ECE 235 - Introduction to Solid State Electronics

Lecture 15: Fermi Distribution, Electron and Hole Density

Prof. Ying Wang
Contact: y.wang@wisc.edu
https://wang.ece.wisc.edu/

Insulators, Semiconductors and Metals

Energy bands and the gaps between them determine the conductivity and other properties of solids.

- **Insulators:** Have a full valence band and a large energy gap (a few eV). Higher energy states are not available.
- **Semiconductors:** Are insulators at T = 0. Have a small energy gap (~ 1 eV) between valence and conduction bands. Higher energy states become available (due to kT) as T increases.
- **Metals:** Have a partly filled band. Higher energy states are available, even at T = 0.

In order to conduct, an electron must have an available state at higher energy.

Semiconductors

The electrons in a filled band cannot contribute to conduction, because with reasonable E fields they cannot be promoted to a higher kinetic energy. Therefore, at T=0, pure semiconductors are actually insulators.
Question: Consider electrons in a semiconductor, e.g., silicon. In a perfect crystal at T=0 the valence bands are filled and the conduction bands are empty $\rightarrow$ no conduction. Which of the following could be done to make the material conductive?

1. Heat the material
2. Shine light on it
3. Add foreign atoms that change the number of electrons

Resistivity vs T

In semiconductors, some electrons can be thermally promoted into the conduction band at high temperature. But in metals, high temperature leads to larger scattering and larger resistance.

Semiconductors (Revisited)

The electrons in a filled band cannot contribute to conduction, because with reasonable E fields they cannot be promoted to a higher kinetic energy. Therefore, at T=0, pure semiconductors are actually insulators.

What Happens to Electron Distribution in a Semiconductor when T is Nonzero?

- Two types of carriers: electrons and holes. Both of them carry conductivity since both conduction and valence bands are partially filled.
- How are electrons and holes distributed in energy bands?

Fermi Distribution Function

- At $T = 0$, we expect all of the atoms in a solid to be in the ground state.
- Probability that an available state at energy E is occupied at finite T:

$$f(E) = \frac{1}{1 + e^{(E-E_F)/kT}}$$

- E_F is called the Fermi energy or the Fermi level.

Typical $f(E)$ Distribution

- If $E \gg E_F$: $f(E) \approx 0$
- If $E \ll E_F$: $f(E) \approx 1$
- If $E = E_F$: $f(E) = 1/2$

Some Observations

1. The energy E in the exponential factor makes it clear why the band gap is so crucial. An increase in the band gap by a factor of 10 will greatly increase the value of $\exp((E - E_F)/kT)$, generally making f so small that the material has to be an insulator.
2. The resistance of a semiconductor is expected to decrease exponentially with increasing temperature. This is approximately true, although not exactly, because the function f is not a simple exponential, and because the band gap does vary somewhat with temperature.

Question 1

To calculate the probability that an energy state above E_F is occupied by an electron. Set T=300K. Determine the probability that an energy level 3kT above the Fermi energy is occupied by an electron.

Band Diagrams (Revisited)

- Conduction band
- Valence band

Band Diagram Representation

Energy plotted as a function of position.

- $E_C \rightarrow$ Conduction band
  - Lowest energy state for a free electron
  - Electrons in the conduction band means current can flow
- $E_V \rightarrow$ Valence band
  - Highest energy state for filled outer shells
  - Holes in the valence band means current can flow
- $E_f \rightarrow$ Fermi Level
  - Shows the likely distribution of electrons
- $E_G \rightarrow$ Band gap
  - Difference in energy levels between E_C and E_V
  - No electrons (e^-) in the bandgap (only above E_c or below E_v)
  - E_G = 1.12 eV in Silicon

Electron vs Hole

- When electrons move into the conduction band, they leave behind vacancies in the valence band. These vacancies are called holes. Because holes represent the absence of negative charges, it is useful to think of them as positive charges.
- Whereas the electrons move in a direction opposite to the applied electric field, the holes move in the same direction as the field.

Electrons in the conduction band and holes in the valence band contribute to conductivity.

Equilibrium Distribution of Carriers

- Obtain $n(E)$ by multiplying $g_c(E)$ and $f(E)$.

Density of States

Density of States, $g_c(E)$: the density of states per unit energy.

Equilibrium Carrier Concentrations

- Integrate $n(E)$ over all the energies in the conduction band to obtain n:

$$n = \int_{E_c}^{\text{top of conduction band}} g_c(E) f(E) dE$$

- Integrate $p(E)$ over all the energies in the valence band to obtain p:

$$p = \int_{\text{bottom of valence band}}^{E_v} g_v(E)[1 - f(E)] dE$$

Electron Concentration in a Semiconductor

- Integrate $n(E)$ over all the energies in the conduction band to obtain n:

$$n = \int_{E_c}^{\text{top of conduction band}} g_c(E) f(E) dE$$

$$n = N_c e^{-(E_c - E_F)/kT} \quad \text{where} \quad N_c = 2\left(\frac{2\pi m^* kT}{h^2}\right)^{3/2}$$

Hole Concentration in a Semiconductor

- Integrate $p(E)$ over all the energies in the valence band to obtain p:

$$p = \int_{\text{bottom of valence band}}^{E_v} g_v(E)[1 - f(E)] dE$$

$$p = N_v e^{-(E_F - E_v)/kT} \quad \text{where} \quad N_v = 2\left(\frac{2\pi m^* kT}{h^2}\right)^{3/2}$$

Intrinsic Carrier Concentration

Intrinsic carrier means that $n = p = n_i$.

$$
\begin{aligned}
np &= (N_c e^{-(E_c - E_F)/kT})(N_v e^{-(E_F - E_v)/kT}) \\
&= N_c N_v e^{-(E_c - E_v)/kT} = N_c N_v e^{-E_G/kT} = n_i^2 \\
n_i &= \sqrt{N_c N_v} e^{-E_G/2kT}
\end{aligned}
$$

	Si	Ge	GaAs
$N_c (cm^{-3})$	2.8×10^{19}	1.04×10^{19}	4.7×10^{17}
$N_v (cm^{-3})$	1.04×10^{19}	6.0×10^{18}	7.0×10^{18}

Intrinsic Fermi Level, E_i

- To find E_F for an intrinsic semiconductor, use the fact that $n = p$:

$$N_c e^{-(E_c - E_F)/kT} = N_v e^{-(E_F - E_v)/kT}$$

$$E_F = \frac{E_c + E_v}{2} + \frac{kT}{2} \ln\left(\frac{N_v}{N_c}\right) \equiv E_i$$

