UNIT – 02 KINEMATICS

TWO MARKS AND THREE MARKS:

01. Explain what is meant by Cartesian coordinate system?

At any given instant of time, the frame of reference with respect to which the position of the object is described in terms of position coordinates (x, y, z) (i.e., distances of the given position of an object along the x, y, and z–axes.) is called **"Cartesian coordinate system"**

02. Define a vector. Give examples

It is a quantity which is described by both magnitude and direction. Geometrically a vector is a directed line segment. **Examples** Force, velocity, displacement, position vector, acceleration, linear momentum and angular momentum

03. Define a scalar. Give examples

It is a property which can be described only by magnitude. In physics a number of quantities can be described by scalars. **Examples** Distance, mass, temperature, speed and energy

04. Write a short note on the scalar product between two vectors.

The scalar product (or dot product) of two vectors is defined as the product of the magnitudes of both the vectors and the cosine of the angle between them. $\vec{A}.\vec{B} = AB\cos\theta$. Here, A and B are magnitudes of $\vec{A}$ and $\vec{B}$.

Properties

The product quantity $\vec{A}$ and $\vec{B}$ is always a scalar. The scalar product is commutative.

05. Write a short note on vector product between two vectors.

The vector product or cross product of two vectors is defined as another vector having a magnitude equal to the product of the magnitudes of two vectors and the sine of the angle between them. $\vec{C} = \vec{A} \times \vec{B} = (AB \sin\theta)\hat{n}$

06. How do you deduce that two vectors are perpendicular?

If two vector $\vec{A}$ and $\vec{B}$ are perpendicular to each other their scalar product. $\vec{A}.\vec{B} = 0$, because $\cos 90^0 = 0$.

07. Define displacement and distance.

Distance is the actual path length travelled by an object in the given interval of time during the motion. It is a positive scalar quantity.

Displacement the shortest distance between these two positions of the object and its direction is from the initial to final position of the object, during the given interval of time. It is a vector quantity.

08. Define velocity and speed.

Velocity: The rate of change of displacement of the particle.

Velocity = Displacement / time taken. Unit: ms^{-1}. Dimensional formula: LT^{-1}

Speed: The distance travelled in unit time. It is a scalar quantity.

09. Define acceleration.

The acceleration of the particle at an instant is equal to rate of change of velocity. It is a vector quantity. SI Unit: ms^{-2}. Dimensional formula: $M^0L^1T^{-2}$

10. What is the difference between velocity and average velocity?

Velocity is the rate at which the position changes. But the average velocity is the displacement or position change per time ratio.

11. Define a radian?

The length of the arc divided by the radius of the arc. One radian is the angle subtended at the center of a circle by an arc that is equal in length to the radius of the circle.

12. Define angular displacement and angular velocity.

Angular displacement: The angle described by the particle about the axis of rotation in a given time is called angular displacement. The unit of angular displacement is radian.

Angular velocity (ω)

The rate of change of angular displacement is called angular velocity.
The unit of angular velocity is radian per second (rad s^{-1}).

13. What is non uniform circular motion?

If the speed of the object in circular motion is not constant, then we have non-uniform circular motion. For example, when the bob attached to a string moves in vertical circle.

14. Write down the kinematic equations for angular motion.

1. $\omega = \omega_0 + \alpha t$ 2. $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ 3. $\omega^2 = \omega_0^2 + 2\alpha\theta$ 4. $\theta = \frac{(\omega + \omega_0)t}{2}$

15. Write down the expression for angle made by resultant acceleration and radius vector in the non uniform circular motion.

$\tan\theta = \frac{a_t}{\frac{v^2}{r}}$, θ = Resultant acceleration angle, $\frac{v^2}{r}$ = Centripetal acceleration

a_t = Resultant acceleration

16. What is meant by Right handed Cartesian co-ordinate system?

The *x, y* and *z* axes are drawn in anticlockwise direction then the coordinate system is called as "right– handed Cartesian coordinate system".

17. What is point mass?

The mass of any object be assumed to be concentrated at a point. Then this idealized mass is called "point mass". It has no internal structure like shape and size.

18. Define linear motion. Give an example.

An object is said to be in linear motion if it moves in a straight line.

Examples

i) An athlete running on a straight track
ii) A particle falling vertically downwards to the Earth.

19. Define circular motion. Give an example.

Circular motion is defined as a motion described by an object traversing a circular path.

Examples

1) The whirling motion of a stone attached to a string
2) The motion of a satellite around the Earth

20. Define rotational motion. Give an example.

If any object moves in a rotational motion about an axis, the motion is called 'rotation'.

Examples

i) Rotation of a disc about an axis through its center
ii) Spinning of the Earth about its own axis.

21. What is meant by motion in one dimension?

One dimensional motion is the motion of a particle moving along a straight line

Examples

i) Motion of a train along a straight railway track.
ii) An object falling freely under gravity close to Earth.

22. What is meant by motion in two dimensions?

If a particle is moving along a curved path in a plane, then it is said to be in two dimensional motion.

Examples

i) Motion of a coin on a carrom board.
ii) An insect crawling over the floor of a room.

23. What is meant by motion in three dimensions?

A particle moving in usual three dimensional space has three dimensional motion

Examples

i) A bird flying in the sky. ii) Random motion of a gas molecule.
iii) Flying of a kite on a windy day.

24. Define magnitude of a vector.

The length of a vector is called magnitude of the vector. It is always a positive quantity. Sometimes the magnitude of a vector is also called 'norm' of the vector. Denoted by 'A'

25. What is meant by equal vectors?

Two vectors $\vec{A}$ and $\vec{B}$ are said to be equal when they have equal magnitude and same direction and represent the same physical quantity.

26. Define parallel and anti-parallel vectors.

Two vectors $\vec{A}$ and $\vec{B}$ act in the same direction along the same line or on **parallel lines**, then the angle between them is 0^0

Two vectors $\vec{A}$ and $\vec{B}$ are said to be anti–parallel when they are in opposite directions along the same line or on parallel lines. Then the angle between them is 180^0

27. Define work done by a force.

The work done by a force $\vec{F}$ to move an object through a small displacement $\overrightarrow{dr}$ is defined as, $\vec{W} = \vec{F}.\ \overrightarrow{dr}$. $W = F\, dr \cos\theta$

28. What is retardation?

If the velocity is decreasing with respect to time then the acceleration.

29. What is instantaneous velocity?

The instantaneous velocity at an instant *t* or simply 'velocity' at an instant *t* is defined as limiting value of the average velocity as $\Delta t \to 0$, evaluated at time *t*.

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

30. Define momentum

The linear momentum or simply momentum of a particle is defined as product of mass with velocity. It is denoted as '$\vec{p}$'. Momentum is also a vector quantity.

31. What is meant by velocity of one object with respect to another?

When two objects A and B are moving with different velocities, then the velocity of one object A with respect to another object B is called relative velocity of object A with respect to B.

32. Write the kinetic equations for linear motion.

i) $v = u + at$ ii) $s = ut + \frac{1}{2} at^2$ iii) $v^2 = u^2 + 2as$ iv) $s = \frac{(u+v)t}{2}$

33. What is meant by projectile?

When an object is thrown in the air with some initial velocity and then allowed to move under the action of gravity alone, the object is known as a projectile

34. Give some examples for projectile motion.

1. An object dropped from window of a moving train
2. A bullet fired from a rifle.
3. A ball thrown in any direction.
4. A javelin or shot put thrown by an athlete.
5. A jet of water issuing from a hole near the bottom of a water tank.

