FINM2002 Derivatives
FINM6041 Applied Derivatives
Lecture 10 - Exotic Options and MBS

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Hull *et al.*: Chapters 22

Australian National University

Review of Previous Lecture

- Last week
  - Delta hedging with Black Scholes Model
    - European options on non-dividend paying assets
    - European options on assets paying a dividend yield q
  - Delta of a portfolio
  - Other Greek Letters

Lecture Overview

- In today's class
  - Exotic Options
    - Asian options
    - Lookback options
    - Shout options
    - Chooser options
    - Barrier options
    - Binary options
    - Binomial-tree on Exotic options
    - Structured deposits (not required)
  - Mortgage Backed Securities (MBS)

1. Exotic Options

- Plain vanilla (standard) options
  - The options that we have discussed in the previous lectures
  - Standardized, well-defined properties and trade actively
- Exotic options
  - Non-standard
  - Created by financial engineers in recent years
  - Traded over the counter (OTC)
  - Modest trading volumes and relatively low liquidity
- Market demand for exotic options and non-standard products
  - Tax, accounting, legal or regulatory reasons
  - Customized designed products
  - Designed by investment banks to appear more attractive

1. Exotic Options

- Different types of exotic options
  - Asian options
  - Lookback options
  - Shout options
  - Chooser options
  - Barrier options
  - Binary options
  - Other exotic options

1.1 Asian Options

- Asian options: payoff depends on the average price of the underlying asset during the life of the option
- Asian Average Price options pay
  - Max $(S_{average} - X, 0)$ for call
  - Max $(X - S_{average}, 0)$ for put
- Often used to hedge the exposure over a certain period that one must buy or sell the underlying asset every day or more frequently than the available expiration dates for regular options or futures

1.1 Asian Options

- Example
  - Consider a 90-day Asian average price call option on a stock
  - Strike price is \$35
  - The payoff is based on the average price of the underlying stock determined at the end of each 30-day period
  - The stock price at 30th, 60th and 90th day of the option was \$30.65, \$36.9 and \$38.49, respectively
- The average stock price used to determine the option payoff

$$S_{average} = (30.65 + 36.9 + 38.49)/3 = \$35.35$$

- The payoff of this Asian call option

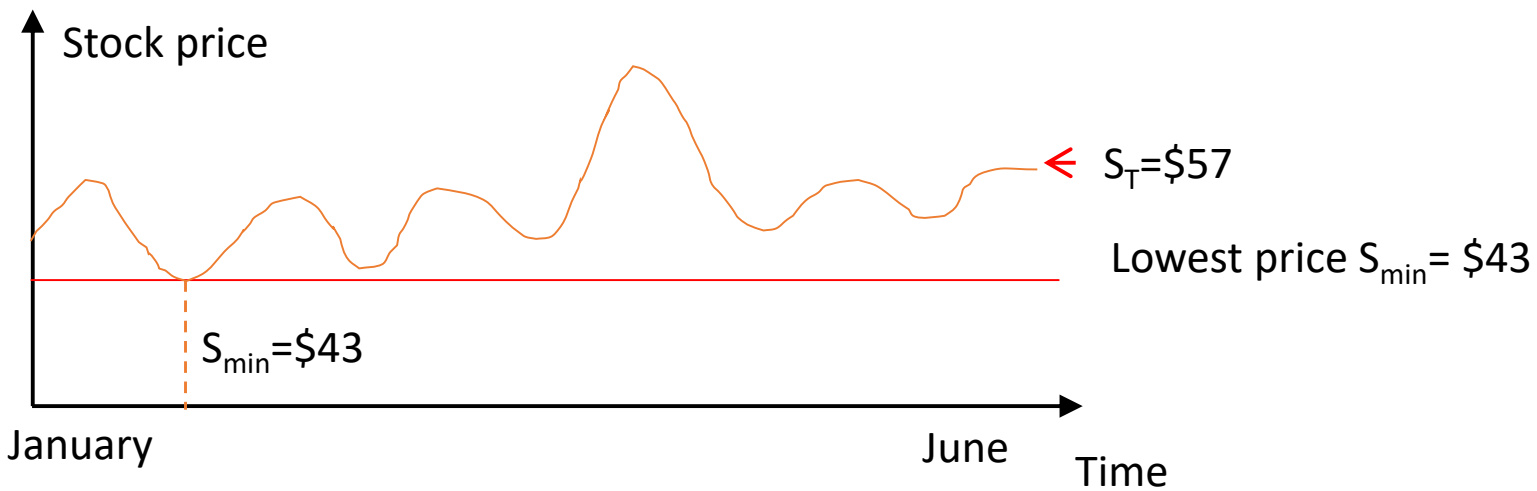
$$\text{Max}\,(S_{average} - X, 0) = 35.35 - 35 = \$0.35$$

1.2 Lookback Options

- Lookback options: payoff depends on the maximum or minimum asset price reached during the life of the option
- Lookback call
  - Allows a buyer to buy stock at the lowest observed price in some interval of time
  - Payoff $S_T - S_{min}$ at time T
- Lookback put
  - Allows a buyer to sell stock at the highest observed price in some interval of time
  - Payoff $S_{max} - S_T$ at time T

