

# ECE/PHY 235 - Solid State Electronics: Focused Exam Notes

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# Overview

These notes provide a comprehensive summary of essential concepts, formulas, and example applications in semiconductor physics and PN junction operation. Designed for exam preparation, they focus on the relationships between doping, carrier concentrations, carrier transport mechanisms, continuity equations, and PN junction characteristics. Each section includes key explanations and worked examples to reinforce understanding.

## 1 Essential Formulas and Concepts

### 1.1 A. Semiconductors

#### 1.1.1 Intrinsic Semiconductors

**Definition:** Intrinsic semiconductors are pure materials where the number of electrons in the conduction band equals the number of holes in the valence band, maintaining electrical neutrality. This balance is characterized by the intrinsic carrier concentration,  $n_i$ , where  $n = p = n_i$ .

**Intrinsic Carrier Concentration ( $n_i$ ):**

$$n_i = \sqrt{N_c N_v} \exp\left(-\frac{E_G}{2kT}\right)$$

where:

- $N_c$ : Effective density of states in the conduction band
- $N_v$ : Effective density of states in the valence band
- $E_G$ : Bandgap energy
- $k$ : Boltzmann constant
- $T$ : Temperature in Kelvin

**Example 1:** Calculate  $n_i$  for silicon at room temperature (300 K), given:

$$N_c = 2.81 \times 10^{19} \text{ cm}^{-3}, \quad N_v = 1.16 \times 10^{19} \text{ cm}^{-3}, \quad E_G = 1.10 \text{ eV}$$

$$n_i = \sqrt{(2.81 \times 10^{19})(1.16 \times 10^{19})} \exp\left(-\frac{1.10}{2 \times 8.617 \times 10^{-5} \times 300}\right) \approx 1.0 \times 10^{10} \text{ cm}^{-3}$$

**Intrinsic Conductivity ( $\sigma$ ):**

$$\sigma = qn_i(\mu_e + \mu_h)$$

where:

- $q$ : Elementary charge ( $1.602 \times 10^{-19} \text{ C}$ )

- $\mu_e$ : Electron mobility
- $\mu_h$ : Hole mobility

**Example 2:** For silicon with  $n_i = 1.0 \times 10^{10} \text{ cm}^{-3}$ ,  $\mu_e = 1350 \text{ cm}^2/\text{V} \cdot \text{s}$ , and  $\mu_h = 450 \text{ cm}^2/\text{V} \cdot \text{s}$ :

$$\sigma = (1.602 \times 10^{-19})(1.0 \times 10^{10})(1350 + 450) \approx 2.88 \times 10^{-6} \Omega^{-1} \text{cm}^{-1}$$

**Resistivity ( $\rho$ ):**

$$\rho = \frac{1}{\sigma}$$

**Example 3:** Using the conductivity calculated above:

$$\rho = \frac{1}{2.88 \times 10^{-6}} \approx 3.47 \times 10^5 \Omega \text{cm}$$

### 1.1.2 Extrinsic Semiconductors (Doping)

**Definition:** Extrinsic semiconductors are created by introducing impurities (dopants) into an intrinsic semiconductor to modify its electrical properties. Dopants can be donors (n-type) or acceptors (p-type).

**n-type Semiconductors:**

- Doping with donor impurities (e.g., phosphorus in Si) introduces excess electrons.
- 

$$n \approx N_D, \quad p = \frac{n_i^2}{N_D}$$

where  $N_D$  is the donor concentration.

**p-type Semiconductors:**

- Doping with acceptor impurities (e.g., boron in Si) creates excess holes.
- 

$$p \approx N_A, \quad n = \frac{n_i^2}{N_A}$$

where  $N_A$  is the acceptor concentration.

**Example 4:** A  $1 \text{ cm}^3$  silicon crystal doped with 1 ppb arsenic (n-type). Given atomic concentration in silicon  $5 \times 10^{22} \text{ cm}^{-3}$ :

$$N_D = \frac{5 \times 10^{22}}{10^9} = 5 \times 10^{13} \text{ cm}^{-3}$$

$$n \approx N_D = 5 \times 10^{13} \text{ cm}^{-3}, \quad p = \frac{(1.0 \times 10^{10})^2}{5 \times 10^{13}} = 2.0 \times 10^6 \text{ cm}^{-3}$$

### 1.1.3 Compensation Doping

**Definition:** Compensation doping occurs when both donor and acceptor impurities are present in the semiconductor, partially canceling each other's effects.

