

MINIMIZING TOTAL TARDINESS

OF 'N' JOBS ON A SINGLE MACHINE WITH RELEASE TIME AND DEADLINE

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Considering a set of 'n' jobs with

- **Processing Time (p_i):**

p_i represents the time required to process job

- **Release Times (r_i):**

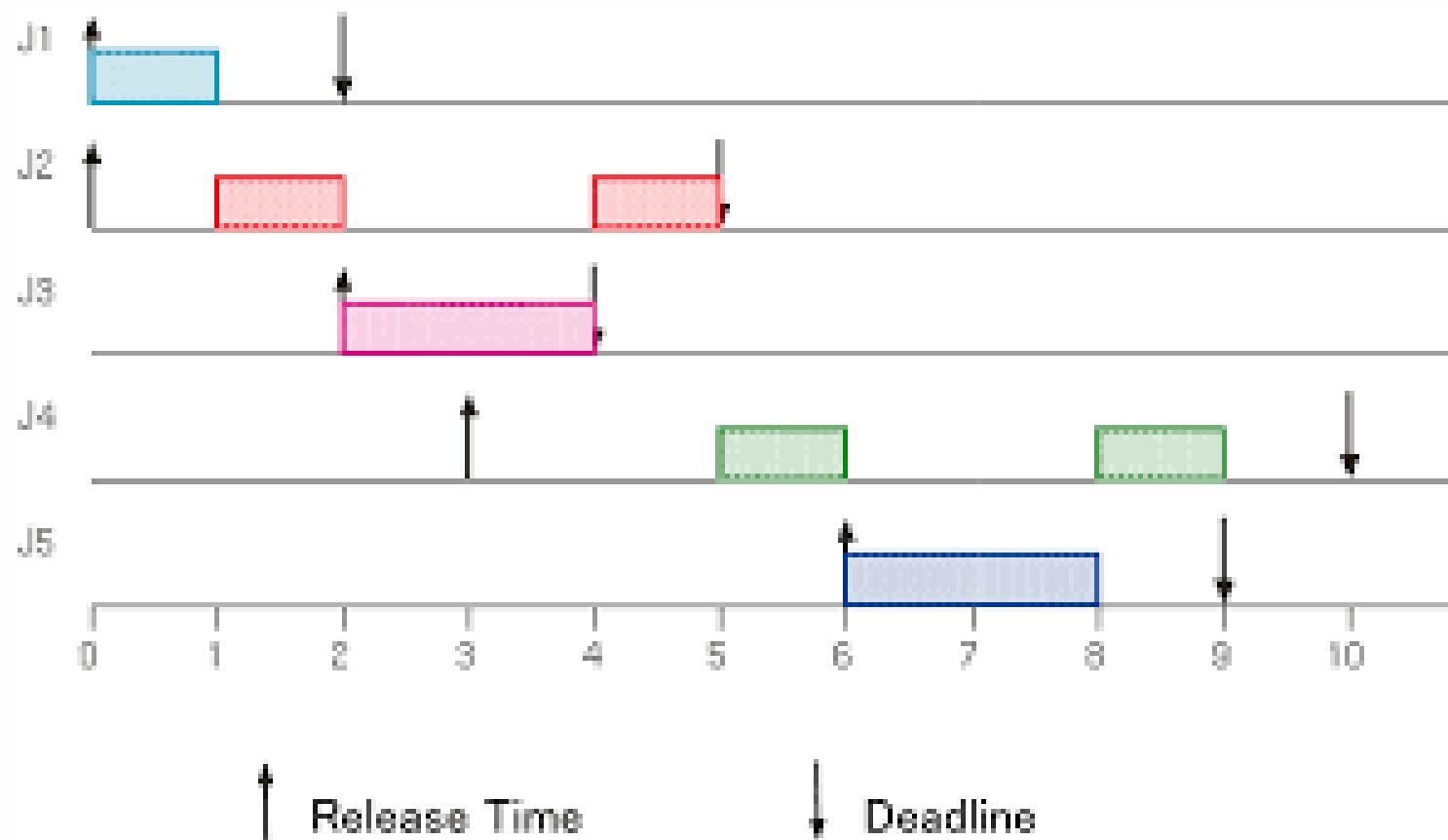
r_i is the earliest time at which job 'i' can start processing

- **Deadlines (d_i):**

d_i is the latest time by which job i should be completed

Objective

The goal is to schedule these jobs on a single machine in a way that **minimizes** the total tardiness/penalty, where the penalty is incurred for each job that is completed after its deadline.



Objective

- Let C_i be the completion time after each job i and T_i be the tardiness of each job i given by $T_i = \max(0, C_i - d_i)$
- The Total Penalty is given by-

$$\sum_{i=1}^n T_i$$

- So our objective is to

$$\text{Minimize } \sum_{i=1}^n T_i$$

Heuristic

- Calculate a **Priority Index (P_i)** for each job:

$$P_i = (\text{Release Time} + \text{Deadline}) - (\text{Processing Time})$$

- **Sort** the jobs based on their Priority Index in ascending order.
- Initialize the schedule with first job in sorted order at its release time
- For each remaining job
 - if $C_i < r_i$, Schedule job at r_i
 - else Schedule job at C_i

Calculating Total Tardiness

- For each scheduled job compute completion time & tardiness.

$$\text{Tardiness}(T_i) = \max(0, \text{Completion_Time}(C_i) - \text{Deadline}(d_i))$$

- Repeat until all jobs are scheduled.

