

# ECE 535: Introduction to Quantum Sensing

## Special Halloween lecture: quantum ghost imaging

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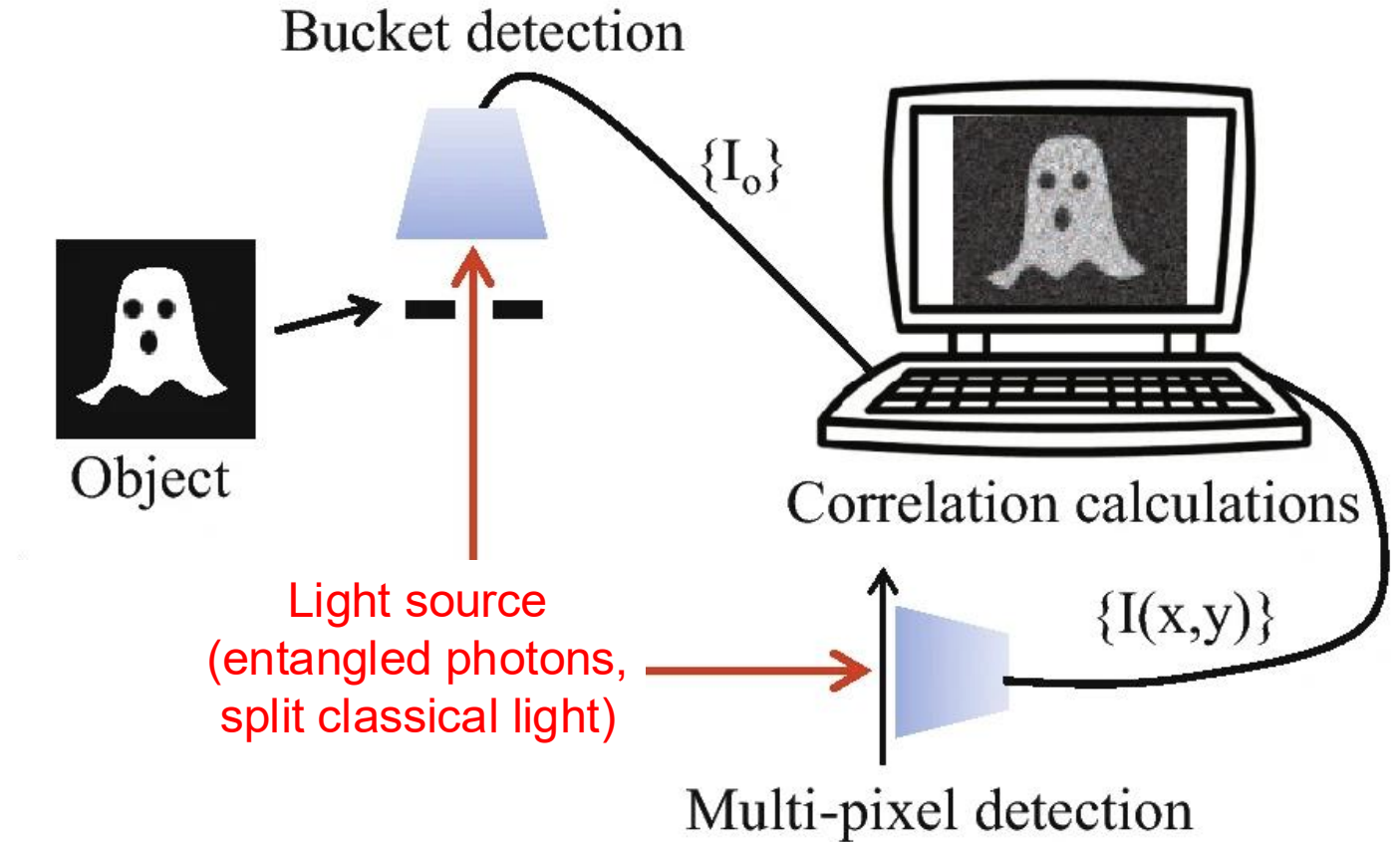
Fall 2025



**WISCONSIN**  
UNIVERSITY OF WISCONSIN-MADISON

# Outline

- **Ghost imaging**: an imaging technique using light that has not physically interacted with the object to be imaged.
  - Not inherently quantum phenomenon, although the first demonstration was done using quantum-entangled photons
- Some fundamentals of imaging
- Properties of light sources in imaging
  - Spatial and temporal coherence
- Quantum light sources, including entangled photons
- Applications in ghost imaging

