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Assignment-2

Q-1 Explain sum of subset Problem. Find all Possible subset of weight of sum to m. Let $n = 8$, $m = 35$ and $w[1:8] = \{5, 10, 12, 13, 15, 17, 18, 20\}$.

Ans Given a set of positive integers, find the combination of number that sum to given value M. Sum of subset. Problem is analogs to knapsack problem in which. we try to fill the knapsack using given set of item to maximize profit. Inequality condition in knapsack. Problem is replace by equality in the sum of subset.

$W = \{w_1, w_2, w_3 \cdots, w_n\}$

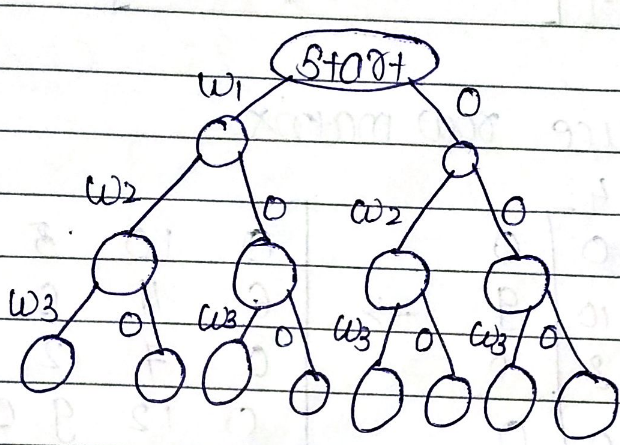
given positive integer M, the sum of subset problem. can be formulated as follows

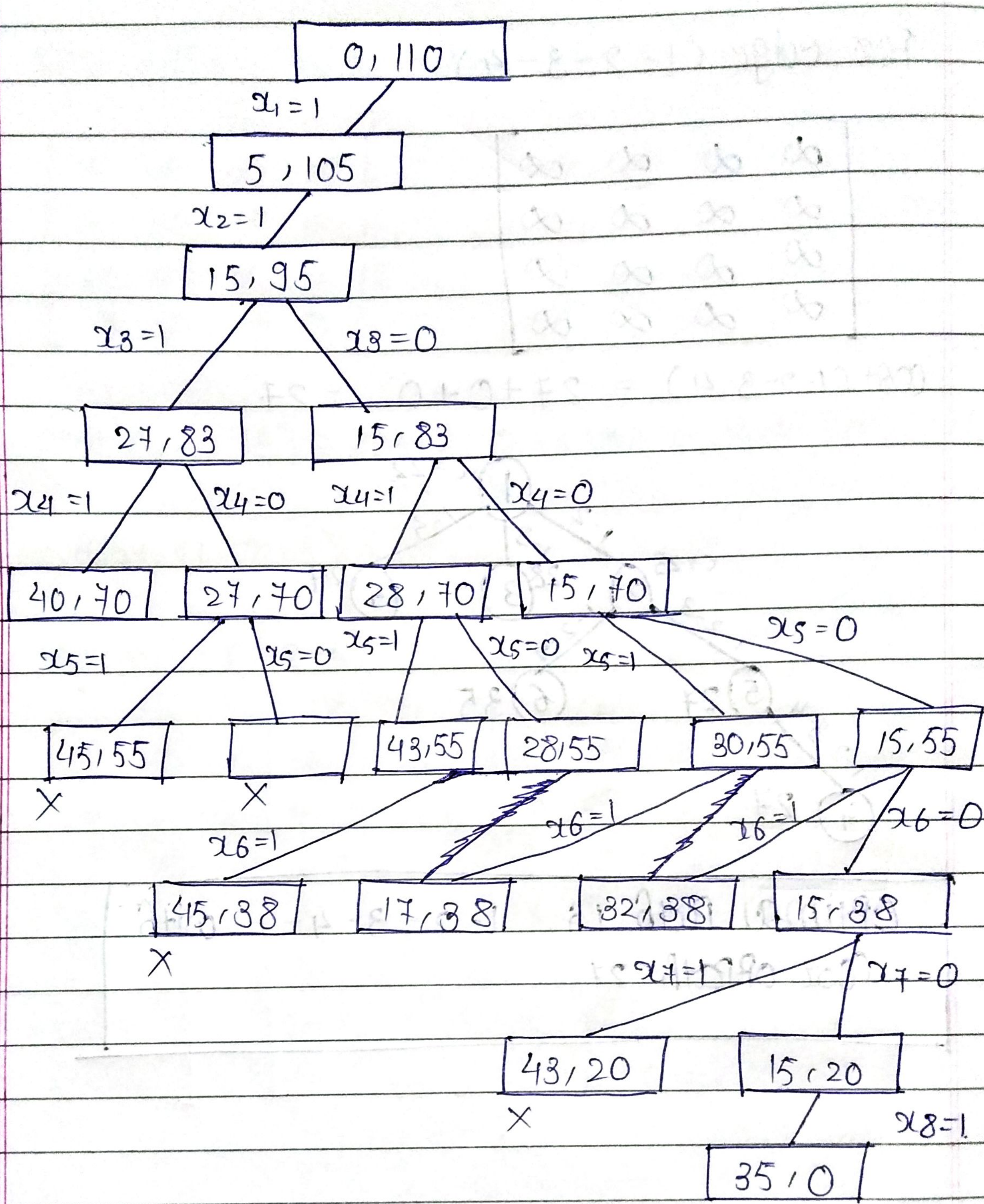
$$\sum_{i=1}^{n} w_i x_i = M \quad \text{where } x_i \in \{0, 1\}$$

