

# Robust video denoising using Low rank matrix completion

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## Abstract

*Most existing video denoising algorithms assume a single statistical model of image noise, e.g. additive Gaussian white noise, which often is violated in practice. In this paper, we present a new patch-based video denoising algorithm capable of removing serious mixed noise from the video data. By grouping similar patches in both spatial and temporal domain, we formulate the problem of removing mixed noise as a low-rank matrix completion problem, which leads to a denoising scheme without strong assumptions on the statistical properties of noise. The resulting nuclear norm related minimization problem can be efficiently solved by many recently developed methods. The robustness and effectiveness of our proposed denoising algorithm on removing mixed noise, e.g. heavy Gaussian noise mixed with impulsive noise, is validated in the experiments and our proposed approach compares favorably against some existing video denoising algorithms.*

## 1. Introduction

With today's advances in sensor design, the image/video is relatively clean for high-end digital cameras at low sensitivities, but it remains noisy for low cost cameras at high sensitivities, e.g., low light condition, high ISO setting and high speed rate. The problem of removing image noise is still of acute and in fact growing importance with the prevalence of webcams and mobile phone cameras. In general, video data tend to be more noisy than single image due to high speed capturing rate of video camera. Video denoising aims at efficiently removing noise from all frames of a video by utilizing information in both spatial and temporal domains. Such an integrated approach is more optimal than independently applying a single-image denoising method on each frame of the video, as there exist high temporal redundancies in a video compared to a single image.

Most existing image/video denoising techniques rely on a single statistical model of image noise, such as i.i.d. Gaus-

sian noise, which is often violated in practice. For example, five major sources of image noise with different statistical distributions have been identified in [14]: fixed pattern noise, amplifier noise, photon shot noise, dark current noise and quantization noise. As a result, the performance of most existing denoising techniques will severely degrade when applied on those real noisy images with noises from multiple sources.

This paper aims at developing a robust video denoising algorithm capable of removing mixed noise from image sequences. The proposed video denoising method is built upon the same methodology "grouping and collaboratively filtering" as many patch-based methods (e.g. [2, 4, 10, 11]) do. Different from existing methods, our proposed algorithm is derived with minimal assumptions on the statistical properties of image noise. The basic idea is to convert the problem of removing noise from the stack of matched patches to a low rank matrix completion problem, which can be efficiently solved by minimizing the nuclear norm ( $\ell_1$  norm of all singular values) of the matrix with linear constraints. It is shown in the experiments that our low rank matrix completion based approach can efficiently remove complex noise mixed from multiple statistical distributions.

### 1.1. Related work

There have been an abundant research literature on image denoising methods. In this section, we will only discuss the most related denoising techniques and interesting readers are referred to [3, 21] for a comprehensive review of the literature. In recent years, patch-based non-local scheme has emerged as one promising approach with very impressive denoising results (e.g. [2, 10, 12, 13, 19]). Although differing from details, these method are built on the same methodology which essentially groups the similar patches together followed by a collaboratively filtering. Take the well-known BM3D ([10]) as a sample. In BM3D, similar image blocks is stacked in a 3D array based on the  $\ell_2$  norm distance function between different patches. Then a shrinkage in 3D transform domain such as wavelet shrinkage or Wiener filter are applied on the 3D block stack. The

denoised image is then synthesized from denoised patches after inverting 3D transform. The result can be further improved by iteratively doing grouping and collaboratively filtering.

Video denoising is different from single image denoising as video sequences usually have very high temporal redundancy which should be effectively used for better performance (*e.g.*, [4, 26, 9, 15, 11, 28]). The basic idea of patch-based image denoising can also be applied on the video by matching similar patches both within the image and over multiple images. The concept of BM3D is generalized to video denoising in [11] by using a predictive search block-matching over time and combined with collaborative Wiener filtering on patch stacks. In [28], a more robust patch matching are proposed by using the depth as a constraint in the matching process and the patch stack is denoising by both PCA (principle component analysis) and Tensor analysis. The idea of sparse coding in a patch dictionary has also been applied on video denoising (*e.g.* [22, 19]), where the denoised image patches are found by seeking for the sparsest solution in a patch dictionary.

Among these patch-based video denoising techniques, most assume data noise is only additive i.i.d. Gaussian noise (*e.g.*, [4, 11, 22]). The image noise mixed with both Gaussian noise and Poisson shot noise are considered in [28]. Regarding impulsive noise, there have been many research works on removing impulsive noise (or salt-and-pepper noise) from a single image (*e.g.* [27, 6]).

