

```
/*
 * For every possible move for the current board state, find the move that has
 * the highest evaluation from evaluate(). Has similar implementation to
 * maxValue(), however, it also has book keeping to record which move had the
 * highest evaluation.
 *
 * Computational complexity analysis:
 *
 *      Let moves.size() = n
 *      Let depth_ = m
 *      Complexity:  $O(n^m)$ 
 *
 * Explanation:
 * minimax() calls minValue which is indirectly recursive with maxValue()
 * for a total of m times. For each of these calls, moves are evaluated for
 * a total of n times. For example, if we assume that  $n = 3$ , and  $m = 2$ , then
 * we have three moves for every level of depth, for a total of 6 moves that
 * need to be processed:
 *
 * m_0: n_0, n_1, n_2
 * m_1: n_0, n_1, n_2
 *
 * which is  $3^2 = 6$ , or  $n^m$ 
 * Therefore, the computational complexity is  $O(n^m)$ 
 *
 * Pre-condition:
 * - The board is not full.
 *
 * Post-condition:
 * - The board is left unchanged from simulations.
 *
 * @return A valid move.
 */
Move minimax();

Move HardComputer::minimax()
{
    map<float_t, Move> valueMoveMap;
    auto moves = gameAnalyzer->findAllPossibleMoves(playerColor_);

    for (const auto& move : moves)
    {
        const auto pipSum = gameAnalyzer->sumPipForMove(move);
        board_->setMove(move, pipSum);
        auto value = minValue(depth_ - 1);
        valueMoveMap[value] = move;
        board_->undoMove();
    }

    return valueMoveMap.rbegin()->second;
}
```

```
float_t HardComputer::minValue(const uint32_t depth)
{
    // Base case: board is full or depth of search reaches 0.
    if (depth == 0 || board_>isBoardFull())
    {
        return evaluate();
    }

    // general case: there are board states to search.
    auto value = INFINITY;
    auto moves = gameAnalyzer_>findAllPossibleMoves(oppositionColor_);

    for (const auto& move : moves)
    {
        const auto pipSum = gameAnalyzer_>sumPipForMove(move);
        board_>setMove(move, pipSum);
        value = min(value, maxValue(depth - 1));
        board_>undoMove();
    }

    return value;
}

float_t HardComputer::maxValue(const uint32_t depth)
{
    // Base case: board is full or depth of search reaches 0.
    if (depth == 0 || board_>isBoardFull())
    {
        return evaluate();
    }

    // general case: there are board states to search.
    auto value = -INFINITY;
    auto moves = gameAnalyzer_>findAllPossibleMoves(playerColor_);
    for (const auto& move : moves)
    {
        const auto pipSum = gameAnalyzer_>sumPipForMove(move);
        board_>setMove(move, pipSum);
        value = max(value, minValue(depth - 1));
        board_>undoMove();
    }

    return value;
}

/*
 * Finds the heuristic value for the current board state. That is,
 * large values for the maximizing player where that player has more
 * occupied cells, and small values for the minimizing player, where
 * that player has less occupied cells.
 */
float_t HardComputer::evaluate() const
{
    const auto whiteCount
= static_cast<float_t>(gameAnalyzer_>countCellsWithColor(WHITE));
    const auto blackCount
= static_cast<float_t>(gameAnalyzer_>countCellsWithColor(BLACK));

    if (playerColor_ == WHITE)
    {
        return whiteCount - blackCount;
    }

    return blackCount - whiteCount;
}
```