* Number are sorted in ascending order, such that $w_1 < w_2 < \cdots < w_n$. The solution is often represented using solution vector X, if the ith item is included, set X_i to 1 else set it to 0. In each iteration, one item is tested. if inclusion of an item does not ~~to~~ violet the constriant of the problem add it.

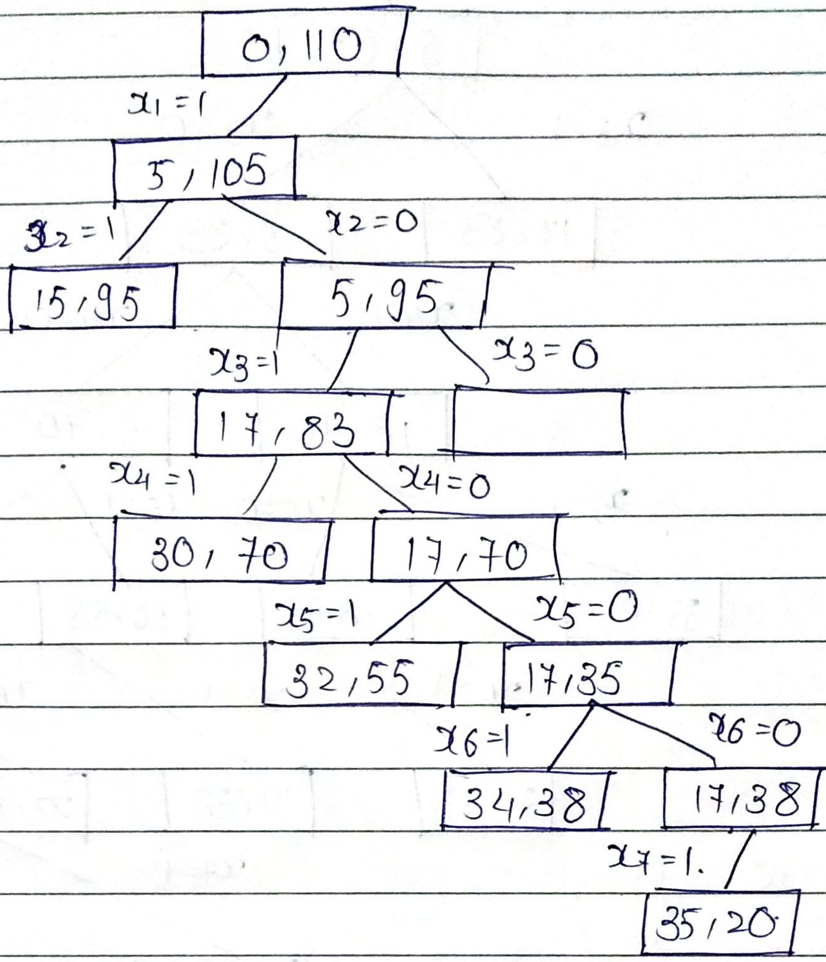
* otherwise, backtrack, remove the previsouly added item and continue the same procedure for all remaining items

* The solution is easily described by state space tree. each left edge denotes the inclusion of ω_i and right edge denote exclusion of ω_i
* Any path from the root to leaf forms a subset. A state space tree for $n=3$ is demonstrated. as follow.