## 1.2. The motivation and our work

The performance of a denoising method is highly dependent on how close the real noise in the given data fits the statistical noise model assumed by the method. Most existing image/video denoising techniques consider the i.i.d additive Gaussian noise model, which often is violated in practice. This fact actually prevents these noise removal techniques from being more widely used in practical applications. According to [14], there are five major sources of image noise: fixed pattern noise, amplifier noise, photon shot noise, impulsive noise and quantization noise. Thus, practical image noise is very likely to be the noise mixed from multiple sources. Most existing noise removers are quite sensitive to the noise model violation as they are heavily tuned for one specific type of noise, *i.e.*, the existence of other types of random noise will severely degrade the performance of the noise remover. Thus, sequentially removing different type of random noise using existing techniques is not a working approach. All these inspire us to develop a robust denoising algorithm capable of removing mixed noise from the given video data.

The basic idea of our proposed approach is to only keep those pixels with high reliability and throw away all other un-reliable pixels. In other words, the used patches stack

in our approach will be incomplete with many missing elements. Since all the matched image patches should have similar underlying image structures, the noiseless and complete version of these matched patches lie in a low dimensional subspace. Thus, if we re-arrange the stack of matched patches to a matrix, such a matrix become a noisy version of a low-rank matrix with many missing elements. As a result, the problem of denoising patch stacks is converted to the problem of recovering a complete low rank matrix from its noisy and incomplete observation.

Recently there have been great progresses on solving the problem of low rank matrix completion. As the rank of matrices is not a convex function, the nuclear norm of matrix is used to approximate the rank of matrices, which leads to a convex minimization problem with many efficient methods available (*e.g.* [5, 18, 23]). In our implementation, we use a fixed point iterative algorithm to find a complete low-rank matrix approximation to the given noisy in-complete matrix by minimizing the nuclear norm of the matrix with linear constraints (See [5, 18] for more details).

The structure of the paper is as follows. In Section 2, we describe our formulation of video denoising based on low rank matrix completion. Section 3 discusses the detailed algorithm of our proposed video denoising. The experimental evaluation of the proposed method are given in Section 4. Section 5 summarizes and concludes the paper.

## 2. Problem formulation and solution overview

Let  $\mathcal{F} = \{f_k\}_{k=1}^K$  be the image sequence with  $K$  frames. each image  $f_k$  is a sum of its underlying clean image  $g_k$  and the noise  $n_k$ :

$$f_k = g_k + n_k. \quad (1)$$

The goal of video denoising is to recover  $\mathcal{G} = \{g_k\}_{k=1}^M$  by removing  $n_k$  from  $f_k$ . To exploit the temporal redundancy in a video, we take a patch-based approach to jointly remove image noise  $n_k$  for all image frames. For each image  $f_k$ , consider one image patch  $p_{j,k}$  of size  $n \times n$  (say  $n = 8$ ) centered at pixel  $j$ . We set this patch as a reference patch and search for patches that are similar to  $p_{j,k}$  in all other images and within the neighborhood of the image  $f_k$  itself. How to find accurate patch matching is not a easy task when there exist significant image noise and we will elaborate this more in Section 3. Temporarily, we assume that  $m$  patches  $\{p_{i,j,k}\}_{i=1}^m$  similar to  $p_{j,k}$  are found in both spatial and temporal domain. If we represent each patch  $p_{i,j,k}$  as a vector  $\mathbf{p}_{i,j,k} \in \mathbb{R}^{n^2}$  by concatenating all columns of the patch, we define a  $n^2 \times m$  matrix  $P_{j,k}$  as follows.

$$P_{j,k} = (\mathbf{p}_{1,j,k}, \mathbf{p}_{2,j,k}, \dots, \mathbf{p}_{m,j,k}). \quad (2)$$

Then, we can re-write (1) in the form of patch matrix:

$$P_{j,k} = Q_{j,k} + N_{j,k}, \quad (3)$$

where  $Q_{j,k}$  denote the patch matrix from the clean image  $g_k$  and  $N_{j,k}$  denote the noise.

The image noise  $n_k$  or  $N_{j,k}$  is usually modeled as some random variable with certain statistical properties. Since there are many noise sources (five noise sources are identified in [14]) with different statistical properties, developing the denoising algorithm using strong statistical characterizations of image noise certainly is not an optimal approach in the presence of noise mixed from multiple statistical distributions. The goal of this paper is then to develop an approach which only assumes minimal statistical properties of image noise.