35. Define Time of flight

The time taken for the projectile to complete its trajectory or time taken by the projectile to hit the ground is called time of flight.

36. What is Horizontal range?

The horizontal distance covered by the projectile from the foot of the tower to the point where the projectile hits the ground is called horizontal range

37. Define maximum height.

The maximum vertical distance travelled by the projectile during its journey is called maximum height.

38. Define horizontal range.

The maximum horizontal distance between the point of projection and the point on the horizontal plane where the projectile hits the ground is called horizontal range (R).

39. Define Time of flight.

The total time taken by the projectile from the point of projection till it hits the horizontal plane is called time of flight.

40. Define Uniform circular motion.

When a point object is moving on a circular path with a constant speed, it covers equal distances on the circumference of the circle in equal intervals of time.

41. Write the assumptions need to study about the projectile motion.

i) Air resistance is neglected.
ii) The effect due to rotation of Earth and curvature of Earth is negligible.
iii) The acceleration due to gravity is constant in magnitude and direction at all points of the motion of the projectile.

42. How are two vectors expressed in a Cartesian system? Explain the addition and subtraction using components.

i) The two vectors $\vec{A}$ and $\vec{B}$ in a Cartesian coordinate system can be expressed as

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k} \qquad , \vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$$

ii) Then the addition of two vectors is equivalent to adding their corresponding x, y and z components.

$$\vec{A} + \vec{B} = (A_x + B_x)\,\hat{\imath} + (A_y + B_y)\,\hat{\jmath} \; + \; (A_z + B_z)\,\hat{k}$$

iii) Similarly the subtraction of two vectors is equivalent to subtracting the corresponding x, y and z components.

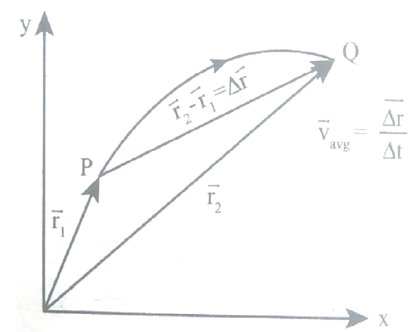
$$\vec{A} - \vec{B} = (A_x - B_x)\,\hat{\imath} + (A_y - B_y)\,\hat{\jmath} \; + \; (A_z - B_z)\,\hat{k}$$

The above rules form an analytical way of adding and subtracting two vectors.

43. Define average velocity and represent it graphically.

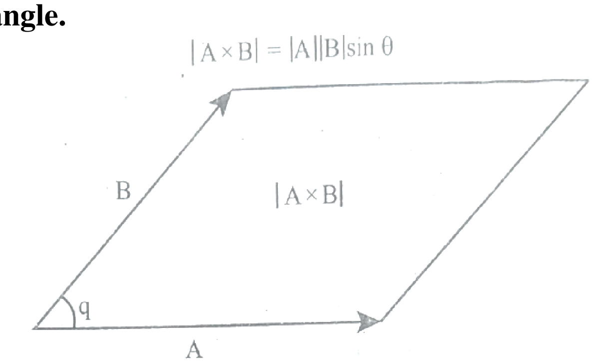
The average velocity is defined as ratio of the displacement vector to the corresponding time interval $\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t}$

It is a vector quantity. The direction of average velocity is in the direction of the displacement vector ($\Delta\vec{r}$).

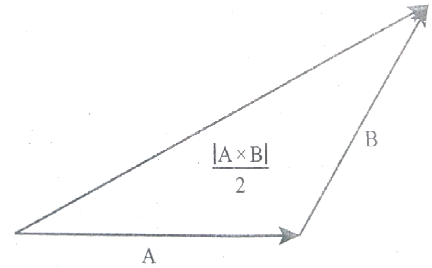


44. Obtain an expression for the area of triangle in terms of the cross product of two vectors representing the two sides of the triangle.

If two vectors $\vec{A}$ and $\vec{B}$ form adjacent sides in a parallelogram, then the magnitude of $|\vec{A} X \vec{B}|$ will give the area of the parallelogram as represented graphically in Figure.



It divide a parallelogram into two equal triangles as shown in the Figure, the area of a triangle with $\vec{A}$ and $\vec{B}$ as sides is $\frac{1}{2}|\vec{A}\,X\vec{B}|$



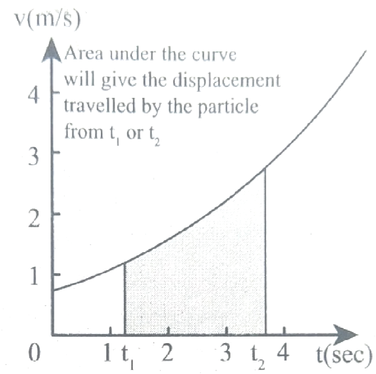
45. What does the slope of 'position – time' graph represent? Which physical quantity is obtained from it?

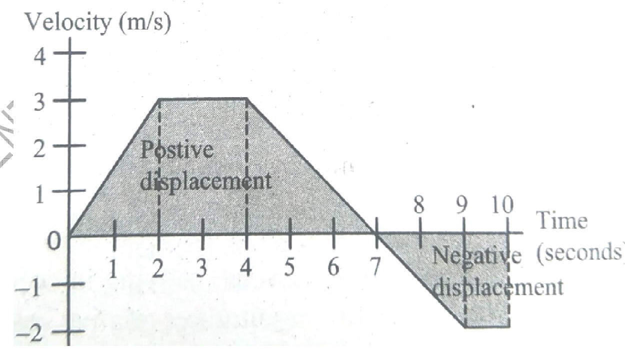
i) Graphically the slope of the position-time graph will give the velocity of the particle. At the same time, if velocity time graph is given, the distance and displacement are determined by calculating the area under the curve. velocity is given by $\frac{dx}{dt} = v$, therefore , $dx = vdt$

ii) By integrating both sides, $\int_{x_1}^{x_2} dx = \int_{x_1}^{x_2} vdt$ integration is equivalent to area under the given curve. So the term $\int_{t_1}^{t_2}$ represents the area under the curve v as a function of time.

iii) Since the left hand side of the integration represents the displacement travelled by the particle from time t_1 to t_2, the area under the velocity time graph will give the displacement of the particle.

iv) If the area is negative, it means that displacement is negative, so the particle has travelled in the negative direction





46. Define the term relative velocity. How can it be obtained vectorially, when the two objects with uniform velocities move in same direction?

i) When two objects A and B are moving with different velocities, then the velocity of one object A with respect to another object B is called relative velocity of object A with respect to B.

ii) Consider two objects A and B moving with uniform velocities V_A and V_B, as shown, along straight tracks in the same direction $\vec{V}_A$, $\vec{V}_B$, with respect to ground.

iii) The relative velocity of object A with respect to object B is $\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$

The relative velocity of object B with respect to object A is $\vec{V}_{BA} = \vec{V}_B - \vec{V}_A$

iv) Thus, if two objects are moving in the same direction, the magnitude of relative velocity of one object with respect to another is equal to the difference in magnitude of two velocities.