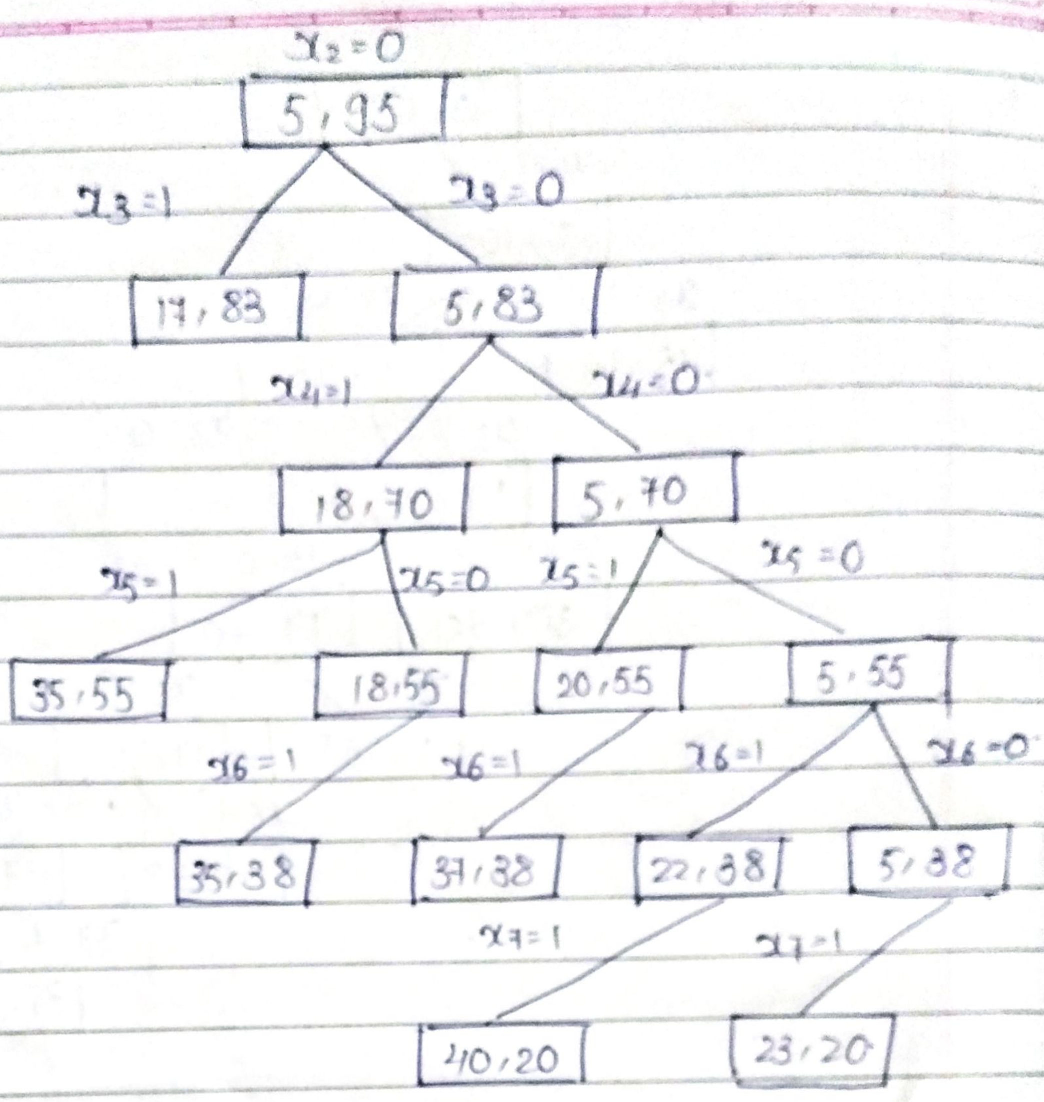




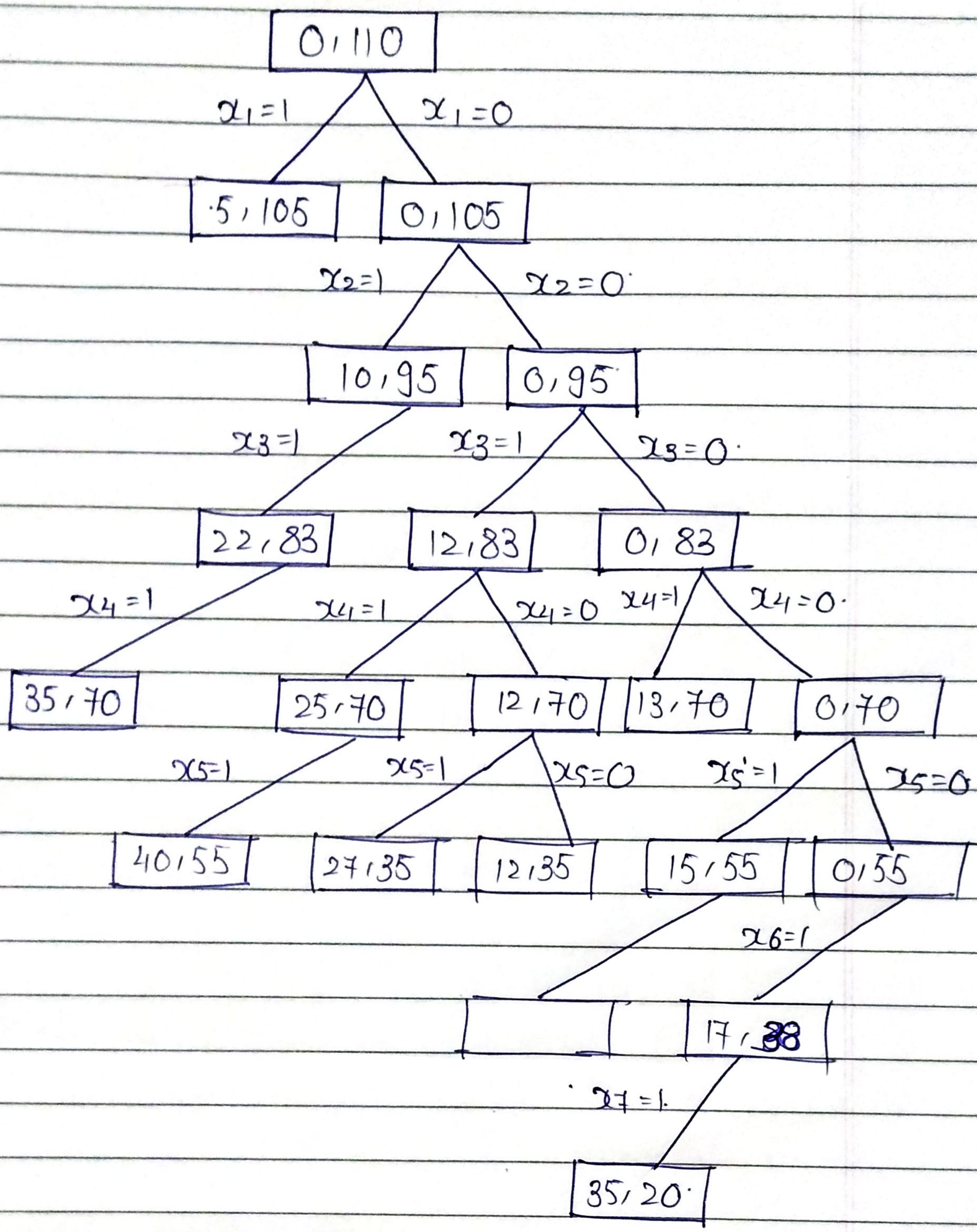
First subset {5, 10, 20}.



Second subset = $\{5, 12, 18\}$.



