

QMCPACK Users Workshop 2019

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Auxiliary-Field quantum Monte Carlo

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Overview

- AFQMC Basics
 - Single particle basis sets, second quantization, etc.
 - Slater determinants, Hubbard-Stratonovich transformation, etc.
- Real vs Orbital Space
 - Advantages/Disadvantages of each approach.
- AFQMC in QMCPACK
 - Workflow, features, examples.
- Tools (Fionn D. Malone)





Useful references

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AFQMC: Projection QMC in Orbital Space Shares many features with DMC

Discrete walker representation of imaginary-time evolving distribution

$$|\Psi_0\rangle = \int f(\phi)|\phi\rangle d\phi$$
 $|\Psi_0\rangle = \sum_{\phi} w_{\phi}|\phi\rangle,$

Short time approximation to imaginary-time propagator with Trotter-like factorization

$$|\Psi_0\rangle \propto \lim_{\tau \to \infty} e^{-\tau \hat{H}} |\Psi_T\rangle;$$

$$|\Psi^{(n+1)}\rangle = e^{-\Delta \tau \hat{H}} |\Psi^{(n)}\rangle,$$

Mixed estimator for the energy. Backward walking or mixed estimator for general observables.

$$\langle \hat{O} \rangle = \lim_{n \to \infty} \frac{\langle \Psi^{(n)} | \hat{O} | \Psi^{(n)} \rangle}{\langle \Psi^{(n)} | \Psi^{(n)} \rangle}$$

$$E_0 = \lim_{n \to \infty} \frac{\langle \Psi_T | \hat{H} | \Psi^{(n)} \rangle}{\langle \Psi_T | \Psi^{(n)} \rangle}$$

Walker Representation

- $(N, N_{\alpha}, N_{\beta})$: # of electrons (total, spin up, spin down)
- $\triangleright |\xi_i\rangle$ (i=1,...,M): Single-particle basis set.
 - Lattice site on a model Hamiltonian, plane waves, LCAO, etc.
 - Molecular orbitals, localized orbitals, etc.
- $\succ c_i, c_i^{\dagger}$: annihilation and creation operators for the basis set .
- $\triangleright |\psi_n\rangle$: Single-particle orbital set.

•
$$|\psi_n\rangle = \sum_{i=1}^M D_{i,n} |\xi_i\rangle$$

In AFQMC, walkers represent Slater determinants. A walker is defined by a weight and the (M, N) Slater matrix $(D_{i,n})$ defining its occupied orbitals.

•
$$|D\rangle = \prod_{n=1}^{N} \psi_n^{\dagger} |0\rangle$$
 Slater Matrix=
$$\begin{bmatrix} D_{1,1} & \cdots & D_{1,N} \\ \vdots & \ddots & \vdots \\ D_{M,1} & \cdots & D_{M,N} \end{bmatrix}$$

Second Quantization

> Real space electronic Hamiltonian:

•
$$\widehat{H} = \widehat{H}_1 + \widehat{H}_2 = -\frac{\hbar^2}{2m} \sum_i \nabla_i^2 + \sum_i V_{ext}(\overrightarrow{r_i}) + \sum_{i < j} V_{e-e}(|\overrightarrow{r_i} - \overrightarrow{r_j}|)$$

- ➤ Given a single particle basis set, the corresponding second quantized Hamiltonian becomes:
 - $\widehat{H} = \sum_{ij} T_{ij} c_i^{\dagger} c_j + \frac{1}{2} \sum_{ijkl} (ik|jl) c_i^{\dagger} c_j^{\dagger} c_l c_k$
 - $\bullet \quad \widehat{H}_2 = \frac{1}{2}\,\widehat{v}_0 + \frac{1}{2}\sum_n \widehat{v}_n^2$
- The Hamiltonian is defined through its matrix elements in the basis:
 - $(ik|jl) = \langle \xi_i \xi_j | \widehat{H}_2 | \xi_k \xi_l \rangle$ = $\int d\vec{r}_1 \vec{r}_2 \xi_i^* (\vec{r}_1) \xi_k (\vec{r}_1) V_{e-e} (|\vec{r}_1 - \vec{r}_2|) \xi_j^* (\vec{r}_2) \xi_l (\vec{r}_2)$
 - $T_{ij} = \langle \xi_i | \widehat{H}_1 | \xi_j \rangle$ = $\int d\vec{r}_1 \; \xi_i^*(\vec{r}_1) \left[-\frac{\hbar^2}{2m} \nabla_i^2 + V_{ext}(\vec{r}_1) \right] \xi_j(\vec{r}_1)$

Walker Propagation - I

- One body propagators are easy!
 - $|D'\rangle = e^{(\sum_{ij} c_i^{\dagger} B_{ij} c_j)} |D\rangle \rightarrow D' = e^B D$
 - Main idea of AFQMC: Represent many-body propagator only with 1-body operators!
- ➤ Hubbard-Stratonovich (HS) transformation:

•
$$e^{-\left(\frac{\tau}{2}\right)\hat{v}^2} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} d\sigma \ e^{-\sigma^2/2} e^{\sigma\sqrt{-\tau} \hat{v}}$$

- > Standard Approximations
 - $e^{-\beta \hat{H}} = \prod_n e^{-\tau \hat{H}}$, $\beta = n\tau$

• $e^{-\tau \hat{H}} = e^{-\frac{\tau}{2}\hat{H}_1}e^{-\tau \hat{H}_2}e^{-\frac{\tau}{2}\hat{H}_1} + O[\tau^2]$

Break-up in timesteps

Trotter approximation

- > AFQMC: Many-body propagator as an integral over 1-body propagators
 - $e^{-\beta \hat{H}} = \prod_n \int d\vec{\sigma} P(\vec{\sigma}) \hat{B}(\vec{\sigma}) + O[\tau^2]$
 - $\hat{B}(\vec{\sigma}) = e^{-\frac{\tau}{2}\hat{H}_1}e^{i\sqrt{\tau}(\vec{\sigma}\cdot\hat{v})}e^{-\frac{\tau}{2}\hat{H}_1}$

Walker Propagation - II

- Efficiency improvements: Similar to DMC
 - Mean-field subtraction: $v_n^{MF} = \frac{\langle \Psi_T | \hat{v}_n | \Psi_T \rangle}{\langle \Psi_T | \Psi_T \rangle}$ $\hat{H}_2 = \left(\frac{1}{2}\hat{v}_0 + \sum_n v_n^{MF} \hat{v}_n \frac{1}{2}\sum_n (v_n^{MF})^2\right) + \frac{1}{2}\sum_n (\hat{v}_n v_n^{MF})^2$
 - Importance sampling and force bias : $v_b = -\frac{\langle \Psi_T | \hat{v} | \phi_n \rangle}{\langle \Psi_T | \phi_n \rangle}$ $\int d\sigma \ e^{-\sigma^2/2} e^{\sigma \hat{v}} \rightarrow \int d\sigma \ \frac{\langle \Psi_T | \hat{B}(\sigma) | \phi \rangle}{\langle \Psi_T | \phi \rangle} e^{-\sigma^2/2} e^{v_b \sigma v_b^2/2} e^{(\sigma v_b)\hat{v}}$ $|\Phi_{AF}\rangle = \frac{1}{N_w} \sum_n w_n |\phi_n\rangle \rightarrow \frac{1}{N_w} \sum_n w_n \frac{|\phi_n\rangle}{\langle \Psi_T | \phi_n\rangle}$
 - Propagation step:

1.
$$|\phi_n\rangle \to \hat{B}(\vec{\sigma})|\phi_n\rangle$$
 "Local Energy Approximation"
2. $w_n \to w_n * \frac{\langle \Psi_T | \hat{B}(\sigma) | \phi \rangle}{\langle \Psi_T | \phi \rangle} e^{v_b \sigma - v_b^2/2} \approx e^{-[E_L(\phi) + E_L(\phi')]/2}$

"Hybrid Propagation"

Walker Propagation - III

Sign problem: Phaseless approximation

•
$$\hat{B}(\vec{\sigma}) = e^{-\frac{\tau}{2}\hat{H}_1}e^{i\sqrt{\tau}(\vec{\sigma}\cdot\hat{v})}e^{-\frac{\tau}{2}\hat{H}_1}$$

