

# Collaborative Path Planning for Multiple Unmanned Aerial Vehicles to Avoid Sudden Threats

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**Abstract**—This paper concerns collaborative path planning for multiple unmanned aerial vehicles to avoid sudden threats. Firstly, using V-diagram and mission assignment model to obtain multi-UAV path planning in a pre-established environment with obstacles. Then, after a sudden threat, the principle of cubic spline second-order continuity and boundary conditions are used for local planning. After that, setting the total cost function to get the most suitable candidate path. Finally, using the principle of crowding mechanism and setting the priority probability, the secondary security screening is performed to get the optimal candidate path. Simulation results show the effectiveness of the proposed method.

**Index Terms**—V-diagram, mission assignment model, cubic spline, total cost function, secondary security screening

## I. INTRODUCTION

With the rapid development of unmanned aerial vehicle technology and the increasing complexity of the operational environment, single unmanned aerial vehicle operations and path planning without considering sudden threats are no longer sufficient [1]. The assignment of multiple unmanned aerial vehicles can greatly improve the efficiency of operations and reduce unnecessary waste of resources [2]. Therefore, the research about multi-UAV technology is becoming more and more important. Then, under the threat of sudden threats, the research on the paths of unmanned aerial vehicles is the same as above, and has received the attention of experts and scholars at home and abroad. Therefore, the planning of the path under the threat of multiple unmanned aerial vehicles is naturally a hot topic of research. Now, there is less literature and research on this aspect. It is difficult to adjust a large number of aircraft tracks and obstacle avoidance paths after a sudden threat. In recent years, the research on multi-aircraft path planning has yielded some results [3].

The results of this research can be divided into three categories, one is heuristic algorithm, the other is mathematical programming algorithm, and the last one is intelligent optimization algorithm [4]. For example, the clustering algorithm has the advantages of strong fault tolerance, fast calculation speed and easy understanding [5]. However, the algorithm is computationally intensive and requires a high initial value [6]. The results obtained by this algorithm are often not optimal solutions. The enumeration method is based on the idea of

transforming multi-stage decision problems into single-stage optimization problems to reduce the difficulty of decision-making problems [7]. However, its algorithm takes a long time [8]. The particle swarm optimization algorithm is a method to simulate the bird group to find food. The algorithm is simple in calculation, good in robustness, strong in searching ability and fast in convergence [9]. However, the algorithm is prone to premature convergence or stagnation [10]. Most of methods use re-planning algorithms to avoid obstacles which can obtain a safe path but take a long time. Another common algorithm is the RRT algorithm, which is fast planning [11]. However, the obtained path is steep and the smoothness is poor [12]. In this paper, using the V-diagram algorithm as the basis, and using the mission assignment model to make decisions on multiple unmanned aerial vehicles, so that paths can be generated. Then, in the case of sudden threats, multiple candidate paths are generated using the properties of the cubic spline second-order continuity and boundary conditions, and some adjustments are made as needed. After that, the total cost function is set, it includes a security cost function, a smoothness cost function, and a consistency cost function. The candidate path corresponding to the minimum value is the selected path. The last one is that using the principle of crowding mechanism and setting the priority probability, the secondary security screening is performed to get the optimal candidate path. The simulation results show that the algorithm has a fast operation speed, good path smoothness and practicability.

The structure of this paper is as follows. Section II introduces the global plan obtained using the V-diagram and the mission assignment model obtained. In section III, Generate candidate paths in case of sudden threats. In section IV, the establishment of the total cost function and the establishment of the secondary security screening model are proposed. Section V gives simulation results. Section VI concludes this paper.

