[This question paper contains 6 printed pages.]

Sr. No. of Question Paper : 1058 **D**

Unique Paper Code : 2342011103

Name of the Paper : Mathematics for Computing

Name of the Course : **B.Sc. (H) Computer Science**

Semester : I

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. The paper has **two** sections. **Section A** is compulsory. Each question is of **5** marks.

3. Attempt any **four** questions from **Section B**. Each question is of **15** marks.

Section A

1. (a) Write the following system of equations in matrix form. Reduce the augmented matrix into row echelon form. (5)

$$x_1 + 3x_2 + x_3 = 1$$

$$-4x_1 - 9x_2 + 2x_3 = -1$$

$$-3x_1 - 6x_3 = -3$$

(b) Define a convex set. Show if $C = \{x_2: 2x_1 + 3x_2 = 7\} \subset R^2$ is a convex set. (5)

(c) Show that the transformation defined by $T(x_1, x_2) = (2x_1 - 3x_2, x_1 + 4, 5x_2)$ is not linear. (5)

(d) Find the characteristic polynomial of the following matrix (5)

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & -3 & 0 \\ 4 & -13 & 1 \end{bmatrix}$$

(e) Let $a = -2\hat{i} + 3\hat{j} + 5\hat{k}$ and $b = \hat{i} + 2\hat{j} + 3\hat{k}$ be two vectors. Find the value of the dot product of these two vectors. (5)

(f) Determine whether or not the vectors (4, 1, –2), (–3, 0, 1) and (1, –2, 1) form a basis of R^3.

(5)

Section B

2. (a) For what values of λ and μ do the following system of equations is consistent. (7)

$$x + y + z = 6$$

$$x + 2y + 3z = 10$$

$$x + 2y + \lambda z = \mu$$

(b) Find the inverse of the following matrix using Gauss Jordan method. (8)

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

3. (a) Determine whether the system has a nonzero solution. (7)

$$x + 2y - 3z = 0$$

$$2x + 5y + 2z = 0$$

$$3x - y - 4z = 0$$

(b) Apply Gram Schmidt orthonormalization process to obtain an orthonormal basis for the subspace of R^4 generated by the vectors. (1, 1, 0, 1), (1, –2, 0, 0), (1, 0, –1, –2). (8)

4. (a) Use the Cayley-Hamilton theorem to find

$(A-2I)(A-3I)$ where $A=\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$. (7)

(b) What is a subspace? Let Y be the set of vectors in R^4 of the form [a, 0, b, 0]. Prove that Y is a subspace of R^4. (8)

5. (a) Calculate the curl and divergence for the following vector field. (7)

$$\vec{F}=x^3y^2\hat{i}+x^2y^3z^4\hat{j}+x^2z^2\hat{k}$$

(b) What is a positive definite matrix? Is the following matrix positive definite? (8)

$$A=\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

6. (a) Let a = [1, 1, 0], b = [3, 2, 1] and c = [1, 0, 2], Find the angle between: a, b and b, c. (3)

(b) If $\phi(x,y,z) = 3x^2y - y^3z^2$, find $\nabla\phi(\text{grad}\phi)$ at the point (1, –2, –1). (4)

(c) Diagonalize the matrix (8)

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 4 & -1 \\ -2 & -4 & 4 \end{bmatrix}$$

7. (a) If V is an inner product space, then show that $< v, au + bw > = a < v, u > + b < v, w >$ where a and b are scalars and v, u, w are vectors in V. (7)

(b) Suppose that three banks in a certain town are competing for investors. Currently, Bank A has 40% of the investors, Bank B has 10%, and Bank C has the remaining 50%. Suppose the towns folk are tempted by various promotional campaigns to switch banks. Records show that each year Bank A keeps half of its investors, with the remainder switching equally to Banks B and C. However, Bank B keeps two-thirds of its investors, with the remainder switching equally to Banks A and C. Finally, Bank C keeps half of its investors, with the remainder switching equally to Banks A and B. Find the distribution of investors after two years. (8)

B.Sc. (Hons.) Computer Science Semester I

Mathematics for Computing : Class Test (01 Feb. 2023)

Time 1 Hr. 15 Min. Max. Marks 50

Answer All Questions.

1 Match the following: [10]

(a)	Solenoidal Vector **V**	1	Irrotational gradient field
(b)	$V^{\perp}$	2	P is symmetric and $P^2 = P$
(c)	Conservative vector field **V**(P)	3	Orthogonal complement of V
(d)	Normal Equations for inconsistent Ax = b	4	Orthogonal Matrix V
(e)	Laplacian of f	5	div **V** = 0
(f)	curl (grad f) = **0**	6	curl **V** = **0**
(g)	Projection matrix P	7	$A^T A\hat{x} = A^T b$
(h)	V is square matrix with orthonormal columns	8	Gradient of a potential function f(P)
(i)	1 is an eigenvalue and all other eigenvalues are less than equal to 1 in magnitude	9	$(A^T W^T W A)\hat{x_W} = A^T W^T W b$
(j)	All eigenvalues less than 1 in magnitude	10	Matrix of a stable system
		11	Markov Matrix
		12	$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$

2 State and prove Schwarz's Inequality. [8+2]

3 Find the best strainght line fit to the measurements: [10]

b = 4 at t = -2, b = 3 at t = -1, b = 1 at t = 0, b = 0 at t = 2,

and projection of b = (4, 3, 1, 0) onto the column space of $A = \begin{bmatrix} 1 & -2 \\ 1 & -1 \\ 1 & 0 \\ 1 & 2 \end{bmatrix}$

4 On the space P_3 of cubic polynomials, what matrix represents d^2/dt^2 ? Construct the 4 by 4 matrix from the standard basis 1, t, t^2 , t^3 and find its nullspace and column space. [3+3+4]

5 Let Π be the plane in $\mathbb{R}^3$ spanned by vectors $x_1 = (1, 2, 2)$ and $x_2 = (-1, 0, 2)$. [10]

(i) Find an orthonormal basis for Π.

(ii) Extend it to an orthonormal basis for $\mathbb{R}^3$.