ECE235- notes for final

I. Introduction

- **Semiconductors**: Materials with conductivity between conductors and insulators (e.g., Silicon (Si), Germanium (Ge)).
- **Importance**: Ability to control electrical properties via doping.

II. Energy Bands and Density of States

A. Energy Bands

- In solids, atomic energy levels split into **energy bands**.
- **Valence band**: Filled with electrons.
- **Conduction band**: Empty in insulators, partially filled in semiconductors.
- **Bandgap (E_g)**: Energy difference between the conduction and valence bands.
- **Direct vs. Indirect Bandgap**:
  - **Direct**: Minimum of conduction band and maximum of valence band occur at the same momentum. Efficient for light emission (e.g., GaAs).
  - **Indirect**: Minimum of conduction band and maximum of valence band occur at different momenta. Less efficient for light emission (e.g., Si).
  - Focus on direct bandgap for problem-solving.

B. Effective Mass (m^*)

- Electrons/holes in a crystal behave as if they have a different mass (m^*) due to interactions with the lattice.
- m^* affects mobility and density of states.

C. Density of States (DOS)

- **Definition**: Number of available energy states per unit volume per unit energy.
- **Effective density of states (N_c, N_v)**: Simplified representation, used in calculations.
- $N_c \propto (m_n^*)^{3/2}$, $N_v \propto (m_p^*)^{3/2}$ (Higher effective mass $\Rightarrow$ higher DOS).

III. Carrier Concentrations and Fermi Level

A. Intrinsic Semiconductors

- Pure materials: electron concentration (n) = hole concentration (p) = intrinsic carrier concentration (n_i).
- **Intrinsic carrier concentration**:

$$n_i = \sqrt{N_c N_v} \exp\left(-\frac{E_g}{2kT}\right)$$

  - N_c: Effective density of states in the conduction band.
  - N_v: Effective density of states in the valence band.
  - E_g: Bandgap energy.
  - k: Boltzmann constant (8.617×10^{-5} eV/K).
  - T: Temperature in Kelvin.
  - **Temperature dependence**: n_i increases exponentially with temperature.

Example 1: Calculating Intrinsic Carrier Concentration

Problem: Calculate n_i for silicon at room temperature (300 K), given:

$$N_c = 2.81 \times 10^{19} \text{ cm}^{-3}, \quad N_v = 1.16 \times 10^{19} \text{ cm}^{-3}, \quad E_g = 1.10 \text{ eV}$$

Solution:

$$n_i = \sqrt{(2.81 \times 10^{19})(1.16 \times 10^{19})} \exp\left(-\frac{1.10}{2 \times 8.617 \times 10^{-5} \times 300}\right)$$

$$n_i = \sqrt{3.2596 \times 10^{38}} \exp(-21.333) \approx 1.0 \times 10^{10} \text{ cm}^{-3}$$

B. Extrinsic Semiconductors (Doping)

- **n-type**: Doped with donor impurities (e.g., P in Si).
  - Donors contribute extra electrons.
  - Electron concentration: $n \approx N_D$ (N_D is donor concentration).

- Hole concentration: $p = n_i^2/N_D$.

- **p-type**: Doped with acceptor impurities (e.g., B in Si).
  - Acceptors create holes.
  - Hole concentration: $p \approx N_A$ (N_A is acceptor concentration).
  - Electron concentration: $n = n_i^2/N_A$.

Example 3: Calculating Carrier Concentrations in n-type Silicon

Problem: A 1 cm^3 silicon crystal doped with 1 ppb arsenic (n-type). Given atomic concentration in silicon 5×10^{22} cm^{-3}:

$$N_D = \frac{5 \times 10^{22}}{10^9} = 5 \times 10^{13} \text{ cm}^{-3}$$

$$n \approx N_D = 5 \times 10^{13} \text{ cm}^{-3}, \quad p = \frac{(1.0 \times 10^{10})^2}{5 \times 10^{13}} = 2.0 \times 10^6 \text{ cm}^{-3}$$

Example 4: Calculating Carrier Concentrations in p-type Silicon

Problem: A silicon crystal doped with 1 ppb boron (p-type). Given atomic concentration in silicon 5×10^{22} cm^{-3}:

$$N_A = \frac{5 \times 10^{22}}{10^9} = 5 \times 10^{13} \text{ cm}^{-3}$$

$$p \approx N_A = 5 \times 10^{13} \text{ cm}^{-3}, \quad n = \frac{(1.0 \times 10^{10})^2}{5 \times 10^{13}} = 2.0 \times 10^6 \text{ cm}^{-3}$$

C. Mass Action Law

- In any semiconductor at thermal equilibrium:

$$np = n_i^2$$

- Applies to both intrinsic and extrinsic semiconductors.

