[This question paper contains 8 printed pages.]

Sr. No. of Question Paper : 1216 **F**

Unique Paper Code : 2342011202

Name of the Paper : Discrete Mathematical Structures

Name of the Course : **B.Sc. (Hons.) Computer Science (NEP-UGCF-2022)**

Semester : II

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Question No. **1 (Section–A)** is compulsory.

3. Attempt any **four** questions from **Section–B.**

4. Parts of a question should be attempted together.

5. Use of simple calculator is allowed.

SECTION A

1. (a) Determine whether the following function is one-to-one and onto from $\mathbb{R}^+$ to $\mathbb{R}^+$

$$f(x) = -3x^2 + 7$$

Also, check whether it is invertible. If invertible, find its inverse. Justify your answer in each case. (5)

(b) Show that $\neg(p \vee (\neg p \wedge q)$ and $(\neg p \wedge \neg q)$ are logically equivalent by developing a series of logical equivalences. (5)

(c) Evaluate 7^{644} mod 645 using Fast Modular exponentiation algorithm. (5)

(d) Prove that if any 14 numbers from 1 to 25 are chosen then one of them will be the multiple of another. (5)

(e) State whether the K_5 graph is/has a

(i) Tree

(ii) Euler Path

(iii) Euler circuit

Justify your answer. (5)

(f) Let a be a numeric function such that (5)

$$a_r = \begin{cases} 2 & 0 \le r \le 3 \\ 2^{-r} + 5 & r \ge 4 \end{cases}$$

(i) Determine S^2a.

(ii) Determine ∇a.

SECTION B

2. (a) Prove that the relation "congruence modulo m" over the set of positive integers is an equivalence relation. (7)

(b) If no three diagonals of a convex decagon meet at the same point inside the decagon, into how many line segments are the diagonals divided by their intersections? (8)

3. (a) Prove the following statement using the Direct Proof method :

If m and n both are perfect squares, then m * n is also a perfect square. (7)

(b) Using the principle of mathematical induction, prove that

$1.2.3 + 2.3.4 + \cdots + n.(n+1).(n+2) = n(n+1)(n+2)/3$ (8)

4. (a) Using the Euclidean algorithm, find the GCD of 1529 and 14039. (7)

(b) The interest for money deposited in a saving bank account is paid at a rate of 0.5% per month, with interest compounded monthly. $50 is deposited in the saving account each month for a period of 3 years, followed by $20 each month for next 2 years. What is the total amount in the account

(i) 4 years after the first deposit?

(ii) 20 years after the first deposit?

Formulate the numeric functions for each. (8)

5. (a) Prove that a tree with n vertices has n – 1 edges. (7)

(b) For the following numeric functions : (8)

$$a_r = 2^r \text{ for all } r$$

$$b_r = \begin{cases} 0 & 0 \le r \le 2 \\ 2^r & r \ge 3 \end{cases}$$

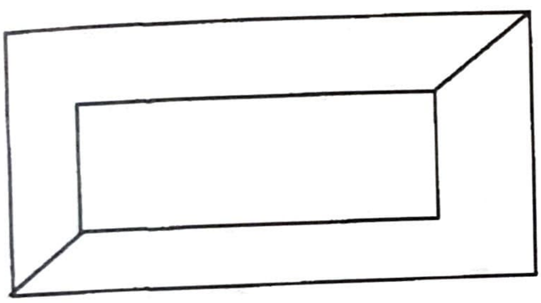
Determine a * b in either sketch or closed form expression.

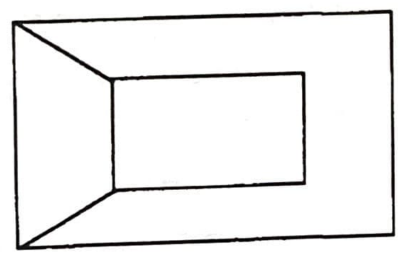
6. (a) In how many ways can a cricket team of eleven be chosen out of a batch of 14 players? How many of them will:

(i) include a particular player?

(ii) exclude a particular player? (7)

(b) Define graph isomorphism. Check whether the following pair of graphs are isomorphic. Give justification in support of your answer. (8)





7\. (a) Is Q_3 a planar graph? If planar, draw it in such a form. Verily your result using Euler formula also. (7)

(b) Draw Hasse Diagram for the relation R on $A = \{1, 2, 3, 4, 5\}$, whose relation matrix is given below

$$\begin{pmatrix} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Is it a totally ordered set? Justify your answer.

(8)

AB–115 Roll No....................

Ist Semester Examination, 2022-23

B.C.A.

DISCRETE MATHEMATICS

BCA 101(L)

Time : 3 Hours] [MAXIMUM MARKS : 80

Note : *Attempt any **two** parts from each Unit. All questions carry equal marks.*

Unit–I

1. (a) Show that the following is a tautology $[p \Rightarrow (\sim q \vee r)] \wedge [\sim q \vee (p \Leftrightarrow \sim r]$

   (b) Show that SVR is a tautology implied by :

   $$(P \vee Q) \wedge (P \Rightarrow R) \wedge (Q \Rightarrow s)$$

   (c) Explain the following terms and also give examples to explain them :

   (i) Universal Quantifier

   (ii) Existential Quantifier

Unit–II

2. (a) If (B, +, ., ′) is a Boolean algebra then prove that the following statements are equivalent :

