Calculus

Sub Code: SC107, Tutorial 4

1. For the following function find a formula for the Riemann sum obtained by dividing the interval $[a, b]$ into n equal subintervals and using the right-hand end point for each c_k (or x_k^*). Then take a limit of this sum as $n \to \infty$ to calculate the area $\int_a^b f(x)dx$ under the curve over $[a, b]$.

   (a) $f(x) = x^2 - x^3$ over the interval $[-1, 0]$.

2. Show that the value of $\int_0^1 \sqrt{x+8}\, dx$ lies between $2\sqrt{2}$ and 3.

3. Use Fundamental theorem to find $\frac{dy}{dx}$ if $y = x \int_2^{x^2} \sin(t^3)dt$.

4. Find the areas of the regions enclosed by lines and curves:

   (a) $y = x^2 - 2x$ and $y = x$

   (b) $x - y^2 = 0$ and $x + 2y^2 = 3$

5. Use Disk method to find the volume of the solid generated by revolving the region in the first quadrant bounded above by the line $y = 2$, below by the curve $y = 2\sin x$, $0 \le x \le \frac{\pi}{2}$, and on the left by the y-axis about the line $y = 2$.

6. Use Washer method to find the volume of the solid generated by revolving the region bounded by the line $y = x + 3$ and the curve $y = x^2 + 1$ about the x-axis.

7. Use the shell method to find the volume of the solid generated by revolving the region bounded by the curves $y = 2 - x^2$ and $y = x^2$ and by the line $x = 0$ about the y-axis.