FINM2002 Derivatives
FINM6041 Applied Derivatives

Lecture 4 – Options Basics

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March 2025

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Review of Previous Lecture

- Interest rates
- Term structure and yield curve
- Spot and forward rates
- Interest rate swaps
- The value of interest rate swaps

Lecture Overview

- In today's class
  - Option basics
  - Stock option properties
  - Option bounds
  - Put-call parity
  - Extensions

1. Option Basics

- Call option, a long position
  - Holder/buyer of call option
  - Have the right but not obligation to buy the underlying asset
  - Payoff $\max(S_T - X, 0)$
- Call option, a short position
  - Writer/seller of call option
  - Obligated to sell the asset to the holder if the holder decides to exercise the option
  - Payoff $\min(X - S_T, 0)$

- Put option, a long position
  - Holder/buyer of put option
  - Have the right but not obligation to sell the asset
  - Payoff $\max(X - S_T, 0)$
- Put option, a short position
  - Writer of put option
  - Obligated to buy the asset from the holder if the holder decides to exercise the option
  - Payoff $\min(S_T - X, 0)$

- The moneyness of options
  - At-the-money (ATM):
    - A call option is at-the-money if the strike price equals the asset price ($S_T = X$).
    - A put option is at-the-money if the strike price equals the asset price ($S_T = X$).
  - In-the-money (ITM):
    - A call option is in-the-money if the strike price is less than the asset price ($S_T > X$).
    - A put option is in-the-money if the strike price is greater than the asset price ($S_T < X$).
  - Out-of-the-money (OTM):
    - A call option is out-of-the-money if the strike price is greater than the asset price ($S_T < X$).
    - A put option is out-of-the-money if the strike price is less than the asset price ($S_T > X$).

1. Option Basics

- Assets Underlying Options
- Stock options
  - Mostly traded on exchanges
  - Mainly American options
- Foreign currency options
  - Mainly over the counter
  - Either European or American options
- Index options
  - Both over the counter and on exchanges
  - European options

1. Option Basics

- Futures options
  - The underlying asset is a futures contract
  - Exchange traded futures often have options trading on them
  - A futures option normally matures just before the delivery period of the futures contract
  - When a call option is exercised, the holder acquires a long position in the underlying futures contract plus a cash amount equal to the excess of the futures price over the strike price
  - When a put option is exercised, the holder acquires a short position in the underlying futures contract plus a cash amount equal to the excess of the strike price over the futures price
  - Will be introduced in detail in Week 8

1. Option Basics

- Dividends & Stock Splits
- Suppose you own N options with a strike price of X
  - No adjustments are made to the option terms for cash dividends
  - When there is an n-for-m stock split

    e.g. 2-for-1 stock split means if you holds 100 shares before the split, after the split you would have 200 shares

    - the strike price is reduced to Xm/n
    - the number of options is increased to Nn/m
  - Stock dividends are handled in a manner similar to stock splits

- Example
  - Consider a call option to buy 100 shares for \$20 per share
  - How should terms be adjusted:
  - Example 1: for a 2-for-1 stock split?
    - the strike price is reduced to $Xm/n = \$20 \times (1/2) = \$10$
    - The number of shares in one option is increased to $Nn/m = 100 \times (2/1) = 200$
  - Example 2: for a 5% stock dividend?
    - the strike price is reduced to $Xm/n = \$20 \times (100/105) = \$19.047$
    - The number of shares in one option is increased to $Nn/m = 100 \times (105/100) = 105$

- Some special forms of options
- Warrants
  - Options that are issued by a company on its own stock
  - Warrants are traded in the same way as stocks
  - When call warrants are exercised, it leads to new stock being issued
  - By offering warrants as part of a financing deal, companies provide investors with the potential for additional returns beyond the initial investment

1. Option Basics

- Executive/employee incentive stock options
  - Issued by a company to its executives as a performance incentive
  - When the option is exercised the company issues more stock
  - Usually out-of-the-money when issued to incentivise executives to increase the share price of the company
  - Usually with a lock period, and cannot be sold by the executive
  - Often last for as long as 10 or 15 years

2. Stock option properties: notation

c	European call option price
p	European put option price
C	American call option price
P	American put option price
X	Strike price
S_0	Stock price today
S_T	Stock price at option maturity
D	Present value of cash dividends during option's life
T	the time to expiry of the option (in years)
r	the riskfree interest rate (continuously compounded)
σ	the volatility (stdev) of returns on the underlying asset

