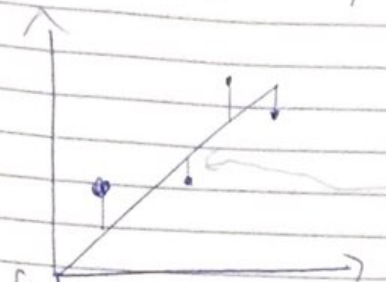
Linear Regression, Least Squares method derivation



each scatter point is (x_i, y_i)

The lines indicate error E_i

~~estimate~~ find best fit line y

$$y = b_1 x_i + b_0,$$

Minimize the square of the error E

$$E_{LR} = \sum_{i=1}^{n} E_i^2 = \sum (y_i - b_1 x_i - b_0)^2.$$

We want to find the minimized E_{LR}, with b_1 & b_0

$$\frac{\partial E_{LR}}{\partial b_1} = \sum \frac{\partial}{\partial b_1} (y_i - b_1 x_i - b_0)^2 = 0$$

$$= \sum -\cancel{2} \cdot (y_i x_i - b_1 x_i^2 - b_0 \cdot x_i) = 0 \quad ①$$

$$\frac{\partial E_{LR}}{\partial b_0} = \sum -\cancel{2} (y_i - b_1 x_i - b_0) = 0 \quad ②$$

We want to solve ①, ② for b_0, b_1

refine ①,

$$①: \sum b_1 x_i^2 + \sum b_0 x_i = \sum y_i x_i$$

$$②: \sum b_1 x_i + \sum b_0 = \sum y_i$$

Turn to matrix form,

$$\begin{bmatrix} \sum x_i^2 & \sum x_i \\ \sum x_i & n \end{bmatrix} \begin{bmatrix} b_1 \\ b_0 \end{bmatrix} = \begin{bmatrix} \sum y_i x_i \\ \sum y_i \end{bmatrix}$$

$$\begin{bmatrix} b_1 \\ b_0 \end{bmatrix} = \begin{bmatrix} \sum x_i^2 & \sum x_i \\ \sum x_i & n \end{bmatrix}^{-1} \begin{bmatrix} \sum y_i x_i \\ \sum y_i \end{bmatrix}$$

$$\begin{bmatrix} b_1 \\ b_0 \end{bmatrix} = \frac{1}{n\sum x_i^2 - (\sum x_i)^2} \begin{bmatrix} n & -\sum x_i \\ -\sum x_i & \sum x_i^2 \end{bmatrix} \begin{bmatrix} \sum y_i x_i \\ \sum y_i \end{bmatrix}$$

$$b_1 = \frac{n\sum y_i x_i - \sum x_i \sum y_i}{n\sum x_i^2 - (\sum x_i)^2} \quad //$$

$$\cancel{b_0 = \frac{-\sum y_i (\sum x_i)^2 + \sum y_i \sum x_i^2}{n(\sum x_i^2 - (\sum x_i)^2}}$$

$$\cancel{b_0 = \sum y_i}$$

Now using ②,

$$\sum_{i=1}^{n} b_0 = \sum_{i=1}^{n} y_i - \sum_{i=1}^{n} b_1 x_i$$

$$\cancel{n} b_0 = \frac{\sum y_i}{\cancel{n}} - b_1 \frac{\sum x_i}{n}$$

$$b_0 = \bar{y} - b_1 \bar{x} \quad //$$

where $\bar{y}$, $\bar{x}$ are averages of the scatters co-ordinates.