

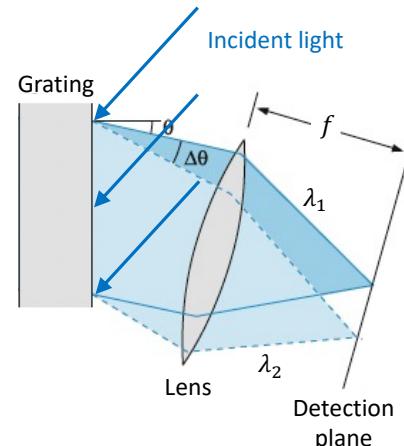
**ECE 535 Fall 2025**  
**Homework #4**  
**Due Friday 11/7 at 11:59 pm on Canvas, in pdf format**

*Guidelines:*

- Please submit a pdf document to Canvas with handwritten solutions, with your approach to each problem and the steps taken clearly laid out and written legibly. In cases where the solution requires plotting, the computer-generated plot should be accompanied by a brief handwritten explanation of your approach. This formatting requirement is worth **5 points** of the point total for each homework.

1. (a) (15 points) Calculate the expectation values  $\langle r \rangle$ , in terms of  $a_0$ , for the 1s, 2s, and 3p states in a singly ionized helium atom ( $\text{He}^+$ , with  $Z = 2$ ) and compare these results with expectation from the Bohr model, where  $r_n = n^2 a_0 / Z$ . Please show your work, but you may use a software to solve this problem.  
(b) (5 points) A  $\text{He}^+$  ion is excited from the ground state with kinetic energy 48.36 eV. Using the Bohr model where  $E_n = -13.6 \text{ eV} \cdot Z^2/n^2$ , what is the change in the radius of the electron? Compare this value to the change in  $\langle r \rangle$  based on your result in (a).
2. (15 points) A diffraction grating has 1000 lines per centimeter and an overall width of 1 cm.
  - (4 points) When monochromatic light of wavelength 589 nm is incident on the grating at  $60^\circ$ , calculate the angles at which the first and second-order maxima occur.
  - (3 points) Estimate the *resolving power* of this grating for the  $m$ -th-order diffraction. Resolving power here is defined as the incident wavelength ( $\lambda$ ) divided by the smallest wavelength separation that can be observed ( $\Delta\lambda_{min}$ ):  $\lambda/\Delta\lambda_{min}$ . Discuss how increasing the number of lines would affect its resolving power.
  - (5 points) The grating is illuminated with emission from sodium at an incident angle of  $60^\circ$ , after which a lens is used to collect and focus the diffracted light onto a detection plane. What is the distance between first-order diffracted lines in sodium (at 589 nm and 589.6 nm) on the detection plane, if the focal length of the lens is 1 m?
  - (3 points) Is the scenario described in (c) sufficient to clearly observe the spectral lines? If not, recommend another way to resolve the lines using this grating.

3. (10 points) A grating spectrometer is used to study the emission spectrum of an element, which has two closely spaced emission lines at 502.1 nm and 502.3 nm. The spectrometer has a grating with 1500 grooves per mm and the grating has a width of 2 cm. The linear dispersion of the spectrometer is 4 mm/nm. The light first passes through an entrance slit before striking the grating. In this problem you will calculate and compare the limits to the spectral resolution of the spectrometer based on the finite size of the grating and the width of the entrance slit.
  - (3 points) The resolution based on the number of grooves illuminated ( $N$ ) in a grating is determined by  $\Delta\lambda_{min} = \lambda/(mN)$ , where  $m$  is the diffraction order. Estimate the resolution



limits due to the grating in this case as a function of  $m$ , by assuming that light is illuminating the entire width of the grating after entering the spectrometer.

- (b) (3 points) Now you are adjusting the entrance slit width to optimize resolution without significantly compromising the intensity of the detected light. The minimum slit width for which you are getting detectable light is 0.3 mm. What is the resolution here based on just the entrance slit width?
- (c) (2 points) Compare the resolutions in parts (a) and (b). Assuming first-order diffraction, which is the limiting factor in the resolution of the spectrometer?
- (d) (2 points) If the incidence angle is  $20^\circ$ , what is the diffraction angle for the order that allows the grating spectrometer to resolve the two spectral lines?
4. (15 points) A Michelson interferometer is implemented to study gravitational waves.
- (a) (3 points) Draw the Michelson interferometer and label the relevant components.
- (b) (6 points) Derive the expression for the intensity  $I$  of the combined beams in the interferometer as a function of the intensity of the initial light source  $I_0$  and the phase difference  $\Delta\phi$  between the two arms of the interferometer. It may be helpful to first represent the electric field of the first beam as  $E_0 \cos(\omega t)$  and that of second beam as  $E_0 \cos(\omega + \Delta\phi)$ .
- (c) (6 points) In order to measure gravitational waves, the output intensity should not fluctuate by more than  $10^{-8} I_0$  due to instability in the interferometer. Determine the required phase stability (in radians) of the interferometer to ensure this condition is met.
5. (10 points) Two lasers, a bragg reflector laser and a ring cavity laser, both emit light at a central wavelength of 780 nm. The bragg reflector lasers have a linewidth of 1 GHz, while the ring cavity laser has a linewidth of 1 kHz. A Michelson interferometer is set up with one arm that can be varied in length.
- (a) (5 points) For each laser, determine the maximum path difference that can be introduced between the two arms of the interferometer before the interference pattern is lost.
- (b) (5 points) Suppose you increase the path length difference in the interferometer gradually from zero: how many bright fringes will you be able to count for each source from the starting point until the visibility is significantly lost?
6. (10 points) In this problem, we will determine how the signal-to-noise ratio of the image is affected by the number patterns used for the image reconstruction in a classical ghost imaging problem.
- (a) (7 points) Use the code provided in class (classical\_ghost\_activity.m) and make the following modifications:
- Instead of using an imaging object from an image file, generate a simple circular mask with the following commands in MATLAB:

```
% Create a circular aperture as our object
object_img = zeros(width, height);
[cx, cy] = meshgrid(1:width, 1:height);
radius = 10;
mask = (cx - width/2).^2 + (cy - height/2).^2 <= radius^2;
object_img(mask) = 1;
```

- Define region of interest (ROI) within the circular aperture (signal) and calculate the average pixel intensity within the ROI as a function of the number of patterns used for the image reconstruction. *Plot the average pixel intensity as a function of the number of patterns.*
  - Define another region of interest outside of the aperture (background) and calculate the standard deviation of the intensities within the ROI as a function of the number of patterns. *Plot the standard deviation as function of the number of patterns.*
  - *Include a representative reconstructed image and indicate where the ROIs are.*
- (b) (3 points) Define the signal to noise (SNR) as the average pixel intensity within the signal ROI divided by the standard deviation in the background ROI. *Plot the SNR as a function of the number of patterns.* Explain the trend of the data.
7. No extra problem this time for graduate students.