MTH 372: Mid Semester Exam

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Instructions

- Show all your work to score full marks. Incomplete explanations will lead to deduction of marks.
- This is a closed book exam.
- No phones or other electronic devices may be used.

Questions

1. Let $X_1, \ldots, X_n$ be i.i.d. with the following pdf

$$f_\theta(x_i) = \frac{e^{-x_i}}{e^{-\theta} - e^{-b}}, \quad \theta < x_i < b, \; b \text{ known.}$$

Answer the following questions

(a) (2 points) Apart from the data itself, find sufficient statistic(s) for θ.

(b) (2 points) Find minimal sufficient statistic(s) for θ.

2. Let $Y_1, \ldots, Y_n$ be i.i.d. with the following pdf

$$f_\theta(y_i) = \frac{1}{2\beta^3}\, y_i^2\, e^{-y_i/\beta}, \quad y_i > 0, \; \beta > 0.$$

Answer the following questions

(a) (2 points) Find method of moments estimator (MME) for β.

(b) (2 points) Find maximum likelihood estimator (MLE) for β.

(c) (1.5 points) Does it belong to a location-scale family. Explain.

3. Let $X \sim$ Exponential (λ), $\lambda > 0$. Consider $\theta = 1/\lambda$

(a) (1.5 points) Consider $T(X) = X^2$, find bias of $T(X)$.

(b) (2 points) Find MSE of $T(X)$.

(c) (2 points) Can we find Cramer-Rao lower bound. If yes, find it. If not, why not.

4. (5 points) Let $X_1, \ldots, X_n$ be i.i.d. from Rayleigh distribution with pdf

$$f_\theta(x_i) = \frac{2}{\theta}\, x_i\, e^{-x_i^2/\theta}, \quad x_i > 0, \quad \theta > 0.$$

Find the UMVUE for θ.