1.2 Lookback Options

- Example
  - Consider a 6-months Lookback call option on a stock
  - Figure shows the stock price movements in the following 6 months
  - The stock price at expiry time T is $S_T = \$57$
  - The lowest stock price during the option life is $S_{min} = \$43$
- We will exercise this Lookback call and buy the stock at S_{min}
- The option payoff is $S_T - S_{min} = 57 - 43 = \$14$

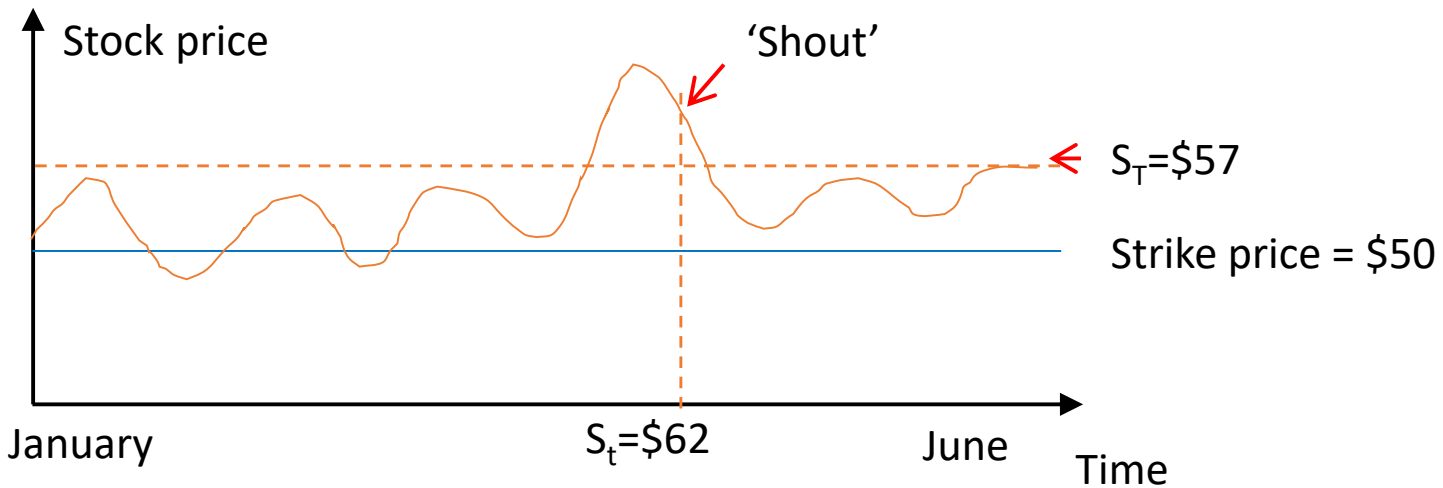


1.3 Shout Options

- Shout option: an European option but the holder can 'shout' to the writer (reserve a price) at one time during the life of the option
- The final payoff at expiry for a shout call option is either
  - Usual option payoff, Max $(S_T - X, 0)$, or
  - $S_t - X$ at time of shout
- Payoff for a shout call option: Max $(S_T - X, S_t - X, 0)$
- Example: $X = \$50$ and the call holder shouts when the underlying asset price $S_t = \$60$. On the expiration date:
  - If $S_T < \$60$, the holder receives a payoff of $S_t - 50 = \$10$
  - if $S_T > \$60$, the holder receives a payoff of $S_T - 50 > \$10$

1.3 Shout Options

- Example
  - Consider a 6-months Shout call option on a stock
  - The strike price is \$50
  - Figure shows the stock price movements in the following 6 months
  - The stock price at expiry time T is $S_T = \$57$
  - The call holder 'shouts' when $S_t = \$62$
- We will exercise this Shout call at expiry, note $S_t > S_T$
- The option payoff is $S_t - X = 62 - 50 = \$12$

