16-17 A= A卷.

一.

1. 过直线 $L_1: \frac{x-1}{1}=\frac{y-2}{0}=\frac{z-3}{-1}$ 且平行于直线 $\begin{cases} x=2t-2 \\ y=t+1 \\ z=t \end{cases}$ 的平面方程是 $\underline{x-3y+z+2=0}$

$L_1: \frac{x-1}{1}=\frac{y-2}{0}=\frac{z-3}{-1}$ $\vec{v_1}=(1,0,-1)$，过 $P_0(1,2,3)$

$L_2: \begin{cases} x=2t-2 \\ y=t+1 \\ z=t \end{cases}$ $\vec{v_2}=(2,1,1)$，过 $P(-2,1,0)$.

π 过 L_1，则其法向量 $\vec{n}\perp\vec{v_1}$，且 $P_0\in\pi$；$\pi\parallel L_2$，则 $\vec{n}\perp\vec{v_2}$

故可取 $\vec{n}=\vec{v_1}\times\vec{v_2}=\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & -1 \\ 2 & 1 & 1 \end{vmatrix}=(1,-3,1) \Rightarrow \pi$ 方程：$(x-1)-3(y-2)+(z-3)=0$

即 $x-3y+z+2=0$.

2. $f(x,y)=\begin{cases} \frac{\sin(x^2y)}{xy} & xy\neq 0 \\ 0 & xy=0 \end{cases}$，则 $\left.\frac{\partial f}{\partial x}\right|_{(0,1)}=\underline{1}$

$$\left.\frac{\partial f}{\partial x}\right|_{(0,1)}=f_x(0,1)=\lim_{\Delta x\to 0}\frac{f(\Delta x,1)-f(0,1)}{\Delta x}=\lim_{\Delta x\to 0}\frac{\frac{\sin(\Delta x^2)}{\Delta x}-0}{\Delta x}=\lim_{\Delta x\to 0}\frac{\sin(\Delta x)^2}{(\Delta x)^2}=1.$$

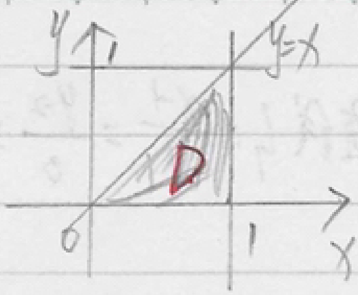
或 $f(x,1)=\begin{cases} \frac{\sin x^2}{x} & x\neq 0 \\ 0 & x=0 \end{cases} \Rightarrow f_x(0,1)=\lim_{x\to 0}\frac{f(x,1)-f(0,1)}{x}=\lim_{x\to 0}\frac{\frac{\sin x^2}{x}-0}{x}=1$

3. $\int_0^1 dy\int_y^1\frac{\tan x}{x}dx=\underline{-\ln\cos 1}$

$\int\frac{\tan x}{x}dx$ 无法求出，故需交换积分次序.

Date

$D: \begin{cases} 0 \le y \le 1 \\ y \le x \le 1 \end{cases} \Rightarrow x\text{-型}: \begin{cases} 0 \le x \le 1 \\ 0 \le y \le x \end{cases}$

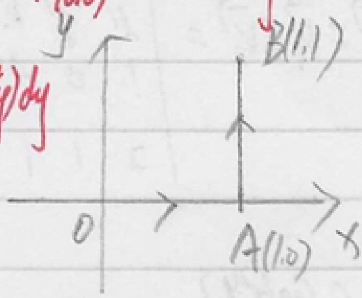


则 $I = \int_0^1 dx \int_0^x \frac{\tan x}{x} dy = \int_0^1 \frac{\tan x}{x} \cdot x\, dx = \int_0^1 \tan x\, dx$

$= -\ln|\cos x| \Big|_0^1 = -\ln\cos 1$

4. 平面上曲线积分 $\int_{(0,0)}^{(1,1)} (x^2+y)dx + (x-2\sin^2 y)dy =$ $\underline{\frac{1}{3}+\frac{1}{2}\sin 2}$

$P = x^2+y$, $Q = x-2\sin^2 y$ 有连续偏导，且 $\frac{\partial P}{\partial y} = 1 = \frac{\partial Q}{\partial x} \Rightarrow \int_{(0,0)}^{(1,1)} Pdx + Qdy$ 与路径无关.



$\therefore I = \int_{OAB} = \int_{OA} (x^2+y)dx + (x-2\sin^2 y)dy + \int_{AB} (x^2+y)dx + (x-2\sin^2 y)dy$

$= \int_0^1 x^2 dx + \int_0^1 (1-2\sin^2 y)dy$

$= \frac{1}{3} + \int_0^1 \cos 2y\, dy = \frac{1}{3} + \frac{1}{2}\sin 2y \Big|_0^1 = \frac{1}{3} + \frac{1}{2}\sin 2$

$\overline{OA}: y=0, x: 0\to 1$; $\overline{AB}: x=1, y: 0\to 1$

5. 若 $x^2 = \sum_{n=0}^{\infty} a_n \cos nx \ (-\pi \le x \le \pi)$，则 $a_2 =$ $\underline{\ 1\ }$

