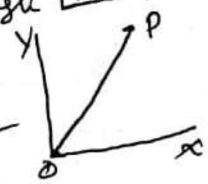
Analytical Geometry – Two Dimensional Formulas TIPS.

1. **Locus of a point**: when a point moves in accordance with a geometrical law, its path is called locus.

2. **Some important loci**

1) A Point P moves such that it is equidistant from the two fixed point is (line joining of two points) is $\perp$r bisector of the line segment AB.

2) A point P moves such that it is equidistant from two fixed lines OX and OY is angle bisector of the angle $\angle XOY$.



3) The locus of a point P which moves equidistant from a fixed point is a circle.

3) **Straight line**. Slope of a straight line is a number that measures its, direction and steepness.

If a line makes an angle θ with x axis then m (slope) $= \tan\theta$.

(or) $m = \dfrac{\Delta y}{\Delta x} = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{\text{Vertical change}}{\text{horizontal change}}$.

when the equation of the line is in general form $ax + by + c = 0$

then $m = -a/b$.

4) In a plane three or more points are said to be collinear if they lie on a same st. line.

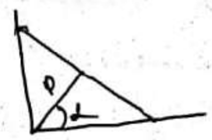
5) The intercept of a line is the point at which the line crosses either the x axis or y axis.

6) Eqn of line 1) Slope intercept form $y = mx + c$

2) Point slope form $y - y_1 = m(x - x_1)$

3) Two point form $\dfrac{y - y_1}{y_2 - y_1} = \dfrac{x - x_1}{x_2 - x_1}$ $\quad m = \dfrac{y_1 - y_2}{x_1 - x_2}$

4) Intercept form $\dfrac{x}{a} + \dfrac{y}{b} = 1$



5.) Normal form $x\cos\alpha + y\sin\alpha = p$.

6) Parametric form $\dfrac{x - x_1}{\cos\theta} = \dfrac{y - y_1}{\sin\theta} = r$ where r is distance between (x_1, y_1) and (x_2, y_2)

7) General form $ax + by + c = 0$

Normal form: comparing $Ax+By+C=0$ and $x\cos\theta + y\sin\theta = P$.

$$\frac{-A}{\sqrt{A^2+B^2}}x + \frac{-B}{\sqrt{A^2+B^2}} = \frac{|c|}{\sqrt{A^2+B^2}}$$

$$\therefore \cos\theta = \frac{-A}{\sqrt{A^2+B^2}},\quad \sin\theta = \frac{-B}{\sqrt{A^2+B^2}},\quad P = \frac{|c|}{\sqrt{A^2+B^2}}.$$

7) Angle between two st. lines. $\tan\theta = \left|\frac{m_1-m_2}{1+m_1\cdot m_2}\right|$ (or) $\frac{a_1}{a_2} = \frac{b_1}{b_2}$

If $\frac{m_1-m_2}{1+m_1m_2}$ is +ve Then θ is acute

1. $m_1 = m_2$ parallel.

If $\frac{m_1-m_2}{1+m_1m_2}$ is −ve then θ obtuse.

2. $m_1\cdot m_2 = -1$ $\perp r$. or $a_1a_2+b_1b_2=0$

If $a_1a_2+b_1b_2 > 0$ then the angle between the lines is obtuse.

If $a_1a_2+b_1b_2 < 0$ " " acute

8) Distance between two points. $d = \sqrt{(x_1-x_2)^2+(y_2-y_1)^2}$

The distance from (x_1,y_1) to $ax+by+c=0$ is

$$d = \left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$$

The distance between the two parallel lines of: $\left|\frac{c_1-c_2}{\sqrt{a^2+b^2}}\right|$

The co-ordinates of the nearest point (foot of the $\perp r$) on the line $ax+by+c=0$ from the point (x_1,y_1)

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = -\frac{(ax_1+by_1+c)}{a^2+b^2}$$

$\therefore$ The co-ordinate of the image of the point (x_1,y_1) w.r.t. the line $ax+by+c=0$ is

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{-2(ax_1+by_1+c)}{a^2+b^2}.$$

9) The family of equation of st. lines through the point of intersection of the two lines $L_1 = a_1x+b_1y+c_1=0$ $L_2 = a_2x+b_2y+c_2=0$

$L_1 + \lambda L_2 = 0.$

10) Pair of st-lines.

Pair of st. line passing through the origin is $ax^2+2hxy+by^2=0$

If $y-m_1x=0$ $y-m_2x=0$ are the two lines.

then $(y-m_1x)(y-m_2x)=0 \Rightarrow y^2-(m_1+m_2)xy+m_1m_2x^2=0$

$\therefore m_1+m_2=\dfrac{-2h}{b}, \quad m_1m_2=a/b.$

11) Angle between pair of st. lines. $\tan\theta=\left|\dfrac{2\sqrt{h^2-ab}}{a+b}\right|$

a) If $h^2-ab>0$ The lines are real and distinct

b) $h^2-ab=0$ The lines are real and coincident

c) $h^2-ab<0$ The lines are imaginary.

d) $h^2-ab=0$ They are IIel or coincident

e) $a+b=0$ they are $\perp$r.

12) Equation of the bisectors of the angle between the lines $ax^2+2hxy+by^2=0$

$$\frac{x^2-y^2}{a-b}=\frac{xy}{h}.$$

13) The general second degree Eqn. $ax^2+2hxy+by^2+2gx+2fy+c=0$ represents a pair of st. line if $abc+2fgh-af^2-bg^2-ch^2=0$

14) Two Straight lines represented by $ax^2+2hxy+by^2+2gx+2fy+c=0$ represents a pair of parallel st. lines then

if $\dfrac{a}{h}=\dfrac{h}{b}=\dfrac{g}{f}$ (or) $bg^2=af^2$

Distance between the two parallel lines $2\sqrt{\dfrac{g^2-ac}{a(a+b)}}$ (or) $2\sqrt{\dfrac{f^2-bc}{b(a+b)}}$

15) Point of intersection of pair of st. line $ax^2+2hxy+by^2+2gx+2fy+c=0$

is $P\left\{\left(\dfrac{hf-bg}{ab-h^2}\right),\left(\dfrac{gh-af}{ab-h^2}\right)\right\}$. h g a h / b f g h b

16) If a line $ax+by+c=0$ cuts the pair of st. line $ax^2+2hxy+by^2+2gx+2fy+c=0$ at two points then the these two points joining with origin then we get pair of st. line. This pair of st is obtaining by homogenising the given two equations.

17) If the two pair of st. lines. $x^2+2axy-y^2=0$, $x^2+2bxy-y^2=0$ are s.t each pair bisect the angles between the other pair if $ab=-1$

EXERCISE-6.1

1) Find the locus of P if for all α, the co-ordinates of a point p is

i) $(9\cos\alpha, 9\sin\alpha)$ 2) $(9\cos\alpha, 6\sin\alpha)$

1) Let $(h, k) = (9\cos\alpha, 9\sin\alpha)$

$h = 9\cos\alpha$ $\quad k = 9\sin\alpha$ $\quad$ W.K.T $\sin^2\alpha + \cos^2\alpha = 1$

$\frac{h}{9} = \cos\alpha$ $\quad \frac{k}{9} = \sin\alpha$ $\quad \frac{k^2}{81} + \frac{h^2}{81} = 1$

$k^2 + h^2 = 81$

$\therefore$ The locus is $x^2 + y^2 = 81$

2) Let $(h, k) = (9\cos\alpha, 6\sin\alpha)$

$h = 9\cos\alpha$ | $k = 6\sin\alpha$ $\quad$ W.K.T $\sin^2\alpha + \cos^2\alpha = 1$

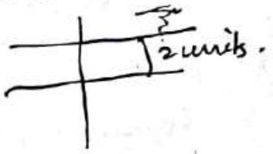
$\frac{h}{9} = \cos\alpha$ | $\frac{k}{6} = \sin\alpha$ $\quad \frac{h^2}{81} + \frac{k^2}{36} = 1$

$\therefore$ The locus is $\frac{x^2}{81} + \frac{y^2}{36} = 1$.

2) Find the locus of a point p that moves at a constant distance of i) Two units from the x axis 2) The line parallel to y axis, which is at 3 units from y axis.

i) Any line parallel to x axis is $y = c$

$y = 2$



2) Any line parallel to y axis is $x = k$

$x = 3$.

3) If θ is a parameter find the equation of the locus of a moving pt whose co-ordinates are $x = a\cos^3\theta$, $y = a\sin^3\theta$.

$x = a\cos^3\theta$ | $y = a\sin^3\theta$

$\frac{x}{a} = \cos^3\theta$ | $\frac{y}{a} = \sin^3\theta$

$\left(\frac{x}{a}\right)^{2/3} = (\cos^3\theta)^{2/3}$ | $\left(\frac{y}{a}\right)^{2/3} = (\sin^3\theta)^{2/3}$

$= \cos^2\theta$ | $= \sin^2\theta$.

W.K.T $\sin^2\theta + \cos^2\theta = 1$

$\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{a}\right)^{2/3} = 1 \Rightarrow x^{2/3} + y^{2/3} = a^{2/3}$.

4) Find the value of K and b if the points P(-3,1) Q(2,b) lies on the locus $x^2 - 5x + ky = 0$ — ①

$\therefore$ P(-3,1) lies on ① $\quad 9 + 15 + k = 0$

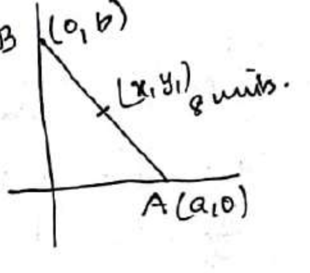
$k = -24$

$\therefore$ The equation becomes $x^2-5x-24y=0$.

Again $Q(2,b)$ lies on ①

$$4-10-24b=0$$
$$-24b=6$$
$$b=\frac{6}{-24}=-\frac{1}{4}$$

5) A straight rod of length 8 units slides with its ends A and B always on the x and y axes resp. find the locus of the mid point of the line segment.



Let $A(a,0)$ $B(0,b)$

Mid point of $AB = \left(\frac{a}{2}, \frac{b}{2}\right) = (x_1, y_1)$

$\Rightarrow \frac{a}{2} = x_1$ | $\frac{b}{2} = y_1$

$a = 2x_1$ | $b = 2y_1$

By theorem $a^2+b^2=8^2$

$$4x_1^2+4y_1^2=64$$
$$x_1^2+y_1^2=16$$

$\therefore$ The locus is $x^2+y^2=16$.

6) Find the equation of the locus of a point s.t the sum of the squares of the distance from the pts $(3,5)$ $(1,-1)$ is equal to 20

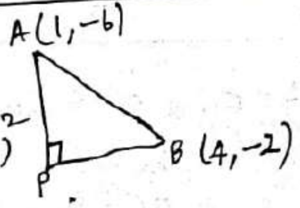
Let $A(3,5)$ $B(1,-1)$ and let $P(x_1,y_1)$ be any point

Given $PA^2+PB^2=20$

$$(x_1-3)^2+(y_1-5)^2+(x_1-1)^2+(y_1+1)^2=20$$
$$x_1^2-6x_1+9+y_1^2-10y_1+25+x_1^2-2x_1+1+y_1^2+2y_1+1=0$$
$$x_1^2+y_1^2-4x_1-4y_1+8=0$$

$\therefore$ The locus is $x^2+y^2-4x-4y+8=0$

7) Find the equation of the locus of the point P s.t the line segment AB joining the pts $A(1,-6)$ $B(4,-2)$ subtends a rt angle at P.



Let $P(x_1,y_1)$ $A(1,-6)$ $B(4,-2)$

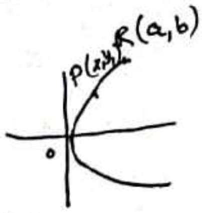
$$PA^2+PB^2=AB^2$$
$$(x_1-1)^2+(y+6)^2+(x_1-4)^2+(y_1+2)^2=(1-4)^2+(-6+2)^2$$
$$2x_1^2+2y_1^2-10x_1+16y_1+32=0$$

$\therefore$ The locus is

$$\boxed{2x^2+2y^2-10x+16y+32=0}$$
$$x^2+y^2-5x+8y+16=0$$

8) If O is the origin and R is variable point on $y^2 = 4x$ then find the equation of of the locus of the mid point of the line segment OR

$y^2 = 4ax$.



Let R (a,b) be any point and P(x₁, y₁) be a mid point of OR (moving pt).

$(x_1, y_1) = \frac{0+a}{2}, \left(\frac{0+b}{2}\right)$

$x_1 = \frac{a}{2}$ | $y_1 = \frac{b}{2}$

$2x_1 = a$ | $2y_1 = b$.

∴ (a, b) be a point on the parabola $y^2 = 4x$

$b^2 = 4a$

Hence $4y_1^2 = 8x_1$

The locus is $y^2 = 2x$.

9) The Co-ordinates of a moving point P are $\left(\frac{a}{2}(\operatorname{cosec}\theta + \sin\theta), \frac{b}{2}(\operatorname{cosec}\theta - \sin\theta)\right)$ where θ is the variable parameter. S.T the Eqn of the locus is $b^2x^2 - a^2y^2 = a^2b^2$

Let $P(x_1, y_1)$ be the moving point.

$x_1 = \frac{a}{2}(\operatorname{cosec}\theta + \sin\theta) \Rightarrow \frac{2x_1}{a} = \operatorname{cosec}\theta + \sin\theta$ — ①

$y_1 = \frac{b}{2}(\operatorname{cosec}\theta - \sin\theta) \Rightarrow \frac{2y_1}{b} = \operatorname{cosec}\theta - \sin\theta$ — ②

$①^2 - ②^2 = \frac{4x_1^2}{a^2} - \frac{4y_1^2}{b^2} = \cancel{\operatorname{cosec}^2\theta} + \cancel{\sin^2\theta} + 2\operatorname{cosec}\theta\sin\theta - \cancel{\operatorname{cosec}^2\theta} - \cancel{\sin^2\theta} + 2\operatorname{cosec}\theta\sin\theta$.

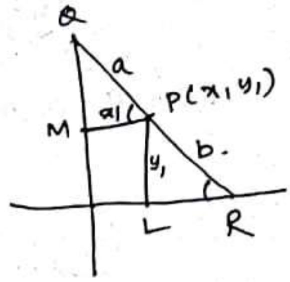
$\frac{4x_1^2}{a^2} - \frac{4y_1^2}{b^2} = 4$

∴ The locus is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

10) If P(2, −7) is a given point and Q is a point on $2x^2 + 9y^2 = 18$ Then find the Eqn of the locus of the mid point PQ.