D. Compensation Doping

- Occurs when both donor and acceptor impurities are present.
- **If** $N_A > N_D$: $p \approx N_A - N_D$, $n = n_i^2/p$ (p-type).
- **If** $N_D > N_A$: $n \approx N_D - N_A$, $p = n_i^2/n$ (n-type).

Example 5: Compensated Doping in Silicon

Problem: An n-type Si semiconductor containing $N_D = 10^{16}$ cm^{-3} phosphorus atoms is doped with $N_A = 10^{17}$ cm^{-3} boron atoms.
Solution:

$$N_A > N_D \Rightarrow \text{p-type}$$

$$p \approx N_A - N_D = 10^{17} - 10^{16} = 9 \times 10^{16}\ \text{cm}^{-3}$$

$$n = \frac{(1.0 \times 10^{10})^2}{9 \times 10^{16}} \approx 1.1 \times 10^{3}\ \text{cm}^{-3}$$

E. Non-degenerate limit

- Assumption: doping is not too high, so the Fermi level E_F is at least a few kT away from the conduction band edge E_C and valence band edge E_V.

F. Fermi Level (E_F)

- Indicates the probability of an energy state being occupied by an electron.
- **Intrinsic**: E_F is near the middle of the bandgap (E_{Fi}).
- **n-type**: E_F shifts up towards the conduction band.
- **p-type**: E_F shifts down towards the valence band.
- **Formulas (Memorize)**:

$$E_F - E_{Fi} = kT \ln\left(\frac{N_D}{n_i}\right) \quad \text{(n-type)}$$

$$E_{Fi} - E_F = kT \ln\left(\frac{N_A}{n_i}\right) \quad \text{(p-type)}$$

Example 6: Fermi Level Shifts in Doped Silicon

Problem: Calculate the position of the Fermi energy with respect to the intrinsic Fermi energy E_{Fi} in intrinsic Si, doped first with $N_D = 10^{16}$ cm^{-3} and then with $N_A = 2 \times 10^{17}$ cm^{-3}. Given $n_i = 1.0 \times 10^{10}$ cm^{-3} at $T = 300$ K.

Solution:

1. **n-type Doping:**

$$E_F - E_{Fi} = kT \ln\left(\frac{N_D}{n_i}\right) = (8.617 \times 10^{-5} \times 300) \ln\left(\frac{10^{16}}{1.0 \times 10^{10}}\right) \approx 0.36 \text{ eV}$$

2. **Compensation Doping (p-type):**

$$N'_A = N_A - N_D = 2 \times 10^{17} - 10^{16} = 1.9 \times 10^{17} \text{ cm}^{-3}$$

$$E_{Fi} - E_F = kT \ln\left(\frac{N'_A}{n_i}\right) = (8.617 \times 10^{-5} \times 300) \ln\left(\frac{1.9 \times 10^{17}}{1.0 \times 10^{10}}\right) \approx 0.43 \text{ eV}$$

IV. Carrier Transport

A. Drift Current

- Due to the influence of an electric field.
- **Mobility** (μ): Measure of how easily carriers move in response to an electric field.
- **Electron Drift Current Density**: $J_{drift,n} = qn\mu_n E$
- **Hole Drift Current Density**: $J_{drift,p} = qp\mu_p E$
- **Conductivity**: $\sigma = q(n\mu_n + p\mu_p)$
  - In intrinsic semiconductor: $\sigma = qn_i(\mu_e + \mu_h)$
- **Resistivity**: $\rho = 1/\sigma$

