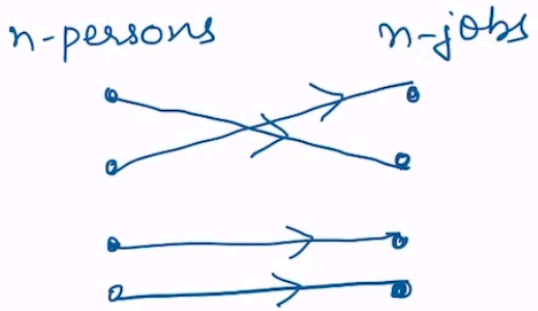
# Assignment Problem

Consider n-jobs that is to be assigned to n-persons in one to one correspondence.

## Mathematical Formulation of Assignment Problem:




Destination

n-Persons

(Resource) Jobs

$$\begin{array}{c|cccccc} & P_1 & P_2 & \cdots & P_j & \cdots & P_n \\ \hline J_1 & c_{11}\,x_{11} & c_{12}\,x_{12} & \cdots & & \cdots & c_{1n}\,x_{1n} \\ J_2 & & & & & & \\ \vdots & & & & x_{ij} & & 0 \\ J_n & & & & & & \end{array}$$

if job is assigned

$c_{11}\,x_{11}$

$c_{11}(1) = c_{11}$

but if job is not assigned

$c_{11}(0) = 0$

$c_{ij}$ = The cost of performing $i^{th}$ job by $j^{th}$ person (Per Unit)

$1 \le i \le n$

$1 \le j \le n$

$$x_{ij} = \begin{cases} 1 ; & \text{if } i^{th} \text{ job is assigned to } j^{th} \text{ person} \\ 0 ; & \text{otherwise} \end{cases}$$

### Mathematical Formulation

$$\text{Min } Z = \sum \sum x_{ij} c_{ij}$$

s.t. $\sum_{j=1}^{n} x_{ij} = 1 \; ; \quad 1 \le i \le n$

$\sum_{i=1}^{n} x_{ij} = 1 \; ; \quad 1 \le j \le n$

$x_{ij} = 0$ or $1 \quad \forall\, i, j$

**Example 1 :** Consider following is an assignment problem, with cost associated given table

Write Math. formulation of given Assig. Problem

$$\begin{array}{c|ccc} & A_1 & A_2 & A_3 \\ \hline R1 & 1 & 2 & 3 \\ R2 & 4 & 5 & 1 \\ R3 & 2 & 1 & 4 \end{array}$$

**Sol$^n$:**

$$x_{ij} = \begin{cases} 1 & \text{if } R_i \text{ is assigned to } A_j \\ 0 & \text{otherwise} \end{cases} \quad ; \quad \begin{array}{l} 1 \le i \le 3 \\ 1 \le j \le 3 \end{array}$$

**Minimization** $Z = \sum\sum x_{ij} c_{ij}$

$$= x_{11} + 2x_{12} + 3x_{13} + 4x_{21} + 5x_{22} + x_{23} + 2x_{31} + x_{32} + 4x_{33}.$$

s.t.

$$x_{11} + x_{12} + x_{13} = 1$$
$$x_{21} + x_{22} + x_{23} = 1$$
$$x_{31} + x_{32} + x_{33} = 1$$

$$x_{11} + x_{21} + x_{31} = 1$$
$$x_{12} + x_{22} + x_{32} = 1$$
$$x_{13} + x_{23} + x_{33} = 1$$

$$x_{ij} = 0 \text{ or } 1 \quad \forall \; i, j$$

## Remark:

- Assignment Problem is a particular case of Transportation problem ( m-sources, n-destinations)

$$
\begin{array}{c|c|c}
 & n\text{-destinations} & \text{Supply} \\
n\text{-Sources} & \begin{bmatrix} c_{11} & \cdots & c_{1n} \\ & & \\ c_{n1} & \cdots & c_{nn} \end{bmatrix} & \begin{matrix} 1 \\ \vdots \\ 1 \end{matrix} \\
\text{Demand} & \begin{matrix} 1 & 1 & \cdots & 1 \end{matrix} &
\end{array}
$$

Formulation of T.P. :

$$\min Z = \sum \sum x_{ij} c_{ij}$$

$$\text{s.t.} \quad \sum x_{ij} = 1 \quad \forall i$$

$$\sum x_{ij} = 1 \quad \forall j$$

$$x_{ij} \geq 0 \; ; \quad x_{ij} \in \{0, 1\}$$

n-resources = n-

**Unbalanced A.P:** $(m \times n)$ problem ; m-jobs ; n-persons $(m \neq n)$

$\Downarrow$

For unbalanced A.P; we first convert this into a balanced A.P.

