

6-5

1. Show that the wave function $\Psi(x, t) = A \sin(kx - \omega t)$ does not satisfy the time-dependent Schrodinger equation.
2. Show that $\Psi(x, t) = A \cos(kx - \omega t) + iA \sin(kx - \omega t)$ satisfies the time-dependent Schrodinger equation.

Solution: Recall the time-dependent Schrodinger equation:

$$-\frac{\hbar^2}{2m}\Psi_{xx} + V\Psi = i\hbar\Psi_t, \quad (1)$$

assuming $V = 0$.

1. We have $\Psi_{xx} = -k^2 A \sin(kx - \omega t)$, and $\Psi_t = -A\omega \cos(kx - \omega t)$. Trivially, plugging back into Equation 1, the LHS is **not equal** to the RHS.
2. We have

$$\begin{aligned} -\frac{\hbar^2}{2m}\Psi_{xx} &= \frac{\hbar^2 k^2 A}{2m} \cos(kx - \omega t) + \frac{\hbar^2 i k^2 A}{2m} \sin(kx - \omega t) \\ i\hbar\Psi_t &= \hbar\omega A \cos(kx - \omega t) + i\hbar\omega A \sin(kx - \omega t). \end{aligned} \quad (2)$$

Upon noticing $\hbar\omega = \frac{\hbar^2 k^2}{2m} + V$, with $V = 0$, the two equations above **are equivalent**

6-9

A particle is in a infinite square well of width L . Calculate the ground-state energy if:

1. The particle is a proton and $L = 0.1$ nm, a typical size for a molecule;
2. the particle is a proton and $L = 1$ fm, a typical size for a nucleus.

Solution: Using $E_1 = \frac{\hbar^2 \pi^2}{2mL^2}$, we have:

1.
$$E_1 = \frac{\hbar^2 \pi^2}{2 * 1.67e-27 \text{ kg} * (0.1\text{nm}^2)} = 3.28e-21 \text{ kg } m^2/s^2 = \boxed{0.021 \text{ eV}} \quad (3)$$

2. Similarly,

$$E_1 = 3.28e-11 \text{ kg } m^2/s^2 = \boxed{205 \text{ MeV}} \quad (4)$$

6-12

A mass of 10^{-6} g is moving with a speed of about 10^{-1} cm/s in a box of length 1cm. Treating this as a one-dimensional infinite square well, calculate the approximate value of the quantum number n

Solution: Equating the kinetic energy of the particle to the energy of n-th level of the box, we have:

$$\begin{aligned} \frac{1}{2}mv^2 &= \frac{n^2 \pi^2 \hbar^2}{2mL^2} \Rightarrow n = \frac{mvL}{\pi \hbar} \\ \Rightarrow n &= \frac{10^{-6} \text{ g} * 0.1 \text{ cm/s} * 1 \text{ cm}}{\pi \hbar} = \boxed{3.02e19} \end{aligned} \quad (5)$$

6-16

The wavelength of light emitted by a ruby laser is 694.3nm. Assuming that the emission of a photon of this wavelength accompanies the transition of an electron from the $n = 2$ level to the $n = 1$ level of an infinite square well, compute L for the well.

Solution: Equating the energy difference between the two energy levels to the energy of the photon emitted, we have:

$$\begin{aligned} \Delta E &= \frac{hc}{\lambda}, \quad \Delta E = E_2 - E_1 = \frac{3\hbar^2\pi^2}{2mL^2} \\ \Rightarrow L &= \sqrt{\frac{3\pi\lambda h}{8mc}} = \left(\frac{3 * 694.3 \text{ nm} * h}{8 * m_e * c} \right) = \boxed{7.95 * e-10 \text{ m}} \end{aligned} \quad (6)$$