S.No.	Physical quantity	Relationship with other physical quantities	Dimensions	Dimensional form
51	Bulk modulus or (compressibility)$^{-1}$	volume × (change in pressure) / (change in volume)	$[L^3][ML^{-1}T^{-2}] / [L^3]$	$[ML^{-1}T^{-2}]$
52	Centripetal acceleration	(velocity)^2 / radius	$[LT^{-1}]^2/[L]$	$[M^0LT^{-2}]$
53	Stefan constant	(energy/area × time) / (temperature)^4	$[ML^2T^{-2}] / [L^2][T][K]^4$	$[ML^0T^{-3}K^{-4}]$
54	Wien constant	wavelength × tempera.	$[L][K]$	$[M^0LT^0K]$
55	Boltzmann constant	energy / temperature	$[ML^2T^{-2}]/[K]$	$[ML^2T^{-2}K^{-1}]$
56	Universal gas constant	Pressure × volume / mole × temperature	$[ML^{-1}T^{-2}][L^3] / [mol][K]$	$[ML^2T^{-2}K^{-1}$ mol
57	Charge	current × time	$[A][T]$	$[M^0L^0TA]$
58	Current density	current / area	$[A]/[L^2]$	$[M^0L^{-2}T^0A]$
59	Voltage, electric potential, electromotive force	work / charge	$[ML^2T^{-2}]/[AT]$	$[ML^2T^{-3}A^{-1}]$
60	Resistance	Potential difference / current	$[ML^2T^{-3}A^{-1}] / [A]$	$[ML^2T^{-3}A^{-2}]$
61	Capacitance	charge / potential difference	$[AT] / [ML^2T^{-3}A^{-1}]$	$[M^{-1}L^{-2}T^4A^2]$
62	Electrical resistivity or (electrical conductivity)$^{-1}$	Resistance × area / length	$[ML^2T^{-3}A^{-2}][L^2] / [L]$	$[ML^3T^{-3}A^{-2}]$
63	Electric field	electrical force / charge	$[MLT^{-2}]/[AT]$	$[MLT^{-3}A^{-1}]$
64	Electric flux	electric field / area	$[MLT^{-3}A^{-1}][L^2]$	$[ML^3T^{-3}A^{-1}]$
65	Electric dipole moment	Torque / electric field	$[ML^2T^{-2}] / [MLT^{-3}A^{-1}]$	$[M^0LTA]$
66	Electric field strength or electric intensity	Potential difference / distance	$[ML^2T^{-3}A^{-1}] / [L]$	$[MLT^{-3}A^{-1}]$

S.No.	Physical quantity	Relationship with other physical quantities	Dimensions	Dimensional formula
67	Magnetic field, magnetic flux density, magnetic induction	$\frac{\text{force}}{\text{current} \times \text{length}}$	$[MLT^{-2}]/[A][L]$	$[ML^0T^{-2}A^{-1}]$
68	Magnetic flux	magnetic field × area	$[MT^{-2}A^{-2}][L^2]$	$[ML^2T^{-2}A^{-1}]$
69	Inductance	$\frac{\text{magnetic flux}}{\text{current}}$	$\frac{[ML^2T^{-2}A^{-1}]}{[A]}$	$[ML^2T^{-2}A^{-2}]$
70	Magnetic dipole moment	Torque/magnetic field or current × area	$[ML^2T^{-2}]/[MT^{-2}A^{-1}]$ or $[A][L^2]$	$[M^0L^2T^0A]$
71	Magnetic field strength, magnetic intensity or magnetic moment density	$\frac{\text{Magnetic moment}}{\text{volume}}$	$\frac{[L^2A]}{[L^3]}$	$[M^0L^{-1}T^0A]$
72	Permittivity constant (of free space)	$\frac{\text{charge} \times \text{charge}}{4\pi \times \text{electric force} \times (\text{distance})^2}$	$\frac{[AT][AT]}{[MLT^{-2}][L]^2}$	$[M^{-1}L^{-3}T^4A^2]$
73	Permeability constant (of free space)	$\frac{2\pi \times \text{force} \times \text{distance}}{\text{current} \times \text{current} \times \text{length}}$	$\frac{[M^0L^0T^0][MLT^{-2}][L]}{[A][A][L]}$	$[MLT^{-2}A^{-2}]$
74	Refractive index	$\frac{\text{Speed of light in vaccum}}{\text{Speed of light in medium}}$	$[LT^{-1}]/[LT^{-1}]$	$[M^0L^0T^0]$
75	Faraday constant	Avogadro constant × elementary charge	$[AT]/[mol]$	$[M^0L^0TA\,mol^{-1}]$
76	Wave number	2π/wavelength	$[M^0L^0T^0]/[T]$	$[ML^{-1}T^0]$
77	Radiant flux,	energy emitted/time	$[ML^2T^{-2}]/[T]$	$[ML^2T^{-3}]$
78	Radiant power			
	Luminosity of radiant flux or radiant intensity	$\frac{\text{Radiant power or radiant flus of source}}{\text{solid angle}}$	$[ML^2T^{-3}]/[M^0L^0T^0]$	$[ML^2T^{-3}]$

S.No	Physical quantity	Relationship with other physical quantity	Dimensions	Dimensional formula
79	Luminous power or luminous flux of source	$\frac{\text{Luminous energy emitted}}{\text{time}}$	$[ML^2T^{-2}]/[T]$	$[ML^2T^{-3}]$
80	Luminous intensity or illuminating power of source	$\frac{\text{Luminous flux}}{\text{Solid angle}}$	$\frac{[ML^2T^{-3}]}{[M^0L^0T^0]}$	$[ML^2T^{-3}]$
81.	Itensity of immin-ating or luminance	$\frac{\text{Luminous intensity}}{(\text{distange})^2}$	$[ML^2T^{-3}]/[L^2]$	$[ML^0T^{-3}]$
82	Relative luminosity	$\frac{\text{Luminous flux of a source of gave wavelength}}{\text{Luminous flux of peak sensitivity wavelength (555 nm)}}$	$\frac{[ML^2T^{-1}]}{[ML^2T^{-3}]}$	$[M^0L^0T^0]$
83	Luminous efficien-cy	$\frac{\text{Total luminous flux}}{\text{Total radiant flux}}$	$[ML^2T^{-3}]/[ML^2T^{-3}]$	$[M^0L^0T^0]$
84	Illuminance or illumination	$\frac{\text{Luminous flux incident}}{\text{are}}$	$[ML^2T^{-3}]/[L^2]$	$[ML^0T^0]$
85	Mass defect	(sum of masses of nucleons)-(mass of the nucleus)	$[M]$	$[ML^0T^0]$
86	Binding energy of nucleus	Mass defect × (speed of light in vacuum)^2	$[M][LT^{-1}]^2$	$[ML^2T^{-2}]$
87	Decay constant	0.693/half life	$[T^{-1}]$	$[M^0L^0T^{-1}]$
88	Resonant frequency	(Inductance × capacitance)$^{-\frac{1}{2}}$	$[ML^2T^{-2}A^{-2}]^{-\frac{1}{2}} \times [M^{-1}L^{-2}T^4A^2]^{-\frac{1}{2}}$	$[M^0L^0A^0T^{-1}]$
89	Quality factor or Q factor of coil	Resonant frequency × inductance / resistance	$\frac{[T^{-1}][ML^2T^{-2}A^{-2}]}{[ML^2T^{-3}A^{-2}]}$	$[M^0L^0T^0]$

S.No	Physical quantity	Relationship with other physical quantities	Dimensions	Dimensional formula
90	Power of lens	$(\text{Focal length})^{-1}$	$[L^{-1}]$	$[M^0L^{-1}T^0]$
91	Magnification	$\frac{\text{Image distance}}{\text{object distance}}$	$[L]/[L]$	$[M^0L^0T^0]$
92	Fluid flow rate	$\frac{(\pi/8)(\text{pressure})\times(\text{radius})^4}{(\text{viscosity coeff.})\times(\text{length})}$	$\frac{[ML^{-1}T^{-2}][L^4]}{[ML^{-1}T^{-1}][L]}$	$[M^0L^3T^{-1}]$
93	Capacitive reactance	(Angular frequency x capacitance)	$[T^{-1}]^{-1}[M^{-1}L^{-2}T^4A^2]^{-1}$	$[ML^2T^{-3}A^{-2}]$
94	Inductive reactance	(Angular frequency X inductance)	$[T^{-1}][ML^2T^{-2}A^{-2}]$	$[ML^2T^{-3}A^{-2}]$

→ To determine the unit of a physical quantity.

example : (i) The dimention formula of density is $[LT^{-1}]$

∴ unit of velocity is ms^{-1} or m/s

(ii) D.F. : force = $[MLT^{-2}]$

unit : force = $kgms^{-2}$

Significant figure

A number of digit used to accurately express a physical quantity is called the S.f.

Rules (i) The number of significant figure does not change on changing the positive of decimal in a number

exam. : for both 1.235 and 123.5

S.f. is 4

(II) All non zeroes are significant figure

exam. :

	S.f.
31795	5
2151	4
231973	8

(III) All zeroes on the right of the non zeroes digit are not significant.

exam. : 27800 – 3

S1750000 – 4

(IV) All zeroes accuring between non zeroes digit are significant.

exam. : 37052 – 5

127051 – 6

(V) Trailing zeroes to the right of the decimal place are significant.

exam. : 12.200 = 5

0.3210 = 4

(VI) If the number is expressed in terms of power of ten than 10 and its power is not included in significant figure.

exam: $3.12 \times 10^4 = 3$

$4.120 \times 10^6 = 4$

(VII) All zeroes on the left side of 1st non zero digit are not included in significant figure.

exam. : 0.007 = 1 , 0.02030 = 4

0.230 = 3

(viii) The number of significant figure in a quantity does not change with unit.

exam. : 0.0432 m = 4.32 cm = 43.2 mm = S.f. 3

		S.f.
(i) 0.001	=	1
(ii) 7.000×10^4	=	4
(iii) 13500	=	3
(iv) 1.3500	=	5

Rounding off

(i) 2.7389

R. 2.74 $(\because 8 > 5)$ (+1)

(ii) 2.7329

R. 2.73 $(\because 2 < 5)$

(iii) 2.7359

R. 2.74 ($\because$ 3 is odd number) (+1)

(iv) 2.7659

R. 2.76 ($\because$ 6 is even number)

Errors in Measurement

→ The difference in true value and measure value of physical quantity is called error of an measurement.