11) If R is any point on the x axis and Q is any point on the y axis and P is variable point on RQ, with RP = b, PQ = a. the eqn of the locus of P.

Let $P(x_1, y_1)$ be the moving point.



In ΔMPQ $\cos\theta = \frac{x_1}{a}$.

ΔLRP $\sin\theta = \frac{y_1}{b}$.

W.K.T $\sin^2\theta + \cos^2\theta = 1$

$$\frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} = 0$$

$\therefore$ The locus is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

12) If the points P(6,2) and Q(-2,1) $R(\alpha, \beta)$ are the vertices of ΔPQR and R is the point on the locus of $y = x^2 - 3x + 4$ then find the eqn of the locus of the centroid of the Δle.

Let $P_1(x_1, y_1)$ be the moving point.

$P(6,2)$ $Q(-2,1)$ $R(\alpha, \beta)$

centroid of the Δle $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$.

$$\left(\frac{6-2+\alpha}{3}, \frac{2+1+\beta}{3}\right) = (x_1, y_1)$$

$$\frac{4+\alpha}{3} = x_1 \Rightarrow \alpha = 3x_1 - 4 \quad \text{—— (1)}$$

$$\frac{3+\beta}{3} = y_1 \Rightarrow \beta = 3y_1 - 3. \quad \text{—— (2)}$$

$\therefore (\alpha, \beta)$ lies on $y = x^2 - 3x + 4$.

$\beta = \alpha^2 - 3\alpha + 4$

Sub (1) and (2) in (a)

$$3y_1 - 3 = (3x_1 - 4)^2 - 3(3x_1 - 4) + 4.$$

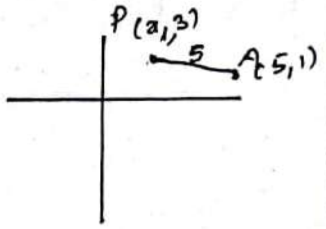
on simplification

$$9x_1^2 - 33x_1 - 3y_1 + 35 = 0$$

$\therefore$ The locus is $9x^2 - 33x - 3y + 35 = 0$.

14) Find the point on the locus of point that are 3 units from x axis and 5 units from (5,1).

Let $P(x_1, 3)$ be the point and $A(5,1)$



$$AB^2 = 5^2$$

$$(x_1-5)^2 + (3-1)^2 = 5^2$$

$$x_1^2 - 10x_1 + \cancel{25} + 4 - \cancel{25} = 0$$

$$x_1^2 - 10x_1 + 4 = 0$$

$$x_1 = \frac{10 \pm \sqrt{100-16}}{2} = \frac{10 \pm 2\sqrt{21}}{2} = 5 \pm \sqrt{21}$$

$\therefore$ The points are $(5+\sqrt{21}, 3)$ $(5-\sqrt{21}, 3)$.

15) The sum of the distance of the moving point from the points $(4,0)$ and $(-4,0)$ is always 10 units

Let $P(x_1, y_1)$ be the moving point $A(4,0)$ $B(-4,0)$

Given $AP + PB = 0$

$$\sqrt{(4-x_1)^2 + y_1^2} + \sqrt{(4+x_1)^2 + y_1^2} = 10$$

$$\sqrt{(4-x_1)^2 + y_1^2} = 10 - \sqrt{(4+x_1)^2 + y_1^2}.$$

Sq. on both sides.

$$(4-x_1)^2 + \cancel{y_1^2} = 100 + (4+x_1)^2 + \cancel{y_1^2} - 20\sqrt{(4+x_1)^2 + y_1^2}.$$

$$\cancel{16} + \cancel{x_1^2} - 8x_1 = 100 + \cancel{16} + 8x_1 + \cancel{x_1^2} - 20\sqrt{(4+x_1)^2 + y_1^2}.$$

$$\overset{5}{\cancel{20}}\sqrt{(4+x_1)^2 + y_1^2} = 100 + 16x_1$$

$$= \cancel{4}(25 + 4x_1)$$

Sq. on both sides.

$$25[(4+x_1)^2 + y_1^2] = 625 + 200x_1 + 16x_1^2$$

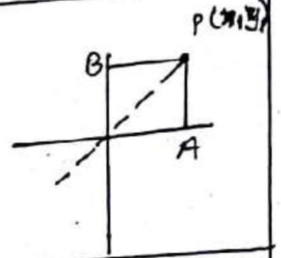
$$25[16 + 8x_1 + x_1^2 + y_1^2] = 625 + 200x_1 + 16x_1^2$$

$$400 + \cancel{200x_1} + 25x_1^2 + 25y_1^2 = 625 + \cancel{200x_1} + 16x_1^2$$

$$9x_1^2 + 25y_1^2 - 225 = 0.$$

$\therefore$ The locus is $9x^2 + 25y^2 = 225 \Rightarrow \frac{x^2}{25} + \frac{y^2}{9} = 1$

EX 6.1 Find the locus of a point which moves such that its distance from x axis is equal to the distance from the y axis.



Let (x_1, y_1) be the moving point

Given $AP = PB$ always.

$x_1 = y_1$

$\therefore$ The locus is $x = y$.

Ex. 6.2. Find the point traced out by the point $(ct, c/t)$ $t \neq 0$ is the parameter and c is a constant.

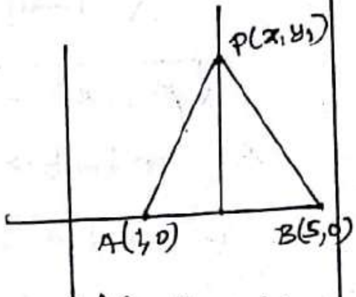
Let $P(x_1 y_1)$ be the moving point $x_1 = ct$, $y_1 = c/t$

$$x_1 y_1 = (ct)\left(\frac{c}{t}\right)$$
$$= c^2$$

$\therefore$ The locus is $xy = c^2$

EX 6.3) Find the locus of a point P moves s.t its distances from two fixed points A(1,0) B(5,0) are always equal.

Let $P(x_1 y_1)$ be the moving point $A(1,0)$ $B(5,0)$



Given $AP = BP$

$$AP^2 = BP^2$$
$$(x_1 - 1)^2 + \cancel{y_1^2} = (x_1 - 5)^2 + \cancel{y_1^2}$$
$$\cancel{x_1^2} - 2x_1 + 1 = \cancel{x_1^2} - 10x_1 + 25$$
$$8x_1 = 24 \Rightarrow x_1 = 3.$$

$\therefore$ The locus is $x = 3$. which is a line $\parallel$el to y axis.

(Also it is a $\perp$r bisector of the line joining (1,0) and (5,0))

6.4) If θ is the parameter find the equation of the locus of a moving point whose co-ordinates are $(a\sec\theta, b\tan\theta)$.

Let $P(x_1 y_1)$ be the moving point

$x_1 = a\sec\theta$ | $y_1 = b\tan\theta$

$\frac{x_1}{a} = \sec\theta$ | $\frac{y_1}{b} = \tan\theta$.

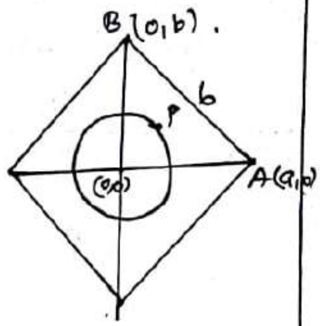
$$\sec^2\theta - \tan^2\theta = \frac{x_1^2}{a^2} - \frac{y_1^2}{b^2}$$
$$1 = \frac{x_1^2}{a^2} - \frac{y_1^2}{b^2}$$

$\therefore$ The locus is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

6.5) A straight rod of the length 6 units, slides with its ends A and B always on the x and y axes respectively. If O is the origin then find the locus of the centroid of ΔOAB.

Let $P(x_1, y_1)$ be the moving point.



Centroid of the $\triangle$le $OAB = \left(\frac{0+a+0}{3}, \frac{0+0+b}{3}\right)$

$\Rightarrow \frac{a}{3} = x_1 \quad | \quad \frac{b}{3} = y_1$

$a = 3x_1 \quad | \quad b = 3y_1.$

W.K.T $OA^2 + OB^2 = AB^2$

$a^2 + b^2 = 6^2$

$9x_1^2 + 9y_1^2 = 36$

$x_1^2 + y_1^2 = 4$

$\therefore$ The locus is $x^2 + y^2 = 4$.

6.6) If θ is the parameter, find the equation of the locus of a moving point whose co-ordinates are $(a(\theta - \sin\theta), a(1-\cos\theta))$

Let $P(x_1, y_1)$ be the moving point.

$x_1 = a(\theta - \sin\theta)$

$\frac{x_1}{a} = (\theta - \sin\theta)$

Seek value of θ and $\sin\theta$.

$\therefore x_1 = a\left(\cos^{-1}\frac{a-y_1}{a} - \frac{\sqrt{2ay_1 - y_1^2}}{a}\right)$

$\therefore$ The locus is $x = a\cos^{-1}\frac{a-y_1}{a} - \sqrt{2ay - y^2}$

$y_1 = a(1-\cos\theta)$

$\frac{y_1}{a} = 1 - \cos\theta$

$\cos\theta = 1 - \frac{y_1}{a}$ ——— (*)

$\theta = \cos^{-1}\left(\frac{a-y_1}{a}\right)$

$\sin\theta = \sqrt{1-\cos^2\theta}$

$= \sqrt{1 - \left(\frac{a-y_1}{a}\right)^2}$

$= \sqrt{\frac{a^2 - (a^2 + y_1^2 - 2ay_1)}{a^2}}$

$= \frac{\sqrt{2ay_1 - y_1^2}}{a}.$

Straight lines.

Ex. 6.7) Find the slope of the st. line passing through the points (5,7) and (7,5). Also find the angle of inclination with x axis.

Slope of the line joining the points $(\overset{x_1, y_1}{5, 7})$ $(\overset{x_2, y_2}{7, 5})$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2}{2} = -1.$

$\tan\theta = -1$ $\therefore$ is obtuse. (it is in II quadrant)

$\theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$.

6.8) Find the equation of the st. line cutting an intercept of 5 from the negative direction of the y axis and is inclined at an angle 150° with x axis.

$c = -5$

$m = \tan 150^\circ = \tan(180-30)$

$= -\tan 30^\circ = -\frac{1}{\sqrt{3}}$.

Eqn of the st. line $y = mx + c$

$y = -\frac{1}{\sqrt{3}}x - 5$

$\sqrt{3}y = -x - 5\sqrt{3}$

$x + \sqrt{3}y + 5\sqrt{3} = 0$.

6.9) S.T the points $(0, -3/2)$ $(1, -1)$ $(2, -1/2)$ are collinear.

$A(0, -3/2)$ $B(1, -1)$ $C(2, -1/2)$

Slope of $AB = \frac{-1 + 3/2}{1} = 1/2$ $m = \frac{y_2 - y_1}{x_2 - x_1}$

Slope $BC = \frac{-1/2 + 1}{1} = 1/2$

$\therefore$ slope of AB = slope of BC. Hence the three points are collinear.

6.10) The pamban Bridge is a railway Bridge of length about 2065m constructed on the palkstrait which connects the Island town of Rameswaram to Mandapam the main land of India. The Bridge is restricted to a uniform speed of only 12.5 m/s. If a train of length 560 m starts at the entry point of the Bridge from Mandapam then

1) Find an equation of the motion of the train.
2) when does the engine touch the Island.
3) when does the last coach cross the entry point of the bridge.
4) what is the time taken by a train to cross the bridge.

Let x axis – time in sec.

y axis – distance in meters.

Length of the train 560 m (negative side of y $\therefore$ y intercept $c = -560$

(7) Slope of the motion = uniform speed

$m = 12.5$ mts/sec.

Eqn. of line $y = mx + c$.

$y = 12.5x - 560$ —— ①

(ie) 1) Eqn of motion

2) When the engine touches the other side of the bridge

$y = 2055, \ c = 0.$

Sub. in ① $2065 = 12.5x$

$x = 165.2$ sec.

3) when $y = 0$ the last coaches cross the entry point

Sub. in ① $0 = 12.5x - 560$

$12.5x = 560$

$x = 44.8$ sec.

4) When $y = 2065$ $x = ?$

$2065 = 12.5x - 560$

$12.5x = 2625$

$x = 210$ sec.

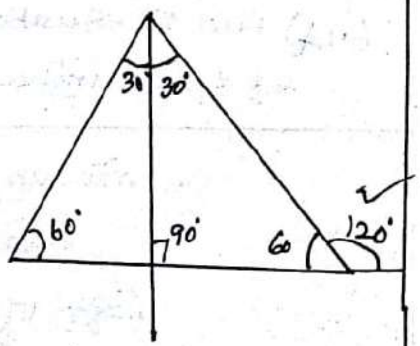
6.11) Find the equations of the st. line making the y intercepts of 7 and angle between the line and the y axis is 30°

y intercept $c = 7$.

$m = \tan 120 = \tan(180 - 60)$

$= -\tan 60^\circ$

$= -\sqrt{3}$.



Eqn. of the line $y = mx + c$

$y = -\sqrt{3}x + 7$

$\sqrt{3}x + y - 7 = 0$

If $m = \tan 60^\circ = \sqrt{3}$

Eqn of line $y = \sqrt{3}x + 7$.

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6.12) The seventh term of an A P is 30 and 10^{th} term is 21.

1) Find the 1st three term of A P

2) Which term of A P is 0

3) Find the relationship between the slope of the st. line and common difference of A.P.

Let x axis be the number of terms and y axis be the value of term

Let (x_1, y_1) and $(x_2\ y_2) = (7, 30), (10, 21)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-9}{3} = -3$$

Eqn. of line $y - y_1 = m(x - x_1)$

$$y - 30 = -3(x - 7)$$

$$y - 30 = -3x + 21$$

$$y = -3x + 51.$$

we can use $\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$ also.

1) Put $x = 1$ $t_1 = -3 + 51 = 48$

$x = 2$ $t_2 = -6 + 51 = 45$

$x = 3$ $t_3 = -9 + 51 = 42$

2) Put $y = 0$ $0 = -3x + 51$

$$3x = 51$$

$$x = 17$$

$t_{17} = 0$

3) clearly slope $= -3$ which is C.D. of AP.

6.13) Find the equation of st. line passing through $(-1, 1)$ and cutting off equal intercepts but opposite in sign with the co-ordinate axis.

∵ intercepts are equal but opposite in sign.

the intercepts are $a, -a$.

