We have $f(\lambda) = \sum_{i=0}^{\lambda} \sum_{j=0}^{i} \sum_{k=0}^{j} \sum_{l=0}^{k} \sum_{m=0}^{l} \sum_{n=0}^{m} (1+n)$

Consider the last summation.

$$\sum_{n=0}^{m} (1+n) = [1+1+1 \dots (m+1) \text{ times}] + [1+2+3 \dots +m]$$
$$= (m+1) + \frac{m(m+1)}{2} = \frac{(m+1)(m+2)}{2} = \binom{m+2}{2}$$

Thus, the second last summation becomes

$$\sum_{m=0}^{l} \binom{m+2}{2} = \binom{2}{2} + \binom{3}{2} + \binom{4}{2} \dots + \binom{l+2}{2}$$

The above sum is equal to coefficient of x^2 in

$(1+x)^2 + (1+x)^3 \dots + (1+x)^{l+2}$

The above sum, being a geometric progression, reduces to

$$(1+x)^2 \frac{[(1+x)^{l+1} - 1]}{(1+x)-1} = \frac{(1+x)^{l+3} - (1+x)^2}{x}$$

Coefficient of x^2 in above $=$ Coefficient of x^3 in $(1+x)^{l+3} - (1+x)^2$

$$= {}^{l+3}C_3 - 0$$
$$= {}^{l+3}C_3 = \binom{l+3}{3}$$

Continuing like this, we get

$$f(\lambda) = \binom{\lambda+7}{7} = \frac{(\lambda+7)!}{\lambda!\,7!}$$ where $\lambda \in N$, $\lambda < 1000$.

Now, $z(\lambda)$ = Highest power of 10 which divides $f(\lambda)$.

= Minimum (Highest power of 2 which divides $f(\lambda)$,
Highest power of 5 which divides $f(\lambda)$).

Let us evaluate the highest power of 5 which divides $f(\lambda)$.

The highest power of 5 which divides $f(\lambda)$

$$= \left\{ \left\lfloor \frac{\lambda+7}{5} \right\rfloor + \left\lfloor \frac{\lambda+7}{25} \right\rfloor + \left\lfloor \frac{\lambda+7}{125} \right\rfloor + \left\lfloor \frac{\lambda+7}{625} \right\rfloor \right\}$$

$$- \left\{ \left\lfloor \frac{\lambda}{5} \right\rfloor + \left\lfloor \frac{\lambda}{25} \right\rfloor + \left\lfloor \frac{\lambda}{125} \right\rfloor + \left\lfloor \frac{\lambda}{625} \right\rfloor \right\}$$

$$- \left\{ \left\lfloor \frac{7}{5} \right\rfloor \right\}$$

(We have gone only till 5^4 because $\lambda < 1000$ and $5^5 = 3125$.)

$$= \left(\left\lfloor \frac{\lambda+2}{5} \right\rfloor - \left\lfloor \frac{\lambda}{5} \right\rfloor \right) + \left(\left\lfloor \frac{\lambda+7}{25} \right\rfloor - \left\lfloor \frac{\lambda}{25} \right\rfloor \right)$$

$$+ \left(\left\lfloor \frac{\lambda+7}{125} \right\rfloor - \left\lfloor \frac{\lambda}{125} \right\rfloor \right) + \left(\left\lfloor \frac{\lambda+7}{625} \right\rfloor - \left\lfloor \frac{\lambda}{625} \right\rfloor \right) \quad \text{— (1)}$$

To obtain maximum value of $z(\lambda)$, the above (Marked as (1)) expression must be maximized.

This is possible if each term in braces ( ) contributes 1.

Thus, $\left\lfloor \frac{\lambda+2}{5} \right\rfloor - \left\lfloor \frac{\lambda}{5} \right\rfloor = 1$

$$\left\lfloor \frac{\lambda+7}{25} \right\rfloor - \left\lfloor \frac{\lambda}{25} \right\rfloor = 1$$

$$\left\lfloor \frac{\lambda+7}{125} \right\rfloor - \left\lfloor \frac{\lambda}{125} \right\rfloor = 1$$

$$\left\lfloor \frac{\lambda+7}{625} \right\rfloor - \left\lfloor \frac{\lambda}{625} \right\rfloor = 1$$

The above implies $\lambda+7 \geq 625$ and $\lambda < 625$

$\Rightarrow \lambda = \{ 618, 619, 620, 621, 622, 623, 624 \}$

Also for $\left\lfloor \frac{\lambda+2}{5} \right\rfloor - \left\lfloor \frac{\lambda}{5} \right\rfloor = 1$, by eliminating some values, we get $\lambda = \{ 618, 619, 623, 624 \}$

For above four values of λ, $z(\lambda)$ attains its maximum value $= 4$, only if maximum power of 2 which divides $f(\lambda)$ is ≥ 4.

This eliminates the possibility of $\lambda = 618$ and $\lambda = 624$.

Thus for $\lambda = 619$ and 623, we get maximum value of $z(\lambda) = 4$.

$\Rightarrow \lambda_{min} = 619$, $\lambda_{max} = 623$

Now, let us evaluate λ_0.

Now, highest power of 10 which divides $f(\lambda)$ would be 0 if

$$\text{Minimum}\left(\begin{array}{l}\text{Highest power of 2 which divides } f(\lambda), \\ \text{Highest power of 5 which divides } f(\lambda)\end{array}\right) = 0.$$

Thus, we should have

Highest power of 5 which divides $f(\lambda) = 0$

Or Highest power of 2 which divides $f(\lambda) = 0$.

Let us find the maximum of λ for which highest power of 5 which divides $f(\lambda) = 0$.

For this to happen, each term in braces ( ) of expression 1 should contribute 0.

$$\Rightarrow \left\lfloor \frac{\lambda+2}{5} \right\rfloor = \left\lfloor \frac{\lambda}{5} \right\rfloor$$

$$\left\lfloor \frac{\lambda+7}{25} \right\rfloor = \left\lfloor \frac{\lambda}{25} \right\rfloor$$

$$\left\lfloor \frac{\lambda+7}{125} \right\rfloor = \left\lfloor \frac{\lambda}{125} \right\rfloor$$

$$\left\lfloor \frac{\lambda+7}{625} \right\rfloor = \left\lfloor \frac{\lambda}{625} \right\rfloor$$

Clearly, $\lambda \geq 975$ and $\lambda + 7 < 1000$

$\Rightarrow 975 \leq \lambda < 993$

Also for $\left\lfloor \frac{\lambda+2}{5} \right\rfloor = \left\lfloor \frac{\lambda}{5} \right\rfloor$, $\lambda = \{975, 976, 977, 980, 981, 982, 985, 986, 987, 990, 991, 992\}$

Note that for $\lambda \geq 992$, highest power of 2 which divides $f(\lambda)$ is always > 0.

Thus $\lambda_0 = 992$.