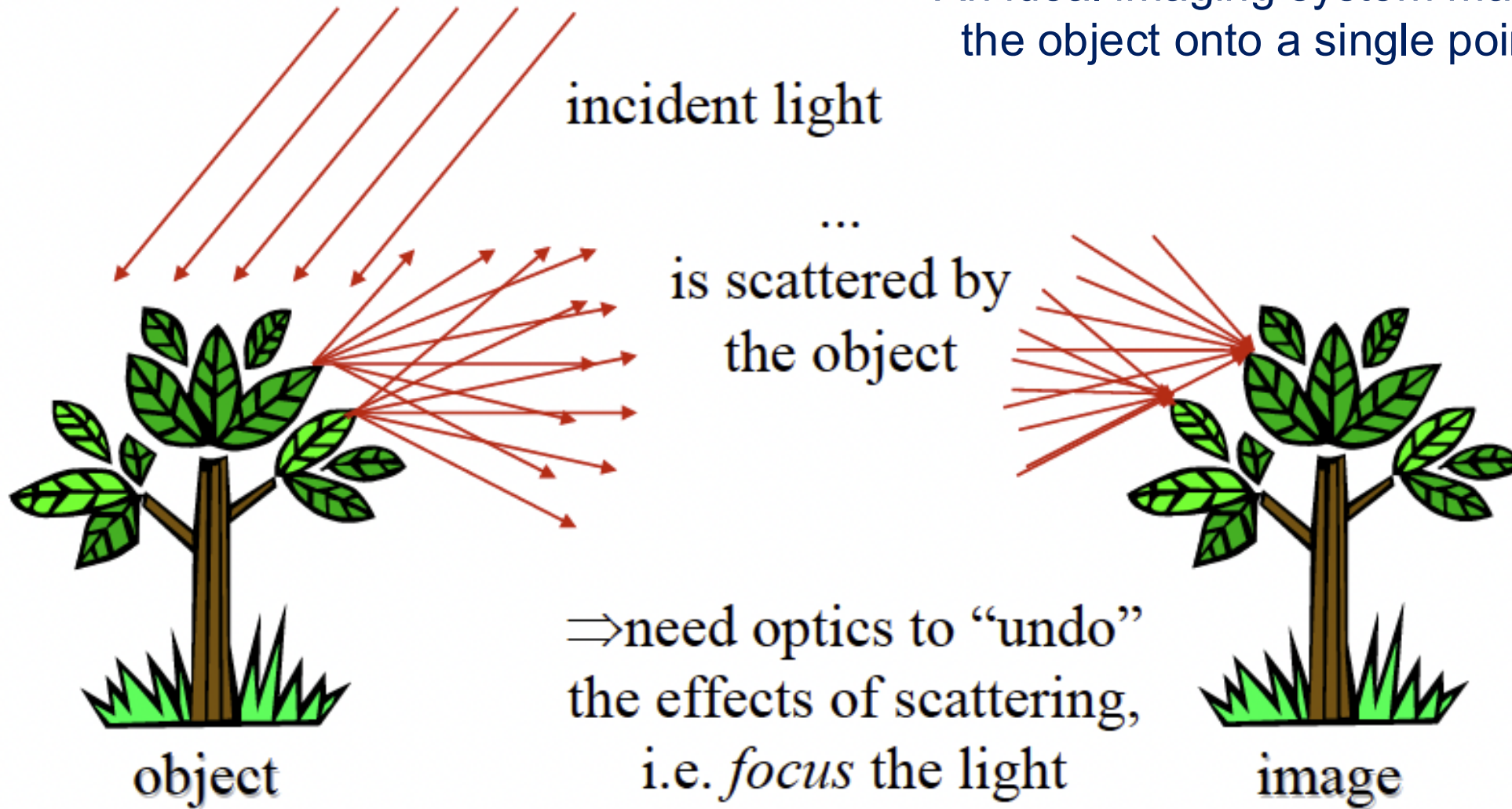


# Imaging problem



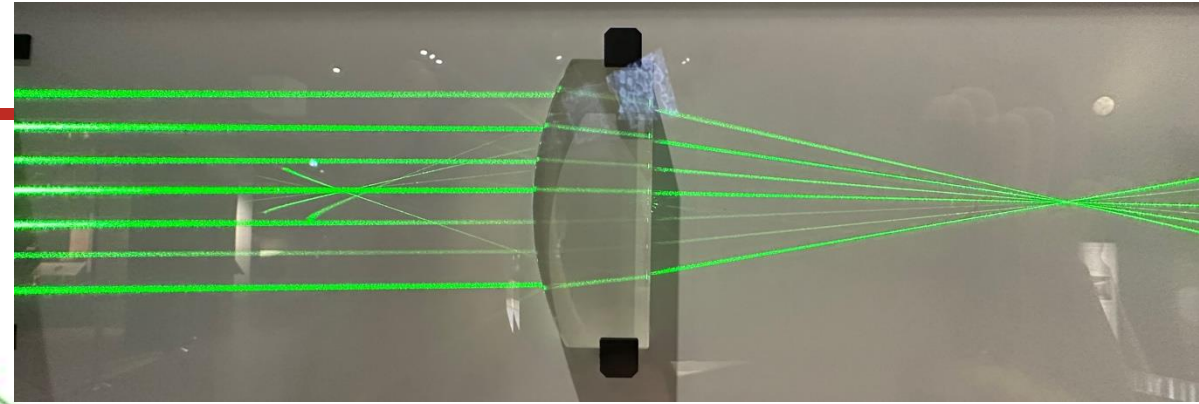
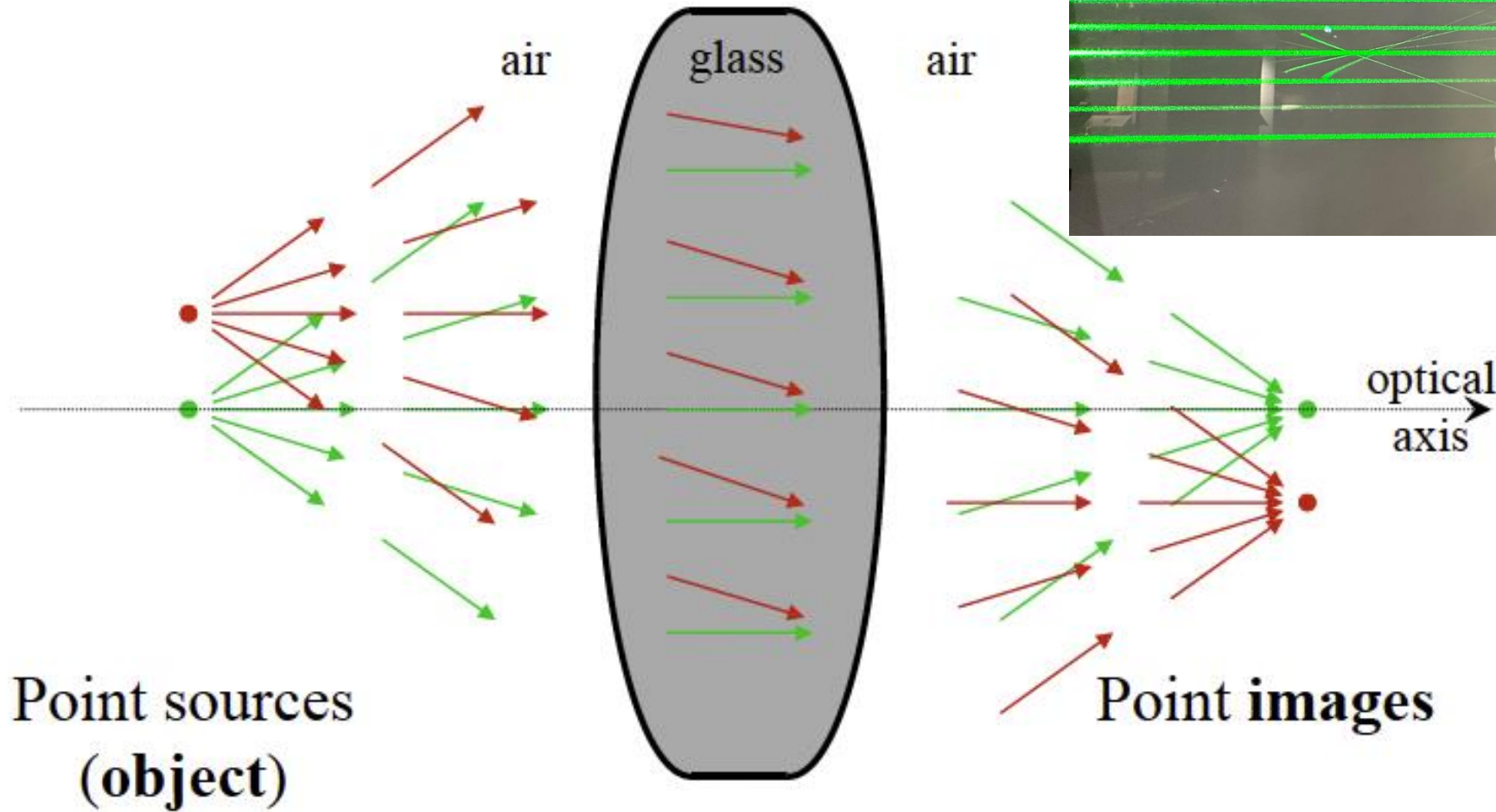
# Why are imaging systems needed?

An ideal imaging system maps each point of the object onto a single point in the image



Credit: Lecture notes on Optics by George Barbastathis (MIT 2.71)

# Ideal lens



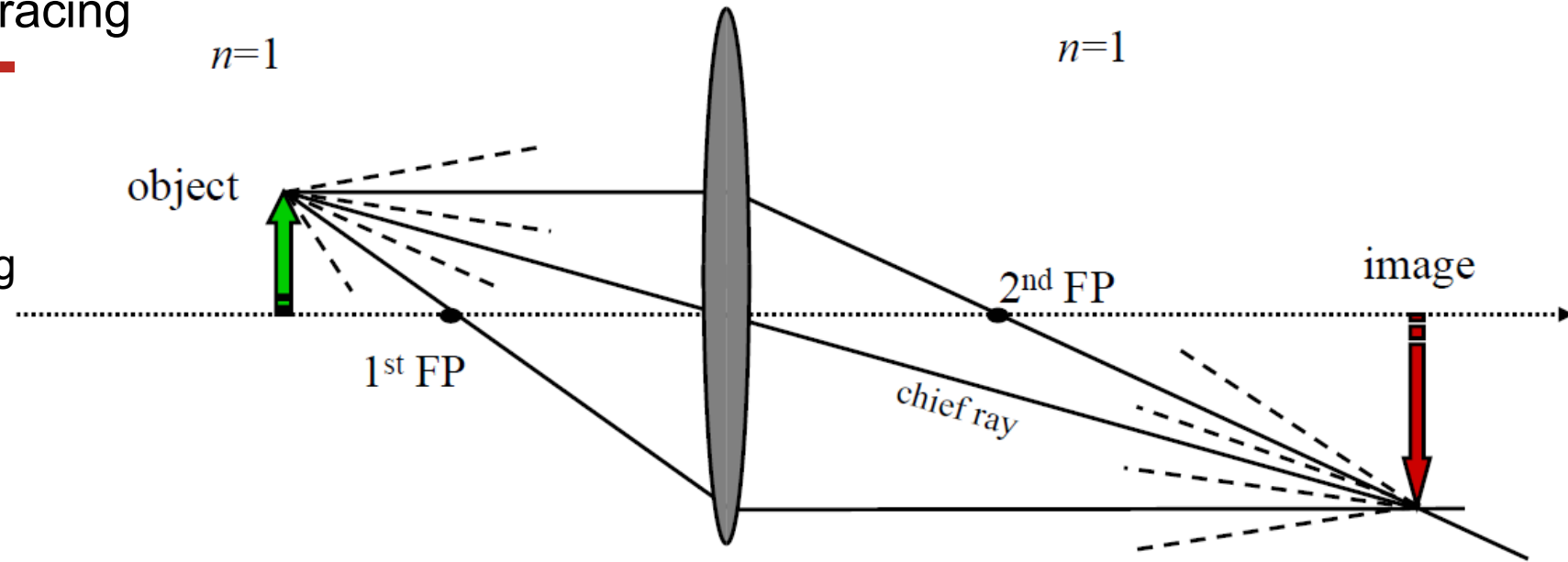
Each point source from the object plane focuses onto a point image at the image plane

Real lenses introduce blur due to aberrations and diffraction

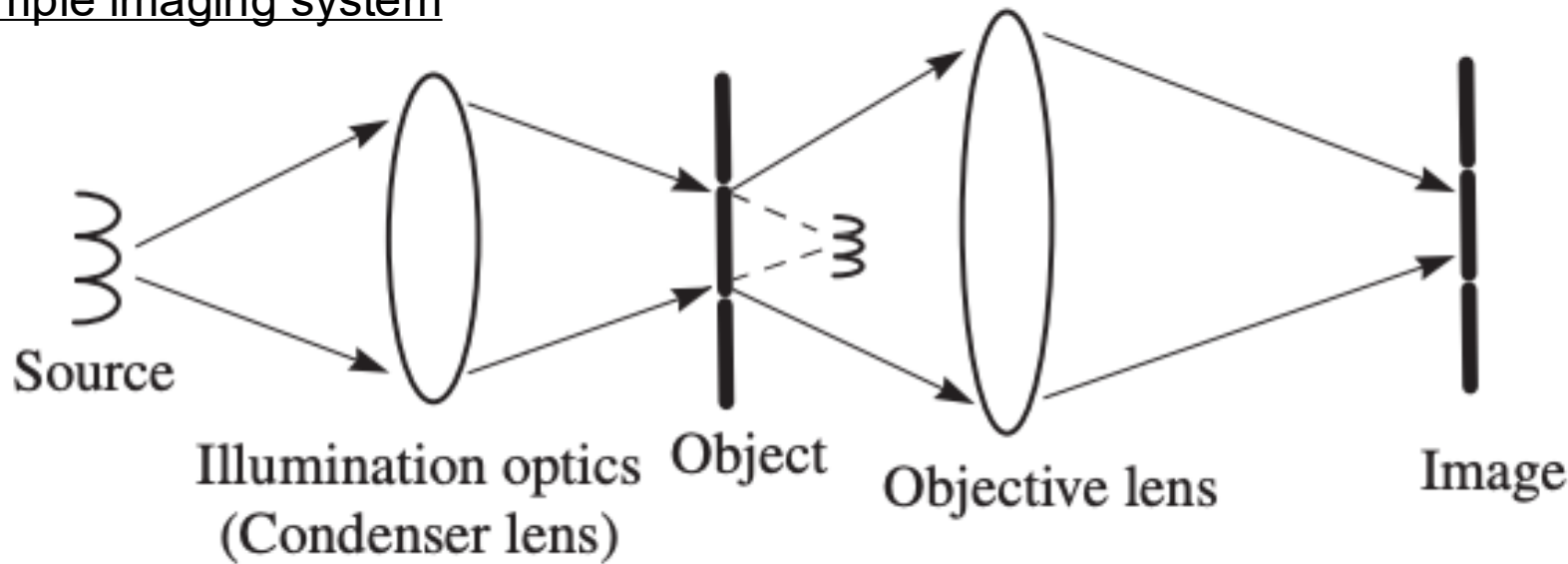
Credit: Lecture notes on Optics by George Barbastathis (MIT 2.71)

## Imaging condition using ray-tracing

- Image point is located at the common intersection of all rays emanating from the corresponding object point
- The two rays passing through the two focal points and the chief ray can be ray-traced directly



