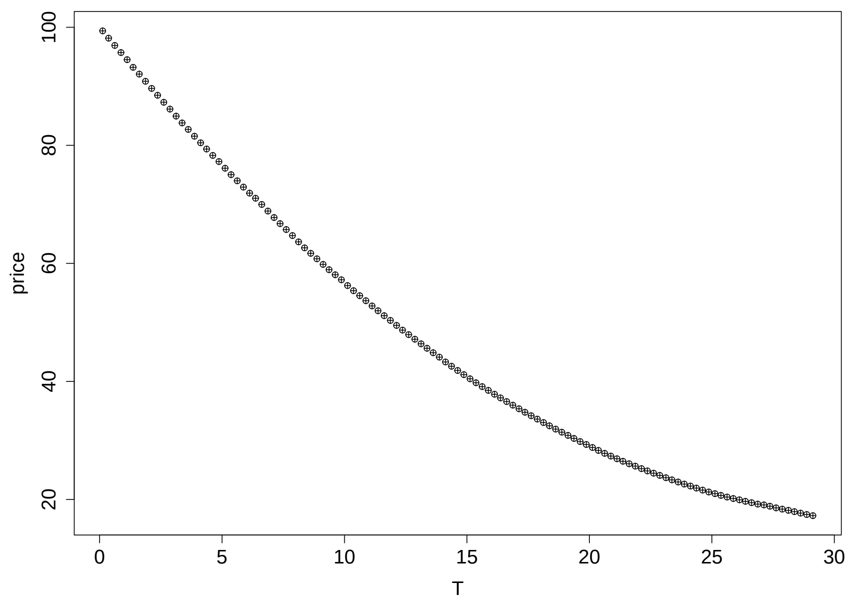
ORIE 4630: Spring Term 2019
Homework #10 Solutions

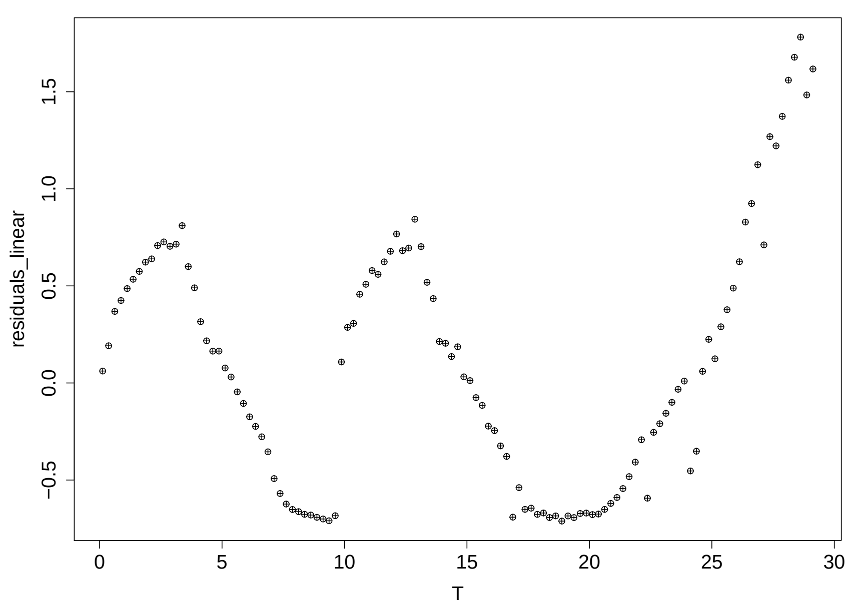
Question 1. [70 points]

