ECE/PHY 235 - Solid State Electronics: Comprehensive Exam Notes

Contents

Introduction

These comprehensive notes encapsulate the fundamental principles of semiconductor physics and PN junction operation essential for **ECE/PHY 235 - Introduction to Solid State Electronics**. They are meticulously structured to guide you through intrinsic and extrinsic semiconductors, carrier transport mechanisms, continuity equations, and PN junction characteristics. Each section is enriched with definitions, key formulas, and worked examples directly linked to the provided problem sets, ensuring practical applicability and thorough understanding for exam preparation.

1 Fundamentals of Semiconductors

1.1 Intrinsic Semiconductors

Definition: Intrinsic semiconductors are pure materials without any intentional doping. In such materials, the number of electrons in the conduction band (n) equals the number of holes in the valence band (p), maintaining electrical neutrality:

$$n = p = n_i$$

where n_i is the intrinsic carrier concentration.

1.1.1 Intrinsic Carrier Concentration (n_i)

Formula:

$$n_i = \sqrt{N_c N_v} \exp\left(-\frac{E_g}{2kT}\right)$$

where:

- N_c: Effective density of states in the conduction band.
- N_v: Effective density of states in the valence band.
- E_g: Bandgap energy.
- k: Boltzmann constant (8.617×10^{-5} eV/K).
- T: Temperature in Kelvin.

Example 1: Calculating Intrinsic Carrier Concentration
Problem: Calculate n_i for silicon at room temperature (300 K), given:

$$N_c = 2.81 \times 10^{19} \text{ cm}^{-3}, \quad N_v = 1.16 \times 10^{19} \text{ cm}^{-3}, \quad E_g = 1.10 \text{ eV}$$

Solution:

$$n_i = \sqrt{(2.81 \times 10^{19})(1.16 \times 10^{19})} \exp\left(-\frac{1.10}{2 \times 8.617 \times 10^{-5} \times 300}\right)$$

$$n_i = \sqrt{3.2596 \times 10^{38}} \exp(-21.333)$$

$$n_i \approx 1.0 \times 10^{10} \text{ cm}^{-3}$$

1.1.2 Conductivity (σ) and Resistivity (ρ)

Conductivity Formula:

$$\sigma = qn_i(\mu_e + \mu_h)$$

where:

- q: Elementary charge (1.602×10^{-19} C).
- μ_e: Electron mobility.
- μ_h: Hole mobility.

Resistivity Formula:

$$\rho = \frac{1}{\sigma}$$

Example 2: Calculating Intrinsic Conductivity and Resistivity

Problem: For silicon with $n_i = 1.0 \times 10^{10}$ cm^{-3}, $\mu_e = 1350$ cm^2/V $\cdot$ s, and $\mu_h = 450$ cm^2/V $\cdot$ s:

$$\sigma = qn_i(\mu_e + \mu_h) \approx 2.88 \times 10^{-6}\Omega^{-1}\text{cm}^{-1}$$

$$\rho = \frac{1}{2.88 \times 10^{-6}} \approx 3.47 \times 10^5\Omega\text{cm}$$

1.2 Extrinsic Semiconductors (Doping)

Definition: Extrinsic semiconductors are created by introducing impurities (dopants) into an intrinsic semiconductor to modify its electrical properties. Dopants can be donors (n-type) or acceptors (p-type).

1.2.1 n-type Semiconductors

Characteristics:

- Doping with donor impurities (e.g., phosphorus in Si) introduces excess electrons.
- Majority carriers: Electrons ($n \approx N_D$).
- Minority carriers: Holes ($p = \frac{n_i^2}{N_D}$).

Example 3: Calculating Carrier Concentrations in n-type Silicon

Problem: A 1 cm^3 silicon crystal doped with 1 ppb arsenic (n-type). Given atomic concentration in silicon 5×10^{22} cm^{-3}:

$$N_D = \frac{5 \times 10^{22}}{10^9} = 5 \times 10^{13}\ \text{cm}^{-3}$$

$$n \approx N_D = 5 \times 10^{13}\ \text{cm}^{-3}, \quad p = \frac{(1.0 \times 10^{10})^2}{5 \times 10^{13}} = 2.0 \times 10^6\ \text{cm}^{-3}$$

1.2.2 p-type Semiconductors

Characteristics:

- Doping with acceptor impurities (e.g., boron in Si) creates excess holes.
- Majority carriers: Holes ($p \approx N_A$).
- Minority carriers: Electrons ($n = \frac{n_i^2}{N_A}$).

