

CS34110

Discrete Mathematics and Graph Theory

**UNIT – I, Module – 1****Lecture 02: Logic**

[ Propositional logic truth table; Converse, inverse, contrapositive; Tautology, contradiction; Logical equivalence; Laws of algebra of propositions ]

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**Propositional logic**

- **Propositional logic:** nature.
  - Atomic propositions and compound propositions of different forms.
  - Fundamental property of compound proposition: its truth value completely determined by truth values of its subpropositions, together with connectives used to connect them.
  - Tautology (compound proposition always TRUE), contradiction (compound proposition always FALSE), contingency (neither tautology nor contradiction).
  - Equivalence: when two compound propositions always having same truth values, regardless of truth values of propositional variables.

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2

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**Compound proposition: Truth table formation**

- Truth table construction of any compound proposition: required for finding truth value of any compound proposition.
- **Truth table:** mathematical tabular representation displaying all possible truth values of any proposition (particularly, compound proposition).
  - Two different approaches to construct truth table of compound proposition.

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3

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Compound proposition: Truth table formation

- A) Separate columns approach::
- Formation steps:
    - ❶ use of separate column to find truth value of each compound expression occurring in compound proposition; ❷ first columns for propositional variables, with rows for all possible combinations of their truth values (i.e., for  $n$  variables,  $2^n$  rows required); ❸ column for each compound expression in "elementary" stages, each truth value determined from previous stages by definitions of connectives as per precedence; ❹ truth value of compound proposition (for each combination of truth values of variables) in final column of truth table.

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Compound proposition: Truth table formation

- A) Separate columns approach::
- Example::  $(p \vee \neg q) \rightarrow (p \wedge q)$ .
- | $p$ | $q$ | $\neg q$ | $p \vee \neg q$ | $p \wedge q$ | $(p \vee \neg q) \rightarrow (p \wedge q)$ |
|-----|-----|----------|-----------------|--------------|--|
| T   | T   | F        | T               | T            | T  |
| T   | F   | T        | T               | F            | F  |
| F   | T   | F        | F               | F            | T  |
| F   | F   | T        | T               | F            | F  |

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Compound proposition: Truth table formation

- B) Sub-columns approach::
- Formation steps:
    - ❶ apart from columns for propositional variables, single final column for compound proposition, with multiple sub-columns corresponding to every variable and every connective in that compound proposition; ❷ rows with all possible combinations of truth values of variables (i.e., for  $n$  variables,  $2^n$  rows required), plus one more row labeled "step" to record step count; ❸ filled-up sub-columns for variables, indicated as step count 1; ❹ truth values entered into remaining sub-columns for connectives, with step count, as per precedence.

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### Compound proposition: Truth table formation

B) Sub-columns approach::

- Example::  $(p \vee \neg q) \rightarrow (p \wedge q)$ .

p	q	$(p \vee \neg q) \rightarrow (p \wedge q)$							
		(p	$\vee$	$\neg$	q)	$\rightarrow$	(p	$\wedge$	q)
T	T	T	T	F	T	T	T	T	T
T	F	T	T	T	F	F	T	F	F
F	T	F	F	F	T	T	F	F	T
F	F	F	T	T	F	F	F	F	F
Step no.		①	④	②	①	⑤	①	③	①

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### Compound proposition: Fundamental property

- Fundamental property of compound proposition: its truth value completely determined by truth values of its subpropositions, together with connectives used to connect them.
- $P(p, q, \dots)$ : denoting proposition constructed from variables  $p, q, \dots$  having truth values TRUE (T) or FALSE (F), connected through connectives.
- Truth value of  $P(p, q, \dots)$ : based on logical operations of truth values of  $p, q, \dots$  and their connectives.

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### Compound proposition: Conditional statement

- Implication.
- Bi-implication.
- Converse of implication.
- Inverse of implication.
- Contrapositive of implication.