   (i) $a \,.\, b' = 0$ (ii) $a + b = b$

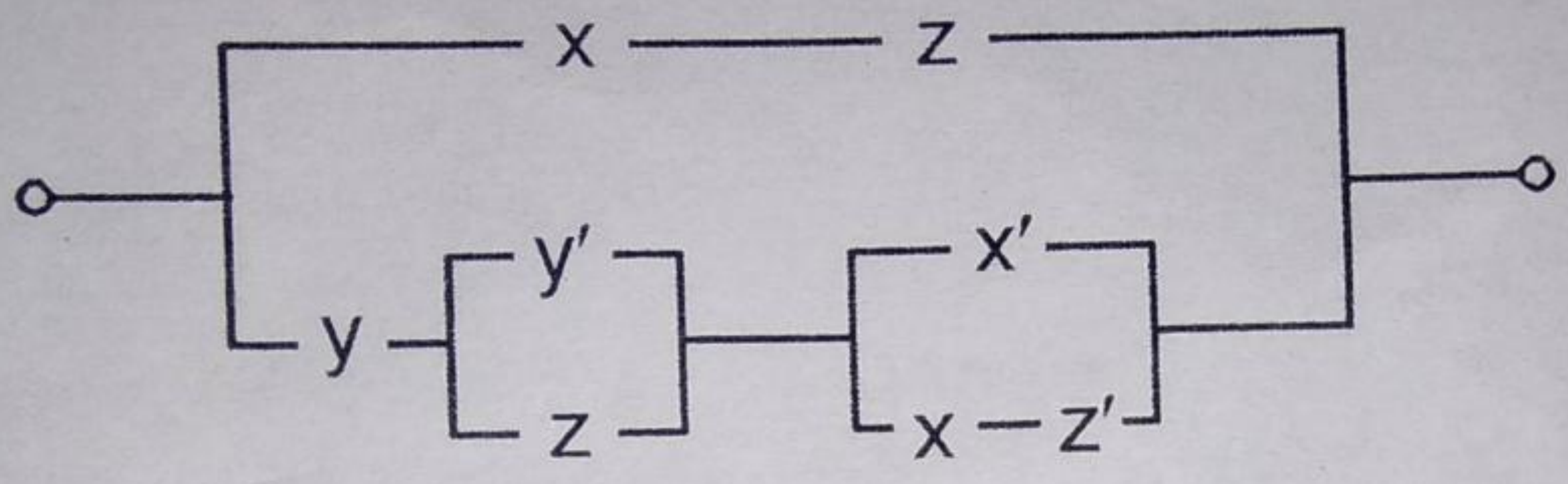
   (iii) $a' + b = 0$ (iv) $a \,.\, b = a$

P.T.O.

(b) Draw the logic circuit of :

$f(a, b, c) = (a + b) \, . \, [a^1 + (c. \, b^1)]$

(c) Replace the following switching circuit by a simpler are :



Unit–III

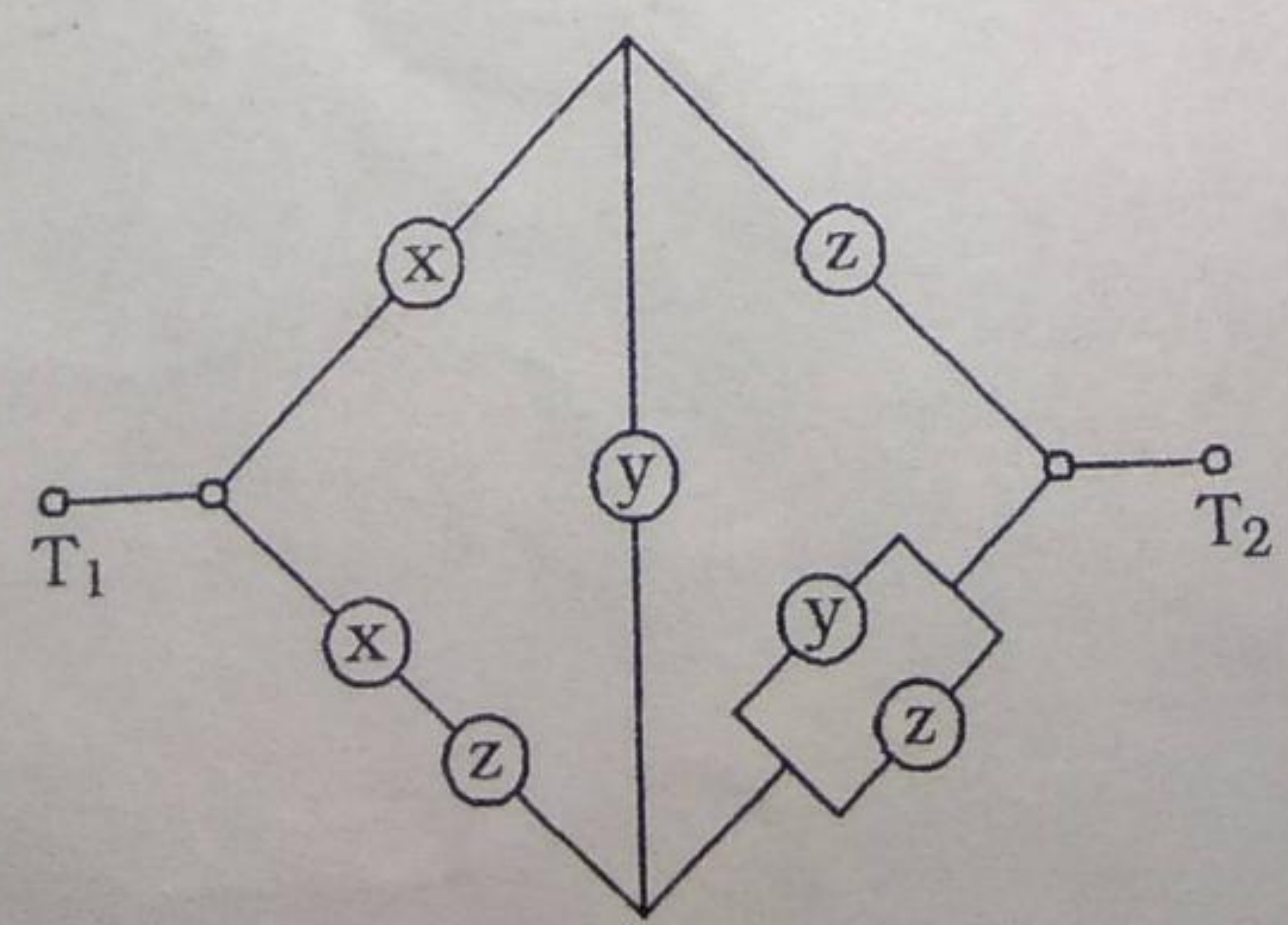
3. (a) Express the following functions into disjunctive normal form :

$f(x, y, z) = (x + y + z) \, . \, (xy + x' \, z)'$

(b) Change the following Boolean function to conjunctive normal form :

$f(x, y, z, t) = (x' \, . \, y + x.y.z'. + x.y'. \, z$
$+ x'. \, y'. \, z'. \, t + t')$

(c) Find the Boolean function of the following circuit and simplify, it possible :



Unit–IV

4. (a) Show that the Relation R = {(x, y) : x − y is divisible by n (n > 1)} defined in the set of positive integers M, where n, x, y ∈ M is a equivalence relation.

(b) Define Partitions and any *one* example of Partitions.