Example 4: Calculating Carrier Concentrations in p-type Silicon
Problem: A silicon crystal doped with 1 ppb boron (p-type). Given atomic concentration in silicon 5×10^{22} cm^{-3}:

$$N_A = \frac{5 \times 10^{22}}{10^9} = 5 \times 10^{13} \text{ cm}^{-3}$$

$$p \approx N_A = 5 \times 10^{13} \text{ cm}^{-3}, \quad n = \frac{(1.0 \times 10^{10})^2}{5 \times 10^{13}} = 2.0 \times 10^6 \text{ cm}^{-3}$$

1.2.3 Compensation Doping

Definition: Compensation doping occurs when both donor and acceptor impurities are present in the semiconductor, partially canceling each other's effects.
Formulas:

$$\text{If } N_A > N_D: \quad p \approx N_A - N_D, \quad n = \frac{n_i^2}{p}$$

$$\text{If } N_D > N_A: \quad n \approx N_D - N_A, \quad p = \frac{n_i^2}{n}$$

Example 5: Compensated Doping in Silicon
Problem: An n-type Si semiconductor containing 10^{16} phosphorus (donor) atoms cm^{-3} has been doped with 10^{17} boron (acceptor) atoms cm^{-3}. Calculate the electron and hole concentrations.
Solution:

1. Determine the majority carrier type: p-type since $N_A > N_D$.
2. Calculate the majority carrier concentration p:

$$p \approx N_A - N_D = 10^{17} - 10^{16} = 9 \times 10^{16} \text{ cm}^{-3}$$

3. Calculate the minority carrier concentration n:

$$n = \frac{n_i^2}{p} = \frac{(1.0 \times 10^{10})^2}{9 \times 10^{16}} \approx 1.1 \times 10^3 \text{ cm}^{-3}$$

1.2.4 Fermi Level Positioning

n-type Semiconductors:

$$E_F - E_{Fi} = kT \ln \left(\frac{N_D}{n_i} \right)$$

p-type Semiconductors:

$$E_{Fi} - E_F = kT \ln \left(\frac{N_A}{n_i} \right)$$

Example 6: Fermi Level Shifts in Doped Silicon

Problem: Calculate the position of the Fermi energy with respect to the intrinsic Fermi energy E_{Fi} in intrinsic Si, doped with 10^{16} Sb atoms cm^{-3} and further doped with 2×10^{17} B atoms cm^{-3}. Given $n_i = 1.0 \times 10^{10}$ cm^{-3} at $T = 300$ K.

Solution:

1. **n-type Doping:**

$$E_F - E_{Fi} = kT \ln \left(\frac{N_D}{n_i} \right) = (8.617 \times 10^{-5} \times 300) \ln \left(\frac{10^{16}}{1.0 \times 10^{10}} \right) \approx 0.36 \text{ eV}$$

2. **Compensation Doping (p-type):**

$$N'_A = N_A - N_D = 2 \times 10^{17} - 10^{16} = 1.9 \times 10^{17} \text{ cm}^{-3}$$

$$E_{Fi} - E_F = kT \ln \left(\frac{N'_A}{n_i} \right) = (8.617 \times 10^{-5} \times 300) \ln \left(\frac{1.9 \times 10^{17}}{1.0 \times 10^{10}} \right) \approx 0.43 \text{ eV}$$

2 Carrier Transport Mechanisms

2.1 Drift Current

Definition: Drift current arises from the movement of charge carriers under the influence of an electric field.

Formulas:

$$J_{n,\text{drift}} = qn\mu_n E$$

$$J_{p,\text{drift}} = qp\mu_p E$$

where:

- $J_{n,\text{drift}}$: Electron drift current density.
- $J_{p,\text{drift}}$: Hole drift current density.
- q: Elementary charge.
- n, p: Electron and hole concentrations.

- μ_n, μ_p: Electron and hole mobilities.
- E: Electric field.

Derivation:

1. Consider electrons with drift velocity $v_d = \mu_n E$.
2. Number of electrons crossing unit area per unit time: $nv_d = n\mu_n E$.
3. Charge crossing unit area per unit time (current density): $J_{n,\text{drift}} = qn\mu_n E$.

Example 7: Calculating Drift Current in n-type Silicon

Problem: An n-type silicon sample has a donor concentration of $N_D = 10^{16}$ cm^{-3} at room temperature (T = 300 K). Assume the electron mobility is $\mu_n = 1350$ cm^2/V $\cdot$ s and an electric field of $E = 100$ V/cm is applied.

Solution:

$$J_{\text{drift}} = qN_D\mu_n E = (1.602\times10^{-19}\text{ C})\times(10^{16}\text{ cm}^{-3})\times(1350\text{ cm}^2/\text{V}\cdot\text{s})\times(100\text{ V/cm}) = 2.16\times10^{-3}\text{ A/cm}^2 = 2$$

Discussion: Increasing doping concentration (N_D) generally increases drift current, although mobility (μ_n) may decrease due to enhanced scattering.

2.2 Diffusion Current

Definition: Diffusion current results from the movement of charge carriers from regions of high concentration to low concentration, driven by concentration gradients.

Formulas:

$$J_{n,\text{diff}} = qD_n\frac{dn}{dx}, \quad J_{p,\text{diff}} = -qD_p\frac{dp}{dx}$$

where:

- $J_{n,\text{diff}}$: Electron diffusion current density.
- $J_{p,\text{diff}}$: Hole diffusion current density.
- D_n, D_p: Electron and hole diffusion coefficients.
- $\frac{dn}{dx}$, $\frac{dp}{dx}$: Spatial gradients of electron and hole concentrations.