Eg:

$$\begin{array}{c|cccc|c} & P_1 & P_2 & P_3 & P_4 & a_i(\text{supply}) \\ \hline J_1 & & & & & \\ J_2 & & & & & \\ J_3 & & & & & \\ \hline \text{Demand} \rightarrow & b_1 = & b_2 = & b_3 = & b_4 = & \end{array}$$

$\sum a_i$

$\sum b_i$

$\sum a_i \neq \sum b_j$

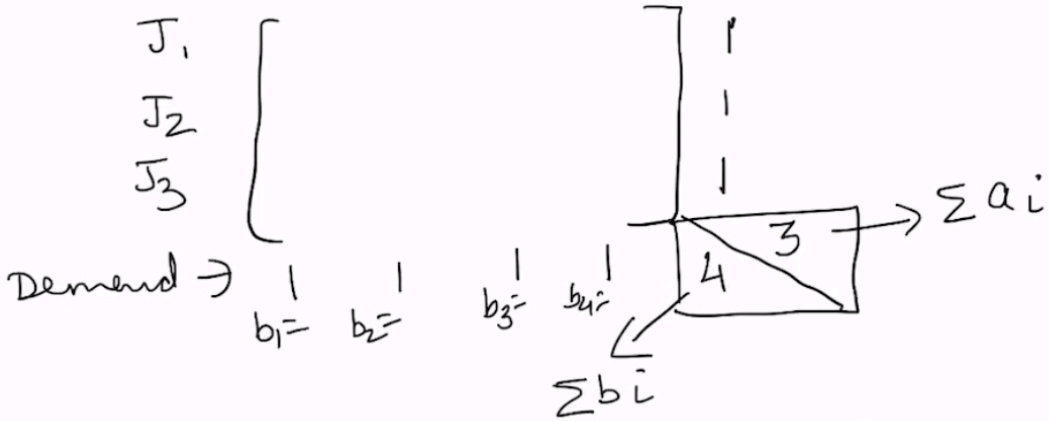
$\sum a_i = \sum b_j$

$\downarrow$

Balanced A.P.

To remove unbalance situation; add a dummy job / or dummy persons

Eg:




$\Sigma a_i \neq \Sigma b_j$

$\Sigma a_i = \Sigma b_j$

↓

Balanced A.P.

To remove unbalance situation; add a dummy job / or dummy persons

Eg:

$$\begin{array}{c|cccc} & P_1 & P_2 & P_3 & P_4 \\ \hline J_1 & 2 & 10 & 8 & 7 \\ J_2 & 1 & 5 & 6 & 7 \end{array}$$

← Unbalanced Assignment

$$\begin{array}{c|cccc} & P_1 & P_2 & P_3 & P_4 \\ \hline J_1 & 2 & 10 & 8 & 7 \\ J_3 & 1 & 5 & 6 & 7 \\ J_3 & 0 & 0 & 0 & 0 \\ J_4 & 0 & 0 & 0 & 0 \end{array}$$

dummy jobs

→ Now this is balanced

↓ we can expect a feasible solution

↓ we ~~w~~ get a feasible solution

# Solution Methods of Assignment Problem

A.P. is a T.P. & T.P is a LPP | (A.P.) is LPP)

## Method 1: Permutation | Enumeration Method:

Eg:

$$\begin{array}{c|ccc} & P_1 & P_2 & P_3 \\ \hline J_1 & \boxed{2} & 5 & 6 \\ J_2 & 7 & \boxed{1} & 8 \\ J_3 & 7 & 2 & 3 \end{array}$$

$3 \times 3$

Possibility of $J_1$ to be assigned $P_1, P_2, P_3$

— — — $J_2$ — — — — $P_2, P_3$

— — — $J_3$ — — — — $P_3$

Total No. of possibilies: $(3)(2)(1) = 3!$

If we have an Assignment Problem of order $(n \times n)$; then we have $n!$ choices.

$n! \rightarrow$ incre.

