FINM2002 Derivatives
FINM6041 Applied Derivatives

Lecture 3 - Interest Rate Contracts and Swaps

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Hull *et al.*: Chapters 4 & 7

Australian
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University

- Interest rates compounding
- The price of forward and futures (different underlying assets)
- The value of forward
- Cross hedging and optimal hedge ratio
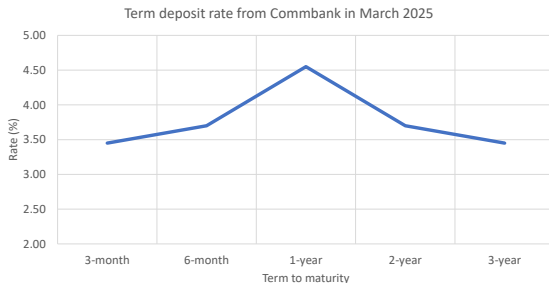- Hedging using index futures

Lecture Overview

- In today's class
  - Interest rates
  - Term structure and yield curve
  - Spot and forward rates
  - Interest rate swaps
  - The value of interest rate swaps

1. Interest Rates

- Interest rates
  - The amount of money a borrower promises to pay to a lender
  - Dependents on the credit risk of the borrower
    - Credit risk: the borrower may default and the interest and principal are not paid as promised
    - Higher credit risk $\rightarrow$ higher interest rate
- Fixed rate v.s. floating rate
  - Fixed rate: interest rate remains constant for the duration of a loan
  - Floating rate (variable rate): interest rate fluctuates over the duration of a loan

2. Term structure and yield curve

Term deposit rate from Commbank in March 2025

Term deposit rate CommBank	
Term to maturity	Deposit rate (%)
3-month	3.45
6-month	3.70
1-year	4.55
2-year	3.70
3-year	3.45

(1) Term structure of interest rates: on the same day, interest rates differ with the terms of maturity

(2) Yield curve: visualized term structure, a plot of annualized yields (Y-axis) against maturity (X-axis)

2. Term structure and yield curve

- Yield
    - Determined by bond price in the mathematical definition
    - A discount rate at which the current price of a bond equals to the present value of all promised future cash flows

$$P_0 = \sum_{t=1}^{T} \frac{CF_t}{(1+y)^t} = \sum_{t=1}^{T} CF_t e^{-y_c t}$$

    - $y = r_f + $ risk premium $+$ default premium $+$ etc.

2. Term structure and yield curve

- Term structure of risk free rate
  - Yields on government zero coupon bonds with different maturities
  - Zeros = government zero coupon bonds
  - Zero rates = yields of government zero coupon bonds
    = risk free rates = spot rates, at a given maturity

$$P = \frac{F}{(1+y)^T} = \frac{F}{(1+r_f)^T}$$
$$P = Fe^{-yT} = Fe^{-r_f T}$$

- Yield curve of zero rates, often used as a reference for fixed rates
- Can be used directly to discount cash flows of risk-free bonds

2. Term structure and yield curve

- Example
    - A two-year Treasury coupon bond
    - Principal $F = \$100$
    - Pays coupon at a rate of 8% p.a. semi-annually
    - The term structure of continuously compounded zero rates

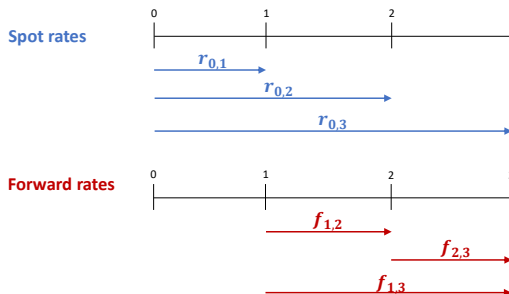| Maturity (years)	Zero rate (%)
0.5	6.0
1.0	6.2
1.5	6.3
2.0	6.2

2. Term structure and yield curve

- The bond pays coupons of

$$100x0.08/2 = \$4, \text{ twice a year}$$

- The price of the bond is

$$P = \sum_{t=1}^{T} CF_t e^{-yt}$$
$$= 4e^{-0.060 \times 0.5} + 4e^{-0.062 \times 1.0}$$
$$+ 4e^{-0.063 \times 1.5} + 104e^{-0.062 \times 2.0}$$
$$= \$103.15$$

