Highlights from ECE 235: Solid-state Physics

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Contents

1 EM wave

1.1 waves

- Traverse wave: oscillation $\perp$ propagation
- Longitudinal wave: oscillation $\|$ propagation
- $v = \lambda f$

1.2 EM wave function

$$\begin{cases} E_x = E_0 \sin(kz - \omega t) \\ B_y = B_0 \sin(kz - \omega t) \end{cases} \quad [1$$

where $k = \frac{2\pi}{\lambda}$ (wave number) , $\omega = 2\pi f = kc$ (dispersion relationship), B_0 : magnetic field amplitude, E_0 : electric field amplitude

1.3 EM Energy flux

Energy flux the energy transferred per unit area per unit time in the direction of wave propagation of an EM wave is defined by the Poynting vecter

$$\vec{S} \equiv \frac{\vec{E} \times \vec{B}}{\mu_0}. \quad [2$$

Where $\mu_0 = 1.25663706126e\text{-}6(N \cdot A^{-2})$ is the vacuum permeability.

- Intensity of EM wave is the magnitude of the Poynting vector:

$$I = \langle S \rangle = \frac{E_0^2}{377\Omega} \quad [3$$

where Ω is ohm. Very unorthodoxy I know, but hey we are in Engineering Hall.

- Specially, when EM wave is emitted from a point light source with power P ,

$$I = \frac{P}{4\pi r^2} = \frac{E_0^2}{377\Omega} \quad [4$$

2 Double slit interference

Consider a double-slit setup, where the first dark line is at an angle θ from the central bright line with a distance Y. Distance from light source to screen is L. Then by trignometry:

$$Y = L \tan \theta. \quad [5$$

When considering constructive/distructive interference,given the separation between two slits is d the path difference between the two slits is

$$\begin{gathered} m\lambda = d \sin \theta \text{ constructive} \\ \left(m + \frac{1}{2}\right)\lambda = d \sin \theta \text{ destructive} \quad m = 0, 1, 2... \end{gathered} \quad [6$$

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3 Photoelectric effect

- Energy of a photon

$$E_p = hf = \frac{hc}{\lambda} = \Phi + E_k \quad [7$$

where Φ is the work function of the material, E_k is the kinetic energy of the emitted electron at the surface of the material. $h = 6.26e\text{-}34$ is the Planck constant, $c = 3e\text{-}8$ m/s is the speed of light, f is the frequency of the photon, and λ is the wavelength of the photon.

- Motion for Photoelectric effect:

$$E_{k,m} + (\text{-}e)V_m = E_{k,d} + (\text{-}e)V_d \quad [8$$

Where $E_{k,m}$ is K.E at the metal surface, V_m is the voltage at the metal, $E_{k,d}$ is the K.E of the electron at the detector, and V_d is the voltage at the detector.

- stopping potential

$$eV_{\text{stop}} = \frac{hc}{\lambda} - \Phi$$ [9

the minimum potential required to stop the emitted electron.

- Threshold frequency& wavelength: set $E_k = 0$:

$$\begin{gathered} \Phi = hf_t = \frac{hc}{\lambda_t} \\ \Rightarrow f_t = \frac{\Phi}{h}, \quad \lambda_t = \frac{hc}{\Phi} \end{gathered}$$ [10

4 Blackbody radiation

- Stefan-Boltzmann law:

$$R = \sigma T^4.$$ [11

Where R is the **power radiated per unit area**, or surface energy density of radiation. T is temprature in Kelvin, $\sigma = 5.67e\text{-}8\left(W \cdot m^{\text{-}2} \cdot K^{\text{-}4}\right)$ is the Stefan-Boltzmann constant.

- Wien's displacement law:

$$\lambda_{\text{max}} T = b$$ [12

where $b = 2.89e\text{-}3(m \cdot K)$ is the Wien's constant, and λ_{max} is the wavelength at which the blackbody **radiation is maximum**, and T is the temprature in Kelvin of the blackbody.

- Rayleigh-Jeans law:

$$\begin{gathered} R(\lambda) = \frac{1}{4} c u(\lambda), \\ u(\lambda) = 8\pi k T \lambda^{-4} \end{gathered}$$ [13

WHere R is radiation power per unit area, or energy density, u is the energy density of radiation, c is the speed of light, and $k = 8.617e\text{-}5\ \text{eV/K} = 1.38e\text{-}23 J \cdot K^{-}1$ is the Boltzmann constatn This law is valid for long wavelength, but it diverges at short wavelength. **This equation is only good for long wavelength.**

- Planck's law:

$$u(\lambda) = \frac{8\pi hc\lambda^{-5}}{e^{hc/\lambda kT} - 1}$$ [14

where $k = 1.38e\text{-}23\left(J \cdot K^{\text{-}1}\right)$ is the Boltzmann constant, h is the Planck constant, T is the temprature in Kelvin of the blackbody.