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9

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### Compound proposition: Converse of implication

- **Converse** of implication  $p \rightarrow q$ : conditional compound proposition  $q \rightarrow p$  of given propositions  $p, q$ .
- Definition of truth value of  $q \rightarrow p$ :
 

Truth table	$p$	$q$	$p \rightarrow q$	$q \rightarrow p$
	T	T	T	T
	T	F	F	T
	F	T	T	F
	F	F	T	T

 if  $p$  to be FALSE and  $q$  to be TRUE,  
 then  $q \rightarrow p$  to become FALSE;  
 otherwise  $q \rightarrow p$  to become TRUE.
- Note: when  $p$  to be TRUE,  $q \rightarrow p$  to be TRUE regardless of truth value of  $q$ .
- Note: alternately, when  $q$  to be FALSE,  $q \rightarrow p$  to be TRUE regardless of truth value of  $p$ .

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10

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### Compound proposition: Converse of implication

- Converse of implication:
  - Converse of implication  $\rightarrow$  not equivalent to given implication.
  - Example::
 

Given: "The home team wins whenever it is raining."

**["q WHENEVER p"]**

= "If it is raining, THEN the home team wins." ["if p, then q"]

$\rightarrow p$  = "It is raining.";  $q$  = "The home team wins."

$\therefore (q \rightarrow p)$  = **"IF** the home team wins, **THEN** it is raining."

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### Compound proposition: Inverse of implication

- **Inverse** of implication  $p \rightarrow q$ : conditional compound proposition  $\neg p \rightarrow \neg q$  of given propositions  $p, q$ .
- Definition of truth value of  $\neg p \rightarrow \neg q$ :
 

Truth table	$p$	$q$	$p \rightarrow q$	$\neg p \rightarrow \neg q$
	T	T	T	T
	T	F	F	T
	F	T	T	F
	F	F	T	T

 if  $p$  to be FALSE and  $q$  to be TRUE,  
 then  $\neg p \rightarrow \neg q$  to become FALSE;  
 otherwise  $\neg p \rightarrow \neg q$  to become TRUE.
- Note: same truth table as converse of implication.
- Inverse of implication  $\rightarrow$  not equivalent to given implication.
- Inverse of implication  $\rightarrow$  logically equivalent to converse of implication of same given implication.

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Compound proposition: Inverse of implication

- Inverse of implication:
- Example::  
Given: "The home team wins whenever it is raining."  
[*q* WHENEVER *p*]  
= "IF it is raining, THEN the home team wins." ["if *p*, then *q*"]  
 $\rightarrow p$  = "It is raining.";  $q$  = "The home team wins."  
 $\therefore (\neg p \rightarrow \neg q)$  = "IF it is NOT raining, THEN the home team does NOT win."  
[Note: "IF it is NOT raining, THEN the home team does NOT win."  $\equiv$  "IF the home team wins, THEN it is raining."]

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13

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Compound proposition: Contrapositive of implication

- **Contrapositive** of implication  $p \rightarrow q$ : conditional compound proposition  $\neg q \rightarrow \neg p$  of given propositions  $p, q$ .
- Definition of truth value of  $\neg q \rightarrow \neg p$ :  
if  $p$  to be TRUE and  $q$  to be FALSE, then  $\neg q \rightarrow \neg p$  to become FALSE;  
otherwise  $\neg q \rightarrow \neg p$  to become TRUE.  
☞ Note: same truth table as given implication.
- Contrapositive of implication  $\rightarrow$  logically equivalent to given implication.

Truth table	$p$	$q$	$p \rightarrow q$	$\neg q \rightarrow \neg p$
	T	T	T	T
	T	F	F	F
	F	T	T	T
	F	F	T	T

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14

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Compound proposition: Contrapositive of implication

- Contrapositive of implication:
- Example::  
Given: "The home team wins whenever it is raining."  
[*q* WHENEVER *p*]  
= "IF it is raining, THEN the home team wins." ["if *p*, then *q*"]  
 $\rightarrow p$  = "It is raining.";  $q$  = "The home team wins."  
 $\therefore (\neg q \rightarrow \neg p)$  = "IF the home team does NOT win, THEN it is NOT raining."

