ECE 601/901 Fall 2024
Practice problems for Exam 2

Exam 2 will be an in-class exam on 11/25. Please prepare a double-sided note sheet and bring your calculator.

Note that these are intended to be supplementary problems to homework and in-class questions; please still review those ahead of the exam. I will be posting solutions to the problems by Saturday morning.

1. In scanning tunneling microscopy (STM), an electrically conductive tip is brought very close to the surface a material to be imaged. When the tip is sufficiently close, electrons can tunnel through the vacuum between the tip and the sample, and a current is measured. For this problem, use the following values for the STM setup: work function of the sample $W = 4.5\ eV$, distance between the tip and the sample surface $d = 1\ nm$, applied voltage between the tip and the sample $V_a = 1\ V$.
   (a) Calculate the probability of the electron tunneling from the tip to the sample.

   The tunneling probability can be calculated using:
   $$T = \frac{1 - E/V_0}{\left(1 - \frac{E}{V_0}\right) + \left(\frac{V_0}{4E}\right)\sinh^2 L\kappa}$$
   where $\kappa = \frac{\sqrt{2m_e(W - eV_a)}}{\hbar} = 9.583 \times 10^9\ m^{-1}$. Therefore $\kappa L = 9.58 \gg 1$ and we can use the simpler form
   $$T \approx \frac{16E}{V_0^2}(V_0 - E)e^{-2\kappa d} = 1.3123 \times 10^{-8}$$

   (b) Qualitatively describe how the probability calculated in (a) is related to the measured current.

   Probability should be proportional to the current, such that $I \propto e^{-2\kappa L}$

   (c) Describe what happens to the measured current if (i) d is doubled or (ii) W is halved.

   The measured current would decrease by a factor of $e^{-3\kappa d}$ if d is doubled. If W is halved, the measured current would increase.

2. Determine the degeneracy of an atomic hydrogen state under the following considerations:
   (a) An H atom with principal quantum number n, but magnetic effects and the electron spin can be ignored.

   n^2. This comes from the fact that for each principal quantum number n, the orbital angular momentum quantum number l can take on n number of integer values from 0 to $n - 1$, and for each l, we can have $2l + 1$ magnetic quantum numbers from $-l$ to l. Summing everything together gives us n^2.

   (b) An H atom with principal quantum number n, but the electron spin must be considered.

$2n^2$ if relativistic and magnetic effects (fine and hyperfine structure) can be ignored. This is due to the fact that the electron spin can have values $m_s = \pm 1/2$.

If we include fine structure, then there is a unique energy for each unique combination of n, l, j values, where $j = l \pm 1/2$. The degeneracy for each state becomes $(2j + 1)$ for all possible values of l and the corresponding j values. This accounts for all the degeneracy involved since l and s are no longer independent and are coupled through j.

3. Using the quantum numbers (n, l, m_l, m_s), write down all possible sets of quantum numbers for the 6f state of atomic hydrogen.

For an atomic hydrogen state designated as $6f$, n is 6 and l is 3.

Given these quantum numbers, m_l can take on values -3, -2 , -1, 0, 1, 2.

$m_s = \pm 1/2$.

Therefore, the possible sets of quantum numbers for the 6f state of atomic hydrogen (n, l, m_l, m_s) are:

(6, 3, -3, -1/2)
(6, 3, -3, +1/2)
(6, 3, -2, -1/2)
(6, 3, -2, +1/2)
(6, 3, -1, -1/2)
(6, 3, -1, +1/2)
(6, 3, 0, -1/2)
(6, 3, 0, +1/2)
(6, 3, +1, -1/2)
(6, 3, +1, +1/2)
(6, 3, +2, -1/2)
(6, 3, +2, +1/2)
(6, 3, +3, -1/2)
(6, 3, +3, +1/2)

4. Describe how a grating spectrometer works (draw a schematic to support your description) and discuss factors that affect its resolution.

Review lecture notes (Topic 2b)

5. Find whether the following atomic transitions are allowed, and, if they are, find the energy and wavelength involved and whether the photon is absorbed or emitted.
   a. (5, 2, 1, ½) → (5, 2, 1, -½)
   b. (4, 3, 0, ½) → (4, 2, 1, -½)
   c. (5, 2, -2, -½) → (1, 0, 0, -½)
   d. (2, 1, 1, ½) → (4, 2, 1, ½)

   Selection rules:
   - Transitions between any values of n are allowed

- $\Delta l = \pm 1$
- $\Delta m_l = 0, \pm 1$
- $\Delta m_s = 0$ (not covered in class, but spin flips are not allowed during a single transition)

The only allowable transition that satisfies these rules is (d), in which a photon is absorbed since the electron is excited from $n = 2$ to $n' = 4$. The associated photon energy and wavelength are 2.55 eV and 487 nm.

7 Calculate the probability of an electron in the ground state of the hydrogen atom actually being inside the nucleus (which has a radius of $1 \times 10^{-15}\ m$). You may use the assumption that the electron's wave function is constant over the entire nucleus.

To calculate the probability of finding the electron inside the nucleus of a hydrogen atom, we'll use the ground state wave function for hydrogen, which is given by:

$$\psi_{1s} = \frac{1}{\sqrt{\pi}} \left(\frac{1}{a_0}\right)^{3/2} e^{-Zr/a_0}$$

where the Bohr radius $a_0 = 5.29177 \times 10^{-11}$ m. Note that if a question like this were to be given on the exam, you would be provided with the expressions for the hydrogen atom wavefunctions.

The probability of the particle being at position r_0 within a differential volume dV is $P(r_0) = |\psi_{1s}(r_0)|^2\ dV$. Since we are asked to assume the wave function is constant over the entire nucleus, we will evaluate $|\psi_{1s}(0)|^2 = \frac{1}{\pi}\left(\frac{1}{a_0}\right)^3$.

The probability of finding the electron inside the nucleus is then the product of this probability density and the volume of the nucleus: $\frac{1}{\pi}\left(\frac{1}{a_0}\right)^3 (\frac{4}{3}\pi r_{nucleus}^3)$, where $r_{nucleus} = 1 \times 10^{-15}\ m$.

Plugging in the numbers, we get that the probability is very small, around 9×10^{-15}.

8. Describe the measurement setups that can allow you to measure the first- and second-order autocorrelation functions for a light source.

Review lecture notes (Topic 3a and b)