1. $Y = \sigma X$, $\sigma > 0$

$X = Y/\sigma \Rightarrow \frac{dx}{dy} = \frac{1}{\sigma}$

$$f(y) = \frac{1}{\Gamma\alpha \beta^{\alpha}} \exp\left[-\frac{1}{\beta\sigma} y\right] \left(\frac{y}{\sigma}\right)^{\alpha-1} \frac{1}{\sigma}$$

$$= \frac{1}{\Gamma\alpha (\beta\sigma)^{\alpha}} \exp\left[-\frac{y}{\beta\sigma}\right] \quad y$$

$\Rightarrow Y \sim \text{Gamma}(\alpha, \beta\sigma)$

Thus Y belongs to scale family.

2) $Y = \sigma X + \theta$

$X = \frac{y-\theta}{\sigma} \Rightarrow \frac{dx}{dy} = \frac{1}{\sigma}$

$$f(y) = \frac{1}{\pi} \frac{1}{1 + \left(\frac{y-\theta}{\sigma}\right)^2} \frac{1}{\sigma}$$

$$= \frac{1}{\pi\sigma} \frac{\sigma^2}{\sigma^2 + (y-\theta)^2} = \frac{1}{\sigma} f\left(\frac{x-\mu}{\sigma}\right)$$

$\Rightarrow Y \sim \text{Cauchy}(\theta, \sigma)$

$\Rightarrow$ Y belongs to the loc-scale fam

③ $y = \sigma X + \theta$

$$\Rightarrow X = \frac{y-\theta}{\sigma} \quad \Rightarrow \frac{dx}{dy} = \frac{1}{\sigma}$$

$$f(y) = \frac{1}{\beta} \frac{e^{-\left[\frac{y-\theta}{\sigma} - \mu\right]\frac{1}{\beta}}}{\left[1 + e^{-\left[\frac{y-\theta}{\sigma} - \mu\right]\frac{1}{\beta}}\right]^2} \frac{1}{\sigma}$$

$$= \frac{1}{\beta\sigma} \frac{e^{-\left(\frac{y-(\theta+\sigma\mu)}{\sigma\beta}\right)}}{\left[1 + e^{-\frac{y-(\theta+\sigma\mu)}{\sigma\beta}}\right]^2}$$

$$= \frac{1}{\sigma} f\left(\frac{x-\theta}{\sigma}\right)$$

Now $\beta' = \beta\sigma$ and $\mu' = \theta + \sigma\mu$. Thus it belongs to loc - scale family. it is logistic distribution.

The logistic distribution does belong to the location - Scale family

$X_i \overset{iid}{\sim} N(\theta, \theta^2)$. ; $\theta > 0$.

For Scale family the transformation required

$$Y = \sigma X \Rightarrow X = Y/\sigma$$

$$\frac{dx}{dy} = \frac{1}{\sigma}$$

$$f_\theta(x_i) = \frac{1}{(2\pi\theta^2)^{1/2}} \exp\left[-\frac{1}{2}\frac{(x_i-\theta)^2}{\theta^2}\right]$$

So, density of Y will be

$$f(y) = \frac{1}{(2\pi\theta^2)^{1/2}} \exp\left[-\frac{1}{2}\frac{1}{\theta^2}\left(\frac{y}{\sigma}-\theta\right)^2\right]\cdot\frac{1}{\sigma}$$

$$= \frac{1}{(2\pi\theta^2)^{1/2}} \exp\left[-\frac{1}{2\theta^2\sigma^2}(y-\theta\sigma)^2\right]\frac{1}{\sigma}$$

$$\Rightarrow Y \sim N[\theta\sigma, (\theta\sigma)^2]$$

Thus, is belongs to the Scale family.

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