Third subset = $\{5, 13, 17\}$.



All possible subsets are:

{10, 12, 13}

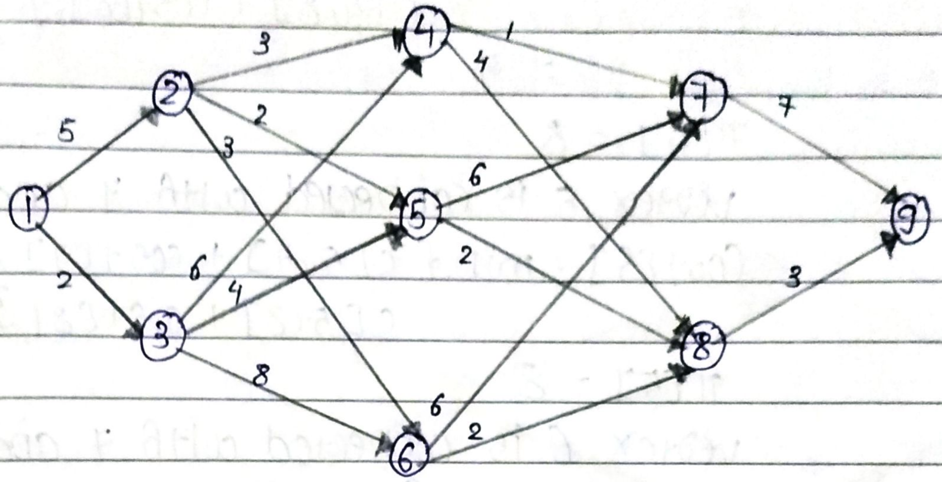
{17, 18}

{5, 13, 17}

{5, 12, 18}

{5, 10, 20}.

