8/28/18 Kengo Kato

STSCI 5080 Homework 1

- Due is 9/6 (Th) in class.
- Write your name and NetID at the top of the first page, along with the assignment number.
- Use only the one side of the paper. Attach your pages with a staple at the top left corner.
- There are five problems. Each problem is worth 10 points.

Problems

1. Two six-sided dice are thrown sequentially, and the face values that come up are recorded.
   (a) List the sample space. (Hint). One example of outcomes is $(1, 2)$.
   (b) List the elements that make up the following event: $A =$the sum of two values is 6.
   (c) Suppose that all the outcomes occur equally likely. What is the probability of event A?

2. Suppose that we know that $P(A) = 1/4, P(B) = 3/8$, and $P(A \cap B) = P(A)P(B)$ holds. Then compute the value of $P(B \cap A^c)$.

3. In a certain city, three newspapers X, Y and Z are published. Suppose that 60 percent of the families in the city subscribe to newspaper X, 40 percent of the families subscribe to newspaper Y and 30 percent to newspaper Z. Suppose also that 20 percent subscribe to both X and Y, 10 percent to both X and Z, 20 percent to both Y and Z, and 5 percent subscribe to X, Y and Z. What percentage of families in the city subscribe to at least one of the three newspapers?

   (Hint). You may use the following identity:

   $$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C).$$

4. Prove the following formula:

   $$P(A_1 \cap A_2 \cap A_3) = P(A_1)P(A_2 \mid A_1)P(A_3 \mid A_1 \cap A_2),$$

   where we assume all of the conditioning events have positive probabilities.

5. Three cards are drawn from an ordinary 52-card deck without replacement (drawn cards are not placed back in the deck). What is the probability that none of the three cards is a heart?

   (Hint). Use the result of Problem 4.

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Solutions STSCI 5080 Homework 1

1. (a) $\Omega = \{(i,j) : i,j = 1,\dots,6\}$.

   (b) $A = \{(1,5),(2,4),(3,3),(4,2),(5,1)\}$.

   (c) Since all the outcomes occur equally likely, we have
$$P(A) = \frac{\text{number of elements of } A}{\text{number of elements of } \Omega} = \frac{5}{36}.$$

2. Partition B as
$$B = (B \cap A) \cup (B \cap A^c),$$
where $B \cap A$ and $B \cap A^c$ are disjoint. Since $P(B \cap A) = P(A \cap B) = P(A)P(B) = \frac{1}{4} \cdot \frac{3}{8} = \frac{3}{32}$, we have
$$P(B \cap A^c) = P(B) - P(B \cap A) = \frac{3}{8} - \frac{3}{32} = \frac{9}{32}.$$

3. Let
$$\begin{aligned} A &= \text{a randomly selected family subscribes to newspaper X},\\ B &= \text{a randomly selected family subscribes to newspaper Y},\\ C &= \text{a randomly selected family subscribes to newspaper Z}. \end{aligned}$$
We want to compute $P(A \cup B \cup C)$, but the hint yields that
$$\begin{aligned} P(A \cup B \cup C) &= P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)\\ &= 0.6 + 0.4 + 0.3 - 0.2 - 0.1 - 0.2 + 0.05 = 0.85. \end{aligned}$$
So the answer is 85 %.

4. Since
$$P(A_2 \mid A_1) = \frac{P(A_1 \cap A_2)}{P(A_1)} \quad \text{and} \quad P(A_3 \mid A_1 \cap A_2) = \frac{P((A_1 \cap A_2) \cap A_3)}{P(A_1 \cap A_2)} = \frac{P(A_1 \cap A_2 \cap A_3)}{P(A_1 \cap A_2)},$$
we have
$$\begin{aligned} &P(A_1)P(A_2 \mid A_1)P(A_3 \mid A_1 \cap A_2)\\ &\quad = P(A_1)\frac{P(A_1 \cap A_2)}{P(A_1)}\frac{P(A_1 \cap A_2 \cap A_3)}{P(A_1 \cap A_2)} = P(A_1 \cap A_2 \cap A_3). \end{aligned}$$

5. Let
$$\begin{aligned} A_1 &= \text{the first card is not a heart},\\ A_2 &= \text{the second card is not a heart},\\ A_3 &= \text{the third card is not a heart}. \end{aligned}$$

We want to calculate $P(A_1 \cap A_2 \cap A_3)$. We know that $P(A_1) = 39/52$. Next, given that the first card is drawn, we are left with 51 cards 38 of which are not hearts, and so

$$P(A_2 \mid A_1) = \frac{38}{51}.$$

Similarly,

$$P(A_3 \mid A_1 \cap A_2) = \frac{37}{50}.$$

Hence,

$$P(A_1 \cap A_2 \cap A_3) = \frac{39}{52} \cdot \frac{38}{51} \cdot \frac{37}{50}.$$