Assignment - 6

#1

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$X_1, X_2, X_3 \dots X_n \Rightarrow$ sample of size n

$$L(X_1, X_2, X_3 \dots X_n) = f(x_1) \cdot f(x_2) \dots f(x_n)$$

$$\Rightarrow \left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_1-\mu)^2}{2\sigma^2}} \right) \cdot \left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_2-\mu)^2}{2\sigma^2}} \right) \dots$$

taking ln on both sides

$$\ln(L) = -\frac{n}{2} \ln(2\pi\sigma^2) + \sum_{i=1}^{n} \left(\frac{(x_i-\mu)^2}{2\sigma^2} \right) \quad \text{— (1)}$$

take partial derivative w.r.t μ of the above eq^n

$$\frac{\partial \ln(L)}{\partial \mu} = 0 + \sum_{i=1}^{n} -\left(\frac{2(x_i - \mu)}{2\sigma^2}\right) = 0$$

$$= \sum_{i=1}^{n} (x_i - \mu) = 0$$

$$n\bar{X} - n\mu = 0$$

$$\bar{X} = \mu$$

hence $\theta_1 = \bar{X}$ is therefore sample mean

Taking derivative w.r.to σ^2 (of eq ①)

$$\frac{\partial \ln(L)}{\partial \sigma^2} = \frac{-n}{2\sigma^2} + \sum_{i=1}^{n} \frac{-(x_i - \mu)^2}{2(\sigma^2)^2} = 0$$

$$\Rightarrow -n + \sum_{i=1}^{n} -\frac{(x_i - \mu)^2}{\sigma^2} = 0$$

$$n = \sum_{i=1}^{n} \frac{(x_i - \mu)^2}{\sigma^2}$$

$$\sigma^2 = \frac{1}{n}\left(\sum_{i=1}^{n} (x_i - \mu)^2\right)$$

hence $\theta_2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \mu)^2$

#2

Binomial distribution $\rightarrow {}^nC_{x_i}\,\theta^{x_i}(1-\theta)^{n-x_i}$

$$L = \prod_{i=1}^{n} {}^nC_{x_i}\,\theta^{x_i}(1-\theta)^{n-x_i}$$

log on both sides

$$\log L = \sum_{i=1}^{n}\left(\log({}^nC_{x_i}) + \log\theta^{x_i} + \log(1-\theta)^{n-x_i}\right)$$

$$\log L = \sum_{i=1}^{n}\log({}^nC_{x_i}) + \log\theta\sum_{i=1}^{n}x_i + \log(1-\theta)\sum_{i=1}^{n}(n-x_i)$$

differentiate wrt θ

$$\frac{d\log(L)}{d\theta} = 0$$

$$\frac{1}{\theta}\sum x_i - \frac{1}{1-\theta}\sum(n-x_i) = 0$$

$$\frac{1}{\theta}\sum x_i - \frac{n^2}{1-\theta} + \frac{1}{1-\theta}\sum x_i = 0$$

$$\frac{1}{\theta(1-\theta)}\sum x_i = \frac{n^2}{1-\theta} \Rightarrow \boxed{\theta = \frac{\sum x_i}{n^2}}$$

Ans