CHAPTER : 4 INVERSE TRIGONOMETRIC FUNCTIONS

Exercise 4.1

1. Find all the values of x such that

(i) $-10\pi \le x \le 10\pi$ and $\sin x = 0$

$\sin\theta = \sin\alpha$
$\theta = n\pi + (-1)^n\alpha$
$n \in R$

Soln:-
$\sin x = 0$
$\sin x = \sin 0$
$x = n\pi + (-1)^n(0)$
$= n\pi, \quad n = 0, \pm1, \pm2 \ldots \pm10.$

(ii) $-8\pi \le x \le 8\pi$ and $\sin x = -1$

Soln
$\sin x = -1$
$\sin x = -\sin \pi/2 = \sin(-\pi/2)$
$x = (4n-1)\frac{\pi}{2}, \quad n = 0, \pm1, \pm2, \pm3, 4$

நா. காமாட்சி, M.Sc., B.Ed., M.Phil., Ph.D.
முதுகலை ஆசிரியர் (கணிதம்)
அரசு மேல்நிலைப்பள்ளி
கோவிந்தவாடி, காஞ்சிபுரம் (Dt)

2. Find the period and amplitude of -

(i) $y = \sin 7x$.

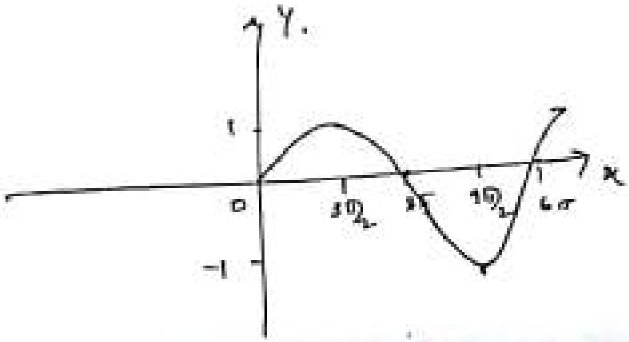
For amplitude use the form $y = a\sin(bx-c)+d$.
amplitude $= |a|$
$a = 1 \quad \therefore |a| = 1$
Period using the formula $\frac{2\pi}{|b|} = \frac{2\pi}{|7|} = \frac{2\pi}{7}$
$\therefore$ amplitude $= 1$
period $= \frac{2\pi}{7}$

(ii) $y = -\sin(\frac{1}{3}x)$
$a = -1 \quad b = 1/3$
$\therefore$ amplitude $|a| = |-1| = 1$
period $= \frac{2\pi}{|b|} = \frac{2\pi}{|1/3|} = 3(2\pi) = 6\pi$

(iii) $y = 4\sin(-2x)$
Here $a = 4 \quad b = -2$
$\therefore$ amplitude $= |4| = 4$
period $= \frac{2\pi}{|b|} = \frac{2\pi}{|-2|} = \frac{2\pi}{2} = \pi$

3) Sketch the graph of $y = \sin(\frac{1}{3}x)$ for $0 \le x \le 6\pi$.

x	$f(x)$
0	0
$3\pi/2$	1
3π	0
$9\pi/2$	-1
6π	0



Scanned by CamScanner

③

4. Find the value of (i) $\sin^{-1}(\sin \frac{2\pi}{3})$

Soln

$y = \sin^{-1}(\sin 2\pi/3)$

$= \sin^{-1}[\sin(\pi - \pi/3)] = \sin^{-1}(\sin \pi/3) = \pi/3$

(ii) $\sin^{-1}[\sin(5\pi/4)]$

$y = \sin^{-1}[\sin(5\pi/4)] = \sin^{-1}[\sin(\pi + \pi/4)] = \sin^{-1}(-\sin \pi/4)$

$= \sin^{-1}[\sin(-\pi/4)] = -\pi/4.$

5. For what value of x does $\sin x = \sin^{-1} x$?

Soln $\sin 0 = 0$

$\sin^{-1}(0) = 0$ $\boxed{\therefore x = 0}$

6) Find the domain of the following.

(i) $f(x) = \sin^{-1}\left(\frac{x^2+1}{2x}\right)$

Soln. Range of $\sin^{-1} x$ is $[-1, 1]$.

$-1 \le \frac{x^2+1}{2x} \le 1.$

Multiply by $2x^2 \ge 0$.

$-2x^2 \le x(x^2+1) \le 2x^2$

$-2x^2 \le x(x^2+1)$

$x(x^2+1) + 2x^2 \ge 0$

$x(x^2+1+2x) \ge 0$

$x(x+1)^2 \ge 0$

$x \ge 0$ $(x+1)^2 \ge 0$

$x = -1.$

$x(x^2+1) - 2x^2 \le 0$

$x[x^2+1-2x] \le 0$

$x(x-1)^2 \le 0$

$x \le 0$ $x = 1.$

$x = 0$ doesn't play with us this time

$\therefore$ solution is $\{-1, 1\}$

(ii) $g(x) = 2\sin^{-1}(2x-1) - \pi/4.$

Soln Range of $\sin^{-1} x$ is $[-1, 1]$.

$-1 \le 2x-1 \le 1$

$-1 \le 2x-1$

$2x-1 \ge -1$ $\Rightarrow$ $2x \ge -1+1$

$2x \ge 0$ $\Rightarrow$ $x \ge 0.$

$$2x-1 \le 1$$
$$2x \le 2$$
$$x \le 1$$

$x \geq 0$ & $x \le 1$ $\therefore$ solution is $[0,1]$.

7) Find the value of $\sin^{-1}\left(\sin\frac{5\pi}{9}\cos\frac{\pi}{9} + \cos\frac{5\pi}{9}\sin\frac{\pi}{9}\right)$

Soln.

$\sin(A+B) = \sin A\cos B + \cos A \sin B$.

$A = \frac{5\pi}{9}$ $B = \pi/9$.

$= \sin^{-1}\left(\sin\left(\frac{5\pi}{9}+\pi/9\right)\right)$

$= \sin^{-1}\left[\sin\left(\frac{6\pi}{9}\right)\right] = \sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right)$

$= \sin^{-1}\left(\sin\left(\pi - \pi/3\right)\right)$

$= \sin^{-1}\left(\sin \pi/3\right) = \pi/3$.

நா. காமாட்சி, M.Sc., B.Ed., M.Phil., Ph.D.
முதுகலை ஆசிரியர் (கணிதம்)
அரசு மேல்நிலைப்பள்ளி
கோவிந்தவாடி, காஞ்சிபுரம் (Dt)

Scanned by CamScanner

Exercise 4.2

1. Find all values of x such that (i) $-6\pi \le x \le 6\pi$ and $\cos x = 0$

Soln
$\cos x = 0$
$\cos x = \cos \pi/2$
$x = (2n+1)\pi/2, \quad n = 0, \pm1, \pm2, \pm3, \pm4, \pm5$.

