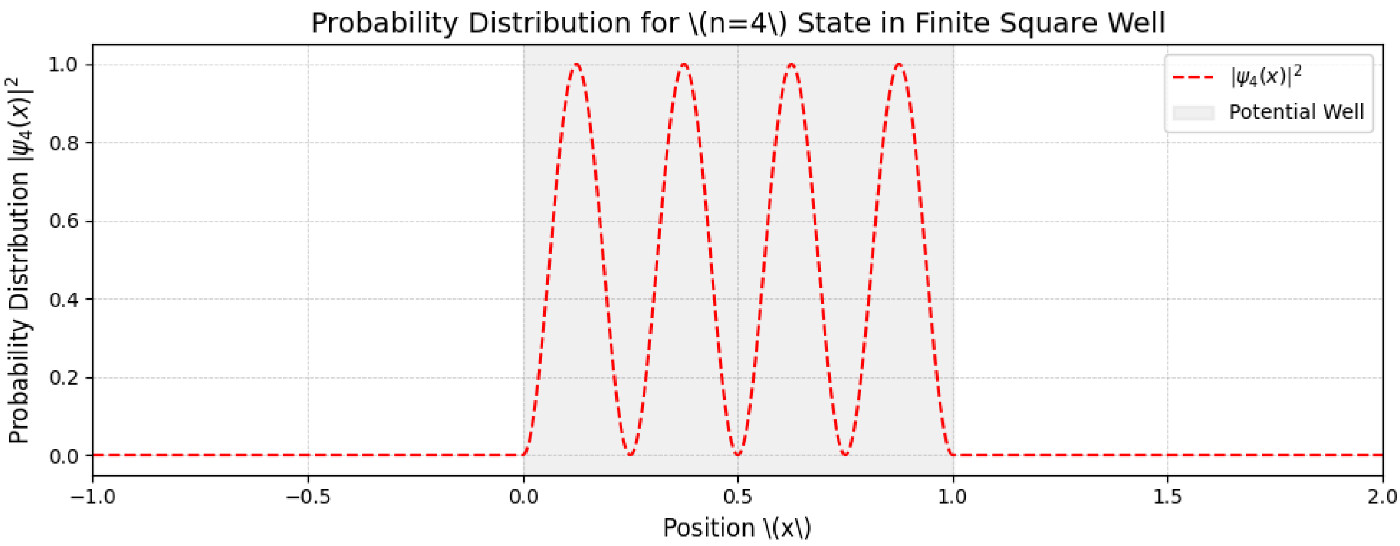
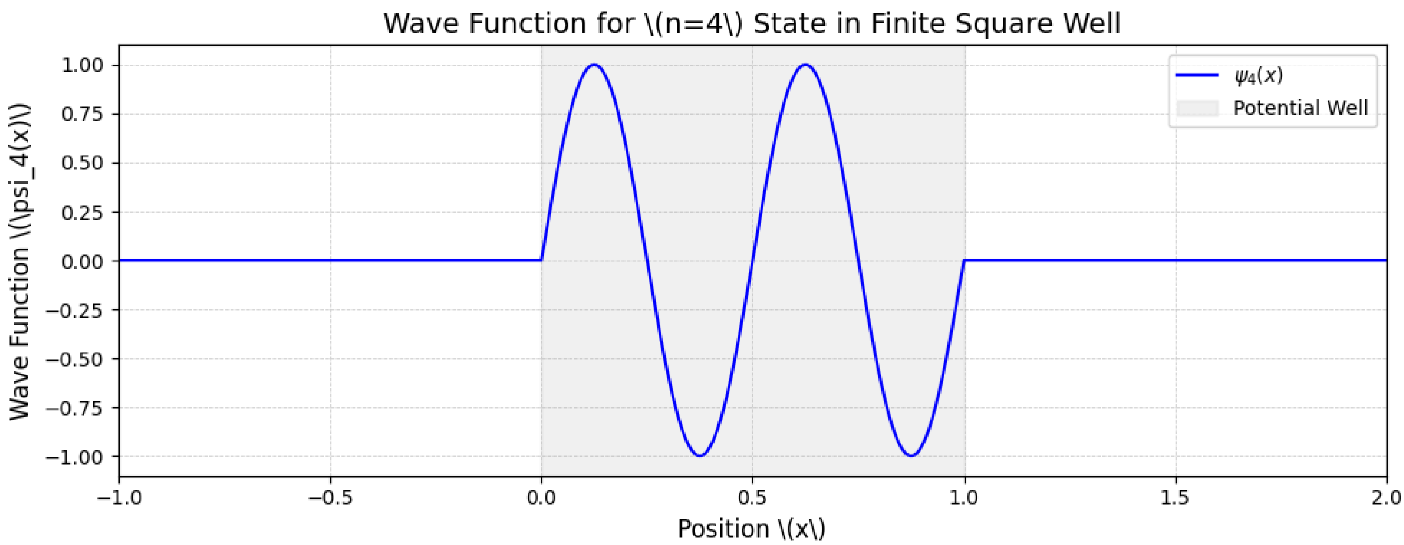
6-23

Sketch (a) the wave function and (b) the probability distribution for the n=4 state for the finite square well potential.





