Computer Vision
Quiz 2 - Feb 9, 2017

Maximum score: 30 **Time: 30 mins**

Name: ______________________________ Roll No: ______________

1. (*10 points*) State the number of degrees of freedom for the camera motion and the corresponding variables in each of the following cases.

1. A fixed focal length camera rigidly mounted on a robot moving on a planar surface.
2. A fixed focal length camera installed on a car running on Delhi roads.
3. A pan-tilt-zoom surveillance camera.

Solution:

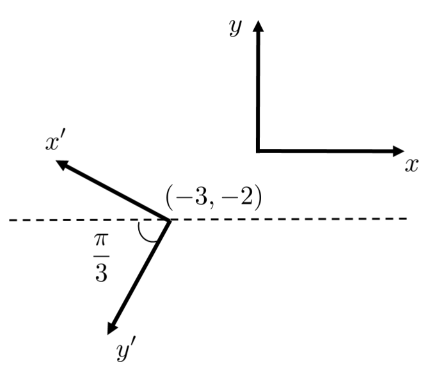
a) 3 dof, one for rotation and two for translation.

b) 6 dof, 3 for rotation and 3 for translation.

c) 3 dof, two for rotation and one for zoom (focal length).

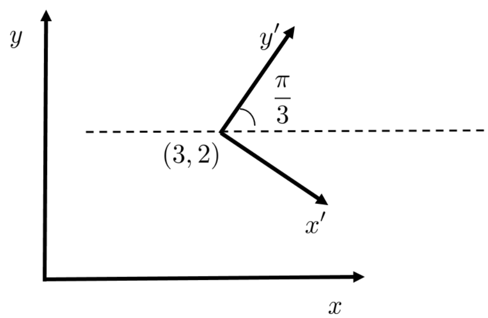
2. (*10 points*) Given a 3D rotation matrix $\mathbf{R}$, how would you find the rotation angle? Which points are invariant to this rotation? Are these invariant points related to the eigenvectors of $\mathbf{R}$? If yes, how?
Solution: A line $\mathbf{l} = [l_1, l_2, l_3]^\top$ that passes through $[x, y, 1]^\top$ is a 3-vector satisfying $\mathbf{l}^\top \mathbf{x} = 0$, i.e., $\mathbf{l}$ is orthogonal to the vector $\mathbf{x}$. Therefore the set of lines that passes through points are the lines lying in the 3D plane with the normal $\mathbf{x}$.

3. (*10 points*) Find the transformation $\mathbf{T}$ that maps points from the xy coordinate frame to the $x'y'$ coordinate frame. Assume that the dotted line is parallel to the x axis.



Solution:
Note: *The next problem is very similar to this one. Please look at that and prepare the solution for this question.*

4. (*10 points*) Find the transformation $\mathbf{T}$ that maps points from the xy coordinate frame to the $x'y'$ coordinate frame. Note that the angle given is between the axes y' and x.
Solution:



Let $\mathbf{T}$ be the transformation that maps points from the xy frame to the $x'y'$ frame. Therefore, for homogeneous coordinates $\mathbf{p}'$ and $\mathbf{p}$, we have:

$$\begin{aligned}\mathbf{p}' &= \mathbf{T}\mathbf{p} \\ &= \begin{bmatrix}\mathbf{R} & \mathbf{t} \\ 0 & 1\end{bmatrix}\mathbf{p} \\ \mathbf{p}_{x'y'} &= \mathbf{R}\mathbf{p}_{xy} + \mathbf{t}\end{aligned}$$

In this representation, the $\mathbf{p}_{xy}$ is first rotated and then translated. Therefore, the point $\mathbf{p}_{xy}$ is mapped to the $x'y'$ frame and then is translated to $[0, 0]^\top$. The vector $\mathbf{t}$ should be in the $x'y'$ frame of reference and is equal to $\mathbf{R}[-3,\ -2]^\top$.
The rotation matrix $\mathbf{R}$ rotates the points by $\frac{\pi}{6}$ in the counter-clockwise direction (since the axes are rotated by $\pi/6$ in the clockwise direction), and is given by

$$\begin{bmatrix}\cos\pi/6 & -\sin\pi/6 \\ \sin\pi/6 & \cos\pi/6\end{bmatrix}$$

Then the final transformation is

$$\mathbf{T} = \begin{bmatrix}\mathbf{R} & \mathbf{R}\begin{bmatrix}-3 \\ -2\end{bmatrix} \\ 0 & 1\end{bmatrix}$$