Eqn of line $\frac{x}{a} + \frac{y}{-a} = 1 \Rightarrow x - y = a.$

∵ it passes through $(-1, 1)$ $-1 - 1 = a$

$a = -2$

∴ Eqn of the line $x - y = -2$

$$x - y + 2 = 0.$$

Q.16) The length of the ⊥r drawn from the origin to a line is 12 and makes an angle 150° with positive direction of x axis. Find the eqn of the line.

$p = 12$

$\alpha = 150^\circ$

$\cos 150 = \cos(180-30)$
$= -\cos 30$
$= -\sqrt{3}/2$

$\sin 150^\circ = \sin(180-30)$
$= \sin 30^\circ$
$= 1/2$

Eqn of line $x\cos\alpha + y\sin\alpha = p$.

$x\cos 150 + y\sin 150^\circ = 12$

$x\left(-\frac{\sqrt{3}}{2}\right) + y\cdot\frac{1}{2} = 12$.

$-\sqrt{3}x + y = 24$

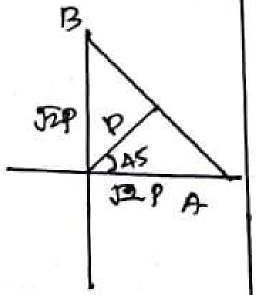
$\sqrt{3}x - y + 24 = 0$.

6.17) Area of the triangle formed by a line with the co-ordinate axes is 36 sq. units. Find the eqn of the line if the ⊥r drawn from the origin to the line makes an angle 45° with +ve the x axis.

$x\cos\alpha + y\sin\alpha = p$.

$x\cos 45 + y\sin 45^\circ = p$.

$\frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}} = p \Rightarrow x + y = \sqrt{2}p$.



x intercept put $y = 0$ $x = \sqrt{2}p$ $\therefore A(\sqrt{2}p, 0)$

y intercept put $x = 0$ $y = \sqrt{2}p$ $B(0, \sqrt{2}p)$

Area of the $\triangle$le $= \frac{1}{2}(\text{base})(\text{height})$

$= \frac{1}{2}(\sqrt{2}p)(\sqrt{2}p) = 36$

$p^2 = 36$

$p = 6$.

$\therefore$ The Eqn. of the line

$x + y = \sqrt{2}\cdot 6$.

$x + y = 6\sqrt{2}$

6.19) Express the equation $\sqrt{3}x - y + 4 = 0$ in the following equivalent form.

1) Slope and intercept form.

2) Intercept form

3) Normal form.

1) slope intercept form $\sqrt{3}x - y + 4 = 0$

$$\sqrt{3}x + 4 = y.$$

slope $m = \sqrt{3}$ y intercept $= 4$.

2). Intercept form $\sqrt{3}x - y = -4$

$$-\frac{\sqrt{3}x}{4} + \frac{y}{4} = 1$$

$$\frac{x}{\left(-4/\sqrt{3}\right)} + \frac{y}{4} = 1.$$

x intercept $= -4/\sqrt{3}$, y intercept $= 4$.

3) Normal form.

$(-\sqrt{3})x + y = 4$ $A = -\sqrt{3}$ $B = 1$

$\therefore \sqrt{A^2+B^2} = \sqrt{3+1} = 2.$

$\div 2.$ $-\frac{\sqrt{3}x}{2} + \frac{y}{2} = 2.$

This is same as $x \cos\alpha + y \sin\alpha = p.$

$\cos\alpha = -\frac{\sqrt{3}}{2}$ $\sin\alpha = \frac{1}{2}$ $p = 2.$

$\alpha = 150 = \frac{5\pi}{6}$

$\therefore x \cos\frac{5\pi}{6} + y \sin\frac{5\pi}{6} = 2.$

6.20) Rewrite $\sqrt{3}x + y + 4 = 0$ into normal form.

Eqn. of st. line in the normal form $x \cos\alpha + y \sin\alpha = p.$

p is always +ve

$$-\sqrt{3}x - y = 4.$$

$\frac{\cos\alpha}{-\sqrt{3}} = \frac{\sin\alpha}{-1} = \frac{p}{4} \Rightarrow \frac{\sqrt{\sin^2\alpha + \cos^2\alpha}}{\sqrt{3+1}} = \frac{1}{2}$

$\Rightarrow \cos\alpha = -\frac{\sqrt{3}}{2}$ $\sin\alpha = -1/2$ $p = 4/2$

$\alpha = 210^\circ = \frac{7\pi}{6}$ $p = 2.$

Eqn. of the line $x \cos\frac{7\pi}{6} + y \sin\frac{7\pi}{6} = 2.$

Ex: 6.22 Find the equations of a parallel line and a perpendicular line passing through the point (1, 2) to the line $3x+4y=7$.

Sol: Any line ||el to $3x+4y=7$ is $3x+4y=k$.

$\because$ it passes through (1, 2) $\quad 3+4=k \Rightarrow k=7$

$\therefore$ Eqn of the line $3x+4y-7=0$.

Any line $\perp$r to $3x+4y-7=0$ is $4x-3y+k=0$

$\because$ it passes through (1, 2) $\quad 4-6+k=0$
$k=2$.

$\therefore$ Eqn of $\perp$r line $4x-3y+2=0$.

Ex. 6.23 Find the distance 1) between two pts (5, 4) (2, 0)

2) from the point (1, 2) to the line $5x+12y-3=0$

3) between the parallel lines $3x+4y-12=0$, $6x+8y+1=0$

1) $(x_1, y_1) = (5,4)$ $\quad d = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$
$(x_2, y_2) = (2,0)$
$= \sqrt{3^2+4^2} = 5$.

2. $\perp$r distance from (1, 2) to $5x+12y-3=0$ is

$$d = \frac{5+24-3}{\sqrt{25+144}} = \frac{\cancel{26}}{\cancel{13}}\ 2$$

3) distance between the two parallel lines $3x+4y-12=0$
$3\cancel{6}x+\cancel{8}4y+\frac{1}{2}=0$

$$d = \left|\frac{c_1-c_2}{\sqrt{a^2+b^2}}\right| = \frac{\frac{1}{2}+12}{\sqrt{9+16}} = \frac{\cancel{25}^{5}}{2.\cancel{5}} = 5/2$$

Ex: 6.24 Find the nearest point on the line $2x+y-5=0$ from the origin.

Given line $2x+y-5=0$ — ①

$\perp$r line to one from the origin is $x-2y+k=0$

$0-0+k=0$

$k=0$

$\therefore x-2y=0$ — ②

Solving ① and ② $\quad 2x+y=5$

$2x-4y=0$

$5y=5$

$y=1$

$\therefore x=2$

$\therefore$ The nearest pt is (2, 1).

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Ex : 6.25 Find the equation of the bisector of the acute angle between the lines $3x+4y+2=0$ and $5x+12y-5=0$

The Eqn. can be written as $3x+4y+2=0$

$-5x-12y+5=0.$

The angle bisectors of the given equations are

$$\frac{3x+4y+2}{\sqrt{9+16}} = \pm \frac{-5x-12y+5}{\sqrt{25+144}}$$

$$39x+52y+26 = \pm -25x-60y+25$$

$\because a_1a_2+b_1b_2 = -15-48 < 0$

Eqn. is $39x+52y+26 = -25x-60y+25$

$64x+112y+1=0.$

Ex: 6.26 Find the points on the line $x+y=5$ that lie at a distance 2 units from $4x+3y-12=0$.

Any point on the line $x+y=5$ is

pnt $x=t$ $y=5-t.$

$\perp r$ distance from $(t, 5-t)$ to $4x+3y-12=0$ is

$$\left|\frac{4t+3(5-t)-12}{\sqrt{16+9}}\right| = 2$$

$$\left|\frac{t+3}{5}\right| = 2 \Rightarrow t+3 = \pm 10.$$

$t = 7, -13.$

When $t=7$, $y=5-7=-2$ $(7,-2)$

When $t=-13$ $y=5+13=18$ $(-13,18)$

Ex 6.27. A straight line passes through a fixed point $(6,8)$ Find the locus of the foot of the $\perp r$ drawn to it from the origin.

Let $(x_1, y_1) = (6,8)$

$P(h,k)$ be a point on the required locus.

Family of equation of st. line passing through the fixed pt (x_1, y_1) is

$y-y_1 = m(x-x_1)$

$y-8 = m(x-6)$

(1) Slope of $O(0,0)$ $P(h,k)$

$$OP = \frac{k-0}{h-0} = \frac{k}{h}.$$

$\because$ OP is $\perp$r to the given line

$$m \cdot \frac{k}{h} = -1 \Rightarrow m = -\frac{h}{k}.$$

$\because$ $P(h,k)$ lies on the line $k-8 = -\frac{h}{k}(h-6)$

$$k(k-8) = -h(h-6)$$

$$h^2+k^2-6h-8k=0.$$

$\therefore$ The locus is $x^2+y^2-2x-8y=0.$

EX 6.28). Find the eqn of the st. lines in the family of lines $y=mx+2$ for which m and the x co-ordinate of the point of intersection of the lines $2x+3y=10$ are integers.

$$mx-y+2=0 \quad \text{—(1)}$$

$$2x+3y-10=0$$

$$\begin{matrix} -1 & 2 & m & -1 \\ 3 & -10 & 2 & 3 \end{matrix}$$

$$\frac{x}{10-6} = \frac{y}{4+10m} = \frac{1}{3m+2}$$

$$x = \frac{4}{3m+2}, \quad y = \frac{4+10m}{3m+2}$$

$\therefore$ m and x co-ordinates are integer

$\frac{4}{3m+2}$ is an integer, $3m+2$ is a divisor of 4. $(\pm1, \pm2, \pm4)$

$3m+2=\pm1$	$3m+2=\pm2$	$3m+2=\pm4$
$3m=-1$ (or) $m=-1$	$m=0$	$3m=4/2$
$m=-1/3$	$m=-4/3$	$3m=-6$
		$m=-2$

$$m = \{-2, -1, 0\}.$$

$\therefore$ The equations are $y=-2x+2$, $y=-x+2$, $y=2$.

EX 6.29) Find the equation of the line through the intersection of the lines $3x+2y+5=0$ $3x-4y+6=0$ and the point $(1,1)$

Any line passing through the point of intersection of lines is of the form $(3x+2y+5)+\lambda(3x-4y+6)=0$.

$\because$ it passes through (1,1) we get $\lambda = -2$

$\therefore$ Eqn of the required line

$$(3x+2y+5)+2(3x-4y+6)=0$$

$$3x-10y+7=0.$$

Ex-6.31. A car rental firm has charges Rs 25 with 1.8 free KM. and Rs 12 for every additional KM. Find the eqn of the cost y to the number of KM x. Also find the cost ~~of~~ to travel 15 KM.

when $0 \le x \le 1.8$ $\quad y = 25$ —— ①

Also Rs 12 for every additional km after 1.8 km.

$\therefore$ The equation becomes $y = 25+12(x-1.8)$ —— ② $\quad x > 1.8$

$$\therefore \quad y = \begin{cases} 25 & 0 \le x \le 1.8 \\ 25+12(x-1.8) & x > 1.8 \end{cases}$$

when $x = 15$ $\quad y = 25+12(15-1.8)$

$$= 183.40$$

13.2×12

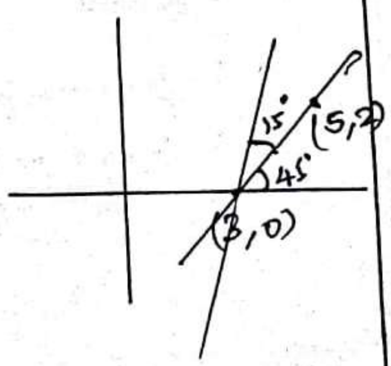
158.4

25

183.4

Ex.6.32 If a line joining two points (3,0) (5,2) is rotated about the point (3,0) in counter clockwise direction through an angle 15° then find the eqn of line of new position.

Slope of the line joining (3,0) (5,2)



$$m = \frac{2}{2} = 1$$

$$\tan\theta = 1 \Rightarrow \theta = 45^\circ$$

For the new position $\theta = 45+15^\circ = 60^\circ$

$$m = \tan 60 = \sqrt{3}.$$

Eqn of line of new position which passes through (3,0)

$$y-0 = \sqrt{3}(x-3)$$

$$y = \sqrt{3}x - 3\sqrt{3}$$

EXERCISE - 6.2

1. Find the equation of the lines passing through the point $(1,1)$ with
   i) y intercept $= -4$ 2) with slope $= 3$ 3) $(-2,3)$
   4 and $\perp r$ from the origin makes an angle $60°$ with x axis.

1) $(x_1, y_1) = (1,1)$ y intercept $= -4$ $\therefore$ point $(0,-4)$

$$\text{slope } m = \frac{-5}{-1} = 5 \qquad \frac{y_1 - y_2}{x_1 - x_2} = m$$

Equ. $y = mx + c$

$y = 5x - 4$

2. $(x_1, y_1) = (1,1)$ $m = 3$

Eqn. $y - y_1 = m(x - x_1)$

$y - 1 = 3(x - 1)$

$= 3x - 3 \Rightarrow 3x - y - 2 = 0$

3) $(x_1 y_1) = (1,1)$ $(x_2 y_2) = (-2,3)$

$$\text{Eqn. } \frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1} \Rightarrow \frac{y-1}{2} = \frac{x-1}{-3}$$

$-3y + 3 = 2x - 2$

$2x + 3y - 5 = 0$

4) P = distance between $(0,0)$ and $(1,1)$

$p = \sqrt{1+1} = \sqrt{2}$

$\alpha = 60°$

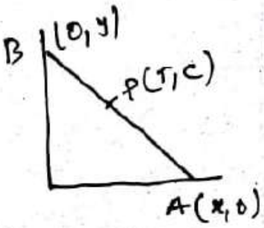
Eqn. $x\cos\alpha + y\sin\alpha = p.$

$x\cos 60° + y\sin 60° = \sqrt{2}$

$$\frac{x}{2} + y\cdot\frac{\sqrt{3}}{2} = \sqrt{2}$$

$x + \sqrt{3}y = 2\sqrt{2}$

2) If $P(r, c)$ are the mid point of the line segment between the co-ordinate axes then S.T $\frac{x}{r} + \frac{y}{c} = 2$.



