

MTH310/520: Submission 1

February 2, 2024

1. (5 points) Let G be a simple connected graph, let $e = xy$ be a cut-edge, and let $H = G \setminus e$. Does H have exactly two components (YES/NO)? Justify your answer.

Solution. Yes.

Since $e = xy$ is the cut edge, x and y belong to different components, say k_1 and k_2 . Without loss of generality let $x \in k_1$ and $y \in k_2$. Let $u \in V(G)$ and P be a x - u path. Then there can be two cases.

1. $e \notin P$. Then P is a x - u path in $G \setminus e$ and hence $u \in k_1$.
2. $e \in P$. Then since $x \in k_1, u \in k_2$.

Thus we conclude that any vertex $u \in V(G)$ lies either in k_1 or in k_2 . Hence the statement.

Rubric: +1 for writing YES. +2 for a correct proof idea. +2 for clarity and preciseness. If someone writes NO, straightforward 0 regardless of the rest of the answer.