• Complex HS transformation leads to sign problem. Same origin as DMC sign problem. Same solution, but in Slater determinant space.

$$E = \frac{\langle \Psi_T | \widehat{H} | \Phi_{AF} \rangle}{\langle \Psi_T | \Phi_{AF} \rangle} = \frac{\sum_n |w_n \langle \Psi_T | \phi \rangle| \ e^{i\theta(\phi)} \ E_L(\phi_n)}{\sum_n |w_n \langle \Psi_T | \phi \rangle| \ e^{i\theta(\phi)}}$$

• Project random walk to real axis. Leads to real weights.

$$w_n \to w_n * \left| \frac{\left\langle \Psi_T \middle| \hat{B}(\sigma) \middle| \phi \right\rangle}{\left\langle \Psi_T \middle| \phi \right\rangle} e^{v_b \sigma - \frac{v_b^2}{2}} \right| * \max(0, \cos(\Delta \theta))$$

$$\Delta \theta = \text{phase of } \frac{\left\langle \Psi_T \middle| \hat{B}(\sigma) \middle| \phi \right\rangle}{\left\langle \Psi_T \middle| \phi \right\rangle}$$

• Importance sampling leads to a seamless application of the approximation.

Observables

Mixed distribution

•
$$\langle \hat{O} \rangle_{M} = \frac{\langle \Psi_{T} | \hat{O} | \Phi_{AF} \rangle}{\langle \Psi_{T} | \Phi_{AF} \rangle} = \frac{\sum_{n} w_{n} O_{M}(\phi_{n})}{\sum_{n} w_{n}}, O_{M}(\phi_{n}) = \frac{\langle \Psi_{T} | \hat{O} | \phi_{n} \rangle}{\langle \Psi_{T} | \phi_{n} \rangle}$$

- $\langle \hat{O} \rangle_{M}$: quadratic in $|\Psi_{T} \Psi_{Exact}|$ for operators that commute with Hamiltonian. Linear for all others.
- Small overhead, but can depend significantly on quality of Ψ_T .
- Back propagation:

•
$$\langle \hat{O} \rangle_{BP} = \frac{\langle \Phi'_{AF} | \hat{O} | \Phi_{AF} \rangle}{\langle \Phi'_{AF} | \Phi_{AF} \rangle} = \frac{\sum_{n} w_{n} O(\phi_{m}, \phi_{n})}{\sum_{n} w_{n}}, O(\phi_{m}, \phi_{n}) = \frac{\langle \phi_{m} | \hat{O} | \phi_{n} \rangle}{\langle \phi_{m} | \phi_{n} \rangle}$$

- $\langle \hat{O} \rangle_{RP}$: quadratic in $|\Psi_T \Psi_{Exact}|$.
- Typically requires an overhead of 50-75%.

Real vs Orbital Space

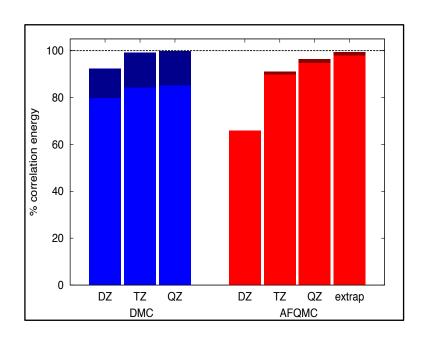
AFQMC

- Direct connection between ab-initio and model Hamiltonians
- Flexible treatment of core electrons:
 - All-e, frozen-core, ECP, NCPP, LAPW, etc.
- Spin-orbit coupling easy to incorporate.
- Typically, smaller bias from phaseless approximation.
- Efficient/simple code.
 - GEMM, QR, Inverse
- Small ab-initio community.
- Basis set error.
 - Error cancellation is "transferred" to the basis.
- Requires 2-electron integrals.
 - M³ memory cost for propagation
 - M⁴ memory cost for energy
 - M²⁻³ memory cost
 - No "direct" algorithm.
 - Forces require xN_{atoms} more memory.