If the data is free of noise and patch matching is also perfect, all column vectors in  $Q_{j,k}$  have similar underlying image structures, the rank of  $Q_{j,k}$  should be low and the variance of each row vector in  $P_{j,k}$  should be very small. In such an ideal case, a good estimation of  $Q_{j,k}$  may be found easily by simply running the standard SVD (singular value decomposition) on  $P_{j,k}$ . In the presence of complex noise, the SVD approach does not yield satisfactory results as it is sensitive to many types of noises. In this paper, we propose a more efficient two-stage approach to robustly estimate  $Q_{j,k}$  which is based on the fact that a complete matrix can be exactly recovered from a small amount of elements under mild conditions ([8]). In other words, since only a small amount of elements are needed to recover the full matrix, we only keep those elements that considered to be very reliable and discard all other elements. In our approach, those matrix elements of  $P_{j,k}$  far away from the mean of its corresponding row vector are considered as highly unreliable elements to be discarded. These elements could be the pixels damaged by impulsive noise, corrupted by Gaussian/Poisson noise with large amplitude or from mismatched patches.

Before we introduce our approach, we first define some notations for the simplicity of discussion. The Frobenius norm of a matrix  $X$  is defined as:

$$\|X\|_F := \left( \sum_{i,j} |x_{i,j}|^2 \right)^{1/2}.$$

The nuclear norm of  $X$  is defined as

$$\|X\|_* := \sum_i (\sigma_i(X)),$$

where  $\sigma_i(X)$  denotes the  $i^{th}$  largest singular value. Let  $X = U\Sigma V^T$  be the SVD for  $X$ . The "soft shrinkage" operator  $D_\tau(X)$  is defined as ([5]):

$$D_\tau(X) = U\Sigma_\tau V^T, \quad (4)$$

where  $\Sigma_\tau = \text{diag}(\max(\sigma_i - \tau, 0))$ . Let  $\Omega$  be an index set and let  $X|_\Omega$  denotes the vector including elements in  $\Omega$  only.

In the first stage of our proposed approach, the reliable elements in  $P_{j,k}$  are identified based on their deviation to the mean of all elements in same row. Let  $\Omega$  denote the index set of all such elements. The main task in the second stage of the proposed approach is then to recover  $Q_{j,k}$  from the incomplete version of  $P_{j,k}$ , denoted by  $P_{j,k}|_\Omega$ , which is actually a matrix completion problem. That is, how to recovering  $Q_{j,k}$  from its noisy and incomplete observation  $P_{j,k}|_\Omega$  under the constraint that the rank of  $Q_{j,k}$  is small. In this paper, we consider the approach to estimate  $Q_{j,k}$  from  $P_{j,k}|_\Omega$  by solving the following minimization problem :

$$\begin{aligned} & \min_Q \|Q\|_* \\ & \text{s.t. } \|Q|_\Omega - P|_\Omega\|_F^2 \leq \#(\Omega)\hat{\sigma}^2, \end{aligned} \quad (5)$$

where  $\|\cdot\|_*$  is the nuclear norm,  $\#(\Omega)$  is the size of the set  $\Omega$ , and  $\hat{\sigma}$  is the estimate of standard deviation of noise from the noisy observations in  $\Omega$ . It is noted that many works on PCA method with missing pixels (e.g. [24, 25]) also can be used to solve the matrix completion problem. The main reason we choose the above minimization approach is for its rigorous mathematical background (e.g. [8, 5]) and the implementation simplicity of some available numerical schemes for solving (5).

### 3. Detailed Algorithm

In this section, we present our two-stage video denoising algorithm in details.

#### 3.1. Patch matching and grouping

Matching similar patches over time is an important problem in video processing with a wide range of applications, e.g., motion estimation, tracking and video compression. There have been extensive research works on efficient patch matching algorithms for motion estimation, (e.g. [17, 20]). Given a reference patch, exhaustive search for similar patches could be very time consuming. As there is a built-in outlier remover in our denoising algorithm, our algorithm is not very sensitive to the accuracy of patch matching. Thus, we adopt a fast three-step hierarchical search algorithm in [17] for its implementation simplicity and computational efficiency.

When the video data is seriously corrupted by image noise, directly applying patch matching algorithms on noisy data is not suitable as the results can be very unreliable. In particular, the performance of patch matching will seriously degrade in the presence of serious impulsive noise. As the pixel corrupted by impulsive noise will be either minimum or maximum intensity (i.e. 0 or 255), the large distortion on pixel value will cause the match score between two patches highly unreliable. Thus, a pre-processing of removing impulsive noise before patch matching will greatly improve the quality of the group of similar patches. In our

implementation, we adopt the adaptive median filter proposed in [16] to identify the pixel corrupted by impulsive noise and replace those damaged pixels by the median of its small neighborhood. In the presence of other types of image noise, the quality of recovered pixel by adaptive mean filter is not good, but adequate for the purpose of patching matching, as observed in our experiments.

