Tutorial 2

1. Find the limits for the following,

   (a) $\lim_{x\to\infty}(\sqrt{x^2+3x}-\sqrt{x^2-2x})$

   (b) $\lim_{x\to\infty}(\sqrt{x^2+x}-\sqrt{x^2-x})$

2. Use formal definitions to prove that,

   (a) $\lim_{x\to 0}\frac{1}{|x|}=\infty$

   (b) $\lim_{x\to 1^-}\frac{1}{1-x^2}=\infty$

3. Find the oblique asymptotes of,

   (a) $f(x)=\frac{x^2+1}{x-1}$

   (b) $f(x)=\frac{x^3+1}{x^2}$

4. $$f(x)=\begin{cases} x, & \text{if } x \text{ is rational} \\ 0, & \text{if } x \text{ is irrational} \end{cases}$$

   (a) Show that f is continuous at $x=0$.

   (b) Also show that f is not continuous at any other real number.

5. Which of the following statements are **true** and which are **false**? If true state why, and if false provide a counter-example.

   (a) If $\lim_{x\to a}f(x)$ exists but $\lim_{x\to a}g(x)$ does not exist, then $\lim_{x\to a}(f(x)+g(x))$ does not exist.

   (b) If neither $\lim_{x\to a}f(x)$, nor $\lim_{x\to a}g(x)$ exists, then $\lim_{x\to a}(f(x)+g(x))$ does not exist.

   (c) If f is continuous at x, then so is $|f|$.

   (d) If $|f|$ is continuous at x, then so is f.

6. Identify the *horizontal*, *vertical* and *oblique* asymptotes for the graph of the following functions,

   (a) $f(x) = \frac{\sqrt{5x^2+7}}{2x+3}$

   (b) $f(x) = \frac{1-4x^3}{3+2x-x^2}$

7. For the following functions classify each discontinuity as a *removable discontinuity*, a *jump discontinuity* or an *infinite discontinuity*,

   (a) $f(x) = \frac{16-x^2}{x+4}$

   (b) $f(x) = \frac{x+12}{x^2-9}$

   (c) $f(x) = \frac{x^3-27}{|x-3|}$

8. Determine the value of **a** so that $f(x) = \frac{x^2+ax+5}{x+1}$ has an oblique asymptote $y = x + 3$.