

## MTH 372 (2025): Tutorial VI

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1. Let  $X_1, \dots, X_n$  be independent random variables with the common distribution.

$$P(X_i \leq x) = \begin{cases} 0 & ; x \leq 0 \\ (x/\beta)^\alpha & ; 0 < x < \beta \\ 1 & ; x \geq \beta. \end{cases}$$

If  $\alpha$  is a known constant  $\alpha_0$ , find an upper confidence limit for  $\beta$  with confidence coefficient 0.95.

2. Find a pivot quantity based on a random sample of size  $n$  from a  $N(\theta, \theta)$  population, where  $\theta > 0$ . Use the pivotal quantity to set up a  $(1 - \alpha)$  confidence interval for  $\theta$ .
3. Let  $X_1, \dots, X_n$  be i.i.d. with  $Uniform(0, \theta)$ . Let  $Y$  be the largest order statistic. Prove that  $Y/\theta$  is a pivotal quantity and show that the interval

$$\{\theta : y \leq \theta \leq \frac{y}{\alpha^{1/n}}\}$$

is the shortest  $(1 - \alpha)$  pivotal interval.