EM wave

waves

- Traverse wave: oscillation $\perp$ propagation
- Longitudinal wave: oscillation $\parallel$ propagation
- $v = \lambda f$

EM wave function

$$\begin{cases} E_x = E_0 \sin(kz - \omega t) \\ B_y = B_0 \sin(kz - \omega t) \end{cases} \quad [1$$

where $k = \frac{2\pi}{\lambda}$ (wave number) , $\quad \omega = 2\pi f = kc$ (dispersion relationship), B_0 : magnetic field amplitude, E_0 : electric field amplitude

EM Energy flux

Energy flux the energy transferred per unit area per unit time in the direction of wave propagation of an EM wave is defined by the Poynting vecter

$$\vec{S} \equiv \frac{\vec{E} \times \vec{B}}{\mu_0}. \quad [2$$

Where $\mu_0 = 1.25663706126e\text{-}6(N \cdot A^{-2})$ is the vacuum permeability.

- Intensity of EM wave is the magnitude of the Poynting vector:

$$I = \langle S \rangle = \frac{E_0^2}{377\Omega} \quad [3$$

  where Ω is ohm. Very unorthodoxy I know, but hey we are in Engineering Hall.

- Specially, when EM wave is emitted from a point light source with power P ,

$$I = \frac{P}{4\pi r^2} = \frac{E_0^2}{377\Omega} \quad [4$$

Double slit interference

Consider a double-slit setup, where the first dark line is at an angle θ from the central bright line with a distance Y. Distance from light source to screen is L. Then by trignometry:

$$Y = L \tan\theta. \quad [5$$

When considering constructive/distructive interference,given the separation between two slits is d the path difference between the two slits is

$$\begin{gathered} m\lambda = d \sin\theta \text{ constructive} \\ \left(m + \frac{1}{2}\right)\lambda = d \sin\theta \text{ destructive} \quad m = 0, 1, 2... \end{gathered} \quad [6$$

Photoelectric effect

- Energy of a photon

$$E_p = hf = \frac{hc}{\lambda} = \Phi + E_k \tag{7}$$

where Φ is the work function of the material, E_k is the kinetic energy of the emitted electron at the surface of the material. $h = 6.26e\text{-}34$ is the Planck constant, $c = 3e\text{-}8\ \mathrm{m/s}$ is the speed of light, f is the frequency of the photon, and λ is the wavelength of the photon.

- Motion for Photoelectric effect:

$$E_{k,m} + (\text{-}e)V_m = E_{k,d} + (\text{-}e)V_d \tag{8}$$

  Where $E_{k,m}$ is K.E at the metal surface, V_m is the voltage at the metal, $E_{k,d}$ is the K.E of the electron at the detector, and V_d is the voltage at the detector.

- stopping potential

$$eV_{\text{stop}} = \frac{hc}{\lambda} - \Phi \tag{9}$$

  the minimum potential required to stop the emitted electron.

- Threshold frequency& wavelength: set $E_k = 0$:

$$\begin{aligned} \Phi &= hf_t = \frac{hc}{\lambda_t} \\ \Rightarrow f_t &= \frac{\Phi}{h}, \quad \lambda_t = \frac{hc}{\Phi} \end{aligned} \tag{10}$$

Blackbody radiation

- Stefan-Boltzmann law:

$$R = \sigma T^4. \tag{11}$$

  Where R is the **power radiated per unit area**, or surface energy density of radiation. T is temprature in Kelvin, $\sigma = 5.67e\text{-}8(W \cdot m^{\text{-}2} \cdot K^{\text{-}4})$ is the Stefan-Boltzmann constant.

- Wien's displacement law:

$$\lambda_{\max} T = b \tag{12}$$

  where $b = 2.89e\text{-}3(m \cdot K)$ is the Wien's constant, and $\lambda_{\max}$ is the wavelength at which the blackbody **radiation is maximum**, and T is the temprature in Kelvin of the blackbody.

