MTH310/520: Submission 4

Time: 15 Minutes, Marks: 5

February 23, 2024

Name and Roll No:

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1. (5 points) Use Cayley's formula to prove that the graph obtained from K_n by deleting an edge has $(n-2)n^{n-3}$ spanning trees.

   Solution. We count number of spanning trees containing any specific edge $e \in E$ and then subtract it from the total number of spanning trees of G. By Cayley's formula, there are n^{n-2} spanning trees of K_n. To find number of spanning tree containing a fixed edge $e \in E$, we construct a bipartite graph $B = (X \cup Y, E)$ where the vertices in X corresponds to the set of all possible spanning trees of K_n and Y corresponds to set of all $\binom{n}{2}$ edges in K_n. We add an edge $(u, v) \in B$ if the edge corresponding to $v \in Y$ is contained in tree corresponding to vertex $u \in X$. Observe that each tree has $n-1$ edges, hence degree of each vertex in X is $n-1$. By symmetry, we conclude that each edge belongs to the same number of trees. Suppose each edge belongs to t many trees. Therefore we have $(n-1)n^{n-2} = t\binom{n}{2}$, simplifying which we obtain $t = 2n^{n-3}$. Therefore, total number of trees excluding a specific edge is $n^{n-2} - 2n^{n-3} = (n-2)^{n-3}$