(1) $X_i \overset{iid}{\sim} P(\lambda)$

The formal def^n of sufficiency is

(Here $T = \Sigma X_i$)

$P(\underset{\sim}{X} = \underset{\sim}{x} \mid T(\underset{\sim}{x}) = t)$: does not depend on θ.

so, $P(\underset{\sim}{X} = \underset{\sim}{x} \mid T(\underset{\sim}{x}) = t)$ or $P(X_1 = x_1, \ldots, X_n = x_n \mid T(\underset{\sim}{x}) = t)$ can be written as

$$P(X_1 = x_1, \ldots, X_n = x_n \mid T(\underset{\sim}{x}) = t) = \frac{P(X_1 = x_1, \ldots, X_n = x_n \cap T(\underset{\sim}{x}) = t)}{P(T(\underset{\sim}{x}) = t)}$$

(In the N^r the intersection is non-zero when $\Sigma x_i = t$)

$$\Rightarrow \quad = \frac{P(X_1 = x_1, \ldots, X_n = x_n)}{P(T(\underset{\sim}{x}) = t)} \quad \text{if } \sum_{i=1} x_i = t$$

(Now, if $X_i \sim P(\lambda)$ then $\Sigma X_i \sim P(n\lambda)$; it can be verified using mgf method).

$$\Rightarrow \quad = \frac{\prod_{i=1}^{n} P(X_i = x_i)}{P(T(\underset{\sim}{x}) = t)}$$

$$= \frac{\prod_{i=1}^{n} \left[e^{-\lambda} \lambda^{x_i} / x_i! \right]}{e^{-n\lambda} (n\lambda)^t / t!}$$

$$= \frac{e^{-n\lambda} \lambda^{\Sigma x_i} / \prod_{i=1}^{n} x_i!}{e^{-n\lambda} (\lambda)^t n^t / t!}$$

$$= \frac{t!}{n^t \prod x_i!} \quad \text{: indep. of } \theta = \lambda$$

Thus $T = \Sigma X_i$ is sufficient for λ.

(2). $X_1, X_2, X_3 \overset{iid}{\sim} B(p)$.

$U = X_1 X_2 + X_3$.

Now $P(U=0) = P(X_1=0, X_2=0, X_3=0) + P(X_1=0, X_2=1, X_3=0)$
$+ P(X_1=1, X_2=0, X_3=0)$

$= q^3 + 2pq^2$

Now, $P(X_1=x_1, X_2=x_2, X_3=x_3 \mid U=u) = \dfrac{P(X_1=x_1, X_2=x_2, X_3=x_3, U=u)}{P(U=u)}$

Now, for $P(X_1=0, X_2=0, X_3=0 \mid U=0) = \dfrac{P(X_1=0, X_2=0, X_3=0)}{P(U=0)}$

$= \dfrac{q^3}{q^3+2pq^2}$

$= \dfrac{q}{q+2p}$; does depend on $\theta = p$

$\Rightarrow U = X_1 X_2 + X_3$ is not sufficient for p.

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[redacted] $X_i \overset{iid}{\sim} P(\lambda)$.

$$P(X_i = x_i) = \frac{e^{-\lambda}\lambda^{x_i}}{x_i!}$$

$$L(\lambda) = \prod_{i=1}^{n} P(X_i = x_i)$$

$$= \frac{e^{-n\lambda}\lambda^{\sum x_i}}{\prod x_i!}$$

$$= \underbrace{\frac{1}{\prod x_i!}}_{\prod_{i=1}^{n} h(x_i)} \; \underbrace{e^{-n\lambda}}_{[c(\theta)]^n} \; \exp\left[\sum x_i \log \lambda\right]$$

Clearly the $L(\lambda)$ likelihood f^n can be written in the form of $L(\theta) = \prod_{i=1}^{n} h(x_i) \; [c(\theta)]^n \; \exp\left[\sum_{j=1}^{k} w_j(\theta) \sum_{i=1}^{n} t_j(x_i)\right]$

Here, $T(X) = \sum X_i$ is sufficient for λ.

(Refer to Q 3 for detailed sol^n.).

$X \sim N(0, \theta)$

Note $\sigma^2 = \theta$.

$$f_\theta(x) = \frac{1}{(2\pi\theta)^{1/2}} \exp\left[-\frac{1}{2}\frac{x^2}{\theta}\right]$$

$$L = f_\theta(x)$$

$$= \frac{1}{(2\pi\theta)^{1/2}} \exp\left[-\frac{1}{2}\frac{x^2}{\theta}\right]$$

$$= \frac{1}{(2\pi\theta)^{1/2}} \exp\left[-\frac{1}{2}\frac{|x|^2}{\theta}\right]$$

$$= g_\theta[T(x)]\, h(x)$$

here $h(x) = 1$

$$g_\theta[T(x)] = \frac{1}{(2\pi\theta)^{1/2}} \exp\left[-\frac{1}{2}\frac{|x|^2}{\theta}\right]$$

Thus by factorization th^m $T(x) = |x|$ is suff for θ

Or. $T(x) = x^2$ is sufficient for θ

Thus, a 1-1 f^n of $T(x) = x^2$ will also be sufficient for θ. And for $T_1(x) = |x|$ there exists one img in the Range. So is a 1-1 f^n of $T(x) = x^2$.

