Theorem (Termination with Bland's rule) If the simplex method uses Bland's rule, it terminates after a finite number of iterations.

Proof.

Assume, for a contradiction, $\exists$ basis matrices $B_1, \ldots, B_k, B_1$ that form a cycle.
We say that an index i is fickle if it is a basic index in some, but not all, bases in the cycle.
Since the solution x remains the same throughout the cycle, i fickle $\Rightarrow x_i = 0$ throughout the cycle.
Let t be the largest fickle index.

(a) $\exists B$ in the cycle s.t. $t \in B$ (meaning t is a basic index for B) and t exits the basis. Let s be the index that enters the basis. $\Rightarrow s$ is fickle $\Rightarrow s < t$.
Denote by $\bar{c}$ the vector of reduced costs and by d the basic direction.

index	$\bar{c}$	d	$\hat{c}$	$\tilde{c}$
t (exits B)(enters $\hat{B}$)	$=0$	<0	<0	<0
s (enters B)	<0	$=1$	$\geq 0^{(+)}$	≥ 0
$i \in B$, $i \neq t$, i fickle	$=0$	$\geq 0^{(*)}$	$\geq 0^{(+)}$	≥ 0
$i \in B$, i not fickle	$=0$	?	$=0$	$=0$
$i \notin B$, $i \neq s$	?	$=0$	?	?

(*) i fickle $\Rightarrow x_i = 0 \Rightarrow -\frac{x_i}{d_i} = 0$. If we had $d_i < 0$, then the pivoting rule would have chosen i to exit the basis since $i < t$.

(b) $\exists \hat{B}$ in the cycle s.t. $t \notin \hat{B}$ and t enters the next basis

Let $\hat{c}$ denote the reduced costs corresponding to $\hat{B}$.

(b) Let $i \neq t$, i fickle. If $i \in \hat{B}$ then $\hat{c}_i = 0$.
Assume $i \notin \hat{B}$. If we had $\hat{c}_i < 0$, then the pivoting rule would have chosen i to enter the basis since $i < t$.
thus $\hat{c}_i \geq 0$.

(c) Let $\tilde{c} = \hat{c} - \bar{c}$

(d) Using the values in the table we show $\tilde{c}'d > 0$:

$$\tilde{c}'d = \tilde{c}_t d_t + \tilde{c}_s d_s + \sum_{\substack{i \in B,\, i \neq t \\ i \text{ fickle}}} \tilde{c}_i d_i + \sum_{\substack{i \in B \\ i \text{ not fickle}}} \tilde{c}_i d_i + \sum_{\substack{i \notin B \\ i \neq s}} \tilde{c}_i d_i > 0$$

To obtain a contradiction we show $\tilde{c}'d = 0$.

$\bar{c}' = \cancel{c'} - c_B' B^{-1} A$ $\qquad$ $\hat{c}' = \cancel{c'} - c_{\hat{B}}' \hat{B}^{-1} A$

$\tilde{c}' = (c_B' B^{-1} - c_{\hat{B}}' \hat{B}^{-1}) A$.

$Ad = 0 \Rightarrow \tilde{c}'d = (\ldots) A d = 0$

We have a contradiction $\square$