### Simple imaging system

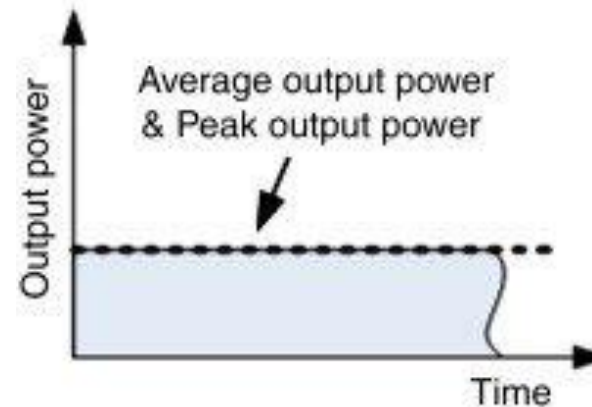
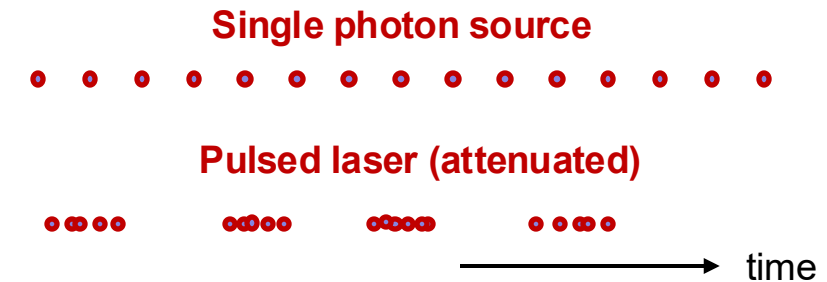
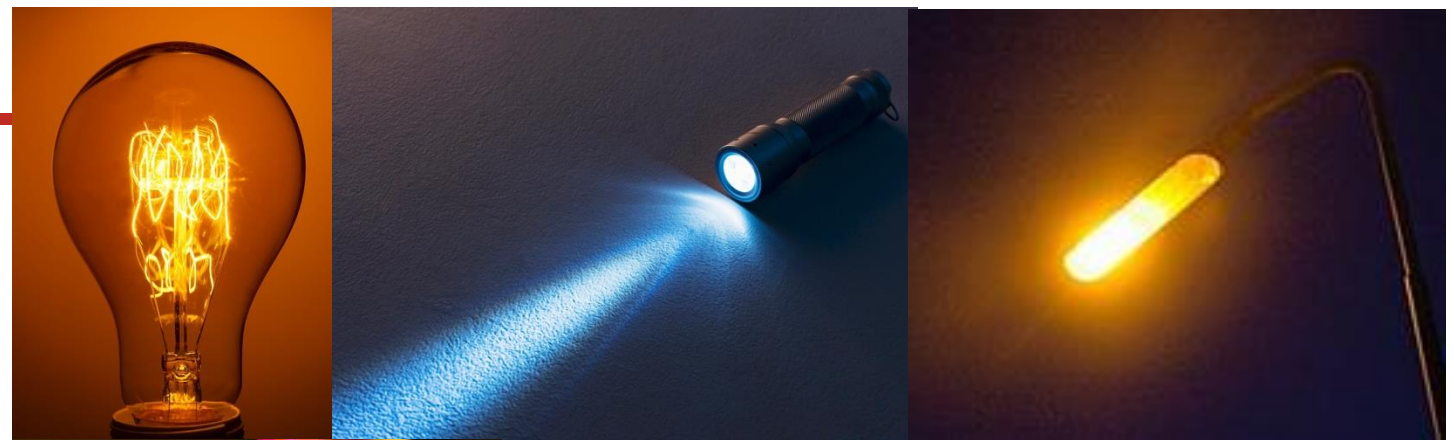


Credits: Lecture notes on Optics by George Barbastathis (MIT 2.71)  
Masud Mansuripur

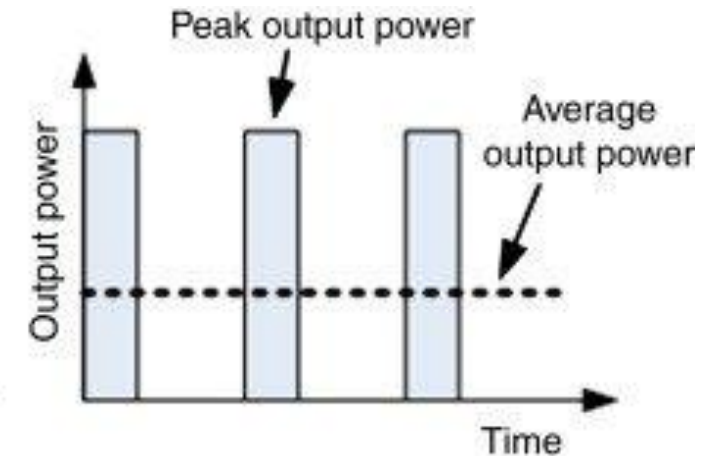
# Types of light sources for imaging

- Thermal sources such as sunlight, light bulb
- Gas discharge lamps
- Light emitting diodes (LEDs)
- Continuous-wave lasers
  - Helium neon (HeNe), argon, diodes
- Pulsed lasers
  - Nd:YAG
- Nonclassical light sources
  - Single-photon sources
  - Squeezed light
  - Entangled photons

How are these light sources categorized?



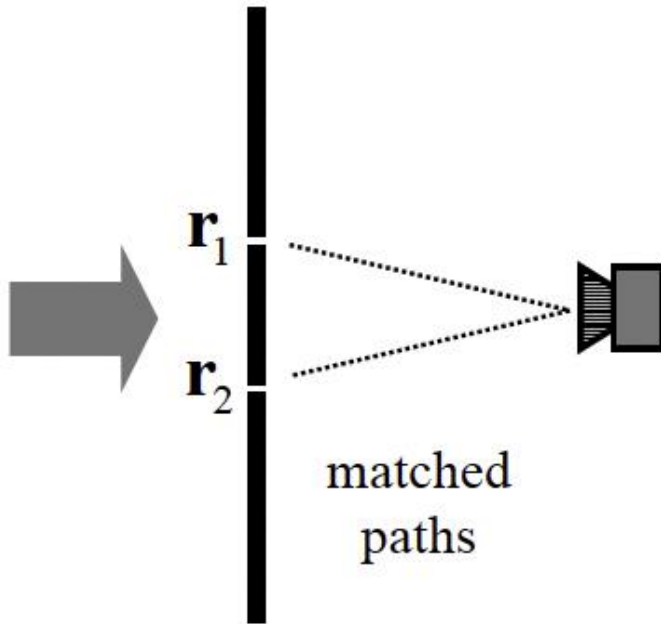
(a) Continuous wave (CW) laser beam



(b) Pulsed laser beam

# Spatial and temporal coherence

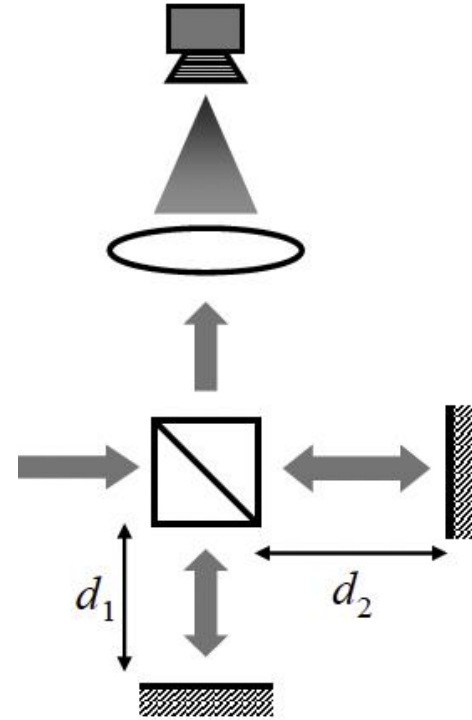
## Spatial coherence



Young interferometer

Waves with equal paths but from different points on the wavefront do not interfere

## Temporal coherence



Michelson interferometer

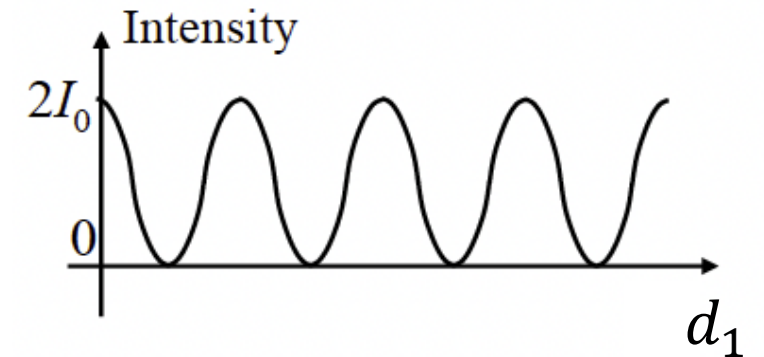
Waves from unequal paths do not interfere

