Mid-Sem Solutions
Computer Vision

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1)

(a) False

$$k=\begin{bmatrix} f_r & 0 & C_x \\ 0 & f & C_y \\ 0 & 0 & 1 \end{bmatrix}$$

x = fX/Z

y = fY/Z

Therefore, x and y are directly proportional to f.

(b) False because det(R) =1

(c) True

Scale -1

Rotation -3

Translation -3

(d) False because affine transformation is a subset of projective transformation. Therefore, any invariants of projective transformation are invariants of affine transformation.

(e) True as principal points are the intersection of the principal axis and the image plane. Epipoles are the intersection of plane with the baseline. In case of translation along principal axis, the baseline coinicides with the principal axis and therefore, the epipoles coinicide with the principal points.

(f) False beacuse the parallel lines in 3D appear parallel only if l_∞ is at its canonical position.i.e. [1,0,0].

(g)True because if the K matrix is known,we can compute angles in 3D using the image of the absolute conic $w = kk^T$

(h) True because k is an upper triangular matrix. Therefore, it is full rank and hence invertible.

(i) False as rank of E is 2.

$$E = t_x R = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \mathrm{R}$$

where,

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

is Rank 2 matrix and R is rank 3 matrix

(j) False as the two pairs should have different directions else they intersect with the same vanishing point.

2)
a)Planar Homography

Degree of Freedom - 8(4 points correspondences)

b)Fundamental Matrix

Degree of Freedom- 7 or 8 points correspondences

H maps points to points ,therefore, each point correspondence imposes constraints along x and y axes.So, 4 point correspondences yield $4 * 2$ .i.e. 8 constraints. In case of fundamental matrix, a point is constrained to lie on a line ,therefore, each point correspondence constraints in one dimension, hence need 8 point matches(more strictly 7 matches).

3)u$\sim$Hv

$$H = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$$

Degree of Freedom -3 3 point Correspondence

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$u_1 = h_{11}v_1 + h_{12}v_2$$

$$u_2 = h_{21}v_1 + h_{22}v_2$$

$$h_{21}u_1v_1 + h_{22}u_1v_2 = h_{11}v_1u_2 + h_{12}v_2u_2$$

Therefore,

$$\begin{bmatrix} -v_1u_2 & -v_2u_2 & u_1v_1 & u_1v_2 \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{12} \\ h21 \\ h_{22} \end{bmatrix} =0$$

4)

T_1 and T_2 should be scaling and shifting in 2D and 3D respectively.
Therefore, T_1 is $3*3$ and t_2 is $4*4$ and we consider points in homogeneous coordinates.

$$\tilde{x} = T_1 x$$

$$center = \tilde{x} = \sum_{i=1}^{n} x_i$$

$$T_1 = \begin{bmatrix} 1/d & 0 & 0 \\ 0 & 1/d & -x/d \\ 0 & 0 & 1 \end{bmatrix}$$

$$average\ distance = d = 1/x \sum_{i=1}^{n} \parallel x_i - \bar{x} \parallel$$

$$\tilde{x_i} = T_1 x_i = x_i/d - \bar{x}/d = 1/d(x_i - \bar{x})$$

$$Average\ distance\ of \tilde{x_c} = 1/x \sum_{i=1}^{n} \parallel \tilde{x_i} \parallel = 1/n \sum_{i=1}^{n} (1/d) \parallel x_i - \bar{x} \parallel = (1/d) * 1/n \sum_{i=1}^{n} \parallel x_i - \bar{x} \parallel = (1/d) * d = 1$$

$$\text{Similarly for } X_i\ T_2 = \begin{bmatrix} 1/d_x & 0 & 0 & \bar{x} \\ 0 & 1/d_x & 0 & 0 \\ 0 & 0 & 1/d_x & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\bar{X} = (1/n) \sum_{i=1}^{n} X_i$$

$$\tilde{x_i} = \tilde{P} \tilde{X_i}$$

$$x_i = P X_i$$

Therefore,

$$x_i * P X_i = 0$$

$$\rightarrow \quad {T_1}^{-1} \tilde{x_i} = {T_1}^{-1} \tilde{P} \tilde{X_i}$$

$$x_i = {T_1}^{-1} \tilde{P} \tilde{X_i}$$

$$x_i = \tilde{P} {T_2}^{-1} \tilde{X_i}$$

$$P {T_2}^{-1} i = {T_1}^{-1} \tilde{P}$$

$$P = {T_1}^{-1} \tilde{P} T_2$$

5) F is rank 2 , therefore, $d_3 = 0$
Last column of V is the null space
Last column of U is the left null space
Therefore, the epipoles are $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$ These are at infinity. Therefore, image planes are coplanar.

6) Please look at practice problem set solution.