Computer Vision - CSE 344/544
Quiz 2 - Feb 17, 2014

Maximum score: 20 **Time: 20 mins**

Name: ______________________ Roll No: ______________

Instructions:

1. You need to solve any four questions out of five. If you solve all five, you get extra credit.
2. Please do not copy. The institute's plagiarism policy is strictly enforced.

1. (*5 points*) Consider a vector $(7, 3, 2)^\top$ which is rotated around the Z axis by $90°$ and then rotated around the Y axis by $90°$ and finally translated by $(4, -3, 7)^\top$. Find the new coordinates of the vector. All rotations are counterclockwise.

$$\cancel{R_z}\ R_z = \begin{bmatrix} \cos 90 & -\sin 90 & 0 \\ \sin 90 & \cos 90 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad R_y = \begin{bmatrix} \cos 90 & 0 & -\sin 90 \\ 0 & 1 & 0 \\ \sin 90 & 0 & \cos 90 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$X = \begin{bmatrix} 7 \\ 3 \\ 2 \end{bmatrix}$$

$$\overset{N}{X} = R_y R_z X + \begin{bmatrix} 4 \\ -3 \\ 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} -3 \\ 7 \\ 2 \end{bmatrix} + \begin{bmatrix} 4 \\ -3 \\ 7 \end{bmatrix} = \begin{bmatrix} -2 \\ 7 \\ -3 \end{bmatrix} + \begin{bmatrix} 4 \\ -3 \\ 7 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 4 \end{bmatrix}$$

2. (*5 points*) Let $\mathbf{A} \in \mathbb{R}^{5\times 3}$ and there exists a 3×5 matrix $\mathbf{C}$ such that $\mathbf{CA} = \mathbf{I}$. Say for some given $\mathbf{b} \in \mathbb{R}^5$, the equation $\mathbf{Ax} = \mathbf{b}$ has at least one solution. Show that this solution is unique.

Same as in assignment 1, Question 7.

3. (*5 points*) Write down the image formation pipeline as a series of linear transformations. For each matrix transformation, write down the form of the matrix and the number of degrees of freedom associated with it.

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} = \begin{bmatrix} f_x & \alpha & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} R_{3\times3} & T_{3\times1} \\ 0\ 0\ 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

3×3 K - intrinsic camera matrix, 5 dof

3×4 Perspective projection matrix. 0 dof

4×4 World to camera Transformation matrix, 6 dof: 3 for rotation, 3 for translation

4×1 3D point in homogen. Co-ordinates

4. (*5 points*) Compute the line that goes through the points with Cartesian coordinates $\mathbf{x}_1 = (1,1)^\top$ and $\mathbf{x}_2 = (3,2)$.

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \times \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix}$$

5. (*5 points*) Find $\mathbf{x}$, the intersection of the two lines $l_1 = (1,2,3)^\top$ and $l_2 = (1,2,1)^\top$. Which point does $\mathbf{x}$ correspond to in $\mathbb{R}^2$?

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \times \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & -3 & 2 \\ 3 & 0 & -1 \\ -2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -4 \\ 2 \\ 0 \end{bmatrix}$$

It is an ideal point, i.e., it corresponds to a point at infinity