Types of Error

The error in measurement can be divided into following three types.

- Systamatic error
- Random error
- Gross error

Systamatic error

→ The error whose causes are known are called systamatic error.

types of systamatic error

(i) instrumental error – These error due to the folty ~~desgin~~ disign of the measuring introment

escam.: zero error.

(ii) ~~List~~ Least count error – The error in measurement due to the limit of resolution of the instrument ~~of it~~ is called least count error.

Example: The least count of an object matter scale is 0.1 cm is the length of object is more than 1.3 cm and less than 1.4 cm than the length cannot be measured accurately.

(III) Constant error - If on taking repeated observation the error remain the same than such an error is called constant error.

(IV) Personal error - This error arise due to the in experience of the observer.

Example: i) Lake of proper setting of the enstrument
ii) Reading an instrument without observing proper percantage.

(V) Error due to external causes - This error generally arise due to change in the external physical condition such as pressure, temperature, wind etc.

example: Increase the velocity of sound along the direction of wind, Increase in length due to increase in temperature etc.

(VI) Error due to imperfection - It arise on account of ignoring certain facts.

example: Radiations in the experiment of heat.

- Random error

Where the error is irregular and Random then It is caused Random error. No specific causes of this error is known as random error. To menimise this error the physical quantity is measured ~~ex~~ several times and finally the mean value is determined.

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- Gross error

The error due to carelessness of the observer is called the gross error.

example: Due to ignoring the source of error, noting the reading wrongly.

Method of Expressing an Error

Errors are represented in three base -
- absolute error
- relative error
- percentage error

1. Absolute error - The difference in the true value of a Physical quantity and the measured value by an observer is called Absolute Error.

Let,

$a_1, a_2, a_3, a_4 ---- a_n$

with the observer value of a physical quantity then the mean value.

$$a_{mean} = a_1 + a_2 + a_3 + ---- a_n$$

$$|\Delta a_1| = |a_{mean} - a_1|$$

$$|\Delta a_2| = |a_{mean} - a_2|$$

$$|\Delta a_3| = |a_{mean} - a_3|$$

$$|$$

$$|$$

$$|\Delta a_n| = |a_{mean} - a_n|$$

Now mean absolute error will be

$$\Delta a_{mean} = |\Delta a_1| + |\Delta a_2| + |\Delta a_3| + --- |\Delta a_n|$$

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Relative Error - The ratio of the mean absolute error to the mean value of a physical quantity is know as relative error.

$$\text{Relative error} = \frac{\Delta a_{mean}}{a_{mean}}$$

Percentage error - Relative error represents in percentage is called percentage error.

$$\text{Percentage error} = \text{Relative error} \times 100\%$$

$$= \frac{\Delta a_{mean}}{a_{mean}} \times 100\%$$

Q. In an experiment the length of object is measured as ~~0.5~~ 2.55 m, 2.52 m, 2.49 m, 2.56 m ~~2.97 m~~ 2.43 m, 2.58 m. find the mean length of the object absolute error in each observation and the percentage error in each ~~element~~ observation and the percentage error in the measurement.

→ $a_1 = 2.55\,m$ $\quad a_4 = 2.56\,m$

$a_2 = 2.52\,m$ $\quad a_5 = 2.48\,m$

$a_3 = 2.49\,m$ $\quad a_6 = 2.58\,m$

$$a_{mean} = a_1 + a_2 + a_3 + a_4 + a_5 + a_6$$
$$= 2.55 + 2.51 + 2.48 + 2.96 + 2.49 + 2.58$$
$$= 2.5235\,m = 2.53$$

$a_{mean} = 2.53$

Now absolute error of each observation

$|\Delta a_1| = |a_{mean} - a_1| = |2.53 - 2.55|$

$= |-0.2| = 0.2$

$|\Delta a_2| = |a_{mean} - a_2| = |2.53 - 2.51|$

$= |0.2| = 0.2$

$|\Delta a_3| = |a_{mean} - a_3| = |2.53 - 2.49|$

$= |0.04| = 0.04$

$|\Delta a_4| = |a_{mean} - a_4| = |2.53 - 2.56|$

$= |-0.03| = |0.03|$

$|\Delta a_5| = |a_{mean} - a_5| = |2.53 - 2.49|$

$= |0.05| = 0.05$

$|\Delta a_6| = |a_{mean} - a_6| = |2.53 - 2.58|$

$= |-0.05| = \cancel{|0.05|}\ 0.05$

∴ Mean absolute error

$$\Delta a_{mean} = \frac{|\Delta a_1| + |\Delta a_2| + |\Delta a_3| + |\Delta a_4| + |\Delta a_5| + |\Delta a_6|}{6}$$

$$= \frac{0.2 + 0.4 + 0.03 + 0.05 + 0.05}{6}$$

$$= \frac{0.21}{6} \approx 0.035\,m$$

∴ Percentage error

$$\frac{\Delta a_{mean}}{a_{mean}} \times 100\%$$

$$\pm \frac{0.035}{2.53} \times 100\%$$

$\pm\ 1.383\%$

$\pm\ 1.38\%$

Q In an experiment time period of simple pendulum is found to have values 2.63 s, 2.56 s, 2.42 s, 2.71 s, 2.80 s. Calculate mean time period, absolute error relative error relative error and percentage error.

→ Given,

$a_1 = 2.63\,s$ $a_4 = 2.71\,s$

$a_2 = 2.56\,s$ $a_5 = 2.80\,s$

$a_3 = 2.42\,s$

$$a_{mean} = \frac{a_1 + a_2 + a_3 + a_4 + a_5}{5}$$

$$= \frac{2.63 + 2.56 + 2.42 + 2.71 + 2.80}{5}$$

$$= \frac{13.12}{5} = 2.62$$

∴ Mean Absolute error of each observation

$|\Delta a_1| = |a_{mean} - a_1| = |2.62 - 2.63|$
$= |-0.01| = 0.01$

$|\Delta a_2| = |a_{mean} - a_2| = |2.62 - 2.56|$
$= |0.06| = 0.06$

$|\Delta a_3| = |a_{mean} - a_3| = |2.62 - 2.42|$
$= |0.20| = 0.20$

$|\Delta a_4| = |a_{mean} - a_4| = |2.62 - 2.71|$
$= |0.09| = 0.09$

$|\Delta a_5| = |a_{mean} - a_5| = |2.62 - 2.80|$
$= |-0.18| = 0.18$

∴ Mean absolute error

$$\frac{|\Delta a_1| + |\Delta a_2| + |\Delta a_3| + |\Delta a_4| + |\Delta a_5|}{5}$$

$$= \frac{0.01 + 0.06 + 0.20 + 0.09 + 0.18}{5}$$

$$= \frac{0.54}{5} = 0.108$$

$$\frac{\Delta a_{mean}}{a_{mean}} = \frac{0.108}{2.62} = 0.042 \times 100 = 4.1\,\%$$

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CHAPTER - 3

VECTOR

Scalar : Those physical quantities which possess the magnetic only but have no direction is called scalar quantities

e.g. - mass, time, temperature, volume, speed etc.

Vector : Those physical quantities which possess both magnitude and direction are called vector quantities.

e.g. - velocity, acceleration, force, torque etc.

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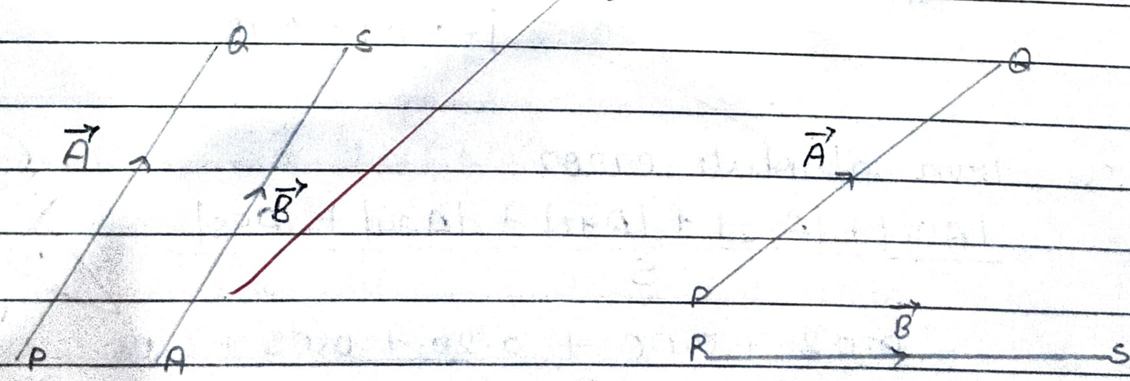
TYPES OF VECTOR

1) **Equal vector** - Two vectors having same magnitude and acting along the same direction are called equal vector

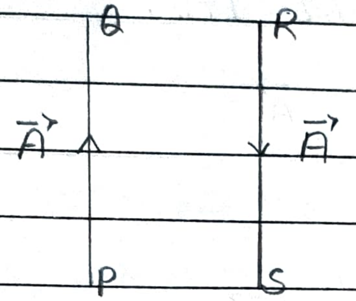


$\overrightarrow{PQ} = \overrightarrow{RS}$ $\qquad\qquad$ $\overrightarrow{PQ} \neq \overrightarrow{RS}$

$\vec{A} = \vec{B}$ $\qquad\qquad$ $\vec{A} \neq \vec{B}$

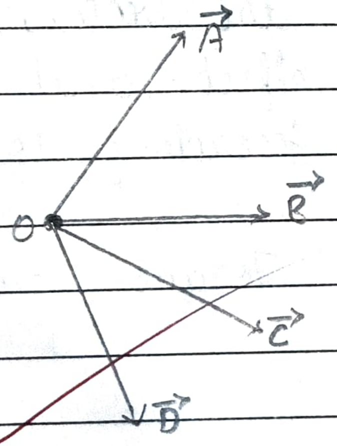
2. Negative vector (or opposite vector)
Two vectors are said to be negative vector if their magnitudes are equal but they act in opposite directions.