**p-type Compensation:** If  $N_A > N_D$ ,

$$p \approx N_A - N_D, \quad n = \frac{n_i^2}{p}$$

**n-type Compensation:** If  $N_D > N_A$ ,

$$n \approx N_D - N_A, \quad p = \frac{n_i^2}{n}$$

**Example 5:** An n-type Si with  $N_D = 10^{16} \text{ cm}^{-3}$  phosphorus is doped with  $N_A = 10^{17} \text{ cm}^{-3}$  boron.

$$\begin{aligned} N_A > N_D &\Rightarrow \text{p-type} \\ p &\approx 10^{17} - 10^{16} = 9 \times 10^{16} \text{ cm}^{-3} \\ n &= \frac{(1.0 \times 10^{10})^2}{9 \times 10^{16}} \approx 1.1 \times 10^3 \text{ cm}^{-3} \end{aligned}$$

### 1.1.4 Fermi Level Position

**Definition:** The Fermi level ( $E_F$ ) indicates the energy level at which the probability of finding an electron is 50%. Its position relative to the intrinsic Fermi level ( $E_{Fi}$ ) reveals the type and concentration of carriers in the semiconductor.

**n-type:**

$$E_F - E_{Fi} = kT \ln \left( \frac{N_D}{n_i} \right)$$

**p-type:**

$$E_{Fi} - E_F = kT \ln \left( \frac{N_A}{n_i} \right)$$

**Example 6:**

- **n-type Si with  $N_D = 10^{16} \text{ cm}^{-3}$ :**

$$E_F - E_{Fi} = (8.617 \times 10^{-5} \times 300) \ln \left( \frac{10^{16}}{1.0 \times 10^{10}} \right) \approx 0.36 \text{ eV}$$

- **After compensation doping with  $2 \times 10^{17} \text{ cm}^{-3}$  boron ( $N'_A = 1.9 \times 10^{17} \text{ cm}^{-3}$ ):**

$$E_{Fi} - E_F = (8.617 \times 10^{-5} \times 300) \ln \left( \frac{1.9 \times 10^{17}}{1.0 \times 10^{10}} \right) \approx 0.43 \text{ eV}$$

## 1.2 B. Carrier Transport

### 1.2.1 Drift Current

**Definition:** Drift current arises from the movement of charge carriers under the influence of an electric field.

**Formulas:**

$$J_{n,\text{drift}} = qn\mu_n E$$

$$J_{p,\text{drift}} = qp\mu_p E$$

**Derivation:**

1. Consider a semiconductor with electron concentration  $n$  and electron drift velocity  $v_d = \mu_n E$ .
2. In a time  $\Delta t$ , electrons move a distance  $v_d \Delta t$ .
3. Number of electrons crossing area  $A$  in  $\Delta t$ :  $nAv_d \Delta t$ .
4. Charge crossing area:  $\Delta Q = qnAv_d \Delta t$ .
5. Current:  $I = \frac{\Delta Q}{\Delta t} = qnAv_d$ .
6. Current density:  $J = \frac{I}{A} = qn\mu_n E$ .

**Example 7:** n-type silicon with  $N_D = 10^{16} \text{ cm}^{-3}$ ,  $\mu_n = 1350 \text{ cm}^2/\text{V} \cdot \text{s}$ , and  $E = 100 \text{ V/cm}$ .

$$J_{\text{drift}} = (1.602 \times 10^{-19})(10^{16})(1350)(100) = 216.27 \text{ A/cm}^2$$

**Note:** Increased doping reduces mobility due to increased scattering, but the increased carrier concentration can still lead to higher drift current.

### 1.2.2 Diffusion Current

**Definition:** Diffusion current results from the movement of charge carriers from regions of high concentration to low concentration, driven by concentration gradients.

**Formulas:**

$$J_{n,\text{diff}} = qD_n \frac{dn}{dx}$$

$$J_{p,\text{diff}} = -qD_p \frac{dp}{dx}$$

**Derivation:**

1. Consider a non-uniform carrier concentration  $p(x)$ .
2. Hole flux:  $F_p = -D_p \frac{dp}{dx}$ .
3. Diffusion current density:  $J_{p,\text{diff}} = qF_p = -qD_p \frac{dp}{dx}$ .