Q-2 Find minimum cost path from 1 to 9 in the given graph using dynamic Programming :



⇒ $cost[j] = \min\{c[j,r] + cost[r]\}$

Number of stages $k = 5$, number of vertices $n = 9$

source $s = 1$ and target $= 9$

$cost[n] = 0 \implies cost[9] = 0$

$P[1] = s \implies P[1] = 1$

$P[k] = t \implies P[5] = 9$

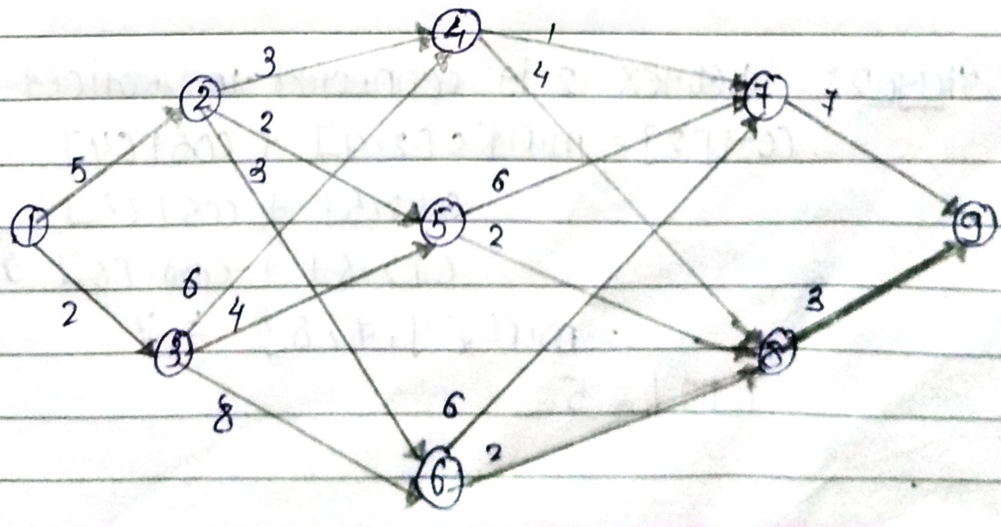
$r = t = 9$

Stage 4: $cost[7] = c[7,9] + cost[9] = 7+0 = 7$

$\pi[7] = 9$

$cost[8] = c[8,9] + cost[9] = 3+0 = 3$

$\pi[8] = 9$



Stage 3: vertex 4 is connected with 7 and 8

cost[4] = min{ c[4,7] + cost[7],
c[4,8] + cost[8] }

= 7.

π[4] = 8

vertex 5 is connected with 7 and 8

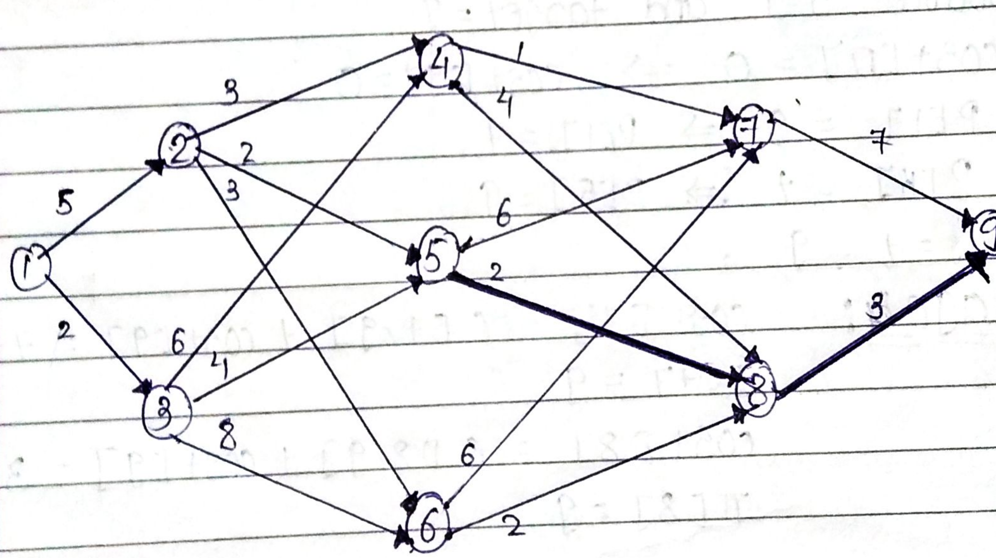
Cost[5] = min{ c[5,7] + cost[7],
c[5,8] + cost[8] }

