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Gödel's Incompleteness Theorem

1. Introduction

Gödel's incompleteness theorems are two theorems of mathematical logic that are concerned with the limits of provability in formal axiomatic theories.
-The first incompleteness theorem states that no consistent system of axioms whose theorems can be listed by an effective procedure is capable of proving all truths about the arithmetic of natural numbers. For any such consistent formal system, there will always be statements about natural numbers that are true, but that are unprovable within the system.
- The second incompleteness theorem, an extension of the first, shows that the system cannot demonstrate its own consistency.

2. Formal systems: completeness, consistency, and effective axiomatization,10 pt

a) Completeness

A set of axioms is complete if, for any statement in the axioms' language, that statement or its negation is provable from the axioms.

b) Consistency

A set of axioms is (simply) consistent if there is no statement such that both the statement and its negation are provable from the axioms, and inconsistent otherwise.

c) Effective axiomatization

A formal system is said to be effectively axiomatized (also called effectively generated) if its set of theorems is a recursively enumerable set.

3. First incompleteness theorem

Incompleteness Theorem: Äny consistent formal system F within which a certain amount of elementary arithmetic can be carried out is incomplete; i.e., there are statements of the language of F which can neither be proved nor disproved in F."

4. Second incompleteness theorem

Gödel's second incompleteness theorem shows that, under general assumptions, this canonical consistency statement Cons(F) will not be provable in F.