Similar to the VBM3D method ([10]), we don't apply patch matching algorithm directly on the raw video data. Instead, a basic (intermediate) estimate of the video data is first obtained by using either some existing denoising technique or using the proposed algorithm, then the patch matching is done by using the intermediately denoised video data, which improves the accuracy of patch matching compared to that using raw un-denoised data.

### 3.2. Denoising patch matrix

For each patch, similar patches are found in both spatial and temporal domain by using the patch matching algorithm described in the previous section to form the matrix  $P_{j,k}$ . The set of missing elements of  $P_{j,k}$  have two subsets: the first subset are those pixels corrupted by impulsive noise using the adaptive median filter based impulsive noise detector ([16]). The second subset includes the pixels whose value differs from the mean of the corresponding row vector by the amount larger than a pre-defined threshold. Then  $\Omega$  is formed by including the index of all remained pixels.

As we discussed in Section 2,  $Q_{j,k}$  is recovered from its incomplete observation  $P_{j,k}|_{\Omega}$  by solving the following minimization problem:

$$\begin{aligned} \min_Q \|Q\|_* \\ \text{s.t. } |Q|_{\Omega} - P|_{\Omega}\|_F^2 \leq \#(\Omega)\hat{\sigma}^2, \end{aligned} \quad (6)$$

where  $\hat{\sigma}$  is the estimate of standard deviation of noise, which is obtained by calculating the average of the variances of all elements  $\in \Omega$  on each row.

Instead of solving (6) directly, we solve its Lagrangian version:

$$\min_Q \frac{1}{2} \|Q|_{\Omega} - P|_{\Omega}\|_F^2 + \mu \|Q\|_*, \quad (7)$$

which is equivalent to (6) for some value of  $\mu$  by the standard duality theory. In this unconstrained formulation (7), the parameter  $\mu$  should be chosen in such a way that the solution of (7) satisfies:

$$|Q|_{\Omega} - P|_{\Omega}\|_F^2 \approx \#(\Omega)\hat{\sigma}^2.$$

Following the heuristic arguments discussed in [7] (See [7] for more details), we set

$$\mu = (\sqrt{n_1} + \sqrt{n_2})\sqrt{p\hat{\sigma}},$$

where  $n_1 \times n_2$  is the size of patch matrix ( $n_1 = n^2$  and  $n_2 = m$  in our case) and  $p$  is the ratio of the number of pixels in  $\Omega$  over the total number of pixels in the patch matrix. .

There are many available efficient algorithms to solve the above minimization (7), the fixed point iterative algorithm (See [5, 18] for more details) is used in our implementation for its simplicity in the implementation. The detailed algorithm is described in Algorithm 1.

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**Algorithm 1** Fixed point iteration for solving the minimization (7)

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1. Set  $Q^{(0)} := 0$ .
2. Iterating on  $k$  till  $\|Q^{(k)} - Q^{(k-1)}\|_F \leq \epsilon$ ,

$$\begin{cases} R^{(k)} = Q^{(k)} - \tau \mathcal{P}_{\Omega}(Q^{(k)} - P), \\ Q^{k+1} = D_{\tau\mu}(R^{(k)}), \end{cases} \quad (8)$$

where  $\mu$  and  $1 \leq \tau \leq 2$  are pre-defined parameters,  $D$  is the shrinkage operator defined in (4) and  $\mathcal{P}_{\Omega}$  is the projection operator of  $\Omega$  defined by

$$\mathcal{P}_{\Omega}(Q)(i, j) = \begin{cases} Q(i, j), & \text{if } (i, j) \in \Omega; \\ 0, & \text{otherwise.} \end{cases}$$

3. Output  $Q := Q^{(k)}$ .
- 

### 3.3. From denoised patches to denoised image

By applying the two-stage algorithm described above on each patch of inputed image frames, we can effectively remove most noises from all patches. The last step is to synthesize the denoised image from these denoised patches. In our implementation, the image patches are sampled with overlapping regions. Thus, each pixel is covered by several denoised patches. Then, the value of each pixel in images is determined by taking the average of denoised patches at this pixel which will suppress the possible artifacts in the neighborhood of the boundaries of patches.

