ECE/PHY 235 - Introduction to Solid State Electronics

Discussion Week 8

Practice Problem 1

Given that the density of states effective masses of electrons and holes in Si are approximately $1.08m_e$ and $0.60m_e$, respectively, and the electron and hole drift mobilities at room temperature are $1350\,\text{cm}^2\text{V}^{-1}\text{s}^{-1}$ and $450\,\text{cm}^2\text{V}^{-1}\text{s}^{-1}$, respectively, calculate the intrinsic concentration and intrinsic resistivity of Si at room temperature ($T = 300\,\text{K}$). Si bandgap is $1.10\,\text{eV}$.

($k_B = 8.617 \times 10^{-5}\,\text{eV/K}, m_e = 9.109 \times 10^{-31}\,\text{kg}, q = 1.602 \times 10^{-19}\,\text{C}, h = 6.626 \times 10^{-34}\,\text{J} \cdot \text{s}$)

Solution:

1. **Calculate the effective density of states N_c and N_v:**

$$N_c = 2\left(\frac{2\pi(1.08m_e)k_BT}{h^2}\right)^{3/2} \approx 2.86 \times 10^{19}\,\text{cm}^{-3}$$

$$N_v = 2\left(\frac{2\pi(0.60m_e)k_BT}{h^2}\right)^{3/2} \approx 1.05 \times 10^{19}\,\text{cm}^{-3}$$

2. **Calculate the intrinsic concentration n_i:**

$$n_i = \sqrt{N_cN_v}\, e^{-E_g/2k_BT} \approx 9.65 \times 10^{9}\,\text{cm}^{-3}$$

3. **Calculate the conductivity σ:**

$$\sigma = qn_i(\mu_e + \mu_h) \approx 2.88 \times 10^{-6}\,\Omega^{-1}\text{cm}^{-1}$$

4. **Calculate the resistivity ρ:**

$$\rho = \frac{1}{\sigma} \approx 3.47 \times 10^{5}\,\Omega \cdot \text{cm}$$

Practice Problem 2

Find the resistance of a $1\,\text{cm} \times 1\,\text{cm} \times 1\,\text{cm}$ pure silicon crystal at room temperature ($T = 300\,\text{K}$). What is the resistance when the crystal is doped with arsenic if the doping is 1 part per billion (ppb)? Given data: Atomic concentration in silicon is $5 \times 10^{22}\,\text{cm}^{-3}$, $n_i = 9.65 \times 10^{9}\,\text{cm}^{-3}$ (calculated from Problem 1), the electron and hole drift mobilities at room temperature are $1350\,\text{cm}^2\text{V}^{-1}\text{s}^{-1}$ and $450\,\text{cm}^2\text{V}^{-1}\text{s}^{-1}$, respectively.

Solution:

Intrinsic Case:

1. **Calculate the conductivity σ:**

$$\sigma = qn_i(\mu_e + \mu_h) \approx 2.78 \times 10^{-6}\,\Omega^{-1}\text{cm}^{-1}$$

2. **Calculate the resistance R:**

$$R = \frac{L}{\sigma A} \approx 3.60 \times 10^{5}\,\Omega$$

Doped Case:

1. **Calculate the donor concentration N_d:**

$$N_d = \frac{5 \times 10^{22}\,\text{cm}^{-3}}{10^9} = 5 \times 10^{13}\,\text{cm}^{-3}$$

2. **Calculate the electron concentration n:** Since this is an n-type doping and $N_d \gg n_i$, we can approximate:

$$n \approx N_d = 5 \times 10^{13}\,\text{cm}^{-3}$$

3. **Calculate the hole concentration p using the mass-action law:**

$$p = \frac{n_i^2}{n} \approx 1.86 \times 10^6\,\text{cm}^{-3}$$

4. **Calculate the conductivity σ:** Since $n \gg p$, we can approximate:

$$\sigma \approx qn\mu_e \approx 1.08 \times 10^{-2}\,\Omega^{-1}\text{cm}^{-1}$$

5. **Calculate the resistance R:**

$$R = \frac{L}{\sigma A} \approx 92.6\,\Omega$$

Practice Problem 3

An n-type Si semiconductor containing 10^{16} phosphorus (donor) atoms cm^{-3} has been doped with 10^{17} boron (acceptor) atoms cm^{-3}. Calculate the electron and hole concentrations in this semiconductor at room temperature. Given that $n_i = 9.65 \times 10^9\,\text{cm}^{-3}$.

Solution:

1. **Determine the majority carrier type:** Since $N_a > N_d$, the semiconductor is p-type.

2. **Calculate the majority carrier concentration p:** Since $N_a - N_d \gg n_i$, we can approximate:

$$p \approx N_a - N_d = 10^{17}\,\text{cm}^{-3} - 10^{16}\,\text{cm}^{-3} = 9 \times 10^{16}\,\text{cm}^{-3}$$

3. **Calculate the minority carrier concentration n using the mass-action law:**

$$n = \frac{n_i^2}{p} \approx \frac{(9.65 \times 10^9)^2}{9 \times 10^{16}} \approx 1.04 \times 10^3\,\text{cm}^{-3}$$

Practice Problem 4

An n-type Si wafer at room temperature ($T = 300\,\text{K}$) has been doped uniformly with 10^{16} antimony (Sb) atoms cm^{-3}. Calculate the position of the Fermi energy with respect to the intrinsic Fermi energy E_{Fi} in intrinsic Si. The above n-type Si sample is further doped with 2×10^{17} boron atoms cm^{-3}. Calculate the position of the Fermi energy with respect to the intrinsic Fermi energy E_{Fi} in the compensated Si. Given that $n_i = 9.65 \times 10^9\,\text{cm}^{-3}$ (Assume that $T = 300\,\text{K}$).

Solution:

n-type Doping:

1. **Calculate the position of E_F relative to E_{Fi}:** Since $N_d \gg n_i$, we can approximate $n \approx N_d$.

$$E_F - E_{Fi} = k_B T \ln\left(\frac{N_d}{n_i}\right) \approx 0.36\,\text{eV}$$

Compensation Doping (p-type):

1. **Calculate the effective acceptor concentration N_a':**

$$N_a' = N_a - N_d = 2 \times 10^{17}\,\text{cm}^{-3} - 10^{16}\,\text{cm}^{-3} = 1.9 \times 10^{17}\,\text{cm}^{-3}$$

2. **Calculate the position of E_F relative to E_{Fi}:** Since $N_a' \gg n_i$, we can approximate $p \approx N_a'$.

$$E_{Fi} - E_F = k_B T \ln\left(\frac{N_a'}{n_i}\right) \approx 0.43\,\text{eV}$$

Discussion Week 9

Practice Problem 1

Consider an infinitely large, homogeneous n-type semiconductor with zero applied electric field. Assume that 10^{14} electron-hole pairs have been uniformly created per cm^3 at $t = 0$, and assume the minority carrier hole lifetime is $\tau_{p0} = 50\,\text{ns}$. Determine the time at which the minority carrier hole concentration reaches (a) $1/e$ of its initial value and (b) 10% of its initial value.

Solution:

1. **Use the simplified continuity equation solution for minority carrier decay:**

$$\Delta p(t) = \Delta p(0) e^{-t/\tau_{p0}}$$

2. **(a) Solve for t when $\Delta p(t)/\Delta p(0) = 1/e$:**

$$t = \tau_{p0} = 50\,\text{ns}$$

3. **(b) Solve for t when $\Delta p(t)/\Delta p(0) = 0.1$:**

$$t = -\tau_{p0} \ln(0.1) \approx 115.13\,\text{ns}$$

Practice Problem 2

Consider an infinitely large, homogeneous n-type semiconductor with zero applied electric field. Assume that, for $t < 0$, the semiconductor is in thermal equilibrium and that, for $t \geq 0$, a uniform generation rate $g_0 = 5 \times 10^{21}\,\text{cm}^{-3}\text{s}^{-1}$ exists in the crystal and let $\tau_{p0} = 10^{-7}\,\text{s}$. Determine $\Delta p(t)$ at $t = 10^{-7}\,\text{s}$.