(c) Define Surjective, Bijective Maps and Countable Sets.

Unit–V

5. (a) A connected graph G is an Euler graph if and only if G is the union of some edges–disjoint circuits.

(b) Define Spanning Tree. Any connected graph with n-vertices and (n–1) edges in a tree.

(c) State and prove Euler's formula for a planar graph.

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This question paper contains 5 printed pages.]

Your Roll No. ....

1726 **A**

MCA / II Sem.

Paper MCA - 202 - DISCRETE MATHEMATICS

(Admissions of 2009 and onwards)

Time : 2 Hours *Maximum Marks* :50

(Write your Roll No. on the top immediately on receipt of this question paper.)

*Attempt **all** questions.*

Parts of a question must be answered together.

1. (a) Find the values of $'c'$ & $'n_c'$ such that

$$n^2 + 3n - 4 = 0\,(n^2 - 2n + 3)$$ 3

(b) Prove or disprove

$$f(n) = \omega(g(n)) \Rightarrow g(n) = 0\,(f(n))$$ 2

(c) Write a recurrence relation for the following :

FACTORIAL (n)

1. if $n = 0$

2. return 1

3. else

4. return n* FACTORIAL $(n-1)$. 2

(d) Can the master's theorem be applied to the recurrence

$T(n) = 9T(n/3) + \lg n$?

If yes, solve it. It not, why not? 3

2. (a) For the formulae

(7P

Obtain

(i) Principal conjunctive normal form.

(ii) Principal disjunctive normal form. 3

$\rightarrow R) \wedge (Q \leftrightarrow R)$

(b) Write the following formula in prefix and suffix form

$P \wedge 7R \rightarrow Q \leftrightarrow P \wedge Q$ 2

(c) Let ii tian (x) and campusite (x) be the statements, " x is an II Tian" and " x stays in campus " respectively. Express each of the following statements using quantifiers, logical connectives and above predicates.

(i) Every II Tian stays in campus

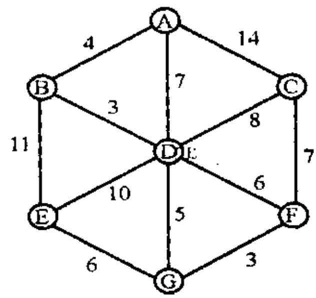
(ii) Some II Tian doesn't stay in campus. 2

(d) Are the following formulae equivalent ?

(i)

(ii) $7(P \leftrightarrow Q)$ and $(P \wedge 7\ \theta) \vee (7PV\theta)$ 3

3. (a) Determine the minimum spanning tree using Prim's Algorithm for the graph given below using 'A' as the initial vertex. 6

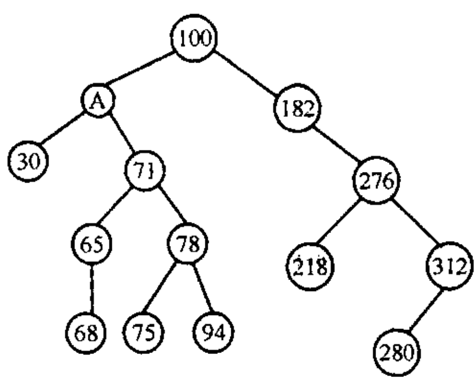


(b) Use Huffman's Algorithm to generate optimal binary prefix codes. The frequency of the characters are given below.

$P \to (\theta \to P)$ and $7P \to (\theta \to P)$

a : 32% , b : 10% , c : 4% , d : 14% , e : 38% and f : 2% 4

4. (a) Given a binary search tree, answer the following questions.



(i) Assuming that tree contains distinct values, what are the valid values for node 'A' ?

(ii) It we want to insert 250, how many comparisons take place ?

(iii) Give the inorder traversal of the tree.

(iv) Show the structure of the tree after deleting 71 from the tree. 4

(b) Prove that for any connected planar graph G ,

$$v - e + r = 2.$$

Is k_4 planar ? What about k_5 ? 4

(c) State the necessary and sufficient condition for an undirected graph to possess an Eulerian path. Does k_4 possesses Eulerian path ? What about k_5 ? 2

5. (a) Is the set of 'N by N' non-singular matrices from a group under matrix multiplication ? Does it form an Abelian group ? 3

(b) How is a group different from a monoid ? Give an example of an algebraic system that forms a monoid but not a group. 2

(c) Consider the following numeric functions.

$$ar = \begin{cases} 0 ; & 0 \le r \le 2 \\ 2^{-r} + 5 ; & r \ge 3 \end{cases}$$

$$br = \begin{cases} 3 - 2^r & ; 0 \le r \le 1 \\ r + 2 & ; \ r \ge 2 \end{cases}$$

Calculate the following :

(i) ab

(ii) $s^4 a$

(iii) ∇a 3

(d) An aircraft takes off after spending 15 minutes on the ground, climbs up at a uniform speed to a cruising altitude of 20,000 feet in 10 minutes, starts to descend uniformly after 110 minutes of flying time, and lands 10 minutes later. Write a numeric function a_r which denotes the altitude of an aircraft in thousands of feet, at the r^{th} minute. 2

PAPER 1 : DISCRETE MATHEMATICS-2018

Time : Three Hours] *[Maximum Marks : 50*

Note : Attempt any two parts from each question. All questions carry equal marks.

UNIT - 1

1. (a) Prove that the following statement is a tautology:

$$(p \Rightarrow q \wedge r) \Rightarrow (\sim r \Rightarrow \sim q)$$

(b) Define the following with example :

(i) Existential quantifiers (ii) Universal quatifiers

(c) Prove that the following are logically equivalent :

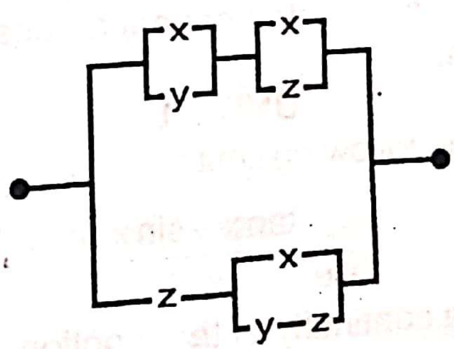
$$p \Rightarrow (q \vee r) \equiv (p \Rightarrow q) \vee (p \Rightarrow r)$$

UNIT - 2

2. (a) For any two elements a and b Boolean algebra B, prove that : (i) $(a + b)' = a' . b'$ (ii) $(a.b)' = a' + b'$

(b) Prove that : $(a + b) + [(a + b'). b]' = 1$

(c) Replace the following circuit by a Simplex one :



UNIT - 3

3. (a) Define minimal Boolean functions and state and prove Boole's theorem.