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15

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### Compound proposition: Tautology

- Tautology**: compound proposition with TRUE as truth value for any truth values of its propositional variables (i.e., compound proposition with only T in final column of its truth table).
- Note: negation of tautology  $\rightarrow$  contradiction.
- Example:: any proposition  $p$ .  
Then,  $(p \vee \neg p)$  = Tautology.
- (Theorem) "Principle of Substitution": If  $P(p, q, \dots)$  is a tautology, then  $P(P_1, P_2, \dots)$  is also a tautology for any propositions  $P_1(p, q, \dots), P_2(p, q, \dots), \dots$

$p$	$\neg p$	$p \vee \neg p$
T	F	T
F	T	T

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### Compound proposition: Contradiction

- Contradiction** (also called absurdity): compound proposition with FALSE as truth value for any truth values of its propositional variables (i.e., compound proposition with only F in final column of its truth table).
- Note: negation of contradiction  $\rightarrow$  tautology.
- Example:: any proposition  $p$ .  
Then,  $(p \wedge \neg p)$  = Contradiction.

$p$	$\neg p$	$p \wedge \neg p$
T	F	F
F	T	F

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### Compound proposition: Equivalence

- Logical equivalence** (notation: ' $\equiv$ '): compound propositions  $P(p, q, \dots)$  and  $Q(p, q, \dots)$  to be **logically equivalent** (or **equivalent** or **equal**), i.e.,  $P(p, q, \dots) \equiv Q(p, q, \dots)$ , if  $P$  and  $Q$  having same truth values in all possible cases (i.e., identical truth tables).
- Example-1:: propositions  
 $P(p, q) = \neg(p \wedge q)$ ,  
 $Q(p, q) = (\neg p \vee \neg q)$ .  
 So,  $P \equiv Q$ .
 

$p$	$q$	$\neg(p \wedge q)$	$p$	$q$	$(\neg p \vee \neg q)$
T	T	F	T	T	F
T	F	T	T	F	T
F	T	T	F	T	T
F	F	T	F	F	T
- Other notations for logical equivalence: ' $\Leftrightarrow$ '.
- Note: symbols ' $\equiv$ ' and ' $\Leftrightarrow$ ': not logical connectives of propositions.

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18

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### Compound proposition: Equivalence

- Logical equivalence:

- Property:: For any propositions

$P(p, q, r)$  and  $Q(p, q, r)$ ,

$P \equiv Q$ , whenever  $P \leftrightarrow Q$

to be tautology.

- Example-1 (contd.):

$p$	$q$	$\neg(p \wedge q)$	$(\neg p \vee \neg q)$	$\neg(p \wedge q) \leftrightarrow (\neg p \vee \neg q)$
T	T	F	F	T
T	F	T	T	T
F	T	T	T	T
F	F	T	T	T

Tautology

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19

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### Compound proposition: Equivalence

- Logical equivalence:

- Example-2 [distributive law of disjunction over conjunction]:

propositions	$p$	$q$	$r$	$q \wedge r$	$p \vee (q \wedge r)$	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$
$P(p, q, r) =$	T	T	T	T	T	T	T	T
$p \vee (q \wedge r),$	T	T	F	F	T	T	T	T
$Q(p, q, r) =$	T	F	T	F	T	T	T	T
$Q(p, q, r) =$	T	F	F	F	T	T	T	T
$(p \vee q) \wedge (p \vee r),$	F	T	T	T	T	T	T	T
$(p \vee q) \wedge (p \vee r),$	F	T	F	F	F	T	F	F
As all possible	F	F	T	F	F	F	T	F
truth values of	F	F	F	F	F	F	F	F

$P$  and  $Q$  agreeing, so,  $P \equiv Q$ .