e.g.



$$\vec{PQ} = -\vec{RS} \;/\; \vec{PQ} \neq \vec{RS}$$
$$\vec{A} = -\vec{A} \;/\; \vec{A} \neq \vec{A}$$

3. Co-initial - The vectors having same initial points are called co-initial vector.



4. Co-planner vectors - The vectors which act in the same plane are called co-~~linear~~ planner vector.

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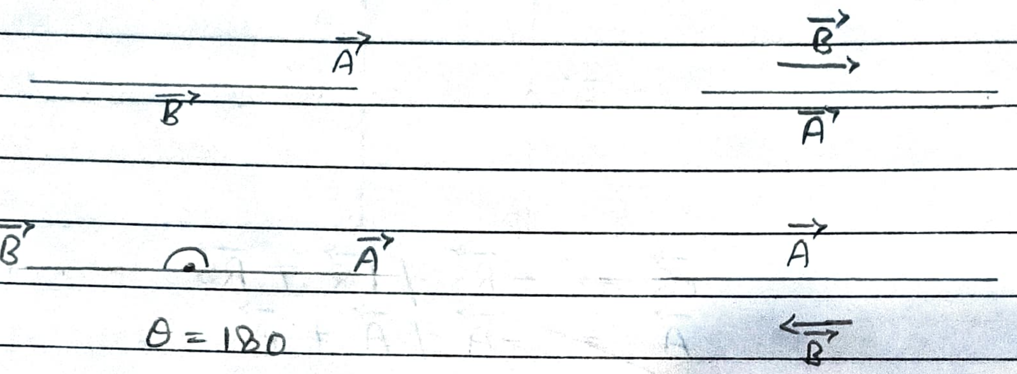
5. Co-linear vectors - vectors acting along a straight line or along parallel lines are called co-linear vectors.

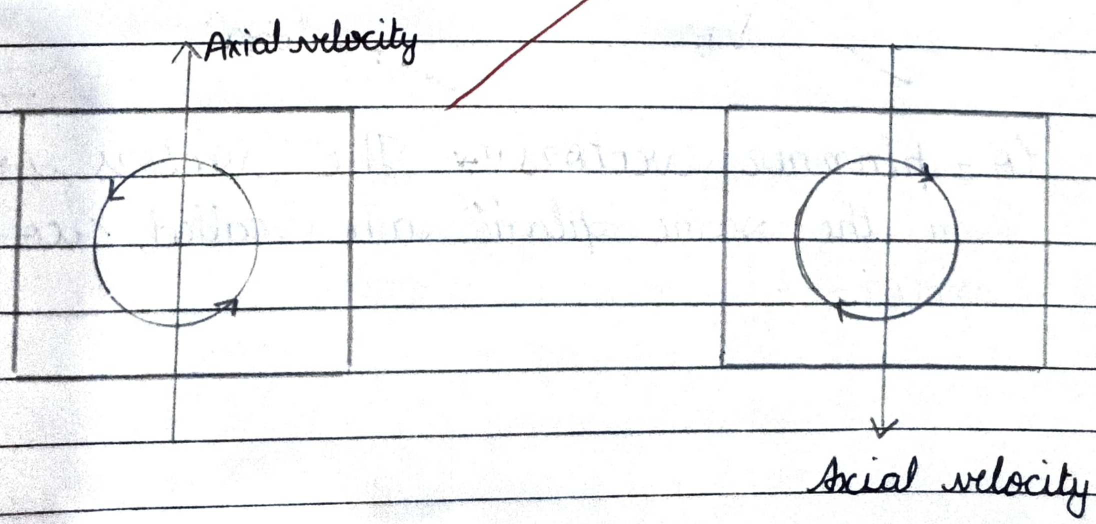
The angle between those vectors is always zero degree or 180°.



6. Polar vector - These are vectors which have a starting point or point of application.

7. Axial vector - These are the vectors which represent rotational effect and all along the axis of rotation According to right hand screw rule.

e.g. - Angular velocity, Torque, angular momentum etc.



8. Free vector - Is there whose initial point are not fixed is called free vector.

9. Position vector - That vector is called position vector which gives the position of particles with respect to the origin at given time.

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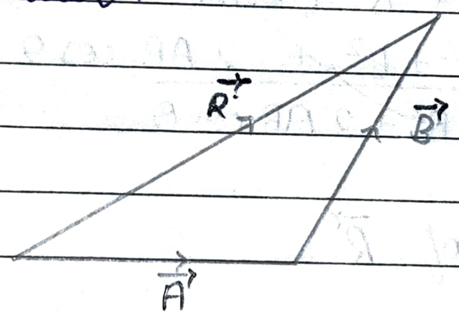
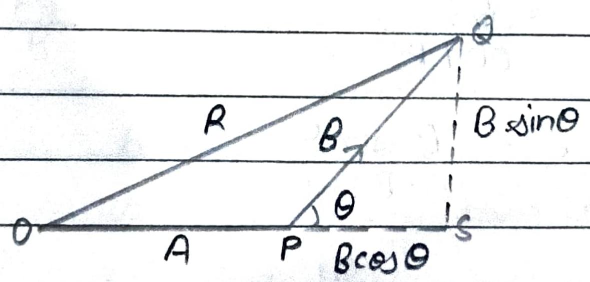
TRIANGLE LAW OF VECTOR OF ADDITION

If two vectors are represented both magnitude and direction by the two adjacent sides of a triangle in one order, then the resultant is represented both in magnitude & direction by the third side of the triangle are taken in opposite order.

$\vec{R} = \vec{A} + \vec{B}$



MAGNITUDE OF $\vec{R}$:-



In Δ QPS

$$\frac{QS}{PQ} = \sin\theta$$

$$\frac{QS}{B} = \sin\theta$$

$QS = B \sin\theta$

Again in $\triangle QPS$

$$\frac{PS}{PQ} = \cos\theta$$

$$\frac{PS}{B} = \cos\theta$$

$$PS = B\cos\theta$$

Now In $\triangle QOS$ by PGT

$$(OQ)^2 = (QS)^2 + (OS)^2$$

$$(OQ)^2 = (QS)^2 + (OP + PS)^2$$

$$R^2 = (B\sin\theta)^2 + (A + B\cos\theta)^2$$

$$R^2 = B^2\sin^2\theta + A^2 + B^2\cos^2\theta + 2AB\cos\theta$$

$$R^2 = A^2 + B^2(\sin^2\theta + \cos^2\theta) + 2AB\cos\theta$$

$$R^2 = A^2 + B^2 + 2AB\cos\theta$$

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

Direction of $\vec{R}$

In $\triangle QOS$

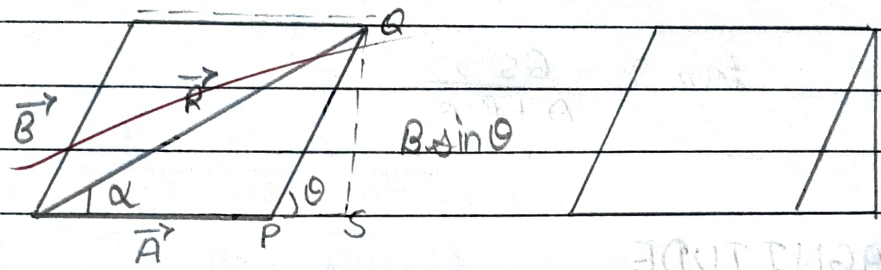
$$\tan\alpha = \frac{QS}{OS}$$

$$= \frac{QS}{OP + PS}$$

$$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$$

PARALLELOGRAM LAW OF VECTOR ADDITION

→ If two vectors are represented both in magnitude and direction by two adjacent sides of parallelo-gram. Then, their resultant vector represented by the diagonal of the parallelogram passing through their common points.



$$\vec{R} = \vec{A} + \vec{B}$$

In parallelogram ∠QPS

$$\frac{QS}{PQ} = \sin\theta$$

$$\frac{QS}{B} = B\sin\theta$$

$$QS = B\sin\theta$$

Again △QPS

$$\frac{PS}{PQ} = \cos\theta$$

$$\frac{PS}{B} = \cos\theta$$

$$PS = B\cos\theta$$

Now in parallelogram ∠QOS by PGT

$$(OQ)^2 = (QS)^2 + (OS)^2$$

$$(OQ)^2 = (B\sin\theta)^2 + (OP + OS)^2$$

$$R^2 = B^2\sin^2\theta + OP^2 + OS^2$$

$$R^2 = B^2\sin^2\theta + A^2 + B^2\cos^2\theta + 2AB\cos\theta$$

$$R^2 = A^2 + B^2 + (\sin^2\theta + \cos^2\theta) + 2AB\cos\theta$$

$$R^2 = A^2 + B^2 + 2AB\cos\theta$$

Direction of $\vec{R}$

~~In A~~ In $\angle QOS$

$$\tan\alpha = \frac{QS}{OS}$$

$$\tan\alpha = \frac{QS}{OP + PS}$$

$$\tan\alpha = \frac{QS}{A + B\cos}$$

MAGNITUDE

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

Direction

$$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$$

Case I - If both vectors are in same direction

$\vec{B}$ $\vec{A}$

i.e. $= \theta = 0$

$$\therefore \quad R = \sqrt{A^2 + B^2 + 2AB\cos 0}$$

$$R = \sqrt{A^2 + B^2 + 2AB \times 1}$$

$$R = \sqrt{(A+B)^2}$$

$$R = A + B$$

and

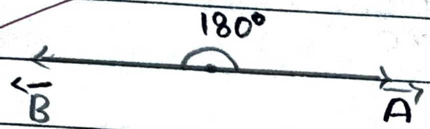
$$\tan\alpha = \frac{B\sin 0}{A + B\cos 0}$$

$$= \frac{B \times 0}{A + B \times 1}$$

$$\tan \alpha = 0$$

$$\alpha = 0$$

Case II – If both vectors are in opposite direction



i.e. $\theta = 180$

$$\therefore R = \sqrt{A^2 + B^2 + 2AB \cos 180}$$

$$R = \sqrt{A^2 + B^2 + 2AB(-1)}$$

$$= \sqrt{A^2 + B^2 - 2AB}$$

$$= \sqrt{(A-B)^2}$$

$$R = A - B$$

and

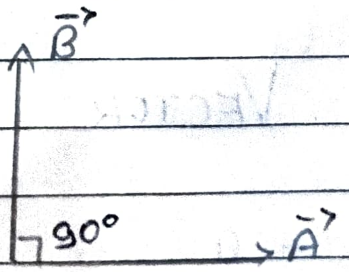
$$\tan \alpha = \frac{B \sin 180}{A + B \cos 180}$$

$$= \frac{B \times 0}{A + B(-1)}$$

$$\tan \alpha = 0$$

$$\alpha = 0$$

Case III – If both vectors are perpendicular to each other.



