Math 2210 - Linear Algebra

Prelim 1 - 20 February 2014 - Classroom PLS 233

Name and NetID: ______________________________

Circle the name of your TA and your discussion session. Jeff Bergfalk Ahmad Rafiqi

1:25-2:15 2:30-3:20 3:35-4:25

INSTRUCTIONS

- This test has 6 problems on 6 pages, worth a total of 100 points. Check if you have all 6 pages with questions.
- If you need more space than you are given under a question, write on the back side of the preceding sheet, but be sure to label your work clearly and point out where your final answer to each question is. You also have a 2-sided page for scratchwork at the end.
- No books or electronic devices allowed. You are allowed a two-sided letter size paper of notes.
- Please **show all your work and justify your answers**.

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1.	______	/ 20
2.	______	/ 15
3.	______	/ 15
4.	______	/ 15
5.	______	/ 15
6.	______	/ 20
Total:	______	/ 100

Academic integrity is expected of all Cornell University students at all times, whether in the presence or absence of members of the faculty.
Understanding this, I declare I shall not give, use, or receive unauthorized aid in this examination.
Please sign below to indicate that you have read and agree to these instructions.

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Signature of Student

1. *(15+5 points)* Let $A = \begin{bmatrix} 1 & 2 & 1 & -2 \\ 1 & 1 & -2 & 1 \\ 1 & -1 & -8 & 7 \end{bmatrix}$, $\mathbf{b}_1 = \begin{bmatrix} 5 \\ 4 \\ 2 \end{bmatrix}$, $\mathbf{b}_2 = \begin{bmatrix} 3 \\ 0 \\ 4 \end{bmatrix}$.

   1). Find all solutions to the systems of equations (1) $A\mathbf{x} = \mathbf{0}$, (2) $A\mathbf{x} = \mathbf{b}_1$, (3) $A\mathbf{x} = \mathbf{b}_2$. Express them in the parametric vector form.

   2). Find the set of all $\mathbf{b}$ such that $A\mathbf{x} = \mathbf{b}$ is consistent. Express the set in the parametric vector form.

2. *(15 points)* Let $A = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$. Either find A^{-1} or show that A^{-1} does not exist.

3. *(7+8 points)* For each of the following statements say if it is true of false; give reasons if it is true, and a counterexample if it is false.

   1). Let $A \in M_{m\times n}$. If $n > m$, then for every $\mathbf{b} \in \mathbb{R}^m$, the solution of $A\mathbf{x} = \mathbf{b}$ is not unique.

   2). Let $A \in M_{n\times n}$. If $A\mathbf{x} = \mathbf{b}$ is consistent for all $\mathbf{b} \in \mathbb{R}^n$, then $A^T\mathbf{x} = \mathbf{b}$ is also consistent for all $\mathbf{b} \in \mathbb{R}^n$. (Hint: Is A invertible?)

4. *(15 points)* Find two 2×2 matrices A and B, neither of which is zero matrix, satisfying (simultaneously) two conditions $A + B = I$, and $AB = 0$.

5. *(7+8 points)* For each of the following statements say if it is true of false; give reasons if it is true, and a counterexample if it is false.

   1). Three vectors $\mathbf{v}_1,\ \mathbf{v}_2,\ \mathbf{v}_3$ cannot span $\mathbb{R}^4$.

   2). Any three linearly independent vectors $\mathbf{v}_1,\ \mathbf{v}_2,\ \mathbf{v}_3$ span $\mathbb{R}^3$.

6. *(10+10 points)* Identify polynomials with their coefficient vectors, for example, identify $ax^2 + bx + c$ with vector $\begin{bmatrix} a \\ b \\ c \end{bmatrix} \in \mathbb{R}^3$, and $ax^3 + bx^2 + cx + d$ with vector $\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \in \mathbb{R}^4$.

   Then we can identify $\mathbb{P}_n$, the set of all polynomials of degree at most n, with $\mathbb{R}^{n+1}$. Prove that the derivative operation D is a linear transformation from $\mathbb{P}_3$ to $\mathbb{P}_2$, and find its standard matrix.

This page is for scratch work; it will not be graded unless you point us here from the page where the question was posed.

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