

UNIT – 03 LAWS OF MOTION

TWO MARKS AND THREE MARKS:

01. Explain the concept of inertia. Write two examples each for inertia of motion, inertia of rest and inertia of direction.

This inability of objects to move on its own or change its state of motion is called inertia. Inertia means resistance to change its state.

Examples:

Inertia of Rest :

- i) Passengers experience a backward push in a sudden start of bus.
- ii) Tightening of seat belts in a car when it stops quickly.

Inertia of Motion:

- i) Passengers experience a forward push during a sudden brake in bus.
- ii) Ripe fruits fall from the trees in the direction of wind.

Inertia of Direction:

- i) A stone moves tangential to Circle.
- ii) When a car moves towards left, we turn to the right.

02. State Newton's second law.

The force acting on an object is equal to the rate of change of its momentum. $\vec{F} = \frac{d\vec{p}}{dt}$

03. Define One Newton

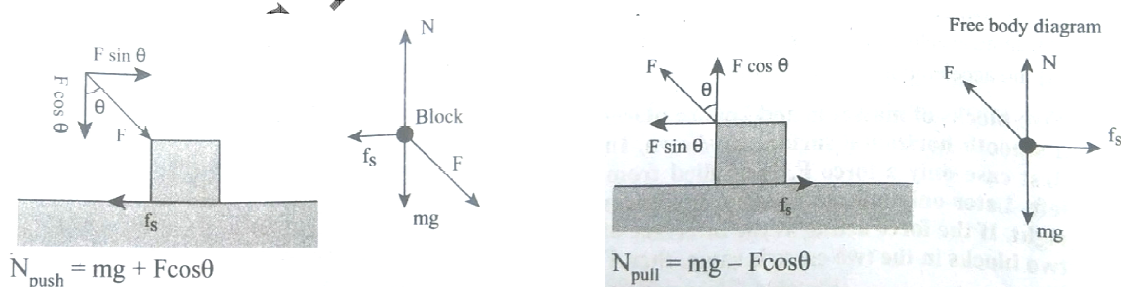
One Newton is defined as the force which acts on 1 kg of mass to give an acceleration 1 m s^{-2} in the direction of the force.

04. Show that impulse is the change of momentum.

The integral $\int_{t_i}^{t_f} F dt = J$ is called the impulse and it is equal to change in momentum of the object.

Proof : If the force is constant over the time interval, then $\int_{t_i}^{t_f} F dt = F \int_{t_i}^{t_f} dt = F(t_f - t_i) = F\Delta t = \Delta p$ is called the impulse and it is equal to change in momentum of the object.

05. Using free body diagram, show that it is easy to pull an object than to push it.



06. Explain various types of friction. Suggest a few methods to reduce friction.

Static Friction (\vec{f}_s):

- 01. Static friction is the force which opposes the initiation of motion of an object on the surface.
- 02. When the object is at rest on the surface, only two forces act on it. They are the downward gravitational force and upward normal force.
- 03. The resultant of these two forces on the object is zero.

Kinetic Friction (\vec{f}_k):

01. When an object slides, the surface exerts a frictional force called **kinetic friction (f_k)**
02. If the external force acting on the object is greater than maximum static friction, the objects begin to slide.
03. The kinetic friction does not depend on velocity.

Rolling Friction:

The force of friction that comes into act when a wheel rolls over a surface.

Methods to reduce friction:

01. By using Lubricants friction
02. By using ball bearings.

07. What is the meaning by ‘pseudo force’?

Centrifugal force is called as a ‘pseudo force’. A pseudo force has no origin. A pseudo force is an apparent force that acts on all masses whose motion is described using non inertial frame of reference such as a rotating reference frame.

08. State the empirical laws of static and kinetic friction.

- i) The magnitude of static frictional force f_s satisfies the following empirical relation.
 $0 \leq f_s \leq \mu_s N_s$ where μ_s is the coefficient of static friction.
- ii) The force of static friction can take any value from zero to $\mu_s N$.
- iii) If the object is at rest and no external force is applied on the object, the static friction acting on the object is zero ($f_s = 0$).
- iv) If the object is at rest, and there is an external force applied parallel to the surface, then the force of static friction acting on the object is exactly equal to the external force applied on the object ($f_s = F_{ext}$). But still the static friction f_s is less than $\mu_s N$.
- v) When object begins to slide, the static friction (f_s) acting on the object attains maximum.
- vi) The static and kinetic frictions depend on the normal force acting on the object.
- vii) The static friction does not depend upon the area of contact.

09. State Newton’s third law.

For every action there is an equal and opposite reaction.

10. What are inertial frames?

01. If an object is free from all forces, then it moves with constant velocity or remains at rest when seen from inertial frames.
02. Thus, there exists some special set of frames in which if an object experiences no force It moves with constant velocity or remains at rest.