(ii) $-5\pi \le x \le 5\pi$, and $\cos x = 1$.

Soln
$\cos x = 1$
$\cos x = \cos 0$
$x = (2n+1)\pi, \quad n = 0, \pm1, \pm2, -3$.

2. State the reason for $\cos^{-1}[\cos(-\pi/6)] \ne -\pi/6$

Soln
$\cos^{-1}[\cos(-\pi/6)] = \cos^{-1}[\cos \pi/6]$ $\quad [\because \cos(-\theta) = \cos\theta]$
$= \pi/6 \ne -\pi/6$.

3. Is $\cos^{-1}(-x) = \pi - \cos^{-1}x$ true? Justify your answer.

Soln. $\cos^{-1}(-x) = \pi - \cos^{-1}x$.
Take $x = \cos\theta \Rightarrow \theta = \cos^{-1}x$
$\cos^{-1}(-x) = \pi - \theta$
$\cos(\pi - \theta) = -\cos\theta = -x \in [-1,1]$
$\pi - \cos^{-1}x = \pi - \cos^{-1}x = \pi - \theta$.
$\pi - \cos^{-1}x = \cos^{-1}(-x)$ is true.

4) Find the principal value of $\cos^{-1}(1/2)$

Soln $y = \cos^{-1}\left(\frac{1}{2}\right)$ $\qquad \cos^{-1}x$ range is $[0, \pi]$.
$\cos y = 1/2 = \cos \pi/3$.
$y = \pi/3 \in [0, \pi]$
Principal value is $\pi/3$

5) Find the value of (i) $2\cos^{-1}(1/2) + \sin^{-1}\left(\frac{1}{2}\right)$

Soln
$y = \cos^{-1}\left(\frac{1}{2}\right)$
$\cos y = \frac{1}{2} = \cos \pi/3$
$y = \pi/3 \in [0, \pi]$

$x = \sin^{-1}\left(\frac{1}{2}\right)$
$\sin x = 1/2 = \sin \pi/6$
$x = \pi/6 \in [0, \pi]$

$2\cos^{-1}\left(\frac{1}{2}\right)+\sin^{-1}\left(\frac{1}{2}\right) = 2\left(\frac{\pi}{3}\right)+\frac{\pi}{6} = \frac{2\pi}{3}+\frac{\pi}{6} = \frac{4\pi+\pi}{6}$ 6

$= 5\pi/6.$

ii) $\cos^{-1}\left(\frac{1}{2}\right)+\sin^{-1}(-1)$

$x = \cos^{-1}\left(\frac{1}{2}\right)$

$\cos x = \frac{1}{2} = \cos\pi/3$

$x = \pi/3$

$y = \sin^{-1}(-1)$

$\sin y = -1 = \sin(-\pi/2)$

$y = -\pi/2.$

$\cos^{-1}\left(\frac{1}{2}\right)+\sin^{-1}(-1) = \pi/3 - \pi/2 = \frac{2\pi-3\pi}{6} = -\pi/6.$

iii) $\cos^{-1}\left[\cos\pi/7\cos\pi/17 - \sin\frac{\pi}{7}\sin\frac{\pi}{17}\right]$

$[\because \cos(A+B) = \cos A\cos B - \sin A\sin B]$

$= \cos^{-1}\left[\cos\left(\frac{\pi}{7}+\frac{\pi}{17}\right)\right]$

$= \cos^{-1}\left[\cos\left(\frac{17\pi+7\pi}{119}\right)\right]$

$= \cos^{-1}\left(\cos\left(\frac{24\pi}{119}\right)\right) = \frac{24\pi}{119}.$

6) Find the domain of i) $f(x) = \sin^{-1}\left(\frac{|x|-2}{3}\right)+\cos^{-1}\left(\frac{1-|x|}{4}\right)$

Soln $-1 \le \sin^{-1} \le 1.$

$-1 \le \frac{|x|-2}{3} \le 1$

$-3 \le |x|-2 \le 3.$

$-3+2 \le |x| \le 3+2$

$-1 \le |x| \le 5$

Since $-1 \le |x|$ is not possible

$|x| \le 5$

$-5 \le x \le 5$

$\therefore x \in [-5, 5].$

நா. காமாட்சி, M.Sc., B.Ed., M.Phil., Ph.D.
முதுகலை ஆசிரியர் (கணிதம்)
அரசு மேல்நிலைப்பள்ளி
கோவிந்தவாடி, காஞ்சிபுரம் (Dt)

Scanned by CamScanner

7

(ii) $g(x) = \sin^{-1}x + \cos^{-1}x$.

Range of $\sin x$ and $\cos x$ is $[-1, 1]$

$-1 \le x \le 1$.

$\therefore x \in [-1, 1]$.

7) For what value of x the inequality $\frac{\pi}{2} < \cos^{-1}(3x-1) < \pi$ holds

Soln

$\frac{\pi}{2} < \cos^{-1}(3x-1) < \pi$

$\cos\frac{\pi}{2} < (3x-1) < \cos\pi$

$0 < 3x-1 < -1$

$0 < 3x < -1+1$

$0+1 < 3x < 0$

$1 < 3x < 0$

$0 < 3x < 1$

$0 < x < \frac{1}{3}$ $\therefore 0 < x < \frac{1}{3}$

8) Find the value of (i) $\cos\left(\cos^{-1}\left(\frac{4}{5}\right) + \sin^{-1}\left(\frac{4}{5}\right)\right)$

We know that $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$.

$\cos\left(\cos^{-1}\left(\frac{4}{5}\right) + \sin^{-1}\left(\frac{4}{5}\right)\right) = \cos\frac{\pi}{2} = 0$.

(ii) $\cos^{-1}\left(\cos\frac{4\pi}{3}\right) + \cos^{-1}\left(\cos\frac{5\pi}{4}\right)$

$\cos\frac{4\pi}{3} = \cos\left(\pi + \frac{\pi}{3}\right) = -\cos\left(\frac{\pi}{3}\right) = \cos\left(-\frac{\pi}{3}\right) = \cos\frac{\pi}{3}$

$\cos\frac{5\pi}{4} = \cos\left(\pi + \frac{\pi}{4}\right) = -\cos\frac{\pi}{4} = \cos\left(-\frac{\pi}{4}\right) = \cos\frac{\pi}{4}$

$\therefore \cos^{-1}\left(\cos\frac{4\pi}{3}\right) + \cos^{-1}\left(\cos\frac{5\pi}{4}\right) = +\frac{\pi}{3} + \frac{\pi}{4}$

$= \cos^{-1}\left(\cos\left(+\frac{\pi}{3}\right)\right) + \cos^{-1}\left(\cos\left(+\frac{\pi}{4}\right)\right)$

$= \frac{+4\pi + 3\pi}{12} = \frac{7\pi}{12}$.