$f(x) = x^2$ 为偶函数，$\therefore a_2 = \frac{2}{\pi}\int_0^{\pi} x^2 \cdot \cos 2x\, dx = \frac{2}{\pi} \cdot \int_0^{\pi} \frac{x^2}{2} d\sin 2x$

即 $a_2 = \frac{1}{\pi}\left(x^2 \cdot \sin 2x \Big|_0^{\pi} - \int_0^{\pi} 2x \sin 2x\, dx \right) = \frac{1}{\pi}\int_0^{\pi} x\, d\cos 2x$

$= \frac{1}{\pi}\left(x\cos 2x \Big|_0^{\pi} - \int_0^{\pi} \cos 2x\, dx \right) = \frac{1}{\pi}(\pi \cdot \cos 2\pi - 0 - 0)$

$= \cos 2\pi = 1$

二.

6. 二元极限 $\lim\limits_{\substack{x\to 0\\ y\to 0}} \frac{3x-y}{x+y}$ (A)

A. 不存在　B. 等于0.　C. 等于$\frac{1}{2}$.　D. 存在, 但不等于0也不等于$\frac{1}{2}$

$\lim\limits_{\substack{x\to 0\\ y=kx}} \frac{3x-kx}{x+kx} = \lim\limits_{x\to 0} \frac{(3-k)x}{(k+1)x} = \frac{3-k}{k+1}$ 与k有关 $(k\neq -1)$.

7. 函数 $z=f(x,y)$ 的全微分 $dz=xdx+ydy$, 则在(0,0)处函数 (B)

A. 取极大值.　B. 取极小值.　C. 不取极值.　D. 无法确定

$dz=xdx+ydy \Rightarrow f_x=x,\ f_y=y.$ 且 $f_x(0,0)=f_y(0,0)=0.$

又 $f_{xx}=1,\ f_{xy}=0,\ f_{yy}=1.$ $A=C=1,\ B=0.$ $AC-B^2>0$, 且 $A>0$. ⇒ (0,0)为极小值点.

8. 设 $\Omega=\{(x,y,z)\mid x^2+y^2\le z,\ 1\le z\le 2\}$. f在Ω上连续, 则 $\iiint\limits_{\Omega} f(z)dv =$ (D)

A. $\pi\int_1^2 z^2\cdot f(z)dz$.　B. $2\pi\int_1^2 f(z)dz$.　C. $2\pi\int_1^2 z\cdot f(z)dz$.　D. $\pi\int_1^2 z\cdot f(z)dz$

$\iiint\limits_{\Omega} f(z)dv \xlongequal{\text{截面法}} \int_1^2 dz \iint\limits_{x^2+y^2\le z} f(z)dz = \int_1^2 f(z)\cdot\pi\cdot z\,dz = \pi\cdot\int_1^2 z\cdot f(z)dz.$

9. 设 $\Sigma: x^2+y^2+z^2=a^2\ (z\ge 0)$. Σ_1为Σ在第一卦限的部分, 则有 (C)

A. $\iint\limits_{\Sigma} x ds = 4\iint\limits_{\Sigma_1} x ds$　　B. $\iint\limits_{\Sigma} y ds = 4\iint\limits_{\Sigma_1} y ds$

C. $\iint\limits_{\Sigma} z ds = 4\iint\limits_{\Sigma_1} z ds$　　D. $\iint\limits_{\Sigma} xyz ds = 4\iint\limits_{\Sigma_1} xyz ds$

Σ关于xoz面、yoz面对称 $\Rightarrow \iint_\Sigma x\,dS=0$，$\iint_\Sigma y\,dS=0$，$\iint_\Sigma xyz\,dS=0$.

z关于x、y变量均为偶函数 $\Rightarrow \iint_\Sigma z\,dS = 4\iint_{\Sigma_1} z\,dS$

10. 若幂级数$\sum_{n=1}^{\infty} a_n(x+2)^n$在$x=0$处收敛，$x=-4$处发散，则幂级数$\sum_{n=1}^{\infty} a_n(x-3)^n$在$x=5$处（ C ）

A.发散　B.绝对收敛　C.条件收敛　D.不能确定

$\sum_{n=1}^{\infty} a_n(x+2)^n$　$x=0$处收敛，即$\sum_{n=1}^{\infty} 2^n a_n$收敛

$x=-4$处发散，即$\sum_{n=1}^{\infty} a_n\cdot(-2)^n$发散

对于$\sum_{n=1}^{\infty} a_n(x-3)^n$　$x=5$处，即$\sum_{n=1}^{\infty} a_n\cdot 2^n$收敛，

若$\sum_{n=1}^{\infty} a_n\cdot 2^n$为绝对收敛，即$\sum_{n=1}^{\infty} |a_n\cdot 2^n|$收敛.

则必有$\sum_{n=1}^{\infty} |a_n\cdot(-2)^n|$也收敛 $\Rightarrow \sum_{n=1}^{\infty} a_n\cdot(-2)^n$收敛，矛盾.

故$\sum_{n=1}^{\infty} a_n\cdot 2^n$为条件收敛.