Time Series

Fundamentals

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What is a Time Series?

- A **time series** is a sequence of numerical **data** points in successive order, usually occurring in uniform intervals.
- **Time series *analysis*** comprises methods for analyzing time series data in order to extract meaningful statistics and other characteristics of the data.
- **Time series *forecasting*** is the use of a model to predict future values based on previously observed values.

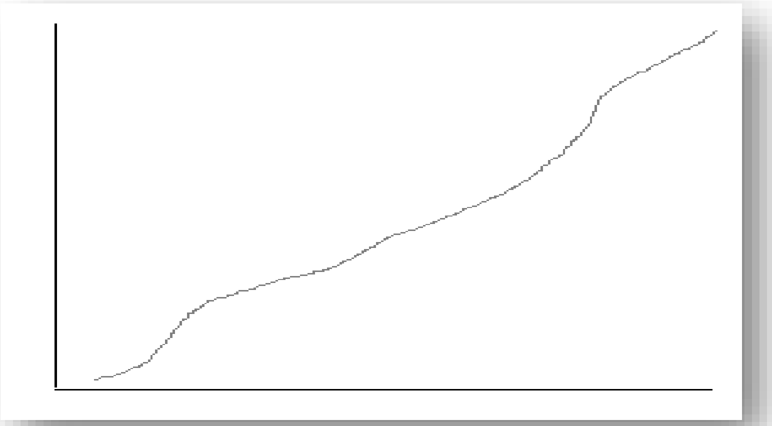
Courtesy: Wikipedia

Assumptions in Time Series Algorithms

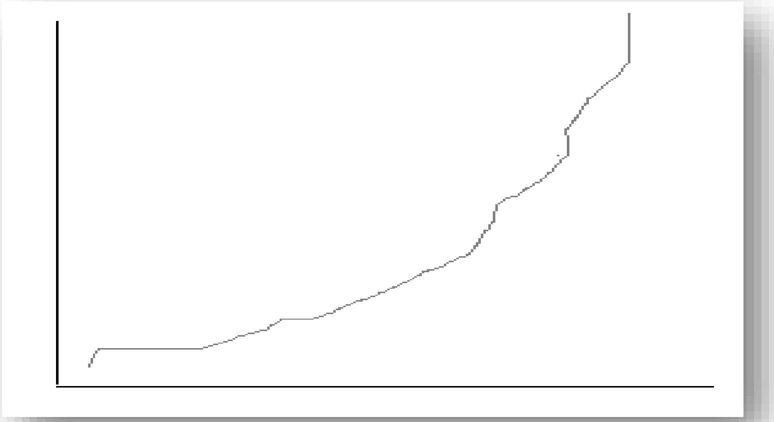
- Consecutive Observations in the series are equally spaced
- Series is indexed on specific period of time. e.g. Weekly, Daily, Yearly etc.
- There aren’t any missing values