2. Stock option properties

	European		American	
Variable	c	p	C	P
S_0	$+$	$-$	$+$	$-$
X	$-$	$+$	$-$	$+$
T	?	?	$+$	$+$
σ	$+$	$+$	$+$	$+$
r	$+$	$-$	$+$	$-$
D	$-$	$+$	$-$	$+$

2. Stock option properties

- American vs. European Options:
  - An American option is worth at least as much as the corresponding European option
  - This is due to the value of being able to exercise the option at the time of your choosing

$$C \geq c$$
$$P \geq p$$

3. Option Bounds

- Ultimately, we want a formula that gives the exact option price
- Start by finding the range in which option prices must lie
- For any option, there is an upper bound on the option price
  - If the option price ever trades above the upper bound $\rightarrow$ arbitrage
- Similarly, every option has a lower bound on option price
  - If the option price ever trades below the lower bound $\rightarrow$ arbitrage

3. Option Bounds - quick preview

European options on non-dividend stocks	
$c \leq S_0$	$p \leq Xe^{-rT}$
$c \geq \max(S_0 - Xe^{-rT}, 0)$	$p \geq \max(Xe^{-rT} - S_0, 0)$
American options on non-dividend stocks	
$C \leq S_0$	$P \leq X$
$C \geq \max(S_0 - Xe^{-rT}, 0)$	$P \geq \max(X - S_0, 0)$

3.1. Option Bounds: upper bound for call options

- Upper bound for call options
  - American or European call option gives the holder the right to buy a stock at the strike price
  - No matter what happens, the option can never be worth more than the stock
  - Hence, the stock price is an upper bound to the call option price
  - If violated, arbitrageurs can buy the stock and sell the call option

$$c \leq S_0 \quad \text{and} \quad C \leq S_0$$

3.2. Option Bounds: upper bound for put options

- Upper bound for put options
  - American or European put option gives the holder the right to sell a stock at the strike price
  - No matter how low the stock price becomes, the option can never be worth more than X

$$p \leq X \quad \text{and} \quad P \leq X$$

  - For European options, it cannot be worth more than X at maturity. Meaning it cannot be worth more than the present value of X today, as it cannot be exercised early

$$p \leq Xe^{-rt}$$

- The lower bound for the price of a European call option on a non-dividend paying stock

$$c \geq \max(S_0 - Xe^{-rT}, 0)$$

- We can prove this by considering the following two portfolios
  - Portfolio A
    - One European call option
    - An amount of cash equal to Xe^{-rT}
  - Portfolio B
    - One share of underlying stock

3.3. Option Bounds: lower bound for European call

	Time 0	Time T If $S_T < X$	Time T If $S_T > X$
Strategy A			
Long call option	c	0	$S_T - X$
Long bond	Xe^{-rT}	X	X
Total payoff		X	S_T

Strategy B			
Long share	S_0	S_T	S_T
Total payoff		S_T	S_T

$$
\begin{aligned}
c + Xe^{-rT} &\geq S_0 \\
c &\geq S_0 - Xe^{-rT} \\
c &\geq \max(S_0 - Xe^{-rT}, 0)
\end{aligned}
$$

3.3. Option Bounds: lower bound for European call

- In portfolio A
  - If $S_T > X$, the call option is exercised at maturity and portfolio A is worth S_T
  - If $S_T < X$, the call option expires worthless and the portfolio is worth X
  - Hence at time T, portfolio A is worth $\max(S_T, X)$.
- Portfolio B is worth S_T at time T
  - Hence portfolio A is always worth at least as much as B at T
  - This must also be true today at $T = 0$
  - Also, the option value cannot be negative (just let it expire)

$$c \geq \max(S_0 - Xe^{-rT}, 0)$$

3.4. Option Bounds: lower bound for European put

- The lower bound for the price of a European put option on a non-dividend paying stock

$$p \geq \max(Xe^{-rT} - S_0, 0)$$

- We can prove this by considering the following two portfolios
  - Portfolio C
    - One European put option
    - One share
  - Portfolio D
    - An amount of cash equal to Xe^{-rT}