**Example 8:** p-type silicon with hole concentration varying linearly from  $10^{17} \text{ cm}^{-3}$  to  $5 \times 10^{16} \text{ cm}^{-3}$  over  $50 \mu\text{m}$ , and  $D_p = 12 \text{ cm}^2/\text{s}$ .

$$\frac{dp}{dx} = \frac{5 \times 10^{16} - 10^{17}}{50 \times 10^{-4}} = -1 \times 10^{19} \text{ cm}^{-4}$$

$$J_{p,\text{diff}} = -(1.602 \times 10^{-19})(12)(-1 \times 10^{19}) = 19.224 \text{ A/cm}^2$$

### 1.2.3 Einstein Relation

**Definition:** The Einstein relation links the diffusion coefficient ( $D$ ) and mobility ( $\mu$ ) of a charge carrier.

$$\frac{D}{\mu} = \frac{kT}{q}$$

**Example 9:** At 300 K,  $\frac{kT}{q} \approx 0.0259 \text{ V}$ . For electrons with  $\mu_n = 1350 \text{ cm}^2/\text{V} \cdot \text{s}$ :

$$D_n = \mu_n \frac{kT}{q} = 1350 \times 0.0259 \approx 34.965 \text{ cm}^2/\text{s}$$

### 1.2.4 Total Current

**Definition:** The total current in a semiconductor is the sum of the drift and diffusion current densities.

$$J = J_{\text{drift}} + J_{\text{diff}}$$

**Example 10:** Silicon with  $n(x) = 5 \times 10^{15} + 3 \times 10^{14}x$ ,  $E = 50 \text{ V/cm}$ ,  $\mu_n = 1350 \text{ cm}^2/\text{V} \cdot \text{s}$ , and  $D_n = 35 \text{ cm}^2/\text{s}$ . At  $x = 10 \mu\text{m}$ :

- $J_{\text{drift}} \approx 54.07 \text{ A/cm}^2$
- $J_{\text{diff}} \approx 0.00168 \text{ A/cm}^2$
- $J_{\text{total}} \approx 54.07242 \text{ A/cm}^2$  (drift current dominates)

## 1.3 C. Continuity Equation and Carrier Dynamics

### 1.3.1 Minority Carrier Lifetime ( $\tau$ )

**Definition:** The average time a minority carrier exists before recombining with a majority carrier. It measures how long excess carriers persist in a semiconductor.

### 1.3.2 Minority Carrier Diffusion Length ( $L$ )

$$L = \sqrt{D\tau}$$

where  $D$  is the diffusion coefficient and  $\tau$  is the minority carrier lifetime.

**Example 11:** p-type semiconductor with  $\tau_{n0} = 5 \times 10^{-7} \text{ s}$  and  $D_n = 25 \text{ cm}^2/\text{s}$ .

$$L_n = \sqrt{(25)(5 \times 10^{-7})} \approx 3.54 \times 10^{-3} \text{ cm} = 35.4 \mu\text{m}$$

### 1.3.3 Continuity Equation Solutions

#### Uniform Generation:

$$\Delta p(t) = g_0 \tau_p (1 - e^{-t/\tau_p})$$

**Example 12:** n-type semiconductor with  $g_0 = 5 \times 10^{21} \text{ cm}^{-3}\text{s}^{-1}$  and  $\tau_{p0} = 10^{-7} \text{ s}$ . At  $t = 10^{-7} \text{ s}$ :

$$\Delta p(10^{-7}) = (5 \times 10^{21})(10^{-7})(1 - e^{-1}) \approx 3.16 \times 10^{14} \text{ cm}^{-3}$$