DMC

- Trivial explicit correlation.
 - Jastrow is almost free!
- No basis set extrapolation. Works at CBS limit.
- Memory friendly.
- Intuitive.
- Hard to simplify.
- Uncontrolled approximation needed for non-local ECP channels.
- Fixed-node error is large.
 - Error cancellation is needed for accurate results.
- Spin-orbit is not trivial (to me!!!).
- Divergent potentials.
 - Forces are noisy!

Real vs Orbital Space



Illustrative DMC – AFQMC comparison

- Cohesive energy of 2-atom unit cell of Carbon in diamond structure
- Bright (dark) region corresponds to a calculation with a single determinant (converged MSD).
 - SD DMC consistently recovers ~80% of the total correlation energy.
 - Fixed node error of ~20%.
 - Converged DMC-MSD almost reaches CBS at TZ.
 - Large basis set error for AFQMC.
 - Requires basis set extrapolation.
 - SD-AFQMC shows small phaseless error.
 - Short determinant expansion recovers remaining error.

AFQMC in QMCPACK: Design Principles

- Code still under active development.
 - Always check developers version for latest functionality.
 - Significant extensions every 3-6 months.
- Core AFQMC algorithms within QMCPACK.
 - Mean-field calculations performed by external codes.
 - PySCF, Molpro, GAMESS, VASP, etc.
 - Basic input: 1-2 electron integrals and trial wave functions.
 - AFQMC code doesn't "know" about atoms, positions, basis sets, pseudopotentials, etc.
- > Focus on efficient, large scale calculations.
 - Multiple levels of parallelization and data distribution.
 - Distribution of integrals over nodes.
 - xml input, hdf5 data, python analysis.

Sample Workflow

- Simple workflow. Relies on converter tools and postprocessing scripts.
 - Many still under development.
 - Contributions are welcome.
 - Properties require custom (simple) post-processing scripts. Examples are offered for charge and spin densities.
- Sample workflow:
 - 1. python scf.py
 - 2. python pyscf_to_afqmc.py [options]
 - 3. qmcpack afqmc.xml
 - 4. qmca [options] sample.scalar.dat
 - 5. python [options] –i sample.scalar.h5 analysis.py

Input Files - I

- Request AFQMC calculation with:
 - <simulation method="afqmc">
- Non-execute blocks must be "named" and are parsed first.
- "Execute" blocks are executed sequentially afterwards.
- ➤ Most decisions are made when pyscf_to_afqmc.py is executed.

```
<AFOMCInfo name="info0">
  <parameter name="NM0">2048</parameter>
  <parameter name="NAEA">256</parameter>
  <parameter name="NAEB">256</parameter>
</AFQMCInfo>
<Hamiltonian name="ham0" info="info0">
  <parameter name="filename">integrals.h5</parameter>
</Hamiltonian>
<Wavefunction name="wfn0" type="MSD" info="info0">
  <parameter name="filetype">ascii</parameter>
  <parameter name="filename">wfn.dat</parameter>
</Wavefunction>
<WalkerSet name="wset0">
  <parameter name="walker_type">closed</parameter>
</WalkerSet>
<Propagator name="prop0" info="info0">
</Propagator>
```

Input Files - II

- > Execute blocks typically require some minimal customization.
- Observables are defined locally in these blocks.
- Wavefunctions used in observables can be independent from the one used for propagation (phaseless constraint and importance sampling).
- > Triple loop structure:
 - blocks: Observables are calculated and data is written to disk. Checkpoint occurs if requested.
 - steps: Branching and load balancing at every step. Orthogonalization occurs with given frequency.
 - substeps: Only propagation.

Hamiltonians

Cholesky Factorization

$$(ik|jl) = M_{ik,jl} = \sum_{i} L_{ik}^n L_{lj}^{n*}.$$

Sparse Representation: (Only option for molecular problems)

- Storage of L_{ik}^n and $M_{ik,il}$ as a sparse matrices. Elements below a certain cutoff are set to zero.
- Automatically incorporates any symmetries present in the system.
- Relies on less efficient sparse linear algebra (compared to dense storage).
- Very large setup cost! Must generate M from L, O[N² M² n_{chol}]

K-Point Representation: (Only for periodic systems with k-points)

- Explicit use momentum conservation. Ideal for small unit cells with lots of k-points.
- Reduction of 1/N_k in memory and computation cost compared to direct algorithm.
- Based on highly efficient batched linear algebra operations.
- Scales poorly with size of the primitive cell. Not practical for large supercells.