Define coherence length

$$L \equiv \frac{c}{n\Delta\nu} = ct_c$$

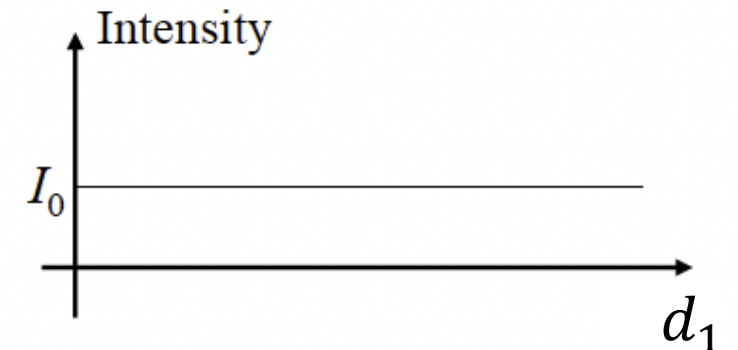
$$d_1 - d_2 \ll L$$

sharp interference fringes



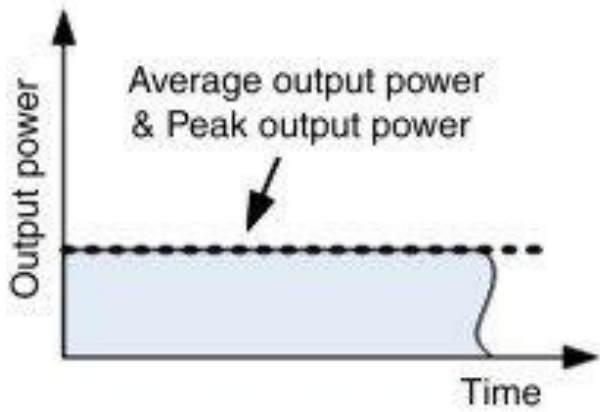
$$d_1 - d_2 \gg L$$

no interference

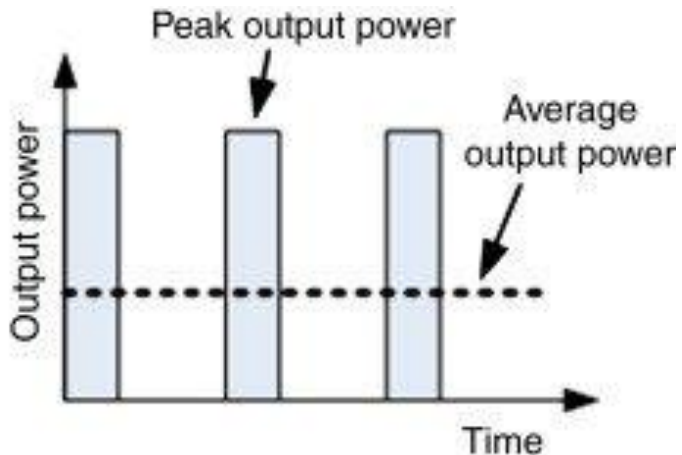


# Let's categorize these classical light sources

- Thermal sources such as sunlight, light bulb
  - Broadband
  - large  $\Delta\omega$
  - poor spatial coherence
- Gas discharge lamps
  - multiple lines
  - each with small  $\Delta\omega$
  - poor spatial coherence
- Light emitting diodes (LEDs)
  - monochromatic
  - small  $\Delta\omega$
  - poor spatial coherence
- Continuous-wave lasers
  - good spatial coherence
- Pulsed lasers
  - larger  $\Delta\omega$

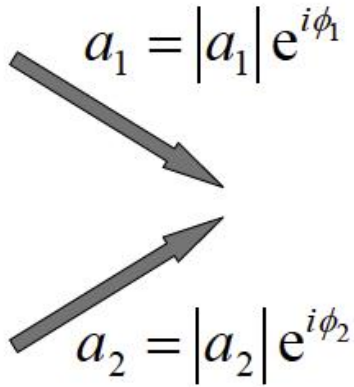


(a) Continuous wave (CW) laser beam



(b) Pulsed laser beam

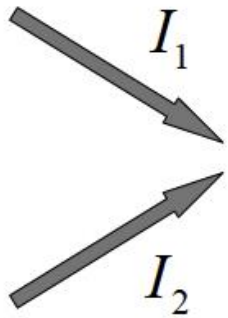
# Imaging with coherent and incoherent beams


$$a_1 = |a_1| e^{i\phi_1}$$
$$a_2 = |a_2| e^{i\phi_2}$$

**Mutually coherent:** superposition field *amplitude* is described by *sum of complex amplitudes*

$$a = a_1 + a_2 = |a_1| e^{i\phi_1} + |a_2| e^{i\phi_2}$$

$$I = |a|^2 = |a_1 + a_2|^2$$


$$I_1$$
$$I_2$$

**Mutually incoherent:** superposition field *intensity* is described by *sum of intensities*

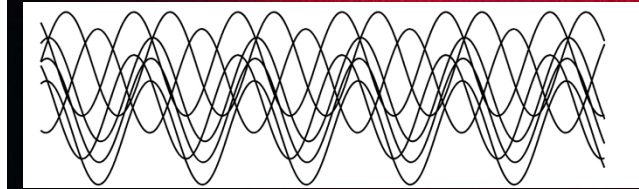
$$I = I_1 + I_2$$

(the phases of the individual beams vary randomly with respect to each other; hence, we would need statistical formulation to describe them properly — *statistical optics*)

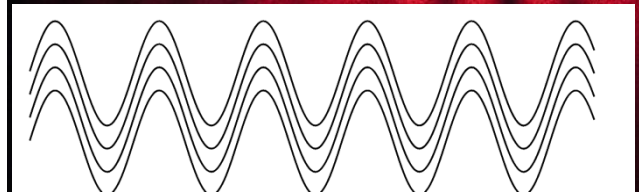
From lecture notes on Optics by George Barbastathis (MIT 2.71)

**Answer on Top Hat:** Is the light source used to generate A more or less coherent than the one for B?

A



B



fineart  
america

At the Michelson detector the fields from the beamsplitter are being summed:

$$E(t) = E_1(t) + E_2(t)$$

A detector is measuring intensity:

$$I = \langle |E(t)|^2 \rangle$$

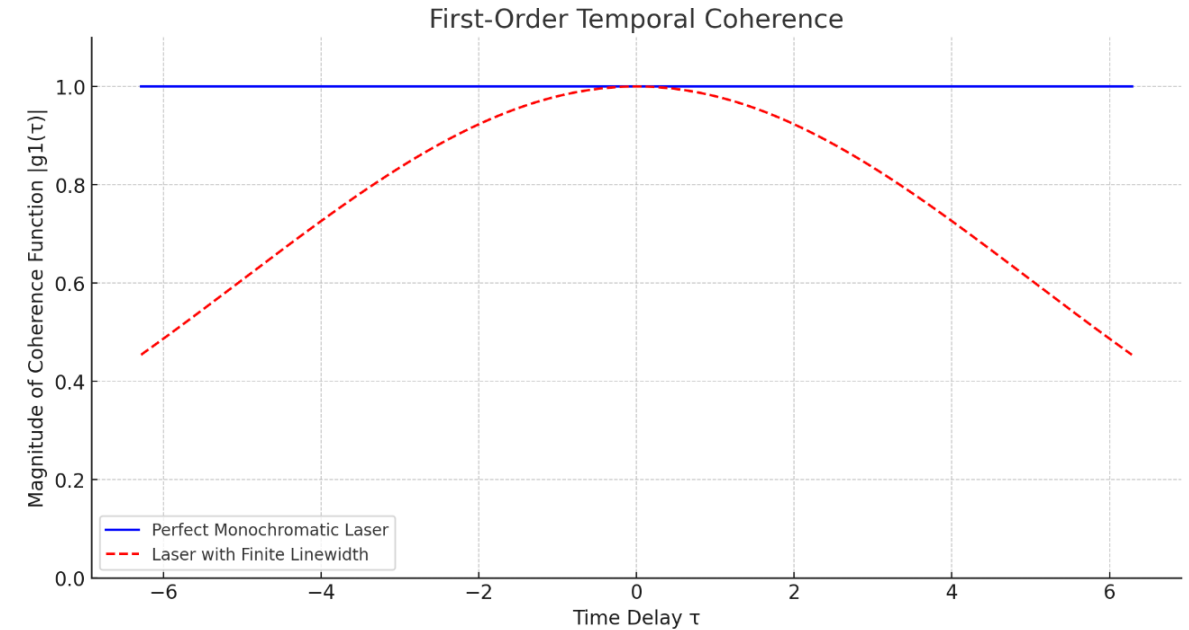
If we assume that the light source can be represented by a plane wave:  $E_0(t) = ae^{i\omega_0 t}$ , then the detector reads:  $I \propto I_0(1 + \cos \Delta\phi)$

where  $\Delta\phi = \omega_0\tau = \frac{\omega_0(d_2 - d_1)}{c}$ .

As for the first-order correlation function:

$$g^{(1)}(\tau) = \frac{\langle E(t)E(t + \tau) \rangle}{\langle |E(t)|^2 \rangle}$$

$g^{(1)}(\tau) = e^{i\omega_0\tau}$  for perfectly monochromatic light



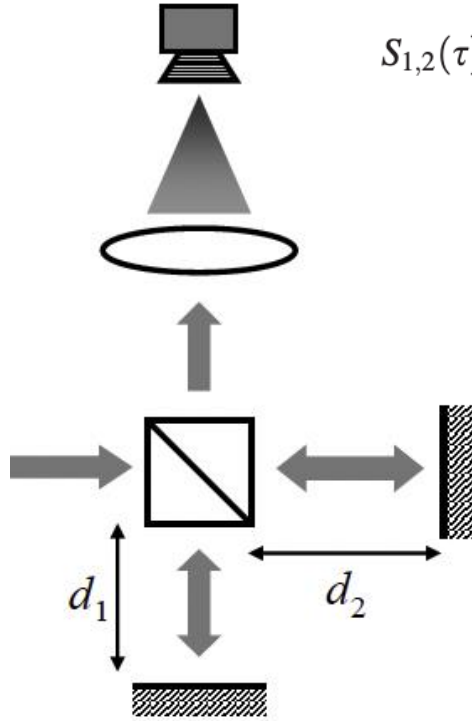
Laser with finite linewidth  $\Delta\omega$ :

No single way to model this, but one representation is:

$E(t) = \text{Re}\{ae^{-\left(\frac{\Delta\omega}{2}\right)^2} e^{i\Delta\omega\tau}\}$ , which represents a wave in which frequencies are distributed in a Gaussian fashion around  $\omega_0$ . Note  $\tau_c \approx (\Delta\omega)^{-1}$ .

