Physics 415
Spring 2025
Homework 5
Due Friday, March 7, 2025

This assignment covers material in Chapter 5 of Reif. I recommend reading through the text and also Lectures Notes 12-16.

Problem 1: (Plastic rod, adapted from Reif 5.14) In a temperature range near absolute temperature T, the tension force F of a stretched plastic rod is related to its length L by the expression

$$F = aT^2(L - L_0) \tag{1}$$

where a and L_0 are positive constants, L_0 being the unstretched length of the rod. When $L = L_0$, the heat capacity C_L of the rod (measured at constant length) is given by the relation $C_L = bT$, where b is a constant.

(a) Write down the fundamental thermodynamic relation for the system, expressing dS in terms of dE and dL.

(b) The entropy $S(T, L)$ of the rod is a function of T and L. Compute $(\partial S/\partial L)_T$.

(c) Knowing $S(T_0, L_0)$, find $S(T, L)$ at *any* other temperature T and length L. (It is most convenient to calculate first the change of entropy with temperature at the length L_0 where the heat capacity is known).

(d) If one starts at $T = T_i$ and $L = L_i$ and stretches the thermally insulated rod quasi-statically until it attain the length L_f, what is the final temperature T_f? Is T_f larger or smaller than T_i?

(e) Calculate the heat capacity $C_L(L, T)$ of the rod when its length is L instead of L_0.

(f) Calculate $S(T, L)$ by writing $S(T, L) - S(T_0, L_0) = [S(T, L) - S(T_0, L)] + [S(T_0, L) - S(T_0, L_0)]$ and using the result of part (e) to compute the first term in square brackets. Show that the final answer agrees with that found in (c).

Problem 2: (Joule-Thomson process for van der Waals gas, adapted from Reif 5.20) In Lecture 15, we demonstrated that the Joule-Thomson (JT) expansion process occurs at constant enthalpy $H = E + pV$. We argued that, in a certain region of the (T, p) plane, the JT process leads to the cooling of a fluid while in the other region warming, the two regions separated by the "inversion curve." Find the inversion curve for the van der Waals gas, which is a non-ideal gas described by the equation of state

$$\left(p + \frac{a}{v^2}\right)(v - b) = RT \quad (T \text{ in degrees}), \tag{2}$$

where $v = V/\nu$ is the molar volume. Express your result in terms of reduced variables $p' = p/p_c$ and $T' = T/T_c$, where $T_c = 8a/27bR$ and $p_c = a/27b^2$. Make a sketch of the inversion curve in the (T', p') plane.

Problem 3: (Heat pump, adapted from Reif 5.22) Refrigeration cycles have been developed for heating buildings. The procedure is to design a device which absorbs heat from the surrounding earth or air outside the house and then delivers heat at a higher temperature to the interior of the building. (Such a device is called a "heat pump.")

(a) If a device is used in this way, operating between the outside absolute temperature T_0 and an interior absolute temperature T_i, what would be the maximum number of kilowatt-hours of heat that could be supplied to the building for every kilowatt-hour of electrical energy needed to operate the device?

(b) Obtain a numerical answer for the case that the outside temperature is 0°C and the interior temperature is 25°C.

Problem 4: (Heat engine, adapted from Reif 5.23) Two identical bodies, each characterized by a heat capacity at constant pressure C which is independent of temperature, are used as heat reservoirs for a heat engine. The bodies remain at constant pressure and undergo no change of phase. Initially, their temperatures are T_1 and T_2, respectively; finally, as a result of the operation of the heat engine, the bodies will attain a common final temperature T_f.

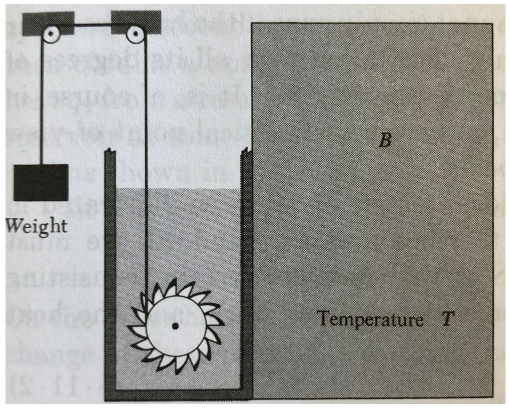
(a) What is the total amount of work W done by the engine? Express the answer in terms of C, T_1, T_2, and T_f.

(b) Use arguments based upon entropy considerations to derive an inequality relating T_f to initial temperatures T_1 and T_2.

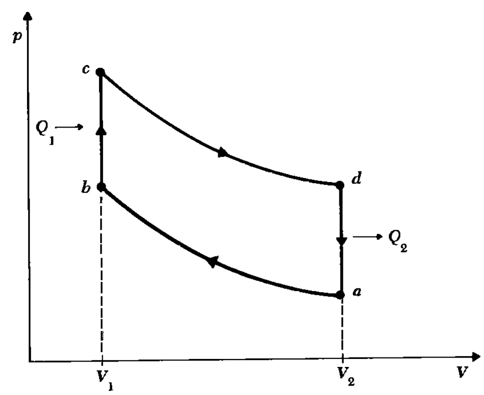
(c) For given initial temperatures T_1 and T_2, what is the maximum amount of work obtainable from the engine?

Problem 5: (Perfect engine, adapted from Reif 5.25) Consider the physical situation illustrated in the figure below, Suppose that under the influence of gravity (g = 980 cm sec^{-2}) the weight, having a mass m = 50 grams, is allowed to descend a distance L = 1 cm before coming to rest on a platform. In this process the weight turns the paddle wheel and raises the temperature of the liquid by a slight amount above its original temperature of 25° C.

Calculate the probability that, as a result of a spontaneous fluctuation, the water gives off its energy to the weight and raises it again so as to restore it to height of 1 cm or more.



Problem 6: (Gasoline engine, adapted from Reif 5.26) A gasoline engine can be approximately represented by the idealized cyclic process *abcd* shown in the figure of pressure p versus volume V of the gas in the cylinder. Here $a \to b$ represents the adiabatic compression of the air-gasoline mixture, $b \to c$ the rise in pressure at constant volume due to the explosion of the mixture (notice there is no "hot reservoir", instead the heat is produced internally by burning the fuel), $c \to d$ the adiabatic expansion of the mixture during which the engine performs useful work, and $d \to a$ the final cooling down of the gas at constant volume (in reality the hot gas is expelled and replaced by a new mixture at lower temperature and pressure).



Assume this cycle to be carried out quasi-statically for a fixed amount of ideal gas having a constant specific heat. Denote the specific heat ratio by $\gamma = c_p/c_V$. Calculate the efficient η (ratio of work performed to heat taken in Q_1) for this process, expressing your answer in terms of V_1, V_2, and γ.

Problem 7: (Second law of thermodynamics and Carnot engines) Consider two heat reservoirs at temperatures T_1 and T_2 with $T_1 > T_2$. We have shown that the maximal efficiency η for a heat engine operating between two such reservoirs is $\eta_{\max} = 1 - T_2/T_1$ (a Carnot engine) and the maximal coefficient of performance κ is $\kappa_{\max} = T_2/(T_1 - T_2)$ (a Carnot refrigerator).

(a) Prove that *if* you had a heat engine with efficiency $\eta > \eta_{\max}$, you could hook it up to an ordinary Carnot refrigerator to make a refrigerator that requires no work input.

(b) Prove that *if* you had a refrigerator with coefficient of performance $\kappa > \kappa_{\max}$, you could hook it up to an ordinary Carnot engine to make an engine that produces no waste heat.