

**ORIE 4630: Spring Term 2019**  
**Homework #6**  
**Due: Tuesday, March 19, 2019**

Students are required to work independently on homework. You should not give or receive help from other students. You should also not receive help from students or former students who took this course in previous years and who may have solutions to similar problems. The solutions you submit should be your own work and not copied from elsewhere.

Homework is due at the end of lecture (12:55pm) on the due date. You will usually have one week to do the assignments. Please don't wait until the homework is nearly due to start. Late homework is not accepted. Also, homework is not accepted by email. You can submit your assignment in lecture or in the drop box in Rhodes Hall.

Please print your name on the front of your homework so that it is legible.

Include your R code, output, graphs, and other work with your homework. This will allow the grader to find any errors you make and to give partial credit.

This assignment uses net returns of certain stocks and indexes from Jan 4, 2006 to Aug 18, 2017. The log gross returns for the stocks and indexes are contained in a csv file named `returns.csv`. This assignment also uses the daily rates of the 3-month treasury bill for the same time period, which are contained in a csv file named `3m_treasury_bill_rate.csv`. You should download these files from the course Blackboard site and put them in your R or Rstudio working directory. The file `returns.csv` has 35 columns: the first column shows the date (Date); the next 30 columns are for the stocks that are the components of the Dow Jones Industrial average (DOW); the final four columns are for the Dow Jones Industrial average index (DOW), the NASDAQ composite index (NASD), the NASDAQ 100 index (NASD100), and the S&P 500 index (SP500). There are 2927 days of log gross returns. The log gross returns are calculated from the adjusted closing prices downloaded from Yahoo. The file `3m_treasury_bill_rate.csv` has 2 columns: the first column shows the date (Date); and the second column shows the annualized rates expressed as percentages. The rates are downloaded from the FRED Economic Data website of the Federal Reserve Bank of St. Louis.

Start R or Rstudio and run the following lines:

```
1 Returns=read.csv("returns.csv")
2 names>Returns)
3 Ret>Returns[ , 2:35]
4 class(Ret)
5 netRet=exp(Ret)-1
6 head(netRet)
7 FRED_rates=read.csv("3m_treasury_bill_rate.csv")
8 names(FRED_rates)
9 rates=FRED_rates[ , 2]
10 mu_f=rates/(100*252)
11 head(mu_f)
12 exRet=netRet-mu_f
13 head(exRet)
```

Line 1 reads the log gross returns into a data frame named `Returns`; line 2 outputs the names of the stocks and indexes to which the log gross returns pertain. If `returns.csv` is not in your working directory, then you need to give a complete path to that file in line 1. In line 3, the log gross returns for the stocks and indexes are stored without the dates in the data frame `Ret`, and in line 5, the net returns are computed and stored in the object `netRet`. Recall that if the net return of an asset is  $R$ , then the log gross return is  $r = \log(1 + R)$ ; thus,  $r = e^R - 1$ . Furthermore, recall that if  $R$  is small, then  $r \sim R$ . Note that the net returns are very similar to the log gross returns. Line 7 reads the annualized three-month treasury bill rates expressed as percentages into a data frame named `FRED_rates`. The rates are stored without the dates in the data frame `rates` in line 9. In line 10, the annualized rates are converted to daily rates and stored as decimals in the object `mu.f`. The rates in `mu.f` are treated as the risk-free rates; the excess returns of the stocks and indexes are computed and stored in the object `exRet` in line 12.

Suppose that interest focuses on the stocks for the eight particular companies: Apple (AAPL), Caterpillar (CAT), Chevron (CVX), General Electric (GE), J. P. Morgan (JPM), McDonald's (MCD), Nike (NKE), and Pfizer (PFE). Furthermore, suppose that the S&P 500 index (SP500) is used as an approximation to the market portfolio. To create data frames containing the net returns and excess returns for these stocks and for the market portfolio, run the following lines:

```
14 stocknames=c("AAPL", "CAT", "CVX", "GE", "JPM", "MCD", "NKE", "PFE")
15 market=c("SP500")
16 stocks_netRet=netRet[ , stocknames]
17 market_netRet=netRet[ , market]
18 stocks_exRet=exRet[ , stocknames]
19 market_exRet=exRet[ , market]
```

