

ADVANCED DC/AC INVERTERS

APPLICATIONS IN RENEWABLE ENERGY



Fang Lin Luo
Hong Ye



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APPLICATIONS IN RENEWABLE ENERGY

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Preface

This book provides knowledge and applications of advanced DC/AC inverters that are both concise and useful for engineering students and practicing professionals. It is well organized in about 300-plus pages and with 250 diagrams to introduce more than 100 topologies of the advanced inverters originally developed by the authors. Some cutting-edge topologies published recently are also illustrated in this book. All prototypes are novel approaches and great contributions to DC/AC inversion technology.

DC/AC inversion technology is one of the main branches in power electronics. It was established in the 1960s and grew fast in the 1980s. DC/AC inverters convert DC power sources to AC power users. It is of vital importance for all industrial applications, including electrical vehicles and renewable energy systems. In recent years, inversion technology has been rapidly developed and new topologies have been published, which largely improved the power factor and increased the power efficiency. One purpose of writing this book is to summarize the features of DC/AC inverters and introduce more than 50 new circuits as well.

DC/AC Inverters can be sorted into two groups: pulse-width modulation (PWM) inverters and multilevel modulation (MLM) inverters. People are familiar with PWM inverters, such as the voltage source inverter (VSI) and current source inverter (CSI). They are very popular in industrial applications. The impedance-source inverter (ZSI) was first introduced in 2003 and immediately attracted many experts of power electronics to this area. Its advantages are so attractive for research and industrial applications that hundreds of papers regarding ZSI have been published in recent years.

All PWM inverters have the same main power circuits, that is, three legs for three-phase output voltage. Multilevel inverters were invented in the 1980s. Unlike PWM inverters, multilevel inverters have different main power circuits. Typical ones are the diode-clamped inverters, capacitor clamped (flying capacitor) inverters, and hybrid H-bridge multilevel inverters. Multilevel inverters overcame the drawbacks of the PWM inverter and opened a broad way for industrial applications.

This book introduces four novel multilevel inverters proposed by the authors: laddered multilevel inverters, super-lift modulated inverters, switched-capacitor inverters, and switched-inductor inverters. They have simple structures with fewer components to implement the DC/AC inversion. They are very attractive to DC/AC inverter designers and have been applied in industrial applications, including renewable energy systems.

This book introduces four methods to manage the switching angles to obtain the lowest THD, which is an important topic for multilevel inverters. The half-height (HH) method is superior to others in achieving low THD

by careful investigation. A MATLAB® program is used to search the best switching angles to obtain the lowest THD. The best switching angles for any multilevel inverter are listed in tables as convenient references for electrical engineers. Simulation waveforms are shown to verify the design.

Due to world energy resource shortage, the development of renewable energy sources is critical. The relevant topics such as energy-saving and power supply quality are also paid much attention. Renewable energy systems require large number of DC/DC converters and DC/AC inverters. In this book, introduction and design examples including analysis and results are given for wind turbine and solar panel energy systems.

The book is organized in 15 chapters. General knowledge is introduced in [Chapter 1](#). Traditional PWM inverters, such as voltage source inverters, current source inverters, and impedance source inverters, are discussed in [Chapters 2 to 5](#). New quasi-impedance source inverters and soft-switching PWM inverters are investigated in [Chapters 6](#) and [7](#), respectively. Multi-level DC/AC inverters are generally introduced in [Chapter 8](#). Trinary H-bridge inverters are specially investigated in [Chapter 9](#). Novel multilevel inverters including laddered multilevel inverters, super-lift modulated inverters, switched capacitor inverters, and switched inductor inverters are introduced in [Chapters 10 to 13](#). Best switching angles to obtain lowest THD for multilevel DC/AC inverters are studied in [Chapter 14](#). Application examples in renewable energy systems are discussed in [Chapter 15](#).

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1

Introduction

DC/AC inverters convert DC source energy for AC users, and are a big category of power electronics. Power electronics is the technology to process and control the flow of electric energy by supplying voltages and currents in a form that is optimally suited for user loads [1]. A typical block diagram is shown in [Figure 1.1](#) [2]. The input power can be AC and DC sources. A general example is that the AC input power is from the electric utility. The output power to load can be AC and DC voltages. The power processor in the block diagram is usually called a *converter*. Conversion technologies are used to construct converters. Therefore, there are four categories of converters [3]:

- AC/DC converters/rectifiers (AC to DC)
- DC/DC converters (DC to DC)
- DC/AC inverters/converters (DC to AC)
- AC/AC converters (AC to AC)

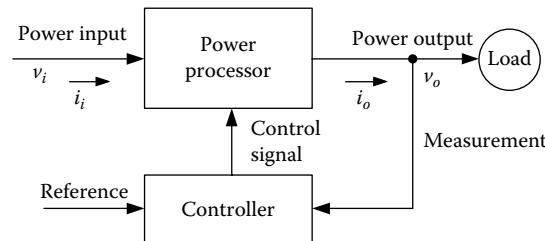
We will use converter as a generic term to refer to a single power conversion stage that may perform any of the functions listed above. To be more specific, in AC to DC and DC to AC conversion, *rectifier* refers to a converter when the average power flow is from the AC to the DC side. *Inverter* refers to the converter when the average power flow is from the DC to the AC side. In fact, the power flow through the converter may be reversible. In that case, as shown in [Figure 1.2](#) [2], we refer to that converter in terms of its rectifier and inverter modes of operation.

