

ECE 535: Introduction to Quantum Sensing

Special Halloween lecture: quantum ghost imaging

Jennifer Choy

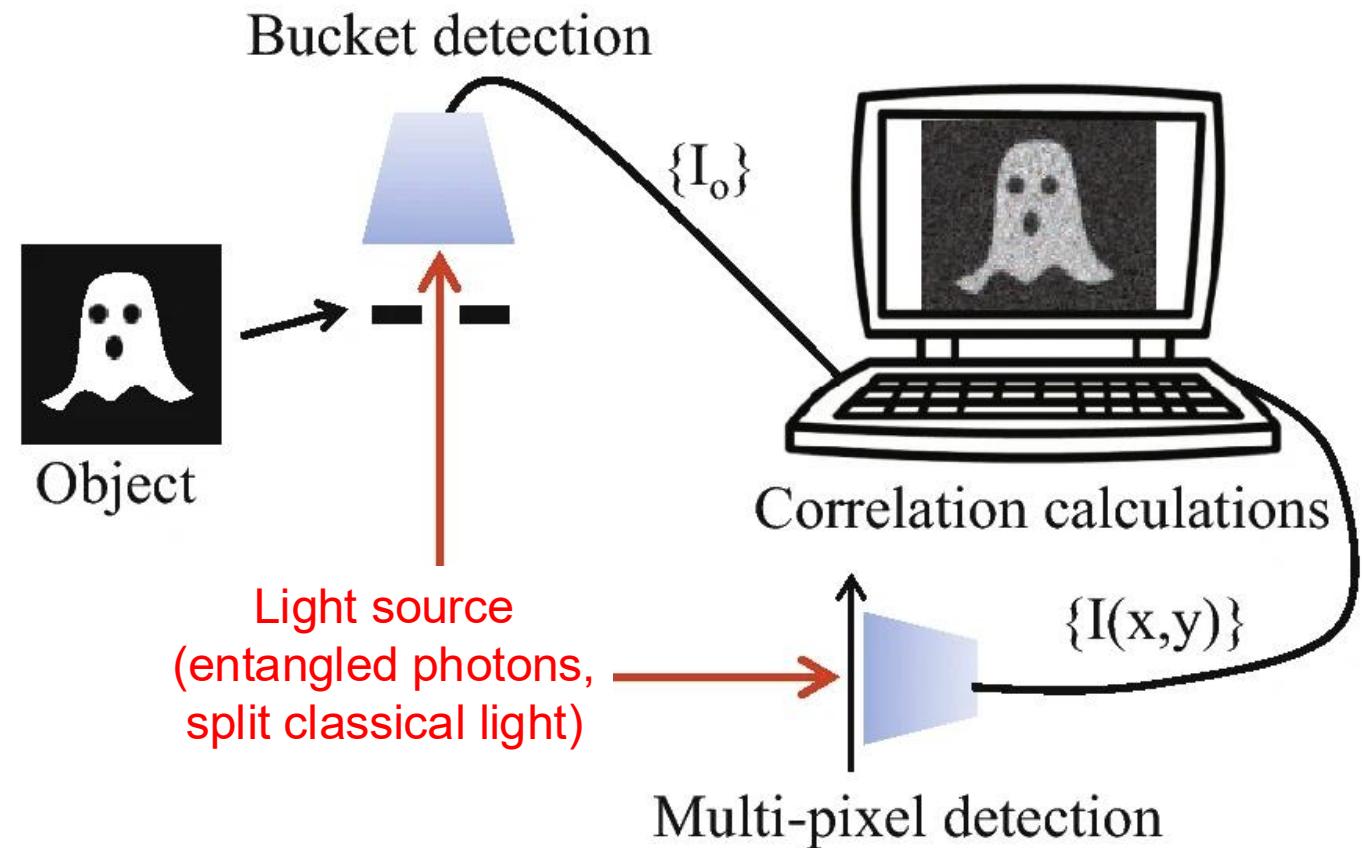
Fall 2025



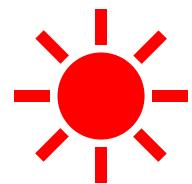
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Outline

- **Ghost imaging:** an imaging technique using light that has not physically interacted with the object to be imaged.
 - Not inherently quantum phenomenon, although the first demonstration was done using quantum-entangled photons
- Some fundamentals of imaging
- Properties of light sources in imaging
 - Spatial and temporal coherence
- Quantum light sources, including entangled photons
- Applications in ghost imaging



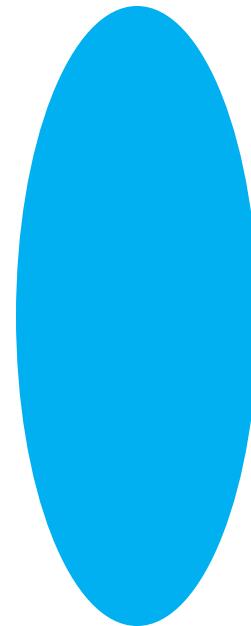
Imaging problem



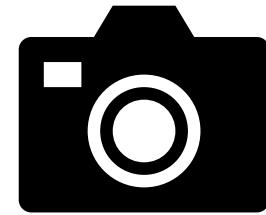
Illumination



Object



Imaging
optics

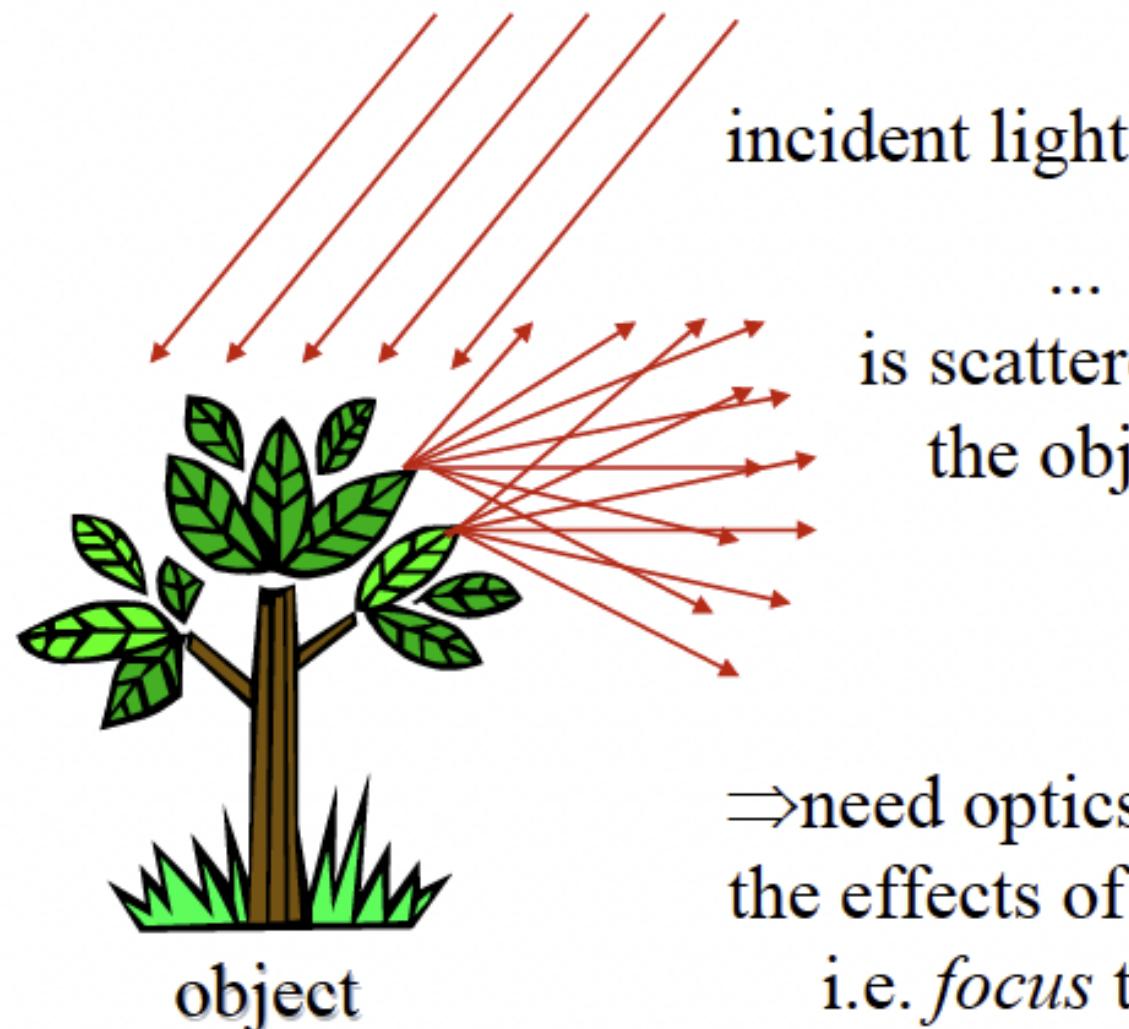


Detection

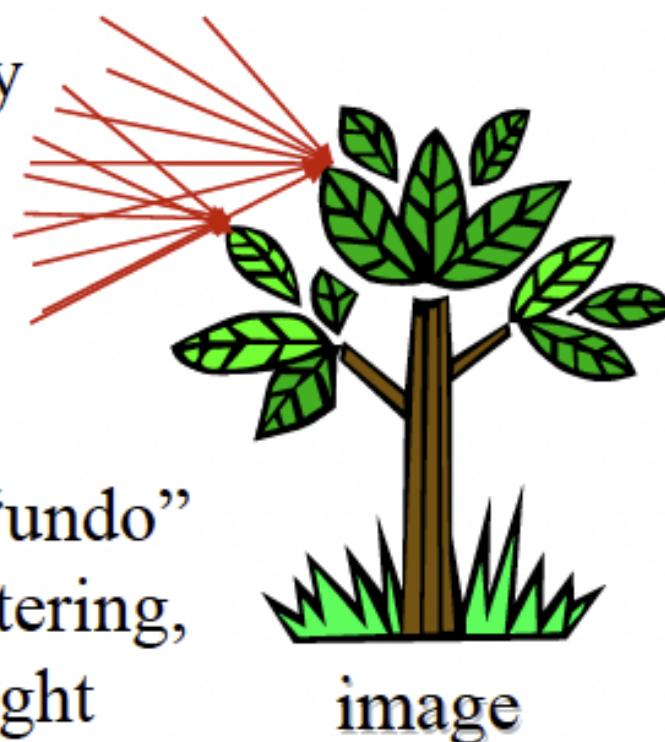


Image

Why are imaging systems needed?