11. Under what condition will a car skid on a leveled circular road?

If the static friction is not able to provide enough centripetal force to turn, the vehicle will start to skid. $\mu < \frac{v^2}{rg}$ (skid)

12. Define impulse.

If a very large force acts on an object for a very short duration, then the force is called impulsive force or impulse.

13. State Newton’s First law.

Every object continues to be in the state of rest or of uniform motion unless there is external force acting on it.

14. Define Inertia of rest, motion and direction.

The inability of an object to change its state of rest is called **inertia of rest**.

The inability of an object to change its direction of motion on its own is called **inertia of direction**.

The inability of an object to change its state of uniform speed on its own is called **inertia of motion**.

15. What is free body diagram? What are the steps to be followed for developing free body diagram?

Free body diagram is a simple tool to analyze the motion of the object using Newton's laws.

The following systematic steps are followed for developing the free body diagram:

1. Identify the forces acting on the object.
2. Represent the object as a point.
3. Draw the vectors representing the forces acting on the object.

16. What is the concurrent force?

A collection of forces is said to be concurrent, if the lines of forces act at a common point.

17. State Lami's theorem.

The magnitude of each force of the system is proportional to sine of the angle between the other two forces. The constant of proportionality is same for all three forces.

18. State the law of conservation of total linear momentum.

If there are no external forces acting on the system, then the total linear momentum of the system (\vec{p}_{tot}) is always a constant vector.

In other words, the total linear momentum of the system is conserved in time.

19. What is the role of air bag in a car?

Cars are designed with air bags in such a way that when the car meets with an accident, the momentum of the passengers will reduce slowly so that the average force acting on them will be smaller.

20. Define frictional force.

Which always opposes the relative motion between an object and the surface where it is placed. If the force applied is increased, the object moves after a certain limit.

21. Define Angle of Friction.

The angle of friction is defined as the angle between the normal force (N) and the resultant force (R) of normal force and maximum friction force (f_s^{max}).

22. Define Angle of repose.

The same as angle of friction. But the difference is that the angle of repose refers to inclined surfaces and the angle of friction is applicable to any type of surface.

23. What are the applications of angle of repose?

01. The angle of inclination of sand trap is made to be equal to angle of repose.
02. Children are fond of playing on sliding board. Sliding will be easier when the angle of inclination of the board is greater than the angle of repose.

24. How does the rolling wheel's work in suitcase?

01. In rolling motion when a wheel moves on a surface, the point of contact with surface is always at rest.
02. Since the point of contact is at rest, there is no relative motion between the wheel and surface. Hence the frictional force is very less.

25. Where does the friction force act?

Walking is possible because of frictional force. Vehicles (bicycle, car) can move because of the frictional force between the tyre and the road. In the braking system, kinetic friction plays a major role.

26. How did the ball bearing reduce kinetic friction?

If ball bearings are fixed between two surfaces, during the relative motion only the rolling friction comes to effect and not kinetic friction.

27. What is the reason for force changes the velocity of the particle?

01. The magnitude of the velocity can be changed without changing the direction of the velocity.
02. In this case the particle will move in the same direction but with acceleration.
03. The direction of motion alone can be changed without changing the magnitude (speed).
04. Both the direction and magnitude (speed) of velocity can be changed. If this happens non circular motion occurs.

28. Define Centripetal force.

If a particle is in uniform circular motion, there must be centripetal acceleration towards the center of the circle. If there is acceleration then there must be some force acting on it with respect to an inertial frame. This force is called centripetal force.

29. How is the centripetal force act in whirling motion?

In the case of whirling motion of a stone tied to a string, the centripetal force on the particle is provided by the tensional force on the string.

In circular motion in an amusement park, the centripetal force is provided by the tension in the iron ropes.

30. How did the car move on circular track?

When a car is moving on a circular track the centripetal force is given by the frictional force between the road and the tyres. Frictional force = mv^2/r m-mass of the car, v-speed of the car r-radius of curvature of track

Even when the car moves on a curved track, the car experiences the centripetal force which is provided by frictional force between the surface and the tyre of the car.

CONCEPTUAL QUESTIONS:

01. Why it is not possible to push a car from inside?

01. When you push on the car from inside, the reaction force of your pushing is balanced out by our body moving backward.
02. The seat behind you pushes against to bring things to static equilibrium. So, we can't push a car from inside.

02. There is a limit beyond which the polishing of a surface increase frictional resistance rather than decreasing it why?

01. Friction arises due to molecular adhesion. For more polishing the molecules of the surface come closer.
02. They offer greater resistance to surface.

03. Can a single isolated force exist in nature? Explain your answer.

No, a single isolated force cannot exist in nature. Because it will be violation of Newton's third law.

04. Why does a parachute descend slowly?

A parachute is a device used to slow down an object that is falling towards the ground. When the parachute opens, the air resistance increases. So the person can land safely.

05. When walking on ice one should take short steps. Why?

To avoid slipping, take smaller steps. Because these steps cause more normal force and there by more friction.

