Lab 4 Solution

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Problem 1 :

To generate Gamma(4,5) from an appropriate proposal distribution using AR method.

```r
## Function to generate gamma distribution from exponential distribution
## using Acceptance-rejection Method

generate_gamma_exponential <- function(alpha, beta, lambda, c){

  # Count to store the number of times required for one acceptance
  count <- 0

  # Checking for acceptance
  while(TRUE){

    # Generating a random sample from Uniform(0,1)
    u_star <- runif(1)

    # Using ITM to generate a random sample from Exponential(lambda)
    expo_sample <- -log(u_star)/lambda

    # Generating a random sample from Uniform(0,1)
    u <- runif(1)

    # Incrementing the number of times the loop ran
    count <- count + 1

    # Checking for condition
    if (u <= dgamma(expo_sample,alpha,beta)/(c*dexp(expo_sample,lambda))){
      # Returning the corresponding sample and no. of times the loop ran
      return(c(expo_sample, count))
    }
```

```
  }
}
```

The function defined in the above code takes four parameters namely α and β , the shape and scale parameters of the Gamma distribution, λ the rate parameter of the exponential distribution and c , denoting the expected number of times the loop ran before an acceptance.

Here, our target distribution is Gamma(4,5) and the proposal distribution is Exponential (λ) , with suitable choice of λ.

```
# The parameters
n <- 1e4
alpha <- 4
beta <- 5

# To mininmize the number of loops, we choose the parameter of the proposal
# ... such that computations become easier
lambda <- beta/alpha

# The theoretical value of c
theoretical_c <- (alpha^alpha)*(exp(1 - alpha))/gamma(alpha)
```

We know that to minimize the computations, we choose c such that $c = max(\frac{\beta^\alpha x^{\alpha-1} e^{-x(\beta-\lambda)}}{\lambda\Gamma(\alpha)})$ for $x > 0$. On solving it we get that c takes its maximum value at $x = \frac{\alpha-1}{\beta-\lambda}$, which it turn takes the minimum value at $\lambda = \frac{\beta}{\alpha}$. On, substituting the required values, we get the theoretical value of $c = \frac{\alpha^\alpha e^{1-\alpha}}{\Gamma(\alpha)}$.

```
# Array to store the generated sample from gamma distribution
gamma_array <- numeric(n)

# Array to store the values of c
count_array <- numeric(n)

# Function call
for (i in 1:n){
  element <- generate_gamma_exponential(alpha, beta, lambda, theoretical_c)
  gamma_array[i] <- element[1]
  count_array[i] <- element[2]
}

# Sample mean
mean(gamma_array)
```

```
[1] 0.7954853
```

```
# Population mean
alpha/beta
```

```
[1] 0.8
```

```
# Sample Variance
var(gamma_array)
```

```
[1] 0.1615055
```

```
# Population Variance
alpha/(beta^2)
```

```
[1] 0.16
```

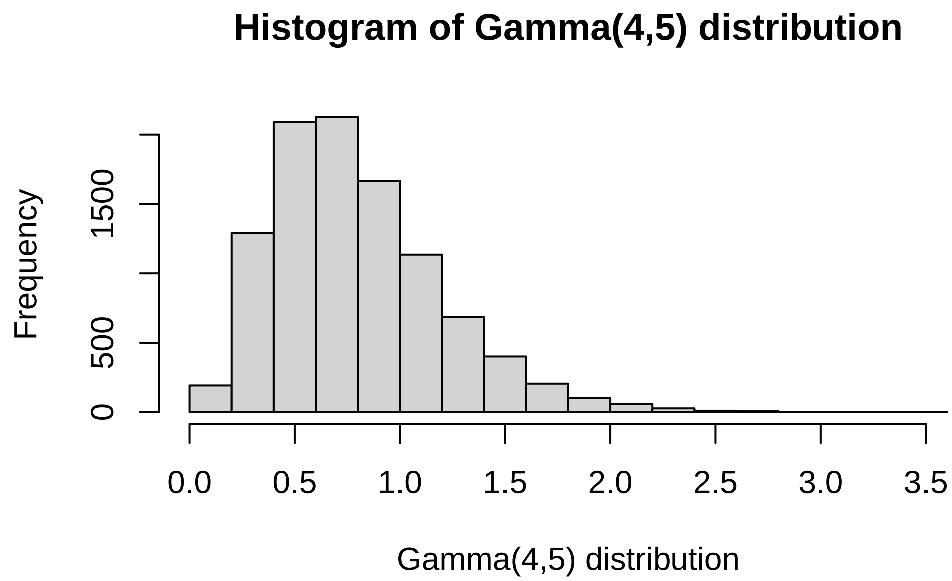
```
# Theoretical Value of c
theoretical_c
```

```
[1] 2.124248
```

