17-18 A(二) 期中

一. 填空题

1. 点 $(2,1,0)$ 到平面 $3x+4y+5z=0$ 的距离为 $\underline{\sqrt{2}}$

$$d=\frac{|3\cdot2+4\cdot1+5\cdot0|}{\sqrt{3^2+4^2+5^2}}=\frac{10}{\sqrt{50}}=\sqrt{2}$$

2. $f(x,y)=\begin{cases}\dfrac{x^3-y^3}{x^2+y^2} & (x,y)\neq(0,0)\\ 0 & (x,y)=(0,0)\end{cases}$，则 $f'_x(0,0)=\underline{1}$，$f'_y(0,0)=\underline{-1}$

$$f'_x(0,0)=\lim_{\Delta x\to0}\frac{f(\Delta x,0)-f(0,0)}{\Delta x}=\lim_{\Delta x\to0}\frac{\frac{(\Delta x)^3-0}{(\Delta x)^2+0}-0}{\Delta x}=\lim_{\Delta x\to0}\frac{(\Delta x)^3}{(\Delta x)^3}=1$$

$$f'_y(0,0)=\lim_{\Delta y\to0}\frac{f(0,\Delta y)-f(0,0)}{\Delta y}=\lim_{\Delta y\to0}\frac{\frac{-(\Delta y)^3}{(\Delta y)^2}}{\Delta y}=-1$$

3. $f(x,y,z)=x^2y+z^2$ 在点 $(1,2,0)$ 处沿向量 $\vec{v}=(1,2,2)$ 的方向导数为 $\underline{2}$

$f'_x=2xy$，$f'_y=x^2$，$f'_y=2z$ $\therefore f'_x(1,2,0)=4$，$f'_y(1,2,0)=1$．$f'_z(1,2,0)=0$

$\vec{v}=(1,2,2)\Rightarrow \cos\alpha=\dfrac{1}{\sqrt{1^2+2^2+2^2}}=\dfrac{1}{3}$，$\cos\beta=\dfrac{2}{3}$，$\cos\gamma=\dfrac{2}{3}$

$$\therefore \left.\frac{\partial f}{\partial \vec{v}}\right|_{(1,2,0)}=f'_x(1,2,0)\cdot\cos\alpha+f'_y(1,2,0)\cos\beta+f'_z(1,2,0)\cdot\cos\gamma=4\cdot\frac{1}{3}+1\cdot\frac{2}{3}+0\cdot\frac{2}{3}=2.$$

4. 曲线 $\begin{cases}x=2t\\ y=t^2-1\\ z=t^3\end{cases}$ 在 $(2,0,1)$ 处的切线方程为 $\underline{\dfrac{x-2}{2}=\dfrac{y}{2}=\dfrac{z-1}{3}}$

$x'(t)=2$，$y'(t)=2t$，$z'(t)=3t^2$．又曲线在 $(2,0,1)$ 对应 $t=1$，$\therefore$ 曲线在 $(2,0,1)$ 处切向量为 $(2,2,3)$

故切线方程为：$\dfrac{x-2}{2}=\dfrac{y-0}{2}=\dfrac{z-1}{3}$。

5. 参数$\sigma>0$，反常积分 $\int_{-\infty}^{+\infty} e^{-\frac{(x-2018)^2}{2\sigma^2}}dx=$ $\underline{\sqrt{2\pi}\sigma}$

已知 $\int_{0}^{+\infty} e^{-x^2}dx=\frac{\sqrt{\pi}}{2}$

$$\int_{-\infty}^{+\infty} e^{-\frac{(x-2018)^2}{2\sigma^2}}dx \xlongequal{令\frac{x-2018}{\sqrt{2}\sigma}=t} \int_{-\infty}^{+\infty} e^{-t^2}d(\sqrt{2}\sigma t+2018)$$

$$=\int_{-\infty}^{+\infty}\sqrt{2}\sigma\cdot e^{-t^2}dt=2\sqrt{2}\sigma\cdot\int_{0}^{+\infty}e^{-t^2}dt=2\sqrt{2}\sigma\cdot\frac{\sqrt{\pi}}{2}=\sqrt{2\pi}\sigma.$$

$$\left(\int_{-\infty}^{+\infty}e^{-t^2}dt=2\int_{0}^{+\infty}e^{-t^2}dt\right)$$