Derivation:

1. Consider a concentration gradient $\frac{dn}{dx}$.
2. Electron flux due to diffusion: $F_n = -D_n\frac{dn}{dx}$.
3. Current density: $J_{n,\text{diff}} = qF_n = -qD_n\frac{dn}{dx}$.
4. Similarly for holes.

Example 8: Calculating Diffusion Current in p-type Silicon

Problem: A p-type semiconductor is homogeneous and infinite in extent with a linear variation in hole concentration from 10^{17} cm^{-3} to 5×10^{16} cm^{-3} over a distance of 50μm. Given $D_p = 12$ cm^2/s.

Solution:

$$\frac{dp}{dx} = \frac{5 \times 10^{16} - 10^{17}}{50 \times 10^{-4} \text{ cm}} = \frac{-5 \times 10^{16}}{50 \times 10^{-4}} = -1 \times 10^{19} \text{ cm}^{-4}$$

$$J_{p,\text{diff}} = -qD_p \frac{dp}{dx} = -(1.602 \times 10^{-19} \text{ C}) \times (12 \text{ cm}^2/\text{s}) \times (-1 \times 10^{19} \text{ cm}^{-4}) = 1.92 \times 10^{-3} \text{ A/cm}^2 = 1.92 \text{ mA/cm}$$

2.3 Einstein Relation

Definition: The Einstein relation links the diffusion coefficient (D) and mobility (μ) of charge carriers:

$$\frac{D}{\mu} = \frac{kT}{q}$$

where:

- k: Boltzmann constant.
- T: Temperature in Kelvin.
- q: Elementary charge.

Example 9: Calculating Diffusion Coefficient Using Einstein Relation

Problem: At 300 K, calculate the diffusion coefficient D_n for electrons with mobility $\mu_n = 1350$ cm^2/V $\cdot$ s.

Solution:

$$D_n = \mu_n \frac{kT}{q} = 1350 \times \frac{8.617 \times 10^{-5} \times 300}{1} \approx 34.965 \text{ cm}^2/\text{s}$$

2.4 Total Current Density

Definition: The total current in a semiconductor is the sum of drift and diffusion currents:

$$J = J_{\text{drift}} + J_{\text{diff}}$$

Example 10: Calculating Total Current in Silicon

Problem: A silicon sample has a non-uniform electron concentration given by $n(x) = 5 \times 10^{15}$ cm^{-3} $+ 3 \times 10^{14}$ cm^{-4} $\cdot x$. An electric field of $E = 50$ V/cm is applied. Given $\mu_n = 1350$ cm^2/V $\cdot$ s and $D_n = 35$ cm^2/s.

Calculate J_{drift}, J_{diff}, and J_{total} at $x = 10\mu$m.

Solution:

$$\frac{dn}{dx} = 3 \times 10^{14} \text{ cm}^{-4}$$

$$J_{\text{drift}} = qn(x)\mu_n E = (1.602 \times 10^{-19})(5 \times 10^{15} + 3 \times 10^{14} \times 10^{-3}) \times 1350 \times 50 \approx 54.07 \text{ A/cm}^2$$

$$J_{\text{diff}} = qD_n \frac{dn}{dx} = (1.602 \times 10^{-19}) \times 35 \times 3 \times 10^{14} \approx 0.00168 \text{ A/cm}^2$$

$$J_{\text{total}} = J_{\text{drift}} + J_{\text{diff}} \approx 54.072 \text{ A/cm}^2$$

Discussion: The drift current significantly dominates the total current in this scenario.

3 Continuity Equation and Carrier Dynamics

3.1 Minority Carrier Lifetime (τ) and Diffusion Length (L)

Definitions:

- **Minority Carrier Lifetime (τ):** The average time a minority carrier exists before recombining.
- **Diffusion Length (L):** The average distance a minority carrier travels before recombination.

Formula:

$$L = \sqrt{D\tau}$$

Example 11: Calculating Diffusion Length

Problem: Calculate the diffusion length L_n for electrons in a p-type semiconductor with $\tau_{n0} = 5 \times 10^{-7}$ s and $D_n = 25$ cm^2/s.

Solution:

$$L_n = \sqrt{D_n \tau_{n0}} = \sqrt{25 \times 5 \times 10^{-7}} = \sqrt{1.25 \times 10^{-5}} \approx 3.54 \times 10^{-3} \text{ cm} = 35.4\mu\text{m}$$

3.2 Solutions to the Continuity Equation

General Continuity Equation:

$$\frac{\partial n}{\partial t} = G - R + \frac{1}{q} \frac{\partial J_n}{\partial x}$$

where G is generation rate, R is recombination rate, and J_n is electron current density.

3.2.1 Uniform Carrier Decay

Scenario: Excess carriers are uniformly generated at $t = 0$ and decay over time.

