Computer Vision - CSE 344/544
Quiz 2 - Feb 17, 2014

Maximum score: 20   **Time: 20 mins**

Name: _____   Roll No: _____

Instructions:

1. You need to solve any four questions out of five. If you solve all five, you get extra credit.

2. Please do not copy. The institute's plagiarism policy is strictly enforced.

1. (*5 points*) Consider a vector $(7, 3, 2)^\top$ which is rotated around the Z axis by $90°$ and then rotated around the Y axis by $90°$ and finally translated by $(4, -3, 7)^\top$. Find the new coordinates of the vector. All rotations are counterclockwise.

$$\begin{bmatrix} \cos 90 & 0 & \sin 90 \\ 0 & 1 & 0 \\ -\sin 90 & 0 & \cos 90 \end{bmatrix} \begin{bmatrix} \cos 90 & -\sin 90 & 0 \\ \sin 90 & \cos 90 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 7 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 7 \\ 3 \\ 2 \end{bmatrix}$$

$$\underbrace{R_Y}_{} \qquad\qquad \underbrace{}_{R_Z} \qquad X \qquad = \qquad R_Y \qquad\qquad R_Z \qquad X$$

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} -3 \\ 7 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 7 \\ 3 \end{bmatrix}$$

$$R_Y \quad R_Z X \qquad\qquad R_Y R_Z X$$

$$R_Y R_Z X + T = \begin{bmatrix} 2 \\ 7 \\ 3 \end{bmatrix} + \begin{bmatrix} 4 \\ -3 \\ 7 \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ 10 \end{bmatrix}$$

2. (*5 points*) Let $\mathbf{A} \in \mathbb{R}^{5\times3}$ and there exists a 3×5 matrix $\mathbf{C}$ such that $\mathbf{CA} = \mathbf{I}$. Say for some given $\mathbf{b} \in \mathbb{R}^5$, the equation $\mathbf{Ax} = \mathbf{b}$ has at least one solution. Show that this solution is unique.

3. (*5 points*) Write down the image formation pipeline as a series of linear transformations. For each matrix transformation, write down the form of the matrix and the number of degrees of freedom associated with it.
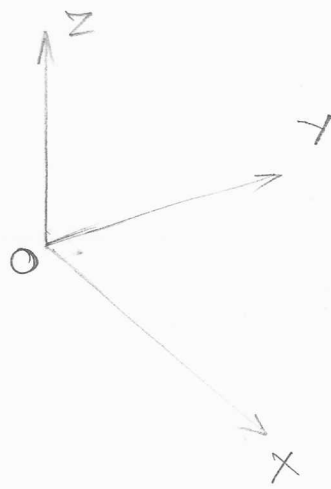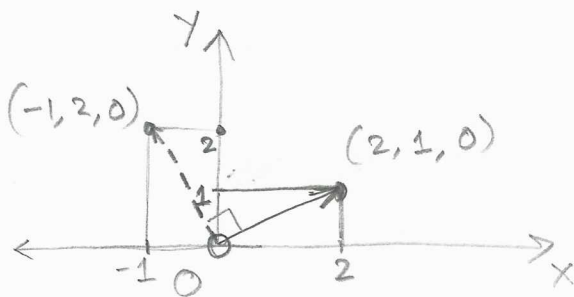
3D co-ordinate frame

Rotation about X

$(0,-1,2)$
$(0,2,1)$
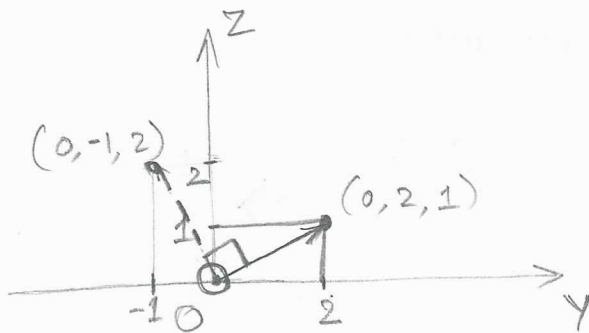
$$R_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix}$$

$\alpha = \pi/2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix}$$
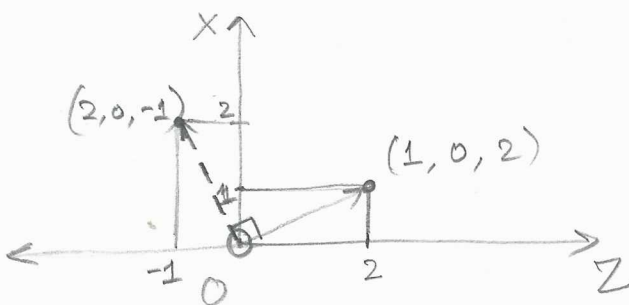
Rotation about Z

$(-1,2,0)$
$(2,1,0)$

$$R_z(\gamma) = \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\gamma = \pi/2$

$$\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}$$

Rotation about Y

$(2,0,-1)$
$(1,0,2)$

$$R_y(\beta) = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}$$

$\beta = \pi/2$

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$$