Computer Vision - CSE 344/544
Mid-Sem Exam - Feb 25, 2014

Maximum score: 100 **Time: 2 hours**

Instructions:

1. There are six questions. Up to 20 extra credit points can be earned by solving additional problems.
2. Please show relevant steps of your calculations to get full credit.
3. There will be **negative marking** for true/false questions. An incorrect answer will cost you twice the points assigned to the question.
4. For true/false questions : true statements are *always* true; for false statements, there exists at least one example for which the statement does not hold.

1. (*20 points*) State whether the following statements are **true** or **false** with appropriate justification.

a) For a pinhole camera, size of the image of a world object decreases as the focal length is increased.

b) Any 3×3 orthogonal matrix is a rotation matrix.

c) A similarity transformation in 3D has seven degrees of freedom.

d) The cross ratio is preserved under a projective transformation, but not under an affine transformation.

e) When there is a translation along the principal axis, the epipoles coincide with the principal points.

f) If $l_\infty = [1, 0, 1]^\top$ for an image, parallel lines in 3D will appear to be parallel in the image.

g) Under affine distortion, angles between intersecting 3D lines can be computed from the image lines if the intrinsic parameters of the camera are known.

h) The intrinsic camera matrix $\mathbf{K}$ is always invertible.

i) When the transformation between a stereo pair of cameras is a pure translation along X axis, the rank of the corresponding essential matrix is strictly less than 2.

j) The line at infinity can be estimated using any two pairs of parallel lines.

2. (*20 points*) For (a) a planar homography $\mathbf{H}_{3\times3}$ and (b) a fundamental matrix $\mathbf{F}_{3\times3}$

- State the number of degrees of freedom.
- State the number of point correspondences required for estimation.
- Why are more point correspondences needed for estimating $\mathbf{F}$ despite having fewer degrees of freedom? Explain.

3. (*20 points*) Let $\mathbf{H}$ be a 2×2 transformation matrix of a 1-D projective space $\mathcal{P}^1$ such that $\mathbf{u} \sim \mathbf{H}\mathbf{v}$, where $\mathbf{u}$ and $\mathbf{v}$ are in $\mathcal{P}^1$ represented using homogeneous co-ordinates. How many degrees of freedom does $\mathbf{H}$ have? How many point correspondences do you need to solve for $\mathbf{H}$? Apply the direct linear transformation (DLT) and write it in the form $\mathbf{A}\mathbf{h} = \mathbf{0}$. Give the structure of $\mathbf{A}$ in terms of u_i and v_j. {*Hint:* Cross-multiply after taking the ratio of rows in LHS and RHS.}

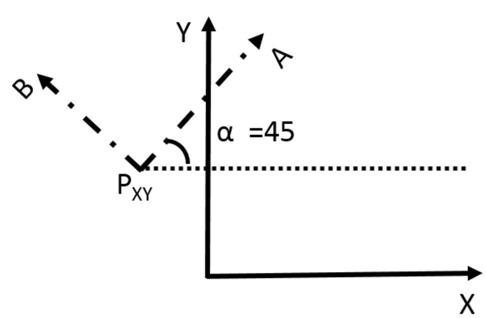
4. (*20 points*) Let $\mathbf{X}_i \in \mathbb{R}^3$ and $\mathbf{x}_i \in \mathbb{R}^2$, $i = 1, \ldots, n$ be 3D-2D *noisy* point correspondences for some camera given by matrix $\mathbf{P}$. Find the transformations $\mathbf{T}_1$ and $\mathbf{T}_2$ such that both sets of transformed points $\{\widetilde{\mathbf{x}}_i = \mathbf{T}_1\mathbf{x}_i\}$ and $\{\widetilde{\mathbf{X}}_i = \mathbf{T}_2\mathbf{X}_i\}$ are centered around the origin and have an average unit distance. Say you estimated a camera matrix $\widetilde{\mathbf{P}}$, such that for $i = 1, \ldots, n$, we have $\widetilde{\mathbf{x}}_i \sim \widetilde{\mathbf{P}}\widetilde{\mathbf{X}}_i$. How are $\mathbf{P}$ and $\widetilde{\mathbf{P}}$ related. We saw that such a transformation of points reduces the variance of the error in estimating $\mathbf{P}$. What is the intuition behind such a claim? {*Hint:* What happens to the noise $\delta\mathbf{x}_i$ when the transformation $\mathbf{T}_1$ is applied to the noisy points?}

5. (*20 points*) Let $\mathbf{F}$ be the fundamental matrix with the singular value decomposition, $\mathbf{F} = \mathbf{U}\mathbf{D}\mathbf{V}^\top$, where

$$\mathbf{U} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \qquad \mathbf{D} = \begin{bmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{bmatrix}, \qquad \mathbf{V} = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 0 & 0 & 1 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \end{bmatrix}$$

Locate the epipoles of this stereo camera system. What can you say about the image planes of this stereo setup?

6. (*20 points*) Find the isometric transformation matrix ${}^{AB}\mathbf{T}_{XY}$ that maps points from the XY co-ordinate frame to the AB co-ordinate frame (See figure below). The angle $\alpha = \pi/4$ and the point $P_{XY} = [-1, 2]^\top$.



What is the inverse transformation ${}^{XY}\mathbf{T}_{AB}$?