Numerical Optimization
Notes: Click Here to View or Download

Unit 1
Study Links

Chapter 1: Click Here to View or Download
Simplex notes: Click Here to View or Download

LPP(Graphically):
Linear Programming 1: Maximization -Extreme/Corner Points - YouTube
Linear Programming 2: Graphical Solution - Minimization Problem - YouTube

LPP(Graphically): Special Cases of Graphical Method

Simplex Method:
Intro to Simplex Method | Solve LP | Simplex Tableau - YouTube
Operation Research | Two Phase Simplex Method | Linear Programming - YouTube
Simplex Special Cases: Read Here

Theory Questions

1. Define optimization problem.
2. Define transportation problem.
3. Compare the following:
   a) Continuous and Discrete Optimization
   b) Constrained and Unconstrained Optimization
   c) Local and Global Optimization
   d) Stochastic and Deterministic Optimization
4. Define Convexity.
5. What are the features of the optimization problem.

Numerical Questions

1. Solve the following using graphical method:
   a) Maximize $Z = 8x + y$

Constraints are:
x + y <=40
2x + y <= 60
x>=0, y>=0

b) Maximize Z = 50x + 15y
Constraints are:
5x + y <= 100
x + y <= 50
x>=0, y>=0

c. Minimize Z = 20x + 10y
Constraints are:
x + 2y <= 40
3x + y >= 30
4x + 3y >= 60
x>=0, y>= 0

d. Maximize Z = 3x + 5y
Constraints are:
x + 5y <= 10
2x + 2y <= 5
x, y >= 0

2. Solve the linear programming problem (any three):

Exercise 5A

1. Ann and Margaret run a small business in which they work together making blouses and skirts.

   Each blouse takes 1 hour of Ann's time together with 1 hour of Margaret's time. Each skirt involves Ann for 1 hour and Margaret for half an hour. Ann has 7 hours available each day and Margaret has 5 hours each day.

   They could just make blouses or they could just make skirts or they could make some of each.

   Their first thought was to make the same number of each. But they get £8 profit on a blouse and only £6 on a skirt.

   (a) Formulate the problem as a linear programming problem.

   (b) Find three solutions which satisfy the constraints.

2. A distribution firm has to transport 1200 packages using large vans which can take 200 packages each and small vans which can take 80 packages each. The cost of running each large van is £40 and of each small van is £20. Not more than £300 is to be spent on the job. The number of large vans must not exceed the number of small vans.

   Formulate this problem as a linear programming problem given that the objective is to **minimise** costs.

3. A firm manufactures wood screws and metal screws. All the screws have to pass through a threading machine and a slotting machine. A box of wood screws requires 3 minutes on the slotting machine and 2 minutes on the threading machine. A box of metal screws requires 2 minutes on the slotting machine and 8 minutes on the threading machine. In a week, each machine is available for 60 hours.

   There is a profit of £10 per box on wood screws and £17 per box on metal screws.

   Formulate this problem as a linear programming problem given that the objective is to **maximise** profit.

4. A factory employs unskilled workers earning £135 per week and skilled workers earning £270 per week. It is required to keep the weekly wage bill below £24 300.

   The machines require a minimum of 110 operators, of whom at least 40 must be skilled. Union regulations require that the number of skilled workers should be at least half the number of unskilled workers.

   If x is the number of unskilled workers and y the number of skilled workers, write down all the constraints to be satisfied by x and y.

3. Solve the following using simplex method:

Maximise $P = x + 2y$

subject to $x + 4y \le 20$

$x + y \le 8$

$5x + y \le 32$

$x \ge 0$

$y \ge 0$

1. Maximise $P = 4x + 6y$

subject to $x + y \le 8$

$7x + 4y \le 14$

$x \ge 0$

$y \ge 0$

4. Prove the convexity of the following:

Practice questions

1. Prove that the following functions are convex

a) $f(x) = e^x$

b) $f(x) = ax^2 + bx + c$ where a is positive

c) $f(x) = x^p$ where p is greater than or equal to 1.

d) $f(x) = |x|$

e) $f(x) = a^x$

f) $f(x) = \sin x$ and $f(x) = \cos x$

5. Solve the following using two-phase method:

Min $Z = 4x_1 + 3x_2$

Such that:

$2x_1 + x_2 >= 10$

$-3x_1 + 2x_2 <= 6$

$x_1 + x_2 >= 6$

$x_1, x_2 >= 0$

Unit 2

Study Links

Chapter 2: Click Here to View or Download
Chapter 9: Click Here to View or Download

Theory Questions

1. What is unconstrained optimization? Give an example.
2. What is non-linear least squares problem.
3. Define the following:
   a) Local minimizer
   b) Global minimizer
   c) Strict local minimizer
   d) Isolated local minimizer
4. State Taylor's theorem.
5. What are the necessary and sufficient conditions for local minima.
6. What is stationary point?
7. State the condition for local minimizer to be global minimizer and vice versa.
8. What are the characteristics of non-smooth problem.
9. What is unimodal problem? Give example.
10. What is solution set?
11. Define the following:
    a) Descent Property
    b) Quadratic Termination Property
    c) Global Convergence
    d) Order of Convergence
    e) Unimodal min function
12. Explain the line search method for unimodal functions.
13. Explain Golden Section method. Write its algorithm.
14. Explain Fibonacci search method. Write its algorithm.
15. Explain the relationship between Golden Section and Fibonacci search method.
16. Explain Steepest Descent and write its algorithm.
17. What are the advantages and disadvantages of Steepest Descent method.
18. Explain Newton's method. Write its algorithm.
19. Explain modified Newton's method.

