ECE/PHY 235 - Introduction to Solid State Electronics

Refined Comprehensive Review Notes

Contact Information

Email: y.wang@wisc.edu
Course Site: `https://wang.ece.wisc.edu/`

Overview

These notes summarize essential topics in solid state electronics, focusing on concepts from band theory to semiconductor physics, doping, carrier transport, and p-n junction behavior. They include both conceptual explanations and relevant equations. Use these notes to:

- Understand the physics behind semiconductor behavior.
- Review key equations and their physical significance.
- Reinforce concepts with example scenarios and practice problems.

Fundamentals of Solid State Physics

Energy Bands

- In solids, individual atomic energy levels spread into energy *bands*.
- **Valence Band (VB):** Highest range of electron energies normally occupied.
- **Conduction Band (CB):** Range of higher energy states into which electrons can be excited to conduct electricity.
- **Band Gap (E_G):** Energy difference between CB minimum (E_C) and VB maximum (E_V).

Material Types:

- **Insulators:** Large E_G (few eV). At typical temperatures, negligible carriers in CB.
- **Semiconductors:** Smaller E_G (1 eV), allowing thermal excitation of electrons from VB to CB, increasing conductivity with T.
- **Metals:** Partially filled band or overlapping bands, abundant carriers even at $T = 0$, hence high conductivity.

Fermi-Dirac Distribution

- Probability an energy state at energy E is occupied by an electron:

$$f(E) = \frac{1}{1 + e^{(E-E_F)/(kT)}}$$

- At $T = 0$, $f(E)$ is a step function at $E = E_F$.
- At $T > 0$, $f(E)$ becomes smoother around E_F. States below E_F are mostly filled; states above E_F are mostly empty.

Intrinsic Semiconductors

- Pure semiconductors with no doping.
- Intrinsic carrier concentration (n_i):

$$n_i = \sqrt{N_C N_V} e^{-E_G/2kT}$$

- At equilibrium: $n = p = n_i$.
- **Intrinsic Fermi Level (E_i):** Often lies near mid-gap.

Effective Density of States

- N_C and N_V represent the effective density of states in the CB and VB:

$$N_C = 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2}, \quad N_V = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2}$$

Relation $np = n_i^2$

Under thermal equilibrium:

$$np = n_i^2$$

This holds for both intrinsic and extrinsic semiconductors, a key relation for analyzing carrier concentrations in doping scenarios.

Extrinsic Semiconductors and Doping

Doping Basics

- **n-type:** Adding donor impurities (e.g., P in Si) introduces extra electrons in the CB.
- **p-type:** Adding acceptor impurities (e.g., B in Si) creates holes in the VB.

Carrier Concentrations in Doped Semiconductors

If N_D (donor concentration) and N_A (acceptor concentration) are introduced:

- **n-type:** $n \approx N_D$, $p = \frac{n_i^2}{N_D}$
- **p-type:** $p \approx N_A$, $n = \frac{n_i^2}{N_A}$

Fermi Level Position

- For n-type: E_F moves closer to E_C.

$$E_F - E_C = kT \ln\left(\frac{N_D}{N_C}\right)$$

- For p-type: E_F moves closer to E_V.

$$E_V - E_F = kT \ln\left(\frac{N_A}{N_V}\right)$$

Carrier Transport Mechanisms

Electrons and Holes as Carriers

- Electrons: Negative charges in conduction band.
- Holes: Positive charges representing the absence of electrons in valence band.
- Under an electric field E, electrons drift opposite to E, holes drift in the same direction as E.

Drift Current

- Caused by an applied electric field $\mathcal{E}$.
- Electron drift current:

$$J_{e,\text{drift}} = ne\mu_e\mathcal{E}$$

- Hole drift current:

$$J_{h,\text{drift}} = pe\mu_h\mathcal{E}$$

- μ_e, μ_h: mobilities, influenced by scattering (phonons, impurities, defects).

Diffusion Current

- Caused by spatial carrier concentration gradients.
- Electrons:

$$J_{e,\text{diff}} = eD_e \frac{dn}{dx}$$

- Holes:

$$J_{h,\text{diff}} = -eD_h \frac{dp}{dx}$$

- Total current:

$$J = J_{\text{drift}} + J_{\text{diff}}$$

Einstein Relation

- Relates diffusion coefficient D and mobility μ:

$$\frac{D}{\mu} = \frac{kT}{q} = V_T$$

Continuity Equations

- Relate changes in carrier concentrations to current flow and generation/recombination:

$$\frac{\partial n}{\partial t} = \frac{1}{q}\frac{\partial J_n}{\partial x} - (R - G)$$

$$\frac{\partial p}{\partial t} = -\frac{1}{q}\frac{\partial J_p}{\partial x} - (R - G)$$

- At steady state ($\partial/\partial t = 0$), spatial gradients in current must balance generation and recombination.

p-n Junction: Formation and Behavior

Formation of the p-n Junction

- Bringing p-type and n-type materials together causes carriers (electrons from n-side, holes from p-side) to diffuse across the junction.
- Recombination near the junction forms a **depletion region** devoid of free carriers but rich in fixed ionized donors and acceptors.
- This depletion region sets up an electric field and a built-in potential V_{bi} that counteracts further diffusion.

Built-In Potential

- The built-in potential arises from the difference in doping on the two sides:

$$V_{bi} = \frac{kT}{q} \ln\left(\frac{N_A N_D}{n_i^2}\right)$$

Equilibrium Conditions

- At equilibrium, diffusion currents of electrons/holes are balanced by their respective drift currents created by the electric field in the depletion region.
- Net current = 0 under no external bias.

