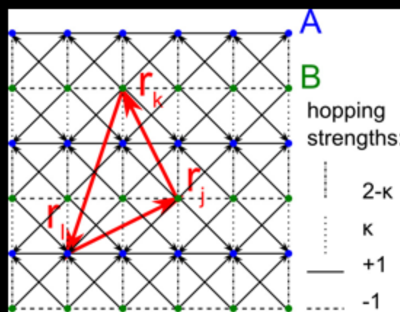
✻ QLT on CI, FCI results. Lect 2.

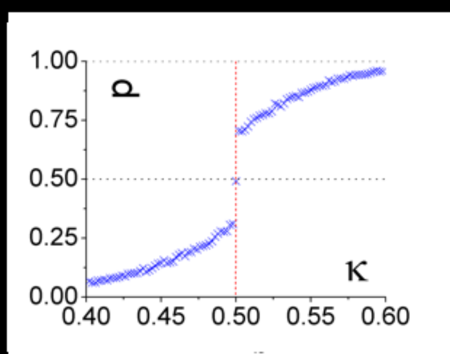
Model



$$H(\kappa) = \sum_{\vec{r}} (-1)^y c^\dagger_{\vec{r}+\hat{x}} c_{\vec{r}} + [1 + (-1)^y (1-\kappa)] c^\dagger_{\vec{r}+\hat{y}} c_{\vec{r}} + (-1)^y \frac{i\kappa}{2} \left[c^\dagger_{\vec{r}+\hat{x}+\hat{y}} c_{\vec{r}} + c^\dagger_{\vec{r}+\hat{x}-\hat{y}} c_{\vec{r}} \right] + \text{h.c.} \quad (2)$$

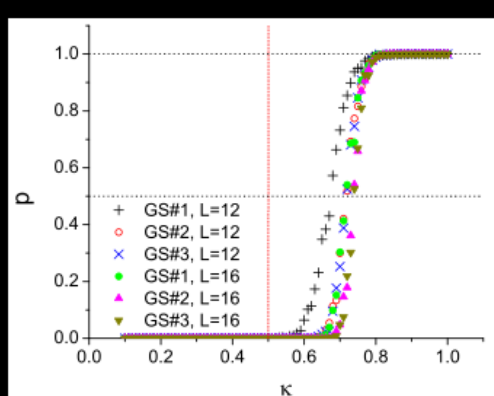
- Topological Quantum Phase Transition at κ=0.5
- κ<0.5 trivial insulator
- κ>0.5 Chern insulator

QLT as the input vector



- Train with two known points:
  κ=0.1 (trivial), κ=1 (topo)
- Smallest triangles (d_c=1) are sufficient in the gapped phases
- Once trained, get PD in 10min on a laptop.
- 99.9% accuracy in the phase verified with 2k test samples.

Model part II: ν=1/3 Fractional Chern Insulator



- VMC wave function: Cube the parent free fermion (parton) wave function
- Free fermion trained NN: Recognizes FCI as a distinct topological phase
- FCI trained NN: 10 min PD for 12x12 with 99.9% accuracy

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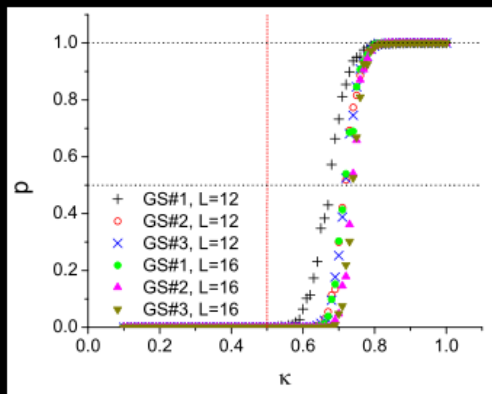

- VMC wave function: Cube the parent free fermion (parton) wave function
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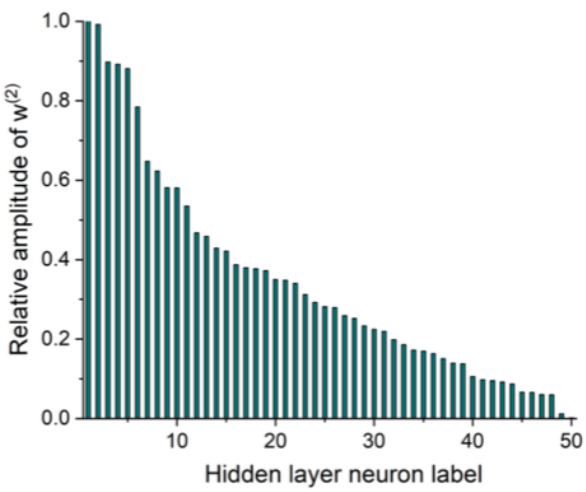
* Interpreting CI results.