Intrinsic Fermi Level:

$$E_i = \frac{E_c + E_v}{2} + \frac{3kT}{4} \ln\left(\frac{m_p^*}{m_n^*}\right) \cong \frac{E_c + E_v}{2}$$

$\rightarrow$ At the middle of E_g

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Lecture 16: Extrinsic Semiconductor and Current

Prof. Ying Wang
Contact: y.wang@wisc.edu
https://wang.ece.wisc.edu/

Intrinsic Semiconductor

- Intrinsic Fermi Level, E_i: At the middle of E_g:

$$E_i = \frac{E_c + E_v}{2} + \frac{3kT}{4} \ln\left(\frac{m_p^*}{m_n^*}\right) \cong \frac{E_c + E_v}{2}$$

- Intrinsic carrier: $n = p = n_i$

$$n_i = \sqrt{N_c N_v} e^{-E_G/2kT}$$

- Material is pure

Extrinsic Semiconductor

- An extrinsic semiconductor is a semiconductor material that has been intentionally doped with impurities to modify its electrical properties.
- Si can be "doped" with other elements to change its electrical properties. If Si is doped with phosphorus (P), each P atom can contribute a conduction electron, so that the Si lattice has more electrons than holes, i.e., it becomes "N type".

n-type Doping in Silicon

- The donor creates a small variation in the lattice potential resulting in an allowed state in the bandgap.

p-type Doping in Silicon

- If Si is doped with Boron (B), each B atom can contribute a hole, so that the Si lattice has more holes than electrons, i.e., it becomes "P type".
- The acceptor creates a small variation in the lattice potential resulting in an allowed state in the bandgap.

Energy to Delocalize the Extra Electron

- The binding energy of an electron in the H atom:

$$E_b = -\frac{m_e e^4}{8\epsilon_0^2 h^2} = -13.6 \text{ eV}$$

  m_e is the mass of electron, ϵ_0 is the permittivity of free space.

- Delocalize electron from As atom in Si:

$$E_b^{As} = E_b \frac{m_e^*}{m_e} \frac{1}{\epsilon_r^2} = -0.032 \text{ eV}$$

  This can be done by using the effective mass of electron, m_e^*, and the relative permittivity $\epsilon_0 \epsilon_r$ of Si. The calculated value of 0.032 eV or 32 meV is comparable to thermal energy $k_B T$ at room temperature, which is 25 meV.

Summary of Charge Carriers

- Intrinsic Semiconductor

Question 1: What type of semiconductor is obtained if silicon is doped with (a) aluminum and (b) phosphorus?

Electron and Hole Concentrations

- Under thermal equilibrium conditions, the product of the conduction-electron density and the hole density is ALWAYS equal to the square of n_i :
$$np = n_i^2$$
- **Intrinsic Semiconductor:** $n = p = n_i$
- **N-type Material:**
$$n \approx N_D$$
$$p \approx \frac{n_i^2}{N_D}$$
- **P-type Material:**
$$p \approx N_A$$
$$n \approx \frac{n_i^2}{N_A}$$

Majority Charge Carriers vs Minority Charge Carriers

- **N-type material:**
$$n \approx N_D$$
$$p \approx \frac{n_i^2}{N_D}$$
- Consider Si with intrinsic carrier concentration of 10^{10} cm^{-3}. It is doped with As (Arsenic) of concentration 10^{15} cm^{-3}.
- **Question:** Concentration of electrons and concentration of holes?
- The carrier with the larger concentration and dominates the conductivity is the major charge carrier.

Drift Current

- Drift current: The current due to the drifting motion of charge carriers under application of an electric field.
- Drift current density is given by:
$$J_e(\text{drift}) = ne\mu_e E$$
$$J_h(\text{drift}) = pe\mu_h E$$
Where μ_e, μ_h are the constants called mobility of electrons and holes, respectively.
$$J(\text{drift}) = J_e(\text{drift}) + J_h(\text{drift})$$

- Consider Si with intrinsic carrier concentration of 10^{10} cm^{-3}. It is doped with As (Arsenic) of concentration 10^{15} cm^{-3}.
- **Question:** Compare the current in intrinsic Si and As doped Si.
  - N-type material:

$$n \approx N_D$$

$$p \approx \frac{n_i^2}{N_D}$$

$$J_e(\text{drift}) = ne\mu_e E$$

$$J_h(\text{drift}) = pe\mu_h E$$

Diffusion Current

- Diffusion current: The directional movement of charge carriers due to a concentration gradient.
- Diffusion current density due to electrons is given by:

$$J_{e(\text{diffusion})} = eD_e \frac{dn}{dx}$$

- Diffusion current density due to holes is given by:

$$J_{h(\text{diffusion})} = -eD_h \frac{dp}{dx}$$

  Where D_e and D_h are the diffusion coefficients for electrons and holes, respectively.

$$J_{\text{diffusion}} = J_e(\text{diffusion}) + J_h(\text{diffusion})$$

Practice

The Fermi-Dirac distribution function f(E) tells you what the probability is of finding an electron in a given energy state E. The following are true/false:

1. f(E) must be between 0 and 1 (T/F)
2. f can be negative (T/F)
3. At high temperatures, f looks like a step function (T/F)
4. At zero temperature, f looks like a step function (T/F)
5. f is always equal to 1/2 at the Fermi level (T/F)
6. As the temperature increases, f gets more "smeared" around the Fermi level (T/F)

Crystalline materials with the following band structures are an insulator, semiconductor, or a metal. Write the material type below each band diagram.

Four graphs correspond to n-type and p-type semiconductors at 0 K and 300 K. Carefully denote each graph with which type of doping and what temperature.

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Lecture 17: Carrier Transport

Prof. Ying Wang
Contact: y.wang@wisc.edu
https://wang.ece.wisc.edu/

Electron and Hole Concentrations

- Under thermal equilibrium conditions, the product of the conduction-electron density and the hole density is ALWAYS equal to the square of n_i:
$$np = n_i^2$$

Extrinsic Fermi Level

- We can now write the Fermi level in terms of the effective density of states N_C and the donor concentration N_d:
- Recall:
$$n = N_C \exp\left[\frac{(E_F - E_C)}{k_B T}\right] \text{ and } n = N_d$$
- Therefore:
$$E_F - E_C = k_B T \ln\left[\frac{N_d}{N_C}\right]$$
- Similarly for shallow acceptors $p = N_A$ and:
$$E_v - E_F = k_B T \ln\left[\frac{N_a}{N_v}\right]$$
- For higher donor concentrations, the smaller the energy difference $(E_F - E_C)$, the Fermi level moves closer to the bottom of the conduction band.
- We know how to obtain n and p, given the doping densities (N_d and N_a) and temperature.

