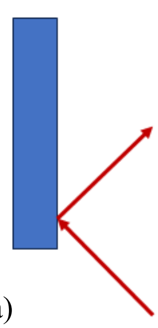
ECE 535 Fall 2025
Homework #2
Due Thursday 10/2 at 11:59 pm on Canvas, in pdf format
Note: Solutions will be posted by Friday night ahead of Exam #1 on 10/6 so please submit on time.

Guidelines:

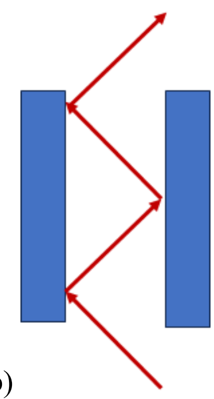
- Please submit a pdf document to Canvas with handwritten solutions, with your approach to each problem and the steps taken clearly laid out and written legibly. In cases where the solution requires plotting, the computer-generated plot should be accompanied by a brief handwritten explanation of your approach. This formatting requirement is worth **5 points** of the point total for each homework.
- Undergraduate students who wish to attempt the extra problem will receive up to 5 additional points for that homework.

1. (15 points) **Photon reflection off a mirror**
   (a) (5 points) A single photon at a wavelength of 1000 nm is incident on a perfect mirror at an angle of 45 degrees, as in the figure below. The mirror starts out at rest and has mass of 1 microgram. After the photon is reflected, what is the direction and speed of the mirror?
   (b) (10 points) Now instead of a single mirror, you have two identical mirrors, each with mass of 1 microgram. The same photon is incident on one of the mirrors at the same angle, but the orientation is such that the photon bounces of the left mirror, then off the right one, and then off the left one again, as in the figure below. After the photon makes all of its reflections, what is the direction and speed of each of the two mirrors?

   (Hint: For both parts, you may assume that the mirror's motion can be described as non-relativistic. This will simplify your conservation formulas for momentum and energy, but *please state any simplifications and justify them*).



(a)



(b)

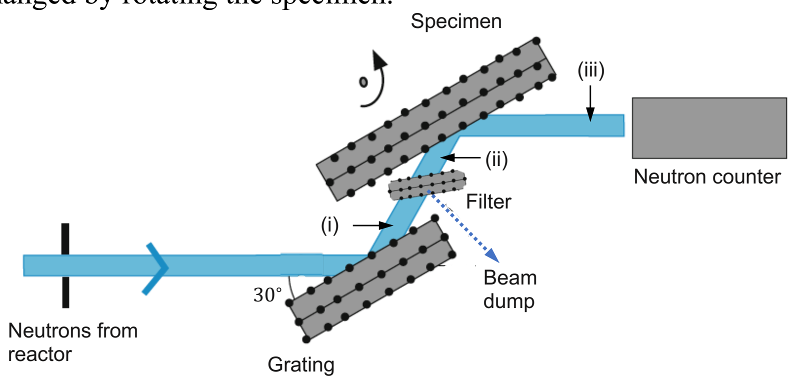
2. (15 points) **Angular momentum transfer by photons**
   (a) (5 points) Left-handed circularly polarized (σ^+) light ~~polarized along the $\hat{y}$ direction~~ propagates along the $\hat{z}$ direction and reflects off a perfect mirror at $z = 0$. What is the electric field and polarization of the reflected light?

   (b) (10 points) A right-handed circularly polarized photon at $\lambda = 632\ nm$ is normally incident on a perfect mirror which is free to rotate. Calculate the change in angular momentum for both the photon and mirror upon reflection. Justify your answer.

3. (15 points) **Neutron spectrometer**
   Neutrons from a nuclear reactor are collimated through a slit before entering the spectrometer setup shown below, which involves neutron diffraction events at three diffracting elements

(grating, filter, and the specimen). In this question, you will consider the effect of each diffracting element, which can be assumed to consist of a different crystal with only one diffracting plane whose spacing is specified by the lattice spacing of the material:

- The grating crystal has a lattice spacing of 5.4 angstrom and is used to select neutron velocities based on the incident angle (30°). The grating-diffracted beam is indicated by (i) in the diagram.
- The filter crystal lets the first-order neutron velocity component pass through and diffracts off higher-order components, resulting in a monochromatic beam in (ii).
- The monochromatic neutrons in (ii) are used to diffract off of the specimen, with the diffracted beam at (iii) measured by a neutron counter. The angle of incidence can be changed by rotating the specimen.