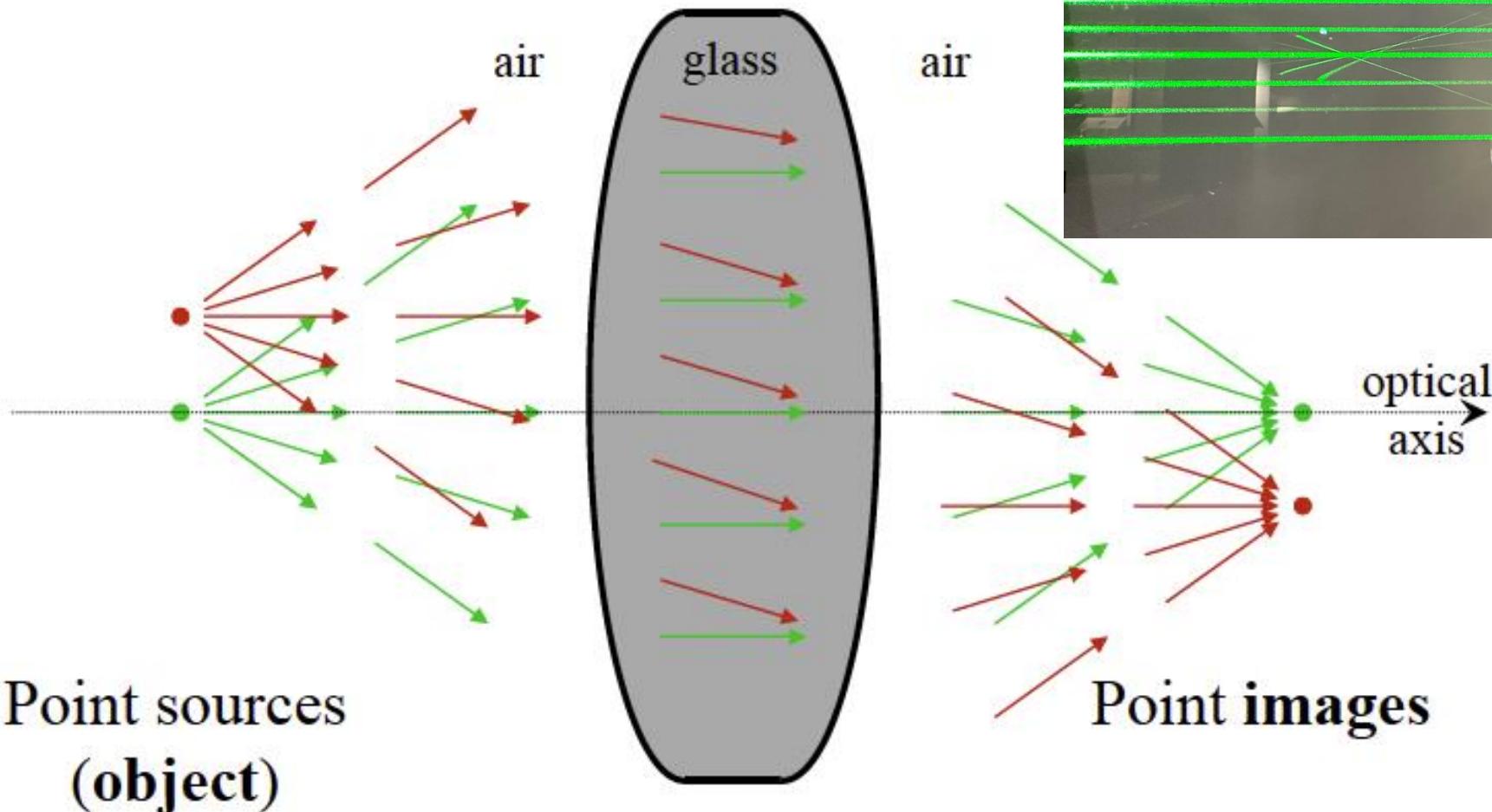
An ideal imaging system maps each point of the object onto a single point in the image



⇒ need optics to “undo”
the effects of scattering,
i.e. *focus* the light

Credit: Lecture notes on Optics by George Barbastathis (MIT 2.71)

Ideal lens



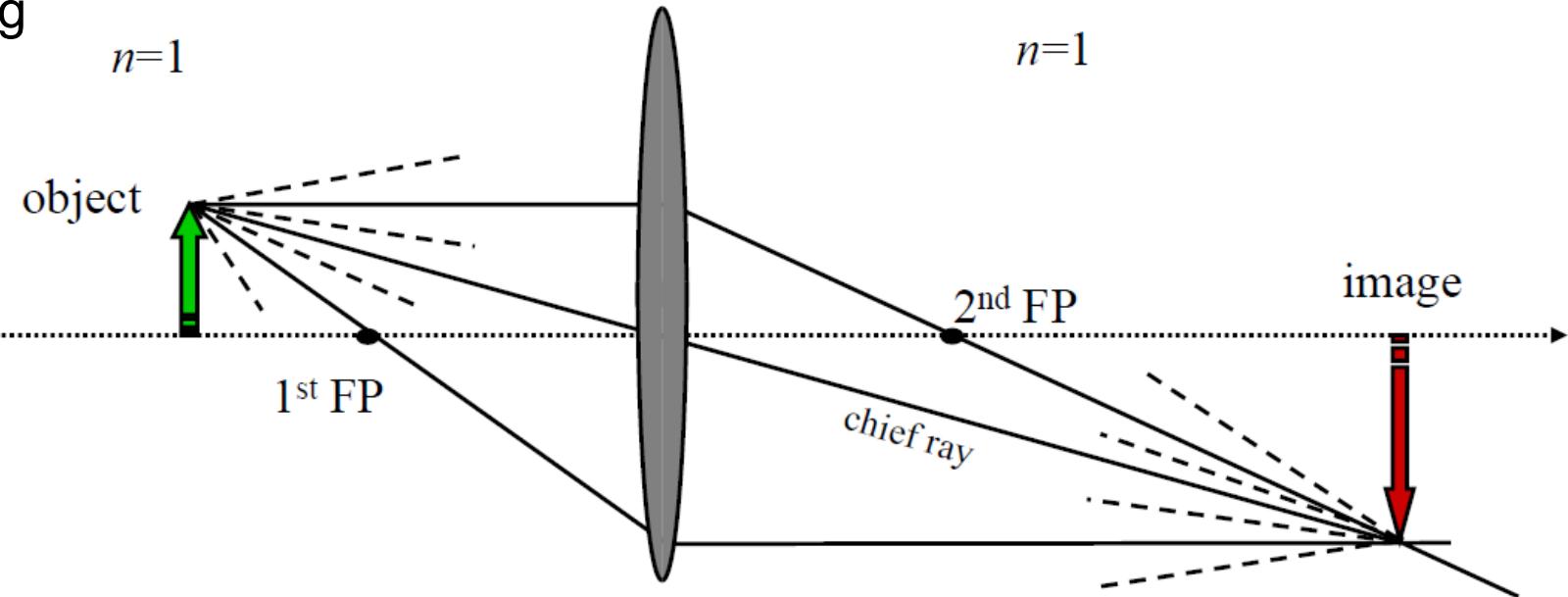
Each point source from the object plane focuses onto a point image at the image plane

Real lenses introduce blur due to aberrations and diffraction

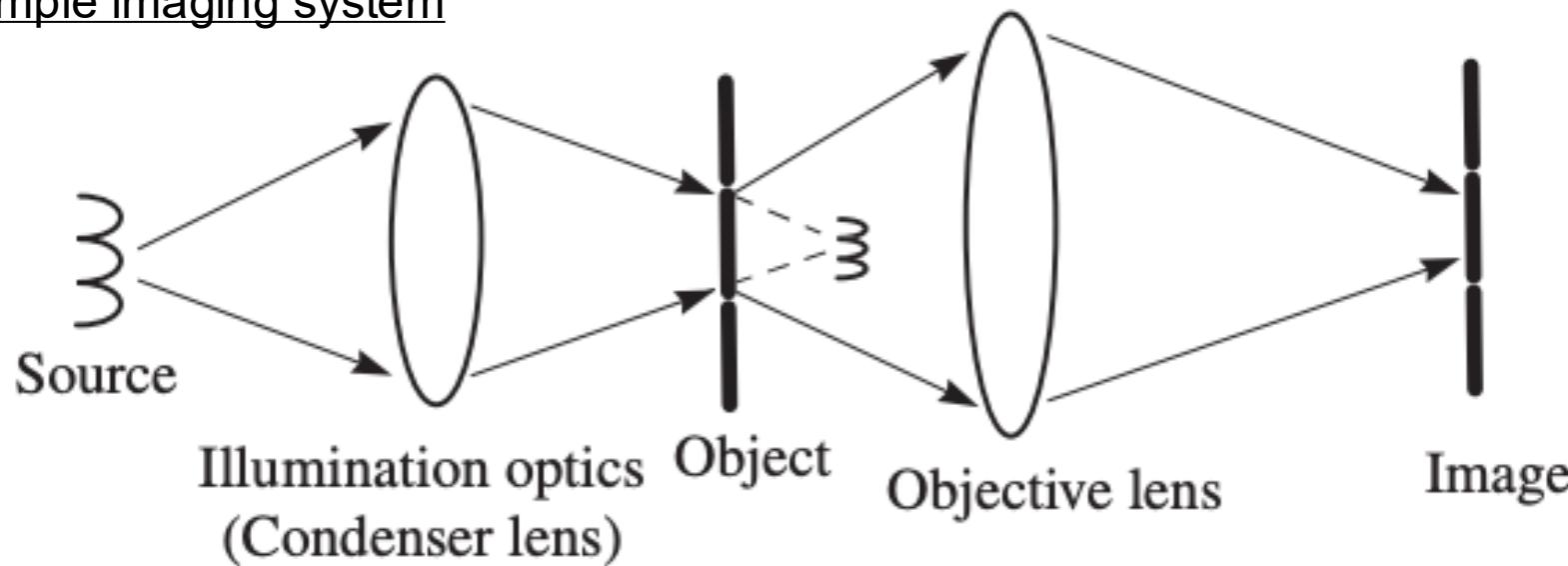
Credit: Lecture notes on Optics by George Barbastathis (MIT 2.71)

Imaging condition using ray-tracing

- Image point is located at the common intersection of all rays emanating from the corresponding object point
- The two rays passing through the two focal points and the chief ray can be ray-traced directly



