1. (5 points) For a rubidium atom in its ground state, calculate the quantum defect for the 5s state given that its energy is -4.18 eV. What is the effective nuclear charge seen by the valence electron?

+2 $E = \frac{-13.6\,eV}{(n-\delta)^2}$

+1 $n = 5$

+1 $\delta = 3.197$

+1 $Z_{eff} \approx 1$ because $E = \frac{-13.6\,eV\, Z_{eff}^2}{n^{*2}}$

where $n^* = n - \delta$

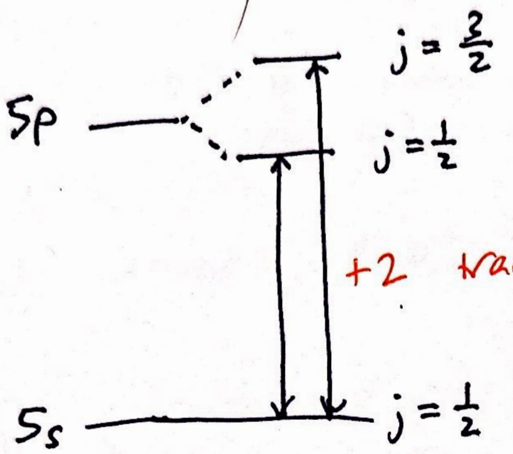
$Z_{eff} = 1.04$

2. (15 points) Consider rubidium 85, an alkali atom with a $5s$ ground state and $I = 5/2$.

(a) (7 points) Illustrate the fine structure associated with the $5p$ states and denote the allowable transitions between $5s - 5p$ (you may ignore hyperfine interactions for now).

$s = \frac{1}{2}, \ l = 1 \rightarrow j = \frac{1}{2}, \frac{3}{2}$ +2 identifying j states

+3 energy diagram



+2 transitions

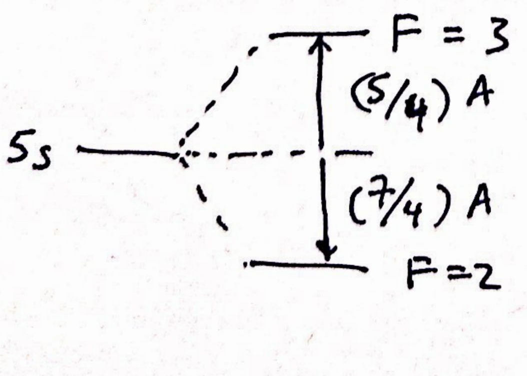
(b) (8 points) Calculate the *frequency splitting* between F states due to hyperfine interaction in the ground state, given $A = h \cdot 1.0119\ GHz$.

Ground state : $l = 0, \ j = \frac{1}{2}, \ s = \frac{1}{2}$, $I = \frac{5}{2}$ $\Big\}$ $F = 2, 3$

+3 identifying F states

$$\Delta E_{HFS} = \left(\frac{A}{2}\right)\left(F(F+1) - I(I+1) - J(J+1)\right)$$

+3 right ΔE_{HFS} relation



Splitting $= 3A = 3.0357$ GHz

+2 splitting

3. (10 points) Determine if the following atomic transitions are allowed or forbidden. For each allowed transition, describe the polarization of the emitted photon. Please also specify a viewing angle for the emission.

Selection rules: Δn anything
$\Delta l = \pm 1$
$\Delta m = 0, \pm 1$

(n, l, m_l)

(a) (4, 2, 0) → (3, 1, 0)

+2

Allowed

$\Delta m = 0$ → linearly polarized emission along quantization axis ($\hat{z}$) can be viewed in transverse direction

(b) (3, 1, 1) → (3, 1, 0)

Not allowed ($\Delta l = 0$)

+2

(c) (6, 2, -1) → (2, 1, 0)

+2

Allowed

$\Delta m = +1$ → circularly polarized emission (σ^+) when viewed along $\hat{z}$

(d) (3, 2, 0) → (2, 1, 1)