## II. COLLABORATIVE PATH PLANNING MODEL BASED ON TASK ASSIGNMENT FOR MULTIPLE UAVS

### A. Multiple unmanned aerial vehicles mission assignment model

The principle of air combat is to protect us and to attack the enemy to the greatest extent. Therefore, we need to consider

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two basic factors, the loss of the enemy and our loss [13]. Therefore, a scheme corresponding to the maximum value of the ratio between the benefit and the loss is adopted as the optimum solution. Generally speaking, the value generated by a path varies according to its target point. The reasonable arrangement of tasks and options should be based on the combination of path and task assignment with minimum probability of discovery. Since the collaborative task assignment is closely related to the collaborative path, this paper assigns the task as part of the objective function of the collaborative path plan. Get the formula as follows [13]:

$$F = \max\left(\frac{\sum_{j=1}^U B_j}{\sum_{i=1}^K S_i}\right) \quad (1)$$

Among them,  $B_j$  is the value obtained when attacking the  $j$ -th target,  $S_i$  is the path cost of the  $i$ -th unmanned aerial vehicle,  $K$  is the number of unmanned aerial vehicles, and  $U$  is the targets number.

$B_j$  is calculated according to the following formula:

$$B_j = \rho_j \nu_j + \gamma_j \lambda_j - \rho_j \nu_j \cdot \prod_{i=1}^K (1 - p_{ij})^{w_{ij}} - \gamma_j \lambda_j \cdot \prod_{i=1}^K (1 - p_{ij})^{w_{ij}} \quad (2)$$

Where:  $p_{ij}$  is the killing probability of the  $i$ -th unmanned aerial vehicle for the  $j$ -th target,  $\nu_j$  is the value of the  $j$ -th target, and  $\lambda_j$  represents the degree of threat to our target by the  $j$ -th target.  $\rho_j$  and  $\gamma_j$  are weight values. When the  $i$ -th unmanned aerial vehicle attacks the  $j$ th target,  $w_{ij}$  is 1 and otherwise  $w_{ij}$  is 0.

$$S_i = \alpha_i S_d(i) + (1 - \alpha_i) S_t(i) + P_i \quad (3)$$

Where:  $\alpha_i$  is the weight coefficient,  $S_d(i)$  is the voyage cost of the  $i$ -th track,  $S_t(i)$  is the threat cost of the  $i$ -th track and  $P_i$  is the constraint function of the  $i$ -th track.

$$S_d(i) = \sum_{t=1}^N l_i(t) \quad (4)$$

Where:  $N$  is the number of segments into which path  $i$  is divided,  $l_i(t)$  is the length of the  $t$ -th path segment of the  $i$ -th path.

$$S_t(i) = \sum_{t=1}^N \sum_{m=1}^M \sum_{q=1}^Q k_q(i) P_{qm}(t) \quad (5)$$

Where:  $M$  is the number of points on the  $t$ -th track segment of the  $i$  th track,  $Q$  is the number of threats,  $P_{qm}(t)$  is the threat probability of the  $q$ -th threat to the  $m$ -th track point on the  $t$ -th track segment of the  $i$ -th trajectory,  $k_q(i)$  is the weight coefficient of the  $q$ -th threat for the  $i$ -th trajectory.

$$P_{qm}(t) = \frac{1}{\sqrt{(x_m - x_q)^2 + (y_m - y_q)^2}} \quad (6)$$

In this equation,  $(x_m, y_m)$  is the coordinate value of  $m$  points,  $(x_q, y_q)$  is the coordinate value of the  $q$ -th threat.

$$P(i) = \eta_1 P_y + \eta_2 P_s(i) + \eta_3 P_t(i) \quad (7)$$

Where:  $\eta_1, \eta_2$  and  $\eta_3$  is the weight coefficient,  $P_y$  is the constraint function of the unmanned aerial vehicle itself, including the deflection angle limit, etc.  $P_s(i)$  is the safety constraint function of the  $i$ -th track,  $P_t(i)$  is the time coordination constraint function of the  $i$ -th track.

$$P_s(i) = \begin{cases} 0, & D_{ij} > L_{safe} \\ \frac{1}{D_{ij}}, & else \end{cases} \quad j = 1, 2, \dots, i-1, i+1, \dots, K \quad (8)$$

Where:  $D_{ij}$  is the Euclidean distance between the  $i$ -th unmanned aerial vehicle and the  $j$ -th unmanned aerial vehicle at the same time,  $L_{safe}$  is set to the safe distance.

$$P_t(i) = \max(\sqrt{(T_i - T_j)}), \quad j = 1, 2, \dots, i-1, i+1, \dots, K \quad (9)$$

Where:  $T_1, T_2, \dots$  and  $T_K$  indicate the time taken for the  $K$ -frame unmanned aerial vehicles to reach the target point.