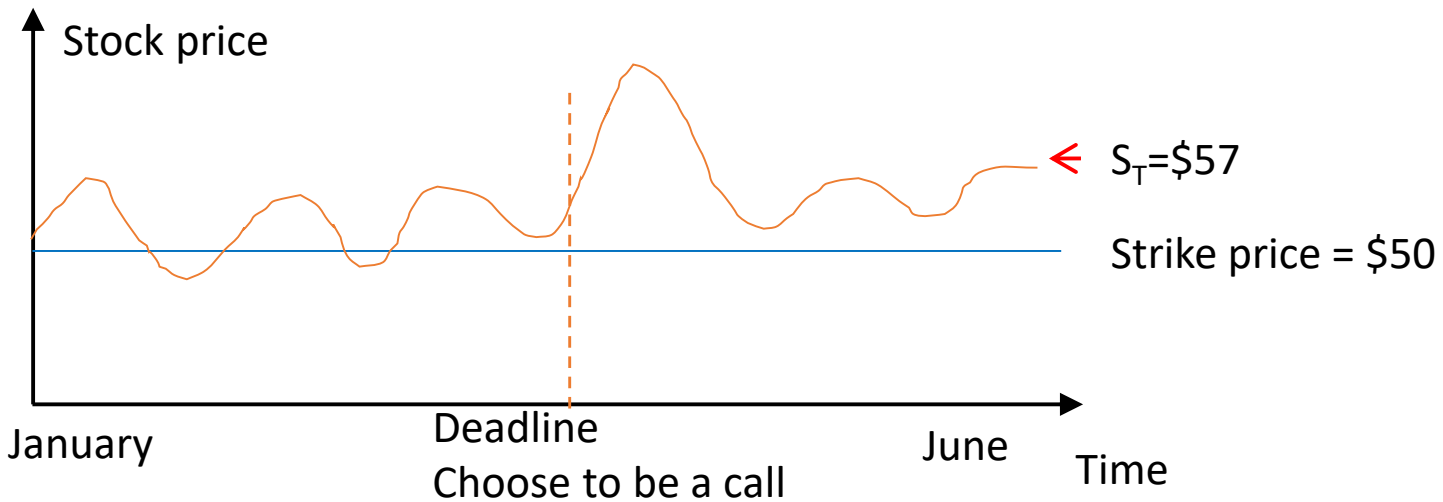


1.4 Chooser Options

- A chooser option gives the holder the ability to
  - Choose whether the option is a call or a put option
  - A certain period prior to the expiration date
  - Also referred to as an "as you like it" option
- The value of the chooser option at the time this choice is made is $max(c, p)$

1.4 Chooser Options

- Example
  - Consider a 6-months Chooser option on a stock
  - The strike price is \$50
  - Figure shows the stock price movements in the following 6 months
  - The stock price at expiry time T is $S_T = \$57$
  - The option decides to make this option as a call before deadline
- We will exercise this call at expiry
- The option payoff is $S_T - X = 57 - 50 = \$7$



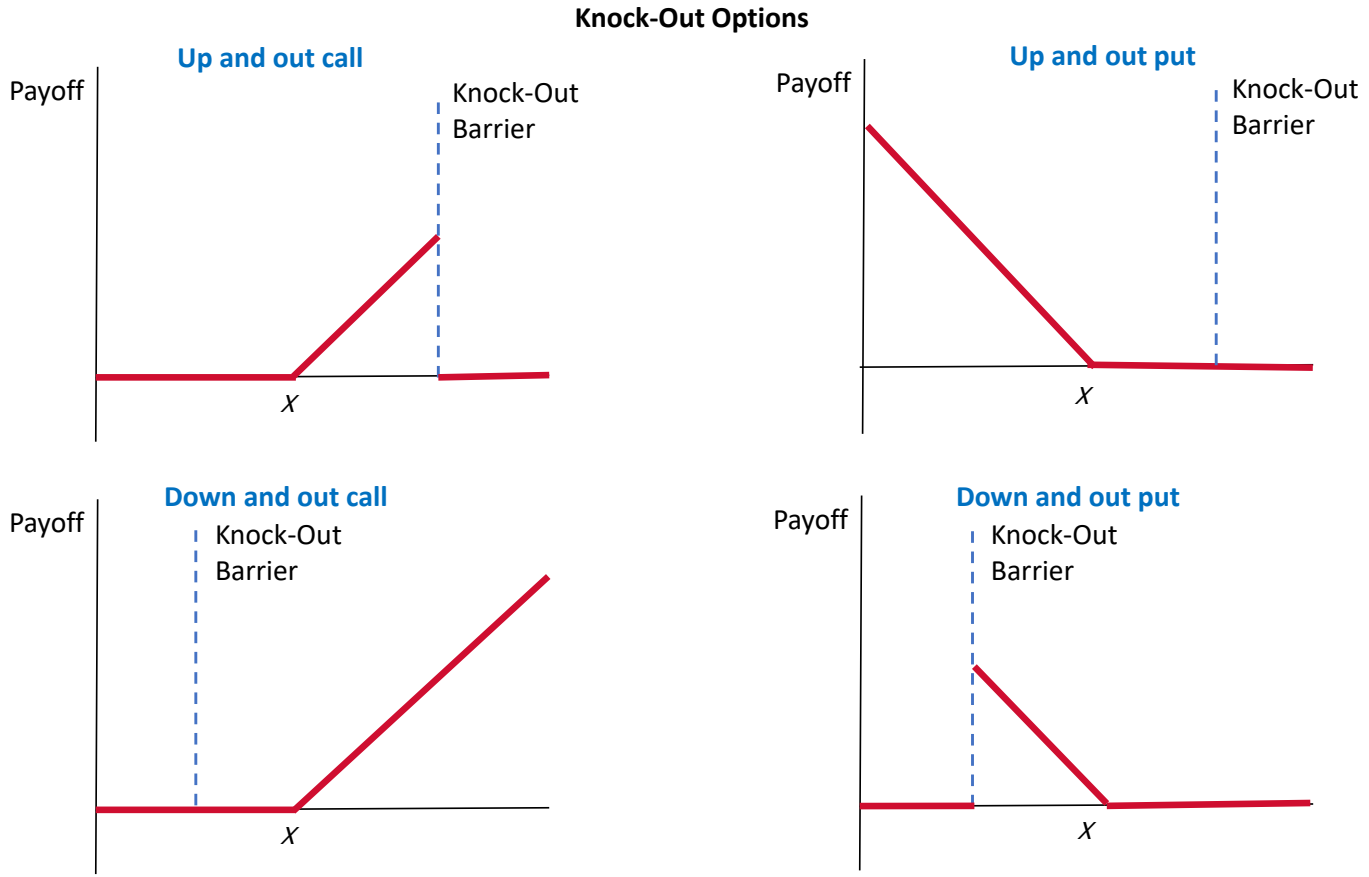
1.5 Barrier Options

- Barrier options: payoff depends on whether the underlying asset's price reaches a certain level before expiry
- Less expensive than the corresponding regular options
- Barrier options are classified as
  - A Knock-Out option *ceases to exist* once the underlying asset price reaches a certain level (barrier) before expiry
  - A Knock-In option *comes into existence* only when the underlying asset price reaches a certain level (barrier) before expiry

