Fall 2018 STSCI 5080 Supplemental Material (9/6)

Some standard distributions

1 Discrete distributions

(1) Bernoulli distribution with success probability p. A random variable X follows the Bernoulli distribution with success probability p if X takes values in $\{0, 1\}$ and has pmf

$$p(x) = \begin{cases} p & \text{if } x = 1 \\ 1 - p & \text{if } x = 0 \\ 0 & \text{otherwise} \end{cases},$$

where $p \in [0, 1]$.

(2) Binomial distribution with parameters n and p. $Bin(n, p)$. Let n be a positive integer and $p \in [0, 1]$. A random variable Y follows the binomial distribution with parameters n and p, $Y \sim Bin(n, p)$ in short, if Y can be written as $Y = X_1 + \cdots + X_n$ for independent Bernoulli trials $X_1, \ldots, X_n$ with success probability p. The variable Y takes values in $\{0, 1, \ldots, n\}$ and has pmf

$$p(k) = \binom{n}{k} p^k (1 - p)^{n-k}, \ k = 0, 1, \ldots, n.$$

(3) Poisson distribution. $Po(\lambda)$. Let $\lambda > 0$. A random variable X follows the Poisson distribution with parameter λ, $X \sim Po(\lambda)$ in short, if X takes values in $\{0, 1, 2, \ldots\}$ and has pmf

$$p(k) = \frac{\lambda^k}{k!} e^{-\lambda}, \ k = 0, 1, 2, \ldots$$

2 Continuous distributions

(1) Uniform distribution on $[a, b]$. $U[a, b]$. Let $a < b$. A random variables X follows the uniform distribution on $[a, b]$, $X \sim U[a, b]$ in short, if X has pdf

$$f(x) = \begin{cases} \frac{1}{b-a} & \text{if } a \le x \le b \\ 0 & \text{otherwise} \end{cases}.$$

The variable X concentrates on $[a, b]$, i.e., $P(X \in [a, b]) = 1$.

(2) Exponential distribution with parameter λ. $Exp(\lambda)$. Let $\lambda > 0$. A random variable X follows the exponential distribution with parameter λ, $X \sim Exp(\lambda)$ in short, if X has pdf

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x \ge 0 \\ 0 & \text{otherwise} \end{cases}.$$

The variable X concentrates on $[0, \infty)$, i.e., $P(X \in [0, \infty)) = 1$.

(3) Standard normal distribution. $N(0,1)$. A random variable X follows the standard normal distribution, $X \sim N(0,1)$ in short, if X has pdf

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}, \ x \in \mathbb{R}.$$

(4) Normal distribution with mean μ and variance σ^2. $N(\mu, \sigma^2)$. Let $\mu \in \mathbb{R}$ and $\sigma > 0$. A random variable Y follows the normal distribution with mean μ and variance σ^2, $Y \sim N(\mu, \sigma^2)$ in short, if Y can be written as $Y = \mu + \sigma X$ for some $X \sim N(0,1)$. The variable Y has pdf

$$f(y) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(y-\mu)^2/(2\sigma^2)}, \ y \in \mathbb{R}.$$

(5) Gamma distribution with shape parameter α and scale parameter β. $Ga(\alpha, \beta)$. Let $\alpha > 0$ and $\beta > 0$. A random variable X follows the gamma distribution with shape parameter α and scale parameter β, $X \sim Ga(\alpha, \beta)$ in short, if X has pdf

$$f(x) = \begin{cases} \frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} e^{-x/\beta} & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases},$$

where $\Gamma(\alpha)$ is the gamma function

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx.$$

The variable X concentrates on $(0, \infty)$, i.e., $P(X \in (0, \infty)) = 1$.

(6) Beta distribution with parameters α and β. $Be(\alpha, \beta)$. Let $\alpha > 0$ and $\beta > 0$. A random variable X follows the beta distribution with parameters α and β, $X \sim Be(\alpha, \beta)$ in short, if X has pdf

$$f(x) = \begin{cases} \frac{1}{B(\alpha,\beta)} x^{\alpha-1} (1-x)^{\beta-1} & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases},$$

where $B(\alpha, \beta)$ is the beta function

$$B(\alpha, \beta) = \int_0^1 x^{\alpha-1} (1-x)^{\beta-1} dx.$$

The variable X concentrates on $(0, 1)$, i.e., $P(X \in (0, 1)) = 1$.