Physics 322, Assignment #1

1. Consider the vector function
$$\boldsymbol{A} = \hat{x}e^{-x} + \hat{y}e^{-y} + \hat{z}e^{-z}.$$
(a) Calculate the divergence of $\boldsymbol{A}$.
(b) Calculate $\int_S \boldsymbol{A} \cdot d\boldsymbol{a}$ for the case of a surface of a cube of side ℓ centered at (x_0, y_0, z_0) and with faces that are parallel to the coordinate planes.

2. Consider the vector function
$$\boldsymbol{A} = yx^2(\hat{x} + \hat{y}) + \hat{z}xyz.$$
(a) Calculate the line integral
$$\oint_C \boldsymbol{A} \cdot d\boldsymbol{\ell},$$
where C is a circle of radius a located in the xy plane and centered at the origin of coordinates.
(b) Verify Stokes' theorem by computing $\int_S (\nabla \times \boldsymbol{A}) \cdot d\boldsymbol{a}$ for the case when the capping surface is taken to be the disk of radius a in the xy plane.

3. (Griffiths 5th ed., 1.40) Compute the divergence of the function
$$\boldsymbol{v} = (r\cos\theta)\hat{r} + (r\sin\theta)\hat{\theta} + (r\sin\theta\cos\phi)\hat{\phi}.$$
Check the divergence theorem for this function, using as your volume the inverted hemispherical bowl of radius R, resting on the xy plane and centered at the origin (shown in Fig 1.40 in the text).

4. (Griffiths 5th ed., 1.43) (a) Find the divergence of the function
$$\boldsymbol{v} = s(2 + \sin^2\phi)\hat{s} + s\sin\phi\cos\phi\hat{\phi} + 3z\hat{z}.$$
(b) Test the divergence theorem for this function, using the quarter-cylinder (radius 2, height 5) as shown in Fig 1.43 in the text. [Note: the quarter-cylinder is aligned along the z axis, with its bottom face at $z = 0$, and it is situated in the xy plane.]
(c) Find the curl of $\boldsymbol{v}$.

5. Suppose $\boldsymbol{r} = (x, y, z)$ and $\boldsymbol{r}' = (x', y', z')$ represent position vectors of points P and P', respectively. Show that
$$\nabla\left(\frac{1}{|\boldsymbol{r} - \boldsymbol{r}'|}\right) = -\frac{\boldsymbol{r} - \boldsymbol{r}'}{|\boldsymbol{r} - \boldsymbol{r}'|^3}$$
and
$$\nabla'\left(\frac{1}{|\boldsymbol{r} - \boldsymbol{r}'|}\right) = \frac{\boldsymbol{r} - \boldsymbol{r}'}{|\boldsymbol{r} - \boldsymbol{r}'|^3},$$
where
$$\nabla = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}, \qquad \nabla' = \hat{x}\frac{\partial}{\partial x'} + \hat{y}\frac{\partial}{\partial y'} + \hat{z}\frac{\partial}{\partial z'}$$
denote differentiation with respect to the unprimed and primed coordinates, respectively.

6. (a) Calculate the electric field (magnitude and direction) at an arbitrary field point P with coordinates (x, y, z) for the following configuration of point charges: charges q at $(x = \pm a, 0, 0)$, and a charge $-2q$ at the origin. Take $q > 0$.
(b) Find the force on a test charge Q located a distance z on the positive z axis due to this configuration of charges. What is the force in the limit that $z \gg a$?