06. When a person walks on a surface, the frictional force exerted by the surface on the person is opposite to the direction of motion. True or false? False.

07. Can the coefficient of friction be more than one?

Yes, $\mu > 1$, friction is stronger than the normal force.

08. Can we predict the direction of motion of a body from the direction of force on it?

In free body diagrams, the force of friction is always parallel to the surface of contact. The force of kinetic friction is always opposite the direction of motion.

09. The momentum of a system of particles is always conserved. True or false? True

10. Why is it dangerous to stand near the open door of moving bus?

It is dangerous to stand near the open door (or) steps while travelling in the bus. When the bus takes a sudden turn in a curved road, due to centrifugal force the person is pushed away from the bus. Even though centrifugal force is a pseudo force, its effects are real.

11. When a cricket player catches the ball, he/she pulls his /her hands gradually in the direction of the ball's motion. Why?

01. If he stops his hands soon after catching the ball, the ball comes to rest very quickly.
02. It means that the momentum of the ball is brought to rest very quickly.
03. So the average force acting on the body will be very large.
04. Due to this large average force, the hands will get hurt.
05. To avoid getting hurt, the player brings the ball to rest slowly

12. A man jumping on concrete floor is more dangerous than in sand floor, why?

01. Jumping on a concrete cemented floor is more dangerous than jumping on the sand.
02. Sand brings the body to rest slowly than the concrete floor, so that the average force experienced by the body will be lesser.

FIVE MARKS QUESTIONS:

01. Prove the law of conservation of linear momentum. Use it to find the recoil velocity of a gun when a bullet is fired from it.

- i) The force on each particle (Newton's second law) can be written as $\vec{F}_{12} = \frac{d\vec{p}_1}{dt}$ and $\vec{F}_{21} = \frac{d\vec{p}_2}{dt}$
- ii) Here \vec{p}_1 is the momentum of particle 1 which changes due to the force \vec{F}_{12} exerted by particle 2. Further \vec{p}_2 is the momentum of particle 2. This changes due to \vec{F}_{21} exerted by particle 1.
- $$\frac{d\vec{p}_1}{dt} = -\frac{d\vec{p}_2}{dt}; \quad \frac{d\vec{p}_1}{dt} + \frac{d\vec{p}_2}{dt} = 0; \quad \frac{d}{dt}(\vec{p}_1 + \vec{p}_2) = 0$$
- iii) It implies that $\vec{p}_1 + \vec{p}_2 = \text{constant vector (always)}$.
- iv) $\vec{p}_1 + \vec{p}_2$ is the total linear momentum of the two particles ($\vec{p}_{\text{tot}} = \vec{p}_1 + \vec{p}_2$). It is also called as total linear momentum of the system. Here, the two particles constitute the system.
- v) If there are no external forces acting on the system, then the total linear momentum of the system (\vec{p}_{tot}) is always a constant vector.

02. What are concurrent forces? State Lami's theorem.

A collection of forces is said to be concurrent, if the lines of forces act at a common point. If they are in the same plane, they are concurrent as well as coplanar forces.

If a system of three concurrent and coplanar forces is in equilibrium, then Lami's theorem states that the magnitude of each force of the system is proportional to sine of the angle between the other two forces. The constant of proportionality is same for all three forces.

$$\frac{|\vec{F}_1|}{\sin \alpha} = \frac{|\vec{F}_2|}{\sin \beta} = \frac{|\vec{F}_3|}{\sin \gamma}$$

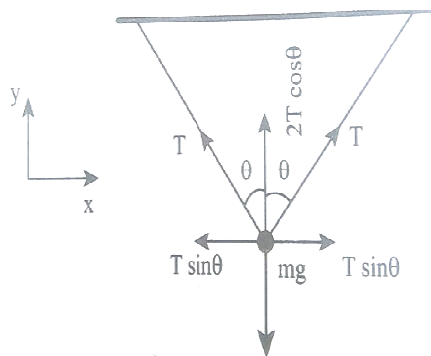
Example:

A baby is playing in a swing which is hanging with the help of two identical chains is at rest. Identify the forces acting on the baby. Apply Lami's theorem and find out the tension acting on the chain.

Solution

The baby and the chains are modeled as a particle hung by two strings as shown in the figure. There are three forces acting on the baby.

- Downward gravitational force along negative y direction (mg)
- Tension (T) along the two strings. These three forces are coplanar as well as concurrent as shown in the following figure



By using Lami's theorem $\frac{T}{\sin(180-\theta)} = \frac{T}{\sin(180-\theta)} = \frac{mg}{\sin(2\theta)}$

Since $\sin(180-\theta) = \sin \theta$ and $\sin(2\theta) = 2\sin \theta \cos \theta$

We get, $\frac{T}{\sin \theta} = \frac{mg}{2\sin \theta \cos \theta}$;

From this, the tension on each string is $T = \frac{mg}{2\cos \theta}$

**03. Explain the motion of blocks connected by a string in i) Vertical motion
ii) Horizontal motion.**

Case 1: Vertical motion

i) Consider two blocks of masses m_1 and m_2 ($m_1 > m_2$) connected by a light and inextensible string that passes over a pulley.

