6-5

1. Show that the wave funciton $\Psi(x,t) = A\sin(kx - \omega t)$ does not satisfy the time-dependent Schrodinger equation.
2. Show that $\Psi(x,t) = A\cos(kx - \omega t) + iA\sin(kx - \omega t)$satisfies the time-dependent Schrodinger equation.

Solution: Recall the time-dependent Schrodinger equation:

$$-\frac{\hbar^2}{2m}\Psi_{xx} + V\Psi = i\hbar\Psi_t, \tag{1}$$

assuming $V = 0$.

1. We have $\Psi_{xx} = -k^2 A\sin(kx - \omega t)$, and $\Psi_t = -A\omega\cos(kx - \omega t)$. Trivially, plugging back into Equation 1, the LHS is **not equal** to the RHS.

2. We have

$$\begin{gathered} -\frac{\hbar^2}{2m}\Psi_{xx} = \frac{\hbar^2 k^2 A}{2m}\cos(kx - \omega t) + \frac{\hbar^2 i k^2 A}{2m}\sin(kx - \omega t) \\ i\hbar\Psi_t = \hbar\omega A\cos(kx - \omega t) + i\hbar\omega A\sin(kx - \omega t). \end{gathered} \tag{2}$$

   Upon noticing $\hbar\omega = \frac{\hbar^2 k^2}{2m} + V$, with $V = 0$, the two equations above **are equivalent**

6-9

A particle is in a infinite square well of width L. Calculate the ground-state energy if:
1. The particle is a proton and $L = 0.1$ nm. a typical size for a molecule;
2. the particle is a proton and $L = 1$fm, a typical size for a nucleus.

Solution: Using $E_1 = \frac{\hbar^2\pi^2}{2mL^2}$, we have:

1. $$E_1 = \frac{\hbar^2\pi^2}{2 * 1.67e\text{-}27 \text{ kg} * (0.1\text{nm}^2)} = 3.28e\text{-}21 \text{ kg } m^2/s^2 = \boxed{0.021 \text{ eV}} \tag{3}$$

2. Similarly,

$$E_1 = 3.28e\text{-}11 \text{ kg } m^2/s^2 = \boxed{205 \text{ MeV}} \tag{4}$$

6-12

A mass of 10^{-6} g is moving with a speed of about 10^{-1}cm/s in a box of length 1cm. Treating this as a one-dimensional infinite square well, calculate the approximate value of the quantum number n

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Solution: Equating the kinetic energy of the particle to the energy of n-th level of the box, we have:

$$\begin{gathered} \frac{1}{2}mv^2 = \frac{n^2\pi^2\hbar^2}{2mL^2} \quad \Rightarrow n = \frac{mvL}{\pi\hbar} \\ \Rightarrow n = \frac{10^{-6}g * 0.1 \text{ cm/s} * 1 \text{ cm}}{\pi\hbar} = \boxed{3.02e19} \end{gathered} \tag{5}$$

6-16

The wavelength of light emitted by a ruby laser is 694.3nm. Assuming that the emission of a photon of this wavelength accompanies the transition of an electron from the $n = 2$ level to the $n = 1$ level of an infinite square well, compute L for the well.

Solution: Equating the energy difference between the two energy levels to the energy of the photon emitted, we have:

$$\Delta E = \frac{hc}{\lambda}, \quad \Delta E = E_2 - E_1 = \frac{3\hbar^2\pi^2}{2mL^2}$$
$$\Rightarrow L = \sqrt{\frac{3\pi\lambda h}{8mc}} = \left(\frac{3 * 694.3 \text{ nm} * h}{8 * m_e * c}\right) = \boxed{7.95 * e\text{-}10 \text{ m}} \tag{6}$$