Numerical Questions

1. Solve for all the critical points of the given function. Then, for each critical point, use the hessian matrix to determine whether the critical point is a local minima, maxima, neither.
   a) $7x^2 + 6xy + 2x + 7y^2 - 22y + 23$
   b) $2x^2 + 2xy + 2x + y^2 - 2y + 5$
2. Find min $2x^2 + 5$ over [2,10] by golden section rule. Take $\varepsilon = 1.2$.
3. Find min $x^2 - 1$ over [1,17] by golden section rule. Take $\varepsilon = 1.7$.
4. Find the min of x^2 in the inteval [-5,15] by taking n = 7 using Fibonacci search method.
5. Find min $2x^2 + 5$ over [2,10] by Fibonacci. Take n = 6.
6. Use the steepest descent method to minimize $f(x_1,x_2) = 3x_1^2 - 4x_1x_2 + 2x_2^2 + 4x_1 + 6$ over $(x_1,x_2) \in R^2$.
7. Use Newton's method to minimize $f(x_1,x_2) = 8x_1^2 - 4x_1x_2 + 5x_2^2$, $(x_1,x_2) \in R^2$.

Unit 3
Study Links

Chapter 9: [Click Here to View or Download]

Theory Questions

1. Explain conjugate gradient method. Write its algorithm.
2. Where the advantages and disadvantages of it.
3. Compare Steepest Descent, Newton's and Conjugate Gradient Method.

Numerical Questions

1. Use the conjugate gradient method to minimize $f(x_1,x_2) = 3x_1^2 - 4x_1x_2 + 2x_2^2 + 4x_1 + 6$, $(x_1,x_2) \in R^2$.
2. Use the conjugate gradient method to minimize $f(x_1,x_2) = 7x_1^2 + 3x_1x_2 + x_2^2 + 13$, $(x_1,x_2) \in R^2$.
3. Use the conjugate gradient method to minimize $f(x_1,x_2) = x_1^2 - x_1x_2 + 5x_2^2 + 6x_1 + 2$, $(x_1,x_2) \in R^2$.

Unit 4
Study Links

Chapter 8(only 8.1): Click Here to View or Download
Class Notes: View Unit 4 Class Notes Here
Important links:
Gradient, Hessian and Jacobian
Gradient and Gradient Hessian Approximation

Theory Questions

1. Explain method of approximating a gradient.
2. Explain method of approximating sparse jacobian.
3. Explain method of approximating hessian.
4. Explain method of approximating sparse hessian.

Numerical Questions

1. Compute the gradient and hessian of the given function:
   $f(x,y) = 5x^2 + 3xy + y^3$

2. Compute the jacobian of the given function:
   $f(x,y) = [\sin x + y \qquad x+\cos y]$

3. Construct quadratic polynomial approximation for:
   a) $f(x) = xe^x$; $x^0 = 0$
   b) $f(x_1,x_2) = x_1e^{x2}$; $x^0 = (0,0)$

4. Construct Taylor series expansion for the function log(1+x) about the point $x^0 = 0$.

Unit 5
Study Links

Chapter 12(only 12.1): [Click Here to View or Download]
Important links:
[Lagrangian Multipliers - Khan Academy]
[Lagrangian Multiplier - Calcworkshop]
[Lagrangian Multiplier - tutorial.math.lamar.edu]
[Lagrangian Inequality Constraints pdf]

Theory Questions

1. Compare constrained and unconstrained optimization.
2. State the optimality conditions for unconstrained minimization problem.
3. Define:
   a) Local Solution
   b) Strict Local Solution
   c) Isolated Local Solution
4. Compare Local and Global Solution.
5. Why smoothness of an objective function and constraint is an important issue in characterizing solution? Justify with the help of an example.
6. Define Active Set.

Numerical Questions

1. Find the maximum and minimum of $f(x,y)=5x-3y$ subject to the constraint $x^2+y^2=136$.
2. Find the maximum and minimum values of $f(x,y,z)=xyz$ subject to the constraint $x+y+z=1$. Assume that $x,y,z\geq 0$.
3. Find the maximum and minimum values of $f(x,y)=4x^2+10y^2$ on the disk $x^2+y^2\leq 4$.
4. Find the dimensions of the box with largest volume if the total surface area is 64 cm^2.