Matrix representation of LT

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$$\mathbb{R}^3 = \left[\begin{pmatrix}1\\0\\0\end{pmatrix}\begin{pmatrix}0\\1\\0\end{pmatrix}\begin{pmatrix}0\\0\\1\end{pmatrix}\right]$$

Q) Find Matrix representation of LT $T : \mathbb{R}^3 \to \mathbb{R}^3$

$$T\begin{pmatrix}a\\b\\c\end{pmatrix} = \begin{pmatrix}2b+c\\a-4b\\3a\end{pmatrix}$$

$$[T]_B^{B'} = ?$$

Solution

$$T\begin{bmatrix}1\\0\\0\end{bmatrix} = \begin{bmatrix}0\\1\\3\end{bmatrix} = [T(v)]_{B'} \qquad B = \begin{pmatrix}1\\0\\0\end{pmatrix}\begin{pmatrix}0\\1\\0\end{pmatrix}\begin{pmatrix}0\\0\\1\end{pmatrix}$$

$$T\begin{pmatrix}0\\0\\1\end{pmatrix} = \begin{bmatrix}1\\0\\0\end{bmatrix} \qquad T\begin{pmatrix}0\\1\\0\end{pmatrix} = \begin{bmatrix}2\\-4\\0\end{bmatrix}$$

$$T\begin{pmatrix}0\\0\\1\end{pmatrix} = \begin{pmatrix}1\\0\\0\end{pmatrix}$$

$$T(v_1) = T\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{bmatrix}0\\1\\3\end{bmatrix}$$

$$\begin{bmatrix}0\\1\\3\end{bmatrix} = \; = 0\begin{bmatrix}1\\0\\0\end{bmatrix} + 1\begin{bmatrix}0\\1\\0\end{bmatrix} + 3\begin{bmatrix}0\\0\\1\end{bmatrix}$$

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$$

$$\begin{bmatrix}0\\1\\3\end{bmatrix}_{B'} = \begin{bmatrix}0\\1\\3\end{bmatrix}$$

$$\begin{bmatrix}3\\-4\end{bmatrix} \; = \begin{bmatrix}3\\-4\end{bmatrix}$$

$$\begin{bmatrix} 3 \\ -4 \\ 3 \end{bmatrix}_{B'} = \begin{bmatrix} 3 \\ -4 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}_{B} = \begin{bmatrix} 1 \\ 9 \\ 3 \end{bmatrix}$$

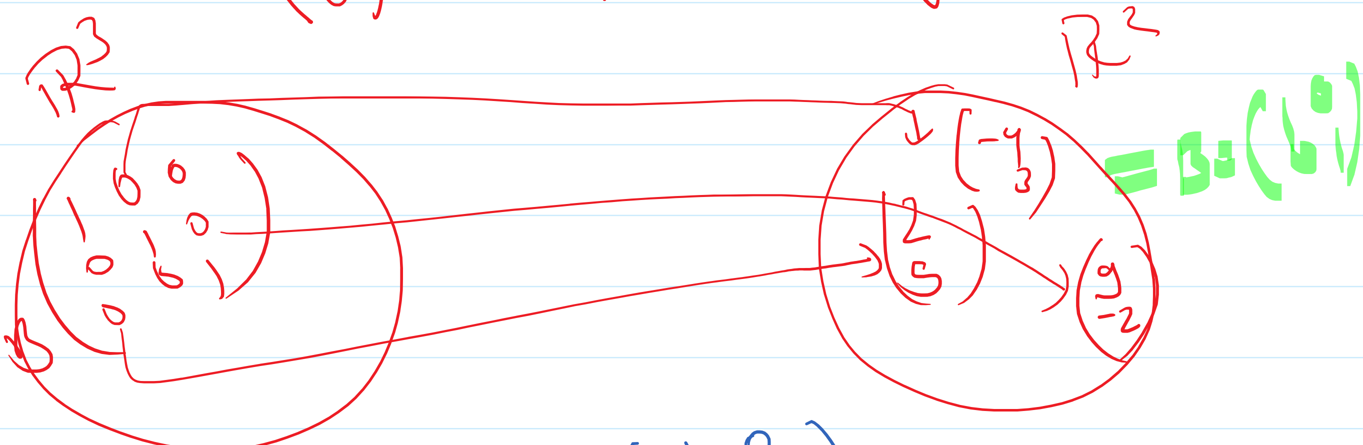
$$[T]_{B}^{B'} = \begin{bmatrix} 0 & 3 & 1 \\ 1 & -4 & 0 \\ 3 & 0 & 0 \end{bmatrix}$$