- We now go one step further and develop an understanding of current flow and carrier dynamics in a semiconductor, an essential ingredient in any semiconductor device.
  - Drift current
  - Diffusion current

Drift Current

- The motion of carriers under the action of an electric field is known as "drift".
- In a semiconductor, electrons and holes are continuously moving with large instantaneous velocities. However, their trajectories are interrupted because of "scattering events".
- With zero electric field, the average displacement of a carrier is zero.
- In the presence of an electric field, a carrier undergoes a net change Δx in its position over a time interval Δt. $\Delta x/\Delta t$ is called the "drift velocity" of the carrier.

Scattering of Carriers

- **Phonons:** Phonons can be thought of as quantum-mechanical "particles" representing lattice vibrations. An electron or a hole can absorb or emit a phonon, gaining or losing energy, accompanied by a change in its momentum.
- **Impurity ions:** An ionized donor or acceptor atom is a disruption in the periodic lattice potential of a semiconductor and is therefore a cause for carrier scattering.
- **Defects:** A semiconductor crystal may have defects, i.e., departures from its periodic structure. These deviations cause a change in the periodic lattice potential and therefore lead to scattering.

Mobility μ

- At low fields (up to a few kV/cm), the drift velocity v_d varies linearly with the electric field E.
- The low-field region is characterized by the "mobility" (μ_n for electrons, μ_p for holes), defined as $\mu = \frac{v_d}{\mathcal{E}}$.
- Units of μ: $\frac{\mathrm{cm}^2}{\mathrm{V\ s}}$

- $\mu = \frac{q\tau}{m^*}$, where m^* is the effective mass and τ is the momentum relaxation time, i.e., the average time interval between successive scattering events (typically 10^{-14} to 10^{-12} s).

Drift Current

- Drift current: The current due to (motion) drifting of charge carrier under application of electric field.
- Drift current density is given by:

$$J_e(\text{drift}) = ne\mu_e E$$

$$J_h(\text{drift}) = pe\mu_h E$$

Where μ_e, μ_h are the constants called mobility of electrons and holes, respectively.

$$J(\text{drift}) = J_e(\text{drift}) + J_h(\text{drift})$$

Example

- For the rectangular silicon bar shown in the figure, $L = 50\mu$m, and the cross-sectional area is $20\mu\text{m}^2$. It is uniformly doped with $N_d = 5 \times 10^{17}$ cm^{-3}. At $T = 300$ K and with an applied voltage of 5 V, find the following:

  1. Electric field
  2. Current density
  3. Total current
  4. Resistance of the bar

  Given: $\mu_n = 400$ cm^2/V-s for $N_d = 5 \times 10^{17}$ cm^{-3} at $T = 300$ K.

Assuming all donors to be ionized, $n = p + N_d^+ \approx N_d = 5 \times 10^{17}$ cm^{-3}. Assume that the metal-semiconductor contacts serve as a perfect source or sink for the carriers. The applied voltage appears across the semiconductor, resulting in a uniform field and causing a drift current.

$$\mathcal{E} = -\frac{dV}{dx} = -\frac{V_0}{L} = -\frac{5 \text{ V}}{50 \times 10^{-4} \text{ cm}} = -1 \text{ kV/cm} \equiv \mathcal{E}_0$$

$$\frac{1}{q}\frac{dE_c}{dx} = \mathcal{E}_0 \rightarrow \int_{x=0}^{L} dE_c = -q\frac{V_0}{L}L = -qV_0 = -5 \text{ eV}$$

$\mathcal{E}_0$ is sufficiently low $\rightarrow$ we can use $v_d = \mu\mathcal{E}$.

$$\begin{aligned} J = J_n + J_p &= q(n\mu_n + p\mu_p)\mathcal{E} \approx qN_d\mu_n\mathcal{E} \\ &= 1.6 \times 10^{-19} \text{ C} \times 5 \times 10^{17}\frac{1}{\text{cm}^3} \times 400\frac{\text{cm}^2}{\text{V-s}} \times \left(-10^3\frac{\text{V}}{\text{cm}}\right) \\ &= -3.2 \times 10^4\frac{\text{A}}{\text{cm}^2} \end{aligned}$$

Diffusion

- Consider a group of particles confined to a narrow region at $t = t_0$, with randomly assigned initial velocities.
- The particles are subjected to random scattering events.
- As time advances, the distribution function becomes more uniform, i.e., its peak reduces, and it spreads in space.
- The particles "diffuse".

Diffusion (cont.)

- The process of diffusion is described by Fick's law:

$$F_x = -D\frac{d\eta}{dx}$$

where F_x is the flux (number of particles crossing a unit area in a unit time), η is the particle concentration, and D is the diffusion coefficient.

- In a semiconductor, diffusion of electrons and holes is described by:

$$F_n = -D_n\frac{dn}{dx}, \quad F_p = -D_p\frac{dp}{dx}$$

- Unit of D $\rightarrow \frac{\mathrm{cm}^2}{\mathrm{s}}$

The Total Electron and Hole Current Densities

- Flux due to drift:

$$\mathcal{F}_n^{\mathrm{drift}} = -n\mu_n\mathcal{E}$$
$$\mathcal{F}_p^{\mathrm{drift}} = +p\mu_p\mathcal{E}$$

- Flux due to diffusion:

$$\mathcal{F}_n^{\mathrm{diff}} = -D_n\frac{dn}{dx}$$
$$\mathcal{F}_p^{\mathrm{diff}} = -D_p\frac{dp}{dx}$$

- Total flux $\left[(\mathrm{cm}^2 - \mathrm{s})^{-1}\right]$:

$$\mathcal{F}_n = \mathcal{F}_n^{\mathrm{drift}} + \mathcal{F}_n^{\mathrm{diff}}$$
$$\mathcal{F}_p = \mathcal{F}_p^{\mathrm{drift}} + \mathcal{F}_p^{\mathrm{diff}}$$

- Current density ($\mathrm{A/cm}^2$):

$$J_n = -q\mathcal{F}_n = qn\mu_n\mathcal{E} + qD_n\frac{dn}{dx}$$
$$J_p = +q\mathcal{F}_p = qp\mu_p\mathcal{E} - qD_p\frac{dp}{dx}$$

ECE/PHY 235 - Introduction to Solid State Electronics

Lecture 18: Carrier Transport II

Prof. Ying Wang
Contact: y.wang@wisc.edu
https://wang.ece.wisc.edu/

Drift Current

- The motion of carriers under the action of an electric field is known as "drift".
- In a semiconductor, electrons and holes are continuously moving with large instantaneous velocities. However, their trajectories are interrupted because of "scattering events".
- With zero electric field, the average displacement of a carrier is zero.
- In the presence of an electric field, a carrier undergoes a net change Δx in its position over a time interval Δt. $\Delta x/\Delta t$ is called the "drift velocity" of the carrier.