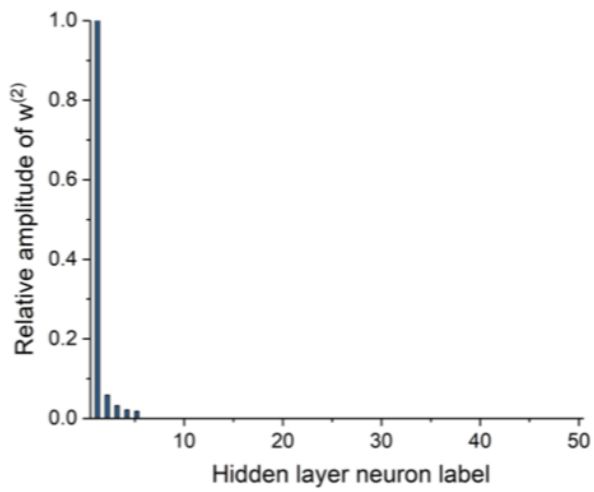
$$y = \sigma\left[\sum_j w_j^{(2)} \sigma\left(\sum_i w_{ji}^{(1)} x_i + b_j^{(1)}\right) + b^{(2)}\right]$$

Ranked Plot of $w_j^{(2)}$

before training

after training

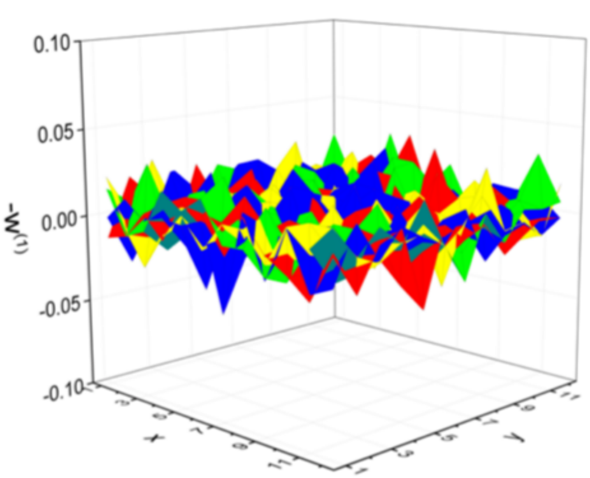


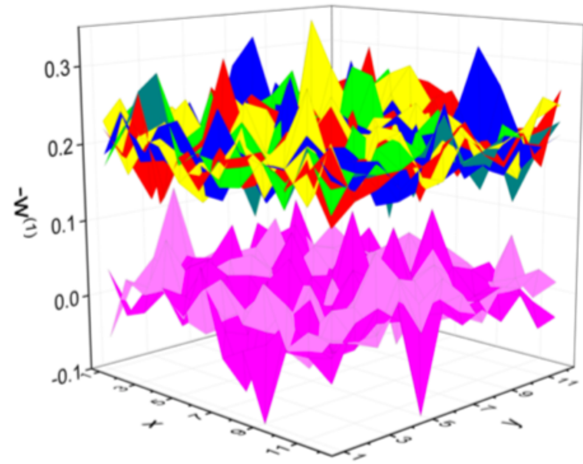


HiddenLayer Weights w_{1i} associated with each QLT input x_i

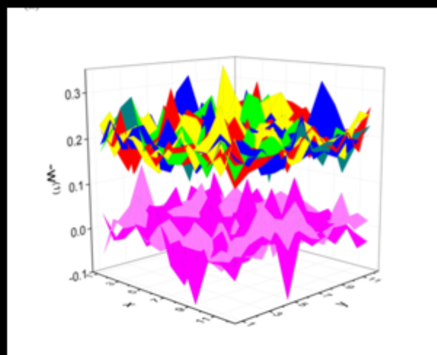
before training

after training





Interpreting ML of Chern Insulator



$$w^{(2)}_{j_{max}} \max[\, \bar{w}^{(1)}_{j_{max}} \sum_{i=\vec{r},\pm\hat{x},\pm\hat{y}} \mathrm{Im} P_{\vec{r}\pm\hat{y},\vec{r}} P_{\vec{r},\vec{r}\pm\hat{x}} P_{\vec{r}\pm\hat{x},\vec{r}\pm\hat{y}} + b^{(1)}_{j_{max}}, 0] + b^{(2)} > 0$$

