POST OPTIMALITY ANALYSIS

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roblem: A company wants to produce three products A, B, C. The unit profits on these products are Rs. 4, Rs. 6 and Rs. 2 respectively. These products require two types of resources, manpower and raw material. The LP model formulated for determining the optimal product mix is as follows:

$x_i \rightarrow$ units corresponding to products A, B, C resp.

LP Model:

Maximize $Z = 4x_1 + 6x_2 + 2x_3$

s.t.

$x_1 + x_2 + x_3 \leq 3$ (Manpower Constraint)

$x_1 + 4x_2 + 7x_3 \leq 9$ (Raw Material Constraint)

$x_1 \geq 0$; $x_2 \geq 0$; $x_3 \geq 0$

(*)

LPP:

Maximize $Z = c_1x_1 + c_2x_2 + c_3x_3$

s.t.

$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 \leq b_1$

$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 \leq b_2$

$x_1, x_2, x_3 \geq 0$

The company wants to know further, what is optimal solution if there is a:

1. Change in objective function coefficients

case 1: Profit for product A is changed from 4 to 8.

this change $\longrightarrow$ Max $Z = 8x_1 + 6x_2 + 2x_3$

s.t. given constraint

Any other case; Max $Z = c_1x_1 + c_2x_2 + c_3x_3$

2. Change in right hand side of constraints (change in resources/ demand)

R.H.S. of 1st constraint is changed to (say) 7

or R.H.S. of 2nd constraint - - - - - - - 8

this change $\longrightarrow$ Max $Z = c_1x_1 + c_2x_2$
s.t. given constraint

Any other case; Max $Z = c_1x_1 + c_2x_2 + c_3x_3$

2. Change in right hand side of constraints (change in resources/ demand)

R.H.S. of 1st constraint is changed to (say) 7

or R.H.S. of 2nd constraint - - - - - - - 8

3. Change in coefficients of decision variables (consumption of resources)

- change in 'a_{ij}'

4. Addition of constraint (addition of resource)

$4x_1 + 2x_2 + x_3 \leq 10$ (Machine 1) Addition of Constraint

5. Addition of a product or a variable (addition of a variable)

A, B, C, D, E

Max $Z = c_1x_1 + c_2x_2 + c_3x_3 + c_4x_4 + c_5x_5$

s.t. constraints

Starting :

LPP Model $\xrightarrow{\text{Solved}}$ Post optimality Analysis

Simplex
Big M
Two-Phase
Dual Simplex method

change.

B.V.	x_1	x_2	----	x_n	s_1	----	s_r	X_B
Z	z_1-c_1	z_2-c_2	------				z_r-c_r	$C_B X_B$
Basic variables	$B^{-1}A_1$	$B^{-1}A_2$	----------				$B^{-1}A_r$	x_{B_1} ⋮ x_{B_r}

change in Cost
$z_j-c_j \rightarrow$ disturbe

$B^{-1}b$ (feasibility

change in R.H.S. of constraints.

$(B^{-1}b)$

change in Coeff^n matrix

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Changes in objective function coefficients-Post Optimality Analysis

Saturday, 21 March 2020 9:57 AM

LPP: Optimize $Z = CX$ s.t.
(Max./Min) $AX = b$; $X \geq 0$

If there is any change in 'C'
$\Rightarrow$ change in $Z_j - C_j$ in simplex table

B.V.	x_1 x_2 --- s_1 ... s_4	X_B (solution)
		$C_B X_B$ ← cost involved
Z	Z_1-C_1 Z_2-C_2	
	$B^{-1}A_1$ $B^{-1}A_2$	$B^{-1}b$

$$\left\{\begin{array}{ll} Z_j - C_j \geq 0 & (\text{for Max.}) \\ Z_j - C_j \leq 0 & (\text{for Min}) \end{array}\right\}$$ change in C may be change optimality condition

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Example 1: Given the LPP

$$\text{Maximize } Z = 3x_1 + 5x_2$$
$$\text{s.t. } x_1 \le 4$$
$$x_2 \le 6$$
$$3x_1 + 2x_2 \le 18 \; ; \; x_1, x_2 \ge 0$$

and the optimal simplex table is ;