$$\theta = 90°$$

$$\therefore R = \sqrt{A^2 + B^2 + 2AB\cos 90}$$
$$R = \sqrt{A^2 + B^2 + 2AB + 0}$$
$$R = \sqrt{A^2 + B^2}$$
and
$$\tan\alpha = \frac{B\sin 90}{A + B\cos 90}$$
$$= \frac{B \times 1}{A + B \times 0}$$
$$\tan\alpha = \frac{B}{A}$$

- $\vec{A} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$
- $\vec{B} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

1. ADDITION OF VECTOR

$$\vec{A} + \vec{B} = (a_1\hat{i} + a_2\hat{j} + a_3\hat{k}) + (b_1\hat{i} + b_2\hat{j} + b_3\hat{k})$$
$$= (a_1 + b_1)\hat{i} + (a_2 + b_2)\hat{j} + (a_3 + b_3)\hat{k}$$

2. SUBTRACTION OF VECTOR

$$\vec{A} - \vec{B} = (a_1\hat{i} + a_2\hat{j} + a_3\hat{k}) - (b_1\hat{i} + b_2\hat{j} + b_3\hat{k})$$
$$= (a_1 - b_1)\hat{i} + (a_2 - b_2)\hat{j} + (a_3 - b_3)\hat{k}$$

3. MAGNITUDE OF VECTOR

$$|\vec{A}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

$$|\vec{B}| = \sqrt{b_1^2 + b_2^2 + b_3^2}$$

4. UNIT OF VECTOR

$$\hat{A} = \frac{\vec{A}}{|A|} = \frac{a_1\hat{i} + a_2\hat{j} + a_3\hat{k}}{\sqrt{a_1^2 + a_2^2 + a_3^2}}$$

$$\hat{B} = \frac{\vec{B}}{|B|} = \frac{b_1\hat{i} + b_2\hat{j} + b_3\hat{k}}{\sqrt{b_1^2 + b_2^2 + b_3^2}}$$

5. MULTIPLY BY CONSTANT

$$3\vec{A} = 3(a_1\hat{i} + a_2\hat{j} + a_3\hat{k})$$
$$= 3a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

6. PRODUCT OF VECTOR

- Dot product / Scalar product

•	$\hat{i}$	$\hat{j}$	$\hat{k}$
$\hat{i}$	1	0	0
$\hat{j}$	0	1	0
$\hat{k}$	0	0	1

$$\vec{A} \cdot \vec{B} = (a_1\hat{i} + a_2\hat{j} + a_3\hat{k}) \cdot (b_1\hat{i} + b_2\hat{j} + b_3\hat{k})$$
$$= (a_1 \times b_1)(\hat{i}\cdot\hat{i}) + (a_2 \times b_2)(\hat{j}\cdot\hat{j}) + (a_3 \times b_3)(\hat{k}\cdot\hat{k})$$
$$= (a_1b_1)1 + (a_2b_2)1 + (a_3b_3)1$$
$$= a_1b_1 + a_2b_2 + a_3b_3$$

- Cross product / vector product

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$$(a_2 b_3 - b_2 a_3)\hat{i} - (a_1 b_3 - b_1 a_3)\hat{j} + (a_1 b_2 - b_1 a_2)\hat{k}$$

7. ANGLE BETWEEN TWO VECTOR

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|A||B|}$$

$$\sin\theta = \frac{|\vec{A}\times\vec{B}|}{|A||B|}$$

If two vectors are perpendicular to each other.

i.e. $\theta = 90$

$$\frac{\vec{A}\cdot\vec{B}}{|A||B|} = \cos 90$$

$$\frac{\vec{A}\cdot\vec{B}}{|A||B|} = 0$$

$$\vec{A}\cdot\vec{B} = 0$$

Q. If $\vec{A} = 2\hat{i} + \hat{j} - 5\hat{k}$

$\vec{B} = 3\hat{i} + 4\hat{j} + 7\hat{k}$

$\vec{C} = \hat{i} + \hat{j} + \hat{k}$

Find

1) $2\vec{A} + 3\vec{B}$
2) $\vec{C} - 5\vec{A} + \vec{B}$
3) $|A|$ and $|B|$ and $|C|$
4) Unit vector of $\vec{B}$
5) $\vec{A} \cdot \vec{C}$ and $\vec{B} \cdot \vec{A}$
6) $\vec{B} \times \vec{C}$ and $\vec{A} \times \vec{B}$
7) $\cos\theta$ between $\vec{A}$ and $\vec{B}$
8) $\sin\theta$ between $\vec{C}$ and $\vec{A}$

Solution :

1. $2\vec{A} + 3\vec{A}$

$2(2\hat{i} + \hat{j} - 5\hat{k}) + 3(3\hat{i} + 4\hat{j} + 7\hat{k})$

$4\hat{i} + 2\hat{j} - 10\hat{k} + 9\hat{i} + 12\hat{j} + 21\hat{k}$

$13\hat{i} + 140\hat{j} + 11\hat{k}$

2. $\vec{C} - 5\vec{A} + \vec{B}$

$(\hat{i} + \hat{j} + \hat{k}) - 5(2\hat{i} + \hat{j} - 5\hat{k}) + (3\hat{i} + 4\hat{j} + 7\hat{k})$

$\hat{i} + \hat{j} + \hat{k} - 10\hat{i} - 5\hat{j} + 25\hat{k} + 3\hat{i} + 4\hat{j} + 7\hat{k}$

$-6\hat{i} + 0\hat{j} + 33\hat{k}$

$-6\hat{i} + 33\hat{k}$

3. $|A|$ and $|B|$ and $|C|$

$|A| = \sqrt{2^2 + 1^2 + (-5)^2}$

$= \sqrt{4 + 1 + 25}$

$= \sqrt{30}$

$|B| = \sqrt{3^2 + 4^2 + 7^2}$

$= \sqrt{9 + 16 + 49} = \sqrt{74}$

$|C| = \sqrt{1^2 + 1^2 + 1^2}$

$= \sqrt{1 + 1 + 1}$

$= \sqrt{3}$

4. Unit vector of $\vec{B}$

$$\hat{B} = \frac{\hat{B}}{|B|}$$

$$= \frac{3\hat{i} + 4\hat{j} + 7\hat{k}}{\sqrt{74}}$$

5. $\vec{A} \cdot \vec{C}$ and $\vec{B} \cdot \vec{A}$ and $\vec{B} \cdot \vec{C}$

$\vec{A} \cdot \vec{C} = (2\hat{i} + \hat{j} - 5\hat{k}) \cdot (\hat{i} + \hat{j} + \hat{k})$

$(2 \times 1) + (1 \times 1) + (-5 \times 1)$

$2 + 1 - 5$

-2

$\vec{B} \cdot \vec{A} = (3\hat{i} + 4\hat{j} + 7\hat{k}) \cdot (2\hat{i} + \hat{j} - 5\hat{k})$

$(3 \times 2) + (4 \times 1) + (7 \times (-5))$

$6 + 4 + (-35)$

$10 + (-35)$

$10 - 35$

-25

$\vec{B} \cdot \vec{C} = (3\hat{i} + 4\hat{j} + 7\hat{k}) \cdot (\hat{i} + \hat{j} + \hat{k})$

$(3 \times 1) + (4 \times 1) + (7 \times 1)$

$3 + 4 \times 7$

14

6. $\vec{B} \times \vec{C}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 7 \\ 1 & 1 & 1 \end{vmatrix}$$

$(4 - 7)\,\hat{i} - (3 - 7)\,\hat{j} + (3 - 4)\,\hat{k}$

$(-3)\,\hat{i}\ 4\hat{j} - 1\hat{k}$

$-3\hat{i} + 4\hat{j} - 1\hat{k}$

$\vec{A} \times \vec{B}$

$\hat{i}$	$\hat{j}$	$\hat{k}$
2	1	-5
2	4	7

$(7-(-20))\hat{i} - ((-15)-14)\hat{j} + (8-3)\hat{k}$

$-27\hat{i} - (-1)\hat{j} + 8\hat{k}$

$-27 + 1\hat{j} + 8\hat{k}$

7. $\cos\theta$ between $\vec{A}$ and $\vec{B}$

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|A||B|}$$

$\vec{A}\cdot\vec{B} = (2\hat{i} + \hat{j} - 5\hat{k})\cdot(3\hat{i} + 4\hat{j} + 7\hat{k})$

$6 + 4 - 35$

-25

We have $= |A| = \sqrt{30}$

$|B| = \sqrt{74}$

2	30
2	15
5	5
	1

2	74
37	37
	1

$$\cos\theta = \frac{-25}{\sqrt{30}\sqrt{74}}$$

$$= \frac{-25}{\sqrt{2\times3\times5\times2\times3\times7}}$$

$$\cos\theta = \frac{-25}{2\sqrt{3\times5\times37}}$$

$$\cos\theta = \frac{-2}{2\sqrt{555}}$$

8. $\sin\theta$ between $\vec{C}$ and $\vec{A}$

$$\sin\theta = \frac{|\vec{C}\times\vec{A}|}{|C||A|}$$

$\hat{i}$	$\hat{j}$	$\hat{k}$
2	1	-5
1	1	1

$$\vec{C} \times \vec{A} = (1-(-5))\hat{i} - ((-5)-2)\hat{j} + (2-1)\hat{k}$$

$$-6\hat{i} + 7\hat{j} - \hat{k}$$

$$|\vec{C} \times \vec{A}| = \sqrt{(-6)^2 + (7)^2 + (-1)^2}$$

$$= \sqrt{36 + 49 + 1}$$

$$\sqrt{86}$$

~~sin θ =~~

we know $|C| = \sqrt{3}$

$|A| = \sqrt{30}$

Now, $\sin\theta = \dfrac{\sqrt{86}}{\sqrt{3}\sqrt{30}}$

$$= \sqrt{\frac{86}{3 \times 30}}$$

$$= \sqrt{\frac{86}{3 \times 30}}$$

$$= \sqrt{\frac{86}{90}}$$

$$\sin\theta = \sqrt{\frac{43}{45}}$$

Q. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$. Than prove that $\vec{a}$ and $\vec{a}$ mutually perpendicular.