47. Write the expression for the magnitude and direction of the relative velocity

i) Consider the velocities $\vec{V}_A$ and $\vec{V}_B$ at an angle θ between their directions. The relative velocity of A with respect to B, $\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$

Then, the magnitude and direction of $\vec{V}_{AB}$ is given by $v_{AB} = \sqrt{v_A^2 + v_B^2 - 2v_A v_B \cos\theta}$

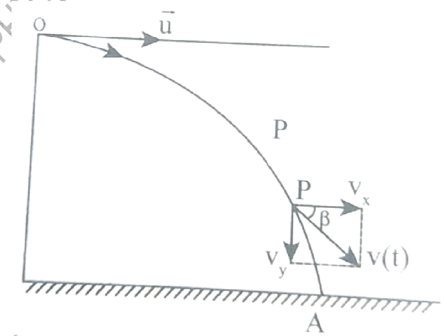
and $\tan\beta = \frac{v_B \sin\theta}{v_B - v_A \cos\theta}$ (Here β is angle between $\vec{V}_{AB}$ and $\vec{V}_B$)

ii) When θ = 0, the bodies move along parallel straight lines in the same direction, We have $v_{AB} = (v_A - v_B)$ in the direction of $\vec{V}_A$. Obviously $v_{BA} = (v_B + v_A)$ in the direction of $\vec{V}_B$.

iii) When θ = 180, the bodies move along parallel straight lines in opposite directions, We have $v_{AB} = (v_A + v_B)$ in the direction of $\vec{V}_A$. Similarly, $v_{BA} = (v_B - v_A)$ in the direction of $\vec{V}_B$.

iv) If the two bodies are moving at right angles to each other, then θ = 90. The magnitude of the relative velocity of A with respect to B = $V_{BA} = \sqrt{v_A^2 + v_B^2}$

48. Derive the expression for a body projected horizontally?



i) At any instant t, the projectile has velocity components along both *x*-axis and *y*-axis. The resultant of these two components gives the velocity of the projectile at that instant *t*, as shown in Figure.

ii) The velocity component at any *t* along horizontal (*x*-axis) is $v_x = u_x + a_x t$

Since, $u_x = u$, $a_x = 0$, we get $v_x = u$.

iii) The component of velocity along vertical direction (*y*-axis) is $v_y = u_y + a_y t$

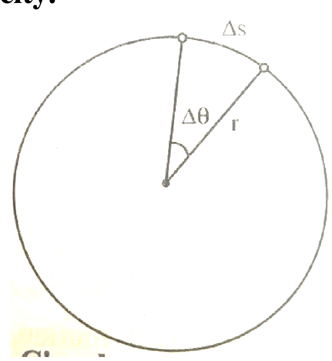
Since, $u_y = 0$, $a_y = g$, we get $v_y = gt$

Hence the velocity of the particle at any instant is $\vec{v} = u\hat{i} + gt\hat{j}$

The speed of the particle at any instant *t* is given by $v = \sqrt{v_x^2 + v_y^2}$

$v = \sqrt{u^2 + g^2 t^2}$

49. Derive the relation between linear velocity and angular velocity.



1) Consider an object moving along a circle of radius r. In a time Δ*t*, the object travels . An arc distance Δ*s* as shown in Figure. The corresponding angle subtended is Δθ

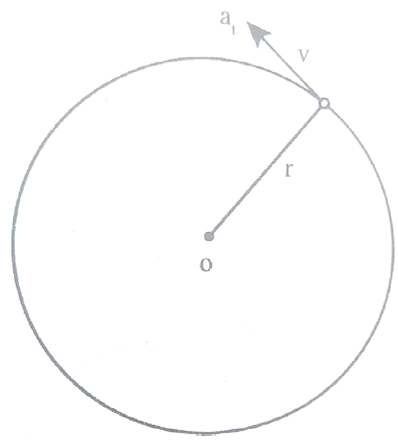
2) The Δs can be written in terms of Δθ as, $\Delta s = r\Delta\theta$

In a time Δ*t*, we have $\frac{\Delta s}{\Delta t} = r\frac{\Delta\theta}{\Delta t}$ In the limit $\Delta t \to 0$,

the above equation becomes $\frac{ds}{dt} = r\omega$ ----------(1)

3) Here $\frac{ds}{dt}$ is linear speed (*v*) which is tangential to the circle and ω is angular speed. So equation (1) becomes $v = r\omega$. Which gives the relation between linear speed and angular speed.

50. Find the expressions tangential acceleration.



i) In general the relation between linear and angular velocity is given by $\vec{v} = \vec{\omega} x \vec{r}$. For circular motion equation reduces to equation v = rω. since $\vec{\omega}$ and $\vec{r}$ are perpendicular to each other. Differentiating the equation v = rω with respect to time, we get (since r is constant) $\frac{dv}{dt} = r\frac{d\omega}{dt} = r\,\alpha$

ii) Here $\frac{dv}{dt}$ is the tangential acceleration and is denoted as a_t.
$\frac{d\omega}{dt}$ is the angular acceleration . Then eqn.
$\vec{v} = \vec{\omega} x \vec{r}$ becomes $a_t = r\,\alpha$

iii) The tangential acceleration a_t experienced by an object is circular motion as shown in Figure.

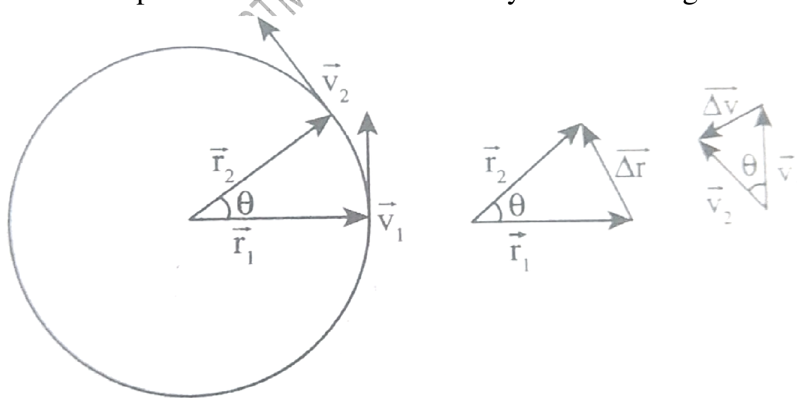
51. Derive an expression for the centripetal acceleration of a body moving in a circular path of radius 'r' with uniform speed.

i) The centripetal acceleration is derived from a simple geometrical relationship between position and velocity vectors.

ii) Let the directions of position and velocity vectors shift through the same angle θ in a small interval of time Δt,

iii) For uniform circular motion, r = $|\vec{r_1}| = |\vec{r_2}|$ and v = $|\vec{v_1}| = |\vec{v_2}|$. If the particle moves from position vector $\vec{r_1}$ to $\vec{r_2}$, the displacement is given by $\Delta\vec{r} = \vec{r_2} - \vec{r_1}$ and the change in velocity from $\vec{v_1}$ to $\vec{v_2}$ is given by $\Delta\vec{v} = \vec{v_2} - \vec{v_1}$.