Simple imaging system

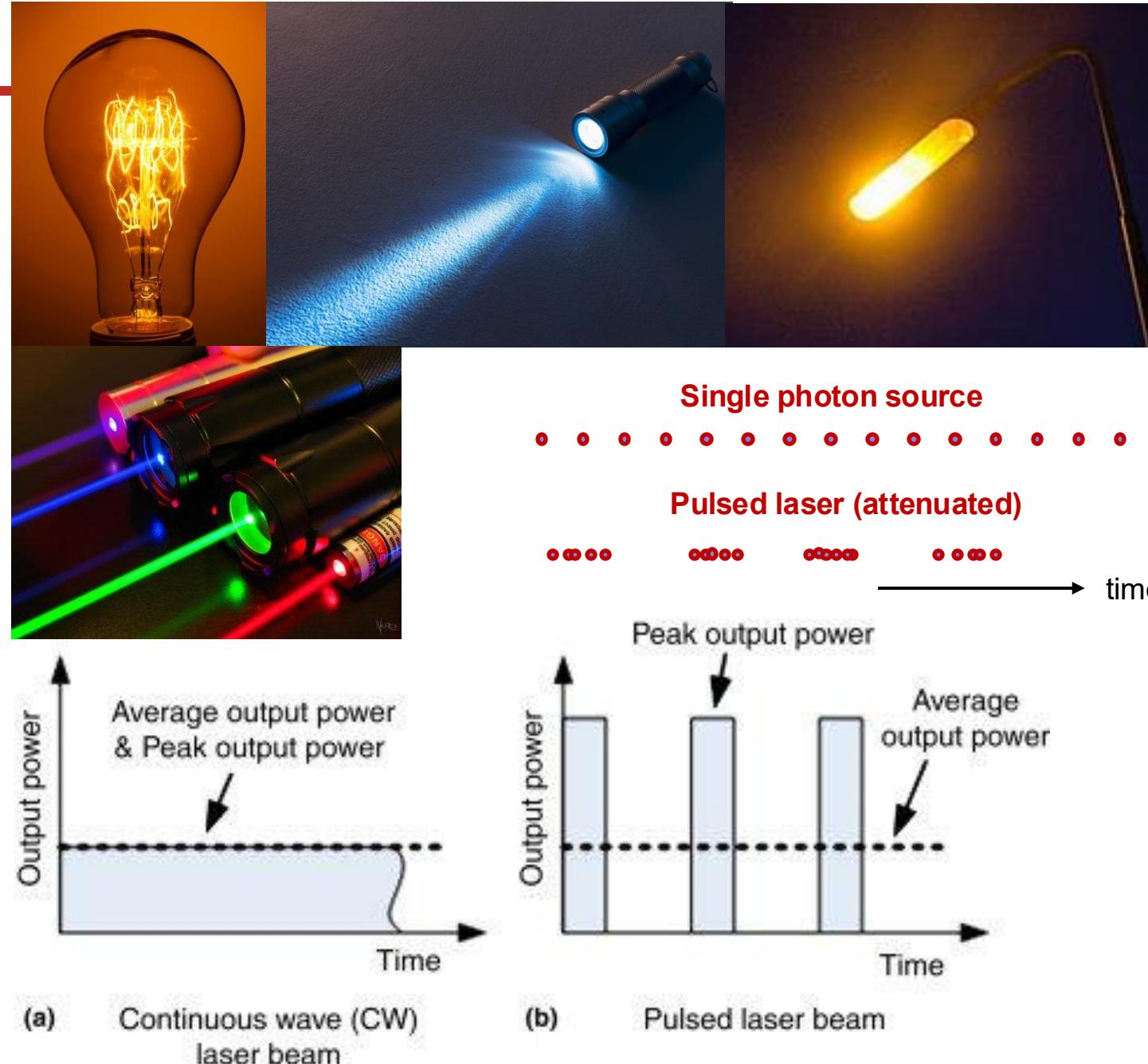


Credits: Lecture notes on Optics by George Barbastathis (MIT 2.71)
Masud Mansuripur

Types of light sources for imaging

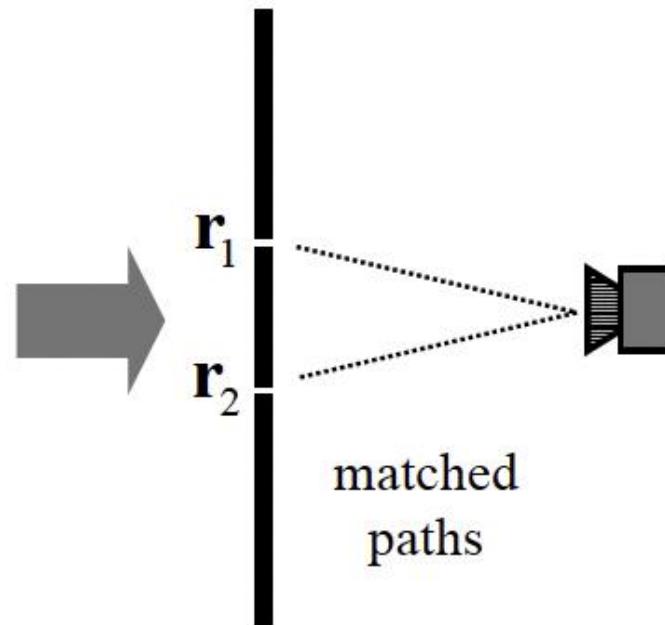
- Thermal sources such as sunlight, light bulb
- Gas discharge lamps
- Light emitting diodes (LEDs)
- Continuous-wave lasers
 - Helium neon (HeNe), argon, diodes
- Pulsed lasers
 - Nd:YAG
- Nonclassical light sources
 - Single-photon sources
 - Squeezed light
 - Entangled photons

How are these light sources categorized?



Spatial and temporal coherence

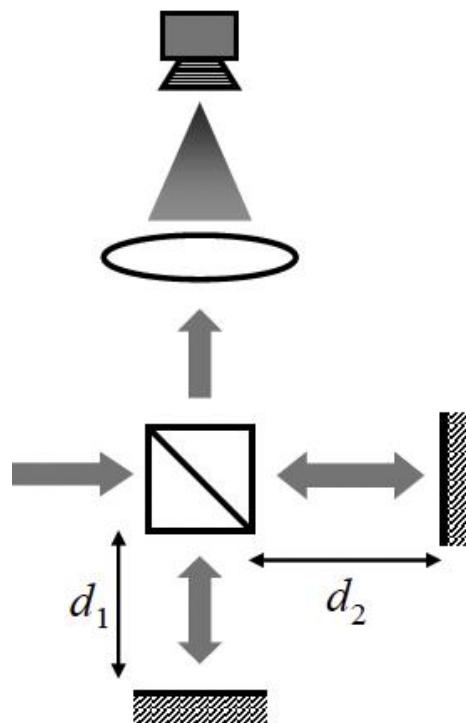
Spatial coherence



Young interferometer

Waves with equal paths but from different points on the wavefront do not interfere

Temporal coherence



Michelson interferometer

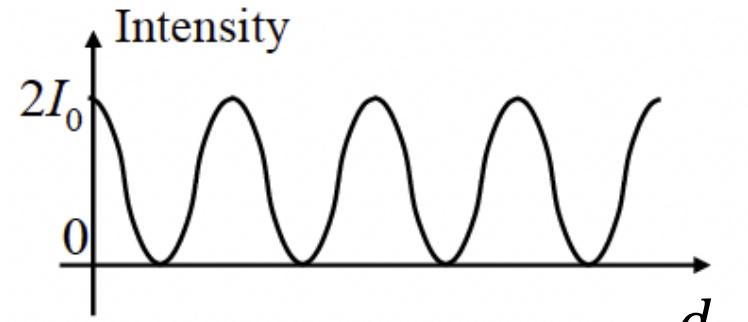
Waves from unequal paths do not interfere

Define coherence length

$$L \equiv \frac{c}{n\Delta\nu} = ct_c$$

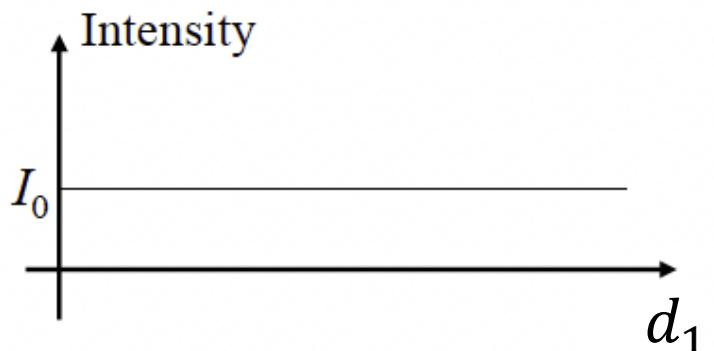
$$d_1 - d_2 \ll L$$

sharp interference fringes



$$d_1 - d_2 \gg L$$

no interference



Let's categorize these classical light sources

- Thermal sources such as sunlight, light bulb

Broadband
large $\Delta\omega$

poor spatial
coherence



- Gas discharge lamps

multiple lines,
each with small
 $\Delta\omega$

poor
spatial
coherence



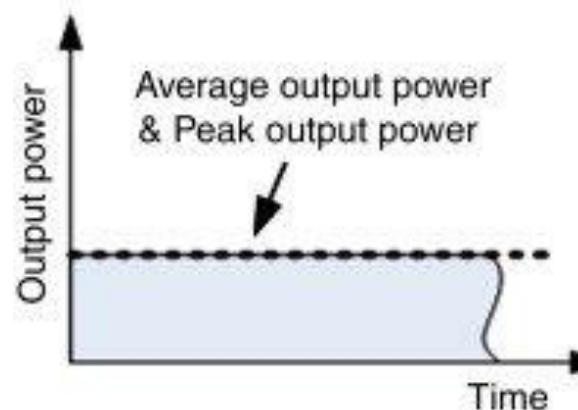
- Light emitting diodes (LEDs)

monochromatic
small $\Delta\omega$

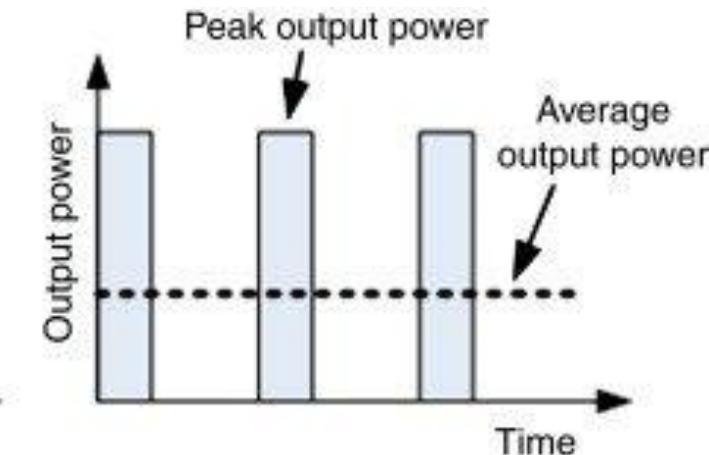
poor
spatial
coherence

- Continuous-wave lasers

good
spatial
coherence



(a) Continuous wave (CW)
laser beam



(b) Pulsed laser beam



- Pulsed lasers

larger $\Delta\omega$

Imaging with coherent and incoherent beams

$$a_1 = |a_1| e^{i\phi_1}$$

$$a_2 = |a_2| e^{i\phi_2}$$

Mutually coherent: superposition field *amplitude* is described by *sum of complex amplitudes*