## 4. Experiments

In this section, we evaluate the performance of the proposed method on several video samples corrupted by different types of mixed noise. All the video data used in the experiments can be downloaded from the website ([1]). By default, we use  $K = 50$  image frames, set patch size to be  $8 \times 8$  pixels, sample references patches with sample interval  $4 \times 4$  pixels. Set the range of image intensity to  $[0, 255]$ . For each reference patch, 5 most similar patches are used in each image frame based on  $\ell_1$  norm distance function. Thus, totally 250 patches are stacked for the reference patch

and the column dimension of the matrix in the matrix completion algorithm is 250. The threshold used in selecting reliable pixels from patch matrix is chosen to be  $2\bar{\sigma}$ , where  $\bar{\sigma}$  is an estimation of standard deviation of noise which can be obtained in a similar way as  $\hat{\sigma}$ . In the matrix completion algorithm, the stopping criterion is either the tolerance  $\epsilon \leq 10^{-5}$  or the maximal number of iterations 30 being reached, whichever is reached first.

#### 4.1. Input image data with mixed noise

Since some types of image noises are either much smaller than other noise such as quantization noise or can be pre-calibrated such as fixed pattern noise, we synthesize  $n_k$  in the experiments by the summation of the following three representative type of noise

$$n_k = n_k^g + n_k^p + n_k^i, \quad (9)$$

where  $n_k^g \sim \mathcal{N}(0, \sigma^2 I)$  denote the Gaussian noise (amplifier noise) with zero mean and pixel independent variance  $\sigma^2 I$ ,  $n_k^p$  denotes the Poisson noise (shot noise) with zero mean and variance  $\kappa g_k$ ,  $n_k^i$  denotes impulsive noise by dead pixels, converter or transmission errors and etc. The impulsive noise is modeled as:

$$f_k(i, j) = \begin{cases} \{0, 255\}^3 & \text{with prob. } s, \\ (g_k + n_k^g + n_k^p)(i, j), & \text{with prob. } 1 - s, \end{cases} \quad (10)$$

where  $\{0, 255\}^3$  means that intensity value is either 0 or 255 for all the three channels (color videos are used in our experiments). Noisy image frames with different mixed noise levels are then synthesized by different configurations of the parameters of the above three types of noise:  $(\sigma, \kappa, s)$ .

An experiment is carried out in this section to investigate how the performance of our algorithm varies with respect to different noise levels. As the single Gaussian image noise is not the main focus of this paper, we fixed the Gaussian noise level at  $\sigma = 10$  in this experiment, and varied the Poisson noise level  $\kappa$  in the range of  $[5, 30]$  and impulsive noise level  $s$  in the range of  $[10\%, 40\%]$  respectively. See Fig. 4.1 (a) for the illustration of one frame, its noisy version and initial denoised result. The results are measured by its PSNR value defined by

$$\text{PSNR}(f^r) = 10 \log_{10} \frac{255^2}{\|f - f^r\|_2^2}, \quad (11)$$

where  $f$  is the original data and  $f^r$  is the recovered result.

The PSNR values of one denoised images are listed in Table 1. It is seen from the table that for a fixed Poisson noise level, the PSNR value of the recovered image decreases very little when the impulsive noise level  $s$  increases from 10% to 40%, which implies that our algorithm is quite robust to impulsive noise. In contrast, the PSNR values of



Figure 1. From top to bottom are two original image frames of two video sequences, the corresponding noisy image frames and the initial denoised results after running median filter. The noise level is ( $\sigma = 20, \kappa = 5, s = 30\%$ ) on the left, ( $\sigma = 30, \kappa = 15, s = 20\%$ ) on the right .

$s \setminus \kappa$	5	10	20	30
10%	30.59	28.41	26.28	24.88
20%	30.51	28.37	26.22	24.87
30%	29.83	28.18	25.69	24.36
40%	29.15	27.49	25.03	24.03

Table 1. the PSNR values of the denoised images with respect to different noise levels of Poisson noise and impulsive noise. Gaussian noise level is fixed at  $\sigma = 10$ .

recovered image will decrease noticeably when the impulsive noise level is fixed and the Poisson noise level  $\kappa$  increases from 5 to 30.

#### 4.2. Comparison to other denoising approaches

We applied our proposed denoising method on several videos with different mixed noise levels. The results are compared to that of two existing video denoising methods: one is the VBM3D method ([10]) using the authors' executable code from their website; the other is the PCA-based method by ([28]). it is noted that a depth constrained patch matching is used in [28] to form stacks of patches of high quality. As this paper focuses on how to denoise the

stack of patches, we only implemented the denoising part of [28] and use our own patch matching algorithm to generate stacks of patches. The parameters involved in both methods are set according to the ground truth of noise levels.