iv) The magnitudes of the displacement Δr and of Δv satisfy the following relation
$\frac{\Delta r}{r} = -\frac{\Delta v}{v} = \theta$



v) Here the negative sign implies that Δv points radially inward, towards the center of the circle. $\Delta v = v\left(\frac{\Delta r}{r}\right)$ then,
$a = \frac{\Delta v}{\Delta t} = \frac{v}{r}\left(\frac{\Delta v}{\Delta t}\right)$; $= -\frac{v^2}{r}$

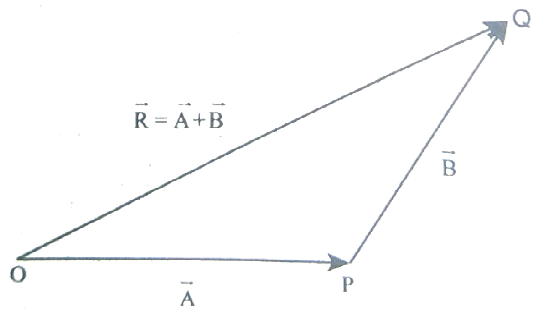
vi) For uniform circular motion $v = \omega r$, where ω is the angular velocity of the particle about the center. Then the centripetal acceleration can be written as
a = $-\omega^2 r$

உங்கள் பாதையை நீங்களே தேர்ந்தெடுங்கள்
ஏனெனில் வேறு எவராலும் உங்கள் கால்களைக்
கொண்டு நடக்க முடியாது.

FIVE MARKS QUESTIONS

01. Explain in detail the triangle law of addition.

1) Represent the vectors $\vec{A}$ and $\vec{B}$ by the two adjacent sides of a triangle taken in the same order. Then the resultant is given by the third side of the triangle taken in the opposite order.



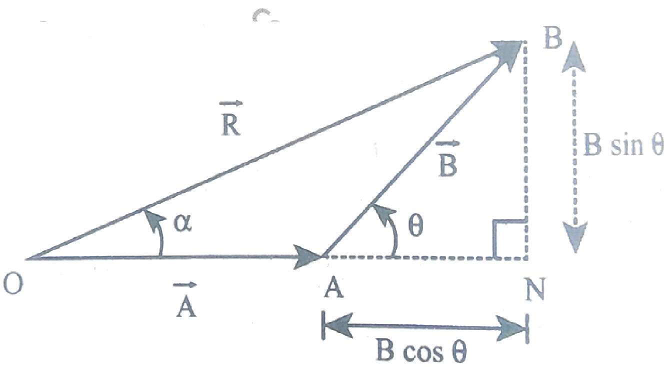
2) The head of the first vector $\vec{A}$ is connected to the tail of the second vector $\vec{B}$. Let θ be the angle between $\vec{A}$ and $\vec{B}$. Then $\vec{R}$ is the resultant vector connecting the tail of the first vector $\vec{A}$ to the head of the second vector $\vec{B}$.

3) The magnitude of $\vec{R}$ (resultant) is given geometrically by the length of $\vec{R}$ (OQ) and the direction of the resultant vector is the angle between $\vec{R}$ and $\vec{A}$

Thus we write $\vec{R} = \vec{A} + \vec{B}$. $\because \overrightarrow{OQ} = \overrightarrow{OP} + \overrightarrow{PQ}$

Magnitude of resultant vector

4) Consider the triangle ABN, which is obtained by extending the side OA to ON. ABN is a right angled triangle.



$\text{Cos}\,\theta = \frac{AN}{B}$ $\therefore AN = B\,\text{Cos}\,\theta$ *and*

$\text{Sin}\,\theta = \frac{BN}{B}$ $\therefore BN = B\,\text{Sin}\,\theta$

For ΔOBN,

we have $OB^2 = ON^2 + BN^2$

$\Rightarrow R^2 = (A + B\,\text{Cos}\,\theta)^2 + (B\,\text{Sin}\,\theta)^2$

$\Rightarrow R^2 = A^2 + B^2\cos^2\theta + 2AB\cos\theta + B^2\sin^2\theta$

$\Rightarrow R^2 = A^2 + B^2(\cos^2\theta + \sin^2\theta) + 2AB\cos\theta$

$\Rightarrow R = \sqrt{A^2 + B^2 + 2AB\,\text{Cos}\theta}$

which is the magnitude of the resultant of *A* and *B*

Direction of resultant vectors:

5) If θ is the angle between $\vec{A}$ and $\vec{B}$, then $|\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2AB\,\text{Cos}\theta}$

If $\vec{R}$ makes an angle α with $\vec{A}$, then in ΔOBN,

$$\tan\alpha = \frac{BN}{ON} = \frac{BN}{OA+AN}$$

$$\tan\alpha = \left(\frac{B\,\text{Sin}\theta}{A+B\,\text{Cos}\theta}\right)$$

$$\boldsymbol{\alpha = \tan^{-1}\left(\frac{B\,\text{Sin}\theta}{A+B\,\text{Cos}\theta}\right)}$$

02. Discuss the properties of scalar and vector products.

Properties of scalar products

1) The product quantity $\vec{A} \cdot \vec{B}$ is always a scalar. It is positive if the angle between the vectors is acute (i.e., $< 90°$) and negative if the angle between them is obtuse (i.e. $90° < \theta < 180°$).
2) The scalar product is commutative, i.e. $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$
3) The vectors obey distributive law i.e. $\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$
4) The angle between the vectors $\theta = \cos^{-1}\left[\frac{\vec{A} \cdot \vec{B}}{AB}\right]$
5) The scalar product of two vectors will be maximum when $\cos\theta = 1$, i.e. $\theta = 0^0$, i.e., when the vectors are parallel; $(\vec{A} \cdot \vec{B})_{max} = AB$
6) The scalar product of two vectors will be minimum, when $\cos\theta = -1$, i.e. $\theta = 180^0$ $(\vec{A} \cdot \vec{B})_{min} = -AB$ when the vectors are anti-parallel.
7) If two vectors $\vec{A}$ and $\vec{B}$, are perpendicular to each other than their scalar Product $\vec{A} \cdot \vec{B} = 0$, because $\cos 90^0 = 0$. Then the vectors $\vec{A}$ and $\vec{B}$. are said to be mutually orthogonal.
8) The scalar product of a vector with itself is termed as self–dot product and is given by $(\vec{A})^2 = \vec{A} \cdot \vec{A} = AA\cos\theta = A^2$. Here angle $\theta = 0°$

   The magnitude or norm of the vector $\vec{A}$ is $|\vec{A}| = A = \sqrt{\vec{A} \cdot \vec{A}}$
9) In case of a unit vector $\hat{n}$, $\hat{n} \cdot \hat{n} = 1 \times 1 \times \cos 0 = 1$.

   For example, $\hat{\imath} \cdot \hat{\imath} = \hat{\jmath} \cdot \hat{\jmath} = \hat{k} \cdot \hat{k} = 1$
10) In the case of orthogonal unit vectors $\hat{\imath}, \hat{\jmath}$ and $\hat{k}$, $\hat{\imath} \cdot \hat{\jmath} = \hat{\jmath} \cdot \hat{k} = \hat{k} \cdot \hat{\imath} = 1.1 \cos 90^0 = 0$
11) In terms of components the scalar product of $\vec{A}$ and $\vec{B}$ can be written as

    $\vec{A} \cdot \vec{B} = (A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}) \cdot (B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k})$

    $= A_xB_x + A_yB_y + A_zB_z$ with all other terms zero.

