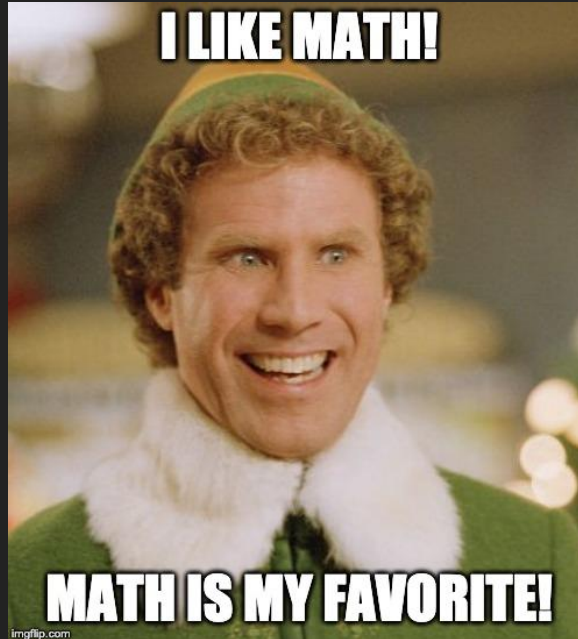
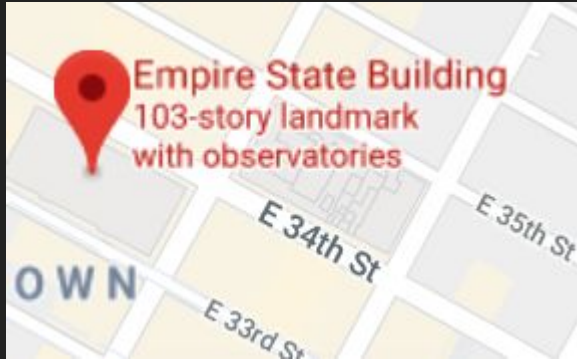


