Harsh Tiwari
230101 0043

Q Test for consistency and solve

i) $2x-3y+7z=5$, $3x+y-3z-13$, $2x+19y-47z=32$.

Sol^n $A\cdot X = B$

$$\begin{bmatrix} 2 & -3 & 7 \\ 3 & 1 & -3 \\ 2 & 19 & -47 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 13 \\ 32 \end{bmatrix}$$

$[A:B]$

$$\left[\begin{array}{ccc:c} 2 & -3 & 7 & 5 \\ 3 & 1 & -3 & 13 \\ 2 & 19 & -47 & 32 \end{array}\right]$$

$R_1 = R_1 - R_2$

$$\left[\begin{array}{ccc:c} -1 & -4 & 10 & -8 \\ 3 & 1 & -3 & 13 \\ 2 & 19 & -47 & 32 \end{array}\right]$$

$R_2 \Rightarrow R_2 + 3R_1$
$R_3 = R_3 + 2R_1$

$$\left[\begin{array}{ccc:c} -1 & -4 & 10 & 8 \\ 0 & -11 & -27 & -11 \\ 0 & 11 & -27 & 32 \end{array}\right]$$

$R_3 \rightarrow R_3 + R_2$

$$\left[\begin{array}{ccc:c} -1 & -4 & 10 & 8 \\ 0 & -11 & -27 & -11 \\ 0 & 0 & 0 & 5 \end{array}\right]$$

$\rho(A) = 2$
$\rho(A:B) = 3$

$\rho(A) \neq \rho(A:B)$
It is inconsistent

Harsh Tiwari
2301010043

$2x - y + 3z = 8 \qquad -x + 2y + z = 4 \qquad 3x + y - 4z = 0$

$$\begin{bmatrix} 2 & -1 & 3 \\ -1 & 2 & 1 \\ 3 & 1 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \\ 0 \end{bmatrix}$$

$$[A:B] \begin{bmatrix} 2 & -1 & 3 & : & 8 \\ -1 & 2 & 1 & : & 4 \\ 3 & 1 & -4 & : & 0 \end{bmatrix}$$

$= R_1 + R_2$

$$\begin{bmatrix} 1 & 1 & 4 & : & 12 \\ -1 & 2 & 1 & : & 4 \\ 3 & 1 & -4 & : & 0 \end{bmatrix}$$

$= R_2 + R_1$

$= R_3 - 3R_1$

$$\begin{bmatrix} 1 & 1 & 4 & : & 12 \\ 0 & 3 & 5 & : & 16 \\ 0 & -2 & -16 & : & -36 \end{bmatrix}$$

$= R_2 + R_3$

$$\begin{bmatrix} 1 & 1 & 4 & : & 12 \\ 0 & 1 & -11 & : & -20 \\ 0 & -2 & -16 & : & -36 \end{bmatrix}$$

$= R_3 + 2R_2$

$$\begin{bmatrix} 1 & 1 & 4 & : & 12 \\ 0 & 1 & -11 & : & 20 \\ 0 & 0 & -33 & : & -76 \end{bmatrix}$$

$$D = \begin{vmatrix} 2 & -1 & 3 \\ -1 & 2 & 1 \\ 3 & 1 & -4 \end{vmatrix}$$

$2(-8-1) + 1(4-3) + 3(-1-6)$

$-18 + 1 - 21 = -38$

$$Dx = \begin{bmatrix} 8 & 1 & 3 \\ 4 & 2 & 1 \\ 0 & 1 & -4 \end{bmatrix}$$

$x = \frac{\cancel{-76}}{\cancel{-32}}\ 2 \qquad x = 2u$

$$Dy = \begin{bmatrix} 2 & 8 & 3 \\ -1 & 4 & 1 \\ 3 & 0 & 4 \end{bmatrix} = 2(16) - 8(7) + 3$$

$(+32 + 56 - 36$

$y = \frac{\cancel{52}}{-38}\ \frac{26}{19}\ \frac{-26}{19}\ 52$

$$Dz = \begin{bmatrix} 2 & -1 & 8 \\ -1 & 2 & 4 \\ 3 & 1 & 0 \end{bmatrix} = 2(-4) + 1(-12) + 2(-7)$$

$-8 - 12 - 56$

$Dz = \frac{-\cancel{76}}{-38} + 2 \qquad D - 76$

$\rho(A) = 3$

$\rho(A:B) = 3$

Since; $\rho(A) = \rho(A:B) = n = 3 \Rightarrow$ It is consistent and have unique solⁿ.