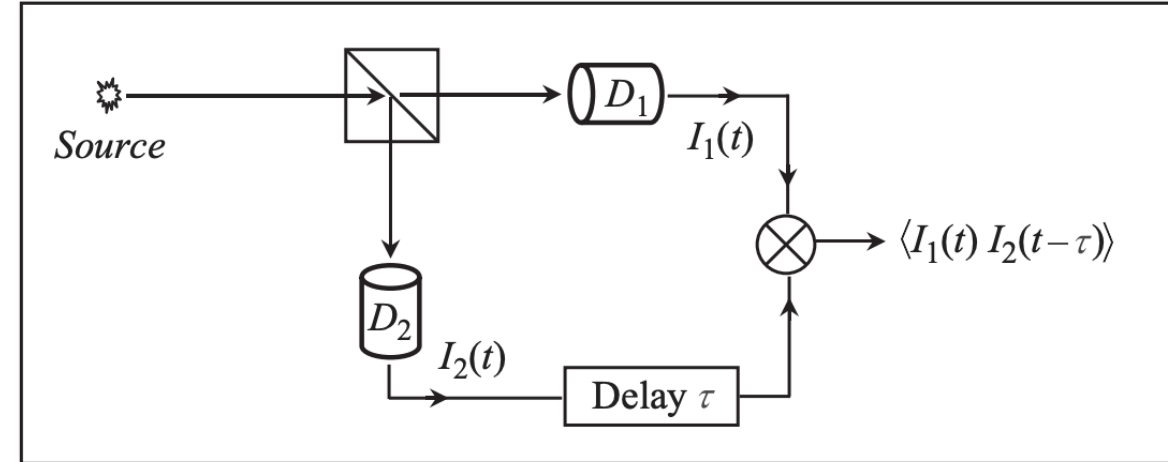
$$g^{(1)}(\tau) = e^{-\left(\frac{\Delta\omega\tau}{2}\right)^2} e^{i\omega_0\tau}$$

$$I \propto I_0(1 + |g^{(1)}(\tau)| \cos \Delta\phi)$$



Michelson interferometer

$$\begin{aligned}
 S_{1,2}(\tau) &= \frac{1}{T} \int_0^T \left\{ \frac{1}{2} [a(t) \pm a(t - \tau)] \right\}^2 dt \\
 &= \frac{1}{2T} \int_0^T a^2(t) dt \pm \frac{1}{2T} \int_0^T a(t) a(t - \tau) dt \\
 &= \frac{1}{2} \left[ \langle I \rangle \pm \frac{1}{2} \sum_n A_n^2 \Delta f \cos(2\pi f_n \tau) \right].
 \end{aligned}$$



Hanbury Brown and Twiss experiment

$$\langle I(t)I(t + \tau) \rangle = \frac{1}{T} \int_0^T I(t)I(t + \tau) dt = I_0^2 + \frac{1}{2} \sum_{m=1}^{2M} |\hat{I}_m|^2 \cos(2\pi m \Delta f \tau).$$

Second-order autocorrelation function

$$g^{(1)}(\tau) = \frac{\langle E(t)E(t + \tau) \rangle}{\langle |E(t)|^2 \rangle}$$

$$g^{(2)}(\tau) = \frac{\langle I(t)I(t + \tau) \rangle}{\langle I(t) \rangle^2}$$

# Single-photon sources, a form of nonclassical light

## Attenuated classical source



## Single photon source



time

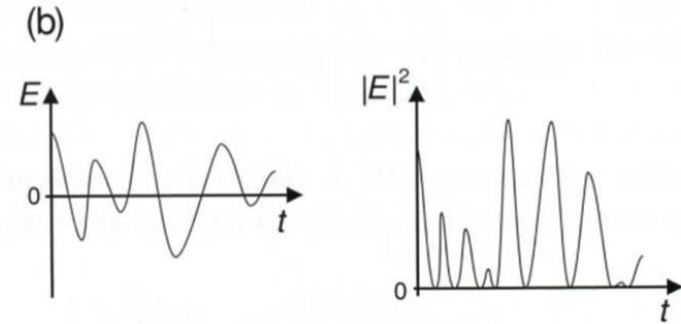
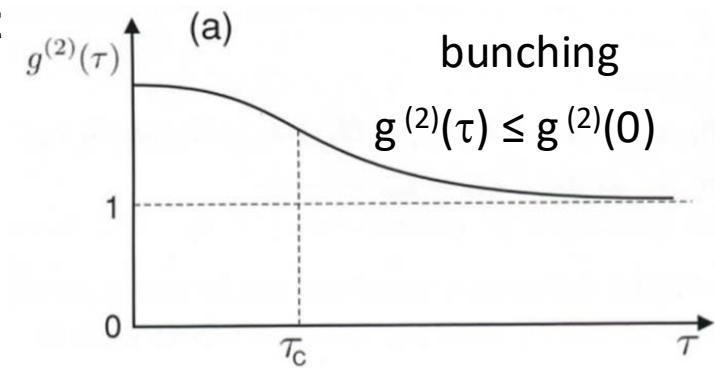
- Low (or zero) probability of emitting more than one photon at a time
- Quantum emitters: atoms (atomic transitions), quantum dots, etc.

**Figure of merit:  
second-order intensity autocorrelation function**

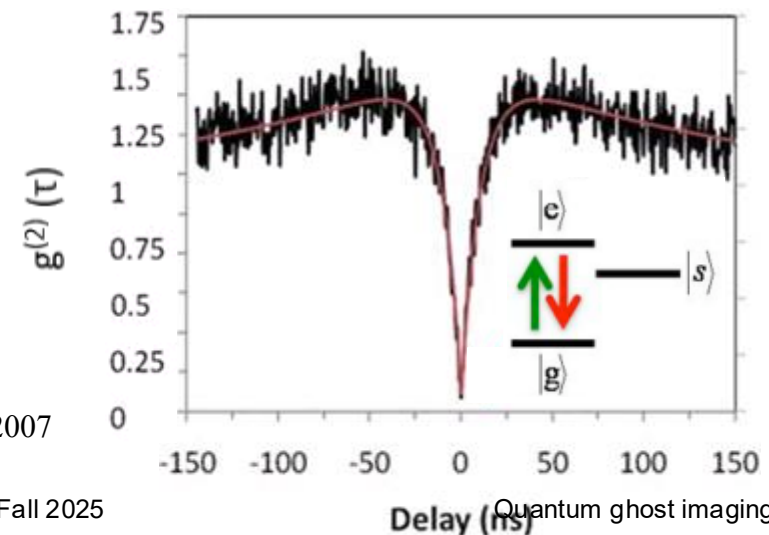
$$g^{(2)}(\tau) = \frac{\langle I(t)I(t + \tau) \rangle}{\langle I(t) \rangle^2}$$

Refs: Novotny and Hecht, "Principles of Nano-Optics," 2007

## Classical light



## Single Photon Emission



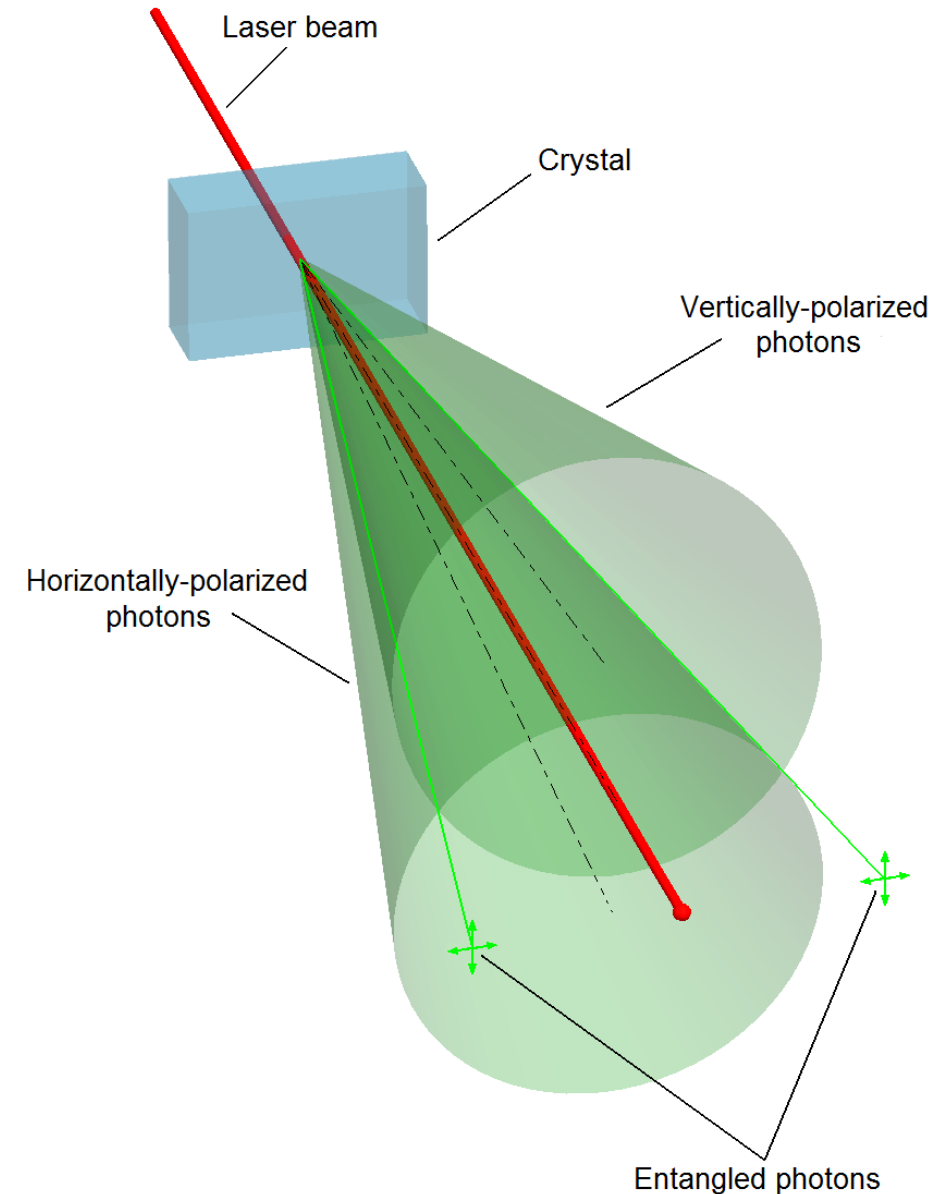
anti-bunching  
 $g^{(2)}(0) = 0$   
 (ideal)  
 $g^{(2)}(0) < 1/2$