# Math!



# Where is the Empire State Building?



34th and 5th

40.7484° N, 73.9857° W

# Cartesian Coordinate System

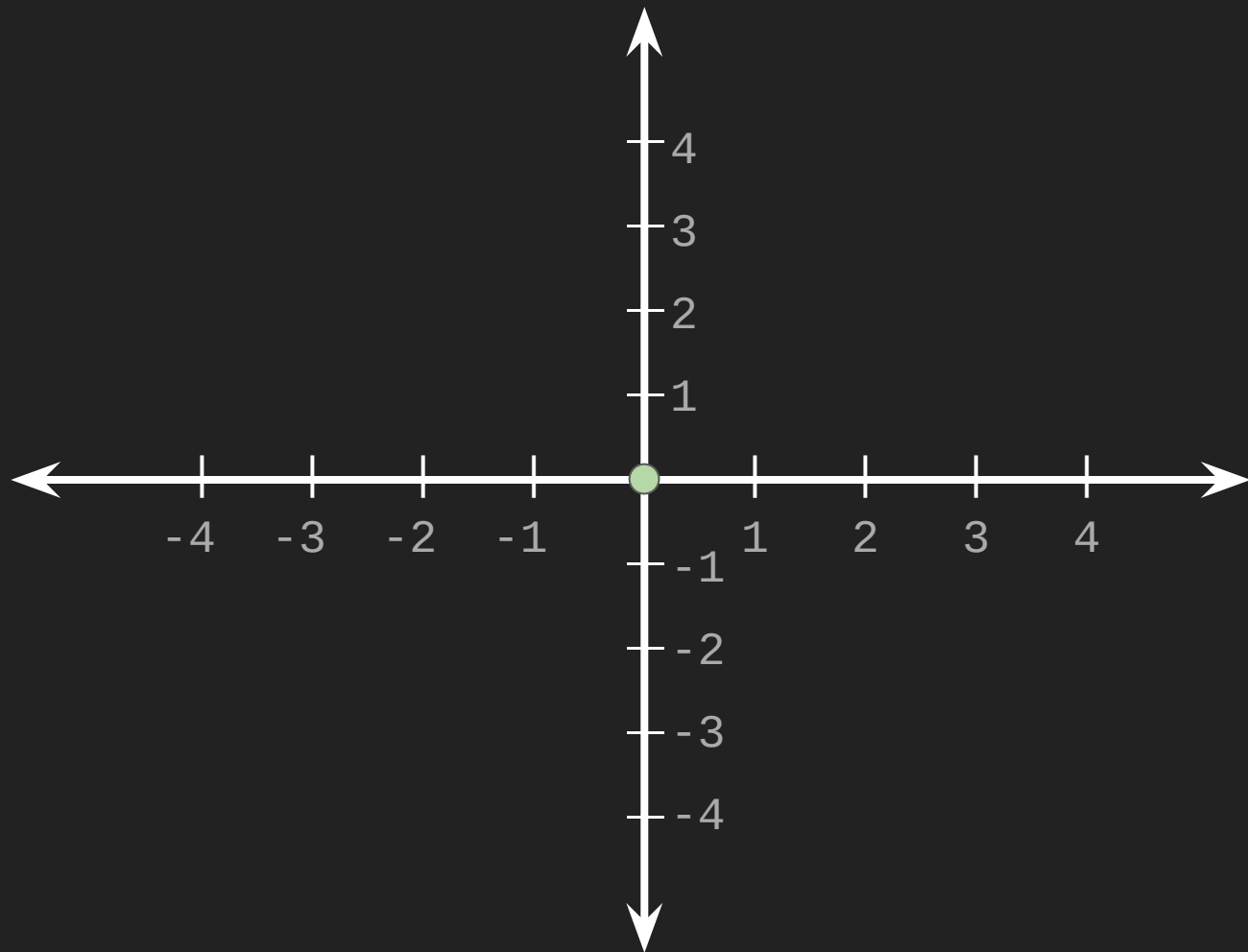


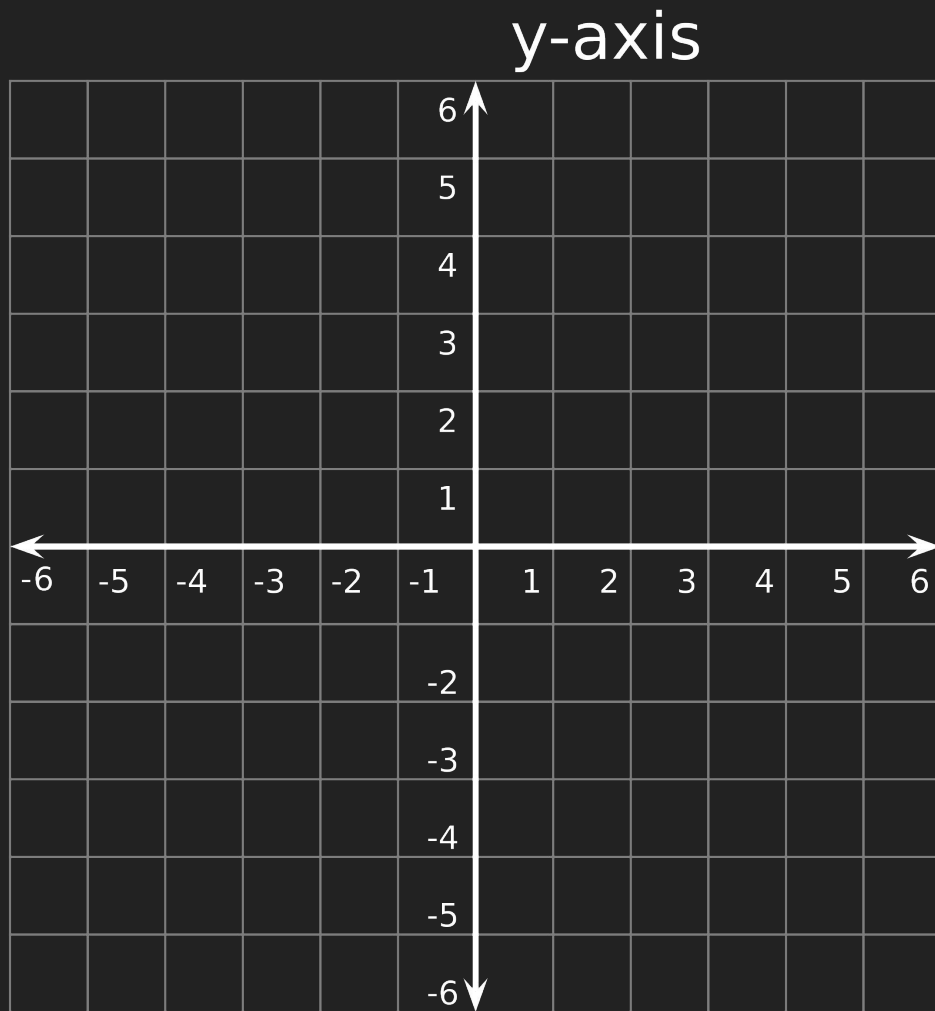
## René Descartes

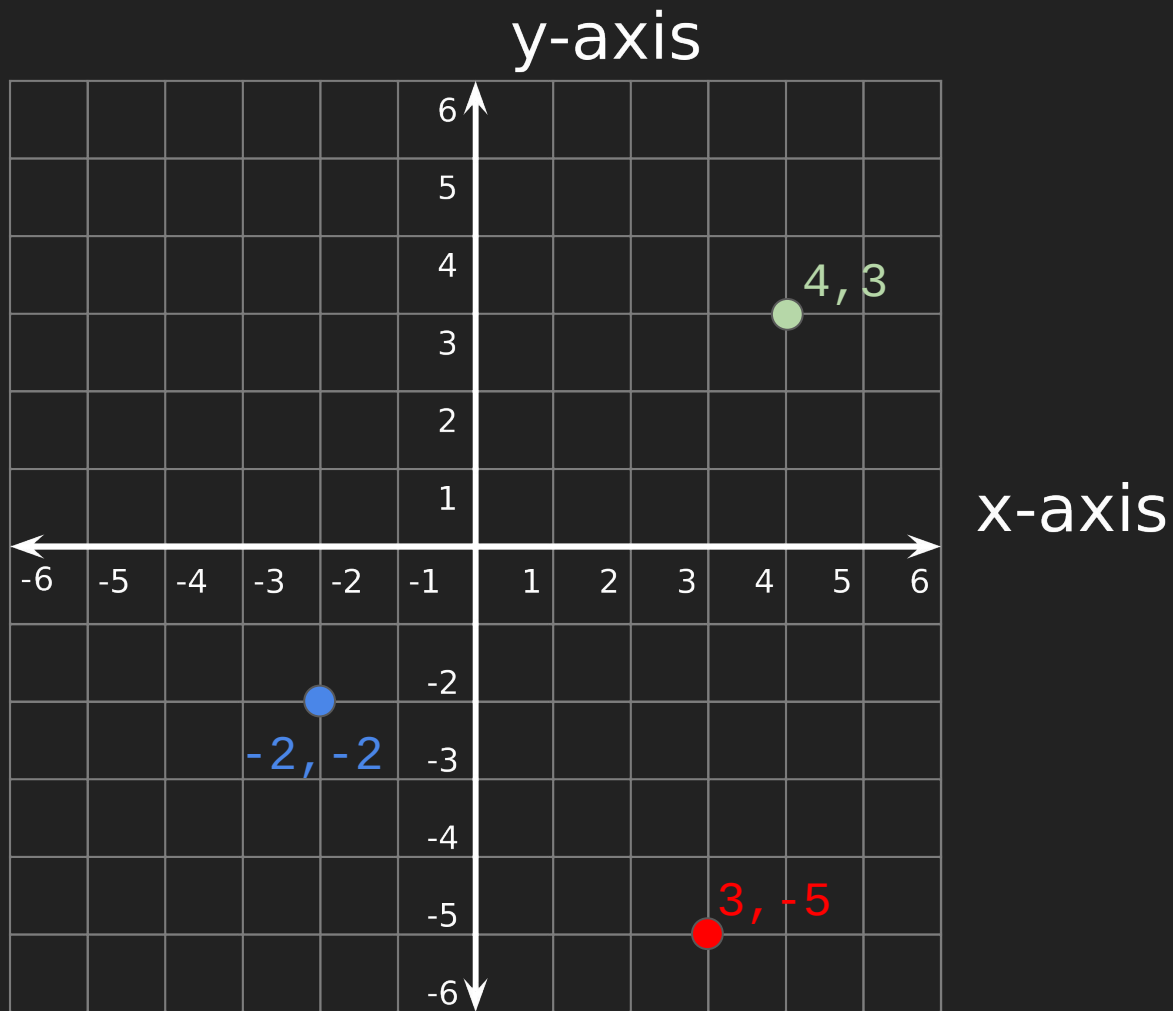
Not just a mathematician and scientist but also a philosopher:

“I think, therefore I am.”









