Trewartha Undergraduate Research Proposal

Quantum Error Mitigation in Bosonic Codes:
A Study of Gottesman-Kitaev-Preskill Codes Under Pure Loss

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October 2025

Objective / Abstract

Can software alone reduce the measurement errors of today's quantum devices without hardware changes? Zero-Noise Extrapolation (ZNE) is a practical error-mitigation technique that estimates the noise-free value of an observable by measuring it at several controlled noise levels and extrapolating to the zero-noise limit. While ZNE has been demonstrated for discrete qubits, its application to bosonic codes—and in particular to Gottesman–Kitaev–Preskill (GKP) encodings under pure loss (the dominant decoherence mechanism in superconducting cavities)—has not been systematically explored.

This thesis presents a comprehensive theoretical and numerical study of ZNE for GKP codes subject to pure loss. Using a custom Python simulator, we evaluate single- and two-qubit logical observables under realistic loss conditions. The key finding is that ZNE successfully recovers ideal expectation values with high fidelity in shallow-to-moderate loss regimes: at moderate photon loss ($\sim$ 18%), recovery is complete within numerical error; even at significant loss ($\sim$ 33%), approximately 96% of the ideal value is restored.

This project makes two primary contributions. First, it provides (to our knowledge) the first quantitative demonstration that ZNE can extend the useful operating regime of GKP-encoded qubits under pure loss. Second, it delivers a self-contained pedagogical treatment of the underlying theory and simulation methodology, from foundational concepts through implementation details, significantly lowering the barrier to entry for future researchers. The documented, open-source framework enables direct reproduction and extension of these results.

1 Introduction

While holding immense promise and world-changing potential, quantum processors are limited by environmental noise that degrades computation and biases observable estimates. Full quantum error correction (QEC) is the long-term solution, but near-term devices lack the resources to implement it at scale. **Quantum error mitigation (QEM)** techniques such as **Zero-Noise Extrapolation (ZNE)** offer a pragmatic alternative: by purposefully varying a noise-related control parameter, fitting the resulting trend, and extrapolating to zero noise, one can reduce bias in expectation values without adding qubits.

This project applies ZNE to **bosonic Gottesman–Kitaev–Preskill (GKP) codes**, which encode a qubit in a harmonic oscillator using a periodic lattice in phase space. GKP codes are naturally resilient to small displacement errors but remain sensitive to **pure loss**—photon loss that is the dominant decoherence channel in many quantum platforms (e.g., superconducting cavities, optical/microwave cavities and photonic platforms). The central research question is: *Can ZNE, in concert with Petz recovery, reliably improve logical observables for GKP codes subject to pure loss, and over what ranges of loss x and code energy $\bar{n}$?*

Recent work by Zheng *et al.* (2025) [6] characterized GKP performance under pure loss and amplification channels but did not investigate error-mitigation overlays. Prior demonstrations of ZNE have focused on discrete-variable codes [5]. The present study applies ZNE specifically to GKP codes under pure loss and quantifies its effectiveness, thereby addressing an unexplored direction at the intersection of bosonic encoding and error mitigation.

Practical significance: If successful, this software-only approach would allow research labs to extract clearer, more reliable measurements from existing quantum devices, effectively extending the useful operating regime of GKP-encoded processors without requiring additional physical resources. This could lower the energy cost (mean photon number) needed to achieve target logical fidelities in applications ranging from quantum simulation to optimization.

Pedagogical contribution: Beyond the research findings, this thesis is designed to serve as a comprehensive reference for junior researchers entering this field. The text provides a self-contained treatment starting from undergraduate quantum mechanics, progressing through harmonic oscillators and phase-space formalism, GKP lattice construction, the pure-loss channel, and culminating in the complete simulation methodology. Combined with documented, open-source code, this work significantly lowers the barrier to entry and reproduction cost for future students and researchers, enabling them to build directly upon these results without reconstructing the foundational infrastructure.

