

Machine Learning

## Linear Algebra review (optional)

# Matrices and vectors

Matrix: Rectangular array of numbers:

Dimension of matrix: number of rows x number of columns

### Matrix Elements (entries of matrix)

$$A = \begin{bmatrix} 1402 & 191 \\ 1371 & 821 \\ 949 & 1437 \\ 147 & 1448 \end{bmatrix}$$

$$A_{ij} = "i,j$$
 entry" in the  $i^{th}$  row,  $j^{th}$  column.

$$A_{11} = |462|$$
 $A_{12} = |9|$ 
 $A_{32} = |437|$ 
 $A_{41} = |47|$ 



**Vector:** An n x 1 matrix.

$$y = \begin{pmatrix} 460 \\ 232 \\ 315 \\ 178 \end{pmatrix}$$

$$y_i = i^{th}$$
 element

### 1-indexed vs 0-indexed:

$$y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} \leftarrow$$
1-indexed

$$y = \begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \end{bmatrix} \leftarrow$$

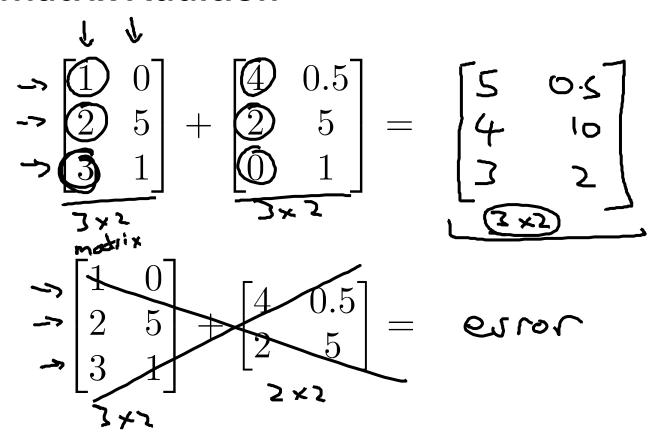


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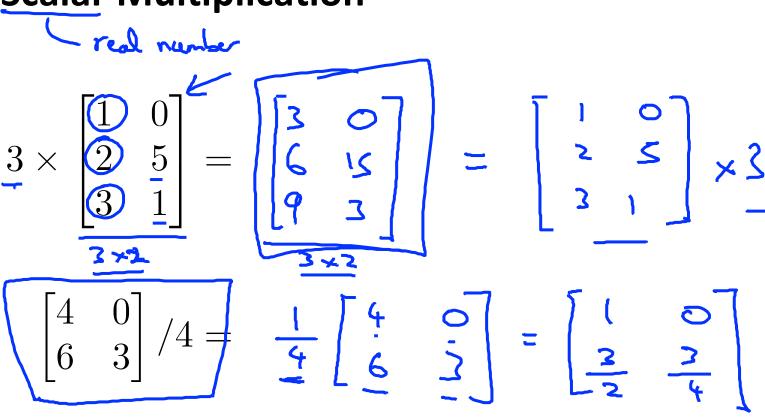
# Linear Algebra review (optional)

Addition and scalar multiplication

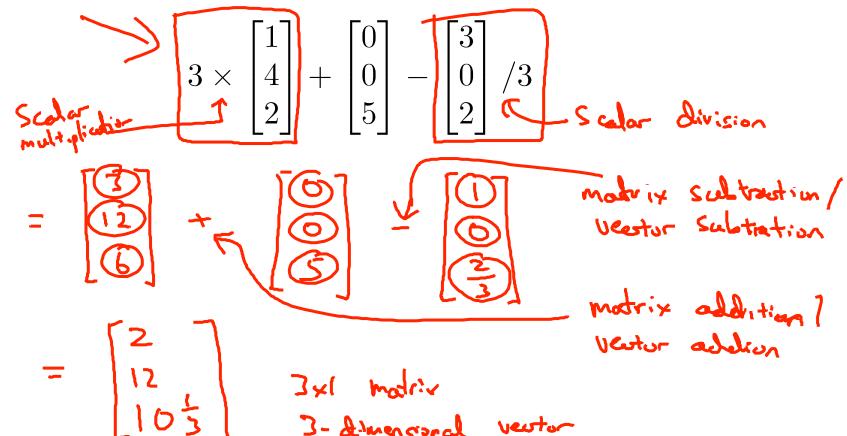
### **Matrix Addition**



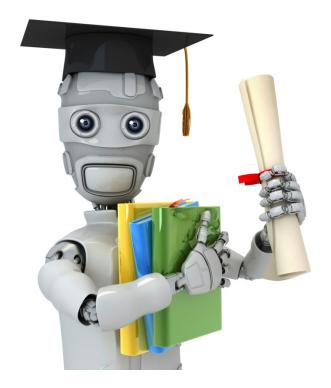
## **Scalar Multiplication**



## **Combination of Operands**



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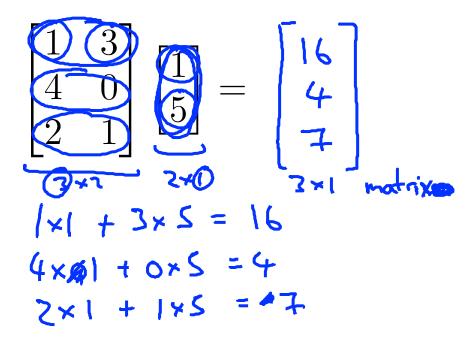