## B. Global Planning Based on V-diagram

Voronoi diagram generation methods mainly include divide and conquer, incremental construction, indirect method and so on. Among them, the indirect method is the most commonly used in the Voronoi diagram generation method [14]. In this paper, the indirect method is used to construct the Voronoi diagram. First threat sources are generated. Then, the growth points sharing the same side in the adjacent Voronoi polygons are connected to each other to obtain the obtained triangle (Delaunay triangle). Then, the vertical line of each side of each triangle is made in the corresponding triangulation, thus forming a Voronoi polygon with the vertices of each triangle as the generation point. In short, Voronoi diagram consists of a set of continuous polygons consisting of vertical bisectors connecting the lines of two adjacent points.

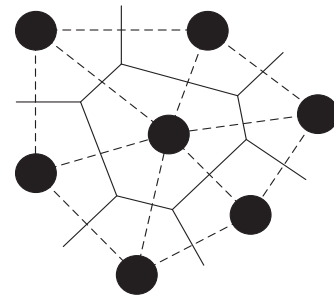


Fig. 1. V-diagram with delaunay triangulation

Where:

The dotted line indicates delaunay triangulation.  
The solid line indicates V-diagram.

### III. CANDIDATE PATH GENERATION

When unmanned aerial vehicles encounter a sudden threat, the original path is no longer applicable. In order to avoid sudden threats and reduce the amount of calculation, this paper adopts the method of local planning. A cluster of candidate paths is generated by using cubic spline second-order continuity and boundary conditions, and the obtained candidate paths are adjusted according to actual conditions to ensure that an effective optimal candidate path is obtained.

$$y = a(x - x_s)^3 + b(x - x_s)^2 + c(x - x_s) + y_s \quad (10)$$

The range of  $x$  is  $x \in (x_s, x_e)$ . Where:  $A(x_s, y_s)$  is the starting point of the candidate path,  $O(x_e, y_e)$  is the end point of the candidate path. The parameters  $a$ ,  $b$  and  $c$  in this paper are obtained by boundary conditions. The parameters in this paper are:

$$\begin{cases} c = 0; \\ a = \frac{c\Delta x + 2(y_s - y_m)}{\Delta x^3} \\ b = \frac{3(y_m - y_s) - 2c\Delta x}{\Delta x^2} \end{cases} \quad (11)$$

Where:  $\Delta x = x_m - x_s$ ,  $x_m$  is the central abscissa of the sudden threat. Different  $y_m$  values will result in different candidate paths, so  $N$  different  $y_m$  values have  $N$  different candidate paths.

In this paper, the function formulas of any two candidate paths are linked together. It is found that the obtained intersection points are not related to  $y_m$ , only related to  $\Delta x$ ,  $x_s$ ,  $y_s$ , so it is proved that all candidate paths pass the point  $A$  and the another point. The another point is treated as a point  $O$ .

However, when the sudden threat and the starting point of the candidate path have an angle  $\alpha$  with the  $x$ -axis, the candidate path is adjusted, and the adjustment formula is:

$$\begin{cases} x_0 = (x - x_s) \cos \alpha - (y - y_s) \sin \alpha + x_s \\ y_0 = (x - x_s) \sin \alpha + (y - y_s) \cos \alpha + y_s \end{cases} \quad (12)$$

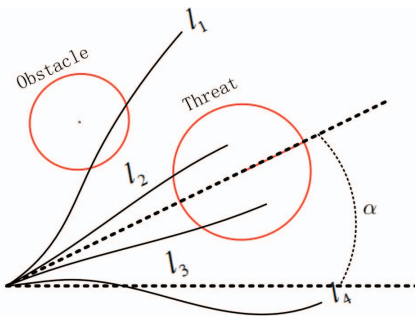


Fig. 2. Candidate path adjustment diagram

### IV. CHOICE OF CANDIDATE PATHS

The security cost function, the smoothness cost function, and the coherency cost function are considered to obtain a suitable candidate path. However, due to the multiple threats in this paper, the obtained candidate paths may appear cross-over.