2301010043

(iv) $4x - y = 12$, $-x + 5y - 2z = 0$, $-2x + 4z = -8$

& $(A \vdots B)$ $\begin{bmatrix} 4 & -1 & 0 & \vdots & 12 \\ -1 & 5 & -2 & \vdots & 0 \\ -2 & 0 & 4 & \vdots & -8 \end{bmatrix}$

$D = 4(20) + 1(-4-4)$
$= 80 - 8 = 72$

$R_2 \leftrightarrow R_2$

$$\begin{bmatrix} -1 & 5 & -2 & \vdots & 0 \\ 4 & -1 & 0 & \vdots & 12 \\ -2 & 0 & 4 & \vdots & -8 \end{bmatrix}$$

$R_2 = R_2 + 2R_3$,

$$\begin{bmatrix} -1 & 5 & -2 & \vdots & 0 \\ 0 & -1 & 8 & \vdots & -4 \\ -2 & 0 & 4 & \vdots & -8 \end{bmatrix}$$

$R_3 \to R_3 - 2R_1$

$$\begin{bmatrix} 4 & 5 & -2 & \vdots & 0 \\ 0 & -1 & 8 & \vdots & -4 \\ 0 & -10 & 8 & \vdots & -8 \end{bmatrix}$$

$R_3 = R_3 - 10R_2$

$$\begin{bmatrix} -1 & 5 & -2 & \vdots & 0 \\ 0 & -1 & 8 & \vdots & -4 \\ 0 & 0 & -62 & \vdots & 32 \end{bmatrix}$$ $P(A) = (A \vdots B) = B$

It is consistent & having unique solⁿ.

$Dx = \begin{bmatrix} 12 & -1 & 0 \\ 0 & 5 & -2 \\ -8 & 0 & 4 \end{bmatrix} = 12(20) + 1(-16), 240 - 16 = 224,$

$x = \frac{224}{72}$

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$D_y = \begin{bmatrix} 4 & 12 & 0 \\ -1 & 0 & -2 \\ 2 & -8 & 4 \end{bmatrix} = 4(-16) - 12\ (-4+4) = -48 \quad y = \frac{-48}{72}$

$D_z \quad \begin{bmatrix} 4 & -1 & 12 \\ -1 & 5 & 0 \\ 2 & 0 & -8 \end{bmatrix} = 4(-40) + 1(8) + 12(-10) = -272 \quad z = \frac{-272}{72}$

$9 \quad -160 + 4 \quad -120$

$= 280$

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Q for what values of λ R is the given system of equations

$x+y+z=6$, $x+2y+3z=10$, $x+2y+\lambda z=\mu$ has

i) no solution ii) unique solⁿ iii) Infinite number of solutions

Sol $[A:B]$ $\begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 1 & 2 & 3 & : & 10 \\ 1 & 2 & \lambda & : & \mu \end{bmatrix}$

$R_3 = R_3 - R_2$

$$\begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 1 & 2 & 3 & : & 10 \\ 0 & 0 & \lambda-3 & : & \mu-10 \end{bmatrix}$$

$R_2 = R_2 - R_1$

$$\begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 0 & 1 & 2 & : & 4 \\ 0 & 0 & \lambda-3 & : & \mu-10 \end{bmatrix}$$

ii) for unique soln

$\lambda \neq 3$ & $\mu \neq 10$

i) for no solⁿ :

$\lambda = 3$ & $\mu \neq 10$

iii) Infinite number of solⁿ

$\lambda = 3$, $\mu = 10$

Q for what values of λ the given eqⁿ $x+y+z=1$, $x+y+4z=\lambda$, $x+4y+10z=\lambda^2$ have a solⁿ & solve them completly in each case.

