Date

17-18 A(一)期中

一.填空题

1. $\lim\limits_{n\to\infty}\left(\frac{1}{n^2+n+1}+\frac{2}{n^2+n+2}+\frac{3}{n^2+n+3}+\cdots+\frac{n}{n^2+n+n}\right)=$ $\underline{\frac{1}{2}}$

$\frac{\frac{n(n+1)}{2}}{n^2+2n}=\frac{1+2+\cdots+n}{n^2+n+n}\le\frac{1}{n^2+n+1}+\cdots+\frac{n}{n^2+n+n}\le\frac{1+2+\cdots+n}{n^2+n+1}=\frac{\frac{n(n+1)}{2}}{n^2+n+1}$

又 $\lim\limits_{n\to\infty}\frac{\frac{n(n+1)}{2}}{n^2+2n}=\lim\limits_{n\to\infty}\frac{n^2+n}{2(n^2+2n)}=\frac{1}{2}$ $\lim\limits_{n\to\infty}\frac{\frac{n(n+1)}{2}}{n^2+n+1}=\lim\limits_{n\to\infty}\frac{n^2+n}{2(n^2+n+1)}=\frac{1}{2}$ ∴原式 $=\frac{1}{2}$

2. $\lim\limits_{n\to\infty}\sin(\pi\sqrt{n^2+1})=$ $\underline{0}$

$\sin(\pi\sqrt{n^2+1})=\sin[(\pi\sqrt{n^2+1}-n\pi)+n\pi]=(-1)^n\cdot\sin(\pi\sqrt{n^2+1}-n\pi)$

而 $\sin(\pi\sqrt{n^2+1}-n\pi)=\sin\frac{\pi^2(n^2+1)-n^2\pi^2}{\pi\sqrt{n^2+1}+n\pi}=\sin\frac{\pi^2}{\pi(\sqrt{n^2+1}+n)}=\sin\frac{\pi}{\sqrt{n^2+1}+n}$

∴ $\lim\limits_{n\to\infty}\sin(\pi\sqrt{n^2+1}-n\pi)=0\Rightarrow\lim\limits_{n\to\infty}\underbrace{(-1)^n}_{\text{有界}}\cdot\underbrace{\sin(\pi\sqrt{n^2+1}-n\pi)}_{\text{无穷小}}=\lim\limits_{n\to\infty}\sin(\pi\sqrt{n^2+1})=0$

3. $f(x)=x(x+1)(x+2)\cdots(x+2017)$. 则 $f'(0)=$ $\underline{2017!}$

$f'(0)=\lim\limits_{x\to0}\frac{f(x)-f(0)}{x-0}=\lim\limits_{x\to0}(x+1)(x+2)\cdots(x+2017)=1\cdot2\cdots2017=2017!$

4. $y=f(x)$ 由 $e^{2x+y}-\cos(xy)=e-1$ 确定. 则曲线 $y=f(x)$ 在 $(0,1)$ 处切线方程为 $\underline{y=-2x+1}$

两边对 x 求导: $e^{2x+y}\cdot(2+y')+\sin(xy)\cdot(y+xy')=0$

代 $x=0, y=1$, 得 $e\cdot(2+y'(0))=0\Rightarrow y'(0)=-2$.

∴切线: $y-1=-2(x-0)$ 即 $y=-2x+1$

KOKUYO

9. $f(x)$ 有任意阶导数，且 $f'(x)=[f(x)]^3$. 则当 n 为正整数时. $f^{(n)}(x)=($ $)$

A. $1\cdot3\cdot5\cdots(2n-1)\cdot[f(x)]^{2n+1}$ B. $1\cdot3\cdot5\cdots(2n+1)[f(x)]^{2n+1}$

C. $1\cdot3\cdot5\cdots(2n-1)[f(x)]^{2n-1}$ D. $1\cdot3\cdot5\cdots(2n+1)[f(x)]^{2n-1}$

$f'(x)=[f(x)]^3$. $\therefore f''(x)=3\cdot[f(x)]^2\cdot f'(x)=3\cdot[f(x)]^5=1\cdot(2\cdot2-1)\cdot[f(x)]^{2\cdot2+1}$

$f'''(x)=3\cdot5\cdot[f(x)]^4\cdot f'(x)=3\cdot5\cdot[f(x)]^7=1\cdot3\cdot(2\cdot3-1)[f(x)]^{2\cdot3+1}$

$\therefore f^{(n)}(x)=1\cdot3\cdot5\cdots(2n-1)[f(x)]^{2n+1}$

10. $f(x)$ 有连续的导函数. $f(0)=0$, 且 $f'(0)=b$. 若 $F(x)=\begin{cases}\dfrac{f(x)+a\sin x}{x} & x\neq0\\ A & x=0\end{cases}$ $x=0$ 处连续.

