ORIE 4630: Spring Term 2019
Homework #5 Solutions

Question 1. [10 points]

i) $r_5 = -0.005385$; $R_5 = -0.005370$; these two values are nearly the same; $\log(1+x) \sim x$ for $|x|$ small, so $r_5 = \log(1 + R_5) \sim R_5$. Also, $r_2 = -0.002551$; $R_2 = -0.002548$.
ii) Output from line 9:

```
> mu_R
         IBM           GE          NKE
0.0003617045 0.0001891787 0.0007575329
```

Output from line 11:

```
> cov_R
             IBM           GE          NKE
IBM 0.0001836376 0.0001328161 0.0001082511
GE  0.0001328161 0.0003501520 0.0001542188
NKE 0.0001082511 0.0001542188 0.0002965468
```

iii) 0.0007575
iv) 0.0001836
v) 0.0001328
vi) Output from command `cor(R)`:

```
> cor(R)
          IBM        GE       NKE
IBM 1.0000000 0.5237713 0.4638790
GE  0.5237713 1.0000000 0.4785886
NKE 0.4638790 0.4785886 1.0000000
```

The correlation between General Electric (GE) and Nike (NKE) is 0.4786.

Question 2. [10 points]

i) Output from line 20:

```
> rbind(c("sigma_P"), round(sqrt(result$value), 7))
     [,1]
[1,] "sigma_P"
[2,] "0.0135527"
```

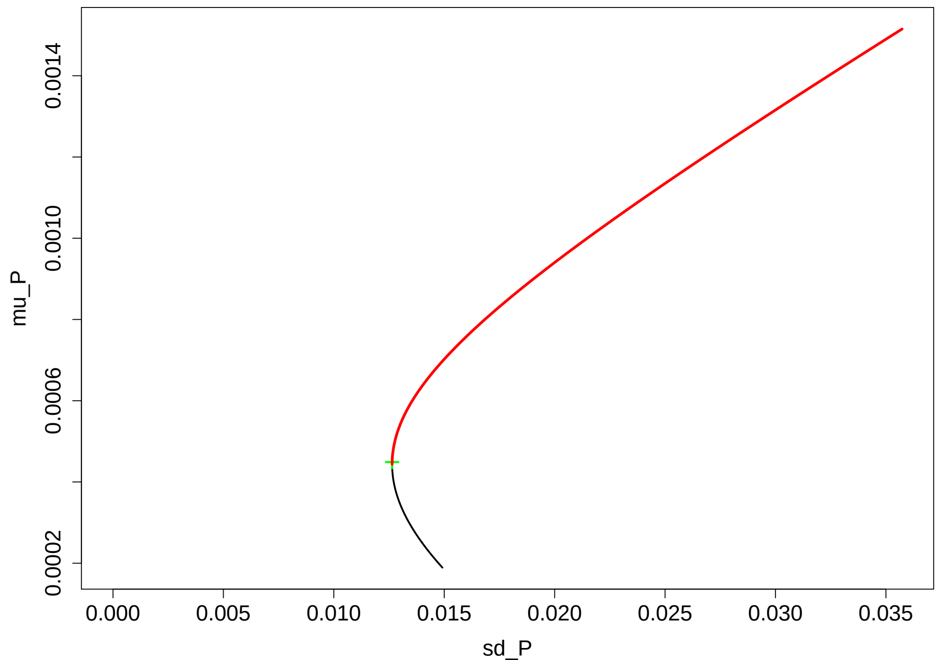
Output from line 21

```
> rbind(c("w_1", "w_2", "w_3"), round(result$solution, 7))
     [,1]        [,2]         [,3]
[1,] "w_1"       "w_2"        "w_3"
[2,] "0.5541076" "-0.1087325" "0.5546249"
```

ii) $\sigma_P = 0.01355$
iii) 0.55411
iv) Yes. The smallest-risk portfolio requires shorting General Electric (GE), as its weight -0.108733 is negative.