Scanned by CamScanner

Exercise 4.3

(8)

1. Find the domain of following functions

(i) $\tan^{-1}\sqrt{9-x^2}$ (ii) $\frac{1}{2}\tan^{-1}(1-x^2)-\pi/4$.

Soln

$9-x^2 \geq 0$

$9 \geq x^2$.

$x^2 \leq 9$

$x \leq \pm 3$.

domain $[-3,3]$

(ii) Range of $\tan^{-1}x$ is R

$-\infty < 1-x^2 < \infty$

$-\infty < -x^2 < \infty$

$-\infty < x < \infty$

$x \in R$.

domain $= R$

நா. காமாட்சி, M.Sc., B.Ed., M.Phil., Ph.D.
முதுகலை ஆசிரியர் (கணிதம்)
அரசு மேல்நிலைப்பள்ளி
கோவிந்தவாடி, காஞ்சிபுரம் (Dt)

2) Find the value of (i) $\tan^{-1}(\tan 5\pi/4)$.

Soln $\tan^{-1}(\tan(\frac{5\pi}{4})) = \tan^{-1}[\tan(\pi+\pi/4)]$

$= \tan^{-1}[\tan\pi/4] = \pi/4$.

(ii) $\tan^{-1}(\tan(-\pi/6)) = -\pi/6$.

3) Find the value of

(i) $\tan[\tan^{-1}(\frac{7\pi}{4})] = 7\pi/4$.

We know that $\tan(\tan^{-1}(x)) = x$.

(ii) $\tan[\tan^{-1}[1947]] = 1947$.

(iii) $\tan[\tan^{-1}(-0.2021)] = -0.2021$.

4) Find the value of

(i) $\tan[\cos^{-1}(\frac{1}{2})-\sin^{-1}(\frac{-1}{2})]$

Soln $\tan\left[\cos^{-1}\left(\frac{1}{2}\right) - \sin^{-1}\left(\frac{-1}{2}\right)\right]$

$= \tan\left[\cos^{-1}\frac{1}{2} + \sin^{-1}\frac{1}{2}\right]$ $\left[\because \sin^{-1}(-x) = -\sin^{-1}x\right]$

$= \tan\left(\frac{\pi}{2}\right) = \infty.$ $\left[\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}\right]$

(ii) $\sin\left[\tan^{-1}\left(\frac{1}{2}\right) - \cos^{-1}\left(\frac{4}{5}\right)\right]$

$a = \tan^{-1}\frac{1}{2}$.

$\tan a = \frac{1}{2} > 0$. (lies in I & III quadrant]

$b = \cos^{-1}\left(\frac{4}{5}\right)$

$\cos b = \frac{4}{5} > 0$. (lies in I & IV quadrant only].

$\sin\left(\tan^{-1}\left(\frac{1}{2}\right) - \cos^{-1}\left(\frac{4}{5}\right)\right] = \sin(a-b]$

$= \sin a \cos b - \cos a \sin b$ $\left[\because \sin(A-B) = \sin A\cos B - \cos A \sin B\right]$

$= \frac{1}{\sqrt{5}}\left(\frac{4}{5}\right) - \frac{2}{\sqrt{5}}\cdot\frac{3}{5}$

$= \frac{4}{5\sqrt{5}} - \frac{6}{5\sqrt{5}} = \frac{-2}{5\sqrt{5}}$.

நா. காமாட்சி, M.Sc., B.Ed., M.Phil., Ph.D.
முதுகலை ஆசிரியர் (கணிதம்)
அரசு மேல்நிலைப்பள்ளி
கோவிந்தவாடி, காஞ்சிபுரம் (Dt)

(iii) $\cos\left[\sin^{-1}\left(\frac{4}{5}\right) - \tan^{-1}\left(\frac{3}{4}\right)\right]$

Soln $a = \sin^{-1}\left(\frac{4}{5}\right)$

$\sin a = \frac{4}{5} > 0$ (lies in I & II quadrant only]

$\cos a = \pm\frac{3}{5}$

opp = 4
Hyp = 5
Adj = $\sqrt{5^2-4^2} = \sqrt{25-16}$
$= \sqrt{9} = 3$.

$b = \tan^{-1}\left(\frac{3}{4}\right) \Rightarrow \tan b = \frac{3}{4}$.

$\sin b = \frac{3}{5}$

opp = 3
adj = 4
Hyp = $\sqrt{4^2+3^2} = \sqrt{16+9}$
$= \sqrt{25} = 5$

$\cos\left[\sin^{-1}\left(\frac{4}{5}\right) - \tan^{-1}\left(\frac{3}{4}\right)\right]$

$= \cos(a-b) = \cos a \cos b + \sin a \sin b$

$= \left(\frac{3}{5}\right)\left(\frac{4}{5}\right) + \left(\frac{4}{5}\right)\left(\frac{3}{5}\right) = \frac{12}{25} + \frac{12}{25} = \frac{24}{25}$.

Scanned by CamScanner

(10)

Exercise 4.4

1. Find the principal value of

(i) $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$

Soln $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = \alpha$

$\sec\alpha = \frac{2}{\sqrt{3}} = \sec \pi/6$

$\alpha = \pi/6$.

(ii) $\cot^{-1}(\sqrt{3})$

soln $\cot^{-1}(\sqrt{3}) = \alpha$.

$\sqrt{3} = \cot\alpha$

$\cot \pi/6 = \cot\alpha$

$\alpha = \pi/6$

(iii) $\operatorname{cosec}^{-1}(-\sqrt{2})$

soln $\operatorname{cosec}^{-1}(-\sqrt{2}) = \alpha$

$\operatorname{cosec}\alpha = -\sqrt{2} = -\operatorname{cosec}(\pi/4) = \operatorname{cosec}(-\pi/4)$

$\alpha = -\pi/4$.

2) Find the value of

(i) $\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$

soln $x = \tan^{-1}(\sqrt{3})$

$\tan x = \sqrt{3} = \tan \pi/6$

$x = \pi/6$

$y = \sec^{-1}(-2)$

$\sec y = -2 = -\sec \pi/3$

$\sec y = \sec(-\pi/3)$

$\sec y = \sec(\cancel{0} + \pi/3)$

$y = (\pi/3)$

$[\because \sec(-\theta) = \sec\theta]$

$\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2) = \pi/6 - \pi/3$

$= \frac{\pi - 2\pi}{6} = -\pi/6$.