$8! = 40{,}320$

**Method 2:** Treat A.P. as T.P.

Basic cells in T.P. $(m \times n)$ : $m+n-1$

Basic cells in A.P. $(n \times n)$ : $n+n-1 = \underline{\underline{2n-1}}$

(we will get a degenerate solution ; after applying NW | LCM | VAM then we follow by UV-Method)

Eg:

| | $P_1$ | $P_2$ | $P_3$ | supply |
|---|---|---|---|---|
| $J_1$ | 4 | 1 6 | 8 | 1 |
| $J_2$ | 7 | 8 | 1 1 | 1 |
| $J_3$ | 1 2 | 3 | 5 | 1 |
| Demand | 1 | 1 | 1 | |

← LCM

here $n \times n$ $(3 \times 3)$

Basic cells $= x_{12}, x_{23}, x_{31}$

desired Basic cells : $2n-1$

$= 2(3)-1 = \underline{\underline{5}}$

but we got 3 B.V.

⇓

we always get a degenerate BFS from initial BFS.

↓ Apply UV Method to find optimal solution

## Method 3: (Simplex Method)

$$\text{Min } Z = \sum\sum x_{ij} c_{ij} \quad \text{s.t.} \sum x_{ij} = 1$$

$$\sum x_{ij} = 1 \quad ; x_{ij} = 0 \text{ or } 1$$

← Balanced

Here the $(A)_{\underline{n+n},\, n^2}$

⇒ As decision variables are $n^2$ so we have $n^2$ columns;

| B.V. | - - - - - - - - | $X_B$ |
|---|---|---|
| ← | | |

## Method 4: (Hungarian Method)

# Hungarian Method to Solve Assignment Problem

Friday, 17 April 2020 12:18 AM

**Problem:** → Suppose there are 5-Machine to perform 5-tasks (jobs). The time taken by each Machine to perform a particular task is given in following table. Find the assignment schedule to minimize total time.

$$
\begin{array}{c|ccccc}
 & J_1 & J_2 & J_3 & J_4 & J_5 \\
\hline
M_1 & 3 & 9 & 2 & 3 & 7 \\
M_2 & 6 & 1 & 5 & 6 & 6 \\
M_3 & 8 & 4 & 7 & 10 & 3 \\
M_4 & 2 & 5 & 4 & 2 & 1 \\
M_5 & 9 & 6 & 2 & 4 & 5
\end{array}
$$

**Solution:** Row - Minimum

→ select least quantity ... row and subtract it from remaining.

$$
\begin{array}{c|ccccc}
 & J_1 & J_2 & J_3 & J_4 & J_5 \\
\hline
M_1 & 1 & 7 & 0 & 1 & 5 \\
M_2 & 5 & 0 & 4 & 5 & 5 \\
M_3 & 5 & 1 & 4 & 7 & 0 \\
M_4 & 1 & 4 & 3 & 1 & 0 \\
M_5 & 7 & 4 & 0 & 2 & 3
\end{array}
$$

Column - Minima

$$
\begin{array}{c|ccccc}
 & J_1 & J_2 & J_3 & J_4 & J_5 \\
\hline
M_1 & 0 & 7 & 0 & 0 & 5 \\
M_2 & 4 & 0 & 4 & 4 & 5 \\
M_3 & 4 & 1 & 4 & 6 & 0 \\
M_4 & 0 & 4 & 3 & 0 & 0 \\
M_5 & 6 & 4 & 0 & 1 & 3
\end{array}
$$

← After column Minima step; every row & every column should have atleast one zero.

← After Row & Column Minima

⇓

we ca

$$M_5 \begin{bmatrix} 9 & 6 & 2 & 4 & 5 \end{bmatrix}$$

**Solution:** Row - Minimum → select least quantity in each row and subtract it from remaining.

$$\begin{array}{c|ccccc} & J_1 & J_2 & J_3 & J_4 & J_5 \\ \hline M_1 & 1 & 7 & 0 & 1 & 5 \\ M_2 & 5 & 0 & 4 & 5 & 5 \\ M_3 & 5 & 1 & 4 & 7 & 0 \\ M_4 & 1 & 4 & 3 & 1 & 0 \\ M_5 & 7 & 4 & 0 & 2 & 3 \end{array}$$

Column - Minima

$$\begin{array}{c|ccccc} & J_1 & J_2 & J_3 & J_4 & J_5 \\ \hline M_1 & 0 & 7 & 0 & 0 & 5 \\ M_2 & 4 & 0 & 4 & 4 & 5 \\ M_3 & 4 & 1 & 4 & 6 & 0 \\ M_4 & 0 & 4 & 3 & 0 & 0 \\ M_5 & 6 & 4 & 0 & 1 & 3 \end{array}$$

← After column Minima step; every row & every column should have atleast one zero.