The scope is deliberately focused:

- **Noise Model:** Pure loss only.
- **System Size:** 1–2 logical qubits.
- **Loss Regime:** Shallow-to-moderate loss ($x \lesssim 0.5$).
- **Methodology:** Simulation and theoretical analysis (no hardware implementation).

2 Methodology

2.1 Simulation Framework (Completed)

A simulation framework was implemented in Python to model the entire physical process. This includes the generation of finite-energy square-lattice GKP codewords, the application of a pure-loss Gaussian channel, and information recovery using a mathematically principled 'best-possible' decoder (**Petz (transpose) map**). The code makes a qubit-level measurement (logical Pauli $\langle \bar{X} \rangle$, $\langle \bar{Z} \rangle$, and $\langle \bar{X} \otimes \bar{X} \rangle$) across grids in both code energy (mean photon number $\bar{n}$) and loss (decay depth $x = -\ln \eta$). Numerical truncation uses a Fock cutoff n_{cut}, and all reported results are converged in n_{cut}. The framework has been validated against no-loss and high-loss limits and published benchmarks.

2.2 ZNE Protocol (Completed for Key Cases)

For a fixed physical loss level x, the logical observable is evaluated at multiple finite-energy code parameters (mean photon numbers $\bar{n}$), which serves as a controllable robustness knob. The results are then extrapolated to the ideal-code limit of $\bar{n} \to \infty$. GKP codes exhibit empirically monotonic trends in relevant expectation values versus $\bar{n}$, a crucial property that enables reliable extrapolation. Reported recovery percentages include uncertainties derived from the extrapolation fit.

2.3 Analytical Program (Planned)

The thesis will include a dedicated analytical chapter. The goal is to derive the conditions under which the trend of an observable is monotone in $\bar{n}$ for the pure-loss channel with Petz recovery. This analysis aims to produce a closed-form expression or bound for the derivative $\partial\mathbb{E}[O]/\partial\bar{n}$ and to connect the curvature of the trend to extrapolation error, thereby providing a rigorous theoretical foundation for the numerical results.

2.4 Pedagogical Deliverables (Planned)

A primary goal of the thesis is to serve as a self-contained pedagogical resource. It will guide the reader from foundational concepts to advanced topics: (1) harmonic oscillator and phase-space basics; (2) GKP lattice construction and finite-energy approximations; (3) the pure-loss channel (from both Kraus and master-equation perspectives); and (4) a "theory-to-computation" cookbook detailing the simulation methodology. The text will assume only a background in undergraduate quantum mechanics.

3 Project Demand

This thesis is demanding for an undergraduate project due to three factors:

- **Graduate-Level Content:** The project requires mastery of topics beyond the standard undergraduate curriculum, including open quantum systems, the bosonic stabilizer formalism, and functional analysis for operator theory.
- **Infrastructure Development:** The work involved building a simulator from scratch, including its validation and documentation, rather than merely analyzing data from existing tools.
- **Dual Deliverables:** The project will produce both a concise research paper for a technical audience and a comprehensive, pedagogical thesis for an interdisciplinary one.

4 Funding Impact

Grant support will primarily fund the student's time, enabling a deeper and more complete project scope. The funding will allow for an increase in dedicated research hours from approximately 10 to 20 per week, and will enable focused work over holidays.

Without Funding (≈10 hrs/wk)	**With Funding (≈20 hrs/wk)**
Core single- & two-qubit ZNE results	Expanded two-qubit suite with error analysis
Minimal pedagogical chapters	Full pedagogical chapters and appendices
Partial analytical sketches	Complete analytical section with proofs/bounds
Unpolished code repository	Public, documented repository with tutorials

A minor portion of the funds will cover AI API expenses used for literature triage and code debugging, with all technical contributions independently verified. Computational time is provided by the department.

References

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[5] T. Giurgica -Tiron, Y. Hindy, R. LaRose, A. Mari, and W. J. Zeng, "Digital zero-noise extrapolation for quantum error mitigation," *Proc. 2020 IEEE Int. Conf. on Quantum Computing and Engineering (QCE)*, pp. 306–316 (2020). doi:10.1109/QCE49297.2020.00045.

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