Squaring both side

$$|\vec{a} + \vec{b}|^2 = |\vec{a} - \vec{b}|^2$$

$$\cancel{a^2} + b^2 + 2\vec{a}\cdot\vec{b} = \cancel{a^2} + \cancel{b^2} - 2\vec{a}\cdot\vec{b}$$

$$2\vec{a}\cdot\vec{b} + 2\vec{a}\cdot\vec{b} = 0$$

$$4\vec{a}\cdot\vec{b} = 0$$

$$\vec{a}\cdot\vec{b} = \frac{0}{4}$$

$$\vec{a}\cdot\vec{b} = 0$$

Q. What will be the relation between $\vec{a}$ and $\vec{b}$ if $[\vec{a}+\vec{b}]$ and $[\vec{a}-\vec{b}]$ mutually perpendicular.

$(\vec{a}+\vec{b})$ and $(\vec{a}\cdot\vec{b})$

According to question

$$(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b})=0$$

$$a^2-b^2=0$$

$$a^2=b^2$$

$$a=b$$

Q If $\vec{a}+\vec{b}=\vec{c}$ and $a^2+b^2=c^2$ then prove that $\vec{a}$ & $\vec{b}$ are perpendicular

→ Given $\vec{a}+\vec{b}=\vec{c}$ —①

$a^2+b^2=c^2$ —②

Proof from eq ①

$$\vec{a}+\vec{b}=\vec{c}$$

$$(\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b})=\vec{c}\cdot(\vec{a}+\vec{b})$$

$$a^2+\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{b}+b^2=\vec{c}\cdot\vec{c}$$

$$(a^2+b^2)+2\vec{a}\cdot\vec{b}=c^2$$

from eq ⑪

$$\cancel{c^2}+2\vec{a}\cdot\vec{b}=\cancel{c^2}$$

$$2\vec{a}\cdot\vec{b}=0$$

$$\vec{a}\cdot\vec{b}=0$$

$$\therefore \vec{a}\perp\vec{b}$$

Akhaply 18/07/2023

CHAPTER - 3

MOTION IN STRAIGHT LINE

Mechanics - The branch of Physics which deals with the study of forces and motion of the materials objects and their mutual relationship is called mechanics.

Mechanics can be broadly classified into two branches:

(i) Statics - It is branch of mechanics which deals with the study of objects at rest.

(ii) Dynamics - It is that branch of machines which deal with the study of motion of object taking into consideration the factors which cause motion.

* FRAME OF REFERENCE - The frame with respect to which the rest or motion of an object is studied is called frame of reference.

(i) ~~Interval~~ Inertial frame of reference - It is the frame of reference which is either at rest or in the state of uniform motion.

(ii) Non-inirtial frame of reference - It is that frame of reference to an inertial frame of reference.

OBJECT IN REST AND IN MOTION

A body is said to be at rest if it does not changes its possition with respect to other stationary object or frame with time.
a body is said to be in motion when it changes its position with respect to other stationary objects or frame with time

POINT OBJECT - If the distance travelled by a moving object is much larger than its size, then the object is considered as a point object.

example :

(i) The distance travelled by cricket ball on the ground is very large as compared to this its size. Therefore, the cricket ball may be considered as a point object.

(ii) The earth revolves around the sun in elliptical orbit. The size of earth is negligible as compared to the distance travelled by it. Therefore, in this elliptical motion the earth may be treated as a point object.

Actually point object is a mathematical concept to simplify the problems.

MOTION IN ONE, TWO AND THREE DIMENSION

One dimension motion : When body is confined to move in a straight motion line, it is said to excute motion in one-dimension.

example : Train running on railway track car moving on a straight road rock.

Two dimension motion : When a body perform motion in a place and its side way motion is not ignored, then its motion is said to be two-dimensional motion.

example : An insect crawling on the ground, motion of billiard ball, sailing of boat in a river, circular motion etc.

Three-dimensional motion : When a body performs motion in space it is said to execute three dimensional motion.

example : Aeroplane, flying bird or kite, motion of molecules of gas etc.

DISTANCE : The total length of path travelled by a particls / body in the given interval of time is called distance.

Unit : cm and m are the CGS and SI unit of distance.

Dimensional formula : Its dimensional formula is $[M^0LT^0]$.

Distance is a scalar quantity.

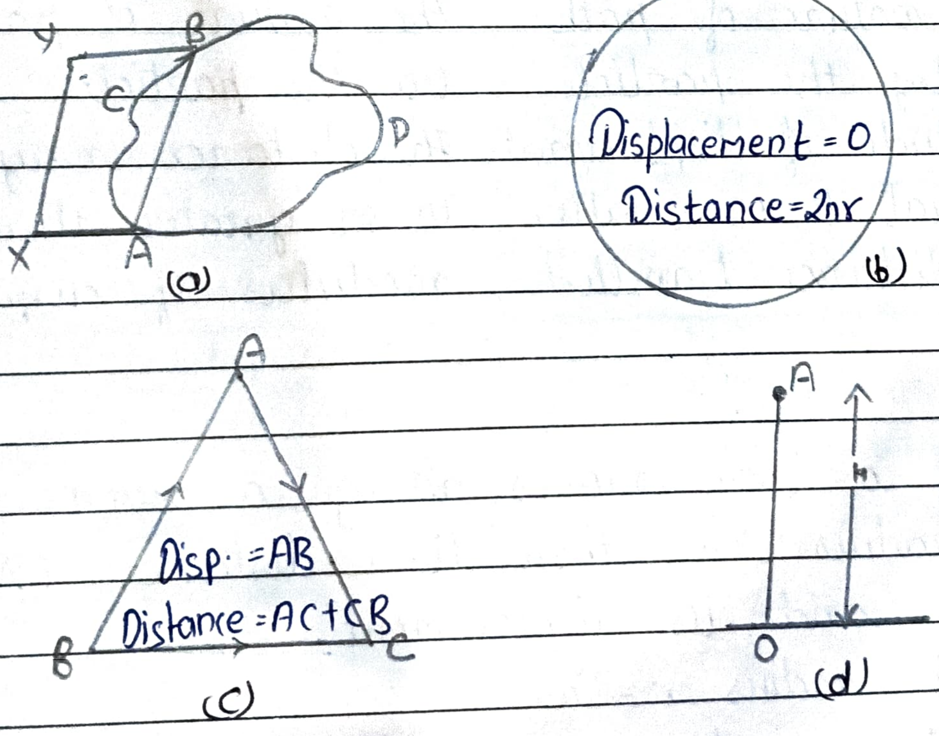
DISPLACEMENT : The change of position of a moving body in a particular direction is called displacement.

Or

The distance travelled by a body in a particular direction is called displacement.

Unit : Basically displacement in length. Therefore, its CGS and SI units are respectivily cm and m

Dimensional formula : Its dimensional formula is $[M^0LT^0]$. Displacement is a vector quantity



Representation of distance and Displacement

No.	Displacement	Distance
1.	It is the different between the position co-ordinates of the particles in the given time interval.	It is the length of the actual path followed by the particle in the given time interval
2.	It is vector quantity.	It is a scalar quantity.
3.	The displacement may be positive, negative or even zero.	The distance is always positive.
4.	The displacement is independent of the nature of path followed by the particle.	The distance depends upon the nature of path followed by the particle.
5.	The magnitude of displacement may be equal to or smaller than the distance travelled.	The distance may be equal to or greater than the modulus of displacement.

Example : If a body moves along a semi-circular path of radius 7m, then find distance travelled by the body and its displacement.

Sol. Given, radius $r = 7m$.

Distance = Length of semi-circular path

$= \frac{1}{2} \times$ circumference of circle

$= \frac{1}{2} \times 2\pi r = \pi r$

$= \frac{22}{7} \times 7 = 22m.$

Displacement = Diameter of circle

$= 2r = 2 \times 7 = 14$ cm.

SPEED

It is defined as the distance travelled by the object per unit time.

Or

The time rate of change of position of a body is known as its velocity.

i.e. $\text{speed} = \dfrac{\text{Distance travelled}}{\text{Time taken}}$

If an object travels distance s in time t, then its speed

$$V = \frac{S}{t}$$

Unit : Using $v = \frac{S}{t}$, the unit of $v = \dfrac{\text{unit of } s}{\text{unit of } t}$

$\therefore$ Unit of v in CGS system $= \dfrac{cm}{sec.}$

It is written as cm per sec, cm/s or cms^{-1}

Similarly, the unit of v in SI system is written as metre per sec, m/s or ms^{-1}.

Dimensional formula : Dimensional formula of

$$V = \frac{\text{dimensional formula of } s}{\text{dimensional formula of } t}$$

$$= \frac{[L]}{[T]} = [LT^{-1}] = [M^{\circ}LT^{-1}].$$

Speed is a scalar quantity.

Example - if a body travels a distance of 60m in sec, than what will be its speed?