- ii) Let the tension in the string be T and acceleration a . When the system is released, both the blocks start moving, m_2 vertically upward and m_1 downward with same acceleration.

The gravitational force m_1g on mass m_1 is used in lifting the mass m_2 .

Applying Newton's second law for mass m_2 ,

$$T\hat{j} - m_2g\hat{j} = m_2a\hat{j}$$

- iii) The left hand side of the above equation is the total force that acts on m_2 and the right hand side is the product of mass and acceleration of m_2 in y direction.

By comparing the components on both sides, we get

$$T - m_2g = m_2a \text{ ----- (1)}$$

Similarly, applying Newton's second law for mass m_1

$$T\hat{j} - m_1g\hat{j} = -m_1a\hat{j}$$

As mass m_1 moves downward ($-\hat{j}$), its acceleration is along $(-\hat{j})$

- iv) By comparing the components on both sides, we get

$$T - m_1g = -m_1a \quad ; \quad m_1g - T = m_1a \text{ ----- (2)}$$

Adding equations (1) and (2), we get

$$m_1g - m_2g = m_1a + m_2a \quad ; \quad (m_1 - m_2)g = (m_1 + m_2)a \text{ ----- (3)}$$

From equation (3), the acceleration of both the masses is $a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)g$ ----- (4)

If both the masses are equal ($m_1 = m_2$), from equation (4) $a = 0$

v) This shows that if the masses are equal, there is no acceleration and the system as a whole will be at rest. To find the tension acting on the string, substitute the acceleration from the equation (4) into the equation (1).

$$T - m_2g = m_2\left(\frac{m_1 - m_2}{m_1 + m_2}\right)g \quad ; \quad T - m_2g + m_2\left(\frac{m_1 - m_2}{m_1 + m_2}\right)g \text{ --- (5)}$$

By taking m_2g common in the RHS of equation (5)

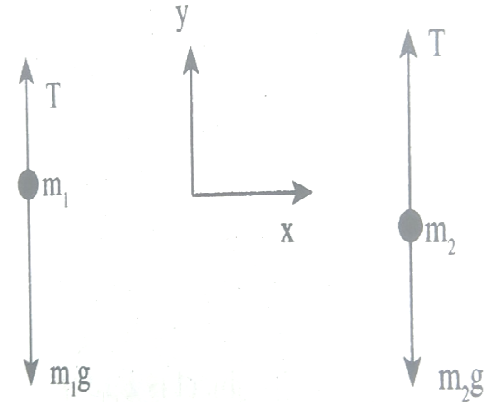
$$T = m_2g \left(1 + \frac{m_1 - m_2}{m_1 + m_2}\right) \quad ;$$

$$T = m_2g \left(\frac{m_1 + m_2 + m_1 - m_2}{m_1 + m_2}\right)$$

$$T = \left(\frac{2m_1m_2}{m_1 + m_2}\right)g$$

Case 2: Horizontal motion

- i) In this case, mass m_2 is kept on a horizontal table and mass m_1 is hanging through a small pulley. Assume that there is no friction on the surface.
- ii) As both the blocks are connected to the un-stretchable string, if m_1 moves with an acceleration a downward then m_2 also moves with the same acceleration a horizontally. The forces acting on mass m_2 are (i) Downward gravitational force (m_2g)
- (ii) Upward normal force (N) exerted by the surface (iii) Horizontal tension (T) exerted by the string



The forces acting on mass m_1 are

- (i) Downward gravitational force (m_1g)
- (ii) Tension (T) acting upwards

The free body diagrams for both the masses

Applying Newton's second law for m_1

$$T\hat{j} - m_1g\hat{j} = m_1a\hat{j}$$

By comparing the components on both sides of the above equation,

$$T - m_1g = -m_1a \text{ ----- (1)}$$

Applying Newton's second law for m_2

$$T\hat{i} - m_2a\hat{i}$$

By comparing the components on both sides of above equation,

$$T = m_2a \text{ -----(2)}$$

There is no acceleration along y direction for m_2 .

$$N\hat{j} - m_2g\hat{j} = 0, \text{ By comparing the components on both sides of the above equation}$$

$$N - m_2g = 0 ; N = m_2g \text{ ----- (3)}$$

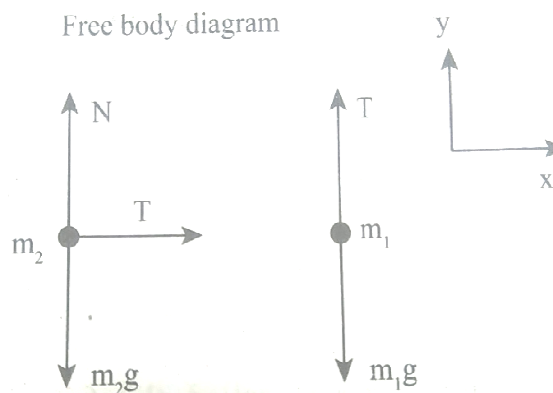
By substituting equation (2) in equation (1), we can find the tension T

$$m_2a - m_1g = -m_1a; m_2a + m_1a = m_1g ; a = \frac{m_1}{m_1+m_2}g \text{ -----(4)}$$

Tension in the string can be obtained by substituting equation (4) in equation (2)

$$T = \frac{m_1m_2}{m_1+m_2}g$$

Comparing motion in both cases, it is clear that the tension in the string for horizontal motion is half of the tension for vertical motion for same set of masses and strings.



04. Briefly explain the origin of friction. Show that in an inclined plane, angle of friction is equal to angle of repose.

- i) If a very gentle force in the horizontal direction is given to an object at rest on the table, it does not move.
- ii) It is because of the opposing force exerted by the surface on the object which resists its motion.
- iii) This force is called the frictional force which always opposes the relative motion between an object and the surface where it is placed.
- iv) Consider an inclined plane on which an object is placed. Let the angle which this plane makes with the horizontal be θ . For small angles of θ , the object may not slide down.
- v) As θ is increased, for a particular value of θ , the object begins to slide down. This value is called angle of repose. Hence, the angle of repose is the angle of inclined plane with the horizontal such that an object placed on it begins to slide.
- vi) Consider the various forces in action here. The gravitational force mg is resolved into components parallel ($mg \sin \theta$) and perpendicular ($mg \cos \theta$) to the inclined plane.
- vii) The component of force parallel to the inclined plane ($mg \sin \theta$) tries to move the object down. The component of force perpendicular to the inclined plane ($mg \cos \theta$) is balanced by the Normal force (N).

$$N = mg \cos \theta \text{ -----(1)}$$

When the object just begins to move, the static friction attains its maximum value,

$$f_s = f_s^{\max} = \mu_s N. \text{ This friction also satisfies the relation}$$

$$f_s^{\max} = \mu_s mg \sin \theta \text{ ----- (2)}$$

Equating the right hand side of equations (1) and (2), we get

$$(f_s^{\max}) / N = \sin \theta / \cos \theta$$

From the definition of angle of friction, we also know that $\tan \theta = \mu_s$
in which θ is the angle of friction

05. State Newton's three laws and discuss their significance.

Newton's First Law

- i) Every object continues to be in the state of rest or of uniform motion (constant velocity) unless there is external force acting on it.
- ii) This inability of objects to move on its own or change its state of motion is called inertia. Inertia means resistance to change its state.

Newton's Second Law

- i) The force acting on an object is equal to the rate of change of its momentum $\vec{F} = \frac{d\vec{p}}{dt}$
- ii) In simple words, whenever the momentum of the body changes, there must be a force acting on it. The momentum of the object is defined as $\vec{p} = m\vec{v}$. In most cases, the mass of the object remains constant during the motion. In such cases, the above equation gets modified into a simpler form $\vec{F} = \frac{d(m\vec{v})}{dt} = m \frac{d\vec{v}}{dt} = m\vec{a}$. $\vec{F} = m\vec{a}$

Newton's third law

- i) Newton's third law assures that the forces occur as equal and opposite pairs. An isolated force or a single force cannot exist in nature.
- ii) Newton's third law states that for every action there is an equal and opposite reaction.
- iii) Here, action and reaction pair of forces do not act on the same body but on two different bodies.
- iv) Any one of the forces can be called as an action force and the other the reaction force. Newton's third law is valid in both inertial and non-inertial frames.
- v) These action-reaction forces are not cause and effect forces. It means that when the object 1 exerts force on the object 2, the object 2 exerts equal and opposite force on the body 1 at the same instant.

06. Explain the similarities and differences of centripetal and centrifugal forces.

Centripetal force	Centrifugal force
It is a real force which is exerted on the body by the external agencies like gravitational force, tension in the string, normal force etc.	It is a pseudo force or fictitious force which cannot arise from gravitational force, tension force, normal force etc.
Acts in both inertial and non-inertial frames	Acts only in rotating frames (non-inertial frame)
It acts towards the axis of rotation or center of the circle in circular motion	It acts outwards from the axis of rotation or radially outwards from the center of the circular motion
$ F_{cp} = m\omega^2 r = \frac{mv^2}{r}$	$ F_{cf} = m\omega^2 r = \frac{mv^2}{r}$
Real force and has real effects.	Pseudo force but has real effects
Origin of centripetal force is interaction between two objects	Origin of centrifugal force is inertia. It does not arise from interaction.
In inertial frames centripetal force has to be included when free body diagrams are drawn.	In an inertial frame the object's inertial motion appears as centrifugal force in the rotating frame. In inertial frames there is no centrifugal force. In rotating frames, both centripetal and centrifugal force have to be included when free body diagrams are drawn.