则常数 $A=($ C. $)$

A. a. B. b C. $a+b$ D. 0

$F(x)$ $x=0$ 连续. $\therefore \lim\limits_{x\to0}F(x)=\lim\limits_{x\to0}\dfrac{f(x)+a\sin x}{x}=F(0)$. 即 $\lim\limits_{x\to0}\dfrac{f(x)}{x}+a=A$.

又 $f(0)=0$ $f'(0)=b$. 则 $f'(0)=\lim\limits_{x\to0}\dfrac{f(x)-f(0)}{x-0}=\lim\limits_{x\to0}\dfrac{f(x)}{x}=b$.

$\therefore a+b=A$.

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5. $f(x)$可微，$y=f(x)\cdot e^{f(x)}$，则$dy=$ <u>$f'(x)\cdot e^{f(x)}\cdot(1+f(x))\,dx$</u>

$y'=f'(x)\cdot e^{f(x)}+f(x)\cdot e^{f(x)}\cdot f'(x)=f'(x)\cdot e^{f(x)}(1+f(x)) \Rightarrow dy=y'dx$

二.选择题.

6. 已知$\lim\limits_{x\to\infty}\left(\frac{x^2}{x+1}-ax-b\right)=0$. a.b为常数.则(C)

A $a=1.\ b=1$.　B $a=-1.\ b=1$.　C. $a=1.\ b=-1$　D. $a=-1.\ b=-1$

$\lim\limits_{x\to\infty}\left(\frac{x^2}{x+1}-ax-b\right)=\lim\limits_{x\to\infty}\frac{x^2-(ax+b)(x+1)}{x+1}=\lim\limits_{x\to\infty}\frac{(1-a)x^2-(a+b)x-b}{x+1}=0$

$\therefore\begin{cases}1-a=0\\a+b=1\end{cases}\Rightarrow\begin{cases}a=1\\b=-1\end{cases}$

7. $x\to2$时. $f(x)=\frac{x^2-4}{x-2}e^{\frac{1}{x-2}}$的极限(D)

A 等于4.　B 等于0.　C为∞.　D不存在但也不为∞

$x\neq2$时. $f(x)=(x+2)\cdot e^{\frac{1}{x-2}}$　$x\to2^+$时 $\frac{1}{x-2}\to+\infty$　$e^{\frac{1}{x-2}}\to+\infty$　$f(2^+)=+\infty$

$x\to2^-$时. $\frac{1}{x-2}\to-\infty$. $e^{\frac{1}{x-2}}\to0$　$f(2^-)=4\cdot0=0$.

8. $x\to0$时. $(1-\cos x)\ln(1+x^2)$是比$x\sin x^n$高阶的无穷小，而$x\sin x^n$是比$(e^{x^2}-1)$高阶的无穷小.

则正整数$n=$(B.)

A. 1　B 2.　C 3.　D 4.

$\lim\limits_{x\to0}\frac{(1-\cos x)\ln(1+x^2)}{x\cdot\sin x^n}=0=\lim\limits_{x\to0}\frac{\frac{1}{2}x^2\cdot x^2}{x^{1+n}}=\lim\limits_{x\to0}\frac{1}{2}x^{3-n}=0\Rightarrow 3-n>0$. 即$n<3$

$\lim\limits_{x\to0}\frac{x\cdot\sin x^n}{e^{x^2}-1}=0=\lim\limits_{x\to0}\frac{x\cdot x^n}{x^2}=\lim\limits_{x\to0}x^{n-1}=0\Rightarrow n-1>0$ 即$n>1$

$\Rightarrow n=2$