Notes on Physics 415 Statistical and Thermal Physics

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Contents

1 Basic Statistical Methods: Binomial distribution and the Emergence of Gaussian

1.1 Random walk

- ***Example:*** We introduce important ideas from Probability via an example of **1D random walk:**

> Consider a drunkard walking along a straight line, starting from Origin $x = 0$, and taking randoms steps of length l at regular intervals. Each step is independent of the last. He takes a probability p of steping to the left, and $1 - p$ to step to the right. After taking N steps, what is the probability that the walker is at position $x = ml$?

Let $P_N(m)$ be the position $x = ml$ of the drunkard after N steps; denote $n_1 =$ number of steps to the left, $n_2 = N - n_1$ number of steps to the right. Notice that $-N \leq m \leq N, N = n_1 + n_2, m = n_1 - n_2$.

Then the number of walking combinations, indexed with either direction left (n_1) or right (n_2), is given by the binomial coefficient:

$$\binom{N}{n_1} = \frac{N!}{n_1!(N - n_1)!} = \frac{N!}{n_1!\, n_2!} = \binom{N}{n_2}. \tag{1}$$

Then the probability of the walker taking n_1 steps to the left and n_2 steps to the right is given by the **binomial distribution:**

$$P_N(n_1) = \frac{N!}{n_1! n_2!}\, p^{n_1} q^{n_2}. \tag{2}$$

Noticing $n_1 = \left(\frac{N+m}{2}\right)$ and $n_2 = \left(\frac{N-m}{2}\right)$:

$$P_N(m) = \frac{N!}{\left[\frac{N+m}{2}\right]!\, \left[\frac{N-m}{2}\right]!}\, p^{\frac{N+m}{2}} (1 - p)^{\frac{N-m}{2}}. \tag{3}$$

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1.2 General Notions from probability:

Let X be a random variable, taking N possible values $x_1, x_2, ..., x_N$ with associated probabilities $P(x_1), P(x_2),$

- Mean: