Explainable AI (CSE615)

Quiz-1, Date: Feb 3, 2025

Max Time: 30 mins | Winter 2025 | Max marks: 20

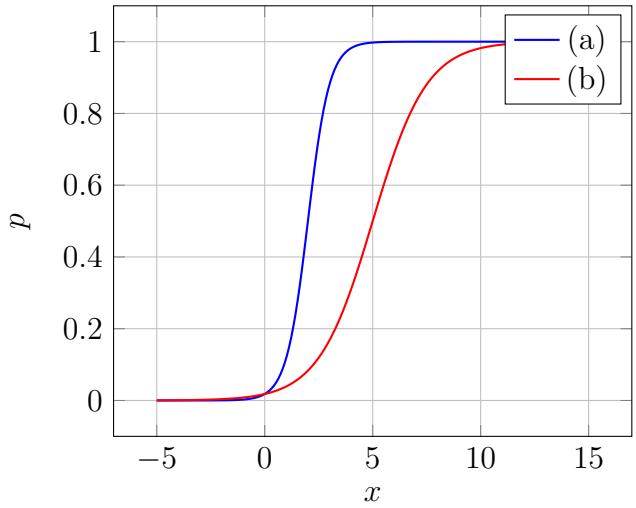
1. Match the following coefficients with the plots mentioned below. Briefly explain your approach to the answer.

   i $\beta_0 = -4$, $\beta_1 = 2$.

   ii $\beta_0 = -4$, $\beta_1 = 0.8$.

[4]

Plot of p versus x for logistic regression model



Solution - (a) - (i)
(b) - (ii)
Reasoning -

$$\log\left(\frac{p}{1-p}\right) = \beta_o + \beta_1 x$$

$$\frac{p}{1-p} = \exp(\beta_o + \beta_1 x)$$

$$p = \frac{1}{1+\exp(-(\beta_o + \beta_1 x))}$$

And then, as we increase the value of β_1, the curve becomes steeper.

2. Each person in the 1069 samples is either an Instagram user or not. The probability p, that a person is an Instagram user depends on their sex, which has only two values ($x = 1$ for women and $x = 0$ for men). So there are two possible values for p, p_{women} and p_{men}. Let log odds for women be,

$$log\left(\frac{\hat{p}_{women}}{1-\hat{p}_{women}}\right) = 0.4507 \tag{1}$$

and for men be,

$$log\left(\frac{\hat{p}_{men}}{1-\hat{p}_{men}}\right) = -0.2419 \tag{2}$$

Calculate coefficients ($\beta_i's$) of the linear equation expressing the logit function for men and women. Also, find the odds ratio ($odds_{women}/odds_{men}$). [4]

Solution -
Since $x = 1$ for women and $x = 0$ for men.

$$log\left(\frac{\hat{p}_{women}}{1-\hat{p}_{women}}\right) = \beta_o + \beta_1 \tag{3}$$

$$log\left(\frac{\hat{p}_{men}}{1-\hat{p}_{men}}\right) = \beta_o \tag{4}$$

Substituting and getting values -

$$\beta_o = -0.2419$$
$$\beta_1 = 0.6926$$
$$(odds_{women}/odds_{men}) = e^{0.6926} = 1.999$$

3. Suppose a logistic regression model for predicting disease presence ($Y = 1$) has the following equation:

$$logit(p) = -2 + 1.2x_1 + 0.8x_2$$

where x_1 is age (in years) and x_2 is BMI (Body Mass Index). For a patient with $x_1 = 40$ and $x_2 = 25$, calculate the predicted probability of disease presence. [4]

Solution
Probability, p, of disease presence =

$$\frac{1}{1+e^{-(-2+1.2*40+0.8*25)}} = \frac{1}{1+e^{-66}}$$

4. Construct the rules for the following data using the sequential covering algorithm. You can create a rule based on the data patterns you observe and don't have to use a decision tree. Explain the reason to pick a rule and the steps in the algorithm. [8]

S.No.	Age	Income	Loan Amount	Credit Risk (Target)
1	25	Low	15,000	High Risk
2	45	Medium	10,000	Low Risk
3	30	High	25,000	High Risk
4	50	Medium	8,000	Low Risk
5	22	Low	18,000	High Risk
6	35	High	20,000	High Risk
7	55	Medium	5,000	Low Risk
8	28	High	30,000	High Risk

Table 1: Customer Credit Risk Data

Provide an appropriate explanation and description for each rule.

Solution -

- If Income = High, then Credit Risk = High Risk
- If Age $<$ 30 and Income = Low, then Credit Risk = High Risk
- If Income = Medium, then Credit Risk = Low Risk

Explanation - We know the algorithm as -

- Choose a class to cover (e.g., "High Risk").
- Find a rule that covers as many uncovered instances of that class as possible without covering other classes.
- Remove the instances covered by the rule.
- Repeat until all instances of that class are covered.
- Move to the next class and repeat.

Step 1: Target class = "High Risk"

For High Risk entries we notice:

- All customers with high Income have High Risk.
- All low income customers younger than age 30 have high risk.

If Income = High, then Credit Risk = High Risk

- Covers: Row 3, Row 6, and Row 8.

If Age < 30 and Income = Low, then Credit Risk = High Risk

- Covers: Row 1, Row 5.

Step 2: Target class = "Low Risk"

For Low Risk entries we notice:

- All have Medium income.

If Income = Medium, then Credit Risk = Low Risk.

- Covers: Row 2, Row 4, and Row 7.