ORIE 4630: Spring Term 2019
Homework #8 Solutions

Question 1. [**24 points**]

Output from lines 17 to 24:

```
> a = BS_price_sim(80,  0.2,  75,  1.25,  0.03,  1e7,  4630)
> a[1]
[1] 11.34699
> a[2]
[1] 14.34952
> a[1]  + c(-1, 1) * qnorm(0.95) * a[2] / sqrt(a[3])
[1] 11.33953 11.35446
> b = BS_price_formula(80, 0.2, 75, 1.25, 0.03)
> b
[1] 11.34635
> a[1] - b
[1] 0.0006387393
> a[1] - b + c(-1, 1) * qnorm(0.95) * a[2] / sqrt(a[3])
[1] -0.006825141  0.008102620
```

i) 11.34699

ii) 14.3495

iii) $(11.3395, 11.3545)$

iv) 11.3464

v) 0.0006388

vi) $(-0.006825, 0.008103)$. The confidence interval provides no evidence against the veracity of the Black-Scholes formula; since 0 is in the 90% confidence interval, the null hypthesis that the discounted expected value is the same as the value produced by the Black-Scholes formula is not rejected in a test at the 10% level.

vii) $P = C + \exp(-rT)K - S_0 = 14.3495 + \exp\{-(0.03)(1.25)\}75 - 80 = 6.5891$

Question 2. [**22 points**]

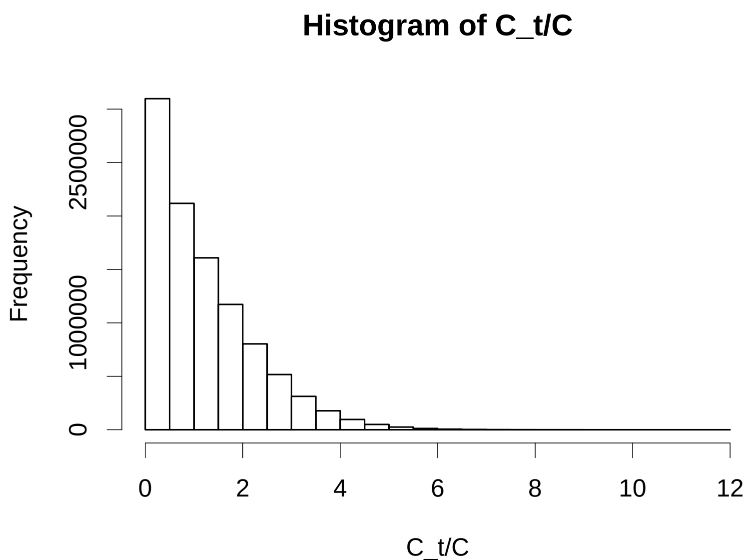
Output from lines 32 to 42:

```
> C = BS_price_formula(80, 0.2, 75, 1.25, 0.03)
> C
[1] 11.34635
> C_t = BS_price_t_sim(0.75, 80, 0.055, 0.2, 75, 1.25, 0.03, 1e7, 4630)
> par(lwd = 2); par(cex = 2)
> hist(C_t / C)
> hist(log(C_t / C))
> par(lwd = 1); par(cex = 1)
> mean(C_t / C); sd(C_t / C)
```

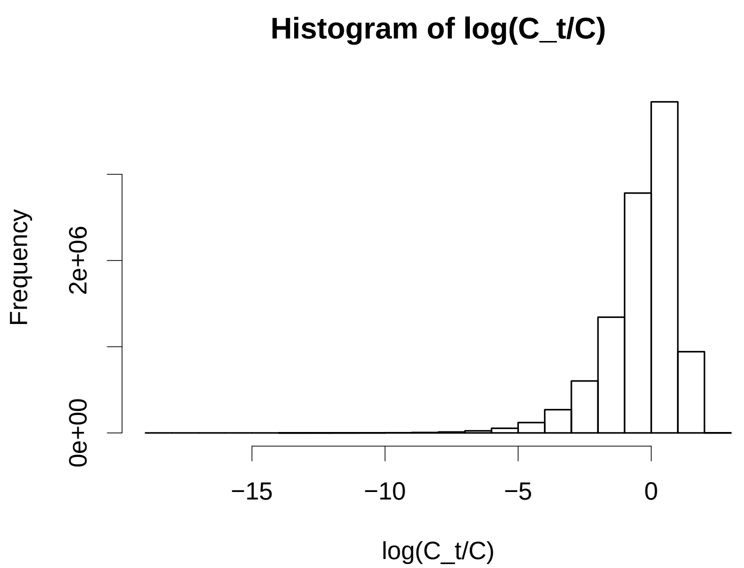
```
[1] 1.205904
[1] 1.044239
> mean(log(C_t / C)); sd(log(C_t / C))
[1] -0.367839
[1] 1.349826
> ind = as.numeric(C_t <= C)
> mean(ind)
[1] 0.521598
```

