CS34110 Discrete Mathematics and Graph Theory

UNIT – II, Module – 1

Lecture 08: Discrete Structures

[Discrete structures; Matrix; Set; Set builder; Universal set; Set inclusion; Cardinality; Finite set, infinite set; Power set]

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Discrete structures

- Discrete structures: part of mathematics devoted to study discrete objects.
  - Discrete structures importance: to represent discrete objects.
  - Examples: (i) sets (collections of objects) and multisets;
    (ii) combinations (built from sets; unordered collections of objects to be used in counting);
    (iii) relations (sets of ordered pairs representing relationships between objects);
    (iv) graphs (sets of vertices and edges connecting vertices) and trees as special forms of graphs;

(contd. to next slide)

Discrete structures

- Discrete structures:
  - Examples contd.
    (v) sequences (ordered lists of elements, as well as special type of functions expressing relationships among elements);
    (vi) matrices.

Matrices

- Matrix: rectangular array of numbers, usually presented in form of rows and columns of 'elements' or 'entries'.

$$\mathbf{A}_{m\times n} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} = [a_{ij}],\ i = 1, \dots, m,\ j = 1, \dots, n$$

($m \times n$ to be read as m-by-n.)

- Property:: Row matrix: any row of $\mathbf{A}$, as $1 \times n$ matrix.
- Property:: Column matrix: any column of $\mathbf{A}$ being $m \times 1$ matrix.
- Property:: Square matrix: having same number of rows as columns.
- Benefit:: to express relationships between elements in sets.

Matrices

- Matrix:
  - Equality property:: Equality of matrices $\mathbf{A}_{m\times n}$ and $\mathbf{B}_{p\times q}$: $\mathbf{A} = \mathbf{B}$, if and only if — (i) $\mathbf{A}$ and $\mathbf{B}$ having same number of rows (i.e., $m = p$), (ii) $\mathbf{A}$ and $\mathbf{B}$ having same number of columns (i.e., $n = q$), and (iii) corresponding entries in every position of $\mathbf{A}$ and $\mathbf{B}$ to be equal.
  - Addition property:: Sum of matrices $\mathbf{A}_{m\times n} = [a_{ij}]$ and $\mathbf{B}_{m\times n} = [b_{ij}]$: $m \times n$ matrix, denoted as $\mathbf{A} + \mathbf{B}$, having (i, j)-th element as $a_{ij} + b_{ij}$. In other words, $\mathbf{A} + \mathbf{B} = [a_{ij} + b_{ij}]$.
  - Property:: No addition of matrices of different sizes [reason: both not having entries in some of their positions].

Matrices

- Matrix:
  - Multiplication property:: Product of matrices $\mathbf{A}_{m\times p} = [a_{ik}]$ and $\mathbf{B}_{p\times n} = [b_{kj}]$: $m \times n$ matrix, denoted as $\mathbf{A} \cdot \mathbf{B} = [c_{ij}]$, with (i, j)-th element $c_{ij} = \sum_{k=1}^{p} (a_{ik} \cdot b_{kj})$.
  - Property:: Matrix multiplication associative, not commutative.
  - Property:: For matrices $\mathbf{A}_{m\times n}$ and $\mathbf{B}_{p\times q}$ — (i) $\mathbf{A} \cdot \mathbf{B}$ defined if $n = p$, with product size $m \times q$; (ii) $\mathbf{B} \cdot \mathbf{A}$ defined if $q = m$, with product size $p \times n$; (iii) $\mathbf{A} \cdot \mathbf{B}$ and $\mathbf{B} \cdot \mathbf{A}$ of same size (but not necessarily equal), if $m = n = p = q$ (i.e., $\mathbf{A}$, $\mathbf{B}$, their product all square matrices).

Matrices

- Matrix:
  - Property:: Zero-one matrix: matrix with all entries either 0 or 1.
  - Identity property:: Identity matrix of order n: $n \times n$ zero-one matrix, denoted as $\mathbf{I}_n = [\delta_{ij}]$, ($\delta$ = Kronecker delta), with (i, j)-th element

$$\delta_{ij} = \begin{cases} 1, & \text{if } i = j \\ 0, & \text{if } i \neq j \end{cases}$$

  - Property:: For matrix $\mathbf{A}_{m \times n}$ — $\mathbf{A} \cdot \mathbf{I}_n = \mathbf{I}_m \cdot \mathbf{A} = \mathbf{A}$.
  - Power property:: r-th power of square matrix $\mathbf{A}_{n \times n}$:

$\mathbf{A}^0 = \mathbf{I}_n$ when $r = 0$, $\mathbf{A}^r = \underbrace{\mathbf{A} \cdot \mathbf{A} \cdot \mathbf{A} \cdot \ldots \cdot \mathbf{A}}_{r \text{ times}}$ for any $r > 0$

Matrices

- Matrix:
  - Property:: Main diagonal (also termed principal diagonal, primary diagonal, major diagonal, leading diagonal and good diagonal) of square matrix $\mathbf{A}_{n \times n} = [a_{ij}]$: entries a_{ii} for all $i = 1, \ldots, n$.
  - Transpose property:: Transpose of matrix $\mathbf{A}_{m \times n} = [a_{ij}]$: $n \times m$ matrix, denoted as $\mathbf{A}^t = [a'_{ij}]$, with (i, j)-th element $a'_{ij} = a_{ji}$ for $i = 1, \ldots, n$ and $j = 1, \ldots, m$. In other words, transpose of matrix to be obtained by interchanging rows and columns of given matrix.

Matrices

- Matrix:
  - Symmetry property:: Square matrix $\mathbf{A}_{n \times n} = [a_{ij}]$ to be called symmetric matrix, if $\mathbf{A} = \mathbf{A}^t$, i.e., $a_{ij} = a_{ji}$ for all $i = 1, \ldots, n$ and all $j = 1, \ldots, m$.
  - Property:: Symmetric square matrix to fulfill symmetry property with respect to main diagonal of square matrix.
  - Diagonal property:: Square matrix $\mathbf{A}_{n \times n} = [a_{ij}]$ to be called diagonal matrix, if $a_{ij} = 0$ when $i \neq j$, for all $i = 1, \ldots, n$ and all $j = 1, \ldots, m$.

Matrices

- Matrix:
  - Join property:: Join of zero-one matrices $\mathbf{A}_{m\times n} = [a_{ij}]$ and $\mathbf{B}_{m\times n} = [b_{ij}]$: $m \times n$ zero-one matrix, denoted as $\mathbf{A} \vee \mathbf{B}$, with (i,j)-th element as $a_{ij} \vee b_{ij}$, where
    $$a_{ij} \vee b_{ij} = \begin{cases} 1, & \text{if } a_{ij} = 1 \text{ or } b_{ij} = 1 \\ 0, & \text{otherwise} \end{cases} \quad \text{for } i = 1, \dots, m,\ j = 1, \dots, n$$
  - Meet property:: Meet of zero-one matrices $\mathbf{A}_{m\times n} = [a_{ij}]$ and $\mathbf{B}_{m\times n} = [b_{ij}]$: $m \times n$ zero-one matrix, denoted as $\mathbf{A} \wedge \mathbf{B}$, with (i,j)-th element as $a_{ij} \wedge b_{ij}$, where
    $$a_{ij} \wedge b_{ij} = \begin{cases} 1, & \text{if } a_{ij} = b_{ij} = 1 \\ 0, & \text{otherwise} \end{cases} \quad \text{for } i = 1, \dots, m,\ j = 1, \dots, n$$

Matrices

- Matrix:
  - Boolean product property:: Boolean product of zero-one matrices $\mathbf{A}_{m\times p} = [a_{ik}]$ and $\mathbf{B}_{p\times n} = [b_{kj}]$: $m \times n$ zero-one matrix, denoted as $\mathbf{A} \odot \mathbf{B} = [c_{ij}]$, with (i,j)-th element $c_{ij} = \bigvee_{k=1}^{p}(a_{ik} \wedge b_{kj}) = (a_{i1} \wedge b_{1j}) \vee (a_{i2} \wedge b_{2j}) \vee \cdots \vee (a_{ip} \wedge b_{pj})$.
  - Boolean power property:: r-th Boolean power of zero-one square matrix $\mathbf{A}_{n\times n}$:
    $\mathbf{A}^{[0]} = \mathbf{I}_n$ when $r = 0$, $\mathbf{A}^{[r]} = \underbrace{\mathbf{A} \odot \mathbf{A} \odot \mathbf{A} \odot \dots \odot \mathbf{A}}_{r \text{ times}}$ for any $r > 0$.
  - Property:: Matrix Boolean product associative.

