MTH 372: Quiz I

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Instructions

- Show all your work to score full marks.
- You can use the following information
  1. The pdf of X from Beta (α, β) is given by

$$f(X) = \frac{x^{(\alpha-1)}(1-x)^{(\beta-1)}}{B(\alpha,\beta)} \quad 0 < x < 1, \;\; \alpha > 0, \;\; \beta > 0.$$

1. (5 points) Let $X_1, \ldots, X_n$ be iid from Beta $(\alpha, \beta), \alpha > 0, \beta > 0$. Answer the following
   (a) Does it belong to the exponential family of distributions.
   (b) Find sufficient statistic(s) for $\boldsymbol{\theta} = (\alpha, \beta)$.
   (c) Find minimal sufficient statistic(s) for $\boldsymbol{\theta} = (\alpha, \beta)$.

2. (2.5 points) Let X be an observation from the pdf

$$f_\theta(x) = \left(\frac{\theta}{2}\right)^{|x|} (1-\theta)^{1-|x|}, x = -1, 0, 1; \; 0 \leq \theta \leq 1.$$

   Is X a complete sufficient statistic.

3. (2.5 points) Let $X \sim f_\mu(x) = e^{-(x-\mu)}, \;\; \mu < x < \infty$.
   Does it belong to the location – scale family? Show the results mathematically.