- Rayleigh-Jeans law:

$$\begin{aligned} R(\lambda) &= \frac{1}{4} c u(\lambda), \\ u(\lambda) &= 8\pi k T \lambda^{-4} \end{aligned} \tag{13}$$

WHere R is radiation power per unit area, or energy density, u is the energy density of radiation, c is the speed of light, and $k = 8.617e\text{-}5 \text{ eV/K} = 1.38e\text{-}23 J \cdot K^{-}1$ is the Boltzmann constatn This law is valid for long wavelength, but it diverges at short wavelength. **This equation is only good for long wavelength.**

- Planck's law:

$$u(\lambda) = \frac{8\pi hc\lambda^{-5}}{e^{hc/\lambda kT} - 1} \quad [14$$

where $k = 1.38e\text{-}23\left(J \cdot K^{\text{-}1}\right)$ is the Boltzmann constant, h is the Planck constant, T is the temprature in Kelvin of the blackbody.

Energy of radiation

For an ideal blackbody, the energy radiated within a certain wavelength range is found by integrating Equation 14 over the range of wavelength.

$$U = \int_{\lambda_1}^{\lambda_2} u(\lambda) \, \mathrm{d}\lambda \quad [15$$

- It is often times easier to use mid-point approximation to handle the above integration:

$$U \approx u(\lambda)\Delta\lambda \quad [16$$

Where $\lambda = \frac{\lambda_2 - \lambda_1}{2}$ is the mid-point of the wavelength range, and $\Delta\lambda$ is the width of the wavelength range.

Wavelike properties of particles

De broglie Hypothesis

$$f = \frac{E}{h} \quad , \lambda = \frac{h}{p} \quad [17$$

Where E is the total energy, p is the momentum, and λ is the wavelength of the particle. $h = 6.63e\text{-}34 J \cdot s$ is the Planck constant.

- For a particle of zero rest energy,

$$E = pc = hf = \frac{hc}{\lambda}, \quad [18$$

where p is the momentum of the particle.

- For a moving particle,

$$E = \frac{1}{2}mv^2 = \frac{p^2}{2m} \quad [19$$

Wavefunction for particles

$$\Psi(x,t) = A\sin(kx - \omega t) \quad \text{or} \quad Ae^{i(kx - \omega t)} \quad [20$$

- probability density of the particle is

$$p(x,t) = |\Psi|^2 \equiv \Psi^* \Psi \quad [21$$

Uncertainty Principle

$$\Delta x \Delta p \geq \frac{\hbar}{2}, \quad \Delta E \Delta t \geq \frac{\hbar}{2} \quad [22$$

Where x is position, p is momentum, E is energy, t is time, and $\hbar = \frac{h}{2\pi} = 1.05e\text{-}34 J \cdot s$ is the reduced Planck constant.

Min. Energy of Particle in a box

$$E = \frac{p^2}{2m} \geq \frac{\hbar^2}{2mL^2} \quad [23$$

Schrodinger's equation

Time-dependent Schrodinger's equation in 1D

1D Schrodinger's equation in position basis:

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t) + V(x,t)\Psi(x,t) = i\hbar\frac{\partial}{\partial t}\Psi(x,t) \quad [24$$

Time-independent Schrodinger's equation in 1D

Via separation of variable, set $\Psi(x,t) = \psi(x)\varphi(t)$, and noticing $f = \frac{E}{h}$, we have

$$\begin{gathered} -\frac{\hbar}{2m}\frac{\mathrm{d}^2}{\mathrm{d}x^2}\psi(x) + V(x)\psi(x) = E\psi(x) \\ \text{time variation of wavefunction: } \varphi(t) = e^{-iEt/\hbar} \end{gathered} \quad [25$$

- Probability density is thus simplified to

$$p(x) = |\Psi(x,t)|^2 = |\psi(x)|^2 \quad [26$$

- Normalization condition:

$$\int_{-\infty}^{\infty} |\psi(x)|^2 \,\mathrm{d}x = 1 \quad [27$$

Infinite potential well- particle in a box

- For a particle in a box of length L , where $V(x) = 0$ for $0 < x < L$, and $V(x) = \infty$ otherwise, the wavefunction is found by

$$\begin{gathered} -\frac{\hbar^2}{2m}\frac{\mathrm{d}^2}{\mathrm{d}x^2}\psi(x) = E\psi(x) \\ \Rightarrow \psi_n(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right). \end{gathered} \quad [28$$

  Noticing boundary values, the following is obtained:

$$E_n = \frac{\hbar^2 k_n^2}{2m} = n^2 \frac{\hbar^2 \pi^2}{2mL^2} = n^2 E_1$$ [29

where $k = 2\frac{\pi}{\lambda}; k^2 = \left(\frac{p}{\hbar}\right)^2 = \frac{2mE}{\hbar^2}$

