

# RBE550

# Motion Planning

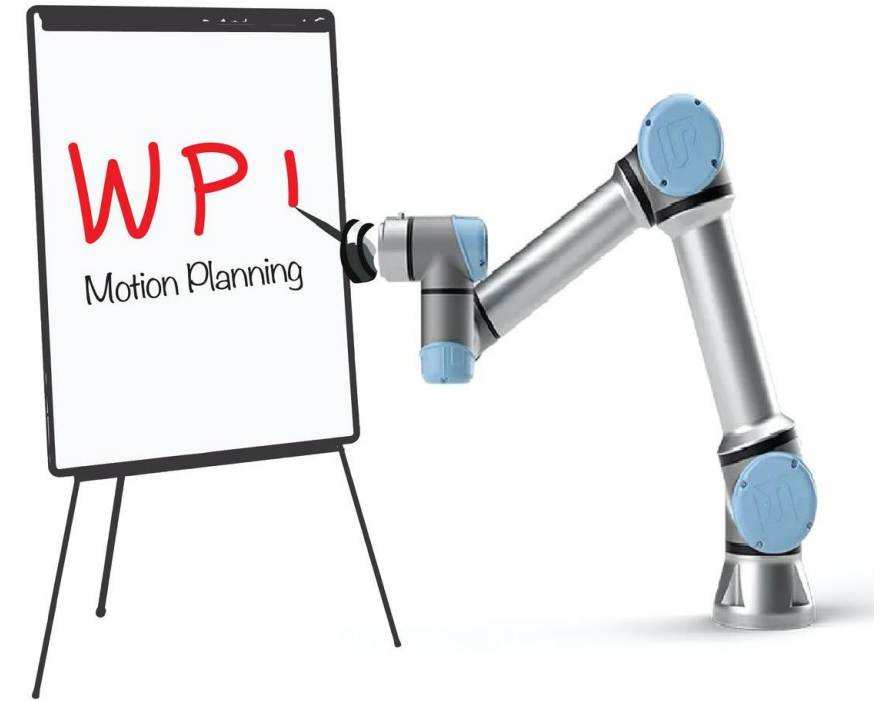
# Introduction

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Constantinos Chamzas

[www.cchamzas.com](http://www.cchamzas.com)

[www.elpislab.org](http://www.elpislab.org)



# Today's Overview

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- Instructor/Lab Introduction
- Introduction to Motion Planning
- Examples of Motion Planning in Real Robots
- Course Logistics

# Prof. Constantinos Chamzas

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- **Assistant Professor, WPI**
  - Joined WPI Fall2023!
  - RBE Faculty, CS&EE background
  - Ph.D. in CS at Rice University
  - Teaching RBE550 Motion Planning
  - Teaching RBE577 ML for Robotics



**Constantinos Chamzas**

Office: Unity Hall 271

Office Hours: Mondays 4:00-5:00 p.m.

Email : [cchamzas@wpi.edu](mailto:cchamzas@wpi.edu)

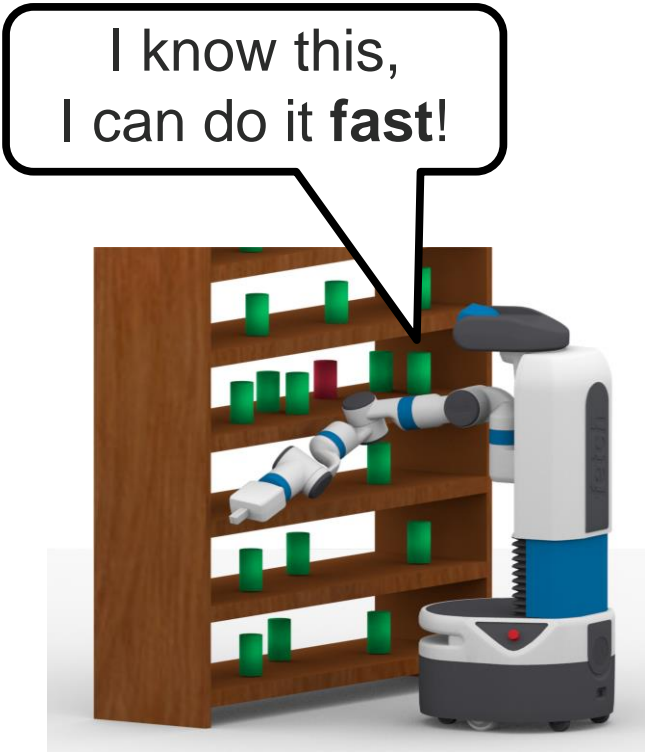
Website: [cchamzas.com](http://cchamzas.com)

Lab Website: [elpsilab.org](http://elpsilab.org)

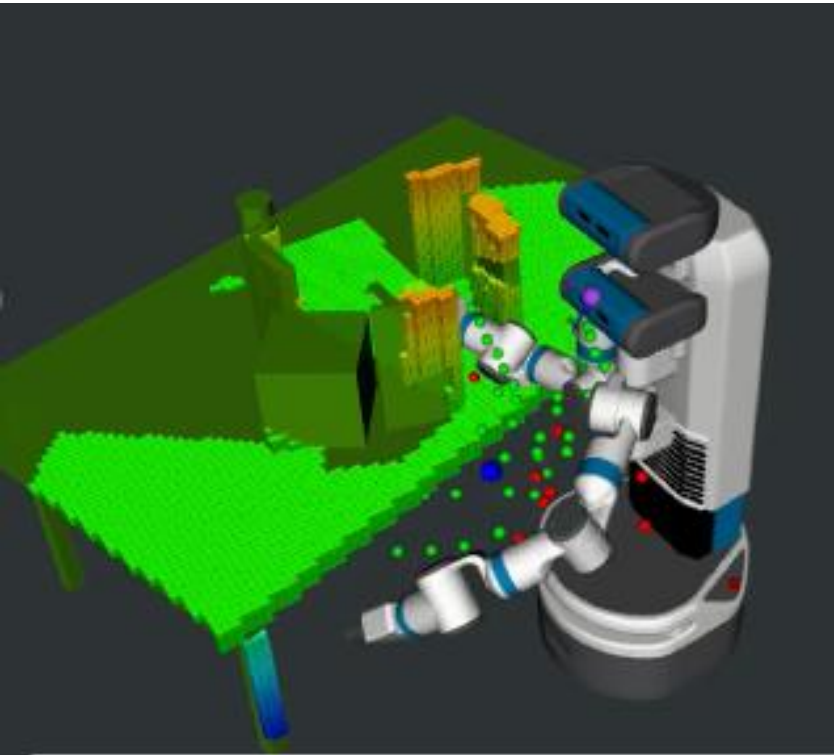
# ELPIS Lab

## Efficient Learning and Planning for Intelligent Systems

### Research Directions/Projects



Learning For Planning Efficiency



Planning Robustly with uncertainty



Vision-Based Planning



# Available Hardware

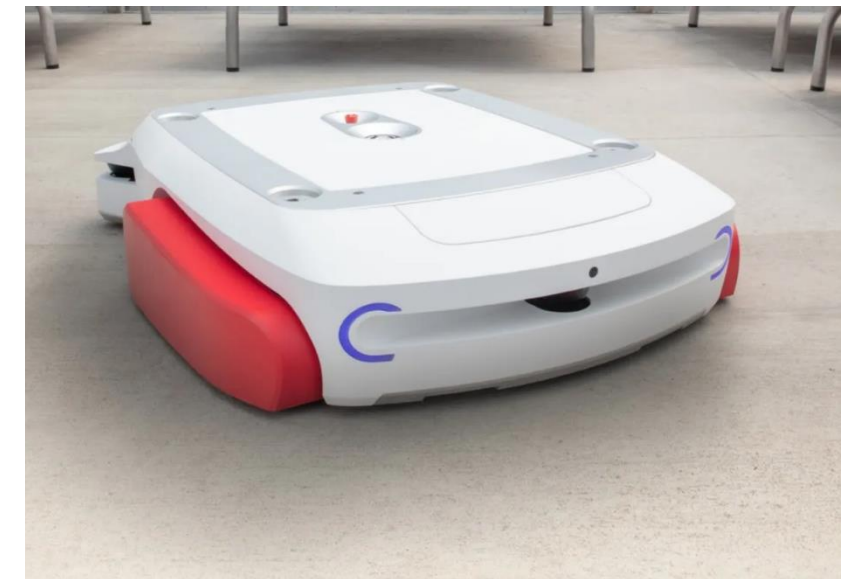
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Universal Robotics UR-10  
6-Dof Industrial Manipulator