Matrices

- Matrix:
  - Inverse property:: For two square matrices $\mathbf{A}_{n\times n}$ and $\mathbf{B}_{n\times n}$, $\mathbf{B}$ to be called inverse of $\mathbf{A}$, denoted by $\mathbf{B} = \mathbf{A}^{-1}$, when $\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A} = \mathbf{I}_n$.
  - Invertible property:: For two square matrices $\mathbf{A}_{n\times n}$ and $\mathbf{B}_{n\times n}$, $\mathbf{A}$ to be called invertible, when inverse of $\mathbf{A}$ existing, i.e., when $\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A} = \mathbf{I}_n$.

Set theory

- Set theory: branch of mathematics (particularly, mathematical logic) on study of sets.
- Number theory: branch of pure mathematics on study of integers (including sequences and series) and arithmetic functions.

Set

- Set: unordered collection of distinct elements (called members).
  - Property:: Set to contain its elements.
  - Property:: $a \in \mathbf{A}$: a is an element of set $\mathbf{A}$.
  - Property:: $a \notin \mathbf{A}$: a not an element of $\mathbf{A}$.
  - Property:: Roster method: description approach of set, in which — (i) all members listed between braces; (ii) some members listed between braces, followed by ellipses ($\cdots$) to express general pattern of elements.
  - Property:: Set builder notation: description approach of set of form $\{a \mid a \text{ has property } P\}$, i.e., "set of all a such that a has property P."

Set

- Set:
  - Property:: Empty set (also called null set): set with no elements; denoted by $\emptyset$, or $\{\,\}$.
  - Example:: $\mathbb{N} = \{0,1,2,3,\ldots\}$: set of all natural numbers.[Ref: standard ISO 80000-2] Roster method
    [No universally-accepted definition of "set of all whole numbers"]
  - Example:: $\mathbb{Z} = \{\ldots,-2,-1,0,1,2,\ldots\}$: set of all integers.
  - Example:: $\mathbb{Z}^+ = \{1,2,3,\ldots\}$: set of all positive integers.
  - Example:: $\mathbb{R}$: set of all real numbers.
  - Example:: $\mathbb{R}^+$: set of all positive real numbers.
  - Example:: $\mathbb{Q} = \{{}^{p}/_{q} \mid p \in \mathbb{Z}, q \in \mathbb{Z}, q \neq 0\}$: set of all rational numbers. Set builder notation
  - Example:: $\mathbb{C}$: set of all complex numbers.

Set

- Set:
  - Property:: Singleton set: set with exactly one element.
  - Property:: Universal set (also called universe): $\mathcal{U}$ = set containing all elements (or objects) under consideration as per context.
  - Property:: Interval: set of all real numbers between two real numbers a and b ($a \leq b$), with or without a and b. Four forms —
    i. $[a, b] = \{x \mid a \leq x \leq b\}$: closed interval from a to b.
    ii. $[a, b) = \{x \mid a \leq x < b\}$.
    iii. $(a, b] = \{x \mid a < x \leq b\}$.
    iv. $(a, b) = \{x \mid a < x < b\}$: open interval from a to b.

Set

- Set: examples.
  - Example-1:: Set $\mathbf{V}$ of all vowels in the English alphabet. Then,
    $$\mathbf{V} = \{\mathrm{a, e, i, o, u}\}.$$
  - Example-2:: Set $\mathbf{A} = \{x \mid x$ is odd positive integer, $x < 10\}$. Then,
    $$\mathbf{A} = \{1, 3, 5, 7, 9\}.$$
  - Example-3:: Set $\mathbf{B} = \{x \in \mathbb{Z}^+ \mid x < 100\} = \{1, 2, 3, \cdots, 99\}$.
  - Example-4:: Set also possible to group together seemingly unrelated elements: $\mathbf{C} = \{$a, 2, Fred, New Jersey$\}$.
  - Example-5: Unit intervals $[0, 1]$, $[0, 1)$, $(0, 1]$, $(0, 1)$: 1st being closed, last being open, 2nd & 3rd being mix of open/close.

