MTH310/520: Submission 7

Time: 15 Minutes, Marks: 5

April 11, 2024

Name and Roll No:

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1. (5 points) Use network-flows to prove König-Egerváry Theorem.

Solution. Let $G = (X \cup Y, E)$ be the given bipartite graph. We form a network as follows: take two new vertices s and t. Add edges (s, u) for all $u \in X$ and (v, t) for all $v \in Y$, and add capacity 1. For all the edges in E, direct it from X to Y and add capacity infinity. Observe that since the edges are of infinite capacity in G, the max possible flow in the network is n, where n is the number of vertices in X and Y.

Let $[S, T]$ be a minimum cut in the network. Let $X_S = S \cap X, X_T = T \cap X, Y_S = S \cap Y, Y_T = T \cap Y$. Clearly, the minimum cut consists of edges from s to X_S, or Y_S to t because otherwise the cut would be of infinite capacity. This further implies that the vertices $X_T \cup Y_S$ defines a min cut and therefore, it suffices to conclude that size of a min cut is equal to size of a minimum cover.

By max flow min cut theorem, max flow=max matching=min cut=min cover.

Rubric: +2 for construction of a proper network. +3 for the rest of the proof that uses max flow min cut theorem properly.