← After Row & Column Minima ⇓ we call it as Reduced Matrix.

Now we find Assignment.

$$\begin{array}{c|ccccc} & J_1 & J_2 & J_3 & J_4 & J_5 \\ \hline M_1 & \boxed{0} & 7 & \cancel{0} & \cancel{0} & 5 \\ M_2 & 4 & \boxed{0} & 4 & 4 & 5 \\ M_3 & 4 & 1 & 4 & 6 & \boxed{0} \\ M_4 & \cancel{0} & 4 & 3 & \boxed{0} & \cancel{0} \\ M_5 & 6 & 4 & \boxed{0} & 1 & 3 \end{array}$$

$$Z_{Min} = \sum\sum c_{ij} x_{ij}$$
$$= \sum\sum c_B x_B \quad (x_B \to 1)$$
$$= \sum\sum c_{ij}$$
$$= 3+1+3+2+2$$
$$= 11$$

So the assignment time = 11 (hours | units).

# Hungarian Method Steps

To solve an Assignment Problem (Minimization) follow the given steps:

**Step 1:** No. of sources = No. of destination ; if not then it is unbalanced assignment problem so before proceeding add a dummy row or dummy column and make it as balanced A.P.

**Step 2:** Select smallest element in each row and subtract the same from each row elements,

**Step 3:** Select smallest element in each column and subtract the same from each column elements.

**Step 4:** In reduced matrix (obtained from step 2 & 3), find assignment;

(a) Start Examining row; if there is a unique zero in row then allocate it ( $\boxed{\phantom{0}}$ ) else move to next row.

(b) Once all rows are covered in previous step; repeat

process for column; if there is a unique zero in column then allocate it (□).

(c) If each row and each column has a unique allocation then it is optimal; otherwise move to next step.

**Step 5:** Draw minimum no. of horizontal and/or vertical lines to cover all zeros of reduced matrix (either randomly or by following procedure)

(a) Mark (√) rows that do not have any assigned zero.

(b) Mark (√) columns that have zeros in the marked rows.

(c) Mark (√) rows that have assigned zeros in the marked columns.

(d) Repeat steps until chain is complete.

(e) Draw lines through unmarked rows and marked columns.

This will gives us desired minimum no. of lines.

**Step 6:** In new reduced matrix obtained in step 5,

(a) Mark (√) rows that do not have any assigned zero (or ☐)

(b) Mark (√) columns that have zeros in the marked rows.

(c) Mark (√) rows that have assigned zeros in the marked columns.

(d) Repeat steps until chain is complete.

(e) Draw lines through unmarked rows and marked columns

This will gives us desired minimum no. of lines.

Step 6: In new reduced matrix obtained in step 5,

(a) Find smallest element of the reduced matrix not covered by any of the lines.

(b) Subtract this element from all the uncovered elements and add the same to all the elements lying at the intersection of any two lines.

Step 7: Go to step 4 and repeat procedure until an optimum solution is attained.

The optimal no. of assignments is equal to n; if

**Problem :** A departmental head has four subordinates, and four tasks to be performed. The subordinates differ in efficiency, and the tasks differ in their intrinsic difficulty; His estimate, of the time each man would take to perform each task, is given in the matrix

Men

| Tasks | E | F | G | H |
|---|---|---|---|---|
| A | 18 | 26 | 17 | 11 |
| B | 13 | 28 | 14 | 26 |
| C | 38 | 19 | 18 | 15 |
| D | 19 | 26 | 24 | 10 |

← Allocate one task to one man so as to min total time.