$$-4.84 \times \max[-0.208 \sum_{d_{\triangle jkl}=1} \mathrm{Im} P_{jk} P_{kl} P_{lj} + 3.73, 0] + 9.03 > 0$$

$$\Leftrightarrow \frac{4\pi}{N} \sum_{d_{\triangle jkl}=1} -\mathrm{Im} P_{jk} P_{kl} P_{lj}/2 > 0.4,$$

□ QLT for transport on DQMC data

Consider

$$\Lambda_{xx}(\vec{r}_1,\vec{r}_2;\omega_n=0) \equiv \int d\tau \, \langle \hat{j}_x(\vec{r}_1,\tau)\,\hat{j}_x(\vec{r}_2,0)\rangle$$

where $\hat{j}_x(\vec{r}_1,\tau) = e^{H\tau}\hat{j}_x(\vec{r}_1)e^{-H\tau}$

$$\hat{j}_x(\vec{r}_1) = -i\,[H(\vec{r}_1),\hat{x}]$$

$$\rho_s \propto \left[\Lambda_{xx}(q_x\to 0,\ q_y=0,\ \omega_n=0) - \Lambda_{xx}(q_x=0,\ q_y\to 0,\ \omega_n=0)\right]$$

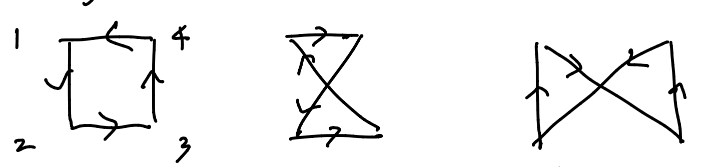
For a gapped SC consider $H' = -\Pi$, $\Pi \equiv |G\rangle\langle G|$

$$\begin{aligned}
\Lambda_{yx}(\vec{r}_1,\vec{r}_2;\omega_n=0) &= \langle G|\hat{j}_x(\vec{r}_1)(1-\Pi)\hat{j}_x(\vec{r}_2)|G\rangle \\
&= \mathrm{Tr}\left[\Pi\,\hat{j}_x(\vec{r}_1)\,(1-\Pi)\,\hat{j}_x(\vec{r}_2)\right] \\
&= \sum_{\vec{r}_3\vec{r}_4} P_{\vec{r}_2\vec{r}_4}P_{\vec{r}_4\vec{r}_1}P_{\vec{r}_1\vec{r}_3}P_{\vec{r}_3\vec{r}_2}(x_1-x_4)(x_2-x_3) \\
&\quad - \sum_{\vec{r}_4} P_{\vec{r}_2\vec{r}_4}P_{\vec{r}_4\vec{r}_1}P_{\vec{r}_1\vec{r}_2}(x_1-x_4)(x_2-x_1)
\end{aligned}$$

$$P_{r'r} \equiv \langle G|c^\dagger_{r'}c_r|G\rangle$$

<u>QLT</u>: $L^{\triangle}_{jkl} \equiv \tilde{P}_{jk}|_\alpha\,\tilde{P}_{kl}|_\beta\,\tilde{P}_{lj}|_\gamma$

$$L^{\square}_{jklm} \equiv \tilde{P}_{jk}|_\alpha\,\tilde{P}_{kl}|_\beta\,\tilde{P}_{lm}|_{\gamma'}\,\tilde{P}_{mj}|_\delta .$$



L_{1234} $\quad L_{1423}$ $\quad L_{1342}$

- Negative U Hubbard Model.

$$H = -\sum_{\langle ij \rangle, \sigma} \left(c_{j\sigma}^{\dagger} c_{i\sigma} + h.c. \right) - \mu \sum_i \left(n_{i\uparrow} + n_{i\downarrow} \right)$$