Types of Trends

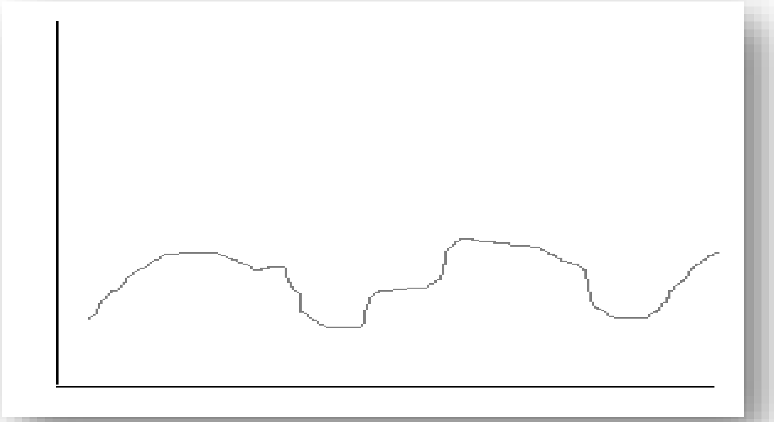
Linear



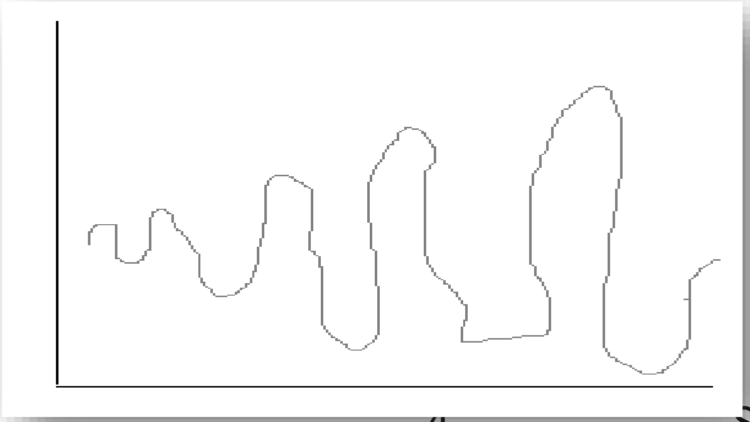
Rapid Growth



Periodic



Varying Variance



Some Transformations

- log: The log() function can linearize the rapid growth trend. It can also stabilize the varying variance series. It is only for positive values.
- diff: The diff() function can remove the linear trends. It can also remove periodic trends.
- Box-Cox Transformation:

```
y = (x**lmbda - 1) / lmbda  if lmbda != 0
    log(x)                  if lmbda == 0
```

Resampling

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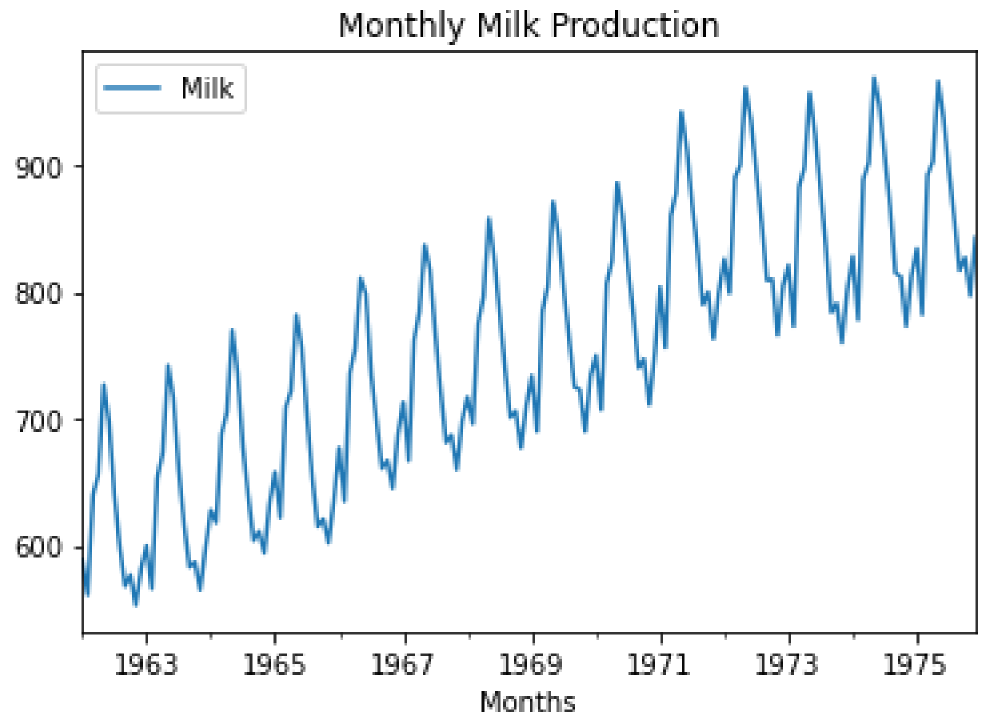
Resampling

- In order to make the data consistent with time, resampling can be done.
- e.g. We may be given monthly data and want to change it into quarterly data

