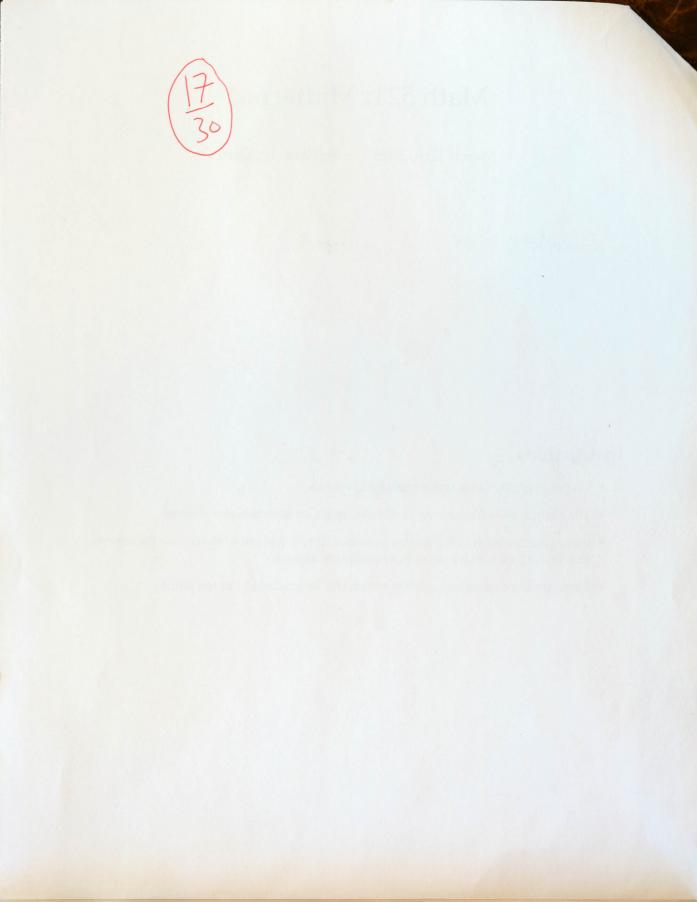
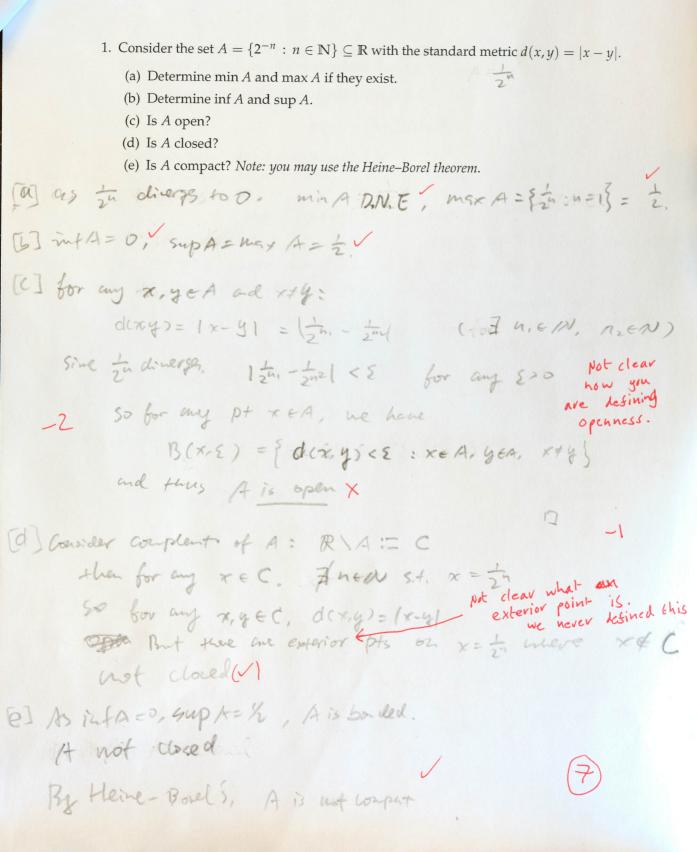
## Math 521: Midterm 1

March 12th, 2025 9:57am-10:42am

## **Instructions**

- Do not open the exam until instructed to do so.
- The exam is closed book—no textbooks, notes, or smartphones allowed.
- Write your answers in the spaces provided. If you run out of space, use the reverse side of the paper. Extra paper is available on request.
- There are three questions, each of which will be graded out of ten points.





- 2. Let S and T be two non-empty subsets of  $(0, \infty)$ . Define  $R = \{st : s \in S, t \in T\}$  to be the set of all products of elements from S and T.
  - (a) Suppose that S and T are bounded. Prove that

$$\sup R = (\sup S)(\sup T).$$

(b) Prove that if  $\sup S = \infty$  then  $\sup R = \infty$ . [a] Pf: consider urbitrary element reR. r=st dses, tteT. Shee 5,7 and bonded, SESUPS (ASES), LESUPT (ALET) so v= St & sups. sups. sups. sups is an apper brack · Consider whitray upper bond in of R. r < m Suppose for contradictor un < Sup 5. Sup T. then I E>> st m= sups. sup I - C The details of this don't work. You probably wanted m=sup Ssup T-E chose tet st. t > SupTles st>sups.supT in your 48 this not sufficient Chose St S s.t. 5> sup 5/25 m=rt = (Sup 7) Sup ) = to (Sup 5)(Sup 7) < \ Sup 5 Sup 7 ,2

=> m Z Saps . Sup T => Sup R= (Sup 5) (Sup T) and obtain a intradiction.

(b) given sup S = 00. case 1 T bound, then Sup T>0 as T = (0,00)

then by (9), sup R = Sup S sap T = 00 Case 2 7 not borded sup T = 0 This result from (a) was only true for bounded sets. then by (9) sup R = sups sup T = 00 It loss not apply

a useful symbol to indicate limiting Lehavior But you can't do arithmetic with it.

- 3. Let  $s \in \mathbb{R}$ , and let  $(s_n)$  be a sequence where  $s_n \neq s$  for all n.
  - (a) Suppose that  $\limsup \left| \frac{s_{n+1}-s}{s_n-s} \right| = L$ , where  $L \in [0,1)$ . Prove that  $\lim_{n\to\infty} s_n = s$ .
  - (b) Suppose that  $\liminf \left| \frac{s_{n+1}-s}{s_n-s} \right| = 0$ . Does  $(s_n)$  always converge to s? Either prove the result or find a counterexample.

(a) to proce lim 5n=5, Suffice to proce | Sn-5 | < 8 (4 n, 3 8>0)

Given linsup | 5/2-5 |= L, where osl <1 True, but more detail tout for you that segment Sn-5 converges needed about how you know this.

and 1600 lines Su-5 =0 as wanted why does it converge to 0?

(5) consider (Sh) = (1) /2. and S = D

When inf (Snei-5) = limite ( +12.1/2 -1) = of Not true.

But the is an alternating sequel and owes not comerge to o; an conter example that true.

It imvages to 0.

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