Example

Jobs	r_i	p_i	d_i
A	0	3	10
B	2	2	8
C	1	4	12
D	3	1	6
E	0	5	15

Calculating Priority Indexes-

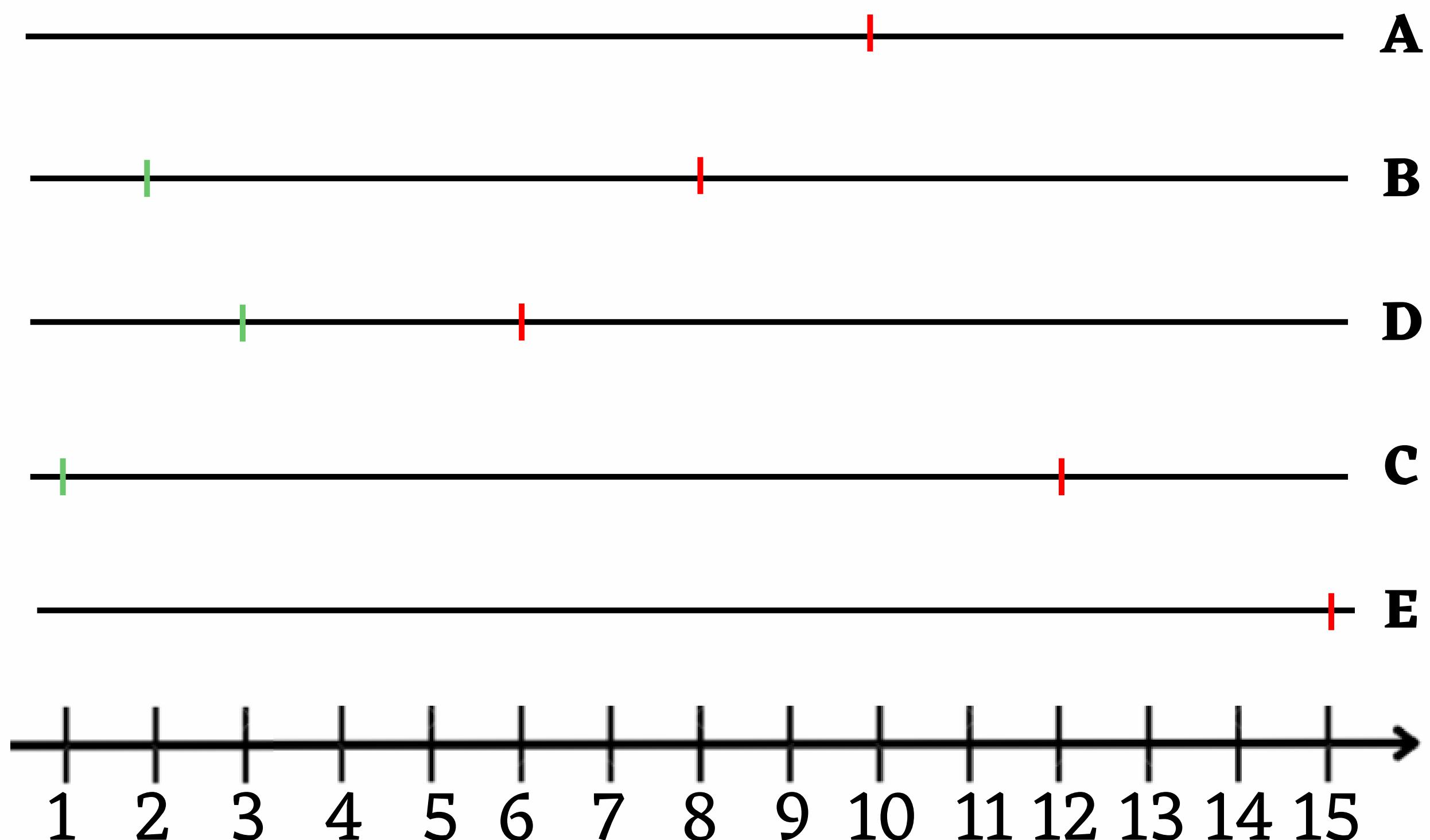
- $\text{PI}(A) = (10 + 0) - 3 = 7$
- $\text{PI}(B) = (8 + 2) - 2 = 8$
- $\text{PI}(C) = (12 + 1) - 4 = 9$
- $\text{PI}(D) = (6 + 3) - 1 = 8$
- $\text{PI}(E) = (15 + 0) - 5 = 10$

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Final Schedule : A → B → D → C → E

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Final Schedule : A -> B -> D -> C -> E



Calculating Total Tardiness

- Tardiness(A) = $\max(0, \) =$
- Tardiness(B) = $\max(0, \) =$
- Tardiness(D) = $\max(0, \) =$
- Tardiness(E) = $\max(0, \) =$
- Tardiness(C) = $\max(0, \) =$

Total Tardiness = + + + + =

Time Complexity

- Calculating PI (Priority Index)
- Sorting Jobs
- Selecting each job

Total Complexity =

THANK YOU !

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