Sol. Given, $s = 60$ m and $t = 45$

$$\because V = \frac{S}{t}$$

$\therefore\ v = \frac{60}{4} = 15\ m/s$

Velocity - the distance travelled per unit by a morning body in a particular direction is knows as its velocity.

if x be the displacement of body in H time t, then velocity $v = \frac{H}{t}$

unit = unit of velocity = $\frac{\text{unit of displacement}}{\text{unit of time}}$

$\therefore$ unit of velocity in CGS system

$= \frac{cm}{s} = cms^{-1}$

and SI system $= \frac{m}{s}\ ms^{-2}$

if a body travelled a distance 50m in ~~5 sec~~, a particular direction in 5 sec, then its velocity $= \frac{50}{5} = 10\ ms^{-1}$ velocity is an vector quantity

Dimentional formula →

velocity = $\frac{\text{displacement}}{\text{time}}$

$\therefore$ Dimensional formula of velocity

$= \frac{[L]}{[T]} = [LT^{-1}] = [M^0LT^{-4}]$

FORMULA

1. $\frac{d}{dx} \sin x = \cos x$

2. $\frac{d}{dx} \cos x = \sin x$

3. $\frac{d}{dx} \tan x = \sec^2 x$

4. $\frac{d}{dx} \sec x = \sec x \tan x$

5. $\frac{d}{dx} \cot x = \operatorname{cosec}^2 x$

6. $\frac{d}{dx} \operatorname{cosec} x = \cot x \operatorname{cosec} x$

7. $\frac{d}{dx} \log x = \frac{1}{x}$

8. $\frac{d}{dx} e^x = e^x$

① x^7

let $y = x^7$

$$\frac{dy}{dx} = \frac{dx^7}{dx}$$

$$= 7x^{7-1}$$

$$= 7x^6$$

(2) $x^3 + 2x^2 + 7x - 9$

Let $y = x^3 + 2x^2 + 7x - 9$

$$\frac{dy}{dx} = \frac{d}{dx}(x^3 + 2x^2 + 7x - 9)$$

$$= \frac{d}{dx}x^3 - \frac{d}{dx}2x^2 + \frac{d}{dx}7x - \frac{d9}{dx}$$

$$= \frac{d}{dx}x^3 + 2\frac{dx^2}{dx} + \frac{7dx}{dx} - \frac{d9}{dx}$$

$$= 3x^2 + 2 \times 2x + 7 \times 1 = 0$$

$$= 3x^2 + 4x + 7$$

(3) $5x^4 + 6x^{3/2} + 9x$

Let $y = 5x^4 + 6x^{3/2} + 9x$

$$\frac{dy}{dx} = \frac{d}{dx}(5x^4 + 6x^{3/2} + 9x)$$

$$= \frac{d}{dx}5x^4 - \frac{d}{dx}6x^{3/2} + \frac{9d}{dx}x$$

$$\frac{5d}{dx}x^4 + 6\frac{dx^{3/2}}{dx} + 9 - \frac{d}{dx}x$$

$$5 \times 3 + 6 \times 3 \quad x^{1/2} + 9 \times 1$$

$$20^{x3} + 9 \times ^1{2} + 9$$

(4) $y = \sin x = \cos x$

$$\frac{dy}{dx} = \frac{d}{dx}(\sin x - \cos x)$$

$$\frac{d}{dx}\sin x - \frac{d}{dx}\cos x$$

$$\cos x - (-\sin x)$$

$$\cos x + \sin x$$

(5) $\sqrt{x} + \frac{1}{\sqrt{x}}$

Let $y = \sqrt{x} = \frac{1}{\sqrt{x}}$

$x\frac{1}{2} + \frac{1}{x^{1/2}}$

$x^{\frac{1}{2}} + x^{-\frac{1}{2}}$

$\frac{dy}{dx} = \frac{d}{dx} x^{1/2} + \frac{d}{dx} x^{-\frac{1}{2}}$

$= -\frac{1}{2} x^{\frac{1}{2}-1} + \left(\frac{-1}{2}\right) x^{-1/2-1}$

$= \frac{1}{2} x^{-\frac{1}{2}} - \frac{1}{2} x^{-\frac{3}{2}}$

(6) $y = e^{x} + 5 \log x + \tan x$

$\frac{dy}{dx} = \frac{d}{dx} (e^{x} + 5\log + \tan x)$

$\frac{d}{dx} e^{x} + \frac{d}{dx} 5 \log \times \frac{d}{dx} \tan x$

$\frac{de^{x}}{dx} + \frac{5d}{dx} \text{clig} \times 1 \frac{d}{dx} \tan x$

$e^{x} + 5 \times \frac{1}{x} + \text{Se}^{2} x$

Differential by product.

formula - $\frac{d}{dx} I.II = I\left(\frac{d}{dx} II\right) + II\left(\frac{d}{dx} I\right)$

Sol. $\frac{d}{dx} x^3 \sin x = x^3 \left(\frac{d}{dx}\sin x\right) + \sin x \left(\frac{dx^3}{dx}\right)$

$= x^3 (\cos x) + \sin x \, 3x^2$

$y = e^x \sin + \log x \tan x$

$\frac{dy}{dx} = \left(\frac{d}{dx} e^x \sin x\right) + \left(\frac{d}{dx} \underset{I}{\log x} \underset{II}{\tan x}\right)$

$\left(e^x \frac{d}{dx} \sin x + \sin x \frac{d}{dx} e^x\right) + \left(\log x \frac{d}{dx} \tan x + \frac{d x \frac{d}{}\log x}{dx}\right)$

$= (e^x \cos x + \sin e^x) + (\log x \sec^2 + \tan x + \frac{1}{x})$

By divide Rule

formula - $\frac{d}{dx}\frac{I}{II} = \frac{II \frac{d}{dx} I - I \frac{d}{dx} II}{II^2}$

exa. $\frac{d}{dx} \cdot \frac{\sin x}{x^3} = \frac{x^3 \left(\frac{d}{dx}\sin x\right) - \sin x \left(\frac{d}{dx} x^3\right)}{(x^3)^2}$

$= \frac{x^3 \cos x - \sin x \, 3x^2}{x^6}$

Ques 1 $\quad y = \frac{2\log x}{e^x}$

Sol. $\frac{d}{dx}\frac{I}{II} = \frac{II \frac{d}{dx} I - I \frac{d}{dx} II}{II^2}$

$\frac{d}{dx} = \frac{2\log x}{e^x} = \frac{e^x \left(\frac{d}{dx} 2\log x\right) - \log x \left(\frac{d}{dx} e^x\right)}{e^x}$

CHAIN RULE

① $y = \log \sin x$

$$\frac{d}{dx} y = \frac{d}{dx} \log \sin x$$

$$\frac{d}{dx} \log x = \frac{1}{x} = \frac{1}{\sin x} \frac{d}{dx} \sin x$$

$$= \frac{1}{\sin x} \times \cos x$$

$$= \cot x$$

② $y = \sin x^2$

$$\frac{dy}{dx} = \frac{d}{dx} \sin x^2$$

$$\cos x^2 \frac{dx^2}{dx}$$

$$\cos x^2 . 2x$$

③ $y = \log \log \tan x$

$$\frac{dy}{dx} = \frac{d}{dx} \log (\log \tan x)$$

$$\frac{1}{\log \tan x} \frac{d}{dx} \log (\tan x)$$

$$\frac{1}{\log \tan x} \times \frac{1}{\tan x} \times \frac{d}{dx} \tan x$$

$$\frac{1}{\log \tan x} \times \frac{1}{\tan x} \times \sec^2 x$$

INTEGRATION :-

$$\int x^n\,dx = \frac{x^{n+1}}{n+1}$$

ex. $$\int x^2\,dx = \frac{x^2+1}{2+1}$$

$$= \frac{x^3}{3}$$

ex. $$\int 5x^{\frac{1}{2}}\,dx = 5\int x^{\frac{1}{2}}\,dx$$

$$= 5\left(\frac{x^{\frac{1}{2}}+1}{\frac{1}{2}+1}\right)$$

$$= 5\,\frac{x^{\frac{3}{2}}}{\frac{3}{2}}$$

$$= 5 \times \frac{2}{3} \times x^{\frac{3}{2}}$$

$$= \frac{10}{3}\,x^{\frac{3}{2}}$$

★ Formula :-> $$\int x^n\,dx = \frac{x^{n+1}}{n+1}$$

ex. $$\int dx = x$$

ex. $$\int dt = t$$

ex. $$\int 2\cdot dx = 2\int dx = 2x$$

(3) $I = \int (x^2+3)\,dx$

$I = \int x^2 dx + \int 3dx$

$I = \frac{x^4}{4} + 3x$

(1) $\int x^2\,dx = \frac{x^{n+1}}{n+1}$

(2) $\int dx = x$

(3) $\int \sin x\,dx = -\cos x$

(4) $\int \cos x\,dx = \sin x$

(5) $\int \sec^2 dx = \tan x$

(6) $\int \sec x \tan dx = \sec x$

(7) $\int \frac{1}{x} dx = \log x$

(8) $\int e^x dx = e^x$

ex. $\int \left(\sin x + 2e^x + \frac{1}{x}\right) dx$

$= \int \sin x\,dx + \int 2e^x dx + \int \frac{1}{x} dx$

$= -\cos x + 2e^x + \log x$

ex. $\frac{x \cos x + 5x^3 + 1}{x}$

$$= \frac{x\cos x}{x} + \frac{5x^3}{x} + \frac{1}{x}$$

$$= \cos x + 5x^2 + \frac{1}{x}$$

$$= \sin x + \frac{5x^3}{3} + \frac{1}{\log x}$$

① $$\int_b^a x^x\,dx = \left(\frac{x^{n+1}}{n+1}\right)_a^b$$

$$= \left(\frac{b^{n+1}}{n+1} - \frac{a^{n+1}}{n+1}\right)$$

ex. $$\int_1^2 x^2\,dx = \left[\frac{x^3}{3}\right]^2$$

$$= \left[\frac{2^3}{3} - \frac{1^3}{3}\right]$$

$$= \left[\frac{8}{3} - \frac{1}{3}\right] = \frac{8-1}{3}$$

$$= \frac{7}{3}$$

- Uniform Motion → when an object travels equal distance in equal intervals of time, Then its motion is said to be uniform motion.

- UNIFORM SPEED → An object is said to be moving with uniform speed, if it covers equal distance in equal intervals of time, however, small These intervals may be.

- UNIFORM VELOCITY → An object is said to be moving with uniform velocity if it covers equal displacements in equal intervals of time, however small These intervals may be.

- NON-UNIFORM MOTION & VARIBLE MOTION → When an object covers equal distance in equal intervals of time, however small These intervals may be.