π[5] = 8

vertex 6 is connected with 7 and 8

Cost[6] = min{ c[6,7] + cost[7],
c[6,8] + cost[8] }

π[6] = 8



Stage 2:- vertex 2 is connected to vertices 4, 5, 6

cost[2] = min{ c[2,4] + cost[4]
c[2,5] + cost[5]
c[2,6] + cost[6] }

= min{9, 7, 8} = 7

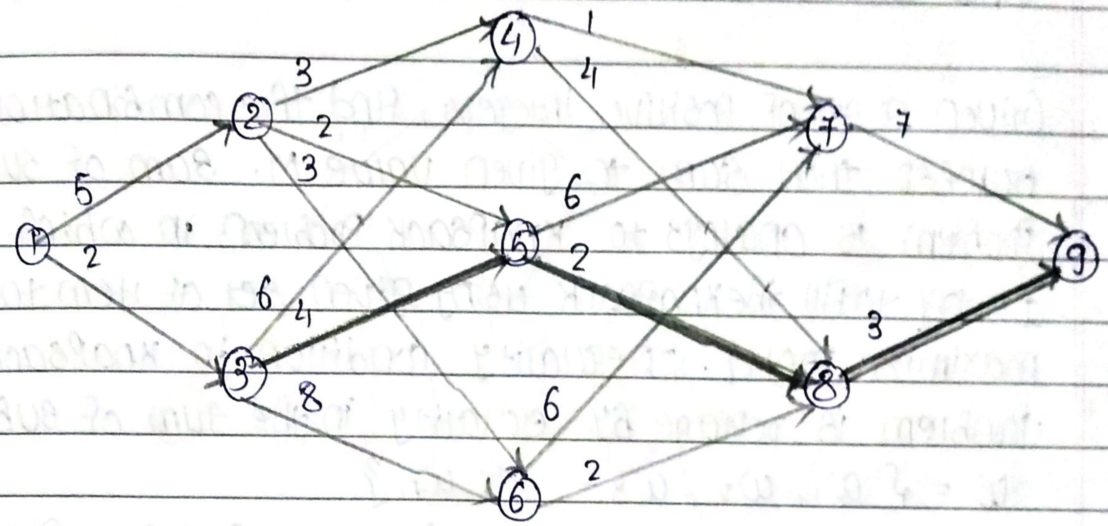
π[2] = 5

Vertex 3 is connected with 4, 5, 6

$Cost[3] = \min\{c[3,4] + cost[4], c[3,5] + cost[5], c[3,6] + cost[6]\}$

$= \min\{13, 9, 13\} = 9$

$\pi[3] = 5$



Stage 1: Vertex 1 is connected to vertices 2 and 3

$Cost[1] = \min\{c[1,2] + Cost[2], c[1,3] + cost[1,3]\}$

$= \min\{12, 11\}$

$= 11$

$\pi[1] = 3$

Trace the solution

$\pi[1] = 3$; $\pi[3] = 5$, $\pi[5] = 8$, $\pi[8] = 9$

minimum cost path is : 1 - 3 - 5 - 8 - 9

Q-3 Find the minimum cost of tour using Branch and Bound.

$$M = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 10 & 5 & 0 \\ 2 & 15 & 20 & 9 & 10 \\ 3 & 6 & 13 & 8 & 8 \\ 4 & 0 & 12 & 9 & 0 \end{array}$$