$$a = a_1 + a_2 = |a_1| e^{i\phi_1} + |a_2| e^{i\phi_2}$$

$$I = |a|^2 = |a_1 + a_2|^2$$

$$\begin{matrix} I_1 \\ I_2 \end{matrix}$$

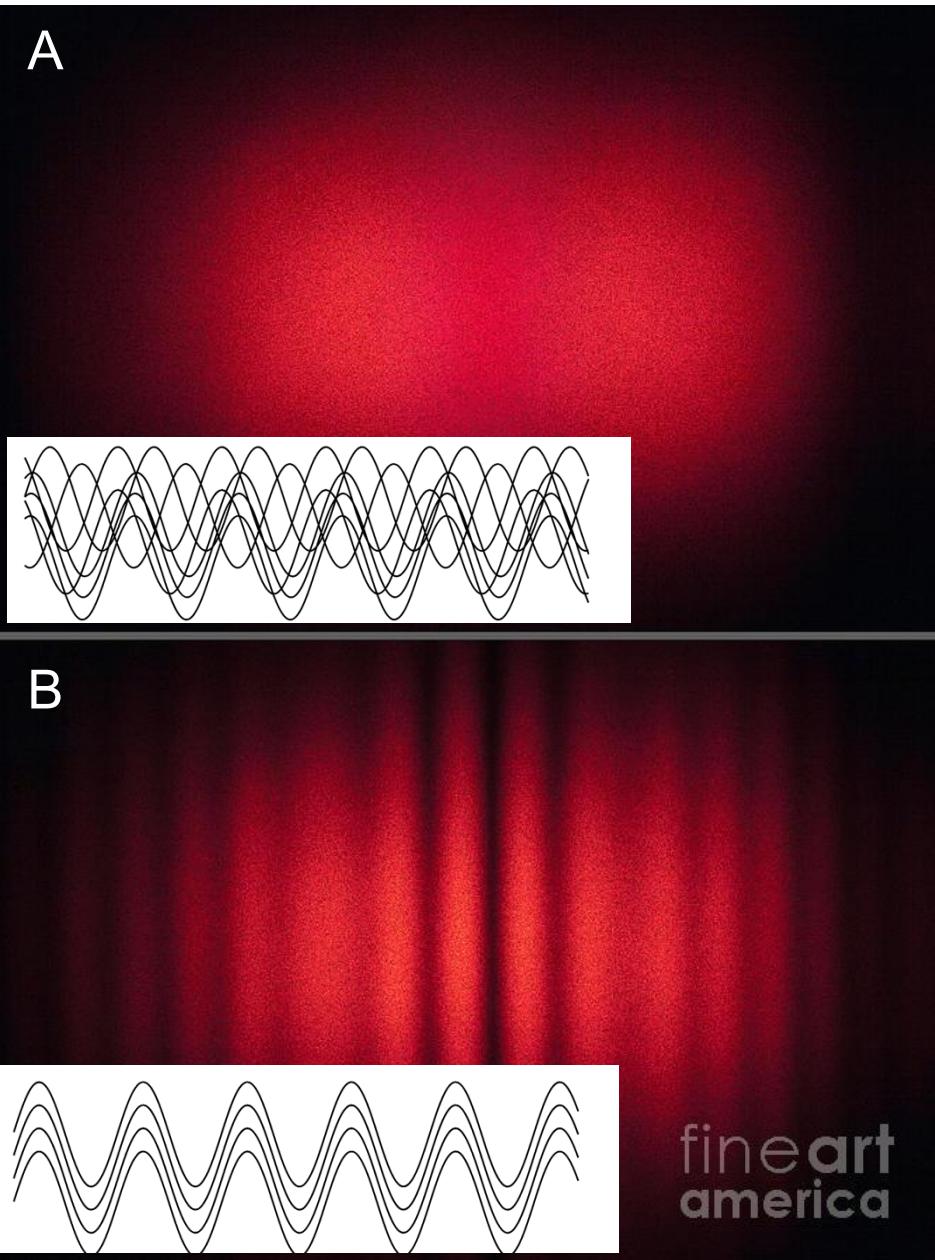
Mutually incoherent: superposition field *intensity* is described by *sum of intensities*

$$I = I_1 + I_2$$

(the phases of the individual beams vary randomly with respect to each other; hence, we would need statistical formulation to describe them properly — *statistical optics*)

From lecture notes on Optics by George Barbastathis (MIT 2.71)

Answer on Top Hat: Is the light source used to generate A more or less coherent than the one for B?



fineart
america

At the Michelson detector the fields from the beamsplitter are being summed:

$$E(t) = E_1(t) + E_2(t)$$

A detector is measuring intensity:

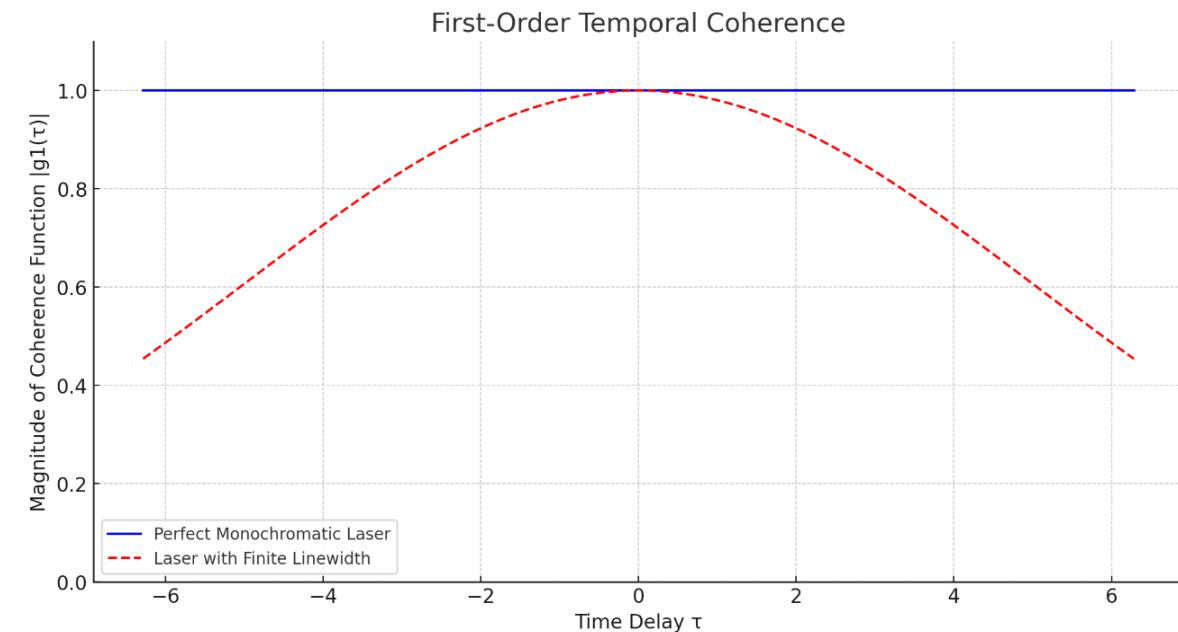
$$I = \langle |E(t)|^2 \rangle$$

If we assume that the light source can be represented by a plane wave: $E_0(t) = ae^{i\omega_0 t}$, then the detector reads: $I \propto I_0(1 + \cos \Delta\phi)$ where $\Delta\phi = \omega_0 \tau = \frac{\omega_0(d_2 - d_1)}{c}$.

As for the first-order correlation function:

$$g^{(1)}(\tau) = \frac{\langle E(t)E(t + \tau) \rangle}{\langle |E(t)|^2 \rangle}$$

$g^{(1)}(\tau) = e^{i\omega_0 \tau}$ for perfectly monochromatic light



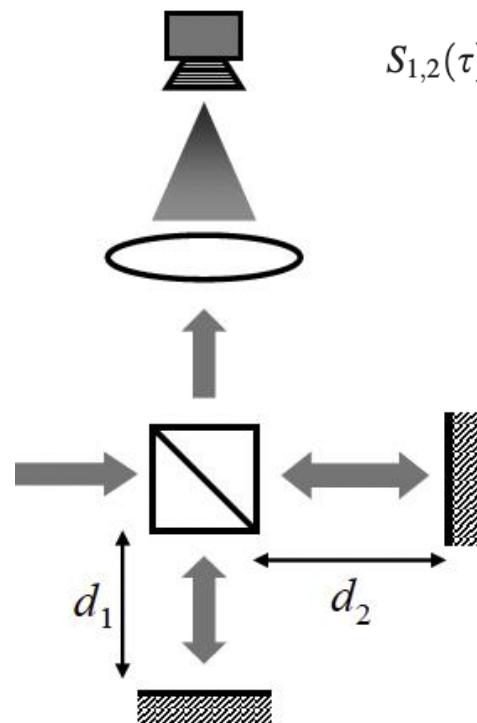
Laser with finite linewidth $\Delta\omega$:

No single way to model this, but one representation is:

$E(t) = Re\{ae^{-\left(\frac{\Delta\omega}{2}\right)^2} e^{i\Delta\omega t}\}$, which represents a wave in which frequencies are distributed in a Gaussian fashion around ω_0 . Note $\tau_c \approx (\Delta\omega)^{-1}$.

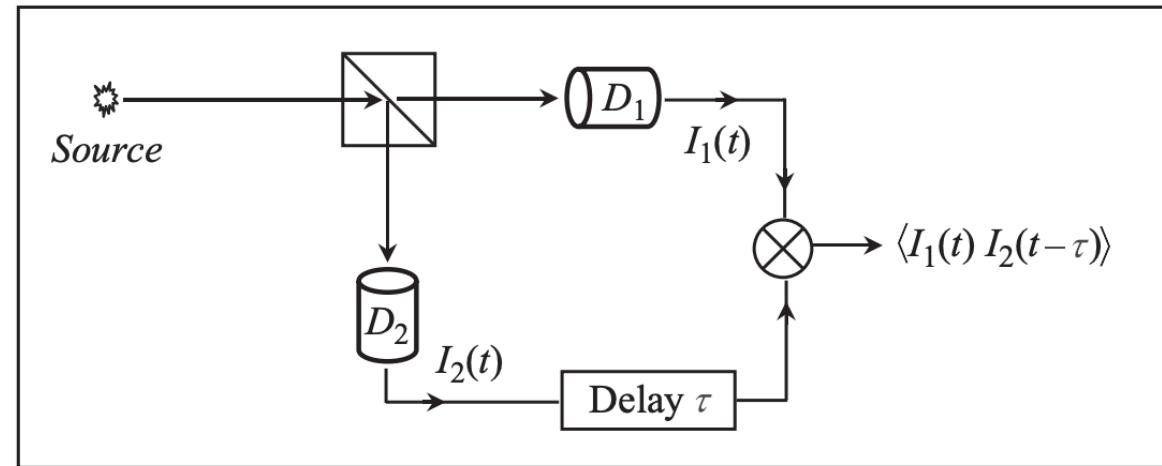
$$g^{(1)}(\tau) = e^{-\left(\frac{\Delta\omega\tau}{2}\right)^2} e^{i\omega_0\tau}$$

$$I \propto I_0(1 + |g^{(1)}(\tau)| \cos \Delta\phi)$$



Michelson interferometer

$$\begin{aligned}
 S_{1,2}(\tau) &= \frac{1}{T} \int_0^T \left\{ \frac{1}{2} [a(t) \pm a(t - \tau)] \right\}^2 dt \\
 &= \frac{1}{2T} \int_0^T a^2(t) dt \pm \frac{1}{2T} \int_0^T a(t) a(t - \tau) dt \\
 &= \frac{1}{2} \left[\langle I \rangle \pm \frac{1}{2} \sum_n A_n^2 \Delta f \cos(2\pi f_n \tau) \right].
 \end{aligned}$$



Hanbury Brown and Twiss experiment

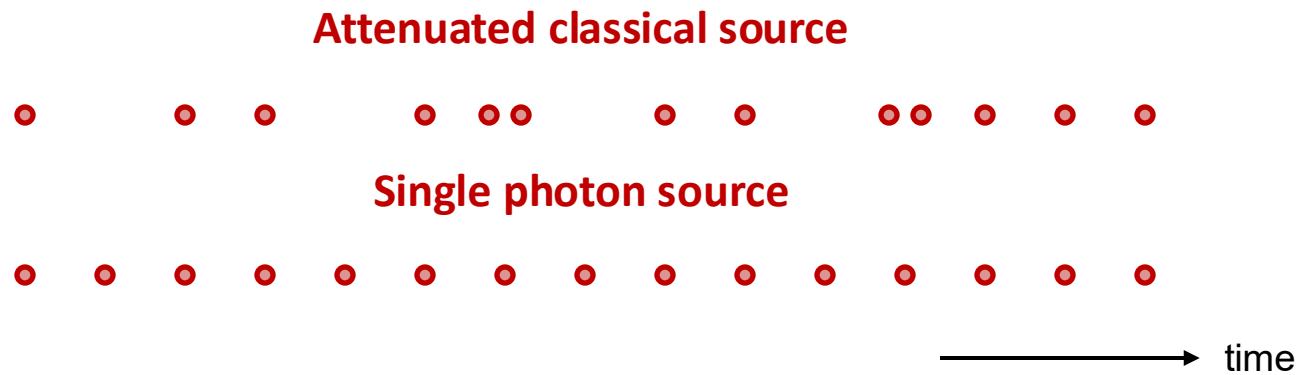
$$\langle I(t) I(t + \tau) \rangle = \frac{1}{T} \int_0^T I(t) I(t + \tau) dt = I_0^2 + \frac{1}{2} \sum_{m=1}^{2M} |\hat{I}_m|^2 \cos(2\pi m \Delta f \tau).$$

