

CS34110 Discrete Mathematics and Graph Theory

UNIT – III, Module – 1

Lecture 23: Graphs & Trees

[Graph theory; Graph, vertex, edge; Null graph; Simple graph; Directed graph; Complete graph; Connected graph; Graph applicability]

Graph theory

- Graph theory: study of graph, one type of discrete structure, consisting of vertices and edges, with edges connecting vertices of that graph.
 - Different kinds of graphs: (i) based on directions of edges; (ii) based on possibility of multiple edges connecting same pair of vertices; (iii) based on loops allowed.

Graph

- Graph: discrete structure, represented as $\mathcal{G} = (\mathbf{V}, \mathbf{E})$, consisting of **nonempty** set of vertices \mathbf{V} and set of edges \mathbf{E} , in which each edge $e \in \mathbf{E}$ connecting one or two vertices (of \mathbf{V}).
- Property:: Graph \mathcal{G} : also called linear complex, 1-complex, one-dimensional complex.
- Property:: **Vertex** (in \mathcal{G}): also called node, point, junction, 0-cell, 0-simplex.
- Property:: **Edge** (in \mathcal{G}): also called line, arc, branch, element, 1-cell, 1-simplex.

Graph

- Graph:
 - Property:: $e = \{u, v\}$, $u, v \in \mathbf{V}$: edge e (without direction) connecting vertices u and v ; e said to become **incident** with (or on) u and v .
 - Property:: $\{u, v\}$: unordered pair of vertices representing **endpoints** of undirected edge $e \in \mathbf{E}$; $\{u, v\} = \{v, u\}$.
 - Property:: **Finite graph**: graph $\mathcal{G} = (\mathbf{V}, \mathbf{E})$ with finite vertex set \mathbf{V} and finite edge set \mathbf{E} .
 - Property:: **Infinite graph**: graph $\mathcal{G} = (\mathbf{V}, \mathbf{E})$ with either infinite vertex set \mathbf{V} , or infinite number of edges in \mathbf{E} , or both.
 - Property:: **Vertex graph** (also **null graph**): graph with $\mathbf{V} \neq \emptyset$, but $\mathbf{E} = \emptyset$.

Graph

- Graph:
 - Property:: Isolated vertex: vertex in graph with no incident edge.
 - Property:: **Self loop**: edge connecting vertex to itself in graph $\mathcal{G} = (\mathbf{V}, \mathbf{E})$, i.e., $e = \{v, v\}$, $e \in \mathbf{E}$, $v \in \mathbf{V}$.
 - Property:: **Parallel edges**: two distinct edges $e_1, e_2 \in \mathbf{E}$, where $e_1 \neq e_2$, connecting same vertex pair in graph $\mathcal{G} = (\mathbf{V}, \mathbf{E})$, i.e., $e_1 = \{u, v\}$, $e_2 = \{u, v\}$, $e_1 \neq e_2$, $e_1, e_2 \in \mathbf{E}$, $u, v \in \mathbf{V}$.
 - Property:: **Simple graph**: graph, with each edge connecting two distinct vertices (i.e., **no self loop**) and no two edges connecting same pair of vertices (i.e., **no parallel edges**).

Graph

- Graph:
 - Special example:: all 11 simple undirected graphs with 4 vertices in V .

Disconnected graphs

Connected graphs

[Ref. Frank Harary, *Graph Theory*, Addison-Wesley, Reading, MA, USA, 1969.]

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Graph

- Graph:
 - Property:: **Multigraph**: graph, with multiple parallel edges; multiplicity (of edge between specified pair of vertices) = count of multiple distinct edges for same pair of vertices.
 - Property:: **Pseudograph**: graph, with possibility of loops and multiple parallel edges, i.e., multigraph, with loops.
 - Property:: **Connected graph**: graph, in which every vertex to be reachable from every other vertex by traveling along edges.
 - Property:: **Disconnected graph**: graph, in which not possible to reach any vertex from any other vertex by traveling along edges.

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Graph

- Graph:
 - Property:: **Complete undirected graph**: simple undirected graph, with exactly one edge between each pair of distinct vertices.
 - Property:: **Directed graph** (or **digraph**): $\mathcal{G} = (V, E)$, consisting of nonempty V and set of directed edges E , in which each directed edge $e = (u, v)$, $u, v \in V$, $e \in E$, where (u, v) = ordered pair of vertices u and v , edge e starting at u and ending at v , and $(u, v) \neq (v, u)$.
 - Property:: **Arc**: directed edge.

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Graph

- Graph:
 - Property:: **Simple directed graph**: directed graph with no loops and no multiple directed edges.
 - Property:: **Complete directed graph**: simple directed graph, with exactly one directed edge between each pair of distinct vertices.
 - Property:: **Directed multigraph**: directed graph with multiple directed edges from one vertex to second (also possibly same) vertex.

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Graph

- Graph:

Graph Terminology			
Type	Edges	Multiple Edges Allowed?	Loops Allowed?
Simple graph	Undirected	No	No
Multigraph	Undirected	Yes	No
Pseudograph	Undirected	Yes	Yes
Simple directed graph	Directed	No	No
Directed multigraph	Directed	Yes	Yes
Mixed graph	Directed and undirected	Yes	Yes

[Ref: Kenneth H. Rosen, *Discrete Mathematics and its Applications*, Eighth edition, McGraw-Hill Education, 2019.]