3. Spot and Forward Rates

- **Spot rate** is the annualized interest rate for the period from today (time 0) to a future date T
- The interest rates in a term structure are spot rates
- **Forward rate** is the annualized interest rate for a period from a future date t to another future date T
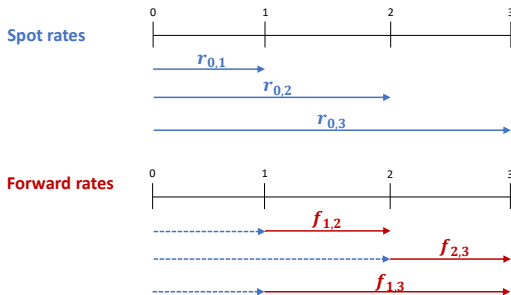- Explains why the spot rates vary with maturities in term structure

3. Spot and Forward Rates

- Forward rates are determined by the spot rates

$$(1 + r_{0,1})(1 + f_{1,2}) = (1 + r_{0,2})^2$$
$$(1 + r_{0,2})^2(1 + f_{2,3}) = (1 + r_{0,3})^3$$
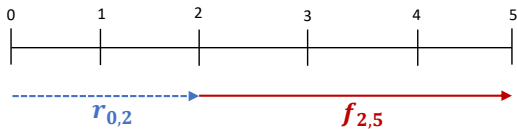$$(1 + r_{0,1})(1 + f_{1,3})^2 = (1 + r_{0,3})^3$$

3. Spot and Forward Rates

- Forward rates are determined by the spot rates
- A general formula for forward rate ($t < T$)

$$(1 + r_{0,t})^t \times (1 + f_{t,T})^{(T-t)} = (1 + r_{0,T})^T$$

$$f_{t,T} = \left[\frac{(1 + r_{0,T})^T}{(1 + r_{0,t})^t} \right]^{\frac{1}{T-t}} - 1$$

3. Spot and Forward Rates

- Example: given the term structure of Zeros
- If we want to borrow \$5000 in two years for a period of 36 months
- What is the forward rate for the required loan?

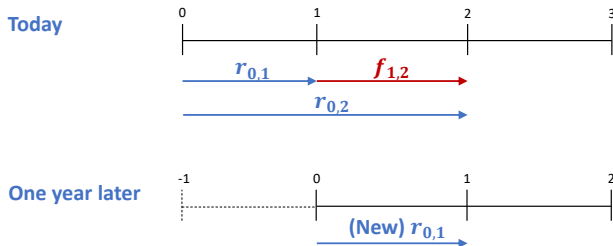| Maturity (years)	Yield (%)
1	1.9
2	2.1
3	2.3
4	2.5
5	2.7

$$f_{2,5} = \left[\frac{(1 + r_{0,T})^T}{(1 + r_{0,t})^t} \right]^{\frac{1}{T-t}} - 1 = \left[\frac{(1 + r_{0,5})^5}{(1 + r_{0,2})^2} \right]^{\frac{1}{5-2}} - 1 = 3.10\%$$

3. Spot and Forward Rates

- Forward rate today does not equal to the spot rate a year later
- E.g. $f_{1,2} \neq$ new $r_{0,1}$
- Forward rate reflects today's market expectation about the interest rate a year later
- The realized interest rate could differ from the expectations

3. Spot and Forward Rates

- Forward rate $f_{t,T}$ on a security that commences in year t and matures in year T

- Discrete

$$f_{t,T} = \left[\frac{(1 + r_{0,T})^T}{(1 + r_{0,t})^t} \right]^{\frac{1}{T-t}} - 1$$

- Continuous

$$e^{r_{0,T} \cdot T} = e^{r_{0,t} \cdot t} e^{r_{t,T} \cdot (T-t)}, \quad T > t$$

$$f_{t,T} = \frac{r_{0,T} \cdot T - r_{0,t} \cdot t}{T - t}$$

3. Spot and Forward Rates

- Example: 10.8% for 3 years, 11% for 4 years, continuously compounded, then forward rate between 3 and 4 years:

$$
\begin{aligned}
f_{3,4} &= \frac{r_{0,T} \cdot T - r_{0,t} \cdot t}{T - t} \\
&= \frac{11\% \times 4 - 10.8\% \times 3}{4 - 3} \\
&= 11.6\% \ p.a.
\end{aligned}
$$