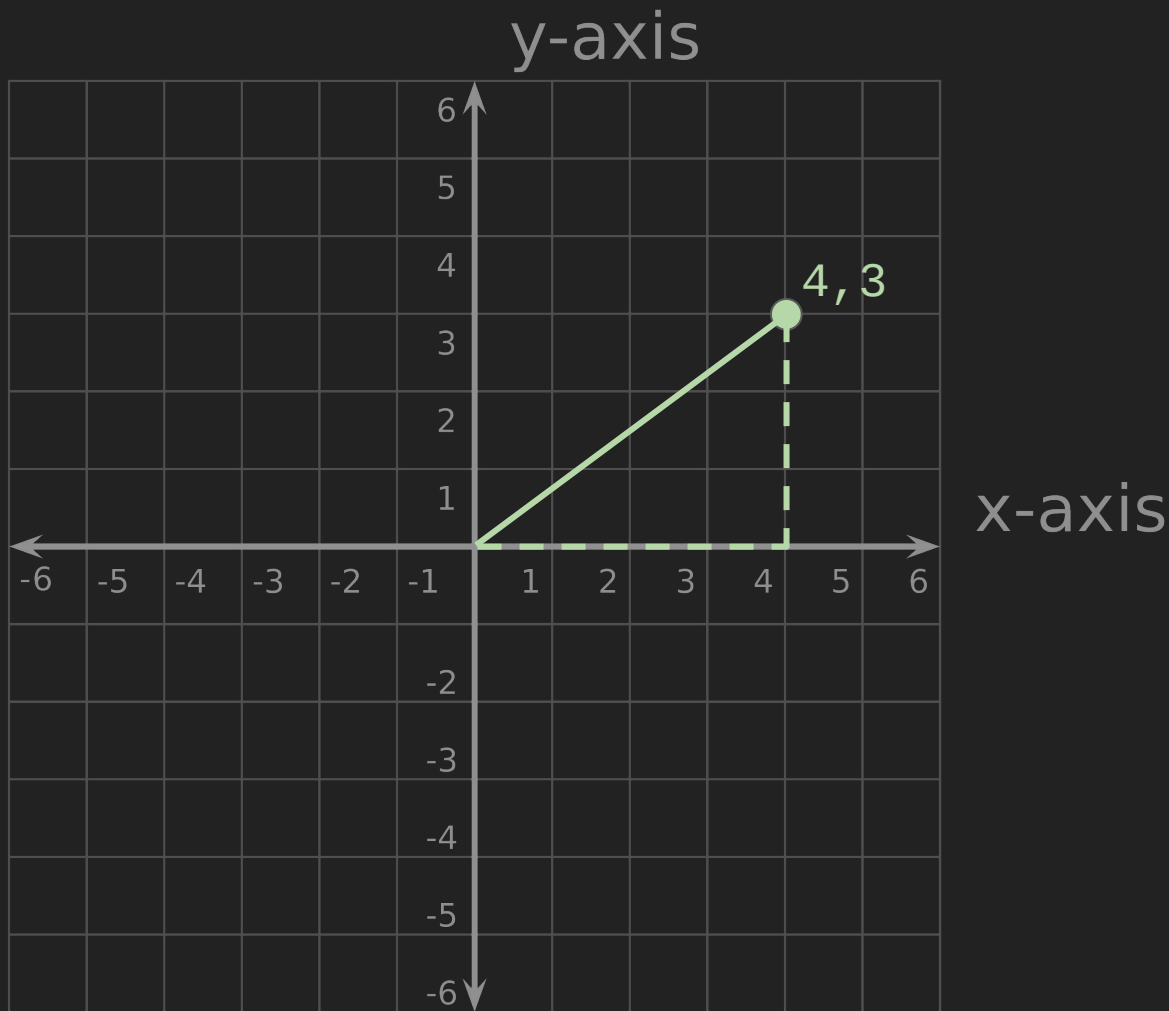
$$A^2 + B^2 = C^2$$

$$4^2 + 3^2 = C^2$$

$$16 + 9 = C^2$$

$$25 = C^2$$

5





# A point is a location on a plane.

(we can't exactly math a point)

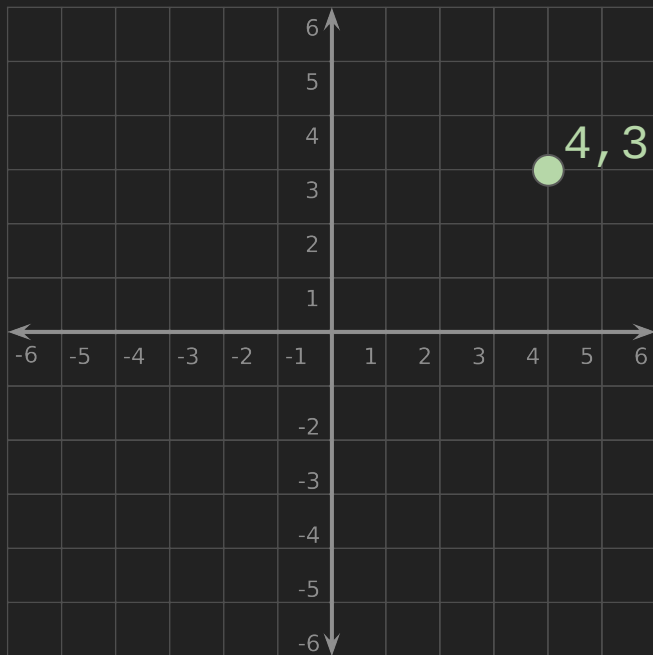
Empire State Building + The Brooklyn Bridge = ?

2 \* Pace Plaza = ?