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Graph

- Graph:
 - Property:: **Labeled undirected graph**: undirected graph, with each vertex to be assigned (user-defined) unique name or label (i.e., no two vertices to have same label).
 - Property:: **Unlabeled undirected graph**: undirected graph, with no assigned (or user-defined) vertex distinctions.
 - Property:: **(p,q) graph**: undirected graph $\mathcal{G} = (V, E)$, with $|V| = p$ and $|E| = q$.
 - Property:: **Trivial graph**: undirected graph $\mathcal{G} = (V, E)$, with single vertex (i.e., $|V| = 1$) and no edge (i.e., $|E| = 0$), i.e., $(1, 0)$ graph.

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Graph

- Graph:
- Property:: **Weighted undirected graph**: undirected graph, with each edge to be assigned (user-defined) numeric value, typically nonnegative value, representing some meaningful information.
- Property:: **Weight of edge e in undirected graph**: (user-defined) numeric value, typically of nonnegative nature, assigned to edge e of given graph.

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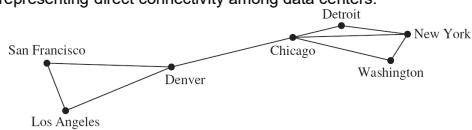
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Graph

- Graph examples:
- Example-1:: Data centers' network: communication link connecting data centers geographically apart.
- Data centers' simple network graph model: simple graph, representing direct connectivity among data centers.

[Ref: Kenneth H. Rosen, *Discrete Mathematics and its Applications*, Eighth edition, McGraw-Hill Education, 2019.]

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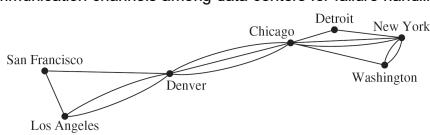
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- Graph examples:
- Example-2:: Data centers' fault-tolerant network graph model: multigraph, representing direct connectivity plus additional communication channels among data centers for failure handling.

[Ref: Kenneth H. Rosen, *Discrete Mathematics and its Applications*, Eighth edition, McGraw-Hill Education, 2019.]

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- Graph examples:
- Example-3:: Data centers' fault-tolerant & self-diagnostic network graph model: pseudograph, representing multiple communication channels among data centers, and feedback communication channel for diagnostic purposes.

[Ref: Kenneth H. Rosen, *Discrete Mathematics and its Applications*, Eighth edition, McGraw-Hill Education, 2019.]

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- Graph examples:
- Example-4:: Data centers' directed network graph model: directed multigraph, representing single duplex behavior of communication channels (based on data traffic) among data centers.

[Ref: Kenneth H. Rosen, *Discrete Mathematics and its Applications*, Eighth edition, McGraw-Hill Education, 2019.]

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Graph

- Graph examples:
- Example-5:: Social network's friendship graph model: simple graph, to represent friendship or acquaintance between two people (either in real world or in virtual world via social networking sites).

[Ref: Kenneth H. Rosen, *Discrete Mathematics and its Applications*, Eighth edition, McGraw-Hill Education, 2019.]

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Graph

- Graph examples:
 - Example-6: Software designer's dependency graph model: simple directed graph, to represent dependency between different modules (i.e., parts of software), like web browser here.

[Ref: Kenneth H. Rosen, *Discrete Mathematics and its Applications*, Eighth edition, McGraw-Hill Education, 2019.]

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Graph

- Graph examples:
 - Example-7: Software designer's program dependency graph model: simple directed graph, to represent dependency among statements of program.

$S_1 \quad a := 0$
 $S_2 \quad b := 1$
 $S_3 \quad c := a + 1$
 $S_4 \quad d := b + a$
 $S_5 \quad e := d + 1$
 $S_6 \quad e := c + d$

[Ref: Kenneth H. Rosen, *Discrete Mathematics and its Applications*, Eighth edition, McGraw-Hill Education, 2019.]

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Summary

- Focus: Graph fundamentals.
- Graph theory.
- Graph, vertex, edge, with properties.
- Finite, infinite graph.
- Null graph.
- Self loop, parallel edges.
- Simple graph, with examples.
- Multigraph, pseudograph.
- Directed graph, arc.
- Simple directed graph, directed multigraph.

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Summary

- Complete undirected, directed graph.
- Practical graph examples.

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References

1. [Ros19] Kenneth H. Rosen, *Discrete Mathematics and its Applications*, Eighth edition, McGraw-Hill Education, 2019.
2. [Lip07] Seymour Lipschutz and Marc Lars Lipson, *Schaum's Outline of Theory and Problems of Discrete Mathematics*, Third edition, McGraw-Hill Education, 2007.
3. [Wes01] Douglas Brent West, *Introduction to Graph Theory*, Second edition, Prentice-Hall, 2001.
4. [Deo74] Narsingh Deo, *Graph Theory with Applications to Engineering and Computer Science*, Prentice-Hall, 1974.
5. [Har69] Frank Harary, *Graph Theory*, Addison-Wesley, Reading, MA, USA, 1969.

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Further Reading

- Graph:: [Ros19]:673; [Deo74]:1,3.
- Finite, infinite graph:: [Ros19]:673; [Deo74]:6-7.
- Null graph:: [Deo74]:9.
- Self loop, parallel edges:: [Ros19]:674; [Deo74]:1,2.
- Simple graph:: [Ros19]:674; [Deo74]:2.
- Multigraph, pseudograph:: [Ros19]:674,675.
- Directed graph:: [Ros19]:675; [Deo74]:194-195.
- Simple directed graph:: [Ros19]:675; [Deo74]:197.
- Complete undirected, directed graphs:: [Ros19]:688; [Deo74]:32,197.
- Graph applicability:: [Ros19]:673-675,676-682; [Deo74]:3-6.

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