Physics 415
Spring 2025
Homework 7
Due Monday, March 24, 2025

This assignment covers material in Chapter 7 of Reif. I recommend reading through the text and also Lectures Notes 22-24. Note the assignment due date.

Problem 1: (Ultra-relativistic ideal gas) Consider an "ultra-relativistic" ideal gas, in which each particle has energy momentum relation $\varepsilon = cp$, where $p = |\mathbf{p}|$ is the magnitude of the particle's momentum and c is the speed of light. For this problem, work in the canonical ensemble at temperature T.

(a) Calculate the partition function Z and Helmholtz free energy $F = -T \ln Z$ for this system.

(b) Find the average pressure p (and hence equation of state), average energy E, and entropy S of the ultra-relativistic ideal gas.

(c) Make an estimate of the "thermal de Broglie" wavelength λ_{th} of an ultra-relativistic gas particle at temperature T.

Problem 2: (Multi-component gas, adapted from Reif 7.1) Consider a homogeneous mixture of inert monatomic ideal gases at absolute temperature T in a container of volume V. Let there be ν_1 moles of gas 1, ν_2 moles gas 2, ..., and ν_k moles of gas k.

(a) By considering the classical partition function of this system, derive the equation of state, i.e., find an expression for its total mean pressure p.

(b) How is this total pressure p of the gas related to the pressure p_i which the i-th gas would produce if it alone occupied the entire volume at this temperature.

Problem 3: (Ideal gas in an external field, adapted from Reif 7.2) An ideal monatomic gas of N particles, each of mass m, is in thermal equilibrium at absolute temperature T. The gas is contained in a cubical box of side L, whose top and bottom sides are parallel to the earth's surface. The effect of the earth's uniform gravitational field on the particles should be considered, the acceleration due to gravity being g.

(a) What is the average kinetic energy of a particle?

(b) What is the average potential energy of a particle?

Problem 4: (Entropy of mixing, adapted from Reif 7.4) A thermally insulated container is divided into two parts by a thermally insulated partition. Both parts contain ideal gases which have equal heat capacities C_V. One of these parts contains ν_1 moles of gas at a temperature T_1 and pressure p_1; the other contains ν_2 moles of gas at a temperature T_2 and pressure p_2. The partition is now removed and the system is allowed to come to equilibrium.

(a) Find the final pressure.

(b) Find the change ΔS of total entropy if the gases are different.

(c) Find ΔS if the gases are identical.

Problem 5: (Non-ideal gas, adapted from Reif 7.6) Consider a gas which is *not* ideal so that molecules *do* interact with each other. This gas is in thermal equilibrium at the absolute temperature T. Suppose that the translational degrees of freedom of this gas can be treated classically. What is the mean kinetic energy of (center-of-mass) translation of a molecule in this gas?

Problem 6: (Harmonic oscillator, adapted from Reif 7.9) A very sensitive spring balance consists of a quartz spring suspended from a fixed support. The spring constant is α, i.e., the restoring force of the spring is $-\alpha x$ if the spring is stretched by an amount x. The balance is at a temperature T in a gravitational field g.

(a) If a very small object of mass M is suspended from the spring, what is the mean resultant elongation $\bar{x}$ of the spring?

(b) What is the magnitude $\overline{(x-\bar{x})^2}$ of the thermal fluctuations of the object about its equilibrium position?

(c) It becomes impracticable to measure the mass of an object when the fluctuations are so large that $[\overline{(x-\bar{x})^2}]^{1/2} = \bar{x}$. What is the minimum mass M which can be measured with this balance?

Problem 7: (Linear and non-linear oscillations, adapted from Reif 7.10) A system consists of N very weakly interacting particles at a temperature T sufficiently high so that classical statistical mechanics is applicable. Each particle has mass m and is free to perform one-dimensional oscillations about its equilibrium position. Calculate the heat capacity of this system of particles at this temperature in each of the following cases:

(a) The force effective in restoring each particle to its equilibrium position is proportional to its displacement x from its equilibrium position.

(b) The restoring force is proportional to x^3.