Let $A(x,0)$ $B(0,y)$ $P(r,c)$

$$\frac{x+0}{2} = r \Rightarrow x = 2r \qquad \frac{y+0}{2} = c \Rightarrow y = 2c$$

$A=(2r,0) \quad B(0,2c)$

Eqn. of AB $\dfrac{y-y_1}{y_2-y_1} = \dfrac{x-x_1}{x_2-x_1}$

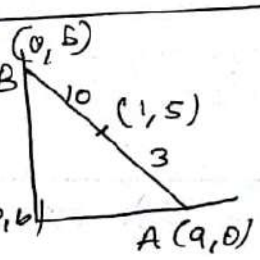
$$\frac{y}{2c} = \frac{x-2r}{-2r}$$

$$\frac{\cancel{2r}}{-\cancel{2r}} = \frac{x}{-2r} - \frac{y}{2c} \qquad \frac{x}{2r}+\frac{y}{2c}=1$$

$$\frac{x}{r}+\frac{y}{c}=2.$$

3) Find the equation of the line passing through the point (1,5) and also divides co-ordinates axes in the ratio 3:10

Let $A(a,0)$ $B(0,b)$ $P(1,5)$ divides AB with ratio 3:10



$$\overset{3}{}\qquad\overset{10}{}$$
$A(a,0)$ $P(1,5)$ $B(0,b)$

$\therefore\ 1 = \dfrac{0+10a}{13} \Rightarrow 10a = 13$

$$a=\frac{13}{10}$$

$5 = \dfrac{3b}{13} \Rightarrow 3b = 5$

$$b=\frac{5}{3}$$

$\therefore$ Eqn. of the line $\dfrac{x}{a}+\dfrac{y}{b}=1 \Rightarrow \dfrac{x}{13/10}+\dfrac{y}{5/3}=1$

$$\frac{10x}{13}+\frac{3y}{5} = 50x+3y=65$$

4) If P is the length of the $\perp$r from the origin to the line whose intercepts on the axes are a and b. S.T $\dfrac{1}{p^2}=\dfrac{1}{a^2}+\dfrac{1}{b^2}$

Eqn. of the line be $\dfrac{x}{a}+\dfrac{y}{b}=1 \Rightarrow \dfrac{x}{a}+\dfrac{y}{b}-1=0$

length of the $\perp$r from (0,0) to $\dfrac{x}{a}+\dfrac{y}{b}-1=0$ is

$$\left|\frac{-1}{\sqrt{1/a^2+1/b^2}}\right| = P \qquad \Rightarrow 1 = P\left(\sqrt{1/a^2+1/b^2}\right)$$

$$1 = p^2\left(1/a^2+1/b^2\right)$$

$$\frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}$$

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5) The normal boiling point of water is $100^\circ c$ or $212^\circ F$ and the freezing point of water is $0^\circ c$ or $32 F$. 1) Find the linear relationship between c and F° 2) the value of c for 98.6F
3) For $38^\circ c$ the value of F

$x_1 = 100^\circ c$ $\quad y_1 = 212 F^\circ$

$x_2 = 0^\circ c$ $\quad y_2 = 32 F^\circ$ $\quad x = c^\circ$

Two point form $\quad y = F^\circ$

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

$$\frac{F-32}{212-32} = \frac{c-0}{100-0}$$

$$\frac{F-32}{180} = \frac{c}{100}$$

$$F-32 = \frac{180c}{100}$$

$$F-32 = 1.8c$$

$$F = 32 + 1.8c$$
$$= 33.8c$$

ii) Find the value of c for $98.6^\circ F$

$$F = 98.6$$
$$c = \frac{5}{9}(98.6-32) = \frac{5}{9}(66.6)$$
$$= 5 \times 7.4$$
$$= 37^\circ$$

iii) $F = (1.8)(38) + 32$
$$= 100.4^\circ$$

6) A jet was launched from a place P in constant speed to hit a target. At the 15^{th} second it was 1400 m away from the target and at the 18^{th} second 800m away. 1) find the distance between the place and target. 2) The distance covered by it in 15 sec 3) time taken to hit the target.

At 15^{th} sec. $d = 1400$ $\quad x$ – Time

18^{th} sec – $d = 800$ $\quad y$ – distance.

Speed = $\frac{\text{distance}}{\text{Time}}$ $\quad \frac{d-1400}{15 5} = \frac{d-800}{18 6}$

$$6d - 8400 = 5d - 4000$$

$$d = 4400 \text{ m}.$$

2) Distance at the end of 15^{th} sec $d - 1400 = 4400 - 1400$ @

$= 3000$ mts.

(iii) Time taken to hit the target

$T_1 = 15$ $D_1 = 1400$

$T_2 = 18$ $D_2 = 800$

$$\frac{T - T_1}{T_2 - T_1} = \frac{D - D_1}{D_2 - D_1} \Rightarrow \frac{T-15}{3} = \frac{D - 1400}{-600}$$

$$\frac{T-15}{3} = \frac{0 - 1400}{-600\ 200} = 7$$

$$T - 15 = 7$$

$$T = 15 + 7 = 22.$$

7) Population of a city in the year 2005 and 2010 are 1,35,000 and 1,45,000 resp find the approximately population in the year 2015.

$x_1 = 0$ $y_1 = 1,35,000$

$x_2 = 5$ $y_2 = 1,45,000$

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} \Rightarrow \frac{x}{5} = \frac{y - 1,35,000}{10,000}$$

when $x = 10$, $\frac{\cancel{10}\,2}{\cancel{5}} = \frac{y - 135000}{10,000}$

$20,000 = y - 135000$

$y = 1,55,000$

The population in 2015 is 1,55,000

8) Find the equation of the line if the ⊥r drawn from the origin makes an angle 30° with x axis and its length is 12.

$\alpha = 30°$ $\quad p = 12$.

Eqn of line $\quad x\cos\alpha + y\sin\alpha = p$

$$x\cos 30 + y\sin 30 = 12$$

$$x \cdot \frac{\sqrt{3}}{2} + y \cdot \frac{1}{2} = 12$$

$$\sqrt{3}x + y - 24 = 0.$$

9) Find the equation of the st. lines passing through (8, 3) and having intercepts whose sum is 1.

$$a + b = 1$$

$$b = 1 - a.$$

Eqn of line $\quad \frac{x}{a} + \frac{y}{1-a} = 1$

$\because$ it passes through (8, 3)

$$\frac{8}{a} + \frac{3}{1-a} = 1 \Rightarrow \frac{8(1-a)+3a}{a(1-a)} = 1$$

$$8 - 8a + 3a = a - a^2$$

$$a^2 - 6a + 8 = 0$$

$$(a-4)(a-2) = 0$$

$$a = 2, 4.$$

when $a = 2$ $\quad \frac{x}{2} + \frac{y}{1-2} = 1$

$$\frac{x}{2} - y = 1 \Rightarrow x - 2y - 2 = 0$$

$a = 4$ $\quad \frac{x}{4} + \frac{y}{1-4} = 1 \Rightarrow \frac{x}{4} - \frac{y}{3} = 1$

$$\Rightarrow \frac{3x - 4y}{12} = 1 \text{ (or) } 3x - 4y - 12 = 0$$

10) S.T the points (1,3) (2,1) (1/2, 4) are collinear by using 1) concept of slope 2) using st. line 3) any other method

$A(1,3) \quad B(2,1) \quad C(1/2, 4)$

slope of $AB = \frac{3-1}{1-2} = \frac{2}{-1} = -2$ $\qquad \therefore$ slope of AB = slope of BC

" $\quad BC = \frac{1-4}{2-1/2} = \frac{-\cancel{3} \times 2}{\cancel{3}} = -2$ $\qquad \therefore$ A B C are collinear.

Distance between $AB = \sqrt{(1-2)^2 + (3-1)^2} = \sqrt{1+4} = \sqrt{5}$

$$BC = \sqrt{(2-\tfrac{1}{2})^2 + (1-4)^2} = \sqrt{\frac{9}{4} + 9} = \frac{\sqrt{45}}{2} = \frac{3\sqrt{5}}{2}$$

$$CA = \sqrt{(1-\tfrac{1}{2})^2 + (3-4)^2} = \sqrt{\tfrac{1}{4}+1} = \frac{\sqrt{5}}{2}$$

$$AB + CA = \sqrt{5} + \frac{\sqrt{5}}{2} = \frac{3\sqrt{5}}{2} = BC.$$

$\therefore$ ABC are collinear.

A(1,3) B(2,1) C(½,4)

$$\text{Area of the } \triangle = \frac{1}{2}\left\{(1-6) + (8-\tfrac{1}{2}) + (\tfrac{3}{2}-4)\right\} \qquad \frac{1}{2}\begin{vmatrix} 1 & 3 \\ 2 & 1 \\ \tfrac{1}{2} & 4 \\ 1 & 3 \end{vmatrix}$$

$$= \frac{1}{2}\left\{-5 + \frac{15}{2} - \frac{5}{2}\right\}$$

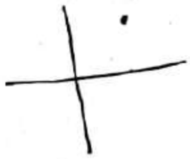
$$= \frac{1}{2}\left\{\frac{-10+15-5}{2}\right\} = 0$$

$\therefore$ ABC are collinear.

11) A straight line is passing through the point A(1,2) with slope 5/12. Find points on the which are 13 units away from A.

Let A(1, 2) B(x_1 y_1) given AB = 13 m = 5/12

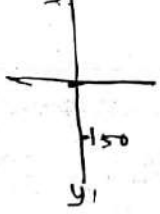
$$AB = \sqrt{(x_1-1)^2}$$



12) A 150m long train is moving with constant velocity 12.5 m/s
1) Find the equation of motion. 2) Time taken to cross the pole
3) The time taken to cross the bridge of length 850m?

Length of the train c = 150 m.

m = 12.5 m/s.



1) Eqn of motion $y = mx - c.$

$y = 12.5x - 150.$

2) Time taken to cross the pole

$y = 0 \Rightarrow 12.5x = 150$

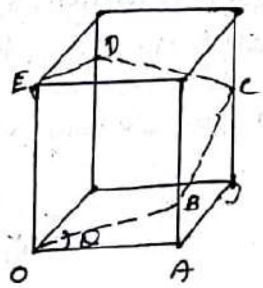
$x = \frac{150}{12.5} = 12$ sec.

3) Time taken to cross the bridge

$850 = 12.5x - 150 \Rightarrow x = \frac{1000}{12.5} = 80$ sec.

15) In a shopping mall there is a hall of cuboid shape with dimension 800 × 800 × 720 units which needs to be added the facility of an escalator in a path shown by the dotted line in the figure. i) Find the minimum total length of escalator 2) the heights at which the escalator changes its direction. 3) slope of the escalator at the turning points.

1) $OA = 800 \quad AB = \frac{1}{4} \times 720 = 180$



$$OB^2 = OA^2 + AB^2$$
$$= 800^2 + 180^2$$
$$= (20\times 40)^2 + (9\times 20)^2$$
$$= 20^2(40^2 + 9^2)$$
$$= 20^2(1600+81)$$
$$OB^2 = 20^2 \times 1681$$
$$OB = 20 \times 41 = 820.$$

Like this $BC = CD = DE = 820$

$\therefore$ Total length $= 820 \times 4 = \underline{3280\ m.}$

2) The heights at which the escalator changes its direction

I st step = 180 m.

IInd step = 360 m

IIIrd step = 540 m.

3) slope $= \tan\theta = \frac{180}{800}$

$= \frac{9}{40}.$

14) A family is using Liquefied petroleum gas (LPG) of weight 14.2 Kg for consumption (Full weight 29.5 kg includes empty cylinder tare weight of 15.3 kg) If it is use with constant rate then it lasts for 24 days. Then the new cylinder is replaced 1) Find the eqn relating the quantity of gas cylinder to the days 2) Draw the graph for first 96 days.

$x_1 = 0 \qquad y_1 = 0$

$x_2 = 14.2 \qquad y_2 = 24$

Eqn. $\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$

$\frac{y}{24} = \frac{x}{14.2}$

$24x - 14.2y = 0 \Rightarrow 12x - 7.1y = 0$

6.13) The quantity demanded of a certain type of compact disk is 22,000 units when a unit price is Rs 8. The customer will not buy the disk at a unit price of 30 or higher. on the other side the manufacturer will not market any disk if the price is Rs 6 or lower. However if the price Rs 14 the manufacturer can supply 24,000 units. Assume that the quantity demanded and quantity supplied are linearly proportional to the price i) Find the demand Eqn. 2) Supply Eqn. 3) The market Equilibrium quantity and price 4) quantity and demand when supply price is Rs10.

1) For Demand function $(x_1, y_1) = (22, 8)$ x axis - units in thous. y axis - price.

$(x_2, y_2) = (0, 30)$

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1} \Rightarrow \frac{y-8}{22} = \frac{x-22}{-22}$$

$$= -x+22$$

$$y_D = -x+30 \text{ (demand)}$$

2) For supply fn $(x_1, y_1) = (0, 6)$

$(x_2, y_2) = (24, 14)$

$$\frac{y-6}{14-6} = \frac{x-0}{24} \Rightarrow y-6 = \frac{8x}{24}$$

$$y_S = \frac{x}{3} + 6.$$

3) At the market equilibrium

$y_D = y_S$

$-x+30 = \frac{x}{3} + 6.$

$-3x+90 = x+18$

$72 = 4x \quad x = 18 \quad$ and $y = \frac{18}{3} + 6 = 12$

4) For demand when $y = 10$ $\quad y_D = -x+30$

$10 = -x+30 \quad x = 20 \quad \therefore$ demand is 20,000 units.

For supply $y = 10$ $\quad 10 = \frac{x}{3} + 6 \Rightarrow 30 = x+18$

$x = 12$

$\therefore x = 12{,}000$ units.

EXERCISE – 6.3.

1. S.T the lines are $3x+2y+9=0$, $12x+8y-15=0$ are parallel.

$3x+2y=9=0$	$12x+8y-15=0$
$m_1 = -\frac{3}{2}$	$m_2 = -\frac{12}{8} = -\frac{3}{2}$

$\therefore m_1 = m_2$ the two lines are parallel.

2) Find the equation of st line parallel to $5x-4y+3=0$ and having x intercept 3.

Any line $\parallel^{el}$ to $5x-4y+3=0$ is of the form

$$5x-4y+K=0.$$

x intercept $= 3 \Rightarrow$ $5x=-K$

$$x = \frac{-K}{5} = 3$$

$\therefore K = -15$

$\therefore$ The equation of the line is $5x-4y-15=0$.