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### Compound proposition: Equivalence

- Logical equivalence:

- Example-3 [conditional-disjunction equivalence]: propositions

$P(p, q) = p \rightarrow q,$

$Q(p, q) = (\neg p \vee q).$

So,  $P \equiv Q$ .

$p$	$q$	$p \rightarrow q$	$p$	$q$	$(\neg p \vee q)$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	F	F	T

- Property:: **Identity**: logically equivalent propositions.

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### Compound proposition: Equivalence

- Logical equivalence  $\rightarrow$  **Laws of algebra of propositions.**
  - Proof of each law based on proving equivalence either by truth table of given propositions, or by showing their bi-implication as tautology.

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### Laws of algebra of propositions

Idempotent laws:	(I.1a) $p \vee p \equiv p$	(I.1b) $p \wedge p \equiv p$
Identity laws:	(I.2a) $p \vee F \equiv p$	(I.2b) $p \wedge T \equiv p$
Domination laws:	(I.3a) $p \vee T \equiv T$	(I.3b) $p \wedge F \equiv F$
Associative laws:	(I.4a) $(p \vee q) \vee r \equiv p \vee (q \vee r)$	(I.4b) $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
Commutative laws:	(I.5a) $p \vee q \equiv q \vee p$	(I.5b) $p \wedge q \equiv q \wedge p$
Distributive laws:	(I.6a) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$	(I.6b) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
Involution law:	(I.7) $\neg\neg p \equiv p$ (also called Double negation law)	
Negation laws:	(I.8a) $p \vee \neg p \equiv T$	(I.8b) $p \wedge \neg p \equiv F$
Complement laws:	(I.9a) $\neg T \equiv F$	(I.9b) $\neg F \equiv T$
De Morgan's laws:	(I.10a) $\neg(p \vee q) \equiv \neg p \wedge \neg q$	(I.10b) $\neg(p \wedge q) \equiv \neg p \vee \neg q$

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### Laws of algebra of propositions

Absorption laws:	(I.11a) $p \vee (p \wedge q) \equiv p$	(I.11b) $p \wedge (p \vee q) \equiv p$
Implication rules:	(I.12a) $p \rightarrow q \equiv \neg p \vee q$	(I.12b) $\neg(p \rightarrow q) \equiv p \wedge \neg q$
	(I.13a) $p \vee q \equiv \neg p \rightarrow q$	(I.13b) $p \wedge q \equiv \neg(p \rightarrow \neg q)$
	(I.14) $p \rightarrow q \equiv \neg q \rightarrow \neg p$	
	(I.15a) $(p \rightarrow q) \vee (p \rightarrow r) \equiv p \rightarrow (q \vee r)$	(I.15b) $(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$
Bi-implication rules:	(I.16a) $(p \rightarrow r) \vee (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$	(I.16b) $(p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \vee q) \rightarrow r$
	(I.17a) $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$	(I.17b) $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$
	(I.18) $p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$	
	(I.19) $\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$	

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### Laws of algebra of propositions

Exclusive disjunction rules:	(I.20) $p \oplus q \equiv (p \wedge \neg q) \vee (\neg p \wedge q)$	
	(I.21a) $p \oplus q \equiv \neg(p \leftrightarrow q)$	(I.21b) $p \leftrightarrow q \equiv \neg(p \oplus q)$
	(I.22) $p \oplus q \equiv \neg((p \wedge q) \vee (\neg p \wedge \neg q))$	

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### Laws of algebra of propositions

