ECE 535 Quantum Sensing

Lectures by Prof. Jennifer Choy.

not sure what this class is about...

Contents

1 Wave particle Nature and Matter waves

2 Diffraction and the nunncertainty principle

2.1 REsolution limits in electron and light microscopy

2.2 Diffraction of matter waves

2.3 Defining wave functions

- Probablistic interpretation WE model a particle as wave $\psi(x)$. The pdf:

$$P(x) = |\psi(x)|^2, \quad \int_{-\infty}^{\infty} P(x)\,\mathrm{d}x = 1. \tag{1}$$

- Wave function EM wave : $\vec{p} = \hbar\vec{k}, E = \hbar\omega$:

$$E \propto E_0 \exp\left(\frac{i}{\hbar(\vec{p}\cdot\vec{r} - E\omega t)}\right) \tag{2}$$

For a particle in 1D (Fourier Transform):

$$\begin{gathered} \psi(x,t) \propto \exp(i\hbar(p_0 x - Et)); \quad E_0 = \frac{p^2}{2}m \\ \Psi(x,t) \propto \int_{-\infty}^{\infty} \psi\,\mathrm{d}t \end{gathered} \tag{3}$$

2.4 The Uncertainty Principle

- Uncertainty Principle

$$\Delta p \Delta x \geq \frac{\hbar}{2}; \quad \Delta E \Delta t \geq \frac{\hbar}{2} \tag{4}$$

- The uncertainty principle is derived form the wave nature of matter The wave function is not normalized. Fix this by defining a **wave packet** that localizes the matter wave within a spatial interval Δx . Take analogy from light waves: a gaussian beam:

$$E(r,z,t) = E_0 \exp(-r^2/w^2)\exp(i(kz - \omega t)) \tag{5}$$

where w is the beam waist, a function of z .

A wave packet comprises of superposition of monochromatic waves with frequencies ω_j in interval $\Delta\omega$.

$$\psi(x,t) \tag{6}$$

To find time dependence of the peak:

$$0 = \frac{\mathrm{d}\omega}{(\mathrm{d}k)_{k_0}}t - x_{\max} \Rightarrow x_{\max} = \frac{\mathrm{d}\omega}{(\mathrm{d}k)_{k_0}}t = \frac{\hbar k_0}{m}t \tag{7}$$

where we read off the particle velocity $v = \frac{\hbar k_0}{m}t$. Eqiuvalant to group velocity of a wave packet.

---

3 CLassical and quantum limits to measuremetns

3.1 Zero-point energy

Recall Heisenburg Uncertainty Principle:

$$\Delta E \Delta t \geq \frac{\hbar}{2}. \tag{8}$$

In the case for a harmonic oscillator with parabollic potential

$$V(x) = \frac{1}{2} m \omega^2 x^2, \tag{9}$$

uncertainty principle implies a minimum energy (zero-point energy), which is the ground state energy of the quantum harmonic oscillator:

$$E_0 = \frac{1}{2} \hbar \omega. \tag{10}$$

This is observed in experiments:

- Electromagnetic field in a cavity: even in the absence of photons, there are fluctuations in the electric and magnetic fields. We can quantify this with the energy density

$$u = \varepsilon_0 \frac{\langle E^2 \rangle}{2} \Rightarrow E_{\text{rms}} = \sqrt{\langle E^2 \rangle} = \sqrt{\frac{\hbar \omega}{2 \varepsilon_0 V}} \tag{11}$$

  where V is the volume of the cavity.

Zero point energies of charges in objects (molecules, small particles, etc.) induce elemetromagnetic fluctuations that interact at small distances. This is the Casimir effect. On a microscopic scale, this is the van der Waals interaction.

4 Select solutions to Schrodinger equation

4.1 The Schrodinger Equation

$$H\psi = E\psi. \tag{12}$$

4.1.1 The free particle

The solution to the time-independent Schrodinger equation for a free particle is

$$\psi(x) = A \exp(ikx) + B \exp(-ikx), \quad k = \frac{\sqrt{2mE}}{\hbar}. \tag{13}$$

4.1.2 THe infinite potential well

this is so boring. check your 731 notes.