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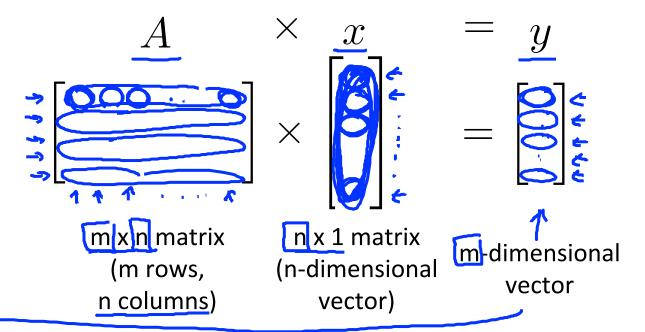
# Linear Algebra review (optional)

Matrix-vector multiplication

## **Example**

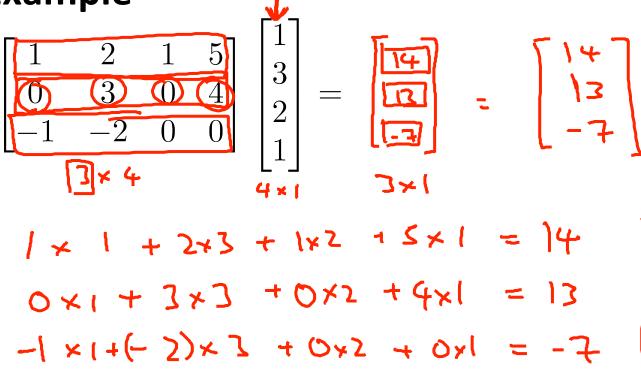


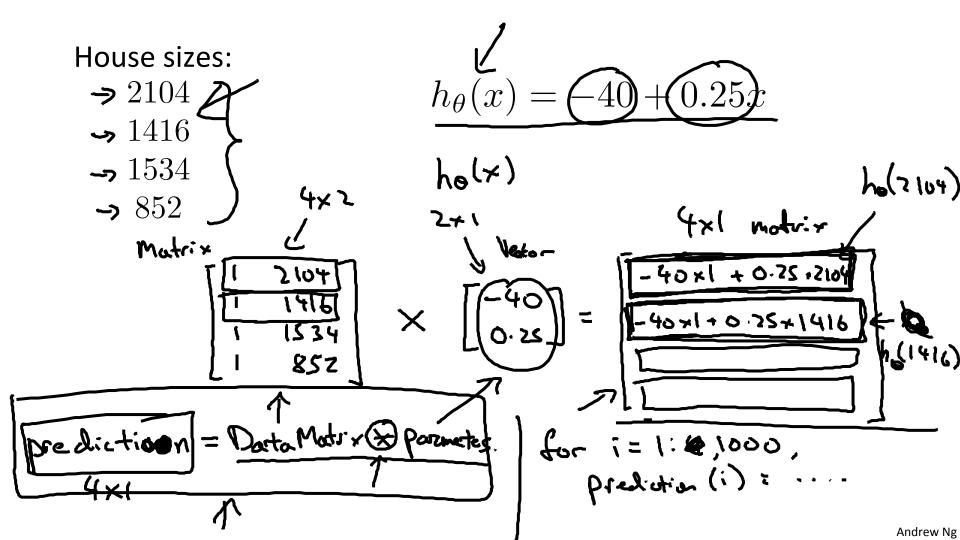
### **Details:**



To get  $y_i$ , multiply  $\underline{A}$ 's  $i^{th}$  row with elements of vector x, and add them up.

## **Example**





```
1  % Initialize a 3 by 2 matrix
2  A = [1, 2; 3, 4; 5, 6]
3
4  % Initialize a 2 by 1 matrix
5  B = [1; 2]
6
7  % We expect a resulting matrix of (3 by 2)*(2 by 1) = (3 by 1)
8  mult_AB = A*B
9
% Make sure you understand why we got that result
```

## ear Algebra ew (optional)



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# Matrix-matrix multiplication

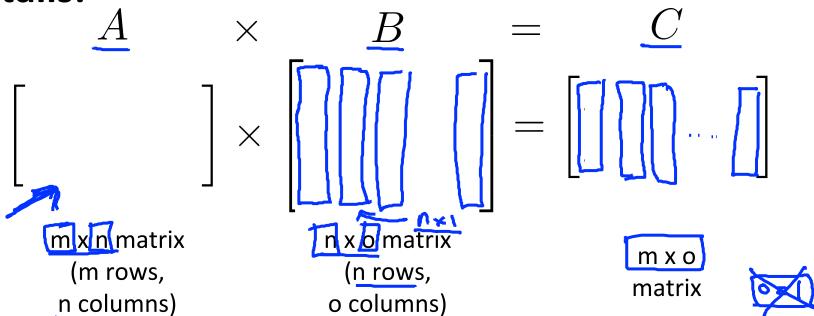
### **Example**

$$\begin{bmatrix} 1 & 3 & 2 \\ 4 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 0 & 1 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 10 \\ 2 & 14 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & 2 \\ 4 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & 2 \\ 4 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 10 \\ 14 \end{bmatrix}$$

