Multi-Agent Systems Assignment 2

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1 Question 1

1.1 Q 1.1

Q 1: Write down the pay-off matrix.

With 2 diners and 2 conditions: there would be a 2x2 pay-off matrix. As seen below.

		B	
		C	F
A	C	0,0	-5, 5
	F	5,-5	0, 0

To find the pay-offs for (Cheap, Cheap) for example, we have:

- Value(10) - Total cost(10 + 10)/2 = Utility(0)

This result within the cell (Cheap, Cheap) is the same for both Alice and Bob as the game is symmetric. Following on from this, the pay-off for (Cheap, Expensive) for Alice is:

- Value(10) - Total cost(10 + 20)/2 = Utility(-5)

For Bob the pay-off for (Cheap, Expensive) is:

- Value(20) - Total cost(10 + 20)/2 = Utility(5)

1.2 Q 1.2

Q2: Assuming that they both order simultaneously and without coordinating, what will they order and why?

Both Alice and Bob will attempt to maximise their utility and go for the more expensive order. Considering the best response for each player, there is no pure Nash Equilibrium. Hence, without co-ordinating both will end up at (0, 0) cell. This situation demonstrates that if both Alice and Bob try to obtain the largest possible utility (Expensive, Expensive), they would find themselves in disadvantageous positions. This indicates that each player would think that it would seem more advantageous to order more expensive food (strictly dominant strategy) as the total cost is split. However, this leads to all players paying more than they would have preferred in the first place. This can be avoided if Alice and Bob are able to communicate and, for example, agree on (Cheap, Cheap) as in reality (Cheap, Cheap) is preferable to (Expensive, Expensive) if they would like to save money.

1.3 Q 1.3

Q3: Alice is quite the romantic type and gets an additional s Euro's worth of pleasure if they happen to pick the same meal (either both cheap or both expensive). Bob, on the other hand, is a bit of a contrarian and gets an additional amount of pleasure (also equivalent to s Euro) when they happen to favour different meal choices. Assume that 0 ¡ s 2. How does this change the pay-off matrix and the Nash equilibrium (or equilibria) of this game?

After re-calculating the utilities with consideration of increased pleasure, the new pay-off matrix looks like the following:

		B: C	B: E
A	C	4, 2	-3, 5
	E	3, -1	0, -2

The new pay-off matrix is different to the original as now, it is evident that Alice, the romantic, prefers the same meals as Bob while Bob the contrarian prefers the meals that are different to Alice. Additionally, it seems that Bob has the potential to get the most enjoyment out of the meal if he goes Expensive while Alice goes Cheap. If they were to coordinate, and if Bob would concede, (Cheap, Cheap) would be a good option.

2 Question 2: Hawk versus Dove

Two animals are in conflict over some resource worth v > 0. Simultaneously, they choose whether to behave like hawks (H) or doves (D). Hawks are willing to fight over the good, whereas doves are not. So if one animal chooses hawk and the other dove, the hawk gets everything leaving nothing for the dove. If both behave like doves, they split the resource equally. If however, both adopt a hawk strategy, they fight and on average get half of the food. The fighting however comes at a cost (c $\leq$ v) to both of them.

2.1 Write down the pay-off matrix

	Hawk	Dove
Hawk	$\frac{v}{2} - c, \frac{v}{2} - c$	$v, 0$
Dove	$0, v$	$\frac{v}{2}, \frac{v}{2}$

Pay-off matrix Hawk versus Dove.

2.2 Determine the Nash equilibria for this game and discuss how they change as the cost of aggression (c) increases.

The hawk versus dove game has varying Nash Equilibria, depending on the value for c. If

$$v/2 - c > 0$$

, then there is a pure NE where both players stick to playing hawk. However, if

$$v/2 - c \leq 0$$

, then there are multiple Nash Equilibria. The pure NE are when one player plays hawk and the other player plays dove: <H, D> and <D, H>. This is because in this scenario, if the row player chooses hawk, it is in the column player's best interest to play dove (and vice versa) because getting 0 utility is better than getting < 0 utility.

However, to get the mixed Nash Equilibrium we have to find the expected utility for player(i) of playing either hawk or dove as a pure strategy and determine what the expected utility will be for the other player. Therefore, we will say that player(i) will play a certain strategy with probability q. This gives us the following calculations:

- First we calculate the expected utility for playing hawk or dove:
  1. EU(h) $= q(v/2 - c) + (1 - q)(v)$
  2. EU(d) $= q(0) + (1 - q)(v/2)$
- Then, we set these equal to each other:
  1. $q(v/2 - c) + (1 - q)(v) = (1 - q)(v/2)$
- This formula simplifies to:
  1. $v/2 = cq$
- Then, finally, to get q we have to divide both sides by c:
  1. $q = v/2c$

The above calculations give us the probability that player(i) plays hawk in a mixed Nash Equilibrium strategy (The probability of player(i) playing dove is equal to this probability.). Therefore, the mixed Nash Equilibrium is for both players to play both hawk and dove with probability $v/2c$.