Second-order autocorrelation function

$$g^{(1)}(\tau) = \frac{\langle E(t) E(t + \tau) \rangle}{\langle |E(t)|^2 \rangle}$$

$$g^{(2)}(\tau) = \frac{\langle I(t) I(t + \tau) \rangle}{\langle I(t) \rangle^2}$$

Classical light

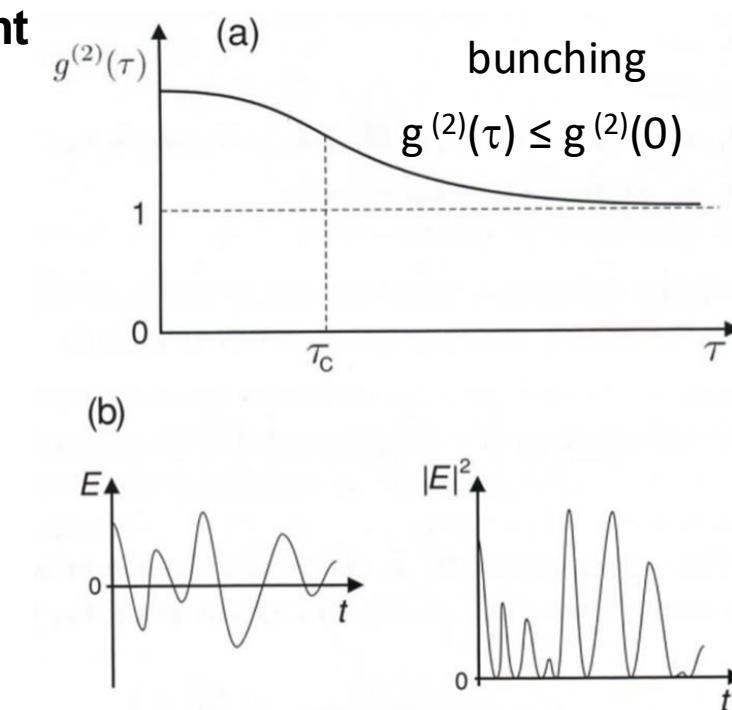


- Low (or zero) probability of emitting more than one photon at a time
- Quantum emitters: atoms (atomic transitions), quantum dots, etc.

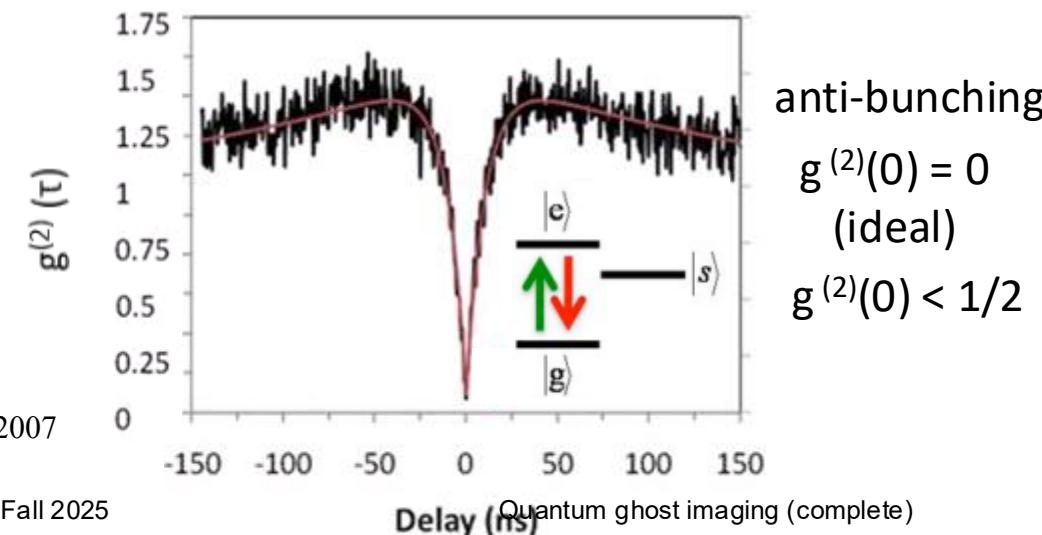
**Figure of merit:
second-order intensity autocorrelation function**

$$g^{(2)}(\tau) = \frac{\langle I(t)I(t + \tau) \rangle}{\langle I(t) \rangle^2}$$

Refs: Novotny and Hecht, "Principles of Nano-Optics," 2007



Single Photon Emission

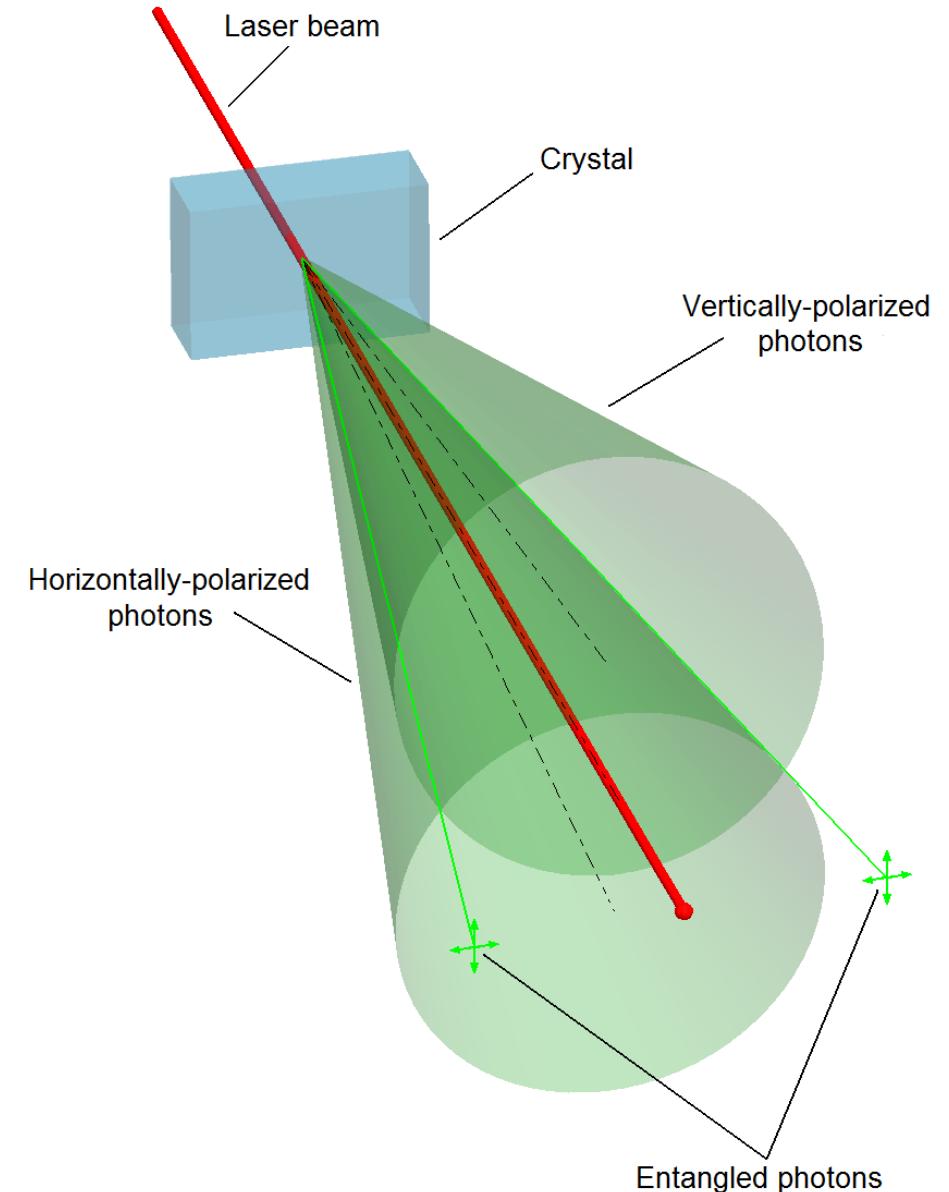


Another type of nonclassical light: entangled photons

- Two or more photons are entangled if they are correlated such that the state of one photon cannot be described without considering the state of the other photon(s), even if they are spatially separated.
- Properties of light that can be entangled
 - Polarization
 - Momentum and position
 - Frequency / energy
 - ...

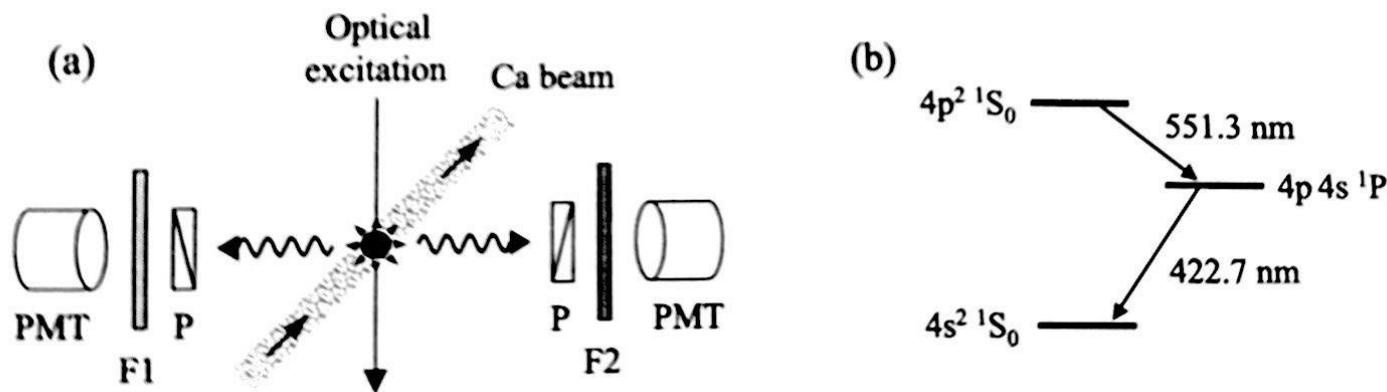
$$\psi_{\pm}^{pos} = \frac{1}{\sqrt{2}} (\psi_{1,0}\psi_{2,0} \pm \psi_{1,1}\psi_{2,1})$$

$$\psi_{\pm}^{neg} = \frac{1}{\sqrt{2}} (\psi_{1,0}\psi_{2,1} \pm \psi_{1,1}\psi_{2,0})$$