Quantum ghost imaging (complete)

## Another type of nonclassical light: entangled photons

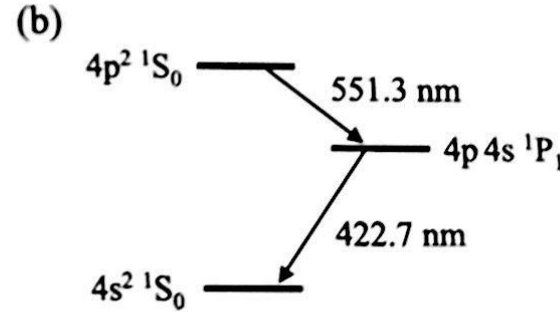
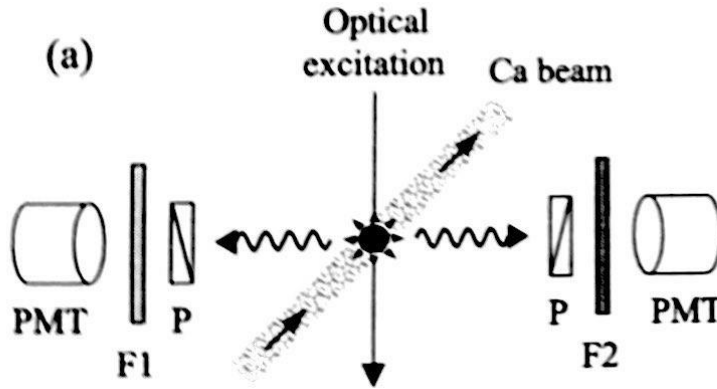
- Two or more photons are entangled if they are correlated such that the state of one photon cannot be described without considering the state of the other photon(s), even if they are spatially separated.
- Properties of light that can be entangled
  - Polarization
  - Momentum and position
  - Frequency / energy
  - ...

$$\psi_{\pm}^{pos} = \frac{1}{\sqrt{2}} (\psi_{1,0}\psi_{2,0} \pm \psi_{1,1}\psi_{2,1})$$
$$\psi_{\pm}^{neg} = \frac{1}{\sqrt{2}} (\psi_{1,0}\psi_{2,1} \pm \psi_{1,1}\psi_{2,0})$$



# Different ways of getting entangled photons

## Using atomic transitions



Question: A correlated pair of photons is generated by a nonlinear crystal using a laser at 502 nm. If the wavelength of one of the photons is 820 nm, what is the wavelength of the other photon?

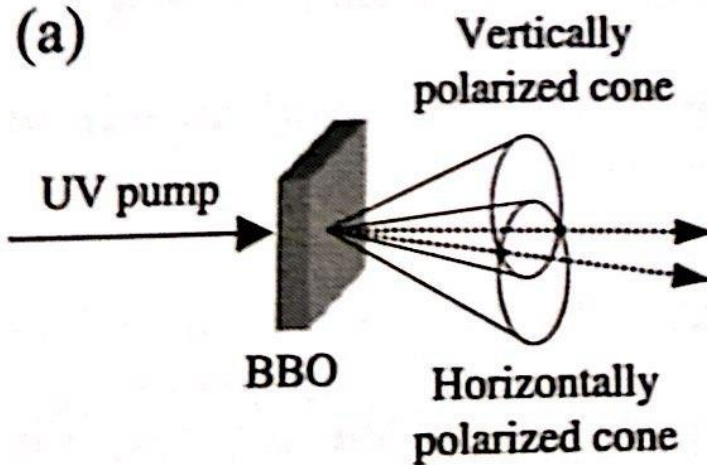
$$\omega_3 = \omega_1 + \omega_2$$

$$\frac{1}{\lambda_3} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}$$

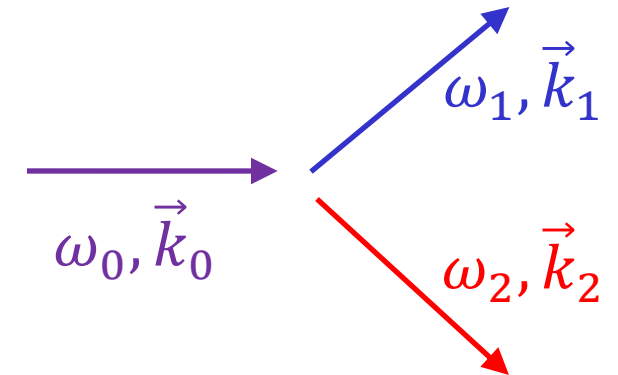
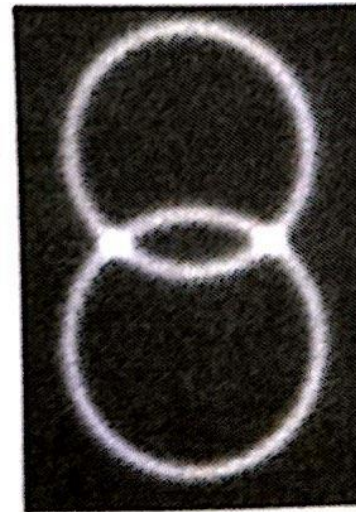
$$\lambda_1 = 820 \text{ nm}; \lambda_3 = 502 \text{ nm}$$

$$\lambda_2 = 1295 \text{ nm}$$

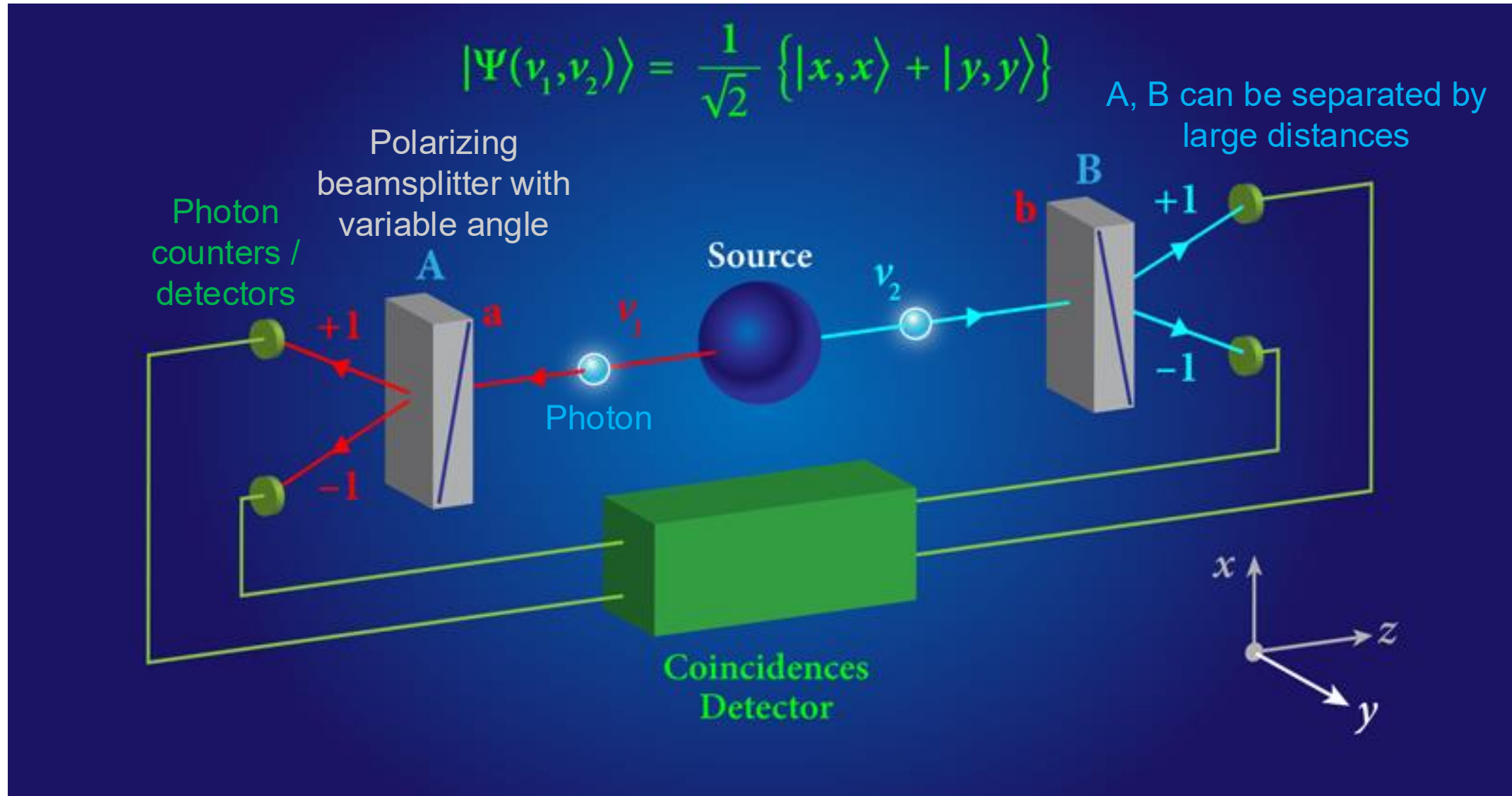
## Using nonlinear optics (shown here is parametric down conversion)



(b)