(ii) $\sin^{-1}(-1) + \cos^{-1}\left(\frac{1}{2}\right) + \cot^{-1}(2)$.

$x = \sin^{-1}(-1)$

$\sin x = -1 = \sin(-\pi/2)$

$x = -\pi/2$.

Scanned by CamScanner

11

$y = \cos^{-1}\left(\frac{1}{2}\right)$

$\cos y = \frac{1}{2} = \cos \pi/3$

$y = \pi/3$.

$z = \cot^{-1}(2)$

$\cot z = 2$

$z = \cot^{-1}(2)$. is constant.

$\sin^{-1}(-1) + \cos^{-1}\left(\frac{1}{2}\right) + \cot^{-1}(2)$

$= -\pi/2 + \pi/3 + \cot^{-1}(2)$

$= \frac{-3\pi + 2\pi}{6} + \cot^{-1}(2) = -\frac{\pi}{6} + \cot^{-1}(2)$

3) $\cot^{-1}(1) + \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) - \sec^{-1}(-\sqrt{2})$

Soln

$x = \cot^{-1}(1)$

$\cot x = 1 = \cot \pi/4$

$x = \pi/4$

$y = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

$\sin y = -\frac{\sqrt{3}}{2} = \sin(-\pi/3)$

$y = -\pi/3$

$z = \sec^{-1}(-\sqrt{2})$ ~~sec~~

$\sec z = -\sqrt{2}$

$\sec z = -\sec \pi/4 = \sec(-\pi/4) = \sec \pi/4$

$z = +\pi/4$ $(\sec(-\theta) = \sec\theta)$

$\cot^{-1}(1) + \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) - \sec^{-1}(-\sqrt{2})$

$= \pi/4 - \pi/3 - \pi/4 = -\pi/3$.

நா. காமாட்சி, M.Sc., B.Ed., M.Phil., Ph.D.
முதுகலை ஆசிரியர் (கணிதம்)
அரசு மேல்நிலைப்பள்ளி
கோவிந்தவாடி, காஞ்சிபுரம் (Dt)

Exercise 4.5 (12)

1. Find the value if it exists If not, give the reason for non existence

(i) $\sin^{-1}(\cos\pi)$ (ii) $\tan^{-1}(\sin(-5\pi/2))$ (iii) $\sin^{-1}(\sin 5)$

Soln (i) $\sin^{-1}(\cos\pi) = \sin^{-1}(-1) = -\sin^{-1}(1) = -\pi/2$

(ii) $\tan^{-1}(\sin(-5\pi/2)) = \tan^{-1}[-\sin 5\pi/2]$

$= \tan^{-1}[-\sin(2\pi+\pi/2)] = \tan^{-1}(-\sin \pi/2)$

$= \tan^{-1}(-1) = -\tan^{-1}(1) = -\pi/4.$

(iii) $\sin^{-1}(\sin 5)$

$-\pi/2 \le \sin^{-1} x \le \pi/2$ ~~$\cancel{\sin(\pi/2)} \le 2\pi$~~.

$-3\pi/2 \le 5 \le 2\pi$

$-\pi/2 \le 5-2\pi \le 0 \le \pi/2$

$\sin(5-2\pi) = \sin 5$

$\sin^{-1}(\sin 5) = 5-2\pi$

நா. காமாட்சி, M.Sc., B.Ed., M.Phil., Ph.D.
முதுகலை ஆசிரியர் (கணிதம்)
அரசு மேல்நிலைப்பள்ளி
கோவிந்தவாடி, காஞ்சிபுரம் (Dt)

2) Find the value of the expression in terms of x with the help of a reference triangle

(i) $\sin(\cos^{-1}(1-x))$

$= \sin[\cos^{-1}(\text{side of adj}/\text{side of Hyp})]$ $\left[\because \cos\left(\frac{adj}{Hyp}\right) = \frac{1-x}{1}\right.$ adj $= 1-x$ Hyp $= 1$

$= \frac{opp}{Hyp} = \frac{\sqrt{2x-x^2}}{1} = \sqrt{2x-x^2}.$

opp $= \sqrt{1^2-(1-x)^2}$
$= \sqrt{1-(1+x^2-2x)}$
$= \sqrt{1-1-x^2+2x}$
$= \sqrt{2x-x^2}\,]$

(ii) $\cos(\tan^{-1}(3x-1))$

$= \cos[\text{side of opp}/\text{side of adj}]$

$= \frac{Adj}{Hyp} = \frac{1}{\sqrt{9x^2-6x+2}}$

$[\because$ opp $= 3x-1$
adj $= 1$
Hyp $= \sqrt{(3x-1)^2+1^2}$
$= \sqrt{9x^2+1-6x+1}$
$= \sqrt{9x^2-6x+2}\,]$

(iii) $\tan(\sin^{-1}(x+1/2)) = \tan\left(\sin^{-1}\left(\frac{2x+1}{2}\right)\right)$

$\tan[\text{side of opp}/\text{side of Hyp}]$

$= \frac{opp}{adj} = \frac{2x+1}{\sqrt{3-4x-4x^2}}$

$[\because$ opp $= 2x+1$ Hyp $= 2$.
adj $= \sqrt{2^2-(2x+1)^2}$
$= \sqrt{4-(4x^2+1+4x)}$
$= \sqrt{4-4x^2-1-4x}$
$= \sqrt{3-4x-4x^2}\,]$

(iii) $2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$.

