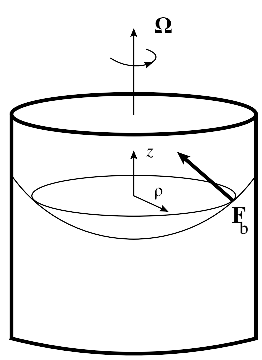
Physics 311
Spring 2024
Homework 9
Due Friday, April 26, 2024

This assignment covers material in Chapter 9 of Taylor. I recommend reading through the text and also Lectures Notes 33-35.

Problem 1: (Accelerating frame, adapted from Taylor 9.2) A donut-shaped space station (outer radius R) arranges for artificial gravity by spinning on the axis of the donut with angular velocity ω. Sketch the forces on, and accelerations of, an astronaut standing in the station (a) as seen from an inertial frame outside the station and (b) as seen in the astronaut's personal rest frame (which has a centripetal acceleration $A = \omega^2 R$ as seen in the inertial frame). What angular velocity is needed if $R = 40$ meters and the apparent gravity is to equal the usual value of about 10 m/s^2? (c) What is the percentage difference between the perceived g at a six-foot astronaut's feet (R = 40 m) and at his head ($R = 38$ m)? (c) What is the percentage difference between the perceived g at a six-foot astronaut's feet ($R = 40$ m) and at his head ($R = 38$ m)?

Problem 2: (Centrifugal force, adapted from Taylor 9.14) In this problem you practice using non-inertial reference frames to derive that the surface of the water in a rotating bucket forms a paraboloid. Work in cylindrical coordinates as illustrated in the figure:



a) What are the EOM of a water droplet with mass m at the surface in the non-inertial frame of the bucket? Which of the inertial forces is non-zero?

b) Since the drop is not accelerating in the frame of the bucket, what is the direction of the sum of the gravitational and centrifugal forces, $\mathbf{F}_\mathrm{g} + \mathbf{F}_\mathrm{cent}$? Recalling that conservative forces are the negative gradients of potentials, what can you conclude about the surface of the water in terms of the potential associated to $\mathbf{F}_\mathrm{g} + \mathbf{F}_\mathrm{cent}$?

c) Use the result of (b) to show the shape of the water's surface is a paraboloid.

Problem 3: (Coriolis deflection) A particle is thrown up vertically with initial speed v_0, reaches a maximum height and falls back to the ground. Show that the Coriolis deflection when it again reaches the ground is opposite in direction, and four times greater in magnitude, than the Coriolis deflection when it is dropped at rest from the same maximum height.

Problem 4: (Coriolis force and spinning objects, adapted from Taylor 9.30) The Coriolis force can produce a torque on a spinning object. To illustrate this, consider a horizontal hoop of mass m and radius r spinning with angular velocity ω about its vertical axis at colatitude θ. Show that the Coriolis force due to the earth's rotation produces a torque of magnitude $m\omega\Omega r^2 \sin\theta$ directed to the west, where Ω is the earth's angular velocity. This torque is the basis of the gyrocompass.

Problem 5: (Foucault pendulum, adapted from Taylor 9.34) At a point P on the earth's surface, an enormous perfectly flat and frictionless platform is built. The platform is exactly horizontal — that is, perpendicular to the local free-fall acceleration $\mathbf{g}_P$. Find the equation of motion for a puck sliding on the platform and show that it has the same form as that for the Foucault pendulum except that the pendulum's length ℓ is replaced by the earth's radius R. What is the frequency of the puck's oscillations and what is that of its Foucault precession? In this problem you may neglect the centrifugal force (as we did in our analysis of the Foucault pendulum in class, the reason being that the centrifugal force produces an effect higher order in the Earth's angular velocity Ω).