Tutorial ~~12~~

1. Show that $f(z) = |z|^2$ is differentiable only at $z = 0$ and nowhere else. So it is nowhere analytic.

2. Prove that an analytic function whose real part is constant is a constant function.

3. Prove that an analytic function whose modulus is constant is a constant function.

4. Find the principal value of the argument for the following,

   (a) $1 - i$

   (b) $3 + 4i$

   (c) $-\pi - \pi i$

   (d) $-5 + 5i$

5. Find,

   (a) $\sqrt[3]{1+i}$

   (b) $\sqrt[4]{-4}$

6. Show that an analytic function is independent of $\overline{z}$.

7. Show that
   $$f(z) = |Re(z)Im(z)|^{\frac{1}{2}}$$
   satisfies the CR equation at the origin but is not differentiable at the origin.

8. Sketch the following sets in the complex plane and decide whether they are open, closed or a domain,

   (a) $S = \{z | \, |z-1| < 1 \text{ or } |z+1| < 1\}$

   (b) $|arg(z)| < \frac{\pi}{4}$

9. Show that the function,

$$f(x) = \begin{cases} \frac{x^3(1+i)-y^3(1-i)}{x^2+y^2} & z \neq 0 \\ 0 & z = 0 \end{cases}$$

   satisfies the Cauchy-Riemann equations at $z = 0$ but $f'(0)$ doesn't exist.

10. Show that the derivative of a real valued function $f(z)$ of a complex variable z at any point is either zero or it doesn't exist.