Soln W.K.T $2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2}$

$$2\tan^{-1}(\cos x) = \tan^{-1}\left(\frac{2\cos x}{1-\cos^2 x}\right) = \tan^{-1}\left(\frac{2\cos x}{\sin^2 x}\right)$$

$$= \tan^{-1}\left(2\frac{\cos x}{\sin^2 x}\right) = \tan^{-1}(2\operatorname{cosec} x)$$

$$\cancel{2}\frac{\cos x}{\sin^2 x} = \cancel{2}\operatorname{cosec} x$$

$$\frac{\cos x}{\sin^2 x} = \frac{1}{\sin x}$$

$$\cot x = 1 = \cot \pi/4.$$

$$x = \pi/4$$

(iv) $\cot^{-1}x - \cot^{-1}(x+2) = \pi/12, \quad x > 0$

Soln $\tan^{-1}\frac{1}{x} - \tan^{-1}\frac{1}{x+2} = \frac{\pi}{12} = \frac{180}{12} = 15^\circ$

$$\tan^{-1}\left(\frac{\frac{1}{x} - \frac{1}{x+2}}{1+\frac{1}{x}\cdot\frac{1}{x+2}}\right) = 15^\circ$$

நா. காமாட்சி, M.Sc., B.Ed., M.Phil., Ph.D.
முதுகலை ஆசிரியர் (கணிதம்)
அரசு மேல்நிலைப்பள்ளி
கோவிந்தவாடி, காஞ்சிபுரம் (Dt)

$$\tan^{-1}\frac{\frac{(x+2)-(x)}{x(x+2)}}{\frac{x(x+2)+1}{x(x+2)}} = 15^\circ$$

$$\frac{\cancel{x}+2-\cancel{x}}{x^2+2x+1} = \tan 15^\circ = \frac{\sqrt3-1}{\sqrt3+1}$$

$$\frac{2}{(x+1)^2} = \frac{\sqrt3-1}{\sqrt3+1}\,\frac{(\sqrt3+1)}{(\sqrt3+1)} = \frac{(\sqrt3)^2-1^2}{(\sqrt3+1)^2}$$

$$\frac{2}{(x+1)^2} = \frac{2}{(\sqrt3+1)^2}.$$

Compare both sides

$$\boxed{x = \sqrt3}$$

3) Find the value of (i) $\sin^{-1}\left(\cos\left(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)\right)$

Soln

$$\sin^{-1}\left[\cos\left(\sin^{-1}\frac{\sqrt{3}}{2}\right)\right] = \sin^{-1}\left[\cos \pi/3\right] = \sin^{-1}\left[\frac{1}{2}\right]$$

$$= \frac{\pi}{6}$$

(ii) $\cot\left[\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{4}{5}\right]$

$\cot\left[\sin^{-1}\frac{3}{5} + \cos^{-1}\frac{3}{5}\right]$

$\cot\left(\frac{\pi}{2}\right) = 0$ $\left[\because \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}\right]$

$\sin^{-1}\frac{4}{5} = \frac{opp}{hyp}$

$adj = \sqrt{5^2 - 4^2}$

$= \sqrt{25-9} = \sqrt{16}$

$= 3$

$\cos^{-1}\frac{adj}{hyp} = \cos^{-1}\frac{3}{5}$

(iii) $\sin^{-1}\frac{3}{5} = a$ & $\cot^{-1}\frac{3}{2} = b$

$\sin a = 3/5$ $\cot b = 3/2 \Rightarrow \tan b = \frac{2}{3}$

$\cos a = \sqrt{1-\sin^2 a} = \sqrt{1-9/25} = \sqrt{\frac{16}{25}} = \frac{4}{5}$.

(iii) $\tan\left(\sin^{-1}\frac{3}{5} + \cot^{-1}\frac{3}{2}\right) = \tan(a+b)$

$$= \frac{\tan a + \tan b}{1 - \tan a \tan b} = \frac{3/4 + 2/3}{1 - \frac{3}{4}\times\frac{2}{3}}$$

$\left[\because \tan a = \frac{\sin a}{\cos a} = \frac{3/5}{4/5} = \frac{3}{4}\right]$

$$= \frac{\frac{9+8}{12}}{\frac{12-6}{12}} = \frac{17}{6}.$$

நா. காமாட்சி, M.Sc., B.Ed., M.Phil., Ph.D.
முதுகலை ஆசிரியர் (கணிதம்)
அரசு மேல்நிலைப்பள்ளி
கோவிந்தவாடி, காஞ்சிபுரம் (Dt)

4) Prove that (i) $\tan^{-1}\frac{2}{11} + \tan^{-1}\frac{7}{24} = \tan^{-1}\frac{1}{2}$

Soln

$$\tan^{-1}A + \tan^{-1}B = \tan^{-1}\left(\frac{A+B}{1-AB}\right)$$

$$\tan^{-1}\left(\frac{2}{11}\right) + \tan^{-1}\left(\frac{7}{24}\right) = \tan^{-1}\left(\frac{\frac{2}{11} + \frac{7}{24}}{1 - \frac{2}{11}\cdot\frac{7}{24}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{48+77}{11\times 24}}{\frac{264-14}{11\times 24}}\right) = \tan^{-1}\left(\frac{125}{250}\right)$$

$$= \tan^{-1}\left(\frac{1}{2}\right).$$

Scanned by CamScanner

15

(ii) $\sin^{-1}\frac{3}{5} - \cos^{-1}\frac{12}{13} = \sin^{-1}\frac{16}{65}$

Soln.

$x = \sin^{-1}\frac{3}{5}$

$\sin x = \frac{3}{5}$

$\cos x = \frac{4}{5}$

$y = \cos^{-1}\frac{12}{13}$

$\cos y = \frac{12}{13}$

$\sin y = \frac{5}{13}$

$\sin x = \frac{3}{5} = \frac{opp}{hyp}$

$adj = \sqrt{5^2-3^2} = \sqrt{25-9} = \sqrt{16} = 4$

$\cos y = \frac{12}{13} = \frac{adj}{hyp}$

$opp = \sqrt{13^2-12^2} = \sqrt{169-144} = \sqrt{25} = 5.$

$\sin^{-1}\left(\frac{3}{5}\right) - \cos^{-1}\left(\frac{12}{13}\right) =$

Let

$\sin(x-y) = \sin x\cos y - \cos x\sin y$

$= \frac{3}{5}\left(\frac{12}{13}\right) - \frac{4}{5}\left(\frac{5}{13}\right) = \frac{36}{65} - \frac{20}{65} = \frac{16}{65}$

$\sin(x-y) = \frac{16}{65}$

$(x-y) = \sin^{-1}\left(\frac{16}{65}\right)$

5) Prove that $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \tan^{-1}\left(\frac{x+y+z-xyz}{1-xy-yz-zx}\right)$

Soln

$\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \tan^{-1}\left(\frac{x+y}{1-xy}\right) + \tan^{-1}z$

$= \tan^{-1}\left(\frac{\frac{x+y}{1-xy} + z}{1 - \left(\frac{x+y}{1-xy}\right)z}\right)$

நா. காமாட்சி, M.Sc., B.Ed., M.Phil., Ph.D.
முதுகலை ஆசிரியர் (கணிதம்)
அரசு மேல்நிலைப்பள்ளி
கோவிந்தவாடி, காஞ்சிபுரம் (Dt)

$= \tan^{-1}\left(\frac{\frac{x+y+z(1-xy)}{1-xy}}{\frac{(1-xy)-(xz+yz)}{1-xy}}\right) = \tan^{-1}\left(\frac{x+y+z-xyz}{1-xy-yz-xz}\right)$