Deep Learning Workstation:  
4xA6000 Nvidia GPU,  
28-core Intel Xeon



Iron Ox Grover  
Wheeled robot with  
1000-pound lift



# How you can join

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- Take RBE550 (Motion Planning) and perform well
- We are looking for:
  - Proficiently in algorithms and coding (Python/C++) is desirable
  - At least 2 semester commitment
- Ways to learn more and get involved:
  - Submit your application at [elpislab.org/join](http://elpislab.org/join)
  - Drop by my office UH271



## What operating system are you using

Linux

Windows

MacOs

## What operating system are you using

Linux

0%

Windows

0%

MacOs

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## What operating system are you using

Linux

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Windows

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MacOs

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# Motion Planning Introduction

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# Disclaimer and Acknowledgments

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*The slides are a compilation of work based on notes and slides from Constantinos Chamzas, Lydia Kavraki, Jane Li, Zak Kingston, Howie Choset, David Hsu, Greg Hager, Mark Moll, G. Ayorkor Mills-Tetty, Hyungpil Moon, Zack Dodds, Nancy Amato, Steven Lavalle, Seth Hutchinson, George Kantor, Dieter Fox, Vincent Lee-Shue Jr., Prasad Narendra Atkar, Kevin Tantiseviand, Bernice Ma, David Conner, Morteza Lahijanian, Erion Plaku, students taking comp450/comp550 at Rice University, and students Taking RB550 at Worcester Polytechnic Institute.*

# A Simple Robotic Task

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**Task:** Place all objects on top shelf



Fetch Robot

# A Simple Robotic Task

World Model



**Task:** Place all objects on top shelf

Planner

Trajectory



Perception



Real World

Controller

# This Class

World Model



Planner

Trajectory



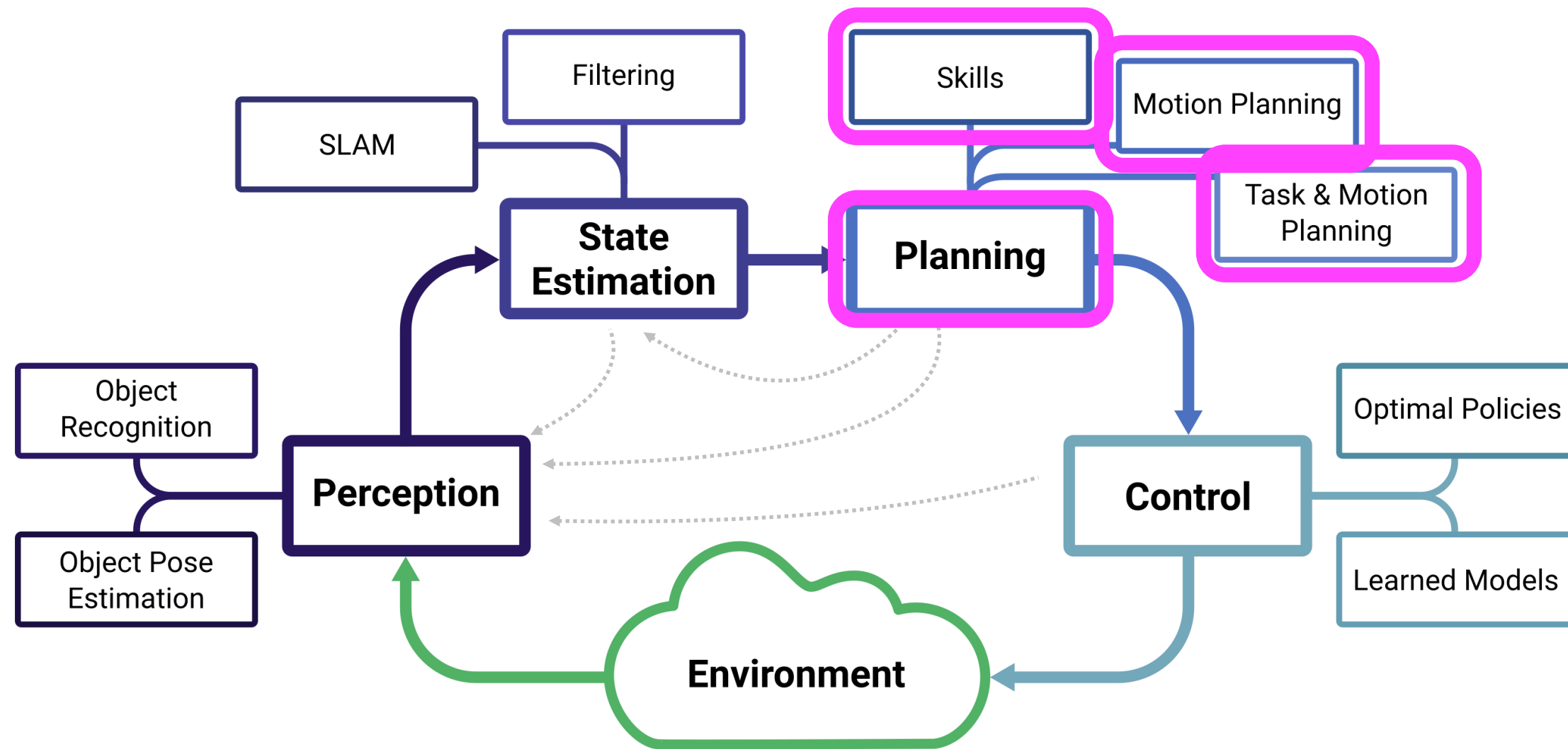
Perception



Controller

Real World

*our class  
mostly on this*

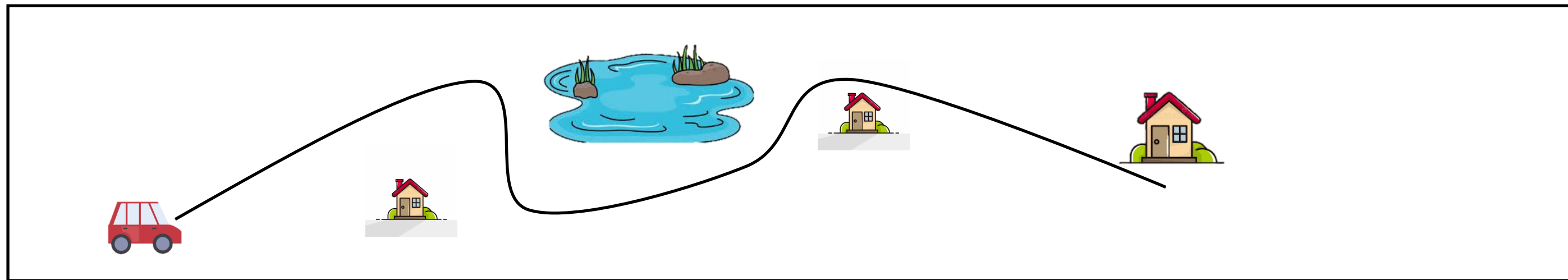




# What is the difference between Planning and Control

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- In a nutshell: The planner designs the path and the controller follows it
  - Planner finds reference motion (plan) from current robot pose to goal pose – *path finding, trajectory optimization, or trivial interpolation/PD*
  - Controller reactively (policy) sends motor commands to follow the reference motion – *inverse dynamics, PD control, Riccati (RBE 502)*