- Specially, the energy levels can be also expressed in terms of hc and mc^2:

$$E_1 = \frac{(hc)^2}{8mc^2L^2}; \quad E_n = \frac{n^2(hc)^2}{8mc^2L^2}$$ [30

- Normalization condition in box of length L:

$$\int_0^L |\psi(x)|^2 \, \mathrm{d}x = 1$$ [31

Appendix

1. Useful integral for probability of wavefunction

$$\int_{-\infty}^{\infty} e^{-a(x+b)^2} \, \mathrm{d}x = \sqrt{\frac{\pi}{a}}$$ [32

2. Useful constants:
   - hc = 1240 eV nm.
   - For an electron: $mc^2 = 0.511\text{MeV} = 5.11e5$ eV

Past homework

3.17

By Stefan-Boltzmann Law, set total power $P = \kappa R$ and initial tempreture T_0 , we have

$$R = \sigma T^4 \Rightarrow P = \kappa\sigma T^4$$
$$\frac{P'}{P} = \frac{T'^4}{T^4} = \frac{(2T_0)^4}{T_0^4} = 16)$$ [33

$\boxed{\text{Power increases by a factor of 16.}}$

3.19

- (a)

Let initial tempreture be T_0 and the new tempreture be T'. By Wien's Law, we have

$$\lambda T_0 = 2.898e^{-3}\ m \cdot K \quad \Rightarrow T_0 = \frac{2.898e\text{-}3}{27e\text{-}6})K = 107.33K.$$ [34

Using Stefan-Boltzmann Law to find the new tempreture.

$$\frac{P'}{P} = \frac{(T')^4}{T_0^4} = 2 \quad \Rightarrow T' = \sqrt[4]{2T_0^4} = \sqrt[4]{2 \times 107.33^4} = \boxed{127.63K}$$ [35

- (b) By Wien's law,

$$\lambda = \frac{2.898e\text{-}3}{T'} = \frac{2.898e\text{-}3}{127.63}m = \boxed{22.7\mu m}$$ [36

3.24

- (a)

Energy quantization shows:

$$E = hf = \frac{hc}{\lambda}.$$ [37

Given $\lambda \in (380, 750)$nm,

$$\frac{hc}{750\text{nm}} < E < \frac{hc}{380\text{nm}} \quad \Rightarrow \boxed{E \in (1.653, 3.542)\text{eV}}$$ [38

- (b)

$$E = hf = 4.136 \times 10^{-15} \times 100 \times 10^6 \text{ eV} = \boxed{4.136 \times 10^-7 \text{ eV}}$$ [39

3.25

- (a)

By the photoelectric effect equation, at therashold wavelength, we have

$$\Phi = hf_t = h\frac{c}{\lambda_t} \quad \Rightarrow \lambda_t = \frac{hc}{\Phi} = \frac{1.24e\text{-}6}{4.87}m = \boxed{2.546e\text{-}7m}$$ [40

- (b)

As suggested on Piazza, we use mid-point approximation to approximate the integrated energy density of sunlight from 0nm to 254.6nm by using the intensity at $254.6/2 = 127.3$ nm as constant density:

$$u(127.3\text{nm}) \times (254.6 \text{ nm}) = \frac{8\pi hc(127.3e\text{-}9m)^{-5}}{e^{hc/(k\times 5800K\times 127.3e\text{-}9m)} - 1} \times (254.6e\text{-}9m) \approx 1.23e\text{-}4 \quad J/m^3$$ [41

Energy density is thus approximately

$$R' = \frac{c}{4}(1.23e\text{-}4) \quad J/m^3$$ [42

Total energy is given by

$$R = \sigma T^4 = \sigma \times 5800K^4 \approx 6.42e7 \quad W/m^2$$ [43

Thus the maximal fractional power is

$$\boxed{\frac{R'}{R} \approx 1.4e\text{-}4} \quad [44$$

3.26

- (a)

Using the photoelectric equation,we can find threshold freq and wavelength, f_t, λ_t as follows,

$$\Phi = hf_t = \frac{hc}{\lambda_t}$$

$$\Rightarrow f_t = \frac{\Phi}{h} = \frac{1.9eV}{4.136e\text{-}15eV \cdot s} = \boxed{4.59e4 \quad \text{Hz}}, \quad [45$$

$$\lambda_t = \frac{hc}{\Phi} = \frac{1240 \text{ eV} \cdot \text{nm}}{1.9 \text{ eV}} = \boxed{653 \text{ nm}}$$