3. Spot and Forward Rates

- Forward rate agreement (FRA)
    - An agreement that a certain rate will apply to a certain principal during a certain future time period
    - Over the counter interest rate derivatives
    - Two parties agree to guarantee each other a given interest rate at a future date
    - A notional principal is used but not exchanged
    - One party compensates the other party on settlement date

4. Interest Rate Swaps

- Common reference for floating rate
- LIBOR (fully stopped in 2023)
  - London InterBank Offered Rate
  - Large banks and financial institutions quote their wholesale rate
  - LIBOR is the average quoted rates every day
  - On a variety of short term maturities and currencies
- Replacement
  - The bank bill swap rate (BBSW) in Australia
  - Secured overnight financing rate (SOFR) in US

4. Interest Rate Swaps

- Why do we want to do trades / swaps?

- Absolute comparative advantage

  - *The Wealth of Nations* (Adam Smith, 1776)
  - An economy can produce a particular good or service at a lower cost (or higher quantity at same cost) than its trading partners

- Relative comparative advantage

  - *On the Principles of Political Economy and Taxation* (David Ricardo, 1817)
  - An economy can produce a particular good or service at a lower opportunity cost (or higher quantity at same opportunity cost) than its trading partners
  - Opportunity cost: the value of the next-best alternative when a decision is made

	Days to produce one unit		Opportunity cost	
	Cloth	Wine	Cloth	Wine
England	12	10	$\frac{12}{10}$ Wine	$\frac{10}{12}$ Cloth
Portugal	4	8	$\frac{4}{8}$ Wine	$\frac{8}{4}$ Cloth
Diff	8	2		

- To produce one unit of Cloth and Wine
  - England needs 12 days to produce Cloth and 10 days for Wine
  - Portugal needs 4 days to produce Cloth and 8 days for Wine
  - Portugal has *absolute comparative advantage* over England in both Cloth and Wine

- Comparative advantage when opportunity cost is lower
  - Portugal has *relative comparative advantage* in Cloth than England
  - England has *relative comparative advantage* in Wine than Portugal
  - We can also tell from the differences

4. Interest Rate Swaps

	Days to produce one unit		Opportunity cost	
	Cloth	Wine	Cloth	Wine
England	12	10	$\frac{12}{10}$ Wine	$\frac{10}{12}$ Cloth
Portugal	4	8	$\frac{4}{8}$ Wine	$\frac{8}{4}$ Cloth
Diff	8	2		

- Suppose each country needs one unit of Cloth and Wine
- Strategy A: Produce by each country and no trade
  - England spent $12 + 10 = 22$ days
  - Portugal spent $4 + 8 = 12$ days
  - Total time spent $= 34$ days
- Strategy B: Produces items with relative comparative advantage good and then trade
  - England produce 2 units of Wine, spent 20 days
  - Portugal produce 2 units of Cloth, spent 8 days
  - Trade and total time spent $= 28$ days, saved 6 days

4. Interest Rate Swaps

- Interest rate swaps
    - An agreement to exchange a floating rate cash flow stream for a fixed rate cash flow stream, or vice versa
    - Used for both liabilities or investments
    - Subject to default risk
    - Benefits
        - Change interest rate exposure from floating to fixed or vice versa
        - Reduce the costs of borrowing

4. Interest Rate Swaps

- Example
- Company A can borrow:
    - Fixed: 7%
    - Floating: LIBOR $+1\%$
    - But really wants a floating rate
- Company B can borrow:
    - Fixed: 10%
    - Floating: LIBOR $+ 2\%$
    - But really wants a fixed rate
- Both companies need to borrow $10 million
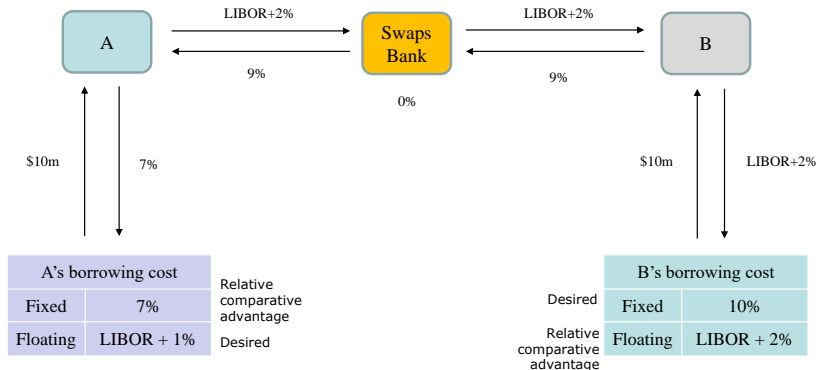- How can we design a swap that gets both parties what they want, and makes them better off?

