Multi-Agent Systems Assignment 3

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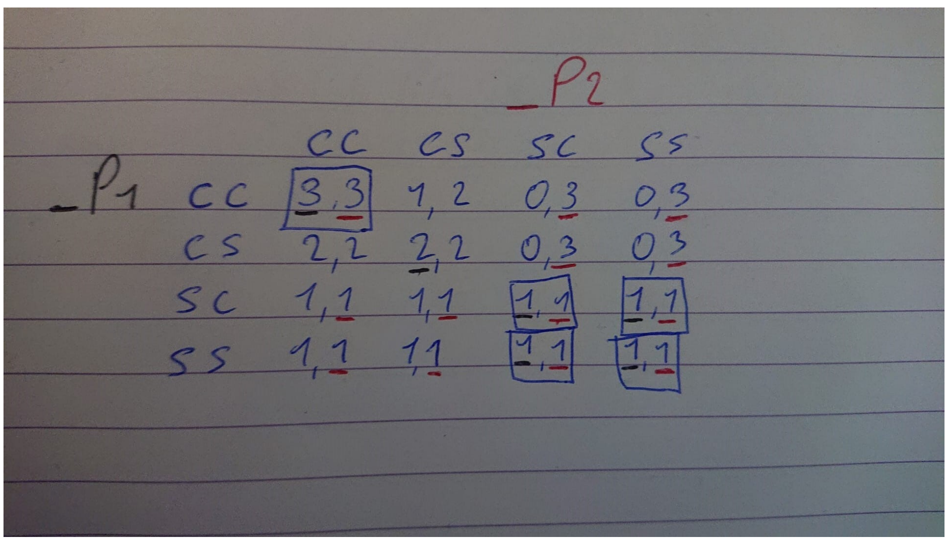
1 Question 1

1.1 Q 1.1

Looking at the given example, the strategy set for both players consists of : $S_1 = S_2 =$ (CC, CS, SC, SS)
From this strategy set we can create the normal form of this game as seen below:

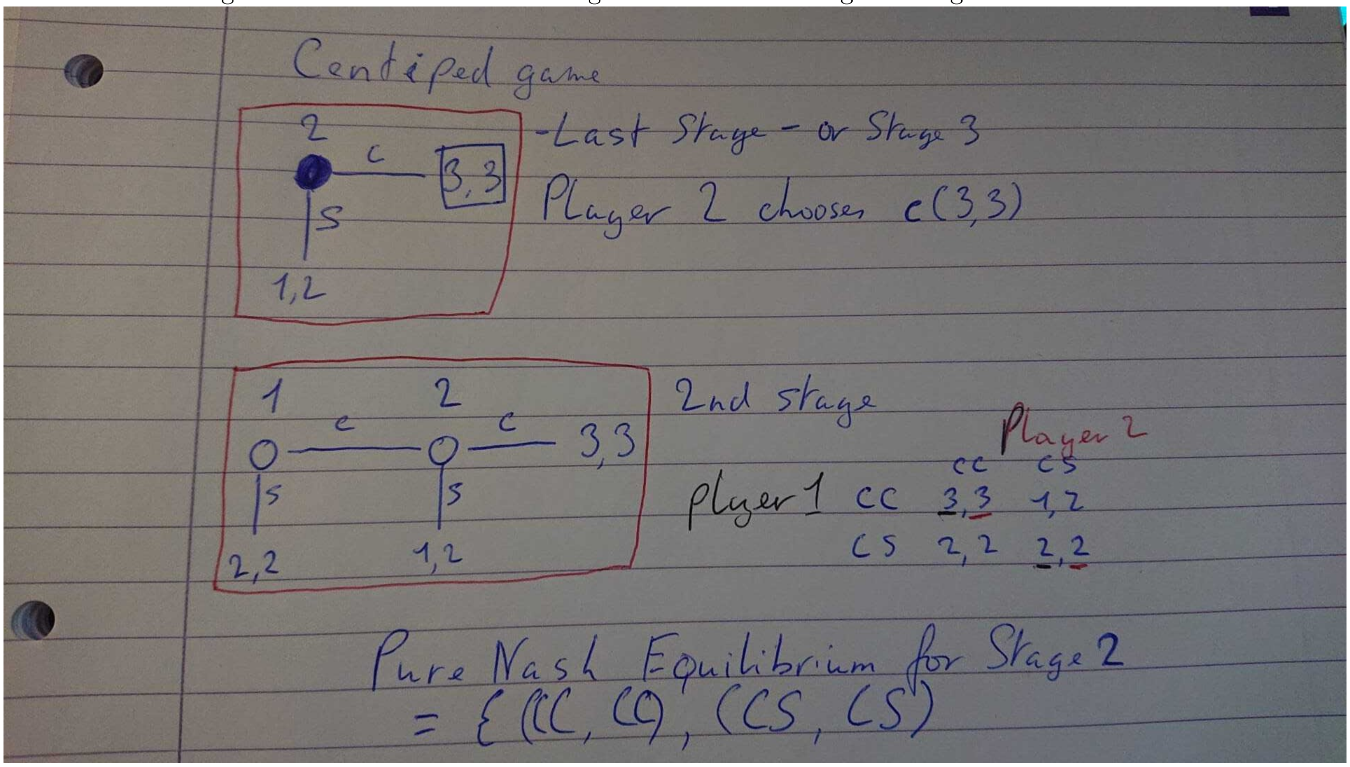


		P2			
		CC	CS	SC	SS
P1	CC	3,3	1,2	0,3	0,3
	CS	2,2	2,2	0,3	0,3
	SC	1,1	1,1	1,1	1,1
	SS	1,1	1,1	1,1	1,1

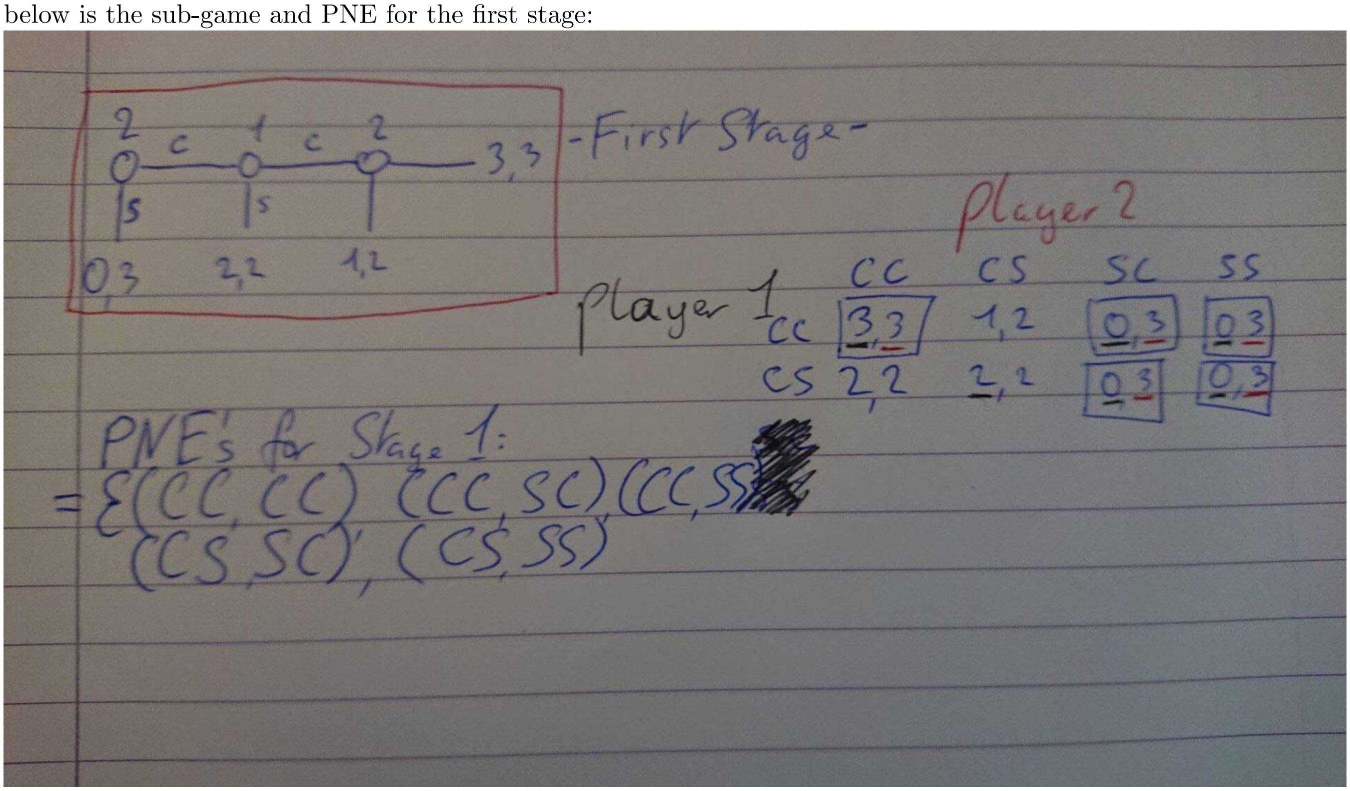
The Nash Equilibria from the normal form are: NE = (CC, CC), (SC, SC), (SC, SS), (SS, SC), (SS, SS) as seen above. Where player 1 is the first element in all brackets.

