**Highlights from ECE 235: Solid-state Physics**

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**Contents**

## 1 EM wave

### 1.1 waves

- Traverse wave: oscillation $\perp$ propagation
- Longitudinal wave: oscillation $\parallel$ propagation
- $v = \lambda f$

### 1.2 EM wave function

$$\begin{cases} E_x = E_0 \sin(kz - \omega t) \\ B_y = B_0 \sin(kz - \omega t) \end{cases} \qquad [1$$

where $k = \frac{2\pi}{\lambda}$ (wave number) , $\omega = 2\pi f = kc$ ( dispersion relationship), $B_0$ : magnetic field amplitude, $E_0$ : electric field amplitude

### 1.3 EM Energy flux

Energy flux the energy transferred per unit area per unit time in the direction of wave propagation of an EM wave is defined by the Poynting vecter

$$\vec{S} \equiv \frac{\vec{E} \times \vec{B}}{\mu_0}. \qquad [2$$

Where $\mu_0 = 1.25663706126e\text{-}6(N \cdot A^{-2})$ is the vacuum permeability.

- Intensity of EM wave is the magnitude of the Poynting vector:

$$I = \langle S \rangle = \frac{E_0^2}{377\Omega} \qquad [3$$

where $\Omega$ is ohm. Very unorthodoxy I know, but hey we are in Engineering Hall.

- Specially, when EM wave is emitted from a point light source with power $P$ ,

$$I = \frac{P}{4\pi r^2} = \frac{E_0^2}{377\Omega} \qquad [4$$

## 2 Double slit interference

Consider a double-slit setup, where the first dark line is at an angle $\theta$ from the central bright line with a distance $Y$. Distance from light source to screen is $L$. Then by trignometry:

$$Y = L \tan \theta. \qquad [5$$

When considering constructive/distructive interference,given the separation between two slits is $d$ the path difference between the two slits is

$$\begin{gathered} m\lambda = d \sin\theta \text{ constructive} \\ \left(m + \frac{1}{2}\right)\lambda = d \sin\theta \text{ destructive} \quad m = 0, 1, 2... \end{gathered} \qquad [6$$

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## 3 Photoelectric effect

- Energy of a photon

$$E_p = hf = \frac{hc}{\lambda} = \Phi + E_k \qquad [7$$

where $\Phi$ is the work function of the material, $E_k$ is the kinetic energy of the emitted electron at the surface of the material. $h = 6.26e\text{-}34$ is the Planck constant, $c = 3e\text{-}8$ m/s is the speed of light, $f$ is the frequency of the photon, and $\lambda$ is the wavelength of the photon.

- Motion for Photoelectric effect:

$$E_{k,m} + (\text{-}e)V_m = E_{k,d} + (\text{-}e)V_d \qquad [8$$

Where $E_{k,m}$ is K.E at the metal surface, $V_m$ is the voltage at the metal, $E_{k,d}$ is the K.E of the electron at the detector, and $V_d$ is the voltage at the detector.

- stopping potential

$$eV_{\text{stop}} = \frac{hc}{\lambda} - \Phi \tag{9}$$

the minimum potential required to stop the emitted electron.

- Threshold frequency& wavelength: set $E_k = 0$:

$$\begin{aligned} \Phi = hf_t = \frac{hc}{\lambda_t} \\ \Rightarrow f_t = \frac{\Phi}{h}, \quad \lambda_t = \frac{hc}{\Phi} \end{aligned} \tag{10}$$

## 4 Blackbody radiation

- Stefan-Boltzmann law:

$$R = \sigma T^4. \tag{11}$$

Where $R$ is the **power radiated per unit area**, or surface energy density of radiation. T is temprature in Kelvin, $\sigma = 5.67e\text{-}8(W \cdot m^{-2} \cdot K^{-4})$ is the Stefan-Boltzmann constant.

- Wien's displacement law:

$$\lambda_{\text{max}} T = b \tag{12}$$

where $b = 2.89e\text{-}3(m \cdot K)$ is the Wien's constant, and $\lambda_{\text{max}}$ is the wavelength at which the blackbody **radiation is maximum**, and T is the temprature in Kelvin of the blackbody.

