Math 421, Section 1
Homework 2
(Name)

Problem 1. Prove that for any $x, y \in \mathbb{N}$, if x is odd and y is odd then $x + y$ is even.

Solution: (Type your solution to problem 1 here.) $\square$

Problem 2. Prove that for any $x \in \mathbb{N}$, if x is odd then x^3 is odd.

Solution: (Type your solution to problem 2 here.) $\square$

Problem 3. Using induction, prove that for all $n \in \mathbb{N}$ we have

$$1 + 3 + 5 + \cdots + (2n - 1) = n^2.$$

Solution: (Type your solution to problem 3 here.) $\square$

Problem 4. Compute the following sum:

$$\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \cdots + \frac{1}{(2n-1)(2n+1)}.$$

Prove that your answer is true for all $n \in \mathbb{N}$ using induction.

Solution: (Type your solution to problem 4 here.) $\square$

Problem 5. Prove the following statements for all $a, b \in \mathbb{R}$:

(a) $-a + (-b) = -(a + b)$.
(b) If $a, b \neq 0$ then $a^{-1} \cdot b^{-1} = (ab)^{-1}$.

Carefully justify every step using properties of $\mathbb{R}$ stated in lecture.

Solution: (Type your solution to problem 5 here.) $\square$

Problem 6. Prove the following statements for all $a, b, c, d \in \mathbb{R}$:

(a) If $a < b$ and $c < d$ then $a + c < b + d$.
(b) If $0 < a < b$ and $0 < c < d$ then $ac < bd$.

Carefully justify every step using properties of $\mathbb{R}$ stated in lecture.

Solution: (Type your solution to problem 6 here.) $\square$