Question 3. [10 points]

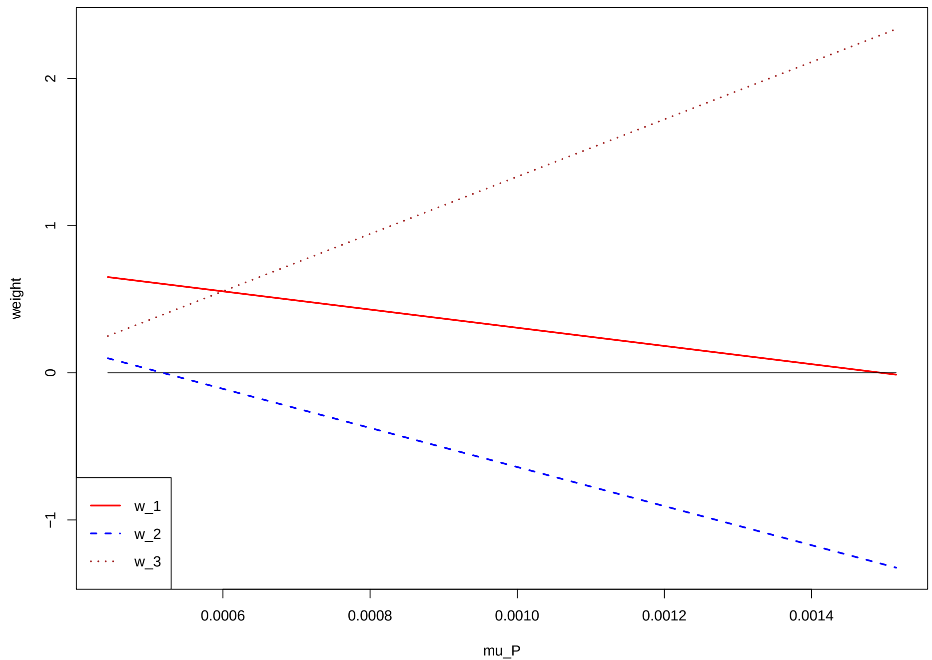
i)




```
> rbind(c("mu_MV"), round(mu_P[ind_MV], 7))
     [,1]
[1,] "mu_MV"
[2,] "0.0004433"
> rbind(c("sd_MV"), round(sd_P[ind_MV], 7))
     [,1]
[1,] "sd_MV"
[2,] "0.0126405"
> rbind(c("w_1_MV", "w_2_MV", "w_3_MV"), round(weights[ind_MV,], 7))
     [,1]        [,2]        [,3]
[1,] "w_1_MV"    "w_2_MV"    "w_3_MV"
[2,] "0.6511004" "0.0994791" "0.2494205"
```

ii) The expected return of the minimum-variance portfolio is $\mu_{MV} = 0.0004433$.
iii) The standard deviation of the minimum-variance portfolio is $\sigma_{MV} = 0.012641$.
iv) For the minimum-variance portfolio, the weights are 0.651100 for IBM (IBM), 0.099479 for General Electric (GE), and 0.249421 for Nike (NKE).

Question 4. [5 points]



Question 5. [10 points]

i) Output from line 59:

```
> rbind(c("sigma_P"), round(sqrt(result$value), 7))
     [,1]
[1,] "sigma_P"
[2,] "0.0137271"
```

Output from line 60:

```
> rbind(c("w_1", "w_2", "w_3"), round(result$solution, 7))
     [,1]        [,2] [,3]
[1,] "w_1"       "w_2" "w_3"
[2,] "0.3979829" "0"   "0.6020171"
```

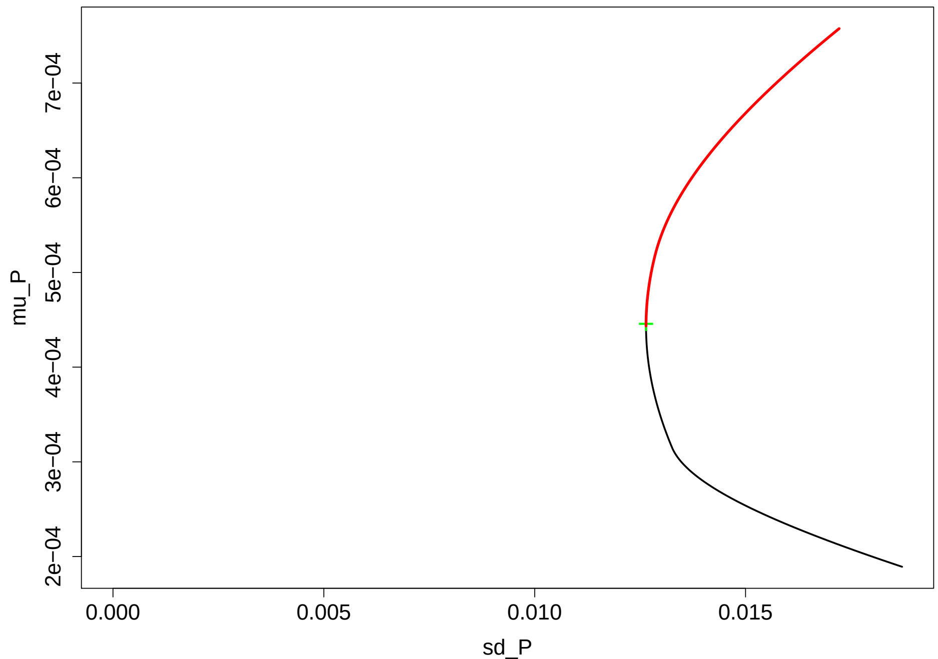
ii) $\sigma_P = 0.013727$.