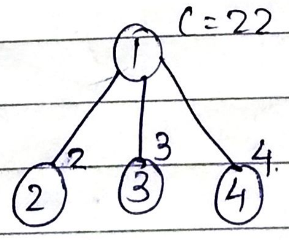
Ans First we reduce row matrix

$$\begin{array}{c|cccc|c} & 1 & 2 & 3 & 4 & \\ \hline 1 & 0 & 10 & 5 & 0 & 0 \\ 2 & 15 & 20 & 9 & 10 & 9 \\ 3 & 6 & 13 & 8 & 8 & 6 \\ 4 & 0 & 12 & 9 & 0 & 0 \\ \hline & & & & & 15 \end{array} \rightarrow \begin{bmatrix} 0 & 10 & 5 & 0 \\ 6 & 11 & 0 & 1 \\ 0 & 7 & 2 & 2 \\ 0 & 12 & 9 & 0 \end{bmatrix}$$

Now column reduction matrix

$$\begin{array}{c|cccc|c} & 1 & 2 & 3 & 4 & \\ \hline 1 & 0 & 10 & 5 & 0 & \\ 2 & 6 & 11 & 0 & 1 & \\ 3 & 0 & 7 & 2 & 2 & \\ 4 & 0 & 12 & 9 & 0 & \\ \hline & 0 & 7 & 0 & 0 & 7 \end{array} \rightarrow \begin{bmatrix} 0 & 3 & 5 & 0 \\ 6 & 4 & 0 & 1 \\ 0 & 0 & 2 & 2 \\ 0 & 5 & 9 & 0 \end{bmatrix} - M_1$$

Total cost reduction $= 15 + 7 = 22$



For (1-2) $\begin{bmatrix} \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & 1 \\ 0 & \infty & 2 & 2 \\ 0 & \infty & 9 & 0 \end{bmatrix} = M_2$

cost (1-2) = cost (1) + Reduce matrix + $M_1[1][2]$

= 22 + 0 + 3

$\boxed{\text{cost } (1-2) = 25}$

For (1-3) $\begin{bmatrix} \infty & \infty & \infty & \infty \\ 6 & 4 & \infty & 1 \\ \infty & 0 & \infty & 2 \\ 0 & 5 & \infty & 0 \end{bmatrix} = M_3$ ~~already reduce~~

$\begin{bmatrix} \infty & \infty & \infty & \infty \\ 5 & 3 & \infty & 0 \\ \infty & 0 & \infty & 2 \\ 0 & 5 & \infty & 0 \end{bmatrix} = M_3$

cost (1-3) = 22 + 1 + 5

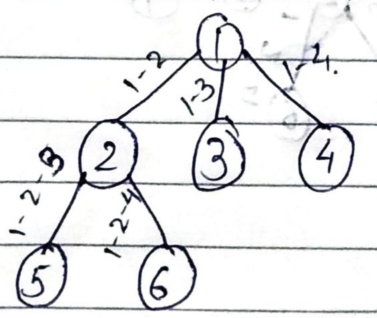
= $\boxed{28}$

For edge (1-4): $\begin{bmatrix} \infty & \infty & \infty & \infty \\ 6 & 4 & 0 & \infty \\ 0 & 0 & 2 & \infty \\ \infty & 5 & 9 & \infty \end{bmatrix} 5 \rightarrow \begin{bmatrix} \infty & \infty & \infty & \infty \\ 6 & 4 & 0 & \infty \\ 0 & 0 & 2 & \infty \\ \infty & 0 & 4 & \infty \end{bmatrix} = M_4$

cost (1-4) = 22 + 5 + $M_1[1][4]$

= 27.

Among (1-2) (1-3) (1-4), (1-2) have minimum cost.



Now root matrix is M_2.

For edge (1-2-3)

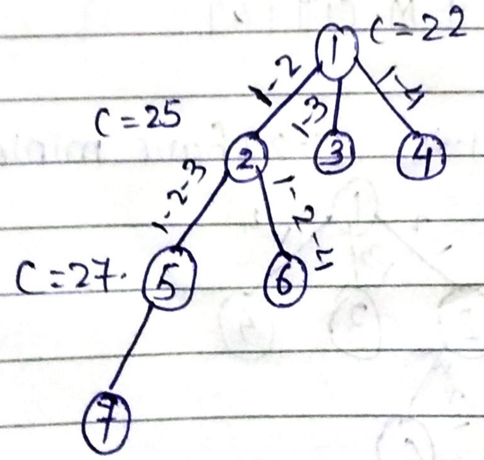
$$\begin{bmatrix} \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & 2 \\ 0 & \infty & \infty & 0 \end{bmatrix} \begin{matrix} \\ \\ 2 \\ \\ \end{matrix} \rightarrow \begin{bmatrix} \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & 0 \\ \infty & \infty & \infty & 0 \end{bmatrix} = M5$$

cost (1-2-3) = cost (1-2) + Reducematrix + M2[2][3]
= 25 + 2 + 0
cost (1-2-3) = 27.