- AVERAGE SPEED → AVERAGE SPEED is defined as The ratio of The total distance travelled by The object is The total time taken. Instantaneous Speed. Therefore, The instantaneous Speed of The object is The first differential coeffient of distance travelled

- Variable Motion in Straight line :
  Acceleration Motion.
  When an object excutes motion along a straight line with variable velocity Then its motion is said to be ~~velocity~~ variable motion in straight line.

- Accelration →
  The time rate of change of velocity of an object is known as its accelration.

$$\text{Acceleration} = \frac{\text{change in velocity}}{\text{Time in interval}}$$

- If v_1 and v_2 The velocity of an object at time t_1 and t_2 respectivally Then the acceleration.

$$a = \frac{v_2 - v_1}{t_2 - t_1}$$

- Unit → Unit of Acceleration

$$\frac{\text{unit of change in velocity}}{\text{unit of time}}$$

- In CGS System

$$\text{unit of acceleration} = \frac{cm/s}{s} = cm/s^2$$

- In SI System
  Unit of Acceleration $= \frac{m/s}{s} = m/s^2$

DIMENSIONAL FORMULA

$$\frac{\text{Dimensional formula of change in velocity}}{\text{Dimensional formula of time}}$$

$$\frac{[LT^{-1}]}{[T]} = [LT^{-2}]$$

$$[M^0LT^{-2}]$$

NATURE → Accelaration is a vector quantity

Types of Acceleration.

(1) Uniform Acceleration

The Acceleration of an object is said to be uniform accelration if its velocity changes by equal Amount in equal intervals of time have so even small The time interval may be

EQUATION OF MATTER

If u = Intial velocity
v = final velocity
a = Acceleration
t = Time
s = displacement

- $v = u + at$
- $s = ut + \frac{1}{2}at^2$
- $v^2 = u^2 + 2as$

① $v = u + at$

Acceleration = $\frac{\text{change in velocity}}{\text{time}}$

Acceleration → final velocity − Intial velocity

$$a = \frac{v-u}{t}$$

$$at = v - u$$

$$\boxed{u + at = v}$$

② $s = ut + \frac{1}{2}at^2$

displacement = Average velocity × time

$s = v_{av} \times t$ —— ①

Now,

Average velocity = $\frac{\text{Initial velocity} + \text{final velocity}}{2}$

$\therefore V_{av} = \frac{u+v}{2}$

For 1^{st} eq. of Motion

$v = u + at$

$V_{av} = \frac{u + u + at}{2}$

$V_{av} = \frac{2u + at}{2}$

$\frac{\cancel{2}u}{\cancel{2}} + \frac{at}{2}$

$V_{av} = u + \frac{1}{2}at$

put in eq. 1

$S = (u + \frac{1}{2}at) \times t$

$$\boxed{S = ut + \frac{1}{2}at^2}$$

③ $v^2 = u^2 + 2as$

From I^{st} eq. of Motion

$$v = u + at$$
$$v - u = at$$
$$\frac{v-u}{a} = t \quad \text{—— ①}$$

Now, from 2^{nd} eq. of Motion

$$s = ut + \frac{1}{2} at^2$$

from eq. ①

$$s = u \times \left(\frac{v-u}{a}\right) + \frac{1}{2} a \left(\frac{v-u}{a^2}\right)^2$$

$$\frac{uv - u^2}{a} + \frac{v^2 + u^2 - 2uv}{2a}$$

$$\frac{2(uv - u^2) + (v^2 + u^2 - 2uv)}{2a}$$

$$s = \frac{\cancel{2uv} - 2u^2 + v^2 + u^2 - \cancel{2ux}}{2a}$$

$$s = \frac{2uv - 2u^2 + v^2 + u^2 - 2ux}{2a}$$

$$2as = v^2 - u^2$$

$$\boxed{u^2 + 2as = v^2}$$

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EQUATION OF MOTION BY CALCULAS METHOD

$$\frac{dv}{dt} = a$$

$$dv = a\,dt$$

as integration with limit when $t_1 = 0$ Then $v_1 = u$ and when $t_2 = t$ Then $v^2 = v$

$$\int_u^v dv = \int_0^t a\,dt$$

$$\int_u^v dv = a \int_0^t dt$$

$$[v]_u^v = a\,[t]_0^t$$

$$[v-u] = a[t-0]$$

$$v - u = at$$

$$\boxed{v = u + at}$$

(2) $S = ut + \frac{1}{2}at^2$

$$\frac{ds}{dt} = v$$

$$ds = v\,dt$$

By first eq. of motion

$$v = u + at$$

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$$ds = (v + at)\, dt$$

on intergrating with The limit

when $t_1 = 0$ Then $s_1 = 0$ and when $t_2 = t$ Then $s_2 = s$

$$\int_0^s ds = \int_0^t (v + at)\, dt$$

$$\int_0^s ds = \int_0^t u\, dt + \int_0^t at\, dt$$

$$\int_0^s ds = a \int_0^t dt + a \int_0^t t\, dt$$

$$[s]_0^s = v[t]_0^t + a\left[\frac{t^2}{2}\right]_0^t$$

$$[s - 0] = v[t - 0] + a\left[\frac{t^2}{2} - \frac{0^2}{2}\right]$$

$$s = vt + \frac{at^2}{2}$$

$$\boxed{s = vt + \frac{1}{2}at^2}$$

(3) $v^2 = u^2 + 2as$

$$\frac{dv}{dt} = a$$

$$\frac{ds}{dt} \quad \frac{dv}{ds} = a$$

$$v \quad \frac{dv}{ds} = a \qquad \left\{ \frac{ds}{dt} = v \right\}$$

$vdv = ads$ on integrating with in limit

∴ when $v_1 = u$ $s_1 = 0$ and when $v_2 = v$

Then $s_2 = s$

$$\int_v^u v\,dv = \int_0^s a\,ds$$

$$\int_u^v v\,du = a \int_0^s ds$$

$$\left[\frac{v^2}{2}\right]_u^v = a\,[s]_0^s$$

$$\left[\frac{v^2}{v} - \frac{v^2}{2}\right] = a\,[s-0]$$

$$\frac{v^2 - u^2}{2} - as$$

$$v^2 = u^2 = 2as$$

$$\boxed{v^2 = u^2 + 2as}$$

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DISPLACEMENT IN n^{th} SECOND :

We have,
displacement in second $(t = n)$

$\therefore \; S_n = un + \frac{1}{2} an^2 \rightarrow (1)$

and $\longrightarrow (x-1)$ second $(t = n-1)$

$S_{n-1} = u(n-1) + \frac{1}{2} a (n-2)^2$

$= un - u + \frac{1}{2} a [n^2 + 1 - 2n]$

$= un - u + \frac{1}{2} an^2 + \frac{1}{2} a - \frac{1}{2} a - \frac{1}{\cancel{2}} a \times \cancel{2} n$

$S_{n-1} = un - u + \frac{1}{2} an^2 + \frac{1}{2} a - an \; - (II)$

Now displacement n^{th} sec

$S_{n^{th}} = S_n - S_{n-1}$

for eq (I) eq. (II)

$S_{n^{th}} = (un + \frac{1}{2} an^2) - (un - u + \frac{1}{2} an^2 + \frac{1}{2} a - an)$

$= un + \frac{1}{2} an^2 - un + u - \frac{1}{2} an^2 - \frac{1}{2} a + an$

$S_{n^{th}} = u - \frac{1}{2} a + an$

$S_{n^{th}} = u - \frac{1}{2} a [1 - 2n]$

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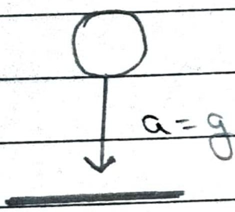
EQUATION OF MOTION UNDER GRAVITY:-

(CASE I)

When body moves downwards

ie $a = g$



$\therefore$ ① $v = u + gt$

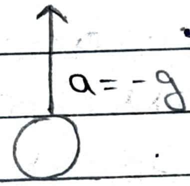
② $s = ut + \frac{1}{2}gt^2$

③ $v^2 = u^2 + 2gs$

(CASE II)

When body move upward

ie $a = -g$



$\therefore$ ① $v = u + (-g)t$

$v = u - gt$

② $s = ut - \frac{1}{2}gt^2$

③ $v^2 = u^2 + 2(-g)s$

$v^2 = u^2 - 2gs$

CHAPTER ⇒

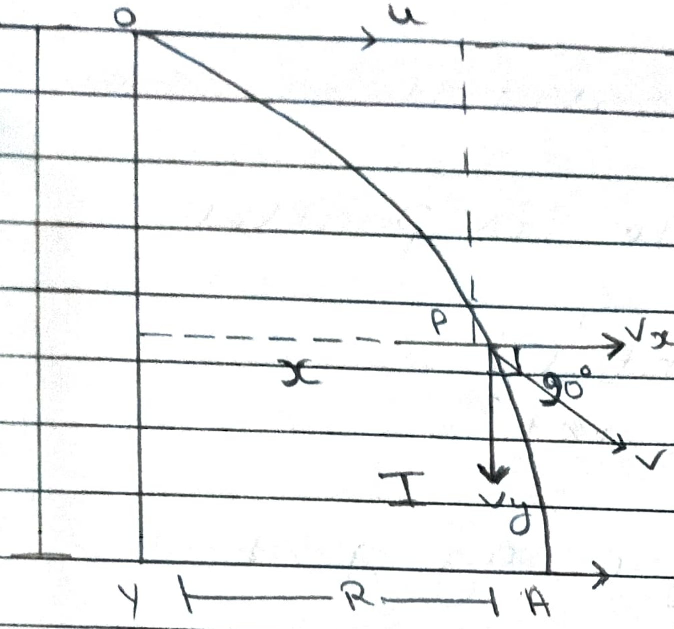
MOTION IN A PLAN

PROJECTILE MOTION ⇒ When a body Through with an initial velocity in The direction other Than vertical Then The body travelled in a curve path in a vertical plane under The excelaration due to gravity and fall on The ground at some other point This type of motion is called projectile Motion.

Ex.→ Motion of bulate fired by The gun, bom droped by an aeroplane etc.