1.5 Barrier Options - Knock-Out

- The four types of Knock-Out options are
  - An up-and-out call option is a regular European call option that ceases to exist as soon as the asset price reaches a barrier level which is greater than the asset price at the time option is initiated
  - A down-and-out call option is a regular European call option that ceases to exist as soon as the asset price reaches a barrier level which is below the asset price at the time option is initiated
  - An up-and-out put option is a regular European put option that ceases to exist as soon as the asset price reaches a barrier level which is greater than the asset price at the time option is initiated
  - A down-and-out put option is a regular European put option that ceases to exist as soon as the asset price reaches a barrier level which is below the asset price at the time option is initiated

1.5 Barrier Options - Knock-Out

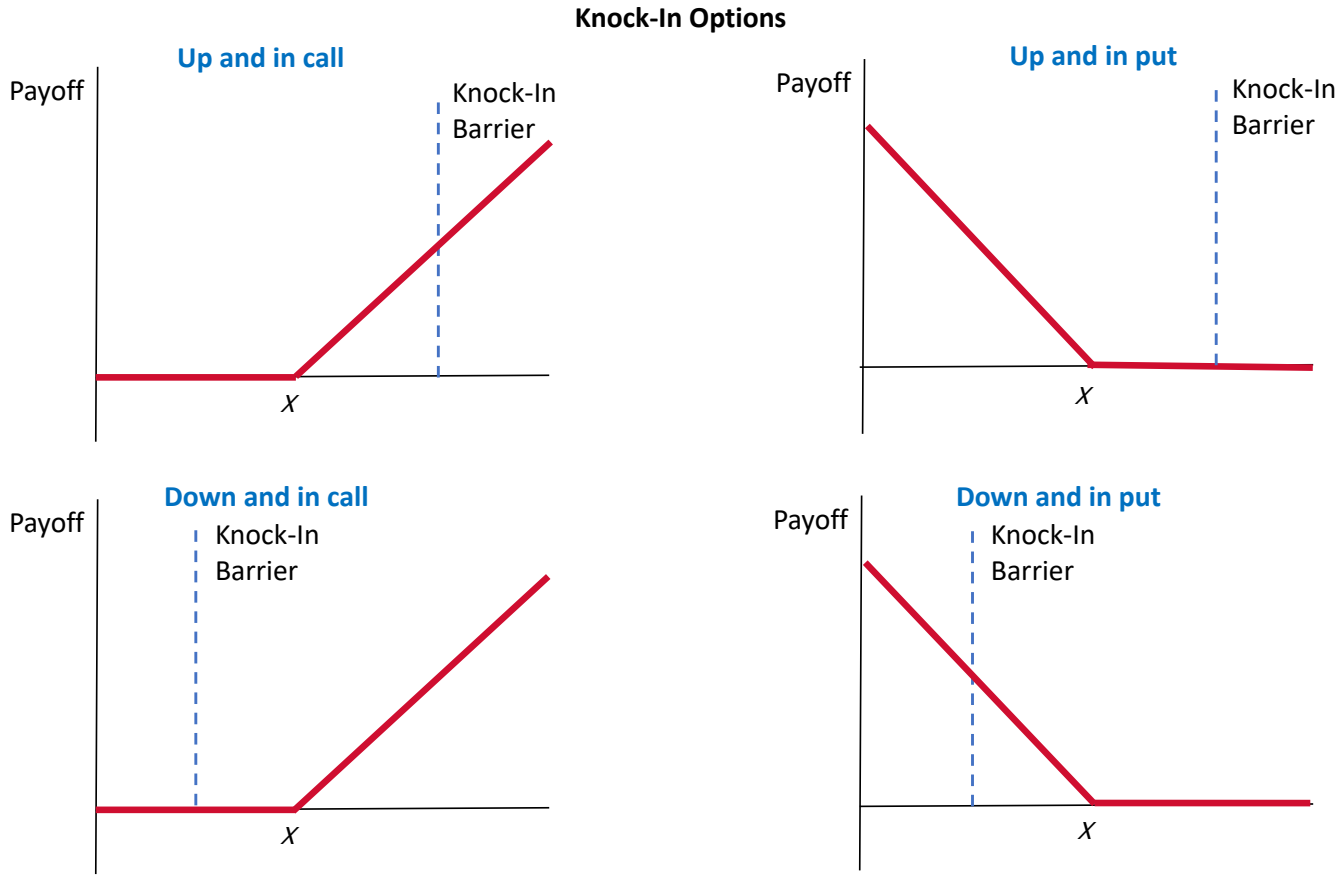


- Once the underlying asset price reaches the knock-Out barrier, the option ceases to exist. The option value remains zero even if the price should move back within the barrier before the expiry.