+2

Allowed

$\Delta m = +1$ → circularly polarized emission (σ^-) when viewed along $\hat{z}$

(e) (3, 2, 1) → (3, 1, 0)

+2

Allowed

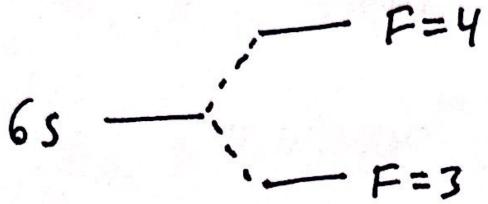
$\Delta m = -1$ → circularly polarized emission (σ^+) when viewed along $\hat{z}$

4. (25 points) Atomic clocks

(a) (5 points) A cesium atomic clock, using Cs-133 atoms in the $6s$ state with $I = 7/2$, operates on the ground state hyperfine transition at 9.192631770 GHz. The measured linewidth of the transition is 1 Hz. Draw the hyperfine energy structure in the ground state.

$n=6,\ s=\frac{1}{2},\ j=\frac{1}{2},\ I=\frac{7}{2} \rightarrow F=3,4$

+3



+2

(b) (5 points) Estimate the contributions of shot noise as a function of the number of atoms (N) used in each measurement cycle to the frequency uncertainty of the clock.

Fractional uncertainty $\sigma_\nu \propto \frac{1}{\sqrt{N}}$

(c) (5 points) Estimate the contributions of quantum projection noise as a function of the number of atoms (N) used in each measurement cycle to the frequency uncertainty of the clock.

Outcomes either $F=3$ or 4

Fractional uncertainty $\sigma_\nu \propto \frac{1}{\sqrt{Np(p-1)}} = \frac{2}{\sqrt{N}}$

$p=\frac{1}{2}$

(d) (7 points) For an optical clock using strontium atoms, the transition frequency is 429.228×10^{12} Hz while the excited state lifetime is 150 s. Calculate the improvement in clock uncertainty compared to the cesium clock above (hint: calculate the quality factors of both transitions).

uncertainty $\propto Q^{-1}$ where $Q = \frac{\nu_0}{\Delta\nu}$

$Q_{Cs} = 9.19 \times 10^9$

$Q_{Sr} = 8.091 \times 10^{17}$ +3

Sr clock has $\sim 8.8 \times 10^{+7}$x improved uncertainty +1

+3

$\Delta\nu\,\tau \gtrsim \frac{1}{2(2\pi)} \rightarrow \Delta\nu_{Sr} = \frac{1}{4\pi(150\,s)} = 5.305 \times 10^{-4}$ Hz

(e) (3 points) For the strontium clock, what is the minimum timing error, in seconds, in 1 day?

$\sigma_{Sr} \sim 1.2359 \times 10^{-18}$

Uncertainty/error in 1 day: $\sigma_{Sr}\left(\frac{60\,s}{min}\right)\left(\frac{60\,min}{hr}\right)\left(\frac{24\,hr}{day}\right)$

$= 1.0678 \times 10^{-13}$ s / day

5. (a) (4 points) Given the amplitude-phase relationship uncertainty $\Delta n \Delta\phi \geq 1/2$, what is the minimum phase uncertainty of a 1064-nm laser with a power output of 100 W?

$$E = \frac{hc}{\lambda} = 1.87\times10^{-19}\ \text{J/photon} \quad +1$$

$$\dot{N} = \frac{100\text{W}}{1.87\times10^{-19}\text{J}} = 5.35\times10^{20}\ \text{photons/s} \quad +1$$