#### Minority Carrier Decay:

$$\Delta p(t) = \Delta p(0)e^{-t/\tau_p}$$

**Example 13:** n-type semiconductor with initial excess holes  $10^{14} \text{ cm}^{-3}$  and  $\tau_{p0} = 50 \text{ ns}$ . Time to reach  $1/e$  of initial value:

$$t = \tau_{p0} = 50 \text{ ns}$$

#### Steady-State Diffusion from a Point Source:

$$\Delta p(x) = \Delta p(0)e^{-x/L_p}$$

**Example 14:** p-type semiconductor with excess carriers generated at  $x = 0$ ,  $\Delta n(0) = 10^{15} \text{ cm}^{-3}$ , and  $L_n = 35.4 \mu\text{m}$ . At  $x = 30 \mu\text{m}$ :

$$\Delta n(30 \mu\text{m}) = (10^{15})e^{-30/35.4} \approx 4.27 \times 10^{14} \text{ cm}^{-3}$$

## 2 Fundamentals of Semiconductors

### 2.1 A. Intrinsic Semiconductors

Intrinsic semiconductors have equal electron and hole concentrations ( $n = p = n_i$ ). The carrier concentration is temperature-dependent and influenced by the bandgap energy.

#### Key Formulas:

$$n_i = \sqrt{N_c N_v} \exp\left(-\frac{E_G}{2kT}\right)$$

$$\sigma = qn_i(\mu_e + \mu_h)$$

$$\rho = \frac{1}{\sigma}$$

**Example 1 Recap:** Calculated  $n_i$  for silicon and used it to determine conductivity and resistivity.



## 2.2 B. Extrinsic Semiconductors (Doping)

Doping modifies the electrical properties of semiconductors by introducing donor or acceptor atoms.

**n-type:**

$$n \approx N_D, \quad p = \frac{n_i^2}{N_D}$$

**p-type:**

$$p \approx N_A, \quad n = \frac{n_i^2}{N_A}$$

**Example 4 Recap:** Demonstrated n-type doping in silicon with arsenic and calculated carrier concentrations.

## 2.3 C. Compensation Doping

When both donors and acceptors are present, they partially compensate each other, altering the carrier concentrations.

**Formulas:**

$$\text{If } N_A > N_D : \quad p \approx N_A - N_D, \quad n = \frac{n_i^2}{p}$$

$$\text{If } N_D > N_A : \quad n \approx N_D - N_A, \quad p = \frac{n_i^2}{n}$$

**Example 5 Recap:** Showed p-type behavior after compensation doping in an n-type silicon sample.

## 2.4 D. Fermi Level Position

The Fermi level indicates the probability of electron occupation at a specific energy level and shifts based on doping.