- (b,c,d) The stopping potential can be found as follows,

$$eV_0 = \frac{hc}{\lambda} - \Phi \quad \Rightarrow V_0 = \frac{hc}{\lambda e} - \frac{\Phi}{e}. \quad [46$$

For $\lambda = 300\text{nm}$:

$$V_0 = \frac{1240 \text{ eV} \cdot \text{nm}}{e \times 300e\text{-}9m} - \frac{1.9 \text{ eV}}{e} = \boxed{2.23V} \quad [47$$

For $\lambda = 400$ nm,

$$V_0 = \frac{1}{e} \frac{1240\text{eV} \cdot \text{nm}}{400\text{nm}} - 1.9\text{eV} = \boxed{1.20V} \quad [48$$

3.28

- (a)

$$f_t = \frac{\Phi}{h} = \frac{4.22 \text{ eV}}{4.14e\text{-}15 \text{ eV} \cdot s} = \boxed{1.02 \times 10^{15} \text{ Hz}} \quad [49$$

- (b)

$$f = \frac{c}{\lambda} = \frac{3e8}{560e_{\text{-}9}} \text{ Hz} = \boxed{5.36 \times 10^{14} \text{ Hz} < f_t} \quad [50$$

Frequency is less than the threshold frequency, so **no** photoelectrons are emitted.

3.31

Consider the photoelectric effect equation for $n = 60$ photons,

$$E = n\frac{hc}{\lambda} = \frac{60 \times 6.63e\text{-}34 \times 3e8}{550e\text{-}-9}J = \boxed{2.17e\text{-}17J}.$$ [51

3.32

- (a)

$$\Phi = \frac{hc}{\lambda} = \frac{1240}{653}\ \text{eV} = \boxed{1.9\ \text{eV}}$$ [52

- (b)

$$E_k = \frac{hc}{\lambda} - \Phi = \frac{1240}{300}\ \text{eV} - 1.9\ \text{eV} = \boxed{2.23\ \text{eV}}$$ [53

3.42

Consider the stopping potential function for both cases, we have

$$eV = \frac{hc}{\lambda} - \Phi$$
$$\Rightarrow \begin{cases} V_1 = \frac{1}{e}\frac{hc}{\lambda_1} - \Phi \\ V_2 = \frac{1}{e}\frac{hc}{\lambda_2} - \Phi \end{cases}$$ [54

Where $V_1 = 0.52V$, $\lambda_1 = 450$ nm; $V_2 = 1.9V$, $\lambda_2 = 300$ nm.

Solving Equation 54 for h and Φ:

$$\begin{cases} 0.52V = \frac{1}{e}\frac{hc}{450\text{nm}} - \Phi \\ 1.9V = \frac{1}{e}\frac{hc}{300\text{nm}} - \Phi \end{cases} \Rightarrow \boxed{\begin{cases} h = 6.6376e\text{-}34J \cdot s \quad \text{(a good approximation!)} \\ \Phi = 2.24\ \text{eV} \end{cases}}$$ [55

3.48

A 100-W beam of light is shone onto a blackbody of mass $2e\text{-}3$ *kg for* 10^4 *sec. The blackbody is initially at rest in a frictionless space.* ***(a)****Compute the total energy and momentum absorbed by the blackbody from the light beam.* ***(B)****calculate the blackbody's velocity at the end of the period of illumination;* ***(C)****Compute the final kinetic energy of the blackbody. Why is the latter less than the total energy of the absorbed photons?*

- (A)

Energy absorbed:

$$E = \int_T P\,\mathrm{d}t = 100J/s * 10^4 s = 10^6 J.$$ [56

Momentum of photon is calculated by:

$$p = \frac{E}{c} = \frac{10^6 J}{3 * 10^8 m/s} = 3.33 * 10^-3 N \cdot s. \quad [57$$

Conservation of momentum tells us that the blackbody will have the same momentum as the photons, so the total momentum absorbed is $3.33 * 10^-3 N \cdot s$.