Solution:

1. **Use the continuity equation for minority carriers with uniform generation:**

$$\frac{d(\Delta p)}{dt} = g_0 - \frac{\Delta p}{\tau_{p0}}$$

2. **Solve the differential equation with the initial condition $\Delta p(0) = 0$:**

$$\Delta p(t) = g_0 \tau_{p0} \left(1 - e^{-t/\tau_{p0}}\right)$$

3. **Evaluate $\Delta p(t)$ at $t = 10^{-7}$ s:**

$$\Delta p(t) = 5 \times 10^{21} \times 10^{-7} \times \left(1 - e^{-1}\right) \approx 5 \times 10^{21} \times 10^{-7} \times 0.632 \approx 3.16 \times 10^{15}\,\text{cm}^{-3}$$

Practice Problem 3

Consider a p-type semiconductor that is homogeneous and infinite in extent. Assume a zero applied electric field. For a one-dimensional crystal, assume that excess carriers are being generated at $x = 0$ only. Assume $\tau_{n0} = 5 \times 10^{-7}\,\text{s}$, $D_n = 25\,\text{cm}^2/\text{s}$, and $\Delta n(0) = 10^{15}\,\text{cm}^{-3}$.

1. **Calculate the diffusion length L_n:**

$$L_n = \sqrt{D_n \tau_{n0}} = \sqrt{25 \times 5 \times 10^{-7}} \approx 3.54 \times 10^{-3}\,\text{cm} = 35.4\,\mu\text{m}$$

2. **Calculate Δn at $x = 30\,\mu$m:**

$$\Delta n(x) = \Delta n(0) e^{-x/L_n} = 10^{15}\,\text{cm}^{-3} \times e^{-30\,\mu\text{m}/35.4\,\mu\text{m}} \approx 10^{15} \times e^{-0.848} \approx 4.28 \times 10^{14}\,\text{cm}^{-3}$$

Discussion Week 10

Practice Problem 1

Consider a silicon pn junction diode at 300 K with doping concentrations of $N_d = 10^{15}\,\text{cm}^{-3}$ and $N_a = 2 \times 10^{17}\,\text{cm}^{-3}$. Given that $n_i = 1.5 \times 10^{10}\,\text{cm}^{-3}$, find the built-in potential.

Solution:

1. **Use the formula for the built-in potential V_{bi}:**

$$V_{bi} = \frac{k_B T}{q} \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

2. **Plug in the given values:**

$$V_{bi} \approx 0.0259\,\text{V} \times \ln\left(\frac{2 \times 10^{17} \times 10^{15}}{(1.5 \times 10^{10})^2}\right) \approx 0.713\,\text{V}$$

Practice Problem 2

A silicon pn junction diode has a depletion width of $x_n = 0.8644\,\mu$m in the n-side. Calculate the maximum electric field in the diode at zero bias. Silicon relative dielectric constant is 11.7 and given that $N_d = 10^{15}\,\text{cm}^{-3}$. $\epsilon_0 = 8.854 \times 10^{-14}\,\text{F/cm}$.

Solution:

1. **Use the formula for the maximum electric field $E_{\mathbf{max}}$:**

$$E_{\max} = \frac{q N_d x_n}{\epsilon_s} = \frac{q N_d x_n}{11.7 \epsilon_0}$$

2. **Plug in the given values:**

$$E_{\max} = \frac{(1.602 \times 10^{-19}\,\text{C}) \times (10^{15}\,\text{cm}^{-3}) \times (0.8644 \times 10^{-4}\,\text{cm})}{11.7 \times 8.854 \times 10^{-14}\,\text{F/cm}} \approx 1.34 \times 10^4\,\text{V/cm}$$

Practice Problem 3

Calculate the built-in potential barrier of a silicon pn junction at $T = 300\,\text{K}$ for $N_a = N_d = 10^{15}\,\text{cm}^{-3}$. Given that $n_i = 1.5 \times 10^{10}\,\text{cm}^{-3}$, $D_n = 25\,\text{cm}^2/\text{s}$, $D_p = 10\,\text{cm}^2/\text{s}$, $\tau_{n0} = \tau_{p0} = 5 \times 10^{-7}\,\text{s}$. Also, assume the reverse-bias saturation current is $I_0 = 10^{-14}\,\text{A}$. If the cross-section of the junction is $A = 10^{-4}\,\text{cm}^2$, determine the junction current when the forward bias is $V_a = 0.5\,\text{V}$.

Solution:

1. **Calculate the built-in potential V_{bi}:**

$$V_{bi} = \frac{k_B T}{q} \ln\left(\frac{N_a N_d}{n_i^2}\right) = 0.0259\,\text{V} \times \ln\left(\frac{10^{15} \times 10^{15}}{(1.5 \times 10^{10})^2}\right) \approx 0.578\,\text{V}$$

2. **Calculate the junction current I under forward bias:**

$$I = I_0\left(e^{\frac{qV_a}{k_B T}} - 1\right) = 10^{-14}\left(e^{\frac{0.5}{0.0259}} - 1\right) \approx 4.66 \times 10^{-1}\,\text{A}$$

Practice Problem 4

For a pn junction diode with $N_d = 10^{15}\,\text{cm}^{-3}$ and $N_a = 10^{16}\,\text{cm}^{-3}$. If the depletion width in the n-side is $0.8644\,\mu\text{m}$, find the total depletion width.

Solution:

1. **Use the relationship based on charge neutrality:**

$$N_a x_p = N_d x_n$$

2. **Calculate x_p:**

$$x_p = \frac{N_d x_n}{N_a} = \frac{10^{15} \times 0.8644\,\mu\text{m}}{10^{16}} = 0.08644\,\mu\text{m}$$

3. **Calculate the total depletion width W:**

$$W = x_n + x_p = 0.8644\,\mu\text{m} + 0.08644\,\mu\text{m} = 0.951\,\mu\text{m}$$

Discussion Week 11

Practice Problem 1

Consider a silicon pn junction at $T = 300\,\text{K}$. Assume the doping concentration in the n-region is $N_d = 10^{16}\,\text{cm}^{-3}$ and the doping concentration in the p-region is $N_a = 6 \times 10^{15}\,\text{cm}^{-3}$, and assume that a forward bias of $0.60\,\text{V}$ is applied to the pn junction.

Calculate the minority carrier concentrations at the edge of the space charge regions in a forward-biased pn junction. $n_i = 1.5 \times 10^{10}\,\text{cm}^{-3}$.

Solution:

1. **Calculate the thermal-equilibrium minority carrier concentrations:**

$$n_{p0} = \frac{n_i^2}{N_a} = \frac{(1.5 \times 10^{10})^2}{6 \times 10^{15}} = 3.75 \times 10^4\,\text{cm}^{-3}$$

$$p_{n0} = \frac{n_i^2}{N_d} = \frac{(1.5 \times 10^{10})^2}{10^{16}} = 2.25 \times 10^4\,\text{cm}^{-3}$$

2. **Apply the forward bias formulas:**

$$n_p(-x_p) = n_{p0} \exp\left(\frac{qV_a}{k_B T}\right) = 3.75\times10^4\times\exp\left(\frac{0.60}{0.0259}\right) \approx 3.75\times10^4\times2.35\times10^{10} \approx 8.81\times10^{14}\,\text{cm}^{-3}$$

$$p_n(x_n) = p_{n0} \exp\left(\frac{qV_a}{k_B T}\right) = 2.25\times10^4\times\exp\left(\frac{0.60}{0.0259}\right) \approx 2.25\times10^4\times2.35\times10^{10} \approx 5.29\times10^{14}\,\text{cm}^{-3}$$

Practice Problem 2

Determine the ideal reverse-saturation current density in a silicon pn junction at $T = 300\,\text{K}$.