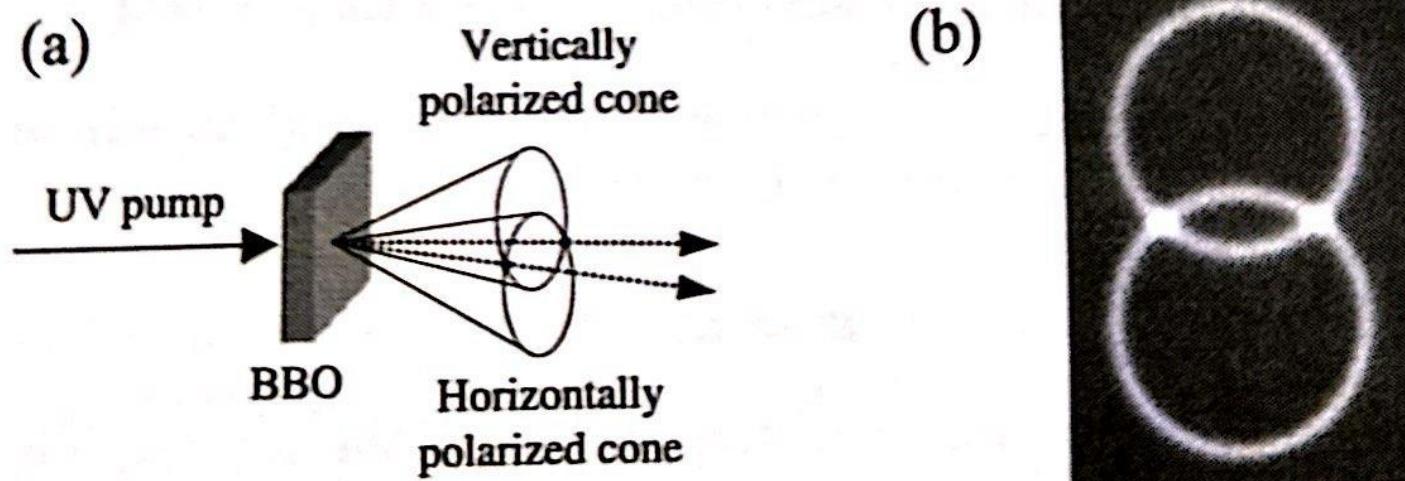


Different ways of getting entangled photons

Using atomic transitions



Using nonlinear optics (shown here is parametric down conversion)



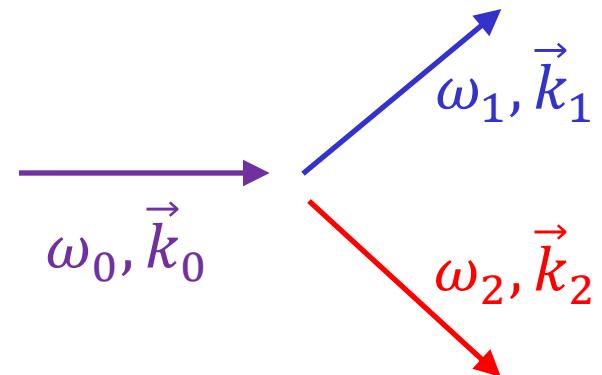
Question: A correlated pair of photons is generated by a nonlinear crystal using a laser at 502 nm. If the wavelength of one of the photons is 820 nm, what is the wavelength of the other photon?

$$\omega_3 = \omega_1 + \omega_2$$

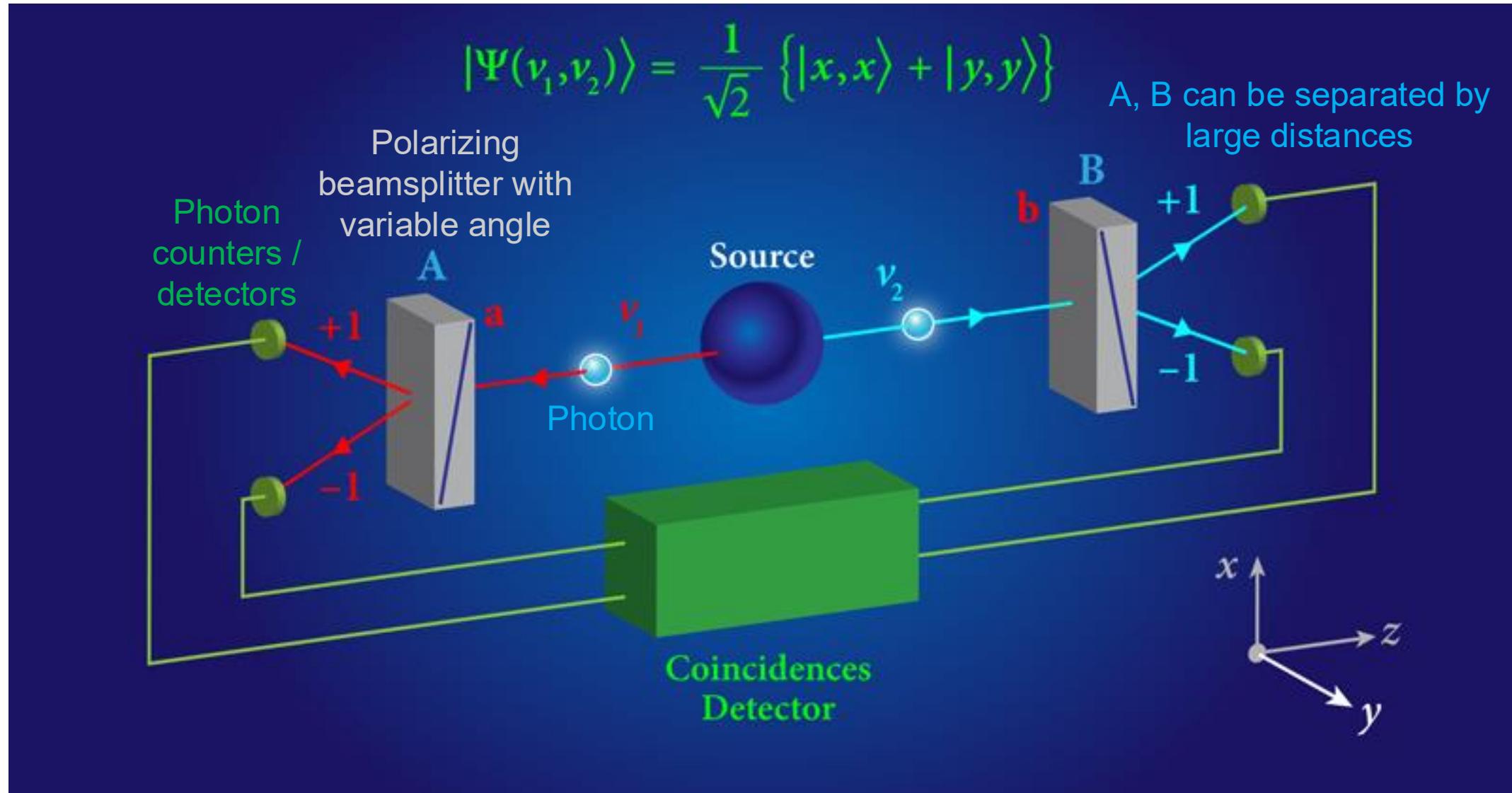
$$\frac{1}{\lambda_3} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}$$

$$\lambda_1 = 820 \text{ nm}; \lambda_3 = 502 \text{ nm}$$

$$\lambda_2 = 1295 \text{ nm}$$



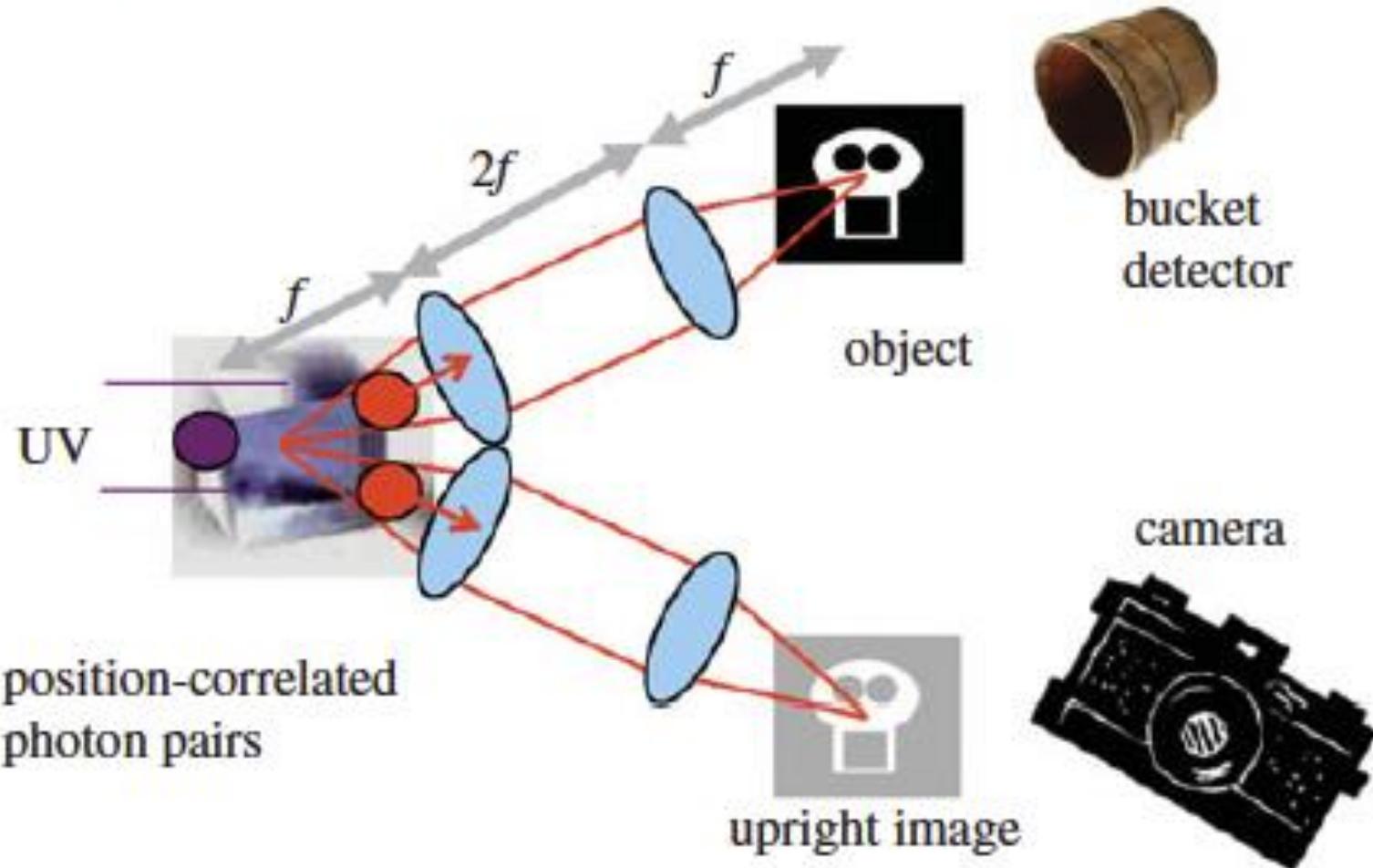
The Bell test is used to evaluate entanglement



Nice review article from Alain Aspect: <https://physics.aps.org/articles/v8/123>

Leveraging correlation of photons in ghost imaging

Concept



Some advantages of ghost imaging:

- Reduced exposure to light-sensitive sample
- Can use one wavelength to interact with sample, and another wavelength for imaging
- Can image through scattering medium (fog, etc)