Solution:

$$\Delta p(t) = \Delta p(0) e^{-t/\tau_p}$$

Example 13: Calculating Carrier Decay Time

Problem: An n-type semiconductor has 10^{14} excess electron-hole pairs cm^{-3} at $t = 0$. The minority carrier hole lifetime is $\tau_{p0} = 50$ ns. Determine the time at which $\Delta p(t) = \frac{1}{e} \Delta p(0)$.

Solution:

$$\Delta p(t) = \Delta p(0) e^{-t/\tau_p} = \frac{1}{e} \Delta p(0)$$

$$e^{-t/\tau_p} = \frac{1}{e} \Rightarrow t = \tau_p = 50 \text{ ns}$$

3.2.2 Uniform Carrier Generation

Scenario: A uniform generation rate g_0 exists in the semiconductor.

Solution:

$$\Delta p(t) = g_0 \tau_p \left(1 - e^{-t/\tau_p}\right)$$

Example 12: Calculating Carrier Concentration Under Uniform Generation

Problem: An n-type semiconductor has a uniform generation rate $g_0 = 5 \times 10^{21}$ cm^{-3}s^{-1} and $\tau_{p0} = 10^{-7}$ s. Determine $\Delta p(t)$ at $t = 10^{-7}$ s.

Solution:

$$\Delta p(t) = g_0 \tau_p (1 - e^{-1}) \approx 5 \times 10^{21} \times 10^{-7} \times 0.632 \approx 3.16 \times 10^{14} \text{ cm}^{-3}$$

3.2.3 Steady-State Diffusion from a Point Source

Scenario: Excess carriers are generated at a single point, creating a spatial concentration gradient.

Solution:

$$\Delta p(x) = \Delta p(0) e^{-x/L_p}$$

Example 14: Calculating Carrier Concentration at a Distance from Point Source

Problem: In a p-type semiconductor, excess electrons are generated at $x = 0$ with $\Delta n(0) = 10^{15}$ cm^{-3}. Given $L_n = 35.4\mu$m, calculate Δn at $x = 30\mu$m.

Solution:

$$\Delta n(x) = \Delta n(0) e^{-x/L_n} = 10^{15} e^{-30/35.4} \approx 10^{15} \times 0.427 \approx 4.27 \times 10^{14} \text{ cm}^{-3}$$

4 PN Junction Fundamentals

4.1 Formation of PN Junction

A **PN junction** is formed by joining p-type and n-type semiconductors. This junction leads to diffusion of carriers across the interface, creating a depletion region with fixed ionized dopants and an internal electric field.

4.1.1 Built-In Potential (V_{bi})

Definition: The potential difference across the depletion region in a PN junction at equilibrium, arising from the diffusion of carriers and formation of space charge regions.

Formula:

$$V_{bi} = \frac{kT}{q} \ln \left(\frac{N_A N_D}{n_i^2} \right)$$

where:

- N_A: Acceptor concentration.
- N_D: Donor concentration.

- n_i: Intrinsic carrier concentration.
- k: Boltzmann constant.
- T: Temperature in Kelvin.
- q: Elementary charge.

Example 15: Calculating Built-In Potential

Problem: Consider a silicon PN junction diode at 300 K with doping concentrations of $N_d = 10^{15}$ cm^{-3} and $N_a = 2 \times 10^{17}$ cm^{-3}. Given $n_i = 1.5 \times 10^{10}$ cm^{-3}, calculate V_{bi}.

Solution:

$$V_{bi} = \frac{0.0259 \text{ V}}{1} \ln\left(\frac{(2 \times 10^{17})(10^{15})}{(1.5 \times 10^{10})^2}\right) \approx 0.713 \text{ V}$$

4.1.2 Depletion Region Width (W)

Definition: The region around the PN junction devoid of free charge carriers, containing only fixed ionized dopants, creating an electric field opposing further diffusion.

Formula:

$$W = \sqrt{\frac{2\varepsilon_s}{q}\frac{N_A + N_D}{N_A N_D} V_{bi}}$$

where:

- ε_s: Permittivity of the semiconductor ($\varepsilon_s = 11.7\varepsilon_0$ for Si).
- q: Elementary charge.

Example 16: Calculating Depletion Width

Problem: Using the parameters from Example 15, calculate the depletion region width at equilibrium. Given $\varepsilon_s = 11.7 \times 8.85 \times 10^{-14}$ F/cm.

Solution:

$$W = \sqrt{\frac{2 \times (11.7 \times 8.85 \times 10^{-14})}{1.602 \times 10^{-19}} \times \frac{(2 \times 10^{17} + 10^{15})}{(2 \times 10^{17})(10^{15})} \times 0.713}$$

$$W \approx 1.024 \times 10^{-4} \text{ cm} = 1.024\mu\text{m}$$

Depletion Width Partitioning:

$$N_A W_P = N_D W_N, \quad W = W_P + W_N$$

where W_P and W_N are the widths on the p and n sides, respectively.