In line 14, the eight stock symbols are stored in the object `stocknames`, and in line 15, the symbol of the S&P 500 index is stored in the object `market`. The net returns for the eight stocks are stored in the data frame `stocks_netRet` in line 16, and the net returns for the market portfolio are stored in the data frame `market_netRet` in line 17. In line 18, the excess returns for the stocks are stored in the data frame `stocks_exRet`, and in line 19, the excess returns for the market are stored in the data frame `market_exRet`.

Recall that there are two ways to estimate the  $\beta$ s of the stocks. Consider, in particular, the stock of Apple (AAPL). The first way to estimate  $\beta_{\text{AAPL}}$  is to regress the net returns of Apple (AAPL) against the net returns of the market, where, for the regression, both intercept and a slope are included; thus,

$$\hat{\beta}_{\text{AAPL}} = \frac{\sum_{i=1}^n (R_{\text{AAPL},t} - \bar{R}_{\text{AAPL}})(R_{\text{SP500},t} - \bar{R}_{\text{SP500}})}{\sum_{i=1}^n (R_{\text{SP500},t} - \bar{R}_{\text{SP500}})^2}.$$

To perform the linear regression for all eight stocks, run the following lines:

```
20 fit_netRet=lm(as.matrix(stocks_netRet) ~ market_netRet)
21 round(fit_netRet$coefficients, 5)
```

The eight regressions are performed and the results are stored in the object `fit_netRet` in line 20; note that in line 20, for the regressions to be performed, the class of the object `fit_netRet` must be converted from `data frame` to `matrix` by using the function `as.matrix()`. The coefficients from the eight regressions, i.e., the eight intercepts and slopes, are output in line 21.

To describe the second way to estimate the  $\beta$ s, consider again the stock of Apple (AAPL). The alternative estimate of  $\beta_{\text{AAPL}}$  is obtained by regressing the excess returns of Apple (AAPL) against the excess returns of the market, where, for the regression, the intercept is excluded; thus,

$$\hat{\beta}_{\text{AAPL}} = \frac{\sum_{i=1}^n R_{\text{AAPL},t}^* R_{\text{SP500},t}^*}{\sum_{i=1}^n (R_{\text{SP500},t}^*)^2},$$

where, at time  $t$ , the excess return for Apple (AAPL) is  $R_{\text{AAPL},t}^* = R_{\text{AAPL},t} - \mu_{f,t}$  and the excess return of the market is  $R_{\text{M},t}^* = R_{\text{M},t} - \mu_{f,t}$ .

To perform the linear regression for all eight stocks, run the following lines:

```
22 fit_exRet=lm(as.matrix(stocks_exRet) ~ market_exRet -1)
23 round(fit_exRet$coefficients, 5)
```

The eight regressions are performed and the results are stored in the object `fit_exRet` in line 22. Note that in line 22, for the regressions to be performed, the class of the object `fit_exRet` must be converted from `data frame` to `matrix` by using the function `as.matrix()`; furthermore, the command `-1` is included in line 22 so that the intercept is excluded. The coefficients from the eight regressions, i.e., the eight slopes, are output in line 23.

One way to check whether a stock is mispriced according to the CAPM is to fit a more elaborate regression model. Consider again the stock of Apple (AAPL), and consider the regression of the excess returns of Apple (AAPL) against the excess returns of the market, where the regression now includes an intercept  $\alpha_{\text{AAPL}}$  and a slope  $\beta_{\text{AAPL}}$ . According to the CAPM, the intercept  $\alpha_{\text{AAPL}}$  should be 0, so one way to see if there is evidence that the excess returns of Apple (AAPL) fail to follow the CAPM is to check whether there is evidence against the null hypothesis  $H_0 : \alpha_{\text{AAPL}} = 0$  in favor of the alternative hypothesis  $H_A : \alpha_{\text{AAPL}} \neq 0$ .