iii) The weight assigned to General Electric (GE) in the smallest-risk portfolio is 0.

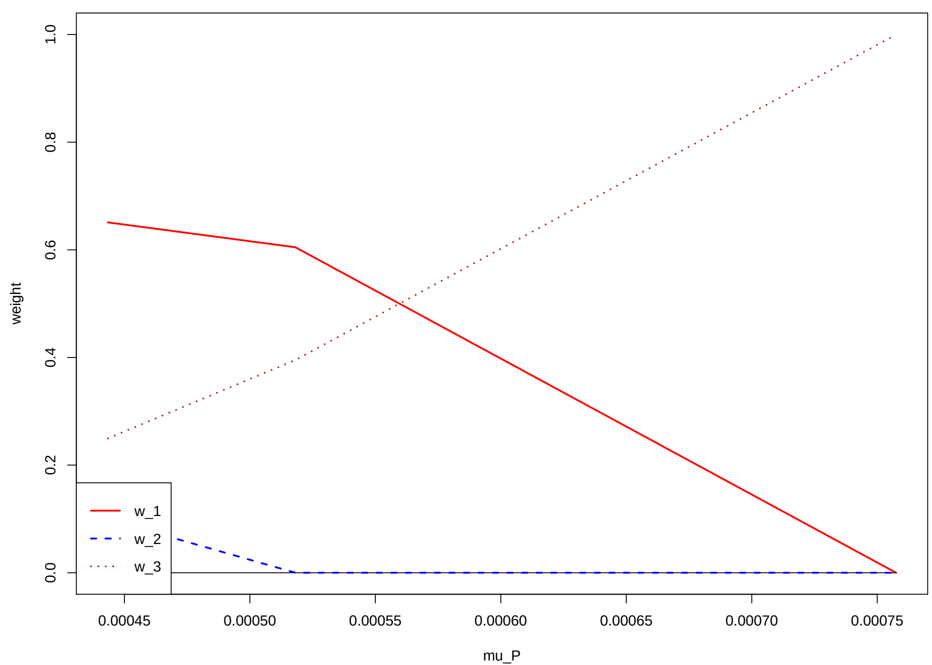
iv) No. The smallest-risk portfolio requires no shorting as all the weights are non-negative.

Question 6. [10 points]

i)



ii)



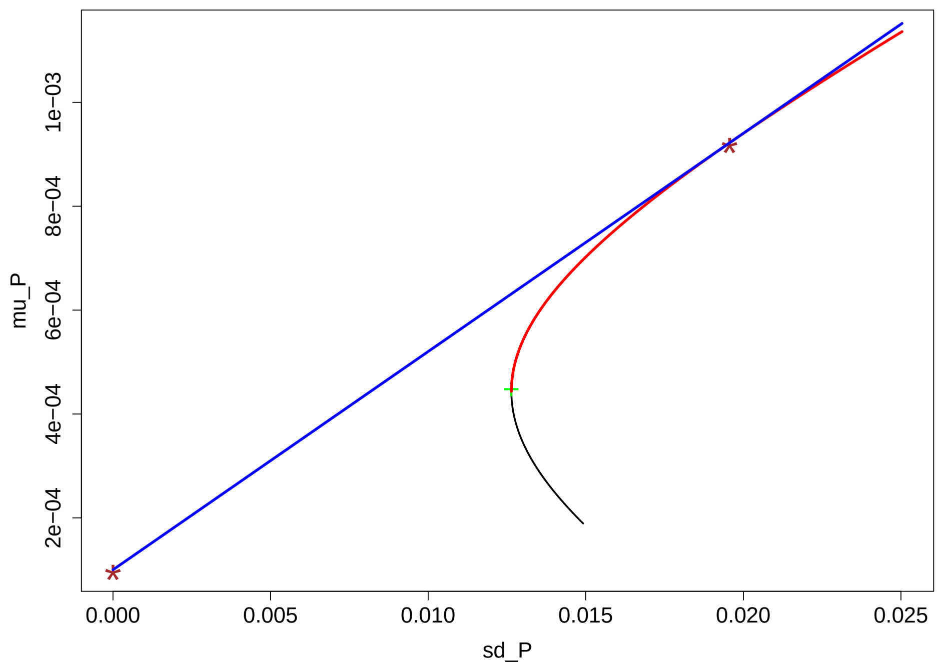
iii) Output from line 82:

```
> rbind(c("w_1_MV", "w_2_MV", "w_3_MV"), round(weights[ind_MV,], 7))
     [,1]        [,2]      [,3]
[1,] "w_1_MV"    "w_2_MV"  "w_3_MV"
[2,] "0.6510915" "0.09946" "0.2494485"
```

