1 Random Variables

1.1 Discrete random variable

Discrete random variables are random variables that can take on a countable number of values. It comes naturally from discrete, finite sample spaces. (As briefly discussed in sec.discreteSampleSpace)

For $A = \{k_1, k_2, ..., \}$ s.t. random variable$X \in A$, or $P(X \in A) = 1$, X is a random variable, with possible values $k_1, k_2, ...$ and $P(X = k_n) > 0$

1.1.1 Probability Mass Function (pmf)

The PMF is a function that defines the probability distribution for a discrete random variable. It gives the probability of the random variable taking on each possible value. The PMF, denoted as $P(X = x)$, satisfies two conditions:

$$P(X = x) \geq 0 \text{ for all x in the domain of } X \quad (1)$$

$$\sum_x P(X = x) = 1 \quad (2)$$

Example: For a fair six-sided die, the PMF would be $P(X = x) = \frac{1}{6}$ for $x = 1, 2, 3, 4, 5, 6$. Or more elegantly,

$$\begin{aligned} P(X = 1) &= \frac{1}{6} \\ P(X = 2) &= \frac{1}{6} \\ P(X = 3) &= \frac{1}{6} \\ P(X = 4) &= \frac{1}{6} \\ P(X = 5) &= \frac{1}{6} \\ P(X = 6) &= \frac{1}{6} \end{aligned} \quad (3)$$

1.2 continuous Random Variables

Not rigorously defined in this class, but a continuous random variable is one that can take on any value in a range. The probability of a continuous random variable taking on a specific value is 0. It came natually from continuous sample spaces.The probability is assigned to intervals of values, and they are assigned by the **probability density function**.

1.2.1 Probability Density Function (pdf)

continuous r.v are defined in this class by having a probability density function.