3.4. Option Bounds: lower bound for European put

	Time 0	Time T If $S_T < X$	Time T If $S_T > X$
Strategy C			
Long put option	p	$X - S_T$	0
Long share	S_0	S_T	S_T
Total payoff		X	S_T

Strategy D			
Long bond	Xe^{-rT}	X	X
Total payoff		X	X

$$
\begin{aligned}
p + S_0 &\geq Xe^{-rT} \\
p &\geq Xe^{-rT} - S_0 \\
p &\geq \max(Xe^{-rT} - S_0, 0)
\end{aligned}
$$

3.4. Option Bounds: lower bound for European put

- In portfolio C:
  - If $S_T < X$, the put option is exercised at maturity and portfolio C is worth X
  - If $S_T > X$, the put option expires worthless and the portfolio is worth S_T
  - Hence at time T, portfolio C is worth $\max(S_T, X)$
- Assuming the cash in Portfolio D is invested at the riskfree interest rate, portfolio D is worth X at time T
  - Portfolio C is always worth at least as much as D at T
  - This must also be true today
  - Also, the option value cannot be negative (just let it expire)

$$p \geq \max(Xe^{-rT} - S_0, 0)$$

3.5. Option Bounds: lower bound for American call

- Possible that an American option will be exercised early
- Never early exercise American call on non-dividend paying stock
  - No income is sacrificed
  - We delay paying the strike price (earn interest)
  - Holding the call provides insurance against the stock price falling below strike price.
  - If you simply think that the stock is currently overpriced. you are better off selling the option than exercising it
- Therefore, lower bound on American call on a non-dividend paying stock is identical to lower bound on European call

$$C \geq \max(S_0 - Xe^{-rT}, 0)$$

- American puts are very often exercised early
- Lower bound on an American put option is how much you would get, if it was in-the-money (ITM) and you exercised it today

$$P \geq \max(X - S_0, 0)$$

3. Option Bounds - Summary[1]

European options on non-dividend stocks	
$c \leq S_0$	$p \leq Xe^{-rT}$
$c \geq \max(S_0 - Xe^{-rT}, 0)$	$p \geq \max(Xe^{-rT} - S_0, 0)$
American options on non-dividend stocks	
$C \leq S_0$	$P \leq X$
$C \geq \max(S_0 - Xe^{-rT}, 0)$	$P \geq \max(X - S_0, 0)$

[1] Will be provided in exam as part of formula sheet

4. Put-Call Parity

- Put-call parity
  - Option pair: calls and puts that are written on the same stock, having same strike and time to expiry
  - A relationship between the prices of the option pair
  - If we know the price of a call, we can derive the price of a put
  - And vice versa: if we know the price of a put, we can derive the price of a call

4. Put-Call Parity

- Consider the following two portfolios
  - Portfolio E: European call on a stock $+PV$ of the strike price in cash
  - Portfolio F: European put on the stock $+$ the stock
- Both are worth $\max(S_T, X)$ at the maturity of the options.
- They must therefore be worth the same today. This means

$$c + Xe^{-rT} = p + S_0$$

4. Put-Call Parity

	Time 0	Time T If $S_T < X$	Time T If $S_T > X$
Strategy E			
Long call option	c	0	$S_T - X$
Long bond	Xe^{-rT}	X	X
Total payoff		X	S_T

Strategy F			
Long put option	p	$X - S_T$	0
Long share	S_0	S_T	S_T
Total payoff		X	S_T

$$c + Xe^{-rT} = p + S_0$$

5. Extensions

- The Impact of Dividends
- European options on stocks with $D > 0$
- D is the present value of cash dividends during option's life
- Lower bound

$$c \geq \max(S_0 - Xe^{-rt} - D, 0)$$

$$p \geq \max(Xe^{-rt} - S_0 + D, 0)$$

- Put-Call Parity

$$c + Xe^{-rT} = p + S_0 - D$$

5. Extensions

- American options, $D = 0$

$$S_0 - X < C - P < S_0 - Xe^{-rT}$$

- American options, $D > 0$

$$S_0 - D - X < C - P < S_0 - Xe^{-rT}$$

- You should know these formulas in this extension section, and be able to use them in an exam for simple calculation questions, but the derivation steps are not required

14. Conclusion

- In today's class
  - Option basics
  - Stock option properties
  - Option bounds
  - Put-call parity
  - Extensions
- Next week
  - Option trading strategies
  - Binomial tree approach to value options