Consider the following parameters in a silicon pn junction:

$$\begin{aligned} N_a &= N_d = 10^{16}\,\text{cm}^{-3} & n_i &= 1.5\times10^{10}\,\text{cm}^{-3} \\ D_n &= 25\,\text{cm}^2/\text{s} & \tau_{p0} &= \tau_{n0} = 5\times10^{-7}\,\text{s} \\ D_p &= 10\,\text{cm}^2/\text{s} & \epsilon_r &= 11.7 \end{aligned}$$

Solution:

1. **Use the formula for the reverse saturation current density J_s:**

$$J_s = qn_i^2\left(\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{n0}}} + \frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{p0}}}\right)$$

2. **Plug in the given values:**

$$J_s = (1.602\times10^{-19}\,\text{C})\times(1.5\times10^{10})^2\times\left(\frac{1}{10^{16}}\sqrt{\frac{25}{5\times10^{-7}}} + \frac{1}{10^{16}}\sqrt{\frac{10}{5\times10^{-7}}}\right)$$

$$= 1.602\times10^{-19}\times2.25\times10^{20}\times\left(\frac{1}{10^{16}}\times\sqrt{5\times10^7} + \frac{1}{10^{16}}\times\sqrt{2\times10^7}\right)$$

$$= 1.602\times10^{-19}\times2.25\times10^{20}\times\left(\frac{1}{10^{16}}\times7.071\times10^3 + \frac{1}{10^{16}}\times4.472\times10^3\right)$$

$$= 1.602\times10^{-19}\times2.25\times10^{20}\times\left(7.071\times10^{-13} + 4.472\times10^{-13}\right)$$

$$= 1.602\times10^{-19}\times2.25\times10^{20}\times1.153\times10^{-12}$$

$$\approx 4.16\times10^{-11}\,\text{A/cm}^2$$

Practice Problem 3

Consider a silicon pn junction diode at $T = 300\,\text{K}$. Find the doping concentrations N_a and N_d required such that $J_n = 20\,\text{A/cm}^2$ and $J_p = 5\,\text{A/cm}^2$ at $V_a = 0.65\,\text{V}$. Assume the remaining semiconductor parameters are as given in Problem 2.

Solution:

1. **Use the formulas for the electron and hole current density components:**

$$J_n = q\sqrt{\frac{D_n}{\tau_{n0}}}\frac{n_i^2}{N_a}\left[e^{\frac{qV_a}{k_BT}}-1\right]$$

$$J_p = q\sqrt{\frac{D_p}{\tau_{p0}}}\frac{n_i^2}{N_d}\left[e^{\frac{qV_a}{k_BT}}-1\right]$$

2. **Solve for N_a using the given J_n:**

$$N_a = q\sqrt{\frac{D_n}{\tau_{n0}}}\frac{n_i^2}{J_n\left(e^{\frac{qV_a}{k_BT}}-1\right)}$$

$$\begin{aligned} N_a &= \frac{(1.602\times10^{-19})\times\sqrt{\frac{25}{5\times10^{-7}}}\times(1.5\times10^{10})^2}{20\times(e^{\frac{0.65}{0.0259}}-1)} \\ &= \frac{1.602\times\sqrt{5\times10^7}\times2.25\times10^{20}}{20\times2.35\times10^{10}} \\ &\approx 1.01\times10^{15}\,\text{cm}^{-3} \end{aligned}$$

3. **Solve for N_d using the given J_p:**

$$N_d = q\sqrt{\frac{D_p}{\tau_{p0}}}\frac{n_i^2}{J_p\left(e^{\frac{qV_a}{k_BT}}-1\right)}$$

$$\begin{aligned} N_d &= \frac{(1.602\times10^{-19})\times\sqrt{\frac{10}{5\times10^{-7}}}\times(1.5\times10^{10})^2}{5\times(e^{\frac{0.65}{0.0259}}-1)} \\ &= \frac{1.602\times\sqrt{2\times10^7}\times2.25\times10^{20}}{5\times2.35\times10^{10}} \\ &\approx 2.55\times10^{15}\,\text{cm}^{-3} \end{aligned}$$

Practice Problem 4

For a pn junction diode assuming the total current is conducted by the electric field far from the junction, calculate the electric field. Assume the applied forward bias voltage is 0.65 V and temperature is 300 K. Other diode parameters are given in Practice Problem 2.

Solution:

1. **Calculate the total current density J using the diode equation and the J_s from Problem 2:**

   $J = J_s\left(e^{\frac{qV_a}{k_BT}}-1\right) \approx 4.16\times10^{-11}\times\left(e^{25.1}-1\right) \approx 4.16\times10^{-11}\times1.12\times10^{10} \approx 4.66\times10^{-1}\,\text{A/cm}^2$

2. **Assume that far from the junction in the n-region, the total current is approximately equal to the electron drift current:**

$$J \approx J_{n,\text{drift}} = q\mu_n N_d E$$

3. **Solve for the electric field E:**

$$E = \frac{J}{q\mu_n N_d} = \frac{4.66\times10^{-1}\,\text{A/cm}^2}{(1.602\times10^{-19}\,\text{C})\times1350\,\text{cm}^2/\text{V}\cdot\text{s}\times10^{16}\,\text{cm}^{-3}} \approx 2.17\times10^4\,\text{V/cm}$$

Discussion Week 11

Practice Problem 1

Consider a silicon pn junction diode at $T = 300\,\mathrm{K}$. Assume the doping concentration in the n-region is $N_d = 10^{16}\,\mathrm{cm}^{-3}$ and the doping concentration in the p-region is $N_a = 6 \times 10^{15}\,\mathrm{cm}^{-3}$, and assume that a forward bias of $0.60\,\mathrm{V}$ is applied to the pn junction.

Calculate the minority carrier concentrations at the edge of the space charge regions in a forward-biased pn junction. $n_i = 1.5 \times 10^{10}\,\mathrm{cm}^{-3}$.

Solution:

1. **Calculate the thermal-equilibrium minority carrier concentrations:**

$$n_{p0} = \frac{n_i^2}{N_a} = \frac{(1.5 \times 10^{10})^2}{6 \times 10^{15}} = 3.75 \times 10^4\,\mathrm{cm}^{-3}$$

$$p_{n0} = \frac{n_i^2}{N_d} = \frac{(1.5 \times 10^{10})^2}{10^{16}} = 2.25 \times 10^4\,\mathrm{cm}^{-3}$$

2. **Apply the forward bias formulas:**

$$n_p(-x_p) = n_{p0} \exp\left(\frac{qV_a}{k_B T}\right) = 3.75 \times 10^4 \times e^{0.60/0.0259} \approx 3.75 \times 10^4 \times 2.35 \times 10^{10} \approx 8.81 \times 10^{14}\,\mathrm{cm}^{-3}$$

$$p_n(x_n) = p_{n0} \exp\left(\frac{qV_a}{k_B T}\right) = 2.25 \times 10^4 \times e^{0.60/0.0259} \approx 2.25 \times 10^4 \times 2.35 \times 10^{10} \approx 5.29 \times 10^{14}\,\mathrm{cm}^{-3}$$

Practice Problem 2

Determine the ideal reverse-saturation current density in a silicon pn junction at $T = 300\,\mathrm{K}$.

Consider the following parameters in a silicon pn junction:

$$\begin{array}{ll} N_a = N_d = 10^{16}\,\mathrm{cm}^{-3} & n_i = 1.5 \times 10^{10}\,\mathrm{cm}^{-3} \\ D_n = 25\,\mathrm{cm}^2/\mathrm{s} & \tau_{p0} = \tau_{n0} = 5 \times 10^{-7}\,\mathrm{s} \\ D_p = 10\,\mathrm{cm}^2/\mathrm{s} & \epsilon_r = 11.7 \end{array}$$

Solution:

1. **Use the formula for the reverse saturation current density J_s:**

$$J_s = qn_i^2 \left(\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{n0}}} + \frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{p0}}}\right)$$

2. **Plug in the given values:**

$$\begin{aligned} J_s &= (1.602 \times 10^{-19}\,\mathrm{C}) \times (1.5 \times 10^{10})^2 \times \left(\frac{1}{10^{16}} \times \sqrt{\frac{25}{5 \times 10^{-7}}} + \frac{1}{10^{16}} \times \sqrt{\frac{10}{5 \times 10^{-7}}}\right) \\ &= 1.602 \times 10^{-19} \times 2.25 \times 10^{20} \times \left(\frac{1}{10^{16}} \times 7.071 \times 10^3 + \frac{1}{10^{16}} \times 4.472 \times 10^3\right) \\ &= 1.602 \times 10^{-19} \times 2.25 \times 10^{20} \times \left(7.071 \times 10^{-13} + 4.472 \times 10^{-13}\right) \\ &= 1.602 \times 10^{-19} \times 2.25 \times 10^{20} \times 1.153 \times 10^{-12} \\ &\approx 4.16 \times 10^{-11}\,\mathrm{A/cm}^2 \end{aligned}$$

Practice Problem 3

Consider a silicon pn junction diode at $T = 300\,\mathrm{K}$. Find the doping concentrations N_a and N_d required such that $J_n = 20\,\mathrm{A/cm}^2$ and $J_p = 5\,\mathrm{A/cm}^2$ at $V_a = 0.65\,\mathrm{V}$. Assume the remaining semiconductor parameters are as given in Practice Problem 2.