C_B	B.V.	x_1	x_2	s_1	s_2	s_3	X_B (solution)
	Z	0	0	0	3	1	36
0	s_1	0	0	1	2/3	−1/3	2
5	x_2	0	1	0	1	0	6
3	x_1	1	0	0	−2/3	1/3	2

(i) Discuss the effect on the optimality of solution if objective function is changed to $3x_1 + 3x_2$

(ii) Discuss the effect on the optimality of solution if objective function is changed to $3x_1 + x_2$

Solution: (i) Given $Z = 3x_1 + 3x_2$ (New)

C_B	B.V.	x_1	x_2	s_1	s_2	s_3	X_B
		0	0	0	1	1	24
0	s_1	0	0	1	2/3	−1/3	2
3	x_2	0	1	0	1	0	6
3	x_1	1	0	0	−2/3	1/3	2

$$z_j - c_j = C_B(B^{-1}A_j) - C_j$$
$$Z = C_B X_B = (0\ 3\ 3)\begin{pmatrix}2\\6\\2\end{pmatrix}$$
$$= 18 + 6 = 24$$

$z_j - c_j \ge 0$; optimal ; $x_1 = 2$; $x_2 = 6$; $Z_{max} = 24$

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Maximize $Z = 3x_1 + 5x_2$

s.t. $x_1 \le 4$

$x_2 \le 6$

$3x_1 + 2x_2 \le 18$; $x_1, x_2 \ge 0$

5 will be 3

(i)

	B.V.	x_1	x_2	s_1	s_2	s_3	
		0	0	0	1	1	24
0	s_1	0	0	1	2/3	−1/3	2
5	x_2	0	1	0	1	0	6
3	x_1	1	0	0	−2/3	1/3	2

$Z = C_B X_B$

$= (0\ 5\ 3)\begin{pmatrix}2\\6\\2\end{pmatrix}$

$= 24$

and the optimal simplex table is ;

C_B	B.V.	x_1	x_2	s_1	s_2	s_3	X_B (solution)
	Z	0	0	0	3	1	36
0	s_1	0	0	1	2/3	−1/3	2
5	x_2	0	1	0	1	0	6
3	x_1	1	0	0	−2/3	1/3	2

$Z_j - C_j \ge 0$ (given problem max.)

Optimality intact

$Z_{max} = 24$; at $x_1 = 2$; $x_2 = 6$

(i) Discuss the effect on the optimality of solution if objective function is changed to $3x_1 + 3x_2$

(ii) Discuss the effect on the optimality of solution if objective function is changed to $3x_1 + x_2$

(ii)

	B.V.	x_1	x_2	s_1	s_2	s_3	X_B
		0	0	0	−1	1	12
0	s_1	0	0	1	$\boxed{2/3}$	−1/3	2
1	x_2	0	1	0	1	0	6
3	x_1	1	0	0	−2/3	1/3	2
	Z	0	0	3/2	0	1/2	15
	s_2	0	0	3/2	1	−1/2	3
	x_2	0	1	−3/2	0	1/2	3
	x_1	1	0	1	0	0	4

$Z_j - C_j \ge 0$; optimal

$Z_{max} = 15$; $x_1 = 4$; $x_2 = 3$.

Maximize $Z = 4x_1 + 6x_2 + 2x_3$ s.t.

$$x_1 + x_2 + x_3 \le 3$$
$$x_1 + 4x_2 + 7x_3 \le 9$$
$$x_1, x_2, x_3 \geq 0$$

and its optimal table is

	B.V.	x_1	x_2	x_3	s_1	s_2	X_B (Solution)
	Z	0	0	6	10/3	2/3	16
4	x_1	1	0	-1	4/3	-1/3	1
6	x_2	0	1	2	-1/3	1/3	2

(i) Find the range of cost coefficient (in objective function) of x_1 so that current optimal solution remains optimal.