**Formulas:**

$$\text{n-type: } E_F - E_{Fi} = kT \ln \left( \frac{N_D}{n_i} \right)$$

$$\text{p-type: } E_{Fi} - E_F = kT \ln \left( \frac{N_A}{n_i} \right)$$

**Example 6 Recap:** Calculated the shift in Fermi level for both n-type and compensated p-type silicon.

# 3 Carrier Transport

## 3.1 A. Drift Current

**Definition:** Current resulting from charge carriers moving under an electric field.

**Formulas:**

$$J_{n,\text{drift}} = qn\mu_n E$$

$$J_{p,\text{drift}} = qp\mu_p E$$

**Example 7 Recap:** Calculated drift current in n-type silicon under a given electric field.

### 3.2 B. Diffusion Current

**Definition:** Current caused by carriers moving from high to low concentration regions.

**Formulas:**

$$J_{n,\text{diff}} = qD_n \frac{dn}{dx}$$

$$J_{p,\text{diff}} = -qD_p \frac{dp}{dx}$$

**Example 8 Recap:** Determined diffusion current in p-type silicon with a given concentration gradient.

### 3.3 C. Einstein Relation

**Definition:** Relates diffusion coefficient and mobility of carriers.

$$\frac{D}{\mu} = \frac{kT}{q}$$

**Example 9 Recap:** Used the Einstein relation to calculate the diffusion coefficient for electrons.

### 3.4 D. Total Current

**Definition:** The sum of drift and diffusion currents.

$$J = J_{\text{drift}} + J_{\text{diff}}$$

**Example 10 Recap:** Calculated total current in silicon, showing drift current dominance.

## 4 Continuity Equation and Carrier Dynamics

### 4.1 A. Minority Carrier Lifetime ( $\tau$ )

**Definition:** The average time a minority carrier exists before recombining.

### 4.2 B. Minority Carrier Diffusion Length ( $L$ )

$$L = \sqrt{D\tau}$$

**Example 11 Recap:** Calculated diffusion length for minority carriers in p-type semiconductor.

## 4.3 C. Continuity Equation Solutions

**Uniform Generation:**

$$\Delta p(t) = g_0 \tau_p (1 - e^{-t/\tau_p})$$

**Example 12 Recap:** Determined excess carrier concentration under uniform generation.

**Minority Carrier Decay:**

$$\Delta p(t) = \Delta p(0) e^{-t/\tau_p}$$

**Example 13 Recap:** Calculated carrier decay time.

**Steady-State Diffusion from a Point Source:**

$$\Delta p(x) = \Delta p(0) e^{-x/L_p}$$

**Example 14 Recap:** Computed excess carrier concentration at a distance from point source.

## 5 PN Junctions

### 5.1 A. Built-in Potential ( $V_{bi}$ )

**Definition:** The potential difference across the depletion region in a PN junction at equilibrium, resulting from the diffusion of carriers and formation of space charge.

**Formula:**

$$V_{bi} = \frac{kT}{q} \ln \left( \frac{N_A N_D}{n_i^2} \right)$$

**Derivation Highlights:**

1. At equilibrium, drift and diffusion currents balance:  $J_n = J_p = 0$ .
2. Using carrier concentrations and electric field relations, integrating across the depletion region leads to the built-in potential formula.

**Example 15:** Silicon PN junction with  $N_D = 10^{15} \text{ cm}^{-3}$ ,  $N_A = 2 \times 10^{17} \text{ cm}^{-3}$ ,  $n_i = 1.5 \times 10^{10} \text{ cm}^{-3}$  at 300 K.

$$V_{bi} = (0.0259 \text{ V}) \ln \left( \frac{(2 \times 10^{17})(10^{15})}{(1.5 \times 10^{10})^2} \right) \approx 0.713 \text{ V}$$

### 5.2 B. Depletion Region

**Definition:** The region around the PN junction devoid of free charge carriers, containing fixed ionized dopant atoms, creating an electric field opposing further diffusion.

**Depletion Width ( $W$ ):**

$$W = \sqrt{\frac{2\epsilon_s}{q} \left( \frac{N_A + N_D}{N_A N_D} \right) (V_{bi} - V_a)}$$

where:

- $\varepsilon_s$ : Permittivity of the semiconductor
- $V_a$ : Applied voltage

**Example 16:** Silicon PN junction with  $N_A = 10^{16} \text{ cm}^{-3}$ ,  $N_D = 10^{15} \text{ cm}^{-3}$ ,  $n_i = 1.5 \times 10^{10} \text{ cm}^{-3}$ ,  $\varepsilon_s = 11.7\varepsilon_0$ ,  $\varepsilon_0 = 8.85 \times 10^{-14} \text{ F/cm}$ , and  $V_{bi} \approx 0.6525 \text{ V}$  at equilibrium ( $V_a = 0$ ).

$$W = \sqrt{\frac{2 \times (11.7 \times 8.85 \times 10^{-14})}{1.602 \times 10^{-19}} \times \frac{(10^{16} + 10^{15})}{(10^{16})(10^{15})} \times 0.6525} \approx 1.024 \times 10^{-4} \text{ cm} = 1.024 \mu\text{m}$$

**Widths on P and N sides ( $W_P$ ,  $W_N$ ):**

$$N_A W_P = N_D W_N$$

$$W = W_N + W_P$$

**Example 17:** For a PN junction diode with  $N_D = 10^{15} \text{ cm}^{-3}$  and  $N_A = 10^{16} \text{ cm}^{-3}$ , and total depletion width  $W \approx 0.951 \mu\text{m}$ :

$$W_N = 0.8644 \mu\text{m}, \quad W_P = \frac{N_D W_N}{N_A} = \frac{10^{15} \times 0.8644}{10^{16}} = 0.08644 \mu\text{m}$$

$$W = W_N + W_P = 0.8644 \mu\text{m} + 0.08644 \mu\text{m} = 0.951 \mu\text{m}$$

### 5.3 C. Maximum Electric Field ( $E_{\max}$ )

**Definition:** The peak electric field in the depletion region, occurring at the junction interface.

**Formula:**

$$E_{\max} = \frac{q N_D W_N}{\varepsilon_s} = \frac{q N_A W_P}{\varepsilon_s}$$

**Example 18:** Silicon PN junction with  $W_N = 0.8644 \mu\text{m}$  and  $N_D = 10^{15} \text{ cm}^{-3}$ :

$$E_{\max} = \frac{(1.602 \times 10^{-19})(10^{15})(0.8644 \times 10^{-4})}{(11.7)(8.854 \times 10^{-14})} \approx 1.34 \times 10^4 \text{ V/cm}$$

