Physics 731: Assignment #3

1. (a) In class, we calculated the even parity eigenstates and their associated energy eigenvalues for a particle of mass m bound in the finite square well:

$$V = \begin{cases} 0, & |x| < a \\ V_0, & |x| > a, \end{cases}$$

in which $V_0 > 0$. Determine the odd parity eigenfunctions and their associated energy eigenvalues for this potential, and discuss the limiting behavior as $V_0 \to 0$ and $V_0 \to \infty$.
(b) Find accurate numerical values for the bound state energy eigenvalues of a particle in the above finite square well potential, in which the parameter

$$R \equiv \left(\frac{2mV_0a^2}{\hbar^2}\right)^{1/2} = 4.$$

Find the solutions graphically (with reasonable precision) and numerically.

2. Show that for (spinless) particles moving in one dimension, the energy spectrum of bound states is always non-degenerate. (*Hint*: Assume the opposite is true, and show that there is a contradiction.)

3. (a) Use the Hermite generating function,

$$g(y,t) = e^{-t^2+2ty} = \sum_{n=0}^{\infty} \frac{t^n}{n!} H_n(y),$$

to prove the following expressions:

$$\begin{aligned} H_n(y) &= e^{y^2/2}\left(y - \frac{d}{dy}\right)^n e^{-y^2/2} \\ H_n'(y) &= 2nH_{n-1}(y) \\ H_{n+1}(y) &= 2yH_n(y) - 2nH_{n-1}(y), \end{aligned}$$

(b) and to evaluate

$$\int_{-\infty}^{\infty} dy\, e^{-y^2} H_n(y) H_{n'}(y).$$

4. Using wavefunctions, compute $\langle n'|p|n\rangle$ for the eigenstates of the one-dimensional simple harmonic oscillator (with frequency ω and mass m) to show that

$$\langle n'|p|n\rangle = i\sqrt{\frac{m\omega\hbar}{2}}\left(\sqrt{n+1}\,\delta_{n',n+1} - \sqrt{n}\,\delta_{n',n-1}\right).$$

Do this explicitly in two ways using (i) position space eigenfunctions, and (ii) momentum space eigenfunctions.

5. For (i) the ground state, and (ii) the first excited state, calculate the probability that a particle of mass m in the one-dimensional simple harmonic oscillator with frequency ω is farther from the origin than the classical turning points (where $E = V$).

6. **[S1r, S2 2.17, S3 2.20]** Show that for the one-dimensional simple harmonic oscillator,

$$\langle 0|e^{ikx}|0\rangle = \exp[-k^2\langle 0|x^2|0\rangle/2],$$

where x is the position operator.