(ii) Find the range of cost coefficient of x_3 so that current solution remains optimal

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Solution: (i)

To calculate Z-row

	B.V.	x_1	x_2	x_3	x_4	x_5	$C_B X_B$
	Z	0	0	Z_3-C_3	Z_4-C_4	Z_5-C_5	
C_1	x_1	1	0	-1	$4/3$	$-1/3$	1
6	x_2	0	1	2	$-1/3$	$1/3$	2

$$Z_3-C_3 = C_B(B^{-1}A_3)-C_3$$
$$= \begin{pmatrix} C_1 & 6 \end{pmatrix}\begin{pmatrix} -1 \\ 2 \end{pmatrix} - 2$$
$$= -C_1 + 12 - 2 = -C_1 + 10$$

$Z_3 - C_3 \geq 0$

$\Rightarrow -C_1 + 10 \geq 0 \Rightarrow C_1 \leq 10$ — (1)

$$Z_4 - C_4 = \frac{4}{3}C_1 - 2$$

$\therefore \frac{4}{3}C_1 - 2 \geq 0$

$\Rightarrow C_1 \geq \frac{6}{4} = 3/2$ — (2)

$$Z_5 - C_5 = -\frac{C_1}{3} + 2$$

$\Rightarrow C_1 \leq 6$ — (3)

$$\boxed{3/2 \leq C_1 \leq 6}$$

(ii) $Z_3 - C_3 = C_B(B^{-1}A_3) - C_3$

$$= \begin{pmatrix} 4 & 6 \end{pmatrix}\begin{pmatrix} -1 \\ 2 \end{pmatrix} - C_3 = -4 + 12 - C_3$$

As $Z_3 - C_3 \geq 0$ maintain optimality; $8 - C_3 \geq 0$

$$\boxed{C_3 \leq 8}$$

Change in R.H.S. of Constraint/ change in resource or demand – Post Optimality Analysis

Monday, 23 March 2020 9:14 AM

LPP: Optimize $Z = CX$ s.t. (Max./Min) $AX = b$; $X \geq 0$] — (*) optimal solution

'b' ; 'b' $= (b_1, b_2 \ldots b_m)^T$ ← Any change

$X_B = B^{-1} b$ ← may correspond to change in X_B

Ist possibility $X_B \geq 0$ $(B^{-1} b \geq 0)$ → feasible

$\therefore Z = C_B X_B$

2nd possibility $X_B = B^{-1} b \leq 0$ (in all / or in some situation)

then feasibility has disturbed

Apply Dual Simplex Method ;

Example 8: Let the LPP is

Maximize $Z = 4x_1 + 6x_2 + 2x_3$ s.t.

$$x_1 + x_2 + x_3 \le 3$$
$$x_1 + 4x_2 + 7x_3 \le 9$$
$$x_1, x_2, x_3 \ge 0$$

$$A = \begin{array}{c} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 4 & 7 \end{pmatrix} \\ \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \end{array}$$

and its optimal table is

B.V.	x_1	x_2	x_3	s_1	s_2	X_B (Solution)
Z	0	0	6	10/3	2/3	16
x_1	1	0	-1	4/3	-1/3	1
x_2	0	1	2	-1/3	1/3	2

$$B.V. = (x_1 \quad x_2)$$

$$B = \begin{pmatrix} 1 & 1 \\ 1 & 4 \end{pmatrix}$$

$$B^{-1} = \begin{pmatrix} 4/3 & -1/3 \\ -1/3 & 1/3 \end{pmatrix}$$

(i) Discuss the effect of discrete change in the availability of resources from $(3,9)^T$ to $(3,10)^T$.

$b_1 = 3$
b_2 has changed from 9 to 10

(ii) Discuss the effect of discrete change in availability of resources from $(3,9)^T$ to $(9,3)^T$.

(iii) Which resource should be increased (or decreased) to get best marginal increase in the values of the objective function.

B.V.	x_1	x_2	x_3	s_1	s_2	X_B (Solution)
Z	0	0	6	10/3	2/3	16
x_1	1	0	-1	4/3	-1/3	1
x_2	0	1	2	-1/3	1/3	2

$\begin{pmatrix} -1/3 & 1/3 \end{pmatrix}$

(i) Discuss the effect of discrete change in the availability of resources from $(3,9)^T$ to $(3,10)^T$.

} $b_1 = 3$, b_2 has changed from 9 to 10

(ii) Discuss the effect of discrete change in availability of resources from $(3,9)^T$ to $(9,3)^T$.

} → R.H.S. of both constraint are changed.

(iii) Which resource should be increased (or decreased) to get best marginal increase in the values of the objective function.