Set

- Set:
  - Equality property:: Equality of sets $\mathbf{A}$ and $\mathbf{B}$: $\mathbf{A} = \mathbf{B}$, if and only if $\forall x\big((x \in \mathbf{A}) \leftrightarrow (x \in \mathbf{B})\big)$, i.e., $\mathbf{A}$ and $\mathbf{B}$ containing same elements.
  - Inclusion property:: $\mathbf{A}$ to be subset of $\mathbf{B}$ (and, equivalently, $\mathbf{B}$ to be superset of $\mathbf{A}$): $\mathbf{A} \subseteq \mathbf{B}$ ($\equiv \mathbf{B} \supseteq \mathbf{A}$) if and only if $\forall x\big((x \in \mathbf{A}) \rightarrow (x \in \mathbf{B})\big)$, i.e., every element of $\mathbf{A}$ also element of $\mathbf{B}$.
  - Strict inclusion property:: $\mathbf{A}$ to be proper subset of $\mathbf{B}$ (equivalently, $\mathbf{B}$ to be proper superset of $\mathbf{A}$): $\mathbf{A} \subset \mathbf{B}$ ($\equiv \mathbf{B} \supset \mathbf{A}$) if and only if $\forall x\big((x \in \mathbf{A}) \rightarrow (x \in \mathbf{B})\big) \wedge \exists x\big((x \in \mathbf{B}) \wedge (x \notin \mathbf{A})\big)$.

Set

- Set:
  - Relation between equality and inclusion properties:: $\mathbf{A} = \mathbf{B}$, if and only if $(\mathbf{A} \subseteq \mathbf{B}) \wedge (\mathbf{B} \subseteq \mathbf{A})$.
  - Inclusion transitivity property :: If $(\mathbf{A} \subseteq \mathbf{B}) \wedge (\mathbf{B} \subseteq \mathbf{C})$, then $\mathbf{A} \subseteq \mathbf{C}$.
  - Property:: For every set $\mathbf{S}$, (i) $\emptyset \subseteq \mathbf{S}$, (ii) $\mathbf{S} \subseteq \mathbf{S}$.
  - Property:: $\forall x((x \in \mathbf{S}) \rightarrow P(x)) \equiv \forall x \in \mathbf{S}\, (P(x))$. [shorthand
  - Property:: $\exists x((x \in \mathbf{S}) \wedge P(x)) \equiv \exists x \in \mathbf{S}\, (P(x))$. forms]
  - Property:: Truth set of predicate P based on set theory: truth set of P $= \{x \in \mathfrak{D} \mid P(x) \equiv \mathrm{T}\}$, where $\mathfrak{D}$ = domain of discourse.

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Set

- Set: inclusion property examples.
  - Example-1:: for sets $\mathbf{A} = \{1, 3, 4, 7, 8, 9\}$, $\mathbf{B} = \{1, 2, 3, 4, 5\}$, $\mathbf{C} = \{1, 3\}$, then $\mathbf{C} \subseteq \mathbf{A}$ and $\mathbf{C} \subseteq \mathbf{B}$. Reason: elements of $\mathbf{C}$, viz. 1 and 3, also members of both $\mathbf{A}$ and $\mathbf{B}$.
    But $\mathbf{B} \nsubseteq \mathbf{A}$, due to some elements of $\mathbf{B}$, viz. 2 and 5, not belonging to $\mathbf{A}$. Similarly, $\mathbf{A} \nsubseteq \mathbf{B}$.
  - Example-2:: for sets $\mathbf{A} = \{x \mid x \text{ is odd positive integer}, x < 10\}$, and $\mathbf{B} = \{x \mid x \text{ is positive integer}, x < 10\}$, $\mathbf{A} \subseteq \mathbf{B}$.
  - Example-3:: for sets $\mathbb{N}$ and $\mathbf{C} = \{x \in \mathbb{Z} \mid x^2 < 100\}$, $\mathbf{C} \nsubseteq \mathbb{N}$, due to -1 belonging to $\mathbf{C}$, but not in $\mathbb{N}$.

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Set

- Set: inclusion property theorems & proofs.
  - Property:: ⟨Theorem⟩: For every set $\mathbf{S}$, (i) $\emptyset \subseteq \mathbf{S}$, (ii) $\mathbf{S} \subseteq \mathbf{S}$.

Proof of (i): For $\emptyset \subseteq \mathbf{S}$, to show that $\forall x\big((x \in \emptyset) \rightarrow (x \in \mathbf{S})\big)$ to be TRUE in domain of discourse $= \mathcal{U}$. [Vacuous proof method to be used]
$(x \in \emptyset) \equiv$ FALSE, for any arbitrary $x \in \mathcal{U}$, as no elements in $\emptyset$.
So, in implication with antecedent having truth value always FALSE, truth value of implication to be always TRUE, irrespective of truth value of subsequent, as per definition of implication.
So, $\big((x \in \emptyset) \rightarrow (x \in \mathbf{S})\big) \equiv$ TRUE for arbitrary x.
$\forall x \in \mathcal{U}\big((x \in \emptyset) \rightarrow (x \in \mathbf{S})\big) \equiv$ TRUE. $\therefore \emptyset \subseteq \mathbf{S}$. ■

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Set

- Set: inclusion property theorems & proofs.
  - Property:: ⟨Theorem⟩ proof contd.