i)

```
> n
[1] 117
```

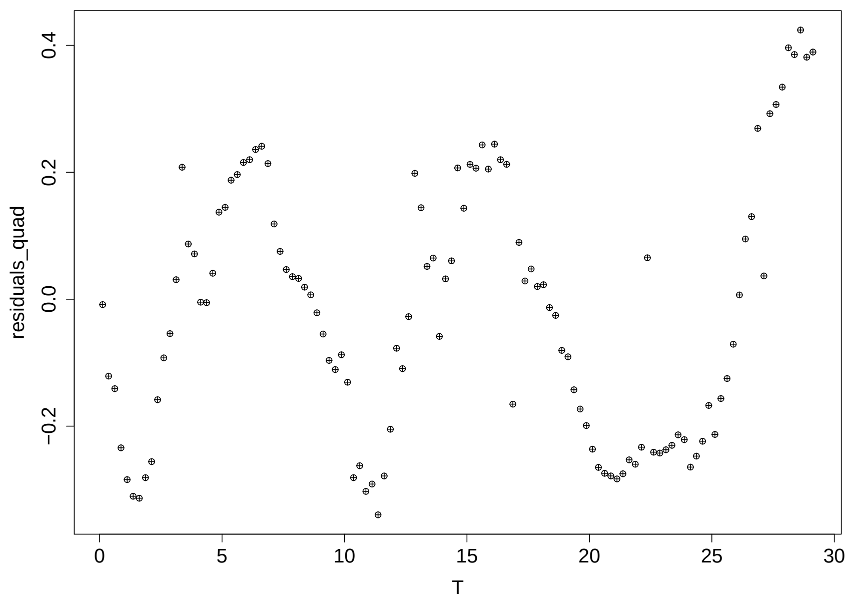


ii)



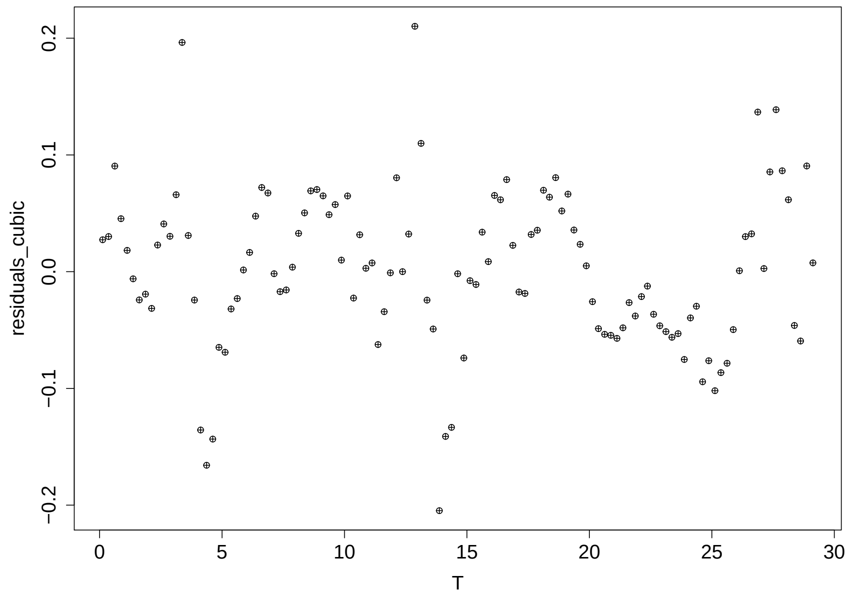
```
> c(min(residuals_linear),max(residuals_linear))
[1] -0.7114548  1.7815853
> coef_linear
      theta0       theta1
0.0531871810 0.0007457895
Residual standard error: 0.6482 on 115 degrees of freedom
```

iii)



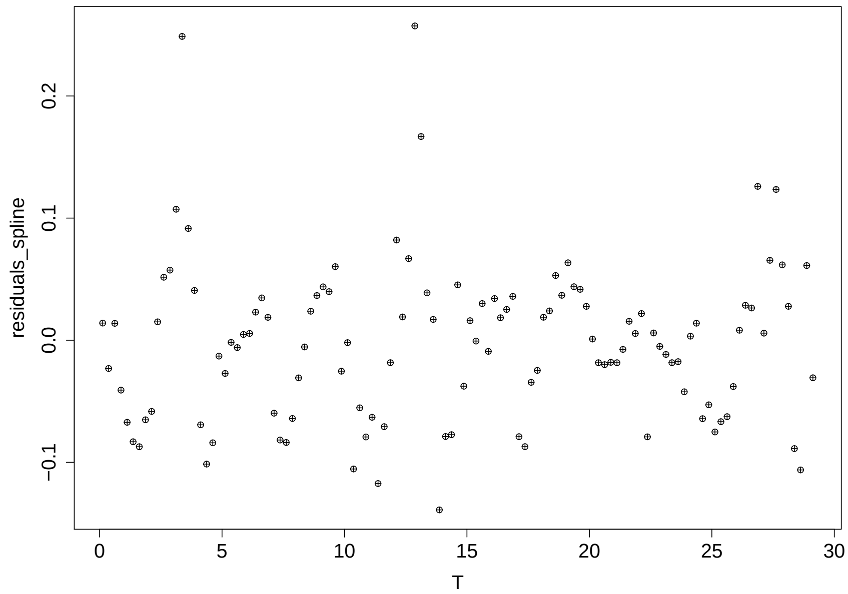

```
> c(min(residuals_quad),max(residuals_quad))
[1] -0.3396536  0.4241376
> coef_quad
       theta0        theta1        theta2
 4.749227e-02  2.403233e-03 -7.542393e-05
Residual standard error: 0.2024 on 114 degrees of freedom
```