Example 2: Calculating Intrinsic Conductivity and Resistivity

Problem: For silicon with $n_i = 1.0 \times 10^{10}$ cm^{-3}, $\mu_e = 1350$ cm^2/V $\cdot$ s, and $\mu_h = 450$ cm^2/V $\cdot$ s:

$$\sigma = qn_i(\mu_e + \mu_h) \approx 2.88 \times 10^{-6}\Omega^{-1}\text{cm}^{-1}$$

$$\rho = \frac{1}{2.88 \times 10^{-6}} \approx 3.47 \times 10^5\Omega\text{cm}$$

Example 7: Calculating Drift Current in n-type Silicon

Problem: An n-type silicon sample has a donor concentration of $N_D = 10^{16}$ cm^{-3} at room temperature ($T = 300$ K). Given electron mobility $\mu_n = 1350$ cm^2/V $\cdot$ s and an electric field $E = 100$ V/cm, calculate the drift current density.
Solution:

$$\begin{aligned} J_{\text{drift}} &= qN_D\mu_n E = (1.602 \times 10^{-19}\text{ C}) \times (10^{16}\text{ cm}^{-3}) \times (1350\text{ cm}^2/\text{V}\cdot\text{s}) \times (100\text{ V/cm}) \\ &= 2.16 \times 10^{-3}\text{ A/cm}^2 = 216\text{ mA/cm}^2 \end{aligned}$$

B. Diffusion Current

- Due to concentration gradients (carriers move from high to low concentration).
- **Diffusion Coefficient (D)**: Measure of how quickly carriers diffuse.
- **Electron Diffusion Current Density**: $J_{diff,n} = qD_n\frac{dn}{dx}$
- **Hole Diffusion Current Density**: $J_{diff,p} = -qD_p\frac{dp}{dx}$

Example 8: Calculating Diffusion Current in p-type Silicon

Problem: A p-type semiconductor has a linear variation in hole concentration from 10^{17} cm^{-3} to 5×10^{16} cm^{-3} over a distance of 50μm. Given hole diffusion coefficient $D_p = 12$ cm^2/s, calculate the diffusion current density.
Solution:

$$\frac{dp}{dx} = \frac{5 \times 10^{16} - 10^{17}}{50 \times 10^{-4}\text{ cm}} = \frac{-5 \times 10^{16}}{50 \times 10^{-4}} = -1 \times 10^{19}\text{ cm}^{-4}$$

$$J_{\text{diff}} = -qD_p\frac{dp}{dx} = -(1.602\times10^{-19}\text{ C})\times12\text{ cm}^2/\text{s}\times(-1\times10^{19}\text{ cm}^{-4}) = 1.92\times10^{-3}\text{ A/cm}^2 = 1.92\text{ mA/cm}$$

C. Einstein Relation

- Relates diffusion coefficient (D) and mobility (μ):

$$\frac{D}{\mu} = \frac{kT}{q}$$

Example 9: Calculating Diffusion Coefficient Using Einstein Relation

Problem: At 300 K, calculate the diffusion coefficient D_n for electrons with mobility $\mu_n = 1350$ cm^2/V $\cdot$ s.
Solution:

$$D_n = \mu_n\frac{kT}{q} = 1350 \times \frac{8.617 \times 10^{-5} \times 300}{1} \approx 34.965\text{ cm}^2/\text{s}$$

D. Total Current

- Total current density is the sum of drift and diffusion current densities:

$$J = J_{drift} + J_{diff}$$

Example 10: Calculating Total Current in Silicon

Problem: Silicon with $n(x) = 5\times10^{15}+3\times10^{14}x$ cm^{-3}, an electric field $E = 50$ V/cm, electron mobility $\mu_n = 1350$ cm^2/V $\cdot$ s, and diffusion coefficient $D_n = 35$ cm^2/s. Calculate J_{drift}, J_{diff}, and J_{total} at $x = 10\mu$m.
Solution:

$$n(10\mu\text{m}) = 5\times10^{15} + 3\times10^{14}\times10^{-3} = 5.3\times10^{15}\ \text{cm}^{-3}$$

$$J_{\text{drift}} = qn\mu_n E = (1.602\times10^{-19})\times5.3\times10^{15}\times1350\times50 \approx 5.74\times10^{-3}\ \text{A/cm}^2 = 5.74\ \text{mA/cm}^2$$

$$\frac{dn}{dx} = 3\times10^{14}\ \text{cm}^{-4}$$

$$J_{\text{diff}} = qD_n\frac{dn}{dx} = (1.602\times10^{-19})\times35\times3\times10^{14} \approx 1.68\times10^{-3}\ \text{A/cm}^2 = 1.68\ \text{mA/cm}^2$$

$$J_{\text{total}} = J_{\text{drift}} + J_{\text{diff}} = 5.74\ \text{mA/cm}^2 + 1.68\ \text{mA/cm}^2 = 7.42\ \text{mA/cm}^2$$

V. Continuity Equation and Carrier Dynamics

A. Generation and Recombination

- Focus on **excess carriers**: carriers in addition to the thermal equilibrium concentrations.
- **Minority carrier lifetime (τ)**: Average time a minority carrier exists before recombining.