07. Briefly explain 'centrifugal force' with suitable examples.

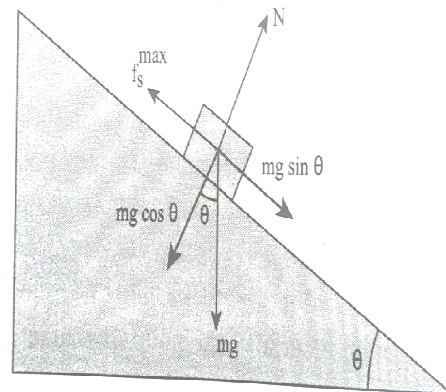
- Consider the case of a whirling motion of a stone tied to a string. Assume that the stone has angular velocity ω in the inertial frame (at rest).
- If the motion of the stone is observed from a frame which is also rotating along with the stone with same angular velocity ω then, the stone appears to be at rest.
- This implies that in addition to the inward centripetal force $-m\omega^2 r$ there must be an equal and opposite force that acts on the stone outward with value $+m\omega^2 r$.
- So the total force acting on the stone in a rotating frame is equal to zero ($-m\omega^2 r + m\omega^2 r = 0$).
- This outward force $+m\omega^2 r$ is called the centrifugal force.

08. Briefly explain 'Rolling Friction'.

- One of the important applications is suitcases with rolling on coasters. Rolling wheels makes it easier than carrying luggage.
- When an object moves on a surface, essentially it is sliding on it. But wheels move on the surface through rolling motion.
- In rolling motion** when a wheel moves on a surface, **the point of contact with surface is always at rest.**
- Since the point of contact is at rest**, there is no relative motion between the wheel and surface. Hence the **frictional force is very less.** At the same time if an object moves **without a wheel**, there is a relative motion between the object and the surface.
- As a result **frictional force is larger.** This makes it **difficult to move the object.**
- Ideally in pure rolling, motion of the point of contact with the surface should be at rest, but in practice it is not so.
- Due to the elastic nature of the surface at the point of contact there will be some deformation on the object at this point on the wheel or surface.
- Due to this deformation, there will be minimal friction between wheel and surface. It is called 'rolling friction'.** In fact, 'rolling friction' is much smaller than kinetic friction.

09. Describe the method of measuring Angle of Repose.

- Consider an inclined plane on which an object is placed. Let the angle which this plane makes with the horizontal be θ . For small angles of θ , the object may not slide down.
- As θ is increased, for a particular value of θ , the object begins to slide down. This value is called angle of repose. Hence, the angle of repose is the angle of inclined plane with the horizontal such that an object placed on it begins to slide.
- Consider the various forces in action here. The gravitational force mg is resolved into components parallel ($mg \sin \theta$) and perpendicular ($mg \cos \theta$) to the inclined plane.
- The component of force parallel to the inclined plane ($mg \sin \theta$) tries to move the object down. The component of force perpendicular to the inclined plane ($mg \cos \theta$) is balanced by the Normal force (N).



$$N = mg \cos \theta \quad \text{-----(1)}$$

When the object just begins to move, the static friction attains its maximum value,

$$f_s = f_s^{\max} = \mu_s N. \text{ This friction also satisfies the relation}$$

$$f_s^{\max} = \mu_s mg \sin \theta \quad \text{----- (2)}$$

Equating the right hand side of equations (1) and (2), we get

$$(f_s^{\max}) / N = \sin \theta / \cos \theta$$

From the definition of angle of friction, we also know that $\tan \theta = \mu_s$

10. Explain the need for banking of tracks.

- In a leveled circular road, skidding mainly depends on the coefficient of static friction μ_s . The coefficient of static friction depends on the nature of the surface which has a maximum limiting value.
- To avoid this problem, usually the **outer edge of the road is slightly raised compared to inner edge**
- This is **called banking of roads or tracks**. This introduces an inclination, and the angle is called banking angle.
- Let the surface of the road make angle θ with horizontal surface. Then the normal force makes the same angle θ with the vertical.
- When the car takes a turn, there are two forces acting on the car:
 - Gravitational force mg (downwards)
 - Normal force N (perpendicular to surface)
- We can resolve the normal force into two components. $N \cos \theta$ and $N \sin \theta$
- The component $N \cos \theta$ balances the downward gravitational force ' mg ' and component $N \sin \theta$ will provide the necessary centripetal acceleration. By using Newton second law

$$N \cos \theta = mg ; N \sin \theta = \frac{mv^2}{r}$$

$$\text{By dividing the equations we get, } \tan \theta = \frac{v^2}{rg}$$

$$v = \sqrt{rg \tan \theta}$$

Need Banking of tracks:

1) The banking angle θ and radius of curvature of the road or track determines the Safe speed of the car at the turning. If the speed of car exceeds this safe speed, then it starts to skid outward but frictional force comes into effect and provides an additional centripetal force to prevent the outward skidding.

