# TUTORIAL ~~07~~

1. Find the first order partial derivatives of the following functions at the point $(x, y)$ from the first principles/definitions.
   (i) $f(x, y) = x^2 + y^2 + x$
   (ii) $f(x, y) = \sin(2x + 3y)$

2. Show that the function
$$f(x,y) = \begin{cases} (x+y)\sin\frac{1}{x+y} & ; x+y \neq 0 \\ 0 & ; x+y = 0 \end{cases}$$
is continuous at $(0, 0)$ but its partial derivatives $f_x$ and $f_y$ do not exist at $(0, 0)$.

3. Show that the function
$$f(x,y) = \begin{cases} \frac{xy}{x^2+2y^2} & ; (x,y) \neq (0,0) \\ 0 & ; (x,y) = (0,0) \end{cases}$$
is not continuous at $(0, 0)$ but its partial derivatives $f_x$ and $f_y$ exist at $(0, 0)$.

4. Show that the function
$$f(x,y) = \begin{cases} \frac{xy}{\sqrt{x^2+y^2}} & ; (x,y) \neq (0,0) \\ 0 & ; (x,y) = (0,0) \end{cases}$$
has partial derivatives $f_x(0, 0)$ and $f_y(0, 0)$, but the partial derivatives are not continuous at $(0, 0)$.

5. Let $f(x, y) = x^2 - xy + y^2 - y$. Find the direction $\mathbf{u}$ and the value of $D_{\mathbf{u}}f(1, -1)$ for which
   (i) $D_{\mathbf{u}}f(1, -1)$ is largest
   (ii) $D_{\mathbf{u}}f(1, -1)$ is smallest
   (iii) $D_{\mathbf{u}}f(1, -1)$=0
   (iv) $D_{\mathbf{u}}f(1, -1)$=4
   (v) $D_{\mathbf{u}}f(1, -1)$=-3

6. Is there a direction $\mathbf{u}$ in which the rate of change of $f(x, y) = x^2 - 3xy + 4y^2$ at $P(1, 2)$ equals 14? Give reason for your answer.

7. The derivative of $f(x, y, z)$ at a point $P$ is greatest in the direction of $\vec{v} = \vec{i} + \vec{j} - \vec{k}$. In this direction the value of the derivative is $2\sqrt{3}$.
   (i) What is $\nabla f$ at $P$? Give reason for your answer.
   (ii) What is the derivative of $f$ at $P$ in the direction of $\vec{i} + \vec{j}$?