Solution:

1. **Use the formulas for the electron and hole current density components:**

$$J_n = q\sqrt{\frac{D_n}{\tau_{n0}}}\frac{n_i^2}{N_a}\left[e^{\frac{qV_a}{k_BT}} - 1\right]$$

$$J_p = q\sqrt{\frac{D_p}{\tau_{p0}}}\frac{n_i^2}{N_d}\left[e^{\frac{qV_a}{k_BT}} - 1\right]$$

2. **Solve for N_a using the given $J_n = 20\,\mathbf{A/cm}^2$:**

$$N_a = \frac{q\sqrt{\frac{D_n}{\tau_{n0}}}n_i^2\left[e^{\frac{qV_a}{k_BT}} - 1\right]}{J_n}$$

$$N_a \approx \frac{(1.602\times10^{-19})\times\sqrt{\frac{25}{5\times10^{-7}}}\times(1.5\times10^{10})^2\times(2.35\times10^{10})}{20} \approx 1.01\times10^{15}\,\mathrm{cm}^{-3}$$

3. **Solve for N_d using the given $J_p = 5\,\mathbf{A/cm}^2$:**

$$N_d = \frac{q\sqrt{\frac{D_p}{\tau_{p0}}}n_i^2\left[e^{\frac{qV_a}{k_BT}} - 1\right]}{J_p}$$

$$N_d \approx \frac{(1.602\times10^{-19})\times\sqrt{\frac{10}{5\times10^{-7}}}\times(1.5\times10^{10})^2\times(2.35\times10^{10})}{5} \approx 2.55\times10^{15}\,\mathrm{cm}^{-3}$$

Practice Problem 4

For a pn junction diode assuming the total current is conducted by the electric field far from the junction, calculate the electric field. Assume the applied forward bias voltage is 0.65 V and temperature is 300 K. Other diode parameters are given in Practice Problem 2.

Solution:

1. **Calculate the total current density J using the diode equation and the J_s from Practice Problem 2:**

$$J = J_s\left(e^{\frac{qV_a}{k_BT}} - 1\right) \approx 4.16\times10^{-11}\times\left(e^{25.1} - 1\right) \approx 4.16\times10^{-11}\times1.12\times10^{10} \approx 4.66\times10^{-1}\,\mathrm{A/cm}^2$$

2. **Assume that far from the junction in the n-region, the total current is approximately equal to the electron drift current:**

$$J \approx J_{n,\mathrm{drift}} = q\mu_n N_d E$$

3. **Solve for the electric field E:**

$$E = \frac{J}{q\mu_n N_d} = \frac{4.66\times10^{-1}\,\mathrm{A/cm}^2}{(1.602\times10^{-19}\,\mathrm{C})\times25\,\mathrm{cm}^2/\mathrm{s}\times10^{16}\,\mathrm{cm}^{-3}} \approx 1.16\times10^{3}\,\mathrm{V/cm}$$

Discussion Week 11

Practice Problem 1

Consider a silicon pn junction diode at $T = 300\,\mathrm{K}$. Assume the doping concentration in the n-region is $N_d = 10^{16}\,\mathrm{cm}^{-3}$ and the doping concentration in the p-region is $N_a = 6 \times 10^{15}\,\mathrm{cm}^{-3}$, and assume that a forward bias of $0.60\,\mathrm{V}$ is applied to the pn junction.

Calculate the minority carrier concentrations at the edge of the space charge regions in a forward-biased pn junction. $n_i = 1.5 \times 10^{10}\,\mathrm{cm}^{-3}$.

Solution:

1. **Calculate the thermal-equilibrium minority carrier concentrations:**

$$n_{p0} = \frac{n_i^2}{N_a} = \frac{(1.5 \times 10^{10})^2}{6 \times 10^{15}} = 3.75 \times 10^4\,\mathrm{cm}^{-3}$$

$$p_{n0} = \frac{n_i^2}{N_d} = \frac{(1.5 \times 10^{10})^2}{10^{16}} = 2.25 \times 10^4\,\mathrm{cm}^{-3}$$

2. **Apply the forward bias formulas:**

$$n_p(-x_p) = n_{p0} \exp\left(\frac{qV_a}{k_B T}\right) = 3.75 \times 10^4 \times e^{0.60/0.0259} \approx 3.75 \times 10^4 \times 2.35 \times 10^{10} \approx 8.81 \times 10^{14}\,\mathrm{cm}^{-3}$$

$$p_n(x_n) = p_{n0} \exp\left(\frac{qV_a}{k_B T}\right) = 2.25 \times 10^4 \times e^{0.60/0.0259} \approx 2.25 \times 10^4 \times 2.35 \times 10^{10} \approx 5.29 \times 10^{14}\,\mathrm{cm}^{-3}$$

Practice Problem 2

Determine the ideal reverse-saturation current density in a silicon pn junction at $T = 300\,\mathrm{K}$.

Consider the following parameters in a silicon pn junction:

$$\begin{aligned} N_a &= N_d = 10^{16}\,\mathrm{cm}^{-3} & n_i &= 1.5 \times 10^{10}\,\mathrm{cm}^{-3} \\ D_n &= 25\,\mathrm{cm}^2/\mathrm{s} & \tau_{p0} &= \tau_{n0} = 5 \times 10^{-7}\,\mathrm{s} \\ D_p &= 10\,\mathrm{cm}^2/\mathrm{s} & \epsilon_r &= 11.7 \end{aligned}$$

Solution:

1. **Use the formula for the reverse saturation current density J_s:**

$$J_s = q n_i^2 \left(\frac{1}{N_a} \sqrt{\frac{D_n}{\tau_{n0}}} + \frac{1}{N_d} \sqrt{\frac{D_p}{\tau_{p0}}} \right)$$

2. **Plug in the given values:**

$$J_s = (1.602 \times 10^{-19}\,\mathrm{C}) \times (1.5 \times 10^{10})^2 \times \left(\frac{1}{10^{16}} \times \sqrt{\frac{25}{5 \times 10^{-7}}} + \frac{1}{10^{16}} \times \sqrt{\frac{10}{5 \times 10^{-7}}} \right)$$

$$= 1.602 \times 10^{-19} \times 2.25 \times 10^{20} \times \left(\frac{1}{10^{16}} \times 7.071 \times 10^3 + \frac{1}{10^{16}} \times 4.472 \times 10^3 \right)$$

$$= 1.602 \times 10^{-19} \times 2.25 \times 10^{20} \times 1.153 \times 10^{-12} \approx 4.16 \times 10^{-11}\,\mathrm{A/cm}^2$$

Practice Problem 3

Consider a silicon pn junction diode at $T = 300\,\mathrm{K}$. Find the doping concentrations N_a and N_d required such that $J_n = 20\,\mathrm{A/cm^2}$ and $J_p = 5\,\mathrm{A/cm^2}$ at $V_a = 0.65\,\mathrm{V}$. Assume the remaining semiconductor parameters are as given in Practice Problem 2.

Solution:

1. **Use the formulas for the electron and hole current density components:**

$$J_n = q\sqrt{\frac{D_n}{\tau_{n0}}}\frac{n_i^2}{N_a}\left[e^{\frac{qV_a}{k_BT}} - 1\right]$$

$$J_p = q\sqrt{\frac{D_p}{\tau_{p0}}}\frac{n_i^2}{N_d}\left[e^{\frac{qV_a}{k_BT}} - 1\right]$$

2. **Solve for N_a using the given $J_n = 20\,\mathbf{A/cm^2}$:**

$$N_a = \frac{q\sqrt{\frac{D_n}{\tau_{n0}}}n_i^2\left[e^{\frac{qV_a}{k_BT}} - 1\right]}{J_n} \approx 1.01 \times 10^{15}\,\mathrm{cm^{-3}}$$

3. **Solve for N_d using the given $J_p = 5\,\mathbf{A/cm^2}$:**

$$N_d = \frac{q\sqrt{\frac{D_p}{\tau_{p0}}}n_i^2\left[e^{\frac{qV_a}{k_BT}} - 1\right]}{J_p} \approx 2.55 \times 10^{15}\,\mathrm{cm^{-3}}$$

Practice Problem 4

For a pn junction diode assuming the total current is conducted by the electric field far from the junction, calculate the electric field. Assume the applied forward bias voltage is $0.65\,\mathrm{V}$ and temperature is $300\,\mathrm{K}$. Other diode parameters are given in Practice Problem 2.