iv)



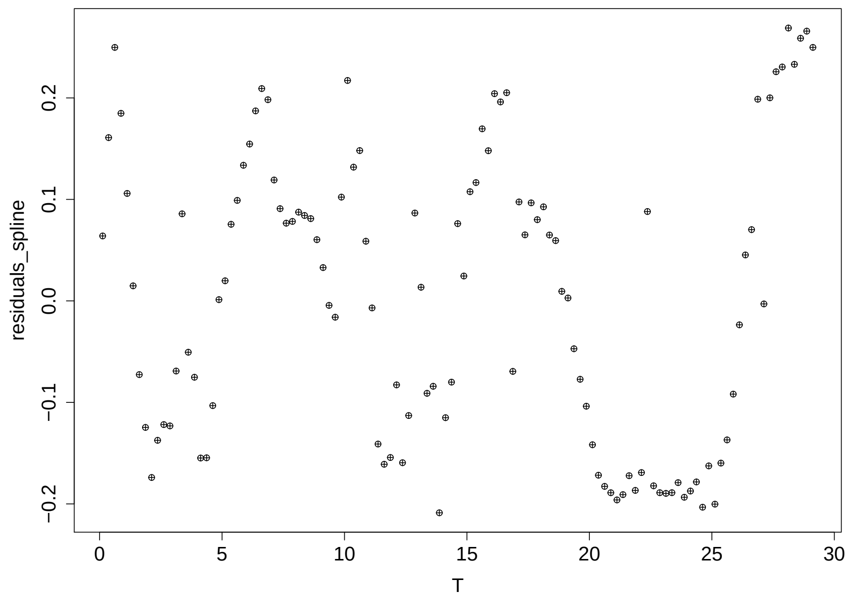

```
> c(min(residuals_cubic),max(residuals_cubic))
[1] -0.2047242  0.2101936
> coef_cubic
       theta0        theta1        theta2        theta3
 5.044677e-02  9.005464e-04  8.260529e-05 -4.327209e-06
Residual standard error: 0.06862 on 113 degrees of freedom
```

v)



```
> c(min(residuals_spline),max(residuals_spline))
[1] -0.1389449  0.2573912
> coef_spline
       theta0        theta1        theta2        theta3
 4.933844e-02  1.671003e-03 -2.981353e-05 -1.931640e-04
Residual standard error: 0.06619 on 113 degrees of freedom
```