**Solution:**

**Step 1:** (Row-Minima) →

$$\begin{bmatrix} 7 & 15 & 6 & 0 \\ 0 & 15 & 1 & 13 \\ 23 & 4 & 3 & 0 \\ 9 & 16 & 14 & 0 \end{bmatrix}$$

**Step 2 :** (Column-Minima)

$$\begin{bmatrix} 7 & 11 & 5 & 0 \\ 0 & 11 & 0 & 13 \\ 23 & 0 & 2 & 0 \\ 9 & 12 & 13 & 0 \end{bmatrix}$$

**Step 3:** Reduced Matrix (Do allocation)

$$\begin{bmatrix} 7 & 11 & 5 & 0 \\ 0 & 11 & 0 & 13 \\ 23 & 0 & 2 & 0 \\ 9 & 12 & 13 & 0 \end{bmatrix}$$

↓ Reduced Matrix

Men

| Tasks | E | F | G | H |
|---|---|---|---|---|
| A | 18 | 26 | $\boxed{17}$ | 11 |
| B | $\boxed{13}$ | 28 | 14 | 26 |
| C | 38 | $\boxed{19}$ | 18 | 15 |
| D | 19 | 26 | 24 | $\boxed{10}$ |

← Allocate one task to one man so as to min total time.

Solution:

Step 1: (Row-Minima) →

$$\begin{bmatrix} 7 & 15 & 6 & 0 \\ 0 & 15 & 1 & 13 \\ 23 & 4 & 3 & 0 \\ 9 & 16 & 14 & 0 \end{bmatrix}$$

Step 2: (Column-Minima)

$$\begin{bmatrix} 7 & 11 & 5 & 0 \\ 0 & 11 & 0 & 13 \\ 23 & 0 & 2 & 0 \\ 9 & 12 & 13 & 0 \end{bmatrix}$$

↓ Reduced Matrix (after Row & Colum Minima stg)

Step 3: Reduced Matrix (Do allocation)

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} E & & & H \end{array} \\ \begin{bmatrix} 7 & 11 & 5 & \boxed{0} \\ \boxed{0} & 11 & \cancel{0} & 13 \\ 23 & \boxed{0} & 2 & \cancel{0} \\ 9 & 12 & 13 & \cancel{0} \end{bmatrix} \end{array}$$

Last row has not allocation & third column has no allocation

Step 4: New Reduced Matrix

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} E & F & G & H \end{array} \\ \begin{bmatrix} 2 & 6 & \boxed{0} & \cancel{0} \\ \boxed{0} & 11 & \cancel{0} & 18 \\ 23 & \boxed{0} & 2 & 5 \\ 4 & 7 & 8 & \boxed{0} \end{bmatrix} \end{array}$$

← choose least element not covered under lines (here it is 5)

optimal solution:

| Task | Men |
|---|---|
| A → | G |
| B → | E |
| C → | F |
| D → | H |

Total time: optimal value

$$\text{Min } Z = 17 + 13 + 19 + 10$$

$$= 59 \text{ man-hours.}$$

$(x_{ij} = 1, 0)$ B.V.

$(c_{ij} x_{ij})$

(Unbalanced Assignment Problem)

**Problem :** A departmental head has four subordinates, and four tasks to be performed. The subordinates differ in efficiency, and the tasks differ in their intrinsic difficulty; His estimate, of the time each man would take to perform each task, is given in the matrix

| Tasks | Men 1 | 2 | 3 |
|---|---|---|---|
| I | 9 | 26 | 15 |
| II | 13 | 27 | 6 |
| III | 35 | 20 | 15 |
| IV | 18 | 30 | 20 |

← Allocate one task to one man so as to min total time.

**Solution:→**

$$\begin{array}{c|cccc} \text{Task} & \multicolumn{4}{c}{\text{Men}} \\ & 1 & 2 & 3 & 4 \\ \hline \text{I} & 9 & 26 & 15 & 0 \\ \text{II} & 13 & 27 & 6 & 0 \\ \text{III} & 35 & 20 & 15 & 0 \\ \text{IV} & 18 & 30 & 20 & 0 \end{array}$$

**Solution:→**

| Task | Men 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| I | 9 | 26 | 15 | 0 |
| II | 13 | 27 | 6 | 0 |
| III | 35 | 20 | 15 | 0 |
| IV | 18 | 30 | 20 | 0 |

**Step 1 (Row Minima)**

[ same ]

**Step 2 (Column Minima)**

$$\begin{bmatrix} \boxed{0} & 6 & 9 & \cancel{0} \\ 4 & 7 & \boxed{0} & \cancel{0} \\ 26 & \boxed{0} & 9 & \cancel{0} \\ 9 & 10 & 14 & \boxed{0} \end{bmatrix}$$

⇑ (Reduced Matrix)

**Optimal Assignment is**

I → 1

II → 3

III → 2

IV → 4 (Dummy)

(task 4 remains incomplete or task 4 is not assigned to given persons 1, 2, 3)

∴ Min time (optimal) = 35 hours.

