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1. recall stokes theorem $\iint_S \left(\nabla \times \vec{F}\right) \cdot \vec{n} \, \mathrm{d}S = \oint_C \vec{F} \cdot \mathrm{d}\vec{r}$.
   Let the surface bound by C be a simple disk $x^2 + y^2 \leq 4, z = 7$. Unit normal vector of this surface is $\vec{n} = (0, 0, 1)$

$$\nabla \times \vec{F} = \det \begin{pmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3x \ln(z) & 2yz^2 & \sqrt{xy + e^x} \end{pmatrix} = \begin{pmatrix} \frac{x}{2\sqrt{xy+e^x}} - 4yz \\ \frac{3x}{z} - \frac{y+e^x}{2\sqrt{xy+e^x}} \\ 0 \end{pmatrix}$$

   Thus, $\left(\nabla \times \vec{F}\right) \cdot \vec{n} = 0$. This implies that $\iint_S \left(\nabla \times \vec{F}\right) \cdot \vec{n} \, \mathrm{d}S = 0$

   $\boxed{\text{Thus the result of the given line integral is 0}}$

2. Choose S as the triangle with vertices $(1, 0, 0), (0, 1, 0), (0, 0, 1)$ as suggested, and S follows the equation $x + y + z = 1$. Normal vector to S is $\vec{n} = (1, 1, 1)$

$$\nabla \times \vec{F} = \det \begin{pmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x + y^2 & y + z^2 & z + x^2 \end{pmatrix} = \begin{pmatrix} -2z \\ -2x \\ -2y \end{pmatrix}$$

   Thus, $\left(\nabla \times \vec{F}\right) \cdot \vec{n} = -\frac{2}{\sqrt{3}}(x + y + z)$.

$$\begin{aligned} \iint_S \left(\nabla \times \vec{F}\right) \cdot \vec{n} \, \mathrm{d}S &= -2 \iint_S x + y + z \, \mathrm{d}A \\ &= -2 \iint_S x + y + 1 - x - y \, \mathrm{d}A \\ &= -2 \iint_S 1 \, \mathrm{d}A \\ &= -2 \times \text{Area}_{\text{triangle}} \\ &= -2\left(\frac{1}{2} \times \sqrt{1^2 + 1^2} \times \sqrt{1^2 + 1^2} \sin\left(\frac{\pi}{3}\right)\right) \\ &= -2\frac{\sqrt{3}}{2} = \boxed{\text{-sqrt(3)}} \end{aligned}$$

3. $\boxed{\text{Re}(z) = \sqrt{2}; \quad \text{Im} = -\pi}$

4. (a)

$$(2 + i) + \left(\sqrt{3} + 8i\right) = \boxed{\left(2 + \sqrt{3}\right) + 9i}$$

   (b)

$$(3 - 6 - 6i) = \boxed{(-3 - 6i)}$$

(c)

$$16 + 2i - 24i - 3i^2 = \boxed{(19 - 22i)}$$

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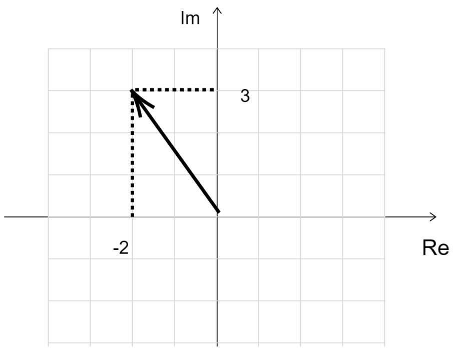
5. $\boxed{|z| = \sqrt{(-2)^2 + 3^2} = \sqrt{13}}$



6. recall triangle inequality $|z_1 + z_2| \leq |z_1| + |z_2|$.

$$|3 + \cos(5)i| \leq |3| + |\cos(5)i| = 3 + |\cos(5)| \leq 4$$
$$|3 + \cos(5)i| = \sqrt{3^2 + \cos^2(5)} \geq \sqrt{9 - 1} = \sqrt{8} \geq 2$$

$\boxed{\text{therefore } 2 \leq |3 + \cos(5)i| \leq 4}$

7. $\boxed{z^* = 3 - 8i}$

8. (a)

$$\frac{(1-i)(1-i)}{(1-i)(1+i)} = \frac{1 - 2i - 1}{2} = \boxed{0 + i(-1)}$$

(b)

$$\frac{(1+i)\left(1 - \sqrt{2}i\right)}{\left(1 + \sqrt{2}i\right)\left(1 - \sqrt{2}i\right)} = \frac{1 - i\sqrt{2} + i + \sqrt{2}}{3} = \frac{1 + \sqrt{2} + \left(1 - \sqrt{2}\right)i}{3} \boxed{= \frac{1 + \sqrt{2}}{3} + i\frac{1 - \sqrt{2}}{3}}$$

(c)

$$-i - 1 - 4 = \boxed{-5 + i(-1)}$$

9. let $z = a + bi; w = c + di$. It follows that

$$(zw)^* = (ac - bd) - i(ad + bc)$$
$$z^*w^* = (a - bi)(c - di) = (ac - bd) - i(ad + bc)$$

$$\boxed{\Rightarrow (zw)^* = z^*w^*}$$