1.1 Symbols and Factors Used in This Book

We list the factors and symbols used in this book here. If no specific description is given, the parameters follow the meaning stated here.

1.1.1 Symbols Used in Power Systems

For instantaneous values of variables such as voltage, current, and power that are functions of time, the symbols used are lowercase letters v , i , and p ,

**FIGURE 1.1**

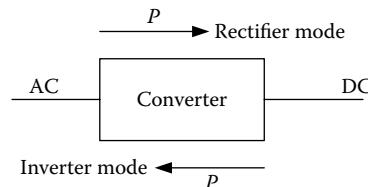
The block diagram of a power electronics system.

respectively. They are functions of time operating in the time domain. We may or may not explicitly show that they are functions of time, for example, using v rather than $v(t)$. The uppercase symbols V and I refer to their average value in DC quantities and a root-mean-square (rms) value in AC quantities, computed from their instantaneous waveforms. They generally refer to an average value in DC quantities and a root-mean-square (rms) value in AC quantities. If there is a possibility of confusion, the subscript *avg* or *rms* is used. The average power is always indicated by P .

Usually, the input voltage and current are represented by v_{in} and i_{in} (or v_1 and i_1), and the output voltage and current are represented by v_O and i_O (or v_2 and i_2). The input and output powers are represented by P_{in} and P_O . The power transfer efficiency (η) is defined as $\eta = P_O/P_{in}$.

Passive loads such as resistor R , inductor L , and capacitor C are generally used in circuits. We use R , L , and C to indicate their symbols and values as well. All these parameters and their combination Z are linear loads since the performance of the circuit constructed by these components is described by a linear differential equation. Z is the impedance of a linear load. If the circuit consists of a resistor R , an inductor L , and a capacitor C connected in series, the impedance Z is represented by

$$Z = R + j\omega L - j \frac{1}{\omega C} = |Z| \angle \phi \quad (1.1)$$

**FIGURE 1.2**

AC-to-DC converters.

where R is the resistance measured by Ω , L is the inductance measured by H , C is the capacitance measured by F , ω is the AC supply angular frequency measured by rad/s, and $\omega = 2\pi f$, where f is the AC supply frequency measured by Hz. For the calculation of Z , if there is no capacitor in the circuit, the term $j\frac{1}{\omega C}$ is omitted (do not take $C = 0$ and $j\frac{1}{\omega C} = >\infty$). The absolute impedance $|Z|$ and the phase angle ϕ are determined by

$$\begin{aligned}|Z| &= \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \\ \phi &= \tan^{-1} \frac{\omega L - \frac{1}{\omega C}}{R}\end{aligned}\tag{1.2}$$

Example 1.1

A circuit has a load with a resistor $R = 20 \Omega$, an inductor $L = 20 \text{ mH}$, and a capacitor $C = 200 \mu\text{F}$ in series connection. The voltage supplying frequency $f = 60 \text{ Hz}$. Calculate the load impedance and the phase angle.

Solution:

From Equation (1.1), the impedance Z is

$$\begin{aligned}Z &= R + j\omega L - j\frac{1}{\omega C} = 20 + j120\pi \times 0.02 - j\frac{1}{120\pi \times 0.0002} \\ &= 20 + j(7.54 - 13.26) = 20 - j5.72 = |Z| \angle \phi\end{aligned}$$

From Equation (1.2), the absolute impedance $|Z|$ and phase angle ϕ are

$$\begin{aligned}|Z| &= \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2} = \sqrt{20^2 + 5.72^2} = 20.8 \Omega \\ \phi &= \tan^{-1} \frac{\omega L - \frac{1}{\omega C}}{R} = \tan^{-1} \frac{-5.72}{20} = -17.73^\circ\end{aligned}$$

If a circuit consists of a resistor R and an inductor L connected in series, the corresponding impedance Z is represented by

$$Z = R + j\omega L = |Z| \angle \phi\tag{1.3}$$

The absolute impedance $|Z|$ and phase angle ϕ are determined by

$$\begin{aligned}|Z| &= \sqrt{R^2 + (\omega L)^2} \\ \phi &= \tan^{-1} \frac{\omega L}{R}\end{aligned}\tag{1.4}$$

We define the circuit time constant τ as

$$\tau = \frac{L}{R}\tag{1.5}$$

If a circuit consists of a resistor R and a capacitor C connected in series, the impedance Z is represented by

$$Z = R - j \frac{1}{\omega C} = |Z| \angle \phi\tag{1.6}$$

The absolute impedance $|Z|$ and phase angle ϕ are determined by

$$\begin{aligned}|Z| &= \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2} \\ \phi &= -\tan^{-1} \frac{1}{\omega CR}\end{aligned}\tag{1.7}$$

