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Tutorial - 4

Solution 1

Let $\alpha = (x_1, x_2, x_3)$ & $\beta = (y_1, y_2, y_3)$

Q.& $\alpha, \beta \in V_3(F)$ & let $a \in F$

By Question.

$f(x_1, x_2, x_3) = f(x_2, x_3)$

$f(y_1, y_2, y_3) = (y_2, y_3)$

$f(x_1, x_2, x_3) + f(y_1, y_2, y_3) = (x_2, x_3) + (y_2, y_3)$

$f(x_1, x_2, x_3) + f(y_1, y_2, y_3) = (x_2 + y_2, x_3 + y_3)$

$f(\alpha) + f(\beta) = (x_2 + y_2, x_3 + y_3)$ — (1)

$f(\alpha + \beta) = f(x_1 + y_1, x_2 + y_2, x_3 + y_3) = (x_2 + y_2, x_3 + y_3)$ — (2)

from eq.-n (1) & (2)

$f(\alpha + \beta) = f(\alpha) + f(\beta)$

Hence 1st equation proved.

Let $\alpha \in V_3(F)$ & $a \in R$

Now,

$$a\alpha = a(x_1, x_2, x_3)$$

$$a\alpha = (ax_1, ax_2, ax_3)$$

$$f(a\alpha) = f(ax_1, ax_2, ax_3)$$

$$f(a\alpha) = (ax_2, ax_3)$$

$$f(a\alpha) = a(x_2, x_3)$$

$$f(a\alpha) = a f(\alpha)$$

Hence IInd condition is proved

So, it is a linear transform

Solution 2

Let, $\alpha = (x_1, y_1)$ & $B = (x_2, y_2)$
where $\alpha, B \in V_2(R)$

$f(\alpha + B) = f[(x_1, y_1) + (x_2, y_2)]$

$f(\alpha + B) = f(x_1 + x_2, y_1 + y_2)$

from question

$f(\alpha + B) = (x_1 + x_2, y_1 + y_2, 0)$ — ①

$f(\alpha) + f(B) = f(x_1, y_1) + f(x_2, y_2)$

$f(\alpha) + f(B) = (x_1, y_1, 0) + (x_2, y_2, 0)$

$f(\alpha) + f(B) = (x_1 + x_2, y_1 + y_2, 0)$ — ②

from eq^n ① & ②

$f(\alpha + B) = f(\alpha) + f(B)$

Hence 1^st condition proved.

let, $a \in F$

$a\alpha = a(x_1, y_1)$

$a\alpha = (ax_1, ay_1)$

$f(a\alpha) = f(ax_1, ay_1)$

$f(a\alpha) = (ax_1, ay_1, 0)$

$f(a\alpha) = (ax_1, ay_1, a0)$ [because $0 \times a = 0$, $a \in R$]

$f(a\alpha) = a(x_1, y_1, 0)$

$f(a\alpha) = a f(\alpha)$

Hence condition IInd proved it is a linear transform.

Solution 3 :-

Let, $\alpha = (x_1, x_2, x_3)$ & $\beta = (y_1, y_2, y_3)$

where $\alpha, \beta \in V_3(R)$

Then, $f(\alpha+\beta) = f(x_1+y_1, x_2+y_2, x_3+y_3)$

According to question

$f(\alpha+\beta) = (x_3+y_3, x_1+y_1+x_2+y_3)$ — (1)

~~$f(\alpha+\beta)$~~ =

$f(\alpha) = f(x_1, x_2, x_3) = (x_3, x_1+x_2)$

$f(\beta) = f(y_1, y_2, y_3) = (y_3, y_1+y_2)$

$f(\alpha)+f(\beta) = (x_3, x_1+x_2) + (y_3, y_1+y_2)$

$f(\alpha)+f(\beta) = (x_3+y_3, x_1+x_2+y_1+y_2)$ — (2)

from eq^n (1) & (2)

$f(\alpha+\beta) = f(\alpha) + f(\beta)$

Hence condition 1st proved.

So, let $a \in F$

$a\alpha = a(x_1, x_2, x_3)$

$a\alpha = (ax_1, ax_2, ax_3)$

$f(a\alpha) = f(ax_1, ax_2, ax_3)$

$f(a\alpha) = (ax_3, ax_1 + ax_2)$

$f(a\alpha) = a(x_3, x_1 + x_2)$

$f(a\alpha) = a f(\alpha)$

Hence condition 2nd proved.

So, it is a linear transform.