Example: Solve the given problem using Hungarian Method

Deficit Cities

| Surplus Cities | a | b | c | d | e |
|---|---|---|---|---|---|
| A | 85 | 75 | 65 | 125 | 75 |
| B | 90 | 78 | 66 | 132 | 78 |
| C | 75 | 66 | 57 | 114 | 69 |
| D | 80 | 72 | 60 | 120 | 72 |
| E | 76 | 64 | 56 | 112 | 68 |

Determine optimum Assignment Schedule.

The given quantity is distance

Solution: Reduced Matrix (after Row / Column Minima Step)

| | a | b | c | d | e |
|---|---|---|---|---|---|
| A | 2 | 2 | 0 | 4 | 0 |
| B | 6 | 4 | 0 | 10 | 2 |
| C | 0 | 1 | 0 | 1 | 2 |
| D | 2 | 4 | 0 | 4 | 2 |
| E | 2 | 0 | 0 | 0 | 2 |

least elt not covered under lines is 2.

New Reduced Matrix

| | | | | | |
|---|---|---|---|---|---|
| 2 | 2 | 2 | 4 | 0 | ✓ |
| 4 | 2 | 0 | 8 | 0 | ✓ |
| 0 | 1 | 2 | 1 | 2 | ✓ |
| 0 | 2 | 0 | 2 | 0 | ✓ |
| 2 | 0 | 2 | 0 | 2 | |
| ✓ | ✓ | ✓ | | | |

| | | | | |
|---|---|---|---|---|
| 2 | 1 | 2 | 3 | 0 |
| 4 | 1 | 0 | 7 | 0 |
| 0 | 0 | 2 | 0 | 2 |
| 0 | 1 | 0 | 1 | 0 |
| 3 | 0 | 3 | 0 | 3 |

Alternate Solution

One Possibility is

| | | | | |
|---|---|---|---|---|
| 2 | 1 | 2 | 3 | 0 |
| 4 | 1 | 0 | 7 | 0 |
| 0 | 0 | 2 | 0 | 2 |
| 0 | 1 | 0 | 1 | 0 |
| 3 | 0 | 3 | 0 | 3 |

$A \to e$, $B \to c$, $C \to b$, $D \to a$, $E \to d$

$Z = 399$ Km

other Possibility is

| | | | | |
|---|---|---|---|---|
| 2 | 1 | 2 | 3 | 0 |
| 4 | 1 | 0 | 7 | 0 |
| 0 | 0 | 2 | 0 | 2 |
| 0 | 1 | 0 | 1 | 0 |
| 3 | 0 | 3 | 0 | 3 |

$A \to e$, $B \to c$, $C \to d$, $D \to a$, $E \to b$

$Z = 399$ Km.

- Maximization Case in Assignment Problem
- Restricted Assignment Problem

**Problem:** (Maximization A.P.) Consider a maximization A.P.;

Zones

| Sales Engineer | A | B | C | D |
|---|---|---|---|---|
| P | 140 | 112 | 98 | 154 |
| Q | 90 | 72 | 63 | 99 |
| R | 110 | 88 | 77 | 121 |
| S | 80 | 64 | 56 | 88 |

Each sales Engineer can choose one zones !!.

**Solution:** → Convert Maximization problem into Minimization
Subtract all enteries from maximum entry available in table

Sales Engg.

$$\begin{bmatrix} 14 & 42 & 56 & 0 \\ 64 & 82 & 91 & 55 \\ 44 & 66 & 77 & 33 \\ 74 & 90 & 98 & 66 \end{bmatrix}$$

← This is Minimization problem.

Engineer

$$\begin{array}{c} Q \\ R \\ S \end{array}\begin{bmatrix} 90 & 72 & 63 & 77 \\ 110 & 88 & 77 & 121 \\ 80 & 64 & 56 & 88 \end{bmatrix}$$

**Solution :→** Convert Maximization problem into Minimization

Subtract all entries from maximum entry available in table

$$\text{Sales Engg.}\quad \begin{array}{c} \\ P \\ Q \\ R \\ S \end{array}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 14 & 42 & 56 & 0 \\ 64 & 82 & 91 & 55 \\ 44 & 66 & 77 & 33 \\ 74 & 90 & 98 & 66 \end{bmatrix} \end{array}$$

← This is Minimization problem.

