STSCI 5080
Probability Models and Inference
Lecture 25: Review for the Final Exam

December 4, 2018

Date and Place

- Dec 9 (Sunday) 2PM-4:30PM at Phillips Hall 219.

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- Hope there will be a clock on the front of the room.

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The final exam will ask questions about the third goal.

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- Problem 5 in the practice final will be also asked in the final. So be prepared.

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- Find the MLE of...
- Find, with or without a justification, a confidence interval for...
- Find, with or without a justification, a test...

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- Even if you can not verify the derivation of the MLE, you should continue other questions.

General guidance

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- Don't forget χ^2- and t-distributions.

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- Bring yourself (most important).

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- The total number of points is 100.

Some useful abbreviations

- If you feel tedious to write “likelihood function”, “log likelihood function”, or “confidence interval”, “asymptotic level”, you can just write: “lik. func.”, “log lik. func.”, “CI”, or “asy. level”.
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For example,

$$\left[\widehat{\lambda} \pm \frac{1.96\sqrt{\widehat{\lambda}}}{\sqrt{n}}\right] = \left[\widehat{\lambda} - \frac{1.96\sqrt{\widehat{\lambda}}}{\sqrt{n}}, \widehat{\lambda} + \frac{1.96\sqrt{\widehat{\lambda}}}{\sqrt{n}}\right]$$

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is a CI for λ in $Po(\lambda)$ with asy. level 95%.

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- Is the MLE always asymptotically normal?

MLE based CIs with asy. level $1-\alpha$

Suppose that $\sqrt{n}(\widehat{\theta}-\theta) \xrightarrow{d} N(0,\sigma^2(\theta))$.

- The first method is to plug in $\widehat{\theta}$ for $\sigma(\theta)$ and use

$$\left[\widehat{\theta}-\frac{z_{\alpha/2}\sigma(\widehat{\theta})}{\sqrt{n}},\widehat{\theta}+\frac{z_{\alpha/2}\sigma(\widehat{\theta})}{\sqrt{n}}\right].$$

- The second method is to find a variance stabilizing transformation $g(\theta)$ such that $\sqrt{n}\{g(\widehat{\theta})-g(\theta)\} \xrightarrow{d} N(0,1)$ and use

$$\left[g^{-1}\left(g(\widehat{\theta})-z_{\alpha/2}/\sqrt{n}\right),g^{-1}\left(g(\widehat{\theta})+z_{\alpha/2}/\sqrt{n}\right)\right].$$

- $z_{\alpha/2}=1.96$ for $\alpha=0.05$.

Testing

For the testing problem,

$$H_0 : \theta = \theta_0 \quad \text{vs.} \quad H_1 : \theta \neq \theta_0,$$

a test with asymptotic level α is given by

$$\left| \frac{\sqrt{n}(\widehat{\theta} - \theta_0)}{\sigma(\theta_0)} \right| > z_{\alpha/2} \Rightarrow \text{reject } H_0.$$

If $\alpha = 0.05$, we can choose $z_{\alpha/2} = 1.96$.

θ_0 **is known.**

Don't forget to add $|\cdot|$ (absolute value sign)!

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- You only have to decide the case under which you reject H_0.

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I will teach:

STSCI 6940 Advanced Probability for Statisticians

mainly for 1st and 2nd year Ph.D. students in Stats.

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- In more short term...
- Happy exam day! It's SUNDAY! Wow!