Quiz- solution (BIO525)

15/4/25
Total marks: 15

Multiple Choice Questions (1-5) ***(1 mark each)*****: One or more options may be correct. Select all correct options. No partial marking.**

Only right answers are given 1 mark.
If a student has marked 2 options, and 1 is correct, 0.5 mark is given.
Otherwise, if 3 or more options are marked for a question, ZERO is given.

1- In the optimization problem: Maximize Z = 3x + 2y subject to constraints, here x and y are?

a) Parameters
b) Constraints
c) Decision variables
d) Objective coefficients

Answer: c

2- In constrained optimization using Lagrange multipliers, what condition must be satisfied at the optimal point?

a) The gradient of the constraint function and objective function must be parallel.
b) The gradient of the constraint function must be zero.
c) The gradient of the constraint function and objective function must be perpendicular.
d) The objective function must be linear.

Answer: a

3- In Genetic Algorithm, what does a chromosome typically represent?

a. Data structure to store neural weights
b. A binary representation of a possible solution
c. A list of all fitness values
d. A type of selection method

Answer: b

4- What does the term elitism refer to in the context of Genetic Algorithms?

a. Choosing random solutions for the next generation
b. Discarding the best solution to encourage diversity
c. Carrying the best solutions unchanged to the next generation
d. Removing all low-fitness individuals immediately

Answer: c

5- In binary-encoded Genetic Algorithms, what does crossover operator generally involve?

a. Reversing the chromosome
b. Random swapping of populations
c. Exchanging parts of parent chromosomes
d. Rotating gene values

Answer: c

6- **True or False:** The Hamilton-Jacobi-Bellman (HJB) equation provides a necessary, but not sufficient, condition for optimality in continuous-time control problems. Support your answer with valid argument. ***(1 mark) [If no justification is provided, 0 mark will be given].***

False: HJB equation provide both necessary and sufficient condition for optimality in continuous-time control problems, if a smooth value function solution V(x,t) to the HJB equation exists and the optimal control u*(x,t) derived from it is well-defined and admissible. It characterizes the value function of the optimal control problem completely.

OR True: The HJB equation doesn't always guarantee sufficient conditions for optimality in general. It only provides sufficient conditions if a smooth value function solution exists, and the associated control is admissible. Without this, the equation is only necessary.

Full marks (1 mark) if the student has given either False or True with valid explanation.
Half mark (0.5 mark) if reasoning is partially correct.
ZERO marks if only "True" or "False" is written without reasoning.

7- (a) Define the *Principle of Optimality* in the context of dynamic programming. ***(1 mark)***
(b) Explain how this principle is demonstrated using a real-world or engineering example. ***(1 mark)***

Solution: The **Principle of Optimality** states that: (1 mark)

"Any optimal policy has the property that, whatever the current state and decision, the remaining decisions must constitute an optimal policy with regard to the state resulting from the current decision." – Richard Bellman
In other words, any subproblem of an optimal solution is itself optimal.

Example (1 mark): In robot path planning, suppose a robot must move from point A to D via B and C. If the path A→B →C→D is optimal, then the subpath B→C→D must also be optimal.
Thus, once the robot reaches point B, the remaining journey from B onward should still follow the optimal policy.

Full marks (1 mark) will be given if you have rewritten the optimal principle with the same meaning but different wording.
Other examples correctly explained are given full marks (1 mark).
Partially correct answer got 0.5 mark for each part, if applicable.

8- A single-celled organism of mass m moves in a medium and aims to minimize its energy expenditure while moving along an optimal trajectory. The energy cost is proportional to the kinetic energy, and the motion is described by **x(t)**, which represents the organism's position over time. Goal here is to minimize the total kinetic energy. Boundary conditions: $x(0) = 0,\ x(T) = 32$ with $T = 8$.

a. Identify the Lagrangian ***(1 mark)***.

b. Find the trajectory that minimizes the total energy expenditure and interpret your result ***(2 marks).***

Solution:

In this case, we're dealing with an optimization problem where the organism wants to minimize its energy expenditure. The energy expenditure is proportional to kinetic energy, which is given by the expression:

$$K.E. = \frac{1}{2}m\dot{x}^2$$

Here, m is the mass of the organism, $\dot{x}(t)$ is the velocity of the organism $= \frac{dx}{dt}$

Since goal is to minimize the Kinetic energy, the Lagrangian is given by the kinetic energy.

$L = \frac{1}{2}m\dot{x}^2$ *(1 mark). If only explained well, but missed the exact expression, 0.5 given.*

Euler Lagrange equation:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0 \quad \text{(0.5 mark)}$$

$$\frac{\partial L}{\partial x} = 0, \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = 0 \rightarrow m\ddot{x} = 0$$

$$x = At + B$$

Solving and putting the boundary conditions: (1 mark for full steps)

$$x = 4\,t.$$

Interpretation: The solution represents a linear trajectory, indicating that the organism moves with a constant velocity of 4 units per time. The optimal strategy to minimize energy expenditure is to move at a steady pace without any acceleration or deceleration, as changes in velocity would increase the kinetic energy and thus the total energy cost. (0.5 mark)

9- You are managing the population of a biological species x(t) over a fixed time interval [0, T]. The population evolves following logistic growth with carrying capacity K and intrinsic growth rate r and is subject to a control effort u(t) which reduces the population. Note: r, K and T are constants.

The dynamics of the system are given by:

$$\frac{dx(t)}{dt} = \boxed{} - u(t)x(t)$$

Your objective is to **maximize** the population at final time, while **minimizing** the total quadratic cost of control over [0, T].

(a) Write the **objective functional** $J(u)$ to be maximized. Briefly explain. ***(1 mark)***
(b) Fill in the boxed term in the dynamics equation. ***(1 mark)***
(c) Using Pontryagin's Maximum Principle, write the necessary condition for the optimal control strategy. ***(2 marks)***

Solution:

- Objective functional:

$$\max_{u(t)} \left[x(T) - \int_0^T u(t)^2\, dt\right]$$ 0.5 mark

x(T): rewards ending with a large population. $\int_0^T u(t)^2\, dt$ penalizes excessive use of control, with a quadratic cost term. 0.5 mark

b) Fill in the boxed term (1 mark)

$$\frac{dx(t)}{dt} = rx(t)\left(1 - \frac{x(t)}{K}\right) - u(t)x(t)$$

Where $rx(t)\left(1 - \frac{x(t)}{K}\right)$ is the logistic growth equation

c) PMP (2 marks for all steps and conditions)

Hamiltonian, H = $-u^2 + \lambda\left[rx\left(1 - \frac{x}{K}\right) - ux\right]$ 0.5 marks

Where λ is the adjoint variable

PMP necessary conditions-

State equation:

$$\frac{dx(t)}{dt} = rx(t)\left(1 - \frac{x(t)}{K}\right) - u(t)x(t)$$

0.5 marks

Adjoint equation:

$$\frac{d\lambda}{dt} = -\frac{\partial H}{\partial x} = -\lambda r + \frac{2\lambda rx}{K} + \lambda u$$

0.5 marks

Optimality conditon:

$\frac{\partial H}{\partial u}$ = 0 implies $-2u - \lambda x = 0$

u* = $-\lambda(t)x(t)/2$

$$\frac{\partial^2 H}{\partial u^2} = -2 < 0$$

0.5 marks

Transversality condition: $\lambda(T) = \frac{\partial \Phi}{\partial t}\big|_T = 1$