The weights for the minimum-variance portfolio reported in Question 3iv) are already non-negative, so this minimum-variance portfolio does not change in Question 6, as it meets the criterion imposed.

Question 7. [10 points]

i)




```
> rbind(c("mu_T"), round(mu_P[ind_T], 7))
     [,1]
[1,] "mu_T"
[2,] "0.000922"
> rbind(c("sd_T"), round(sd_P[ind_T], 7))
     [,1]
[1,] "sd_T"
[2,] "0.019562"
> rbind(c("w_1_T", "w_2_T", "w_3_T"), round(weights[ind_T, ], 7))
     [,1]        [,2]         [,3]
[1,] "w_1_T"     "w_2_T"      "w_3_T"
[2,] "0.3548266" "-0.5365235" "1.1816969"
```

ii) The expected return of the tangency portfolio is $\mu_T = 0.000922$.
iii) The standard deviation of the tangency portfolio is $\sigma_T = 0.019562$.
iv) For the tangency portfolio, the weights are 0.354827 for IBM (IBM), -0.536524 for General Electric (GE), and 1.181697 for Nike (NKE). Since the weight on General Electric (GE) is negative, it is shorted in the tangency portfolio.

Question 8. [10 points]

i) Output from line 119:

```
> rbind(c("w_F", "w_T"), round(c(1-w, w), 7))
     [,1]        [,2]
[1,] "w_F"       "w_T"
[2,] "0.3917407" "0.6082593"
```

Output from line 120:

```
> rbind(c("sd_O"), round(sd_O, 7))
     [,1]
[1,] "sd_O"
[2,] "0.0118988"
```

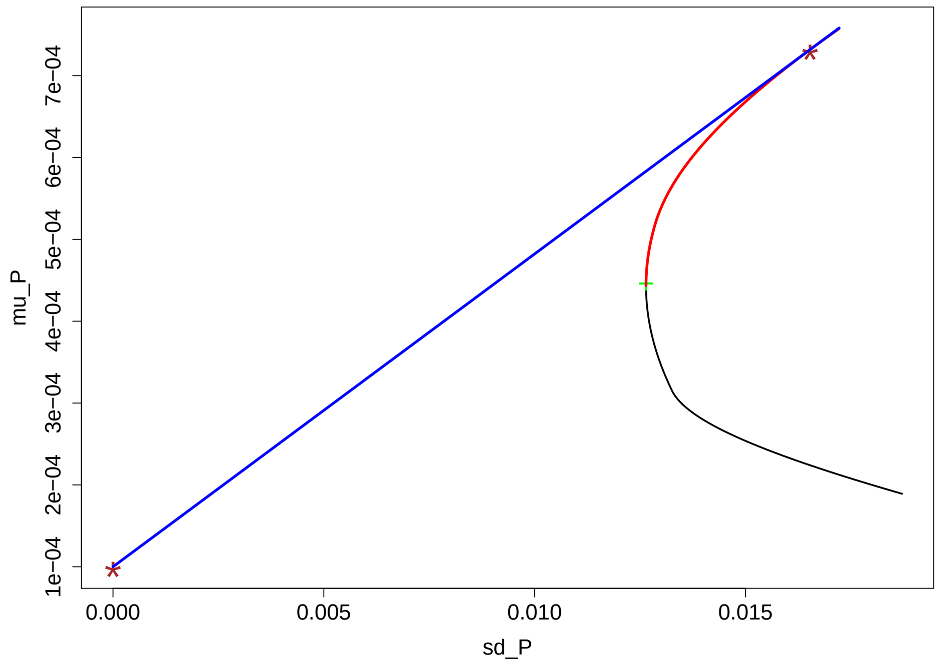
Output from line 121

```
> rbind(c("w_F_O", "w_1_O", "w_2_O", "w_3_O"), round(c(1-w, w*weights[ind_T,]), 7))
     [,1]        [,2]        [,3]         [,4]
[1,] "w_F_O"     "w_1_O"     "w_2_O"      "w_3_O"
[2,] "0.3917407" "0.2158266" "-0.3263454" "0.7187781"
```