3) Find the Co... distance between the line $4x+3y+4=0$ and a point 1) $(-2,4)$ 2) $(7,-3)$

The distance from (x_1,y_1) to the line $ax+by+c=0$ is

$$d = \frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}$$

1) The distance from $(-2,4)$ to the line $4x+3y+4=0$

$$d = \frac{-8+12+4}{\sqrt{16+9}} = 8/5$$

2) The distance from $(7,-3)$ to the line $4x+3y+4=0$ is

$$d = \frac{28-9+4}{\sqrt{16+9}} = \frac{23}{5}.$$

4) Write the equation of the lines through the point $(1,-1)$

i) Parallel to $x+3y-4=0$ 2) Perpendicular to $3x+4y=6$.

1) Any line parallel to $x+3y-4=0$ is of the form $x+3y+K=0$

$\therefore$ it passes through $(1,-1)$ $1-3+K=0$

$K=2$.

$\therefore$ Eqn. of the line $x+3y+2=0$

2) Any line $\perp r$ to $3x+4y=6$ is of the form $4x-3y+K=0$

$\therefore$ it passes through $(1,-1)$ $4+3+K=0$

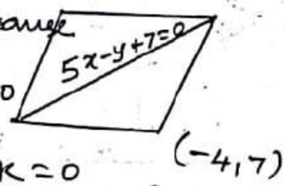
$K=-7$

$\therefore$ Eqn of the line $4x-3y-7=0$.

5) If $(-4,7)$ is one vertex of a Rhombus and if the equation of one diagonal is $5x-y+7=0$ find the equation of another diagonal.

Eqn. of one diagonal is $5x-y+7=0$.

$\because$ the diagonals of a rhombus are at $\perp$ angle



Eqn of diagonal is of the form $x+5y+k=0$

$\because$ it passes through $(-4,7)$ $\quad -4+35+k=0$

$k=-31$

$\therefore$ Eqn. of another diagonal $x+5y-31=0$.

6) Find the equation of the lines passing through the point of intersection of the lines $4x-y+3=0$ and $5x+2y+7=0$ i) through the point $(-1,2)$ ii) Parallel to $x-y+5=0$ 3) $\perp$r to $x-2y+1=0$.

Point of intersection of $\quad 4x-y+3=0$

$5x+2y+7=0$

$$\begin{matrix} -1 & 3 & 4 & -1 \\ 2 & 7 & 5 & 2 \end{matrix} \qquad \frac{x}{-7-6} = \frac{y}{15-28} = \frac{1}{8+5}$$

$$x = \frac{-13}{13} = 1, \quad y = \frac{-13}{13} = 1. \quad (1,1)$$

Eqn of the

1) the line joining of $(\overset{x_1}{1},\overset{y_1}{1})$ $(\overset{x_2}{-1},\overset{y_2}{2})$ is $\dfrac{y-y_1}{y_2-y_1} = \dfrac{x-x_1}{x_2-x_1}$

$$\frac{y-1}{1} = \frac{x-1}{-2}$$

$$-2y+2 = x-1$$

$$x+2y-3=0$$

2) Any line parallel to $x-y+5=0$ is of the form $x-y+k=0$.

$\because$ it passes through $(1,1)$ $\quad 1-1+k=0 \quad k=0$

$\therefore$ the required line is $x-y=0$

3) Any line $\perp$r to $x-2y+1=0$ is of the form $2x+y+k=0$

$\because$ it passes through $(1,1)$ $\quad 2+1+k=0$

$k=-3$.

$\therefore$ Eqn of the required line $2x+y-3=0$

7) Find the equations of two straight lines which are $\parallel$el to the line $12x+5y+2=0$ and at a unit distance from the point $(1,-1)$

Any line $\parallel$el to $12x+5y+2=0$ is of the form $12x+5y+k=0$.

$\perp$r distance from $(1,-1)$ to this line $\pm\dfrac{12-5+k}{\sqrt{144+25}} = 1$

$$\frac{7+k}{13} = 1 \Rightarrow 7+k = \pm 13$$
$$k = 6, -20$$

$\therefore$ The required lines are $12x+5y+6=0$, $12x+5y-20=0$.

8) Find the equations of st. lines which are $\perp r$ to the line $3x+4y-6=0$ are at a distance of 4 units from $(2,1)$

Any line $\perp r$ to $3x+4y-6=0$ is of the form $4x-3y+k=0$.

$\perp r$ distance from $(2,1)$ to $4x-3y+k=0$ is $\pm \frac{8-3+k}{\sqrt{16+9}} = 4$

$$\frac{5+k}{5} = \pm 4$$
$$5+k = \pm 20$$
$$k = 15, -25$$

$\therefore$ The required lines are $4x-3y+15=0$

$4x-3y-25=0$.

9) Find the equation of a st. line parallel to $2x+3y=10$ and which is such that the sum of its intercepts on the axes is 15.

Any line parallel to $2x+3y=10$ is of the form $2x+3y=k$.

x intercept put $y=0$ $\quad 2x=k$

$x = k/2$

y intercept put $x=0$ $\quad 3y=k$

$y = k/3$

Sum of the intercepts $\frac{k}{2}+\frac{k}{3}=15$

$$\frac{3k+2k}{6} = 15 \Rightarrow \cancel{5}k = 6 \times \cancel{15}^{3}$$
$$k = 18$$

$\therefore$ Eqn of the required line $2x+3y=18$

10) Find the lengths of the perpendicular and the co-ordinates of the foot of the $\perp r$ from $(-10,-2)$ to the line $x+y-2=0$.

Length of the $\perp r$ from (x_1, y_1) to $ax+by+c=0$ is

$$d = \pm \frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}$$

Length of the $\perp r$ from $(-10,-2)$ to the line $x+y-2=0$ is

$$d = \pm \frac{-10-2-2}{\sqrt{1+1}} = \frac{+14}{\sqrt{2}} = \frac{7\times 2}{\sqrt{2}} = 7\sqrt{2}$$

Foot of the $\perp r$ is given by

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = -\frac{ax_1+by_1+c}{a^2+b^2}$$

$$\frac{x+10}{1} = \frac{y+2}{1} = -\frac{(-10-2-2)}{2}$$

$$= 7$$

$x = 7-10$ | $y+2 = 7$
$= -3$ | $y = 7-2 = 5$

$\therefore$ Foot of the $\perp r$ $(-3, 5)$

11) If P_1 and P_2 are the lengths of the $\perp rs$ from the origin to the st line ~~$x\cos x$~~ $x\sec\theta + y\,\mathrm{cosec}\,\theta = 2a$ and $x\cos\theta - y\sin\theta = a\cos 2\theta$ then P.T $p_1^2 + p_2^2 = a^2$.

Lengths of the $\perp r$ from the origin to $x\sec\theta + y\,\mathrm{cosec}\,\theta = 2a$ is

$$P_1 = \left|\frac{-2a}{\sqrt{\sec^2\theta + \mathrm{cosec}^2\theta}}\right| = \frac{2a}{\sqrt{\sec^2\theta + \mathrm{cosec}^2\theta}}.$$

Lengths of the $\perp r$ from the origin to $x\cos\theta - y\sin\theta = a\cos 2\theta$ is

$$P_2 = \left|\frac{-a\cos 2\theta}{\sqrt{\cos^2\theta + \sin^2\theta}}\right| = a\cos 2\theta.$$

$$P_1^2 + P_2^2 = \frac{4a^2}{\sec^2\theta + \mathrm{cosec}^2\theta} + a^2\cos^2 2\theta.$$

$$= \frac{4a^2}{\frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta}} + a^2\cos^2 2\theta.$$

$$= \frac{4a^2\sin^2\theta\cos^2\theta}{\sin^2\theta + \cos^2\theta} + a^2\cos^2 2\theta$$

$$= 4a^2\sin^2\theta\cos^2\theta + a^2\cos^2 2\theta$$

$$= a^2(2\sin\theta\cos\theta)^2 + a^2\cos^2 2\theta$$

$$= a^2\sin^2 2\theta + a^2\cos^2 2\theta$$

$$= \underline{a^2}$$

12) Find the distance between the Parallel lines.

1) $12x+5y-7=0$ $\quad 12x+5y+7=0$

2) $3x-4y+5=0$ $\quad 6x-8y-15=0$.

Distance between the two parallel lines $12x+5y-7=0$, $12x+5y+7=0$.

$$d = \left|\frac{c_1-c_2}{\sqrt{a^2+b^2}}\right| = \left|\frac{7+7}{\sqrt{144+25}}\right| = \frac{14}{13}.$$

Distance between the parallel lines $3x-4y+5=0$, $6x-8y-15=0$

$3x-4y-15/2=0$

$$d = \left|\frac{5+15/2}{\sqrt{9+16}}\right| = \frac{25}{2\times5} = \frac{25}{10}\ \ 5/2$$

13) Find the family of straight lines i) Perpendicular to 2) Parallel to $3x+4y-12=0$.

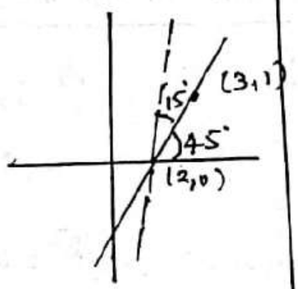
Any line parallel to $3x+4y-12=0$ is $3x+4y+k=0 \ \forall k \in R$

Any line $\perp r$ to $3x+4y-12=0$ is $4x-3y+k=0 \ \forall k \in R$.

14) If the line joining two points $A(2,0)$ and $B(3,1)$ is rotated about A in anticlockwise direction through an angle of 15° then find the equation of the line of new position.

Slope of the line joining $(2,0)$ $(3,1)$

$$m = \frac{y_2-y_1}{x_2-x_1} = \frac{1-0}{3-2} = 1$$



$\tan\theta = 1 \Rightarrow \theta = 45^\circ$.

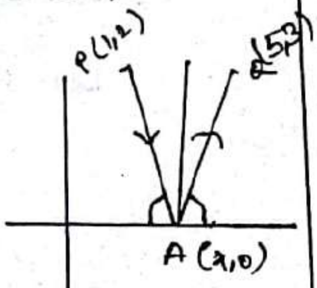
In the new position $\theta = 45^\circ+15^\circ = 60^\circ$

$\tan\theta = \tan 60^\circ = \sqrt{3}$.

$\therefore$ The equation of the line (in new position) $y-0=\sqrt{3}(x-2)$

$\sqrt{3}x - y - 2\sqrt{3} = 0$

15. A ray of light coming from the point $(1,2)$ is reflected at a point A on the x-axis and it passes through the point $(5,3)$. Find the co-ordinates of the point A.



slope of AP $= \frac{2-0}{1-x}$

slope of AQ $= \frac{3-0}{5-x}$.

Slopes are equal but opposite direction

$$\frac{2}{1-x} = -\frac{3}{5-x} \Rightarrow 10-2x = -3+3x.$$

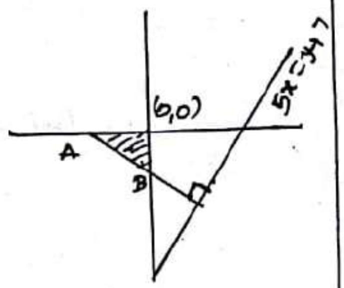
$13 = 5x \Rightarrow x = 13/5$

$\therefore A = (13/5, 0)$

16) A line is drawn $\perp r$ to $5x = y+7$. Find the equation of the line if the area of the Δle formed by this line with co-ordinate axes is 10 sq. units.

Given line $y = 5x - 7$

$$5x - y - 7 = 0. \quad \text{--- (1)}$$



Any line $\perp r$ to (1) is

$$x + 5y + k = 0.$$

A = x intercept. put $y = 0$, $x = -k$.

$\therefore A(-k, 0)$.

y intercept put $x = 0$, $5y = -k$

$y = -k/5$

$\therefore B = (0, -k/5)$

Area of the Δle AOB $= \frac{1}{2} \cdot (-k)\left(-\frac{k}{5}\right) = 10.$

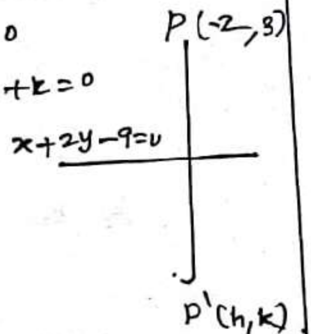
$k^2 = 100$

$k = \pm 10$

$\therefore$ Eqn. of the line $x + 5y + 10 = 0$

$x + 5y - 10 = 0.$

(17) Find the image of the point $(-2, 3)$ about the line $x + 2y - 9 = 0$



Any line $\perp r$ to $x + 2y - 9 = 0$ is $2x - y + k = 0$

$\therefore$ This line passes through $(-2, 3)$ $-4 - 3 + k = 0$

$k = 7$.

Eqn. of $\perp r$ line $2x - y + 7 = 0$

$$x + 2y - 9 = 0.$$

$$\begin{array}{r} 4x - 2y + 14 = 0 \\ \hline (+) \quad 5x = -5 \end{array} \quad x = -1.$$

$$-2 - y + 7 = 0$$

$$y = 5$$

$\therefore$ The foot of the $\perp r$ $(-1, 5)$

Let $P'(h, k)$ be $\frac{h-2}{2} = -1$ $\quad \frac{k+3}{2} = 5$

$h = 0 \quad k = 7$

$\therefore P' = (0, 7)$

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19) Find atleast two equations of the st-lines in the family of the lines $y=5x+b$ for which b and the x coordinate of the point of intersection of the lines with $3x-4y=6$ are integers

$5x-y+B=0$ — ①　　　$b=B$.

$3x-4y+6$

Point of intersection.

$-1 \quad B \quad 5 \quad -1$

$-4 \quad -6 \quad 3 \quad -4$

$$\frac{x}{+6+4B} = \frac{y}{3B-30} = \frac{1}{-20+3}.$$

$$x = \frac{+6+4B}{-17} \qquad y = \frac{3B-30}{-17}. \Rightarrow \cancel{3B-30 = \pm 17, \pm 34},$$

$=$.

(ie) $\dfrac{6+4B}{-} = -17, \ -34$

$4B = -23$

$B = -\dfrac{23}{4}$

$\therefore \ y = 5x - \dfrac{23}{4}$

$4y = 20x - 23$

$20x - 4y - 23 = 0$

$6+4B = -34$

$4B = -40$

$B = -10$.

(or) $y = 5x - 10$

$5x - y - 10 = 0$

20) Find all the equations of the straight lines in the family of the lines $y = mx - 3$ for which m and the x co-ordinate of the point of intersection of the lines $x-y-6=0$ are integers.

$mx - y - 3 = 0$ — ①

$x - y - 6 = 0$ — ②

Point of intersection:

$-1 \quad -3 \quad m \quad -1$

$-1 \quad -6 \quad 1 \quad -1$

$$\frac{x}{6-3} = \frac{y}{-3+6m} = \frac{1}{-m+1}$$

$\therefore \ x = \dfrac{3}{1-m}$ $\because$ x is an integer $1-m$ be a divisor of 3.