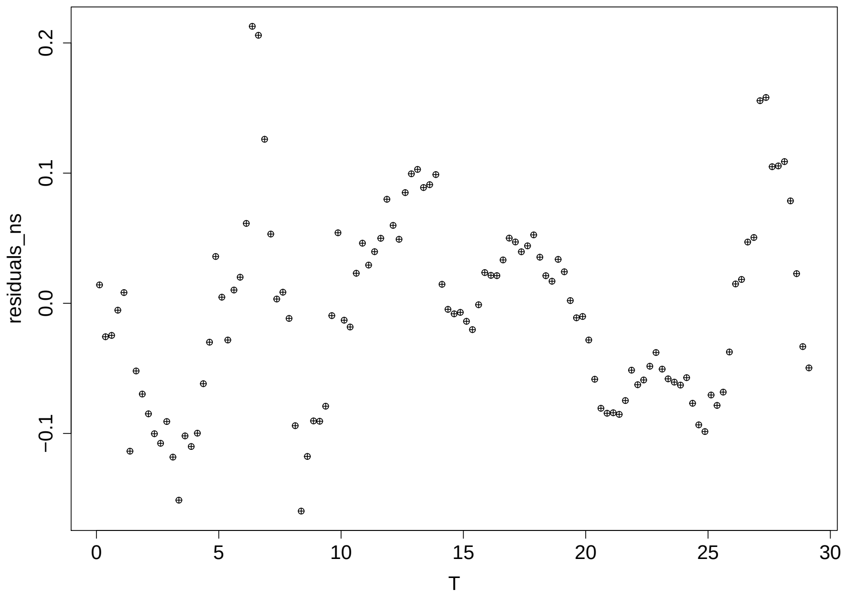
vi)




```
> c(min(residuals_spline),max(residuals_spline))
[1] -0.2087603  0.2688594
> coef_spline
       theta0        theta1        theta2        theta3
 0.0535837953 -0.0024421375  0.0006574093 -0.0007536731
Residual standard error: 0.1451 on 113 degrees of freedom
```

The knot 15.8740 seems preferable compared to the knot 3.6219: the range of the residuals $(-0.139, 0.257)$ for 15.8740 is shorter than is the range $(-0.209, 0.269)$ for 3.6219; the residual standard error 0.06619 for 15.8740 is smaller than is the residual standard error 0.1451 for 3.6219.

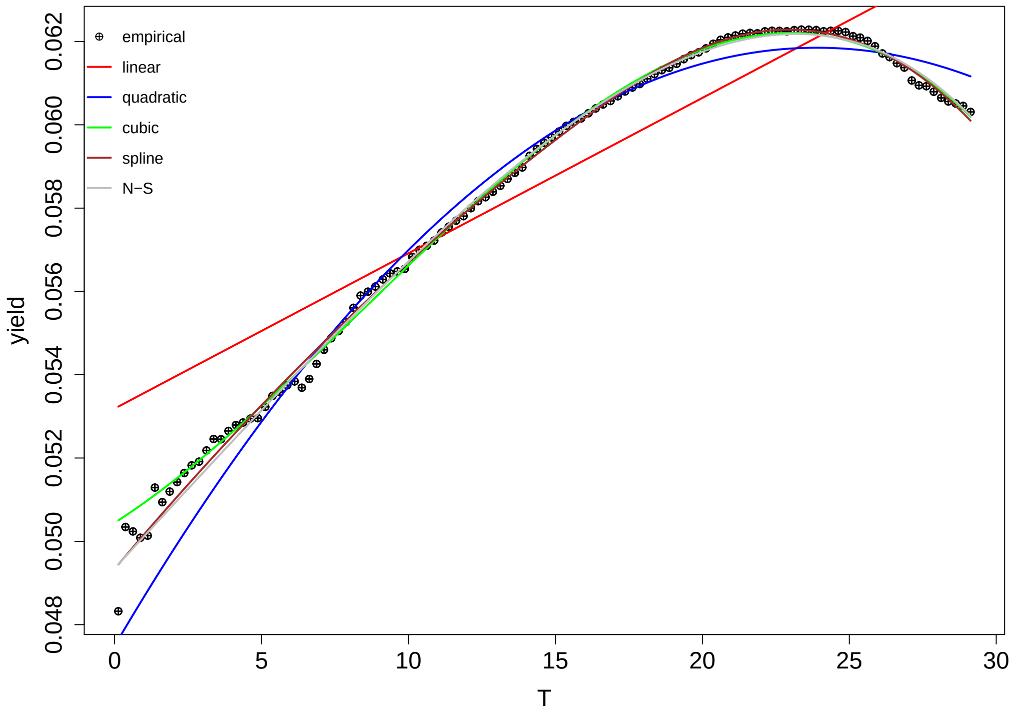
vii)