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Sol^n: $[A:B]$ $\left[\begin{array}{ccc:c} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & \lambda \\ 1 & 4 & 10 & \lambda^2 \end{array}\right]$ $R_2 = R_2 - R_1$, $R_3 = R_3 - R_1$

$R_3 = R_3 - 3R_2$

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 1 \\ 0 & -1 & 3 & \lambda-1 \\ 0 & 0 & 0 & \lambda^2 - 3\lambda + 2 \end{array}\right]$$

for system having sol^n

$$-\lambda^2 - 3\lambda + 2 = 0$$

$$\lambda^2 - 2\lambda - \lambda + 2$$

$$\lambda(\lambda - 2) - 1(\lambda - 2)$$

$$(\lambda - 1)(\lambda - 2) = 0$$

$$\lambda = 1, \lambda = 2$$

Q find the sol^n of the system of eq^n $x + 3y - 2z = 0$, $2x - y + 4z = 0$, $x + 1y + 4z = 0$

$A = \begin{bmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 1 & -11 & 14 \end{bmatrix}$ $R_2 \Rightarrow R_2 - 2R_1$, $R_3 \Rightarrow R_3 - R_1$ $\begin{bmatrix} 1 & 3 & -2 \\ 0 & -7 & 8 \\ 0 & -14 & 16 \end{bmatrix}$

$R_3 = R_3 - 2R_2$

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & -7 & 8 \\ 0 & 0 & 0 \end{bmatrix}$$

$P(A)$

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Q find for what values of λ the given eq^n $3x+y-\lambda z=0$, $4x-2y-3z=0$, $2x+4y+\lambda z=0$, may posses on-trivial sol^n & Solve them completly in each case.

So A $\begin{bmatrix} 3 & 1 & -\lambda \\ 4 & -2 & -3 \\ 2\lambda & 4 & \lambda \end{bmatrix}$

$\rightarrow (A)$ $\begin{bmatrix} 3 & 1 & -\lambda \\ 4 & -2 & -3 \\ 2\lambda & 4 & \lambda \end{bmatrix}$

$3(-2\lambda+12) - 1(4\lambda+6\lambda) - \lambda(16-4\lambda)$

$-6\lambda+36-10\lambda-16\lambda-4\lambda^2-0$

$-32\lambda+36-4\lambda^2-0$

$4\lambda^2+32\lambda-36=0$

$\lambda^2+9\lambda-\lambda-9=0$

$\lambda(\lambda+9)-1(\lambda+9)=0$

$(\lambda+9)(\lambda-1)=0$

$\lambda=-9, \lambda=1$

for $\lambda = -9$ & 1 the System of eq has non-trivial sol^n or infinite sol^n.

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Are the following sets of vector the only independent or dependent

i) [1 0 0] , [1 1 0] , [1 1 1]

Solⁿ $v = c_1[100] + c_2[110] + c_3[x1 . 11]$

$$c_1 + c_2 + c_3 = 0$$
$$c_2 + c_3 = 0$$
$$c_3 = 0$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \text{Rank} = 3 = n = \text{trivial . Sol}^n$$

linear Independent

ii) [7 -3 . 1] , [-56 24 - 88 48]

$$v = c_1(7 \ -3 \ 11 \ -6] + c_2[-56 \ -24 \ -88 \ 48]$$

$$7c_1 - 56c_2 = 0$$
$$-3c_1 + 24c_2 = 0$$
$$11c_1 - 82c_2 = 0$$
$$-6c_1 + 48c_2 = 0$$

$$Y = \begin{bmatrix} 7 & -56 \\ -3 & 24 \\ 11 & -88 \\ -6 & 48 \end{bmatrix}$$

$R_2 \to R_2 + 3R_4 + R_3 \to R_3 - 11R_4$

$$= \begin{vmatrix} 7 & -56 \\ 0 & 0 \\ 0 & 0 \\ 1 & -8 \end{vmatrix}$$

$R_1 = R_1 - 7R_4 \qquad R_4 \leftrightarrow R_1$

$$\begin{vmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & -8 \end{vmatrix} = \begin{vmatrix} 1 & -8 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{vmatrix}$$