ii) The standard deviation of the optimal portfolio is $\sigma_O = 0.011899$.
iii) The weight assigned to the risk-free asset in the optimal portfolio is 0.3917407.
iv) The weights in the optimal portfolio are 0.215827 for IBM (IBM), -0.326345 for General Electric (GE), and 0.718778 for Nike (NKE).

Question 9. [15 points]

i)




```
> rbind(c("mu_T"), round(mu_P[ind_T], 7))
     [,1]
[1,] "mu_T"
[2,] "0.0007318"
> rbind(c("sd_T"), round(sd_P[ind_T], 7))
     [,1]
[1,] "sd_T"
[2,] "0.0165291"
> rbind(c("w_1_T", "w_2_T", "w_3_T"), round(weights[ind_T, ], 7))
     [,1]        [,2]    [,3]
[1,] "w_1_T"     "w_2_T" "w_3_T"
[2,] "0.0649139" "0"     "0.9350861"
```

ii) The expected return of the tangency portfolio is $\mu_T = 0.000732$.

iii) The standard deviation of the tangency portfolio is $\sigma_T = 0.016529$.

iv) The weights for the tangency portfolio The weights are 0.064914 for IBM (IBM), 0 for General Electric (GE), and 0.935086 for Nike (NKE).

v) The two tangency portfolios are different, since the tangency portfolio from Question 7 has a negative weight on General Electric (GE).

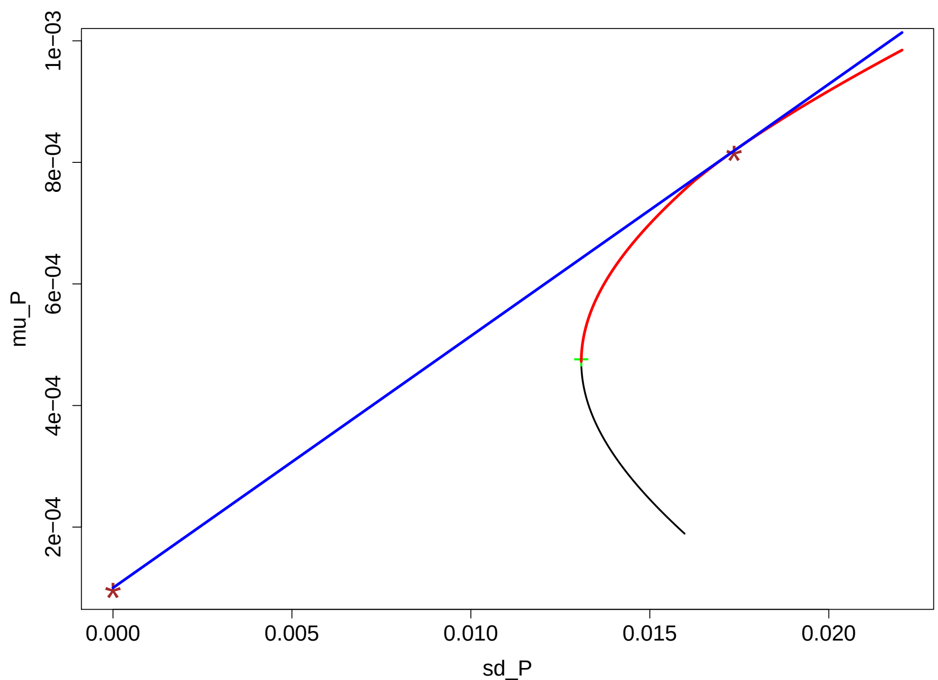
vi) Output from line 120:

```
> rbind(c("sd_O"), round(sd_O, 7))
     [,1]
[1,] "sd_O"
[2,] "0.0130802"
```