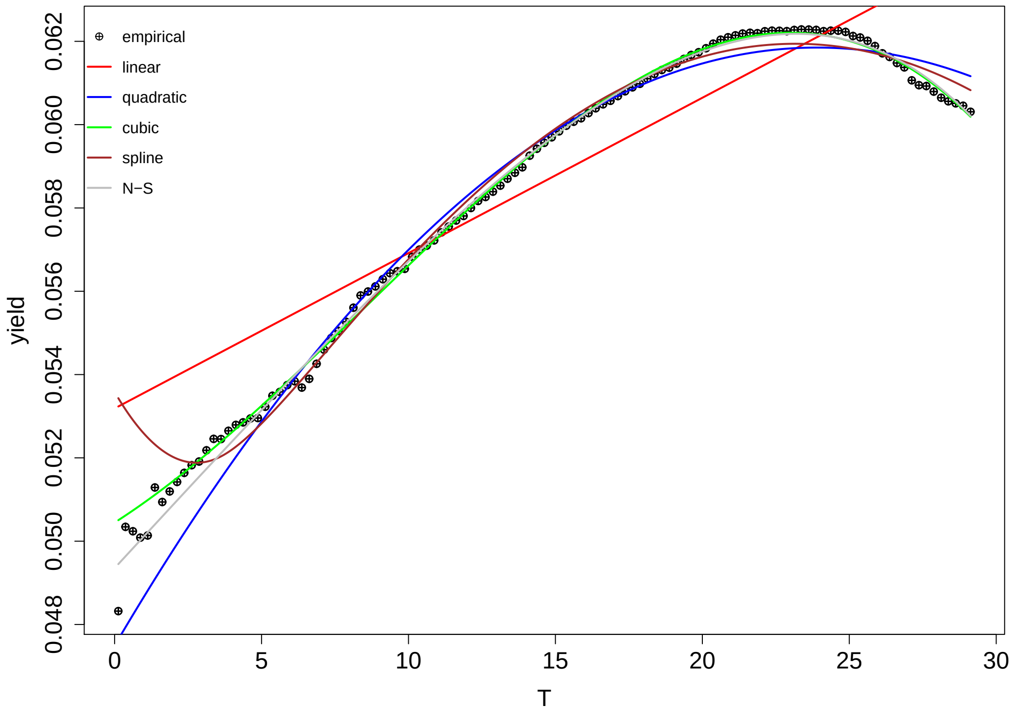

```
> c(min(residuals_ns),max(residuals_ns))
[1] -0.1596089  0.2127549
> coef_ns
[1] -0.002848560  0.052202053 -0.001531349 -0.058635980
> sqrt(sum((price-pricehat)^2)/(n-length(coef_ns)))
[1] 0.07413617
```

viii) The residual standard error is smallest (0.06619) for the quadratic spline model with knot 15.8740; however, the range of the residuals is shortest $(-0.160, 0.213)$ for the Nelson-Siegel model. Typically, the criterion of smallest residual standard error is used to measure goodness-of-fit, so the quadratic spline model with knot 15.8740 would be preferred. A close second choice for providing best fit would be the cubic model, which has residual standard error 0.06862.

ix)
Plot for knot 15.8740:



Plot for knot 3.6219:



x)

Results for maturity 2 years:

