1 Notes on Quantum computation and quantum information by Nielsen and Chuang

2 Chapter 2: Linear algebra

2.I vector space

- C^n: space of all n-tuple complex numbers (c numbers)

i.e. $(z_1, z_2, z_3, ..., z_n)$

- a vector space is closed under scalar multiplicationa nd addition

2.II Dirac notation

Symbols	Meaning
$\lvert v\rangle$	ket, a vector in vec space
$\langle v\rvert$	bra, a dual vector in vec space; the complex transpose of ket $\langle v\rvert = \left(\lvert v\rangle^*\right)^T$
$\langle v\vert w\rangle$	inner product of $\lvert v\rangle$ and $\lvert w\rangle$
$\lvert\varphi\rangle \otimes \lvert\psi\rangle$	tensor product of $\lvert\varphi\rangle$ and $\lvert\psi\rangle$ abbriviates as $\lvert\varphi\rangle\lvert\psi\rangle$
$\boldsymbol{A}^*$	complex conjugate of $\boldsymbol{A}$
$\boldsymbol{A}^T$	transpose of $\boldsymbol{A}$
$\boldsymbol{A}^\dagger$	hermitian conjugate of $\boldsymbol{A}$ i.e. $\boldsymbol{A}^\dagger = (\boldsymbol{A}^*)^T$ $$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^\dagger = \begin{pmatrix} a^* & c^* \\ b^* & d^* \end{pmatrix}$$
$\langle\varphi\vert\boldsymbol{A}\vert\psi\rangle$	inner product betweeen $\lvert\varphi\rangle$ and $\boldsymbol{A}\lvert\psi\rangle$

2.III Span

a set of bec $|v_1\rangle, |v_2\rangle, ..., |v_n\rangle$ spans the vector space if any vector in the space can be written as

$$|v\rangle = \sum_i a_i |v_i\rangle$$

for some complex numbers a_i

2.IV Linear Independence

a set of non-zero vectors $|v_1\rangle, |v_2\rangle, ..., |v_n\rangle$ are liinearly dependent if there exists a set of complex numbers $a_1, a_2, ..., a_n$, s.t.

$$a_1|v_1\rangle + a_2|v_2\rangle + ... + a_n|v_n\rangle = 0$$

If the only solution to the above equation is $a_1 = a_2 = ... = a_n = 0$, then the vectors are **linearly independent**

2.V Linear operators

A linear operator A is any linear function that

$$A\left(\sum_i a_i |v_i\rangle\right) = \sum_i a_i A(|v_i\rangle)$$

It is convention to write $A(|v_i\rangle) = A|v_i\rangle$

- Identity Operator $I_V : I_V|v\rangle \equiv |v\rangle$. It is convinent to write I if no confution arises.
- zero operator $0|v\rangle \equiv 0$
- composition of linear operators A and B is AB

We observe that the above is equivalent to the matrix representation of linear transformations.

In other words, for a linear operator $A : V \to W$, and suppose $|v_1\rangle, |v_2\rangle, ..., |v_m\rangle$

3 Inner product