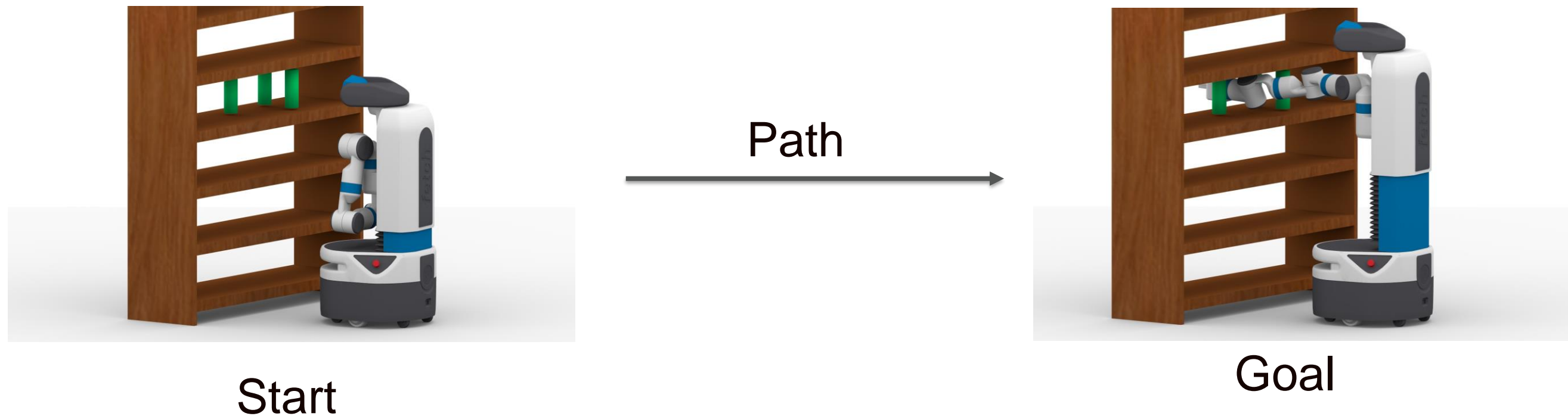
\* Planning is not strictly required e.g., (OSC, Model-Free DRL), e.g., control directly to the goal

\*\*For some methods/applications the boundary between planning and control is blurry e.g., MPC

# What is Motion Planning?

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- Research field that computes the trajectories for a robot to follow



# What is Motion Planning?

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- Research field that computes the trajectories for a robot to follow



Start



Goal

# Formally: The General Motion Planning

- Let  $x \in \mathcal{X}$  denote the state and state-space
- Let  $u \in \mathcal{U}$  denote the control and control-space of a robot
- Let  $f$  denote the forward dynamics of the robot such that:

$$x(\mathcal{T}) = x(0) + \int_0^{\mathcal{T}} f(x(t), u(t)) dt \quad (1)$$

- Let  $\mathcal{X}_{\text{obs}} \subset \mathcal{X}$  denote the invalid state space, e.g., validating kinematic constraints, collisions, etc.
- Then the valid state space is denoted as  $\mathcal{X}_{\text{free}} = \mathcal{X} \setminus \mathcal{X}_{\text{obs}}$
- The start state as  $x_{\text{start}} \in \mathcal{X}_{\text{free}}$
- The goal region as:  $X_{\text{goal}} \subseteq \mathcal{X}_{\text{free}}$

## The Motion planning problem:

Find a time  $\mathcal{T}$  and a set of controls  $u : [0, \mathcal{T}] \rightarrow \mathcal{U}$  such that the motion described by (1) satisfies

$$x(0) = x_{\text{start}}$$

$$x(\mathcal{T}) \in \mathcal{X}_G$$

$$x(t) \in \mathcal{X}_{\text{free}}.$$

# Geometric Motion Planning or Path Planning

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- If the quasi-static assumption holds (geometric position of the robot fully specifies its state) then the motion planning problem simplifies to:
  - Let  $x \in \mathcal{X}$  denote the configuration and C-space of the robot
  - Let  $\mathcal{X}_{\text{obs}} \subset \mathcal{X}$  denote the obstacle c-space (collisions, and violated joint limits)
  - Then the free C-space is denoted as  $\mathcal{X}_{\text{free}} = \mathcal{X} \setminus \mathcal{X}_{\text{obs}}$
  - The start state as  $x_{\text{start}} \in \mathcal{X}_{\text{free}}$
  - The goal state as:  $\mathbf{x}_{\text{goal}} \subseteq \mathcal{X}_{\text{free}}$

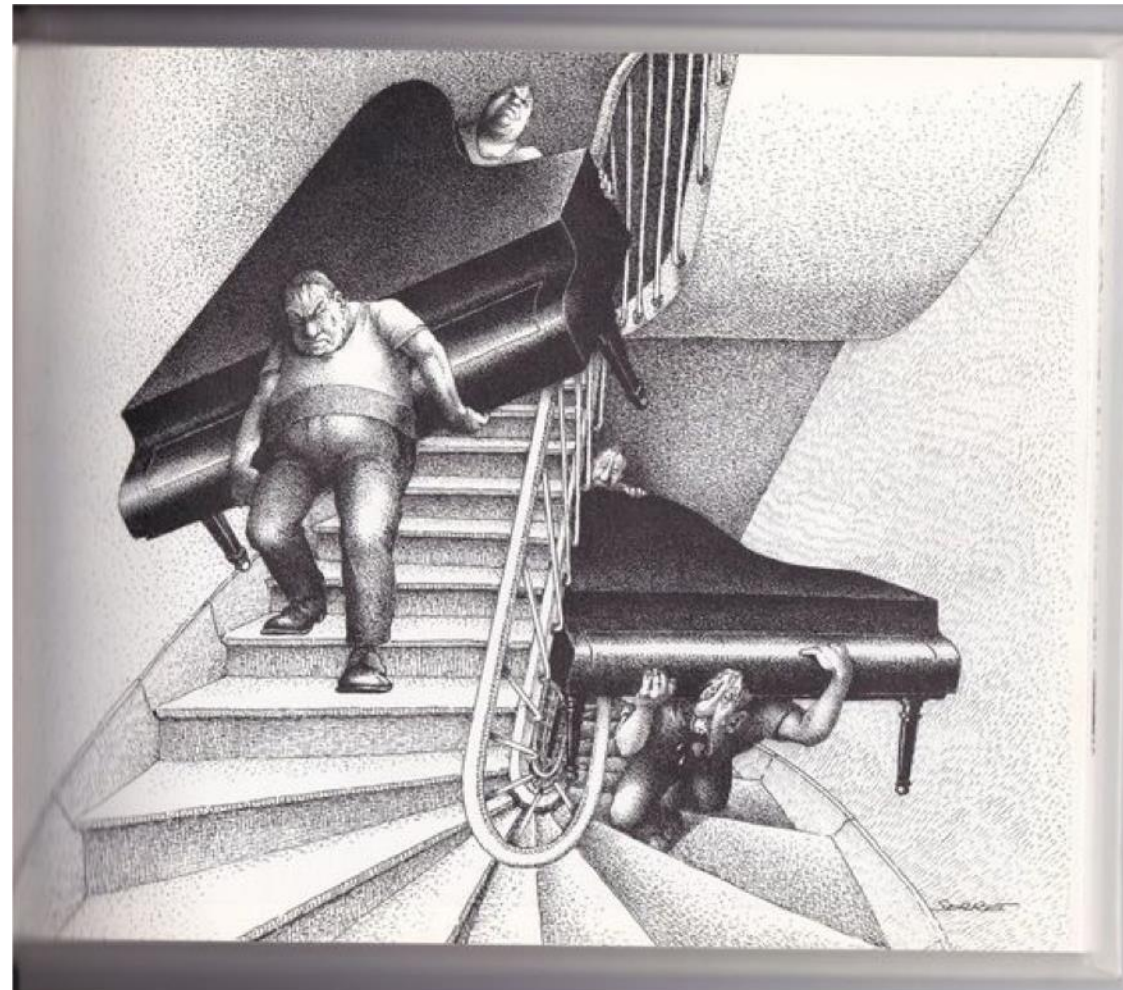
The geometric motion planning problem is defined as, finding a path  $\sigma$  such that:

$$\sigma(0) = \mathbf{x}_{\text{start}}, \sigma(1) \in \mathbf{x}_{\text{goal}}, \quad \sigma(t) \in \mathcal{X}_{\text{free}} \quad \forall t \in [0, 1]$$