We define the circuit time constant τ as

$$\tau = RC\tag{1.8}$$

Summary of the Symbols

Symbol	Explanation (Measuring Unit)
C	capacitor/capacitance (F)
f	frequency (Hz)
i, I	instantaneous current, average/rms current (A)
L	inductor/inductance (H)
R	resistor/resistance (Ω)
p, P	instantaneous power, rated/real power (W)
q, Q	instantaneous reactive power, rated reactive power (VAR)
s, S	instantaneous apparent power, rated apparent power (VA)
v, V	instantaneous voltage, average/rms voltage (V)
Z	impedance (Ω)
ϕ	phase angle (degree, or radian)
η	efficiency (percents%)
τ	time constant (second)
ω	angular frequency (radian/sec), $\omega = 2\pi f$

1.1.2 Factors and Symbols Used in AC Power Systems

The input AC voltage can be single-phase or three-phase voltages. They are usually a pure sinusoidal wave function. For a single-phase input voltage $v(t)$, the function can be expressed as [4]:

$$v(t) = \sqrt{2}V \sin \omega t = V_m \sin \omega t \quad (1.9)$$

where v is the instantaneous input voltage, V is its root-mean-square (rms) value, V_m is its amplitude, ω is the angular frequency, $\omega = 2\pi f$, and f is the supply frequency. Usually, the input current may not be a pure sinusoidal wave, depending on the load. If the input voltage supplies a linear load (resistive, inductive, capacitive loads, or their combination) the input current $i(t)$ is not distorted, but may be delayed in a phase angle ϕ . In this case, it can be expressed as

$$i(t) = \sqrt{2}I \sin(\omega t - \phi) = I_m \sin(\omega t - \phi) \quad (1.10)$$

where i is the instantaneous input current, I is its root-mean-square value, I_m is its amplitude, and ϕ is the phase-delay angle. We define the power factor (PF) as

$$PF = \cos \phi \quad (1.11)$$

PF is the ratio of the real power (P) to the apparent power (S). We have the relation $S = P + jQ$, where Q is the reactive power. The power vector diagram is shown in [Figure 1.3](#). We have the following relations between the powers:

$$S = VI^* = \frac{V^2}{Z^*} = P + jQ = |S| \angle \phi \quad (1.12)$$

$$|S| = \sqrt{P^2 + Q^2} \quad (1.13)$$

$$\phi = \tan^{-1} \frac{Q}{P} \quad (1.14)$$

$$P = S \cos \phi \quad (1.15)$$

$$Q = S \sin \phi \quad (1.16)$$

If the input current is distorted, it consists of harmonics. Its fundamental harmonic can be expressed as

$$i_1 = \sqrt{2}I_1 \sin(\omega t - \phi_1) = I_{m1} \sin(\omega t - \phi_1) \quad (1.17)$$

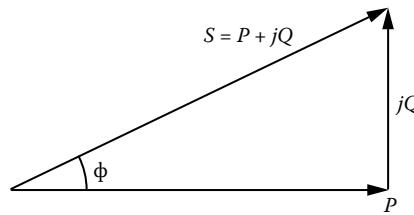


FIGURE 1.3
Power vector diagram.

where i_1 is the fundamental harmonic instantaneous value, I_1 its rms value, I_{m1} its amplitude, and ϕ_1 its phase angle. In this case, the displacement power factor (DPF) is defined as

$$DPF = \cos \phi_1 \quad (1.18)$$

Correspondingly, the power factor is defined as

$$PF = \frac{DPF}{\sqrt{1+THD^2}} \quad (1.19)$$

where THD is the total harmonic distortion. It can be used to measure both voltage and current waveforms. It is defined as

$$THD = \frac{\sqrt{\sum_{n=2}^{\infty} I_n^2}}{I_1} \quad \text{or} \quad THD = \frac{\sqrt{\sum_{n=2}^{\infty} V_n^2}}{V_1} \quad (1.20)$$

where I_n or V_n is the amplitude of the nth order harmonic.

The harmonic factor (HF) is a variable that describes the weighted percentage of the nth order harmonic with reference to the amplitude of the fundamental harmonic V_1 . It is defined as

$$HF_n = \frac{I_n}{I_1} \quad \text{or} \quad HF_n = \frac{V_n}{V_1} \quad (1.21)$$

$n = 1$ corresponds to the fundamental harmonic. Therefore, $HF_1 = 1$. The total harmonic distortion (THD) can be written as

$$THD = \sqrt{\sum_{n=2}^{\infty} HF_n^2} \quad (1.22)$$

A pure sinusoidal waveform has THD = 0.