  Proof of (ii): For $\mathbf{S} \subseteq \mathbf{S}$, to show that $\forall x\big((x \in \mathbf{S}) \rightarrow (x \in \mathbf{S})\big)$ to be TRUE in domain of discourse $= \boldsymbol{\mathcal{U}}$. [Direct proof method to be used]

  Let $P(x)$ be $(x \in \mathbf{S})$, $Q(x)$ be $\big((x \in \mathbf{S}) \rightarrow (x \in \mathbf{S})\big) = (P(x) \rightarrow P(x))$.

  For any arbitrary $x \in \boldsymbol{\mathcal{U}}$, either $P(x) \equiv$ TRUE or $P(x) \equiv$ FALSE, based on when $(x \in \mathbf{S})$ and $(x \notin \mathbf{S})$ respectively.

  In either case, $Q(x) \equiv$ TRUE, as per definition of implication.

  So, $\big((x \in \mathbf{S}) \rightarrow (x \in \mathbf{S})\big) \equiv$ TRUE for arbitrary x.

  $\forall x \in \boldsymbol{\mathcal{U}}\big((x \in \mathbf{S}) \rightarrow (x \in \mathbf{S})\big) \equiv$ TRUE. $\therefore \mathbf{S} \subseteq \mathbf{S}$. ■

Set

- Set:
  - Property:: Cardinality of set $\mathbf{S}$: $|\mathbf{S}| = n$, where $n =$ distinct elements belonging to $\mathbf{S}$, and n being nonnegative integer.
  - Property:: $|\emptyset| = 0$.
  - Property:: Same cardinality of sets $\mathbf{A}$ and $\mathbf{B}$: $|\mathbf{A}| = |\mathbf{B}|$, if and only if one-to-one correspondence (i.e., bijective) function from $\mathbf{A}$ to $\mathbf{B}$.
  - Property:: $|\mathbf{A}| \leq |\mathbf{B}|$ for sets $\mathbf{A}$ and $\mathbf{B}$, i.e., cardinality of $\mathbf{A}$ less than or same as cardinality of $\mathbf{B}$, if one-to-one (i.e., injective) function from $\mathbf{A}$ to $\mathbf{B}$.
  - Property:: $|\mathbf{A}| < |\mathbf{B}| \equiv (|\mathbf{A}| \leq |\mathbf{B}|) \wedge (|\mathbf{A}| \neq |\mathbf{B}|)$.

Set

- Set:
  - Property:: Finite set $\mathbf{S}$: if $\mathbf{S} = \emptyset$, or if one-to-one correspondence (i.e., bijective) function from members of $\mathbf{S}$ to $\{1, \ldots, n\}$, for some $n \in \mathbb{N}$, and then $|\mathbf{S}| = n$.
  - Property:: Infinite set: if not finite set.
  - Property:: Power set of $\mathbf{S}$: $\boldsymbol{\mathcal{P}}(\mathbf{S}) =$ set of all subsets of $\mathbf{S}$, including $\emptyset$ and $\mathbf{S}$ itself.
  - Property:: $|\boldsymbol{\mathcal{P}}(\mathbf{S})| = 2^n$, if $|\mathbf{S}| = n$.
  - Property:: $\boldsymbol{\mathcal{P}}(\emptyset) = \{\emptyset\}$. $\boldsymbol{\mathcal{P}}\big(\boldsymbol{\mathcal{P}}(\emptyset)\big) = \boldsymbol{\mathcal{P}}(\{\emptyset\}) = \big\{\emptyset, \{\emptyset\}\big\}$.