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$R_1 + R_4$

$$\begin{vmatrix} 7 & -56 \\ -3 & 24 \\ 11 & -88 \\ 1 & -8 \end{vmatrix}$$

$\rho(X) < 2$ = Infinite Sol^n
$\therefore$ The vector is linearly dependent

iv) $[-1, 5, 0]$, $[16\ 8\ -3]$ $[-64\ 56\ 9]$

$$V = c_1[-1\ 5\ 0] + c_2[16\ 8\ -3] + c_3[-64\ 56\ 9]$$

$$c_1 + 16c_2 - 64c_3 = 0$$
$$5c_1 + 8c_2 + 56c_3 = 0$$
$$-3c_2 + 9c_3 = 0$$

$$V = \begin{vmatrix} -1 & 16 & -64 \\ 5 & 8 & 56 \\ 0 & -3 & 9 \end{vmatrix}$$

$$|V| = -1(72 + 168) - 5(144 - 192)$$
$$= -240 + 240 = 0$$

$|V| \neq 0$ = Rank $(V) < 3$ is
$\therefore$ System has Infinite Sol^n

iv) $[1\ -1\ 1]$, $[1\ 1\ -1]$, $[-1\ 1\ 1]$ $[0\ 1\ 0]$

Sol^n $V = c_1[1\ -1\ 1] + c_2[1\ 1\ -1] + c_3[-1\ 1\ 1] + c_4[0\ 1\ 0]$

$$c_1 + c_2 - c_3 = 0$$
$$c_1 + c_2 + c_3 + c_4 = 0$$
$$c_1 - c_2 + c_3 = 0$$

No of Unknown = 4

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$$\begin{vmatrix} 1 & 1 & -1 & 0 \\ -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 0 \end{vmatrix} \quad R_3 \to R_3 + R_2 \quad \begin{vmatrix} 1 & 1 & -1 & 0 \\ -1 & 1 & 1 & 1 \\ 0 & 0 & 2 & 1 \end{vmatrix} \quad R_3 \to R_2 + R_1 \quad \begin{vmatrix} 1 & 1 & 1 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 2 & 1 \end{vmatrix}$$

Rank $= 3 \neq n$

$\to$ Infinite Sol^n $\to$ Linearly dependent

$[2 \ -4]$ $[1 \ 9]$ $[3 \ 5]$

$$V = c_1(2,-4) + c_2[1 \ 9] + c_3[3 \ 5]$$

$$2c_1 + c_2 + 3c_3 = 0$$
$$-4c_1 + 9c_2 + 5c_3 = 0$$

$$\begin{vmatrix} 2 & 1 & 3 \\ -4 & 9 & 5 \end{vmatrix} \quad R_3 \to R_2 + 2R_1 = \begin{vmatrix} 2 & 1 & 3 \\ 0 & 11 & 11 \end{vmatrix}$$

No of unknown $= 3$, Rank $\to 2$

$\therefore$ Infinite Sol^n

$\therefore$ Linearly dependent

$[3 \ -2 \ 0 \ 4]$, $[5 \ 0 \ 0 \ 1]$ $[-6 \ 1 \ 0 \ 1]$ $[2 \ 0 \ 0 \ 3]$

Sol^n $c_1[3 \ -2 \ 0 \ 4] + c_2[5 \ 0 \ 0 \ 1] + c_3[-6 \ 1 \ 0 \ 1] + c_4[2 \ 0 \ 0 \ 1]$

$$3c_1 + 5c_2 - 6c_3 + 2c_4 = 0$$
$$-2c_1 + c_3 = 0$$
$$4c_1 + c_2 + c_3 + c_4 = 0$$

$$\begin{vmatrix} 3 & 5 & -6 & 2 \\ -2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 4 & 1 & 1 & 1 \end{vmatrix} = V$$

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$|V| = 0$

$=$ Rank $< 4 \neq$ no of unknown

$=$ Infinite Sol^n non trivial sol^n

Linearly dependent