(a) $f_\theta(x) = \theta x^{-2}$

$$L = \prod_{i=1}^{n} \frac{\theta}{x_i^2} \qquad 0<\theta \le x < \infty$$

$$= \theta^n \prod_{i=1}^{n} \frac{1}{x_i^2} I_{(\theta,\infty)}(x_i)$$

$$= \theta^n \prod_{i=1}^{n} \frac{1}{x_i^2} I_{(\theta,\infty)}(x_{(1)})$$

$$= \underbrace{\prod_{i=1}^{n} \frac{1}{x_i^2}}_{h(x)} \quad \underbrace{\theta^n I_{(\theta,\infty)}(x_{(1)})}_{g_\theta[T(x)]}$$

Thus, by factorization th^m.

$T(x) = X_{(1)}$ is sufficient for θ.

(b) $$E(X) = \int_\theta^\infty \frac{x\,\theta}{x^2}\,dx$$

$$= \theta \ln x \Big|_\theta^\infty$$

$$= \theta(\infty - \ln\theta)$$

$$= \infty$$

Thus, method of moment est$^{\underline{s}}$ does not exist. The method fails here.

(5) $X_i \overset{iid}{\sim} P(\lambda)$

Here the unknown parameter is λ. We have to find estimator for λ using MOM.

By the method, for 1-parameter unknown we need 1 eqn. We need to equate first order popn moment to first order sample moment.

Thus

$$E(X) = \bar{X} \quad \text{will be the eqn.}$$

in Poisson we know that

$$E(X) = \lambda$$

$$\Rightarrow \quad \hat{\lambda}_{MOM} = \bar{X}$$

Now, $\lambda > 0$ thus we need its estimator to be positive so

$$\hat{\lambda}_{MOM} = \bar{X} \quad \text{if } \bar{X} > 0.$$