Diffusion Current

- Consider a group of particles confined to a narrow region at $t = t_0$, with randomly assigned initial velocities.
- The particles are subjected to random scattering events.
- As time advances, the distribution function becomes more uniform, i.e., its peak reduces, and it spreads in space.
- The particles "diffuse".

The Total Electron and Hole Current Densities

- Flux due to drift:
$$\mathcal{F}_n^{\text{drift}} = -n\mu_n\mathcal{E}$$
$$\mathcal{F}_p^{\text{drift}} = +p\mu_p\mathcal{E}$$
- Flux due to diffusion:
$$\mathcal{F}_n^{\text{diff}} = -D_n\frac{dn}{dx}$$
$$\mathcal{F}_p^{\text{diff}} = -D_p\frac{dp}{dx}$$

- Total flux $\left[(\text{cm}^2-\text{s})^{-1}\right]$:

$$\mathcal{F}_n = \mathcal{F}_n^{\text{drift}} + \mathcal{F}_n^{\text{diff}}$$

$$\mathcal{F}_p = \mathcal{F}_p^{\text{drift}} + \mathcal{F}_p^{\text{diff}}$$

- Current density (A/cm^2):

$$J_n = -q\mathcal{F}_n = qn\mu_n\mathcal{E} + qD_n\frac{dn}{dx}$$

$$J_p = +q\mathcal{F}_p = qp\mu_p\mathcal{E} - qD_p\frac{dp}{dx}$$

Generation and Recombination

- The processes of generation and recombination of electron-hole pairs take place continuously.
- The rates of generation and recombination depend on several factors:
  - Band structure of the semiconductor
  - Presence of light of an appropriate wavelength
  - Defects and impurity atoms in the semiconductor
  - Electron and hole densities
  - Temperature

Direct Generation and Recombination

- **Direct recombination:** An electron from the conduction band combines directly with a hole in the valence band, thus destroying an electron-hole pair.
- The energy lost by the electron may be transferred to a photon (light) in "radiative" recombination or to a phonon (lattice vibration) in "non-radiative" recombination.
- **Direct generation:** An electron from the valence band goes directly to the conduction band, thus generating an electron-hole pair.
- The energy required for the transition may be supplied by a photon (photo-generation) or a phonon (thermal generation).

Indirect Generation and Recombination (G-R)

- In indirect G-R, the transitions from the conduction band to the valence band (and vice versa) take place through a "G-R centre", with an energy level E_T located in the forbidden gap.
- The G-R center could be due to a defect in the crystal or an impurity atom.
- Recombination or generation takes place in two steps.

Continuity Equations

- Two processes can change the number of electrons and holes:
  - Carrier transport governed by $\mathcal{F}_n$ and $\mathcal{F}_p$

$$\mathcal{F}_n = \mathcal{F}_n^{\text{drift}} + \mathcal{F}_n^{\text{diff}}$$

$$\mathcal{F}_p = \mathcal{F}_p^{\text{drift}} + \mathcal{F}_p^{\text{diff}}$$

  - Generation and recombination of electron-hole pairs

The continuity equations serve to relate these phenomena: transistor, solar cell, sensor....

Continuity Equations (cont.)

- Assume that there are no variations of n, p, ψ in the y and z directions $\rightarrow \mathcal{F}_n$ and $\mathcal{F}_p$ in the y and z directions are zero.
- The number of electrons in the box can change because of the following factors:
  - Flux $\mathcal{F}_n$ (if positive) brings electrons into the box at the rate $\mathcal{F}_n \times A$.
  - Flux $\mathcal{F}_n(x + \Delta x)$ removes electrons from the box at the rate $\mathcal{F}_n(x + \Delta x) \times A$.
  - EHPs disappear from the box due to recombination at the rate $(R - G)(x)(A\Delta x)$

  We can relate the above factors to $\frac{\partial n}{\partial t}$:

$$(A\Delta x)\frac{\partial n}{\partial t} = \mathcal{F}_n(x)A - \mathcal{F}_n(x + \Delta x)A - (R - G)A\Delta x$$

$$\rightarrow \frac{\partial n}{\partial t} = -\frac{\mathcal{F}_n(x + \Delta x) - \mathcal{F}_n(x)}{\Delta x} - (R - G)$$

Continuity Equations (cont.)

$$\frac{\partial n}{\partial t} = -\frac{\mathcal{F}_n(x+\Delta x) - \mathcal{F}_n(x)}{\Delta x} - (R - G)$$

In the limit $\Delta x \to 0$, we get:

$$\frac{\partial n}{\partial t} = -\frac{\partial \mathcal{F}_n}{\partial x} - (R - G)$$

Similarly, for holes:

$$\frac{\partial p}{\partial t} = -\frac{\partial \mathcal{F}_p}{\partial x} - (R - G)$$

These equations are called the "continuity equations" for electrons and holes. We can rewrite the continuity equations in terms of the current densities $J_n = -q\mathcal{F}_n$ and $J_p = +q\mathcal{F}_p$:

$$\frac{\partial n}{\partial t} = \frac{1}{q}\frac{\partial J_n}{\partial x} - (R - G), \quad \frac{\partial p}{\partial t} = -\frac{1}{q}\frac{\partial J_p}{\partial x} - (R - G)$$

The general problem is very complex and needs to be solved numerically.