    The magnitude of vector $|\vec{A}|$ is given by $|\vec{A}| = A = \sqrt{A_x^2 + A_y^2 + A_z^2}$

Properties of vector (cross) product.

i) The vector product of any two vectors is always another vector whose direction is perpendicular to the plane containing these two vectors, i.e., orthogonal to both the vectors $\vec{A}$ and $\vec{B}$, even though the vectors $\vec{A}$ and $\vec{B}$ may or may not be mutually orthogonal.

ii) The vector product of two vectors is not commutative, i.e., $\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$ But., $\vec{A} \times \vec{B} = -[\vec{B} \times \vec{A}]$. Here it is worthwhile to note that $|\vec{A} \times \vec{B}| = |\vec{B} \times \vec{A}| = AB\sin\theta$.
i.e. in the case of the product vectors $\vec{A} \times \vec{B}$ and $\vec{B} \times \vec{A}$, the magnitudes are equal but directions are opposite to each other

iii) The vector product of two vectors will have maximum magnitude when $\sin\theta = 1$, i.e., $\theta = 90^0$ i.e., when the vectors $\vec{A}$ and $\vec{B}$, are orthogonal to each other.
$(\vec{A} \times \vec{B})_{max} = AB\hat{n}$

iv) The vector product of two non–zero vectors will be minimum when $\sin\theta = 0$, *i.e.*, $\theta = 0^0$ *or* 180^0 $[\vec{A} \times \vec{B}]_{min} = 0$ i.e., the vector product of two non–zero vectors vanishes, if the vectors are either parallel or anti-parallel.

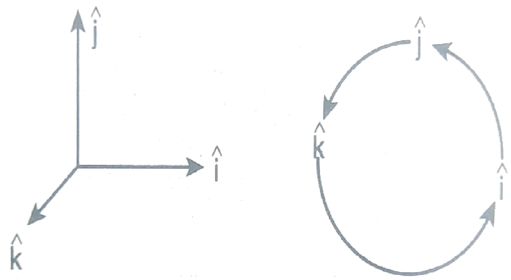
v) The self–cross product, i.e., product of a vector with itself is the null vector
$\vec{A} \times \vec{A} = AA\sin\theta\,\hat{n} = \vec{0}$ In physics the null vector $\vec{0}$ is simply denoted as zero.

vi) The self–vector products of unit vectors are thus zero. $\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = \vec{0}$

vii) In the case of orthogonal unit vectors, $\hat{\imath}, \hat{\jmath}, \hat{k}$ in accordance with the right hand screw rule: $\hat{\imath} \times \hat{\jmath} = \hat{k}$, $\hat{\jmath} \times \hat{k} = \hat{\imath}$, $\hat{k} \times \hat{\imath} = \hat{\jmath}$.

Also, since the cross product is not commutative,

$\hat{\jmath} \times \hat{\imath} = -\hat{k}$, $\hat{k} \times \hat{\jmath} = -\hat{\imath}$ and $\hat{\imath} \times \hat{k} = -\hat{\jmath}$



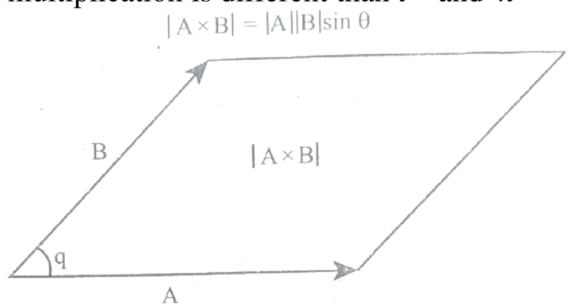
viii) In terms of components, the vector product of two vectors $\vec{A}$ and $\vec{B}$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ A_X & A_Y & A_Z \\ B_X & B_Y & B_Z \end{vmatrix}$$

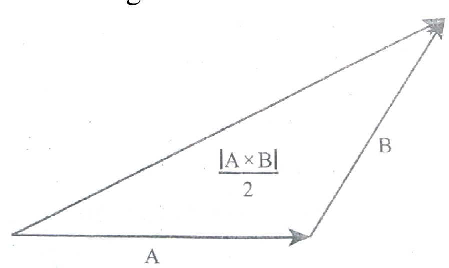
$$= \vec{\imath}\,(A_yB_z - A_zB_y) + \vec{\jmath}\,(A_zB_x - A_xB_z) + \vec{k}\,(A_xB_y - A_yB_x)$$

Note that in the $\vec{\jmath}^{\,th}$ component the order of multiplication is different than $\vec{\imath}^{\,th}$ and $\vec{k}^{th}$ components.

ix) If two vectors $\vec{A}$ and $\vec{B}$ form adjacent sides in a parallelogram, then the magnitude of $|\vec{A} \times \vec{B}|$ will give the area of the parallelogram as represented graphically in Figure.



x) It divide a parallelogram into two equal triangles as shown in the Figure, the area of a triangle with $\vec{A}$ and $\vec{B}$ as sides is $\frac{1}{2}|\vec{A} \times \vec{B}|$ number of quantities used in Physics are defined through vector products. Particularly physical quantities representing rotational effects like torque, angular momentum, are defined through vector products.



03. Derive the kinematic equations of motion for constant acceleration.

Consider an object moving in a straight line with uniform or constant acceleration ‘a’. Let u be the velocity of the object at time t = 0, and v be velocity of the body at a later time t.

Velocity - time relation

1) The acceleration of the body at any instant is given by the first derivative of the velocity with respect to time, $a = \frac{dv}{dt}$ or dv = a .dt

Integrating both sides with the condition that as time changes from 0 to t, the velocity changes from u to v. For the constant acceleration,

$$\int_u^v dv = \int_0^t a\,dt$$

$$= a\int_u^v dt \implies [v]_u^v = a\,[t]_0^t \quad \text{--------(1)}$$

v – u = at (or) v = u + at

Displacement – time relation

2) The velocity of the body is given by the first derivative of the displacement with respect to time. $v = \frac{ds}{dt}$ or ds = v dt and since v = u + at We get $ds = (u + at)dt$

Assume that initially at time t = 0, the particle started from the origin. At a later time t, the particle displacement is s. Further assuming that acceleration is time independent, we have $\int_0^s ds$

$$=\int_0^t u dt + \int_0^t at dt \quad \text{or } s = ut + \tfrac{1}{2} at^2 \quad \text{--------------(2)}$$

Velocity – displacement relation

3) The acceleration is given by the first derivative of velocity with respect to time.

$$a = \frac{dv}{dt} = \frac{dv}{ds}\frac{ds}{dt} = \frac{dv}{ds} v$$

[since ds / dt = v where s is distance traversed]

This is rewritten as $a = \frac{1}{2}\frac{dv^2}{ds}$ or $ds = \frac{1}{2a} d(v^2)$

