Quiz 1 Rubric Q3

$X \sim f_\mu(x) = e^{-(x-\mu)} \quad \mu < x < \infty$

$Y = \sigma X + \theta$

$\frac{y-\theta}{\sigma} = x \Rightarrow \frac{dx}{dy} = \frac{1}{\sigma}$

1 mark — By transformation Theorem distribution / pdf of Y is given by →

$$f(y) = \left|\frac{dx}{dy}\right| \cdot f(x)$$

$$= \frac{1}{\sigma} \cdot e^{\left[\frac{y-\theta}{\sigma} - \mu\right]}$$

$$= \frac{1}{\sigma} \cdot e^{-\left(\frac{y-\theta-\mu\sigma}{\sigma}\right)}$$

1 mark

$$= \frac{1}{\sigma} \cdot e^{-\left(\frac{y-(\theta+\mu\sigma)}{\sigma}\right)}$$

$$= \frac{1}{\sigma} \cdot f_X\left[(y-\theta)/\sigma\right]$$

0.5 — Thus, Y belongs to the loc-scale family

4(a)

$Y = \sigma X + \theta$

$\frac{y-\theta}{\sigma} = x \Rightarrow \frac{dx}{dy} = \frac{1}{\sigma}$

$f(y) = \frac{1}{\sigma} \exp\left[-\left\{\left(\frac{y-\theta}{\sigma}\right) - \mu\right\}\right]$

$= \frac{1}{\sigma} \exp\left[-\left\{\frac{y-(\theta+\mu\sigma)}{\sigma}\right\}\right]$

$= \frac{1}{\sigma} e^{-\left[\frac{y-(\theta+\mu\sigma)}{\sigma}\right]}$

$= \frac{1}{\sigma} \cdot f\left(\frac{x-\theta}{\sigma}\right)$

at $x = \mu$

$y = \sigma\mu + \theta$

$\sigma\mu + \theta < y < \infty$

$E(Y) = \sigma E(X) + \theta = \theta + \sigma(1+\mu) = \theta + \sigma + \sigma\mu$

$V(Y) = V(\sigma X + \theta) = \sigma^2 V(X) = \sigma^2(1) = \sigma^2$

Thus, Y belongs to the loc-scale family