6) If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$ show that $x+y+z = xyz$.

soln $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi.$

$\tan^{-1}\left(\frac{x+y}{1-xy}\right) + \tan^{-1}z$

$\tan^{-1}\left(\frac{\frac{x+y}{1-xy} + z}{1 - \left(\frac{x+y}{1-xy}\right)\cdot z}\right) = \pi$

$$\tan^{-1}\left(\frac{\frac{x+y+z(1-xy)}{1-xy}}{\frac{1-xy-(xz+yz)}{1-xy}}\right)=\pi$$

$$\frac{x+y+z-xyz}{1-xy-xz-yz}=\tan\pi=0.$$

$$x+y+z-xyz=0$$

$$\boxed{x+y+z=xyz}$$

7) Prove that $\tan^{-1}x+\tan^{-1}\frac{2x}{1-x^2}=\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$ $|x|<\frac{1}{\sqrt{3}}$.

Soln

$$\tan^{-1}x+\tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

$$=\tan^{-1}\left[\frac{x+\frac{2x}{1-x^2}}{1-x\left(\frac{2x}{1-x^2}\right)}\right]=\tan^{-1}\left(\frac{\frac{x(1-x^2)+2x}{\cancel{1-x^2}}}{\frac{1-x^2-2x^2}{\cancel{1-x^2}}}\right)$$

$$=\tan^{-1}\left(\frac{x-x^3+2x}{1-3x^2}\right)$$

if $3x^2<1$
$x^2<\frac{1}{3}$
$|x|<\frac{1}{\sqrt{3}}$

$$=\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$$

நா. காமாட்சி, M.Sc., B.Ed., M.Phil., Ph.D.
முதுகலை ஆசிரியர் (கணிதம்)
அரசு மேல்நிலைப்பள்ளி
கோவிந்தவாடி, காஞ்சிபுரம் (Dt)

8) Simplify $\tan^{-1}\frac{x}{y}-\tan^{-1}\left(\frac{x-y}{x+y}\right)$

Soln

$$\tan^{-1}\left(\frac{x}{y}\right)-\tan^{-1}\left(\frac{x-y}{x+y}\right)$$

$$=\tan^{-1}\left(\frac{\frac{x}{y}-\frac{x-y}{x+y}}{1+\frac{x}{y}\cdot\left(\frac{x-y}{x+y}\right)}\right)=\tan^{-1}\left(\frac{\frac{x(x+y)-y(x-y)}{\cancel{y(x+y)}}}{\frac{y(x+y)+x(x-y)}{\cancel{y(x+y)}}}\right)$$

$$=\tan^{-1}\left(\frac{x^2+\cancel{xy}-\cancel{xy}+y^2}{\cancel{xy}+y^2+x^2-\cancel{xy}}\right)=\tan^{-1}\left(\frac{x^2+y^2}{x^2+y^2}\right)=\tan^{-1}(1)$$

$$=\pi/4.$$

17

9) Solve: (i) $\sin^{-1}\frac{5}{x} + \sin^{-1}\frac{12}{x} = \frac{\pi}{2}$.

Soln $\sin^{-1} 5/x = \pi/2 - \sin^{-1}\frac{12}{x}$

$\pi/2 - (\cos^{-1} 5/x) = \pi/2 - \sin^{-1}\frac{12}{x}$

$\cos^{-1} 5/x = \sin^{-1}\frac{12}{x}$

$\cos\theta = 5/x$ $\quad \sin\theta = \frac{12}{x}$

$\sin^2\theta + \cos^2\theta = 1$

$\left(\frac{5}{x}\right)^2 + \left(\frac{12}{x}\right)^2 = 1$

$\frac{25}{x^2} + \frac{144}{x^2} = 1$

$\frac{169}{x^2} = 1$

$x^2 = 169$

$x = \pm 13.$

(ii) $2\tan^{-1}x = \cos^{-1}\left(\frac{1-a^2}{1+a^2}\right) - \cos^{-1}\left(\frac{1-b^2}{1+b^2}\right)$ $a > 0$.

Soln
W.K.T $2\tan^{-1}x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

நா. காமாட்சி, M.Sc., B.Ed., M.Phil., Ph.D.
முதுகலை ஆசிரியர் (கணிதம்)
அரசு மேல்நிலைப்பள்ளி
கோவிந்தவாடி, காஞ்சிபுரம் (Dt)

$\cos^{-1}\left(\frac{1-a^2}{1+a^2}\right) = 2\tan^{-1}a$

$\cos^{-1}\left(\frac{1-b^2}{1+b^2}\right) = 2\tan^{-1}b$

RHS
$\cos^{-1}\left(\frac{1-a^2}{1+a^2}\right) - \cos^{-1}\left(\frac{1-b^2}{1+b^2}\right) = 2\tan^{-1}a - 2\tan^{-1}b$

$= 2\left[\tan^{-1}a - \tan^{-1}b\right]$

$= 2\left(\tan^{-1}\frac{a-b}{1+ab}\right)$

LHS = RHS

$\cancel{2}\tan^{-1}\left(\frac{a-b}{1+ab}\right) = \cancel{2}\tan^{-1}x$

$x = \frac{a-b}{1+ab}.$ $\quad a > 0.$

18

16) Find the number of solution of the equation $\tan^{-1}(x-1) + \tan^{-1}x + \tan^{-1}(x+1) = \tan^{-1}3x$.

Soln $\tan^{-1}(x-1) + \tan^{-1}x + \tan^{-1}(x+1)$

$= \tan^{-1}(x-1) + \tan^{-1}(x+1) + \tan^{-1}x$

$= \tan^{-1}\left(\frac{x-1+x+1}{1-(x-1)(x+1)}\right) + \tan^{-1}x$

$= \tan^{-1}\left(\frac{2x}{1-(x^2-1)}\right) + \tan^{-1}x$

$= \tan^{-1}\left(\frac{2x}{1-x^2+1}\right) + \tan^{-1}x = \tan^{-1}\left(\frac{2x}{2-x^2}\right) + \tan^{-1}x$

$= \tan^{-1}\left(\frac{\frac{2x}{2-x^2} + x}{1 - \frac{2x}{2-x^2}\cdot x}\right) = \tan^{-1}\left(\frac{\frac{2x+2x-x^3}{\cancel{2-x^2}}}{\frac{2-x^2-2x^2}{\cancel{2-x^2}}}\right)$

$= \tan^{-1}\left(\frac{4x-x^3}{2-3x^2}\right)$

Given LHS = RHS

$\cancel{\tan^{-1}}\left(\frac{4x-x^3}{2-3x^2}\right) = \cancel{\tan^{-1}}3x$

$\frac{4x-x^3}{2-3x^2} = 3x$

$4x - x^3 = 6x - 9x^3$

$8x^3 = 2x$

$8x^3 - 2x = 0$

$2x(x^2-1) = 0$

$x = 0$ $\quad$ $x^2 = 1$

$x = \pm 1.$

number of solutions are 0, 1, −1.