To perform the linear regression for all eight stocks, run the following lines:

```
24 fit_exRet=lm(as.matrix(stocks_exRet) ~ market_exRet)
25 summary(fit_exRet)
```

Note that the command `-1` which was included in line 22 is omitted from line 24, so the function `lm()` in line 24 performs the eight regressions with an intercept and slope being fit for each stock. The `summary()` function in line 25 causes the results of the eight regressions to be output in detail. In particular, the output from `summary()` gives, for the regression for each stock  $j$ , the  $p$ -value for testing the null hypothesis  $H_0 : \alpha_j = 0$  against the alternative hypothesis  $H_A : \alpha_j \neq 0$  as well as the value of  $R_j^2$ , the coefficient of determination. Recall that, in simple linear regression, the coefficient of determination is the proportion of variability in the response that is explained by the predictor variable.

Recall that the security characteristic line for stock  $j$  is

$$R_{j,t} = \mu_{f,t} + \beta_j(R_{\text{M},t} - \mu_{f,t}) + \epsilon_{j,t},$$

i.e.,  $R_{j,t}^* = \beta_j R_{\text{M},t}^* + \epsilon_{j,t}$ , where  $\epsilon_{j,t}$  has mean 0 and variance  $\sigma_{\epsilon,j}^2$ . It is also assumed that  $\text{Corr}(\epsilon_{j,t}, \epsilon_{j',t}) = 0$  for any two stocks  $j$  and  $j'$ . Taking expectations in the security characteristic line for stock  $j$  implies that

$$\mu_j - \mu_f = \beta_j(\mu_{\text{M}} - \mu_f),$$

i.e.,  $E(R_{j,t}^*) = \beta_j E(R_{M,t}^*)$ ; the expected excess return for stock  $j$  is  $\beta_j$  times the expected excess return of the market. To see whether this consequence of the security characteristic line actually holds, it is useful to compare the sample mean  $\overline{R_j^*}$  with  $\hat{\beta}_j \overline{R_M^*}$  for each stock  $j$ .

To make this comparison, run the following lines:

```
26 fit_exRet=lm(as.matrix(stocks_exRet) ~ market_exRet -1)
27 beta=fit_exRet$coefficients
28 round(apply(stocks_exRet, 2, mean), 6)
29 round(beta*mean(market_exRet), 6)
30 round(apply(stocks_exRet, 2, mean)-beta*mean(market_exRet), 6)
```

In lines 26 and 27, the  $\beta$ s are estimated by using the second method already employed in lines 22 and 23, and the estimates are stored in the object **beta**. Line 28 computes and outputs the  $\overline{R_j^*}$ s; line 29 computes and outputs  $\hat{\beta}_j \overline{R_M^*}$  for each stock  $j$ . The difference between the two estimates of the expected excess returns, i.e.,  $\overline{R_j^*} - \hat{\beta}_j \overline{R_M^*}$ , is computed for each stock  $j$  in line 30. You can judge whether the estimate from the security characteristic line, namely  $\hat{\beta}_j \overline{R_M^*}$ , is close to the estimate  $\overline{R_j^*}$  for each stock  $j$ .

Suppose that the excess return on a given day is projected to be  $R_M^* = 0.003$ . Then the forecast excess return for stock  $j$  is  $R_j^* = \beta_j(0.003)$ , which is estimated by  $\hat{R}_j^* = \hat{\beta}_j(0.003)$ . To make this prediction for each stock  $j$ , run the following line

```
31 round(beta*0.003, 6)
```

