

B.Tech. 3rd Semester (ME) (G-Scheme) Examination,

December-2024

MATHEMATICS-III

Paper-BSC-ME-203-G

PDE, Probability and Statistics

Time allowed : 3 hours ]

[ Maximum marks : 75

*Before answering the questions, candidate should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after the examination.*

*Note : Attempt five questions in all ; selecting one question from each unit. Question No. 1 is compulsory.*

1. (a) Solve  $\frac{\partial^2 z}{\partial x^2} + z = 0$ , given that  $x = 0$ ,  $z = e^y$  and

$$\frac{\partial z}{\partial x} = 1.$$

- (b) Classify the following equations :

$$\frac{\partial^2 u}{\partial x^2} + 4 \frac{\partial^2 u}{\partial x \partial y} + 4 \frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial x} + 2 \frac{\partial u}{\partial y} = 0$$

- (c) Prove that  $\frac{d}{dx} J_0(x) = -J_1(x)$ .

- (d) Express  $2 - 3x + 4x^2$  in terms of Legendre function.
- (e) A bag contains 7 white, 6 red and 8 black balls. Two balls are drawn at random. Find the probability that both will be white.
- (f) Three cities A, B, C are equidistant from each other. A motorist travels from A to B at 30 km/hr, from B to C at 40 km/hr, from C to A at 50 km/hr. Determine the average speed.  $6 \times 2.5 = 15$

## Unit-I

2. (a) Solve the differential equation

$$x^2 (y - z) p + y^2 (z - x) q = z^2 (x - y). \quad 8$$

- (b) Solve  $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial y} - 6 \frac{\partial^2 z}{\partial y^2} = y \cos x.$  7

3. (a) Solve  $(D - 3D' - 2)^3 z = 6 e^{2x} \sin(3x + y).$  8

- (b) Find the deflection of a vibrating string of unit length having fixed ends with initial velocity zero and initial deflection  $f(x) = k(\sin x - \sin 2x).$  7

## Unit-II

4. (a) Solve by the method of separation of variables :

$$\frac{\partial u}{\partial x} = 2 \frac{\partial u}{\partial t} + u, \text{ where } u(x, 0) = 6 e^{-3x}. \quad 8$$

(b) Show that

$$\int_0^1 x^3 J_0(x) dx = 2 J_0(1) - 3 J_1(1). \quad 7$$

5. (a) Prove that

$$(n+1) P_{n+1}(x) = (2n+1) x P_n(x) - n P_{n-1}(x). \quad 7$$

(b) Prove that  $(1 - 2xz + x^2)^{-\frac{1}{2}}$  is a solution of the

$$\text{equation } z \frac{\partial^2 (zv)}{\partial x^2} + \frac{\partial}{\partial x} \left[ (1-x^2) \frac{\partial v}{\partial x} \right] = 0. \quad 8$$

### Unit-III

6. (a) A speaks truth in 60% cases and B in 70% cases. In what percent (%) of cases are they likely to contradict to each other in stating the same fact? 7

(b) Find defective bulbs are accidentally mixed with twenty good ones. It is not possible to just look at a bulb and tell whether or not it is defective. Find the probability distribution of the number of defective bulbs, if four bulbs are drawn at random from this lot. 8

7. (a) The probability that a man aged 60 will live to be 70 is 0.65. What is the probability that out of 10 men now 60, at least 7 would live to be 70? 7



- (b) If the probability of a bad reaction from a certain injection is 0.001, Determine the chances that out of 2000 individuals more than two will get a bad reaction. 8

### Unit-IV

8. (a) Find the mean, median and mode for the following: 7

*Mid value* : 15 20 25 30 35 40 45 50 55

*Frequency* : 2 22 19 14 3 4 6 1 1

- (b) Find the rank correlation coefficient between x and y from the given data : 8

x: 78 89 97 69 59 79 68 57

y: 125 137 156 112 107 138 123 108

9. (a) The first four moments about the working mean 28.5 of a distribution are 0.294, 7.144, 42.409 and 454.98. Calculate the moments about the mean and hence skewness and kurtosis of the distribution. 7
- (b) Samples of sizes 10 and 14 were taken from two normal populations with S.D. 3.5 and 5.2. The sample means were found to be 20.3 and 18.6. Test whether the means of the two populations are the same at 5% level. 8