Solution:

1. **Calculate the total current density J using the diode equation and the J_s from Practice Problem 2:**

$$J = J_s\left(e^{\frac{qV_a}{k_BT}} - 1\right) \approx 4.16\times10^{-11}\times\left(e^{25.1} - 1\right) \approx 4.16\times10^{-11}\times1.12\times10^{10} \approx 4.66\times10^{-1}\,\mathrm{A/cm^2}$$

2. **Assume that far from the junction in the n-region, the total current is approximately equal to the electron drift current:**

$$J \approx J_{n,\mathrm{drift}} = q\mu_n N_d E$$

3. **Solve for the electric field E:**

$$E = \frac{J}{q\mu_n N_d} = \frac{4.66 \times 10^{-1}\,\mathrm{A/cm^2}}{(1.602 \times 10^{-19}\,\mathrm{C}) \times 25\,\mathrm{cm^2/s} \times 10^{16}\,\mathrm{cm^{-3}}} \approx 1.16 \times 10^{3}\,\mathrm{V/cm}$$

The Drift and Diffusion Current

Problem 1

An n-type silicon sample has a donor concentration of $N_d = 10^{16}\,\text{cm}^{-3}$ at room temperature ($T = 300\,\text{K}$). Assume the electron mobility is $\mu_n = 1350\,\text{cm}^2/\text{V}\cdot\text{s}$.

1. **Derive the expression for drift current density $J_{\textbf{drift}}$.**
2. **If an electric field of $E = 100\,\textbf{V/cm}$ is applied, calculate the drift current density.**
3. **Discuss how $J_{\textbf{drift}}$ changes if the mobility is reduced due to increased doping to $N_d = 10^{18}\,\textbf{cm}^{-3}$ (assume $\mu_n = 400\,\textbf{cm}^2/\textbf{V}\cdot\textbf{s}$).**

Solution:

1. **Expression for Drift Current Density:**

$$J_{\text{drift}} = nq\mu_n E$$

where:

- $n \approx N_d$ for n-type semiconductor.
- q is the elementary charge ($1.602 \times 10^{-19}\,\text{C}$).
- μ_n is the electron mobility.
- E is the applied electric field.

2. **Calculate Drift Current Density:**

$$J_{\text{drift}} = (10^{16}\,\text{cm}^{-3}) \times (1.602 \times 10^{-19}\,\text{C}) \times (1350\,\text{cm}^2/\text{V}\cdot\text{s}) \times (100\,\text{V/cm})$$

$$J_{\text{drift}} = 1.602 \times 10^{-19} \times 10^{16} \times 1350 \times 100 = 2.166 \times 10^{-6}\,\text{A/cm}^2$$

3. **Effect of Increased Doping:** When doping increases to $N_d = 10^{18}\,\text{cm}^{-3}$ and mobility decreases to $\mu_n = 400\,\text{cm}^2/\text{V}\cdot\text{s}$:

$$J_{\text{drift}} = (10^{18}\,\text{cm}^{-3}) \times (1.602 \times 10^{-19}\,\text{C}) \times (400\,\text{cm}^2/\text{V}\cdot\text{s}) \times (100\,\text{V/cm})$$

$$J_{\text{drift}} = 1.602 \times 10^{-19} \times 10^{18} \times 400 \times 100 = 6.408 \times 10^{-1}\,\text{A/cm}^2$$

Discussion: Increasing doping concentration N_d from 10^{16} to $10^{18}\,\text{cm}^{-3}$ increases carrier concentration by a factor of 100. Although mobility decreases from 1350 to 400 $\text{cm}^2/\text{V}\cdot\text{s}$ (a factor of approximately 3.375 reduction), the overall drift current density J_{drift} increases significantly due to the substantial increase in carrier concentration. The final J_{drift} increases from 2.166×10^{-6} to $6.408 \times 10^{-1}\,\text{A/cm}^2$, demonstrating the dominant effect of carrier concentration over mobility in this scenario.

Problem 2

A p-type silicon sample has a hole concentration of $10^{17}\,\text{cm}^{-3}$. The diffusion coefficient for holes is $D_p = 12\,\text{cm}^2/\text{s}$.

1. **If the hole concentration varies linearly from $10^{17}\,\textbf{cm}^{-3}$ to $5\times10^{16}\,\textbf{cm}^{-3}$ over a distance of $50\,\mu\textbf{m}$, calculate the diffusion current density.**

2. **Compare this to the drift current density if an electric field of 10^3 V/cm is applied and the mobility for p-type silicon is $\mu_p = 450\,\mathrm{cm}^2/\mathrm{V}\cdot\mathrm{s}$.**

Solution:

1. **Calculate the Diffusion Current Density:**

$$\frac{dp}{dx} = \frac{5\times10^{16} - 10^{17}}{50\times10^{-4}\,\mathrm{cm}} = \frac{-5\times10^{16}}{5\times10^{-3}} = -10^{19}\,\mathrm{cm}^{-4}$$

$$J_{p,\mathrm{diff}} = -qD_p\frac{dp}{dx} = -(1.602\times10^{-19}\,\mathrm{C})\times 12\times(-10^{19}\,\mathrm{cm}^{-4}) = 19.224\,\mathrm{A/cm}^2$$

2. **Calculate the Drift Current Density:**

$$J_{p,\mathrm{drift}} = pq\mu_p E = (10^{17}\,\mathrm{cm}^{-3})\times(1.602\times10^{-19}\,\mathrm{C})\times(450\,\mathrm{cm}^2/\mathrm{V}\cdot\mathrm{s})\times(10^3\,\mathrm{V/cm})$$

$$J_{p,\mathrm{drift}} = 10^{17}\times1.602\times10^{-19}\times450\times10^3 = 7.209\,\mathrm{A/cm}^2$$

Comparison: The diffusion current density $J_{p,\mathrm{diff}} \approx 19.224\,\mathrm{A/cm}^2$ is significantly larger than the drift current density $J_{p,\mathrm{drift}} \approx 7.209\,\mathrm{A/cm}^2$ under the given conditions.

Problem 3

A silicon sample has a non-uniform electron concentration given by $n(x) = 5\times10^{15}\,\mathrm{cm}^{-3} + 3\times10^{18}\,\mathrm{cm}^{-4}\cdot x$. The sample is also subjected to an electric field of $E = 50\,\mathrm{V/cm}$. Assume $\mu_n = 1350\,\mathrm{cm}^2/\mathrm{V}\cdot\mathrm{s}$ and diffusion coefficient $D_n = 35\,\mathrm{cm}^2/\mathrm{s}$.