(b) Express the following into disjunctive normal form :

$$f(x, y, z) = (x + y)(x + y')(x' + z)$$

(c) Write the following function into conjunctive normal form :

$$f(x, y, z) = x.y' + x.z + x.y$$

UNIT - 4

4. (a) If A, B, C, are any three non-empty sets, then prove that

$$A \times (B \cup C) = (A \times B) \cup (A \times C)$$

(b) Show that the relation R defined by $xRy = x^y = y^x$ where $x, y \in I$ is an equivalence relation.

(c) Define the following : (i) Denumerable set (ii) Bijection mapping (iii) Domain and Range of a relation

UNIT - 5

5. (a) Write short notes on the following :

(i) Hamitonian graph (ii) Chromatic number

(iii) Sanning tree

(b) Show that tree with n vertices has (n –1) edges.

(c) Define the following with examples : (i) Binary tree (ii) Planar graph (iii) Isomorphisms in graphs

PAPER 2 : CALCULUS & STATISTICAL METHOD

2018

rks : 50

G/86/20

Online Assignment, 2020

B.C.A. Part –I

BCA-101

Paper –I

(Discrete Mathematics)

Time : 3 Hours] [Max. M. : 80

Note: Attempt any one parts from each unit. All questions carry equal marks.

1. (a) What do you understand by Quantifiers ? Explain its types.

   (b) Show that $p \Rightarrow (q \wedge r) \equiv (p \Rightarrow q) \wedge (p \Rightarrow r)$ is a tautology

2. (a) In a Boolean algebra $(B, +, \cdot, ')$ prove that

   $a \cdot b + [(a + b') \cdot b]' = 1.$

   (b) Draw the logic circuit of $x \cdot y + y \cdot z'$.

3. (a) Change the following boolean function into disjunctive normal form

   $$F(x, y, z) = x \cdot y' + x \cdot z + x \cdot y.$$

   (b) Draw a binomial net for the flow function

   $$x \cdot y + x' \cdot y'.$$

4. (a) If A, B, C are any three non-empty sets, then prove that

   $$A \times (B \cap C) = (A \times B) \cap (A \times C).$$

   (b) If R is an equivalence relation in the set A,

5. (a) Define the following :

   (i) Cycle, (ii) Circuit,

   (iii) Path, (iv) Walk.

   (b) Prove that the sum of the degree of all vertices in a graph G is equal to twice the number of edges in G.

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This question paper contains 8+3 printed pages]

Your Roll No.................

2142

B.Sc.(Hons.)/III C

MATHEMATICS—Paper XII (*ii*)

(Discrete Mathematics)

(Admissions of 2009 and onwards)

Time : 3 Hours *Maximum Marks : 75*

(Write your Roll No. on the top immediately on receipt of this question paper.)

All six questions are compulsory.

Do any *two* parts from each question.

1. (*a*) Let P and Q be ordered sets. Prove that $(a_1, b_1) \longrightarrow\!\!< (a_2, b_2)$ in $P \times Q$ if and only if ($a_1 = a_2$ and $b_1 \longrightarrow\!\!< b_2$) or ($b_1 = b_2$ and $a_1 \longrightarrow\!\!< a_2$). 6

(*b*) (*i*) Let P be an ordered set and $x, y \in P$. Prove that the following are equivalent :

I. $x \leq y$;

P.T.O.

II. $\downarrow x \subseteq \downarrow y$;

III. $(\forall\ Q \in \mathcal{O}(P))\ y \in Q \Rightarrow x \in Q$.

(*ii*) Draw the diagram of $M_3 \times 2$, where 2 denotes chain of two elements. 4, 2

(*c*) (*i*) Let f be a monomorphism from the lattice L into the lattice M. Show that L is isomorphic to a sublattice of M.

(*ii*) Let P be an ordered set and S, T $\subseteq$ P. Assume that $\vee S$, $\vee T$, $\wedge S$ and $\wedge T$ exist in P. Prove that if $S \subseteq T$, then $\vee S \leq \vee T$ and $\wedge S \geq \wedge T$. 4, 2

2. (*a*) Let B be a Boolean algebra. Prove that an ideal M in B is maximal if and only if for any $b \in B$, either $b \in M$ or $b' \in M$, but not both, hold. 6½

(*b*) (*i*) Find the conjunctive normal form of

$$(x_1 + x_2 + x_3)\ (x_1 x_2 + x_1' x_3)'.$$

(*ii*) Let L be a lattice. Prove that the following are equivalent :

$(\mathrm{D})\ (\forall a, b, c \in \mathrm{L}), a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c);$

$(\mathrm{D})'\ (\forall p, q, r \in \mathrm{L}), p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r).$

3½,3

(*c*) (*i*) Use Karnaugh diagram to simplify the following polynomial :

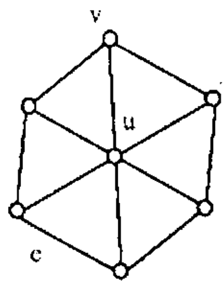
$$p = (x_1 + x_2)(x_1 + x_3) + x_1\ x_2\ x_3.$$

(*ii*) Determine the symbolic representation of the circuit given by :

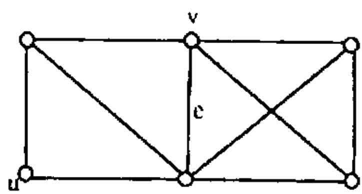
$p = (x_1 + x_3)'\ (x_1' + (x_2 + x_3)(x_2' + x_3'))$ using six gates. 3½,3

3. (a) (i) Define subgraph of a given graph. For each of the graphs below, draw pictures of the subgraphs $G \setminus \{e\}$, $G \setminus \{v\}$, $G \setminus \{u\}$

(a)

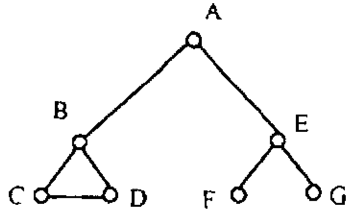


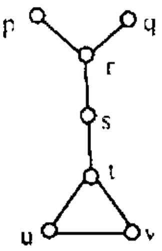
(b)