- (B)

By exploiting momentum, we find the terminal velocity by:

$$\begin{aligned} p = mv \quad &\Rightarrow \Delta v = \frac{\Delta p}{m} = \frac{3.33 * 10^-3 N.s}{2 * 10^-3 \text{ kg}} = 1.67 m/s \\ &\Rightarrow v_t = \Delta v - v_i = \boxed{1.67 m/s.} \end{aligned} \quad [58$$

- (C)

$$T = \frac{1}{2} m v^2 = \frac{1}{2} * 2e\text{-}3 * 1.67^2 J = \boxed{2.78 * 10^{-3} J} \quad [59$$

Kinetic energy being less than the total energy absorbed is due to the fact that the blackbody is a perfect absorber, and the difference in energy is lost to **increase its internal heat** and **radiate into space**.

3.51

Determine the fraction of the energy radiated by the Sun in the visible region of the spectrum(350 nm to 700 nm). Assume that the Sun's surface temperature is 5800K.

-

By Planck's law, the energy radiated by the Sun in the visible region is found by the following integration:

$$U_v = \int_{350 \text{ nm}}^{700 \text{ nm}} \frac{8\pi hc \lambda^{-5}}{e^{hc/\lambda kT} - 1} d\lambda. \quad [60$$

We approximate this integral using mid-point approximation, with $\lambda = (700 + 350)/2 \text{ nm} = 525 \text{ nm}$ and interval $\Delta\lambda = 350\text{nm}$:

$$U_v \approx u(\lambda)\Delta\lambda = \frac{8\pi hc (525 \text{ nm})^{-5}}{e^{hc/525 \text{ nm} \cdot k \cdot 5800K} - 1} \times 350 \text{ nm} = 0.389 J/m^3. \quad [61$$

Then, using Rayleigh-Jeans Equation:

$$R_v = \frac{c}{4} U_v = 2.92e7 W/m^2, \quad [62$$

while total energy radiated by Sun is

$$R = \sigma T^4 = \sigma \times 5800 K^4 \approx 6.42e7 \quad W/m^2. \quad [63$$

Thus the fraction of energy radiated in the visible region is:

$$\boxed{\frac{R_v}{R} = 0.455.} \tag{64}$$

Other Problem (1)

If a person of mass 70kg walks at the speed of $5\ \text{km/hr}$*, what is their DeBroglie wavelength? Do you think it would be possible to observe the person's wavelike properties in experiment (compare it to the conditions of double slit experiment)? Explain your reasoning.*

•

$$\lambda = \frac{h}{mv} = \frac{h}{70\ \text{kg} \cdot 5\ \text{km/h}} = \boxed{6.815e\text{-}36\ m.} \tag{65}$$

This wavelength is of magnitudes smaller than what is typically observed in double slit experiments, which are on the order of 10^-10m or bigger. It is thus nearly impossible to detect.

Other Problem (2)

You are given the task of constructing a double slip experiment for electrons of energy of 5 eV (converting this into velocity).

1. *If you wish the first dark line of the interference patter to occur at* $5°$*, what must the separation between the slits be?*
2. *How far from the slits must the detector plane be located, if the first dark line on each side of the central maximum is to be seperated by 1cm?*

•

$$T = \frac{1}{2}mv^2 \quad \Rightarrow v = \sqrt{\frac{2T}{m}} = \sqrt{\frac{2 \times 5eV}{9.11e\text{-}31\ \text{kg}}} = 1.326e6\ m/s. \tag{66}$$

We need to find the wavelength of our propagating electrons:

$$\lambda = \frac{h}{mv} = \frac{6.63e\text{-}34}{9.11e\text{-}31 \times 1.326e6} = 5.485e\text{-}10m. \tag{67}$$

For the first dark fringe to occur at $5°$, we have:

$$d\sin(\theta) = \frac{1}{2}\lambda \quad \Rightarrow d = \frac{\lambda}{2\sin(\theta)} = \frac{5.485e\text{-}10m}{2\sin(5°)}$$
$$\boxed{d = 3.146e\text{-}9m}. \tag{68}$$

•

$$Y = L\tan\theta \Rightarrow L = \frac{Y}{\tan(\theta)} = \frac{0.5\text{cm}}{\tan(5°)} = \boxed{5.71\ \text{cm}} \tag{69}$$

Other Problem (3)

A particle moving in one dimension between rigid walls separated by a distance L has the wave function $\psi(x) = A\sin\left(\pi\frac{x}{L}\right)$. Since the particle must remain etween the walls, what must be the value of A?