Question 1: Continuity Equations

Consider an n-type silicon sample with $N_d = 10^{17}$ cm^{-3}. Light is (continuously) incident on its surface, resulting in an optical generation rate shown in the figure. (We are assuming here that the light is entirely absorbed in a very thin region near the semiconductor surface ($x = 0$) and does not penetrate deeper). Assume that, as a result of the incident light, the excess minority carrier concentration (i.e., $p - p_0$) at $x = 0$ is maintained at $\Delta p_1 = 10^{10}$ cm^{-3}. Solve the continuity equation for holes and obtain $\Delta p(x)$. ($T = 300$ K)

Since only one end of the semiconductor is perturbed, we expect a region with a deviation from equilibrium conditions. We do not know at this point the extent of this region. We expect $p(x \to \infty) = p_0$, i.e., $\Delta p(x \to \infty) \equiv p(x \to \infty) - p_0 = 0$. At the surface ($x = 0$), EHPs are continuously generated; therefore, we expect some excess hole concentration there, i.e., $p(0) = p_0 + \Delta p_1$. We assume a steady-state situation in which all quantities have settled to their steady-state forms, not varying with time.

- Continuity equation for holes:

$$\frac{\partial p}{\partial t} = -\frac{\partial \mathcal{F}_p}{\partial x} - (R - G) = 0$$

- Because of diffusion and recombination, the excess hole concentration decreases from Δp_1 at $x = 0$ to 0 at $x = \infty$.

$$\to \mathcal{F}_p \approx \mathcal{F}_p^{\text{diff}} = -D_p\frac{\partial p}{\partial x} = -D_p\frac{\partial (p_0 + \Delta p)}{\partial x} = -D_p\frac{\partial \Delta p}{\partial x}$$

$$-\frac{\partial \mathcal{F}_p}{\partial x} - (R - G) = 0 \rightarrow -\frac{\partial}{\partial x}\left(-D_p \frac{\partial \Delta p}{\partial x}\right) - \frac{\Delta p}{\tau_p} = 0$$

$$\rightarrow \frac{\partial^2 \Delta p}{\partial x^2} - \frac{\Delta p}{D_p \tau_p} = 0$$

The quantity $\sqrt{D_p \tau_p}$ has units of $\sqrt{\frac{\mathrm{cm}^2}{\mathrm{s}} \times \mathrm{s}} = \mathrm{cm}$ and is called the "hole diffusion length" L_p. With this definition, we have:

$$\frac{\partial^2 \Delta p}{\partial x^2} = \frac{\Delta p}{L_p^2} \rightarrow \Delta p(x) = Ae^{-x/L_p} + Be^{x/L_p}$$

Using the boundary conditions, i.e., $\Delta p(0) = \Delta p_1$ and $\Delta p(\infty) = 0$, we get:

$$\Delta p(x) = \Delta p_1 e^{-x/L_p}$$

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Lecture 19: PN Junction I

Prof. Ying Wang
Contact: y.wang@wisc.edu
https://wang.ece.wisc.edu/

p-n Junction Diodes

P-n junctions are responsible for injecting and collecting charge carriers, which is necessary for the operation of diodes, transistors, and other devices. (Useful circuit elements (one-way valve), Light emitting diodes (LEDs), Light sensors (imagers))

p-n Junctions

Bring p-type and n-type material into contact.

p-n Junctions (cont.)

- All the h^+ from the p-type side and e^- from the n-type side undergo diffusion $\rightarrow$ move towards the opposite side (less concentration).
- When the carriers get to the other side, they become minority carriers.
- Recombination $\rightarrow$ The minority carriers are quickly annihilated by the large number of majority carriers.

- All the carriers on both sides of the junction are depleted from the material leaving.
  - Only charged, stationary particles (within a given region)
  - A net electric field
- This area is known as the depletion region (depleted of carriers) $\rightarrow$ Size of the depletion region depends on the diffusion length.

Charge Density

The remaining stationary charged particles results in areas with a net charge.

Electric Field across the PN Junction

- Areas with opposing charge densities creates an E-field.
- Total areas of charge are equal (but opposite).
- E-field is the integral of the charge density.
- Poisson's Equation:
$$\frac{dE}{dx} = \frac{\rho(x)}{\epsilon}$$
  ϵ is the permittivity of Silicon.
$$E = -\frac{dV}{dx}$$
$$E_{\max} = -\frac{qN_A}{\epsilon}x_p = -\frac{qN_D}{\epsilon}x_n$$

Potential

- E-field sets up a potential difference
- Potential is the negative of the integral of the E-field.
$$\frac{d\psi}{dx} = -E(x)$$

Built-In Potential

- Integrate the E-field within the depletion region.
- Use the Einstein Relation:
$$V_{bi} = \frac{kT}{q}\ln\left(\frac{N_A N_D}{n_i^2}\right)$$

Band Diagram

- Line up the Fermi levels.
- Draw a smooth curve to connect them.

p-n Junction - No Applied Bias

- Any e^- or h^+ that wanders into the depletion region will be swept to the other side via the E-field.
- Some e^- and h^+ have sufficient energy to diffuse across the depletion region.
- If no applied voltage: $I_{\text{drift}} = I_{\text{diff}}$

ECE/PHY 235 - Introduction to Solid State Electronics

Lecture 19: PN Junction II

Prof. Ying Wang
Contact: y.wang@wisc.edu
https://wang.ece.wisc.edu/

PN Junction in Equilibrium: Current Densities

- The diffusion currents can be expected to be substantial since there is a large change in n or p between the p-side and the n-side.
- In equilibrium, the drift and diffusion currents are equal and opposite for electrons as well as holes, i.e.,

$$J_n^{\text{diff}} = -J_n^{\text{drift}}, \quad J_p^{\text{diff}} = -J_p^{\text{drift}}$$

- Qualitatively, we can see that the diffusion and drift currents will be in opposite directions:
  - **Electrons:**
  $$\mathcal{F}_n^{\text{diff}} : \leftarrow, \quad \mathcal{E} : \leftarrow, \quad \mathcal{F}_n^{\text{drift}} : \rightarrow$$
  - **Holes:**
  $$\mathcal{F}_p^{\text{diff}} : \rightarrow, \quad \mathcal{E} : \leftarrow, \quad \mathcal{F}_p^{\text{drift}} : \leftarrow$$

PN Junction Under Bias

- In this PN junction example, with $V_a \approx 0.6$ V, a substantial current flows.
- When V_a is increased further, the current increases rapidly.
- When a reverse bias (i.e., $V_a < 0$ V) is applied, the diode blocks conduction, i.e., the current is negligibly small. This is called "rectifying" behavior.