# Motion Planning Historically- The Piano's Movers Problem

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Picture credit: <https://www.pinterest.com/pin/178736678931496070/>



# Motion Planning is Hard

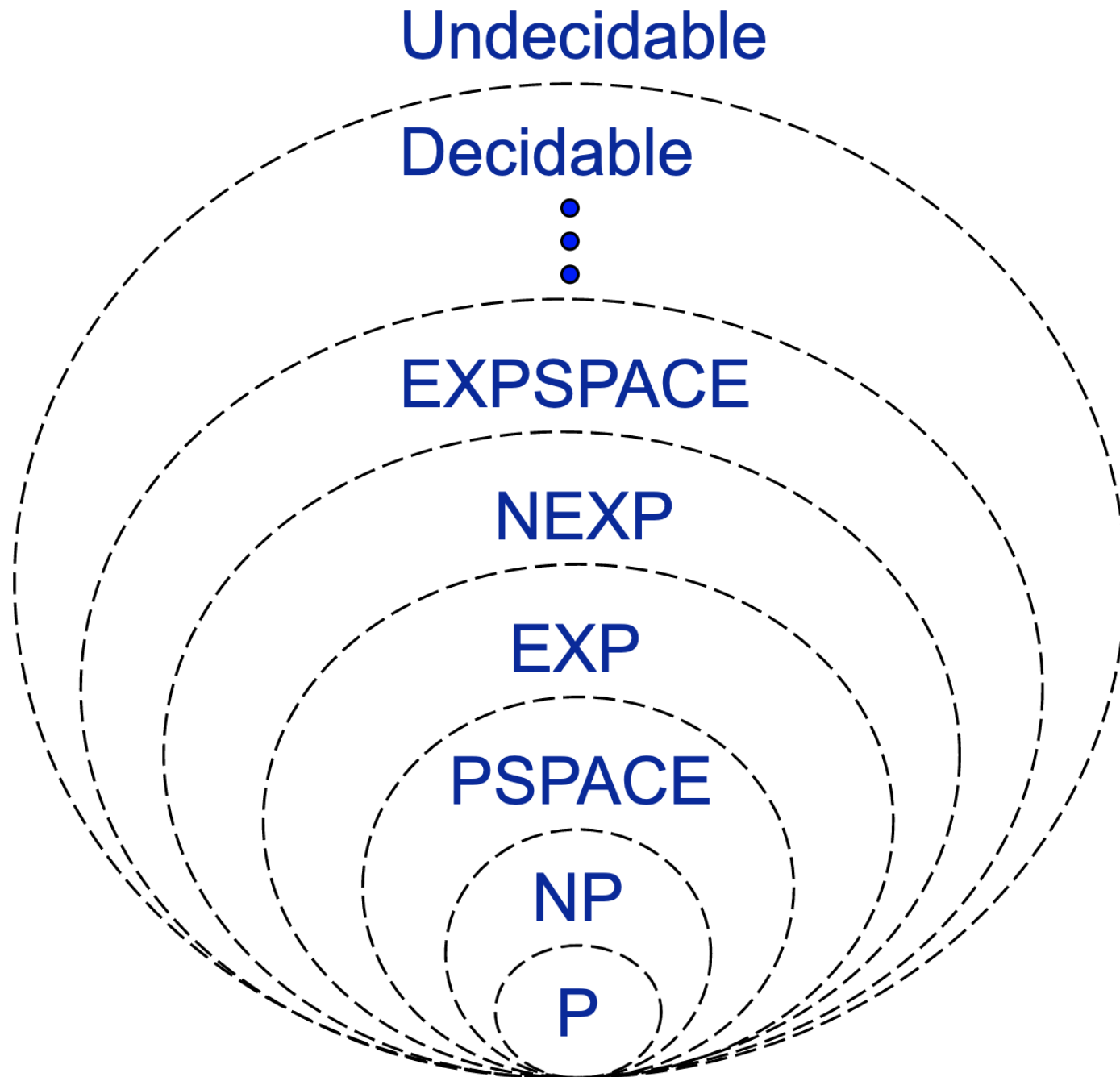
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Problem	Complexity
Sofa Mover (3 DOF)	$O(n^{2+\epsilon})$ not implemented
Piano Mover (6 DOF)	Polynomial – no practical algorithm known
n Disks in the Plane	NP-hard
n Link Planar Chain	PSPACE-Complete
Generalized Mover	PSPACE-Complete
Shortest Path for a Point in 3D	NP-hard
Curvature Constrained Point in 2D	NP-hard
Simplified Coulomb Friction	Undecidable



# Computational Complexity

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$$\text{PSPACE} \subset \text{EXPSpace}$$