4) Integrating the above equation, using the fact when the velocity changes from u^2 to v^2, displacement changes from 0 to s, we get

$$\int_0^s ds$$

$$= \int_u^v \frac{1}{2a} d(v^2) \; ; \; s = \frac{1}{2a}(v^2 - u^2) \; ; \; v^2 = u^2 + 2as \quad \text{------------(3)}$$

5) We can also derive the displacement s in terms of initial velocity u and final velocity v. From the equation (1) we can write, at = v – u

Substitute this in equation (2), we get

$$s = ut + \tfrac{1}{2}(v - u)t$$

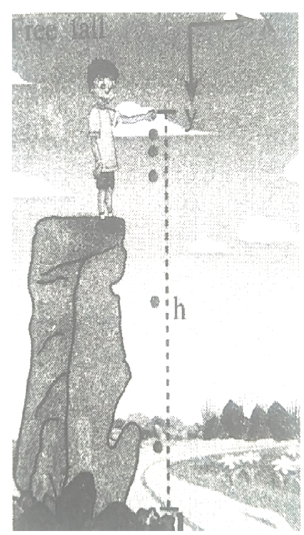
$$s = \frac{(u+v)t}{2} \quad \text{-----------------(4)}$$

The equations (1), (2), (3) and (4) are called kinematic equations of motion, and have a wide variety of practical applications.

Kinematic equations

$v = u + at$; $s = ut + \frac{1}{2} at^2$; $v^2 = u^2 + 2as$; $s = \frac{(u+v)t}{2}$

04. Derive the equations of motion for a particle (a) falling vertically (b) projected vertically



Case (1): A body falling from a height h

1) Consider an object of mass m falling from a height h. Assume there is no air resistance. For convenience, let us choose the downward direction as positive y-axis as shown in the Figure.
2) The object experiences acceleration 'g' due to gravity which is constant near the surface of the Earth. We can use kinematic equations to explain its motion. We have

The acceleration $\vec{a} = g\hat{j}$

By comparing the components, we get

$A_x = 0$, $a_y = g$, $a_z = 0$ Let us take for simplicity, $a_y = a = g$

3) If the particle is thrown with initial velocity 'u' downward which is in negative y axis, then velocity and position at of the particle any time t is given by

$$v = u + gt \text{ -------------------- (1)}$$
$$y = ut + \tfrac{1}{2} gt^2 \text{ -------------- (2)}$$

4) The square of the speed of the particle when it is at a distance y from the hill-top, is

$$v^2 = u^2 + 2gy \text{ --------------(3)}$$

Suppose the particle starts from rest. Then u = 0

5) Then the velocity v, the position of the particle and v2 at any time t are given by (for a point y from the hill-top)

$$v = gt \text{ ------------ (4)}$$
$$y = \tfrac{1}{2} gt^2 \text{ -----------(5)}$$
$$v^2 = 2gy \text{ ------------(6)}$$

6) The time (t = T) taken by the particle to reach the ground (for which y = h), is given by using equation (5),

$$h = y = \tfrac{1}{2} gT^2 \text{ -----------(7)}$$
$$T = \sqrt{\frac{2h}{g}} \text{ ------------(8)}$$

The equation (8) implies that greater the height(h), particle takes more time(T) to reach the ground. For lesser height(h), it takes lesser time to reach the ground.

The speed of the particle when it reaches the ground (y = h) can be found using equation (6), we get $v_{ground} = \sqrt{2gh}$ ------------- (9)

7) The above equation implies that the body falling from greater height (h) will have higher velocity when it reaches the ground.

The motion of a body falling towards the Earth from a small altitude (h << R), purely under the force of gravity is called free fall. (Here R is radius of the Earth)

Case (ii): A body thrown vertically upwards

1) Consider an object of mass m thrown vertically upwards with an initial velocity u.
   Let us neglect the air friction.
2) In this case we choose the vertical direction as positive y axis as shown in the Figure then the acceleration a = – g (neglect air friction) and g points towards the negative y axis.
3) The kinematic equations for this motion are,

$$v = u - gt \text{ -------------------- (10)}$$
$$y = ut - \tfrac{1}{2} gt^2 \text{ -------------- (11)}$$

The velocity and position of the object at any time t are,

$$v^2 = u^2 - 2gy \text{ --------------(12)}$$

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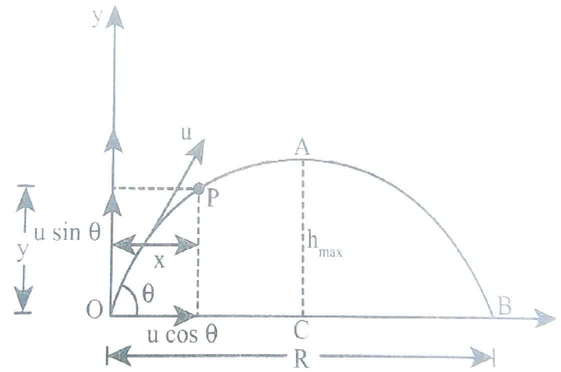
தோல்வி வந்தால் பொறுமை அவசியம்
எதிர்ப்பு வந்தால் துணிவு அவசியம்
எது வந்தாலும் நம்பிக்கை அவசியம்.

05. Derive the equation of motion, range and maximum height reached by the particle thrown at an oblique angle θ with respect to the horizontal direction.

i) Consider an object thrown with initial velocity $\vec{u}$ at an angle θ with the horizontal.
then, $\vec{u} = u_x\hat{\imath} + u_y\hat{\jmath}$ where $u_x = u \cos\theta$ is the horizontal component and $u_y = u \sin\theta$ the vertical component of velocity.



ii) u_x remains constant throughout the motion. u_y changes with time under the effect of acceleration due to gravity. First it decreases, becomes zero at the maximum height, after which it again increases till the prose reach be ground.

iii) Hence after the time t, the velocity along horizontal motion

$$v_x = u_x + a_x t = u_x = u\cos\theta$$

The horizontal distance travelled by projectile in time t is $s_x = u_x t + \frac{1}{2} a_x t^2$

Here, $s_x = x$, $u_x = u\cos\theta$, $a_x = 0$

iv) Thus, $x = u\cos\theta \cdot t$ **or** $t = \dfrac{x}{u\cos\theta}$ -------------- (1)

Next, for the vertical motion $v_y = u_y + a_y t$ Here $u_y = u\sin\theta$, $a_y = -g$ (acceleration due to gravity acts opposite to the motion).

Thus, $v_y = u\sin\theta - gt$ -------------------(2)

The vertical distance travelled by the projectile in the same time *t* is

$s_y = u_y t + \frac{1}{2} a_y t^2$ Here, $s_y = y$, $u_y = u\sin\theta$, $a_x = -g$

Then, $y = u\sin\theta\, t - \frac{1}{2} g t^2$ ---------------- (3)

v) Substitute the value of *t* from equation (1) in equation (3), we have

$$y = u\sin\theta \frac{x}{u\cos\theta} - \frac{1}{2} g \frac{x^2}{u^2\cos^2\theta}$$

$$y = x\tan\theta - \frac{1}{2} g \frac{x^2}{u^2\cos^2\theta} \quad \text{------------------(4)}$$

Thus the path followed by the projectile is an inverted parabola.