It is of interest to check the assumption that  $\rho_{j,j'} = 0$  for any two different stocks  $j$  and  $j'$ , where  $\rho_{j,j'} = \text{Corr}(\epsilon_{j,t}, \epsilon_{j',t})$ . Recall that, for stock  $j$ , the model disturbance term  $\epsilon_{j,t}$  is estimated by the residual  $\hat{\epsilon}_{j,t} = R_{j,t}^* - \hat{\beta}_j R_{M,t}^*$ . The assumption can be checked by using the sample correlation between the  $\hat{\epsilon}_{j,t}$ s and the  $\hat{\epsilon}_{j',t}$ s to estimate  $\text{Corr}(\epsilon_{j,t}, \epsilon_{j',t})$ . If the assumption is valid, then the off-diagonal elements of the sample correlation matrix should be close to 0. In addition, a formal test can be used to test the null hypothesis  $H_0 : \rho_{j,j'} = 0$  against the alternative hypothesis  $H_A : \rho_{j,j'} \neq 0$ . To output the sample correlation matrix and perform the test for Apple (AAPL) and General Electric (GE), run the following lines:

```
32 res=fit_exRet$residuals
33 head(res)
34 round(cor(res), 5)
35 cor.test(res[,1], res[,4])
```

In line 32, the residuals from the regression conducted in line 26 are computed and stored in the matrix **res**. The first six residuals for each stock are shown in line 33. Line 34 outputs the sample correlation matrix of the residuals. The test of the null hypothesis  $H_0 : \rho_{1,4} = 0$  against the alternative hypothesis  $H_0 : \rho_{1,4} \neq 0$  is output in line 35. Note that, from line 14, stock 1 is Apple (AAPL) and stock 4 is General Electric (GE).

The security characteristic line also implies: i)  $\sigma_j^2 = \beta_j^2 \sigma_M^2 + \sigma_{\epsilon,j}^2$ , where  $\sigma_j^2 = \text{Var}(R_{j,t}^*)$  and  $\sigma_M^2 = \text{Var}(R_{M,t}^*)$ ; and ii)  $\sigma_{j,j'} = \beta_j \beta_{j'} \sigma_M^2$ , where  $\sigma_{j,j'} = \text{Cov}(R_{j,t}^*, R_{j',t}^*)$ . To see whether consequences i) and ii) of the security characteristic line actually hold, it is useful to compare the two estimates of the covariance matrix of the excess returns of the stocks: the sample covariance matrix of the excess returns of the stocks; and the estimate of the covariance matrix derived from implications i) and ii) of the security characteristic line, i.e., the estimate that has  $\hat{\sigma}_j^2 = \hat{\beta}_j^2 \hat{\sigma}_M^2 + \hat{\sigma}_{\epsilon,j}^2$  and  $\hat{\sigma}_{j,j'} = \hat{\beta}_j \hat{\beta}_{j'} \hat{\sigma}_M^2$ . To output and compare these two estimates by taking their ratio, run the following lines:

```

36 cov_est1=cov(stocks_exRet)
37 round(cov_est1, 6)
38 cov_est2=t(beta) %*% beta * var(market_exRet) + diag(diag(cov(res)))
39 round(cov_est2, 6)
40 round(cov_est1/cov_est2, 4)

```

The sample covariance matrix is computed and stored in the object `cov_est1` in line 36, and it is output in line 37. The estimate of the covariance matrix based on the security characteristic line is computed and stored in the object `cov_est2` in line 38, and it is output in line 39. Note that, for line 38, the object `beta` is a row vector; the function `t()` takes the transpose. The function `diag()`, when operated on a matrix, creates a vector of the diagonal elements of the matrix; thus, `diag(cov(res))` is a row vector consisting of the sample variances of the residuals. The function `diag()`, when operated on a vector, creates a diagonal matrix whose diagonal elements are the elements of the vector; thus, `diag(diag(cov(res)))` is a diagonal matrix whose diagonal elements are the sample variances of the residuals. The element-by-element ratios of the two estimates of the covariance matrix are output in line 40.