Therefore, after considering the cost function of the candidate path itself, it is necessary to combine the other environmental information for secondary security screening.

#### A. Security cost function

Safety is a primary consideration for path planning. Unmanned aerial vehicles are threatened by sudden threats and known obstacles. Therefore, the setting function  $R$  represents the relationship between the candidate path and the threat being received. By calculating the Euclidean distance between the sample point on the candidate path and the threat, when the distance is greater than the threat radius,  $R = 0$ , or  $R = 1$ . The obtained data is not discriminative, so the discrete Gaussian convolution is introduced to optimize the  $R$  function, and the formula is:

$$F_{sa}(i) = \sum_{k=-N}^N \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(k-i)^2}{2\sigma^2}} R[k+i] \quad (13)$$

In this formula,  $F_{sa}(i)$  is the security cost function of the  $i$ -th candidate path, and the number of candidate paths is  $2N + 1$ .  $\sigma$  is the standard deviation of collision risk, determining the effective range of collision detection

#### B. Smoothness cost function

Smoothness is also an important factor in path planning. Poor path smoothness will affect the flight efficiency of the UAV, and more serious accidents will occur. Smoothness depends on the curvature of the path. Using this property, the smoothness cost function formula is:

$$F_{sm}(i) = \int |k_i(x)| dx \quad (14)$$

Where:  $k_i(x)$  is the curvature of the  $i$ -th candidate path at the  $x$  position, and the upper and lower lines of the integral are the start and end points of the candidate path. The smoothness cost function can indicate the degree of bending of the path to a certain extent, and the more curved the path is more dangerous.

#### C. Consistency cost function

Both the security cost function and the smoothness cost function consider only the characteristics of the candidate path itself, and do not consider the relationship between the candidate path and the original path. The consistency cost function takes this problem into consideration. Using the Euclidean distance between the candidate path sample points and the corresponding original path sample points and the difference between the corresponding angles, the consistency cost function is obtained as follows:

$$F_{co}(i) = \frac{\lambda_1}{x_s - x_m} \int_{x_m}^{x_s} \sqrt{(x_0 - x_y)^2 + (y_0 - y_y)^2} dx + \lambda_2 \Delta \theta_i \quad (15)$$

Where:  $\Delta \theta_i = \frac{1}{T} \sum_{t=1}^T \|\theta_1(t) - \theta_2(t)\|$ ,  $T$  is the number of sample points,  $\theta_1(t)$  is the deflection angle of the original path and the  $x$ -axis,  $\theta_2(t)$  is the deflection angle of the tangent and the  $x$ -axis on the candidate path,  $x_y$  and  $y_y$  are the

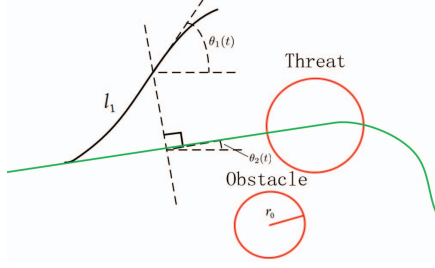


Fig. 3. Schematic diagram of deflection angles  $\theta_1(t)$  and  $\theta_2(t)$

corresponding point on the original path,  $\lambda_1$  and  $\lambda_2$  are weight coefficients.

In Fig.3, the black line is one of the candidate paths, and the green line is the original path.

#### D. Total cost function

The above obtains three cost functions, which are weighted and summed to obtain the minimum value, and the candidate path corresponding to the value is the most suitable candidate path to avoid the sudden threat. The formula is as follows:

$$F_z(i) = \mu_1 F_{sa}(i) + \mu_2 F_{sm}(i) + \mu_3 F_{co}(i) \quad (16)$$

Where:  $\mu_1 + \mu_2 + \mu_3 = 1$ ,  $\mu_1$ ,  $\mu_2$  and  $\mu_3$  are the weights of the security cost function, the smoothness cost function and the consistency cost function, respectively.