Maximum height (h_{max})

The maximum vertical distance travelled by the projectile during its journey is called maximum height. This is determined as follows:

For the vertical part of the motion, $v_y^2 = u_y^2 + 2a_y s$

Here, $u_y = u\sin\theta$, $a = -g$, $s = h_{max}$, and at the maximum height $v_y = 0$

Here, $(0)^2 = u^2 \sin^2 = gh_{max}$ (or) $\mathbf{h_{max} = \dfrac{u^2\sin^2\theta}{2g}}$

Horizontal range (R)

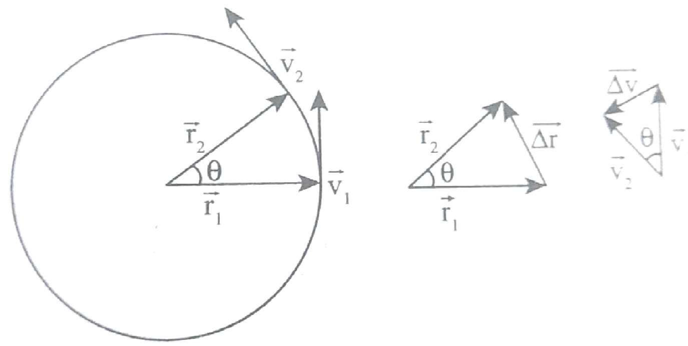
The maximum horizontal distance between the point of projection and the point on the horizontal plane where the projectile hits the ground is called horizontal range (R). This is found easily since the horizontal component of initial velocity remains the same. We can write Range R = Horizontal component of velocity x time of flight

$= u\cos\theta \times T_f$

$$R = u\cos\theta \times \frac{2u\sin\theta}{g} = \frac{2u^2\sin\theta\cos\theta}{g} \quad \mathbf{R = \frac{u^2\sin 2\theta}{g}}$$

06. Derive the expression for centripetal acceleration.

When a body is in uniform circular motion. Its speed remains constant, but its velocity changes continuously due to the change in its direction. Hence the motion is accelerated.



A body undergoing uniform circular motion is acted upon by an acceleration which is directed along the radius towards the centre of the circular path. The acceleration is called centripetal acceleration.

i) The centripetal acceleration is derived from a simple geometrical relationship between position and velocity vectors.

ii) Let the directions of position and velocity vectors shift through the same angle θ in a small interval of time Δt,

iii) For uniform circular motion, $r = |\vec{r_1}| = |\vec{r_2}|$ and $v = |\vec{v_1}| = |\vec{v_2}|$. If the particle moves from position vector $\vec{r_1}$ to $\vec{r_2}$, the displacement is given by $\Delta\vec{r} = \vec{r_2} - \vec{r_1}$ and the change in velocity from $\vec{v_1}$ to $\vec{v_2}$ is given by $\Delta\vec{v} = \vec{v_2} - \vec{v_1}$.

iv) The magnitudes of the displacement Δr and of Δv satisfy the following relation

$$\frac{\Delta r}{r} = -\frac{\Delta v}{v} = \theta$$

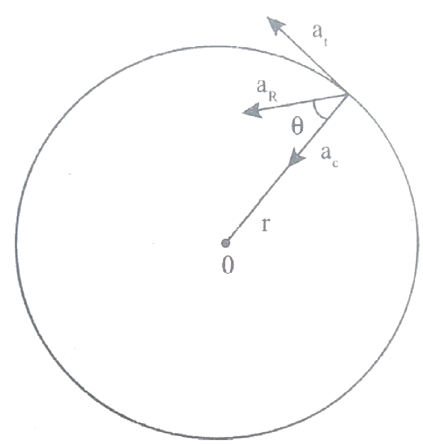
v) Here the negative sign implies that Δv points radially inward, towards the center of the circle. $\Delta v = v\left(\frac{\Delta r}{r}\right)$ then,

$$a = \frac{\Delta v}{\Delta t} = \frac{v}{r}\left(\frac{\Delta v}{\Delta t}\right) ; \; = -\frac{v^2}{r}$$

vi) For uniform circular motion $v = \omega r$, where ω is the angular velocity of the particle about the center. Then the centripetal acceleration can be written as

$$a = -\omega^2 r$$

07. Derive the expression for total acceleration in the non uniform circular motion.



1) Consider a particle moving along a circular path of radius r with a variable speed v. As the speed of the particle changes so acceleration has a tangential component, $a_t = \frac{dv}{dt} r\alpha$; $a_t = r\alpha$

2) As the direction of motion changes continuously, so the acceleration has a radial component

(i.e.) Centripetal acceleration $a_c = \frac{v^2}{r}$

3) The resultant acceleration is obtained by vector sum of centripetal and tangential acceleration.

4) The magnitude of this resultant acceleration is given by $a_R = \sqrt{a_t^2 + \left(\frac{v^2}{r}\right)^2}$

08. Define the term motion and explain the different types of motion.

An object is said to be in motion if it changes its position with respect to its surroundings with the passage of time.

a) Linear motion

An object is said to be in linear motion if it moves in a straight line.

Examples

1) An athlete running on a straight track
2) A particle falling vertically downwards to the Earth.

b) Circular motion

Circular motion is defined as a motion described by an object traversing a circular path.

Examples

1) The whirling motion of a stone attached to a string
2) The motion of a satellite around the Earth

c) Rotational motion

If any object moves in a rotational motion about an axis, the motion is called 'rotation'.

Examples

i) Rotation of a disc about an axis through its center
ii) Spinning of the Earth about its own axis.

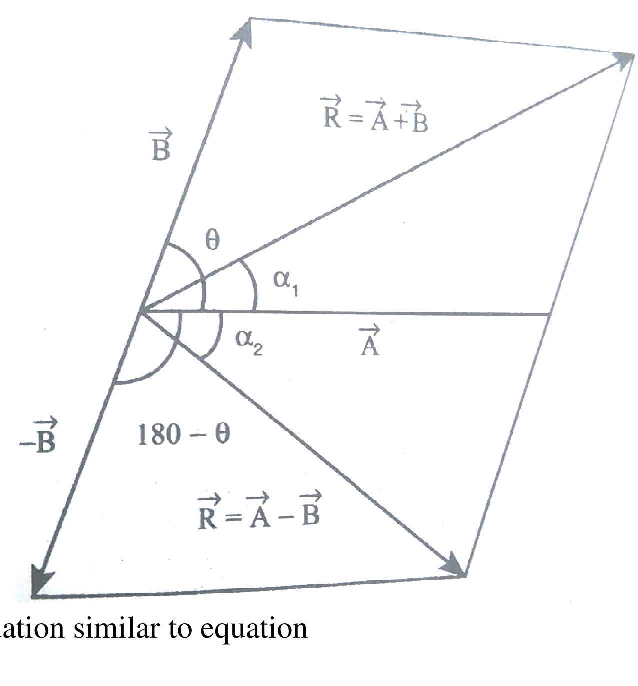
d) Vibratory motion

If an object or particle executes a to–and– fro motion about a fixed point, it is said to be in vibratory motion.

Examples

i) Vibration of a string on a guitar ii) Movement of a swing

09. Explain the subtraction of vectors.



i) For two non-zero vectors $\vec{A}$ and $\vec{B}$ which are inclined to each other at an angle θ, the difference $\vec{A} - \vec{B}$ is obtained as follows. First obtain $-\vec{B}$ as in Figure. The angle between $\vec{A}$ and $-\vec{B}$ is 180 – θ. The difference $\vec{A} - \vec{B}$ is the same as the resultant o $\vec{A}$ and $-\vec{B}$.