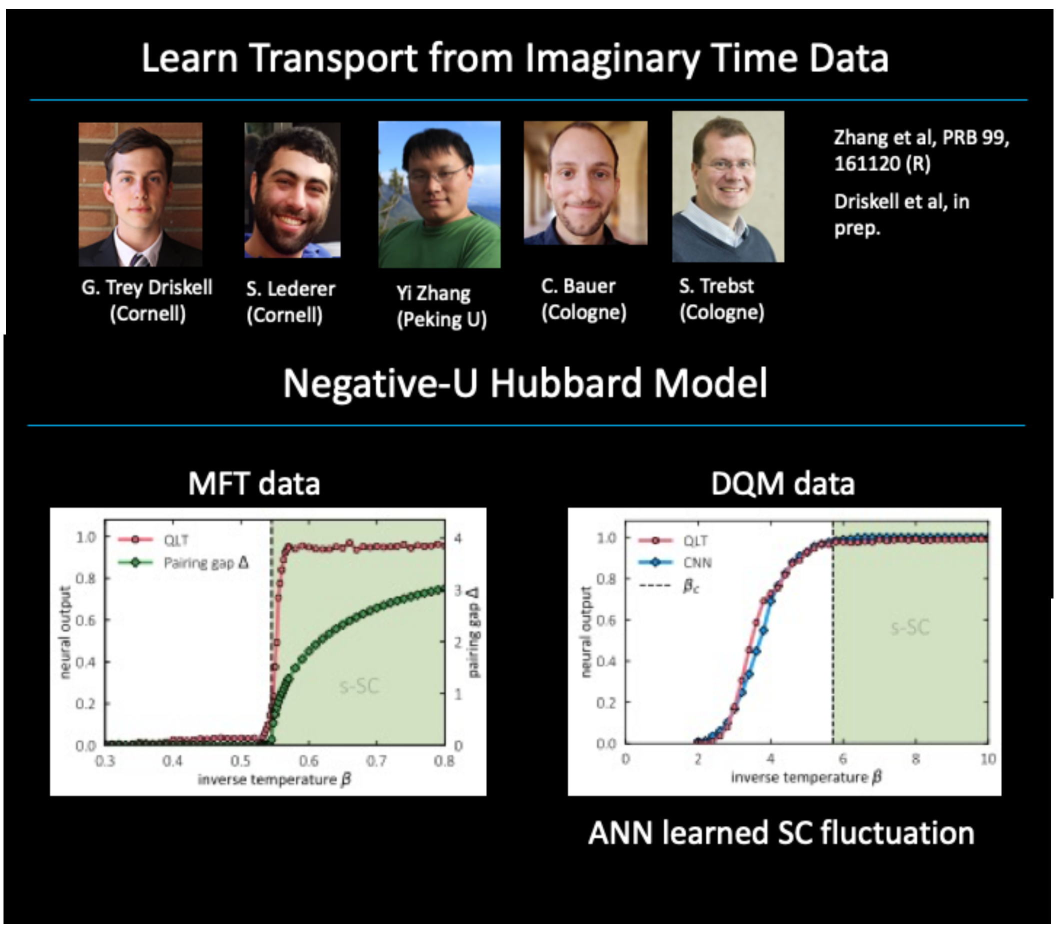
$$+ U \sum_i \left(n_{i,\uparrow} - \tfrac{1}{2} \right)\left(n_{i,\downarrow} - \tfrac{1}{2} \right)$$

$$U = -|U|$$

Approach for data acquisition

① Mean-field theory (BCS)

② sign-problem free DQMC.



- Spin-fermion model: QC spin fluct. mediated SC.