Solution:

(i) $\hat{X}_B = B^{-1}\hat{b} = \begin{pmatrix} 4/3 & -1/3 \\ -1/3 & 1/3 \end{pmatrix} \begin{pmatrix} 3 \\ 10 \end{pmatrix} = \begin{pmatrix} 2/3 \\ 7/3 \end{pmatrix} \geq 0$

$\therefore$ Current solution is feasible

$Z = C_B \hat{X}_B = (4 \;\; 6) \begin{pmatrix} 2/3 \\ 7/3 \end{pmatrix} = \frac{50}{3}$; $Z_{max} = \frac{50}{3}$; at $x_1 = 2/3$; $x_2 = 7/3$

$$Z = C_B \hat{X}_B = (4\ \ 6)\begin{pmatrix} 2/3 \\ 7/3 \end{pmatrix} = \frac{50}{3}\ ;$$

$Z_{max} = \frac{50}{3}$; at $x_1 = 2/3$; $x_2 = 7/3$

(ii)

$$\hat{X}_B = B^{-1}\hat{b} = \begin{pmatrix} 4/3 & -1/3 \\ -1/3 & 1/3 \end{pmatrix}\begin{pmatrix} 9 \\ 3 \end{pmatrix} = \begin{pmatrix} 11 \\ -2 \end{pmatrix} \longrightarrow$$ so feasibility is disturbed

B.V.	x_1	x_2	x_3	s_1	s_2	X_B
Z	0	0	6	10/3	2/3	16
x_1	1	0	-1	4/3	-1/3	11
x_2	0	1	2	$\boxed{-1/3}$	1/3	-2
	0	10	26	0	4	12
x_1	1	4	7	0	1	3
s_1	0	-3	-6	1	-1	6

Apply Dual Simplex Method

Remove: Most-ve x_{Bi}.

Enter: $\left\{ \min \left| \frac{Z_j - C_j}{\alpha_j} \right| ;\ \alpha_j < 0 \right\}$

Now $Z_j - C_j \geq 0$; and $x_{Bi} \geq 0$ feasible;

$\therefore$ Current table is optimal & feasible

$Z = 12$; at $x_1 = 3$

$x_2 = 0$

$x_3 = 0$

$x_2 = 0$
$x_3 = 0$

(iii) R.H.S. of first constraint 'b₁' range

$$\hat{X}_B = B^{-1}\hat{b} = \begin{pmatrix} 4/3 & -1/3 \\ -1/3 & 1/3 \end{pmatrix} \begin{pmatrix} b_1 \\ 9 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{4}{3}b_1 - 3 \\ -\frac{b_1}{3} + 3 \end{pmatrix}$$

Put $\hat{X}_B \geq 0$

$$\Rightarrow \frac{4}{3}b_1 - 3 \geq 0 \quad ; \quad -\frac{b_1}{3} + 3 \geq 0$$

$$b_1 \geq 9/4 \quad ; \quad b_1 \leq 9$$

$$\boxed{\frac{9}{4} \leq b_1 \leq 9}$$

$$= \begin{pmatrix} \frac{4}{3} b_1 - 3 \\ -\frac{b_1}{3} + 3 \end{pmatrix}$$

Put $\hat{X}_B \geq 0$

$$\Rightarrow \quad \frac{4}{3} b_1 - 3 \geq 0 \quad ; \quad -\frac{b_1}{3} + 3 \geq 0$$

$$b_1 \geq 9/4 \quad ; \quad b_1 \leq 9$$

$$\boxed{\frac{9}{4} \leq b_1 \leq 9} \quad \text{— (1)}$$

Similarly we obtain; range on 'b_2' that is R.H.S of 2nd constraint

$$\boxed{9 \leq b_2 \leq 12} \quad \text{— (2)}$$

economical interpretation; As dual (shadow prices) are calculated based on R.H.S. of constraint;

so; In optimal table given in LPP;

First dual var. $y_1 = 10/3$; second dual var. $= 2/3$

Change in the coefficients a_{ij}'s – Post Optimality Analysis

Monday, 23 March 2020 6:01 PM

Consider a LPP; Opt (Max./Min) $Z = CX$

$$\left.\begin{aligned} AX &= b \;;\; X \geq 0 \\ \text{or} \quad (A_1 \; A_2 \cdots A_n) X &= b \end{aligned}\right\} \text{—} (*)$$

or $(A)_{m \times n} \rightarrow$ real matrix of rank m

Now if we change a column / or some entry in matrix A.