1.5 Barrier Options - Knock-In

- The four types of Knock-In options
  - An up-and-in call option is a regular European call option that starts to exist as soon as the asset price reaches a barrier level. The barrier level is greater than the asset price when the option is initiated
  - A down-and-in call option is a regular European call option that starts to exist as soon as the asset price reaches a barrier level. The barrier level is below the asset price when the option is initiated
  - An up-and-in put option is a regular European put option that starts to exist as soon as the asset price reaches a barrier level. The barrier level is greater than the asset price when the option is initiated
  - A down-and-in put option is a regular European put option that starts to exist as soon as the asset price reaches a barrier level. The barrier level is below the asset price when the option is initiated

1.5 Barrier Options - Knock-In



- Once the underlying asset price reaches the knock-In barrier, the option comes into existence and works as a regular option. Otherwise the option value remains zero.

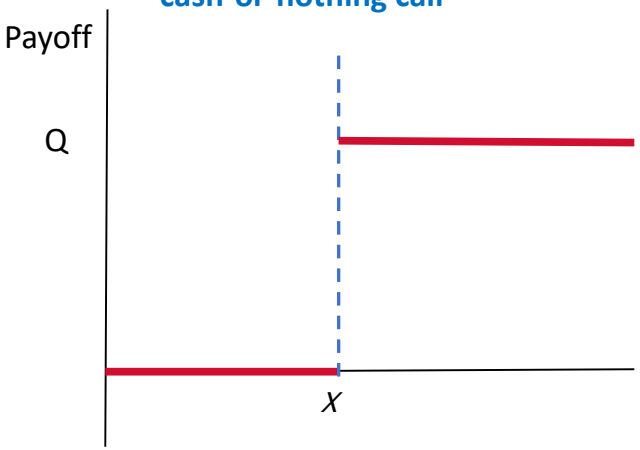
1.6 Binary options

- Binary options: discontinuous payoffs which depend on whether a condition is true or false
  - Cash-or-nothing put and call
  - Asset-or-nothing put and call
  - Usually short expiration period

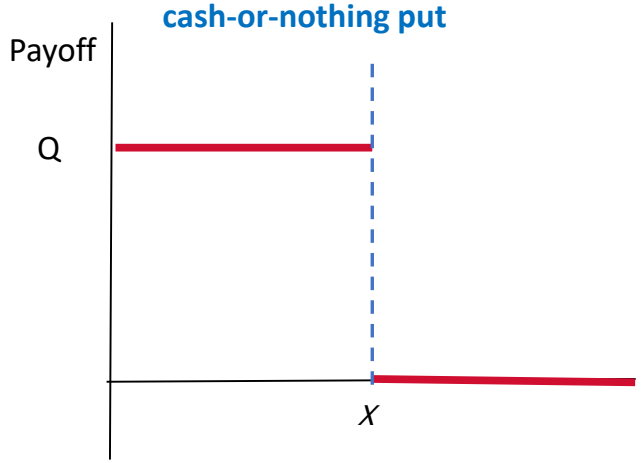
1.6 Binary options

Binary Options

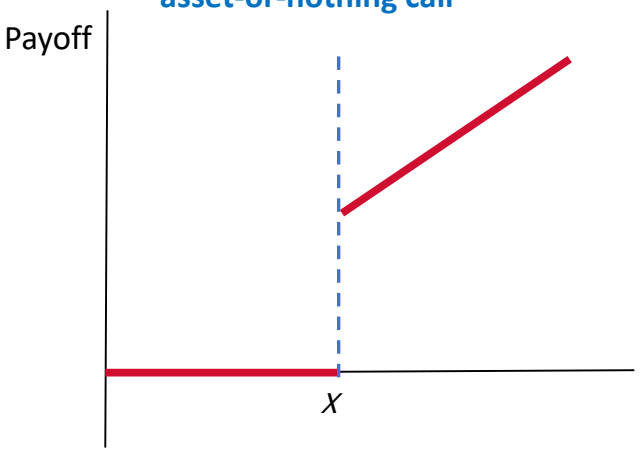
cash-or-nothing call



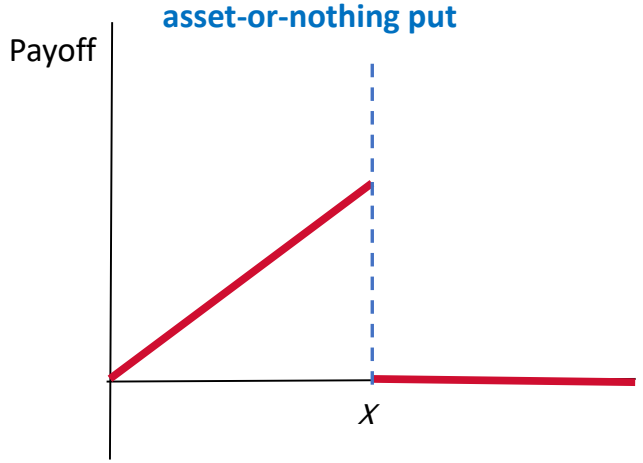
cash-or-nothing put



asset-or-nothing call



asset-or-nothing put



1.6 Binary options

- Cash-or-nothing call
  - If $S_T < X$, option pays nothing
  - If $S_T > X$, option pays a fixed amount of cash Q
  - Option price today $= Qe^{-rT}N(d_2)$
- Cash-or-nothing put
  - If $S_T < X$, option pays a fixed amount of cash Q
  - If $S_T > X$, option pays nothing
  - Option price today $= Qe^{-rT}N(-d_2)$
- Asset-or-nothing call
  - If $S_T < X$, option pays nothing
  - If $S_T > X$, option physically delivers the underlying asset (pays S_T)
  - Option price today $= S_0e^{-qT}N(d_1)$
- Asset-or-nothing put
  - If $S_T < X$, option physically delivers the underlying asset (pays S_T)
  - If $S_T > X$, option pays nothing
  - Option price today $= S_0e^{-qT}N(-d_1)$