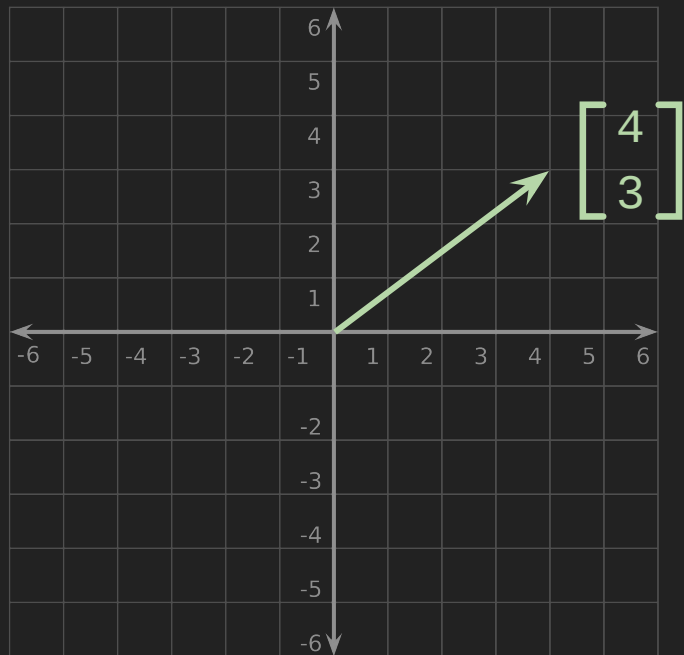
Rotate Carnegie Hall  $45^0$  = ?

# Vectors!

# Point

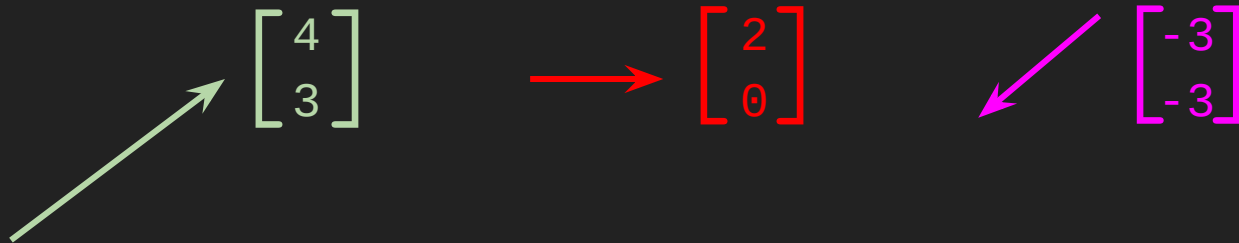


# Vector



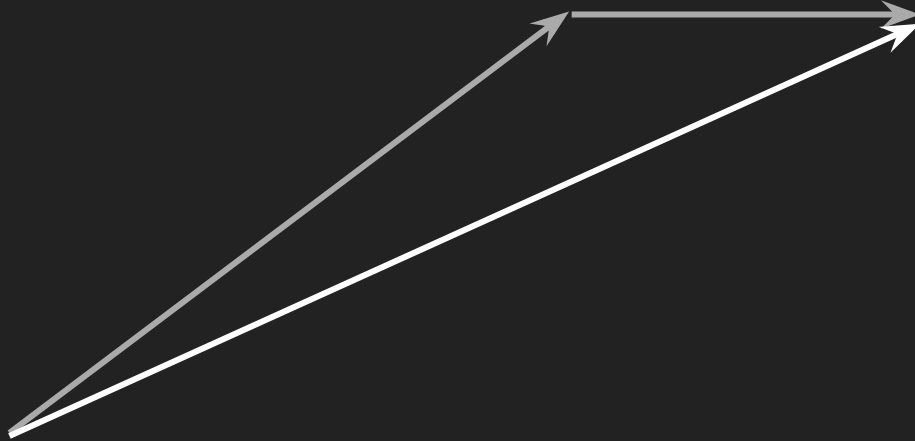
# Vectors

Vectors have a direction and magnitude (length).  
Can be thought of as a displacement.



# Vectors

We can add vectors.



# Vectors

We can multiply a vector by a scalar.

$$2 * \begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 8 \\ 6 \end{bmatrix}$$

# Vectors

We can rotate a vector.



$$\begin{bmatrix} 4 \\ 0 \end{bmatrix}$$



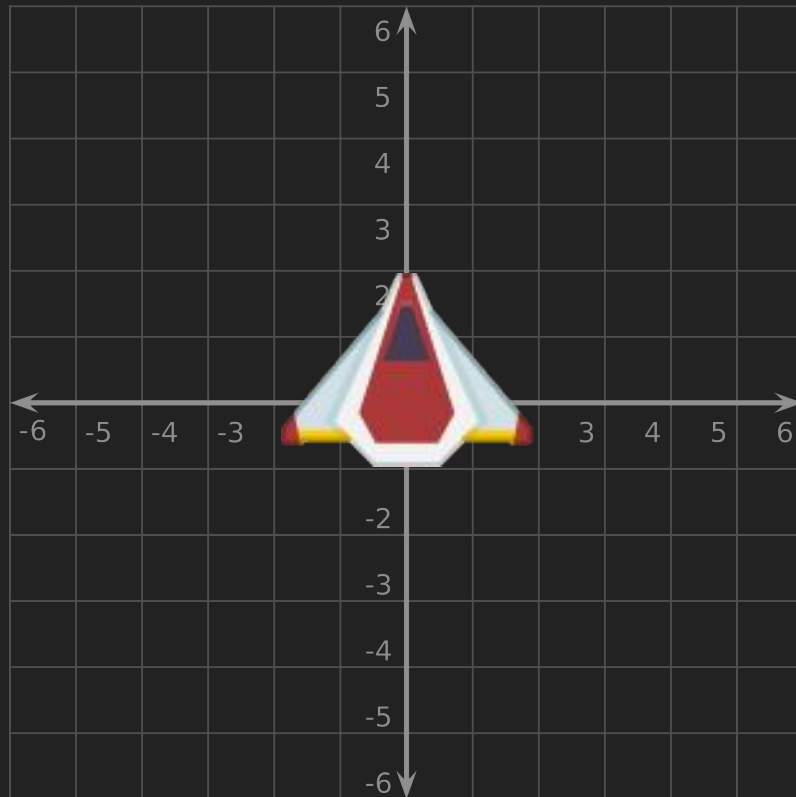
$$\begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