Two possibilities arise;

Case 1; changed $a_{ij} \notin B$ (Basis Matrix)

Case 2; changed $a_{ij} \in B$ (Basis matrix)

Case 2 : Restart from beginning with new values

Example 1:

Let LPP; Max $Z = 3x_1 + 4x_2 + x_3 + 7x_4$

s.t. $8x_1 + 3x_2 + 4x_3 + x_4 \le 7$

$2x_1 + 6x_2 + x_3 + 5x_4 \le 3$

$x_1 + 4x_2 + 5x_3 + 2x_4 \le 8$

$x_1, x_2, x_3, x_4 \ge 0$

The optimal table for given LPP is

B.V.	x_1	x_2	x_3	x_4	s_1	s_2	s_3	X_B
Z	0	$169/38$	$1/2$	0	$1/38$	$53/38$	0	$83/19$
x_1	1	$9/38$	$1/2$	0	$\frac{5}{38}$	$\frac{-1}{38}$	0	$16/19$
x_4	0	$21/19$	0	1	$\frac{-1}{19}$	$8/38$	0	$5/19$
s_3	0	$59/38$	$9/2$	0	$\frac{-1}{38}$	$\frac{-15}{38}$	1	$126/19$

Find the effect of change;

(i) If the coefficient of x_2 in second constraint is changed to 2.

(ii) If the coefficient of x_1 in the first constraint is changed to 4.

Solution: (i) Given B.V. $= (x_1, x_4, s_3)$

$$\therefore B = \begin{pmatrix} 8 & 1 & 0 \\ 2 & 5 & 0 \\ 1 & 2 & 1 \end{pmatrix} \qquad B^{-1} = \begin{pmatrix} \frac{5}{38} & \frac{-1}{38} & 0 \\ \frac{-1}{19} & 8/38 & 0 \\ \frac{-1}{38} & \frac{-15}{38} & 1 \end{pmatrix}$$

Now x_2 is not a basic variable

Solution : (i) Given $B.V. = (x_1, x_4, s_3)$

$$\therefore B = \begin{pmatrix} 8 & 1 & 0 \\ 2 & 5 & 0 \\ 1 & 2 & 1 \end{pmatrix} \qquad B^{-1} = \begin{pmatrix} \frac{5}{38} & \frac{-1}{38} & 0 \\ \frac{-1}{19} & 8/38 & 0 \\ \frac{-1}{38} & \frac{-15}{38} & 1 \end{pmatrix}$$

Now x_2 is not a basic variable so only column of x_2 in simplex table will take place

$\therefore$ Calculate Scalars

$$\alpha^2 = B^{-1}A_2 = \begin{pmatrix} 5/38 & \frac{-1}{38} & 0 \\ \frac{-1}{19} & 8/38 & 0 \\ \frac{-1}{38} & \frac{-15}{38} & 1 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix} = \begin{pmatrix} 13/38 \\ 5/19 \\ 119/38 \end{pmatrix}$$

Earlier in problem this entry is 6

Calculate Net-evaluation

$$Z_2 - C_2 = C_B(B^{-1}A_2) - C_2 = (3 \;\; 7 \;\; 0)\begin{pmatrix} 13/38 \\ 5/19 \\ 119/38 \end{pmatrix} - 4$$

$$= -43 < 0$$

$$\alpha = B^{-1} A_2 = \begin{pmatrix} /38 & 38 & \\ -\frac{1}{19} & 8/38 & 0 \\ -\frac{1}{38} & -\frac{15}{38} & 1 \end{pmatrix} \begin{pmatrix} \\ 2 \\ 4 \end{pmatrix} = \begin{pmatrix} 5/19 \\ 119/38 \end{pmatrix}$$

Earlier in problem this entry is 6

Calculate Net-evaluation

$$Z_2 - C_2 = C_B (B^{-1} A_2) - C_2 = (3 \;\; 7 \;\; 0) \begin{pmatrix} 13/38 \\ 5/19 \\ 119/38 \end{pmatrix} - 4$$

$$= -\frac{43}{38} < 0$$

(this disturb optimality)

So we enter x_2 variable in basis and remove some variable (by min Ratio Rule) and carry simplex / row-operations to find optimal solution.

