HARSH (88028)

Date: / /20
Page:

A2 - (i) $T(n) = 2T\left(\frac{n}{4}\right) + \sqrt{n}$

we know that Master Theorem applies to recurrences of the following form:

$$T(n) = aT(n/b) + f(n)$$

where $a \geq 1$ and $b > 1$ are constants and $f(n)$ is an asymptotically positive func.

Given, $T(n) = 2T(n/4) + \sqrt{n}$

Let the base four representation of n be

$$n = \sum_{k=0}^{\lfloor \log_4 n \rfloor} d_k 4^k$$

$$T(n) = \sum_{j=0}^{\lfloor \log_4 n \rfloor} 2^j \left[\sqrt{\sum_{k=j}^{\lfloor \log_4 n \rfloor} d_k 4^{k-j}} \right]$$

$$T(n) \leq \sum_{j=0}^{\log_4 N} 2j \sqrt{\sum_{k=j}^{\lfloor \log_4 n \rfloor} 3 \times 4^{k-j}} = \sum_{j=0}^{\lfloor \log_4 n \rfloor} 2j \sqrt{4^{\lfloor \log_4 n + 1 - j \rfloor} - 1}$$

88028

Date: / /20

Page:

$$T(n) < \sum_{j=0}^{\lceil \log_4 n \rceil} 2^j \sqrt{4^{\lceil \log_4 n \rceil + 1 - j}} = \sum_{j=0}^{\lceil \log_4 n \rceil} \sqrt{4^{\lceil \log_4 n \rceil + 1}}$$

$$= (\lceil \log_4 n \rceil + 1) \times 2^{\lceil \log_4 n \rceil + 1}$$

$$T(n) \geq \sum_{j=0}^{\lfloor \log_4 n \rfloor} 2^j \sqrt{4^{\lfloor \log_4 N \rfloor - j}} = \sum_{j=0}^{\lfloor \log_4 n \rfloor} \sqrt{4^{\lfloor \log_4 n \rfloor}}$$

$$T(n) = (\lfloor \log_4 n \rfloor + 1) \times 2^{\lfloor \log_4 n \rfloor}$$

Joining the dominant terms of the upper and the lower bound we obtain the asymptotics

$$\lfloor \log_4 n \rfloor \times 2^{\lfloor \log_4 n \rfloor} \in \Theta(\log_4 n \times 4^{\frac{1}{2} \log_4 n})$$

$$= \Theta(\log n \times \sqrt{n})$$

Observe that there is a lower order term

$$2^{\lfloor \log_4 n \rfloor} \in \Theta(4^{\frac{1}{2} \log_4 n}) = \Theta(\sqrt{n})$$

88028

Q3 - (i) Insertion Algorithms :

1. If it is first element, it is already sorted.
2. Pick the next element.
3. Compare with all the elements in sorted ~~sub-list~~ sub-list.
4. Shift all the elements in sorted sub-list that is greather than the value to be sorted.
5. Insert the value.
6. Repeat until list is sorted.

Given | 5 | 2 | 7 | 4 | 9 | 3 | 6 | 1 | 8 |

Applying the above Algorithm,

- | 5 | 2 | . . . . . . | 8 | [5 > 2 ∴ 5 is moved forward.
- | 2 | 5 | 7 | . . . . . | 8 | [7 > 5, so no changes.
- | 2 | 5 | 7 | 4 | . . . . | 8 | [7 > 4 so both are interchaged.
- | 2 | 5 | 4 | 7 | 9 | . . . . | 8 | [First: 5 > 4 so 5 goes forward

88.28

Date: / /20

Page:

- |2|4|5|7|9|3| ... |8|
- |2|4|5|7|9|3| ... |8| [9 > 3, 9 moves forward]
- |2|4|5|7|3|9| ... |8|
- |2|3|4|5|7|9|6|1|8|
- |2|3|4|5|6|7|9|1|8|
- |1|2|3|4|5|6|7|9|8|

The sorted order is –

|1|2|3|4|5|6|7|8|9|

88028

Date: / /20
Page:

A5- Given equation - $a_x - 4a_{x-1} + 4a_{x-2} = 3x + 2^x$ ... (i)