Monthly Data



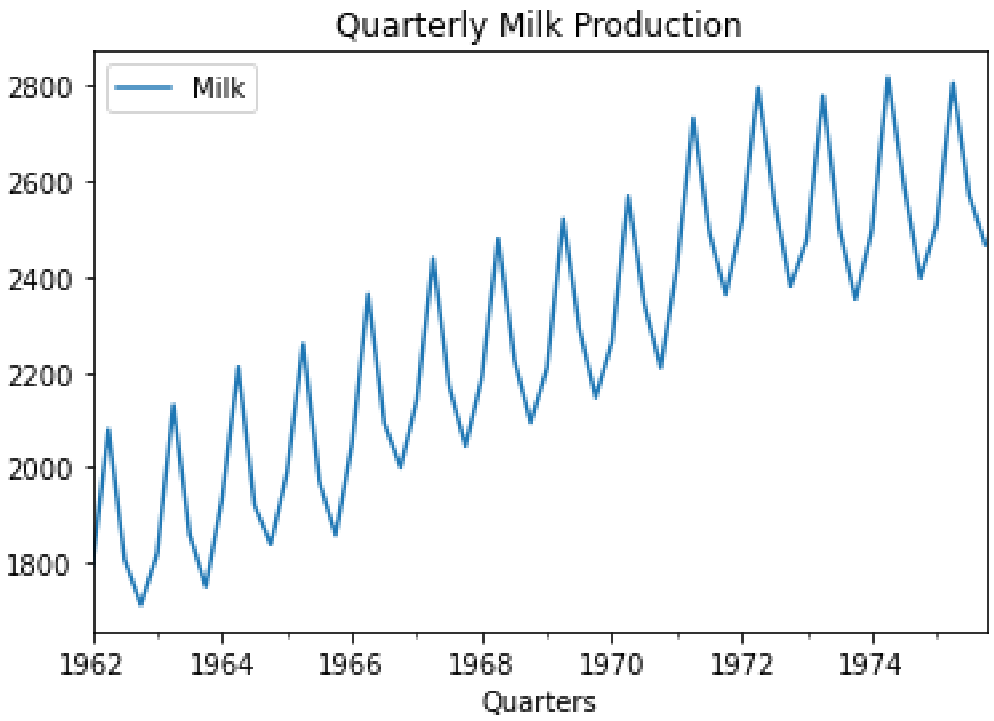
Month	1962-01	1962-02	1962-03	1962-04	...	1975-09	1975-10	1975-11	1975-12
Milk	589	561	640	656	...	817	827	797	843

Resampling Example

```
downsampled  = df.resample('Q').sum()
```

```
downsampled.index.rename('Quarter', inplace=True)
downsampled.plot()
plt.title("Quarterly Milk Production")
plt.xlabel("Quarters")
plt.show()
```

Quarterly Data



Quarter	1962Q1	1962Q2	1962Q3	1962Q4	...	1975Q1	1975Q2	1975Q3	1975Q4
Milk	1790	2080	1807	1712	...	2508	2806	2571	2467

$\underbrace{35 = 25 + 5}+5$ or

$35 = 5 \times 7$

Components of Time Series

- **Trend**: Indicates a long term increase or decrease in the data. It may be linear or non-linear.
- **Seasonal**: Seasonality is a pattern observed with regular intervals of time. e.g. Sale of woolen clothes increases in winter and is relatively low in other seasons.
- **Cyclic**: Data exhibits rise and fall not in regular time intervals. e.g. Recession and Boom
- **Random**: This is an error component. Also called irregular component.

