

Quiz 1 Rubric Q3

$$X \sim f_X(x) = e^{-(x-\mu)} \quad \mu < x < \infty$$

$$Y = \sigma X + \theta$$

$$\frac{y-\theta}{\sigma} = x \Rightarrow \frac{dx}{dy} = \frac{1}{\sigma}$$

1 mark By transformation Theorem distribution/pdf of Y is given by \rightarrow

$$f(y) = \left| \frac{dy}{dx} \right| \cdot f(x)$$

$$= \frac{1}{\sigma} \cdot e^{\left[\frac{y-\theta}{\sigma} - \mu \right]}$$

$$= \frac{1}{\sigma} \cdot e^{\left(\frac{y-\theta-\mu\sigma}{\sigma} \right)}$$

$$= \frac{1}{\sigma} \cdot e^{-\left(\frac{y-(\theta+\mu\sigma)}{\sigma} \right)}$$

$$= \frac{1}{\sigma} \cdot f_X \left[\frac{(y-\theta)}{\sigma} \right]$$

0.5 Thus, Y belongs to the loc-scale family



IIIT-D SUPPLEMENTARY SHEET

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पाठ्यक्रम

Course Title

पृष्ठ संख्या

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4(a) $Y = \sigma X + \theta$

$$\frac{y - \theta}{\sigma} = x \Rightarrow \frac{dx}{dy} = \frac{1}{\sigma}$$

$$f(y) = \frac{1}{\sigma} \exp\left[-\frac{1}{\sigma}\left(\frac{y - \theta}{\sigma} - \mu\right)\right]$$

$$= \frac{1}{\sigma} \exp\left[-\frac{y - (\theta + \mu\sigma)}{\sigma}\right]$$

$$= \frac{1}{\sigma} e^{-\left[\frac{y - (\theta + \mu\sigma)}{\sigma}\right]}$$

$$= \frac{1}{\sigma} f\left(\frac{y - \theta}{\sigma}\right)$$

$$\begin{cases} E(Y) = \sigma E(X) + \theta = \theta + \sigma(1 + \mu) = \theta + \sigma + \sigma\mu \\ V(Y) = V(\sigma X + \theta) = \sigma^2 V(X) = \sigma^2(1) = \sigma^2 \end{cases}$$

Thus, Y belongs to the loc-scale family.