

## 1. Hidden Markov model

Hidden Markov Model (HMM) is a statistical Markov model in which the system being modeled is assumed to be a Markov process – call it  $X$  – with unobservable ("hidden") states. HMM assumes that there is another process  $Y$  whose behavior "depends on"  $X$ .

## 2. Definition

Let  $X_n$  and  $Y_n$  be discrete-time stochastic processes and  $n \geq 1$ . The pair  $(X_n, Y_n)$  is a hidden Markov model if

- a)  $X_n$  is a Markov process whose behavior is not directly observable ("hidden");
- b)  $P(Y_n \in A | X_1 = x_1, \dots, X_n = x_n) = P(Y_n \in A, X_n = x_n)$

### a) Terminology

The states of the process  $X_n$  are called hidden states, and  $P(Y_n \in A, X_n = x_n)$  is called emission probability or output probability.

## 3. Examples

### 1) Drawing balls from hidden Urns

In its discrete form, a hidden Markov process can be visualized as a generalization of the urn problem with replacement (where each item from the urn is returned to the original urn before the next step). [6] Consider this example: in a room that is not visible to an observer there is a genie. The room contains urns  $X_1, X_2, X_3, \dots$  each of which contains a known mix of balls, each ball labeled  $y_1, y_2, y_3, \dots$ . The genie chooses an urn in that room and randomly draws a ball from that urn. It then puts the ball onto a conveyor belt, where the observer can observe the sequence of the balls but not the sequence of urns from which they were drawn. The genie has some procedure to choose urns; the choice of the urn for the  $n$ -th ball depends only upon a random number and the choice of the urn for the  $(n-1)$ -th ball. The choice of urn does not directly depend on the urns chosen before this single previous urn; therefore, this is called a Markov process.

### 2) Weather guessing game

Consider two friends, Alice and Bob, who live far apart from each other and who talk together daily over the telephone about what they did that day. Bob is only interested in three activities: walking in the park, shopping, and cleaning his apartment. The choice of what to do is determined exclusively by the weather on a given day. Alice has no definite information about the weather, but she knows general trends. Based on what Bob tells her he did each day, Alice tries to guess what the weather must have been like. Alice believes that the weather operates as a discrete Markov chain. There are two states, "Rainy" and "Sunny", but she cannot observe them directly, that is, they are hidden from her. On each day, there is a certain chance that Bob will perform one of the following activities, depending on the weather: "walk",

shop", or "clean". Since Bob tells Alice about his activities, those are the observations. The entire system is that of a hidden Markov model (HMM).

#### 4. Structural architecture

The hidden state space is assumed to consist of one of  $N$  possible values, modelled as a categorical distribution. (See the section below on extensions for other possibilities.) This means that for each of the  $N$  possible states that a hidden variable at time  $t$  can be in, there is a transition probability from this state to each of the  $N$  possible states of the hidden variable at time  $T + 1$ , for a total of  $N^2$  transition probabilities. Note that the set of transition probabilities for transitions from any given state must sum to 1. Thus, the  $N \times N$  matrix of transition probabilities is a Markov matrix. Because any one transition probability can be determined once the other  $(N - 1)$  transition parameters.

In addition, for each of the  $N$  possible states, there is a set of emission probabilities governing the distribution of the observed variable at a particular time given the state of the hidden variable at that time. The size of this set depends on the nature of the observed variable. For example, if the observed variable is discrete with  $M$  possible values, governed by a categorical distribution, there will be  $(M-1)$  separate parameters, for a total of  $N \times (M-1)$  emission parameters over all hidden states. On the other hand, if the observed variable is an  $M - 1$  dimensional vector distributed according to an arbitrary multivariate Gaussian distribution, there will be  $M$  parameters controlling the distribution. 
$$N \left( M + \frac{M(M+1)}{2} \right) = \frac{NM(M+3)}{2} = O(NM^2) \text{ emission parameters.}$$

#### 5. Applications

Computational finance  
 Single-molecule kinetic analysis  
 Cryptanalysis  
 Speech recognition, including Siri  
 Speech synthesis  
 Part-of-speech tagging  
 Document separation in scanning solutions  
 Machine translation  
 Partial discharge  
 Gene prediction  
 Handwriting recognition  
 Alignment of bio-sequences  
 Time series analysis  
 Activity recognition  
 Protein folding  
 Sequence classification  
 Metamorphic virus detection  
 DNA motif discovery  
 DNA hybridization kinetics  
 Chromatin state discovery  
 Transportation forecasting  
 Solar irradiance variability