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Experiment No.	4
Aim	To implement dynamic algorithms.
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Theory/Experiment:

Dynamic Programming :

Dynamic Programming is mainly an optimization over plain recursion. Wherever we see a recursive solution that has repeated calls for same inputs, we can optimize it using Dynamic Programming.

The idea is to simply store the results of subproblems, so that we do not have to re-compute them when needed later. This simple optimization reduces time complexities from exponential to polynomial.

The approach of solving problems using dynamic programming algorithm has following steps :

1. Characterize the structure of an optimal solution.
2. Recursively define the value of an optimal solution.
3. Compute the value of an optimal solution, typically in a bottom-up fashion.
4. Construct an optimal solution from computed information.

Matrix Chain Multiplication :

The main aim is that for the given dimension of a sequence of matrices in an array $dim[]$, where the dimension of the i th matrix is $(dim[i-1] * dim[i])$, the task is to find the most efficient way to multiply these matrices together such that the total number of element multiplications is minimum.

Two matrices of size $m*n$ and $n*p$ when multiplied, they generate a matrix of size $m*p$ and the number of multiplications performed are $m*n*p$.

Now, for a given chain of N matrices, the first partition can be done in $N-1$ ways. For example, sequence of matrices A, B, C and D can be grouped as $(A)(BCD)$, $(AB)(CD)$ or $(ABC)(D)$ in these 3 ways.

So a range $[i, j]$ can be broken into two groups like $\{[i, i+1], [i+1, j]\}, \{[i, i+2], [i+2, j]\}, \dots, \{[i, j-1], [j-1, j]\}$.

Each of the groups can be further partitioned into smaller groups and we can find the total required multiplications by solving for each of the groups.

The minimum number of multiplications among all the first partitions is the required answer.

Algorithm :

MCM(dim)

for $i = 0$ to $n - 1$

$c[i][j] = 0;$

 for $j = 0$ to $n - 1$

$c[i][j] = 0;$

 for $k = 0$ to $n - 2$

$d = d + 1;$

$j = d;$

 for $i = 1$ to $n - d$

 if ($i == j$)

$dp[i][j] = 0;$

$paranthesis[i][j] = 0;$

 else

 for $m = i + 1$ to j

$min = dp[i][m] + dp[m+1][j] + dim[i-1]*dim[m]*dim[j];$

$dp[i][j] = min;$

Code :

```
#include <stdio.h>
#include <stdlib.h>
#include <time.h>
// Print paranthesis recursive function that divides matrices sequence into two parts
void printParenthesis(int i, int j, int n, int paranthesis[][10], char *matrices)
{
    if(i < 0 || j > n) return;

    if(i == j) {
        printf("%c",matrices[i-1]);
        return;
    }

    printf("(");
    printParenthesis(i,paranthesis[i][j],n,paranthesis,matrices);
    printParenthesis(paranthesis[i][j] + 1,j,n,paranthesis,matrices);
    printf(")");

}

int main()
{
    int n; //n = 10
    printf("Enter total no. of Dimensions of Matrices:");
    scanf("%d",&n);
    int *dim = (int *)malloc(sizeof(int) * n);
    //Randomly generated number between 15 and 46
    printf("Randomly Generated Dimension:\n");
    for(int i = 0; i < n; i++) {
        dim[i] = rand() % 46 + 15;
        printf("Dimension at %d index: %d\n",i,dim[i]);
    }
    int dp[n][n], paranthesis[n][n];
    for(int i = 0 ; i < n ; i++) {
        for(int j = 0 ; j < n; j++) {
            dp[i][j] = 0;
            paranthesis[i][j] = 0;
        }
    }
    clock_t start, end;
```

```

double cpu_time_used;
start = clock();
int d = 0;
for(int k = 0; k < n - 1; k++) {
    d = d + 1;
    int j = d;
    for(int i = 1 ; i <= n - d; i++, j++) {
        if(i == j) {
            dp[i][j] = 0;
            paranthesis[i][j] = 0;
        }
        else {
            int m = i;
            int min = dp[i][m] + dp[m+1][j] + dim[i-1]*dim[m]*dim[j];
            paranthesis[i][j] = i;
            for(m = i + 1; m < j; m++) {
                if(min > dp[i][m] + dp[m+1][j] + dim[i-1]*dim[m]*dim[j]) {
                    min = dp[i][m] + dp[m+1][j] + dim[i-1]*dim[m]*dim[j];
                    paranthesis[i][j] = m;
                }
            }
            dp[i][j] = min;
        }
    }
}

printf("\nAuxillary table:\n");
for(int i = 0 ; i < n ; i++) {
    for(int j = 0 ; j < n; j++) {
        printf("%d\t",dp[i][j]);
    }
    printf("\n");
}

printf("\nTable for storing value of k:\n");
for(int i = 0 ; i < n ; i++) {
    for(int j = 0 ; j < n; j++) {
        printf("%d\t",paranthesis[i][j]);
    }
    printf("\n");
}

