

Math 421, Section 1
Practice Midterm 1
Fall 2024

First name: _____ **Last name:** _____

Instructions:

- This exam contains 3 problems, and there are a total of 30 points available.
- Show all your work in the space provided. You may also use the backs of pages.
- No outside resources are allowed, including notes, calculators, textbooks, etc.

Question	Points	Score
1	10	
2	10	
3	10	
Total:	30	

1. (a) (7 points) Prove the following statement: For all $a, b, c, d \in \mathbb{R}$ with $b, d \neq 0$, we have

$$(a \cdot b^{-1}) + (c \cdot d^{-1}) = (ad + bc) \cdot (bd)^{-1}.$$

Make sure each step uses a property of \mathbb{R} proved in lecture or homework. (You do not need to cite each fact by name; just show your logic one step at a time.)

- (b) (3 points) Negate the statement in part (a).

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2. (10 points) Let $f : A \rightarrow A$ be a function. Suppose that there exists a function $g : A \rightarrow A$ such that $(g \circ f)(x) = x$ and $(f \circ g)(x) = x$ for all $x \in A$. Prove that f is bijective.

3. (a) (2 points) State the definition of the set $A \subseteq \mathbb{R}$ is *open*.
(b) (4 points) Prove or disprove: The set $(-\infty, 1)$ is open.
(c) (4 points) Prove or disprove: The set $(-\infty, 1]$ is open.

Extra paper