# Transformations

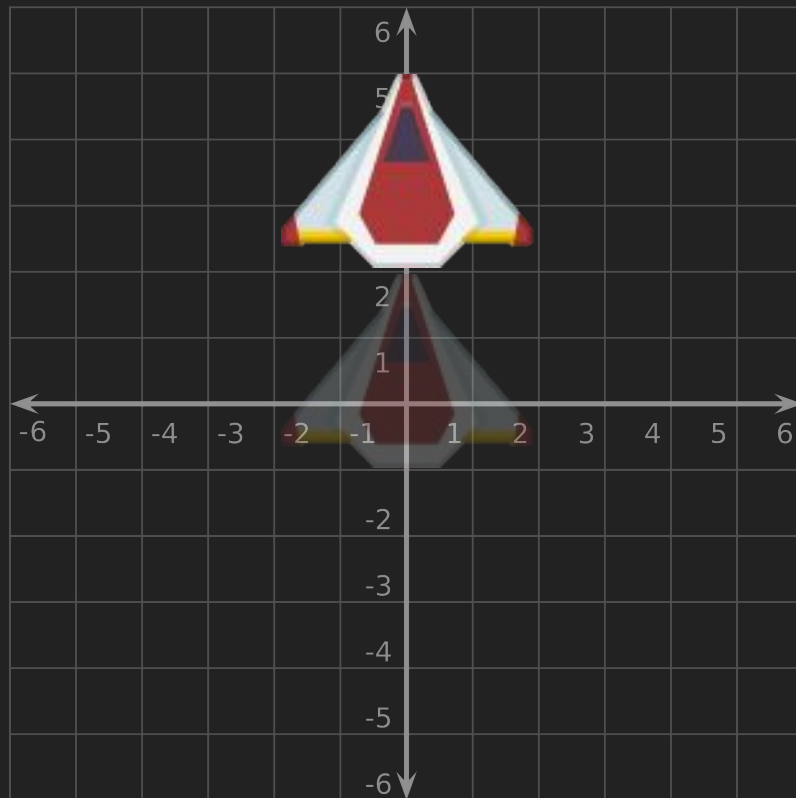
Translation, Rotation and Scale



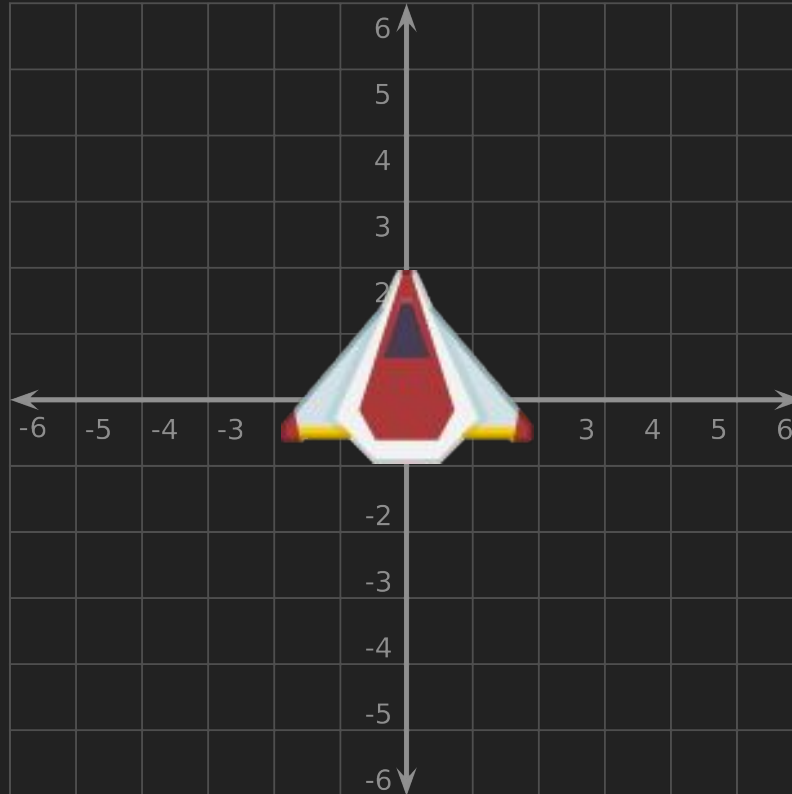
# Translation



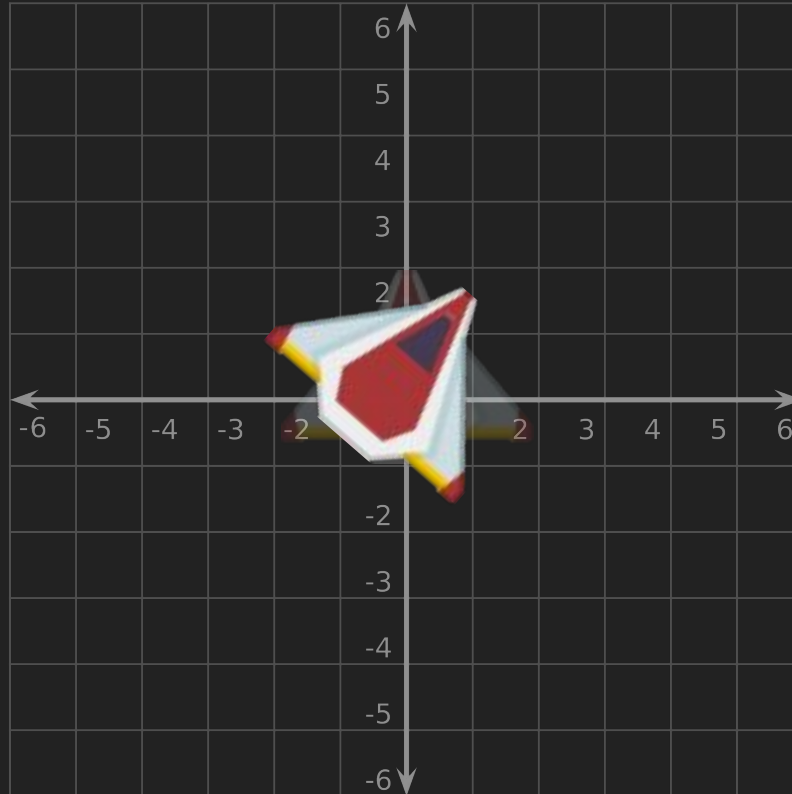
# Translation



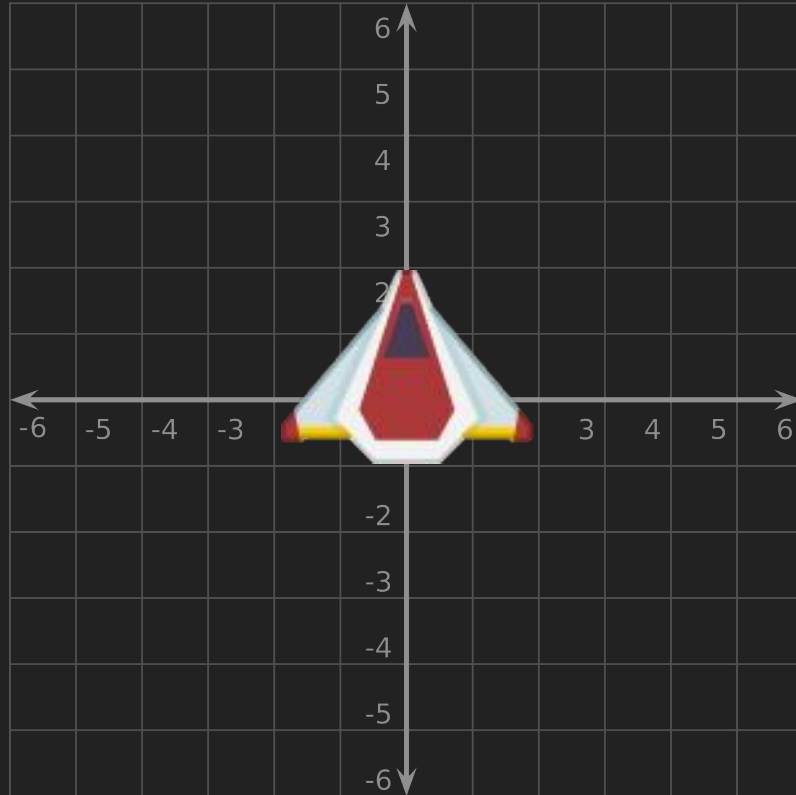
# Rotation



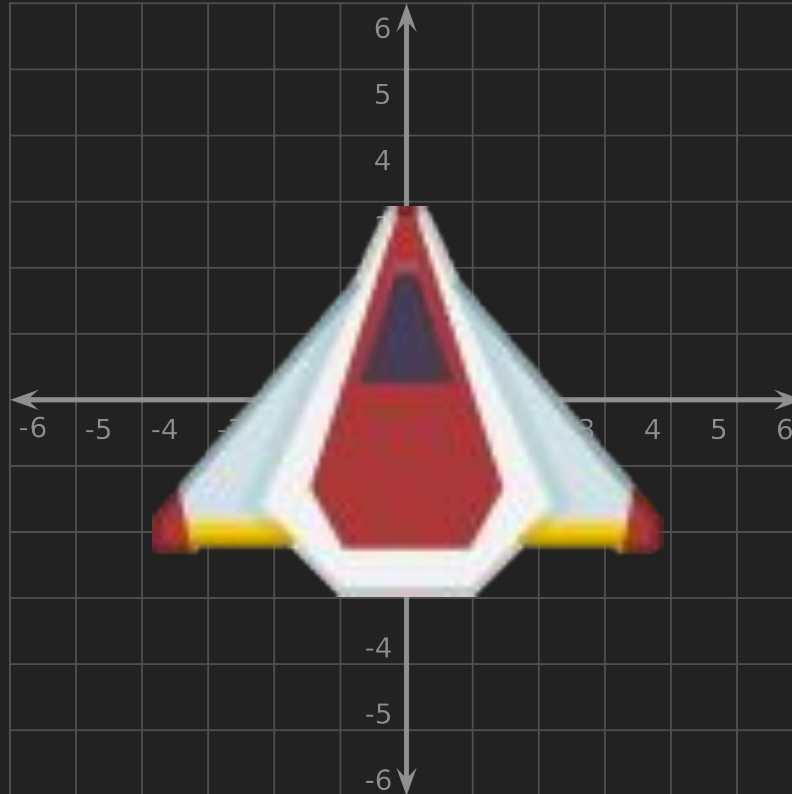
# Rotation



# Scale

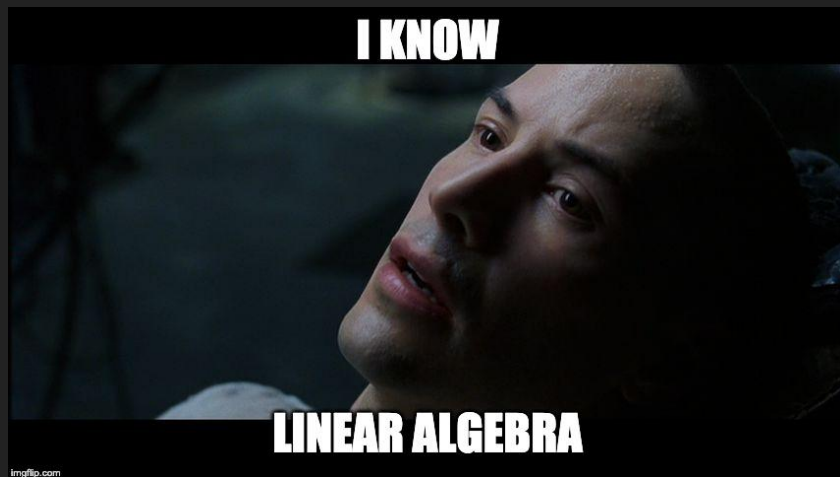


# Scale



# How do we do that stuff?

## Matrices!



# m-by-n Matrix

m rows -by- n columns

2x3 matrix

$$\begin{bmatrix} 1 & 2 & 0 \\ 4 & 3 & 2 \end{bmatrix}$$

3x2 matrix

$$\begin{bmatrix} 1 & 2 \\ 0 & 3 \\ 4 & 3 \end{bmatrix}$$

3x3 matrix

$$\begin{bmatrix} 1 & 2 & 7 \\ 0 & 3 & 2 \\ 4 & 3 & 1 \end{bmatrix}$$



We can treat a 2D Vector as a  
**2x1 matrix.**

(this will come in handy later)

$$\begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \end{bmatrix}$$

# Matrix Operations

(and rules)

# Matrix Addition

Add their corresponding entries.  
They must be the same size!

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} + \begin{bmatrix} E & F \\ G & H \end{bmatrix} = \begin{bmatrix} A+E & B+F \\ C+G & D+H \end{bmatrix}$$

# Matrix Addition

Add their corresponding entries.  
They must be the same size!

$$\begin{bmatrix} 1 & 0 \\ 4 & 3 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ 5 & 6 \end{bmatrix}$$

# Matrix Subtraction

Subtract their corresponding entries.

