Computer Vision
Quiz 2 Solutions

Maximum score: 30

1. (*4 points*) For each statement below, please indicate whether it is *true* or *false* with appropriate justification.

1. k-means always converges to the same minimum of the objective function regardless the initialization.
   False. k-means converges to local optimum because the objective function is non-convex.

2. Given the window size (bandwidth parameter), the output of mean shift clustering/segmentation does not depend on its initialization.
   True. Mean shift iterations start from each data point, thus the output is always deterministic.

2. (*6 points*) What is the gradient of a grayscale image? How do you compute the direction of the gradient in an image? What kind of a mask would you use if you want to achieve sharpening of an image?
Solution: The gradient is given by the vector $\nabla I = [\nabla_x I \;\; \nabla_y I]^\top = [\frac{\partial I}{\partial x} \;\; \frac{\partial I}{\partial y}]^\top$ and the direction is computed as $\theta = \arctan\left(\frac{\nabla_y I}{\nabla_x I}\right)$. Let H be the 3×3 averaging mask and F be the identity mask of the same size, then the corresponding image sharpening filter mask would have the form

$$F + \alpha(F - F * H) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \frac{\alpha}{9} \begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

where α controls the degree of sharpening.

3. (*10 points*) State the number of degrees of freedom for the camera motion and the corresponding variables in each of the following cases.

1. A fixed focal length camera rigidly mounted on a robot moving on a planar surface.

2. A constant focal length camera installed on a car running on Delhi roads (with potholes, that is).

3. A pan-tilt-zoom surveillance camera.

Solution: You may also consider the intrinsic parameters if you wish.

1. 3 dof, one for rotation and two for translation.

2. 6 dof, 3 for rotation and 3 for translation.

3. 3 dof, two for rotation and one for zoom (focal length).

4. (*10 points*) Given a 3D rotation matrix $\mathbf{R}$, how would you find the rotation angle? Which points are invariant to this rotation? How are these invariant points related to the eigenvectors of $\mathbf{R}$.
Solution: $\theta = \arccos((trace(\mathbf{R}) - 1)/2)$. The points invariant to the rotation are the ones that lie along the axis of rotation. The axis of rotation is given by the eigenvector of $\mathbf{R}$ that has a corresponding eigenvalue of 1.