$$S = S_\psi + S_\varphi + S_\lambda$$

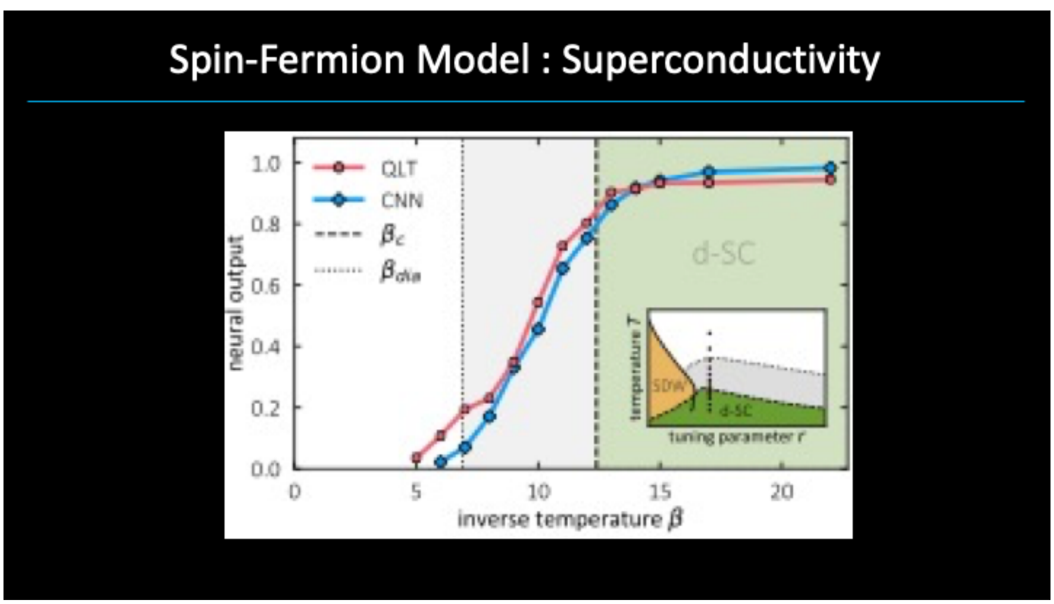
$$S_\psi = -\int_{\tau,r,r'} \sum_{\sigma,\alpha} \left[(\partial_\tau - \mu)\delta_{rr'} - t_{\alpha rr'}\right] \psi^\dagger_{\alpha r s} \psi_{\alpha r' s}$$

: two flavor of fermions $\Rightarrow$ sign-problem free.

$$S_\varphi = \int_{\tau,r} \frac{1}{2c^2}(\partial_\tau \vec{\varphi})^2 + (\nabla \vec{\varphi})^2 + \frac{r}{2}\vec{\varphi}^2 + \frac{u}{4}(\vec{\varphi}^2)^2$$

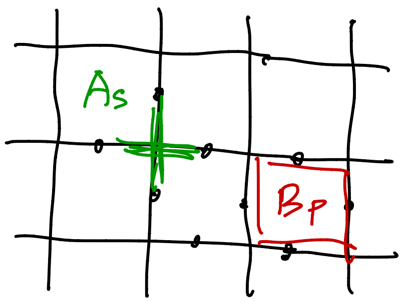
$\vec{\varphi}$: easy-plane SDW o.p. at $\vec{Q} = (\pi, \pi)$

$$S_\lambda = \lambda \int_{\tau,r} e^{iQ\cdot r_i} \vec{\varphi}_r \cdot (\psi^\dagger_{ars} \vec{\sigma}_{ss'} \psi_{brs'} + h.c.)$$



□ Z_2 QSL: Deconfinement Transition.

$$H_{2D} = -J_x \sum_s A_s - J_z \sum_p B_p - h_x \sum_j \sigma_j^x - h_z \sum_j \sigma_j^z$$



↑ ↑ external field breaks exact solvability.

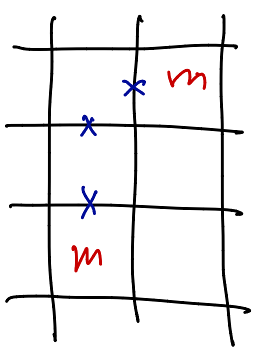
$[A_s, B_p] = 0$ $\quad A_s = \prod_{j \in s} \sigma_j^x$, $\quad B_p = \sum_{j \in p} \sigma_j^z$

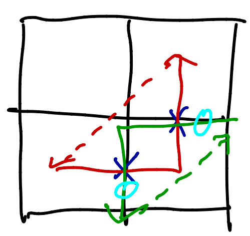
For $h_x = h_z = 0$, set all $A_s = B_p = 1$.

