Math 421, Section 1
Homework 1

Problem 1 (De Morgan's laws)**.** Let A and B be statements. Use a truth table to prove the following:

(a) "Not (A and B)" is equivalent to "(not A) or (not B)".
(b) "Not (A or B)" is equivalent to "(not A) and (not B)".

Problem 2 (The distributive property)**.** Let A, B, and C be statements. Use a truth table to prove the following:

(a) "A and (B or C)" is equivalent to "(A and B) or (A and C)".
(b) "A or (B and C)" is equivalent to "(A or B) and (A or C)".

Problem 3. Let A and B be statements. If we know that A implies B, which one of the following can we conclude?

(a) A cannot be false.
(b) A and B are both true.
(c) If A is false, then B is false.
(d) B cannot be false.
(e) If B is false, then A is false.
(f) If B is true, then A is true.
(g) At least one of A and B is true.

Problem 4. Negate the following sentences:

(a) If there is a job worth doing, then it is worth doing well.
(b) Every cloud has a silver lining.
(c) For every complex problem, there is an answer that is clear, simple, and wrong.

Problem 5. Let A, B, and C be statements. Negate the following sentences:

(a) At least one of A and B are true.
(b) Both A and B are false.
(c) At least two of A, B, and C are false.

Problem 6. Let X be a set, and let $P(x)$ be a statement about elements x in X. Negate the following sentences:

(a) For every x in X, there is a y in X not equal to x, for which $P(y)$ is true.
(b) If $P(x)$ and $P(y)$ are both true, then $x = y$.