Set

- Set:
  - Property:: $|\mathbb{N}| = \aleph_0$ (read: aleph-null).
    $\aleph_0 = 1^{st}$ infinite (or, transfinite) cardinal $= |\omega_0|$, ω_0 = set of all countable ordinals. (Ordinal = numbers used to order/enumerate infinite set; Cardinal = to measure cardinality or size of infinite sets.)
  - Property:: Cardinality of continuum $= |\mathbb{R}| = \mathfrak{c}$ (Fraktur 'c') $= 2^{\aleph_0} = \aleph_1$ (read: aleph-one). $\aleph_1 = 2^{nd}$ infinite cardinal $= |\omega_1|$, ω_1 = set of all uncountable ordinals.
  - Property:: ⟨Cantor's theorem⟩: For any (finite/infinite) set $\mathbf{S}$, $|\mathbf{S}| < |\mathcal{P}(\mathbf{S})|$.

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Set

- Set: size examples.
  - Example-1:: for sets $\mathbf{A} = \{1, 3, 4, 7, 8, 9\}$, $\mathbf{B} = \{1, 2, 3, 4, 5\}$, $\mathbf{C} = \{1, 3\}$, respective cardinalities: $|\mathbf{A}| = 6$, $|\mathbf{B}| = 5$, $|\mathbf{C}| = 2$.
  - Example-2:: for sets $\mathbb{N}$ and $\mathbb{Z}$, sizes of $\mathbb{N}$ and $\mathbb{Z}$ to be infinite.
  - Example-3:: for set $\mathbf{S} = \{0, 1, 2\}$, $\mathcal{P}(\mathbf{S})$ to be set of subsets of $\mathbf{S}$, i.e. $\mathcal{P}(\mathbf{S}) = \{\emptyset, \{0\}, \{1\}, \{2\}, \{0,1\}, \{0,2\}, \{1,2\}, \mathbf{S}\}$, and size of $\mathcal{P}(\mathbf{S}) = |\mathcal{P}(\mathbf{S})| = 2^{|\mathbf{S}|} = 8$.

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Summary

- Focus: Matrix, set as discrete structures.
- Discrete structures.
- Matrix representation, equality, addition, multiplication.
- Types of matrix: square, zero-one, identity, power, transpose, symmetric, diagonal, inverse, invertible.
- Join, meet, Boolean product and Boolean power of matrices.
- Set, and its representation through roster method and set builder notation.
- Standard sets of number theory and calculus.
- Singleton set and universal set.

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Summary

- Focus: Matrix, set as discrete structures (contd.).
- Set inclusion property, and related theorems with examples.
- Cardinality of set, and related theorems.
- Finite set and infinite set.
- Power set, and related theorems with examples.

References

1. [Ros21] Kenneth H. Rosen, Kamala Krithivasan, *Discrete Mathematics and its Applications*, Eighth edition, McGraw-Hill Education, 2021.
2. [Ross12] Kenneth A. Ross, Charles R. B. Wright, *Discrete Mathematics*, Fifth edition, Pearson Education, 2012.
3. [Mot15] Joe L. Mott, Abraham Kandel, Theodore P. Baker, *Discrete Mathematics for Computer Scientists and Mathematicians*, Second edition, Pearson Education, 2015.
4. [Lip07] Seymour Lipschutz, Marc L. Lipson, *Schaum's Outline of Theory and Problems of Discrete Mathematics*, Third edition, McGraw-Hill Education, 2007.

Further Reading

- Matrix properties:: [Ros21]:188-195.
- Set, representations of set:: [Ros21]:122-123.
- Set nature:: [Ros21]:123-124.
- Set inclusion:: [Ros21]:125-127.
- Cardinality of set:: [Ros21]:127-128,180-181.
- Finite set, infinite set, power set:: [Ros21]:127-128.

Lecture Exercises: Problem 1 [Ref: Gate 2015, Q.2, p.1 (Set2)]

The cardinality of the power set of $\{0, 1, 2, \cdots, 10\}$ is ______.

Lecture Exercises: Problem 1 Ans

- Let $\mathbf{A} = \{0, 1, 2, \cdots, 10\}$.
- Then, $|\mathbf{A}| = 11$.
- So, according to definition, power set of $\mathbf{A}$ is given by $\mathcal{P}(\mathbf{A}) = \{\emptyset, \{0\}, \{1\}, \{2\}, \cdots, \{10\}, \{0,1\}, \{0,2\}, \cdots, \mathbf{A}\}$.
- $|\mathcal{P}(\mathbf{S})| = 2^n$, if $|\mathbf{S}| = n$
- Hence, cardinality of $\mathcal{P}(\mathbf{A})$ is given by $|\mathcal{P}(\mathbf{S})| = 2^{|\mathbf{A}|} = 2^{11} = 2048$.