$\exists$ excitations: spinon at site $s \Leftrightarrow A_s = -1$ (e)

vison in plaquette $p \Leftrightarrow B_p = -1$ (m)

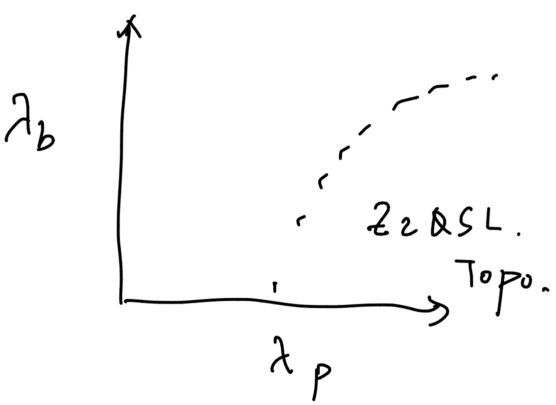
String of σ^x: create a pair of visons





$\langle \sigma_j^x \, \sigma_j^z, \, \sigma_j^x, \, \sigma_j^z \rangle$

QLT: $\prod_{j \in C_e} \sigma_j^z \prod_{k \in \tilde{C}_m} C_k^x$



$\lambda_b = h_z \Delta\tau$

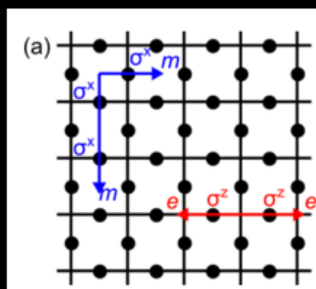
$\lambda_p = J_z \Delta\tau$

$$\beta H_{3D} = -\lambda_b \sum_j S_j - \lambda_p \sum_p \prod_{j \in p} S_j$$

* QLT based ML of Z_2 QSL Results

Kitaev Model under field

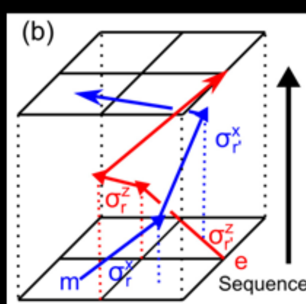
$$H_{2D} = -J_x \sum_s A_s - J_z \sum_p B_p - h_x \sum_b \sigma_b^x - h_z \sum_b \sigma_b^z$$



- Finite region of Z2 spin liquid with finite correlation length
- Spinons and Visons
- Mutual statistics

Quantum Loop Topography for Z2 QSL

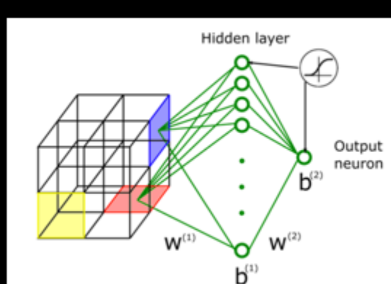
$$H_{2D} = -J_x \sum_s A_s - J_z \sum_p B_p - h_x \sum_b \sigma_b^x - h_z \sum_b \sigma_b^z$$



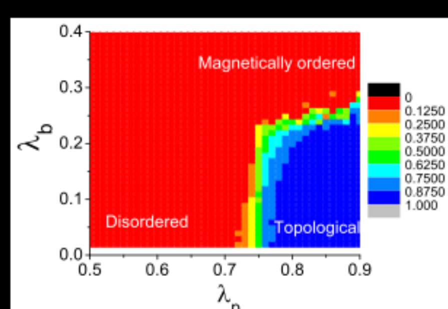
- QLT designed to probe mutual statistics

$$\left\langle \sigma_r^x \sigma_{r'}^z \sigma_{r'}^x \sigma_r^z \right\rangle = \mathrm{tr}\left[\rho \sigma_r^x \sigma_r^z \sigma_{r'}^z \sigma_{r'}^x\right]$$

Interpret the ML of Deconfinement



$$H_{3D} = -\lambda_b \sum_b S_b - \lambda_p \sum_p \prod_{j \in p} S_j$$

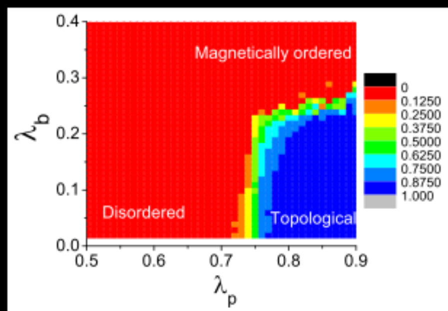


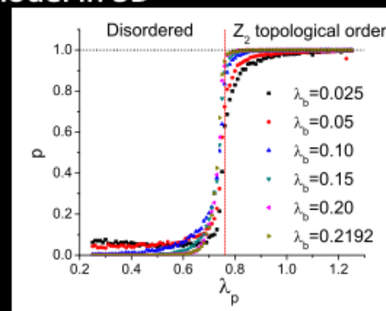
Yi Zhang , R. Melko &E-AK, PRB, 96, 245119 (2017)