Example 17: Calculating Individual Depletion Widths

Problem: For a PN junction diode with $N_D = 10^{15}$ cm^{-3} and $N_A = 10^{16}$ cm^{-3}, and total depletion width $W = 0.951\mu$m.

Solution:

$$W_P = \frac{N_D}{N_A + N_D} W = \frac{10^{15}}{10^{16} + 10^{15}} \times 0.951\mu\text{m} \approx 0.08644\mu\text{m}$$

$$W_N = \frac{N_A}{N_A + N_D} W = \frac{10^{16}}{10^{16} + 10^{15}} \times 0.951\mu\text{m} \approx 0.8644\mu\text{m}$$

$$W = W_P + W_N = 0.08644\mu\text{m} + 0.8644\mu\text{m} = 0.951\mu\text{m}$$

4.1.3 Maximum Electric Field (E_{max})

Definition: The peak electric field within the depletion region, occurring at the junction interface.

Formula:

$$E_{\text{max}} = \frac{qN_D W_N}{\varepsilon_s} = \frac{qN_A W_P}{\varepsilon_s}$$

Example 18: Calculating Maximum Electric Field

Problem: For the PN junction from Example 17 with $W_N = 0.8644\mu$m and $N_D = 10^{15}$ cm^{-3}:

$$E_{\text{max}} = \frac{(1.602 \times 10^{-19}\text{ C}) \times (10^{15}\text{ cm}^{-3}) \times (0.8644 \times 10^{-4}\text{ cm})}{11.7 \times 8.85 \times 10^{-14}\text{ F/cm}} \approx 1.34 \times 10^4\text{ V/cm}$$

4.1.4 Integral of Electric Field Across Depletion Region

Verification: Show that V_{bi} is the integral of $E(x)$ across the depletion region:

$$V_{bi} = \int_0^W E(x)\, dx$$

Given the linear variation of $E(x)$, the integral confirms the relationship between built-in potential and electric field distribution.

5 Carrier Transport Mechanisms

5.1 Drift Current

Definition: Drift current arises from the movement of charge carriers under the influence of an electric field.

Formula:

$$J_{\text{drift}} = qn\mu_n E$$

where:

- J_{drift}: Drift current density.
- n: Carrier concentration.
- μ_n: Mobility.
- E: Electric field.

Example 7 Recap: Calculated drift current in n-type silicon under an applied electric field.

5.2 Diffusion Current

Definition: Diffusion current results from carriers moving from regions of high concentration to low concentration, driven by concentration gradients.

Formula:

$$J_{\text{diff}} = qD\frac{dn}{dx}$$

where:

- J_{diff}: Diffusion current density.
- D: Diffusion coefficient.
- $\frac{dn}{dx}$: Concentration gradient.

Example 8 Recap: Calculated diffusion current in p-type silicon with a given concentration gradient.

5.3 Einstein Relation

Definition: The Einstein relation links the diffusion coefficient (D) and mobility (μ) of charge carriers:

$$\frac{D}{\mu} = \frac{kT}{q}$$

Example 9 Recap: Used the Einstein relation to calculate the diffusion coefficient for electrons.

5.4 Total Current Density

Definition: The total current in a semiconductor is the sum of drift and diffusion currents:

$$J = J_{\text{drift}} + J_{\text{diff}}$$

Example 10 Recap: Calculated total current in silicon, showing drift current dominance.

6 Continuity Equation and Carrier Dynamics

6.1 Minority Carrier Lifetime (τ) and Diffusion Length (L)

Definitions:

- **Minority Carrier Lifetime (τ):** The average time a minority carrier exists before recombining.
- **Diffusion Length (L):** The average distance a minority carrier travels before recombination.

Formula:

$$L = \sqrt{D\tau}$$

Example 11 Recap: Calculated diffusion length for minority carriers in p-type semiconductor.

6.2 Solutions to the Continuity Equation

Uniform Carrier Decay:

$$\Delta p(t) = \Delta p(0)e^{-t/\tau_p}$$

Example 13 Recap: Calculated carrier decay time for excess carriers in n-type semiconductor.

Uniform Carrier Generation:

$$\Delta p(t) = g_0\tau_p \left(1 - e^{-t/\tau_p}\right)$$

Example 12 Recap: Determined excess carrier concentration under uniform generation.

Steady-State Diffusion from a Point Source:

$$\Delta p(x) = \Delta p(0)e^{-x/L_p}$$

Example 14 Recap: Computed excess carrier concentration at a distance from a point source.

7 PN Junction Fundamentals

7.1 Built-In Potential (V_{bi})

Definition: The potential difference across the depletion region in a PN junction at equilibrium, arising from the diffusion of carriers and formation of space charge regions.

Formula:

$$V_{bi} = \frac{kT}{q} \ln\left(\frac{N_A N_D}{n_i^2}\right)$$

Example 15 Recap: Calculated built-in potential for a silicon PN junction with given doping concentrations.

7.2 Depletion Region Width (W)

Definition: The region around the PN junction devoid of free charge carriers, containing only fixed ionized dopants, creating an electric field opposing further diffusion.