```

```
int i = 1, j = n - 1;
char matrices[9] = "ABCDEFGHI";
printf("\nOptimal cost for multiplication of 10 Matrices : %d\n",dp[1][9]);
printf("\nOptimal Paranthesizing : ");
printParenthesis(i, j, n, paranthesis,matrices);
end = clock();
cpu_time_used = ((double) (end - start)) / CLOCKS_PER_SEC;
printf("\n\nTime taken for calculating optimal cost and paranthesization is:
%fs",cpu_time_used);
return 0;
}
```

Output :

```
Command Prompt
D:\>cd Assignments
D:\Assignments>cd DAA Assignments
D:\Assignments\DAA Assignments>cd daa codes
D:\Assignments\DAA Assignments\daa codes>gcc MCM.c
D:\Assignments\DAA Assignments\daa codes>a
Enter total no. of Dimensions of Matrices:10
Randomly Generated Dimension:
Dimension at 0 index: 56
Dimension at 1 index: 36
Dimension at 2 index: 47
Dimension at 3 index: 19
Dimension at 4 index: 48
Dimension at 5 index: 53
Dimension at 6 index: 39
Dimension at 7 index: 25
Dimension at 8 index: 21
Dimension at 9 index: 53

Auxillary table:
0 0 0 0 0 0 0 0 0
0 0 94752 70452 121524 175180 199557 203186 204957 264100
0 0 0 32148 64980 116736 146433 155382 162621 202689
0 0 0 0 42864 95665 122436 128459 134862 184585
0 0 0 0 0 48336 87609 106134 116109 137256
0 0 0 0 0 0 99216 115275 117306 170730
0 0 0 0 0 0 0 51675 63882 122871
0 0 0 0 0 0 0 0 20475 63882
0 0 0 0 0 0 0 0 0 27825
0 0 0 0 0 0 0 0 0 0

Table for storing value of k:
0 0 0 0 0 0 0 0 0 0
0 0 1 1 3 3 3 3 1 3
0 0 0 2 3 3 3 3 3 8
0 0 0 0 3 3 3 3 3 3
0 0 0 0 0 4 5 6 7 8
0 0 0 0 0 0 5 5 5 8
0 0 0 0 0 0 0 6 6 8
0 0 0 0 0 0 0 0 7 8
0 0 0 0 0 0 0 0 0 8
0 0 0 0 0 0 0 0 0 0

Optimal cost for multiplication of 10 Matrices : 264100

Optimal Paranthesizing : ((A(BC))((((DE)F)G)H)I))

Time taken for calculating optimal cost and paranthesization is: 0.049000s
D:\Assignments\DAA Assignments\daa codes>
```

Time Complexity : $O(n^3)$

CONCLUSION :

By performing this experiment, I understood the concept of matrix chain multiplication which is used to find an optimal cost and parenthesization order in which the matrix should be multiply. The time complexity of this algorithm is $O(n^3)$, where n is the number of matrices, but it can be reduced to $O(n^2)$ using memoization.