

MTH 377/577 Convex Optimization

Practice Quiz

Instructions: Answer all questions. Maximum Marks: 25

1. State whether the following statements are true/false. For true statements, provide a short proof. For False, provide a counter example.

- (a) The set of all real numbers R is both open and closed. (3)
- (b) A cone is always a convex set. (2)
- (c) The function $f(x) = |x - 3|$ is convex. (2)
- (d) The set $C = A \cup B$ where A, B are convex is always convex. (3)

2. Let $\{x_i\}_{i \in \{1,2,3,\dots\}}$ be a sequence where $x_i = i + 1$ for all $i = 1, 2, \dots$. Let $\{y_i\}_{i \in \{1,2,\dots\}}$ be a sequence where $y_i = -i$ for all $i = 1, 2, \dots$. Consider the set $Z = \{z_i \in R | z_i = x_i + y_i, \forall i = 1, 2, \dots\}$. Is Z bounded? Does it have an infimum or a supremum? (3)

3. Is the following set of vectors linearly independent? (2)

$$v_1 = (2, 1, 1, 5), v_2 = (2, 2, 1, 1), v_3 = (3, -1, 6, 1), v_4 = (1, 1, 1, -1)$$

4. Consider a set $A = \{(x_1, x_2) \in R^2 | x_1^2 + x_2^2 \leq 25\}$ and $f : A \rightarrow R$ where $f(x) = d(x, 0)$ for all $x \in A \subseteq R^2$. We assume that 0 denotes the origin $(0, 0)$ in R^2 . Can you use the Weierstrass theorem to state whether there is a maxima or minima of f in A ? Why/why not? (4)

5. Write down the convex hull for the following set: (3)

$$A = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$$

6. Consider a set $A = \{\theta_1 x_1 + \theta_2 x_2 | x_1 = (4, 5), x_2 = (1, 3); \forall \theta_1, \theta_2 \geq 0\}$. Is A a cone? Is it convex? (3)