ORIE 4630: Spring Term 2019
Test #1 Solutions

February 21, 2019

1.

i) $102 - 100 = 2$

ii) $\dfrac{P_1 - 100}{100} = 0.03 \Rightarrow P_1 = 100(0.03) + 100 = 103$

iii) $1 + R_1 = 0.98 \Rightarrow R_1 = 0.98 - 1 = -0.02$

iv) $P_2 = 100(1 - 0.03)(1.05) = 101.85$

v) $r_2(2) = \log \dfrac{108}{100} = 0.077$

2.

i) $N(0.05, (0.1)^2)$, i.e., the Normal distribution having mean $100(0.0005) = 0.05$ and standard deviation $\sqrt{100}(0.01) = 0.1$

ii) $\mathrm{LogNormal}(0.05, (0.1)^2)$, i.e., the lognormal distribution having mean 0.05 and standard deviation 0.1

iii) $2000 \exp\left(0.05 + \dfrac{(0.1)^2}{2}\right) = 2113.08$

iv) $2000 \exp(0.05) = 2102.54$

v) $\log \dfrac{x}{2000} = \texttt{qnorm}(\texttt{0.05}, \texttt{0.05}, \texttt{0.1}) \Rightarrow x = 2000 \exp\{\texttt{qnorm}(\texttt{0.05}, \texttt{0.05}, \texttt{0.1})\} = 1783.65$

3.

i) Log normal model is contradicted; the mean of the returns appears to be increasing over time, which contradicts the assumption of constant mean.

ii) Log normal model is supported; the points in the normal probability plot fall on a straight line, which is consistent with the distribution of the returns being normal.

iii) Log normal model is supported; the p-value of the Ljung-Box test is large, so there is no evidence against the null hypothesis that the five autocorrelations $\rho(1), \ldots, \rho(5)$ are all 0. In particular, the independence assumption is supported.

iv) Log normal model is contradicted; the p-value of the Jarque-Bera test is very small (essentially 0), which provides strong evidence against the null hypothesis that the distribution of the returns has skewness 0 and kurtosis 3.

v) Log normal model is supported; the mean and the variance of the returns appear to be constant over time.

vi) Log normal model is contradicted; r_t appears to be correlated with r_{t-1}, which contradicts the independence assumption.

vii) Log normal model is supported; there are no significant autocorrelations, so therre is no evidence against the assumption of independence.

viii) Log normal model is supported; the returns appear to have skewness 0 and kurtosis 3, which agrees with the skewness and the kurtosis of the normal distribution.

ix) Log normal model is contradicted; the normal probability plot shows the convex-concave pattern, so the distribution of the returns appears to have much heavier tails than the normal distribution has.

x) Log normal model is contradicted; there is evidence of volatility clustering, so the assumption that the variance of the returns is constant over time appears to be violated.

xi) Log normal model is supported; the p-value of the Shapiro-Wilk test is large, so there is no evidence against the null hypothesis of normality.

xii) Log normal model is contradicted; there appears to be significant autocorrelations at lags 2, 3, and 4, which contradicts the assumption of independence.

xiii) Log normal model is contradicted; when the location-scale Student's t-distribution is fit to the returns, the estimated degrees of freedom is 3.055, which indicates that the distribution of the returns has much heavier tails than the normal distribution has.

xiv) Log normal model is contradicted; the normal probability plot has a concave appearance which indicates that the distribution of the returns is skewed to the right.

4.

i) 0.010354

ii) $0.010354 \pm 1.960(0.00025051) = (0.009863, 0.010845)$

iii) $0.010354 \times \sqrt{\dfrac{2.9512}{2.9512-2}} = 0.018238$

iv) 0.00037455

v) No! $\dfrac{0.00037455}{0.00023426} = 1.5989 < 1.645$

5.

i) $r_t - \mu_t = \phi(r_{t-1} - \mu) + \epsilon_t$, or, equivalently, $r_t = (1-\phi)\mu + \phi r_{t-1} + \epsilon_t$; $\epsilon_t \sim \text{WhiteNoise}(0, \sigma_\epsilon^2)$.

ii) $(-0.0348)^2 = 0.001211$

iii) The ACF plot based on the residuals shows estimates $\hat{\rho}(h)$, where $\rho(h) = corr(\epsilon_t, \epsilon_{t+h})$; since $\hat{\rho}(2)$ exceeds the standard error bars, the ACF plot indicates that $\rho(2)$ is non-zero. Consequently, the ACF plot suggest that the ϵ_ts are not white noise, so the model is refuted.

iv) $H_0 : \rho(1) = \cdots = \rho(10) = 0$ and H_A : not all $\rho(1), \ldots, \rho(10)$ are 0, where $\rho(h) = corr(\epsilon_t, \epsilon_{t+h})$.

v) The p-value 0.3569 is large, so there is no evidence against the null hypothesis; thus, there is no evidence that the ϵ_ts are not white noise, and the model is supported.

vi) $0.0003 + (-0.0348)(-0.02182) = 0.00106$

vii) For the first model, AIC $= -15022.41$, while for the second model AIC $= -15026.47$; since AIC is smaller for the second model, the second model would be preferred.