$$\text{P} \subset \text{EXP}$$

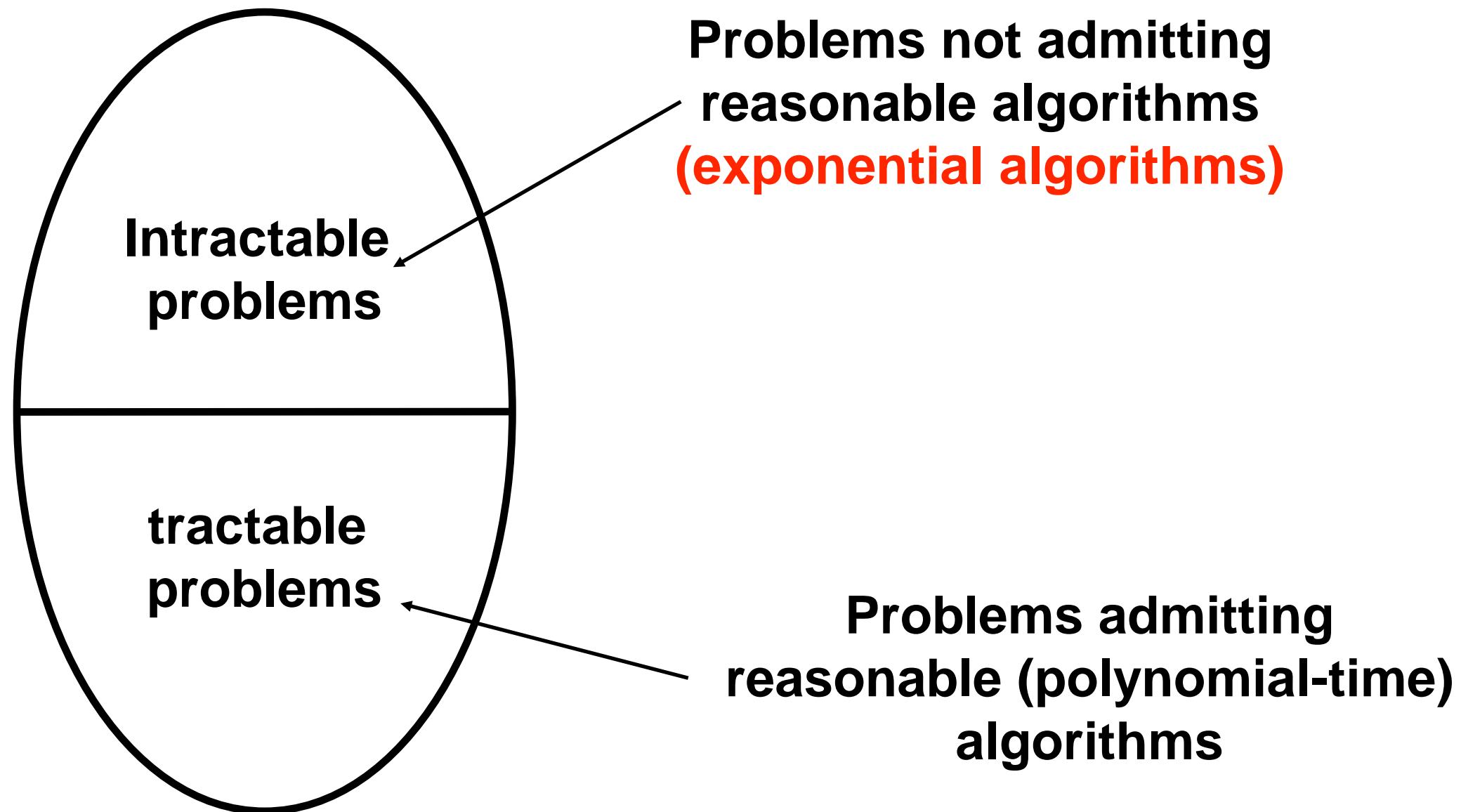
$$\text{PSPACE} = \text{NPSPACE}$$

$$\text{P} \subseteq \text{NP} \subseteq \text{PSPACE}$$

$$\text{P} =? \text{NP}$$

# Computational Tractability

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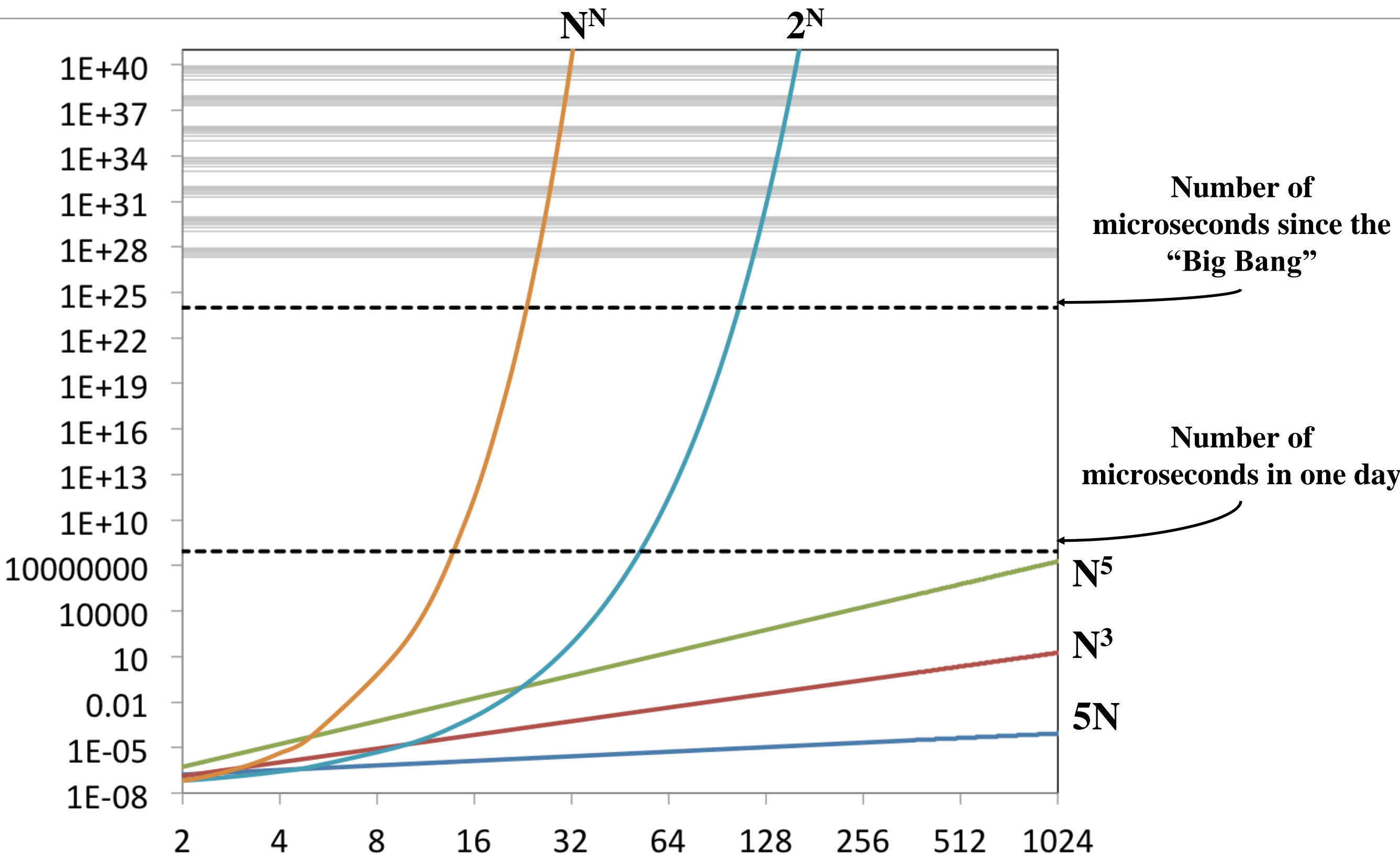
**Polynomial in terms of  $N$  (size of the input)**

# Functions

Function \ N	10	50	100	300	1000
5N	50	250	500	1500	5000
N <sup>2</sup>	100	2500	10000	90000	1 million (7 digits)
N <sup>3</sup>	1000	125000	1 million (7 digits)	27 million (8 digits)	1 billion (10 digits)
2 <sup>N</sup>	1024	a 16-digit number	a 31-digit number	a 91-digit number	a 302-digit number
N <sup>N</sup>	10 billion (11 digits)	an 85-digit number	a 201-digit number	a 744-digit number	unimaginably large

[The number of microseconds since the ‘Big Bang’ has 24 digits ]

# Growth Rates



# Some extensions/variations of the basic problem

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- Optimal Planning
- Non-Holonomic Constraints
- Anytime Planning
- Multiple robots
- Movable objects
- Kinodynamic constraints
- Stability constraints
- Constrained-Based Motion Planning
- Dynamical unknown environments
- Integration of planning and control
- Integration with higher-level planning
- Planning under Uncertainty

# Examples of Motion Planning in Real Robots

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# Roomba: vacuum cleaning robot