2) At the same time, if the speed of the car is little lesser than safe speed, it starts to skid inward and frictional force comes into effect, which reduces centripetal force to prevent inward skidding.

3) However if the speed of the vehicle is sufficiently greater than the correct speed, then frictional force cannot stop the car from skidding.

11. Calculate the centripetal acceleration of Moon towards the Earth?

- The centripetal acceleration is given by $a = \frac{v^2}{r}$. This expression explicitly depends on Moon's speed which is non trivial. We can work with the formula $\omega^2 R_m = a_m$
- a_m is centripetal acceleration of the Moon due to Earth's gravity.
 ω is angular velocity.

R_m is the distance between Earth and the Moon, which is 60 times the radius of the Earth.

$$R_m = 60R = 60 \times 6.4 \times 10^6 = 384 \times 10^6 \text{ m}$$

As we know the angular velocity $\omega = \frac{2\pi}{T}$ and $T = 27.3 \text{ days} = 27.3 \times 24 \times 60 \times 60 \text{ second} = 2.358 \times 10^6 \text{ sec}$

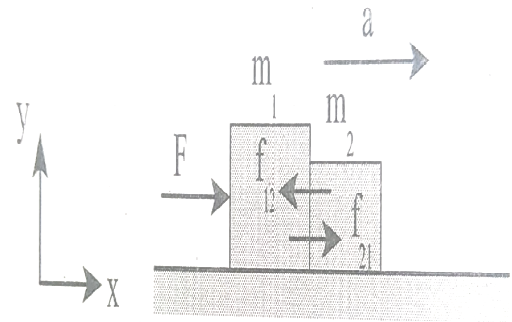
By substituting these values in the formula for acceleration

$$a_m = \frac{(4\pi^2)(384 \times 10^6)}{(2.358 \times 10^6)^2} \quad a_m = 0.00272 \text{ ms}^{-1}$$

The centripetal acceleration of Moon towards the Earth is $a_m = 0.00272 \text{ ms}^{-1}$

12. How will you confirm Newton's third law by the way of two bodies in contact on a horizontal surface?

- i) Consider two blocks of masses m_1 and m_2 ($m_1 > m_2$) kept in contact with each other on a smooth, horizontal frictionless surface as shown
- ii) By the application of a horizontal force F , both the blocks are set into motion with acceleration 'a' simultaneously in the direction of the force F .



- iii) To find the acceleration \vec{a} , Newton's second law has to be applied to the system
(combined mass $m = m_1 + m_2$)

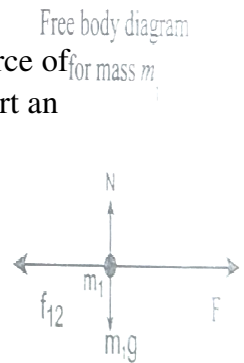
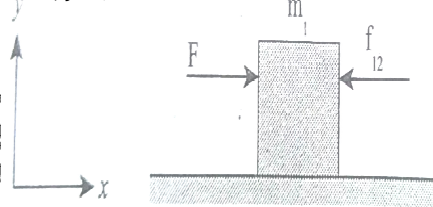
$\vec{F} = m\vec{a}$, If we choose the motion of the two masses along the positive x direction, $F\hat{i} = ma\hat{i}$

By comparing components on both sides of the above equation

$$F = ma \text{ where } m = m_1 + m_2$$

The acceleration of the system is given by $a = \frac{F}{m_1 + m_2}$ ----- (1)

- iv) The force exerted by the block m_1 on m_2 due to its motion is called force of contact (\vec{f}_{21}). According to Newton's third law, the block m_2 will exert an equivalent opposite reaction force (\vec{f}_{12}) on block m_1



$$F\hat{i} - f_{12}\hat{i} = m_1a\hat{i}$$

By comparing the components on both sides of the above equation, we get

$$F - f_{12} = m_1a ; f_{12} = F - m_1a \text{ ----- (2)}$$

Substituting the value of acceleration from equation (1) in (2) we get,

$$f_{12} = F - m_1 \left(\frac{F}{m_1 + m_2} \right) ; f_{12} = F \left[1 - \frac{m_1}{m_1 + m_2} \right]$$

$$f_{12} = \frac{Fm_2}{m_1 + m_2} \text{ ----- (3)}$$

Equation (3) shows that the magnitude of contact force depends on mass m_2 which provides the reaction force. Note that this force is acting along the negative x direction.

In vector notation, the reaction force on mass m_1 is given by $\vec{f}_{12} = \frac{Fm_2}{m_1 + m_2} \hat{i}$

- v) For mass m_2 there is only one force acting on it in the x direction and it is denoted by \vec{f}_{21} . This force is exerted by mass m_1 .