(ii) Show that the following graphs are isomorphic :

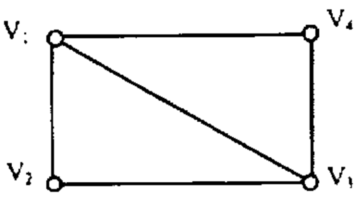
(a)

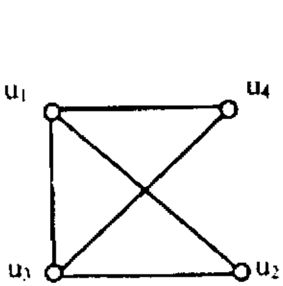


(b) 

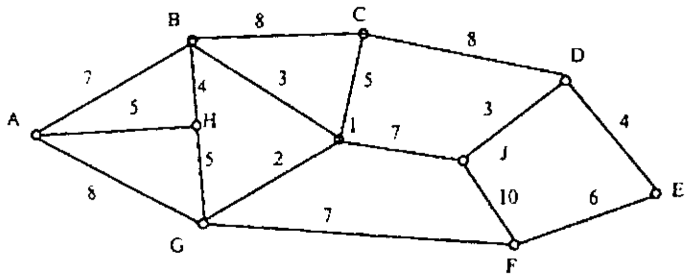
3,3

(b) Find the adjacency matrices A_1 and A_2 of the graphs G_1 and G_2 given below. Find a permutation matrix 'P' such that $A_2 = PA_1P^T$. 6

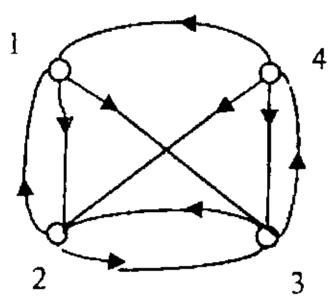




(c) Apply first form of Dijkstra's algorithm to find the shortest path from A to E in the following graph. Write steps. 6



4. (a) Let A be the adjacency matrix of the digraph : 6



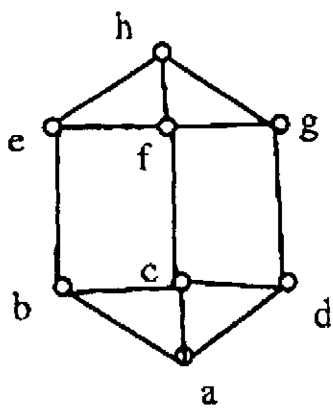
(i) Find A;

(*ii*) Is the digraph strongly connected ? Explain.

(*iii*) Is the digraph Eulerian ? Explain.

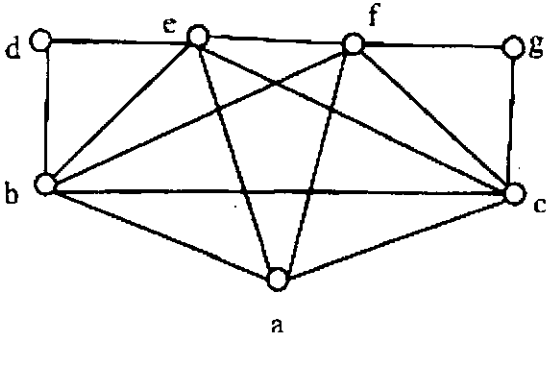
(*b*) Prove that a tree with more than one vertex has at least two leaves. Hence prove that any edge added to a tree must produce a cycle. 6

(*c*) Solve the Chinese Postman problem for the following graph : 6

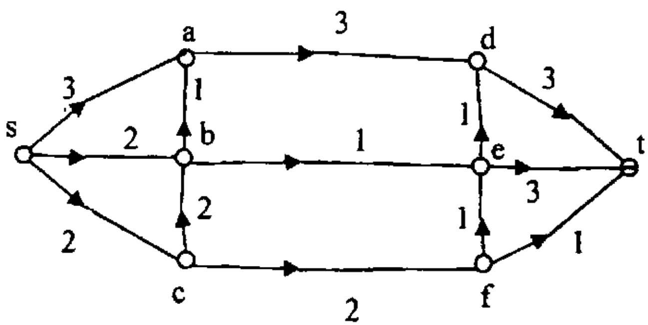


5. (*a*) Let G be a connected planar graph with $V \geq 3$ vertices and E edges. Then prove that $E \leq 3V - 6$. Hence show that K_5 is not planar. 6½

(*b*) (*i*) Compute $\chi(G)$ for the graph shown. Explain your answer and exhibit a $\chi(G)$ coloring.



(*ii*) Is $\chi(K_{m,n})$ the minimum of m and n ? Give reasons. 4,2½ (*c*) Find the maximum flow for the network shown and verify the answer by finding a cut whose capacity equals the value of the flow. 6½



6. (*a*) (*i*) Define $S^{-1}a$, ∇a and Δa . For the numeric function '*a*' given by

$$a_r = \begin{cases} 0, & 0 \le r \le 2 \\ 2^{-r} + 5, & r \ge 3 \end{cases}$$

show that $S^{-1}(\nabla a) = \Delta a$

(*ii*) Let 'a' and 'b' be the numeric functions such that :

$$a_r = 2^r, \; r \geq 0$$

and $$b_r = \begin{cases} 0, & 0 \leq r \leq 2 \\ 2^r, & r \geq 3 \end{cases}$$

Find the convolution $a * b$. 3½, 3

(*b*) (*i*) Find a simple expression for the generating function of the discrete numeric function $0 \times 1, 1 \times 2, 2 \times 3, 3 \times 4 \ldots$

(*ii*) Determine the discrete numeric function corresponding to the generating function :

$$A(z) = \frac{1}{5 - 6z + z^2}.$$ 3½,3

(*c*) (*i*) The solution of the recurrence relation

$$a_r = Aa_{r-1} + B3^{r-1}, r \geq 1 \text{ is } a_r = C2^r + D3^{r-1}, r \geq 0.$$

Given that $a_0 = 19$ and $a_1 = 50$, determine the constants A, B, C and D

(*ii*) Solve the recurrence relation $a_r + 6a_{r-1} + 9a_{r-2} = 3$, given that $a_0 = 0$ and $a_1 = 1$.

3½,3

This question paper contains 7 printed pages]

Your Roll No.................

