Topics/Goals: Cauchy integral formulas, Taylor series, Poles, Cauchy residue theorem and its applications in evaluating improper integrals
Due on: **May 1st, 2024**. Since our class ends after May 3rd, **please keep a copy of your HW 13**. The detailed rubrics will be posted on Canvas.

1. Let C be the circle centered at i with radius 2 oriented in the counter clockwise direction. Using the **Cauchy integral formula**, evaluate the integral

$$\int_C \frac{z^3 + e^{z^2}}{z - 1 - i} dz$$

(Hint: apply the theorem in the lecture note for $z_0 = 1+i \in D = \{|z-i| < 2\}$ and $f(z) = z^3 + e^{z^2}$.)

2. Let C be the circle centered at $2i$ with radius 3 oriented in the counter clockwise direction. Using the **Cauchy integral formula**, evaluate the integral

$$\int_C \frac{e^z + e^{z^3}}{z^2} d\xi$$

3. Let C be the circle centered at $3i$ with radius 5 oriented in the counter clockwise direction. Using the **Cauchy integral formula**, evaluate the integral

$$\int_C \frac{z^{2024} + 4z}{(z+1)^3} d\xi$$

4. The complex function $f(z) = \cos(z)$ is defined to be the analytic series

$$f(z) = \cos(z) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} z^{2n}$$

Using the **Cauchy integral formula** and the above **Taylor series**, evaluate the integral

$$\int_{|z|=3} \frac{\cos(z)}{z^5} dz$$

(the contour is in the positive direction)

5. Let

$$f(z) = \frac{1}{1+z^2}$$

(a) Find all the poles of the above function f and evaluate their corresponding residues.
(b) Using the **Cauchy residue theorem**, evaluate the improper integral

$$\int_0^{\infty} \frac{1}{1+x^2} dx$$

Clearly and carefully justify your answers.

6. Let

$$f(z) = \frac{1}{(1+z^2)^2}$$

(a) Find all the poles of the above function f and evaluate their corresponding residues.
(b) Using the **Cauchy residue theorem**, evaluate the improper integral

$$\int_0^{\infty} \frac{1}{(1+x^2)^2} dx$$

Clearly and carefully justify your answers.

7. Let

$$f(z) = \frac{z^2}{(z^2+1)(z^2+4)}$$

(a) Find all the poles of the above function f and evaluate their corresponding residues.
(b) Using the **Cauchy residue theorem**, evaluate the improper integral

$$\int_0^\infty \frac{x^2}{(x^2+1)(x^2+4)} dx$$

Clearly and carefully justify your answers.