Q.1 (a) Find a curve with a positive derivation through the point (1,1) whose length integral is

$$L = \int_1^4 \sqrt{1+\frac{1}{4x}}\, dx. \quad —(1)$$

Arc length of a curve $y = f(x)$ from a to b is

$$L = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

comparing with (1)

$$\left(\frac{dy}{dx}\right)^2 = \frac{1}{4x} \Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$

$$\Rightarrow y = \sqrt{x}$$

The curve is $y = \sqrt{x}$ from (1,1) to (4,2)

(b) How many such curves are there! Give reason for your answer.

- Only one. Because, the interval (1,1) is fixed and we are searching for a function in the interval only.

Q.2 Find the length of the curve $y = \int_0^x \sqrt{\cos 2t}\, dt$

from $x = 0$ to $x = \frac{\pi}{4}$.

$$S(x) = \int_0^x \sqrt{\cos 2t}\, dt$$

By fundamental theorem of calculus,

$$\frac{ds}{dx} = \sqrt{\cos 2x}$$

$$\Rightarrow \sqrt{1 + \left(\frac{ds}{dx}\right)^2} = \sqrt{1 + \cos 2x}$$

$$L = \int_0^{\pi/4} \sqrt{1 + \left(\frac{ds}{dx}\right)^2}\, dx$$

$$= \int_0^{\pi/4} \sqrt{1 + \cos 2x}\, dx$$

$$= \int_0^{\pi/4} \sqrt{\cancel{1} + 2\cos^2 x - \cancel{1}}\, dx$$

$$= \sqrt{2} \int_0^{\pi/4} \cos x\, dx$$

$$= \sqrt{2}\, (\sin x)_0^{\pi/4}$$

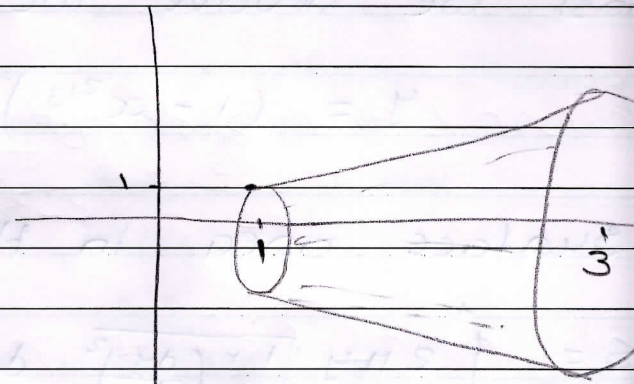
$$= \sqrt{2}\left(\frac{1}{\sqrt{2}} - 0\right)$$

$$= \boxed{1}$$

Q3 Find the surface area of the cone frustum generated by revolving the line segment $y = \frac{x}{2} + \frac{1}{2}$, $1 \le x \le 3$, about the x-axis.

$$S = \int_1^3 2\pi y \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \int_1^3 2\pi\left(\frac{x}{2}+\frac{1}{2}\right)\frac{\sqrt{5}}{2}\, dx$$



$$= \sqrt{5}\,\pi \int_1^3 \left(\frac{x}{2}+\frac{1}{2}\right) dx$$

$$= \frac{\sqrt{5}\,\pi}{2}\left[\frac{x^2}{2}+x\right]_1^3$$

$y = x/2 + 1/2$

$\Rightarrow \frac{dy}{dx} = \frac{1}{2}$

$\Rightarrow \sqrt{1+\left(\frac{dy}{dx}\right)^2} = \sqrt{1+\frac{1}{4}}$

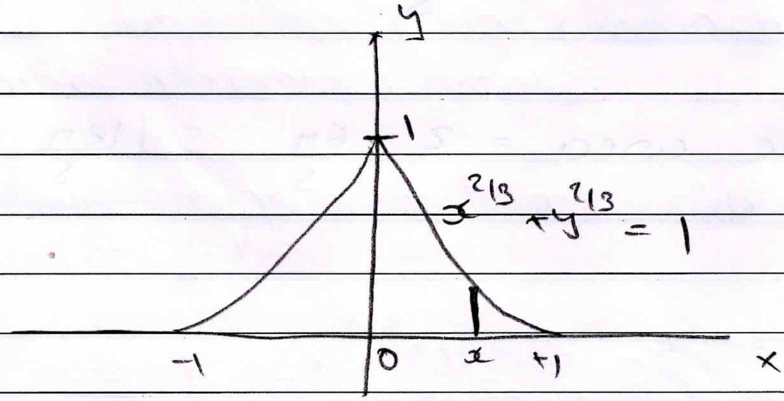
$= \sqrt{5}/2$

$$= \frac{\sqrt{5}}{2}\pi\left[\frac{9}{2}+3-\frac{1}{2}-1\right]$$

$$= \frac{\sqrt{5}}{2}\pi\left[4+2\right]$$

$$= \boxed{3\sqrt{5}\,\pi}$$

Q.4 Find the area of the surface generated by revolving the upper half portion of the astroid $x^{2/3}+y^{2/3}=1$ about the x-axis.



First, we sevolve the first quadsate position,

$$y = (1 - x^{2/3})^{3/2} \quad ; \; 0 \le x \le 1$$

Susface asea in the ~~first quadsate~~ sight side of y-axis:

$$S = \int_0^1 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx$$

$$= \int_0^1 2\pi (1 - x^{2/3})^{3/2} \cdot x^{-1/3} \, dx$$

$$(1 - x^{2/3}) = u$$

$$= \int_1^0 2\pi u^{3/2} \cdot \frac{-3}{2} du$$

$$= -3\pi \int_1^0 u^{3/2} du$$

$$= -3\pi \frac{2}{5} \left[u^{5/2} \right]_1^0$$

$$= -\frac{6\pi}{5} [0 - 1] = \frac{6\pi}{5}$$

$$y = (1 - x^{2/3})^{3/2}$$

$$\frac{dy}{dx} = -\frac{3}{2}(1 - x^{2/3}) \frac{2}{3} x^{-1/3}$$

$$= -(1 - x^{2/3}) x^{-1/3}$$

$$\left(\frac{dy}{dx}\right)^2 = (1 - x^{2/3}) x^{-2/3}$$

$$= x^{-2/3} - x^{2/3} \cdot x^{-2/3}$$

$$= x^{-2/3} - 1$$

$$\sqrt{1 + \left(\frac{dy}{dx}\right)^2} = \sqrt{\cancel{1} + x^{-2/3} - \cancel{1}}$$

$$= \sqrt{x^{-2/3}}$$

$$= x^{-1/3}$$

Total susface asea $= 2 \times \frac{6\pi}{5} = \boxed{\frac{12\pi}{5}}$