In the first experiment, the video data are seriously corrupted by significant mixed noise level:

$$(\sigma = 30, \kappa = 15, s = 20\%).$$

One frame, its noisy version and the initial denoised result by median filter are shown in Fig. 4.1 (b). It is noted that both the VBM3D method ([10]) and the PCA-based method by ([28]) have no built-in impulsive noise remover. Thus, we not only run the unmodified version of both methods on the test data, but also run the modified version of both methods on the test data with a pre-process of removing impulsive noise. The adaptive medium filter method ([16]) is used to remove impulsive noise before applying these two methods. The denoising results are shown in Fig. 2. On the contrary, our proposed algorithm does not use existing impulsive noise remover to remove impulsive noise. Instead, we use the existing impulsive noise detector to detect those damaged pixels which are further refined by a census along the row in the matched patch stack.

Clearly, neither VBM3D method nor PCA-based method is robust to impulsive noise or outliers, as shown in Fig. 2 (a)–(b). With the pre-processing of removing impulsive noise, the results from these two methods are greatly improved, as shown in Fig. 2 (c)–(d). However, as there exist other types of image noises in addition to impulsive noise, the detection accuracy and the estimation accuracy of damaged pixels unavoidably decreases. The outliers caused by the inaccurate detection and estimate of damaged pixels will degrade their performances of deblurring patch stacks. On the contrary, it is seen from Fig. 2 that the proposed approach using low-rank matrix yields most visually pleasant result which is also validated by their PSNR values with 21.5dB, 21.4 dB, and 22.5 dB for the results from BM3D method (Fig. 2 (c)), PCA-based method (Fig. 2 (d)) and our proposed method (Fig. 2 (e)) respectively.

In the second experiments, we only compare our results to that from the methods with impulsive noise pre-processing. Total three data sequences are tested. Each data sequences corrupted by different types of noise. Fig. 3 compared the denoised results for data corrupted by mixed noise with dominant Gaussian noise ( $\sigma = 50, \kappa = 5, s = 10\%$ ), Fig. 4 showed the denoised results for data corrupted by mixed noise with dominant Poisson noise ( $\sigma = 20, \kappa = 25, s = 10\%$ ) and Fig. 5 shows the denoised results for data corrupted by mixed noise with dominant impulsive noise ( $\sigma = 20, \kappa = 5, s = 30\%$ ). It is seen from three figures that overall our proposed algorithm yielded most visually pleasant denoised results. The results from VBM3D method tend to smooth out image details and there are still many noticeable noise left in the results from PCA-based method.

## 5. Conclusions

In this paper, we proposed a robust patch-based algorithm to remove mixed noise from video data. By formulating the video denoising problem to a low-rank matrix completion problem, our proposed algorithm does not assume any specific statistical properties of image noise and is robust to patch matching error. The effectiveness of our proposed algorithm is also validated in various experiments and our method compared favorably against two state-of-art algorithms. In future, we would like to study most robust low-rank matrix completion algorithm with respect to Poisson-type noises. Also, we are interested in investigating the extension of our work to single image denoising.

## ACKNOWLEDGMENTS

The authors would like to thank the anonymous reviewers for the useful comments and suggestions. This work is partially supported by various NUS ARF grants.

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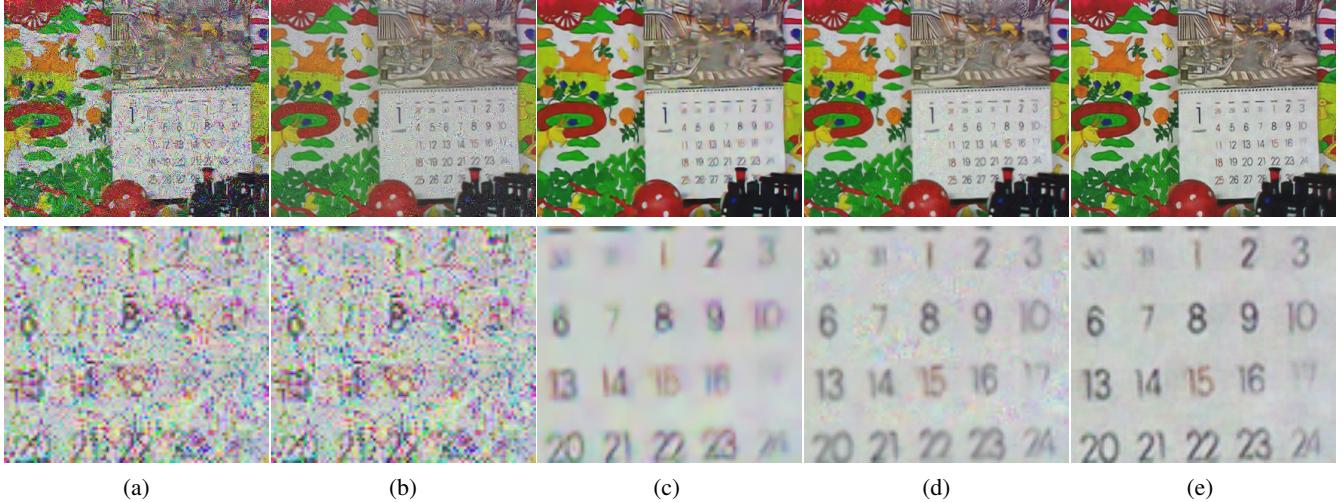