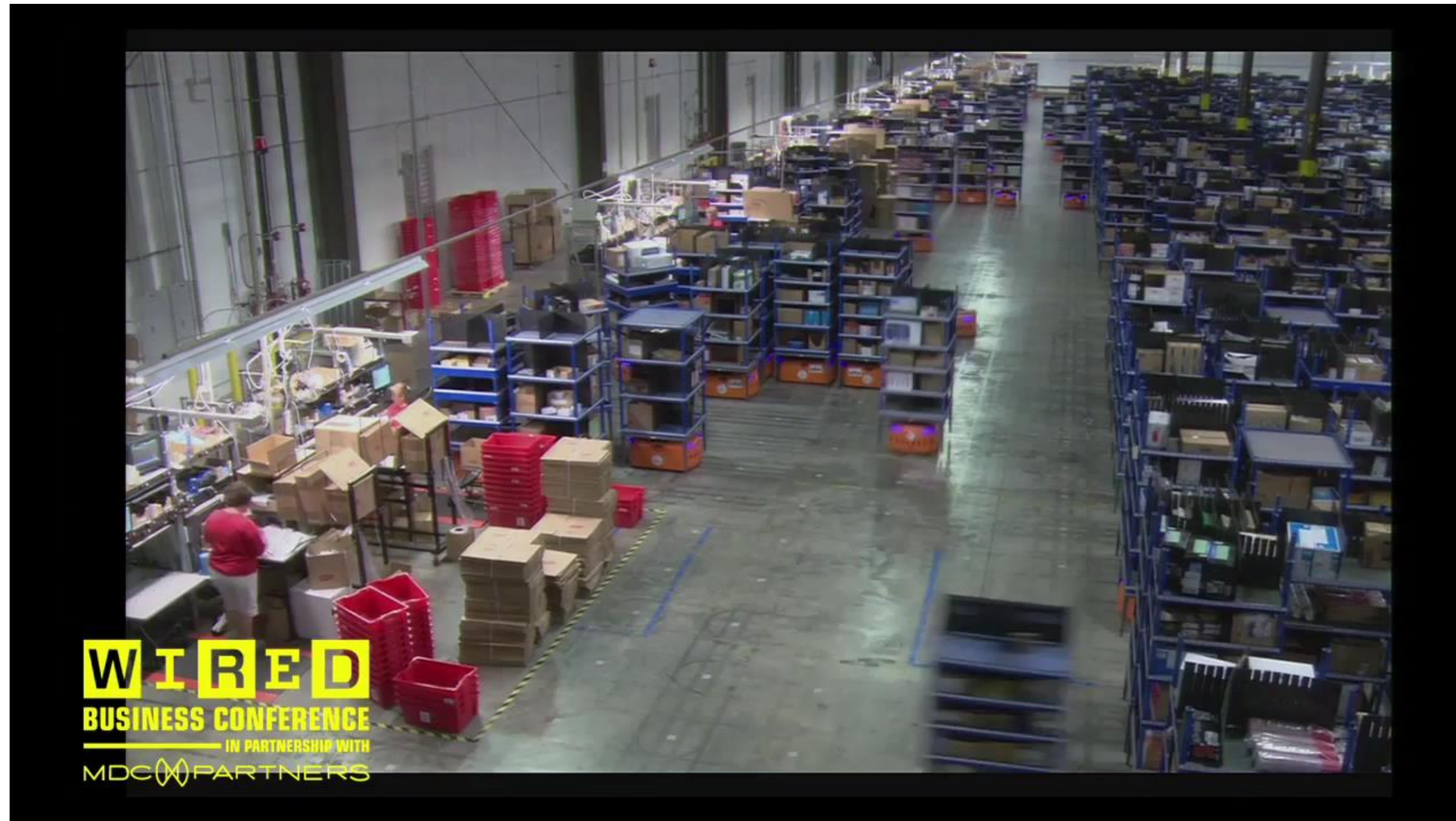
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# Amazon warehouse robots

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# Industrial Automation

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# Waymo Autonomous Car

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# Planning to Pick up an Object is still Hard!

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**Such problems can take up to 15 seconds to compute**

# Failures are common in robotics (even in 2024)

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- Check these spectacular videos:  
<https://www.youtube.com/user/bostondynamics>
- Check these spectacular failures:  
<https://www.cnn.com/videos/business/2021/08/19/boston-dynamics-robots-falling-down-orig.cnn-business/video/playlists/business-news/>

# Have you seen a robot helping with household chores?

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Motion Planning is still a very active research area!



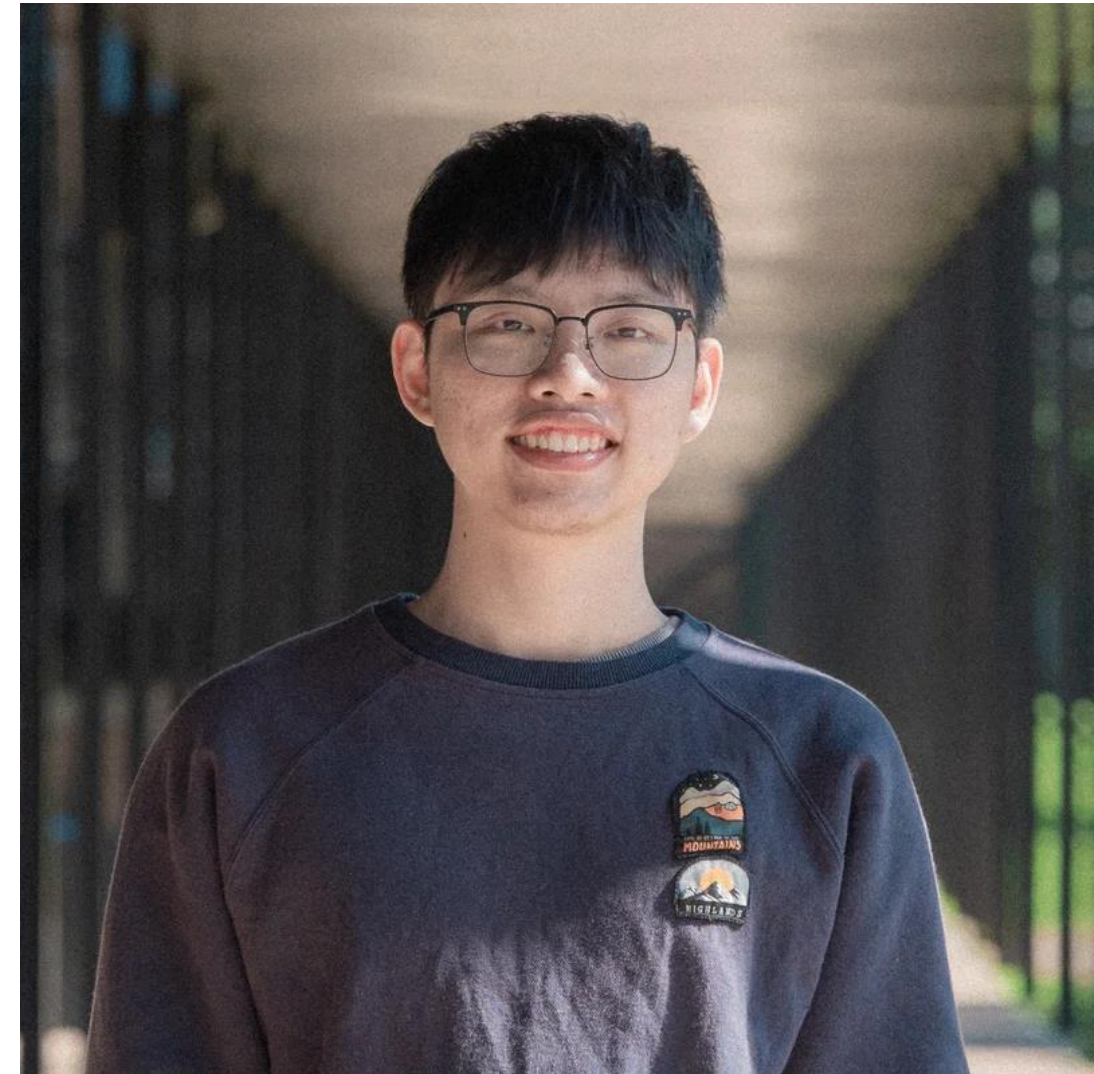
# Course Logistics

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# Teaching Assistant

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- Name: Mr. Zhuoyun Zhong
- Email: [zzhong3@wpi.edu](mailto:zzhong3@wpi.edu),
- Office Hours: 11am-12pm Fridays
- Zoom link: <https://wpi.zoom.us/my/zhuoyunzhong>
- Office Location: 200A Unity Hall




# The Open Motion Planning Library

[←](#) [→](#) [↻](#) [🔒](#) [https://ompl.kavrakilab.org](#) [☆](#) [📁](#) [📄](#) [📄](#) [📄](#) [📄](#) [📄](#)

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## The Open Motion Planning Library



**OMPL used in ROS/MoveIt**  
OMPL is the default planning library in MoveIt and has been used for [many robots](#). Here, OMPL is used to plan footsteps for NASA's Robonaut2 aboard the International Space Station.