Tensor Hyper-Contraction

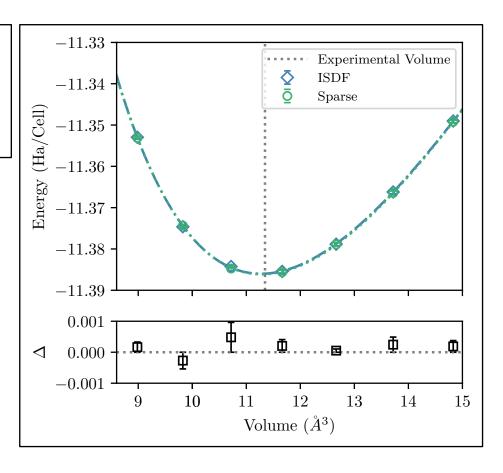
$$(ik|jl) = \sum \phi_{i,\mu}^* \phi_{k,\mu} M_{\mu,\nu} \phi_{j,\nu}^* \phi_{l,\nu}$$

- Factorize integral tensor as a contraction of 2nd-order tensors. Quadratic memory requirements. Cubic computational cost.
- Popularized in QC by Todd Martinez. Made practical in PBC by Lin Lin.
- Ideal for large supercells at the Gamma point or large molecular problems with no symmetry.
- Does not currently benefit from symmetries in the system.
- Large prefactor, specially for small system sizes. Currently problematic for heavier atoms.

Cholesky vs THC

	a_0 (Å)	B_0 (GPA)	$\Delta E \text{ (eV/atom)}$
HF	3.527	507	5.36
MP2	3.545	436	7.91
CCSD	3.539	463	7.04
AFQMC (Sparse)	3.561(2)	441	-
AFQMC (ISDF)	3.559(2)	442	6.95(19)
Experiment	3.553	455	7.55

- THC (ISDF) is only moderately slower even though it does not make use of any symmetries.
- Only practical choice for large unit cells.
 - Actual calculations are done in a supercell representation.
- Same accurate answer regardless of approach.



Hamiltonians

Systems without K-Point Symmetry

Representation	Memory		Computation	
	HS Potential	Energy	HS Potential	Energy
Sparse Cholesky	s x _c M ³	s' N ² M ²	s x _c M ³	s'N ² M ²
Dense Cholesky	x _c M³	x _c NM ²	x _c M ³	$x_c N^2 M^2$
THC	x_{μ}^2 M ²	$x_{\mu}^2~M^2$	x_{μ} M ²	$x_{\mu}^2 M^2$

Systems with K-Point Symmetry (N_k: # kpoints)

Representation	Memory		Computation	
	HS Potential	Energy	HS Potential	Energy
Sparse Cholesky	s x _c M ³	s' N ² M ²	s x _c M ³	s'N ² M ²
Dense Cholesky	$x_c N_k^2 m^3$	x _c N _k ² nm ²	$x_c N_k^2 m^3$	$x_c N_k^3 n^2 m^2$

• Always useful to use K-Point representation.

• Dense representation is generally more efficient.

s,s': Sparsity of corresponding structures

 x_{μ} : THC prefactor, typically 5-20.

x_c: Cholesky prefactor, typically 5-20.

Orthogonal vs Non-Orthogonal Determinants

$$|\Phi_T\rangle = \sum_i c_i |D_i\rangle$$

Orthogonal (PHMSD)

$$\langle D_i | D_j \rangle = \delta_{ij}$$

- Robust and well-controlled.
- Some form of CI typically involved in construction.
 - Truncated-CI, CASCI, selected CI, etc.
- Size consistency problems.
 - Possibly fatal for bulk.
- Fast evaluation algorithms:
 - Cost proportional to rank of the update
- Lower memory overhead for large expansions.