Figure 2. the image frames and their zoomed region are shown in the first and the second row. (a): denoised frame by the VBM3D method without impulsive noise pre-processing; (b) denoised frame by the PCA-based method without impulsive noise pre-processing; (c): denoised frame by the VBM3D method with impulsive noise pre-processing; (d): denoised frame by the PCA-based method with impulsive noise pre-processing; (e): denoised frame by our low-rank matrix completion algorithm.

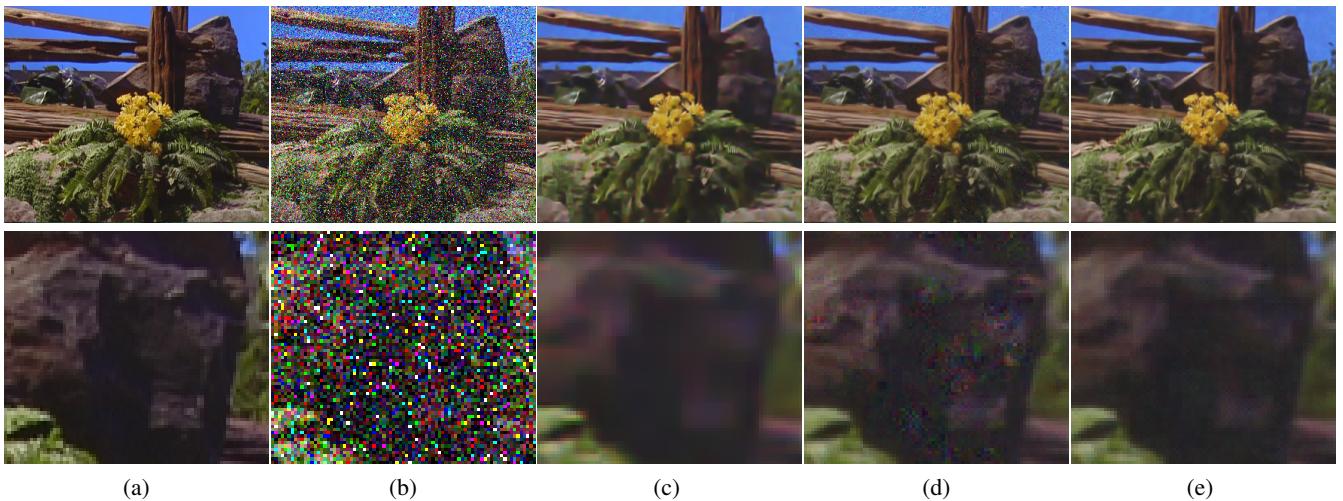


Figure 3. the image frames and their zoomed region are shown in the first and the second row. (a): original frame; (b) noisy frame with ( $\sigma = 50, \kappa = 5, s = 10\%$ ); (c): denoised frame by the VBM3D method; (d): denoised frame by the PCA-based method (e): denoised frame by our low-rank matrix completion algorithm.