(ii) Coefficient of x_1 in first constraint is changed to 4.

$$\text{Old } B = \begin{pmatrix} 8 & 1 & 0 \\ & & \end{pmatrix} \qquad \text{New } B = \begin{pmatrix} 4 & 1 & 0 \\ 2 & 5 & 0 \end{pmatrix}$$

carry simplex row operations to find optimal ...

(ii) Coefficient of x_1 in first constraint is changed to 4.

$$\text{Old } B = \begin{pmatrix} 8 & 1 & 0 \\ 2 & 5 & 0 \\ 1 & 2 & 1 \end{pmatrix} \qquad \text{New } B = \begin{pmatrix} 4 & 1 & 0 \\ 2 & 5 & 0 \\ 1 & 2 & 1 \end{pmatrix}$$

$$\text{New } B^{-1} = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}$$

B.V.	x_1	x_2	x_3	x_4	s_1	s_2	s_3	X_B
Z	Z_1-C_1	Z_2-C_2	-	-	-	-	-	$C_B X_B$
	$B^{-1}A_1$	$B^{-1}A_2$	-	-	-	-	-	$B^{-1}b$

$$Z_j - C_j = C_B(B^{-1}A_j) - C_j$$

Fill entries in above table with new B^{-1}; and carry simplex iterations / row-operations

Addition of variable - Post Optimality Analysis

Tuesday, 24 March 2020 10:48 PM

Addition of a new variable in a given LPP

Problem: Suppose there is a X-company which manufactures two products and the LPP is given as;

$$\text{Max } Z = 3x_1 + 5x_2$$
$$\text{s.t. } x_1 \le 4 \quad \text{(Machine 1)}$$
$$3x_1 + 2x_2 \le 8 \quad \text{(Machine 2)}$$
$$x_1, x_2 \ge 0$$

Now company wishes to introduce another product (Launch) with cost/profit 7 per unit and constraint's coefficients as $(1,2)^T$

It is given optimal table of LPP as;

B.V.	x_1	x_2	s_1	s_2	X_B
Z	9/2	0	0	5/2	45
s_1	1	0	1	0	4
x_2	3/2	1	0	1/2	9

B^{-1}

B

$x_2 = 9$ units.
$x_1 = 0$ units.

If z3 - c3 would've been greater than 0 after addition of variable , result wouldn't have changed

Solution:

New LPP:

$$\text{Max } Z = 3x_1 + 5x_2 + 7x_3$$
$$\text{s.t. } x_1 + x_3 \le 4$$
$$3x_1 + 2x_2 + 2x_3 \le 8$$
$$x_1, x_2, x_3 \ge 0 \quad (*)$$

finding new

C_B	B.V.	x_1	x_2	x_3	s_1	s_2	X_B	$Z_3 - C_3$
	Z	9/2	0	-2	0	5/2	45	
0	s_1	1	0	$\boxed{1}$	1	0	4	
5	x_2	3/2	1	1	0	1/2	9	

$B^{-1}A_3$

$$\alpha^3 = B^{-1}A_3 = \begin{pmatrix} 1 & 0 \\ 0 & 1/2 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$Z_3 - C_3 = C_B(B^{-1}A_3) - C_3 = (0\ 5)\begin{pmatrix} 1 \\ 1 \end{pmatrix} - 7$$
$$= 5 - 7 = -2 < 0$$

B.V.	x_1	x_2	x_3	s_1	s_2	X_B
Z	13/2	0	0	2	5/2	53
x_3	1	0	1	1	0	4
x_2	1	1	0	-1	1/2	5

$Z_j - C_j \ge 0$ optimal..

$Z_{max} = 53$ units.

$x_2 = 5$ units.
$x_3 = 4$ units.

Addition of Constraint-Post Optimality Analysis

Tuesday, 24 March 2020 11:18 PM

LPP: opt $Z = CX$ s.t. $AX = b$; $X \geq 0$

$(A)_{m \times n} \longrightarrow \text{Rank}(A) = m$

$\downarrow$ we choose

$B = (\quad)_{m \times m}$

Now if add a constraint; $(A)_{(m+1) \times n}$

$\downarrow$

$B = (\quad)_{(m+1) \times (m+1)}$

Problem: $\text{Max } Z = c_1 x_1 + c_2 x_2 + c_3 x_3$

s.t. $a_{11} x_1 + a_{12} x_2 + a_{13} x_3 \leq b_1$ (constraint of 1st deptt

(,, ,, other deptt.)