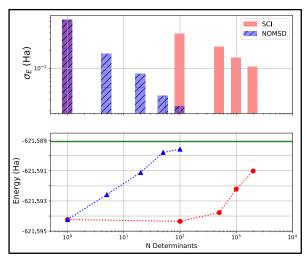
Non-Orthogonal (NOMSD)

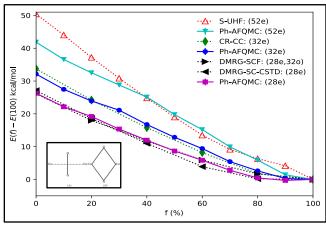
$$\langle D_i | D_j \rangle \neq \delta_{ij}$$

- Not common/routine in QC community.
- Several algorithms:
 - Iterative HF: 1-det at a time
 - Resonating HF: N-dets simult.
 - Complicated optimization problem.
- Size consistency problem less severe.
- Fastest convergence in MSD ansatz.
- Linear cost with #dets.
 - Both computation and memory.
 - Large memory overhead ultimately makes the approach impractical for large systems.
- Different wavefunctions can be combined in a single execution.
 - NOMSD for propagation, PHMSD for energy evaluation.

Systematic Improvable Wavefunctions

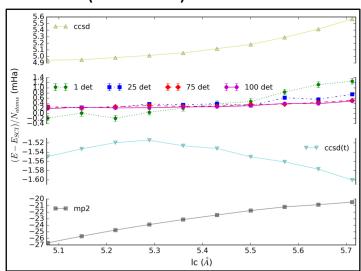
NOMSD vs PHMSD





Cu₂O₂ with NOMSD

Si (diamond) with NOMSD



- NOMSD offer great promise for compact and accurate trial wavefunctions.
- Mostly unexplored territory.
 - Great opportunity!

Observables

- Slowly expanding list of observables.
 - Currently: energy, static 1-RDM
 - 1-RDM can be used to calculate many properties: spin and charge densities, occupancies, momentum distribution, magnetic moments, etc.
 - In development: dynamic 1-RDM, 2-RDM, contractions of the 1-/2-RDM, forces.
- Back propagation (in principle) available for any observable.
 - Currently limited to 1-RDMs.

```
<Estimator name="back_propagation">
  <parameter name="nsteps">500</parameter>
  <parameter name="naverages">5</parameter>
  <parameter name="ortho">25</parameter>
  <parameter name="block_size">10</parameter>
  </Estimator>
```

Data Distribution

- Large data structures can be distributed over multiple memory spaces (nodes, GPUs, etc).
 - "nnodes" parameter in wavefunction and propagator.
 - Distributed algorithms are automatically chosen if nnodes > 1.
 - Enables calculations on large systems.
 - Some communication overhead. Typically modest.
 - Does not change the number of walkers, only static data layout.
- Walker sets can be shared over multiple cores in a node.
 - Use "ncores" parameter in the execution block.
 - Operations on the walker set are distributed over all cores in the group.
 - nWalkers defines the number of walkers per "working" group.
 - Improved time to solution.
 - Allows 1 walker per node, useful for large systems.

Warning: Use with caution!

GPUs

- AFQMC code is actively being ported to GPU.
 - Currently: Production level GPU port for K-Point dense Hamiltonian.
 - 15-40x speed-up for DP, 10-25x speed-up for SP compared to good CPU.
 - Larger performance gains for large systems.
 - Mixed precision build recommended for GPUs.
 - Memory is the more important resource!
 - No real build on GPUs yet (no molecules or gamma point).
 - Summit/Sierra enable AFQMC calculations on systems with 250 atoms.
 - Bcc Fe with 6x6x6 k-point grid.
 - C (diamond) with 5x5x5 k-point grid.
 - AFM-NiO with 3x3x3 k-point grid including back propagation and magnetic moment.

Upcoming Features



- Full GPU coverage (3-6 months)
- Finite-T algorithm (3 months)
- Spin-Orbit coupling and non-collinear magnetism (6 months)
- Forces (6-12 months)
- VASP interface (3 months)
- K-Point THC (6-12 months)