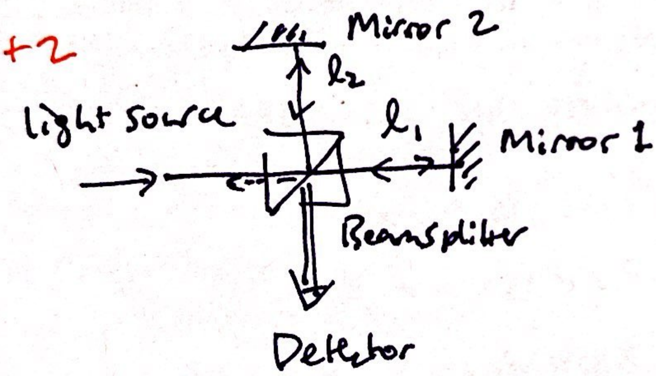
$$\Delta\phi \geq \frac{1}{2}\frac{1}{\sqrt{5.35\times10^{20}}} \rightarrow \Delta\phi \gtrsim 3.4\times10^{-11}\ \text{radians}$$

+2

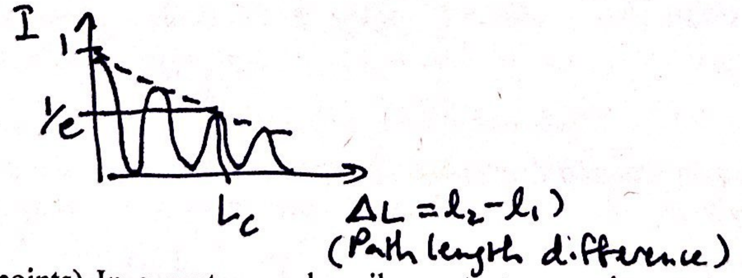
(b) (4 points) You are to construct an interferometric measurement using the laser. In a few sentences, describe a measurement you can do to quantify how long the path-length of the interferometer can be given the coherence of the laser. Please also include a sketch of the measurement and label all components in your sketch.

+1 Describe Mach-Zehnder interferometer

+2



+1 Observed interference

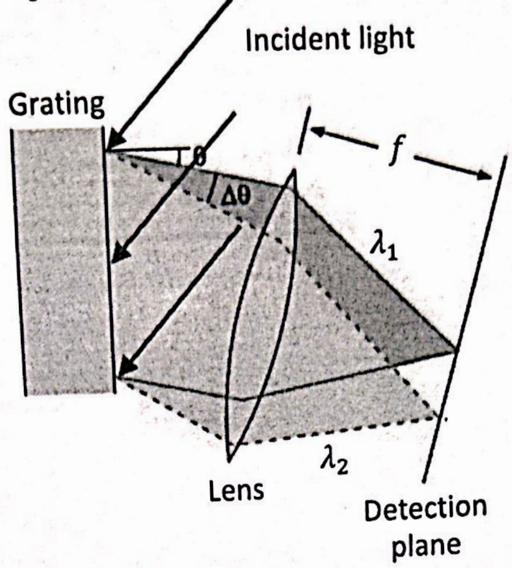


(c) (2 points) In a sentence, describe a strategy to improve the phase sensitivity of an interferometer constructed using this light source.

- use squeezed light + trade off amplitude noise

- increase intensity

6. (a) (8 points) Consider the following setup for analyzing the spectrum of an incident light collected from excited sodium vapor in a glass cell:



Suppose the grating has 1200 grooves per mm and it is illuminated with emission from sodium at an incident angle of 30°. What is the distance between two lines in sodium (at 588.9 nm and 589.5 nm) on the detection plane, if the focal length of the lens is 1 m?

Periodicity of grating $d = (1200/\text{mm})^{-1} = 8.33 \times 10^{-7}$ m +2

+2 $\lambda_{1,2} = d[\sin(\underset{30°}{\alpha}) - \sin(\beta_{1,2})]$

+2 $\beta_1 = -11.93° = -0.2082$ radians

$\beta_2 = -11.97° = -0.2089$ radians

+2 $\Delta x = f \Delta\beta = 0.736$ mm

(b) (2 points) In a sentence or two, *qualitatively* describe processes that relate to the atom's properties as well as its environment that affect the width of each emission line.

- Temperature of gas (Doppler broadening)
- Pressure / density of gas

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