Thus, $T_1(x) = |x|$ will be suff for θ

i) $L = \prod_{i=1}^{n} \frac{\theta}{(1+x_i)^{1+\theta}}$

$$= \theta^n \frac{1}{\prod_{i=1}^{n}(1+x_i)} \frac{1}{\prod_{i=1}^{n}(1+x_i)^{\theta}}$$

$$= \theta^n \frac{1}{\prod_{i=1}^{n}(1+x_i)^{\theta}} \frac{1}{\prod(1+x_i)}$$

$$= g_\theta(T(x))\, h(x)$$

$$h(x) = \frac{1}{\prod_{i=1}^{n}(1+x_i)}$$

$$g_\theta(T(x)) = \frac{\theta^n}{\prod_{i=1}^{n}(1+x_i)^{\theta}}$$

Thus, by factorization thm

$T(x) = \prod_{i=1}^{n}(1+x_i)$ is sufficient for θ.

ii. Now, $U = \sum_{i=1}^{n} \log(1+x_i) = \sum_{i=1}^{n} \log T(x)$ is a

1-1 f^n of T. (As $x_i > 0$)

Thus, $U = \sum \log(1+x_i)$ is also sufficient for θ.

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$$f_\theta(x) = \begin{cases} \frac{1}{\sigma} e^{-\left(\frac{x-\mu}{\sigma}\right)} & \mu < x < \infty \\ 0 & o/w \end{cases}$$

$$I_{(\mu,\infty)}(x) = \begin{cases} 1 & \text{if } \mu < x < \infty \\ 0 & o/w \end{cases}$$

$$L(\theta) = \prod_{i=1}^{n} f_\theta(x_i)$$

$$= \prod_{i=1}^{n} \frac{1}{\sigma} e^{-\left(\frac{x_i-\mu}{\sigma}\right)} I_{(\mu,\infty)}(x_i)$$

$$= \frac{1}{(\sigma)^n} e^{\frac{n\mu}{\sigma}} e^{-\frac{\Sigma x_i}{\sigma}} \prod_{i=1}^{n} I_{(\mu,\infty)}(x_i)$$

$$= \left(\frac{e^{\mu/\sigma}}{\sigma}\right)^n e^{-\frac{\Sigma x_i}{\sigma}} I_{(\mu,\infty)}(x_{(1)})$$

$$= g_\theta[T(x)] \cdot h(x)$$

ere, $h(x) = 1$

$$g_\theta[T(x)] = \left(\frac{e^{\mu/\sigma}}{\sigma}\right)^n e^{-\frac{\Sigma x_i}{\sigma}} I_{(\mu,\infty)}(x_{(1)})$$

Thus, by the factorization th^m

$T(x) = \left(\sum_{i=1}^{n} x_i, x_{(1)}\right)$ are suff for $\theta = (\mu, \sigma)$

$$f_\theta(x_i) = \frac{1}{\Gamma\alpha\, \beta^\alpha} x_i^{\alpha-1} e^{-x_i/\beta}$$

$$L(\theta) = \prod_{i=1}^{n} \frac{1}{\Gamma\alpha\, \beta^\alpha} x_i^{\alpha-1} e^{-x_i/\beta}$$

$$= \frac{1}{(\Gamma\alpha\, \beta^\alpha)^n} \prod_{i=1}^{n} x_i^{\alpha-1} e^{-\frac{\Sigma x_i}{\beta}}$$

$$= \frac{1}{(\Gamma\alpha\, \beta^\alpha)^n} \prod_{i=1}^{n} x_i^{\alpha} e^{-\frac{\Sigma x_i}{\beta}} \prod_{i=1}^{n} \frac{1}{x_i}$$

$$= g_\theta(T(x)) \cdot h(x)$$

$$h(x) = \prod_{i=1}^{n} \frac{1}{x_i}$$

$$g_\theta(T(x)) = \frac{1}{(\Gamma\alpha\, \beta^\alpha)^n} \prod_{i=1}^{n} x_i^{\alpha} e^{-\frac{\Sigma x_i}{\beta}}$$

Thus by factorization Th^{m}

$T(x) = \left(\prod_{i=1}^{n} x_i, \sum_{i=1}^{n} x_i\right)$ is a suff. statisti

for $\theta = (\alpha, \beta)$

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