1.

Given the width of the depletion region W as $W = \sqrt{\frac{2\varepsilon_s(N_A + N_D)}{qN_AN_D}V_{bi}}$, where N_A, N_D are the doping concentrations, V_{bi} is the built-in potential, and ε_s is the permittivity of the semiconductor. A silicon PN junction has $N_A = 10^{16}\text{cm}^3$, $N_D = 10^{15}\text{cm}^3$, and the intrinsic carrier concentration $n_i = 1.5 * 10^{10}\text{cm}^-3$. The permittivity of silicon is $\varepsilon_s = 11.7\varepsilon_0$, and $\varepsilon_0 = 8.85 * 10^{-14}\ \text{F}/\text{cm}$.

- Calculate the built-in potential V_{bi}
- Calculate the depletion region width at equilibrium.

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a.

using the relation

$$V_{\text{bi}} = \frac{kT}{q}\ln\left(\frac{N_AN_D}{n_i^2}\right), \tag{1}$$

where , taking $T = 300K$, $k = 1.38 * 10^{-23}\ \text{J}/\text{K}$, and $q = 1.6 * 10^{-19}C$, we have:

$$\frac{N_AN_D}{n_i^2} = \frac{10^{16} * 10^{15}}{(1.5 * 10^{10})^2} = 4.44e10, \quad \frac{kT}{q} = 0.0259V \tag{2}$$

Thus

$$\boxed{V_{\text{bi}} = 0.0259\ln(4.44 * 10) = 0.635V.} \tag{3}$$

b.

Notice $N_A + N_D = 1.1 * 10^{16}\text{cm}^{-3}, N_AN_D = 10^{31}\text{cm}^{-6}$,

Using

$$\begin{aligned} W &= \sqrt{\frac{2\varepsilon_s(N_A + N_D)}{qN_AN_D}V_{bi}} \\ &= \sqrt{\frac{2 * 1.035e\text{-}12 * 1.1e16}{1.6e\text{-}19 * 10^{31}} * 0.635V}\ \boxed{= 9.48e\text{-}5\ \text{cm}} \end{aligned} \tag{4}$$

2.

Felectric FIeld in the Depletion Region.

The expression for the maximum electric field in the depletion region is $E_{\max} = \frac{qN_AW_P}{\varepsilon_s} = \frac{qN_DW_N}{\varepsilon_s}$, where W_P, W_N are the widths of the depletion region on the P and N sides, respectively.

- Using the information from problem 1, calculate the maximum electric field at equilibrium.
- Show that the built-in potential V_{bi} is the integral of the electric field across the depletion region:

$$V_{\text{bi}} = \int_0^W E(x)\,\mathrm{d}x \tag{5}$$

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a.

Using

$$W = W_N + W_P, \tag{6}$$

with charge neutrality condition:

$$qN_A W_P = qN_D W_N \Rightarrow \frac{W_P}{W_N} = \frac{N_D}{N_A} = 0.1. \tag{7}$$

So

$$\begin{aligned} W &= W_N + W_P = 1.1 W_N \\ \Rightarrow W_N &= \frac{W}{1.1} = \frac{9.48 * 10^{-5}}{1.1} = 8.64e\text{-}5 \text{ cm}, \\ W_P &= 0.1 W_N = 8.64e\text{-}6 \text{ cm} \end{aligned} \tag{8}$$

Using

$$E_{\max} = \frac{qN_A W_P}{\varepsilon_s} \tag{9}$$

We have

$$E_{\max} = \frac{1.6 * 10^{-19} C * \left(10^{16} \text{cm}^{-3} (8.64 * 10^{-6})\right)}{1.035 * 10^{-12} \frac{F}{\text{cm}}} \boxed{= 1.334 * 10^4 V/ \text{ cm}} \tag{10}$$

b.

Using definition that

$$E(x) = -\frac{\mathrm{d}V}{\mathrm{d}x}, \tag{11}$$

we have

$$\int_0^W E(x) \, \mathrm{d}x = V(0) - V(W). \tag{12}$$

Since P-side is at a lower potential thatn the N-side in equilibrium, we have

$$V_{\text{bi}} = V(N) - V(P) = V(0) - V(W) = \int_0^W E(x) \, \mathrm{d}x \tag{13}$$

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