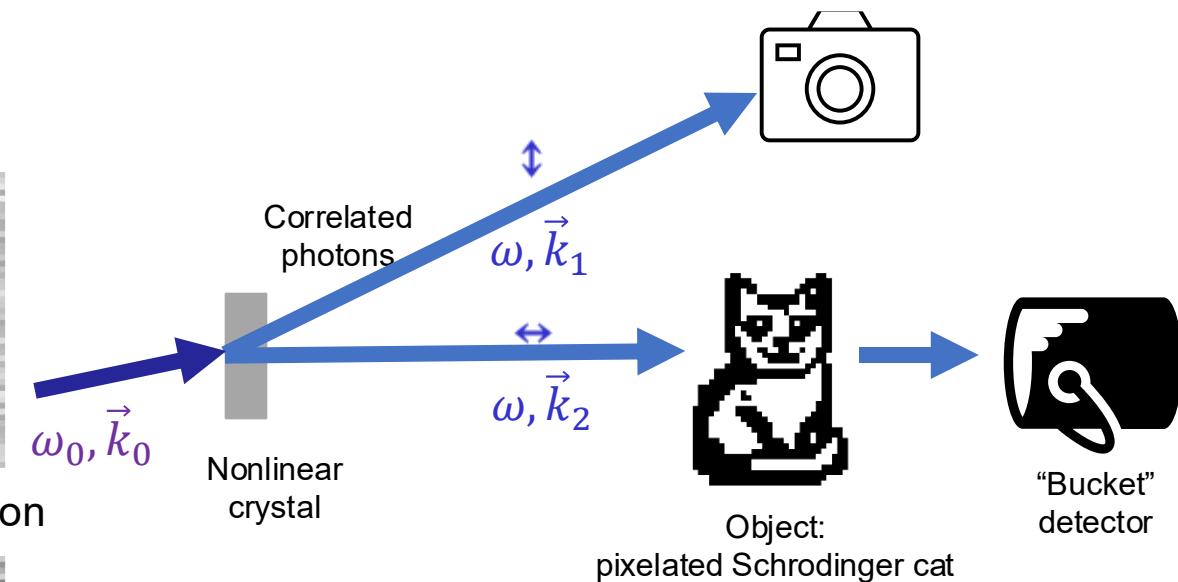
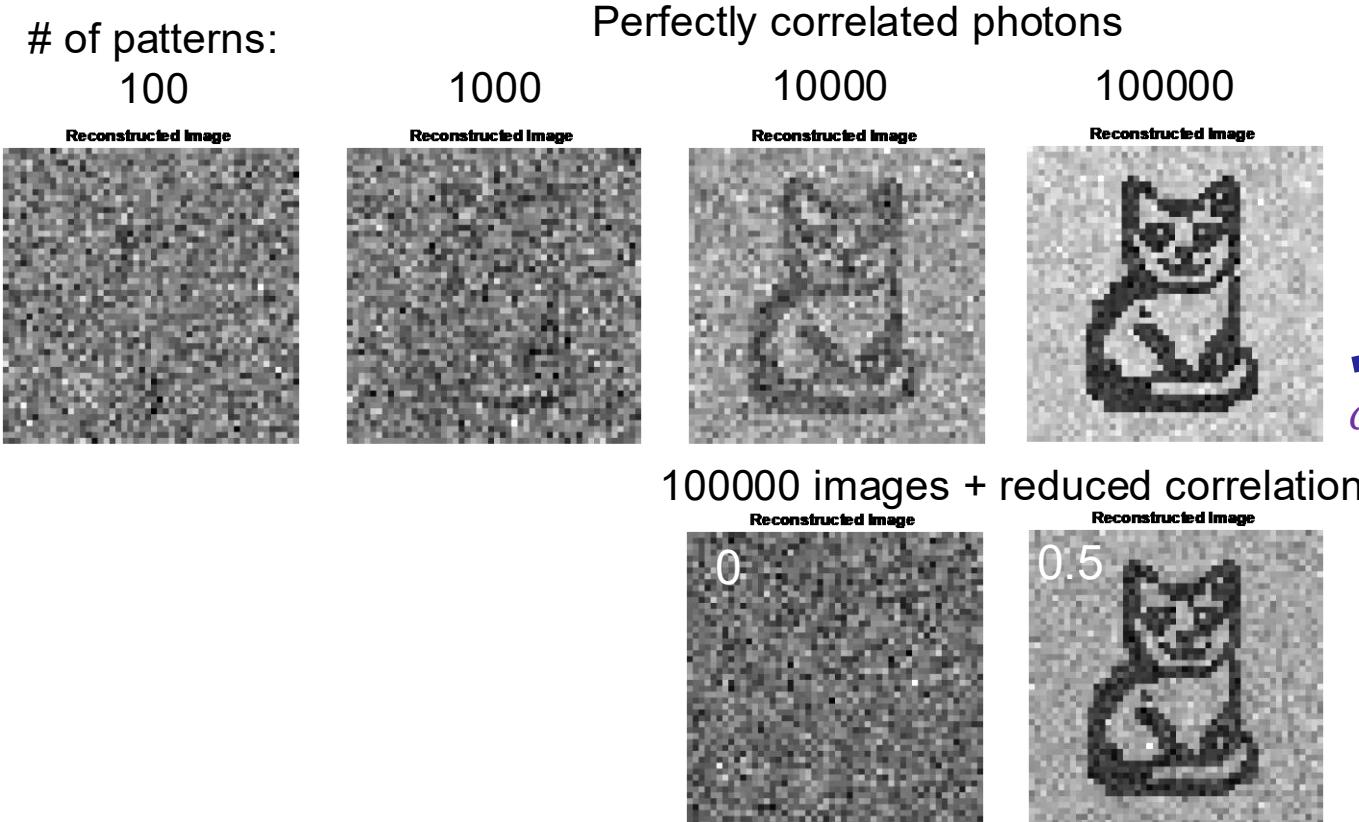
Let's simulate ghost imaging in MATLAB

quantum_ghost_activity.m

Change num_patterns: scales with the number of entangled photons used for the measurement

Change correlationDegree between 0 and 1: how correlated the photons are

What is the image you see?

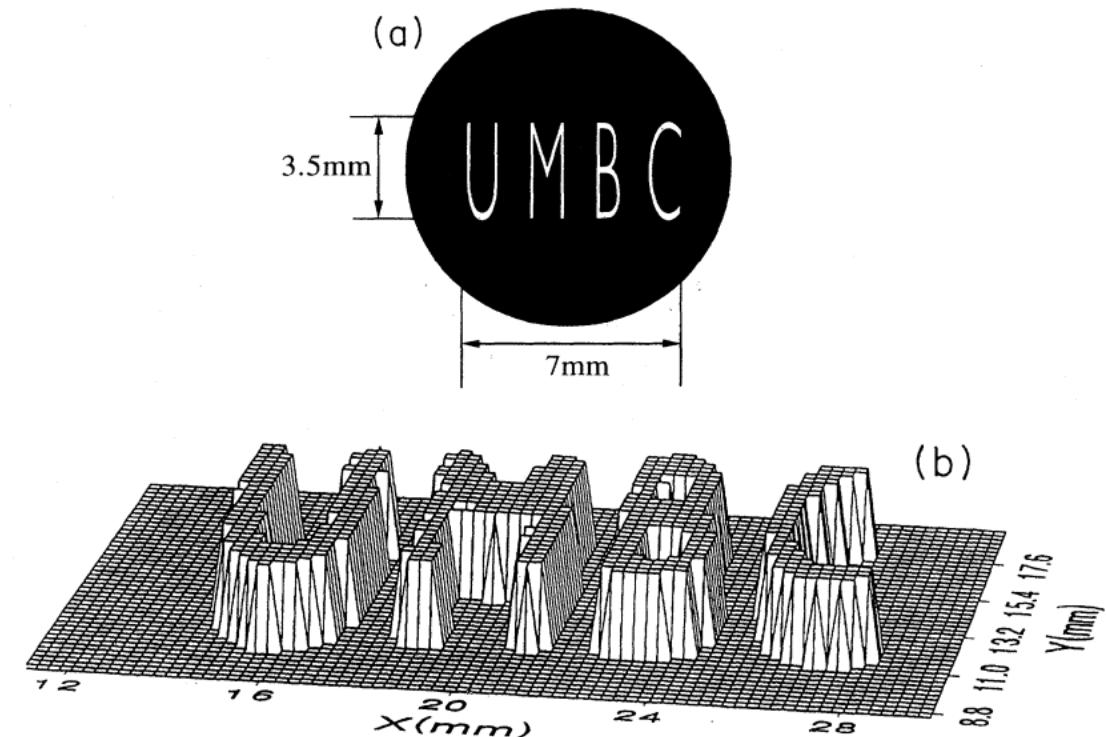
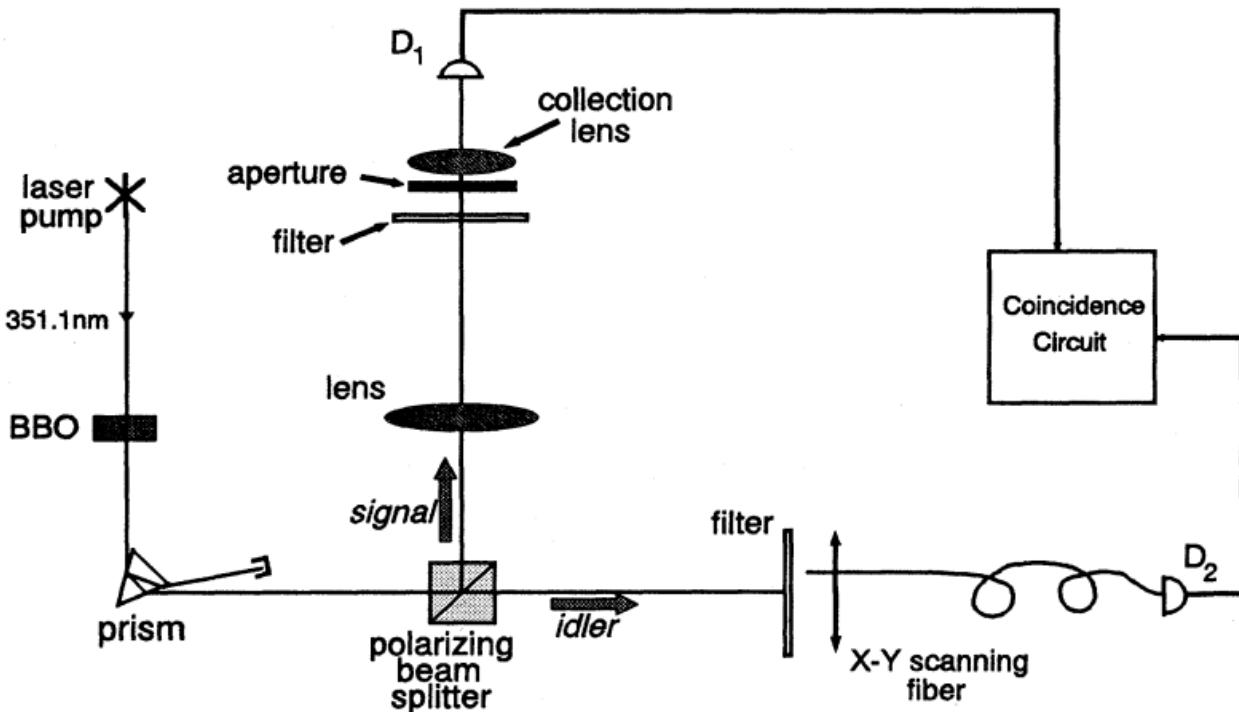


