Topics/Goals: Evaluate the line integrals using Stokes theorem, basic properties of complex numbers
Due on: April 17th, 2024

1. Evaluate

$$\int_C \vec{F} \cdot d\vec{r}$$

where C is the boundary (in the counter clockwise direction) of the circle

$$\{x^2 + y^2 = 4, \quad z = 7\}$$

and

$$\vec{F} = (3x \ln(z), 2yz^2, \sqrt{xy + e^x})$$

2. Evaluate

$$\int_C \vec{F} \cdot d\vec{r}$$

where C is the triangle with vertices $(1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 1)$ (in the counter clockwise direction) and

$$\vec{F} = (x + y^2, y + z^2, z + x^2)$$

(Hint: Choose the surface S to be the triangle, then S can be written as $z = 1 - x - y$, which is a graph of a function. Then use the Stokes theorem)

3. Find the real part and imaginary part of the complex number

$$z = \sqrt{2} - \pi i$$

4. Evaluate the following complex numbers

(a)

$$(2 + i) + (\sqrt{3} + 8i)$$

(b)

$$(3 - i) - (6 + 5i)$$

(c)

$$(8 + i)(2 - 3i)$$

5. Sketch the complex $z = -2 + 3i$ on the complex plane $\mathbb{C}$. Find the absolute value of z.

6. Using the triangle inequality, show that

$$2 \leq |3 + \cos(5)i| \leq 4$$

7. Find the complex conjugate of $z = 3 + 8i$

8. Write the following complex numbers in the form $x + iy$

(a)

$$\frac{1 - i}{1 + i}$$

(b)

$$\frac{1 + i}{1 + \sqrt{2}i}$$

(c)

$$i^3 + i^2 - 4$$

(Hint: Multiply both sides of $i^2 = -1$ by i, we get $i^3 = -i$)

9. Let $z, w \in \mathbb{C}$. Show that the complex conjugate of zw is $\bar{z}\bar{w}$