MTH 377/577 Convex Optimization
Problem set 1

1. Is the set of vectors $\{(a, b) \in R^2 : b = 3a + 1\}$ a vector space?
2. Find the rank of the given matrix:

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 4 \\ 1 & 4 & 1 \end{bmatrix}$$

3. Is the set of all $x \in R^3$ such that $||x|| = 1$ a subspace of R^3?
4. Are the following vectors linearly independent?
   (a) $v_1 = (2, 6, 3), v_2 = (1, -1, 4), v_3 = (3, 2, 1), v_4 = (2, 5, 4)$
   (b) $v_1 = (1, -1, 3, -1), v_2 = (1, -1, 4, 2), v_3 = (1, -1, 5, 7)$
5. Find a basis for the subspace of R^n spanned by the following vectors:
$$\{(1, 3, 3), (-3, -9, -9), (1, 5, -1), (2, 7, 4), (1, 4, 1)\}$$
6. Is the set of possible distances between a point on the circle $x^2 + y^2 = 1$ and a point on the circle $x^2 + y^2 = 100^2$ bounded?
7. Are the following sets open or closed?
   (a) The empty set ϕ
   (b) The complement of $\{x = 5\}$ in R.
8. Suppose f is a real valued continuous function and its domain $D \subset R$ is convex at all points except at $x = 1$. Does f have a maxima or minima?
9. Is the set of points $\{x^k\}_{k=1,2,\ldots}$ where $x^k = k^{\frac{1}{2}}$ for each $k = 1, 2, \ldots$ a closed set?
10. Every affine set is convex. True/False?

1 Indicative answers

1. This is not a vector space. It does not contain the zero vector, and is not closed under either addition or scalar multiplication.
2. 3
3. No. It does not include zero vector and it is not closed under either addition or scalar multiplication.
4. (a) No., (b) Yes.
5. There may be various answers. For example: $\{(1, 3, 3), (1, 5, -1)\}$
6. Yes.
7. (a) both open and closed; (b) open.
8. May or may not have. Since domain is non-convex, we cannot apply Weierstrass' theorem.
9. No. The sequence does not converge to a limit point.
10. True.