நா. காமாட்சி, M.Sc., B.Ed., M.Phil., Ph.D.
முதுகலை ஆசிரியர் (கணிதம்)
அரசு மேல்நிலைப்பள்ளி
கோவிந்தவாடி, காஞ்சிபுரம் (Dt)

Exercise 4.6

Choose the correct or the most suitable answer from the given four alternatives.

1. The value of $\sin^{-1}(\cos x)$ $0 \le x \le \pi$ is

Soln $\sin^{-1}(\cos x) = \sin^{-1}(\sin(\pi/2 - x)) = \pi/2 - x$

Ans (3) $\pi/2 - x$.

2. If $\sin^{-1}x + \sin^{-1}y = \frac{2\pi}{3}$ Then $\cos^{-1}x + \cos^{-1}y$ is equal to.

Soln let $\sin^{-1}x + \cos^{-1}x + \cos^{-1}y + \sin^{-1}y = \pi/2 + \pi/2 = \pi$

$= 2\pi/3 + \cos^{-1}x + \cos^{-1}y = \pi$

$\cos^{-1}x + \cos^{-1}y = \pi - 2\pi/3 = \frac{3\pi - 2\pi}{3} = \pi/3$

Ans (2) $\pi/3$

3. $\sin^{-1}\frac{3}{5} - \cos^{-1}\frac{12}{13} + \sec^{-1}\frac{5}{3} - \operatorname{cosec}^{-1}\frac{13}{12}$ is equal to.

$= \sin^{-1}\frac{3}{5} - \sec^{-1}\frac{13}{12} + \cos^{-1}\frac{3}{5} - \operatorname{cosec}^{-1}\frac{13}{12}$

$= (\sin^{-1}\frac{3}{5} + \cos^{-1}3/5) - (\sec^{-1}\frac{13}{12} + \operatorname{cosec}^{-1}\frac{13}{12})$

$= \pi/2 - \pi/2 = 0$

Ans (3) 0.

4) If $\sin^{-1}x = 2\sin^{-1}\alpha$ has a solution then.

$-\pi/2 \le 2\sin^{-1}\alpha \le \pi/2$

$-\pi/4 \le \sin^{-1}\alpha \le \pi/4$

$\sin(-\pi/4) \le \alpha \le \sin\pi/4$

$-\frac{1}{\sqrt{2}} \le \alpha \le \frac{1}{\sqrt{2}}$

$|\alpha| \le \frac{1}{\sqrt{2}}$.

Ans (1) $|\alpha| \le \frac{1}{\sqrt{2}}$

நா. காமாட்சி, M.Sc., B.Ed., M.Phil., Ph.D.
முதுகலை ஆசிரியர் (கணிதம்)
அரசு மேல்நிலைப்பள்ளி
கோவிந்தவாடி, காஞ்சிபுரம் (Dt)

5) $\sin^{-1}(\cos x) = \pi/2 - x$ is valid for

$\cos x = \sin(\pi/2 - x)$.

$\cos x \in [0, \pi]$ $\therefore 0 \le x \le \pi$

Ans (1) $0 \le x \le \pi$

6) If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{3\pi}{2}$. The value of

$x^{2017} + y^{2018} + z^{2019} - \frac{9}{x^{101} + y^{101} + z^{101}}$ is

Given $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = 3\pi/2$ $\quad$ $\left(\because \sin^{-1}(1) = \pi/2 \quad \pi/2 + \pi/2 + \pi/2 = 3\pi/2 \right)$

$\therefore x = y = z = 1$

$x^{2017} + y^{2018} + z^{2019} - \frac{9}{x^{101} + y^{101} + z^{101}}$

$= 1+1+1 - \frac{9}{1+1+1} = 3 - \frac{9}{3} = 3 - 3 = 0$

Ans (1) 0

7) If $\cot^{-1}x = \frac{2\pi}{5}$ for some $x \in R$ The value of $\tan^{-1}x$ is

Soln $\tan^{-1}x + \cot^{-1}x = \pi/2$.

$\tan^{-1}x = \pi/2 - \cot^{-1}x = \pi/2 - \frac{2\pi}{5} = \frac{5\pi - 4\pi}{10} = \frac{\pi}{10}$

Ans (3) $\pi/10$

8) The domain of the function defined by $f(x) = \sin^{-1}\sqrt{x-1}$ is

Soln $f(x) = \sin^{-1}\sqrt{x-1}$

Soln $\sqrt{x-1} \geq 0 \quad -1 \leq \sqrt{x-1} \leq 1$

$\therefore 0 \leq \sqrt{x-1} \leq 1$

$0 \leq x-1 \leq 1$

$1 \leq x \leq 2$

$x \in [1,2]$.

நா. காமாட்சி, M.Sc., B.Ed., M.Phil., Ph.D.
முதுகலை ஆசிரியர் (கணிதம்)
அரசு மேல்நிலைப்பள்ளி
கோவிந்தவாடி, காஞ்சிபுரம் (Dt)

Ans (1) $[1,2]$

9) If $x = 1/5$ the value of $\cos(\cos^{-1}x + 2\sin^{-1}x)$ is

Soln $\cos[\cos^{-1}x + \sin^{-1}x + \sin^{-1}x] = \cos(\pi/2 + \sin^{-1}x)$ $\quad$ $[\because \cos(90+\theta) = -\sin\theta]$

$= -\sin(\sin^{-1}x)$

$= -x = -\frac{1}{5}$ $\quad$ Ans (4) $-\frac{1}{5} = x$.

10) $\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{2}{9}$ is equal to.

Soln $\tan^{-1}\left(\frac{\frac{1}{4} + \frac{2}{9}}{1 - \frac{1}{4}\cdot\frac{2}{9}}\right) = \tan^{-1}\left(\frac{\frac{9+8}{36}}{\frac{36-2}{36}}\right) = \tan^{-1}\left(\frac{17}{34}\right)$

$= \tan^{-1}\left(\frac{1}{2}\right)$ Ans (4) $\tan^{-1}\frac{1}{2}$. (21)

11) If the function $f(x) = \sin^{-1}(x^2-3)$ then x belongs to

Soln $-1 \le x^2-3 \le 1$

$-1 \le x^2-3 \le 1$

$-1+3 \le x^2 \le 1+3 \Rightarrow 2 \le x^2 \le 4$

$\pm\sqrt{2} \le x \le \pm 2$.

