Computer Vision
Quiz 1 - Solutions

1. (*10 points*) State True or False. Justify your answer.

a) If $\mathbf{B}, \mathbf{C}$ and $\mathbf{D}$ are matrices, then $\mathbf{BC} = \mathbf{BD}$ implies $\mathbf{C} = \mathbf{D}$.

b) The intrinsic camera matrix $\mathbf{K}$ is always invertible.

c) Every 3×3 orthogonal matrix is a rotation matrix.

d) Collinearity is preserved under projective transformations but not under affine transformations.

e) For a VGA resolution (640×480) CCD array, the principal point is always at $(320, 240)$.

Solution:

a) False. This will not hold if columns of $\mathbf{C}$ and $\mathbf{D}$ both lie in the null space of $\mathbf{B}$.

b) True. $\mathbf{K}$ is an upper triangular matrix, therefore non-singular.

c) False. Projective transformations are the most general type of transformations. This implies any invariant property of Projective transformations would be invariant for a more restrictive transformation, e.g., affine.

d) Fale. The principal point is the projection of the camera center on the CCD array. In general, it would depend on the relative position of the CCD array w.r.t. the camera center.

2. (*10 points*) Given $\mathbf{u} \in \mathbb{R}^n$ and $\mathbf{u}^\top \mathbf{u} = 1$, let $\mathbf{P} = \mathbf{u}\mathbf{u}^\top$ (outer product) and $\mathbf{Q} = \mathbf{I} - 2\mathbf{P}$. Justify the following statements: (a) $\mathbf{P}^2 = \mathbf{P}$ (b) $\mathbf{P}^\top = \mathbf{P}$ (c) $\mathbf{Q}^2 = \mathbf{I}$.

Solution:

(a)

$$\mathbf{P}^2 = \mathbf{PP} = \mathbf{u}^\top \mathbf{u}\mathbf{u}^\top \mathbf{u} = \mathbf{u}\mathbf{u}^\top.$$

(b)

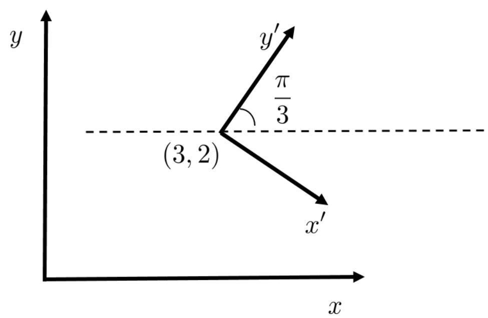
$$\mathbf{P}^\top = (\mathbf{u}\mathbf{u}^\top)^\top = \mathbf{u}\mathbf{u}^\top = \mathbf{P}.$$

(c)

$$\mathbf{Q}^2 = (\mathbf{I} - 2\mathbf{P})(\mathbf{I} - 2\mathbf{P}) = \mathbf{I} - 4\mathbf{P} + 4\mathbf{P}^2 = \mathbf{I} - 4\mathbf{P} + 4\mathbf{P} = \mathbf{I}$$

3. (*10 points*) Find the transformation $\mathbf{T}$ that maps points from the xy coordinate frame to the $x'y'$ coordinate frame. Note that the angle given is between the axes y' and x.

Solution:



Let $\mathbf{T}$ be the transformation that maps points from the xy frame to the $x'y'$ frame. Therefore, for homogeneous coordinates $\mathbf{p}'$ and $\mathbf{p}$, we have:

$$\begin{aligned} \mathbf{p}' &= \mathbf{T}\mathbf{p} \\ &= \begin{bmatrix} \mathbf{R} & \mathbf{t} \\ 0 & 1 \end{bmatrix} \mathbf{p} \\ \mathbf{p}_{x'y'} &= \mathbf{R}\mathbf{p}_{xy} + \mathbf{t} \end{aligned}$$

In this representation, the $\mathbf{p}_{xy}$ is first rotated and then translated. Therefore, the point $\mathbf{p}_{xy}$ is mapped to the $x'y'$ frame and then is translated to $[0,0]^\top$. The vector $\mathbf{t}$ should be in the $x'y'$ frame of reference and is equal to $\mathbf{R}[-3,\ -2]^\top$.
The rotation matrix $\mathbf{R}$ rotates the points by $\frac{\pi}{6}$ (since the axes are rotated by $\pi/6$ in the clockwise direction), and is given by

$$\begin{bmatrix} \cos\pi/6 & -\sin\pi/6 \\ \sin\pi/6 & \cos\pi/6 \end{bmatrix}$$

Then the final transformation is

$$\mathbf{T} = \begin{bmatrix} \mathbf{R} & \mathbf{R}\begin{bmatrix} -3 \\ -2 \end{bmatrix} \\ 0 & 1 \end{bmatrix}$$