General Tips:

- **Identify the System:** Isolated? Fixed T? Fixed T & μ? Gas? Spins? Oscillators? Solid? Radiation?
- **Identify the Process:** Quasi-static? Adiabatic? Isothermal? Isobaric? Isochoric? Cyclic? Free Expansion? Throttling?
- **Identify the Goal:** Calculate $\Omega, S, T, p, E, F, H, G, Z, \mathcal{Z}$, work W, heat Q, efficiency η, COP K, distribution $P(x)$, average $\bar{O}$, fluctuations ΔO?

Thermal Physics Quick Reference Guide

(Keywords $\rightarrow$ Concepts $\rightarrow$ Key Equations $\rightarrow$ HW/Lecture Ref)

1. Basic Probability & Combinatorics

- **Keywords:** Probability, dice, coins, random walk (steps), combinations, arrangements.
- **Concept:** Calculating probability for discrete outcomes, Binomial distribution.
- **Key Equations:**
  - $P(\text{Event}) = (\#\text{ Favorable})/(\#\text{ Total})$
  - Binomial: $P_N(n) = \binom{N}{n} p^n q^{N-n}$
  - Mean: $\bar{n} = Np$
  - Variance: $\sigma^2 = Npq$
- **References:**
  - Phase 1 Review
  - HW1 (Probs 1a, 1b, 1c), HW2 (Prob 4a - combinatorics)
  - Lecture 1

2. Phase Space, $\Omega(E)$, $\omega(E)$, Entropy S

- **Keywords:** Phase space, microstates, density of states, number of states, entropy (statistical def), isolated system, classical gas, oscillators, spins.
- **Concept:** Relating macroscopic state (fixed E) to number of microscopic configurations $\Omega(E)$ or density $\omega(E)$. $S = \ln \Omega$.
- **Key Equations:**
  - $\Omega(E) = (\text{Phase Space Vol})/h^S$ (Classical)
  - $\Omega(E) = \#\text{ Quantum States}$ (Quantum)
  - $\omega(E) = d\Phi/dE$ or $\Omega(E) = \omega(E)\delta E$
  - $S = \ln \Omega$ (using $k_B = 1$)

    - Classical Ideal Gas (3D): $\omega(E) \propto V^N E^{3N/2-1}$
    - N Oscillators (Classical): $\omega(E) \propto E^{N-1}$
    - N Spins: $\Omega(E) = \binom{N}{n_1(E)}$
- **References:**
    - Phase 1 Review
    - HW2 (Probs 1b, 2, 3, 4a, 4b), HW3 (Prob 1a)
    - Lectures 3, 4

3. Temperature T, Equilibrium, Heat Flow

- **Keywords:** Temperature (statistical def), thermal equilibrium, heat flow direction, maximize entropy, coupled systems, negative temperature.
- **Concept:** Defining T via S, condition for thermal equilibrium, Second Law driving towards equilibrium.
- **Key Equations:**
    - $1/T = (\partial S/\partial E)_{V,N}$
    - Equilibrium: $T_1 = T_2$
    - Maximize $S_{total} = S_1 + S_2$
    - Heat Flow: Driven by T difference, towards $\Delta S_{tot} \geq 0$.
- **References:**
    - Phase 2 Review
    - HW3 (Probs 1a, 1b, 1c, 2a)
    - Lectures 7, 8, 9

4. First Law, Work, Heat, Processes

- **Keywords:** First Law, internal energy (E), heat (Q), work (W), quasi-static, reversible, irreversible, isothermal, adiabatic, isobaric, isochoric, pV diagram, cycle.
- **Concept:** Energy conservation, path dependence of Q, W, state function E. Calculating $W, Q, \Delta E$ for specific processes.
- **Key Equations:**
    - $dE = Q - W$ (or $\Delta E = Q - W$)
    - Quasi-static work: $W = \int pdV$ (or $W = -\int FdL$)
    - E depends only on T for Ideal Gas. $dE = C_V dT$.
    - Adiabatic ($Q = 0$): $\Delta E = -W$. Ideal Gas: $pV^\gamma = \text{const}$.
    - Isothermal ($dT = 0$): Ideal Gas $\Delta E = 0 \implies Q = W$.
    - Isochoric ($dV = 0$): $W = 0 \implies \Delta E = Q$.
    - Isobaric ($dp = 0$): $W = p\Delta V$. Ideal Gas $Q = C_p \Delta T$.
    - Cycle: $\Delta E = 0 \implies Q_{net} = W_{net}$. W_{net} = Area enclosed.