二.选择题

6. $L_1:\begin{cases}y+z=1\\x=0\end{cases}$，$L_2=\begin{cases}x-z=1\\y=0\end{cases}$，$L_1$与$L_2$的位置关系（ C ）

A.相交于一点　B.平行.　C.异面　D.重合

$L_1:\begin{cases}y+z=1\\x=0\end{cases}\Rightarrow\frac{x}{0}=\frac{y+1}{1}=\frac{z}{1}$，其方向向量 $\vec{s_1}=(0,1,1)$，$P_1(0,-1,0)\in L_1$

$L_2:\begin{cases}x-z=1\\y=0\end{cases}\Rightarrow\frac{x}{1}=\frac{y}{0}=\frac{z+1}{1}$，其方向向量 $\vec{s_2}=(1,0,1)$，$P_2(0,0,-1)\in L_2$

$\vec{s_1}$与$\vec{s_2}$不共线，故B×，D×.

$$(\vec{s_1},\vec{s_2},\overrightarrow{P_1P_2})=\begin{vmatrix}0&1&1\\1&0&1\\0&1&-1\end{vmatrix}=1\cdot(-1)\cdot\begin{vmatrix}1&1\\0&-1\end{vmatrix}+1\cdot\begin{vmatrix}1&0\\0&1\end{vmatrix}=1+1=2\neq0$$

$\therefore L_1$与L_2异面.

7. 二元函数$f(x,y)$在(x_0,y_0)某邻域内有定义，则下列说法不正确的是(B)

A. 若f在(x_0,y_0)可微，则f在(x_0,y_0)处偏导数存在. ✓

B. 若f在(x_0,y_0)处偏导数都存在，则f在(x_0,y_0)连续. ×

C. 若f的偏导数f_x, f_y在(x_0,y_0)连续，则f在(x_0,y_0)处可微 ✓

D. 若f在(x_0,y_0)处可微，则f在(x_0,y_0)处连续 ✓

如: $f(x,y)=\begin{cases}\dfrac{xy}{x^2+y^2} & x^2+y^2\neq 0\\ 0 & x^2+y^2=0\end{cases}$，$f'_x(0,0)=\lim\limits_{\Delta x\to 0}\dfrac{f(\Delta x,0)-f(0,0)}{\Delta x}=0=f'_y(0,0)$.

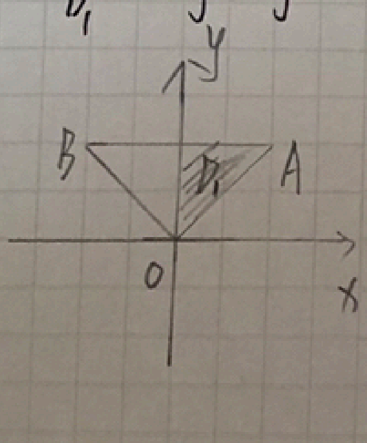
但$\lim\limits_{\substack{x\to 0\\ y\to 0}} f(x,y)=\lim\limits_{\substack{x\to 0\\ y\to 0}}\dfrac{xy}{x^2+y^2}$不存在 $\left(\lim\limits_{\substack{x\to 0\\ y=kx}}\dfrac{kx^2}{x^2+k^2x^2}=\dfrac{k}{1+k^2}\text{与}k\text{有关}\right)$. ∴ $f(x,y)$ $(0,0)$处不连续.