Suppose there are  $N$  stocks that follow the security characteristic line, and consider a portfolio constructed from the  $N$  stocks that puts weight  $w_j$  on stock  $j$  ( $j = 1, \dots, N$ ). Then the excess return of the portfolio  $R_{P,t}^* = R_{P,t} - \mu_{f,t}$  satisfies the relationship

$$R_{P,t}^* = \beta_P R_{M,t}^* + \epsilon_{P,t}, \quad \beta_P = \sum_{j=1}^N w_j \beta_j, \quad \epsilon_{P,t} = \sum_{j=1}^N w_j \epsilon_{j,t}.$$

By the assumptions of the security characteristic line,  $\epsilon_{P,t}$  has mean 0 and variance  $\sigma_{\epsilon,P}^2 = \sum_{j=1}^N w_j^2 \sigma_{\epsilon,j}^2$ . Recall that, since  $\sigma_j^2 = \beta_j^2 \sigma_M^2 + \sigma_{\epsilon,j}^2$  for stock  $j$ , the market component of risk is  $\beta_j^2 \sigma_M^2$ , and the unique component of risk is  $\sigma_{\epsilon,j}^2$ . Similarly,  $\sigma_P^2 = \beta_P^2 \sigma_M^2 + \sigma_{\epsilon,P}^2$ ; for the portfolio,  $\beta_P^2 \sigma_M^2$  is the market component of risk and  $\sigma_{\epsilon,P}^2$  is the unique component of risk. The market component of risk for the portfolio can be estimated by  $\hat{\beta}_P^2 \hat{\sigma}_M^2$ , where  $\hat{\beta}_P = \sum_{j=1}^N w_j \hat{\beta}_j$ , and the unique component of risk can be estimated by  $\hat{\sigma}_{\epsilon,P}^2 = \sum_{j=1}^N w_j^2 \hat{\sigma}_{\epsilon,j}^2$ . In the case of the eight stocks considered, the components of risk for the portfolio that has equal weights, i.e.,  $w_j = 1/8$  ( $j = 1, \dots, 8$ ), can be estimated by running the following lines:

```

41 w=as.matrix(rep(1, 8)/8)
42 beta_P=beta[,1:8] %*% w
43 beta_P
44 systematic_component_P=beta_P^2 * var(market_exRet)
45 systematic_component_P
46 unique_component_P=t(w) %*% diag(diag(cov(res))) %*% w
47 unique_component_P
48 diag(cov(res))
49 sigma_P=sqrt(systematic_component_P+unique_component_P)
50 sigma_P
51 sqrt(diag(cov_est2))

```

In line 41, the weights are stored in the column vector `w`. The estimate  $\hat{\beta}_P$  is computed and stored in line 42, and it is output in line 43. The estimate of the systematic component of risk for the portfolio is computed in line 44 and output in line 45; the estimate of the unique component of risk for the portfolio is computed in line 46 and output in line 47. The estimates of the unique component of risk for the stocks is output in line 48. The estimate

of the overall risk of the portfolio is computed and stored in line 49, and it is output in line 50. The estimates of risk for the stocks are output in line 51.

Consider again the more elaborate version of the security characteristic line for  $N$  stocks that allows for  $\alpha$ :  $R_{j,t}^* = \alpha_j + \beta_j R_{M,t}^* + \epsilon_{j,t}$ , where  $\epsilon_{j,t}$  has mean 0 and variance  $\sigma_{\epsilon,j}^2$ , and  $\text{Corr}(\epsilon_{j,t}, \epsilon_{j',t}) = 0$  for  $j \neq j'$  ( $j, j' = 1, \dots, N$ ). Furthermore, consider a portfolio constructed from the  $N$  stocks that puts weight  $w_j$  on stock  $j$  ( $j = 1, \dots, N$ ). Then the excess return of the portfolio  $R_{P,t}^* = R_{P,t} - \mu_{f,t}$  satisfies the relationship

$$R_{P,t}^* = \alpha_P + \beta_P R_{M,t}^* + \epsilon_{P,t}, \quad \alpha_P = \sum_{j=1}^N w_j \alpha_j, \quad \beta_P = \sum_{j=1}^N w_j \beta_j, \quad \epsilon_{P,t} = \sum_{j=1}^N w_j \epsilon_{j,t}.$$

By the assumptions of this version of the security characteristic line,  $\epsilon_{P,t}$  has mean 0 and variance  $\sigma_{\epsilon,P}^2 = \sum_{j=1}^N w_j^2 \sigma_{\epsilon,j}^2$ . As before,  $\sigma_P^2 = \beta_P^2 \sigma_M^2 + \sigma_{\epsilon,P}^2$ ; for the portfolio,  $\beta_P^2 \sigma_M^2$  is the market component of risk, and  $\sigma_{\epsilon,P}^2$  is the unique component of risk.

