MTH310/520: Submission 10

Time: 15 Minutes, Marks: 5

April 23, 2024

Name and Roll No:

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1. (5 points) A graph is an interval graph if it's vertices can be mapped to intervals on the real line so that two vertices are adjacent if and only if their corresponding intervals intersect. Show that for such graphs, $\chi(G) = \omega(G)$, and likewise for the complement of such graphs.

Solution. Let $\chi(G) = k$ and I be the interval that gets the color k by greedy coloring algorithm. Then the left end point of I is overlapped with a set of $k-1$ pairwise intersecting intervals because otherwise I could use reuse any color from $\{1, \ldots, k-1\}$. This implies $\omega(G) \geq k$. On other hand, $\omega(G) \leq \chi(G) = k$. Thus the statement holds.

Let $\chi(G) = k$. Then each color class defines an independent set in $\overline{G}$, thus they get the same color. Consider a greedy algorithm that finds a maximum independent set by ordering the intervals by their left endpoints and selecting closest possible interval that does not overlaps with the current interval. The right endpoint of each interval induces a clique. Therefore, we find k cliques which covers the whole vertex set in G. This in turn gives k independent sets of $\overline{G}$ each of which gets the same color. Therefore, size of a maximum clique in $\overline{G}$ is at least as large as $\chi(\overline{G})$. On other hand $\chi(\overline{G}) \geq \omega(\overline{G})$ trivially holds.

Rubric: +2.5 for each part