- Rayleigh-Jeans law:

$$\begin{aligned} R(\lambda) = \frac{1}{4} cu(\lambda), \\ u(\lambda) = 8\pi kT\lambda^{-4} \end{aligned} \tag{13}$$

WHere $R$ is radiation power per unit area, or energy density, $u$ is the energy density of radiation, $c$ is the speed of light, and $k = 8.617e\text{-}5 \text{ eV/K} = 1.38e\text{-}23J \cdot K^{-}1$ is the Boltzmann constatn This law is valid for long wavelength, but it diverges at short wavelength. **This equation is only good for long wavelength.**

- Planck's law:

$$u(\lambda) = \frac{8\pi hc\lambda^{-5}}{e^{hc/\lambda kT} - 1} \tag{14}$$

where $k = 1.38e\text{-}23(J \cdot K^{-1})$ is the Boltzmann constant, $h$ is the Planck constant, $T$ is the temprature in Kelvin of the blackbody.

### 4.1 Energy of radiation

For an ideal blackbody, the energy radiated within a certain wavelength range is found by integrating Equation 14 over the range of wavelength.

$$U = \int_{\lambda_1}^{\lambda_2} u(\lambda) \, \mathrm{d}\lambda \tag{15}$$

- It is often times easier to use mid-point approximation to handle the above integration:

$$U \approx u(\lambda)\Delta\lambda \tag{16}$$

Where $\lambda = \frac{\lambda_2 - \lambda_1}{2}$ is the mid-point of the wavelength range, and $\Delta\lambda$ is the width of the wavelength range.

## 5 Wavelike properties of particles

### 5.1 De broglie Hypothesis

$$f = \frac{E}{h} \quad , \lambda = \frac{h}{p} \tag{17}$$

Where $E$ is the total energy, p is the momentum, and $\lambda$ is the wavelength of the particle. $h = 6.63e\text{-}34 J \cdot s$ is the Planck constant.

- For a particle of zero rest energy,

$$E = pc = hf = \frac{hc}{\lambda}, \quad [18$$

  where p is the momentum of the particle.

- For a moving particle,

$$E = \frac{1}{2} m v^2 = \frac{p^2}{2m} \quad [19$$

### 5.2 Wavefunction for particles

$$\Psi(x,t) = A\sin(kx - \omega t) \quad \text{or} \quad Ae^{i(kx - \omega t)} \quad [20$$

- probability density of the particle is

$$p(x,t) = |\Psi|^2 \equiv \Psi^* \Psi \quad [21$$

### 5.3 Uncertainty Principle

$$\Delta x \Delta p \geq \frac{\hbar}{2}, \quad \Delta E \Delta t \geq \frac{\hbar}{2} \quad [22$$

Where x is position, p is momentum, E is energy, t is time, and $\hbar = \frac{h}{2\pi} = 1.05e\text{-}34 J \cdot s$ is the reduced Planck constant.

#### 5.3.1 Min. Energy of Particle in a box

$$E = \frac{p^2}{2m} \geq \frac{\hbar^2}{2mL^2} \quad [23$$

## 6 Schrodinger's equation

### 6.1 Time-dependent Schrodinger's equation in 1D

1D Schrodinger's equation in position basis:

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t) + V(x,t)\Psi(x,t) = i\hbar\frac{\partial}{\partial t}\Psi(x,t) \quad [24$$

### 6.2 Time-independent Schrodinger's equation in 1D

Via separation of variable, set $\Psi(x,t) = \psi(x)\varphi(t)$, and noticing $f = \frac{E}{h}$, we have

$$\begin{gathered} -\frac{\hbar}{2m}\frac{\mathrm{d}^2}{\mathrm{d}x^2}\psi(x) + V(x)\psi(x) = E\psi(x) \\ \text{time variation of wavefunction: } \varphi(t) = e^{-iEt/\hbar} \end{gathered} \quad [25$$

- Probability density is thus simplified to

$$p(x) = |\Psi(x,t)|^2 = |\psi(x)|^2 \quad [26$$

### 6.3 Infinite potential well- particle in a box

- For a particle in a box of length $L$ , where $V(x) = 0$ for $0 < x < L$, and $V(x) = \infty$ otherwise, the wavefunction is found by

$$\begin{gathered} -\frac{\hbar^2}{2m}\frac{\mathrm{d}^2}{\mathrm{d}x^2}\psi(x) = E\psi(x) \\ \Rightarrow \psi_n(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right). \end{gathered} \quad [27$$

  Noticing boundary values, the following is obtained:

$$E_n = \frac{\hbar^2 k_n^2}{2m} = n^2\frac{\hbar^2\pi^2}{2mL^2} = n^2 E_1 \quad [28$$

  where $k = 2\frac{\pi}{\lambda}; k^2 = \left(\frac{p}{\hbar}\right)^2 = \frac{2mE}{\hbar^2}$

## 7 Appendix

1. Useful integral for probability of wavefunction

$$\int_{-\infty}^{\infty} e^{-a(x+b)^2}\,\mathrm{d}x = \sqrt{\frac{\pi}{a}} \qquad [29$$