Optical imaging by means of two-photon quantum entanglement

T. B. Pittman, Y. H. Shih, D. V. Strekalov, and A. V. Sergienko

Department of Physics, University of Maryland Baltimore County, Baltimore, Maryland 21228

(Received 22 December 1994)



We don't actually need quantum mechanics for ghost imaging

PHYSICAL REVIEW A 77, 041801(R) (2008)

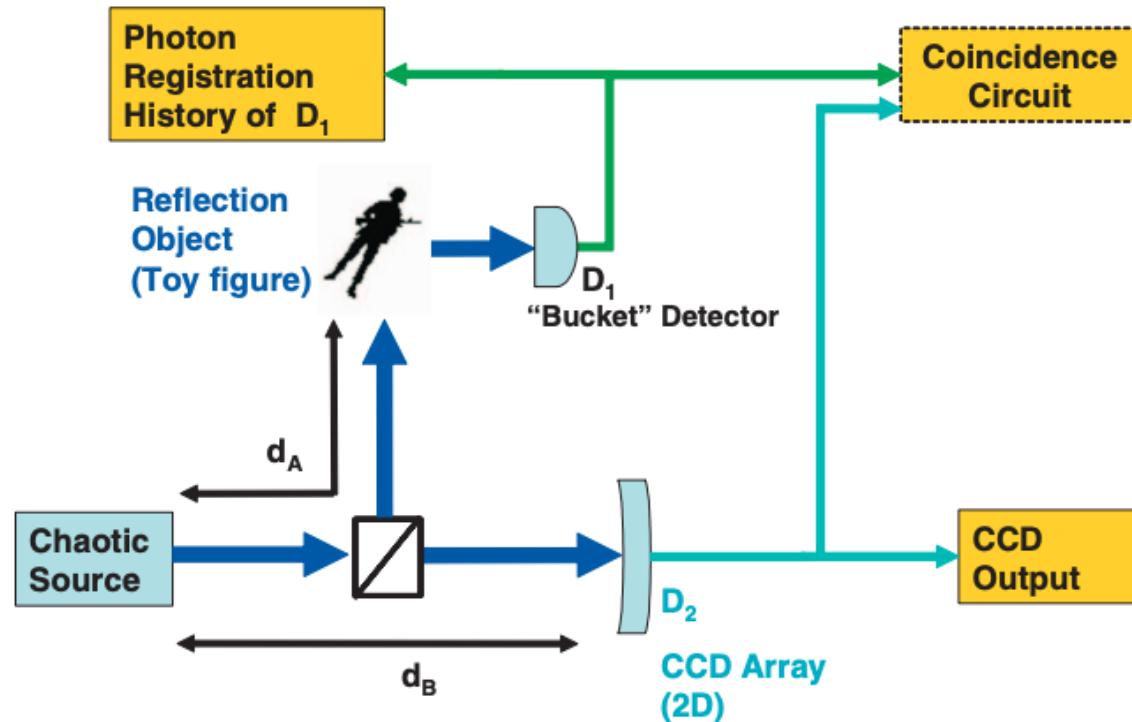
Ghost-imaging experiment by measuring reflected photons

Ron Meyers,¹ Keith S. Deacon,¹ and Yanhua Shih²

¹*U.S. Army Research Laboratory, Adelphi, Maryland 20783, USA*

²*Department of Physics, University of Maryland, Baltimore County, Baltimore, Maryland 21250, USA*

(Received 8 March 2007; revised manuscript received 12 June 2007; published 8 April 2008)



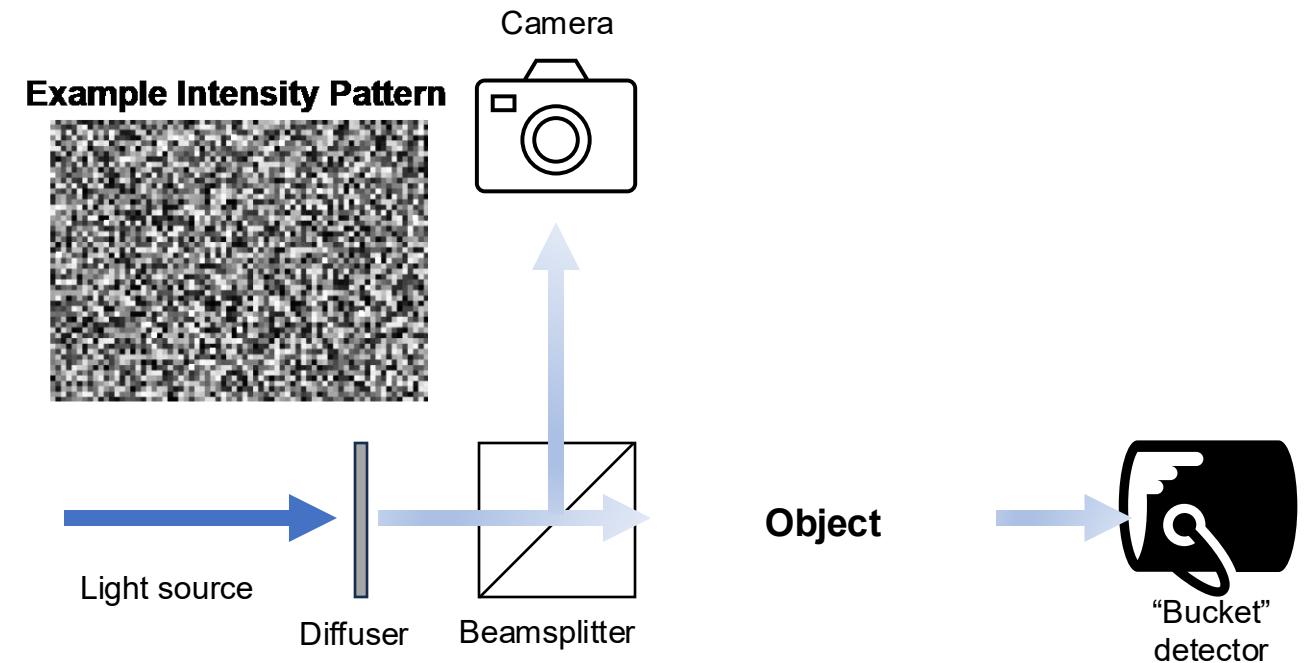
Let's now simulate classical ghost imaging

$$I_{ghost}(x, y) = \sum_{i=1}^N [I_{bucket,i} \times I_{pattern,i}(x, y)]$$

Classical_ghost_activity.m

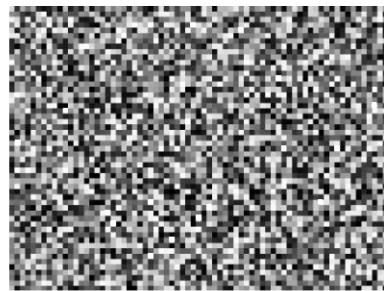
Change num_patterns: number of speckled patterns used for the imaging

What image do you see now?



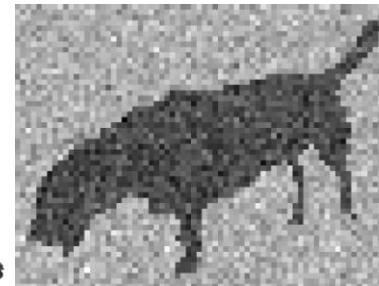
Classical ghost imaging

Example Intensity Pattern

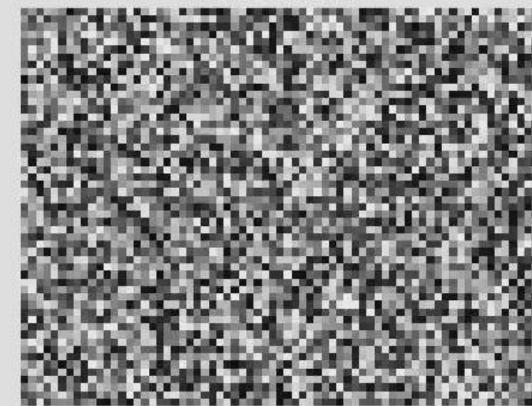


Camera

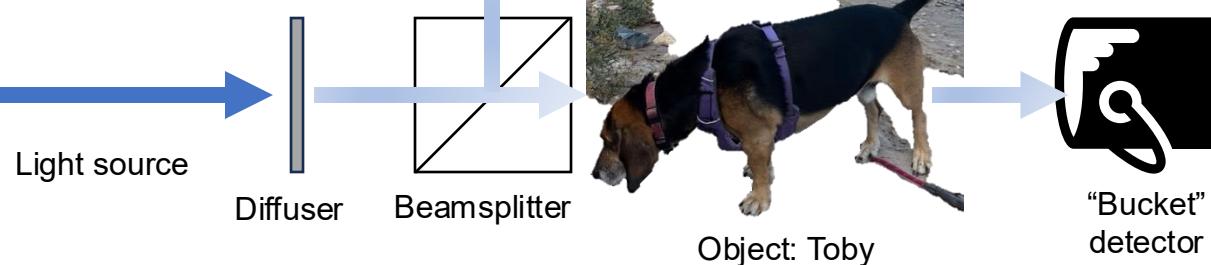
(50000 patterns)
Reconstructed Image



Pattern 1

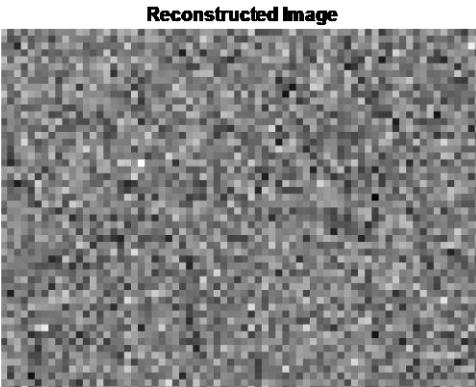


Reconstruction (1 patterns)

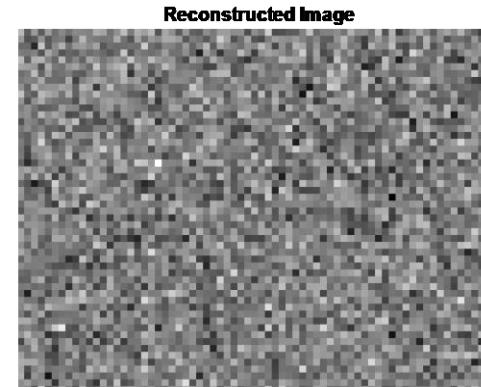


of patterns:

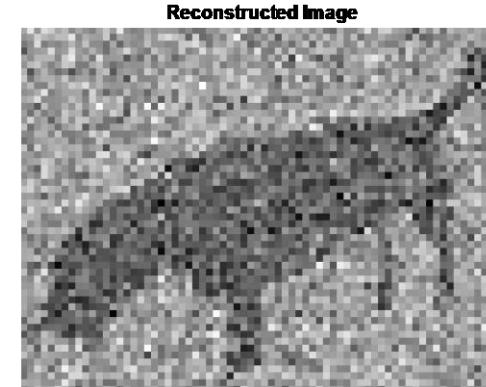
100



1000



10000



100000

