MTH 377/577 Convex Optimization
Problem set 3

1. Let $A = \{(x,y) \in R \times R^2 | x \in R, y \in R^2; y = (x, x^2)\}$ and $B = \{(x,y) \in R \times R^2 | x \in R, y \in R^2; y = (x, \frac{x}{2})\}$. Write down the partial sum of A and B. Is it convex?
   Ans. Partial sum of A, B is $P = \{(x, 2x, x^2 + \frac{x}{2}) \in R^3 | x \in R\}$. Yes, it is convex. (show on your own. hint: let $(x, 2x, x^2 + \frac{x}{2}), (z, 2z, z^2 + \frac{z}{2}) \in P$, pick any $\theta \in [0,1] \ldots$)

2. Let $f : R \to R$. Define $f(x) = \begin{cases} x^{\frac{1}{3}} & x \geq 0 \\ -x^{\frac{1}{2}} & x < 0 \end{cases}$ Is f convex? Is it quasi-convex? Ans. f is not convex, but it is quasiconvex. Every sub-level set is convex (write down some sub-level sets to demonstrate: $[-1,1]$ when $\alpha = 1$,$[-8,8]$ for $\alpha = 2$ etc.).

3. Let $\succeq$ be a strict binary relation defined over the set $\{a, b, c, d, e\} \in R^5$. Suppose $\succeq$ is reflexive and symmetric, but is not acyclic. Is $\succeq$ an ordering? Why/why not?
   Ans. No, $\succeq$ is not an ordering: suppose $a \succeq b, b \succeq c, c \succeq a$ but not $a \succeq c$. Then there is no order over a, b, c.

4. Let $f_1, f_2, f_3, \ldots f_n$ be convex functions. Is $f(x) = \max\{f_1(x), f_2(x), \ldots, f_n(x)\}$ convex?
   Ans. Yes. try to show om your own.

5. Convert the following LP into standard form and write down its dual:

$$\begin{array}{rc} max & x_1 + 2x_2 \\ s.t. & x_1 + \frac{8}{3}x_2 \leq 4 \\ & x_1 + x_2 = 2 \\ & 2x_1 \geq 3 \\ & x_1 \geq 0 \end{array}$$

   Ans. Follow procedure as discussed in lecture.