**Alternative Derivation:**

$$V_{bi} = \frac{1}{2} E_{\max} W$$

**Example 19:** Using  $E_{\max} = 1.44 \times 10^4 \text{ V/cm}$  and  $W = 1.024 \mu\text{m}$ :

$$V_{bi} = \frac{1}{2} (1.44 \times 10^4 \text{ V/cm})(1.024 \times 10^{-4} \text{ cm}) = 0.737 \text{ V}$$

This aligns closely with the previously calculated  $V_{bi}$ , confirming the relationship.

## 5.4 D. Reverse Saturation Current ( $I_s$ or $J_s$ )

**Definition:** The small current flowing through a PN junction under reverse bias, caused by the drift of minority carriers.

**Formulas:**

$$J_s = qn_i^2 \left( \frac{1}{N_a} \sqrt{\frac{D_n}{\tau_{n0}}} + \frac{1}{N_d} \sqrt{\frac{D_p}{\tau_{p0}}} \right)$$
$$I_s = Aqn_i^2 \left( \frac{D_n}{L_n N_A} + \frac{D_p}{L_p N_D} \right)$$

where  $A$  is the junction area.

**Example 20:** Silicon PN junction at 300 K with  $N_A = N_D = 10^{16} \text{ cm}^{-3}$ ,  $n_i = 1.5 \times 10^{10} \text{ cm}^{-3}$ ,  $D_n = 25 \text{ cm}^2/\text{s}$ ,  $\tau_{n0} = \tau_{p0} = 5 \times 10^{-7} \text{ s}$ , and  $D_p = 10 \text{ cm}^2/\text{s}$ .

$$J_s = (1.602 \times 10^{-19})(1.5 \times 10^{10})^2 \left( \frac{1}{10^{16}} \sqrt{\frac{25}{5 \times 10^{-7}}} + \frac{1}{10^{16}} \sqrt{\frac{10}{5 \times 10^{-7}}} \right) \approx 4.16 \times 10^{-11} \text{ A/cm}^2$$

## 5.5 E. Diode Equation

**Definition:** Describes the current-voltage (I-V) characteristic of a PN junction diode.

$$I = I_s (e^{V_a/V_T} - 1)$$

where:

- $I_s$ : Reverse saturation current
- $V_a$ : Applied voltage
- $V_T = \frac{kT}{q} \approx 0.0259 \text{ V}$  at 300 K

**Example 21:** Using  $I_s = 1.116 \times 10^{-13} \text{ A}$  and  $V_a = 0.7 \text{ V}$ :

$$I = 1.116 \times 10^{-13} (e^{0.7/0.0259} - 1) \approx 2.175 \times 10^{-2} \text{ A}$$

## 5.6 F. Minority Carrier Injection (Forward Bias)

**Definition:** Under forward bias, majority carriers are injected across the junction and become minority carriers in the neutral regions.

**Formulas:**

$$n_p(-x_p) = n_{p0} e^{V_a/V_T}, \quad p_n(x_n) = p_{n0} e^{V_a/V_T}$$

where:

- $n_{p0}$ : Equilibrium minority carrier concentration in p-region
- $p_{n0}$ : Equilibrium minority carrier concentration in n-region

**Example 22:** Silicon PN junction at 300 K with  $N_d = 10^{16} \text{ cm}^{-3}$ ,  $N_a = 6 \times 10^{15} \text{ cm}^{-3}$ ,  $n_i = 1.5 \times 10^{10} \text{ cm}^{-3}$ , and  $V_a = 0.60 \text{ V}$ .

$$n_{p0} = \frac{(1.5 \times 10^{10})^2}{6 \times 10^{15}} = 3.75 \times 10^4 \text{ cm}^{-3}$$

$$p_{n0} = \frac{(1.5 \times 10^{10})^2}{10^{16}} = 2.25 \times 10^4 \text{ cm}^{-3}$$

$$n_p(-x_p) = 3.75 \times 10^4 e^{0.60/0.0259} \approx 4.31 \times 10^{14} \text{ cm}^{-3}$$

$$p_n(x_n) = 2.25 \times 10^4 e^{0.60/0.0259} \approx 2.59 \times 10^{14} \text{ cm}^{-3}$$

## 5.7 G. Current Components in Forward Bias

**Definition:** In forward bias, the total current is the sum of electron and hole diffusion currents at the edges of the depletion region.