Classical Decomposition

- There are two types of classical decompositions:
  - Additive
  - Multiplicative
- We assume here that the seasonal component is constant from year to year.
- Suppose that we have *m* seasonal periods. Then there are m seasonal values which are called *seasonal indices.*

Notations

- y_t: Value in time series at time t
- $\widehat{T_t}$: Trend-cycle component (Moving Average) calculated for time t
- $\widehat{S_t}$: Seasonal Index for time t

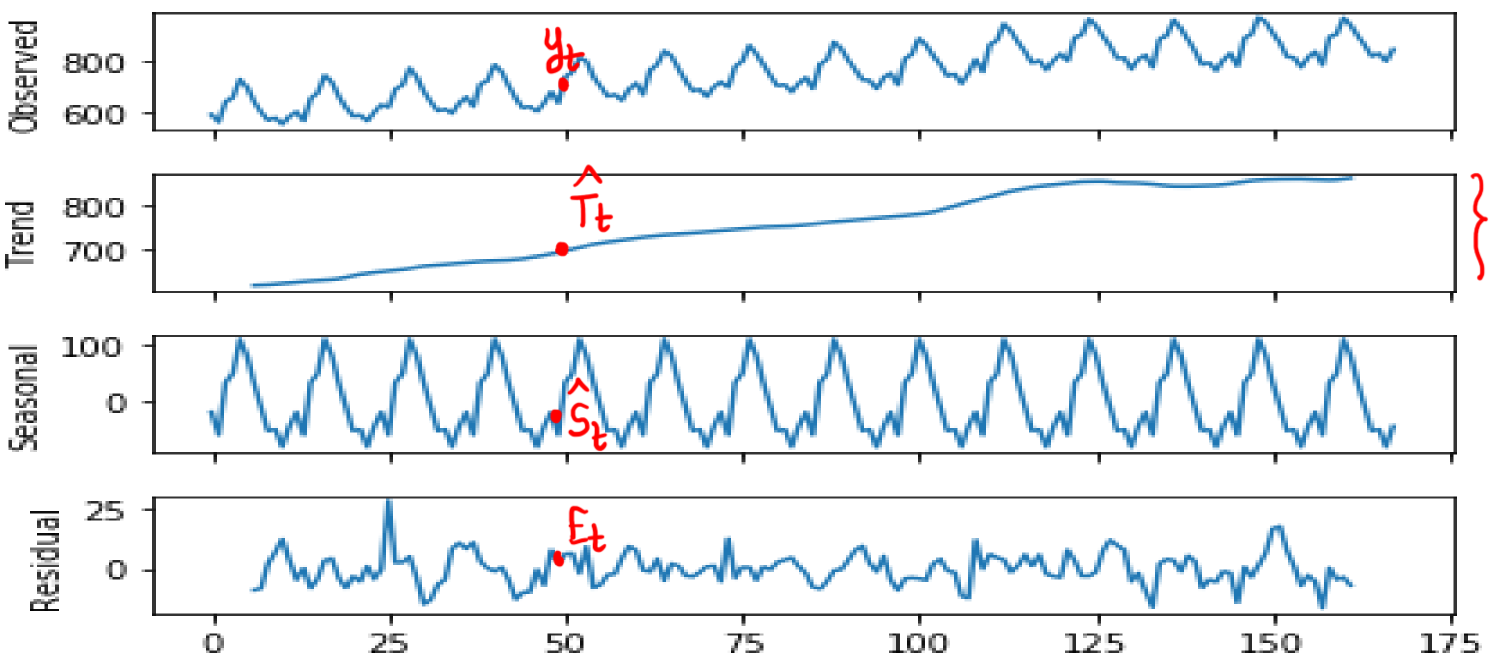
Additive Decomposition

1. If m is even number, then centered MA is calculated otherwise non-centered MA is calculated.
2. Calculate the de-trended series, y_t-$\widehat{T_t}$
3. For estimating the seasonal component for each month, a simple average is calculated for detrended values for that particular month. It is denoted by $\widehat{S_t}$.
4. The random component is calculated by subtracting seasonal and trend-cycle components. $\widehat{E_t} = \widehat{y_t}$-$\widehat{T_t}$-$\widehat{S_t}$

