ECE 535 Fall 2025
Homework #1 (60 points)
Due 9/17 at 11:59 pm on Canvas, in pdf format

Guidelines:

- Please submit a pdf document to Canvas with handwritten solutions, with your approach to each problem and the steps taken clearly laid out and written legibly. In cases where the solution requires plotting, the computer-generated plot should be accompanied by a brief handwritten explanation of your approach. This formatting requirement is worth **5 points** of the point total for each homework.
- Undergraduate students who wish to attempt the extra problem will receive up to 5 additional points for that homework.

1. (10 points) **Defining the second and the meter** The SI defines the meter in terms of the speed of light and the second. Today, the second is realized using the cesium (Cs)-133 hyperfine transition, but in the future optical transitions may be used instead. In this problem you will compare the cesium microwave frequency to an optical frequency (strontium (Sr) around 698 nm) for measurements of length and time.

(a) (3 points) Look up the value of the speed of light in vacuum, using a resource like the NIST database of constants (CODATA; https://physics.nist.gov/cuu/Constants/index.html). Using this SI defining constant, calculate the frequency of the optical transition of the Sr-87 optical clock, at 698.445 nm. Write your answer in THz.

(b) (3 points) Compute the ratio between the Sr clock frequency and the Cs clock frequency (also found using CODATA) that is used to define the second. Approximately how many ticks from the Cs clock occur during one oscillation of the Sr optical clock?

(c) (2 points) Suppose you want to measure the time it takes light tuned to the strontium (Sr) optical clock to travel 1 meter. Compare the number of cycles (of Sr vs Cs clock) within this time.

(d) (2 points) Explain why it may be advantageous to use an optical standard for length and time measurements.

2. (15 points) **Planck's radiation law**

(a) (8 points) Using data provided in this spreadsheet, fit the measured spectral emission from the sun to the Planck's radiation law and estimate the temperature of the sun T_{sun}. **Note that the sun's radiation only reaches earth over a small solid angle and so you may have to modify the expression for $\frac{dI}{d\lambda}$.** Plot the data and fitted curve on the same figure and compare your estimate of the sun's temperature with reported values on the web.

(b) (2 points) Explain the deviations between the spectral emission data from the sun and the Planck's radiation law.

(c) (5 points) The sun has a radius $R(= 7 \times 10^8\, m)$ while the radius of the earth is $r(= 6.37 \times 10^6\, m)$. The mean distance between the sun and the earth is $L(= 1.5 \times 10^{11}\, m)$. One can assume that both the sun and the earth absorb all electromagnetic radiation and that earth has reached a steady-state temperature T_{earth} that does not change with time. Find an approximate expression for the temperature T_{earth} of the earth in terms of $r, R, L,$ and T_{sun}. How does it compare with your answer in (a).

3. (15 points) **Johnson noise thermometry** You are to determine the temperature of a wire by taking measurements of the thermal noise voltage V_{rms} across this wire with known resistance $R = 150\,\Omega$ over a bandwidth $\Delta\nu = 1000\, Hz$. If the average RMS voltage was $V_{rms} = 2.1 \times 10^{-6}\, V$.

   (a) (2 points) What is the noise power associated with the measured V_{rms} and resistance R?

   (b) (3 points) What is the temperature of the wire based on the measured V_{rms}?

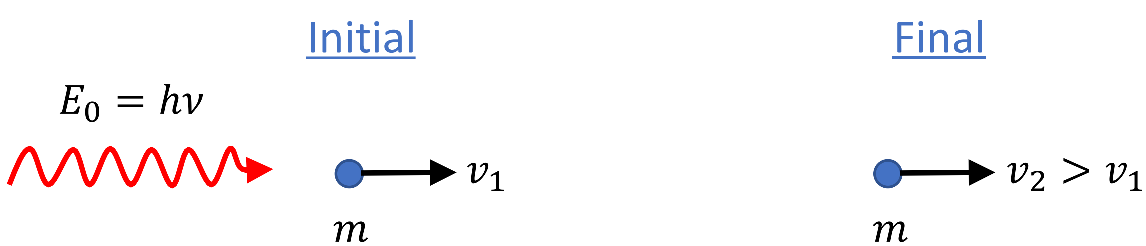
   (c) (6 points) Explain why the Johnson noise in a wire, $P = k_B T(\Delta\nu)$, scales linearly with the temperature T and measurement bandwidth $\Delta\nu$, whereas Stefan–Boltzmann law, $\frac{P}{A} = \sigma T^4$, scales as T^4 and independent of any measurement bandwidth.

   (d) (4 points) Does the model of the Johnson noise work at optical frequencies (say 600 nm)? Justify your answer.

4. (5 points) Calculate the de Broglie wavelength for the following:
   (a) An electron that has been accelerated through a potential difference of 54 V.
   (b) A 10-MeV electron.

5. (10 points) A laser pointer is normally incident on a small spherical particle (where the particle size is much smaller than the beam size) and is fully absorbed without loss by the particle, causing it to rotate. Provide a schematic drawing of the interaction, including the coordinate system, and write a valid expression for the electric field of the laser based on your drawing.

6. (10 points) **Extra problem for graduate students: can a free electron absorb a photon?**
   (a) (8 points) Consider a scenario in which a free electron with velocity v_1 absorbs a photon with energy $h\nu$ and increases its velocity to v_2. Evaluate whether it is possible for both energy and momentum to be conserved in this scenario.



   (b) (2 points) In a sentence or two, describe the condition under which the absorption of a photon by an electron is allowed.