$m = 0, 2, -2, \ldots -2.$

$\therefore \ y+3=0$ | $y = 2x-3$, $2x-y-3=0$ | $y = -2x-3$, $2x+y+3=0$.

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EXERCISE – 6.4

1. Find the combined Eqn of st. lines whose separate Eqns are $x-2y-3)=0$ and $x+y+5=0$.

Combined Eqn. $(x-2y-3)(x+y+5)=0$

$x^2-xy-2y^2+2x-13y-15=0$

2) S.T $4x^2+4xy+y^2-6x-3y-4=0$ represents a pair of st. line.

$a=4,\ h=2,\ b=1,\ g=-3\ f=-3/2\ c=-4.$

Condition $abc+2fgh-af^2-bg^2-ch^2=0$.

$4\cdot1(-4)+\cancel{2}\left(\frac{-3}{\cancel{2}}\right)(-3)(2)-\cancel{4}\cdot\frac{9}{\cancel{4}}-9+16$

$=-\cancel{16}+\cancel{18}-\cancel{9}-\cancel{9}+\cancel{16}$

$=0$ ∴ The given equation represents pair of St. line.

3) S.T $2x^2+3xy-2y^2+3x+y+1=0$ represents a pair of perpendicular line.

$a=2,\ h=3/2\ b=-2\ g=3/2\ f=1/2\ c=1.$

Condition $abc+2fgh-af^2-ch^2-bg^2=0$.

$=2(-2)\cdot1+\cancel{2}\cdot\frac{1}{\cancel{2}}\cdot\frac{3}{2}\cdot\frac{3}{2}-2\cdot\frac{1}{4}+2\cdot\frac{9}{4}-1\cdot\frac{9}{4}$

$=-4+\cancel{\frac{9}{4}}-\frac{1}{4}+\frac{18}{4}-\cancel{\frac{9}{4}}$.

$=-4+\frac{\cancel{16}}{\cancel{4}}=0$ ∴ it represents a st. line.

$a+b=2-2=0$ ∴ they are $\perp$r.

4) S.T the equation $2x^2-xy-3y^2-6x+19y-20=0$ represents a pair of intersecting lines. Show further that the angle between them is $\tan^{-1}(5)$

$a=2\ h=-\frac{1}{2}\ b=-3\ g=-3\ f=\frac{19}{2},\ c=-20$

Condition: $abc+2fgh-af^2-bg^2-ch^2=0$

$2(-3)(20)+\cancel{2}\cdot\frac{19}{\cancel{2}}\cdot(-3)\left(-\tfrac{1}{2}\right)-\cancel{2}\,\frac{19^2}{\cancel{4}_2}+3\cdot9+20\times\frac{1}{\cancel{4}_2}^{10}$

$\dfrac{+240+57-361+54+10}{\cancel{2}\,2}=0$ ∴ it represents pair of st. line.

Point of intersection $\left(\frac{hf-bg}{ab-h^2}, \frac{hg-af}{ab-h^2}\right)$

$$=\left(\frac{11}{5}, \frac{14}{5}\right)$$

$$\tan\theta = \left|\frac{2\sqrt{h^2-ab}}{a+b}\right| = \left|\frac{2\sqrt{\frac{1}{4}+6}}{-1}\right| = \frac{\cancel{2}\times\frac{5}{\cancel{2}}}{1} = 5$$

$$\theta = \tan^{-1}(5)$$

6) Find the equation of the pair of st. lines passing through the pt (1,3) and ⊥r to $2x-3y+1=0$, $5x+y-3=0$

Any line ⊥r to $2x-3y+1=0$ is $3x+2y+k=0$

∵ it passes through (1,3) $\quad 3+6+k=0$
$\quad k=-9$

∴ ⊥r line $3x+2y-9=0$

Any line ⊥r $5x+y-3=0$ is of the form $x-5y+k=0$

∵ it passes through (1,3) $\quad 1-15+k=0$
$\quad k=14$

∴ ⊥r line $x-5y+14=0$

Combined Eqn of the lines $(3x+2y-9)(x-5y+14)=0$

$$3x^2-13xy-10y^2+33x+73y-126=0$$

7) Find the separate Equations of the following pair of st lines

1) $3x^2+2xy-y^2=0$

$3x^2+3xy-xy-y^2=0$

$3x(x+y)-y(x+y)=0$

$(3x-y)(x+y)=0 \Rightarrow 3x-y=0$
$\quad x+y=0$

2) $6(x-1)^2+8(x-1)(y-2)-3(x-1)(y-2)-4(y-2)^2=0.$

$6(x-1)^2-3(x-1)(y-2)+8(x-1)(y-2)-4(y-2)^2=0$

$3(x-1)[2(x-1)-(y-2)]+4(y-2)(2(x-1)-(y-2)]=0$

$(3(x-1)+4(y-2))(2(x-1)-(y-2))=0$

$(3x+4y-11)(2x-y)=0$

$\Rightarrow 3x+4y-11=0$

$2x-y=0.$

3) $2x^2 - xy - 3y^2 - 6x + 19y - 20 = 0$

Consider $2x^2 - xy - 3y^2 = 2x^2 - 3xy + 2xy - 3y^2 = 0$

$x(2x-3y) + y(2x-3y) = 0$

$(2x-3y)(x+y) = 0.$

$\therefore 2x^2 - xy - 3y^2 - 6x + 19y - 20 = (2x - 3y + l)(x + y + m)$

$l + 2m = -6$

$l - 3m = 19 \quad m = -5,\ l = 4$

$\therefore (2x - 3y + 4)(x + y - 5) = 0$

$2x - 3y + 4 = 0,\ x + y - 5 = 0$

8) The slope of one of the st. lines $ax^2 + 2hxy + by^2 = 0$ is twice that of the other. ST $8h^2 = 9ab.$

Condition $m_1 + m_2 = \frac{-2h}{b}$ — ①

$m_1 \cdot m_2 = a/b.$ — ②

Given that $m_1 = 2m_2$

Sub. in ① $2m_2 + m_2 = -\frac{2h}{b}$

$3m_2 = -\frac{2h}{b} \Rightarrow m_2 = -\frac{2h}{3b}.$

Sub. in ②

and $2m_2 \cdot m_2 = +\frac{a}{b}$

$2 \cdot \frac{4h^2}{9b^{\cancel{2}}} = \frac{a}{\cancel{b}}$

$8h^2 = 9ab.$

9) The slope of one of the st. lines $ax^2 + 2hxy + by^2 = 0$ is three times that of the other S.T $3h^2 = ab.$

Condition: $m_1 + m_2 = -\frac{2h}{b}$ — ①, $m_1 \cdot m_2 = \frac{a}{b}$ — ②

Given $m_1 = 3m_2$

Sub. in ① $3m_2 + m_2 = \frac{-2h}{b}$

$^2\cancel{4}m_2 = -\frac{\cancel{2}h}{b}$

$m_2 = -\frac{h}{2b}.$

Sub. in ②

$3m_2 \cdot m_2 = \frac{a}{b}$

$3 \cdot \frac{h^2}{4b^{\cancel{2}}} = \frac{a}{\cancel{b}}$

$3h^2 = 4ab.$

11) Find the value of p and q if the following equation represents a pair of ⊥r lines $6x^2 + 5xy - py^2 + 7x + qy - 5 = 0$

$a = 6,\ h = 5/2,\ b = -p,\ g = 7/2\ f = \frac{q}{2}\ c = -5$

$\underset{=-6}{}$

$\therefore$ They are ⊥r $a + b = 0$

$6 - p = 0 \Rightarrow p = 6.$

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Condition $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

$$6\left(\frac{q}{2}\right)^2 - 6\left(\frac{7}{2}\right)^2 + (-5)\left(\frac{5}{2}\right)^2 - (6)(-6)(-5) - 2\left(\frac{q}{2}\right)\left(\frac{7}{2}\right)\left(\frac{5}{2}\right) = 0.$$

$$6q^2 - 35q - 1139 = 0$$

$$6q^2 - 102q + 67q - 1139 = 0$$

$$6q(q-17) + 67(q-17) = 0$$

$$(q-17)(6q+67) = 0$$

$$q = 17, \quad q = \frac{-67}{6}.$$

12) Find the value of k, if the following equation represents a pair of st. lines, Further find whether these lines are parallel or intersecting $12x^2 + 7xy - 12y^2 - x + 7y + k = 0$.

$a = 12,\ h = \frac{7}{2},\ b = -12,\ g = -\frac{1}{2}\ f = \frac{7}{2}\ c = k.$

condition: $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

$$12(-12)k + \cancel{2}\cdot\frac{7}{\cancel{2}}\cdot\left(-\frac{1}{2}\right)\cdot\frac{7}{2} - 12\cdot\frac{49}{4} + 12\frac{1}{4} - k\cdot\frac{49}{4}$$

$$= -144k - \frac{49}{4} - \frac{582}{4} + \frac{12}{4} - \frac{49k}{2} = 0$$

$$= \frac{-576k - 49 - 582 + 12 - \cancel{49}k}{4} = 0$$

$$-625k = 625$$

$$k = -1$$

$h^2 - ab = \frac{49}{4} + 144 \neq 0$ ∴ They are not parallel.

∴ They intersect.

13) For what value of k does the equation $12x^2 + 2kxy + 2y^2 + 11x - 5y + 2 = 0$ represent two st. lines.

$a = 12\quad h = k,\ b = 2\quad g = \frac{11}{2}\quad f = -\frac{5}{2}\ c = 2.$

condition: $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

$$(12)(2)(2) + \cancel{2}\left(-\frac{5}{\cancel{2}}\right)\left(\frac{11}{2}\right)k - \overset{3}{\cancel{12}}\cdot\frac{25}{\cancel{4}} - \cancel{2}\cdot\frac{121}{\cancel{4}_2} - 2\cdot k^2 = 0$$

$$48 - \frac{55k}{2} - 75 - \frac{121}{2} - 2k^2 = 0$$

$$96 - 55k - 150 - 121 - 4k^2 = 0$$

$$4k^2 + 55k + 175 = 0$$

$$4k^2 + 35k + 20k + 175 = 0$$

$$k(4k+35) + 5(4k+35) = 0$$

$$(4k+35)(k+5) = 0$$

$$k = -5, -\frac{35}{4}$$

14) S.T the equation $4x^2-24xy+16y^2-12x+16y-12=0$ represents a pair of Parallel st. lines. Find the distance between them.

$$9x^2-24xy+16y^2-12x+16y-12=0$$

$a=9,\ h=-12,\ b=16,\ g=-6\ \ f=8\ \ c=-12$

If the given eqn represents a pair of parallel st. line if

$$bg^2=af^2$$

$$16(36) = 9(64)$$

$$16\times4\times9 = 9\times16\times4$$ ∴ it represents a pair of parallel st. line.

Distance between them $= 2\sqrt{\frac{g^2-ac}{a(a+b)}} = 2\sqrt{\frac{36+108}{9(25)}} = \frac{2\times\cancel{12}^{4}}{\cancel{3}\times5} = 8/5$

15) S.T the equation $4x^2+4xy+y^2-6x-3y+4=0$ represents a pair of parallel st. line. Find the distance between them.

$$4x^2+4xy+y^2-6x-3y+4=0$$

$a=4,\ h=2,\ b=1,\ g=-3\ \ f=-3/2\ \ c=-4.$

Condition for pair of parallel st. line $bg^2=af^2$

$$1(9) = \cancel{4}\cdot\frac{9}{\cancel{4}} \Rightarrow 9=9$$

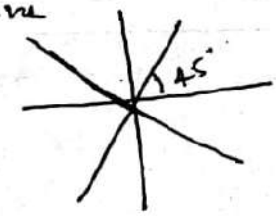
∴ it represents a pair of parallel st. line.

Distance between them $= 2\sqrt{\frac{g^2-ac}{a(a+b)}} = 2\sqrt{\frac{9+16}{4\times5}}$

$$= \frac{\cancel{2}\times5}{\cancel{2}\times\sqrt5} = \sqrt5.$$

16) S.T one of the st. lines $x^2-2kxy-y^2=0$ bisect the angle between the pair of ~~st. line~~ co-ordinate axes if $(a+b)^2=4h^2$

If $ax^2+2hxy+by^2=0$ represents a pair of st. line



$$m_1+m_2=-\frac{2h}{b} \text{ and } m_1\cdot m_2 = a/b$$

Given $m_1=\tan45=1$

$$1+m_2=-\frac{2h}{b} \quad \text{and } 1\cdot m_2=\frac{a}{b}$$

$$1+\frac{a}{b}=-\frac{2h}{b}$$

$$\frac{a+b}{\cancel{b}}=\frac{-2h}{\cancel{b}} \Rightarrow (a+b)^2=4h^2$$

17) If the pair of st. lines $x^2-2kxy-y^2=0$ bisect the angle between the pair of st lines $x^2-2lxy-y^2=0$. S.T the later pair also bisects the angle between the former.

The equation of the bisector of the lines $ax^2+2hxy+by^2=0$ is

$$\frac{x^2-y^2}{a-b}=\frac{xy}{h}.$$

$$x^2-2kxy-y^2=0 \Rightarrow a=1, \ b=-1 \ \ h=-k.$$

Eqn of the bisector of the line $\frac{x^2-y^2}{1+1}=\frac{xy}{-k}$

$$\frac{x^2+y^2}{2}=\frac{xy}{-k}. \quad \text{——} (1)$$

This is given as $x^2-2lxy-y^2=0$

$$x^2-y^2=2lxy$$

$$\frac{x^2-y^2}{2}=\frac{xy}{1/2l} \quad \text{——} (2)$$

From (1) and (2) $-k=\frac{1}{l} \Rightarrow kl=-1.$

Eqn of the bisectors of $x^2-2lxy-y^2=0$

$$\frac{x^2-y^2}{2}=\frac{xy}{-l}$$

$$=\frac{xy}{1/K}$$

$$x^2-y^2=2kxy$$

$x^2-2kxy-y^2=0$ Hence the result.