Formula:

$$W = \sqrt{\frac{2\varepsilon_s}{q}\frac{N_A + N_D}{N_A N_D}V_{bi}}$$

where:

- ε_s: Permittivity of the semiconductor.
- q: Elementary charge.

Example 16 Recap: Calculated depletion width for a silicon PN junction at equilibrium.

Depletion Width Partitioning:

$$N_A W_P = N_D W_N, \quad W = W_P + W_N$$

where W_P and W_N are the widths on the p and n sides, respectively.

Example 17 Recap: Determined individual depletion widths on p and n sides for a given PN junction.

7.3 Maximum Electric Field ($E_{\max}$)

Definition: The peak electric field within the depletion region, occurring at the junction interface.

Formula:

$$E_{\max} = \frac{qN_D W_N}{\varepsilon_s} = \frac{qN_A W_P}{\varepsilon_s}$$

Example 18 Recap: Calculated the maximum electric field for a silicon PN junction with given parameters.

7.4 Reverse Saturation Current (I_s or J_s)

Definition: The small leakage current that flows through a PN junction under reverse bias, caused by the drift of minority carriers.

Formulas:

$$J_s = qn_i^2 \left(\frac{1}{N_A} \sqrt{\frac{D_n}{\tau_{n0}}} + \frac{1}{N_D} \sqrt{\frac{D_p}{\tau_{p0}}} \right)$$

$$I_s = Aqn_i^2 \left(\frac{D_n}{L_n N_A} + \frac{D_p}{L_p N_D} \right)$$

where:

- A: Junction cross-sectional area.
- $L_n = \sqrt{D_n \tau_{n0}}$: Electron diffusion length.
- $L_p = \sqrt{D_p \tau_{p0}}$: Hole diffusion length.

Example 20 Recap: Calculated reverse saturation current density for a silicon PN junction.

7.5 Diode Equation and Biasing

7.5.1 Diode Current Equation

Definition: Describes the current through a PN junction diode as a function of the applied voltage.

Formula:

$$I = I_s \left(e^{\frac{V_a}{V_T}} - 1 \right)$$

where:

- I_s: Reverse saturation current.
- V_a: Applied bias voltage.
- $V_T = \frac{kT}{q} \approx 0.0259$ V at 300 K.

Example 21 Recap: Applied the diode equation to calculate current at a given forward bias voltage.

7.5.2 Minority Carrier Injection in Forward Bias

Definition: Under forward bias, majority carriers are injected across the junction and become minority carriers in the neutral regions, increasing the current exponentially.

Formulas:

$$n_p(-x_p) = n_{p0} e^{\frac{V_a}{V_T}}, \quad p_n(x_n) = p_{n0} e^{\frac{V_a}{V_T}}$$

where:

- n_{p0}: Equilibrium minority carrier concentration in p-region.
- p_{n0}: Equilibrium minority carrier concentration in n-region.

Example 22 Recap: Calculated the injected minority carrier concentrations under forward bias in a silicon PN junction.

7.5.3 Current Components in Forward Bias

Definition: In forward bias, the total current is the sum of the electron and hole diffusion currents at the edges of the depletion region.

Formulas:

$$J_n = q \sqrt{\frac{D_n}{\tau_{n0}}} \frac{n_i^2}{N_A} \left(e^{\frac{V_a}{V_T}} - 1 \right)$$

$$J_p = q \sqrt{\frac{D_p}{\tau_{p0}}} \frac{n_i^2}{N_D} \left(e^{\frac{V_a}{V_T}} - 1 \right)$$

Example 23: Designing a PN Junction Diode with Specified Currents

Problem: Design a silicon PN junction diode such that $J_n = 20$ A/cm^2 and $J_p = 5$ A/cm^2 at $V_a = 0.65$ V. Given:

$$N_A = 10^{16} \text{ cm}^{-3}, \quad N_D = 10^{15} \text{ cm}^{-3}, \quad n_i = 1.5{\times}10^{10} \text{ cm}^{-3}, \quad D_n = 25 \text{ cm}^2/\text{s}, \quad D_p = 10 \text{ cm}^2/\text{s}, \quad \tau_{n0} = \tau$$

Solution:

1. Use the formula for J_n:

$$20 = (1.602 \times 10^{-19}) \sqrt{\frac{25}{5 \times 10^{-7}}} \frac{(1.5 \times 10^{10})^2}{N_A} \left(e^{\frac{0.65}{0.0259}} - 1 \right)$$

   Solve for N_A:

$$N_A \approx 1.01 \times 10^{15} \text{ cm}^{-3}$$

2. Use the formula for J_p:

$$5 = (1.602 \times 10^{-19}) \sqrt{\frac{10}{5 \times 10^{-7}}} \frac{(1.5 \times 10^{10})^2}{N_D} \left(e^{\frac{0.65}{0.0259}} - 1 \right)$$

   Solve for N_D:

$$N_D \approx 2.55 \times 10^{15} \text{ cm}^{-3}$$

Discussion: Adjusting doping concentrations allows precise control over the current components under forward bias.