1.6 Binary options

- Additional intuition (Optional)
- A regular European call option is equivalent to
  - A long position in an asset or nothing call, plus
  - A short position in a cash-or-nothing call,
  - When the fixed cash amount in cash-or-nothing call $Q = X$

$$c = S_0 e^{-qT} N(d_1) - Q e^{-rT} N(d_2)$$

- A regular European put option is equivalent to
  - A short position in an asset or nothing put, plus
  - A long position in a cash-or-nothing put,
  - When the fixed cash amount in cash-or-nothing put $Q = X$

$$p = Q e^{-rT} N(-d_2) - S_0 e^{-qT} N(-d_1)$$

1.6 Binary options

- Example
  - Consider a binary call option on a stock
  - Expires in one year
  - The current stock price is \$14.8
  - The risk free rate is 4% p.a.
  - The stock pays a dividend yield of 8% p.a.
  - The volatility of the stock price is 12% p.a.
- What is the price of this option if the payoff will be
  (1) \$10 cash if the stock price $S_T > \$15$
  (2) One share of the underlying stock if its price $S_T > \$15$

1.6 Binary options

- Given $S_0 = 14.8, X = 15, r = 4\%, q = 8\%, \sigma = 12\%, T = 1$

$$d_1 = \frac{\ln(S_0/X) + (r - q + \sigma^2/2)T}{\sigma\sqrt{T}} \approx -0.39$$

$$d_2 = d_1 - \sigma\sqrt{T} = -0.51$$

- $N(d_1) = 0.3483, N(d_2) = 0.3050$

(1) This is a cash-or-nothing call, its price today

$$Qe^{-rT}N(d_2) = 10^{-4\%\times 1} \times 0.3050 = \$2.93$$

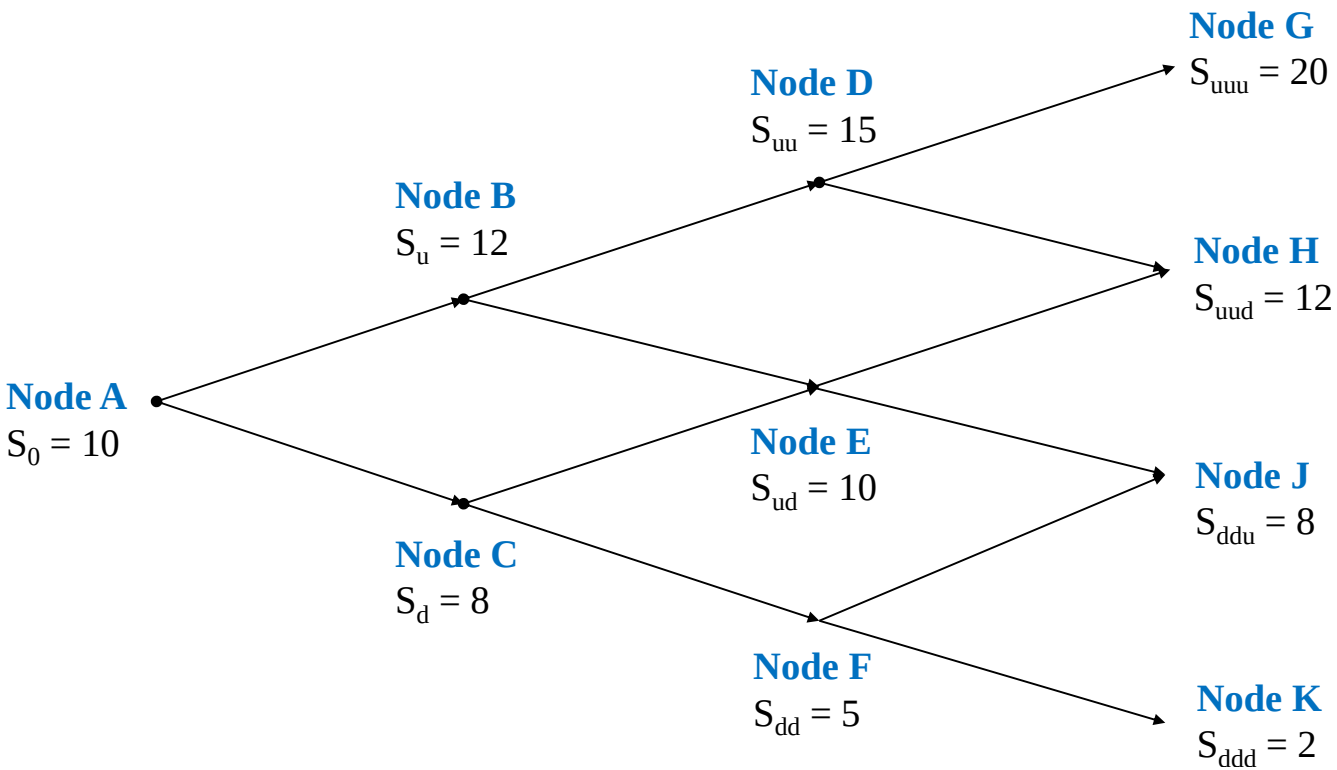
(2) This is an asset-or-nothing call, its price today