Suppose that the weights  $w_1, \dots, w_N$  are chosen so that  $\beta_P = 0$ ; then the portfolio is said to be market neutral. Typically, the  $\beta_j$ s are positive, so some of the weights must be negative to achieve  $\beta_P = 0$ ; thus, shorting must be allowed. An interesting exercise might be to find the portfolio of the  $N$  stocks that maximizes  $\alpha_P$  with  $\beta_P = 0$ . This calculation is a linear programming problem; the linear programming problem can be solved by using the `solveLP()` function in the `linprog` package of R. The function `solveLP()` finds the  $M \times 1$  column vector  $x = (x_1, \dots, x_M)^T$  that maximizes the objective function  $c^T x$  subject to the constraints  $Ax \leq b$  and  $x \geq 0$ ; here,  $c$  is a  $M \times 1$  column vector of constants,  $A$  is a  $k \times M$  matrix of constants, and  $b$  is a  $k \times 1$  column vector of constants. The constraint  $x \geq 0$  means that all of the entries of  $x$  are non-negative, and in the constraint  $Ax \leq b$ , some of the rows can be stipulated to be equalities. Since the vector of weights being sought to form the portfolio must have some negative entries, the identification  $x = w$  cannot be used directly, because of the constraint  $x \geq 0$ . Moreover, bounds are usually imposed on the weights: suppose the bounds are  $w_j \leq b_1$  and  $w_j \geq -b_2$  ( $j = 1, \dots, N$ ). For instance, we might have  $b_1 = b_2 = 1$ . To apply the function `solveLP()`, let  $M = 2N$ , so  $x$  is a  $2N \times 1$  column vector, and take  $w_j = x_j - x_{j+N}$ . The bounds  $w_j \leq b_1$  and  $w_j \geq -b_2$  ( $j = 1, \dots, N$ ) on the weights translate to the inequality constraints  $x_j \leq b_1$  and  $x_{j+N} \leq b_2$  ( $j = 1, \dots, N$ ). The equality constraint  $\sum_{j=1}^N w_j = 1$  translates to  $\sum_{j=1}^N (x_j - x_{j+N}) = 1$ , while the equality constraint  $\sum_{j=1}^N w_j \beta_j = 0$  translates to  $\sum_{j=1}^N (x_j \beta_j - x_{j+N} \beta_j) = 0$ .

To solve the linear programming problem with  $b_1 = b_2 = 1$  for the eight stocks considered, run the following lines:

```
52 install.packages("linprog")
53 library(linprog)
54 fit_exRet=lm(as.matrix(stocks_exRet) ~ market_exRet)
55 coefficients=fit_exRet$coefficients
56 alpha=coefficients[1, ]
57 beta=coefficients[2, ]
58 b1=1; b2=1
59 N=length(alpha)
60 A1=diag(1, nrow=2*N)
61 A2=c(rep(1,N), rep(-1,N))
62 A3=c(beta,-beta)
63 Amat=rbind(A1, A2, A3)
64 bvec=c(rep(b1,N), rep(b2,N),1,0)
```

```

65 cvec=c(alpha, -alpha)
66 const.dir=c(rep("<=", 2*N), "=", "=")
67 result=solveLP(cvec=cvec, bvec=bvec, Amat=Amat, lpSolve=TRUE,
68               const.dir=const.dir, maximum=TRUE, verbose=4)
69 solution=result$solution
70 solution
71 w=solution[1:N]-solution[-(1:N)]
72 w
73 alpha_P=w %*% alpha
74 alpha_P
75 alpha
76 beta_P=w %*% beta
77 beta_P
78 portfolio_risk=sqrt(t(w) %*% diag(diag(cov(res))) %*% w)
79 portfolio_risk
80 stocks_risk=sqrt(diag(beta %*% t(beta) * var(market_exRet) + diag(diag(cov(res)))))
81 rbind(stocknames,round(stocks_risk,5))