Weighted total harmonic distortion (WTHD) is a variable to describe waveform distortion. It is defined as follows:

$$WTHD = \frac{\sqrt{\sum_{n=2}^{\infty} \frac{V_n^2}{n}}}{V_1} \quad (1.23)$$

Note that THD gives an immediate measure of the inverter output voltage waveform distortion. WTHD is often interpreted as the normalized current ripple expected in an inductive load when fed from the inverter output voltage.

Example 1.2:

A load with a resistor $R = 20 \Omega$, an inductor $L = 20 \text{ mH}$, and a capacitor $C = 200 \mu\text{F}$ in series connection is supplied by an AC voltage of 240 V (rms) with frequency $f = 60 \text{ Hz}$. Calculate the circuit current and the corresponding apparent power S , real power P , reactive power Q , and the power factor PF .

Solution:

From Example 1.1, the impedance Z is

$$\begin{aligned} Z &= R + j\omega L - j\frac{1}{\omega C} = 20 + j120\pi \times 0.02 - j\frac{1}{120\pi \times 0.0002} \\ &= 20 + j(7.54 - 13.26) = 20 - j5.72 = 20.8\angle -17.73^\circ \Omega \end{aligned}$$

The circuit current I is

$$I = \frac{V}{Z} = \frac{240}{20.8\angle -17.73^\circ} = 11.54\angle 17.73^\circ \text{ A}$$

The apparent power S is

$$S = VI^* = 240 \times 11.54\angle -17.73^\circ = 2769.23\angle -17.73^\circ \text{ VA}$$

The real power P is

$$P = |S| \cos \phi = 2769.23 \times \cos 17.73^\circ = 2637.7 \text{ W}$$

The reactive power Q is

$$Q = |S| \sin \phi = 2769.23 \times \sin 17.73^\circ = -843.3 \text{ VAR}$$

The power factor is

$$PF = \cos \phi = 0.9525 \text{ Leading}$$

Summary of the Symbols

Symbol	Explanation (Measuring Unit)
DPF	displacement power factor (percent)
HF _n	nth order harmonic factor
i ₁ , I ₁	instantaneous fundamental current, average/rms fundamental current (A)
i _n , I _n	instantaneous nth order harmonic current, average/rms nth order harmonic current (A)
I _m	current amplitude (A)
PF	power factor (leading/lagging percent)
q, Q	instantaneous reactive power, rated reactive power (VAR)
s, S	instantaneous apparent power, rated apparent power (VA)
t	time (second)
THD	total harmonic distortion (percent)
v ₁ , V ₁	instantaneous fundamental voltage, average/rms fundamental voltage (V)
v _n , V _n	instantaneous nth order harmonic voltage, average/rms nth order harmonic voltage (V)
WTHD	weighted total harmonic distortion (percent)
φ ₁	phase angle of the fundamental harmonic (degree, or radian)

1.1.3 Factors and Symbols Used in DC Power Systems

We define the output DC voltage instantaneous value to be v_d and the average value to be V_d (or V_{d0}) [5]. A pure DC voltage has no ripple; it is then called ripple-free DC voltage. Otherwise, a DC voltage is distorted and consists of a DC component and AC harmonics. Its rms value is V_{d-rms} . For a distorted DC voltage, its rms value V_{d-rms} is constantly higher than its average value V_d . The ripple factor (RF) is defined as

$$RF = \frac{\sqrt{\sum_{n=1}^{\infty} V_n^2}}{V_d} \quad (1.24)$$

where V_n is the nth order harmonic. The form factor (FF) is defined as

$$FF = \frac{V_{d-rms}}{V_d} = \frac{\sqrt{\sum_{n=0}^{\infty} V_n^2}}{V_d} \quad (1.25)$$

where V_0 is the 0th order harmonic; that is, the average component V_d . Therefore, we obtain $FF > 1$, and the relation

$$RF = \sqrt{FF^2 - 1} \quad (1.26)$$

The form factor FF and ripple factor RF are used to describe the quality of a DC waveform (voltage and current parameters). For a pure DC voltage, FF = 1 and RF = 0.

Summary of the Symbols

Symbol	Explanation (Measuring Unit)
FF	form factor (percent)
RF	ripple factor (percent)
v_d, V_d	instantaneous DC voltage, average DC voltage (V)
$V_{d-\text{rms}}$	rms DC voltage (V)
v_n, V_n	instantaneous nth order harmonic voltage, average/rms nth order harmonic voltage (V)

1.2 FFT—Fast Fourier Transform

The FFT [6] is a very versatile method of analyzing waveforms. A periodic function with radian frequency ω can be represented by a series of sinusoidal functions:

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos n\omega t + b_n \sin n\omega t) \quad (1.27)$$

where the Fourier coefficients are

$$a_n = \frac{1}{\pi} \int_0^{2\pi} f(t) \cos(n\omega t) d(\omega t) \quad n = 0, 1, 2, \dots, \infty \quad (1.28)$$

$$b_n = \frac{1}{\pi} \int_0^{2\pi} f(t) \sin(n\omega t) d(\omega t) \quad n = 1, 2, \dots, \infty \quad (1.29)$$

In this case, we call the terms with radian frequency ω the fundamental harmonic and the terms with radian frequency $n\omega$ ($n > 1$) higher order harmonics. If we draw the amplitudes of all harmonics in the frequency domain, we can get the spectrum in individual peaks. The term $a_0/2$ is the DC component.