Kitaev Model under field

$$H_{3D} = -\lambda_b \sum_b S_b - \lambda_p \sum_p \prod_{j \in p} S_j$$

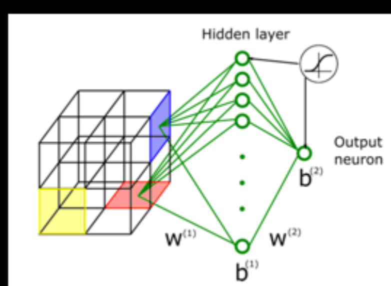
- 2+1D Kitaev Model under field ~Classical Z2 gauge Higgs model in 3D

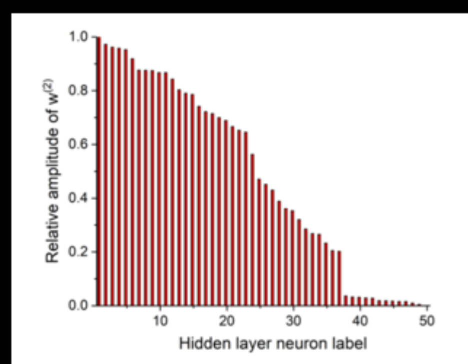




Yi Zhang , R. Melko &E-AK, PRB, 96, 245119 (2017)

Interpret the ML of Deconfinement





The non-linearity of ANN was essential

Yi Zhang , P. Ginsparg, &E-AK, in prep (2019)

□ Interpretability when non-linear,

① Include higher order terms, e.g., $x_i \cdot x_j$

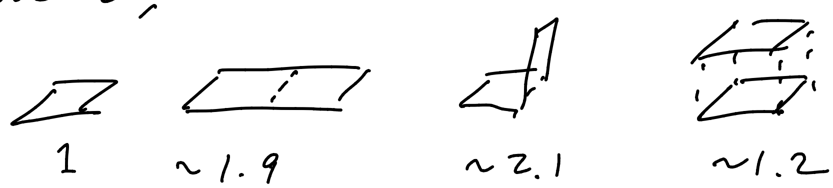
&

Shrink the hidden layer width

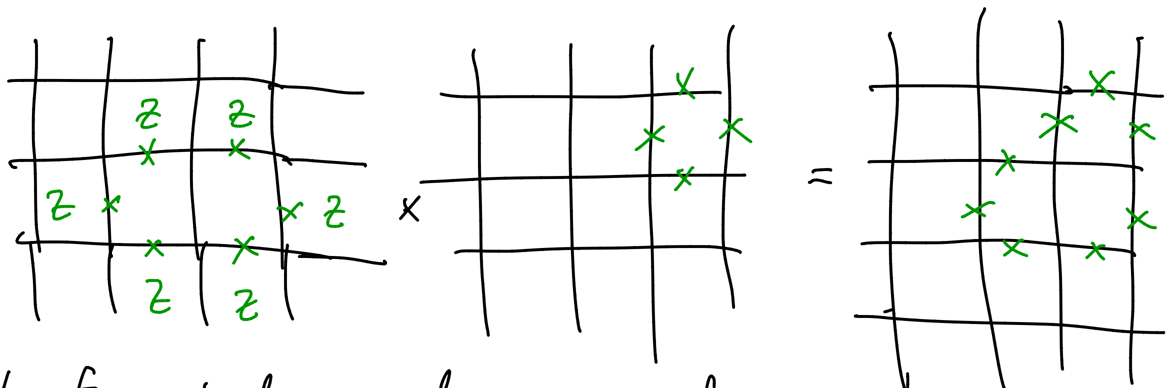
② Keep only the higher order terms with weight $W^{(2)}$

⇒ At width = 3,

$\bar{W}^{(1)}_{j_{max}}$



Note :



⇒ ANN formed larger loops out of local information to learn deconfinement tr.