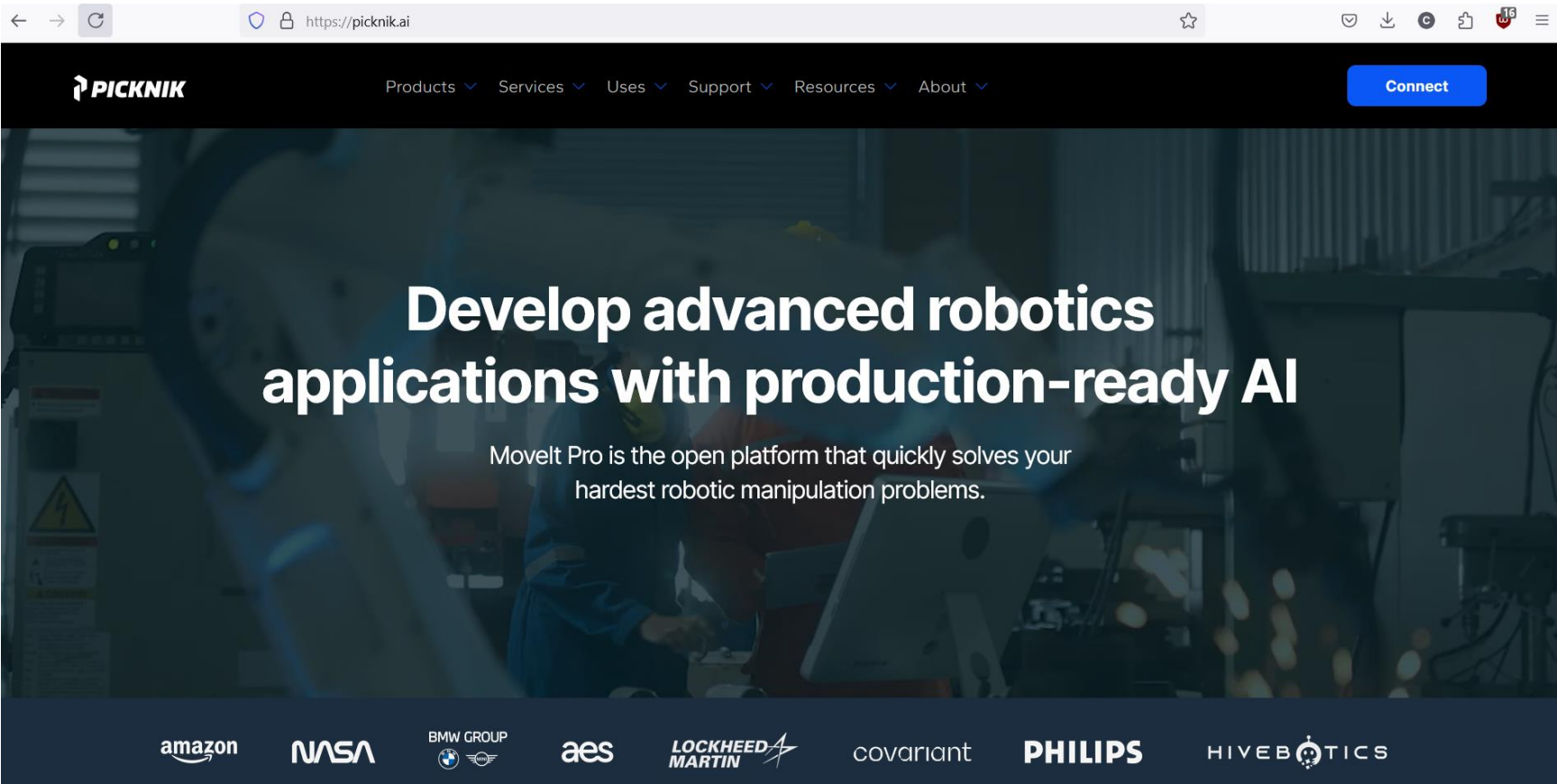
OMPL, the Open Motion Planning Library, consists of many state-of-the-art sampling-based motion planning algorithms. OMPL itself does not contain any code related to, e.g., collision checking or visualization. This is a deliberate design choice, so that OMPL is not tied to a particular collision checker or visualization front end. The library is designed so it can be easily integrated into [systems that provide the additional needed components](#)

**Download version 1.6.0**  
Released: Jan 16, 2023

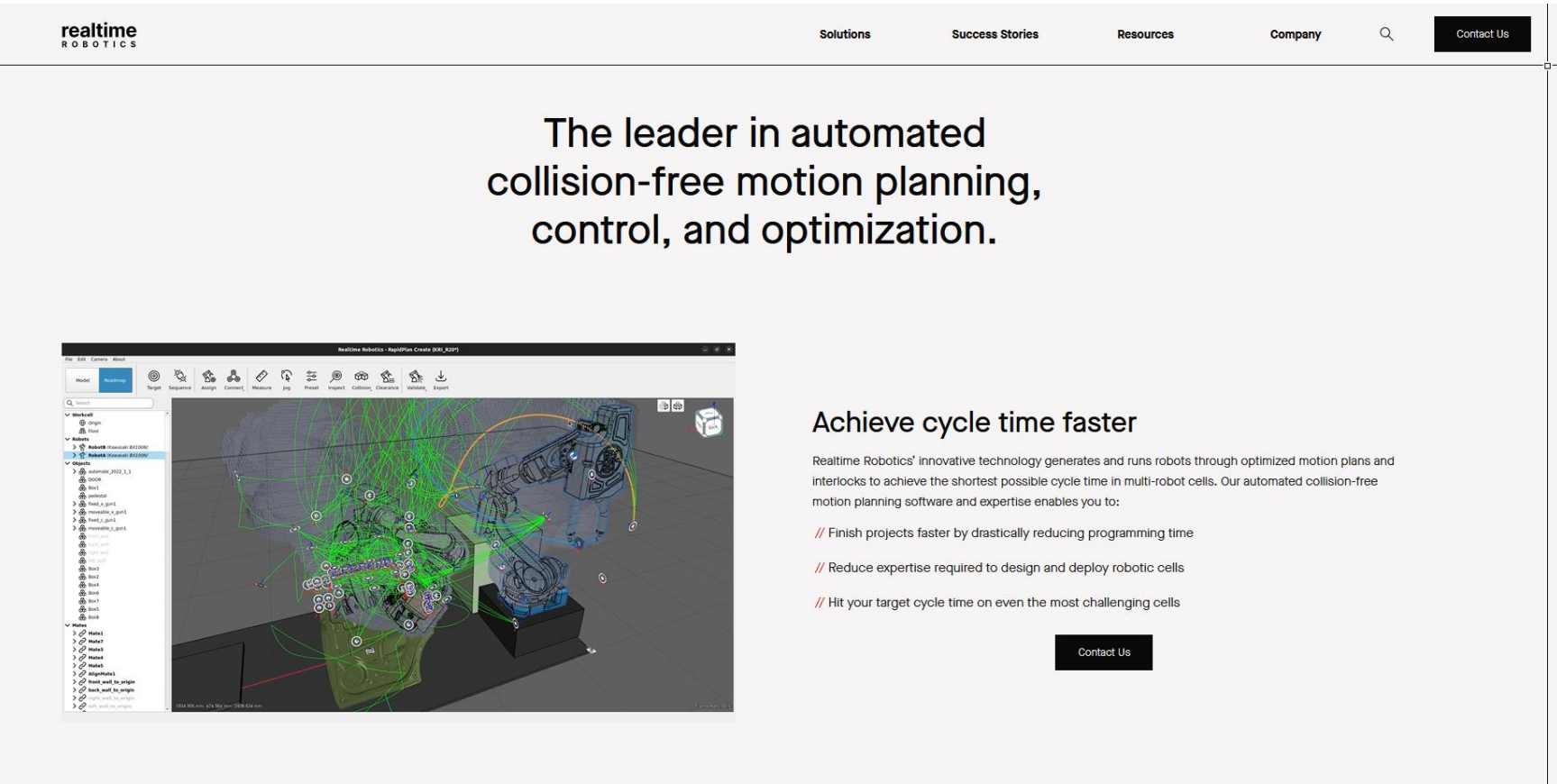
<https://ompl.kavrakilab.org/>



# OMPL In Industry



PickNik (Moveit Maintainers)



Realtime Robotics

# Prerequisites

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- Be familiar with the following:
  - Undergraduate Linear Algebra
  - Kinematics in Robotics
  - Basic Algorithms
  - Proficiency in C++ and Python, or have the ability to pick up quickly!
- Project 0/1 aims to test your preparedness for this class, by covering these topics

# Logistics of the Class

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- CANVAS - we will make heavy use of this tool
- Piazza – use Piazza if you want your questions answered promptly.  
Signup Link: <https://piazza.com/wpi/fall2024/rbe550>
- **Read “Course Information” in Canvas:** It contains details for the grading, the policy for late assignments or projects, the cut-off date for all work to be graded in this class, and what you are expected and not expected to do. Also, a lot of information about the class in general.