Example

```
In [55]: from statsmodels.tsa.seasonal import seasonal_decompose
    ...: from matplotlib import pyplot
    ...: series = df['Milk']
    ...: result = seasonal_decompose(series, model='additive',freq=12)

In [56]: result.plot()
    ...: pyplot.show()
```



$$y_t = \hat{T}_t + \hat{S}_t + E_t$$

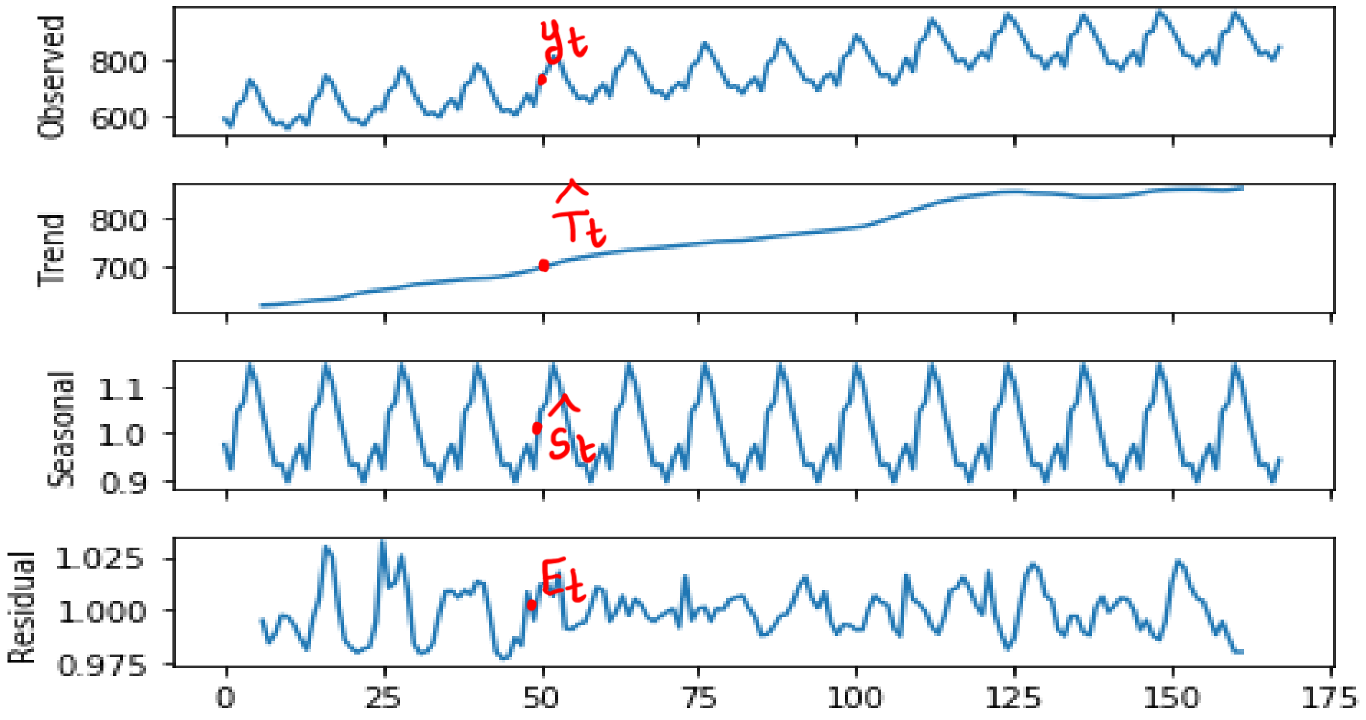
Multiplicative Decomposition

1. If m is even number, then centered MA is calculated otherwise non-centered MA is calculated.
2. Calculate the de-trended series, $y_t/\widehat{T_t}$
3. For estimating the seasonal component for each month, a simple average is calculated for de-trended values for that particular month. It is denoted by $\widehat{S_t}$.
4. The random component is calculated by subtracting seasonal and trend-cycle components. $\widehat{E_t} = \widehat{y_t}/(\widehat{T_t}\widehat{S_t})$

Example

```
In [57]: result = seasonal_decompose(series, model='multiplicative',freq=12)

In [58]: result.plot()
    ...: pyplot.show()
```



$$y_t = \hat{T}_t \hat{S}_t E_t$$

e.g.

$$105 = 7 \times 5 \times 5$$

Questions?