3 Question 3: Investment in Recycling

3.1 1

The best response function for the first country r_1 may be computed in the following way:

$$(10 - r_1 + \frac{r_2}{2})r_1 - 4r_1$$

which can be rewritten in the following way:

$$10 - 2r_1 + \frac{r_2}{2} - 4$$

Now we take the derivative and set it equal to zero which will give us the final best response for the first country r_1:

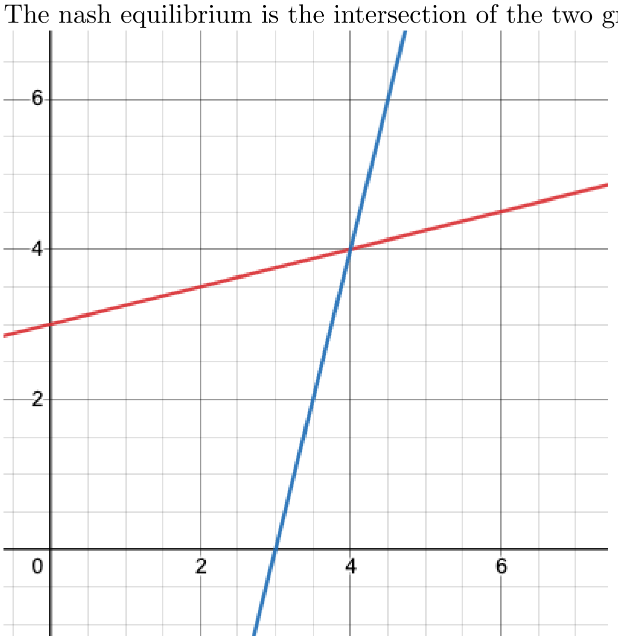
$$r_1 = \frac{r_2}{4} + 3$$

Similarly, the best response function for the second country r_2 is computed in the same way and results in the following function:

$$r_2 = \frac{r_1}{4} + 3$$

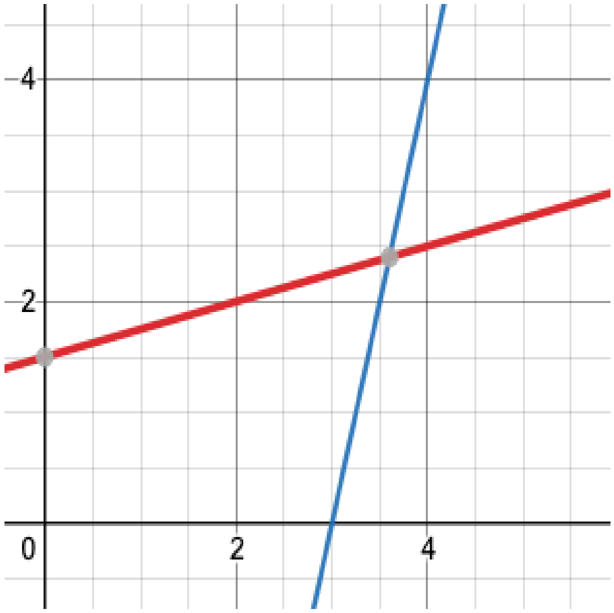
3.2 2

The nash equilibrium is the intersection of the two graphs.



3.3 3

If one of the countries decides to invest less effort into recycling, the nash equilibrium moves to a smaller number and both countries would suffer from a decreased benefit of their investments per hour. This is illustrated in the following figure where the red graph invests less than in the previous example.



4 Question 4: Tragedy of the Commons

In this section, we only need to consider the special case where there are only two players. The utility of two players would be $u_1(x_1, x_2) = x_1(1 - x_1 - x_2)$ and $u_2(x_2, x_1) = x_2(1 - x_1 - x_2)$. To find the Nash equilibrium, we need to find the partial derivative of two utility equations, which is $-2x_1 + 1 - x_2$ and $-2x_2 + 1 - x_1$. Both partial derivatives need to equal to 0 and then we get $x_1 = x_2 = 1/3$. The utilities for both x_1 and x_2 are 1/9.

4.1 2

The Nash equilibrium does not optimise the aggregated utility. For example, we set there are still only player x_1 and x_2. Therefore, their aggregated utility is $x_1(1 - x_1 - x_2) + x_2(1 - x_1 - x_2) = -x_1^2 - x_2^2 + x_1 + x_2 - 2x_1x_2 = -(x_1 + x_2)^2 + (x_1 + x_2)$. We set (x_1+x_2) equivalent to X, now the equation becomes $X - X^2$. In order to find the maximum value, we need to find the point where the derivative equals to 0. So $1 - 2X = 0$. Now we get the x_1 and x_2 sum X=1/2 and the maximum aggregated utility is 1/4 which is bigger than 2/9. Thus, Nash equilibrium will not optimize aggregated utility.

4.2 3

To generalise the question 2 conclusion, we get the aggregated utility of x_n is

$$\sum_{i=1}^{n} u_i(x_i, x_{-i} = \sum_{i=1}^{n}[x_i(1 - \sum_{j=1}^{n} x_j)]$$

In this situation, $\sum_{j=1}^{n} x_j$ can be consider as a constant, so we can the above formula it into

$$\sum_{i=1}^{n} x_i(1 - \sum_{j=1}^{n} x_j)$$

Now, because both $\sum_{i=1}^{n} x_i$ and $\sum_{j=1}^{n} x_j$ are the sum of all players' share, so they are the same thing. Thus, we can replace $\sum_{i=1}^{n} x_i$ and $\sum_{j=1}^{n} x_j$ by X and we get

$$\sum_{i=1}^{n} u_i(x_i, x_{-i}) = X - X^2$$

Same to the question 2, we can get the maximum aggregated utility is 1/4, when the sum share equals to 1/2.