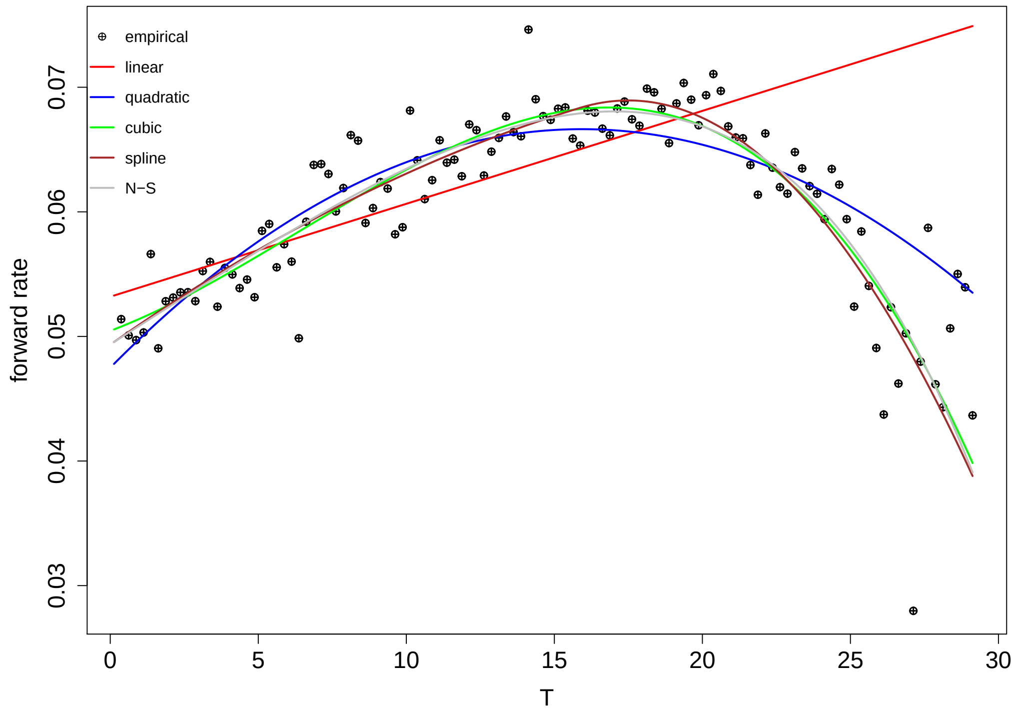
```
> par=1000
> t=2
> yield_linear=coef_linear[1]+coef_linear[2]*t/2
> yield_quad=coef_quad[1]+coef_quad[2]*t/2+coef_quad[3]*t^2/3
> yield_cubic=coef_cubic[1]+coef_cubic[2]*t/2+coef_cubic[3]*t^2/3+coef_cubic[4]*t^3/4
> yield_spline=coef_spline[1]+coef_spline[2]*t/2+coef_spline[3]*t^2/3+
+   (t>knot)*coef_spline[4]*(t-knot)^3/(3*t)
> yield_ns=coef_ns[1]+(coef_ns[2]+
+   coef_ns[3]/coef_ns[4])*(1-exp(-coef_ns[4]*t))/(coef_ns[4]*t)-
+   (coef_ns[3]/coef_ns[4])*exp(-coef_ns[4]*t)
> as.numeric(par*exp(-t*yield_linear))
[1] 897.7479
> as.numeric(par*exp(-t*yield_quad))
[1] 905.2086
> as.numeric(par*exp(-t*yield_cubic))
[1] 902.2193
> as.numeric(par*exp(-t*yield_spline))
[1] 903.0843
> as.numeric(par*exp(-t*yield_ns))
[1] 903.2442
```

Results for maturity 23 years:

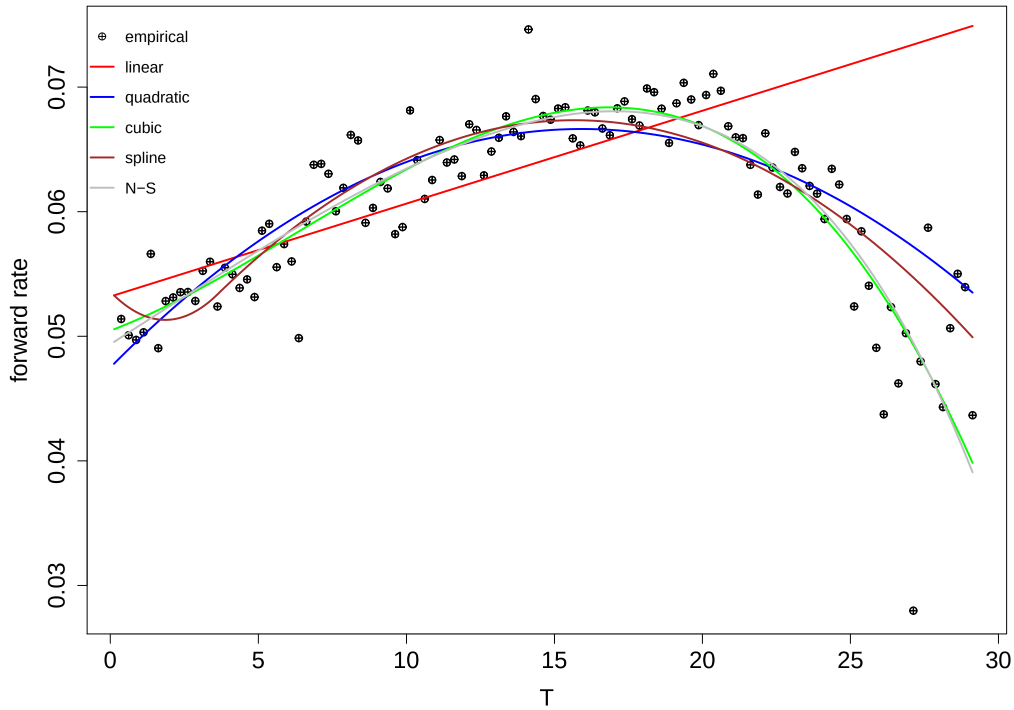
```
> par=1000
> t=23
> yield_linear=coef_linear[1]+coef_linear[2]*t/2
> yield_quad=coef_quad[1]+coef_quad[2]*t/2+coef_quad[3]*t^2/3
> yield_cubic=coef_cubic[1]+coef_cubic[2]*t/2+coef_cubic[3]*t^2/3+coef_cubic[4]*t^3/4
> yield_spline=coef_spline[1]+coef_spline[2]*t/2+coef_spline[3]*t^2/3+
+   (t>knot)*coef_spline[4]*(t-knot)^3/(3*t)
> yield_ns=coef_ns[1]+(coef_ns[2]+
+    coef_ns[3]/coef_ns[4])*(1-exp(-coef_ns[4]*t))/(coef_ns[4]*t)-
+   (coef_ns[3]/coef_ns[4])*exp(-coef_ns[4]*t)
> as.numeric(par*exp(-t*yield_linear))
[1] 241.5771
> as.numeric(par*exp(-t*yield_quad))
[1] 241.211
> as.numeric(par*exp(-t*yield_cubic))
[1] 239.1278
> as.numeric(par*exp(-t*yield_spline))
[1] 238.6991
> as.numeric(par*exp(-t*yield_ns))
[1] 239.2805
```