1.2 Q 1.2

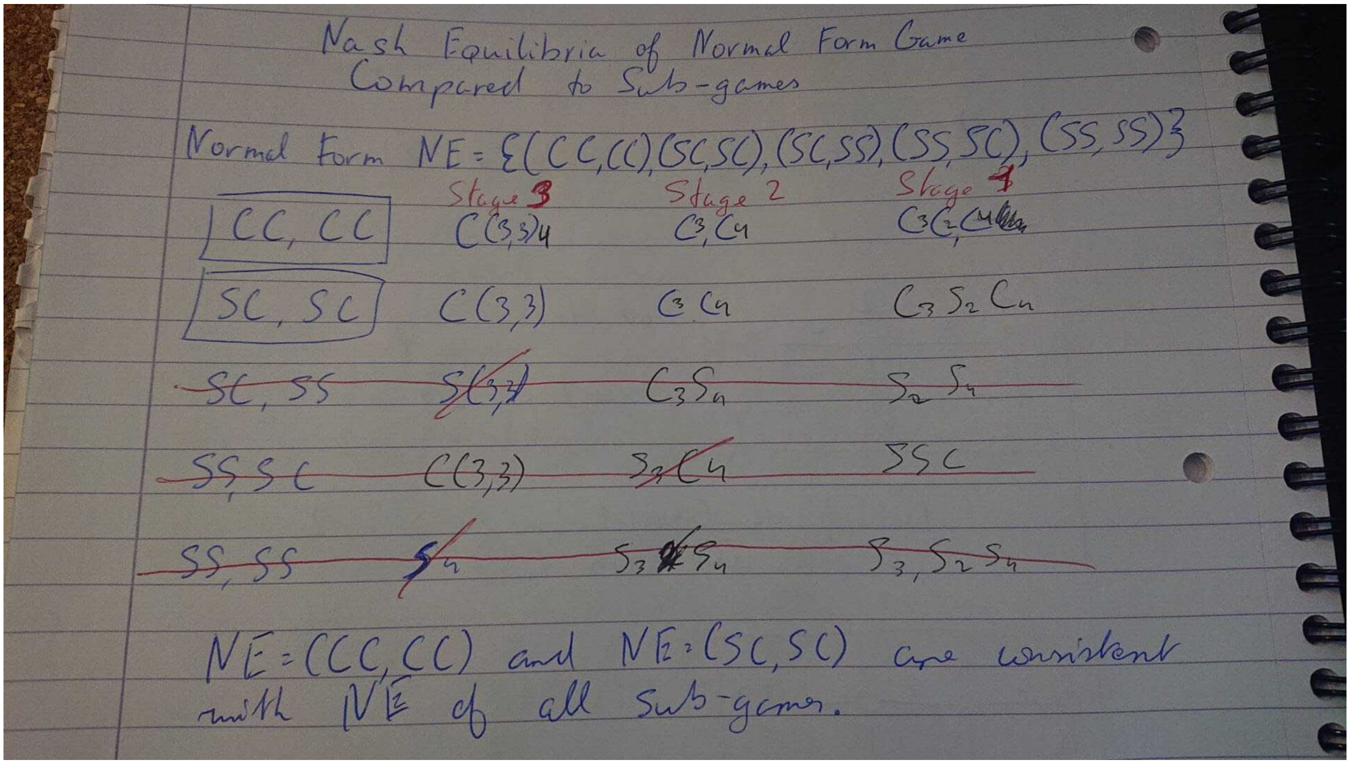
Below are the sub-games and PNE for the last stage and the middle stage of the game:



below is the sub-game and PNE for the first stage:



Below is NE of Normal Form Game compared to the Sub-Games:



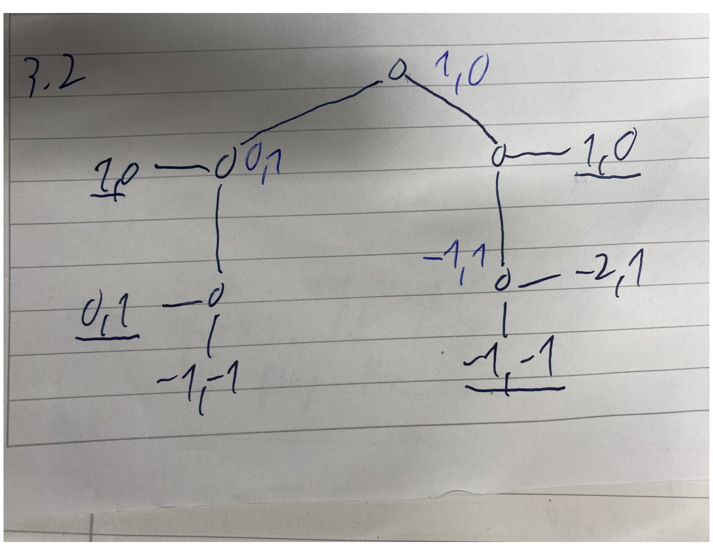
Both NE = (CC, CC) and NE = (SC, SC) are sub-game perfect.

1.3 Q 1.3

If we follow the process of backwards induction, we eventually see that player 2 has two decisions at first option point, both options provide utility of 3. Assuming pure rationality, player 2 is impartial to both options. This blocks player 1 from making most optimal moves. If player 1 was a risk averse player then player 1 could settle for the first stop giving him utility of 1. A similar result would occur if player 1 knew that player 2 is aggressive and would punish player 1 for continuing.

2 Question 2

2.1 Q 2.1



2.2 Q2.2

	H	S
(N,I)	1,0	0,1
(N,F)	1,0	-1,-1
(W,I)	1,0	1,0
(W,F)	1,0	1,0

Table 1: Normal Form Matrix

3 Boss and stealing employee

A boss notices that one of her employers has been stealing company material lately. The material was not all that valuable, so she is inclined to let it pass, preferring to keep the employee around rather than firing him and having to hire and retrain a replacement. Nevertheless she wants the stealing to stop. She is therefore thinking to issue a warning at the next company meeting: the next person caught stealing company property will be fired immediately. She envisages the following game tree with pay-offs (see fig below).

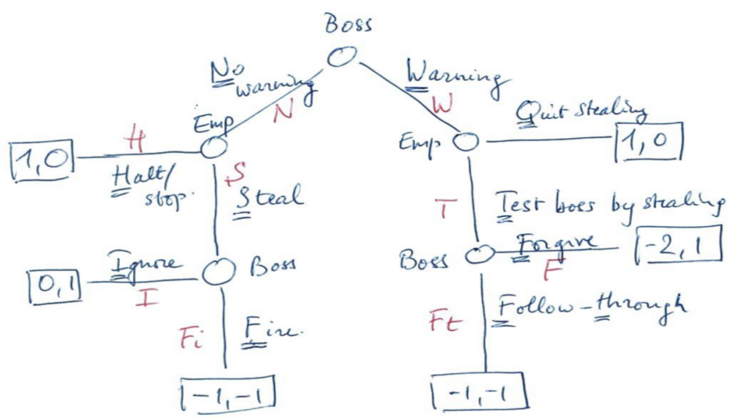


Figure 2: Game tree pay-offs.