B. Continuity Equation

- Describes the conservation of charge:

$$\frac{\partial n}{\partial t} = G - R + \frac{1}{q}\frac{\partial J_n}{\partial x}$$

where G is the generation rate and R is the recombination rate.

- Focus on these solutions:
  1. **Uniform carrier decay**: $\Delta p(t) = \Delta p(0)e^{-t/\tau_p}$
  2. **Uniform carrier generation**: $\Delta p(t) = g_0\tau_p\left(1 - e^{-t/\tau_p}\right)$

3. **Steady-state diffusion from a point source**: $\Delta p(x) = \Delta p(0) e^{-x/L_p}$

Example 12: Calculating Carrier Concentration Under Uniform Generation

Problem: An n-type semiconductor has a uniform generation rate $g_0 = 5 \times 10^{21}$ cm^{-3}s^{-1} and a minority carrier lifetime $\tau_p = 10^{-7}$ s. Determine the excess hole concentration $\Delta p(t)$ at $t = 10^{-7}$ s.
Solution:

$$\Delta p(t) = 5 \times 10^{21} \times 10^{-7} \times \left(1 - e^{-1}\right) \approx 3.16 \times 10^{14} \text{ cm}^{-3}$$

Example 13: Calculating Carrier Decay Time

Problem: An n-type semiconductor has an initial excess hole concentration of 10^{14} cm^{-3} and a minority carrier lifetime $\tau_p = 50$ ns. Determine the time at which $\Delta p(t) = \frac{1}{e}\Delta p(0)$.
Solution:

$$\Delta p(t) = \Delta p(0) e^{-t/\tau_p} = \frac{1}{e}\Delta p(0) \Rightarrow t = \tau_p = 50 \text{ ns}$$

Example 14: Calculating Carrier Concentration at a Distance from Point Source

Problem: In a p-type semiconductor, excess electrons are generated at $x = 0$ with $\Delta n(0) = 10^{15}$ cm^{-3}. Given the diffusion length $L_n = 35.4\mu$m, calculate Δn at $x = 30\mu$m.
Solution:

$$\Delta n(x) = 10^{15} \times e^{-30/35.4} \approx 4.27 \times 10^{14} \text{ cm}^{-3}$$

C. Diffusion Length

- Average distance a minority carrier travels before recombination:

$$L = \sqrt{D\tau}$$

Example 11: Calculating Diffusion Length

Problem: In a p-type semiconductor, the minority carrier (electron) lifetime is $\tau_n = 5 \times 10^{-7}$ s and the diffusion coefficient is $D_n = 25$ cm^2/s. Calculate the diffusion length L_n.
Solution:

$$L_n = \sqrt{D_n \tau_n} = \sqrt{25 \times 5 \times 10^{-7}} = \sqrt{1.25 \times 10^{-5}} \approx 3.54 \times 10^{-3} \text{ cm} = 35.4\mu\text{m}$$

VI. PN Junction Diode

A. Formation and Depletion Region

- Formed by joining p-type and n-type materials.
- **Carrier diffusion**: Electrons diffuse from n to p, holes from p to n.
- **Depletion region (space charge region)**: Forms near the junction, depleted of free carriers, containing only fixed ionized dopants (positive ions in n-side, negative ions in p-side).

B. Built-in Potential (V_{bi})

- Potential difference across the depletion region at equilibrium.
- Arises due to the internal electric field created by the fixed charges.
- **Formula (Memorize)**:

$$V_{bi} = \frac{kT}{q} \ln\left(\frac{N_A N_D}{n_i^2}\right)$$

Example 15: Calculating Built-In Potential

Problem: Consider a silicon PN junction diode at 300 K with doping concentrations of $N_d = 10^{15}$ cm^{-3} and $N_a = 2 \times 10^{17}$ cm^{-3}. Given $n_i = 1.5 \times 10^{10}$ cm^{-3}, calculate V_{bi}.
Solution:

$$V_{bi} = \frac{0.0259\text{ V}}{1} \ln\left(\frac{(2 \times 10^{17})(10^{15})}{(1.5 \times 10^{10})^2}\right) \approx 0.713\text{ V}$$

C. Depletion Width (W)

- Width of the depletion region.
- **Formula (Memorize)**:

$$W = \sqrt{\frac{2\varepsilon_s}{q}\left(\frac{N_A + N_D}{N_A N_D}\right) V_{bi}}$$

  where ε_s is the permittivity of the semiconductor.