4. Interest Rate Swaps

- Examine the comparative advantage of each party

	Fixed	Floating
Company A	7%	L+1%
Company B	10%	L+2%
Difference	3%	1%

- Maximum saving is *difference in differences*: $3\% - 1\% = 2\%$
  - Company A has a (relative) comparative advantage in fixed, but really wants a floating rate
  - Company B has a (relative) comparative advantage in floating, but really wants a fixed rate

A's borrowing cost		
Fixed	7%	
Floating	LIBOR + 1%	

Relative comparative advantage

Desired

B's borrowing cost		
Fixed	10%	
Floating	LIBOR + 2%	

Desired

Relative comparative advantage

- Total saving from swaps = 2%
- Assume the saving is equally shared between A and B, then each company enjoys a 1% saving in their borrowing cost.

4. Interest Rate Swaps

- The cost of borrowing after entered into interest rate swaps:
  - Company A is better off by 1% (i.e., LIBOR +1% - LIBOR)
  - Company B is better off by 1% (i.e., 10% - 9%)
  - Therefore, the total gain (saving) from swap is 2%

	Fixed	Floating
Company A	7%	L+1%
Company B	10%	L+2%
Difference	3%	1%

	Borrowed	Received	Paid out	Net
Company A	-7%	$+9\%$	$-(L+2\%)$	$-$LIBOR
Company B	$-(L+2\%)$	$+(L+2\%)$	-9%	-9%

4. Interest Rate Swaps

- What if swap bank requires 0.5% commission?
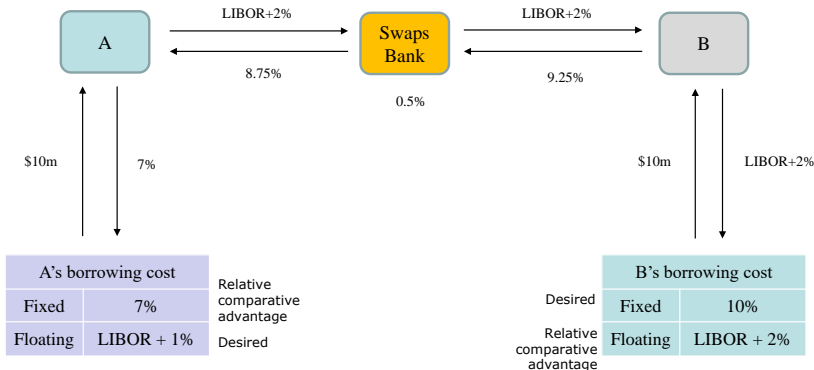 - After deducting the 0.5% as commission, the total gain left

    $$2\% - 0.5\% = 1.5\%$$

  - Assume the commission is evenly assigned to both companies, the gain left for each company is

    $$1.5\%/2 = 0.75\%$$

  - Company A outcome: LIBOR + 0.25% (i.e. LIBOR + 1% - 0.75%)
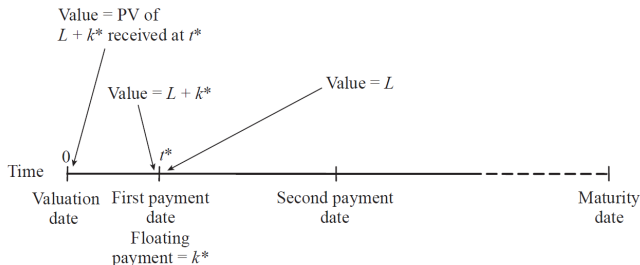 - Company B outcome: 9.25% (i.e. 10% - 0.75%)

4. Interest Rate Swaps

- Total saving from swaps = 2 - 0.5% = 1.5%
- Assume the saving is equally shared between A and B, then each company enjoys a 0.75% saving in their borrowing cost.