```

The package `linprog` is installed in line 52 and loaded in line 53. For each stock, the linear regression, with intercept and slope, of the excess returns of the stock on the excess returns of the market is performed in line 54, and the results are stored in the object `fit_exRet`. In line 55, the coefficients of the regressions are stored in the object `coefficients`. The estimates of the  $\alpha$ s are stored in the object `alpha` in line 56, and the estimates of the  $\beta$ s are stored in the object `beta` in line 57. The numbers  $b_1$  and  $b_2$  that determine the bounds on the weights are specified in line 58. The number of stocks is stored in the object `N` in line 59. The rows of the matrix  $A$  that correspond to inequality constraints are specified and stored in the object `A1` in line 60; the row that corresponds to the equality constraint that the weights sum to 1 is specified and stored in the object `A2` in line 61; and the row that corresponds to the equality constraint that  $\beta_P = 0$  is specified and stored in the object `A3` in line 62. Here,  $k = 2N + 2$ . The  $A$  matrix for the linear programming problem is specified and stored in the object `Amat` in line 63. The  $b$  vector is specified in line 64, and the  $c$  vector is specified in line 65. In line 66, the object `const.dir` specifies that the first  $2N$  rows of `Amat` correspond to inequality constraints and that the last two rows of `Amat` correspond to equality constraints. The linear programming problem is solved by using the function `solveLP()` in lines 67 and 68. The solution to the linear programming problem is stored in line 69, and they are output in line 70. The weights are calculated and stored in line 71, and they are output in line 72. The estimate of  $\alpha_P$  for the portfolio is computed and stored in line 73, and it is output in line 74. For comparison, the estimates of the  $\alpha$ s for the stocks are output in line 75. The estimate of  $\beta_P$  for the portfolio is calculated in line 76, and it is output in line 77. The estimate of the risk for the portfolio,  $\hat{\sigma}_P$ , from the security characteristic line is computed in line 78, and it is output in line 79. For comparison, the estimates of the risks of the stocks from the security characteristic line are computed in line 80, and they are output in line 81 for comparison.

## Questions:

1. [5 points] Adapt and run lines 1 to 19 for the stocks Apple Inc (AAPL), Goldman Sachs Group Inc (GS), McDonald's Corp (MCD), Nike Inc (NKE), Walmart Inc (WMT), and Exxon Mobile Corp (XOM).

i) Report the net return for Goldman Sachs Group Inc (GS) on August 24, 2011.

- ii) Report the risk-free rate, expressed as a decimal daily rate, on August 24, 2011.
- iii) Report the excess return for Goldman Sachs Group Inc (GS) on August 24, 2011.

**2. [10 points]** Adapt and run lines 20 and 21 for the six stocks listed in Question 1. Submit the output.

- i) Report the estimate  $\hat{\beta}_{\text{XOM}}$  for Exxon Mobile Corp (XOM).
- ii) For which of the six stocks is the estimate of  $\beta$  largest? What is the value of  $\hat{\beta}$  for that stock?
- iii) For which of the six stocks is the estimate of  $\beta$  smallest? What is the value of  $\hat{\beta}$  for that stock?
- iv) What is the average of the six estimates of  $\beta$ ?
- v) Which of the six stocks is “most aggressive”? Justify your answer briefly.
- vi) Which of the six stocks is “least aggressive”? Justify your answer briefly.

**3. [5 points]** Adapt and run lines 22 and 23 for the six stocks listed in Question 1. Submit the output.

- i) Report the estimate  $\hat{\beta}_{\text{XOM}}$  for Exxon Mobile Corp (XOM). What is the numerical difference between this estimate and the previous one from Question 2 i)?
- ii) What is the average of the eight estimates of  $\beta$ ?

**4. [15 points]** Adapt and run lines 24 and 25 for the six stocks listed in Question 1. Submit the output for the regression for Exxon Mobile Corp (XOM).

