ECE 535 Fall 2025
Homework #5
Due 11/20 at 11:59 pm on Canvas, in pdf format

Guidelines:

- Please submit a pdf document to Canvas with handwritten solutions, with your approach to each problem and the steps taken clearly laid out and written legibly. In cases where the solution requires plotting, the computer-generated plot should be accompanied by a brief handwritten explanation of your approach. This formatting requirement is worth **5 points** of the point total for each homework.
- Undergraduate students who wish to attempt the extra problem will receive up to 5 additional points for that homework.

1. (10 points) Consider the decay of the 2P orbital in hydrogen for three isotopes: ^{1}H (nuclear spin $I = 1/2$), ^{2}D $(I = 1)$, and ^{3}T $(I = 3/2)$.
   (a) (7 points) *Ignoring spin-orbit and hyperfine interactions*, calculate the wavelengths of emission from each isotope using the Rydberg constant and reduced mass of the electron. What wavelength resolution is needed in a spectrometer to distinguish these lines?
   (b) (3 points) Draw the energy level diagram (but do not calculate the energies) for the hyperfine structure for each isotope.

2. (10 points) The energy of the ground state in lithium is $E = -5.39\ eV$, while that for a Rydberg state of $n = 20$ is $E = -0.034$ eV. Determine the quantum defects for these states and compare their mean Bohr radii. For the Rydberg atom, estimate the effective charge experienced by the electron and justify your answer.

3. (20 points) Consider a rubidium-87 atom in a weak magnetic field.
   (a) (6 points) Draw the energy diagram for transitions between the $5^2S_{1/2}$ and $5^2P_{3/2}$ states. Indicate and label all energy levels and the allowed transitions.
   (b) (7 points) Draw the frequency spectrum (without worrying about the relative intensities of the lines) observed along the applied magnetic field and indicate the polarization of the spectral lines. Calculate the spacing between the spectral lines for an applied field of 1 G, in MHz.
   (c) (7 points) Draw the frequency spectrum (without worrying about the relative intensities of the lines) observed perpendicular to the applied magnetic field and indicate the polarization of the spectral lines. Calculate the spacing between the spectral lines for an applied field of 1 G, in MHz.

4. (15 points) A cesium atomic clock, using Cs-133 atoms in the $6s$ state with $I = 7/2$, operates on the ground state hyperfine transition at 9.192631770 GHz. The clock is a Ramsey interferometer operating at "mid-fringe", where the signal vs frequency has a maximum slope and the atom population is at 50% in each hyperfine ground state. In this linear region, small population noise σ_p maps to frequency noise $\sigma_\nu \approx \Delta\nu\, \sigma_p$. The measured linewidth $(\Delta\nu)$ of the transition is 1 Hz.
   (a) (5 points) Draw the hyperfine energy structure in the ground state.
   (b) (5 points) Estimate the contributions of shot noise as a function of the number of atoms (N) used in each measurement cycle to the frequency uncertainty of the clock. Include the effects of detector quantum efficiency η = 0.85.
   (c) (5 points) Estimate the contributions of quantum projection noise as a function of the number of atoms (N) used in each measurement cycle to the frequency uncertainty of the clock. Assume perfect detection efficiency and that at the end of the experiment, 50% of

the atoms are in the state you are detecting. Assume perfect detection and binomial quantum projection noise with 50% population in the measured state.

5. (10 points) **Extra problem for graduate students**. Hydrogenic ions are atoms that have one electron bound to a nucleus with atomic number $Z \geq 1$. Determine the scaling factor of Z in each of the following, relative to the values in the case of the hydrogen atom:
   Hint: You should not have to use explicit wave functions for this problem. Please consider the dimensions of the quantities of interest. For example, for length, determine the scaling of the characteristic length a_c relative to Z and the Bohr radius a_0.
   (a) (2 points) Expectation values of r, $1/r$, $1/r^3$, where r is the distance between the electron and the nucleus.

   (b) (2 points) Expectation values of the potential energy (V) and total energy (E).

   (c) (1 point) The probability density of finding the electron at the center of the nucleus.

   (d) (3 points) Internal electric ($\mathrm{E_c}$) and magnetic (B_c) fields determined using electrostatic equations.

   (e) (2 point) Calculate the internal magnetic field associated with a tin ion consisting of a single electron, in terms of the magnetic field for a single hydrogen atom.