1.2.1 Central Symmetrical Periodical Function

If the periodic function is a central symmetrical periodic function, all terms with cosine function disappear. The FFT becomes

$$f(t) = \sum_{n=1}^{\infty} b_n \sin n\omega t \quad (1.30)$$

where

$$b_n = \frac{1}{\pi} \int_0^{2\pi} f(t) \sin(n\omega t) d(\omega t) \quad n = 1, 2, \dots, \infty \quad (1.31)$$

We usually call this the odd function. In this case, we call the term with the radian frequency ω the fundamental harmonic, and the terms with the radian frequency $n\omega$ ($n > 1$) higher order harmonics. If we draw the amplitudes of all harmonics in the frequency domain, we can get the spectrum in individual peaks. Since it is an odd function, the DC component is zero.

1.2.2 Axial (Mirror) Symmetrical Periodical Function

If the periodic function is an axial symmetrical periodic function, all terms with sine function disappear. The FFT becomes

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\omega t \quad (1.32)$$

where $a_0/2$ is the DC component and

$$a_n = \frac{1}{\pi} \int_0^{2\pi} f(t) \cos(n\omega t) d(\omega t) \quad n = 0, 1, 2, \dots, \infty \quad (1.33)$$

The term $a_0/2$ is the DC component. We usually call this function the even function. In this case, we call the term with the radian frequency ω the fundamental harmonic, and the terms with the radian frequency $n\omega$ ($n > 1$) higher-order harmonics. If we draw the amplitudes of all harmonics in the frequency domain, we can get the spectrum in individual peaks. Since it is an even function, the DC component is usually not zero.

1.2.3 Nonperiodic Function

The spectrum of a periodic function in the time domain is a discrete function in the frequency domain. For a nonperiodic function in the time domain, it is possible to represent it by Fourier integration. The spectrum is a continuous function in the frequency domain.

1.2.4 Useful Formulae and Data

Some trigonometric formulae are useful for FFT:

$$\sin^2 x + \cos^2 x = 1 \quad \sin x = \cos\left(\frac{\pi}{2} - x\right)$$

$$\sin x = -\sin(-x) \quad \sin x = \sin(\pi - x)$$

$$\cos x = \cos(-x) \quad \cos x = -\cos(\pi - x)$$

$$\frac{d}{dx} \sin x = \cos x \quad \frac{d}{dx} \cos x = -\sin x$$

$$\int \sin x \, dx = -\cos x \quad \int \cos x \, dx = \sin x$$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

Some values corresponding to the special angles are usually used:

$$\sin \frac{\pi}{12} = \sin 15^\circ = 0.2588$$

$$\cos \frac{\pi}{12} = \cos 15^\circ = 0.9659$$

$$\sin \frac{\pi}{8} = \sin 22.5^\circ = 0.3827$$

$$\cos \frac{\pi}{8} = \cos 22.5^\circ = 0.9239$$

$$\sin \frac{\pi}{6} = \sin 30^\circ = 0.5$$

$$\cos \frac{\pi}{6} = \cos 30^\circ = \frac{\sqrt{3}}{2} = 0.866$$

$$\sin \frac{\pi}{4} = \sin 45^\circ = \frac{\sqrt{2}}{2} = 0.7071$$

$$\cos \frac{\pi}{4} = \cos 45^\circ = \frac{\sqrt{2}}{2} = 0.7071$$

$$\tan \frac{\pi}{12} = \tan 15^\circ = 0.2679$$

$$\tan \frac{\pi}{8} = \tan 22.5^\circ = 0.4142$$

$$\tan \frac{\pi}{6} = \tan 30^\circ = \frac{\sqrt{3}}{3} = 0.5774$$

$$\tan \frac{\pi}{4} = \tan 45^\circ = 1$$

$$\tan x = \frac{1}{co - \tan x}$$

$$\tan x = co - \tan\left(\frac{\pi}{2} - x\right)$$

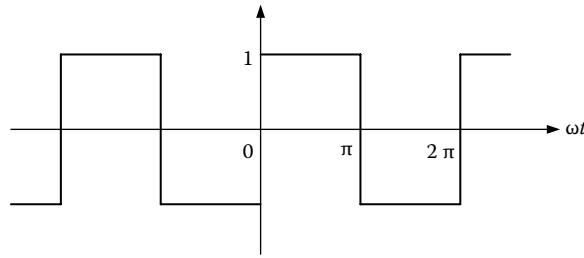


FIGURE 1.4
A waveform.

1.2.5 Examples of FFT Applications

Example 1.3

An odd-square waveform is shown in [Figure 1.4](#). Find the FFT and HF up to the 7th order, and also the THD and WTHD.