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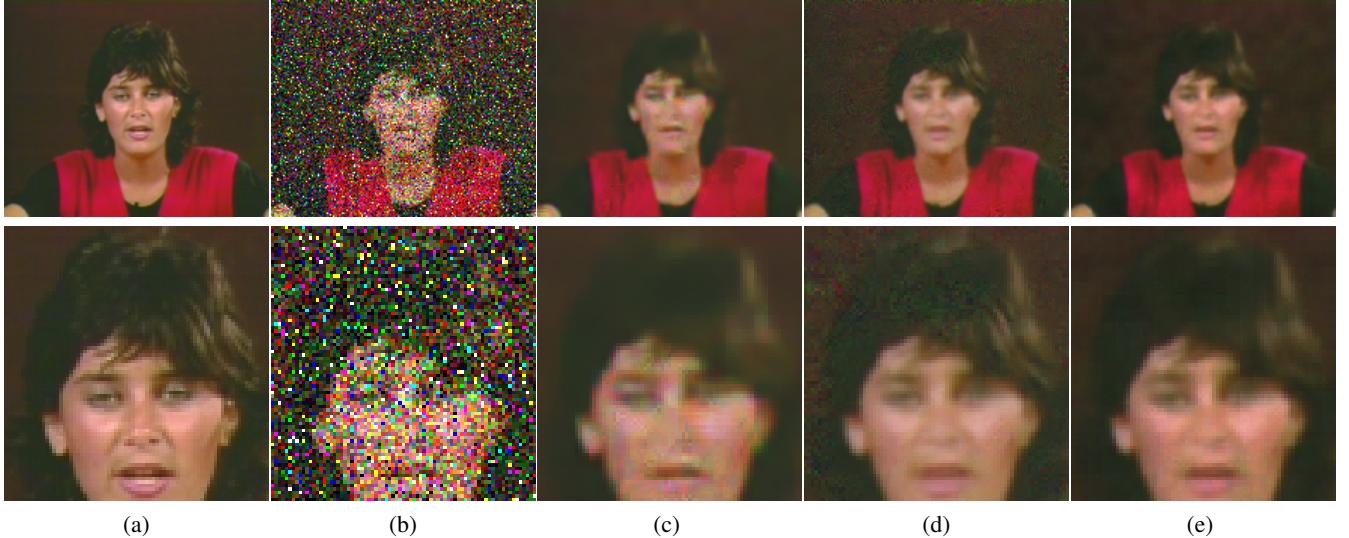


Figure 4. the image frames and their zoomed region are shown in the first and the second row. (a): original frame; (b): noisy frame ( $\sigma = 20, \kappa = 25, s = 10\%$ ); (c): denoised frame by the VBM3D method; (d): denoised frame by the PCA-based method; (e): denoised frame by our low-rank matrix completion algorithm.

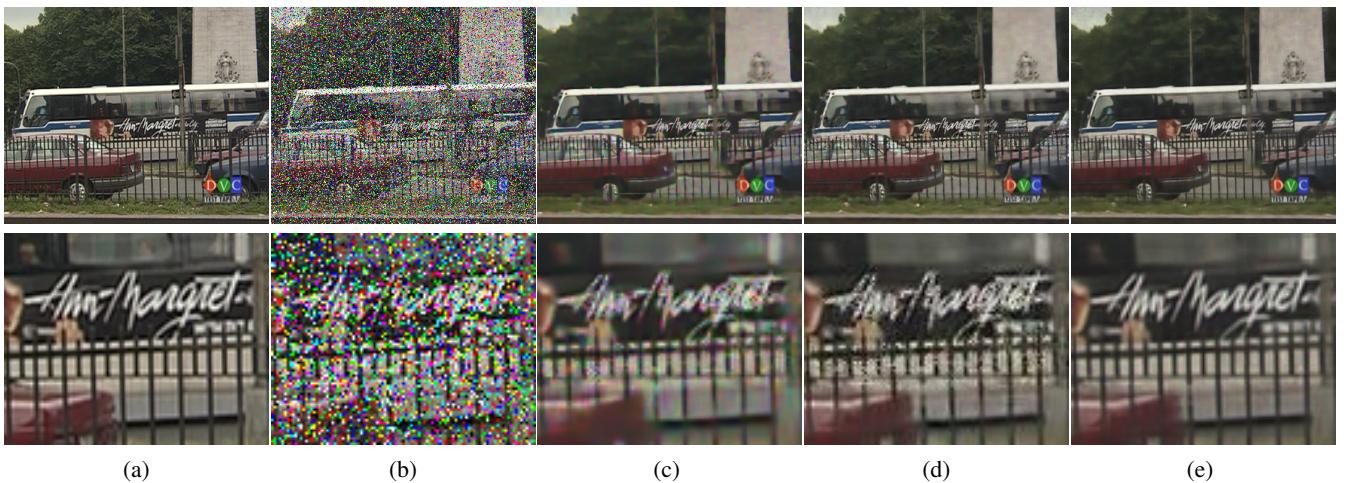


Figure 5. the image frames and their zoomed region are shown in the first and the second row. (a): original frame; (b): noisy frame ( $\sigma = 20, \kappa = 5, s = 30\%$ ); (c): denoised frame by the VBM3D method; (d): denoised frame by the PCA-based method; (e): denoised frame by our low-rank matrix completion algorithm.

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