Voltage Drop Across PN Junction

- In equilibrium, $V_p = V_n$, and we get

$$V^p_{\text{contact}} - V_{\text{bi}} + V^n_{\text{contact}} = 0$$

taking the voltage drop as positive.

- When a bias is applied, we have

$$V^p_{\text{contact}} - V_j + V^n_{\text{contact}} = V_p - V_n = V_a$$

- (1) - (2) gives

$$-V_{\text{bi}} + V_j = -V_a$$

i.e.,

$$V_j = V_{\text{bi}} - V_a$$

Forward Bias

- We can extend the Fermi level concept to describe carrier concentrations sufficiently far from the depletion region.
- $p = N_v e^{-(E_{Fp}-E_v)/kT}$ on the p-side, $n = N_c e^{-(E_c-E_{Fn})/kT}$ on the n-side.
- The Fermi levels are called "quasi Fermi levels" since the situation is almost like equilibrium.
- From the band diagram, we see that

$$E_g = \Delta_p + \Delta_n + qV_{bi}$$

$$E_g = \Delta_p + (E_{Fn} - E_{Fp}) + \Delta_n + q(V_{bi} - V_a)$$

$$\rightarrow E_{Fn} - E_{Fp} = qV_a$$

- It is much easier for electrons to flow to the right and holes to flow to the left with less barrier.

Reverse Bias

- We can extend the Fermi level concept to describe carrier concentrations sufficiently far from the depletion region.
- $p = N_v e^{-(E_{Fp} - E_v)/kT}$ on the p-side, $n = N_c e^{-(E_c - E_{Fn})/kT}$ on the n-side.
- The Fermi levels are called ”quasi Fermi levels” since the situation is almost like equilibrium.
- From the band diagram, we see that

$$E_g = \Delta_p + \Delta_n + qV_{bi}$$

$$E_g = \Delta_p - (E_{Fp} - E_{Fn}) + \Delta_n + q(V_{bi} + V_R)$$

$$\rightarrow E_{Fp} - E_{Fn} = qV_R$$

- It is more difficult for electrons to flow to the right and holes to flow to the left with a higher barrier.

ECE/PHY 235 - Introduction to Solid State Electronics

Lecture 20: PN Junction III

Prof. Ying Wang
Contact: y.wang@wisc.edu
https://wang.ece.wisc.edu/

Current Densities in Forward Bias

Near the junction, although the equilibrium condition is disturbed, we still have

$$J_p^{\text{diff}} \approx -J_p^{\text{drift}}, \quad J_n^{\text{diff}} \approx -J_n^{\text{drift}}$$

Current Densities in Reverse Bias

Near the junction, although the equilibrium condition is disturbed, we still have

$$J_p^{\text{diff}} \approx -J_p^{\text{drift}}, \quad J_n^{\text{diff}} \approx -J_n^{\text{drift}}$$

PN Junction: Derivation of I-V Equation

Definitions:

- p_{p0}: equilibrium hole density in the neutral p-region
- p_{n0}: equilibrium hole density in the neutral n-region
- n_{p0}: equilibrium electron density in the neutral p-region
- n_{n0}: equilibrium electron density in the neutral n-region
- p_{p0} and n_{n0} are majority carrier densities.
- p_{n0} and n_{p0} are minority carrier densities.

Example: $N_a = 5 \times 10^{16}$ cm^{-3}, $N_d = 10^{18}$ cm^{-3} ($T = 300$ K).

$$\rightarrow p_{p0} \approx N_a = 5 \times 10^{16} \text{ cm}^{-3}$$

$$\rightarrow n_{n0} \approx N_d = 10^{18} \text{ cm}^{-3}$$

$$n_{p0} = \frac{n_i^2}{p_{p0}} = \frac{(1.5 \times 10^{10})^2}{5 \times 10^{16}} = 4.5 \times 10^3 \text{ cm}^{-3}$$

$$p_{n0} = \frac{n_i^2}{n_{n0}} = \frac{(1.5 \times 10^{10})^2}{10^{18}} = 2.25 \times 10^2 \text{ cm}^{-3}$$

$$J_p^{\text{diff}} \approx -J_p^{\text{drift}} \rightarrow q\mu_p p\mathcal{E} = qD_p\frac{dp}{dx}, \text{ i.e., } \mathcal{E} = -\frac{d\psi}{dx} = \frac{D_p}{\mu_p}\frac{1}{p}\frac{dp}{dx}$$

$$\frac{D}{\mu} = \frac{kT}{q} \rightarrow \int d\psi = -V_T \int \frac{1}{p}dp \rightarrow \psi|_{x_1}^{x_2} = -V_T \log\frac{p(x_2)}{p(x_1)} \rightarrow \frac{p(x_n)}{p(x_p)} = \exp\left(\frac{\psi(x_p) - \psi(x_n)}{V_T}\right)$$

$$J_p^{\text{diff}} \approx -J_p^{\text{drift}} \rightarrow \frac{p(x_n)}{p(x_p)} = \exp\left(\frac{\psi(x_p) - \psi(x_n)}{V_T}\right)$$

$$J_n^{\text{diff}} \approx -J_n^{\text{drift}} \rightarrow \frac{n(x_n)}{n(x_p)} = \exp\left(\frac{\psi(x_n) - \psi(x_p)}{V_T}\right)$$

$$\psi(x_n) - \psi(x_p) = V_j$$

Low-level injection: $p(x) \approx p_{p0}$ for $x \le x_p$, and $n(x) \approx n_{n0}$ for $x \ge x_n$.

$$V_j = V_{\text{bi}} - V_a$$

$$\frac{p(x_n)}{p_{p0}} = e^{-V_j/V_T} \rightarrow p(x_n) = p_{p0}e^{-V_j/V_T}$$

$$\frac{n_{n0}}{n(x_p)} = e^{V_j/V_T} \rightarrow n(x_p) = n_{n0}e^{-V_j/V_T}$$

Equilibrium: $p(x_n) = p_{n0} = p_{p0} \exp\left(\frac{-V_{\text{bi}}}{V_T}\right)$, $n(x_p) = n_{p0} = n_{n0} \exp\left(\frac{-V_{\text{bi}}}{V_T}\right)$. With bias: $p(x_n) = p_{p0} \exp\left(\frac{-V_{\text{bi}}+V_a}{V_T}\right)$, $n(x_p) = n_{n0} \exp\left(\frac{-V_{\text{bi}}+V_a}{V_T}\right)$.