3.1 Analyse the game using backward induction.

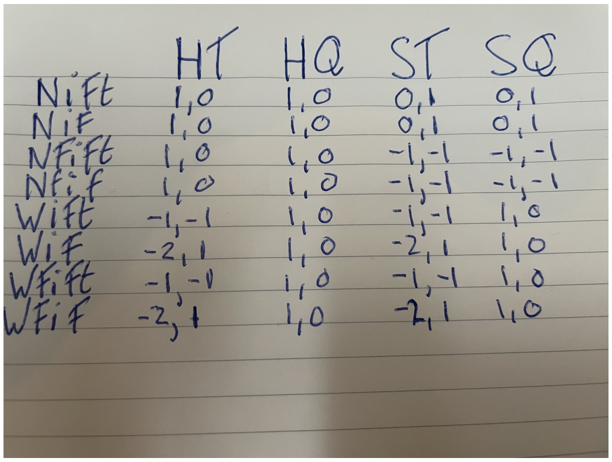
When using backward induction, we start at the leaf nodes and work our way up to the root in order to determine what strategy player 1 (in this case the boss) should play in order to maximize its utility.

If we start with the left side of the decision tree (boss chooses no warning), we can see that the final utility is (-1, -1) or (0, -1). Having this information, we know that the boss will choose (0, -1) at this branch to maximize her utility. The employee now has to decide between (1, 0) and (0, 1) at the branch above and will choose (0, 1) to maximize his utility. So if the boss chooses "No warning" as her first strategy, the utility for both players will result in (0, 1).

Now, if the boss chooses "Warning" first and we end up in the right side of the decision tree, we can see that the final utility is (-1, -1) or (-2, 1). Having this information, we know that the boss will choose (-1, -1) to maximize her utility. The employee now has to decide between (-1, -1) and (1, 0) at the branch above and will choose (1, 0) to maximize his utility.

Finally, we now know that the boss will get 0 utility (0, 1) if she chooses "No warning", and 1 utility (1, 0) if she chooses "Warning". Therefore, using backward induction, we know that to maximize her utility, the boss has to play the strategy to warn the employee at the next company meeting.

3.2 What are the pure actions for the two players (boss and employee)? Construct the normal form matrix.



	HT	HQ	ST	SQ
NiFt	1,0	1,0	0,1	0,1
NiF	1,0	1,0	0,1	0,1
NFiFt	1,0	1,0	-1,-1	-1,-1
NFiF	1,0	1,0	-1,-1	-1,-1
WiFt	-1,-1	1,0	-1,-1	1,0
WiF	-2,1	1,0	-2,1	1,0
WFiFt	-1,-1	1,0	-1,-1	1,0
WFiF	-2,1	1,0	-2,1	1,0

Figure 3: Normal form pay-off matrix.

3.3 Use this matrix to identify all the pure Nash equilibria of the normal form game.

If we look at the normal form matrix in figure 3, we can determine that there are four pure Nash equilibria: (NFiFT, HT), (NFiF, HT), (WFiFT, SQ), and (WFiF, SQ). The strategy (NFiFT, HT) should be interpreted that if the boss uses strategy *N* first, then the employee will use strategy *H* and we end up in (1, 0). If the employee would have used strategy *S* instead of *H*, then the boss will use strategy Fi resulting in (-1, -1). If the boss would have used strategy *W* instead of *N* as a first move, then the employee would use strategy *T*, where upon the boss would use strategy FT resulting in an utility of (-1, -1). Because of this, both the boss and the employee have no incentive to change their strategy because they will both end up with less utility if they do so. Likewise, the other three pure Nash equilibria should be interpreted in a similar manner.

3.4 Determine the subgame-perfect equilibrium (equilibria?) by eliminating all the Nash equilibria that fail to induce a NE in subgames.