Step1 Row | Column Reduced Matrix

$$\begin{bmatrix} 6 & 18 & 24 & \boxed{0} \\ 1 & 3 & 4 & 0 \\ 3 & 9 & 12 & 0 \\ \boxed{0} & 0 & 0 & 0 \end{bmatrix}$$

New Reduced Matrix

$$\begin{bmatrix} 5 & 17 & 23 & \boxed{0} \\ \boxed{0} & 2 & 3 & 0 \\ 2 & 8 & 11 & 0 \\ 0 & \boxed{0} & 0 & 1 \end{bmatrix}$$

→ New Reduced Matrix

$$\begin{bmatrix} 3 & 15 & 21 & \boxed{0} \\ \boxed{0} & 2 & 3 & 2 \\ 0 & 6 & 9 & 0 \\ 0 & \boxed{0} & 0 & 3 \end{bmatrix}$$

$$\Downarrow$$

$$\begin{bmatrix} 3 & 13 & 19 & \boxed{0} \\ 0 & \boxed{0} & 1 & 2 \\ \boxed{D} & 4 & 7 & 0 \\ 2 & 0 & \boxed{0} & 5 \end{bmatrix}$$

Now optimal.

$$\underbrace{P \to D,\ Q \to B,\ R \to A,\ S \to C}$$

$$\text{Max } Z = 154 + 72 + 110 + 56$$

$$= 392$$ ~~units~~ (units (Rs. Profit))

**Problem:** (Restricted A.P.)

In the modification of a plant layout of a factory, four new machines $M_1$, $M_2$, $M_3$ and $M_4$ are to be installed in a Machine shop. There are five vacant places A, B, C, D and E available. Because of limited space, Machine $M_2$ cannot be placed at C and $M_3$ cannot be placed at A. The cost of placing of machine $i$ at place $j$ (in dollars) is shown below.

$$\begin{array}{c|ccccc} & A & B & C & D & E \\ \hline M1 & 9 & 11 & 15 & 10 & 11 \\ M2 & 12 & 9 & - & 10 & 9 \\ M3 & - & 11 & 14 & 11 & 7 \\ M4 & 14 & 8 & 12 & 7 & 8 \end{array}$$

Locations

**Solution:** → Assign, a high cost M to the locations that need to be blocked. → $(M>0)$

$$\begin{array}{c} M1 \\ M2 \\ M3 \\ M4 \end{array} \begin{bmatrix} 9 & 11 & 15 & 10 & 11 \\ 12 & 9 & M & 10 & 9 \\ M & 11 & 14 & 11 & 7 \\ 14 & 8 & 12 & 7 & 8 \end{bmatrix} \xrightarrow{\text{Solve this using}}$$

$$\begin{array}{c} M1 \\ M2 \\ M3 \\ M4 \\ M5 \end{array} \begin{bmatrix} 9 & 11 & 15 & 10 & 11 \\ 12 & 9 & M & 10 & 9 \\ M & 11 & 14 & 11 & 7 \\ 14 & 8 & 12 & 7 & 8 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Now Apply Hungarian Method.

Row Min | Cole. Min

$$\begin{bmatrix} 0 & 2 & 6 & 1 & 2 \\ 3 & 0 & M & 1 & 0 \\ M & 4 & 7 & 4 & 0 \\ 7 & 1 & 5 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(M>0)

$$\begin{bmatrix} \boxed{0} & 2 & 6 & 1 & 2 \\ 3 & \boxed{0} & M & 1 & \cancel{0} \\ M & 4 & 7 & 4 & \boxed{0} \\ 7 & 1 & 5 & \boxed{0} & 1 \\ \cancel{0} & \cancel{0} & \boxed{0} & \cancel{0} & \cancel{0} \end{bmatrix}$$

Optimal Assignment

$M_1 \rightarrow A$

$M_2 \rightarrow B$

$M_3 \rightarrow E$

$M_4 \rightarrow D$

( Location C remains vacant ).

$Z_{\text{Min cost}} = 320$

# The END