18) P.T the st. line joining the origin to the point of intersection $3x^2+5xy-3y^2+2x+3y=0$ and $3x-2y-1=0$ are at right angles

Homogenizing $3x^2+5xy-3y^2+2x+3y=0$ with $3x-2y-1=0$

$$3x-2y=1$$

$$\frac{3x-2y}{1}=1$$

$$3x^2+5xy-3y^2+(2x+3y)\cdot 1=0$$

$$3x^2+5xy-3y^2+(2x+3y)(3x-2y)=0$$

$$3x^2+5xy-3y^2+6x^2+5xy-6y^2=0$$

$9x^2+10xy-9y^2=0 \Rightarrow a=9, \ b=-9$ $\therefore a+b=9-9=0$

$\therefore$ The lines are $\perp$r.

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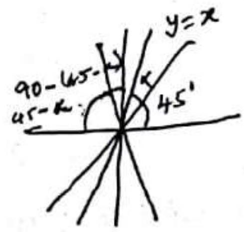
5) P.T The equation to the st. lines through the origin, each of which makes an angle α with the st. lines $y=x$ is $x^2 - 2xy\sec 2\alpha + y^2 = 0$.

Let one line makes an angle α with $y=x$



then $m_1 = \tan(45+\alpha)$

and $m_2 = \tan(45-\alpha)$.

$\therefore$ let the lines are $y = \tan(45+\alpha)x$

$y = \tan(45-\alpha)x$.

$\therefore$ Combined Eqn $\left(y - \tan(45+\alpha)x\right)\left(y - \tan(45+\alpha)x\right) = 0$

$$y^2 - xy\left(\tan(45-\alpha) + \tan(45+\alpha)\right) + x^2\left(\tan(45+\alpha)(\tan(45-\alpha)\right) = 0$$

$$\tan(45+\alpha) + \tan(45-\alpha) = \frac{\sin(45+\alpha)}{\cos(45+\alpha)} + \frac{\sin(45-\alpha)}{\cos(45-\alpha)}$$

$$= \frac{\sin(45+\alpha)\cos(45-\alpha) + \cos(45+\alpha)\sin(45-\alpha)}{\cos(45+\alpha)\cos(45-\alpha)}$$

$$= \frac{\sin(45+\alpha+45-\alpha)}{\frac{1}{2}(\cos 90 + \cos 2\alpha)} = \frac{2\sin 90}{\cos 2\alpha}$$

$$= 2\sec 2\alpha.$$

$$\tan(45+\alpha) \times (\tan(45-\alpha)) = \frac{\tan 45 + \tan\alpha}{1 - \tan 45 \tan\alpha} \times \frac{\tan 45 - \tan\alpha}{1 + \tan 45 \tan\alpha}$$

$$= \frac{1+\tan\alpha}{1-\tan\alpha} \times \frac{1-\tan\alpha}{1+\tan\alpha} = 1$$

$\therefore \quad y^2 - 2xy\sec 2\alpha + x^2 = 0$

10) A $\triangle OPQ$ is formed by the pair of st. lines $x^2 - 4xy + y^2 = 0$ and the line PQ. The equation of PQ is $x+y-2=0$. Find the equation of the median of the $\triangle$le OPQ

Ex. 6.38. If the equation $\lambda x^2 - 10xy + 12y^2 + 5x - 16y - 3 = 0$ Represents a pair of st. line find i) λ and separate equations of the line. ii) point of intersection iii) angle between the lines.

$$\lambda x^2 - 10xy + 12y^2 + 5x - 16y - 3 = 0$$

$$a = \lambda, \quad h = -5, \quad b = 12 \quad g = 5/2 \quad f = -8 \quad c = -3$$

Condition: $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

$$\lambda(12)(-3) + 2(-8)\left(\tfrac{5}{2}\right) - \lambda(-8)^2 - 12\left(\tfrac{5}{2}\right)^2 - (-3)(-5)^2 = 0$$

$$-36\lambda + 200 - 64\lambda - 75 + 75 = 0 \Rightarrow \lambda = 2$$

Separate Eqns Consider $2x^2 - 10xy + 12y^2 = 2x^2 - 4xy - 6xy + 12y^2$

$$= 2x(x - 2y) - 6y(x - 2y)$$

$$= (2x - 6y)(x - 2y)$$

$$2x^2 - 10xy + 12y^2 - 5x - 16y - 3 = (2x - 6y + l)(x - 2y + m) = 0$$

$$-5 = 2m + l$$
$$-16 = -6m - 2l$$ on solving $l = -1$, $m = 3$.

$\therefore$ The separate lines are $2x - 6y - 1 = 0$ $x - 2y + 3 = 0$

Point of intersection.

$$\begin{array}{cccc} -6 & -1 & 2 & -6 \\ -2 & 3 & 1 & -2 \end{array}$$

$$\frac{x}{-18-2} = \frac{y}{-1-6} = \frac{1}{-4+6}$$

$$= \frac{x}{-20} = \frac{y}{-7} = \frac{1}{2}$$

$$x = -\frac{20}{2} = -10$$

$$y = -7/2$$

$$\therefore (x, y) = (-10, -7/2)$$

$$\tan\theta = \left|\frac{2\sqrt{h^2 - ab}}{a+b}\right| = \left|\frac{2\sqrt{25-24}}{2+12}\right| = 1/7$$

$$\theta = \tan^{-1}(1/7)$$

Ex 6.39) A student when walk from his house at an average speed of 6 kmph reaches the school 10 minutes before the school starts. When his average speed is 4 km/hr he reaches the school 5 minutes late. If he starts to school every day at 8.00 AM. then 1) find distance between the school and his house 2) the minimum average speed to reach the school on time and time taken to reach the school. 3) the time the school gate closes 4) pair of st. line of his path of the walk.

Sol: Let x be the time in hr and y be the distance in KM.

$$y = 6\left(x - \frac{10}{60}\right) \Rightarrow y = 6x - 1 \quad \text{—— (1)}$$

$$y = 4\left(x + \frac{5}{60}\right) \Rightarrow y = 4x + \frac{1}{3} \quad \text{—— (2)}$$

Solving (1) and (2) $\quad 6x - 1 = 4x + \frac{1}{3}$

$$2x = \frac{1}{3} + 1 = \frac{4}{3}$$

$$x = \frac{4}{6} = \frac{2}{3} \quad \therefore \; y = 6 \cdot \frac{2}{3} - 1 = 3$$

$\therefore$ Time $= \frac{2}{3}$ hrs $= 40$ min. distance is 3 KMs.

Average speed $= \frac{60}{40} \times 3 = 4.5$ KM.P hr.

School starts at 8.40 AM.

Pair of st line $\therefore (6x - y - 1)(4x - y + \frac{1}{3}) = 0$

$$72x^2 - 30xy + 3y^2 - 6x + 2y - 1 = 0.$$

Ex 6.40) If one of the st. lines of $ax^2 + 2hxy + by^2 = 0$ is $\perp$r to $px + qy = 0$ then S.T $ap^2 + 2hpq + bq^2 = 0$

Let m_1, m_2 are the slope of the lines $ax^2 + 2hxy + by^2 = 0$ then —— (1)

$$m_1 + m_2 = -\frac{2h}{b} \text{ and } m_1 \cdot m_2 = \frac{a}{b}$$

and let m be the slope of the line $px + qy = 0$ —— (2)

then $\quad m = -\frac{p}{q}$

$\therefore$ $px + qy = 0$ is $\perp$r to (1)

$$m \cdot m_1 = -1 \quad \text{and} \quad m \cdot m_2 = -1$$

(ie) $\quad mm_1 + 1 = 0 \quad m \cdot m_2 + 1 = 0$

$$(mm_1 + 1)(m \cdot m_2 + 1) = 0$$

$$(m_1 m_2) m^2 + (m_1 + m_2) m + 1 = 0$$

$$\frac{a}{b}\left(-\frac{p}{q}\right)^2 + \left(-\frac{2h}{b}\right)\left(-\frac{p}{q}\right) + 1 = 0$$

$$\frac{ap^2 + 2hpq + bq^2}{bq^2} = 0 \Rightarrow ap^2 + 2hpq + bq^2 = 0$$

Ex 6.41 S.T the st. lines joining the origin to the points of intersection of $3x - 2y + 2 = 0$ and $3x^2 + 5xy - 2y^2 + 4x + 5y = 0$ are at rt angles.

Homogenizing the equation $3x^2+5xy-2y^2+4x+5y=0$ with

$3x-2y+2=0$

$$3x-2y=-2$$

$$\frac{3x-2y}{-2}=1$$

$$3x^2+5xy-2y^2+(4x+5y)\cdot 1=0$$

$$3x^2+5xy-2y^2+(4x+5y)\left(\frac{3x-2y}{-2}\right)=0$$

$$-6x^2-10xy+4y^2+12x^2-8xy+15xy-10y^2=0$$

$$6x^2-3xy-6y^2=0$$

$$2x^2-xy-2y^2=0 \qquad a=2,\ b=-2$$

$$a+b=2-2=0$$

$\therefore$ The lines are $\perp r$.

Ex. 6.33 Separate the equations $5x^2+6xy+y^2=0$

$$5x^2+6xy+y^2= 5x^2+5xy+xy+y^2=0$$

$$=5x(x+y)+y(x+y)=0$$

$$=(5x+y)(x+y)=0$$

$$5x+y=0,\quad x+y=0$$

Ex 6.34. If exists find the st lines by separating the equations $2x^2+2xy+y^2=0$

We cannot factorise this one. So we can use another method to separate the line.

$$2x^2+2xy+y^2=0$$

$$\div x^2 \quad 2+2\frac{y}{x}+\frac{y^2}{x^2}=0 \quad \text{put } m=\frac{y}{x}$$

$\therefore$ This is another way to separate the lines.

$m^2+2m+2=0$ $\because$ the value of m is imaginary

$\therefore$ They are imaginary lines.

Ex 6.35. Find the equation of the pair of lines through the origin and $\perp r$ to the lines $ax^2+2hxy+by^2=0$.

Let m_1 and m_2 are the slopes of the required lines.

$\therefore$ The lines are $y-m_1x=0$ and $y-m_2x=0$ —— (1)

~~Combined eqn~~

Given that $ax^2+2hxy+by^2=0$

$$m_1+m_2 = \frac{-2h}{b} \qquad m_1m_2 = \frac{a}{b}.$$

$\perp$r lines to one ① are $m_1y+x=0$, $m_2y+x=0$

Combined equation $(m_1y+x)(m_2y+x)=0$

$$m_1 \cdot m_2 y^2 + (m_1+m_2)xy + x^2 = 0$$

$$\frac{a}{b}y^2 - \frac{2h}{b}xy + x^2 = 0$$

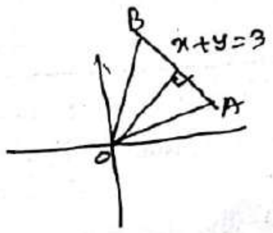
$$ay^2 - 2hxy + bx^2 = 0.$$

Ex 6.36) S.T the lines $x^2-4xy+y^2=0$ and $x+y=3$ form an equilateral triangle

$$x^2-4xy+y^2=0$$

$a=1,\ h=-2,\ b=1$

$$\tan\theta = \left|\frac{2\sqrt{h^2-ab}}{a+b}\right| = \left|\frac{\cancel{2}\sqrt{4-1}}{\cancel{2}}\right| = \sqrt{3}$$



$$\theta = \tan^{-1}(\sqrt{3}) = 60^\circ \Rightarrow \angle AOB = 60$$

The angle bisector of the angle AOB is

$$\frac{x^2-y^2}{a-b} = \frac{xy}{h}.$$

$$x^2-y^2=0$$

$$(x+y)(x-y)=0$$

$$x+y=0,\quad x-y=0$$

The angle bisector $x-y=0$ is $\perp$r to $x+y=3$

(If the angle bisector is $\perp$r to the third side then the $\triangle$le is isosceles) $\therefore \angle ABO = \angle BAO = 60^\circ$

$\therefore$ The $\triangle$le is equilateral $\triangle$le.

Ex 6.37. If the pair of st. lines $x^2-2cxy-y^2=0$ and $x^2-2dxy-y^2=0$ be such that each pair bisects the angle between the other pair P.T $cd=-1$.

Given lines are

$x^2-2cxy-y^2=0$

$a=1,\ h=-c,\ b=-1$

Eqn. of angle bisector $\frac{x^2-y^2}{a-b} = \frac{xy}{h}$

$$\frac{x^2-y^2}{2} = \frac{xy}{-c} \Rightarrow cx^2+2xy-cy^2=0 \quad \text{— ①}$$

This is same as $x^2-2dxy-y^2=0$ — ②

$$\frac{1}{c} = \frac{-\cancel{2}d}{\cancel{2}} = \frac{+1}{-c}$$

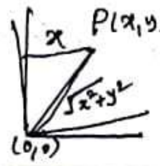
$$\Rightarrow 1 = -cd \text{ or } cd = 1$$

EXERCISE – 6.5 (One Mark)

1. The equation of the locus of the point whose distance from y axis is half the distance from the origin is

1) $x^2+3y^2=0$ 2) $x^2-3y^2=0$ 3) $3x^2+y^2=0$ 4) ✓ $3x^2-y^2=0$

Given $x = \frac{1}{2}\sqrt{x^2+y^2}$



$2x = \sqrt{x^2+y^2} \Rightarrow 4x^2 = x^2+y^2$

$3x^2-y^2=0$

2) Which of the following equation is the locus of $(at^2, 2at)$

a) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ 2) $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ 3) $x^2+y^2=a^2$ 4) ✓ $y^2=4ax$

It is clear that $(at^2, 2at)$ are the parametric Eqn. of parabola $y^2=4ax$

$y^2=4ax$

$4a^2t^2 = 4a \cdot at^2$

3) Which of the following point lie on the locus $3x^2+3y^2-8x-12y+17=0$

1) $(0,0)$ 2) $(-2,3)$ 3) ✓ $(1,2)$ 4) $(0,-1)$.

Try one by one. Substitute one by one. $(0,0) = 17$ $(1,2) = 3+12-8-24+17 = 0$

$(-2,3)$ $12+27+16-36+17 \neq 0$

4) If the point $(8,-5)$ lies on the locus $\frac{x^2}{16}-\frac{y^2}{25}=k$ then the value of k is

1) ~~0~~ 2) 1 3) 2 4) ✓ 3.

Sub. the pt in the given Eqn.

$\frac{\cancel{64}^{4}}{\cancel{16}}-\frac{\cancel{25}^{1}}{\cancel{25}} = k \Rightarrow$ ~~k≠0~~

$k=3$

5) St. line joining the pts $(2,3)$ $(-1,4)$ passes through (α, β) if

1) $\alpha+2\beta=7$, 2) $3\alpha+\beta=9$ 3) ✓ $\alpha+3\beta=11$ 4) $3\alpha+\beta=1$.