The homogenous solution of this equation is obtained by putting RHS equal to 0 i.e,

$$a_x - 4a_{x-1} + 4a_{x-2} = 0$$

The homogenous solution is $a_{x(h)} = (C_1 + C_2 x) \times 2^x$

The equation (i) can be written as

$$(E^2 - 4E + 4) a_x = 3x + 2^x$$

The particular solution is given as

$$a_{x(p)} = \frac{1}{(E^2 - 4E + 4)} \cdot (3x + 2^x)$$

$$= \frac{1}{(E-2)^2} \cdot (3x) + \frac{1}{(E-2)^2} \cdot 2^x$$

$$= 3 \cdot \frac{1}{(1-\Delta)^2} (x) + \frac{x(x-1)}{2!} \cdot 2^{x-2}$$

88.28

Date: / /20

Page:

$$= 3(1-\Delta)^{-2}(\gamma) + \frac{\gamma(\gamma-1)}{2!} \cdot 2^{\gamma-2}$$

$$= 3(1+2\Delta)[\gamma] + \frac{\gamma(\gamma-1)}{2!} \cdot 2^{\gamma-2}$$

$$= 3(\gamma+2) + \gamma(\gamma-1) \cdot 2^{\gamma-3}$$

$$a_{\gamma}(P) = 3(\gamma+2) + \gamma(\gamma-1) \cdot 2^{\gamma-3}$$

$\therefore$ The total solution is,

$$a_{\gamma} = (C_1 + C_2\gamma) \cdot 2^{\gamma} + 3(\gamma+2) + \gamma(\gamma-1) \cdot 2^{\gamma-3}$$

88028

Date: / /20
Page:

A4 - The given set is

$A = \{1, 2, 3, 5, 6, 10, 15, 30\}$

The relation R is such that,

$R = \{(a, b) \mid a \text{ is divisor of } B\}$

- Checking for Reflexive

Let $a \in A$, since a is divisor a

So $(a, a) \in R$

so, R is Reflexive

- Checking for Antisymmetric

Let $a, b \in A$ and $(a, b) \in R$ and $(b, a) \in R$

$(a, b) \in R$ implies a is divisor of b

$(b, a) \in R$ implies b is divisor of a

$\therefore \quad a = b$

so, $(a, b) \in R$ and $(b, a) \in R$

implies $a = b$

88028

Date: / /20

Page:

So, R is anti symmetric

- Checking for transitive

Let $a, b, c \in A$

Also let $(a,b) \in R$ and $(b,c) \in R$

a is divisor of b and b is divisor of c

a is divisor of c

$(a,c) \in R$

Thus $(a,b) \in R$ and $(b,c) \in R$ implies $(a,c) \in R$

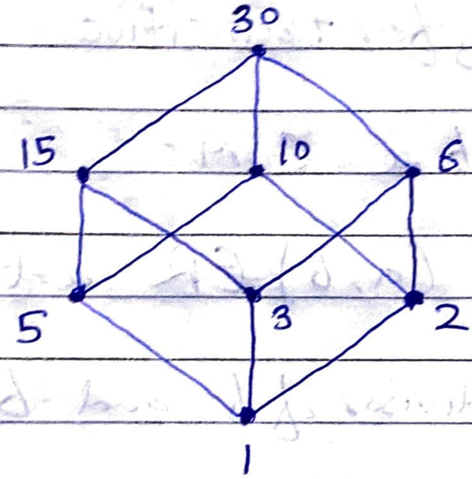
R is transitive

Hence R is a poset

88028

Date: / /20
Page:

→ Hasse Diagram –



The least element a is the element such that $a \leq x$ for all $x \in A$

The greatest element b is the element such that $x \leq b$ for all $x \in A$

Thus 1 is minimal element and 30 is the maximal element.

→ Given $f(x) = x+3$, $g(x) = x-3$ and $h(x) = 4x$ $x \in R$

88628

Date: / /20

Page:

(i) $gof(x)$

$= g(f(x))$

$= g(x+3)$

$= x+3-3$

$= x$ $\therefore gof(x) = x$

(ii) $hof(x)$

$= h(f(x))$

$= h(x+3)$

$= 4(x+3)$

$= 4x+12$ $\therefore hof(x) = 4x+12$

(iii) $fohog(x)$

$= f(h(g(x)))$

88028

Date: / /20

Page:

$= f(h(x-3)))$

$= f(4x-12)$

$= 4x-12+3$

$= 4x-9$

$\therefore \; f \circ h \circ g\,(x) = 4x-9$

88028

Date: / /20
Page:

A6 (ii) Given statement is -

"If you send me the URL, then I will finesh configuring the software."

- Inverse (If not p, then not q)

If you don't send me the URL, then I can't finesh configuring the software.

- Converse (If q, then p)

I will finesh configuring the software, If you send me the URL.

- Contrapositive (If not q, then not p)

I can't finesh configuring the software, If you don't send me the URL.

(iii) Given word "Climate"

The total number of ways these letters can be arranged = 7! = 5040

The number of ways the letters can be arranged such that vowels occur in odd places -

we know that $^nP_r = \frac{n!}{(n-r)!}$

now, there are 3 vowels in 'climate'

$r = 3$

And there are 4 odd places.

$n = 4$

$^4P_3 = \frac{4!}{(4-3)!} = 4! = \boxed{24}$

And corresponding to these 24 ways the other 4 letters may be placed in 4! ways.

$4! = 24$

88028

Date: / /20
Page:

$\therefore$ The total number of required arrangements is

$^4P_3 \times 24$

$= 24 \times 24$

$= 576$ ways