# Projects and grading (RBE550)

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Grade Policies

Category	Percentage of Final Grade
Project0	4%
Project1	6%
Project2	10%
Project3	10%
Project4	10%
Midterm Exam	20%
Final Exam	20%
Final Project	20%

Table 2: Grade Policies

Category	Final Grade
A	$\geq 90\%$
B	$\geq 80\%$
C	$\geq 70\%$
D	$\geq 50\%$
F	$< 50\%$

Table 3: Letter Grade Percentages



# Projects (40%)

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- Constituted of two parts:
  - Theoretical Questions that aim to test different concepts taught in class
  - Programming Component in C++/Python3 that will usually include implementations of planning algorithms, or critical components of planners
- All projects(except Project0) can be completed in pairs of two or individually
- All the written components must be in LaTeX, handwritten or Word documents will **not** be accepted and will be graded with zero (An Overleaf template is provided in “Course Information” document in Canvas)

# Midterm (20%)- Final (20%)

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- Midterm scheduled for March 7 and will be in-class
- Finals scheduled for April 22 and will also be in-class

# Final Project (20%)

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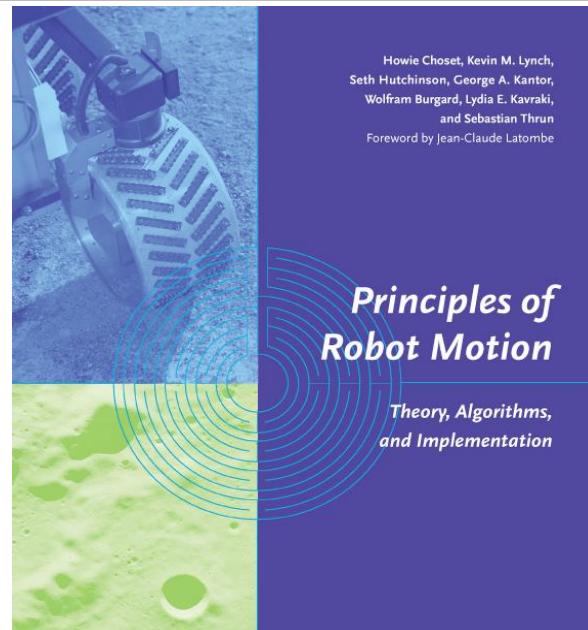
- Will be released in the second half of the semester
- It will be a big project than can be:
  - One of the instructors' proposed projects
  - A research paper reproduction
  - Part of your research

# Tentative Schedule

This schedule is subject to change but we will try to keep as close as possible to the schedule below.

Date	Topic	Instructor	Projects
1/17	Introduction	C.Chamzas	Project0 & Project1 out
1/21	Bug Algorithms, Potential Fields	C.Chamzas	
1/24	Roadmaps and Discrete Search	C.Chamzas	Project0 due
1/28	Configuration Space	C. Chamzas	
1/31	Tree-Based Planners and OMPL	C. Chamzas	Project1 due, Project2 out
2/4	Configuration Space Obstacles	C. Chamzas	
2/7	Probabilistic Roadmap Planners	C. Chamzas	
2/11	Theoretical Issues	C. Chamzas	
2/14	Wellness Day ( <b>No Class</b> )		
2/18	Asymptotically Optimal Planning	C. Chamzas	Project2 due, Project3 out
2/21	Collision Checking	C. Chamzas	
2/25	SPARS/Dynamic Roadmaps	C. Chamzas	
2/28	Kinodynamic Planning I	C. Chamzas	
3/4	Kinodynamic Planning II	C. Chamzas	Project3 due, Project4 out
3/7	Midterm (In Class)	C. Chamzas	
3/11	Spring Break ( <b>No Class</b> )		
3/14	Spring Break ( <b>No Class</b> )		
3/18	Learning and Planning I	C. Chamzas	
3/21	Learning and Planning II	C. Chamzas	Final-Project out
3/25	Midterm & Final Projects Discussion	C. Chamzas	Project 4 due
3/28	Task and Motion Planning	C. Chamzas	Declare Final-Project
4/1	Monday Schedule ( <b>No Class</b> )	C. Chamzas	
4/4	Task and Motion Planning II	C. Chamzas	
4/8	Planning under Uncertainty	C. Chamzas	
4/11	Class Overview	C. Chamzas	
4/15	Mid-Report Presentation	Students	Mid Report due
4/18	TBD	TBD	
4/22	Final Exams (In Class)	C. Chamzas	
4/25	UG Research Show Day ( <b>No Class</b> )		
4/29	Guest Lecture	TBD	
5/2	Guest Lecture	TBD	
5/6	Final-Project Pres.		
5/7	Final-Project Pres. ( <b>Friday Schedule</b> )	Students	Final Report due

# Relevant books



## Principles of Robot Motion: Theory, Algorithms, and Implementations

H. Choset, K.M. Lynch,  
S. Hutchinson, G. Kantor,  
W. Burgard, L.E. Kavraki  
and S. Thrun, MIT Press

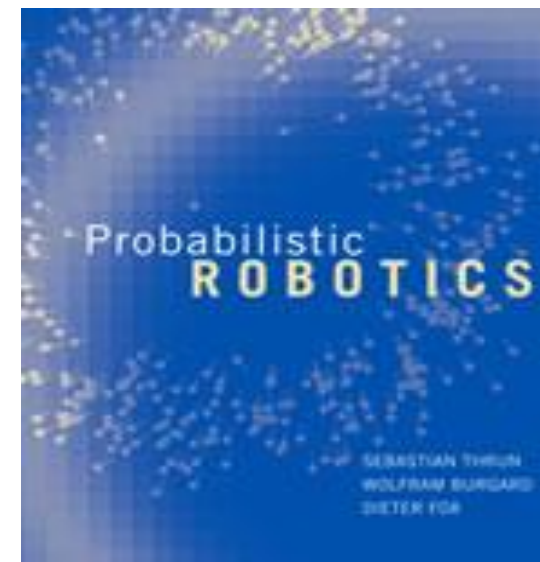


*Planning Algorithms*,  
Steven Lavalle, Cambridge  
University Press, 2006

**Free download:**  
<http://planning.cs.uiuc.edu/>



*Robot Motion Planning*,  
Jean-Claude Latombe,  
Kluwer, 1991.



*Probabilistic Robotics*  
S. Thrun, W. Burgard, D. Fox  
MIT Press, 2006



*Handbook of Robotics*  
B. Siciliano et al  
MIT Press, 2018

# Project 0 is already posted in Canvas

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- Due at **11:59 PM** on **Jan24**
- This project should be done **individually** not in pairs
- It will test your preparedness for the class

# Project 1 released today

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- Due at **11:59 PM** on **Jan 31st**
- This project **can** be done in pairs
- It will also test your preparedness for the class



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# Welcome to RBE550!