ORIE 4630: Spring Term 2019
Homework #7 Solutions

Question 1. [2 points]

$200\left(1+\frac{0.06}{365}\right)^{730} = 225.50;\ 200(1+0.06)^2 = 224.72$

Question 2. [3 points]

$\frac{10,000,000}{(1.03)(1.05)} = 9,246,417.01$

Question 3. [10 points]

$P = 1,200e^{0.04(200/365)} = 1,226.59.$

Question 4. [15 points]

i) $f(2) = (85-90)_+ = (-5)_+ = 0$

ii) $f(3) = (110-90)_+ = (20)_+ = 20$

iii) $\delta = \frac{20-0}{110-85} = \frac{20}{25} = 0.8$

iv) At the beginning of the period, hold 0.8 share at \$100 per share and borrow $\$(0.8)(85)/(1.07) = 68/(1.07)$ at the risk-free rate of 7%.

v) $(0.8)(100) - 68/(1.07) = 17.6/1.07 = 16.4486$

vi) $q = \frac{(1.07)(100)-85}{110-85} = \frac{22}{25} = 0.88$

vii) $\frac{22}{25}(20) + \frac{3}{25}(0) = 17.60$

viii) $\frac{17.60}{1.07} = 16.4486$

Question 5. [15 points]

i) $f(2) = (48-40)_+ = (8)_+ = 8$

ii) $f(3) = (48-60)_+ = (-12)_+ = 0$

iii) $\delta = \frac{0-8}{60-40} = -0.4$

iv) At the beginning of the period, borrow 0.4 share at \$50 per share, i.e., go short 0.4 share, and hold \$24/(1.04) at the risk-free rate of 4%.

v) $(-0.4)(50) + 24/(1.04) = 3.2/1.04 = 3.0769$

vi) $q = \frac{(1.04)(50)-40}{60-40} = \frac{12}{20} = 0.6$

vii) $(0.6)0 + (0.4)8 = 3.2$

viii) $\dfrac{3.2}{1.04} = 3.0769$

Question 6. [15 points]

i) $q_1 = \dfrac{100-90}{110-90} = 0.5$, $q_2 = \dfrac{90-70}{95-75} = 0.8$, $q_3 = \dfrac{110-100}{125-100} = 0.4$

ii) After two periods, the values of the option at nodes 4, 5, 6, and 7 are:
$f(4) = (70-90)_+ = (-20)_+ = 0$, $f(5) = (95-90)_+ = (5)_+ = 5$,
$f(6) = (100-90)_+ = (10)_+ = 10$, $f(7) = (125-90)_+ = (35)_+ = 35$.
After one period, the values of the option at nodes 2 and 3 are:
$f(3) = (0.4)(35) + (0.6)(10) = 20$, $f(2) = (0.8)(5) + (0.2)(0) = 4$
At the start, the value of the option at node 1 is: $f(1) = (0.5)(20) + (0.5)(4) = 12$
Note that $f(1) = (0.5)(0.4)(35) + (0.5)(0.6)(10) + (0.5)(0.8)(5) + (0.5)(0.2)(0)$
$= (.2)(35) + (.3)(10) + (0.4)(5) + (.1)(0) = 12$
The price of the option at the start of the two periods is \$12.

iii) $f(1) = (.2)(28) + (.3)(3) + (0.4)(0) + (.1)(0) = 6.5$
The price of the option at the start of the two periods is \$6.5.

iv) $f(1) = (.2)(11) + (.3)(0) + (0.4)(0) + (.1)(0) = 2.2$
The price of the option at the start of the two periods is \$2.2.

