Computer Vision
Quiz 1 - Jan 21, 2016

Maximum score: 30 **Time: 30 mins**

Name: ______________________________ Roll No: _______________

Instructions:

- Attempt all questions.
- True/False questions without justification will not be awarded any points. Keep in mind that a statement is true only if it is *always* true.
- Do not copy. Institute plagiarism policy is strictly enforced.

1. (*10 points*) State True or False. Justify your answer.

a) The intrinsic camera matrix **K** is always invertible.

Ans. **True**. It is an upper triangular matrix with all its diagonal elements as non-zero. This ensures that the determinant is never 0 which is equivalent to it having 'n' pivots (where n is the dimension of the matrix) which in turn is the only necessary condition for a matrix to be invertible.

b) Every 3×3 orthogonal matrix is a rotation matrix.

Ans. **False**. Determinant of the matrix should be 1 as well.

c) For a VGA resolution (640×480) CCD array, the principal point is always at $(320, 240)$.

Ans. **False**. No, it is not a necessary condition, the principal point depends of the relative position on the camera center and the CCD array.

d) Let $S = \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $R = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$. We define two transformations as $T_1 = SR$ and $T_2 = RS$. T_1 and T_2 are equivalent.

Ans. **True**. Rotation and Scaling are commutative, provided the scaling is uniform in all dimensions.

e) Given a 3×4 camera matrix M, the extrinsic parameters can be recovered by executing linear operations.

Ans. **True**. The camera center is obtained as null(M). By performing RQ decomposition of the first three columns of M, we obtain both the intrinsic(upper triangular) and rotation matrix(orthogonal). The translation vector can be recovered from the last column of M, and the intrinsic matrix and the rotation matrix.

2. (*10 points*) Describe the three components of the image formation pipeline. State the number of degrees of freedom in each component and list the corresponding parameters.

1. Intrinsic Matrix: 5(2: Focal Length- α, β, 2: Principal Offset- C_x, Cy, 1: Skew- s)
2. Perspective Projection Matrix: 0
3. External Matrix: 3(Rotation)+3(Translation)

3. (*10 points*) Find the transformation $\mathbf{T}$ that maps points from the xy coordinate frame to the $x'y'$ coordinate frame. Note that the angle given is between the axes y' and x.
Ans.
We perform a translation first followed by rotation. The points are translated such that $(3, 2) \mapsto (0, 0)$. This gives us the translation vector to be added to the points $\mathbf{t} = [-3, -2]^\top$. The rotation shown in the image is that of rotating the *axes* by a clockwise angle of $\pi/6$. This is equivalent to rotating the *points* in the anti-clockwise direction by $\pi/6$. Therefore the angle of rotation is *positve*. Thus the rotation matrix will be:

$$\mathbf{R} = \begin{bmatrix} cos\theta & -sin\theta & 0 \\ sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

where $\theta = \pi/6$.
Take $\theta = \pi/6$, $\mathbf{t}_x = -3$ and $\mathbf{t}_y = -2$ to get:

$$\mathbf{T} = \begin{bmatrix} cos\theta & -sin\theta & 0 \\ sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & \mathbf{t}_x \\ 0 & 1 & \mathbf{t}_y \\ 0 & 0 & 1 \end{bmatrix} \tag{1}$$

$$\mathbf{T} = \begin{bmatrix} cos\theta & -sin\theta & \mathbf{t}_x cos\theta - \mathbf{t}_y sin\theta \\ sin\theta & cos\theta & \mathbf{t}_x sin\theta + \mathbf{t}_y cos\theta \\ 0 & 0 & 1 \end{bmatrix} \tag{2}$$

$$\mathbf{T} = \begin{bmatrix} cos\theta & -sin\theta & \mathbf{t}_x cos\theta - \mathbf{t}_y sin\theta \\ sin\theta & cos\theta & \mathbf{t}_x sin\theta + \mathbf{t}_y cos\theta \\ 0 & 0 & 1 \end{bmatrix} \tag{3}$$

$$\mathbf{T} = \begin{bmatrix} cos(\pi/6) & -sin(\pi/6) & -3cos(\pi/6) + 2sin(\pi/6) \\ sin(\pi/6) & cos(\pi/6) & -3sin(\pi/6) - 2cos(\pi/6) \\ 0 & 0 & 1 \end{bmatrix} \tag{4}$$

You could very well apply the rotation first and the translation later. However, in that case, $\mathbf{t} \neq [-3, -2]^\top$ and will have to be appropriately adjusted to the new rotated co-ordinate frame. The final transformation $\mathbf{T}$ in either case would be the same as in (4).