#### E. Secondary safety screening

After the most appropriate candidate path is obtained by the total cost function, the path is only applicable to the sudden threat. When multiple sudden threats are encountered, multiple candidate paths are generated. Because the relationship between these different cluster candidate paths is not considered, and the relationship between these candidate paths and other original paths is not considered, crossover may occur. There are two sudden threats in this article. To avoid this phenomenon, a secondary security screening function is set up as follows:

First, consider the relationship between different cluster candidate paths. Simultaneous formula:

$$\begin{cases} y_1 = a_1(x - x_s)^3 + b_1(x - x(s))^2 + c_1(x - x_s) + y_s \\ y_2 = a_2(x - x_s)^3 + b_2(x - x(s))^2 + c_2(x - x_s) + y_s \end{cases} \quad (17)$$

The candidate path  $y_1$  obtained from the total cost function when encountering the first sudden threat, the candidate path  $y_2$  obtained from the total cost function when the second sudden threat is encountered. Then, any two formulas in the above formula are substituted into the following formulas:

$$S_L(r) = y_1 - y_2, \quad r \subseteq (1, 2) \quad (18)$$

If there is a zero point in the formula, it means that there are crossover phenomena between the two candidate paths brought in. Otherwise, there is no crossover phenomenon.

$$F_{ss}(r) = \begin{cases} 1 & S_L(r) \text{ exists zero} \\ 0 & S_L(r) \text{ does not exist zero} \end{cases} \quad (19)$$

Second, consider the relationship between the candidate path and the original path. Simultaneous formula:

$$S_o(r) = y_r - p_j, \quad r \subseteq (1, 2), \quad j \subseteq (1, 2, 3, \dots, W) \quad (20)$$

Where:  $p(j)$  is a path function obtained by smoothing the cubic spline based on the path generated by the V-diagram.  $W$  is the number of paths generated by  $M$  unmanned aerial vehicles obtained through the mission assignment model.

$$F_{so}(r) = \begin{cases} 1 & S_o(r) \text{ exists zero} \\ 0 & S_o(r) \text{ does not exist zero} \end{cases} \quad (21)$$

After that, the result of the logical OR operation of the corresponding candidate path in the  $F_{ss}$  function and the  $F_{so}$  function is obtained.

$$F_{sso}(r) = F_{ss}(r) \parallel F_{so}(r) \quad (22)$$

When  $F_{sso}(r)$  is 1, the corresponding candidate path has an intersection phenomenon with other candidate paths, and the candidate path needs to be discarded to select other candidate paths. Design a priority probability to decide which candidate path to keep preferentially. The probability that the first candidate path is selected is  $P_1$ , and the probability that the second candidate path is selected is  $P_2$ . Then, a candidate path that is preferentially reserved is selected according to the roulette rules. Candidate paths that are not selected need to be replaced by other paths in the cluster of candidate paths in which they are located.

Using the principle of the crowding mechanism: First, the values obtained by the total cost function are used to sort the candidate paths belonging to the same cluster. Then, when the  $F_{sso}(r)$  value of the optimal candidate path is 1, the suboptimal candidate path is selected. Finally, it is repeated to substitute the above equation to determine whether there is still an intersection phenomenon. If this phenomenon exists, the third best candidate path is selected, and so on. Until the qualified candidate path is obtained.

## V. SIMULATION RESULTS AND VERIFICATION

In order to verify the above theory, a simulation experiment was performed below. In this paper, the setting is as follows: three unmanned aerial vehicles are set up as UAV1, UAV2 and UAV3, and the two ground targets are called Target1 and Target2 respectively. Three UAVs were assigned missions to attack two ground targets. The tracks of the UAVs are derived from the v-diagram and the mission assignment model. The blue solid circle is a known obstacle, the red path is the path taken by three unmanned aerial vehicles, as shown in Fig.4, the unit of the coordinate axis is meter.