i) 11.34635

ii) Plot generated by line 36:



iii) Plot generated by line 37:



iv) 1.205904

v) 1.044239

vi) -0.367839

vii) 1.349826

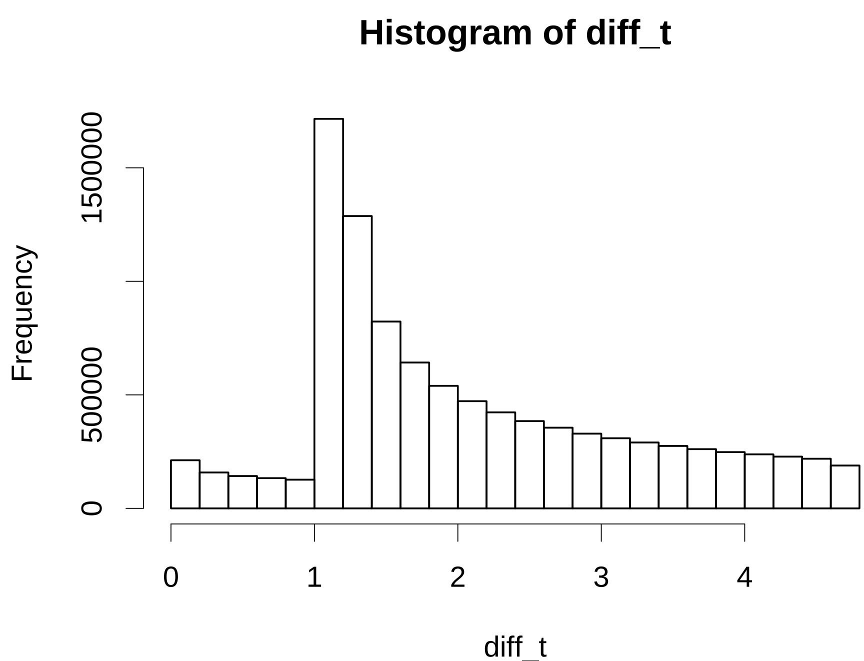
viii) $1 - 0.521598 = 47.8402\%$

Question 3. [15 points]

Output from lines 43 to 57:

```
> mean(diff_t)
[1] 2.077124
> sd(diff_t)
[1] 1.130588
> min(diff_t)
[1] 6.543277e-08
```

i) Plot generated by line 54:



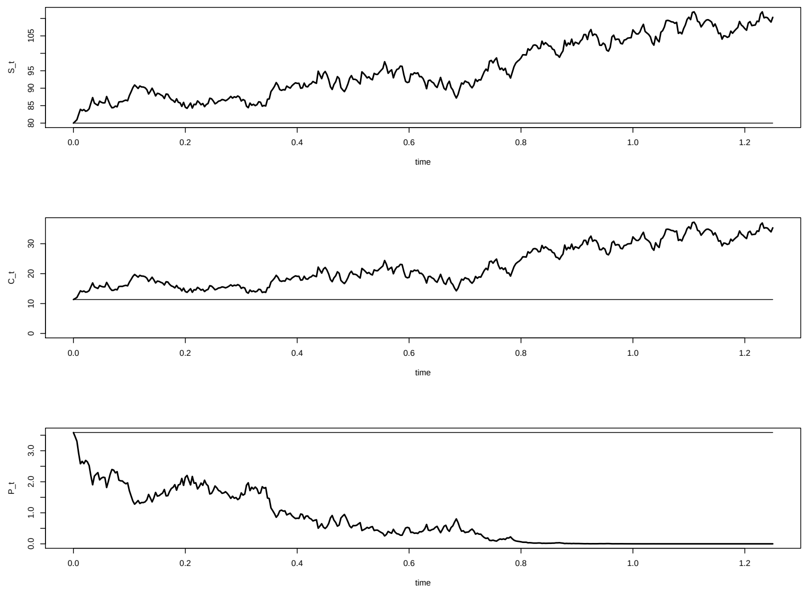
ii) 2.077124

iii) 1.130588

iv) $6.543277 \times 10^{-8} \approx 0$. The difference $C_t - (S_t - K)_+$ is never negative for the simulations, so $C_t \geq (S_t - K)_+$ for $t = 0.75$.