We can write $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$ and using the equation, we have

ii) $|\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2AB\,\mathrm{Cos}\theta}$, we have

$$|\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 + 2AB\,\mathrm{Cos}(180 - \theta)}$$

iii) Since, cos(180 – θ) = –cosθ. we get,

$$|\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 - 2AB\,\mathrm{Cos}\theta}$$

Again from the Figure 2.19, and using an equation similar to equation

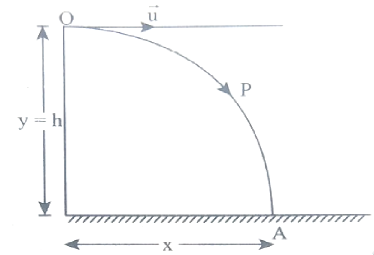
$$\tan\alpha_2 = \frac{B\sin(180-\theta)}{A + B\cos(180-\theta)}$$

iv) But sin(180 – θ) = sinθ, hence we get,

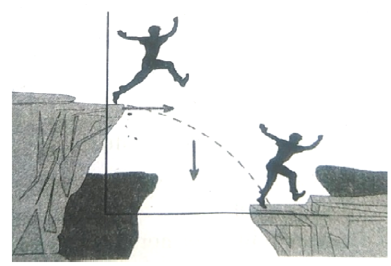
$$\tan\alpha_2 = \frac{B\sin\theta}{A - B\cos\theta}$$

10. Find horizontal range and time of flight projectile in horizontal projection.

Consider a projectile, say a ball, thrown horizontally with an initial velocity $\vec{u}$ from the top of a tower of height h (Figure)



1) As the ball moves, it covers a horizontal distance due to its uniform horizontal velocity u, and a vertical downward distance because of constant acceleration due to gravity g.
2) Thus, under the combined effect the ball moves along the path OPA. The motion is in a 2-dimensional plane. Let the ball take time t to reach the ground at point A,
Then the horizontal distance travelled by the ball is x (t) = x, and the vertical distance travelled is y (t) = y



Motion along horizontal direction

3) The particle has zero acceleration along x direction. So, the initial velocity u_x remains constant throughout the motion. The distance traveled by the projectile at a time t is given by the equation $x = u_x t + \frac{1}{2} at^2$ Since a = 0 along x direction, we have

$$x = u_x t \text{ --------------------(1)}$$

Motion along downward direction

4) Here $u_y = 0$ (initial velocity has no downward component), a = g (we choose the +ve y-axis in downward direction), and distance y at time t

From equation, $y = u_y t + \frac{1}{2} at^2$, we get $y = \frac{1}{2} gt^2$ ----------(2)

Substituting the value of t from equation (1) in equation (2) we have

$$y = \frac{1}{2}\, g \frac{x^2}{u_x^2} = \left(\frac{g}{2u_x^2}\right) x^2$$

$$\mathbf{y = Kx^2} \text{ -------------- (3), where } K = \frac{g}{2u_x^2}$$

5) Equation is the equation of a parabola. Thus, **t**he path followed by the projectile is a parabola (curve OPA in the Figure)

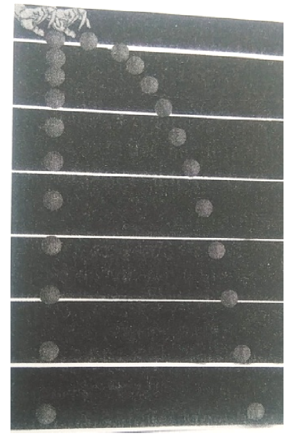
Time of Flight:

h be the height of a tower. Let T be the time taken by the projectile to hit the ground, after being thrown horizontally from the tower.

$s_y = u_y t + \frac{1}{2} at^2$ $s_y = h$, t=T, $u_y = 0$ (i.e no initial vertical velocity)

$$T = \sqrt{\frac{2h}{g}}$$

6) Thus, the time of flight for projectile motion depends on the height of the tower, but is independent of the horizontal velocity of projection.



Horizontal range:

The horizontal distance covered by the projectile from the foot of the tower to the point where the projectile hits the ground is called horizontal range. For horizontal motion, we have $s_x = u_x t + \frac{1}{2} at^2$

Here, $s_x = R$ (range), $u_x = u$, $a = 0$ (no horizontal acceleration)

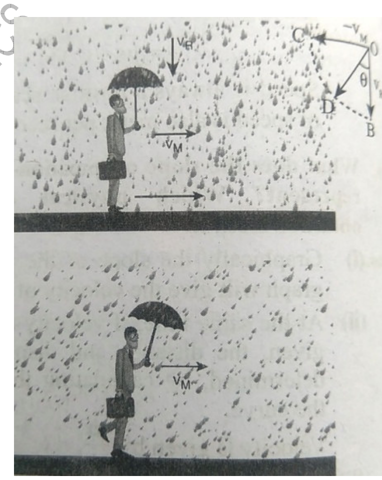
T is time of flight.

Then horizontal range $= uT$.

7) Since the time of flight $T = \sqrt{\frac{2h}{g}}$, we substitute this and we get the horizontal range of the particle as $R = u\sqrt{\frac{2h}{g}}$.

8) The above equation implies that the range R is directly proportional to the initial velocity u and inversely proportional to acceleration due to gravity g.

11. A man moving in rain holds an umbrella inclined to the vertical though the rain drops are falling vertically. Why?



1) Consider a person moving horizontally with velocity $\vec{V}_M$. Let rain fall vertically with velocity $\vec{V}_R$.
2) An umbrella is held to avoid the rain. Then the relative velocity of the rain with respect to the person is,
3) $\vec{V}_{RM} = \vec{V}_R - \vec{V}_M = \vec{OB} + \vec{OC} + \vec{OD}$ which has magnitude $V_{RM} = \sqrt{v_R^2 + v_M^2}$ $\tan\theta = \frac{DB}{OB} = \frac{V_M}{V_R}$ and direction $\theta = \tan^{-1}\left(\frac{V_M}{V_R}\right)$ with the vertical as shown in Figure.
4) In order to save himself from the rain, he should hold an umbrella at an angle θ with the vertical.

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உங்களை வெற்றி பெற
எவரும் பிறக்கவில்லை என்பது பொய்.
பிறரைத் தோற்கடிக்க
நீங்கள் பிறந்து இருக்கிறீர்கள் என்பதே மெய்.
சுயமாக முன்னேறியவர் என்று
எவரும் கிடையாது
நீங்கள் உழைக்கத் தயார் என்றால்
பலர் உங்களை உயர்த்தத் தயாராக இருக்கிறார்கள்.

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PREPARED BY
RAJENDRAN M, M.Sc., B.Ed., C.C.A.,
P. G. ASSISTANT IN PHYSICS,
DEPARTMENT OF PHYSICS,
SRM HIGHER SECONDARY SCHOOL,
KAVERIYAMPOONDI,
THIRUVANNAMALAI DISTRICT.
For your suggestion: mrrkphysics@gmail.com, murasabiphysics@gmail.com