3.48

A 100-W beam of light is shone onto a blackbody of mass $2e\text{-}3$ *kg for* 10^4 *sec. The blackbody is initially at rest in a frictionless space.* ***(a)****Compute the total energy and momentum absorbed by the blackbody from the light beam.* ***(B)****calculate the blackbody's velocity at the end of the period of illumination;* ***(C)****Compute the final kinetic energy of the blackbody. Why is the latter less than the total energy of the absorbed photons?*

- (A)

Energy absorbed:

$$E = \int_T P\,\mathrm{d}t = 100J/s * 10^4 s = 10^6 J. \tag{1}$$

Momentum of photon is calculated by:

$$p = \frac{E}{c} = \frac{10^6 J}{3 * 10^8 m/s} = 3.33 * 10^-3 N \cdot s. \tag{2}$$

Conservation of momentum tells us that the blackbody will have the same momentum as the photons, so the total momentum absorbed is $3.33 * 10^-3 N \cdot s$.

- (B)

By exploiting momentum, we find the terminal velocity by:

$$\begin{aligned} p = mv \quad &\Rightarrow \Delta v = \frac{\Delta p}{m} = \frac{3.33 * 10^-3 N.s}{2 * 10^-3 \text{ kg}} = 1.67 m/s \\ &\Rightarrow v_t = \Delta v - v_i = \boxed{1.67 m/s.} \end{aligned} \tag{3}$$

- (C)

$$T = \frac{1}{2} m v^2 = \frac{1}{2} * 2e\text{-}3 * 1.67^2 J = \boxed{2.78 * 10^{-3} J} \tag{4}$$

Kinetic energy being less than the total energy absorbed is due to the fact that the blackbody is a perfect absorber, and the difference in energy is lost to **increase its internal heat** and **radiate into space**.

3.51

Determine the fraction of the energy radiated by the Sun in the visible region of the spectrum(350 nm to 700 nm). Assume that the Sun's surface temperature is 5800K.

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By Planck's law, the energy radiated by the Sun in the visible region is found by the following integration:

$$U_v = \int_{350 \text{ nm}}^{700 \text{ nm}} \frac{8\pi hc\lambda^{-5}}{e^{hc/\lambda kT} - 1} d\lambda. \tag{5}$$

We approximate this integral using mid-point approximation, with $\lambda = (700 + 350)/2 \text{ nm} = 525 \text{ nm}$ and interval $\Delta\lambda = 350\text{nm}$:

$$U_v \approx u(\lambda)\Delta\lambda = \frac{8\pi hc(525 \text{ nm})^{-5}}{e^{hc/525 \text{ nm} \cdot k \cdot 5800K} - 1} \times 350 \text{ nm} = 0.389 J/m^3. \tag{6}$$

Then, using Rayleigh-Jeans Equation:

$$R_v = \frac{c}{4} U_v = 2.92e7 W/m^2, \tag{7}$$

while total energy radiated by Sun is

$$R = \sigma T^4 = \sigma \times 5800 K^4 \approx 6.42e7 \quad W/m^2. \tag{8}$$

Thus the fraction of energy radiated in the visible region is:

$$\boxed{\frac{R_v}{R} = 0.455.} \tag{9}$$

Other Problem (1)

If a person of mass 70kg walks at the speed of 5 km/hr, *what is their DeBroglie wavelength? Do you think it would be possible to observe the person's wavelike properties in experiment (compare it to the conditions of double slit experiment)? Explain your reasoning.*

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$$\lambda = \frac{h}{mv} = \frac{h}{70 \text{ kg} \cdot 5 \text{ km/h}} = \boxed{6.815e\text{-}36\ m.} \tag{10}$$

This wavelength is of magnitudes smaller than what is typically observed in double slit experiments, which are on the order of $10^{-}10m$ or bigger. It is thus nearly impossible to detect.

Other Problem (2)

You are given the task of constructing a double slip experiment for electrons of energy of 5 eV (converting this into velocity).

1. *If you wish the first dark line of the interference patter to occur at* $5°$, *what must the separation between the slits be?*
2. *How far from the slits must the detector plane be located, if the first dark line on each side of the central maximum is to be seperated by 1cm?*

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$$T = \frac{1}{2} m v^2 \quad \Rightarrow v = \sqrt{\frac{2T}{m}} = \sqrt{\frac{2 \times 5eV}{9.11e\text{-}31 \text{ kg}}} = 1.326e6\ m/s. \tag{11}$$

We need to find the wavelength of our propagating electrons:

$$\lambda = \frac{h}{mv} = \frac{6.63e\text{-}34}{9.11e\text{-}31 \times 1.326e6} = 5.485e\text{-}10m. \tag{12}$$

For the first dark fringe to occur at $5°$, we have:

$$d \sin(\theta) = \frac{1}{2}\lambda \quad \Rightarrow d = \frac{\lambda}{2\sin(\theta)} = \frac{5.485e\text{-}10m}{2\sin(5°)} \\ \boxed{d = 3.146e\text{-}9m}. \tag{13}$$

- $$Y = L \tan\theta \Rightarrow L = \frac{Y}{\tan(\theta)} = \frac{0.5\text{cm}}{\tan(5°)} = \boxed{5.71 \text{ cm}} \tag{14}$$

Other Problem (3)

A particle moving in one dimension between rigid walls separated by a distance L has the wave function $\psi(x) = A\sin\left(\pi \frac{x}{L}\right)$. Since the particle must remain etween the walls, what must be the value of A?

- Using the Noamalized wavefunction in a confined 1-D space, we can jot down the following:

$$\begin{aligned} \int_0^L \psi^*(x)\psi(x)\,\mathrm{d}x = 1 \\ \Rightarrow \int_0^L |\psi(x)|^2\,\mathrm{d}x = \int_0^L A^2 \sin^2\left(\pi\frac{x}{L}\right)\mathrm{d}x = 1 \end{aligned} \tag{15}$$

The above integral yields:

$$\begin{gathered} \left(\frac{1}{2}x + \frac{L}{4\pi}\sin\left(\frac{2\pi}{L}x\right)\right)\Bigg|_0^L = \frac{1}{A^2} \\ \boxed{\Rightarrow A = \sqrt{\frac{2}{L}}} \end{gathered} \tag{16}$$