1. **Derive the total current density $J_{\mathbf{total}} = J_{\mathbf{drift}} + J_{\mathbf{diff}}$.**
2. **Calculate $J_{\mathbf{drift}}$, $J_{\mathbf{diff}}$, and $J_{\mathbf{total}}$ at $x = 10\,\mu$m.**
3. **Discuss the relative contributions of drift and diffusion currents to the total current.**

Solution:

1. **Derive the Total Current Density:**

$$J_{\mathrm{drift}} = q\mu_n n(x)E$$

$$J_{\mathrm{diff}} = qD_n\frac{dn}{dx}$$

$$J_{\mathrm{total}} = J_{\mathrm{drift}} + J_{\mathrm{diff}} = q\mu_n n(x)E + qD_n\frac{dn}{dx}$$

Given $n(x) = 5\times10^{15} + 3\times10^{18}x$, the derivative is:

$$\frac{dn}{dx} = 3\times10^{18}\,\mathrm{cm}^{-4}$$

Thus:

$$J_{\mathrm{total}} = q\mu_n\left(5\times10^{15} + 3\times10^{18}x\right)E + qD_n\times3\times10^{18}$$

2. **Calculate $J_{\textbf{drift}}$, $J_{\textbf{diff}}$, and $J_{\textbf{total}}$ at $x = 10\,\mu$m ($x = 10^{-3}$ cm):**

$$J_{\text{drift}} = (1.602 \times 10^{-19}\,\text{C}) \times 1350 \times \left(5 \times 10^{15} + 3 \times 10^{18} \times 10^{-3}\right) \times 50$$
$$= 1.602 \times 10^{-19} \times 1350 \times (5 \times 10^{15} + 3 \times 10^{15}) \times 50$$
$$= 1.602 \times 10^{-19} \times 1350 \times 8 \times 10^{15} \times 50 \approx 8.656 \times 10^{-2}\,\text{A/cm}^2$$
$$J_{\text{diff}} = (1.602 \times 10^{-19}\,\text{C}) \times 35 \times 3 \times 10^{18} \approx 1.682 \times 10^{-2}\,\text{A/cm}^2$$
$$J_{\text{total}} = 8.656 \times 10^{-2} + 1.682 \times 10^{-2} \approx 1.0358 \times 10^{-1}\,\text{A/cm}^2$$

3. **Relative Contributions:** At $x = 10\,\mu$m, the drift current density $J_{\text{drift}} \approx 8.656 \times 10^{-2}\,\text{A/cm}^2$ is larger than the diffusion current density $J_{\text{diff}} \approx 1.682 \times 10^{-2}\,\text{A/cm}^2$, but both contribute significantly to the total current. The drift current remains the dominant component, but diffusion also plays a notable role due to the non-uniform carrier concentration.

PN Junction

Problem 1

Given the width of the depletion region (W) as:

$$W = \sqrt{\frac{2\varepsilon_s}{q}\frac{(N_A + N_D)}{N_A N_D}V_{bi}}$$

where N_A and N_D are the doping concentrations, V_{bi} is the built-in potential, and ε_s is the permittivity of the semiconductor.

A silicon PN junction has $N_A = 10^{16}\,\text{cm}^{-3}$, $N_D = 10^{15}\,\text{cm}^{-3}$, and the intrinsic carrier concentration $n_i = 1.5 \times 10^{10}\,\text{cm}^{-3}$ at 300 K. The relative permittivity of silicon is $\epsilon_s = 11.7\epsilon_0$ and $\epsilon_0 = 8.85 \times 10^{-14}\,\text{F/cm}$.

1. **Calculate the built-in potential V_{bi}:**

$$V_{bi} = \frac{k_B T}{q} \ln\left(\frac{N_A N_D}{n_i^2}\right)$$

Given $\frac{k_B T}{q} \approx 0.0259\,\text{V}$ at 300 K:

$$V_{bi} = 0.0259 \times \ln\left(\frac{10^{16} \times 10^{15}}{(1.5 \times 10^{10})^2}\right) \approx 0.6525\,\text{V}$$

2. **Calculate the depletion region width at equilibrium:**

$$W = \sqrt{\frac{2 \times 11.7 \times 8.85 \times 10^{-14}\,\text{F/cm}}{1.602 \times 10^{-19}\,\text{C}} \times \frac{10^{16} + 10^{15}}{10^{16} \times 10^{15}} \times 0.6525\,\text{V}}$$
$$W \approx 1.024 \times 10^{-4}\,\text{cm} = 1.024\,\mu\text{m}$$

Problem 2

The expression for the maximum electric field in the depletion region is:

$$E_{\text{max}} = -\frac{qN_A W_p}{\epsilon_s} = \frac{qN_D W_n}{\epsilon_s}$$

where W_p and W_n are the widths of the depletion region on the p and n sides, respectively.

1. **Calculate the maximum electric field at equilibrium:**

$$W_p = \frac{N_D}{N_A + N_D} W = \frac{10^{15}}{10^{16} + 10^{15}} \times 1.024\,\mu\text{m} \approx 0.0931\,\mu\text{m}$$

$$W_n = \frac{N_A}{N_A + N_D} W = \frac{10^{16}}{10^{16} + 10^{15}} \times 1.024\,\mu\text{m} \approx 0.9309\,\mu\text{m}$$

$$E_{\max} = \frac{qN_D W_n}{\epsilon_s} = \frac{1.602 \times 10^{-19} \times 10^{15} \times 0.9309 \times 10^{-4}}{11.7 \times 8.85 \times 10^{-14}} \approx 1.44 \times 10^4\,\text{V/cm}$$

2. **Show that the built-in potential V_{bi} is the integral of the electric field across the depletion region:**

$$V_{bi} = \int_{-W_p}^{W_n} E(x)\,dx$$

Given the linear variation of $E(x)$, the integral confirms that V_{bi} is consistent with the derived expression.

Problem 4 (HW9)

A silicon pn junction diode is fabricated with the following parameters: $N_A = 10^{16}\,\text{cm}^{-3}$, $N_D = 10^{15}\,\text{cm}^{-3}$. The intrinsic carrier concentration is $n_i = 1.5 \times 10^{10}\,\text{cm}^{-3}$ at 300 K. The electron and hole mobility are $\mu_n = 1350\,\text{cm}^2/\text{V}\cdot\text{s}$ and $\mu_p = 480\,\text{cm}^2/\text{V}\cdot\text{s}$ respectively. The junction cross-sectional area is $0.01\,\text{cm}^2$. The electron lifetime in the p-region is $\tau_n = 1\,\mu\text{s}$, and the hole lifetime in the n-region is $\tau_p = 2\,\mu\text{s}$.

1. **Calculate the diffusion coefficient for electrons and holes (based on the Einstein relationship).**

2. **Calculate the built-in voltage across the depletion region.**

3. **Calculate the current at biases of -0.1 V, 0.3 V, and 0.7 V.**

Solution:

1. **Calculate the Diffusion Coefficients:** The Einstein relationship relates the diffusion coefficient D and mobility μ:

$$D = \mu \frac{k_B T}{q}$$

Given $\frac{k_B T}{q} \approx 0.0259\,\text{V}$ at 300 K:

$$D_n = 1350 \times 0.0259 \approx 34.965\,\text{cm}^2/\text{s}$$

$$D_p = 480 \times 0.0259 \approx 12.432\,\text{cm}^2/\text{s}$$

2. **Calculate the Built-in Voltage V_{bi}:**

$$V_{bi} = \frac{k_B T}{q} \ln\left(\frac{N_A N_D}{n_i^2}\right) = 0.0259 \times \ln\left(\frac{10^{16} \times 10^{15}}{(1.5 \times 10^{10})^2}\right) \approx 0.6525\,\text{V}$$

3. **Calculate the Current at Different Biases:**

(a) **Calculate the Reverse Saturation Current I_s:**

$$L_n = \sqrt{D_n \tau_n} = \sqrt{34.965 \times 1 \times 10^{-6}} \approx 5.913 \times 10^{-3}\,\text{cm}$$

$$L_p = \sqrt{D_p \tau_p} = \sqrt{12.432 \times 2 \times 10^{-6}} \approx 4.986 \times 10^{-3}\,\text{cm}$$

$$I_s = Aqn_i^2 \left(\frac{D_n}{L_n N_A} + \frac{D_p}{L_p N_D} \right)$$

$$I_s = 0.01 \times 1.602 \times 10^{-19} \times (1.5 \times 10^{10})^2 \times \left(\frac{34.965}{5.913 \times 10^{-3} \times 10^{16}} + \frac{12.432}{4.986 \times 10^{-3} \times 10^{15}} \right)$$

$$I_s \approx 0.01 \times 1.602 \times 10^{-19} \times 2.25 \times 10^{20} \times \left(1.18 \times 10^{-12} + 2.50 \times 10^{-12}\right) \approx 1.116 \times 10^{-13}\,\text{A}$$

(b) **Apply the Diode Equation:**

$$I = I_s \left(e^{\frac{qV_a}{k_B T}} - 1 \right)$$

- **For $V_a = -0.1$ V:**

$$I = 1.116 \times 10^{-13} \times \left(e^{-0.1/0.0259} - 1\right) \approx -1.092 \times 10^{-13}\,\text{A}$$

- **For $V_a = 0.3$ V:**

$$I = 1.116 \times 10^{-13} \times \left(e^{0.3/0.0259} - 1\right) \approx 1.227 \times 10^{-8}\,\text{A}$$

- **For $V_a = 0.7$ V:**

$$I = 1.116 \times 10^{-13} \times \left(e^{0.7/0.0259} - 1\right) \approx 2.35 \times 10^{-6}\,\text{A}$$

Practice Problem 4

For a pn junction diode assuming the total current is conducted by the electric field far from the junction, calculate the electric field. Assume the applied forward bias voltage is 0.65 V and temperature is 300 K. Other diode parameters are given in Practice Problem 2.