(constraint of Quality check)

$\longrightarrow a_{m+1,1} x_1 - - - - + a_{m+1,3} x_3 \leq, \geq b_{m+1}$ (constraint of Quality check)

Working offline (an error occurred) Details

Example 1:

Let LPP; Max $Z = 3x_1 + 4x_2 + x_3 + 7x_4$

s.t.
$$\begin{cases} 8x_1 + 3x_2 + 4x_3 + x_4 \le 7 \\ 2x_1 + 6x_2 + x_3 + 5x_4 \le 3 \\ x_1 + 4x_2 + 5x_3 + 2x_4 \le 8 \end{cases}$$
$$x_1, x_2, x_3, x_4 \ge 0$$

The optimal table for given LPP:

B.V.	x_1	x_2	x_3	x_4	s	s_2	s_3	X_B
Z	0	169/38	1/2	0	29/38	53/38	0	83/19
x_1	1	9/38	1/2	0	$\frac{5}{38}$	$\frac{-1}{38}$	0	16/19
x_4	0	21/19	0	1	$-\frac{1}{19}$	$\frac{8}{38}$	0	5/19
s_3	0	59/38	9/2	0	$\frac{-1}{38}$	$\frac{-15}{38}$	1	126/1

$x_1 = \frac{16}{19}$; $x_2 = 0$; $x_3 = 0$
$x_4 = 5/19$

What is the effect of addition of a constraint $2x_1 + 3x_2 + x_3 \ge 4$.

(or $2x_1 + 3x_2 + x_3 + 0.x_4 \ge 4$)

Step 1: Check whether the constraint is satisfied with given optimal sol^n or not.

$$\Rightarrow 2\left(\frac{16}{19}\right) + 3(0) + (0) + 0.(5/19) \ngeq 4$$

$$\frac{32}{19} \ngeq 4$$

Note: If constraint satisfy the given solution, there is no need to add. If it does not satisfy then move to step 2.

Step 2:

$\frac{32}{19} \ngeq 4$

Step 2:

	BV	x_1	x_2	x_3	x_4	s_1	s_2	s_3	s_4	X_B	
		0	169/38	1/2	0	29/38	53/38	0	0	83/19	R_1
		0	9/38	1/2	0	5/38	-1/38	0	0	16/19	R_2
previous B.V.	x_1	1	21/19	0	1	-1/19	4/19	0	0	5/19	R_3
	x_4	0	59/38	9/2	0	-1/38	-15/38	1	0	126/19	R_4
	s_3	0	-2	-3	-1	0	0	0	1	-4	R_5
	s_4	0	169/38	1/2	0	29/38	53/38	0	0	83/19	
										16/19	
	x_1		same as above							5/19	
	x_4									126/19	
	s_3										
	s_4	0	$\boxed{-\frac{48}{19}}$	0	0	$\frac{10}{38}$	$-\frac{2}{38}$	0	1	$-\frac{44}{19}$	

B.V. ↗ x_1; B.V. ↗ x_4; ✓ s_3; ✓ s_4

$z_j - c_j \geq 0$

In this step, no entering var.

Note: If constraint satisfied given solution, there is no need to add. If it does not satisfy then move to step 2.

we directly coefficients from the new constraint that has to added.

$R_5 \to R_5 - 2R_2$

(with this operation, maintain the identity column of B.V.)

feasibility is disturbed.

Apply Dual Simplex Method

Remark: Suppose we want to add a constraint

$$2x_1 + 3x_2 + 7x_3 + x_4 = 8$$

$$2x_1 + 3x_2 + 7x_3 + x_4 \leq 8 \qquad \text{or} \qquad 2x_1 + 3x_2 + 7x_3 + x_4 \geq 8 \quad (2)$$

so either this is satisfied with optimal sol^n, (1)

Eg: If (1) satisfies the given optimal sol^n then introduce (2) as addition of constraint in