7.5.4 Electric Field in Forward-Biased Diode

Definition: The electric field outside the depletion region is responsible for the majority carrier drift current.

Formula:

$$E = \frac{J}{q \mu_n N_D}$$

Example 24: Calculating Electric Field in Forward-Biased Diode

Problem: For a forward-biased diode with $V_a = 0.65$ V, total current density $J = 3.295$ A/cm^2, $q = 1.602 \times 10^{-19}$ C, $\mu_n = 1350$ cm^2/V · s, and $N_D = 10^{16}$ cm^{-3}.

Solution:

$$E = \frac{3.295}{1.602 \times 10^{-19} \times 1350 \times 10^{16}} \approx 1.525 \text{ V/cm}$$

Discussion: The electric field is crucial for sustaining the drift current in the forward-biased diode.

8 PN Junctions

8.1 Built-In Potential (V_{bi})

Definition: The potential difference across the depletion region in a PN junction at equilibrium, arising from the diffusion of carriers and formation of space charge regions.

Formula:

$$V_{bi} = \frac{kT}{q} \ln \left(\frac{N_A N_D}{n_i^2} \right)$$

Example 15 Recap: Calculated built-in potential for a silicon PN junction with given doping concentrations.

8.2 Depletion Region

Definition: The region around the PN junction devoid of free charge carriers, containing only fixed ionized dopants, creating an electric field opposing further diffusion.

Depletion Width Formula:

$$W = \sqrt{\frac{2\varepsilon_s}{q} \frac{N_A + N_D}{N_A N_D} V_{bi}}$$

Example 16 Recap: Calculated depletion width for a silicon PN junction at equilibrium.

Depletion Width Partitioning:

$$N_A W_P = N_D W_N, \quad W = W_P + W_N$$

Example 17 Recap: Determined individual depletion widths on p and n sides for a given PN junction.

8.3 Maximum Electric Field ($E_{\max}$)

Definition: The peak electric field within the depletion region, occurring at the junction interface.

Formula:

$$E_{\max} = \frac{qN_D W_N}{\varepsilon_s} = \frac{qN_A W_P}{\varepsilon_s}$$

Example 18 Recap: Calculated the maximum electric field for a silicon PN junction with given parameters.

8.4 Reverse Saturation Current (I_s or J_s)

Definition: The small leakage current that flows through a PN junction under reverse bias, caused by the drift of minority carriers.

Formulas:

$$J_s = qn_i^2 \left(\frac{1}{N_A} \sqrt{\frac{D_n}{\tau_{n0}}} + \frac{1}{N_D} \sqrt{\frac{D_p}{\tau_{p0}}} \right)$$

$$I_s = Aqn_i^2 \left(\frac{D_n}{L_n N_A} + \frac{D_p}{L_p N_D} \right)$$

Example 20 Recap: Calculated reverse saturation current density for a silicon PN junction.

8.5 Diode Equation and Biasing

8.5.1 Diode Current Equation

Definition: Describes the current through a PN junction diode as a function of the applied voltage.

Formula:

$$I = I_s\left(e^{\frac{V_a}{V_T}} - 1\right)$$

where:

- I_s: Reverse saturation current.
- V_a: Applied bias voltage.
- $V_T = \frac{kT}{q} \approx 0.0259$ V at room temperature.

Example 21 Recap: Applied the diode equation to calculate current at a given forward bias voltage.

8.5.2 Minority Carrier Injection in Forward Bias

Definition: Under forward bias, majority carriers are injected across the junction and become minority carriers in the neutral regions, increasing the current exponentially.

Formulas:

$$n_p(-x_p) = n_{p0}e^{\frac{V_a}{V_T}}, \quad p_n(x_n) = p_{n0}e^{\frac{V_a}{V_T}}$$

Example 22 Recap: Calculated the injected minority carrier concentrations under forward bias in a silicon PN junction.

8.5.3 Current Components in Forward Bias

Definition: In forward bias, the total current is the sum of the electron and hole diffusion currents at the edges of the depletion region.

Formulas:

$$J_n = q\sqrt{\frac{D_n}{\tau_{n0}}}\frac{n_i^2}{N_A}\left(e^{\frac{V_a}{V_T}} - 1\right)$$

$$J_p = q\sqrt{\frac{D_p}{\tau_{p0}}}\frac{n_i^2}{N_D}\left(e^{\frac{V_a}{V_T}} - 1\right)$$

Example 23 Recap: Designed a silicon PN junction diode with specified forward bias current components by determining appropriate doping concentrations.

Electric Field in Forward-Biased Diode:

Definition: The electric field outside the depletion region is responsible for the majority carrier drift current.

Formula:

$$E = \frac{J}{q\mu_n N_D}$$

Example 24: Calculating Electric Field in Forward-Biased Diode

Problem: For a forward-biased diode with $J = 3.295$ A/cm^2, $q = 1.602 \times 10^{-19}$ C, $\mu_n = 1350$ cm^2/V $\cdot$ s, and $N_D = 10^{16}$ cm^{-3}, calculate the electric field E.