Q $T : \mathbb{R}^3 \to \mathbb{R}^2$ $\quad T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2x - 4y + 9z \\ 5x + 3y - 2z \end{pmatrix}$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad T\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 9 \\ -2 \end{pmatrix}$$

Solution

$$T\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}, \quad T\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$$



$$\begin{pmatrix} -4 \\ 3 \end{pmatrix} = \text{LC of } \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= -4\begin{pmatrix} 1 \\ 0 \end{pmatrix} + 3\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$= -4\begin{pmatrix} 1 \\ 3 \end{pmatrix} + 3\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} -4 \\ 3 \end{pmatrix}_{B'} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} 2 \\ 5 \end{pmatrix}_{B'} = \begin{pmatrix} 2 \\ 5 \end{pmatrix} \qquad \begin{pmatrix} +9 \\ -2 \end{pmatrix}_{B'} = \begin{pmatrix} 9 \\ -2 \end{pmatrix}$$

$$(T)_{B}^{B'} = \begin{bmatrix} 2 & -4 & 9 \\ 5 & 3 & -2 \end{bmatrix} \quad \text{Ans}$$

Q $T: \mathbb{R}^2 \longrightarrow \mathbb{R}^3$ find MR of LT

$$"\begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

$$B_1 = \left[\begin{pmatrix} 1 \\ 2 \end{pmatrix} \begin{pmatrix} 3 \\ 1 \end{pmatrix}\right] \qquad B_2 = \left[\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}\right]$$

$$T\begin{pmatrix} a \\ b \end{pmatrix} = \begin{bmatrix} a \\ a-b \\ a+b \end{bmatrix} \qquad T\begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \\ 7 \end{pmatrix}$$

$$T\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix} \in \mathbb{R}^3 \qquad T\begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{bmatrix} 3 \\ 3 \\ 4 \end{bmatrix} \in \mathbb{R}^3$$

$$\begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix} = LC \text{ of } B_2 \qquad \begin{pmatrix} 3 \\ 3 \\ 4 \end{pmatrix} = LC \text{ of } B_2$$

$$\begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix} = 2\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + -4\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + 3\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} =$$

$$\begin{pmatrix} 3 \\ 3 \end{pmatrix} = 1\begin{bmatrix} 1 \\ 0 \end{bmatrix} + -2\begin{bmatrix} 1 \\ 1 \end{bmatrix} \; 4\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\begin{pmatrix}3\\3\\4\end{pmatrix} = 1\begin{bmatrix}1\\0\\0\end{bmatrix} + (-2)\begin{bmatrix}1\\1\\0\end{bmatrix} + 4\begin{pmatrix}1\\1\\1\end{pmatrix}$$

$$[T]_{B_2}^{B'} = \Big[[TB_1]_{B_2} \; [TB_1]_{B_2} \Big]$$

$$= \begin{bmatrix}2 & 1\\-4 & -2\\3 & 4\end{bmatrix}$$

$$T(v) = LC \text{ of } T(v) \text{ w.r.t } B_2$$

$$= c_1\begin{bmatrix}1\\0\\0\end{bmatrix} + c_2\begin{bmatrix}1\\1\\0\end{bmatrix} + c_3\begin{bmatrix}1\\1\\1\end{bmatrix}$$

$$(Tv)_{B_2} = \begin{bmatrix}c_1\\c_2\\c_3\end{bmatrix}$$

$$(T(v))_{B_2} = [T]_{B_2}^{B'}[v]_{B_1} = \begin{bmatrix}c_1\\c_2\\c_3\end{bmatrix}$$