5. The Value of Interest Rate Swaps

- The value of interest rate swap
  - Zero when first initiated
  - When time goes on, its value may be positive or negative
  - The value of an interest rate swap is computed as the price difference between a fixed-rate bond and a floating-rate bond
- Price of fixed-rate bond: present value all of the cash flows
- Price of floating-rate bond
  - Equals notional principal *immediately* after next interest payment
  - Plus accrued interest for next interest payment
  - Discount back to today

5. The Value of Interest Rate Swaps

- The notional principal is L
- Next (accrued) interest payment is k^* at t^*
- Immediately before the payment $B_{fl} = L + k^*$
- PV of this single cash flow floating-rate bond is: $(L + k^*)e^{-r^* t^*}$
- Note that r^* is the LIBOR rate for a maturity of t^*

5. The Value of Interest Rate Swaps

- From the point of view of the floating-rate payer:
    - A long position in a fixed-rate bond and
    - A short position in a floating-rate bond
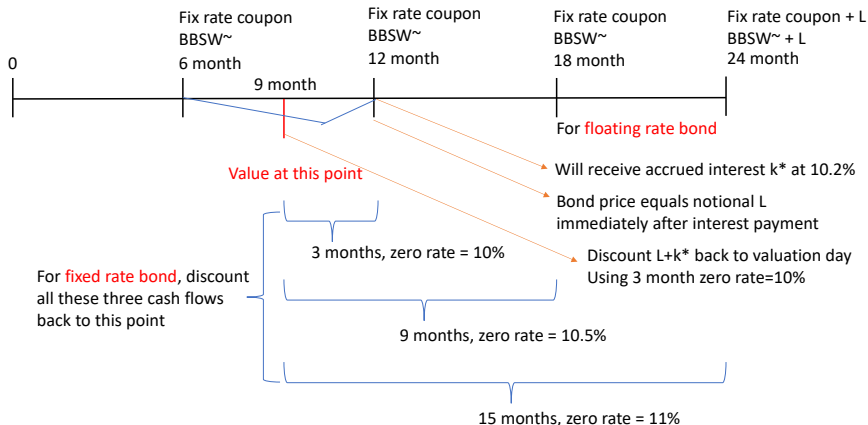
$$V_{swap} = B_{fix} - B_{fl}$$

- From the point of view of the fixed-rate payer:
    - A long position in a floating-rate bond and
    - A short position in a fixed-rate bond

$$V_{swap} = B_{fl} - B_{fix}$$

5. The Value of Interest Rate Swaps

- Example: in a swap agreement, an institution agrees to
  - Pay 6-month BBSW floating rate
  - Receive fixed 8% p.a every 6 months
  - Notional principal is $100 million
  - The 2-year swap has a remaining life of 1.25 years
  - The zero rates with continuous compounding for 3-month, 9-month, and 15-month maturities are 10%, 10.5% and 11% respectively
  - The 6-month BBSW rate at the last payment date was 10.2% with semi-annual compounding

5. The Value of Interest Rate Swaps

- The value of the fixed rate bond is as follows

$$Coupons = \frac{8\%}{2} \times \$100m = \$4m$$

$$B_{fix} = \$4m \times e^{-10\% \times \frac{3}{12}} + \$4m \times e^{-10.5\% \times \frac{9}{12}}$$

$$+ \$104m \times e^{-11\% \times \frac{15}{12}}$$

$$= \$98.328m$$

- The value of the floating rate bond is as follows

$$\text{Accrued interest} = \frac{10.2\%}{2} \times \$100m = \$5.1m$$

$$B_{fl} = \$(100 + 5.1)m \times e^{-10\% \times \frac{3}{12}}$$

$$= \$102.505m$$

5. The Value of Interest Rate Swaps

- The financial institution is paying floating and receiving fixed
- Therefore, the value of the swap is:

$$V_{swap} = B_{fix} - B_{fl}$$
$$= \$98.238m - \$102.505m$$
$$= -\$4.267m$$

- For the counterpart paying fixed and receiving floating, the value of this swap would be $+\$4.267m$ [1]

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[1] In terms of rounding, suggested to be precise in middle steps, and in final result round to two decimal points for dollar values, and two places after 0 for ratios. No penalty for being more precise.

6. Conclusion

- In today's class
    - Interest rates
    - Term structure and yield curve
    - Spot and forward rates
    - Interest rate swaps
    - The value of interest rate swaps
- Next week: options