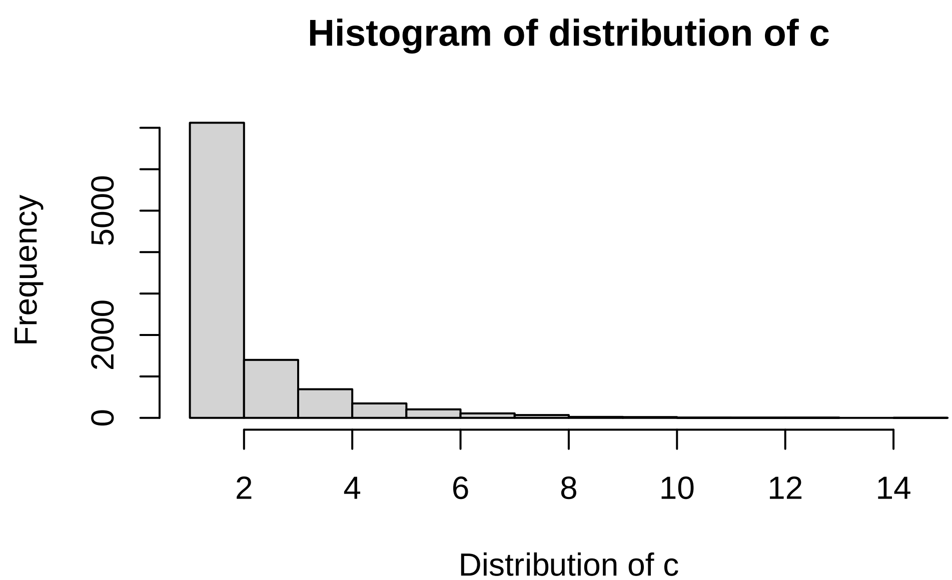
```
# Estimated Value of c
mean(count_array)
```

```
[1] 2.1328
```

```
# Histogram of the generated gamma sample
hist(gamma_array, main = "Histogram of Gamma(4,5) distribution",
     xlab = "Gamma(4,5) distribution", ylab = "Frequency")
```




```
# Histogram of values of c
hist(count_array, main = "Histogram of distribution of c",
     xlab = "Distribution of c", ylab = "Frequency")
```



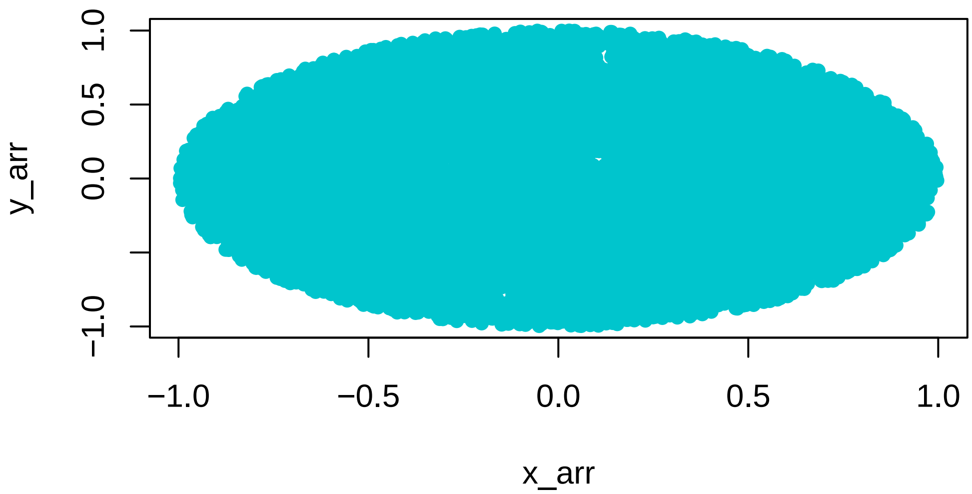
From the above, we can see that the sample mean of the Gamma(4,5) distribution is approximately the same as that of the population mean ($\frac{\alpha}{\beta} = \frac{4}{5} = 0.80$) and the sample variance is close to the population variance ($\frac{\alpha}{\beta^2}$) $= \frac{4}{5^2} = 0.16$. We can also see that the estimated value of c is quite close to the theoretical value of c.

We can see that distribution of c follows a Geometric distribution.

Problem 2

To generate 10^4 samples from a unit circle.

```r
## Generating 1e4 points inside a unit circle centered at origin

# Count of number of points generated
t_count <- 1

# Array to store the x-coordinates of the points generated
x_arr <- numeric(1e4)
# Array to store the y-coordinates of the points generated
y_arr <- numeric(1e4)

# Generating points inside the circle
while(t_count <= 1e4){

  # Drawing uniformly from square with side length 2
  x <- runif(1, min = -1, max = 1)
  y <- runif(1, min = -1, max = 1)

  # Checking if the point lies inside the circle
  if (x^2 + y^2 < 1){
    x_arr[t_count] <- x
    y_arr[t_count] <- y

    # Incrementing the count of points inside circle
    t_count <- t_count + 1
  }
}

## Scatterplot of the points generated
plot(x_arr, y_arr, pch = 16, col = 'turquoise3')
```



From the scatter-plot we can see that the points are generated inside the circle uniformly as there are no visible clustering of the generated point.