MTH 372 (Winter 2025) : Tutorial VI

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1. Let $X_1, \ldots, X_n$ be a random sample from $N(\theta, \sigma^2), \sigma^2$ known. Show that the likelihood ratio test (LRT) for testing $H_0 : \theta \leq \theta_0$ versus $H_1 : \theta > \theta_0$ rejects the null hypothesis if $\dfrac{\sqrt{n}(\overline{X} - \theta_0)}{\sigma} \geq c_1$. Further, find the value of c_1 for $\alpha = 0.05$.

2. In general, LRT depends on a minimal sufficient statistic (MSS). Let $X_1, \ldots, X_n$ be a random sample from Bernoulli(p). Then $T = \sum X_i$ is a MSS for p. The T follows Binomial(n, p). Consider n is known and p is unknown. Using the likelihood function of T perform LRT for $H_0 : p \leq p_0$ versus $H_1 : p \geq p_0$.

3. Let $X_1, \ldots, X_n$ be a random sample whose pdf is given by
$$f_\theta(x) = \frac{1}{\theta}\, e^{-x/\theta}, \;\; 0 < x < \infty, \;\; \theta > 0.$$
Show that the likelihood ratio test (LRT) for testing $H_0 : \theta = \theta_0$ versus $H_1 : \theta \neq \theta_0$ rejects the null hypothesis if $\overline{X} \leq c_1$ or if $\overline{X} \geq c_2$.

4. Let $X_1, \ldots, X_n$ be a random sample from $N(\theta, \sigma^2), \theta, \sigma^2$ unknown. The likelihood ratio test (LRT) for testing $H_0 : \theta \leq \theta_0$ versus $H_1 : \theta \geq \theta_0$ rejects the null hypothesis if $T = \dfrac{\sqrt{n}(\overline{X} - \theta_0)}{S} \geq c_1; T \sim t(n-1)$. Find the value of c_1 for $\alpha = 0.05$.

5. Suppose we have two independent random samples: $X_1, \ldots, X_n$ are exponential (θ) and $Y_1, \ldots, Y_m$ are exponential (μ). Answer the following
   (a) Find the LRT of $H_0 : \theta = \mu$ versus $H_1 : \theta \neq \mu$.
   (b) Show that the above test can be based on the statistic $T = \dfrac{\sum X_i}{\sum X_i + \sum Y_i}$.
   (c) Find the distribution of T when H_0 is true.

6. Find the LRT of $H_0 : \theta \leq 0$ versus $H_1 : \theta > 0$ based on a random sample $X_1, \ldots, X_n$ from a population with probability density function
$$f_{\theta,\lambda}(x) = \frac{1}{\lambda}\; e^{\frac{-(x-\theta)}{\lambda}} I_{(\theta,\infty)}(x).$$
Here the parameters, θ and λ are unknown.