Solution:

1. **Calculate the total current density J using the diode equation and the J_s from Practice Problem 2:**

$$J = J_s \left(e^{\frac{qV_a}{k_B T}} - 1 \right) \approx 4.16 \times 10^{-11} \times \left(e^{25.1} - 1\right) \approx 4.66 \times 10^{-1}\,\text{A/cm}^2$$

2. **Assume that far from the junction in the n-region, the total current is approximately equal to the electron drift current:**

$$J \approx J_{n,\text{drift}} = q\mu_n N_d E$$

3. **Solve for the electric field E:**

$$E = \frac{J}{q\mu_n N_d} = \frac{4.66 \times 10^{-1}}{1.602 \times 10^{-19} \times 25 \times 10^{16}} \approx 1.16 \times 10^{3}\,\text{V/cm}$$

The Drift and Diffusion Current

Problem 1

An n-type silicon sample has a donor concentration of $N_d = 10^{16}\,\text{cm}^{-3}$ at room temperature ($T = 300\,\text{K}$). Assume the electron mobility is $\mu_n = 1350\,\text{cm}^2/\text{V}\cdot\text{s}$.

1. **Derive the expression for drift current density J_{drift}.**
2. **If an electric field of $E = 100\,\text{V/cm}$ is applied, calculate the drift current density.**
3. **Discuss how J_{drift} changes if the mobility is reduced due to increased doping to $N_d = 10^{18}\,\text{cm}^{-3}$ (assume $\mu_n = 400\,\text{cm}^2/\text{V}\cdot\text{s}$).**

Solution:

1. **Expression for Drift Current Density:**

$$J_{\text{drift}} = nq\mu_n E$$

where:

- $n \approx N_d$ for n-type semiconductor.
- q is the elementary charge ($1.602 \times 10^{-19}\,\text{C}$).
- μ_n is the electron mobility.
- E is the applied electric field.

2. **Calculate Drift Current Density:**

$$J_{\text{drift}} = (10^{16}\,\text{cm}^{-3}) \times (1.602 \times 10^{-19}\,\text{C}) \times (1350\,\text{cm}^2/\text{V}\cdot\text{s}) \times (100\,\text{V/cm})$$

$$J_{\text{drift}} = 1.602 \times 10^{-19} \times 10^{16} \times 1350 \times 100 = 2.166 \times 10^{-6}\,\text{A/cm}^2$$

3. **Effect of Increased Doping:** When doping increases to $N_d = 10^{18}\,\text{cm}^{-3}$ and mobility decreases to $\mu_n = 400\,\text{cm}^2/\text{V}\cdot\text{s}$:

$$J_{\text{drift}} = (10^{18}\,\text{cm}^{-3}) \times (1.602 \times 10^{-19}\,\text{C}) \times (400\,\text{cm}^2/\text{V}\cdot\text{s}) \times (100\,\text{V/cm})$$

$$J_{\text{drift}} = 1.602 \times 10^{-19} \times 10^{18} \times 400 \times 100 = 6.408 \times 10^{-1}\,\text{A/cm}^2$$

Discussion: Increasing doping concentration N_d from 10^{16} to $10^{18}\,\text{cm}^{-3}$ increases carrier concentration by a factor of 100. Although mobility decreases from 1350 to 400 $\text{cm}^2/\text{V}\cdot\text{s}$ (a factor of approximately 3.375 reduction), the overall drift current density J_{drift} increases significantly due to the substantial increase in carrier concentration. The final J_{drift} increases from 2.166×10^{-6} to $6.408 \times 10^{-1}\,\text{A/cm}^2$, demonstrating the dominant effect of carrier concentration over mobility in this scenario.

Problem 2

A p-type silicon sample has a hole concentration of $10^{17}\,\text{cm}^{-3}$. The diffusion coefficient for holes is $D_p = 12\,\text{cm}^2/\text{s}$.

1. **If the hole concentration varies linearly from $10^{17}\,\text{cm}^{-3}$ to $5\times10^{16}\,\text{cm}^{-3}$ over a distance of $50\,\mu\text{m}$, calculate the diffusion current density.**

2. **Compare this to the drift current density if an electric field of 10^3 V/cm is applied and the mobility for p-type silicon is $\mu_p = 450\,\mathbf{cm}^2/\mathbf{V}\cdot\mathbf{s}$.**

Solution:

1. **Calculate the Diffusion Current Density:**

$$\frac{dp}{dx} = \frac{5\times 10^{16} - 10^{17}}{50\times 10^{-4}\,\mathrm{cm}} = \frac{-5\times 10^{16}}{5\times 10^{-3}} = -10^{19}\,\mathrm{cm}^{-4}$$

$$J_{p,\mathrm{diff}} = -qD_p\frac{dp}{dx} = -(1.602\times 10^{-19}\,\mathrm{C})\times 12\times(-10^{19}\,\mathrm{cm}^{-4}) = 19.224\,\mathrm{A/cm}^2$$

2. **Calculate the Drift Current Density:**

$$J_{p,\mathrm{drift}} = pq\mu_p E = (10^{17}\,\mathrm{cm}^{-3})\times(1.602\times 10^{-19}\,\mathrm{C})\times(450\,\mathrm{cm}^2/\mathrm{V}\cdot\mathrm{s})\times(10^3\,\mathrm{V/cm})$$

$$J_{p,\mathrm{drift}} = 10^{17}\times 1.602\times 10^{-19}\times 450\times 10^3 = 7.209\,\mathrm{A/cm}^2$$

Comparison: The diffusion current density $J_{p,\mathrm{diff}} \approx 19.224\,\mathrm{A/cm}^2$ is significantly larger than the drift current density $J_{p,\mathrm{drift}} \approx 7.209\,\mathrm{A/cm}^2$ under the given conditions.

Problem 3

A silicon sample has a non-uniform electron concentration given by $n(x) = 5\times 10^{15}\,\mathrm{cm}^{-3} + 3\times 10^{18}\,\mathrm{cm}^{-4}\cdot x$. The sample is also subjected to an electric field of $E = 50\,\mathrm{V/cm}$. Assume $\mu_n = 1350\,\mathrm{cm}^2/\mathrm{V}\cdot\mathrm{s}$ and diffusion coefficient $D_n = 35\,\mathrm{cm}^2/\mathrm{s}$.

1. **Derive the total current density $J_{\mathbf{total}} = J_{\mathbf{drift}} + J_{\mathbf{diff}}$.**
2. **Calculate $J_{\mathbf{drift}}$, $J_{\mathbf{diff}}$, and $J_{\mathbf{total}}$ at $x = 10\,\mu$m.**
3. **Discuss the relative contributions of drift and diffusion currents to the total current.**

Solution:

1. **Derive the Total Current Density:**

$$J_{\mathrm{drift}} = q\mu_n n(x)E$$

$$J_{\mathrm{diff}} = qD_n\frac{dn}{dx}$$

$$J_{\mathrm{total}} = J_{\mathrm{drift}} + J_{\mathrm{diff}} = q\mu_n n(x)E + qD_n\frac{dn}{dx}$$

Given $n(x) = 5\times 10^{15} + 3\times 10^{18}x$, the derivative is:

$$\frac{dn}{dx} = 3\times 10^{18}\,\mathrm{cm}^{-4}$$

Thus:

$$J_{\mathrm{total}} = q\mu_n\left(5\times 10^{15} + 3\times 10^{18}x\right)E + qD_n\times 3\times 10^{18}$$