- **References:**
  - Phase 2 & 3 Reviews
  - HW2 (Prob 5a, 6), HW4 (Probs 3, 4, 5, 6), HW5 (Prob 1d)
  - Lectures 5, 6, 9, 11, 16

5. Thermodynamic Identity & $TdS = Q_{rev}$

- **Keywords:** Thermodynamic identity, exact differential, state function, reversible heat.
- **Concept:** Fundamental relation between state variables for infinitesimal changes between equilibrium states. Link between dS and reversible heat.
- **Key Equations:**
  - $dE = TdS - pdV$ (or $dE = TdS + FdL$)
  - $dS = Q_{rev}/T$
  - $(\partial S/\partial V)_E = p/T$
- **References:**
  - Phase 3 Review
  - HW3 (Prob 3b), HW4 (Prob 3b), HW5 (Prob 1a, 1d)
  - Lecture 9

6. Thermodynamic Potentials (F, H, G) & Maxwell Relations

- **Keywords:** Helmholtz Free Energy (F), Enthalpy (H), Gibbs Free Energy (G), Legendre transform, natural variables, Maxwell relations, $C_p - C_V$.
- **Concept:** Using different potentials ($F(T,V), H(S,p), G(T,p)$) suited for different constraints. Deriving relations from exactness of differentials.
- **Key Equations:**
  - $F = E - TS \implies dF = -SdT - pdV$
  - $H = E + pV \implies dH = TdS + Vdp$
  - $G = H - TS \implies dG = -SdT + Vdp$
  - Maxwell Relations (see Phase 4 summary)
  - $p = -(\partial F/\partial V)_T$, $S = -(\partial F/\partial T)_V$, etc.
  - $C_p - C_V = TV\alpha_p^2/K_T$
- **References:**
  - Phase 4 Review
  - HW5 (Prob 1b, 1c, 1e, 1f)
  - Lectures 12, 13

7. Free Expansion & Joule-Thomson

- **Keywords:** Free expansion, Joule expansion, throttling, Joule-Thomson process, constant energy, constant enthalpy, inversion curve.
- **Concept:** Analyzing specific irreversible expansion processes.
- **Key Equations:**
  - Free Expansion: $\Delta E = 0$. $\mu_J = (\partial T/\partial V)_E = (p - T(\partial p/\partial T)_V)/C_V$.
  - Joule-Thomson: $\Delta H = 0$. $\mu_{JT} = (\partial T/\partial p)_H = (T(\partial V/\partial T)_p - V)/C_p$.
  - Inversion Curve: $\mu_{JT} = 0 \implies T\alpha_p = 1$.
- **References:**
  - Phase 4 Review
  - HW5 (Prob 2)
  - Lectures 14, 15

8. Heat Engines & Refrigerators

- **Keywords:** Heat engine, refrigerator, heat pump, efficiency (η), COP (K), Carnot cycle, Second Law limits.
- **Concept:** Applying First and Second Laws to cyclic devices. Maximum performance limits.
- **Key Equations:**
  - Engine: $W = Q_H - Q_C$. $\eta = W/Q_H \leq 1 - T_C/T_H$.
  - Refrigerator: $Q_H = Q_C + W$. $K = Q_C/W \leq T_C/(T_H - T_C)$.
  - Heat Pump: $K_{heating} = Q_H/W \leq T_H/(T_H - T_C)$.
  - Carnot = Reversible $\implies \Delta S_{tot} = 0 \implies Q_H/T_H = Q_C/T_C$.
- **References:**
  - Phase 4 Review
  - HW5 (Probs 3, 4, 7)
  - Lecture 16

9. Canonical Ensemble (CE)

- **Keywords:** Canonical ensemble, partition function (Z), Boltzmann factor, fixed T, Helmholtz free energy (F), average energy ($\bar{E}$), fluctuations.
- **Concept:** Statistical mechanics at constant T. Z is central.
- **Key Equations:**
  - $P_r = e^{-\beta E_r}/Z$
  - $Z = \sum_r e^{-\beta E_r}$
  - $F = -T \ln Z$
  - $\bar{E} = -\partial(\ln Z)/\partial\beta$
  - $\overline{(\Delta E)^2} = T^2 C_V$

    - For N independent systems: $Z_{total} = (Z_1)^N$ (distinguishable) or $Z = (Z_1)^N/N!$ (identical classical).
- **References:**
    - Phase 5 Review
    - HW6 (Probs 1, 2, 3, 4, 5, 6, 7)
    - Lectures 18, 19, 20, 22, 23, 24, 25