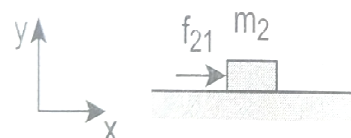
The free body diagram for mass m_2 .

Applying Newton's second law for mass m_2 .

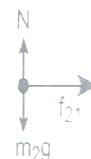
$$f_{21}\hat{i} = m_2a\hat{i}$$

By comparing the components on both sides of the above equation

$$f_{21} = m_2a \text{ ----- (4)}$$



Free body diagram for mass m_2



Substituting for acceleration from equation (1) in equation (4), we get $f_{21} = \frac{Fm_2}{m_1+m_2}$

vi) In this case the magnitude of the contact force is $f_{21} = \frac{Fm_2}{m_1+m_2}$.

The direction of this force is along the positive x direction.

vii) In vector notation, the force acting on mass m_2 exerted by mass m_1 is $\vec{f}_{21} = \frac{Fm_2}{m_1+m_2} \hat{i}$

Note : $\vec{f}_{12} = -\vec{f}_{21}$ which confirms Newton's third law

13. Write the salient features of Static and Kinetic friction.

Static friction	Kinetic friction
It opposes the starting of motion	It opposes the relative motion of the object with respect to the surface
Independent of surface of contact	Independent of surface of contact
μ_s depends on the nature of materials in mutual contact	μ_k depends on nature of materials and temperature of the surface
Depends on the magnitude of applied force	Independent of magnitude of applied force
It can take values from zero to $\mu_s N$	It can never be zero and always equals to $\mu_k N$ whatever be the speed (true $< 10 \text{ ms}^{-1}$)
$f_s^{\max} > f_k$	It is less than maximal value of static friction
$\mu_s > \mu_k$	Coefficient of kinetic friction is less than coefficient of static friction

14. Briefly explain what are all the forces act on a moving vehicle on a leveled circular road?

i) When a vehicle travels in a curved path, there must be a centripetal force acting on it. This centripetal force is provided by the frictional force between tyre and surface of the road.

ii) Consider a vehicle of mass 'm' moving at a speed 'v' in the circular track of radius 'r'. There are three forces acting on the vehicle when it moves

1. Gravitational force (mg) acting downwards
2. Normal force (mg) acting upwards
3. Frictional force (F_s) acting horizontally inwards along the road

iii) Suppose the road is horizontal then the normal force and gravitational force are exactly equal and opposite. The centripetal force is provided by the force of static friction F_s between the tyre and surface of the road which acts towards the center of the circular track, $\frac{mv^2}{r} = F_s$,

the static friction can increase from zero to a maximum value $F_s \leq \mu_s mg$

iv) The static friction would be able to provide necessary centripetal force to bend the car on the road. So the coefficient of static friction between the tyre and the surface of the road determines what maximum speed the car can have for safe turn.

If the static friction is not able to provide enough centripetal force to turn, the vehicle will start to skid.

if $\frac{mv^2}{r} > \mu_s mg$, of $\mu_s < \frac{v^2}{rg}$ (skid)

15. Find i) acceleration ii) speed of the sliding object using free body diagram.

i) To draw the free body diagram, the block is assumed to be a point mass. Since the motion is on the inclined surface, we have to choose the coordinate system parallel to the inclined surface as shown in Figure (b).

ii) The gravitational force mg is resolved into parallel component $mg \sin\theta$ along the inclined plane and perpendicular component $mg \cos\theta$ perpendicular to the inclined surface Figure b.

iii) Note that the angle made by the gravitational force (mg) with the perpendicular to the surface is equal to the angle of inclination θ .

iv) There is no motion (acceleration) along the y axis. Applying Newton's second law in the y direction

$$-mg \cos\theta + N = 0 \text{ (No acceleration)}$$

By comparing the components on both

$$\text{sides, } N - mg \cos\theta = 0$$

$$N = mg \cos\theta$$

v) The magnitude of normal force (N) exerted by the surface is equivalent to $mg \cos\theta$. The object slides (with an acceleration) along the x direction. Applying Newton's second

law in the x direction $mg \sin\theta = ma$

By comparing the components on both sides, we can equate $mg \sin\theta = ma$

The acceleration of the sliding object is $a = g \sin\theta$

vi) Note that the acceleration depends on the angle of inclination θ . If the angle θ is 90 degree, the block will move vertically with acceleration $a = g$. Newton's kinematic equation is used to find the speed of the object when it reaches the bottom. The acceleration is constant throughout the motion.

$$v^2 = u^2 + 2as \text{ along the x direction}$$

vii) The acceleration a is equal to $g \sin\theta$. The initial speed (u) is equal to zero as it starts from rest. Here s is the length of the inclined surface.

$$\text{The speed (v) when it reaches the bottom is } v = \sqrt{2sg \sin\theta}$$