Example: Consider an abrupt, uniformly doped silicon pn junction at $T = 300$ K, with $N_a = 5 \times 10^{16}$ cm^{-3} and $N_d = 10^{18}$ cm^{-3}. Compute the depletion width and the minority carrier densities at the depletion region edges (x_p and x_n) for an applied bias of +0.3 V, +0.6 V, -1 V, -5 V. ($n_i = 1.5 \times 10^{10}$ cm^{-3} for silicon at $T = 300$ K.)

V_a (V)	W(μm)	$\mathcal{E}_m$ (kV/cm)	$n(x_p)$ (cm^{-3})	$p(x_n)$ (cm^{-3})
0.6	0.08	61.3	5.18×10^{13}	2.59×10^{12}
0.3	0.12	90.4	4.83×10^{8}	2.41×10^{7}
0.0	0.15	112.2	4.50×10^{3}	2.25×10^{2}
-1.0	0.22	165.3	$7.68 \times 10^{-14} \approx 0$	$3.84 \times 10^{-15} \approx 0$
-5.0	0.40	293.6	≈ 0	≈ 0

With forward bias, the minority carrier concentrations can increase by several orders of magnitude. With reverse bias, the minority carrier concentrations become very small and can be replaced with zero for all practical purposes.

PN Junction: Derivation of I-V Equation (cont.)

Continuity equation for holes ($x > x_n$):

$$\frac{\partial p(x,t)}{\partial t} = -\frac{1}{q}\frac{\partial J_p}{\partial x} - (R - G) = 0$$

(in DC conditions). In the neutral n-region, $\mathcal{E}$ is small, so $J_p^{\text{drift}} = qp\mu_p\mathcal{E}$ is small $\rightarrow J_p \approx J_p^{\text{diff}} = -qD_p\frac{dp}{dx}$. Also assuming low-level injection, $R - G \approx \frac{\Delta p}{\tau_p} = \frac{p(x)-p_{n0}}{\tau_p}$.

$$\rightarrow D_p\frac{d^2p}{dx^2} - \frac{p - p_{n0}}{\tau_p} = 0 \text{ or } \frac{d^2\Delta p}{dx^2} - \frac{\Delta p}{L_p^2} = 0$$

where $L_p = \sqrt{D_p\tau_p}$ is the hole diffusion length.

Hole continuity equation ($x > x_n$):

$$\frac{d^2\Delta p}{dx^2} - \frac{\Delta p}{L_p^2} = 0$$

Boundary conditions:

$$\Delta p(x_n) = p_{n0}\exp\left(\frac{V_a}{V_T}\right) - p_{n0} = p_{n0}\left[\exp\left(\frac{V_a}{V_T}\right) - 1\right]$$

$$\Delta p(x \rightarrow \infty) = p(x \rightarrow \infty) - p_{n0} = 0$$

$$\rightarrow \Delta p(x) = p_{n0}\left[\exp\left(\frac{V_a}{V_T}\right)-1\right]\exp\left(-\frac{x-x_n}{L_p}\right), \quad x > x_n$$

$$\Delta n(x) = n_{p0}\left[\exp\left(\frac{V_a}{V_T}\right)-1\right]\exp\left(-\frac{x_p-x}{L_n}\right), \quad x < x_p$$

$$\Delta p(x) = p_{n0}\left[\exp\left(\frac{V_a}{V_T}\right)-1\right]\exp\left(-\frac{x-x_n}{L_p}\right)$$

$$\Delta n(x) = n_{p0}\left[\exp\left(\frac{V_a}{V_T}\right)-1\right]\exp\left(-\frac{x_p-x}{L_n}\right)$$

- When $x - x_n = 5L_p$, the exponential factor in $\Delta p(x)$ is $e^{-5} = 0.0067 \rightarrow$ about five minority carrier diffusion lengths, the disturbance caused by the applied bias vanishes.
- Consider the minority carrier concentrations at the depletion region edges:

$$\Delta p = p_{n0}\left[\exp\left(\frac{V_a}{V_T}\right)-1\right] \text{ at } x = x_n$$

$$\Delta n = n_{p0}\left[\exp\left(\frac{V_a}{V_T}\right)-1\right] \text{ at } x = x_p$$

- For forward bias, $\Delta p(x_n)$ and $\Delta n(x_p)$ are positive.
- For reverse bias, $\Delta p(x_n)$ and $\Delta n(x_p)$ are negative.

PN Junction: Current Flow under Forward Bias

$$\Delta p(x) = p_{n0}\left[\exp\left(\frac{V_a}{V_T}\right)-1\right]\exp\left(-\frac{x-x_n}{L_p}\right), \quad x > x_n$$

$$\Delta n(x) = n_{p0}\left[\exp\left(\frac{V_a}{V_T}\right)-1\right]\exp\left(-\frac{x_p-x}{L_n}\right), \quad x < x_p$$

Note that, although $\mathcal{F}_n^{\text{diff}}$ (for $x < x_p$) and $\mathcal{F}_p^{\text{diff}}$ (for $x > x_n$) are in opposite directions, J_n^{diff} (for $x < x_p$) and J_p^{diff} (for $x > x_n$) are in the same direction. In particular, we are interested in $J_n^{\text{diff}}(x_p)$ and $J_p^{\text{diff}}(x_n)$.

$$J_n^{\text{diff}}(x_p) = \frac{qD_n n_{p0}}{L_n}\left(e^{V_a/V_T}-1\right)$$

$$J_p^{\text{diff}}(x_n) = \frac{qD_p p_{n0}}{L_p}\left(e^{V_a/V_T}-1\right)$$

PN Junction: Current Flow under Reverse Bias

$$\Delta p(x) = p_{n0}\left[\exp\left(\frac{V_a}{V_T}\right) - 1\right]\exp\left(-\frac{x - x_n}{L_p}\right), \quad x > x_n$$
$$\Delta n(x) = n_{p0}\left[\exp\left(\frac{V_a}{V_T}\right) - 1\right]\exp\left(-\frac{x_p - x}{L_n}\right), \quad x < x_p$$

Note that, although $\mathcal{F}_n^{\mathrm{diff}}$ (for $x < x_p$) and $\mathcal{F}_p^{\mathrm{diff}}$ (for $x > x_n$) are in opposite directions, J_n^{diff} (for $x < x_p$) and J_p^{diff} (for $x > x_n$) are in the same direction. In particular, we are interested in $J_n^{\mathrm{diff}}(x_p)$ and $J_p^{\mathrm{diff}}(x_n)$.

$$J_n^{\mathrm{diff}}(x_p) = \frac{qD_n n_{p0}}{L_n}\left(e^{V_a/V_T} - 1\right) \approx -\frac{qD_n n_{p0}}{L_n}$$
$$J_p^{\mathrm{diff}}(x_n) = \frac{qD_p p_{n0}}{L_p}\left(e^{V_a/V_T} - 1\right) \approx -\frac{qD_p p_{n0}}{L_p}$$

The currents are much smaller under reverse bias.