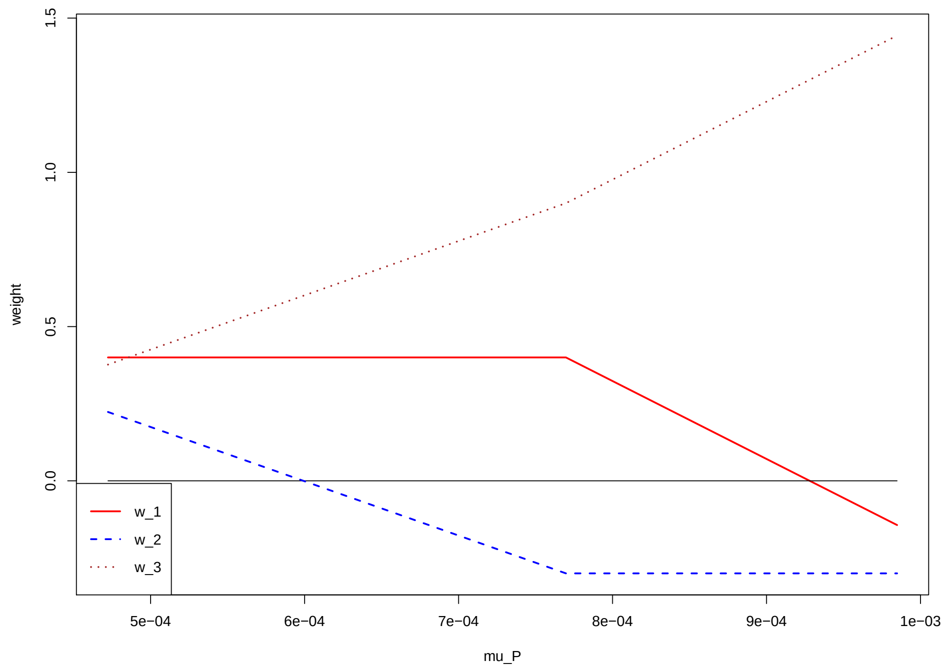
By imposing the constraint, the standard deviation for the optimal portfolio increases: here the standard deviation for the optimal portfolio is $\sigma_O = 0.013080$, while in Question 8, the standard devation is $\sigma_O = 0.011899$. The price for imposing the constraint of no shorting is to increase the risk.

Question 10. [10 points]

i)



ii)




```
> rbind(c("mu_MV"), round(mu_P[ind_MV], 7))
     [,1]
[1,] "mu_MV"
[2,] "0.0004722"
> rbind(c("sd_MV"), round(sd_P[ind_MV], 7))
     [,1]
[1,] "sd_MV"
[2,] "0.013085"
> rbind(c("w_1_MV", "w_2_MV", "w_3_MV"), round(weights[ind_MV,], 7))
     [,1]     [,2]        [,3]
[1,] "w_1_MV" "w_2_MV"    "w_3_MV"
[2,] "0.4"    "0.2235329" "0.3764671"
```

iii) The expected return of the minimum-variance portfolio is $\mu_{MV} = 0.0004722$.
iv) The standard deviation of the minimum-variance portfolio is $\sigma_{MV} = 0.013085$.

```
> rbind(c("mu_T"), round(mu_P[ind_T], 7))
     [,1]
[1,] "mu_T"
[2,] "0.0008191"
> rbind(c("sd_T"), round(sd_P[ind_T], 7))
     [,1]
[1,] "sd_T"
[2,] "0.0173541"
> rbind(c("w_1_T", "w_2_T", "w_3_T"), round(weights[ind_T, ], 7))
```

```
     [,1]        [,2]    [,3]
[1,] "w_1_T"     "w_2_T" "w_3_T"
[2,] "0.2751858" "-0.3"  "1.0248142"
```

v) The expected return of the tangency portfolio is $\mu_T = 0.0008191$.
vi) The standard deviation of the minimum-variance portfolio is $\sigma_T = 0.0173541$.

```
> rbind(c("sd_O"), round(sd_O, 7))
     [,1]
[1,] "sd_O"
[2,] "0.0120664"
```

vii) The standard deviation of the optimal portfolio is $\sigma_O = 0.0120664$.