Solution 4

Given,

$V_1(F) \longrightarrow V_2(F)$

and $V_2(F)$ have zero element

mean let, $\alpha, \beta \in V_1(F)$

$f(\alpha) = 0$, $f(\beta) = 0$
and $f(\alpha + \beta) = 0$

$f(\alpha + \beta) = f(\alpha) + f(\beta)$

Hence condition 1st proved.

let, $a \in F$

$f(a\alpha) = 0$
$f(a\alpha) = a \times 0$ (because $a \times 0 = 0$, $a \in R$)

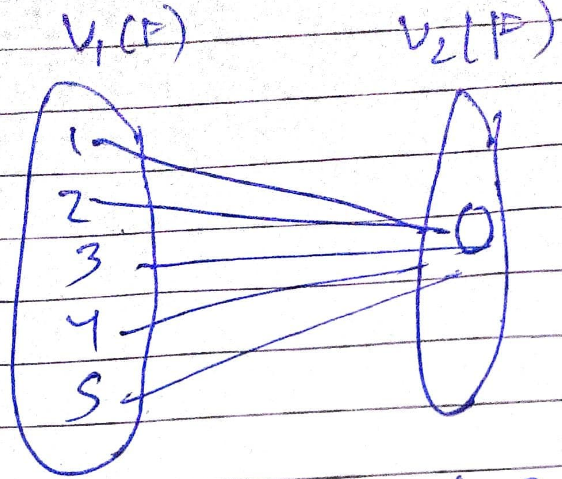
$f(a\alpha) = a f(\alpha)$
Hence condition second is proved.
and this is linear transform.

$f(\alpha_1) = 0$
$f(\alpha_2) = 0$ } according to question.

$f(\alpha_1) = f(\alpha_2)$
so, it is a one-one function.

for each value $V_1(F)$ it image to $V_2(F)$'s 0

mean,



Range of $V_1(F)$ is 0 and it has equal range and co-domain

So, it is onto functio.

Hence, due to linear transform, one-one and onto it is a isomorphism

Solution 5

Let,

$\alpha = (x_1, y_1)$, $\beta = (x_2, y_2)$ where $\alpha, \beta \in V_2(\mathbb{R})$

& let $a \in F$

Then $f(\alpha+\beta) = f((x_1, y_1) + (x_2, y_2))$

$f(\alpha+\beta) = f(x_1+x_2, y_1+y_2)$

Given in question that:
$f(x, y) = (x^3, y^3)$

$f(\alpha+\beta) = f[(x_1+x_2)^3, (y_1+y_2)^3]$ — ①

Now,

$f(\alpha)+f(\beta) = f(x_1, y_1) + f(x_2, y_2)$

$= (x_1^3, y_1^3) + (x_2^3 + y_2^3)$

$= (x_1^3+x_2^3, y_1^3+y_2^3)$ — ②

from eq^n ① & ②

$f(\alpha+\beta) \neq f(\alpha) + f(\beta)$

Solution 6

Let,

$\alpha = (x_1, x_2, x_3)$ & $\beta = (y_1, y_2, y_3)$

$\forall$ & $\alpha, \beta \in V_3(R)$ & let $a \in F$

By Question

$T(x_1, x_2, x_3) = (x_1 - x_2, x_1 + x_3)$

$\alpha + \beta = (x_1, x_2, x_3) + (y_1, y_2, y_3)$

$\alpha + \beta = (x_1 + y_1, x_2 + y_2, x_3 + y_3)$

So, we write above eq-n a.

$T(\alpha + \beta) = T(x_1 + y_1, x_2 + y_2, x_3 + y_3)$ — ①

~~$T(\alpha+\beta) =$~~

$T(\alpha) = T(x_1, x_2, x_3)$

$T(\alpha) = (x_1 - x_2, x_1 + x_3)$ — ②

Similarly, $T(\beta) = (y_1 - y_2, y_1 + y_3)$ — ③

So, $T(\alpha) + T(\beta) = (x_1 - x_2, x_1 + x_3) + (y_1 - y_2, y_1 + y_3)$

$T(\alpha) + T(\beta) = (x_1 + y_1 - x_2 - y_2, x_1 + y_1 + x_3 + y_3)$

$T(\alpha) + T(\beta) = T(\alpha + \beta)$ for eq-n (1)

So, we know that for linear transformation

$T(\alpha + \beta) = T(\alpha) + T(\beta)$

condition 1st is proved.

2nd condition

$\therefore \alpha \in V_3(R)$ & $a \in R$

We $a\alpha = a(x_1, x_2, x_3)$

$a\alpha = (ax_1, ax_2, ax_3)$

$T(a\alpha) = T(ax_1 + ax_2 + ax_3)$

$T(a\alpha) = (ax_1 - ax_2, ax_1 + ax_3)$

$T(a\alpha) = a(x_1 - x_2, x_1 + x_3)$

$T(a\alpha) = a\,T(\alpha)$

Hence proved. IInd condition

Therefore it is linear.