- **Depletion width partitioning**:

$$N_A W_P = N_D W_N, \quad W = W_P + W_N$$

  where W_P is the depletion width on the p-side and W_N is the depletion width on the n-side.

Example 16: Calculating Depletion Width

Problem: Using the parameters from Example 15, calculate the depletion region width W. Given $\varepsilon_s = 11.7 \times 8.85 \times 10^{-14}$ F/cm.
Solution:

$$W = \sqrt{\frac{2 \times (11.7 \times 8.85 \times 10^{-14})}{1.602 \times 10^{-19}} \times \frac{(2 \times 10^{17} + 10^{15})}{(2 \times 10^{17})(10^{15})} \times 0.713} \approx 1.024{\times}10^{-4} \text{ cm} = 1.024\mu\text{m}$$

Example 17: Calculating Individual Depletion Widths

Problem: For a PN junction with $N_D = 10^{15}$ cm^{-3}, $N_A = 10^{16}$ cm^{-3}, and total depletion width $W = 0.951\mu$m:

$$W_N = \frac{N_A}{N_A + N_D} W = \frac{10^{16}}{10^{16} + 10^{15}} \times 0.951\mu\text{m} \approx 0.8644\mu\text{m}$$

$$W_P = \frac{N_D}{N_A + N_D} W = \frac{10^{15}}{10^{16} + 10^{15}} \times 0.951\mu\text{m} \approx 0.08644\mu\text{m}$$

$$W = W_N + W_P = 0.8644\mu\text{m} + 0.08644\mu\text{m} = 0.951\mu\text{m}$$

D. Maximum Electric Field (E_{max})

- The peak electric field within the depletion region, occurring at the junction interface.
- **Formula (Memorize)**:

$$E_{max} = \frac{qN_D W_N}{\varepsilon_s} = \frac{qN_A W_P}{\varepsilon_s}$$

Example 18: Calculating Maximum Electric Field

Problem: For the PN junction from Example 17 with $W_N = 0.8644\mu$m and $N_D = 10^{15}$ cm^{-3}:

$$E_{max} = \frac{(1.602 \times 10^{-19} \text{ C}) \times (10^{15} \text{ cm}^{-3}) \times (0.8644 \times 10^{-4} \text{ cm})}{(11.7 \times 8.85 \times 10^{-14} \text{ F/cm})} \approx 1.34 \times 10^4 \text{ V/cm}$$

VII. PN Junction Under Bias

A. Biasing

- **Forward Bias**:
  - Positive terminal of voltage source connected to p-type, negative to n-type.

- Reduces the depletion region width.
- Lowers the built-in potential barrier.
- Allows significant current flow.

- **Reverse Bias**:
  - Positive terminal of voltage source connected to n-type, negative to p-type.
  - Increases the depletion region width.
  - Raises the built-in potential barrier.
  - Blocks current flow (except for a small leakage current).

B. Forward Bias

- **Minority carrier injection**: Under forward bias, majority carriers are injected across the junction and become minority carriers.
  - **Formulas**:

$$n_p(-x_p) = n_{p0} e^{\frac{V_a}{V_T}}, \quad p_n(x_n) = p_{n0} e^{\frac{V_a}{V_T}}$$

    where V_a is the applied forward bias voltage, $V_T = \frac{kT}{q}$ is the thermal voltage, $n_{p0} = \frac{n_i^2}{N_A}$ is the equilibrium minority carrier concentration in p-region, and $p_{n0} = \frac{n_i^2}{N_D}$ is the equilibrium minority carrier concentration in n-region.

