MTH 372 (Winter 2025): Tutorial I

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1. Let $X_1 \ldots, X_n$ be a random sample from the Poisson distribution with parameter $\lambda > 0$. Using formal definition of sufficiency, verify if $T = \sum_{i=1}^{n} X_i$ is sufficient for λ.

2. Let X_1, X_2, X_3 be iid from Bernoulli distribution with parameter $0 < p < 1$. Using formal definition show that $U = X_1 X_2 + X_3$ is not sufficient for p.

3. Let $X_1 \ldots, X_n$ be a random sample from Poisson$(\lambda), \lambda > 0$. Verify that it belongs to the exponential family. Find sufficient statistic for λ.

4. Let X be one observation from a normal distribution with mean 0 and variance $\sigma^2 = \theta$. Use the factorization theorem to show that $T = |X|$ is a sufficient statistic.

5. Let $X_1, \ldots, X_n$ be a random sample from a Pareto distribution, whose pdf is given by

$$f_\theta(x) = \frac{\theta}{(1+x)^{1+\theta}}, \quad x > 0$$

where $\theta > 0$ is an unknown parameter. Using the factorization theorem, show that $T = \prod_{i=1}^{n}(1 + x_i)$ is a sufficient statistic for θ. Is $U = \sum\limits_{i=1}^{n} \log(1 + x_i)$ a sufficient statistic for θ? Justify your answer.

6. Let $X_1, \ldots, X_n$ be a random sample from the pdf

$$f(x|\mu, \sigma) = \frac{1}{\sigma} e^{-(x-\mu)/\sigma}, \quad \mu < x < \infty, \quad 0 < \sigma < \infty.$$

Find a two dimensional sufficient statistic for (μ, σ).

7. Let $X_1, \ldots, X_n$ be a random sample from a gamma(α, β). Find a two dimensional sufficient statistic for (α, β).

8. Let $X_1, \ldots, X_n$ be a random sample whose pdf is given by

$$f_\theta(x) = exp(-(x - \theta)), \quad \text{if } x > \theta.$$

(a) Use the Indicator function method to find the sufficient statistic for θ.

(b) Show that $Y = \min_{1 \le i \le n} X_i$ is minimal sufficient for θ.