TUTORIAL ~~13~~

1. Integrate the following.
   a. $\int_C Re(z)dz$, where C is the shortest path from $1+i$ to $3+2i$.

   b. $\int_C \bar{z}dz$, where C from 0 along the parabola $y = x^2$ to $1+i$.

   c. $\int_C ze^{z^2}dz$, where C is from 1 along the axes to i.

   d. $\int_C sec^2 z dz$, where C is any path from $\frac{\pi i}{4}$ to $\frac{\pi}{4}$ in the unit disk.

2. If $f(z)$ is analytic in a simply connected domanin D. Prove that $\int_a^b f(z)dz$ is independent of the path in D joining any two points a and b in D.

3. Integrate $\oint_C \frac{4z^2+z+5}{z-3.5}dz$, where C is the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$.

4. Integrate $\oint_C \frac{z^3+\sin z}{(z-i)^3}dz$, where C is the boundary of the square with vertices $\pm 2, \pm 2i$.

5. Integrate $\oint_C \frac{2z^3-3}{z(z-1-i)^2}dz$, where
$$C : \begin{cases} |z| = 2 \text{ anticlockwise} \\ |z| = 1 \text{ clockwise} \end{cases}$$

6. Evaluate $\oint_C \frac{3z^2+z}{z^2-1}dz$, where C is the circle $|z-1| = 1$.

7. Evaluate $\oint_C \frac{e^{zt}}{(z^2+1)^2}dz$, if $t > 0$ and C is the circle $|z| = 3$.

8. Let C denote the right hand half of the circle $|z| = 2$ in the counterclockwise direction. Show that the two parametric representation for C are
   a. $z = r(\theta) = 2e^{i\theta},\ -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$,
   b. $z = \sqrt{4-y^2} + iy,\ -2 \leq y \leq 2$.

   Find the value of the integral
$$I = \int_C \bar{z}dz$$
   using both the parametric representation.

9. Evaluate $\int_C (z+1)^2 dz$, where C is the boundary of the rectangle in anticlockwise direction with vertices at points $a+ib,\ -1+ib,\ -1-ib,\ a-ib$. Do not use the Cauchy's integral theorem.

10. Evaluate the integral $\oint_C \frac{1}{z^2+4}dz$, where $(i)\ C : |z-2i| = 1,\ (ii)\ C : |z+2i| = 1,\ (iii)\ C : |z| = 4$.

11. Let Γ be a smooth curve. Suppose $\Gamma = [a, b]$. Prove that for any integrable function $f : [a, b] \to C$

$$\overline{\int_a^b f(t)dt} = \int_a^b \overline{f(t)}dt$$

What can you deduce about the integrability of $\bar{f}$ if f is integrable?