1478

B.A./B.Sc. (Hons.)/III **A**

MATHEMATICS—Paper XVII & XVIII (III)

(Discrete Mathematics)

Time : 2 Hours *Maximum Marks : 38*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *All* the questions.

Section I

1. Attempt any *two* of the following :

(*a*) Give description of the following graph G(Fig.1)

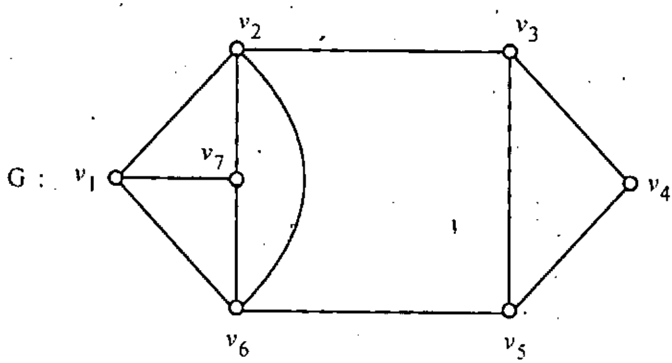


Fig. 1

Check if :

(*i*) G is connected.

(*ii*) $G' : V' = \{v_1, v_2, v_4, v_6\}$

$E' = \{\{v_1, v_2\}, \{v_1, v_4\}, \{v_2, v_6\}, \{v_4, v_6\}\}$

is a subgraph of G. Justify. 3

(*b*) Show that a self-complementary graph must have $4k$ or $4k + 1$ vertices. 3

(*c*) Define an Eulerian path in a graph. Find an Eulerian path in the given graph G (Fig. 2) between vertices A and B : 3

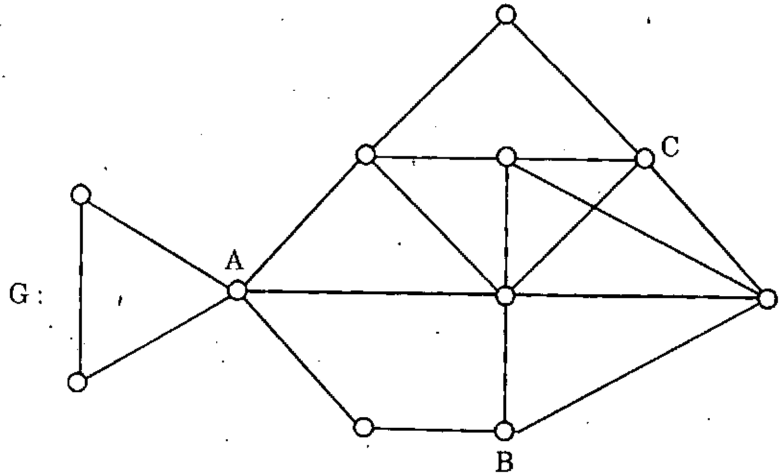


Fig. 2

2. Use nearest neighbourhood method to find a Hamiltonian circuit of minimum length for the following graph (Fig. 3), beginning at the vertex '*a*'. 3½

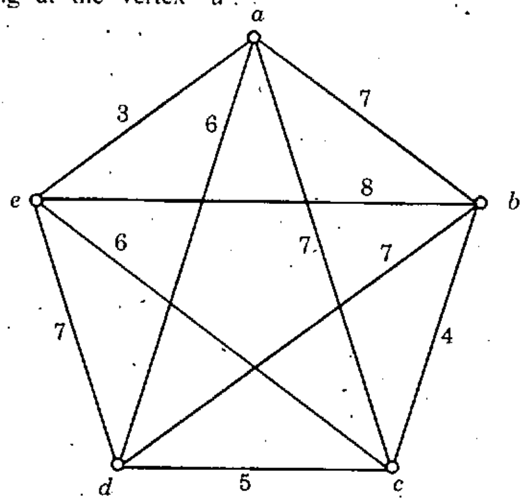


Fig. 3

Section II

3. Attempt any *two* of the following :

(*a*) Represent the following FSM in tabular form : 3

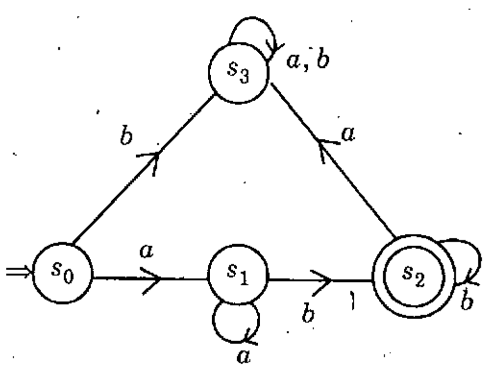


Fig. 4

Also draw the transition diagrams to find out the output sequences corresponding to the input sequences :

(*i*) $a\ a\ b\ a\ a\ b\ b\ a$

(*ii*) $b\ a\ a\ a\ b\ b\ b\ a.$

(*b*) Design a FSM with input symbols as 0, 1 and which accepts sequences having even numbers of 1's. 3

(*c*) Which type/types of strings are accepted by the following FSM : 3

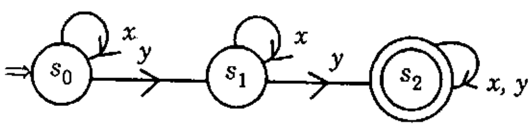


Fig. 5

4. Reduce the following FSM shown in the table, to an equivalent FSM with smallest number of states : 3½

State ↓	Input		Output ↓
	0	1	
A	B	H	0
B	F	D	0

C	A	F	0
D	A	G	0
E	D	B	1
F	C	B	1
G	D	B	1
H	C	A	0

Section III

5. Attempt any *two* of the following :

(*a*) Show that there is no (12, 8, 3, 2, 1) configuration. 3

(*b*) Show that for a (b, v, r, k, λ) configuration

(*i*) $vr(k-1)\lambda = r^2(k-1)^2 + r(k-1)\lambda$

(*ii*) $(k-1)\lambda = (k-1)r - (v-k)\lambda.$ 3

(*c*) Every particle inside a nuclear reactor splits into two particles in each second. Suppose one particle is injected into the reactor every second beginning at $t = 0$. How many particles are there in the reactor at the nth second ? 3

6. From the seven-point plane, construct a code system of 16 words, which can detect upto 3 errors and correct upto 1 error. 3½

