Tutorial 6

1. Solve the following linear differential equations,

   (a) $(1 + y^2)dx = (\tan^{-1}(y) - x)dy$

   (b) $x\,dy = (y + 4)dx$

   (c) $(2y - x^3)dx = x\,dy$

   (d) $3y^2 \frac{dy}{dx} + xy^3 = x$

   (e) $\sin(y)\frac{dy}{dx} = \cos(x)(2\cos(y) - \sin^2(x))$

   (f) $(1 - x^2)\frac{dy}{dx} - xy - axy^2$

2. Find the general solution of the following differential equations,

   (a) $4y'' + 16y' + 52y = 0$

   (b) $4y'' + 4y' + y = 0$

3. (a) Show that there is no solution to the boundary value problem,

   $$y'' + 4y = 0, \quad y(0) = 0, \quad y(\pi) = 1$$

   (b) Show that there are infintely many solutions to the boundary value problem,

   $$y'' + 4y = 0, \quad y(0) = 0, \quad y(\pi) = 0$$

4. Solve the following non-homogeneous equations by the method of undetermined coefficients,

   (a) $y'' + y = 2x + 3e^x$

   (b) $y'' - y' - 6y = e^{-x} - 7\cos(x)$

5. Solve the following differenatial equations by the method of variation of parameters,

   (a) $\frac{d^2y}{dx^2} + y = \csc(x)$

   (b) $(D^2 + 4)y = 2\tan(2x)$

6. Solve,

   (a) $\frac{dy}{dx} = \frac{x-y+1}{x+y-3}$

   (b) $(x + y)(dx - dy) = dx + dy$

   (c) $(2x + 3y - 5)\frac{dy}{dx} + (2x + 3y + 1) = 0$