- Generalization of Laws of algebra of propositions:
  - Associative** law for disjunction: for propositions  $p_1, p_2, p_3, \dots, p_{n-1}, p_n$ ,  
 $(\dots (p_1 \vee p_2) \vee p_3) \vee \dots \vee p_n \equiv p_1 \vee (p_2 \vee (p_3 \vee \dots (p_{n-1} \vee p_n) \dots))$
  - Associative** law for conjunction: for propositions  $p_1, p_2, p_3, \dots, p_{n-1}, p_n$ ,  
 $(\dots (p_1 \wedge p_2) \wedge p_3) \wedge \dots \wedge p_n \equiv p_1 \wedge (p_2 \wedge (p_3 \wedge \dots (p_{n-1} \wedge p_n) \dots))$
  - In same manner, generalization of **distributive** laws for disjunction and conjunction.
  - De Morgan's laws**: for propositions  $p_1, p_2, p_3, \dots, p_{n-1}, p_n$ ,  
 $\neg(p_1 \vee p_2 \vee \dots \vee p_n) \equiv (\neg p_1 \wedge \neg p_2 \wedge \dots \wedge \neg p_n), \neg(\bigvee_{j=1}^n p_j) \equiv \bigwedge_{j=1}^n \neg p_j$   
 $\neg(p_1 \wedge p_2 \wedge \dots \wedge p_n) \equiv (\neg p_1 \vee \neg p_2 \vee \dots \vee \neg p_n), \neg(\bigwedge_{j=1}^n p_j) \equiv \bigvee_{j=1}^n \neg p_j$

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### Laws of algebra of propositions

- Example-1::
  - For **negation** of "You have an iPhone **and** an iPad.":  $p \wedge q$ ,  
 where  $p$ : "You have an iPhone",  $q$ : "You have an iPad".  
 By one of De Morgan's laws in (I.10b),  $\neg(p \wedge q) \equiv \neg p \vee \neg q$ .  
 Here,  $\neg p$ : "It is false that you have an iPhone" = "You don't have an iPhone".  
 Similarly,  $\neg q$ : "It is false that you have an iPad" = "You don't have an iPad".  
 So,  $\neg p \vee \neg q$  = "You **don't** have an iPhone **or** an iPad".

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### Laws of algebra of propositions

- Example-2::
- To prove  $\neg(p \vee (\neg p \wedge q))$  and  $\neg p \wedge \neg q$  to be logically equivalent —
  - $\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg(\neg p \wedge q)$  by De Morgan's law in (I.10a)
  - $\equiv \neg p \wedge (\neg(\neg p) \vee \neg q)$  by De Morgan's law in (I.10b)
  - $\equiv \neg p \wedge (p \vee \neg q)$  by double negation law in (I.7)
  - $\equiv (\neg p \wedge p) \vee (\neg p \wedge \neg q)$  by distributive law in (I.6b)
  - $\equiv F \vee (\neg p \wedge \neg q)$  by negation law in (I.8b)
  - $\equiv (\neg p \wedge \neg q) \vee F$  by commutative law in (I.5a)
  - $\equiv \neg p \wedge \neg q$  by identity law in (I.2a)

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### Compound proposition: Operator '↓'

- Logical operator '↓': (called Sheffer stroke, named after H. M. Sheffer) to represent **NAND** logical operation, with FALSE as truth value for TRUE truth values of its propositional variables.
- Definition of truth value of  $p \downarrow q \equiv p \text{ NAND } q$ :  
if both  $p$  and  $q$  to be TRUE, then  $p \downarrow q$  to become FALSE; otherwise  $p \downarrow q$  to become TRUE.

Truth table	$p$	$q$	$p \downarrow q$
	T	T	F
	T	F	T
	F	T	T
	F	F	T

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### Compound proposition: Operator '↓'

- Logical operator '↓': (called Peirce arrow, named after Charles Sanders Peirce) to represent **NOR** logical operation, with TRUE as truth value for FALSE truth values of its propositional variables.
- Definition of truth value of  $p \downarrow q \equiv p \text{ NOR } q$ :  
if both  $p$  and  $q$  to be FALSE, then  $p \downarrow q$  to become TRUE; otherwise  $p \downarrow q$  to become FALSE.