Section IV

7. Attempt any *two* of the following :

(*a*) Solve the recurrence relation 3

$$a_n = 4a_{n-1} - 4a_{n-2} \ (n \geq 2)$$

$$a_0 = 1, \ a_1 = 3.$$

(*b*) Find the generating function corresponding to the discrete numeric function : 3

$$2, 0, 2, 0, 2, 0, \ldots\ldots .$$

(c) Given that $a_0 = 0$, $a_1 = 1$, $a_2 = 4$, $a_3 = 12$, satisfy the recurrence relation :

$$a_r + c_1 a_{r-1} + c_2 a_{r-2} = 0,$$

determine a_r. 3

8. Solve the recurrence relation :

$$a_n = 4a_{n-1} - 4a_{n-2} + 4^n, \; n \geq 2$$

with initial conditions $a_0 = 2$, $a_1 = 8$. Also, obtain an expression for the corresponding generating functions. 3½

H-98-21 **Roll No. ....**

ANNUAL EXAMINATION, 2021

B.C.A. I

B.C.A. 101

Paper I

(Discrete Mathematics)

Time : 3 Hours] [Maximum Marks : 80

Note : *Attempt any two parts from each unit. All questions carry equal marks.*

Unit-I

1. (a) Prove that :

$(p \Leftrightarrow q) \wedge (q \Leftrightarrow r) \Rightarrow (p \Leftrightarrow r)$ is a tautology.

(b) Show that :

$\sim (p \Rightarrow q) \equiv p \wedge (\sim q)$.

(c) Explain the universal and existential quantifiers and also explain its negation.

Unit-II

2. (a) Prove the following identity in a Boolean algebra $(B, +, ., ')$

$(a + b).(a' + c) = a.c + a'.b \;\; \forall a.b, c \in B$.

(b) Draw the logic circuit for the following expression.

$f \equiv (a + b).(a' + b' + c').(b'.c)$.

(c) Draw a circuit for the following Boolean function and replace it by a simpler one :

$F(x, y, z) = x.z + [y.(y' + z).(x' + x.z')]$

Unit-III

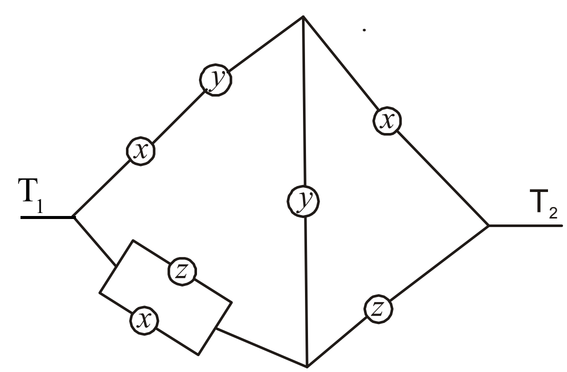
3. (a) Write the following functions into conjunctive normal form

$f(x, y, z) = x.y' + xz + xyz.$

(b) Change the following function to disjunctive normal form :

$f(x, y, z, t) = [x'.y + x.y.z' + xy'z + t]$.

(c) Simplify the following circuit :



Unit-IV

4. (a) Show that the relation "x R $y \Leftrightarrow x - y$ is divisible by 3" where $x, y \in$ I defined in the set of integer I is an equivalence relation.

(b) Let A = {–2, –1, 0, 1, 2} and f : A $\rightarrow$ Z (set of integers) be given by $f(x) = x^2 - 2x - 3$ find (a) the range of f, (b) pre-images of 6, –3, –5.

(c) Let f : A $\rightarrow$ B if function f is one-one onto, then show that f^{-1} is also one-one onto.

Unit-V

5. (a) Show that a complete graph with five vertices is not a planar graph.

(b) Show that a simple graph with n vertices has $\frac{n\,(n-1)}{2}$ maximum number of edges.

(c) Explain the spanning tree of a given graph.

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This question paper contains 4 printed pages.

Your Roll No. ....

Sl. No. of Ques. Paper: **8372** **HC**

Unique Paper Code	**: 32357505**
Name of Paper	**: Discrete Mathematics**
Name of Course	**: Mathematics : DSE for Hons.**
Semester	**: V**
Duration	**: 3 hours**
Maximum Marks	**: 75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any two parts from each questions.

Section I

1 (a) Define 'covering relation' in an ordered set. Prove that if P *and* Q are two ordered sets, then (a_2,b_2) covers (a_1,b_1) in $P\times Q$ if and only if either ($a_1=a_2$ and b_2 covers b_1) or (a_2 covers a_1 and $b_1=b_2$). (6)

(b) Let N_0 be the set of whole numbers equipped with the partial order $\leq$ defined by $m\leq n$ if and only if m divides n. Draw a Hasse diagram and find out maximal and minimal elements, if they exist, for the subset $\{2,3,4,6,10,12,0\}$ of $(N_0, \leq)$. Does it have the smallest and the greatest elements? Justify your answer. (6)

(c) Define an order isomorphism for ordered sets. Show that every order isomorphism is bijective but the converse is not true. (6)

2 (a) Let $(L,\leq)$ be a lattice as an ordered set. Define two binary operations $+$ and $\cdot$ on L by $x+y=x\vee y=\sup\{x,y\}$ and $x\cdot y=x\wedge y=\inf\{x,y\}$. Prove that $(L,+,\cdot)$ is an algebraic lattice. (6.5)

P. T. O.

(b) Let L be a lattice and let $x,y,z\in L$. Prove that

(i) $y\leq z \Rightarrow x\wedge y\leq x\wedge z$ *and* $x\vee y\leq x\vee z$

(ii) $((x\wedge y)\vee(x\wedge z))\wedge((x\wedge y)\vee(y\wedge z))=x\wedge y$ (6.5)

(c) Let $f:L\to K$ be a lattice homomorphism. Show that

(i) If S is a sublattice of L, then $f(S)$ is a sublattice of K.

