

TUTORIAL 03

1. Find the volume of the largest right circular cylinder that can be inscribed in a sphere of radius 5.
2. You operate a tour service that offers the following rates:
Rs 200 per person if 50 people (the minimum number to book the tour) go on the tour. For each additional person, upto a maximum of 80 people total, the rate per person is reduced by Rs 2. It costs Rs 6000 (a fixed cost) plus Rs 32 per person to conduct the tour. How many people doesn't take to maximize your profit?
3. A trucker handed in a ticket at a toll booth showing that in 2 hours she had covered 159 mi on a toll road with speed limit 65 mph. The trucker was cited for speeding. Why?
4. A marathoner ran the 26.2-mi Marathon in 2.2 hours. Show that at least twice the marathoner was running at exactly 11 mph, assuming the initial and final speeds are zero.
5. If $f'(x) = 0$ for all $x \in (a, b)$, then $f(x)$ is constant. Prove it.
6. Show that $|\cos x - 1| \leq |x|$ for all x -values. (*Hint:* Consider $f(t) = \cos t$ on $[0, x]$).
7. If f has a finite 3rd derivative f''' on $[a, b]$ and if $f(a) = f(b) = f'(a) = f'(b) = 0$, prove that $f'''(c) = 0$ for some $c \in (a, b)$.
8. Assume f has a finite derivative in (a, b) and is continuous on $[a, b]$ with $f(a) = f(b) = 0$. Prove that for every real λ there is some $c \in (a, b)$ such that $f'(c) = \lambda f(c)$.
9. Plot a graph of the function $f(x) = \frac{x-4}{x^2}$ and then use derivative to discuss any significant features of the plot.