For edge (1-2-4)

$$\begin{bmatrix} \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty \\ 0 & \infty & 2 & \infty \\ \infty & \infty & 9 & \infty \end{bmatrix} \begin{matrix} \\ \\ \\ 9 \end{matrix} \rightarrow \begin{bmatrix} \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty \\ 0 & \infty & 2 & \infty \\ \infty & \infty & 0 & \infty \end{bmatrix} = M6$$

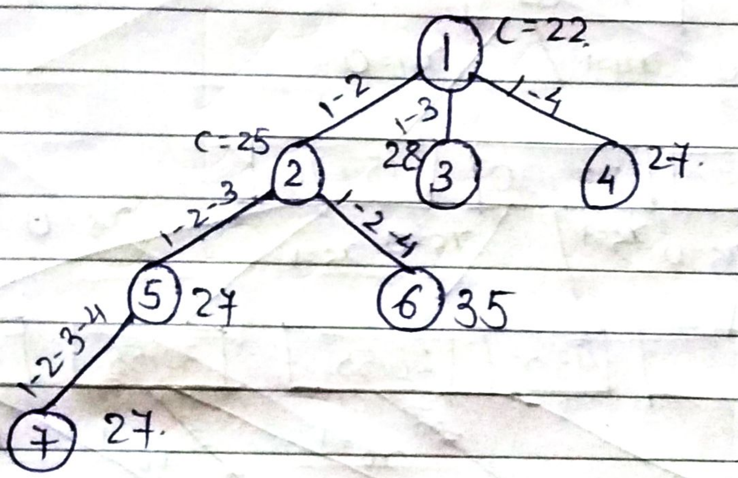
cost (1-2-4) = 25 + 9 + 1 = 35
Among (1-2-3), (1-2-4) minimum cost (1-2-3).



For edge (1-2-3-4)

$$\begin{bmatrix} \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty \end{bmatrix}$$

cost (1-2-3-4) = 27 + 0 + 0 = 27.



Optimal path is 1-2-3-4-1 with cost of path 27.

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Q-4 Explain 15 Puzzle Problem with example.

Ans * 15 Puzzle Problem is the Problem of arranging 15 titels in 4x4 board such that tiles are ordered from left to right and bottom to top such that the bottom right title contain empty space.

1	2	3	4
5	6		8
9	10	7	11
13	14	15	12

initial state

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

Goal state.

* Given the initial state with random distribution of number between 1 to 15, aim is to achive the goal state by moving empty tile. This is NP complete Problem.
* cost Function :- each node in state space tree is associated with some cost. cost Function is applied to each node and the Function help to select next E node. E node is the node being evaluted. The node with min cost should be selected for the further expansion.
* The cost Function is define as, $C(x) = g(x) + h(x)$ where $g(x)$ is the cost of reaching to current state from the initial state and $h(x)$ is the cost of reaching from current state to the answer state.
* cost Function of 15-Puzzle problem is define as the number of tiles on wrong location.

* $C(x) = f(x) + h(x)$, where $f(x)$ is the length from the root node in state space tree, i.e number of moves so far and $h(x)$ is the non-blank tiles which are not in correct locations.

*

1	2	3	4
5	6		8
9	10	7	11
13	14	15	12

up — $C = 1+4$

1	2		4
5	6	3	8
9	10	7	11
13	14	15	12

right — $C = 1+4$

1	2	3	4
5	6	8	
9	10	7	11
13	14	15	12

down — $C = 1+2$

1	2	3	4
5	6	7	8
9	10		11
13	14	15	12

left — $C = 1+4$

1	2	3	4
5		6	8
9	10	7	11
13	14	15	12

* Third configuration has the minimum cost so, remaining 3 state will be cut down and will not be explored further

1	2		4
5	6	3	8
9	10	7	11
13	14	15	12

1	2	3	4
5	6	8	
9	10	7	11
13	14	15	12

1	2	3	4
5	6	7	8
9	10		11
13	14	15	12

1	2	3	4
5		6	8
9	10	7	11
13	14	15	12

$C = 2+1$

1	2	3	4
5	6	7	8
9	10	11	
13	14	15	12

$C = 2+3$

1	2	3	4
5	6	7	8
9	10	15	11
13	14		12

$C = 2+3$

1	2	3	4
5	6	7	8
9		10	11
13	14	15	12

$C = 3+0$

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	