They must be the same size!

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} - \begin{bmatrix} E & F \\ G & H \end{bmatrix} = \begin{bmatrix} A-E & B-F \\ C-G & D-H \end{bmatrix}$$

# Matrix Subtraction

Subtract their corresponding entries.

They must be the same size!

$$\begin{bmatrix} 1 & 0 \\ 4 & 3 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} -1 & -3 \\ 3 & 0 \end{bmatrix}$$

# Transpose

Changes an  $M \times N$  matrix into an  $N \times M$  matrix.  
Rows and Columns are switched.

$$\begin{bmatrix} A & B & C \\ D & E & F \end{bmatrix}$$

$M$

$$\begin{bmatrix} A & D \\ B & E \\ C & F \end{bmatrix}$$

$M^T$

# Transpose

Changes an  $M \times N$  matrix into an  $N \times M$  matrix.  
Rows and Columns are switched.

$$\begin{bmatrix} 1 & 4 & 3 \\ 2 & 3 & 5 \end{bmatrix}$$

$M$

$$\begin{bmatrix} 1 & 2 \\ 4 & 3 \\ 3 & 5 \end{bmatrix}$$

$M^T$



# Scalar Multiplication

Multiply each entry by S.

$$2 \times \begin{bmatrix} 1 & 2 & 0 \\ 2 & 3 & 4 \\ 1 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 4 & 0 \\ 4 & 6 & 8 \\ 2 & 0 & 4 \end{bmatrix}$$

# Matrix Multiplication

(this is where things get interesting)

# Matrix Multiplication

The number of columns in the first matrix must match the number of rows in the second matrix.

$$\begin{bmatrix} A & B & C \\ D & E & F \end{bmatrix} \times \begin{bmatrix} G & H \\ I & J \\ K & L \end{bmatrix} = ?$$

$$2 \times 3 \quad \times \quad 3 \times 2 \quad = \quad ?$$


# Matrix Multiplication

The result is the number of rows in the first matrix by the number of columns in the second matrix.

$$\begin{bmatrix} A & B & C \\ D & E & F \end{bmatrix} \times \begin{bmatrix} G & H \\ I & J \\ K & L \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

$2 \times 3 \quad \times \quad 3 \times 2 \quad = \quad 2 \times 2$

$$\begin{bmatrix} A & B & C \\ D & E & F \end{bmatrix} \times \begin{bmatrix} G & H \\ I & J \\ K & L \end{bmatrix} =$$

$$\begin{bmatrix} A \times G + B \times I + C \times K \end{bmatrix}$$

$$\begin{bmatrix} \boxed{A \ B \ C} \\ D \ E \ F \end{bmatrix} \times \begin{bmatrix} G \ H \\ I \ J \\ K \ L \end{bmatrix} =$$

$$\begin{bmatrix} A \times G + B \times I + C \times K & A \times H + B \times J + C \times L \end{bmatrix}$$

$$\begin{bmatrix} A & B & C \\ D & E & F \end{bmatrix} \times \begin{bmatrix} G & H \\ I & J \\ K & L \end{bmatrix} =$$

$$\begin{bmatrix} A \times G + B \times I + C \times K & A \times H + B \times J + C \times L \\ D \times G + E \times I + F \times K & \end{bmatrix}$$

$$\begin{bmatrix} A & B & C \\ D & E & F \end{bmatrix} \times \begin{bmatrix} G & H \\ I & J \\ K & L \end{bmatrix} =$$

$$\begin{bmatrix} A \times G + B \times I + C \times K & A \times H + B \times J + C \times L \\ D \times G + E \times I + F \times K & D \times H + E \times J + F \times L \end{bmatrix}$$



We can use  
matrix multiplication  
to transform things!



# Identity Matrix

A NxN matrix with 1 on the diagonal and 0 for the other values.

1x1

$$\begin{bmatrix} 1 \end{bmatrix}$$

2x2

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

3x3

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Multiplying by the  
Identity Matrix has no effect.

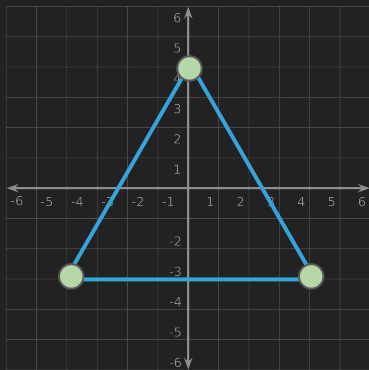
$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 1 * X + 0 * Y \\ 0 * X + 1 * Y \end{bmatrix}$$

# From our example code...

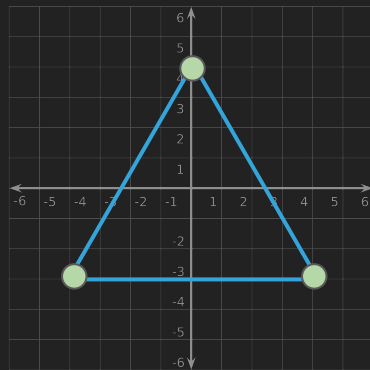
```
modelMatrix = glm::mat4(1.0f);
```

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\times$



$=$



# Scaling

$$\begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} S_x * X + 0 * Y \\ 0 * X + S_y * Y \end{bmatrix} = \begin{bmatrix} S_x * X \\ S_y * Y \end{bmatrix}$$

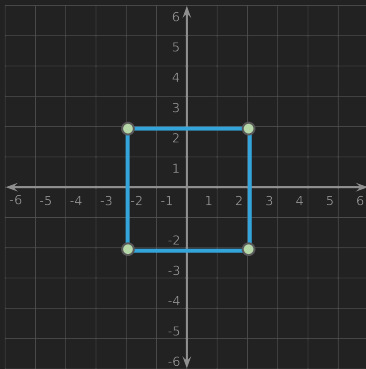
$$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 * 4 + 0 * 3 \\ 0 * 4 + 2 * 3 \end{bmatrix} = \begin{bmatrix} 8 \\ 6 \end{bmatrix}$$