$\therefore [-2,-\sqrt{2}] \cup [\sqrt{2}, 2]$ Ans (3)

12) If $\cot^{-1} 2$ and $\cot^{-1} 3$ are two angles of a triangle then the third angle is

Soln $A+B+C = \pi$ (triangle.

$\cot^{-1} 2 + \cot^{-1} 3 + C = \pi$

$\cot^{-1}\left(\frac{1-2(3)}{2+3}\right) + C = \pi$

$\cot^{-1}\left(\frac{1-6}{5}\right) + C = \pi$

$\cot^{-1}\left(\frac{-5}{5}\right) + C = \pi \Rightarrow \cot^{-1}(-1) + C = \pi$

$\pi/4 + C = \pi$

$C = \pi - \pi/4 = 3\pi/4$ Ans (2)

13) $\sin^{-1}(\tan \pi/4) - \sin^{-1}\left(\sqrt{\frac{3}{x}}\right) = \pi/6$ Then x is a root of the equation.

Soln $\sin^{-1}(1) - \sin^{-1}\left(\sqrt{\frac{3}{x}}\right) = \pi/6$

$\pi/2 - \sin^{-1}\sqrt{\frac{3}{x}} = \pi/6$

$\cos^{-1}\sqrt{\frac{3}{x}} = \pi/6$.

$\sqrt{\frac{3}{x}} = \cos \pi/6 = \frac{1}{2}$

$\frac{\sqrt{3}}{\sqrt{x}} = \frac{1}{2}$

$\sqrt{x} = 2\sqrt{3}$

$x = 4(3) = 12$

$x = 12$.

நா. காமாட்சி, M.Sc., B.Ed., M.Phil., Ph.D.
முதுகலை ஆசிரியர் (கணிதம்)
அரசு மேல்நிலைப்பள்ளி
கோவிந்தவாடி, காஞ்சிபுரம் (Dt)

14) $\sin^{-1}(2\cos^2 x-1)+\cos^{-1}(1-2\sin^2 x) = \sin^{-1}(2\cos^2 x-1)+\cos^{-1}(1-\sin^2 x-\sin^2 x)$

22

$= \sin^{-1}(2\cos^2 x-1)+\cos^{-1}[(\cos^2 x-(1-\cos^2 x)]$

$= \sin^{-1}(2\cos^2 x-1)+\cos^{-1}(\cos^2 x-1+\cos^2 x)$

$= \sin^{-1}(2\cos^2 x-1)+\cos^{-1}(2\cos^2 x-1)$

$= \pi/2$ $[\because \sin^{-1}x+\cos^{-1}x = \pi/2]$

Ans (3)

15) If $\cot^{-1}\sqrt{\sin\alpha}+\tan^{-1}\sqrt{\sin\alpha} = u$ then $\cos 2u$ is equal to

Soln $\cot^{-1}x+\tan^{-1}x = \pi/2$

$\therefore u = \pi/2$

$\cos 2u = \cos 2(\pi/2) = \cos\pi = -1$ Ans (5)

16) If $|x|\le 1$ Then $2\tan^{-1}x - \sin^{-1}\frac{2x}{1+x^2}$ is equal to

Soln $\sin^{-1}\frac{2x}{1+x^2} = 2\tan^{-1}x$ Ans (3)

$\therefore 2\tan^{-1}x - 2\tan^{-1}x = 0$

17) The eqn $\tan^{-1}x-\cot^{-1}x = \tan^{-1}\left(\frac{1}{\sqrt3}\right) = \pi/6$

soln $\tan^{-1}x+\cot^{-1}x = \pi/2$.

Add (2) & (1) $2\tan^{-1}x = \pi/6+\pi/2 = 2\pi/3$.

$\tan^{-1}x = \pi/3$

$x = \sqrt3$ Ans (2)

which is unique solution.

18) If $\sin^{-1}x+\cot^{-1}\left(\frac12\right) = \pi/2$ Then x is equal to.

$\sin^{-1}x+\cos^{-1}\left(\frac{1}{\sqrt5}\right) = \pi/2$

$\cot^{-1}\frac12 = \frac{opp}{adj}$

$Hyp = \sqrt{2^2+1^2} = \sqrt5$

$\cos^{-1}\frac{adj}{Hyp} = \frac{1}{\sqrt5}$

w.k.T $\sin^{-1}x+\cos^{-1}x = \pi/2$

$\therefore x = \frac{1}{\sqrt5}$

Ans. $\frac{1}{\sqrt5}$.

நா. காமாட்சி, M.Sc., B.Ed., M.Phil., Ph.D.
முதுகலை ஆசிரியர் (கணிதம்)
அரசு மேல்நிலைப்பள்ளி
..., காஞ்சிபுரம் (Dt)

19) If $\sin^{-1}\frac{x}{5} + \operatorname{cosec}^{-1}\frac{5}{4} = \frac{\pi}{2}$ then the values of x. 23

$\operatorname{cosec}\alpha = \frac{hyp}{opp} = \frac{5}{4}$

$adj = \sqrt{5^2-4^2}$

$= \sqrt{25-16} = \sqrt{9} = 3$

Soln $\sin^{-1}\frac{x}{5} + \cos^{-1}\frac{3}{5} = \frac{\pi}{2}$

$= \sin^{-1}\frac{x}{5} + \cancel{\frac{\pi}{2}} - \sin^{-1}3/5 = \cancel{\pi/2}$

$\sin^{-1}\frac{x}{5} = \sin^{-1}\frac{3}{5}$

$\frac{x}{\cancel{5}} = \frac{3}{\cancel{5}} \Rightarrow x = 3$ Ans (4)

20) $\sin(\tan^{-1}x)$, $|x|<1$ is equal to.

Soln Let $a = \tan^{-1}x$

$\tan a = x$

w.k.T $1+\tan^2 a = \sec^2 a$

$1+x^2 = \sec^2 a$

$\sec a = \sqrt{1+x^2}$

$\frac{1}{\cos a} = \sqrt{1+x^2}$

$\cos a = \frac{1}{\sqrt{1+x^2}}$

$\sin a = \sqrt{1-\cos^2 a} = \sqrt{1-\frac{1}{1+x^2}} = \sqrt{\frac{\cancel{1}+x^2-\cancel{1}}{1+x^2}}$

$= \frac{x}{\sqrt{1+x^2}}$ Ans (4)