4.1 Energy of radiation

For an ideal blackbody, the energy radiated within a certain wavelength range is found by integrating Equation 14 over the range of wavelength.

$$U = \int_{\lambda_1}^{\lambda_2} u(\lambda) \, \mathrm{d}\lambda$$ [15

- It is often times easier to use mid-point approximation to handle the above integration:

$$U \approx u(\lambda) \Delta\lambda$$ [16

Where $\lambda = \frac{\lambda_2 - \lambda_1}{2}$ is the mid-point of the wavelength range, and $\Delta\lambda$ is the width of the wavelength range.

5 Wavelike properties of particles

5.1 De broglie Hypothesis

$$f = \frac{E}{h} \quad , \lambda = \frac{h}{p}$$ [17

Where E is the total energy, p is the momentum, and λ is the wavelength of the particle. $h = 6.63e\text{-}34 J \cdot s$ is the Planck constant.

- For a particle of zero rest energy,

$$E = pc = hf = \frac{hc}{\lambda}, \tag{18}$$

  where p is the momentum of the particle.
- For a moving particle,

$$E = \frac{1}{2}mv^2 = \frac{p^2}{2m} \tag{19}$$

5.2 Wavefunction for particles

$$\Psi(x,t) = A\sin(kx - \omega t) \quad \text{or } Ae^{i(kx-\omega t)} \tag{20}$$

- probability density of the particle is

$$p(x,t) = |\Psi|^2 \equiv \Psi^* \Psi \tag{21}$$

5.3 Uncertainty Principle

$$\Delta x \Delta p \geq \frac{\hbar}{2}, \quad \Delta E \Delta t \geq \frac{\hbar}{2} \tag{22}$$

Where x is position, p is momentum, E is energy, t is time, and $\hbar = \frac{h}{2\pi} = 1.05e\text{-}34 J \cdot s$ is the reduced Planck constant.

5.3.1 Min. Energy of Particle in a box

$$E = \frac{p^2}{2m} \geq \frac{\hbar^2}{2mL^2} \tag{23}$$

6 Schrodinger's equation

6.1 Time-dependent Schrodinger's equation in 1D

1D Schrodinger's equation in position basis:

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t) + V(x,t)\Psi(x,t) = i\hbar\frac{\partial}{\partial t}\Psi(x,t) \tag{24}$$

6.2 Time-independent Schrodinger's equation in 1D

Via separation of variable, set $\Psi(x,t) = \psi(x)\varphi(t)$, and noticing $f = \frac{E}{h}$, we have

$$\begin{gathered} -\frac{\hbar}{2m}\frac{\mathrm{d}^2}{\mathrm{d}x^2}\psi(x) + V(x)\psi(x) = E\psi(x) \\ \text{time variation of wavefunction: } \varphi(t) = e^{-iEt/\hbar} \end{gathered} \tag{25}$$

- Probability density is thus simplified to

$$p(x) = |\Psi(x,t)|^2 = |\psi(x)|^2 \tag{26}$$

- Normalization condition:

$$\int_{-\infty}^{\infty} |\psi(x)|^2 \,\mathrm{d}x = 1 \tag{27}$$

6.3 Infinite potential well- particle in a box

- For a particle in a box of length L , where $V(x) = 0$ for $0 < x < L$, and $V(x) = \infty$ otherwise, the wavefunction is found by

$$\begin{gathered} -\frac{\hbar^2}{2m}\frac{\mathrm{d}^2}{\mathrm{d}x^2}\psi(x) = E\psi(x) \\ \Rightarrow \psi_n(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right). \end{gathered} \tag{28}$$

  Noticing boundary values, the following is obtained:

$$E_n = \frac{\hbar^2 k_n^2}{2m} = n^2 \frac{\hbar^2 \pi^2}{2mL^2} = n^2 E_1 \tag{29}$$

where $k = 2\frac{\pi}{\lambda}; k^2 = \left(\frac{p}{\hbar}\right)^2 = \frac{2mE}{\hbar^2}$

- Specially, the energy levels can be also expressed in terms of hc and mc^2:

$$E_1 = \frac{(hc)^2}{8mc^2 L^2}; \quad E_n = \frac{n^2 (hc)^2}{8mc^2 L^2} \tag{30}$$

- Normalization condition in box of length L:

$$\int_0^L |\psi(x)|^2 \, \mathrm{d}x = 1 \tag{31}$$

7 Appendix

1. Useful integral for probability of wavefunction

$$\int_{-\infty}^{\infty} e^{-a(x+b)^2} \, \mathrm{d}x = \sqrt{\frac{\pi}{a}} \tag{32}$$

2. Useful constants:
   - hc = 1240 eV nm.
   - For an electron: $mc^2 = 0.511\text{MeV} = 5.11e5$ eV