### **Details:**



The  $\underline{i^{th}}$  column of the matrix C is obtained by multiplying A with the  $i^{th}$  column of B. (for i = 1,2,...,0)

**Example** 

$$\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 0 \\ 3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 9 & 7 \\ 15 & 12 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \times 0 + 3 \times 3 \\ 2 \times 0 + 5 \times 3 \end{bmatrix} = \begin{bmatrix} 9 \\ 15 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \times 1 + 3 \times 2 \\ 2 \times 1 + 5 \times 2 \end{bmatrix} = \begin{bmatrix} 7 \\ 12 \end{bmatrix}$$

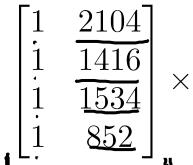
#### House sizes:

$$h_{\theta}(x) = -40 + 0.25x$$

2. 
$$h_{\theta}(x) = 200 + 0.1x$$

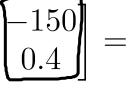
3. 
$$h_{\theta}(x) = (150 + 0.4)$$

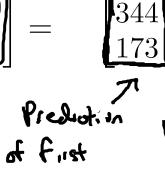
### Matrix

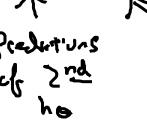












```
% Initialize random matrices A and B
A = [1,2;4,5]
B = [1,1;0,2]
% Initialize a 2 by 2 identity matrix (Diagonal matrix)
I = eve(2)
% The above notation is the same as I = [1,0;0,1]
% What happens when we multiply I*A?
IA = I*A
% How about A*I ?
AI = A*I
% Compute A*B
AB = A*B
% Is it equal to B*A?
BA = B*A
% Note that IA = AI but AB != BA
```



# Linear Algebra review (optional)

Matrix multiplication properties

Let A and B be matrices. Then in general,  $A \times B \neq B \times A$ . (not commutative.)

E.g. 
$$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 2 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 2 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 2 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 2 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 2 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 2 & 2 \end{bmatrix}$$

$$3 \times 5 \times 2$$
 $3 \times (5 \times 2) = (3 \times 5) \times 2$ 

$$3 \times 10 = 30 = 15 \times 2$$

$$A \times (0 \times c) \leftarrow \uparrow$$

$$(A \times B) \times C \leftarrow$$

$$A \times B \times C$$
.

Let 
$$\underline{D} = B \times C$$
. Compute  $A \times D$ .

Let  $\underline{E} = A \times B$ . Compute  $E \times C$ .

A  $\times$  ( $\mathbb{C} \times \mathbb{C}$ )

Some

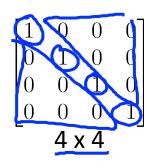
## **Identity Matrix**

Denoted  $\underline{I}$  (or  $I_{n \times n}$ ).

Examples of identity matrices:

$$\begin{bmatrix}
 0 & 0 \\
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For any matrix A,

1×2 = 2×1 = 2



Note: AB + BA in general AI = BA IA

```
A = [1,2,0;0,5,6;7,0,9]

% Transpose A
A_trans = A'

% Take the inverse of A
A_inv = inv(A)
% Inverse of A (In matlab, inv(A); In Octave, pinv(A))

% What is A^(-1)*A?
A_invA = inv(A)*A
```

% Initialize matrix A



Machine Learning

# Linear Algebra review (optional)

# Inverse and transpose

Not all numbers have an inverse.

Matrix inverse:

If A is an m x m matrix, and if it has an inverse,

$$A^{-1} = A^{-1}A = I.$$

Eg. [3 4] [0.4 -0.1] = [1.7]

[2 16] [0.4 -0.1] = [1.7]

12 > (12-1) = 1

Matrices that don't have an inverse are "singular" or "degenerate"

### **Matrix Transpose**

Example: 
$$\underline{\underline{A}} = \underbrace{\frac{1}{1} \cdot \frac{2}{2} \cdot 0}_{2 \times 3}$$

$$\mathbf{B} = \underline{A^T} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \\ 0 \end{bmatrix}$$

Let A be an  $\underline{\mathbf{m}}$   $\underline{\mathbf{x}}$   $\underline{\mathbf{n}}$  matrix, and let  $B = A^T$ . Then B is an  $\underline{\mathbf{n}}$   $\underline{\mathbf{x}}$   $\underline{\mathbf{m}}$  matrix, and

$$B_{\underline{i}\underline{j}} = A_{\underline{j}\underline{i}}.$$

$$B_{12} = A_{21} = 2$$

$$B_{32} = 9$$

$$A_{23} = 9$$