- **Current Components in Forward Bias**:
  - The total current is the sum of electron and hole diffusion currents at the edges of the depletion region.
  - **Formulas**:

$$J_n = q\sqrt{\frac{D_n}{\tau_{n0}}} \frac{n_i^2}{N_A} \left(e^{\frac{V_a}{V_T}} - 1\right)$$

$$J_p = q\sqrt{\frac{D_p}{\tau_{p0}}} \frac{n_i^2}{N_D} \left(e^{\frac{V_a}{V_T}} - 1\right)$$

- **Electric Field in Forward-Biased Diode**: The electric field outside the depletion region is responsible for the majority carrier drift current.
  - **Formula**:

$$E = \frac{J}{q\mu_n N_D}$$

Example 22: Calculating Injected Minority Carrier Concentrations

Problem: Silicon PN junction at 300 K with $N_D = 10^{16}$ cm^{-3}, $N_A = 6 \times 10^{15}$ cm^{-3}, $n_i = 1.5 \times 10^{10}$ cm^{-3}, and $V_a = 0.60$ V. Calculate $n_p(-x_p)$ and $p_n(x_n)$.
Solution:

$$n_{p0} = \frac{n_i^2}{N_A} = \frac{(1.5 \times 10^{10})^2}{6 \times 10^{15}} = 3.75 \times 10^4 \text{ cm}^{-3}$$

$$p_{n0} = \frac{n_i^2}{N_D} = \frac{(1.5 \times 10^{10})^2}{10^{16}} = 2.25 \times 10^4 \text{ cm}^{-3}$$

$$n_p(-x_p) = 3.75 \times 10^4 \times e^{\frac{0.60}{0.0259}} \approx 4.31 \times 10^{14} \text{ cm}^{-3}$$

$$p_n(x_n) = 2.25 \times 10^4 \times e^{\frac{0.60}{0.0259}} \approx 2.59 \times 10^{14} \text{ cm}^{-3}$$

Example 23: Designing a PN Junction Diode with Specified Currents

Problem: Design a silicon PN junction diode at $T = 300$ K such that $J_n = 20$ A/cm^2 and $J_p = 5$ A/cm^2 at $V_a = 0.65$ V, given:

$$D_n = 25 \text{ cm}^2/\text{s}, \quad D_p = 10 \text{ cm}^2/\text{s}, \quad \tau_{n0} = \tau_{p0} = 5 \times 10^{-7} \text{ s}, \quad n_i = 1.5 \times 10^{10} \text{ cm}^{-3}$$

Solution:

1. **Solving for N_A using J_n:**

$$20 = (1.602 \times 10^{-19}) \sqrt{\frac{25}{5 \times 10^{-7}}} \frac{(1.5 \times 10^{10})^2}{N_A} \left(e^{\frac{0.65}{0.0259}} - 1 \right)$$

$$N_A \approx 1.01 \times 10^{15} \text{ cm}^{-3}$$

2. **Solving for N_D using J_p:**

$$5 = (1.602 \times 10^{-19}) \sqrt{\frac{10}{5 \times 10^{-7}}} \frac{(1.5 \times 10^{10})^2}{N_D} \left(e^{\frac{0.65}{0.0259}} - 1 \right)$$

$$N_D \approx 2.55 \times 10^{15} \text{ cm}^{-3}$$

Example 24: Calculating Electric Field in Forward-Biased Diode

Problem: For a forward-biased diode with $J = 3.295$ A/cm^2, $q = 1.602 \times 10^{-19}$ C, $\mu_n = 1350$ cm^2/V $\cdot$ s, and $N_D = 10^{16}$ cm^{-3}, calculate the electric field E.
Solution:

$$E = \frac{3.295}{1.602 \times 10^{-19} \times 1350 \times 10^{16}} \approx 1.525 \text{ V/cm}$$

C. Reverse Bias

- **Reverse saturation current (I_s or J_s)**: Small leakage current that flows under reverse bias, due to drift of minority carriers across the depletion region.

- **Formulas**:

$$J_s = qn_i^2 \left(\frac{1}{N_A}\sqrt{\frac{D_n}{\tau_{n0}}} + \frac{1}{N_D}\sqrt{\frac{D_p}{\tau_{p0}}} \right)$$

$$I_s = Aqn_i^2 \left(\frac{D_n}{L_n N_A} + \frac{D_p}{L_p N_D} \right)$$

  where A is the junction cross-sectional area.