(a) (5 points) Which de Broglie wavelengths can be found in the beam at (i)? What are the corresponding neutron velocities?

(b) (4 points) Assume that there are only two velocity components in (i). Now, you are asked to design a neutron filter which eliminates the higher-order velocity component. You have limited material choices to construct this filter: pyrolitic graphite (lattice constant = 2.46 angstroms) and copper (lattice constant = 3.61 angstroms). Which material would you pick to construct the filter? Justify your response. Please assume here that as long as the diffraction condition is satisfied, 100% of the velocity component will be diffracted. Additionally, any un-diffracted components will pass through the filter without being scattered or absorbed.

(c) (4 points) At what angle should the filter, made of the material you selected in (b), be oriented relative to the incident beam (i)?

(d) (2 points) Describe a measurement procedure to determine the lattice spacing of the specimen. Include a discussion on any limitations of the technique.

4. (15 points) **Determination of the fine structure constant**

(a) (6 points) Suppose we are able to experimentally determine the following in a gamma-emitting atom:

i. The mass difference ΔM of two nuclear energy levels in terms of atomic mass units (amu), which is related to the mass in kilograms kg by the Avogadro's number N_A: $mass\ [amu] = mass\ [kg]\ 1000 N_A$

ii. The wavelength λ (in meters) of the gamma ray emitted when transitioning between the nuclear energy levels.

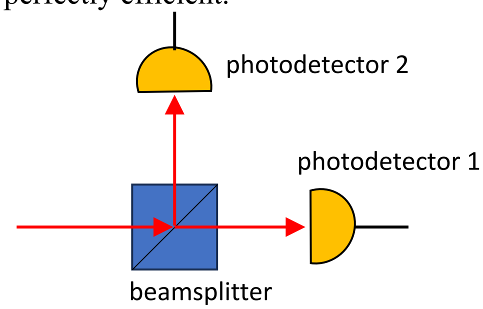
By relating the energy of the gamma photon with the mass difference of the atom in grams, show that measurements of $\Delta M\ [amu]$ and $\lambda\ [m]$ can give us the product of two fundamental constants $N_A \hbar$.

(b) (6 points) $N_A\hbar$ is known as the molar Planck constant and can be used to infer the fine constant α. Write α in terms of $N_A\hbar$, the Rydberg constant R_∞, and the electron mass in atomic mass unit M_e.

(c) (3 points) Use the data in https://physics.nist.gov/cuu/Constants/index.html to estimate the uncertainty with which we can determine α.

5. (10 points) **Quantum projection noise and photon shot noise**

A stream of single photons is being generated at a rate of N (in units of number of photons per second) and sent through a beamsplitter. The probability of each photon taking either path after the beamsplitter is $p = 0.5$. The photons in each path are detected by a photodetector, which can be assumed to be perfectly efficient.



(a) (6 points) Based on information provided through the lecture notes, calculate and plot the uncertainties in the photon detection rate δn at photodetector 1 due to the quantum projection noise and photon shot noise, as a function N (spanning 1 to $10^4\ s^{-1}$). For this problem, consider the two noise sources separately.

(b) (3 points) Now plot $\delta n/\langle n\rangle$ vs N, where $\langle n\rangle$ is the mean photon count rate at photodetector 1. Come up with analytical expressions for $\delta n/\langle n\rangle$ for each of the quantum projection noise and photon shot noise based on your results.

(c) (1 point) Discuss what happens to $\delta n/\langle n\rangle$ if the photodetectors are only 75% efficient.

6. (10 points) **Extra question for graduate students: what is the Doppler shift of a reflected photon?**

(a) (7 points) A photon with frequency ν_0 is normally incident on a perfectly reflective mirror at rest. The mirror, which has mass m, moves at a velocity of v, but the motion of the mirror can be described as relativistic. Using the conservation of energy and momentum, find an expression relating ν_0 to the frequency of the reflected photon (ν'). If you cannot find a closed form solution for ν', simplify the expression as much as possible.

(b) (3 points) The frequency shift you solved for in (a) is the Doppler shift of a reflected photon off a moving mirror. Using relativity, the Doppler shift for a source moving away from the observer is calculated as

$$\nu' = \sqrt{\frac{c - v}{c + v}}\nu_0$$

Compare your answer in (a) with the above formula and justify why the expressions may be different. For your comparison, feel free to use a numerical example (say for $\nu_0 =$ 500 THz, $m = 10^{-35}\ kg$).