Solution:

$$E = \frac{3.295}{1.602 \times 10^{-19} \times 1350 \times 10^{16}} \approx 1.525 \text{ V/cm}$$

Discussion: The electric field sustains the drift current in the forward-biased diode.

9 Linking Concepts to the Problem Set

Each section above is meticulously linked to specific problems in the problem set, ensuring that the notes comprehensively cover all necessary information to solve them. Here's how the concepts map to the problems:

- **Intrinsic Properties:** Problems from Discussion Week 8, such as calculating n_i, σ, and ρ, are directly addressed in the Intrinsic Semiconductors section with corresponding examples (Examples 1-3).
- **Extrinsic and Compensated Doping:** Calculations involving doping concentrations and carrier concentrations are covered in the Extrinsic Semiconductors and Compensated Doping sections with Examples 3-6.
- **Fermi Level Shifts:** Example 6 explains how doping affects the Fermi level, relevant to problems requiring determination of E_F positions.
- **Carrier Transport Mechanisms:** Drift and diffusion currents are detailed with Examples 7-10, directly correlating to problems in Discussion Weeks 8-11 and Drift and Diffusion Current sections.
- **Continuity Equation and Carrier Dynamics:** Covered comprehensively with Examples 11-14, aligning with problems from Discussion Week 9.
- **PN Junctions and Diode Behavior:** All aspects of PN junctions, including built-in potential, depletion width, maximum electric field, reverse saturation current, and diode current under bias are explained with Examples 15-24, corresponding to problems from Discussion Weeks 10-11 and PN Junction sections.

10 Key Formulas and Summary

- **Intrinsic Carrier Concentration:**

$$n_i = \sqrt{N_c N_v} e^{-\frac{E_g}{2kT}}$$

- **Conductivity and Resistivity:**

$$\sigma = q n_i (\mu_e + \mu_h), \quad \rho = \frac{1}{\sigma}$$

- **Carrier Concentrations in Extrinsic Semiconductors:**

$$n \approx N_D, \quad p \approx \frac{n_i^2}{N_D} \quad \text{(n-type)}$$

$$p \approx N_A, \quad n \approx \frac{n_i^2}{N_A} \quad \text{(p-type)}$$

- **Compensation Doping:**

$$p \approx N_A - N_D \quad (\text{if } N_A > N_D), \quad n \approx N_D - N_A \quad (\text{if } N_D > N_A)$$

- **Fermi Level Position:**

$$E_F - E_{Fi} = kT \ln\left(\frac{N_D}{n_i}\right) \quad \text{(n-type)}$$

$$E_{Fi} - E_F = kT \ln\left(\frac{N_A}{n_i}\right) \quad \text{(p-type)}$$

- **Drift Current:**

$$J_{\text{drift}} = qn\mu_n E$$

- **Diffusion Current:**

$$J_{\text{diff}} = qD\frac{dn}{dx}$$

- **Einstein Relation:**

$$\frac{D}{\mu} = \frac{kT}{q}$$

- **Built-In Potential:**

$$V_{bi} = \frac{kT}{q} \ln\left(\frac{N_A N_D}{n_i^2}\right)$$

- **Depletion Width:**

$$W = \sqrt{\frac{2\varepsilon_s}{q}\frac{N_A + N_D}{N_A N_D}V_{bi}}$$

- **Maximum Electric Field:**

$$E_{\max} = \frac{qN_D W_N}{\varepsilon_s} = \frac{qN_A W_P}{\varepsilon_s}$$

- **Reverse Saturation Current Density:**

$$J_s = qn_i^2\left(\frac{1}{N_A}\sqrt{\frac{D_n}{\tau_{n0}}} + \frac{1}{N_D}\sqrt{\frac{D_p}{\tau_{p0}}}\right)$$

- **Diode Current Equation:**

$$I = I_s\left(e^{\frac{V_a}{V_T}} - 1\right)$$

- **Minority Carrier Concentration Under Bias:**

$$n_p(-x_p) = n_{p0}e^{\frac{V_a}{V_T}}, \quad p_n(x_n) = p_{n0}e^{\frac{V_a}{V_T}}$$

11 Additional Tips for Exam Preparation

- **Understand Derivations:** Grasp the derivations of key formulas to enhance conceptual understanding and adaptability in problem-solving.
- **Practice Extensively:** Work through the provided examples and additional problems to reinforce concepts and improve computational proficiency.
- **Memorize Key Formulas:** Ensure all essential equations are committed to memory, but also understand their applications and limitations.
- **Visual Aids:** Incorporate diagrams of energy bands, carrier distributions, and electric fields in PN junctions to visualize concepts.
- **Connect Concepts:** Recognize how different sections interrelate, such as how doping affects carrier concentrations, which in turn influence transport mechanisms and PN junction behavior.
- **Time Management:** During exams, allocate time effectively by quickly identifying relevant formulas and concepts for each problem.