# Scaling

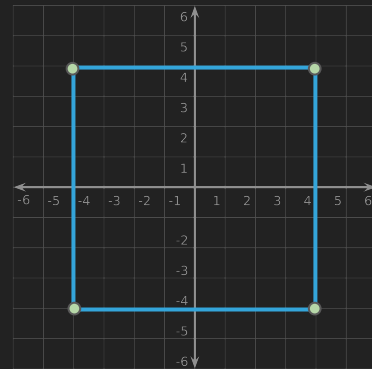
(uniform and non-uniform)

$$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

X

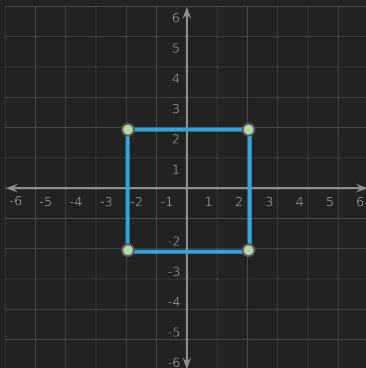


=

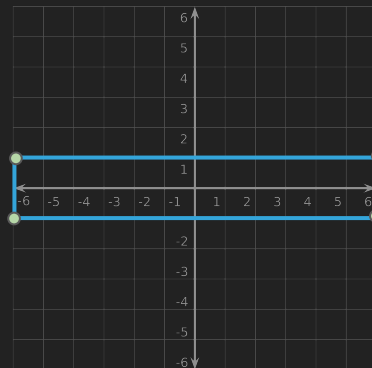


$$\begin{bmatrix} 3 & 0 \\ 0 & 0.5 \end{bmatrix}$$

X



=





# Rotation

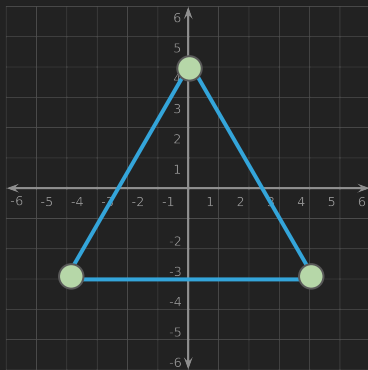
$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} \cos\theta * X + -\sin\theta * Y \\ \sin\theta * X + \cos\theta * Y \end{bmatrix}$$



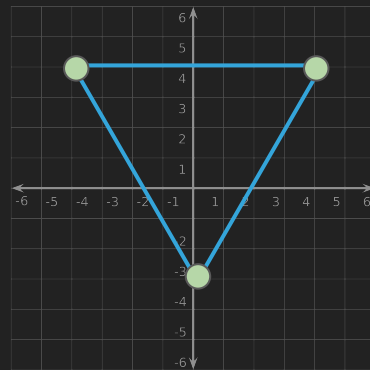
# Rotation

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$\theta = 180^\circ$$



=





# Translation

First we need to represent our 2D vector as a 3D vector.  
(Homogeneous Coordinates)

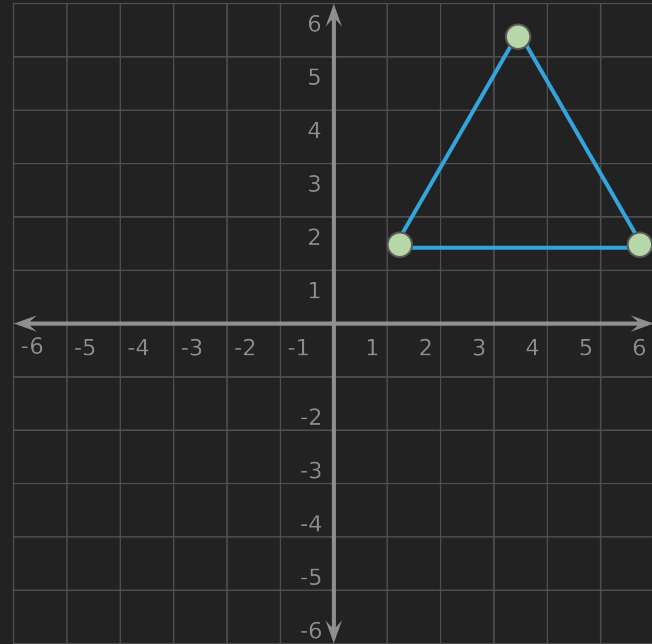
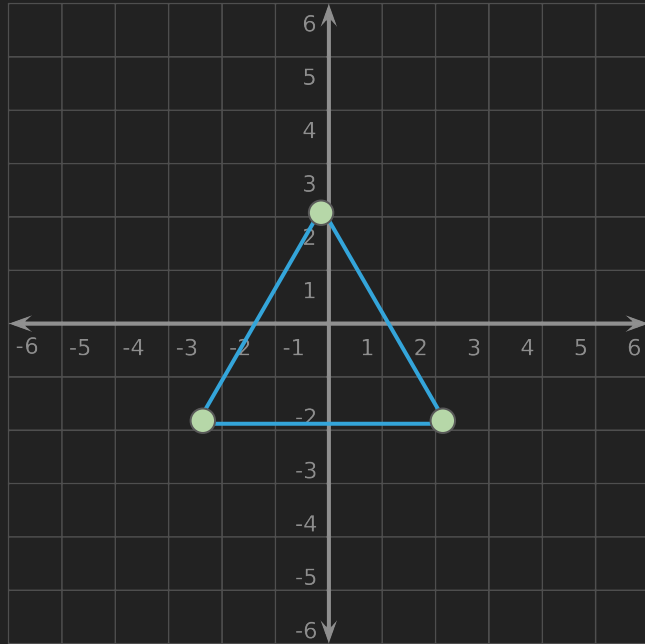
$$\begin{bmatrix} X \\ Y \end{bmatrix} \rightarrow \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

# Translation

$$\begin{bmatrix} 1 & 0 & T_x \\ 0 & 1 & T_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} = \begin{bmatrix} 1 * X + 0 * Y + T_x * 1 \\ 0 * X + 1 * Y + T_y * 1 \\ 0 * X + 0 * Y + 1 * 1 \end{bmatrix}$$

$$\begin{bmatrix} X + T_x \\ Y + T_y \\ 1 \end{bmatrix}$$

# Translation





# mat4?

You may have noticed our code used 4x4 matrices. That is because everything is really 3D and we need to use Homogeneous Coordinates to transform 3D vectors.

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \rightarrow \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

# Let's Code!