For the NE (NFiFT, HT) the boss needs to choose strategy *N* first and then the employee has to choose strategy *H*. This case is not a NE in a subgame because the boss will not choose strategy *N* over strategy *W* because if she does, she will end up with less utility.

For the NE (NFiF, HT) the boss again needs to choose strategy *N* first and, therefore, this NE also does not hold when we look at the subgame decisions.

The NE (WFiFT, SQ) induces the employee to choose strategy *Q* in the subgame at the decision node for the employee and the boss to choose strategy *W* at the first decision node, which they will both do. This shows that (WFiFT, SQ) also induces a NE in subgames and holds as a NE.

The NE (WFiF, SQ) induces the employee to choose strategy *Q* in the subgame at the decision node of the employee. It is clear that the employee will choose this strategy because it will provide him with 0 utility (over -1). Additionally, this NE induces the boss to choose strategy *W* at the start, which she will choose because it provides her with 1 utility (over 0).

3.5 Compare to the solution based on backward induction.

Based on backward induction we found that we will always end up with utility (1, 0) where the boss chooses strategy *W* "Warning", which results in the employee always choosing *Q* "Quit stealing".

Using the subgame-perfect equilibrium we also find that the only NE are for the boss to choose strategy *W* and the employee responding with strategy *Q*.

4 Question 4 Stackelberg's Duopoly Model

4.1 1

In this question, we are requested to use the backward induction to find the optimal quantities for both firms. Because of the backward induction, we should find the optimal quantity q_2 first and then use q_2 to find q_1.The process is as follow:

1. Suppose q_1 is given.
2. Find the optimal $q_2^*(q_1)$ given q_1 according to $\frac{\partial u_2}{\partial q_2}$
3. Find the $q_1^*(q_2)$ based on q_2^* according to $\frac{\partial u_1}{\partial q_1}$

In order to find q_2, we need to calculate the partial derivative for u_2.

$$u_2 = P(q_1, q_2)q_i - cq_i = \alpha - \beta q_2 * 2 - \beta q_1 q_2 - cq_2$$

$$u_2' = \alpha - 2\beta q_1 - \beta q_2 - c$$

In order to find the maximum quantity for q_2, we need to let $u_2' = 0$. Now we get $q_2 = \frac{\alpha - \beta q_1 - c}{2\beta}$. So now, no matter what q_1 equals, q_2 will always equal to $\frac{\alpha - \beta q_1 - c}{2\beta}$.
Next we use the same method to find the maximum quantity for q_1 is when $q_1 = \frac{\alpha - \beta q_2 - c}{2\beta}$.
Let us plug in $q_2 = \frac{\alpha - \beta q_1 - c}{2\beta}$ into the equation and now the equations transform to:

$$q_1 = \frac{\alpha - \beta \frac{\alpha - \beta q_1 - c}{2\beta} - c}{2\beta}$$

Solving the equation, we get $q_1 = \frac{\alpha - c}{3\beta}$ and $q_2 = \frac{\alpha - c}{3\beta}$.

4.2 2

Comparing to the results for the Cournot simultaneous model when the unit cost for two company are the same, the results are the same. The reason is that the the results got from backward induction is also the Nash equilibrium. Just from the information above, we cannot ensure whether there is a "first mover" advantage. However, when unit cost for two company are not same, separately c_1 and c_2. According to the backward induction, we can get:

$$q_1 = \frac{\alpha - \beta \frac{\alpha - \beta q_1 - c_2}{2\beta} - c_1}{2\beta}$$

Solving the equation, we can get $q_1 = \frac{\alpha + c_2 - 2c_1}{3\beta}$ and $q_2 = \frac{\alpha + c_1 - 2c_2}{3\beta}$.
However, if we calculate the best response for Nash equilibrium, the result is also $q_1 = \frac{\alpha + c_2 - 2c_1}{3\beta}$ and $q_2 = \frac{\alpha + c_1 - 2c_2}{3\beta}$.
Thus, the conclusion is comparing to the Cournot (simultaneous) model, there isn't a "first mover" advantage.