

Math 421, Section 1
Midterm 1
Fall 2024

First name: _____ **Last name:** _____

Instructions:

- This exam contains 3 problems, and there are a total of 30 points available.
- Show all your work in the space provided. You may also use the backs of pages.
- No outside resources are allowed, including notes, calculators, textbooks, etc.

| Question | Points | Score |
|----------|--------|-------|
| 1 | 10 | |
| 2 | 10 | |
| 3 | 10 | |
| Total: | 30 | |

1. (10 points) Using induction, prove that for all $n \in \mathbb{N}$ we have

$$\frac{1}{1 \cdot 4} + \frac{1}{4 \cdot 7} + \cdots + \frac{1}{(3n-2)(3n+1)} = \frac{n}{3n+1}.$$

2. Let $f : A \rightarrow B$ be a function. Prove or disprove the following statements:

(a) (5 points) $f(A_1) \cap f(A_2) \subseteq f(A_1 \cap A_2)$ for all $A_1, A_2 \subseteq A$.

(b) (5 points) $f(A_1 \cap A_2) \subseteq f(A_1) \cap f(A_2)$ for all $A_1, A_2 \subseteq A$.

-
3. (a) (3 points) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function and $a, \ell \in \mathbb{R}$ be numbers. State the definition of the *limit* of $f(x)$ as x approaches a is ℓ .
- (b) (7 points) Prove that the function $f(x) = x^2 + 3$ satisfies this definition for $a = 1$ and $\ell = 4$.

Extra paper