Total Current Density

- The total current density is the same throughout the device.
- If there is no G-R in the depletion region, we have
$$J = J_n(x_p) + J_p(x_n)$$
- Using our earlier results for $J_p(x_n)$ and $J_n(x_p)$, we get:
$$J = J_p(x_n) + J_n(x_p) = \left[\frac{qD_p p_{n0}}{L_p} + \frac{qD_n n_{p0}}{L_n}\right]\left(e^{V_a/V_T} - 1\right)$$
- We can now obtain $J_n(x > x_n)$ and $J_p(x < x_p)$ using $J_n(x) + J_p(x) = J$.

PN Junction: Derivation of I-V Equation (cont.)

$$J = \left[\frac{qD_p p_{n0}}{L_p} + \frac{qD_n n_{p0}}{L_n}\right]\left(e^{V_a/V_T} - 1\right)$$
$$\rightarrow I = A \times J = I_s\left(e^{V_a/V_T} - 1\right)$$

with

$$I_s = A\left(\frac{qD_p p_{n0}}{L_p} + \frac{qD_n n_{p0}}{L_n}\right)$$

- This equation is known as the "Shockley diode equation".
- Under reverse bias, with V_R equal to a few V_T or larger, $e^{V_a/V_T} = e^{-V_R/V_T} \approx 0$, and $I \approx -I_s$, i.e., the diode current "saturates" (at $-I_s$).
- I_s is therefore called the "reverse saturation current."

ECE/PHY 235 - Introduction to Solid State Electronics

Lecture 21: PN Junction Practice

Prof. Ying Wang
Contact: y.wang@wisc.edu
https://wang.ece.wisc.edu/

For an abrupt, uniformly doped silicon pn junction diode, $N_a = 10^{17}$ cm^{-3}, $N_d = 2 \times 10^{16}$ cm^{-3}, $\mu_n = 1500$ cm^2/V-s, $\mu_p = 500$ cm^2/V-s, $\tau_n = 2\mu$s, $\tau_p = 5\mu$s, $A = 10^{-3}$ cm^2. Compute the following for a forward bias of 0.65 V at $T = 300$ K:

1. $n(x_p)$ and $p(x_n)$
2. $J_n(x_p)$ and $J_p(x_n)$
3. The diode current I

1.

$$V_{\text{bi}} = V_T \log\left(\frac{N_a N_d}{n_i^2}\right) = (0.0259 \text{ V}) \log \frac{(10^{17})(2 \times 10^{16})}{(1.5 \times 10^{10})^2} = 0.77 \text{ V}$$

2. The equilibrium minority carrier densities are:

$$p_{n0} = \frac{n_i^2}{n_{n0}} \approx \frac{n_i^2}{N_d} = \frac{(1.5 \times 10^{10})^2}{2 \times 10^{16}} = 1.125 \times 10^4 \text{ cm}^{-3}$$

$$n_{p0} = \frac{n_i^2}{p_{p0}} \approx \frac{n_i^2}{N_a} = \frac{(1.5 \times 10^{10})^2}{1 \times 10^{17}} = 2.25 \times 10^3 \text{ cm}^{-3}$$

The minority carrier densities at x_p and x_n are:

$$n(x_p) = n_{p0} e^{V_a/V_T} = 2.25 \times 10^3 \times e^{0.65/0.0259} = 1.8 \times 10^{14} \text{ cm}^{-3}$$

$$p(x_n) = p_{n0} e^{V_a/V_T} = 1.125 \times 10^4 \times e^{0.65/0.0259} = 8.9 \times 10^{14} \text{ cm}^{-3}$$

The minority carrier current densities at x_n and x_p are:

$$J_p(x_n) = \frac{qD_p p_{n0}}{L_p}(e^{V_a/V_T} - 1)$$

$$J_n(x_p) = \frac{qD_n n_{p0}}{L_n}(e^{V_a/V_T} - 1)$$

The diffusion coefficients are:

$$D_p = V_T \mu_p = 0.0259 \times 500 = 12.9 \text{ cm}^2/\text{s}$$

$$D_n = V_T \mu_n = 0.0259 \times 1500 = 38.7 \text{ cm}^2/\text{s}$$

The minority carrier diffusion lengths in the neutral regions are:

$$L_p = \sqrt{D_p \tau_p} = \sqrt{12.9 \times 5 \times 10^{-6}} \text{ cm} = 80.3\mu\text{m}$$

$$L_n = \sqrt{D_n \tau_n} = \sqrt{38.7 \times 2 \times 10^{-6}} \text{ cm} = 88\mu\text{m}$$

The minority carrier current densities at x_n and x_p are:

$$\begin{aligned} J_p(x_n) &= \frac{qD_p p_{n0}}{L_p}(e^{V_a/V_T} - 1) \\ &= \frac{1.6 \times 10^{-19} \times 12.9 \times 1.125 \times 10^4}{80.3 \times 10^{-4}} \times 8.12 \times 10^{10} \\ &= 0.235 \text{ A/cm}^2 \end{aligned}$$

$$\begin{aligned} J_n(x_p) &= \frac{qD_n n_{p0}}{L_n}(e^{V_a/V_T} - 1) \\ &= \frac{1.6 \times 10^{-19} \times 38.7 \times 2.25 \times 10^3}{88 \times 10^{-4}} \times 8.12 \times 10^{10} \\ &= 0.13 \text{ A/cm}^2 \end{aligned}$$

The diode current I is:

$$\begin{aligned} I &= A(J_p(x_n) + J_n(x_p)) \\ &= 10^{-3} \text{ cm}^2 \times (0.235 + 0.13) \text{ A/cm}^2 \\ &= 0.365 \text{ mA} \end{aligned}$$