10. Grand Canonical Ensemble (GCE)

- **Keywords:** Grand canonical ensemble, grand partition function ($\mathcal{Z}$), chemical potential (μ), fugacity (z), grand potential (Φ), fixed T, μ.
- **Concept:** Statistical mechanics at constant T, μ. Useful for variable particle number / quantum gases.
- **Key Equations:**
    - $P_r = e^{-\beta(E_r - \mu N_r)}/\mathcal{Z}$
    - $\mathcal{Z} = \sum_r e^{-\beta(E_r - \mu N_r)} = \sum_N z^N Z_N$
    - $\Phi = -T \ln \mathcal{Z}$
    - $\bar{N} = -(\partial\Phi/\partial\mu)_{T,V}$
    - $p = -\Phi/V$ (if $\Phi \propto V$)
- **References:**
    - Phase 5 Review
    - HW6 (Prob 8), HW9 (Prob 2, 3)
    - Lectures 20, 21, 28

11. Quantum Statistics (BE/FD/MB)

- **Keywords:** Bose-Einstein, Fermi-Dirac, Maxwell-Boltzmann, bosons, fermions, occupation number ($\bar{n}_r$), Pauli exclusion, classical limit, quantum corrections, indistinguishable particles, Gibbs paradox.
- **Concept:** Statistical distributions governing identical particles.
- **Key Equations:**
    - $\bar{n}_r = 1/(e^{\beta(\epsilon_r - \mu)} \mp 1)$ (+FD, -BE)
    - Classical limit ($\bar{n}_r \ll 1$): $\bar{n}_r \approx z e^{-\beta\epsilon_r}$ (MB). Condition $n\lambda_{th}^3 \ll 1$.
    - Entropy: $S = -\sum_r [\bar{n}_r \ln \bar{n}_r \mp (1 \pm \bar{n}_r)\ln(1 \pm \bar{n}_r)]$
    - Classical $Z = (Z_1)^N/N!$.
- **References:**
    - Phase 6 & 7 Reviews
    - HW4 (Prob 2), HW9 (Probs 1, 2, 3, 4), HW10 (Prob 4)
    - Lectures 22, 23, 27, 28, 30, 31

12. Degenerate Fermi Gas

- **Keywords:** Fermi gas, degenerate, Fermi energy (ϵ_F), Fermi temperature (T_F), Pauli pressure, Sommerfeld expansion, heat capacity ($C_V \propto T$), Pauli paramagnetism.
- **Concept:** Behavior of fermions at $T \ll T_F$.
- **Key Equations:**
  - $\epsilon_F = (\hbar^2/2m)(6\pi^2 n/g)^{2/3}$
  - $E_0 = (3/5)N\epsilon_F$, $p_0 = (2/5)n\epsilon_F$
  - $C_V \approx (\pi^2/2)N(T/\epsilon_F)$
  - $\chi_{Pauli} \approx \mu_m^2 g\rho(\epsilon_F)/V \propto N/\epsilon_F$ (T-independent)
- **References:**
  - Phase 7 Review
  - HW10 (Probs 3, 5, 6, 7)
  - Lectures 32, 33

13. Bose-Einstein Condensation (BEC)

- **Keywords:** Bose gas, Bose-Einstein condensation, critical temperature (T_c), condensate fraction (N_0/N), macroscopic occupation, ground state.
- **Concept:** Phase transition in Bose gas at low T where ground state becomes macroscopically populated.
- **Key Equations:**
  - Requires $\mu \to \epsilon_0 (= 0)$ as $T \to T_c$.
  - $T_c \propto n^{2/3}$ (3D box)
  - $N_0/N = 1 - (T/T_c)^{3/2}$ (for $T < T_c$, 3D box)
  - No BEC in ideal 2D gas ($T_c = 0$).
- **References:**
  - Phase 7 Review
  - HW11 (Probs 1, 2)
  - Lecture 34

14. Black-Body Radiation (Photon Gas)

- **Keywords:** Black-body radiation, photon gas, Planck distribution, $\mu = 0$, density of modes, Planck's Law, Stefan-Boltzmann Law, Wien's Law, radiation pressure.
- **Concept:** Equilibrium EM radiation as a gas of non-conserved bosons.
- **Key Equations:**
  - $\bar{n}(\omega) = 1/(e^{\beta\hbar\omega} - 1)$
  - DOS: $g(\omega) \propto V\omega^2$

    - Energy density: $u(\omega) \propto \omega^3/(e^{\beta\hbar\omega} - 1)$ (Planck)
    - Total energy density: $u = \sigma_E T^4$ (Stefan-Boltzmann)
    - Pressure: $p = u/3$
    - Adiabatic expansion: $VT^3 = \text{const}$
    - Emitted Power: $J = \sigma T^4$
- **References:**
    - Phase 7 Review
    - HW11 (Probs 3, 4, 5, 6)
    - Lectures 35, 36

Use this guide to quickly locate relevant principles, equations, and examples when tackling exam problems. Identify keywords in the problem, match them to a concept here, and then check the associated equations and homework/lecture references. Good luck!