Math 421, Section 1
Homework 3
(Name)

Problem 1. Determine whether each of the following functions are injective, surjective, and bijective, and prove your answer.

(a) $f : \mathbb{Z} \to \mathbb{Z}$, $f(x) = 2x$.
(b) $g : \mathbb{R} \to \mathbb{R}$, $g(x) = 2x$.

Solution: (Type your solution to problem 1 here.) $\square$

Problem 2. Let $f : A \to B$ be a function and $A_1, A_2 \subseteq A$ and $B_1, B_2 \subseteq B$ be subsets. Prove the following statements:

(a) $f(A_1 \cup A_2) = f(A_1) \cup f(A_2)$.
(b) $f(A_1 \cap A_2) \subseteq f(A_1) \cap f(A_2)$.
(c) $f^{-1}(B_1 \cup B_2) = f^{-1}(B_1) \cup f^{-1}(B_2)$.
(d) $f^{-1}(B_1 \cap B_2) = f^{-1}(B_1) \cap f^{-1}(B_2)$.

Solution: (Type your solution to problem 2 here.) $\square$

Problem 3. Let $f : A \to B$ be a function. Prove that f is injective if and only if $f(A_1 \cap A_2) = f(A_1) \cap f(A_2)$ for all subsets $A_1, A_2 \subseteq A$.

Solution: (Type your solution to problem 3 here.) $\square$

Problem 4. Let $f : A \to B$ be a function. Prove that the following two statements are equivalent:

(a) The function f is surjective.
(b) For every set C and for any functions $g : B \to C$ and $h : B \to C$ such that $g \circ f = h \circ f$, we have $g = h$.

Solution: (Type your solution to problem 4 here.) $\square$

Problem 5. Let A be a nonempty set and $f : A \to A$ a function. We call f an *involution* if $(f \circ f)(a) = a$ for all $a \in A$. Prove that if $f : A \to A$ is an involution, then f is bijective. What is the inverse function f^{-1} in terms of f?

Solution: (Type your solution to problem 5 here.) $\square$

Problem 6. Prove or disprove the following statements:

(a) The set $\{x \in \mathbb{R} : x \geq 2\}$ is an interval.
(b) The set $\{x \in \mathbb{R} : x \neq 2\}$ is an interval.

(Hint: In order to disprove a statement, you must prove that the negation of the statement is true.)

Solution: (Type your solution to problem 6 here.) $\square$