Solution:

The function $f(t)$ is

$$f(t) = \begin{cases} 1 & 2n\pi \leq \omega t < (2n+1)\pi \\ -1 & (2n+1)\pi \leq \omega t < 2(n+1)\pi \end{cases} \quad (1.34)$$

The Fourier coefficients are

$$b_n = \frac{1}{\pi} \int_0^{2\pi} f(t) \sin(n\omega t) d(\omega t) = \frac{2}{n\pi} \int_0^{n\pi} \sin \theta d\theta = 2 \frac{1 - (-1)^n}{n\pi}$$

or

$$b_n = \frac{4}{n\pi} \quad n = 1, 3, 5, \dots \infty \quad (1.35)$$

Finally, we obtain

$$f(t) = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\sin(n\omega t)}{n} \quad n = 1, 3, 5, \dots \infty \quad (1.36)$$

The fundamental harmonic has an amplitude of $4/\pi$. If we consider the higher order harmonics until the 7th order, that is, $n = 3, 5, 7$, the HFs are

$$HF_3 = 1/3; \quad HF_5 = 1/5; \quad HF_7 = 1/7$$

The *THD* is

$$THD = \frac{\sqrt{\sum_{n=2}^{\infty} V_n^2}}{V_1} = \sqrt{\left(\frac{1}{3}\right)^2 + \left(\frac{1}{5}\right)^2 + \left(\frac{1}{7}\right)^2} = 0.41415 \quad (1.37)$$

The *WTHD* is

$$WTHD = \frac{\sqrt{\sum_{n=2}^{\infty} \frac{V_n^2}{n}}}{V_1} = \sqrt{\left(\frac{1}{3}\right)^3 + \left(\frac{1}{5}\right)^3 + \left(\frac{1}{7}\right)^3} = 0.219 \quad (1.38)$$

Example 1.4

An even-square waveform is shown in [Figure 1.5](#). Find the FFT and HF up to the 7th order, and also the *THD* and *WTHD*.

The function $f(t)$ is

$$f(t) = \begin{cases} 1 & (2n - 0.5)\pi \leq \omega t < (2n + 0.5)\pi \\ -1 & (2n + 0.5)\pi \leq \omega t < (2n + 1.5)\pi \end{cases} \quad (1.39)$$

The Fourier coefficients are

$$a_0 = 0$$

$$a_n = \frac{1}{\pi} \int_0^{2\pi} f(t) \cos(n\omega t) d(\omega t) = \frac{4}{n\pi} \int_0^{\frac{n\pi}{2}} \cos \theta d\theta = \frac{4 \sin \frac{n\pi}{2}}{n\pi}$$

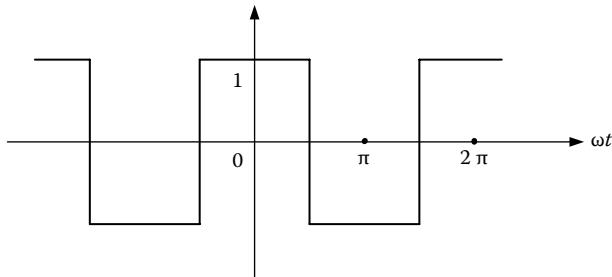


FIGURE 1.5
An even square waveform.

or

$$a_n = \frac{4}{n\pi} \sin \frac{n\pi}{2} \quad n = 1, 3, 5, \dots \infty \quad (1.40)$$

The term $\sin \frac{n\pi}{2}$ is used to define the sign. Finally, we obtain

$$F(t) = \frac{4}{\pi} \sum_{n=1}^{\infty} \sin \frac{n\pi}{2} \cos(n\omega t) \quad n = 1, 3, 5, \dots \infty \quad (1.41)$$

The fundamental harmonic has the amplitude $4/\pi$. If we consider the higher order harmonics until the 7th order, that is, $n = 3, 5, 7$, the HFs are

$$\text{HF}_3 = 1/3; \quad \text{HF}_5 = 1/5; \quad \text{HF}_7 = 1/7$$

The THD is

$$\text{THD} = \frac{\sqrt{\sum_{n=2}^{\infty} V_n^2}}{V_1} = \sqrt{\left(\frac{1}{3}\right)^2 + \left(\frac{1}{5}\right)^2 + \left(\frac{1}{7}\right)^2} = 0.41415 \quad (1.42)$$

The WTHD is

$$\text{WTHD} = \frac{\sqrt{\sum_{n=2}^{\infty} \frac{V_n^2}{n}}}{V_1} = \sqrt{\left(\frac{1}{3}\right)^3 + \left(\frac{1}{5}\right)^3 + \left(\frac{1}{7}\right)^3} = 0.219 \quad (1.43)$$

Example 1.5

An odd-waveform pulse with pulse width x is shown in [Figure 1.6](#). Find the FFT and HF up to the 7th order, and also the THD and WTHD.