8. $f(x,y)$在开区域D内有二阶连续偏导数，且$f'_x(x_0,y_0)=f'_y(x_0,y_0)=0$. 记$A=f''_{xx}(x_0,y_0)$, $B=f''_{xy}(x_0,y_0)$, $C=f''_{yy}(x_0,y_0)$. 则下列为$f(x,y)$在(x_0,y_0)处取极小值的充分条件是(B) (定理9.6.2)

A. $A<0$, $AC-B^2>0$. B. $A>0$, $AC-B^2>0$. C. $A<0$, $AC-B^2<0$. D. $A>0$, $AC-B^2<0$

9. D是xy平面上以$O(0,0)$, $A(1,1)$, $B(-1,1)$为顶点的三角形区域, D_1是D在第一象限部分. 则:
$\iint\limits_D (xy+\cos x\sin y)\,dxdy=$ (A)

A. $2\iint\limits_{D_1}\cos x\sin y\,dxdy$. B. $2\iint\limits_{D_1}(xy+\cos x\sin y)\,dxdy$. C. $2\iint\limits_{D_1}xy\,dxdy$. D. 0.



D关于y轴对称，xy为关于x的奇函数，∴ $\iint\limits_D xy\,dxdy=0$.

而$\cos x\sin y$关于x为偶函数，∴ $\iint\limits_D \cos x\sin y\,dxdy=2\iint\limits_{D_1}\cos x\sin y\,dxdy$

$\Rightarrow \iint\limits_D (xy+\cos x\sin y)\,dxdy=\iint\limits_D xy\,dxdy+\iint\limits_D \cos x\sin y\,dxdy=2\iint\limits_{D_1}\cos x\sin y\,dxdy$.

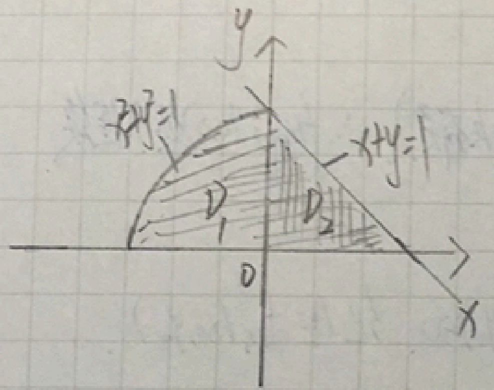
10. f(x,y)是连续函数，则：$\int_0^1 dy\int_{-\sqrt{1-y^2}}^{1-y} f(x,y)dx =$ (D)

A. $\int_0^1 dx\int_0^{x-1} f(x,y)dy + \int_{-1}^0 dx\int_0^{\sqrt{1-x^2}} f(x,y)dy$.

B. $\int_0^1 dx\int_0^{1-x} f(x,y)dy + \int_{-1}^0 dx\int_{-\sqrt{1-x^2}}^0 f(x,y)dy$

C. $\int_0^1 dx\int_0^{x-1} f(x,y)dy + \int_{-1}^0 dx\int_0^{-\sqrt{1-x^2}} f(x,y)dy$

D. $\int_0^1 dx\int_0^{1-x} f(x,y)dy + \int_{-1}^0 dx\int_0^{\sqrt{1-x^2}} f(x,y)dy$.



$\int_0^1 dy\int_{-\sqrt{1-y^2}}^{1-y} f(x,y)dx = \iint_D f(x,y)dxdy$

D如左图，$D=\{(x,y)\mid -\sqrt{1-y^2}\le x\le 1-y,\ 0\le y\le 1\}$.

则 $\iint_D f(x,y)dxdy = \iint_{D_1} f(x,y)dxdy + \iint_{D_2} f(x,y)dxdy$

$D_1=\{(x,y)\mid 0\le y\le\sqrt{1-x^2},\ -1\le x\le 0\}$，$D_2=\{(x,y)\mid 0\le y\le 1-x,\ 0\le x\le 1\}$.

$\therefore \int_0^1 dy\int_{-\sqrt{1-y^2}}^{1-y} f(x,y)dx = \int_0^1 dx\int_0^{1-x} f(x,y)dy + \int_{-1}^0 dx\int_0^{\sqrt{1-x^2}} f(x,y)dy$.