2. **Calculate at $x = 10\,\mu\mathrm{m} = 10^{-3}$ cm:**

$$J_{\text{drift}} = (1.602 \times 10^{-19}\,\mathrm{C}) \times 1350 \times \left(5 \times 10^{15} + 3 \times 10^{18} \times 10^{-3}\right) \times 50$$

$$= 1.602 \times 10^{-19} \times 1350 \times (5 \times 10^{15} + 3 \times 10^{15}) \times 50$$

$$= 1.602 \times 10^{-19} \times 1350 \times 8 \times 10^{15} \times 50 \approx 8.656 \times 10^{-2}\,\mathrm{A/cm^2}$$

$$J_{\text{diff}} = (1.602 \times 10^{-19}\,\mathrm{C}) \times 35 \times 3 \times 10^{18} \approx 1.682 \times 10^{-2}\,\mathrm{A/cm^2}$$

$$J_{\text{total}} = 8.656 \times 10^{-2} + 1.682 \times 10^{-2} \approx 1.0358 \times 10^{-1}\,\mathrm{A/cm^2}$$

3. **Relative Contributions:** At $x = 10\,\mu$m, the drift current density $J_{\text{drift}} \approx 8.656 \times 10^{-2}\,\mathrm{A/cm^2}$ is larger than the diffusion current density $J_{\text{diff}} \approx 1.682 \times 10^{-2}\,\mathrm{A/cm^2}$, but both contribute significantly to the total current. The drift current remains the dominant component, but diffusion also plays a notable role due to the non-uniform carrier concentration.

PN Junction

Problem 1

Given the width of the depletion region (W) as:

$$W = \sqrt{\frac{2\varepsilon_s}{q} \frac{(N_A + N_D)}{N_A N_D} V_{bi}}$$

where N_A and N_D are the doping concentrations, V_{bi} is the built-in potential, and ε_s is the permittivity of the semiconductor.

A silicon PN junction has $N_A = 10^{16}\,\mathrm{cm^{-3}}$, $N_D = 10^{15}\,\mathrm{cm^{-3}}$, and the intrinsic carrier concentration $n_i = 1.5 \times 10^{10}\,\mathrm{cm^{-3}}$ at 300 K. The relative permittivity of silicon is $\epsilon_s = 11.7\epsilon_0$ and $\epsilon_0 = 8.85 \times 10^{-14}\,\mathrm{F/cm}$.

1. **Calculate the built-in potential V_{bi}:**

$$V_{bi} = \frac{k_B T}{q} \ln\left(\frac{N_A N_D}{n_i^2}\right) \approx 0.6525\,\mathrm{V}$$

2. **Calculate the depletion region width at equilibrium:**

$$W = \sqrt{\frac{2 \times 11.7 \times 8.85 \times 10^{-14}}{1.602 \times 10^{-19}} \times \frac{10^{16} + 10^{15}}{10^{16} \times 10^{15}} \times 0.6525} \approx 1.024 \times 10^{-4}\,\mathrm{cm} = 1.024\,\mu\mathrm{m}$$

Problem 2

The expression for the maximum electric field in the depletion region is:

$$E_{\max} = -\frac{qN_A W_p}{\epsilon_s} = \frac{qN_D W_n}{\epsilon_s}$$

where W_p and W_n are the widths of the depletion region on the p and n sides, respectively.

1. **Calculate the maximum electric field at equilibrium:**

$$W_p = \frac{N_D}{N_A + N_D} W = \frac{10^{15}}{10^{16} + 10^{15}} \times 1.024\,\mu\text{m} \approx 0.0931\,\mu\text{m}$$

$$W_n = \frac{N_A}{N_A + N_D} W = \frac{10^{16}}{10^{16} + 10^{15}} \times 1.024\,\mu\text{m} \approx 0.9309\,\mu\text{m}$$

$$E_{\text{max}} = \frac{qN_D W_n}{\epsilon_s} = \frac{1.602 \times 10^{-19} \times 10^{15} \times 0.9309 \times 10^{-4}}{11.7 \times 8.85 \times 10^{-14}} \approx 1.44 \times 10^4\,\text{V/cm}$$

2. **Show that the built-in potential V_{bi} is the integral of the electric field across the depletion region:**

$$V_{bi} = \int_{-W_p}^{W_n} E(x)\,dx$$

Given the linear variation of $E(x)$, the integral confirms that V_{bi} is consistent with the derived expression.

Problem 4 (HW9)

A silicon pn junction diode is fabricated with the following parameters: $N_A = 10^{16}\,\text{cm}^{-3}$, $N_D = 10^{15}\,\text{cm}^{-3}$. The intrinsic carrier concentration is $n_i = 1.5 \times 10^{10}\,\text{cm}^{-3}$ at 300 K. The electron and hole mobility are $\mu_n = 1350\,\text{cm}^2/\text{V}\cdot\text{s}$ and $\mu_p = 480\,\text{cm}^2/\text{V}\cdot\text{s}$ respectively. The junction cross-sectional area is $0.01\,\text{cm}^2$. The electron lifetime in the p-region is $\tau_n = 1\,\mu\text{s}$, and the hole lifetime in the n-region is $\tau_p = 2\,\mu\text{s}$.

1. **Calculate the diffusion coefficient for electrons and holes (based on the Einstein relationship).**

2. **Calculate the built-in voltage across the depletion region.**

3. **Calculate the current at biases of $-0.1\,\text{V}$, $0.3\,\text{V}$, and $0.7\,\text{V}$.**

Solution:

1. **Calculate the Diffusion Coefficients:** The Einstein relationship relates the diffusion coefficient D and mobility μ:

$$D = \mu \frac{k_B T}{q}$$

Given $\frac{k_B T}{q} \approx 0.0259\,\text{V}$ at 300 K:

$$D_n = 1350 \times 0.0259 \approx 34.965\,\text{cm}^2/\text{s}$$

$$D_p = 480 \times 0.0259 \approx 12.432\,\text{cm}^2/\text{s}$$

2. **Calculate the Built-in Voltage V_{bi}:**

$$V_{bi} = \frac{k_B T}{q} \ln\left(\frac{N_A N_D}{n_i^2}\right) = 0.0259 \times \ln\left(\frac{10^{16} \times 10^{15}}{(1.5 \times 10^{10})^2}\right) \approx 0.6525\,\text{V}$$

3. **Calculate the Current at Different Biases:**

(a) **Calculate the Reverse Saturation Current I_s:**

$$L_n = \sqrt{D_n \tau_n} = \sqrt{34.965 \times 1 \times 10^{-6}} \approx 5.913 \times 10^{-3}\,\text{cm}$$

$$L_p = \sqrt{D_p \tau_p} = \sqrt{12.432 \times 2 \times 10^{-6}} \approx 4.986 \times 10^{-3}\,\text{cm}$$

$$I_s = Aqn_i^2 \left(\frac{D_n}{L_n N_A} + \frac{D_p}{L_p N_D} \right)$$

$$I_s = 0.01 \times 1.602 \times 10^{-19} \times (1.5 \times 10^{10})^2 \times \left(\frac{34.965}{5.913 \times 10^{-3} \times 10^{16}} + \frac{12.432}{4.986 \times 10^{-3} \times 10^{15}} \right)$$

$$= 0.01 \times 1.602 \times 10^{-19} \times 2.25 \times 10^{20} \times \left(1.18 \times 10^{-12} + 2.50 \times 10^{-12}\right) \approx 1.116 \times 10^{-13}\,\text{A}$$

(b) **Apply the Diode Equation:**

$$I = I_s \left(e^{\frac{qV_a}{k_B T}} - 1 \right)$$

- **For $V_a = -0.1\,\mathbf{V}$:**

$$I = 1.116 \times 10^{-13} \times \left(e^{-0.1/0.0259} - 1\right) \approx -1.092 \times 10^{-13}\,\text{A}$$

- **For $V_a = 0.3\,\mathbf{V}$:**

$$I = 1.116 \times 10^{-13} \times \left(e^{0.3/0.0259} - 1\right) \approx 1.227 \times 10^{-8}\,\text{A}$$

- **For $V_a = 0.7\,\mathbf{V}$:**

$$I = 1.116 \times 10^{-13} \times \left(e^{0.7/0.0259} - 1\right) \approx 2.35 \times 10^{-6}\,\text{A}$$