$$S_0 e^{-qT}N(d_1) = 14.8e^{-8\%\times 1} \times 0.3483 = \$4.76$$

1.7 Use Binomial-tree to price Exotic options

- How to price exotic options
  - Black-Scholes model is not suitable most of the time
  - Banks use sophisticated models – the core is Binomial-tree model
  - A Binomial-tree model with many steps to numerically approximate the price of exotic options
  - Take the special option conditions into account and compute option payoffs on each price movement path
  - Repetitive simulations with different assumptions on the stock price movements to get a precise estimate
- In this class
  - Make some basic attempt to compute an option's payoff following a given price movement path in a binomial tree

1.7 Use Binomial-tree to price Exotic options



The stock price movements in each node are described in the above figure (all prices are in \$). For example, the price path A-B-D-G means the top path on the binomial tree where the stock price increases three times until Node G

1.7 Use Binomial-tree to price Exotic options

(1) In the price movement path A-B-D-G, what is the payoff of an Asian Average Price call option? The strike price is \$10 and the stock prices at node B, D, and G should be used to compute the average price.

Answer
The average price is (12+15+20)/3=\$15.67.
Exercise the call at Node G and payoff is 15.67-10=\$5.67

(2) In the price movement path A-B-E-J, what is the payoff of a European UP-and-Out call option, with a strike price of \$5 and a Knock-Out Barrier of \$11, expires at the end of the binomial tree?

Answer
Stock price goes above the barrier \$11 at Node B, and therefore the option is knocked out, and the payoff will be zero at Node J

1.8 Structured Deposits

- Application of exotic options – Structured Deposits (optional)
- Three products from China Merchants Bank this week

1. *Structured deposit: 185-day two-layer bull strategy on gold*
   - It's a fixed-term deposit matures in 185 days
   - The deposit interest to be paid depends on the closed-price of gold (Bloomberg goldlnpm index) on expiration date, related to a benchmark price determined today
   - If $S_T > X_{0,benchmark}$, deposit rate is 2.0% p.a.
   - If $S_T < X_{0,benchmark}$, deposit rate is 1.5% p.a.
   - Principal of deposit is covered by government deposit insurance

- Analysis
  - Essentially a fixed-term deposit plus a Binary Cash-or-nothing call
  - A small fraction of interest is used to buy the option for higher $E(r)$
  - Realized return is no longer certain but still positive
  - Pricing of such product involves risk-free asset and exotic options

1.8 Structured Deposits

2. *Structured deposit: 185-day advanced cumulative range on gold*
   - The deposit interest to be paid depends on the daily average gold price during the life of the deposit (185 days), compared to a range of price bounds determined today
   - If $S_{min} < S_{average} < S_{max}$, deposit rate is 2.0% p.a.
   - Otherwise if $S_{average}$ is outside of the price bounds, deposit rate is 1.05% p.a.

- Analysis
  - Essentially a fixed-term deposit, plus a special form of Asian average price option, plus some Butterfly spread strategy, and also some Binary Cash-or-nothing property

1.8 Structured Deposits

3. *Structured deposit: 365-day advanced bear strategy on CSI500 index with take-profit automatic redemption*
    - The deposit interest to be paid depends on the CSI500 index during the life of the deposit, compared to a benchmark determined today
    - On the last trading day of each month (i.e. observation days), if $S_\tau < X$, deposit rate is 3.4% p.a., and the deposit will be redeemed automatically and immediately
    - Otherwise if the condition is not met on any observation day, deposit will mature in 365 days as usual and pay a rate of 0.1% p.a.

- Analysis
    - Essentially a fixed-term deposit, plus a non-standardized form of American put on index, with some Binary Cash-or-nothing property
    - Possibility of automatic early exercise

2. Mortgage-Backed Securities

- Mortgage-Backed Security (MBS)
  - A derivative security and a fixed-income security
  - One type of Asset–Backed Security (ABS)
  - Financial institutions sell part of their mortgage portfolios to investment banks or government institutions
  - Create MBS through securitization and sell to investors
  - Collateralized by a pool of mortgages
  - Main cause of the 2008 subprime crisis

