Physics 415
Spring 2025
Homework 8
Due Monday, April 7, 2025

This assignment covers material in Chapter 7 of Reif. I recommend reading through the text and also Lectures Notes 24-25. Note the assignment due date.

Problem 1: (Paramagnetism, adapted from Reif 7.13) In Lecture 19 we showed that the average moment of a spin-1/2 paramagnet at temperature T in a field H is

$$\overline{\mu_z} = \frac{g\mu_B}{2} \tanh\left(\frac{g\mu_B H}{2T}\right). \tag{1}$$

For general spin J, we showed in Lecture 24 that

$$\overline{\mu_z} = g\mu_B J B_J\left(\frac{g\mu_B H}{T}\right), \tag{2}$$

where B_J is the Brillouin function

$$B_J(x) = \frac{1}{J}\left\{\left(J+\frac{1}{2}\right)\coth\left[\left(J+\frac{1}{2}\right)x\right] - \frac{1}{2}\coth\left(\frac{x}{2}\right)\right\}. \tag{3}$$

Show that (2) reduces to (1) for $J = 1/2$.

Problem 2: (Paramagnetism, adapted from Reif 7.14) Consider an assembly of non-interacting magnetic atoms with a density of n atoms per unit volume at a temperature T and describe the situation *classically*. Then each magnetic moment $\boldsymbol{\mu}$ can make any arbitrary angle θ with respect to a given direction (call it the z direction). In the absence of a magnetic field, the probability that this angle lies between θ and $\theta + d\theta$ is simply proportional to the solid angle $2\pi \sin\theta d\theta$ enclosed in this range. In the presence of a magnetic field H in the z direction, this probability must further be proportional to the Boltzmann factor $e^{-\beta E}$, where E is the magnetic energy of the moment $\boldsymbol{\mu}$ making this angle θ with the z axis. Use this result to calculate the expression for the mean magnetic moment per unit volume M_z.

Problem 3: (Paramagnetism, adapted from Reif 7.15) Consider the quantum mechanical expression (2) for M_z (obtained by multiplication by density n) in the limit where the spacing between the magnetic energy levels is small compared to T, i.e., where $\eta \equiv g\mu_B H/T \ll 1$. Assume further that the angle θ between $\mathbf{J}$ and the z axis is almost continuous, i.e., that J is so large that the possible values of $\cos\theta = m/J$ are very closely spaced; to be specific assume that J is large enough that $J\eta \gg 1$. Show that in this limit the general expression (2) for $\overline{M_z}$ does approach the classical expression derived in the preceding problem.

Problem 4: (Kinetic theory, adapted from Reif 7.20) An ideal monatomic gas is in thermal equilibrium at room temperature T so that the molecular velocity distribution is Maxwellian.

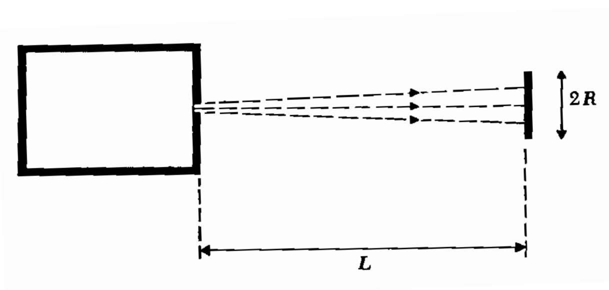
(a) If v denotes the speed of a molecule, calculate $\overline{(1/v)}$. Compare this with $1/\bar{v}$.

(b) Find the probability that a gas molecule has energy in the range between ϵ and $\epsilon + d\epsilon$.

Problem 5: (Kinetic theory, adapted from Reif 7.21) What is the most probable kinetic energy $\tilde{\epsilon}$ of molecules having a Maxwellian velocity distribution? Is it equal to $\frac{1}{2}m\tilde{v}^2$, where $\tilde{v}$ is the most probable speed of the molecules?

Problem 6: (Kinetic theory, adapted from Reif 7.24) A thin-walled vessel of volume V, kept at constant temperature, contains a gas which slowly leaks out through a small hole of area A. The outside pressure is low enough that leakage back into the vessel is negligible. Find the time required for the pressure in the vessel to decrease to $1/e$ of its original value. Express your answer in terms of A, V, and the mean molecular speed $\bar{v}$.

Problem 7: (Kinetic theory, adapted from Reif 7.31) An enclosure contains gas at a pressure p and has in one of its walls a small hole of area A through which gas molecules pass into a vacuum by effusion. In this vacuum, directly in front of the hole at a distance L from it, there is suspended a circular disk of radius R. It is oriented so that the normal to its surface points toward the hole (see figure). Assuming that the molecules in the effusing beam get scattered elastically from this disk, calculate the force exerted on the disk by the molecular beam.



Problem 8: (Kinetic theory, drag coefficient) Consider an ideal gas of particles with mass m and density n in a cylindrical container kept at an equilibrium temperature T. The cylinder is closed with a movable piston of area A. Use the Maxwell distribution to calculate the drag coefficient $\eta = -\partial F/\partial u$, where F is the force exerted by the ideal classical gas on the piston moving with a low velocity u, assuming that collisions of the gas particles with the piston are elastic. In this problem we ignore the friction between the piston and walls of the container.