Physics 415
Spring 2025
Homework 3
Due Friday, February 14, 2025

This assignment covers material in Chapter 3 of Reif. I recommend reading through the text and also Lectures Notes 7-9.

Problem 1: (Negative temperature, adapted from Reif 3.2) In lecture it was argued that the temperature T, defined in relation to entropy S according to

$$\frac{1}{T} = \frac{\partial S}{\partial E}, \tag{1}$$

is positive, $T > 0$ (this was deduced from assumed properties of the density of states in a macroscopic system). The inequality holds in those system where the allowed energies are unbounded, $E_{\text{min}} \leq E < \infty$. However, there are certain unique features in systems with a *finite* range of allowed energies $E_{\text{min}} \leq E \leq E_{\text{max}}$ (and hence a finite number of total accessible states); in this problem you investigate some of these properties.

Consider a system of N localized non-interacting spin-1/2 particles, each with magnetic moment μ, located in an external magnetic field H. This system was discussed in Homework 2, Problem 4 (Reif 2.4).

a) Using the expression for $S = \ln \Omega(E)$ (you may simply quote the result from Homework 2) and the definition $1/T = \partial S/\partial E$, find the relation between the absolute temperature T and the total energy E of the system.

b) Make a sketch of $S(E)$ and $T(E)$. Under what circumstances is T negative?

c) Suppose the spin system (System 1) at a negative temperature $T_1 < 0$ is placed in thermal contact with another system (System 2) at a positive temperature $T_2 > 0$. In what direction will the heat flow? *Hint:* Consider the evolution of the total entropy $S_1 + S_2$ of the combined system.

d) The total magnetic moment M of this system is related to its energy E. Use the result of part (a) to find M as a function of H and the absolute temperature T.

Problem 2: (Coupled systems of spins, adapted from Reif 3.3) Consider two spin systems A and A' placed in an external field H. System A consists of N decoupled localized particles of spin-1/2 and magnetic moment μ. Similarly, System A' consists of N' decoupled localized particles of spin-1/2 with magnetic moments μ'. The two systems are initially isolated with respective total energies $bN\mu H$ and $b'N'\mu' H$. They are then placed in thermal contact with each other. Suppose that $|b| \ll 1$ and $|b'| \ll 1$, so that the simplified approximations obtained in Homework 2, Problem 4c can be used for the densities of states of the two systems.

a) In the most probable situation corresponding to the final thermal equilibrium, how is the energy $\tilde{E}$ of the system A related to the energy $\tilde{E}'$ of system A'?

b) What is the value of the energy $\tilde{E}$ of system A?

c) What is the heat Q absorbed by system A in going from the initial situation to the final situation when it is in equilibrium with A'?

d) What is the probability $P(E)dE$ that A has its final energy in the range between E and $E + dE$?

e) What is the dispersion $(\Delta^* E)^2 = \overline{(E - \bar{E})^2}$ of the energy E of the system A in its final equilibrium situation?

f) What is the value of the relative energy spread $|\Delta^* E/\tilde{E}|$ in the case when $N' \ll N$?

Problem 3: (Ideal gases, adapted from Reif 3.5, 3.6) A system consists of N_1 molecules of type 1 and N_2 molecules of type 2 confined within a box of volume V. The molecules are supposed to interact very weakly so that they constitute an ideal gas mixture.

a) How does the total number of states $\Omega(E)$ in the range between E and $E + \delta E$ depend on the volume V of this system? You may treat the problem classically.

b) Use this result to find the equation of state of this system, i.e., to find its mean pressure $\overline{p}$ as a function of V and T.

c) As an application of the result in (b), consider the following scenario: A glass bulb contains air at room temperature and at a pressure of 1 atmosphere. It is placed in a chamber filled with helium gas at 1 atmosphere and at room temperature. A few months later, the experimenter happens to read in a journal article that the particular glass of which the bulb is made is quite permeable to helium, although not any other gases. Assuming that equilibrium has been attained by this time, what gas pressure will the experimenter measure inside the bulb when he goes back to check?