Final Project – Due 05/05/2025

- *The final project should be handed in as a term paper*
- *Keep the main text (with main answers + background info) in prose form and put all calculations in the Appendix (handwritten appendix is ok).*
- *main text: 2-3 pages (including title + name) 12pt, 1.5 line spacing, 1in margin + figures (not counting towards the 3 page limit).*
- *The itemized presentation a)-d) is a guide to you, and should not appear in the main text of the term paper (it may appear in the appendix if you wish). For the main text, choose your own free form of scientific writing, making sure that all aspects of the problem listed in a)-d) are covered.*
- *Upload to Canvas by 05/05/2025, 11 59 pm.*

Superconducting Qubits [10 + 10 + 10 + 10 = 40 points]

Info: The fundamental degree of freedom in superconducting quantum devices is the superconducting phase $\phi \in [-\pi, \pi)$. No knowledge of superconductivity is needed to solve this problem set, simply regard ϕ as the position of a particle on a ring.

The entire problem is based on the Hamiltonian

$$\hat{H} = 4E_C\left[\hat{N} - n_g\right]^2 - E_J^{(1)}\cos(\hat{\phi}) - \frac{E_J^{(2)}}{2}\cos(2\hat{\phi}). \quad (1)$$

where $[\hbar\hat{N}, \hat{\phi}] = -i\hbar$. The operator $\hat{N}$ is called the "charge" operator, and $\hbar\hat{N}$ plays a role analogous to the (angular) momentum of a particle on a ring. We can thus represent it as $\hbar\hat{N} = -i\hbar\partial_\phi$. Here, $E_J^{(1)}, E_J^{(2)}, E_C$ are positive constants with dimension of energy and $n_g \in \mathbb{R}$ is a dimensionless number (background charge).

a) Solve the problem in the limit $E_C \gg E_J^{(1)}, E_J^{(2)}$.

i) Consider $E_J^{(1)} = E_J^{(2)} = 0$. Plot the levels as a function of n_g. Calculate and interpret the probability density to find the particle at position ϕ for these eigenstates.

ii) Now switch on $E_J^{(1)}, E_J^{(2)} \ll E_C$: Calculate the leading correction to the ground state energy at $n_g = 0$. Where in n_g space is the effect of the perturbation strongest?

iii) Consider the vicinity of $n_g = 1/2$ and choose the lowest two states as a qubit.

* Derive an effective, 2×2 Hamiltonian describing the dynamics of the qubit.
* Calculate and plot the probability to find the particle with charge $N \in \mathbb{Z}$ in this regime. Consider the probability that the ground state has charge 0 and plot it as a function of $n_g - 1/2$ for $E_J^{(1)} = 0.08E_C$, $E_J^{(2)} = 0$.

b) Solve the problem in the limit $E_C \ll E_J^{(1)}, E_C \ll E_J^{(2)}$ and $n_g = 0$.

i) Calculate all energy levels to leading order in $E_J^{(1,2)}/E_C$ as well as the wave function of the lowest three levels.

ii) Calculate and plot the probability to find the particle at position ϕ for these three states using $E_C = (E_J^{(1)} + 2E_J^{(2)})/800$.

iii) Calculate the expectation value and variance for the charge in the ground state and plot it as a function of $\sqrt{8E_C/(E_J^{(1)} + 2E_J^{(2)})}$.

iv) Calculate the subleading correction to the first three energy levels.

v) Interpret your result: Are the levels equidistant?

c) Now consider the case $E_J^{(1)} = E_J, E_J^{(2)} = E_J^2/4E_C$ and find an exact solution for the ground state and the ground state energy for arbitrary E_J/E_C at $n_g = 0$. Compare to results from a) and b). Plot the probability density $p(\phi)$ for a particle in the ground state for $E_J = 0.4E_C$, $E_J = 4E_C$ and $E_J = 40E_C$.

d) In your paper, interpret the results from parts a)-c). Include some some background on superconducting quantum devices and explain what a "charge qubit" is and what a "transmon" is. You may use Sec. 2.1.1 of Kjaergaard, Morten, et al. "Superconducting qubits: Current state of play." Annual Review of Condensed Matter Physics 11.1 (2020): 369-395.

You can be very superficial about the physics of superconductors and where the basic Hamiltonian comes from, since all of this is covered in classes about solid state physics and/or quantum devices but was not explained in Physics 531.

Address the following questions, if you wanted to use the lowest two levels in the system to define a qubit:

- Why are subleading corrections to the energy levels important for the qubit discussed in b)?
- Temporal fluctuations of n_g are often a serious problem in superconducting quantum devices. In this respect explain why the qubit of part b) is advantageous as compared to the qubit from part a).