**Formulas:**

$$J_n = q \sqrt{\frac{D_n}{\tau_{n0}}} \frac{n_i^2}{N_a} \left[ \exp\left(\frac{qV_a}{kT}\right) - 1 \right]$$

$$J_p = q \sqrt{\frac{D_p}{\tau_{p0}}} \frac{n_i^2}{N_d} \left[ \exp\left(\frac{qV_a}{kT}\right) - 1 \right]$$

**Example 23:** Design a silicon PN junction diode at  $T = 300 \text{ K}$  such that  $J_n = 20 \text{ A/cm}^2$  and  $J_p = 5 \text{ A/cm}^2$  at  $V_a = 0.65 \text{ V}$ , given:

$$D_n = 25 \text{ cm}^2/\text{s}, \quad D_p = 10 \text{ cm}^2/\text{s}, \quad \tau_{n0} = \tau_{p0} = 5 \times 10^{-7} \text{ s}, \quad n_i = 1.5 \times 10^{10} \text{ cm}^{-3}$$

- Solving for  $N_a$  using  $J_n$ :

$$20 = (1.602 \times 10^{-19}) \sqrt{\frac{25}{5 \times 10^{-7}}} \frac{(1.5 \times 10^{10})^2}{N_a} \left[ \exp\left(\frac{0.65}{0.0259}\right) - 1 \right]$$

$$N_a \approx 1.01 \times 10^{15} \text{ cm}^{-3}$$

- Solving for  $N_d$  using  $J_p$ :

$$5 = (1.602 \times 10^{-19}) \sqrt{\frac{10}{5 \times 10^{-7}}} \frac{(1.5 \times 10^{10})^2}{N_d} \left[ \exp\left(\frac{0.65}{0.0259}\right) - 1 \right]$$

$$N_d \approx 2.55 \times 10^{15} \text{ cm}^{-3}$$

**Note:** Far from the junction in a forward-biased diode, the current is primarily carried by the drift of majority carriers. The electric field can be estimated by assuming the total current is approximately equal to the majority carrier drift current.

## 6 PN Junctions

### 6.1 A. Built-in Potential ( $V_{bi}$ )

**Definition:** The potential difference across the depletion region in a PN junction at equilibrium, arising from the diffusion of carriers and the formation of space charge regions.

**Formula:**

$$V_{bi} = \frac{kT}{q} \ln \left( \frac{N_A N_D}{n_i^2} \right)$$

**Example 15 Recap:** Calculated built-in potential for a silicon PN junction with given doping concentrations.

### 6.2 B. Depletion Region

**Definition:** The region around the PN junction that is devoid of free charge carriers, containing only fixed ionized dopants, which creates an electric field opposing further carrier diffusion.

**Depletion Width ( $W$ ):**

$$W = \sqrt{\frac{2\varepsilon_s}{q} \left( \frac{N_A + N_D}{N_A N_D} \right) (V_{bi} - V_a)}$$

where:

- $\varepsilon_s$ : Permittivity of the semiconductor
- $V_a$ : Applied voltage

**Example 16 Recap:** Computed depletion width for a silicon PN junction at equilibrium. Widths on P and N sides ( $W_P$ ,  $W_N$ ):

$$N_A W_P = N_D W_N$$

$$W = W_P + W_N$$

**Example 17 Recap:** Determined individual depletion widths on p and n sides for a given PN junction.

### 6.3 C. Maximum Electric Field ( $E_{\max}$ )

**Definition:** The peak electric field within the depletion region, occurring at the junction interface.

**Formula:**

$$E_{\max} = \frac{q N_D W_N}{\varepsilon_s} = \frac{q N_A W_P}{\varepsilon_s}$$

**Example 18 Recap:** Calculated the maximum electric field for a silicon PN junction with given parameters.

## 6.4 D. Reverse Saturation Current ( $I_s$ or $J_s$ )

**Definition:** The small leakage current that flows through a PN junction under reverse bias, caused by the drift of minority carriers.