Question 7. [15 points]

i) After two periods, the values of the option at nodes 4, 5, 6, and 7 are:
$f(4) = (114-70)_+ = (44)_+ = 44$, $f(5) = (114-95)_+ = (19)_+ = 19$,
$f(6) = (114-100)_+ = (14)_+ = 14$, $f(7) = (114-125)_+ = (-11)_+ = 0$.
After one period, the values of the option at nodes 2 and 3 are:
$f(3) = (0.4)(0) + (0.6)(14) = 8.4$, $f(2) = (0.8)(19) + (0.2)(44) = 24$
At the start, the value of the option at node 1 is: $f(1) = (0.5)(8.4) + (0.5)(24) = 16.2$
Note that $f(1) = (0.5)(0.4)(0) + (0.5)(0.6)(14) + (0.5)(0.8)(19) + (0.5)(0.2)(44)$
$= (.2)(0) + (.3)(14) + (0.4)(19) + (.1)(44) = 16.2$
The price of the option at the start of the two periods is \$16.2.

ii) $f(1) = (.2)(0) + (.3)(0) + (0.4)(2) + (.1)(27) = 3.5$
The price of the option at the start of the two periods is \$3.5.

iii) $f(1) = (.2)(0) + (.3)(0) + (0.4)(0) + (.1)(20) = 2$
The price of the option at the start of the two periods is \$2.

iv) $K = 114$: $16.2 = 2.2 + 114 - 100$; $K = 97$: $3.5 = 6.5 + 97 - 100$; $K = 90$: $2 = 12 + 90 - 100$

Question 8. [15 points]

i) Since the price changes are proportional and the proportional changes are the same for nodes 1, 2, and 3, we have $q_1 = q_2 = q_3 = \dfrac{0.05 + 0.1}{0.15 + 0.1} = 0.6$.

ii) The value of the option at nodes 4, 5, 6, and 7 are
$f(4) = (105 - 81)_+ = (24)_+ = 24$, $f(5) = (105 - 103.5)_+ = (1.5)_+ = 1.5$,
$f(6) = (105 - 103.5)_+ = (1.5)_+ = 1.5$, $f(7) = (100 - 132.25)_+ = (-32.25)_+ = 0$

The value of the option at node 3 is $f(3) = \dfrac{1}{1.05}\big\{(0.6)(0) + (0.4)(1.5)\big\} = \dfrac{0.6}{1.05}$

The value of the option at node 2 is $f(2) = \dfrac{1}{1.05}\big\{(0.6)(1.5) + (0.4)(24)\big\} = \dfrac{10.5}{1.05}$

The value of the option at node 1 is

$$f(1) = \frac{1}{1.05}\left\{(0.6)(\frac{0.6}{1.05}) + (0.4)(\frac{10.5}{1.05})\right\} = \frac{4.56}{(1.05)^2} = 4.1361$$

The value of the European option at the start of the two periods is \$4.1361.

iii) At node 3, since $(105 - 115)_+ = (-10)_+ = 0$, there is no benefit to exercising the option, so the value of the option is $f(3) = \dfrac{0.6}{1.05}$

At node 2, since $(105-90)_+ = (15)_+ = 15$, the option has more value if it is exercised than if it is held at value $\dfrac{10.5}{1.05} = 10$; thus, $f(2) = 15$

At node 1, the value of the option is

$$f(1) = \frac{1}{1.05}\left\{(0.6)(\frac{0.6}{1.05}) + (0.4)(15)\right\} = \frac{6.66}{(1.05)^2} = 6.0408$$

The value of the American option at the start of the two periods is \$6.0408.

Note that the value of the American option is greater than the value of the European option.

Question 9. [10 points]

$$(1+r)f(j) = q_j f(2j+1) + (1-q_j)f(2j) \geq (s_{2j+1} - K)q_j + (s_{2j} - K)(1-q_j)$$
$$= q_j(s_{2j+1} - s_{2j}) + s_{2j} - K = (1+r)s_j - s_{2j} + s_{2j} - K = (1+r)s_j - K$$

Thus, $f(j) \geq \dfrac{1}{1+r}\big\{(1+r)s_j - K\big\} = s_j - \dfrac{K}{1+r} \geq s_j - K$