The following situation is set: UAV2 and UAV3 encounter two sudden threats during flight, generating two clusters of candidate paths. The red solid circle is a sudden threat, these blue paths wrapped with the sudden threat is candidate paths, as shown in Fig.5, the unit of the coordinate axis is meter.

After generating the candidate paths, the optimal path among the two clusters of candidate paths is obtained by the total cost function and the secondary security screening.



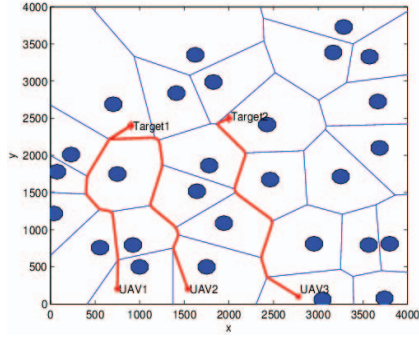


Fig. 4. Simulation rendering with all candidate paths

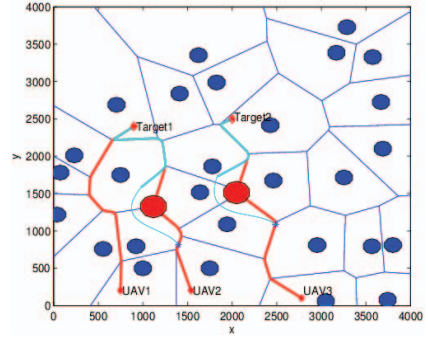


Fig. 7. Simulation rendering

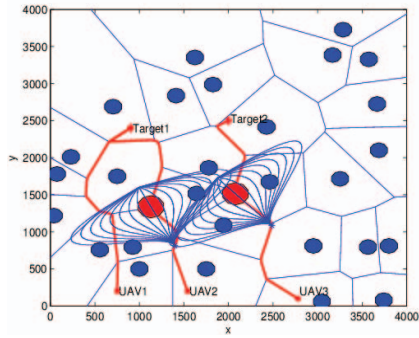


Fig. 5. Simulation rendering with all candidate paths

The cyan path in the candidate path is the selected optimal candidate path, and the thicker cyan path following the optimal candidate path is the path that returns to the original path after the UAV completes the avoidance of the sudden threat, as shown in Fig.6 and Fig.7, the unit of the coordinate axis is meter.

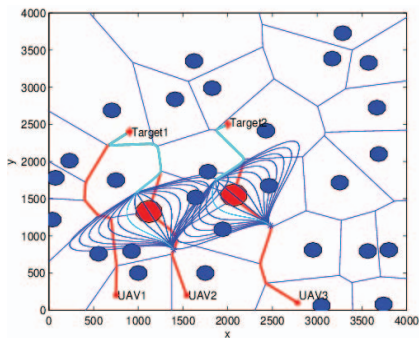


Fig. 6. Simulation rendering with all candidate paths

## VI. CONCLUSION

This paper proposes a local planning algorithm for multi-UAV based on V-diagram and the mission assignment model

for global planning to avoid sudden threats. First, the V-diagram is used for global planning of a single unmanned aerial vehicle. Then, the mission assignment model is used to obtain the global planning path of multiple unmanned aerial vehicles. When a sudden threat is encountered, a cluster of candidate paths is generated according to the cubic spline second-order continuity and boundary conditions, and a total cost function is established to select an appropriate path. When multiple threats are encountered, in order to avoid crossover of the obtained paths, a secondary security screening is performed on the obtained suitable paths. The secondary security screening process is: first the priority probability is set, using roulette to determine the priority reserved path, and then using the crowding mechanism principle to make the other path replaced by other paths in its candidate path cluster. The simulation proves that the proposed algorithm can obtain multiple secure and smooth paths with short time-consuming and high efficiency. Since the threats considered are round and such threats are not representative, the next research direction is the path planning in the presence of threats of other shapes.

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