Truth table	$p$	$q$	$p \downarrow q$
	T	T	F
	T	F	F
	F	T	F
	F	F	T

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### Summary

- Focus: Propositional logic (contd.).
- Propositional logic nature.
- Truth table construction for compound propositions - separate columns approach, and sub-columns approach, with examples.
- Fundamental property of compound propositions.
- Definitions and truth tables of converse, inverse, and contrapositive of implication, with examples.
- Definitions and truth tables of tautology, contradiction, and logical equivalence, with examples.

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### Summary

- Principle of substitution.
- Laws of algebra of propositions, and generalizations of laws, with examples.
- Logical operators Sheffer stroke and Peirce arrow definitions, truth tables.

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### References

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2. [Ross12] Kenneth A. Ross, Charles R. B. Wright, *Discrete Mathematics*, Fifth edition, Pearson Education, 2012.
3. [Mot15] Joe L. Mott, Abraham Kandel, Theodore P. Baker, *Discrete Mathematics for Computer Scientists and Mathematicians*, Second edition, Pearson Education, 2015.
4. [Lip07] Seymour Lipschutz, Marc L. Lipson, *Schaum's Outline of Theory and Problems of Discrete Mathematics*, Third edition, McGraw-Hill Education, 2007.
5. [https://www.cs.odu.edu/~toida/nerzic/content/logic/prop\\_logic/identities/identities.html](https://www.cs.odu.edu/~toida/nerzic/content/logic/prop_logic/identities/identities.html).

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33

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Further Reading

- Truth tables construction of compound propositions:: [Ros21]:11.
- Conditional statements in compound propositions:: [Ros21]:6-9.
- Converse, inverse, and contrapositive of implication:: [Ros21]:9.
- Tautology and contradiction:: [Ros21]:26-27.
- Logical equivalence:: [Ros21]:27-32.
- Laws of algebra of propositions:: [Ros21]:29-30.

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34

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Lecture Exercises: Problem 1 [Ref: Gate 2019, Q.6, p.2 (Set2)]

Which one of the following is NOT a valid identity?

- (a)  $(x \oplus y) \oplus z = x \oplus (y \oplus z)$ .
- (b)  $(x + y) \oplus z = x \oplus (y + z)$ .
- (c)  $x \oplus y = x + y$ , if  $xy = 0$ .
- (d)  $x \oplus y = (xy + x'y')'$ .

Note:  $x' = \neg x$ .

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35

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Lecture Exercises: Problem 1 Ans

- Is  $(x \oplus y) \oplus z \equiv x \oplus (y \oplus z)$ ?
- Is  $(x \vee y) \oplus z \equiv x \oplus (y \vee z)$ ?
- Is  $x \oplus y \equiv x \vee y$ , if  $x \wedge y \equiv F$ ?
- Is  $x \oplus y \equiv \neg((x \wedge y) \vee (\neg x \wedge \neg y))$ ?

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36

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### Lecture Exercises: Problem 2 [Ref: Gate 2018, Q.4, p.2 (Set3)]

Let  $\oplus$  and  $\odot$  denote the Exclusive OR and Exclusive NOR operations, respectively. Which one of the following is NOT CORRECT?

- (a)  $\bar{P} \oplus \bar{Q} = P \odot Q$ .
- (b)  $\bar{P} \oplus Q = P \odot Q$ .
- (c)  $\bar{P} \oplus \bar{Q} = P \oplus Q$ .
- (d)  $(P \oplus \bar{P}) \oplus Q = (P \odot \bar{P}) \odot \bar{Q}$ .

Note:  $\bar{P} = P' = \neg P$ .

Discrete Mathematics

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37

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### Lecture Exercises: Problem 2 Ans

- Is  $\bar{P} \oplus \bar{Q} \equiv P \odot Q$ ?
- Is  $\bar{P} \oplus Q \equiv P \odot Q$ ?
- Is  $\bar{P} \oplus \bar{Q} \equiv P \oplus Q$ ?
- Is  $(P \oplus \bar{P}) \oplus Q \equiv (P \odot \bar{P}) \odot \bar{Q}$ ?

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38

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