Example 20: Calculating Reverse Saturation Current

Problem: Silicon PN junction at 300 K with $N_A = N_D = 10^{16}$ cm^{-3}, $n_i = 1.5 \times 10^{10}$ cm^{-3}, $D_n = 25$ cm^2/s, $\tau_{n0} = \tau_{p0} = 5 \times 10^{-7}$ s, and $D_p = 10$ cm^2/s. Calculate J_s.
Solution:

$$J_s = (1.602\times10^{-19})(1.5\times10^{10})^2 \left(\frac{1}{10^{16}}\sqrt{\frac{25}{5\times10^{-7}}} + \frac{1}{10^{16}}\sqrt{\frac{10}{5\times10^{-7}}} \right) \approx 4.16\times10^{-11}\ \text{A/cm}^2$$

D. Diode Equation

- Describes the current-voltage (I-V) characteristic of a PN junction diode.

- **Formula (Memorize)**:

$$I = I_s \left(e^{\frac{V_a}{V_T}} - 1 \right)$$

  where I_s is the reverse saturation current, V_a is the applied bias voltage, and $V_T = \frac{kT}{q}$ is the thermal voltage (≈ 0.0259 V at 300 K).

Example 21: Applying the Diode Equation

Problem: Using $I_s = 1.116 \times 10^{-13}$ A and $V_a = 0.7$ V, calculate the diode current I.
Solution:

$$I = 1.116 \times 10^{-13} \left(e^{\frac{0.7}{0.0259}} - 1 \right) \approx 2.175 \times 10^{-2}\ \text{A}$$

VIII. Summary of Key Formulas (Prioritized)

- **Intrinsic Carrier Concentration:**

$$n_i = \sqrt{N_c N_v} e^{-\frac{E_g}{2kT}}$$

- **Mass Action Law:**

$$np = n_i^2$$

- **Fermi Level Position:**

$$E_F - E_{Fi} = kT \ln\left(\frac{N_D}{n_i}\right) \quad \text{(n-type)}$$

$$E_{Fi} - E_F = kT \ln\left(\frac{N_A}{n_i}\right) \quad \text{(p-type)}$$

- **Drift Current:**

$$J_{drift,n} = qn\mu_n E \quad \text{(electrons)}$$

$$J_{drift,p} = qp\mu_p E \quad \text{(holes)}$$

- **Diffusion Current:**

$$J_{diff,n} = qD_n \frac{dn}{dx} \quad \text{(electrons)}$$

$$J_{diff,p} = -qD_p \frac{dp}{dx} \quad \text{(holes)}$$

- **Einstein Relation:**

$$\frac{D}{\mu} = \frac{kT}{q}$$

- **Diffusion Length:**

$$L = \sqrt{D\tau}$$

- **Built-in Potential:**

$$V_{bi} = \frac{kT}{q} \ln\left(\frac{N_A N_D}{n_i^2}\right)$$

- **Depletion Width:**

$$W = \sqrt{\frac{2\varepsilon_s}{q}\left(\frac{N_A + N_D}{N_A N_D}\right) V_{bi}}$$

- **Maximum Electric Field:**

$$E_{max} = \frac{qN_D W_N}{\varepsilon_s} = \frac{qN_A W_P}{\varepsilon_s}$$

- **Reverse Saturation Current Density:**

$$J_s = qn_i^2 \left(\frac{1}{N_A}\sqrt{\frac{D_n}{\tau_{n0}}} + \frac{1}{N_D}\sqrt{\frac{D_p}{\tau_{p0}}}\right)$$

- **Diode Current Equation:**

$$I = I_s\left(e^{\frac{V_a}{V_T}} - 1\right)$$

- **Minority Carrier Concentration Under Bias:**

$$n_p(-x_p) = n_{p0} e^{\frac{V_a}{V_T}}, \quad p_n(x_n) = p_{n0} e^{\frac{V_a}{V_T}}$$

- **Current Components in Forward Bias**:

$$J_n = q\sqrt{\frac{D_n}{\tau_{n0}}}\frac{n_i^2}{N_A}\left(e^{\frac{V_a}{V_T}} - 1\right)$$

$$J_p = q\sqrt{\frac{D_p}{\tau_{p0}}}\frac{n_i^2}{N_D}\left(e^{\frac{V_a}{V_T}} - 1\right)$$

- **Electric Field in Forward-Biased Diode**

$$E = \frac{J}{q\mu_n N_D}$$