Junction Under Forward and Reverse Bias

- **Forward Bias (p-side positive w.r.t. n-side):** Lowers the barrier, allows increased injection of minority carriers across the junction, leading to a large forward current.
- **Reverse Bias (p-side negative w.r.t. n-side):** Increases the barrier, reduces carrier injection. Only a small saturation current flows due to minority carriers.

Minority Carrier Distributions

For forward bias V_a:

$$p(x_n) = p_{n0}\left(e^{V_a/V_T} - 1\right) + p_{n0},$$
$$n(x_p) = n_{p0}\left(e^{V_a/V_T} - 1\right) + n_{p0},$$

where p_{n0} and n_{p0} are equilibrium minority concentrations on the n and p sides, respectively.

In practice, we often write:

$$p(x_n) = p_{n0} e^{V_a/V_T}, \quad n(x_p) = n_{p0} e^{V_a/V_T}$$

assuming $e^{V_a/V_T} \gg 1$ for forward bias.

Current-Voltage (I-V) Characteristics

- The diode equation (Shockley equation):

$$I = I_s \left(e^{V_a/V_T} - 1\right)$$

- I_s: reverse saturation current, depends on doping and carrier lifetimes:

$$I_s = A\left(\frac{qD_p p_{n0}}{L_p} + \frac{qD_n n_{p0}}{L_n}\right)$$

Key Length Scales: Diffusion Lengths

- The minority carrier diffusion length $L = \sqrt{D\tau}$ indicates how far carriers diffuse before recombination.
- In analyzing p-n junctions, L_n (for electrons in p-side) and L_p (for holes in n-side) simplify the exponential carrier decay profiles away from the depletion region.

Summary of Key Equations and Concepts

Key Equations

- **Fermi-Dirac Distribution:** $f(E) = \frac{1}{1+e^{(E-E_F)/kT}}$
- **Intrinsic Carrier Concentration:** $n_i = \sqrt{N_C N_V} e^{-E_G/2kT}$
- **Mass Action Law:** $np = n_i^2$
- **Drift Current:** $J_{e,\text{drift}} = ne\mu_e E$, $J_{h,\text{drift}} = pe\mu_h E$
- **Diffusion Current:** $J_{e,\text{diff}} = eD_e \frac{dn}{dx}$, $J_{h,\text{diff}} = -eD_h \frac{dp}{dx}$
- **Einstein Relation:** $\frac{D}{\mu} = \frac{kT}{q}$
- **Built-In Potential:** $V_{bi} = \frac{kT}{q} \ln \left(\frac{N_A N_D}{n_i^2} \right)$
- **Diode Equation:** $I = I_s(e^{V_a/V_T} - 1)$

Important Concepts

- **Energy Band Diagrams:** Understand how E_F, E_C, and E_V shift with doping and bias.
- **Intrinsic vs. Extrinsic:** How doping changes E_F and carrier concentrations.
- **Carrier Transport:** Distinguish between drift (field-driven) and diffusion (gradient-driven) processes.
- **p-n Junction Behavior:** Depletion region formation, equilibrium conditions, and the impact of forward/reverse bias on carrier injection and current.
- **Recombination and Generation:** Underlying processes that maintain steady-state carrier distributions.

Visual Aids (Suggested)

- **Energy Band Diagram for a p-n Junction (No Bias):** - Show E_C, E_V, and E_F on both sides. - Depict the depletion region and bending of bands. - Indicate V_{bi} as the difference in conduction/valence bands across junction.
- **p-n Junction Under Forward Bias:** - Show reduced barrier height and increased minority carrier injection.
- **Drift vs. Diffusion:** - A simple sketch with concentration gradients (for diffusion) and direction of current under an electric field (for drift).

Example Problem

Example: For a silicon p-n junction with $N_a = 10^{17}$ cm^{-3}, $N_d = 2 \times 10^{16}$ cm^{-3}, given $\mu_n, \mu_p, \tau_n, \tau_p, A$, and n_i, find the diode current at $V_a = 0.65$ V.

Approach:

1. Calculate V_{bi} using doping and n_i.
2. Find minority carrier concentrations p_{n0}, n_{p0}.
3. Under forward bias, $p(x_n) = p_{n0}e^{V_a/V_T}$ and $n(x_p) = n_{p0}e^{V_a/V_T}$.
4. Determine D_n, D_p from $D = \mu V_T$ and $L_n = \sqrt{D_n\tau_n}$, $L_p = \sqrt{D_p\tau_p}$.
5. Compute I_s and then find I from the diode equation.

This type of problem consolidates understanding of the diode's I-V behavior, doping effects, and the use of fundamental equations.

Additional Practice Suggestions

- **True/False Checks:** - At $T = 0$, Fermi-Dirac distribution is a perfect step function. (True) - $np = n_i^2$ holds under thermal equilibrium. (True)
- **Band Diagram Identification:** - Sketch and label diagrams for intrinsic, n-type, and p-type semiconductors.
- **Continuity Equation Problems:** - Given generation/recombination rates, solve for steady-state carrier profiles.

Final Tips

- Familiarize yourself with the key equations and their physical significance, not just their forms.
- Practice with a variety of problems to be comfortable with applying these concepts under exam conditions.
- Remember the relationships between E_F, doping, and carrier concentrations.
- Use diagrams and conceptual reasoning to support equation-based solutions.