1) $m(a+bx) = a + bm(x)$

$$m(a+bx) = \frac{1}{N}\sum_{i=1}^{N} a+bx$$

$$= \frac{1}{N}\sum_{i=1}^{N} a + \frac{1}{N}\sum_{i=1}^{N} bx_i$$

$$= \frac{Na}{N} + b \times \frac{1}{N}\sum_{i=1}^{N} x_i$$

$$= a + b\,m(x)$$

2) $cov(X, a+bY) = b \times cov(X,Y)$

$$cov(X, a+bY) = \frac{1}{N}\sum_{i=1}^{N} (x_i - m(x))((a+by_i) - m(a+by))$$

$m(a+bY)$
$= a + bm(Y)$

$$(a+by_i) - (a+bm(y)) = b(y_i - m(y))$$

$$= \frac{1}{N}\sum_{i=1}^{N} (x_i - m(x))(b(y_i - m(y))$$

$$= b \cdot \frac{1}{N}\sum_{i=1}^{N} (x_i - m(x))(y_i - m(y))$$

$$= b \cdot cov(X,Y)$$

3) $cov(a+bx, a+bx) = b^2 cov(x,x)$ $\quad cov(x,x) = s^2$

$$cov(a+bx, a+bx) = \frac{1}{N}\sum_{i=1}^{N} \left((a+bx_i) - (a+bm(x))\right)^2$$

$$= \frac{1}{N}\sum_{i=1}^{N} (b(x_i - m(x))^2$$

$$= b^2 \frac{1}{N}\sum_{i=1}^{N} (x_i - m(x))^2$$

$$= b^2\, cov(x,x)$$

$$s^2 = \frac{1}{N}\sum_{i=1}^{N} (x_i - m(x))^2$$

$$= \frac{1}{N}\sum_{i=1}^{N} (x_i - m(x))(x_i - m(x))$$

$$= cov(x,x)$$

4) if the function is non-decreasing then the median of the non-transformed function is the same as the transformed function.

For quantiles this is the same

$$Q_p(g(x)) = g(Q_p(x))$$

This should generally apply to the IQR due to the quantile being similar.

With the range this should be similar

5) $m(g(x)) \neq g(m(x))$

For example if given 2 values for X of $\{0,2\}$ $m(x) = 1$. If $g(x) = x^2$ which is non-decreasing for $0 \to \infty$

$$m(g(x)) = \frac{0^2 + 2^2}{2} = 2$$

$$g(m(x)) = 1^2 = 1$$

Therefore $m(g(x)) \neq g(m(x))$