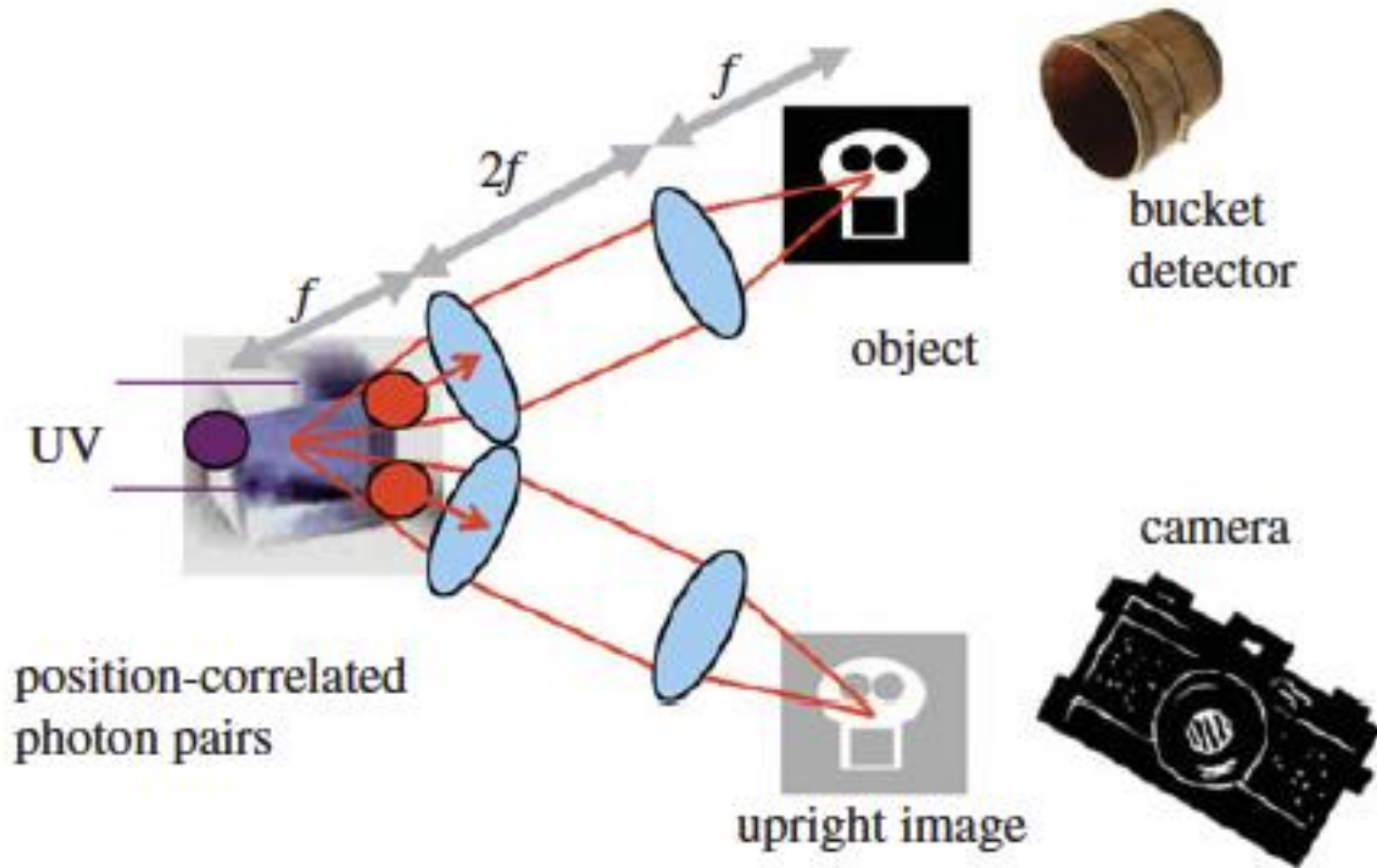
## The Bell test is used to evaluate entanglement



Nice review article from Alain Aspect: <https://physics.aps.org/articles/v8/123>

# Leveraging correlation of photons in ghost imaging

## Concept



Some advantages of ghost imaging:

- Reduced exposure to light-sensitive sample
- Can use one wavelength to interact with sample, and another wavelength for imaging
- Can image through scattering medium (fog, etc)

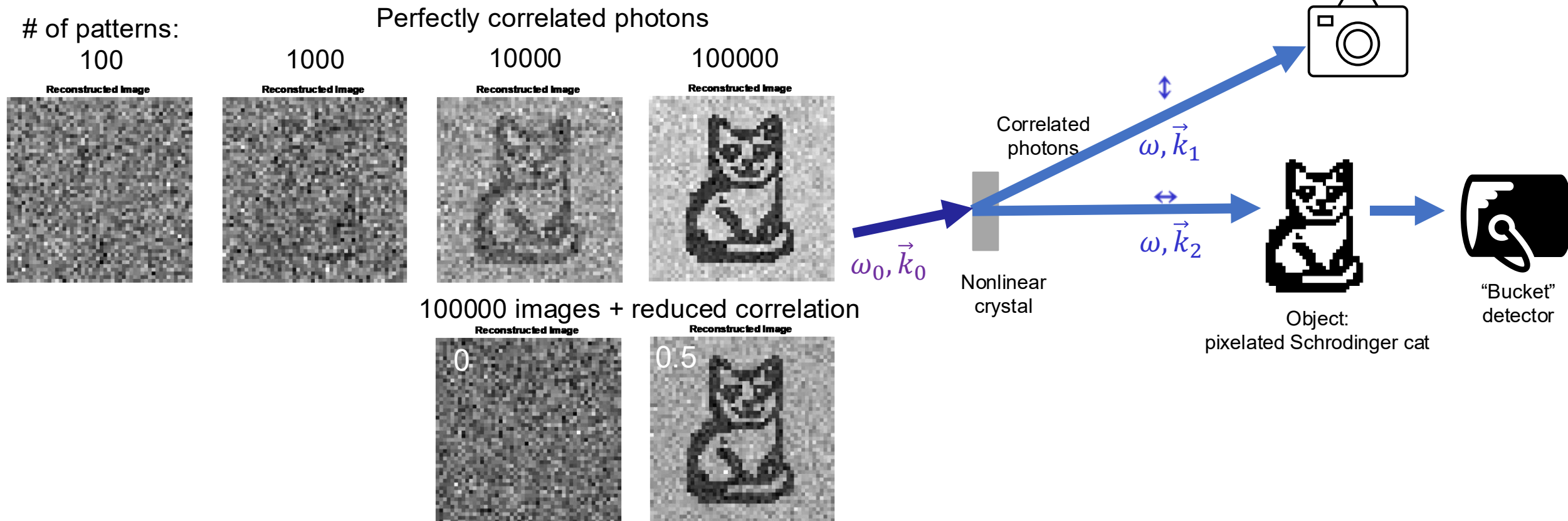
# Let's simulate ghost imaging in MATLAB

quantum\_ghost\_activity.m

Change `num_patterns`: scales with the number of entangled photons used for the measurement

Change `correlationDegree` between 0 and 1: how correlated the photons are

What is the image you see?

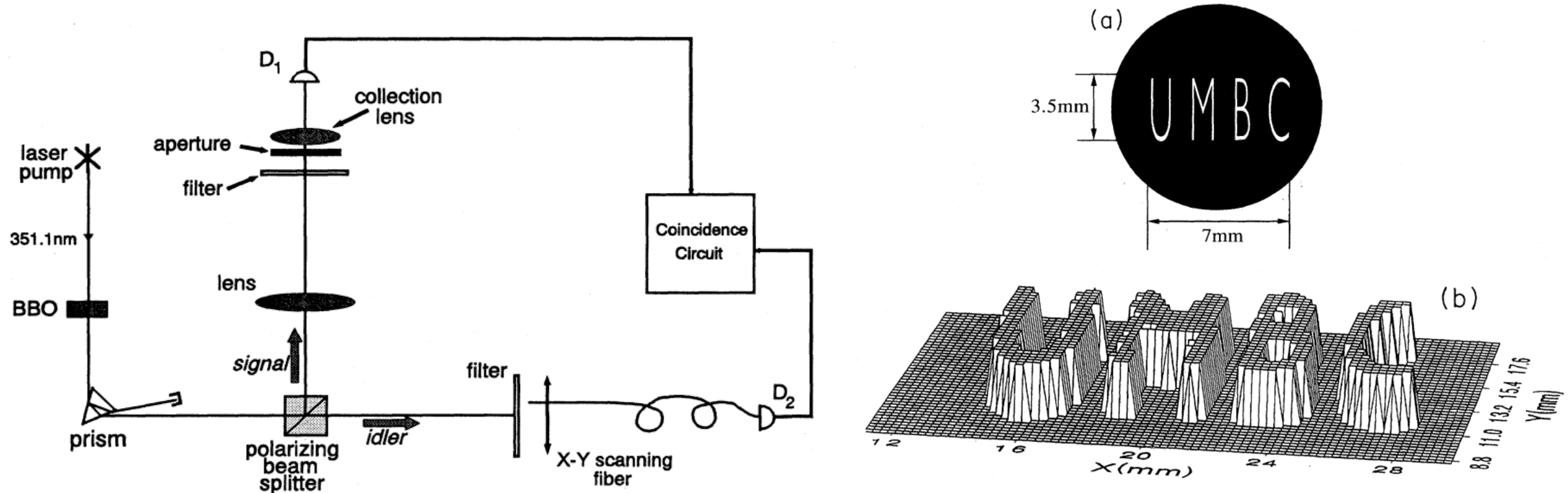


## Optical imaging by means of two-photon quantum entanglement

T. B. Pittman, Y. H. Shih, D. V. Strekalov, and A. V. Sergienko

*Department of Physics, University of Maryland Baltimore County, Baltimore, Maryland 21228*

(Received 22 December 1994)



# We don't actually need quantum mechanics for ghost imaging

PHYSICAL REVIEW A 77, 041801(R) (2008)

