Tutorial ~~8~~

1. Find all the local minima, local maxima and saddle points of the functions,

   (a) $f(x, y) = e^{x^2+y^2-4x}$

   (b) $f(x, y) = \ln(x + y) + x^2 - y$

   (c) $f(x, y) = 1 - \sqrt[3]{x^2 + y^2}$

2. Find the global(absolute) maxim and minima of the function,

   $$f(x, y) = 2x^2 - 4x + y^2 - 4y + 1$$

   on the closed triangular plate bounded by the lines $x = 0$, $y = 2$, $y = 2x$ in the first quadrant.

3. Find two numbers a and b where $a < b$, such that

   $$\int_a^b (6 - x - x^2)dx$$

   has its largest value.

4. A flat circular plate has the shape of the region $x^2 + y^2 \leq 1$. The plate including the boundary where $x^2 + y^2 = 1$ is heated so that the temperature at the point (x, y) is,

   $$T(x, y) = x^2 + 2y^2 - x$$

   Find the temperature at the hottest and coldest points on the plate.

5. Verify that $(0, 0)$ is the only critical point of the function,

   $$f(x, y) = x^2 - 6xy^2 + y^4$$

   and that the Hessian(or discriminant) $f_{xx}f_{yy} - f_{xy}^2 = 0$ at this critical point. Show algebraically that the critical point gives a saddle point.

6. Using Taylor's formula, for any $k \in \mathbb{N}$ and for all $x > 0$ show that

$$x - \frac{1}{2}x^2 + ... - \frac{1}{2k}x^{2k} < \log(1+x) < x - \frac{1}{2}x^2 + ... - \frac{1}{2k+1}x^{2k+1}$$

7. The function $f(x, y) = x^2 - xy + y^2$ is approximated by a first degree Taylor's polynomial about the point $(2, 3)$. Find a square $|x - 2| < \delta$, $|y - 3| < \delta$ with centre at $(2, 3)$ such that the error of approximation is less than or equal to 0.1.