

Topics/Goals: Evaluate the line integrals using Green theorem, computing surface integrals using the definitions

Due on: April 10th, 2024

1. Evaluate

$$\int_C (y^2 + \sin(x))dx + (3xy + y^4)dy$$

where C is the boundary (in the counter clockwise direction) of the disk lying in the first quadrant

$$D = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 < 4, x > 0, y > 0\}$$

2. Evaluate

$$\int_C (x^5 + y^3)dx - (x^3 + y^5)dy$$

where C is the circle $x^2 + y^2 = 4$ in the counter clockwise direction.

3. Evaluate

$$\int_C (3y - e^{\sin(x^2)})dx + (7y + \sqrt{y^3 + 1})dy$$

where C is the circle $x^2 + y^2 = 1$ in the counter clockwise direction.

4. Evaluate

$$\int_C (e^{\sqrt{x+1}} + y^2 + 1)dx + (\sin(y^2 - 1) + x^2)dy$$

where C is the triangular curve consisting of the line segments connecting from $(0, 0)$ to $(2, 0)$, then $(2, 0)$ to $(0, 1)$, and $(0, 1)$ to $(0, 0)$.

5. Evaluate

$$\int_C (y + e^{\sqrt{x}})dx + (2x + \cos(y^2))dy$$

where C is the boundary (in the counter clockwise direction) of the region enclosed by the parabolas $y = x^2$ and $x = y^2$. (Hint: sketch the picture of the region D and see that D is a simple domain in x with $0 \leq x \leq 1$ and $x^2 \leq y \leq \sqrt{x}$)

6. Evaluate the surface integral

$$\int \int_S \vec{F} \cdot d\vec{S}$$

where $\vec{F} = (x, y, z)$ and S is the surface with parametric equations

$$\vec{r}(u, v) = (u + v, u - v, 1 + 2u + v), \quad 0 \leq u \leq 1, \quad 0 \leq v \leq 1$$

7. Evaluate the surface integral

$$\int \int_S \vec{F} \cdot d\vec{S}$$

where $\vec{F} = (y, z, x^2)$ and S is the given as

$$S = \{(x, y, z) : 0 \leq x \leq 1, 0 \leq y \leq 1, z = x^2 + y^2\}$$