(ii) If T is a sublattice of K and $f^{-1}(T)$ is non-empty, then $f^{-1}(T)$ is a sublattice of L. (6.5)

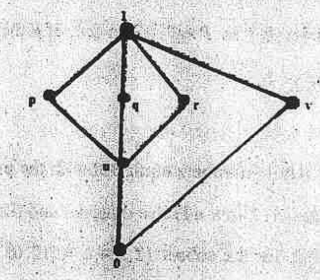
Section II

3 (a) Prove that a lattice L is distributive if and only if $\forall a,b,c\in L$ we have

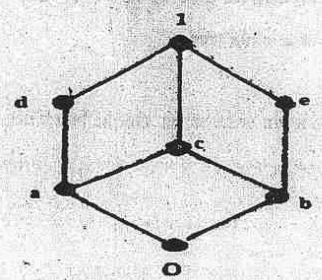
$$(a\vee b=c\vee b \text{ and } a\wedge b=c\wedge b)\Rightarrow a=c.$$

(6)

(b) Use M_3-N_5 Theorem to find if the lattices L_1 and L_2 given below are modular or distributive:



L_1



L_2

(6)

(c) Find the Conjunctive Normal form of

$$(x_1 + x_2 + x_3)(x_1x_2 + x_1'x_3)'$$

(6)

4 (a) Define sectionally complemented lattice. Show that every Boolean Algebra is sectionally complemented.

(6.5)

(b) Find all the prime implicants of $xy'z + x'yz' + xyz' + xyz$ and form the corresponding prime implicant table.

(6.5)

(c) Draw the contact diagram and give the symbolic representation of the circuit given by

$$p = (x_1 + x_2 + x_3)(x_1' + x_2)(x_1x_3 + x_1'x_2)(x_2' + x_3).$$

(6.5)

Section III

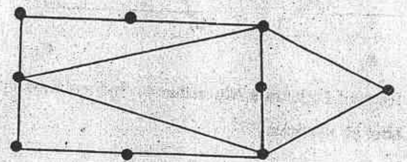
5 (a) (i) Answer the Königsberg bridge problem and explain your answer with graph.

(ii) Draw $K_{3,6}$ and $K_{4,4}$.

(3, 3)

(b) (i) Draw a graph with 5 vertices and as many edges as possible. How many edges does your graph contain. What is the name of this graph and how is it denoted?

(ii) What is bipartite graph? Determine whether the graph given below is bipartite. Give the bipartition sets or explain why the graph is not bipartite.

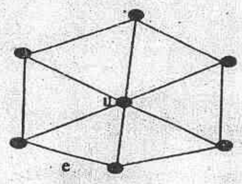


(3, 3)

(c) (i) Draw a graph whose degree sequence is 1,1,1,1,1,1.

(ii) Does there exist a graph G with 28 edges and 12 vertices, each of degree 3 or 4. Justify your answer.

(iii) Draw pictures of the subgraphs G \{e} and G \{u} of the following graph G:



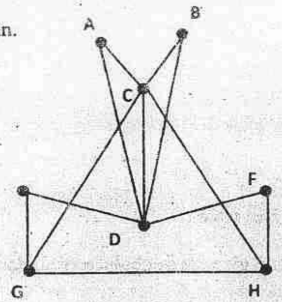
G

(2, 2, 2)

4

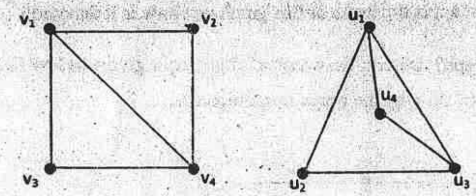
6 (a) (i) Consider the graph G given below. Is it Hamiltonian? If no, explain your answers, if yes find a Hamiltonian cycle.

(ii) Is it Eulerian? Explain.



(4, 2.5)

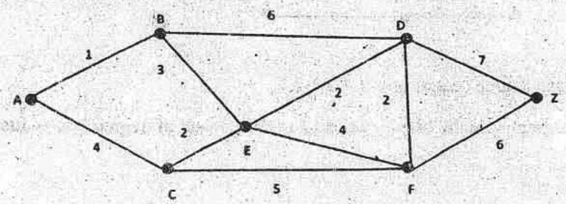
(b) Find the adjacency matrices A_1 and A_2 of the graphs G_1 and G_2 shown below. Find a permutation matrix P such that $A_2 = PA_1P^T$.



G_1 G_2

(6.5)

(c) Apply the first form of Dijkstra's Algorithm to find a shortest path from A to Z in the graph shown. Label all vertices.



(6.5)

1800

I/86—22 Roll No. ....

Annual Examination, 2022

B.C.A. Part I

B.C.A.-101

Paper I

(Discrete Mathematics)

Time : 3 Hours] [Max. Marks : 80

Note : *Attempt any two parts from each unit. All questions carry equal marks.*

Unit-I

1. (a) Show that the following is a tautology :

$$(P \Leftrightarrow q \wedge r) \Rightarrow (\sim r \Rightarrow - P)$$

(b) Show that $[(p \wedge q) \Rightarrow P] \Rightarrow (q \wedge \sim q)$ is a contradiction.

(c) Explain the quantifires and also explain its negations.

Unit-II

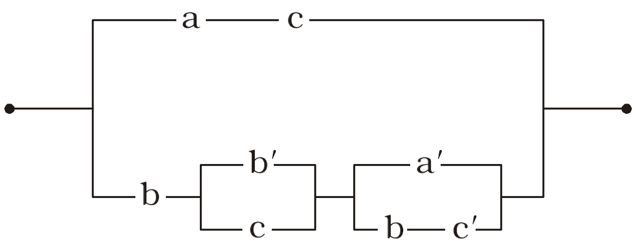
2. (a) In a Boolean algebra (B + ,·, 1) Prove that

$$x.y + y.z + z.x = (x + y)\ (y + z)(z + x)$$

$$\forall\ x, y, z \in B$$

(b) Draw the logic circuit of $f(a, b, c) = (a.b + a.b' + b.c').(b + c)$

(c) Replace the following switching circuit by a simpler are :



Unit-III

3. (a) Find complete disjunctive normal form in three variables and show that its value is :

(b) Write the conjunctive normal form of given boolean function $f(x, y, z) = (x + y)(xy + y'z)'$

(c) Explain wye to delta transformation given an example.

Unit-IV

4. (a) If A, B, C are any three non empty sets, then Prove that : $A \times (B - C) = A \times B - A \times C$

(b) Define the equivalence relation and also give an example.

(c) Let N be the set of natural numbers and E be the set of even number natural numbers. Let $f : N \rightarrow E$ be defined by $f(x) = 2x$, $x \in N$. Show that the map f is a bijection (one-one. onto). Find the formula that defines the inverse function f^{-1}.

Unit-V

5. (a) Show that in any graph the number of odd degree vertices is always even.

(b) Explain simple graph, multigraph. Complete graph and planar graph.

(c) Define the tree and show that a graph is a tree if and only if it is minimally connected.

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