MTH 372 (Winter 2025): Tutorial III

Instructor: Monika Arora

- 1. Let X_1, \ldots, X_n be a random sample from Poisson $(\lambda), \lambda > 0$.
 - (a) Find the method of moments estimator of λ .
 - (b) Find the maximum likelihood estimator (MLE) of λ .
- 2. Let X_1, \ldots, X_n be a random sample from the pdf given by

$$f_{\theta}(x) = \theta x^{-2} \quad 0 < \theta \le x < \infty.$$

- (a) What is a sufficient statistic for θ .
- (b) Find the method of moments estimator of θ .
- (c) Find the maximum likelihood estimator (MLE) of θ .
- 3. Let X_1, \ldots, X_n be a random sample from Binomial(n, p), both are unknown. Find the method of moments estimator of (n, p).
- 4. One observation, X, is taken from a $N(0, \sigma^2)$ population.
 - (a) Find an unbiased estimator of σ^2 .
 - (b) Find the MLE of σ .
 - (c) Discuss how the method of moments estimator of σ might be found.
- 5. Let X_1, \ldots, X_n form a random sample from a uniform distribution on the interval $(\theta, \theta + 1)$, where the value of the parameter θ is unknown, $-\infty < \theta < \infty$. What will be the MLE of θ .
- 6. Find the maximum likelihood estimate of θ for the following two random samples.
 - (a) $X_i \sim \text{Binomial } (3, \theta)$, we observe $(X_1 = 1, X_2 = 3, X_3 = 2, X_4 = 2)$.
 - (b) $X_i \sim \text{Exponential } (\theta)$, we observe $(X_1 = 1.23, X_2 = 3.32, X_3 = 1.98, X_4 = 2.12)$.