xi)

Plot for knot 15.8740:



Plot for knot 3.6219:



Question 2. [30 points]

i)

```
> head(DIS_netRet)
[1]  0.034831600  0.029628050  0.003034144 -0.005854880 -0.023262506  0.023917063
```

ii)

```
> VaR
[1] 11.79496
> ES
[1] 19.78558
```

iii)

```
> VaR
[1] 13.56911
> ES
[1] 18.66057
```

iv)

Results for $B = 500$:

```
> VaR_confidence_interval
       5\%       95\%
 9.853321 13.148487
> ES_confidence_interval
      5\%      95\%
16.96837 22.28676
```

Results for $B = 2000$:

```
> VaR_confidence_interval
       5\%       95\%
 9.894105 13.148487
> ES_confidence_interval
      5\%      95\%
16.99361 22.39708
```

Results for $B = 10000$:

```
> VaR_confidence_interval
       5\%       95\%
 9.868088 13.148487
> ES_confidence_interval
      5\%      95\%
17.11775 22.42722
```

Results for $B = 50000$:

```
> VaR_confidence_interval
       5\%       95\%
 9.868088 13.148487
> ES_confidence_interval
      5\%      95\%
17.03186 22.45051
```

Results for $B = 100000$:

```
> VaR_confidence_interval
       5\%       95\%
 9.868088 13.148487
> ES_confidence_interval
      5\%      95\%
17.02848 22.44658
```

v)

Results for $B = 500$:

```
> VaR_confidence_interval
      5\%      95\%
12.44336 14.67280
> ES_confidence_interval
      5\%      95\%
17.29366 19.93580
```

Results for $B = 2000$:

```
> VaR_confidence_interval
      5\%      95\%
12.48913 14.67371
> ES_confidence_interval
      5\%      95\%
17.41407 19.94340
```

Results for $B = 10000$:

```
> VaR_confidence_interval
      5\%      95\%
12.47845 14.64072
> ES_confidence_interval
      5\%      95\%
17.38640 19.92302
```

Results for $B = 50000$:

```
> VaR_confidence_interval
      5\%      95\%
12.49138 14.63847
> ES_confidence_interval
      5\%      95\%
17.38876 19.92758
```

Results for $B = 100000$:

```
> VaR_confidence_interval
      5\%      95\%
12.49108 14.63327
> ES_confidence_interval
      5\%      95\%
17.38641 19.91684
```