2. Mortgage-Backed Securities

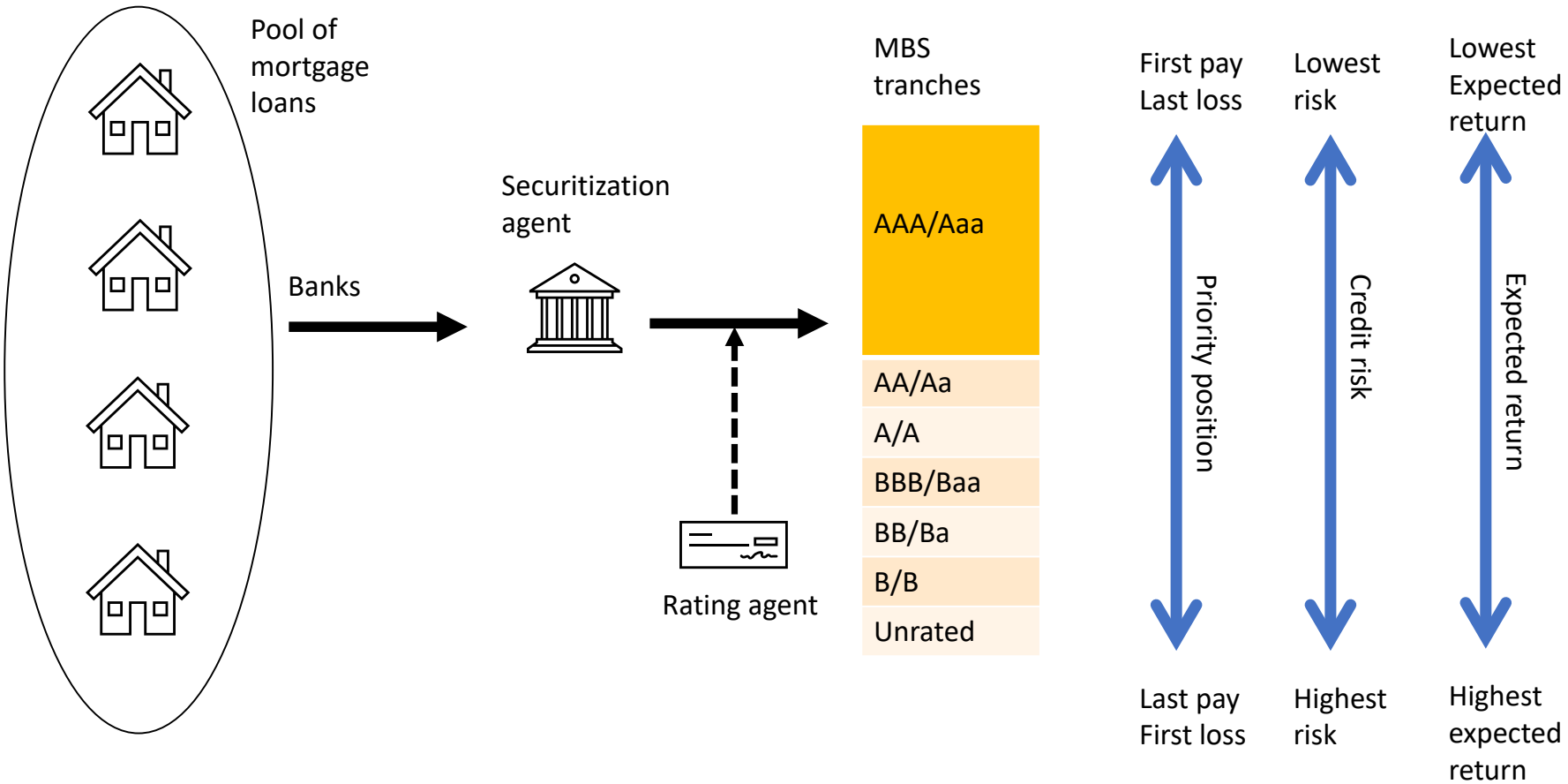
- Participants of MBS
  - Mortgage lending financial institutions
    - e.g. commercial banks
    - Sell mortgages and transfer risk
    - Use the proceeds from the sale to make new loans
  - Securitization agents
    - e.g. Freddie Mac, Fannie Mae, investment banks
    - Buy mortgages from the banks
    - Package pools of mortgages into MBS and sell to investors
  - Investors
    - Retail investors, governments, financial institutions and mutual funds, etc
    - Become the ultimate creditor of the mortgage borrowers
    - Attractive investment flexibility and returns
  - Rating agencies
    - e.g. S&P, Moody's and Fitch

2. Mortgage-Backed Securities

- Two types of MBS
- Pass-through MBS
  - Principal and interests payments are passed through to the investors
  - All investors receive same rate of return and bear same risk
  - Ownership of X% is entitled to X% of the principal and interest cash flows received from the mortgages in the pool
- Collateralized mortgage obligation (CMO)
  - Comprise multiple classes of debt securities (tranches)
  - Different maturities and priorities in the receipt of the principal and the interest
  - Different credit ratings
  - More attractive to different investors

2. Mortgage-Backed Securities

- Collateralized mortgage obligation (CMO)



2. Mortgage-Backed Securities

- The role of MBS in the 2008 Financial Crisis (GFC)
- Sub-prime mortgage crisis
  - Mortgage to low-credit quality borrowers
  - Banks securitized mortgages and sold to investors
  - Investors are unprotected from the default risk
  - Excessive speculation on house price keep raising
  - Aggressive expansion of mortgage and MBS market
  - Mortgage borrowers began to default and lead to a collapse of MBS
- But it was only the start of GFC
  - The real problem was the CDS – more discussion in Lecture 11
  - Over-leveraging on massive derivative products

4. Conclusion

- In today's class
  - Exotic options
  - Mortgage backed securities
- Next week: CDS and other derivatives
- Final exam information
  - May 30, Friday, from 2PM to 4:15PM (2 hours)
  - In-person and closed-book
  - 65% to your final grade
  - Formula sheet will be provided in week 12 and also in exam
  - Details and revision session in week 12