6-28

Compute the expectation value of the x component of the momentum of a particle of mass m in the $n = 3$ level of a one-dimensional infinite infinite square well of width L. Reconcile your answer with the fact that the kinetic energy of the particle in this level is $9\pi^2\hbar^2/2mL^2$

$$\begin{aligned} \langle p_x \rangle &= \int_{-\infty}^{\infty} \psi_3^* x \left(\frac{\hbar}{i} \frac{\partial \psi_3}{\partial x} \right) \mathrm{d}x = \int_0^L \sqrt{\frac{2}{L}} \sin\left(3\pi \frac{x}{L}\right) \left(\frac{\hbar}{i} \frac{\partial}{\partial x} \right)) \sqrt{\frac{2}{L}} \sin\left(3\pi \frac{x}{L}\right) \mathrm{d}x \\ &= \frac{2\hbar}{Li} \int_0^L \sin\left(3\pi \frac{x}{L}\right) \cos\left(3\pi \frac{x}{L}\right) \frac{3\pi}{L} \mathrm{d}x \\ \text{letting } \; u = \frac{3\pi x}{L}, \quad &= \frac{2\hbar}{Li} \int_0^{3\pi} \sin(u) \cos(u) \, \mathrm{d}x = 0 \end{aligned} \tag{1}$$

This result is consistent with the fact that the kinetic energy is non-zero, as the kinetic energy is related to the expectation value of the square of the momentum, not the momentum itself.

6-29

Find (a) $\langle x \rangle$ and (b) $\langle x^2 \rangle$ for the second excited state (n=3) in an infinite square well potential.

Noticing that, for n = 3,

$$\psi_3 = \sqrt{\frac{2}{L}} \sin\left(3\pi \frac{x}{L}\right) \tag{2}$$

- a.

$$\langle x \rangle = \frac{2}{L} \int_0^L \sin^2\left(3\pi \frac{x}{L}\right) x \, \mathrm{d}x = \frac{L}{2} \tag{3}$$

- b.

$$\langle x^2 \rangle = \frac{2}{L} \int_0^L \sin^2\left(3\pi \frac{x}{L}\right) x^2 \, \mathrm{d}x = \frac{L^2}{3} - \frac{L^2}{2\pi^2} \tag{4}$$

6-32

Find $\sigma_x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}, \sigma_p = \sqrt{\langle p^2 \rangle - \langle p \rangle^2}$ and $\sigma_x \sigma_p$ for the ground-state wave function of an infinite square well. Use the fact that $\langle p \rangle = 0$ by symmetry and $\langle p^2 \rangle = \langle 2mE \rangle$ from problem 6-31

First find $\langle x \rangle$, exploiting symmetry of its probability density function:

$$\langle x \rangle = \frac{2}{L} \int_0^L \sin^2\left(\pi \frac{x}{L}\right) x \, \mathrm{d}x = \frac{L}{2} \tag{5}$$

Then find $\langle x^2 \rangle$:

$$\langle x^2 \rangle = \frac{2}{L} \int_0^L \sin^2\left(\pi \frac{x}{L}\right) x^2 \, \mathrm{d}x = \frac{L^2}{3} - \frac{L^2}{2\pi^2} \tag{6}$$

It follows that

$$\sigma_x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2} = \sqrt{\frac{L^2}{3} - \frac{L^2}{2\pi^2} - \left(\frac{L^2}{4}\right)} = \sqrt{\frac{1}{12} - \frac{1}{2\pi^2}}\, L \tag{7}$$

Using the fact that $\langle p \rangle = 0, \quad \langle p^2 \rangle = 2mE = \frac{\hbar^2 \pi^2}{L^2}$

$$\sigma_p = \sqrt{\langle p^2 \rangle - \langle p \rangle^2} = \sqrt{\frac{\hbar^2 \pi^2}{L^2}} = \frac{\hbar \pi}{L} \tag{8}$$

Thus,

$$\sigma_x \sigma_p = \sqrt{\frac{1}{12} - \frac{1}{2\pi^2}}\, \hbar \pi \tag{9}$$