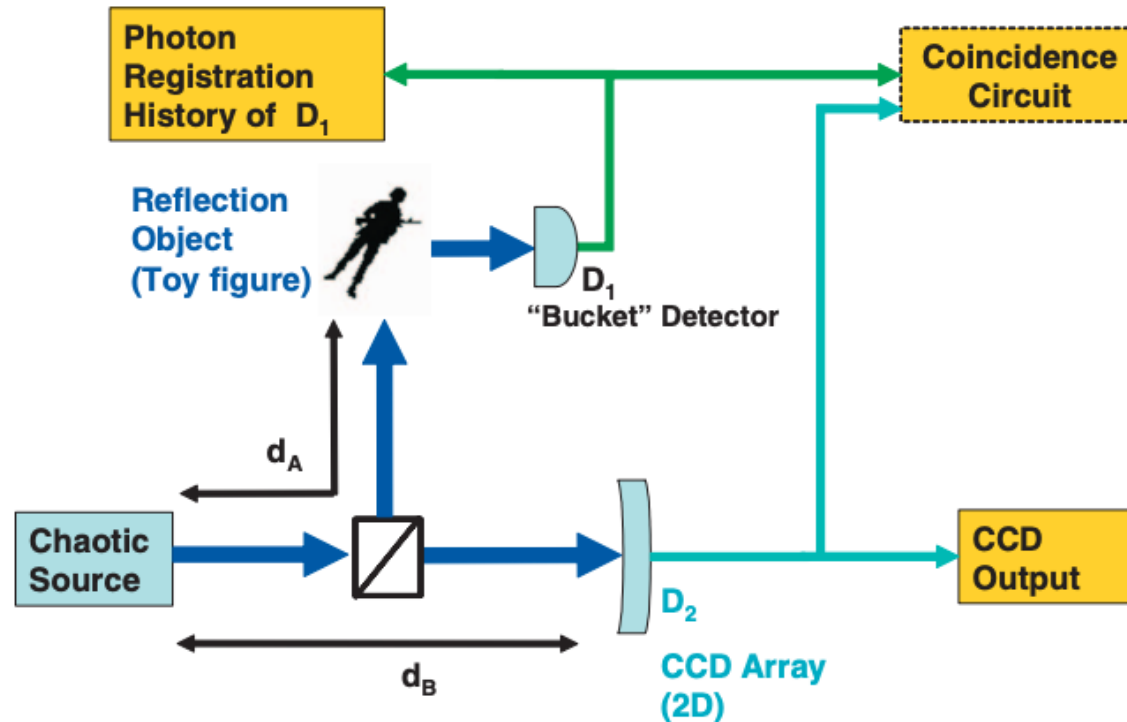
## Ghost-imaging experiment by measuring reflected photons

Ron Meyers,<sup>1</sup> Keith S. Deacon,<sup>1</sup> and Yanhua Shih<sup>2</sup>

<sup>1</sup>U.S. Army Research Laboratory, Adelphi, Maryland 20783, USA

<sup>2</sup>Department of Physics, University of Maryland, Baltimore County, Baltimore, Maryland 21250, USA

(Received 8 March 2007; revised manuscript received 12 June 2007; published 8 April 2008)



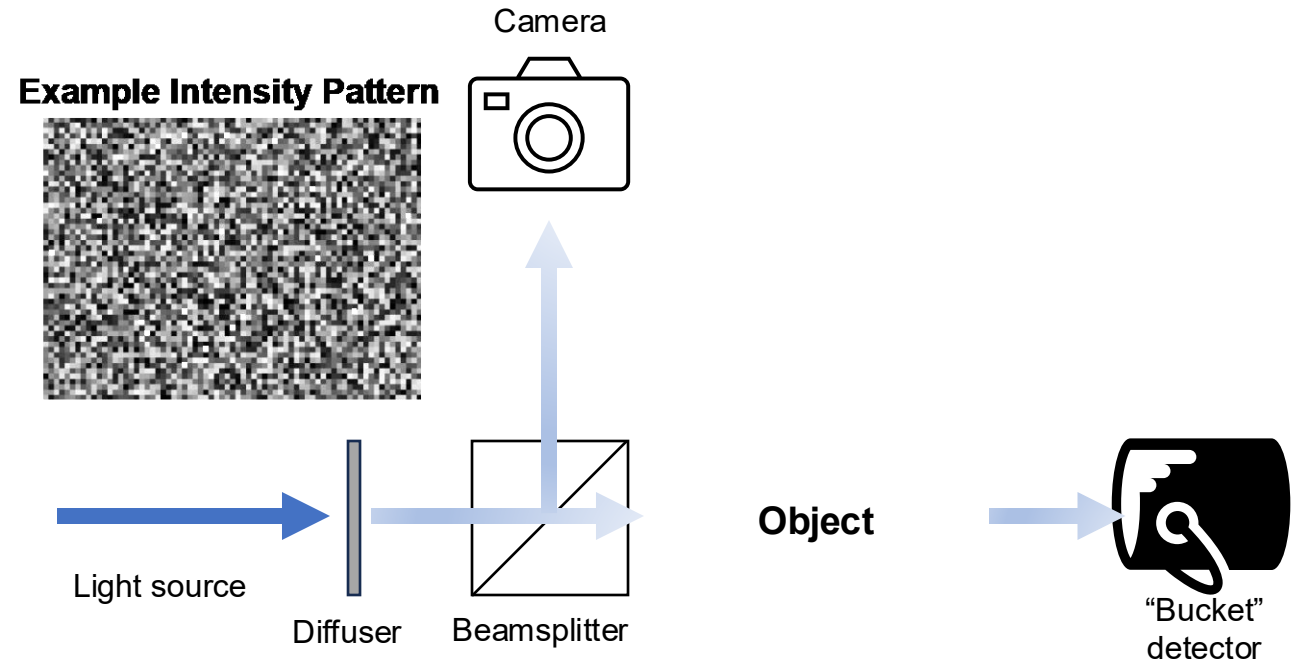
## Let's now simulate classical ghost imaging

$$I_{ghost}(x, y) = \sum_{i=1}^N [I_{bucket,i} \times I_{pattern,i}(x, y)]$$

Classical\_ghost\_activity.m

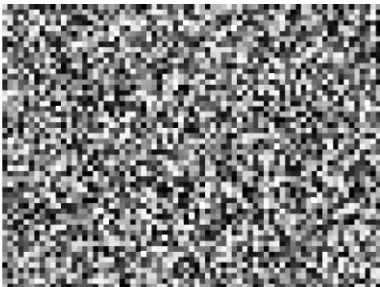
Change `num_patterns`: number of speckled patterns used for the imaging

What image do you see now?

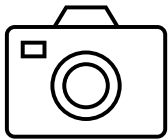


# Classical ghost imaging

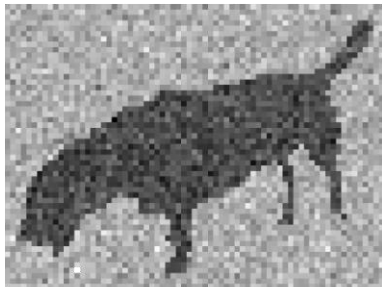
Example Intensity Pattern



Camera



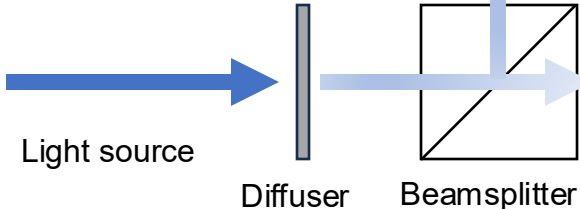
(50000 patterns)  
Reconstructed Image



Light source

Diffuser

Beamsplitter



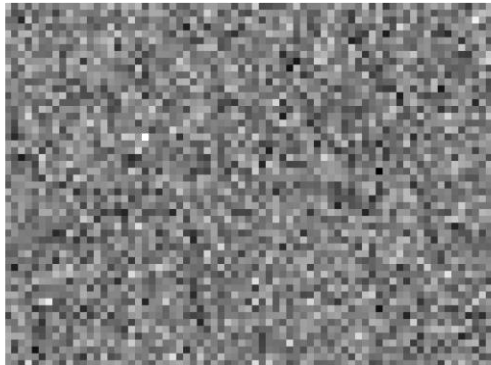
Object: Toby



"Bucket"  
detector

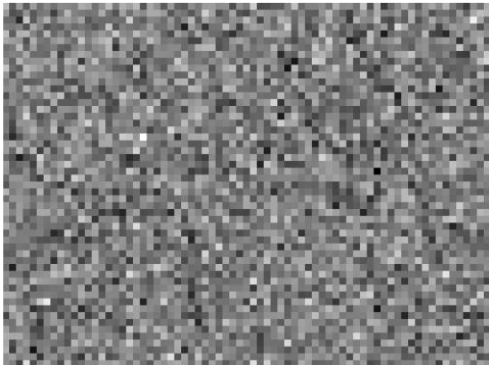
# of patterns:  
100

Reconstructed Image



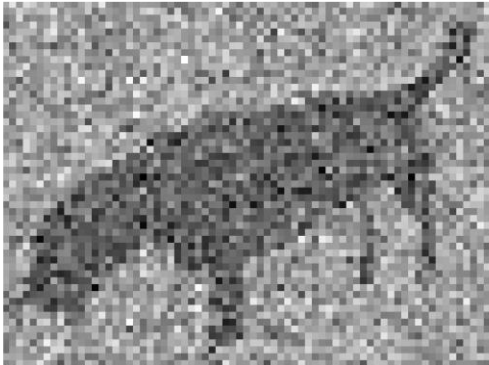
1000

Reconstructed Image



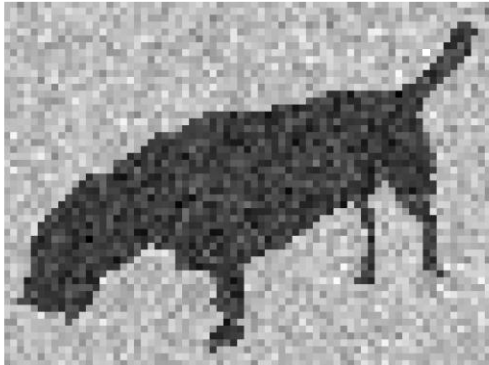
10000

Reconstructed Image

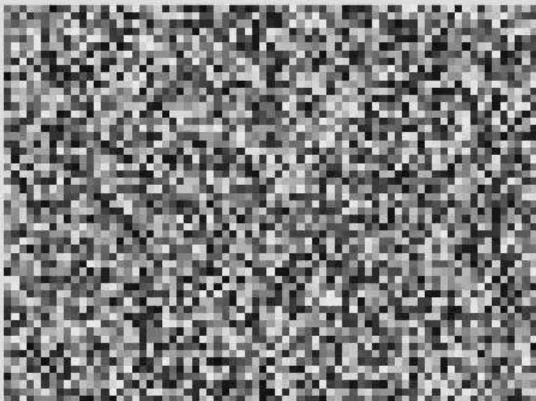


100000

Reconstructed Image



Pattern 1



Reconstruction (1 patterns)