$$T(v) = c_1\begin{bmatrix}1\\0\\0\end{bmatrix} + c_2\begin{bmatrix}1\\1\\0\end{bmatrix} + c_3\begin{bmatrix}1\\1\\1\end{bmatrix}$$

$$[T(v)]_{B_2} = [T]_{B_2}^{B'}(v)_{B_1}$$

$$\left[T\begin{pmatrix}3\\4\end{pmatrix}\right]_{B_2} = \begin{bmatrix}2 & 1\\-4 & -2\\3 & 4\end{bmatrix}\begin{bmatrix}3\\4\end{bmatrix}_{B_1\begin{pmatrix}1 & 3\\2 & 1\end{pmatrix}}$$

$$\begin{pmatrix}3\\4\end{pmatrix} = c_1\begin{bmatrix}1\\2\end{bmatrix} + c_2\begin{bmatrix}3\\1\end{bmatrix}$$

$$\begin{pmatrix}3 \\ \end{pmatrix} = \begin{pmatrix}9/_{6} \\ \end{pmatrix}$$

$$\begin{pmatrix} 3 \\ 4 \end{pmatrix}_{B_1} = \begin{pmatrix} 9/5 \\ 2/5 \end{pmatrix}$$

$$\left[T\begin{pmatrix} 3 \\ 4 \end{pmatrix} \right]_{B_2} = \begin{bmatrix} 2 & 1 \\ -4 & -3 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 9/5 \\ 2/5 \end{bmatrix}$$

$$= \begin{bmatrix} 4 \\ -8 \\ 7 \end{bmatrix}$$

$$T(v) = 4\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} - 8\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + 7\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 \\ -1 \\ 7 \end{bmatrix} \quad \#$$

$3x^3 + x^2 + 1$ $\quad D(3x^3 + x^2 + 1) = \boxed{9x^2 + 2x}$

Q Find MRLT $D : P_3 \to P_3$

Basis of P_3

$\{1, x, x^2, x^3\}$

$$\boxed{D(P(x)) = P'(x)}$$

$5 + x + 2x^2 - 7x^3$

MRLT
$$D(1) = 0$$
$$D(x) = 1$$
$$D(x^2) = 2x$$
$$D(x^3) = 3x^2$$
$= P_3$

$(1 + x^3 - x^3)$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 0 \\ -1 \\ +1 \end{bmatrix}$$

$$MRLT = \left[(D(1))_{\text{Basis of } P_3} , [D(x)]_{\text{Basis of } P_3} \cdots D(x^3) \right]$$

x^3

$$T\begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}_{B\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$D(1) = 0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$D(x) = \boxed{1} \quad \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$D(x^2) = 2x = \begin{bmatrix} 0 \\ 2 \\ 0 \\ 0 \end{bmatrix}$$

$$D(x^3) = 3x^2 = \begin{bmatrix} 0 \\ 0 \\ 3 \\ 0 \end{bmatrix}$$

$$[T]_{B'} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$P_3 = (3x^3 + x^2 + 1) = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 3 \end{bmatrix}$$

$$(Tv)_B = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 9 \\ 0 \end{bmatrix}$$

$$T(v) = 0(1) + 2(x) + 9(x^2) + 0(x^3)$$

$$= 2x + 9x^2$$

Q2 $T : \mathbb{R}^3 \to \mathbb{R}^3$

$$T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{bmatrix} x+z \\ 2y-z \end{bmatrix}$$

$B_2 = \left(\begin{bmatrix} 0 \\ 0 \\ \cdot \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ \cdot \end{bmatrix} \right.$

$$T\begin{pmatrix} \\ y \\ z \end{pmatrix} = \begin{bmatrix} \\ 2y - z \\ y + z \end{bmatrix} \qquad B_2 = \left(\begin{bmatrix} \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} \\ 0 \\ -1 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix} \right)$$

$$B_1 = \left(\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right)$$

MRLT = ?

$v \in \mathbb{R}^3 \qquad v = \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}$

$T(v) =$ by using MRLT