- Using the Noamalized wavefunction in a confined 1-D space, we can jot down the following:

$$\begin{gathered}\int_0^L \psi^*(x)\psi(x)\,\mathrm{d}x = 1 \\ \Rightarrow \int_0^L |\psi(x)|^2\,\mathrm{d}x = \int_0^L A^2\sin^2\left(\pi\frac{x}{L}\right)\mathrm{d}x = 1\end{gathered} \quad [70$$

The above integral yields:

$$\begin{gathered}\left(\frac{1}{2}x + \frac{L}{4\pi}\sin\left(\frac{2\pi}{L}x\right)\right)_0^L\Bigg| = \frac{1}{A^2} \\ \boxed{\Rightarrow A = \sqrt{\frac{2}{L}}}\end{gathered} \quad [71$$

6-5

1. Show that the wave funciton $\Psi(x,t) = A\sin(kx - \omega t)$ does not satisfy the time-dependent Schrodinger equation.
2. Show that $\Psi(x,t) = A\cos(kx - \omega t) + iA\sin(kx - \omega t)$satisfies the time-dependent Schrodinger equation.

Solution: Recall the time-dependent Schrodinger equation:

$$-\frac{\hbar^2}{2m}\Psi_{xx} + V\Psi = i\hbar\Psi_t, \quad [72$$

assuming $V = 0$.

1. We have $\Psi_{xx} = -k^2A\sin(kx - \omega t)$, and $\Psi_t = -A\omega\cos(kx - \omega t)$. Trivially, plugging back into Equation 72, the LHS is **not equal** to the RHS.
2. We have

$$\begin{gathered}-\frac{\hbar^2}{2m}\Psi_{xx} = \frac{\hbar^2k^2A}{2m}\cos(kx - \omega t) + \frac{\hbar^2ik^2A}{2m}\sin(kx - \omega t) \\ i\hbar\Psi_t = \hbar\omega A\cos(kx - \omega t) + i\hbar\omega A\sin(kx - \omega t).\end{gathered} \quad [73$$

   Upon noticing $\hbar\omega = \frac{\hbar^2k^2}{2m} + V$, with $V = 0$, the two equations above **are equivalent**

6-9

A particle is in a infinite square well of width L. Calculate the ground-state energy if:

1. The particle is a proton and $L = 0.1$ nm. a typical size for a molecule;

2. the particle is a proton and $L = 1$fm, a typical size for a nucleus.

Solution: Using $E_1 = \frac{\hbar^2\pi^2}{2mL^2}$, we have:

1.
$$E_1 = \frac{\hbar^2\pi^2}{2 * 1.67e\text{-}27 \text{ kg} * (0.1\text{nm}^2)} = 3.28e\text{-}21 \text{ kg } m^2/s^2 = \boxed{0.021 \text{ eV}} \tag{74}$$

2. Similarly,
$$E_1 = 3.28e\text{-}11 \text{ kg } m^2/s^2 = \boxed{205 \text{ MeV}} \tag{75}$$

6-12

A mass of 10^{-6} g is moving with a speed of about 10^{-1}cm/s in a box of length 1cm. Treating this as a one-dimensional infinite square well, calculate the approximate value of the quantum number n

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Solution: Equating the kinetic energy of the particle to the energy of n-th level of the box, we have:

$$\begin{aligned} \frac{1}{2}mv^2 &= \frac{n^2\pi^2\hbar^2}{2mL^2} \quad \Rightarrow n = \frac{mvL}{\pi\hbar} \\ \Rightarrow n &= \frac{10^{-6}g * 0.1 \text{ cm/s} * 1 \text{ cm}}{\pi\hbar} = \boxed{3.02e19} \end{aligned} \tag{76}$$

6-16

The wavelength of light emitted by a ruby laser is 694.3nm. Assuming that the emission of a photon of this wavelength accompanies the transition of an electron from the $n = 2$ level to the $n = 1$ level of an infinite square well, compute L for the well.

Solution: Equating the energy difference between the two energy levels to the energy of the photon emitted, we have:

$$\begin{aligned} \Delta E &= \frac{hc}{\lambda}, \quad \Delta E = E_2 - E_1 = \frac{3\hbar^2\pi^2}{2mL^2} \\ \Rightarrow L &= \sqrt{\frac{3\pi\lambda h}{8mc}} = \left(\frac{3 * 694.3 \text{ nm} * h}{8 * m_e * c}\right) = \boxed{7.95 * e\text{-}10 \quad \text{m}} \end{aligned} \tag{77}$$