T(\mathbf{x}) = T(T(\mathbf{x}))$.)

Problem 3: Consider the linear map

$$\begin{cases} T\colon \mathbb{R}^4 \to \mathbb{R}^3 \\ \qquad \mathbf{x} \mapsto A\mathbf{x} \end{cases} \quad \text{, where} \quad A = \begin{pmatrix} 0 & 1 & -1 & 2 \\ 1 & 2 & 1 & -1 \\ 1 & -1 & 0 & 1 \end{pmatrix}$$

(a) Compute Image(T). Is T *surjective* (onto)?

(b) Compute Kernel(T). Is T *injective* (one-to-one)?

Recall: $\text{Image}(T) = \{T(\mathbf{x})\colon \mathbf{x} \in \mathbb{R}^4\}$, $\text{Kernel}(T) = \{\mathbf{x} \in \mathbb{R}^4\colon T(\mathbf{x}) = \mathbf{0}\}$.

Problem 4: Consider the matrix

$$A = \begin{pmatrix} 0 & 1 & 2 \\ 2 & 1 & 0 \\ 1 & 0 & 2 \end{pmatrix}$$

Is this matrix *invertible*? If it is, compute its inverse A^{-1}.

Problem 5: Let A and B be any two matrices such that the product AB is defined. Suppose that $\mathrm{Kernel}(A) = \{0\}$. Show that

$$\mathrm{Kernel}(B) = \mathrm{Kernel}(AB) \ .$$

In other words, prove $\mathbf{x} \in \mathrm{Kernel}(B)$ if and only if $\mathbf{x} \in \mathrm{Kernel}(AB)$.

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