Question 4. [15 points]

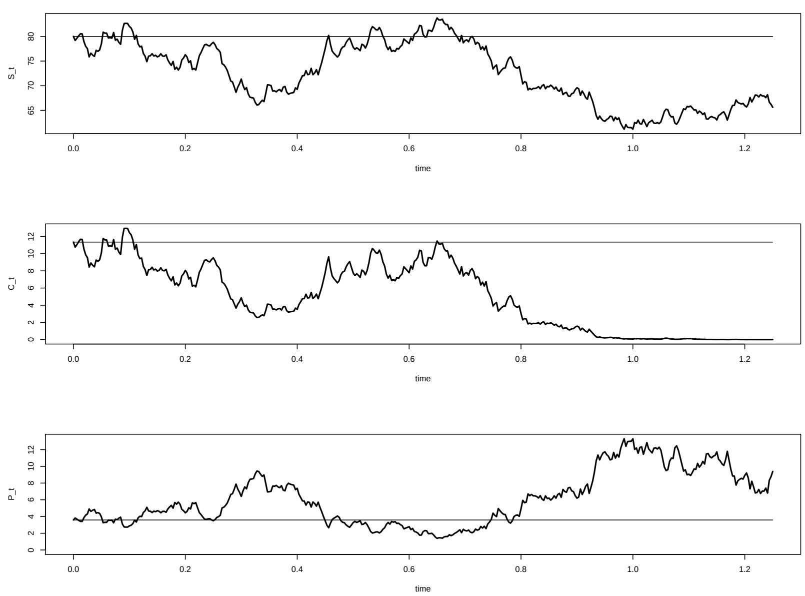
For the seed set equal to 8938:




```
> c(S_t[401],C_t[401],P_t[401])
[1] 110.31717  35.31717   0.00000
```

For $t = 1.5$ and seed 8938: $S_t = 110.3172$; $C_t = 35.3172$; $P_t = 0$.

For the seed set equal to 8957:




```
> c(S_t[401],C_t[401],P_t[401])
[1] 65.606344  0.000000  9.393656
```

For $t = 1.5$ and seed 8957: $S_t = 65.6063$; $C_t = 0$; $P_t = 9.3937$.

Question 5. [24 points]

```
> delta(S=80, T=1.25, t=0, K=75, sigma=0.2, r=0.03)
  [,1]          [,2]        [,3]         [,4]
a "call price" "put price" "call delta" "put delta"
b "11.34635"   "3.58594"   "0.71503"    "-0.28497"
```

i) 11.34635

ii) 0.71503

iii) $0.71503 \times \dfrac{80}{11.34635} = 5.041480$

iv) $\dfrac{1}{0.71503}$ call options replicate the change in price of 1 share of stock, so the number of call options that replicate the change in price of 100 shares of stock is $\dfrac{100}{0.71503} = 139.8543$; the cost of the call options is $(139.8543)(11.34635) = 1586.836$, while the cost of the shares is $(100)(80) = 8,000$.

v) \$250,000 worth of stock is $\dfrac{250,000}{80} = 3125$ shares, so $\dfrac{3125}{0.71503} = 4370.4460$ options are required; the cost of the options would be $(4370.4460)(11.34635) = 49588.610$, which is \$250,000 divided by the leverage 5.041480.

vi) The change in price of 1 share of stock is hedged by $\dfrac{1}{0.28497}$ put options, so the number of put options that replicate the change in price of 100 shares of stock is $\dfrac{100}{0.28497} = 350.9141$; the cost of the put options is $(350.9141)(3.58594) = 1258.357$, while the cost of the shares is $(100)(80) = 8,000$.

vii) \$250,000 worth of stock is $\dfrac{250,000}{80} = 3125$ shares, so $\dfrac{3125}{0.28497} = 10966.0660$ options are required; the cost of the options would be $(10966.0660)(3.58594) = 39323.655$, which is \$250,000 divided by $0.28497 \times \dfrac{80}{3.58594} = 6.35750$, which is the put hedging leverage.