$\overset{x_1\ y_1}{(2,3)}$ $\overset{x_2\ y_2}{(-1,4)}$

$\frac{y-3}{1} = \frac{x-2}{-3} \Rightarrow -3y+9 = x-2$

$x+3y=11$

$\therefore \alpha, \beta$ is a pt on the line $\alpha+3\beta=11$

6) The slope of the line which makes an angle 45° with the line $3x-y=5$ are

1) $(1,-1)$ 2) ✓ $\frac{1}{2}, -2$ 3) $1, \frac{1}{2}$ 4) $2, -\frac{1}{2}$

Let m_1 be the slope of the given line $3x-y=-5$

$$m_1 = 3$$

Let m_2 be the slope of the second line.

$$\theta = 45^\circ$$

$$\tan\theta = \frac{m_1-m_2}{1+m_1 m_2} \Rightarrow \tan 45^\circ = \frac{m_1-m_2}{1+m_1 m_2}$$

$$1 = \frac{m_1-m_2}{1+m_1 m_2}$$

$$1 = \frac{3-m_2}{1+3m_2}$$

$$1+3m_2 = 3-m_2$$

$$4m_2 = 2$$

$$m_2 = 2/4 = 1/2$$

If $\frac{m_2-3}{1+3m_2} = 1$

$$m_2 - 3 = 1+3m_2$$

$$-4 = 2m_2$$

$$m_2 = -2.$$

$\therefore$ The slopes are $1/2, -2$

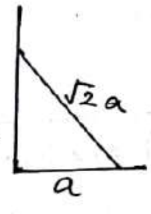
7) Equation of st. line that forms an isosceles triangle with co-ordinates axes in the I quadrant with perimeter $4+2\sqrt{2}$ is

1) $x+y+2=0$ 2) $x+y-2=0$ 3) $x+y-\sqrt{2}=0$ 4) $x+y+\sqrt{2}=0$

Parameter $a+a+\sqrt{2}a = 4+2\sqrt{2}$

$$2a+\sqrt{2}a = 4+2\sqrt{2}$$

$$\cancel{2}\left(a+\frac{a}{\sqrt{2}}\right) = \cancel{2}(2+\sqrt{2})$$



$\therefore$ Eqn of the line $\frac{x}{2}+\frac{y}{2}=1 \Rightarrow a=2.$

$$x+y=2 \Rightarrow x+y-2=0$$

8) The co-ordinates of the four vertices of a quadrilateral are $(-2,4)$ $(-1,2)$ $(1,2)$ $(2,4)$ taken in order. The equation of the line passing through the vertex $(-1,2)$ and dividing the quadrilateral in equal area is

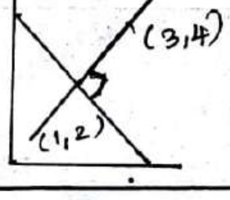
1) $x+1=0$ 2) $x+y=1$ 3) $x+y+3=0$ 4) $x-y+3=0$ ✓

9) The intercepts of the ⊥r bisector of the line segment joining $(1,2)$ $(3,4)$ with co-ordinates axes are

1) $5,-5$ 2) $5,5$ 3) $5,3$ 4) $5,-4$.

Mid point of $A(1,2)$ $B(3,4)$

$$\left(\frac{1+3}{2}, \frac{2+4}{2}\right) = \left(\frac{4}{2}, \frac{6}{2}\right) = (2,3)$$



Eqn of line joining $(1,2)$ $(3,4)$

$$\frac{y-2}{2} = \frac{x-1}{2} \qquad x-y+1=0. \text{ — } ①$$

Any line ⊥r to ① $x+y+k=0$.

∴ it passes through the mid pt of $(1,2)(3,4)$ is $(2,3)$

$$2+3+k=0$$
$$k=-5$$

∴ Eqn of the required line $x+y=5$

$$\frac{x}{5}+\frac{y}{5}=1 \quad \text{intercept } (5,5)$$

10) The equation of the line with slope 2 and the length of the ⊥r from the origin equal to $\sqrt{5}$ is.

1) $x+2y=\sqrt{5}$ 2) $2x+y=\sqrt{5}$ 3) $2x+y=5$ 4) $x+2y-5=0$

Let the required line be $y=2x+c$.

But length of the ⊥r from $(0,0)$ to this line $\frac{c}{\sqrt{1+4}} = \frac{c}{\sqrt{5}}$

$$\therefore \frac{c}{\sqrt{5}} = \sqrt{5} \Rightarrow c=5$$

∴ Eqn. of the line is $y=2x+5$

$$2x-y+5=0.$$

11) A line ⊥r to a line $5x-y=0$ forms a △le with the co-ordinate axes. If the area of the △le is 5 units then its equation is.

1) ✓ $x+5y\pm5\sqrt{2}=0$ 2) $x-5y\pm5\sqrt{2}=0$ 3) $5x+y\pm5\sqrt{2}=0$

4) $5x-y\pm5\sqrt{2}=0$.

any line ⊥r to $5x-y=0$ is $x+5y+k=0$.

x intercept $x=-k$

y intercept, $y=-k/5$. $\Delta = \frac{1}{2}(-k)\left(-\frac{k}{5}\right)=5$

$$k^2=50$$
$$k=\pm5\sqrt{2}$$

∴ The Eqn $x+5y\pm5\sqrt{2}=0$.

12) Equation of St. line ⊥r to the line $x-y+5=0$ through the point of intersection of y axis and the given line is.

1) $x-y-5=0$ 2) ✓ $x+y-5=0$ 3) $x+y+5=0$ 4) $x+y+10=0$.

Any line ⊥r to $x-y+5=0$ is $x+y+k=0$.

Point of intersection of y axis. Put $x=0$ ∴ $y=5$ $(0,5)$

⊥r line passes through $(0,5)$ $0+5+k=0$

$k=-5$ ∴ $x+y-5=0$

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13) If the eqn. of the base opposite to the vertex $(2,3)$ of an equilateral $\triangle$le is $x+y=2$ then the length of the side is 1) $\sqrt{3}/\sqrt{2}$ 2) $2\sqrt{6}$ 3) $\sqrt{6}$ 4) $3\sqrt{2}$

Length of the $\perp$r from $(2,3)$ to $x+y-2=0$

$$d = \frac{2+3-2}{\sqrt{1+1}} = \frac{3}{\sqrt{2}}$$

$$\frac{3}{\sqrt{2}} = \frac{\sqrt{3}}{2}a \Rightarrow \frac{\sqrt{3}\cdot\sqrt{3}}{\sqrt{2}} = \frac{\sqrt{3}}{\sqrt{2}\cdot\sqrt{2}}a$$

$a = \sqrt{6}$

$\therefore$ Length of the sides is $= \sqrt{6}$.

14) The line $(P+2q)x + (P-3q)y = P-q$ for different values of P and q passes through the point.

1) $\left(\frac{3}{2}, \frac{5}{2}\right)$ 2) $\left(\frac{2}{5}, \frac{2}{5}\right)$ 3) $\left(\frac{3}{5}, \frac{3}{5}\right)$ 4) $\left(\frac{2}{5}, \frac{3}{5}\right)$

$P(x+y-1) + q(2x-3y+1) = 0$

$\Rightarrow x+y=1$

$2x-3y=-1$

$3x+3y=3$

$2x-3y=-1$

$5x = 2$

$x = 2/5$

$\therefore y = 1-\frac{2}{5} = 3/5$

$(2/5, 3/5)$

15) The point on the line $2x-3y+5=0$ is equidistance from $(1,2)$ $(3,4)$ is 1) $(7,3)$ 2) $(4,1)$ 3) $(1,-1)$ 4) $(-2,3)$

Let (a,b) be a point $2x-3y+5=0$ $\Rightarrow 2a-3b=-5$

Distance from (a,b) and $(1,2)$, (a,b) and $(3,4)$ are equal.

$(a-1)^2+(b-2)^2 = (a-3)^2+(b-4)^2$

$\Rightarrow 4a+4b=20$

$2a+2b=10$

$2a-3b=-5$

$(-)$ $5b = 15$ $b = 3$.

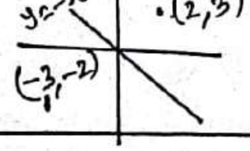
then $2a = -5+9 = 4$

$a = 2$

$\therefore (4,1)$

16) The image of the point $(2,3)$ in the line $y=-x$ is

1) $(-3,-2)$ 2) $(-3,2)$ 3) $(-2,-3)$ 4) $(3,2)$



17) Length of the ⊥r from the origin to the line $\frac{x}{3} - \frac{y}{4} = 1$ is

1) $\frac{11}{5}$ 2) $\frac{5}{12}$ 3) $\frac{12}{5}$ 4) $-\frac{5}{12}$

$$d = \frac{1}{\sqrt{\frac{1}{3^2} + \frac{1}{4^2}}} = \frac{1}{\sqrt{\frac{16+9}{144}}} = \frac{1}{5/12} = 12/5$$

18) The y intercept of the st. line passing through (1,3) and ⊥r to $2x - 3y + 1 = 0$ is

1) $3/2$ 2) $9/2$ 3) $2/3$ 4) $2/9$.

⊥r line $3x + 2y + k = 0$

∵ Passes through (1, 3) $3 + 6 + k = 0$

$k = -9$.

∴ Eqn $3x + 2y - 9 = 0$

y intercept. Put $x = 0$ $2y = 9$

$y = 9/2$.

19) If the two st. lines $x + (2k-7)y + 3 = 0$ and $3kx + 9y - 5 = 0$ are ⊥r then the value of k is

1) $k = 3$ 2) $k = 1/3$ 3) $k = 2/3$ 4) $3/2$.

$$m_1 = -\frac{1}{2k-7} \qquad m_2 = -\frac{3k}{9}$$

$$m_1 \cdot m_2 = -1$$

$$\frac{1}{(2k-7)} \cdot \frac{k}{3} = -1 \Rightarrow \frac{k}{6k-21} = -1$$

$k = -6k + 21$

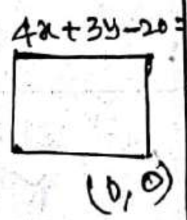
$7k = 21$ $k = 3$.

20) If a vertex of a square is at the origin and its one side lies along $4x + 3y - 20 = 0$ then the area of the square is

1) 20 s.q. units 2) 16 s.u. 3) 25 s.u. 4) 4 sq.u.

Length of the ⊥r from (0,0) to $4x + 3y - 20 = 0$ is

$$= \frac{20}{\sqrt{16+9}} = \frac{20}{5} = 4.$$



Side of the square is = 4

Area = 4×4 = 16.

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21) If the lines intercepted by the equations $6x^2+41xy-7y^2=0$ makes an angle α and β with x axis then $\tan\alpha \tan\beta$ is

1) $-\frac{6}{7}$ 2) $6/7$ 3) $-7/6$ 4) $7/6$.

$$m_1 \cdot m_2 = \frac{a}{b} = \frac{6}{-7} = -\frac{6}{7}$$

22) The area of the $\triangle$le formed by the lines $x^2-4y^2=0$ and $x=a$.

1) $2a^2$ 2) $\frac{\sqrt{3}}{2}a^2$ 3) $\frac{1}{2}a^2$ 4) $\frac{2}{\sqrt{3}}a^2$

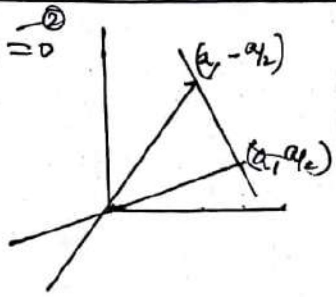
The separate lines are $x+2y=0$ ① , $x-2y=0$ ②

Given line $x=a$ ③

Point of intersection of ① and ③ $2y=-a$

$y=-a/2$

② and ③ $y=a/2$



Area of the $\triangle$le: $\frac{1}{2}\begin{vmatrix} a & -a/2 \\ a & a/2 \\ 0 & 0 \\ a & -a/2 \end{vmatrix} = \frac{1}{2}\left(\frac{a^2}{2}+\frac{a^2}{2}\right) = \frac{1}{2}\cdot\frac{\cancel{2}a^2}{\cancel{2}} = \frac{1}{2}a^2$

23) If one of the lines given by $6x^2-xy+4cy^2=0$ is $3x+4y=0$ then c is equal to 1) -3 2) -1 3) 3 4) 1.

Let the line be $ax+by=0$

$\therefore 6x^2-xy+4cy^2 = (ax+by)(3x+4y)=0$.

$\cancel{4}c=\cancel{4}b$. $3a=b \Rightarrow a=2$

$4a+3b=-1$

$3b=-9 \therefore b=-3$.

$c=-3$

(or)

$$3x+4y\,\overline{\big)\,6x^2-xy+4cy^2}\quad(2x-3y)$$
$$6x^2+8xy$$
$$-9xy+4cy^2$$
$$-9xy-12y^2$$

$4c-12=0$

$c=3$

24) θ is acute angle between the lines $x^2-xy-6y^2=0$ then $\frac{2\cos\theta+3\sin\theta}{4\sin\theta+5\cos\theta}$ is

1) 1 2) $-1/9$ 3) $5/9$ 4) $1/9$.

$a=1$, $h=-1/2$ $b=-6$

$\tan\theta = \left|\frac{2\sqrt{\frac{1}{4}+6}}{-5}\right| = \left|\frac{2\cdot\frac{5}{2}}{-5}\right| = 1$ $\theta=45^\circ$.

$\frac{2\cos 45+3\sin 45}{4\sin 45+5\cos 45} = \frac{\frac{2}{\sqrt{2}}+\frac{3}{\sqrt{2}}}{\frac{4}{\sqrt{2}}+\frac{5}{\sqrt{2}}} = 5/9$.

25) The equation of one of the lines represented by $x^2+2xy\cot\theta-y^2=0$ is

1) $x-y\cot\theta=0$ 2) $x+y\tan\theta=0$ 3) $x\cos\theta+y(\sin\theta+1)=0$ 4) $x\sin\theta+y(\cos\theta+1)=0$.

$x^2+x(2y\cot\theta)+(-y^2)=0$ $a=1$, $b=2y\cot\theta$ $c=-y^2$

$x = \frac{-2y\cot\theta \pm\sqrt{4y^2\cot^2\theta+4y^2}}{2} = \frac{\cancel{2}(-y\cot\theta \pm y\operatorname{cosec}\theta)}{\cancel{2}}$

$\therefore x = -\frac{y\cos\theta}{\sin\theta} - \frac{y}{\sin\theta}$

$x\sin\theta+y(1+\cos\theta)=0$