Computer Vision
Quiz 3 - Solutions

1. (*10 points*) State True or False. Justify your answer.

a) Collinearity is preserved under projective transformations but not under affine transformations.
False. Projective transformations are the most general kind, therefore any invariants under a projective transformation will be an invariant for affinities as well.

b) If $l_\infty = [1\ 0\ 1]^\top$, parallel lines in 3D will appear to be parallel in the image.
False. Parallel lines will appear to be parallel only if $l_\infty = [1\ 0\ 0]^\top$.

c) Under affine distortion, angles between intersecting 3D lines can be computed from the image lines if the intrinsic parameters of the camera are known.
True. Using the intrinsic camera matrix, the direction of the lines could be used to compute the cosine of the angle between the vectors.

d) The line at infinity can be estimated using *any* two pairs of parallel lines.
False. The two pairs should be in different directions.

e) Ratio of lengths are invariant to affine transformation.
False. Ratio of lengths are preserved only if lengths are measured along parallel or collinear lines.

2. (*10 points*) Given a set of image points $\mathcal{X} = \{x_1, x_2, \dots, x_n\}$, find a transformation T, which when applied to the set $\mathcal{X}$ generates a set $\mathcal{Y} = \{y_1, y_2, \dots, y_n\}$ that has zero mean and unit average distance from the origin.
Solution:
Let the points $x_i = [x_{i1}\ x_{i2}\ 1]^\top$ be in the homogeneous representation. We first center the points by applying a translation so that they have zero mean. Let $\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ be the mean of the points in $\mathcal{X}$. The centering transformation will be

$$T_c = \begin{bmatrix} 1 & 0 & -\overline{x}_1 \\ 0 & 1 & -\overline{x}_2 \\ 0 & 0 & 1 \end{bmatrix}.$$

Let $x_i' = T_c x_i$ be the centered points. It is easy to see that $\frac{1}{n}\sum_{i=1}^{n} x_i' = 0$. The average distance of x_i' from the origin is given as $s = \frac{1}{n}\sum_{i=1}^{n} ||x_i'||$. Since we want the average distance to be unity, we shall apply the following transformation T_s to x_i'

$$T_s = \begin{bmatrix} \frac{1}{s} & 0 & 0 \\ 0 & \frac{1}{s} & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Therefore our final transformation is $T = T_s T_c$

$$T = \begin{bmatrix} \frac{1}{s} & 0 & -\overline{x}_1/s \\ 0 & \frac{1}{s} & -\overline{x}_2/s \\ 0 & 0 & 1 \end{bmatrix}$$

where $\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ and $s = \frac{1}{n}\sum_{i=1}^{n}\sqrt{(x_{i1}-\overline{x}_1)^2 + (x_{i2}-\overline{x}_2)^2}$.

3. (*10 points*) How would you remove projective distortion from a given image of a plane? Please write the form of the Homography you would apply to the distorted image. What would you need to know in order to remove the affine distortion?

Solution: Let l^1,l^2 and m^1,m^2 are two pairs of parallel image lines (i.e $l^1||l^2$ and $m^1||m^2$). Due to perspective distortion in the image these two pairs of parallel lines intersect at p^1 and p^2 respectively. We can compute vanishing line $\ell = (\ell_1, \ell_2, \ell_3)^T$ from these two points as $\ell = p^1 \times p^2$. In order to remove perspective distortion this vanishing line must be mapped back to the line at infinity ($\ell^\infty = (0,0,1)^T$).

The form of homography that send back this vanishing line to infinity is give by: $H_p = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ l_1 & l_2 & l_3 \end{pmatrix}$

You can verify this by multiplying $H_p^{-\top}\ell$

$$\begin{bmatrix} 1 & 0 & -\frac{\ell_1}{\ell_3} \\ 0 & 1 & -\frac{\ell_1}{\ell_3} \\ 0 & 0 & \frac{1}{\ell_3} \end{bmatrix} \begin{bmatrix} \ell_1 \\ \ell_2 \\ \ell_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

In order to remove the affine distortion we need to know a pair of orthogonal lines.