

Math 421, Section 1
Midterm 2
Fall 2024

First name: _____ **Last name:** _____

Instructions:

- This exam contains 3 problems, and there are a total of 30 points available.
- Show all your work in the space provided. You may also use the backs of pages.
- No outside resources are allowed, including notes, calculators, textbooks, etc.

Question	Points	Score
1	10	
2	10	
3	10	
Total:	30	

1. (10 points) Suppose that $f : [0, 1] \rightarrow \mathbb{R}$ is a continuous function that satisfies $f(x) \in [0, 1]$ for all $x \in [0, 1]$. Prove that there exists a point $x \in [0, 1]$ such that $f(x) = x^2$.

2. (10 points) Let $A \subseteq \mathbb{R}$ be nonempty and bounded, and define the set

$$B = \{2a : a \in A\}.$$

Prove that $\inf B = 2 \cdot \inf A$.

3. Prove or disprove the following statements:

- (a) (5 points) If $f : \mathbb{R} \rightarrow \mathbb{R}$ is differentiable, then for any constant $c \in \mathbb{R}$ the function $g : \mathbb{R} \rightarrow \mathbb{R}$, $g(x) = f(x) + c$ has derivative $g'(a) = f'(a)$ for all $a \in \mathbb{R}$.
- (b) (5 points) If $f : \mathbb{R} \rightarrow \mathbb{R}$ is differentiable, then for any constant $c \in \mathbb{R}$ the function $g : \mathbb{R} \rightarrow \mathbb{R}$, $g(x) = f(x + c)$ has derivative $g'(a) = f'(a)$ for all $a \in \mathbb{R}$.

Extra paper