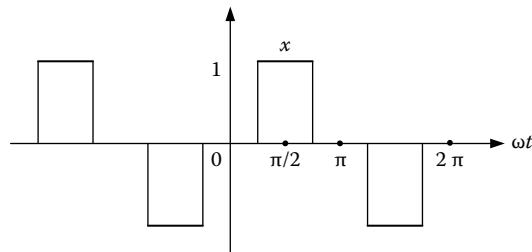


FIGURE 1.6

An odd-waveform pulse.

The function $f(t)$ is in the period $-\pi$ to $+\pi$:

$$f(t) = \begin{cases} 1 & \frac{\pi-x}{2} \leq \omega t < \frac{\pi+x}{2} \\ -1 & -\frac{\pi+x}{2} \leq \omega t < -\frac{\pi-x}{2} \end{cases} \quad (1.44)$$

The Fourier coefficients are

$$\begin{aligned} b_n &= \frac{1}{\pi} \int_0^{2\pi} f(t) \sin(n\omega t) d(\omega t) = \frac{2}{n\pi} \int_{\frac{n\pi-x}{2}}^{\frac{n\pi+x}{2}} \sin \theta d\theta = 2 \frac{\cos(n\frac{\pi-x}{2}) - \cos(n\frac{\pi+x}{2})}{n\pi} \\ &= 2 \frac{2\cos(n\frac{\pi-x}{2})}{n\pi} = \frac{4\sin(\frac{n\pi}{2})\sin(\frac{nx}{2})}{n\pi} \end{aligned}$$

or

$$b_n = \frac{4}{n\pi} \sin \frac{n\pi}{2} \sin \frac{nx}{2} \quad n = 1, 3, 5, \dots \infty \quad (1.45)$$

Finally, we obtain

$$F(t) = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\sin(n\omega t)}{n} \sin \frac{n\pi}{2} \sin \frac{nx}{2} \quad n = 1, 3, 5, \dots \infty \quad (1.46)$$

The fundamental harmonic has the amplitude $\frac{4}{\pi} \sin \frac{x}{2}$. If we consider the higher order harmonics until the 7th order, that is, $n = 3, 5, 7$, the HFs are

$$HF_3 = \frac{\sin \frac{3x}{2}}{3 \sin \frac{x}{2}}; \quad HF_5 = \frac{\sin \frac{5x}{2}}{5 \sin \frac{x}{2}}; \quad HF_7 = \frac{\sin \frac{7x}{2}}{7 \sin \frac{x}{2}}$$

The values of the HFs should be absolute.

If $x = \pi$, the THD is

$$THD = \frac{\sqrt{\sum_{n=2}^{\infty} V_n^2}}{V_1} = \sqrt{\left(\frac{1}{3}\right)^2 + \left(\frac{1}{5}\right)^2 + \left(\frac{1}{7}\right)^2} = 0.41415 \quad (1.47)$$

The WTHD is

$$WTHD = \frac{\sqrt{\sum_{n=2}^{\infty} \frac{V_n^2}{n}}}{V_1} = \sqrt{\left(\frac{1}{3}\right)^3 + \left(\frac{1}{5}\right)^3 + \left(\frac{1}{7}\right)^3} = 0.219 \quad (1.48)$$

Example 1.6

A 5-level odd waveform is shown in Figure 1.7. Find the FFT and HF up to the 7th order, and also the THD and WTHD.

The function $f(t)$ is in the period $-\pi - +\pi$:

$$f(t) = \begin{cases} 2 & \frac{\pi}{3} \leq \omega t < \frac{2\pi}{3} \\ 1 & \frac{\pi}{6} \leq \omega t < \frac{\pi}{3}, \frac{2\pi}{3} \leq \omega t < \frac{5\pi}{6} \\ 0 & \text{other} \\ -1 & -\frac{5\pi}{6} \leq \omega t < -\frac{2\pi}{3}, -\frac{\pi}{3} \leq \omega t < -\frac{\pi}{6} \\ -2 & -\frac{2\pi}{3} \leq \omega t < -\frac{\pi}{3} \end{cases} \quad (1.49)$$

The Fourier coefficients are

$$\begin{aligned} b_n &= \frac{1}{\pi} \int_0^{2\pi} f(t) \sin(n\omega t) d(\omega t) = \frac{2}{n\pi} \left[\int_{\frac{n\pi}{6}}^{\frac{5n\pi}{6}} \sin \theta d\theta + \int_{\frac{n\pi}{3}}^{\frac{2n\pi}{3}} \sin \theta d\theta \right] \\ &= \frac{2}{n\pi} \left[\left(\cos \frac{n\pi}{6} - \cos \frac{5n\pi}{6} \right) + \left(\cos \frac{n\pi}{3} - \cos \frac{2n\pi}{3} \right) \right] = \frac{4}{n\pi} \left(\cos \frac{n\pi}{6} + \cos \frac{n\pi}{3} \right) \end{aligned}$$

or

$$b_n = \frac{4}{n\pi} \left(\cos \frac{n\pi}{6} + \cos \frac{n\pi}{3} \right) \quad n = 1, 3, 5, \dots \infty \quad (1.50)$$

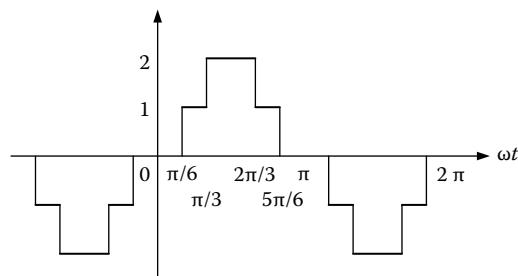


FIGURE 1.7

A five-level odd waveform.