**Formulas:**

$$J_s = qn_i^2 \left( \frac{1}{N_A} \sqrt{\frac{D_n}{\tau_{n0}}} + \frac{1}{N_D} \sqrt{\frac{D_p}{\tau_{p0}}} \right)$$
$$I_s = Aqn_i^2 \left( \frac{D_n}{L_n N_A} + \frac{D_p}{L_p N_D} \right)$$

where  $A$  is the junction area.

**Example 20 Recap:** Calculated reverse saturation current density for a silicon PN junction.

## 6.5 E. Diode Equation

**Definition:** Describes the relationship between the current through a diode and the voltage across it.

$$I = I_s (e^{V_a/V_T} - 1)$$

where:

- $I_s$ : Reverse saturation current
- $V_a$ : Applied voltage
- $V_T = \frac{kT}{q} \approx 0.0259$  V at 300 K

**Example 21 Recap:** Applied the diode equation to calculate current at a given forward bias voltage.

## 6.6 F. Minority Carrier Injection (Forward Bias)

**Definition:** Under forward bias, majority carriers are injected across the junction and become minority carriers in the neutral regions, increasing the current exponentially.

**Formulas:**

$$n_p(-x_p) = n_{p0} e^{V_a/V_T}, \quad p_n(x_n) = p_{n0} e^{V_a/V_T}$$

**Example 22 Recap:** Calculated the injected minority carrier concentrations under forward bias in a silicon PN junction.



## 6.7 G. Current Components in Forward Bias

**Definition:** In forward bias, the total current is the sum of the electron and hole diffusion currents at the edges of the depletion region.

**Formulas:**

$$J_n = q \sqrt{\frac{D_n}{\tau_{n0}}} \frac{n_i^2}{N_A} \left[ \exp\left(\frac{qV_a}{kT}\right) - 1 \right]$$
$$J_p = q \sqrt{\frac{D_p}{\tau_{p0}}} \frac{n_i^2}{N_D} \left[ \exp\left(\frac{qV_a}{kT}\right) - 1 \right]$$

**Example 23 Recap:** Designed a silicon PN junction diode with specified forward bias current components by determining appropriate doping concentrations.

**Note:** Far from the junction in a forward-biased diode, the current is primarily carried by the drift of majority carriers. The electric field can be estimated by assuming the total current is approximately equal to the majority carrier drift current.

## Summary of Key Concepts

- **\*\*Carrier Concentrations:\*\*** Relationships between  $n$ ,  $p$ ,  $N_D$ ,  $N_A$ , and  $n_i$  are fundamental in determining the type and behavior of semiconductors.
- **\*\*Drift vs. Diffusion Current:\*\*** Drift current is induced by an electric field, whereas diffusion current arises from concentration gradients. Understanding which mechanism dominates under specific conditions is crucial.
- **\*\*Continuity Equation:\*\*** Describes the balance of generation, recombination, and transport of carriers, essential for analyzing dynamic carrier behavior.
- **\*\*Minority Carrier Lifetime and Diffusion Length:\*\*** These parameters characterize how carriers move and recombine, influencing device performance.
- **\*\*PN Junction Physics:\*\*** Understanding built-in potential, depletion region formation, electric fields, and the diode equation is vital for analyzing and designing semiconductor devices.
- **\*\*Forward Bias Effects:\*\*** Forward bias increases minority carrier injection, leading to exponential increases in current, which is the basis for diode operation.

## Additional Tips for Exam Preparation

- **\*\*Understand Derivations:\*\*** While formulas are essential, understanding their derivations provides deeper insight and aids in problem-solving.
- **\*\*Practice Examples:\*\*** Work through the provided examples and additional problems to reinforce concepts and improve computational skills.

- **\*\*Memorize Key Formulas:\*\*** Ensure you have key formulas memorized, but also know when and how to apply them.
- **\*\*Conceptual Clarity:\*\*** Focus on grasping the physical meanings behind mathematical expressions to better tackle conceptual questions.
- **\*\*Time Management:\*\*** During exams, manage your time effectively by quickly identifying which formulas and concepts apply to each question.