7.1

7.2

7.8

7.9

7.16

Add-on problem

A free particle of mass m with wave number k_1 is traveling to the right. at $x = 0$, the potential jumps from zero to V_0 and remains at this value for positive x.

- a. If the total energy is equal to $2V_0$, what is the wave number in the region $x > 0$? Express your answer in terms of k_1 and V_0 .

Energy in each region is found as

$$\begin{aligned} E_1 = K_1 + V_1 = \frac{\hbar^2 k_1^2}{2m} \qquad & (x < 0), \\ E_2 = K_2 + V_2 = \frac{\hbar^2 k_2^2}{2m} + V_0 \quad & (x > 0), \end{aligned} \tag{1}$$

where k_1, k_2 are the wavenumbers in region $x < 0, x > 0$ respectively.

Given Total energy $E = 2V_0$, we have:

$$\begin{aligned} 2V_0 = \frac{\hbar^2 k_1^2}{2m} \qquad & \Rightarrow k_1^2 = \frac{4mV_0}{\hbar^2} \\ 2V_0 = \frac{\hbar^2 k_2^2}{2m} + V_0 & \Rightarrow k_2^2 = \frac{2mV_0}{\hbar^2} \end{aligned} \tag{2}$$

Therefore, algebra gives

$$k_2 = \frac{k_1}{\sqrt{2}}. \tag{3}$$

- b. Calculate the reflection coefficient R and the transmission coefficient T at the potential step. Note that $1 + R = T$ (typo?) .

From lecture,

$$R = \left(\frac{k_1 - k_2}{k_1 + k_2}\right)^2, \quad T = \frac{4k_1 k_2}{(k_1 + k_2)^2}, \quad R + T = 1 \tag{4}$$

(it was hinted in problem that $1 + R = T$, but is it a typo? I think it should be $R + T = 1$ as per lecture slides.)

Since we found the relationship between k_1, k_2, we can write

$$
\begin{aligned}
R &= \left(\frac{k_1 - k_1/\sqrt{2}}{k_1 + k_1/\sqrt{2}}\right)^2 = \left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)^2 \approx 0.02943 \\
T &= \frac{4k_1 \cdot k_1/\sqrt{2}}{\left(k_1 + k_1/\sqrt{2}\right)^2} = \frac{4/\sqrt{2}}{\left(1+1/\sqrt{2}\right)^2} \approx 0.9705
\end{aligned} \tag{5}
$$

It can be verified that $R + T = 1$.

- c. If one million particles with wave number k_1 are incident upon the potential step, how many particles are expected to continue along the positive x direction? How does this compare with the classical prediction (which says that all particles go through if their energy is above that of the potential step?)

Number of transmitted particles:

$$
N_T = TN_0 = 0.9705 * 10^6 \approx 9.705 * 10^5 \tag{6}
$$

Number of reflected particles

$$
N_R = RN_0 = 0.02943 * 10^6 \approx 2.943 * 10^4. \tag{7}
$$

However, classically, since each particles carries same energy $2V_0$, all particles should go through.