Type of projectile Motion →
1. Horizontal projectile
2. Angular projectile/→{ oblique projectile

1. Horizonal Projectile → let a projectile be projected horizontaly with velocity small u from a point O at height h above The earth surface let x and y be The distance travelled by The projectile respectively alon horizontal and vertical direction at time t.



(Solution)

(I) Path of projectile

(i) Horizontal displacement of P (x)

$S_x = u_x t + \frac{1}{2} a t_x^2$

We have $S_x = x$, $u_x = u$, $a_x = 0$

$\therefore x = u \times t + \frac{1}{2} \times 0 \times t^2$

$x = ut$

$$t = \frac{x}{4} \quad \text{— (1) eq.}$$

(ii) Vertical disp at P (y)

$S_y = u_y t + \frac{1}{2} a_y t^2$

we have, $S_y = y$; $u_y = 0$, $a_y = g$

$y = 0 \times t + \frac{1}{2} g t^2$

$$y = \frac{1}{2} g t^2$$

From eq (1)

$$y = \frac{1}{2} g \left(\frac{x}{4}\right)^2$$

$$y = \frac{1}{2} g \left(\frac{x}{4}\right)^2$$

$$y = \frac{1}{2} g \frac{x^2}{4^2}$$

$$\boxed{y \propto x^2}$$

were $\frac{1}{2}\frac{g}{u^2}$ = constant

path of projectile is prallel

② Time of flight (T)

$S_y = u_y t + \frac{1}{2} a \times t^2$

By $S_y = h$, $u_y = 0$, $a_y = g$, $t = T$

$\therefore h = 0 \times T + \frac{1}{2} gT^2$

$h = \frac{1}{2} gt^2$

$\frac{2h}{g} = T^2$

$$\boxed{T = \sqrt{\frac{2h}{g}}}$$

③ Horizontal Range (R)

R = Horizontal velocity x time of flight

$R = U_x \times T$

we have $U_x = U$

$S.\ T = \sqrt{\frac{2h}{g}}$

$$\therefore \boxed{R = U\sqrt{\frac{2h}{g}}}$$

④ velocity at any time

(i) Horizontal velocity (v_x)

$$V_x = u_x + a_x t$$

We have, $u_x = u + a_x = 0$

$$\therefore v_x = u + a_x t$$

$$\boxed{V = x = u}$$

(ii) Vertical velocity (v_y)

$$u_y = u_y + a_y t$$

We have

$$u_y = 0, \quad a_y = g$$

$$\therefore v_y = 0 + gt$$

$$\boxed{V_y = gt}$$

Resultant velocity

$$V = \sqrt{v_x^2 + v_x^2 + 2V_x V_x \cos 90^\circ}$$

$$V = \sqrt{v_x^2 + v_y^2} \qquad (\because \cos 90 = 0^\circ)$$

$$\boxed{V = \sqrt{u^2 + g^2 t^2}}$$

Direction

$$\tan\theta = \frac{v^2}{v_x}$$

$$= \left[\tan\theta = \frac{gt}{u}\right]$$

$$= \left[\theta = \tan^{-1}\frac{gt}{u}\right]$$

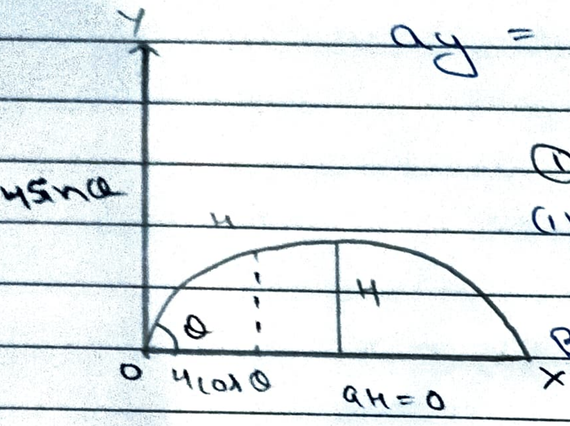
Angular projectile & oblique projectile

$u_x = u\cos\theta$

$u_y = u\sin\theta$

$a_x = 0$

$a_y = -g$



① path of projectile/eq. of trajecta

(i) Horizontal displacement (x)

$S_x = u_x t + \frac{1}{2} a_x t^2$

But $= S_x = x,\ u_x = u\cos\theta$

$\therefore x = u\cos\theta t + \frac{1}{2} \times 0 \times t^2$

$x = u\cos t$

$x/\ u\cos t$

$x/u\cos\theta = t$ —— ①

(ii) vertical displacement (y)

$S_y = u_y t + \frac{1}{2} a_y t^2$

But $S_y = y,\ u_y = u\sin\theta,\ a_y = -g$

$\therefore y = u\sin\theta t + \frac{1}{2}(-g) t^2$

from eq ①

$y = u\sin\theta \times \frac{x}{u\cos\theta} - \frac{1}{2} g \left(\frac{x}{u\cos\theta}\right)^2$

$y = x\tan\theta - \frac{1}{2}\frac{gx^2}{u^2\cos\theta}$

$\boxed{y = Ax + Bx}$ —— (ii)

where $A = \tan\theta$

$B = \frac{1}{2}\cdot\frac{g}{u^2\cos^2\theta}$

(11) The eq^n of parabula
∵ The path of projectile B parabula
let a particle is projected with a velocity small is making an angle θ with The horizontal from a point O on the ground a That θ angle of projection.

(1) Time of flight (T)

$$S_y = u_y t + \frac{1}{2} a_y t^2$$

But $S_y = 0$, $u_y = u\sin\theta$
$a_y = -g$, $t = T$

$$0 = u\sin\theta\, T + \frac{1}{2}(-g)T^2$$

$$0 = u\sin\theta \cdot T^2 - \frac{1}{2} \cdot gT^{-2}$$

$$\frac{1}{2} gT = u\sin\theta$$

$$T = \frac{2u\sin\theta}{g}$$

(2) Horizontal Ray (R)
R = Horizontal velocity × time of Height

$$R = u_x \times T$$

But $u_x = u\cos\theta$

$$T = \frac{2u\sin\theta}{g}$$

$$\therefore R = u\cos\theta \times \frac{2u\sin\theta}{g}$$

$$R = u^2\left(\frac{2\sin\theta\cos\theta}{g}\right)$$

$\because P\,2\sin\theta\ \cos\theta = \sin 2\theta$

$\because \left(R = \dfrac{u^2 \sin 2\theta}{g} \right)$

(iv) MAXIMUM RANGE

$\sin 2\theta = 1 \,(\text{Max})$

$\sin 2\theta = \sin 90^\circ$

$2\theta = 90^\circ$

$\theta = \dfrac{\cancel{90}}{\cancel{2}}$

$\theta = 45^\circ.$

$\because R_{mx} = \dfrac{u^2 \sin(2 \times 45)}{g}$

$R_{mx} = \dfrac{u^2 \sin 90}{g}$

$R_{max} = \dfrac{u^2 \times 1}{g}$

$= \left[R_{max} = \dfrac{u^2}{g} \right]$

UNIFORM CIRCULAR MOTION

- When a particle moves on a circular path on the circumference of the circle it motion is called circular motion.

example - Motion of stone tide type at the one end of a string

The motion of the tips of the hands of the clock.

Terms used in circular motion

1) Time period

- The time taken by the particle moving along a circular path. To complete one revolution is called time period. It is denoted by capital T.

It SI unit is second.

Dimension formula [D]

2) Frequency

- The number of revolution complete per unit time by particle moving on a circular path is known as its frequency.

It is denoted by ν or f or n.

It SI unict is Hertz (H_z) or CPS (cycle per seco.)

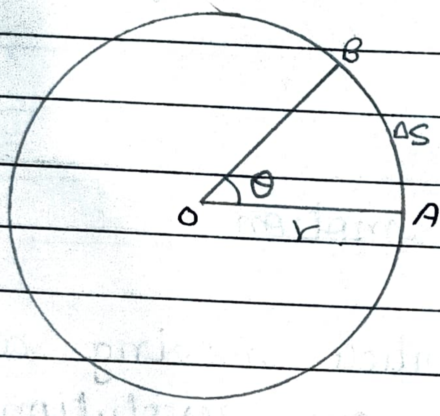
Demension formula is $[T^{-1}]$

3) Relation between time period and frequency

- $T = \frac{1}{\nu}$, $\nu = \frac{1}{T}$, $\nu T = 1$

4) Angular displacement

- The angle subtended by the particle on a circular path at the centre of the circle is called angular displacement. (Director can be determined by right hand rule)



Angular displacement $\theta = \frac{\Delta S}{r}$

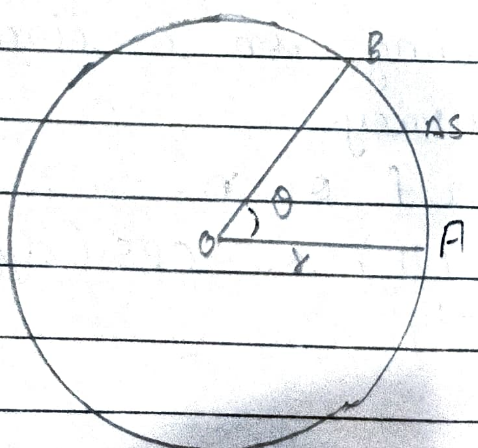
Unit : Radian

D.F. : $[M^0 L^0 T^0]$

Nature : Vector

5) Angular velocity

- The rate of change of angular displacement with time is called angular velocity.
It is denoted by ω (omegha)



Angular velocity = $\frac{\text{angular displace.}}{\text{time internal}}$

$$\omega = \frac{\Delta \theta}{\Delta t}$$

Unit : Radian / sec or radian s^{-1}

D.F. : $[M^0 L^0 T^{-1}]$

Nature : Vector

6) Relation between Time period and velocity.

A particle performing circular motion will angular velocity ω time period T and frequency ν.

$$\omega = \frac{\theta}{t}$$

But in one revoluton

$$\theta = 2\pi$$

and time period

$$t = T$$

$$\therefore \omega = \frac{2\pi}{T}$$

~~Byt~~ But $\frac{1}{T} = \nu$

$$\therefore \omega = 2\pi\nu$$

$$\boxed{\omega = \frac{2\pi}{T} = 2\pi\nu}$$