Finally, we obtain

$$F(t) = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\sin(n\omega t)}{n} \left(\cos \frac{n\pi}{6} + \cos \frac{n\pi}{3} \right) \quad n = 1, 3, 5, \dots \infty \quad (1.51)$$

The fundamental harmonic has the amplitude $\frac{2}{\pi}(1+\sqrt{3})$. If we consider the higher-order harmonics until the 7th order, that is, $n = 3, 5, 7$, the HFs are

$$HF_3 = \frac{2}{3(1+\sqrt{3})} = 0.244; \quad HF_5 = \frac{\sqrt{3}-1}{5(1+\sqrt{3})} = 0.0536; \quad HF_7 = \frac{\sqrt{3}-1}{5(1+\sqrt{3})} = 0.0383$$

The values of the HFs should be absolute.

The THD is

$$THD = \frac{\sqrt{\sum_{n=2}^{\infty} V_n^2}}{V_1} = \sqrt{\sum_{n=2}^{\infty} HF_n^2} = \sqrt{0.244^2 + 0.0536^2 + 0.0383^2} = 0.2527 \quad (1.52)$$

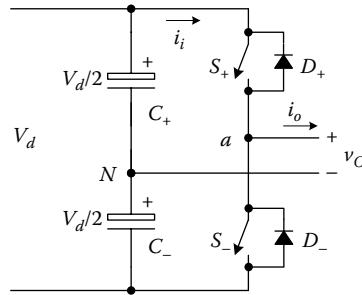
The WTHD is

$$WTHD = \frac{\sqrt{\sum_{n=2}^{\infty} \frac{V_n^2}{n}}}{V_1} = \sqrt{\sum_{n=2}^{\infty} \frac{HF_n^2}{n}} = \sqrt{\frac{0.244^2}{3} + \frac{0.0536^2}{5} + \frac{0.0383^2}{7}} = 0.1436 \quad (1.53)$$

1.3 DC/AC Inverters

DC/AC inverters [1,2] were not widely used in industrial applications before the 1960s because of their complexity and cost. They were used in most fractional horsepower AC motor drives in the 1970s since AC motors have advantages such as lower cost than DC motors, small size, and they are maintenance-free. Because of advances in semiconductor technology, more effective devices such as IGBTs and MOSFETs were produced in the 1980s, and DC/AC inverters began to be widely applied in industrial applications. Currently, DC/AC conversion techniques can be grouped into two categories: pulse width modulation (PWM) and multilevel modulation (MLM). Each category has many circuits that implement the modulation. Using PWM, we can design various inverters such as voltage source inverters (VSIs), current source inverters (CSIs), impedance source inverters (ZSIs), and multistage PWM inverters.

A single-phase half-wave PWM is shown in [Figure 1.8](#).

**FIGURE 1.8**

Single-phase half-wave PWM VSI.

The pulse width modulation (PWM) method is suitable for DC/AC conversion since the input voltage is usually a constant DC voltage (DC link). Pulse phase modulation (PPM) is also possible, but is not so convenient. Pulse amplitude modulation (PAM) is not suitable for DC/AC conversion since the input voltage is usually a constant DC voltage. In PWM operation, all pulses' leading edges start from the beginning of the pulse period, and their trailing edge is adjustable. PWM is the fundamental technique for many types of PWM DC/AC inverters such as VSI, CSI, ZSI, and multistage PWM inverters.

Another group of DC/AC inverters are the multilevel inverters (MLIs). They were invented in the late 1970s. The early MLIs were constructed by diode-clamped and capacitor-clamped circuits. Later, other MLIs were developed.

Three important procedures have to be emphasized in this book:

- To categorize existing inverters
- To introduce updated circuits
- To investigate soft switching methods

1.3.1 Categorizing Existing Inverters

Since the number of inverters is large, we have to sort them systematically. Some circuits have not been precisely named, so their functions cannot be inferred from their names.

1.3.2 Updated Circuits

Many updated DC/AC inverters were developed in recent decades, but not introduced in textbooks. We have to incorporate these techniques in this book and teach students to understand them.

1.3.3 Soft Switching Methods

The soft switching technique has been widely used in switching circuits for a long time. It effectively reduces the power losses of equipment and greatly increases the power transfer efficiency. A few soft switching technique methods will be introduced in this book.

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Introduction

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