- i) Report the estimate  $\hat{\alpha}_{\text{XOM}}$ .
- ii) What proportion of the variability in the excess returns for Exxon Mobile Corp (XOM) is explained by the excess returns of the market?
- iii) Report the  $p$ -value for testing the null hypothesis  $H_0 : \alpha_{\text{XOM}} = 0$  against the alternative hypothesis  $H_A : \alpha_{\text{XOM}} \neq 0$ .
- iv) Is there evidence against the null hypothesis in part iii)? Use the 5% level of significance. Is there evidence against the suitability of the CAPM for Exxon Mobile Corp (XOM)? Justify your answer briefly.
- v) For which of the other five stocks is there evidence that  $\alpha$  is not 0? Use the 5% level of significance.

**5. [10 points]** Adapt and run lines 26 to 30 for the six stocks listed in Question 1. Submit the output from lines 28, 29, and 30.

- i) Report the value of the sample mean excess return for Exxon Mobile Corp (XOM).
- ii) Report the estimate of the mean excess return for Exxon Mobile Corp (XOM) based on the security characteristic line.
- iii) For which stocks is the difference between the two estimates of the mean excess return most extreme? Compare with your answer to Question 4 v). Speculate why the security characteristic line might provide a less satisfactory estimate for these stocks than it provides for the other stocks.

**6. [10 points]** Adapt and run line 31 for the six stocks listed in Question 1 under the assumption that the forecast excess return for the market is 0.009. Submit the output.

- i) Report the predicted excess return for McDonald’s Corp (MCD).



- ii) Would you expect your prediction in part i) to be a satisfactory forecast in light of your answer to Question 4 v)?
- iii) Obtain a forecast based on Question 4. Would you prefer one forecast over the other?

**7. [10 points]** Adapt and run lines 32 to 35 for the six stocks listed in Question 1 so that you perform the test for Goldman Sachs Group Inc (GS) and Nike Inc (NKE).

- i) Submit the output from line 34.
- ii) Submit the results of the test you perform.
- iii) Is there evidence that the assumptions of the security characteristic line are violated for Goldman Sachs Group Inc (GS) and Nike Inc (NKE)? Use the 5% level of significance.

**8. [5 points]** Adapt and run lines 36 to 40 for the six stocks listed in Question 1. Submit the output from lines 37, 39, and 40.

- i) Report the sample covariance between the excess returns for Goldman Sachs Group Inc (GS) and Nike Inc (NKE)).
- ii) Report the estimate of the covariance between the excess returns for Goldman Sachs Group Inc (GS) and Nike Inc (NKE) based on the security characteristic line.
- iii) Report the ratio between your answer in part i) to your answer in part ii).

**9. [15 points]** Adapt and run lines 41 to 51 for the six stocks listed in Question 1.

- i) Report  $\hat{\beta}_P$  for the portfolio.
- ii) Report estimate of the systematic component of risk for the portfolio.
- iii) Report the estimate of the unique component of risk for the portfolio.
- iv) Is the estimate of the unique component of risk for the portfolio less than the estimates of the unique components of risk for each of the stocks? Does diversification appear to reduce the unique component of risk?
- v) Is the estimate of the risk for the portfolio less than the estimates of the risks of the stocks?

**10. [15 points]** Adapt and run lines 52 to 81 for the six stocks listed in Question 1 to handle the case that the weights are constrained to satisfy  $-1 \leq w_j \leq 2$  ( $j = 1, \dots, 8$ ).

- i) Submit the output of the weights for the portfolio. Which stocks are shorted in the portfolio?
- ii) Report the estimate of  $\alpha_P$ . Is the estimate of  $\alpha_P$  larger than the estimates of the  $\alpha$ s for the stocks?
- iii) Report the estimate of  $\beta_P$  for the portfolio.
- iv) Report the estimate of risk for the portfolio. Is the estimate of the risk of the portfolio larger than the estimates of the risk for each of the stocks?
- v) Compare the estimate of risk for the market neutral portfolio to the estimate of risk for the portfolio based on equal weights investigated in Question 9. Does maximizing  $\alpha$  come at a cost? Does maximizing  $\alpha$  in a market neutral portfolio offer an arbitrage opportunity?