10. ORDINARY DIFFERENTIAL EQUATIONS

Differential Equation, Order, Degree THIRVYARUR DT

Order. - highest order derivative present in the DIE.

Degree - Integral power of the highest order derivative CD.E. is expressible in a polynomial form)

. - . - If not expressible as a highest order derivative as the leading term then the degree of D.E. is not defined,

Exercise 1011

1) determine the order, degree (if exists) of the D.E.

U dy xy = cotx order=1 degree=1

(ii) (d3y) 3-3 d2y +5 th +4=0 11 22 + 21 = 25

 $\frac{\left(\frac{d^3y}{dx^3}\right)^{2/3}}{\left(\frac{dx}{dx^2}\right)^{2/3}} = 3\frac{d^2y}{dx^2} = 3\frac{dy}{dx} - 7$   $\frac{d^3y}{dx^3} = 3\frac{d^3y}{dx^2} = 3\frac{dy}{dx} - 7$   $\frac{d^3y}{dx^3} = 3\frac{d^3y}{dx^2} = 3\frac{d^3y}{dx} - 7$   $\frac{d^3y}{dx^3} = 3\frac{d^3y}{dx^2} = 3\frac{d^3y}{dx} - 7$   $\frac{d^3y}{dx^3} = 3\frac{d^3y}{dx^2} = 3\frac{d^3y}{dx} - 7$   $\frac{d^3y}{dx^3} = 3\frac{d^3y}{dx^3} = 3\frac{d^3y}{dx} - 7$   $\frac{d^3y}{dx^3} = 3\frac{d^3y}{dx^3} = 3\frac{d^3y}{dx} - 7$   $\frac{d^3y}{dx^3} = 3\frac{d^3y}{dx} - 7$   $\frac{d^3y}{dx} = 3\frac{d^3y}{dx} - 7$ (d34)2 (3d2 - 522 - 1)3

(ii)  $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 = x \sin\left(\frac{d^2y}{dx^2}\right)$  order=2 This can not be expressed

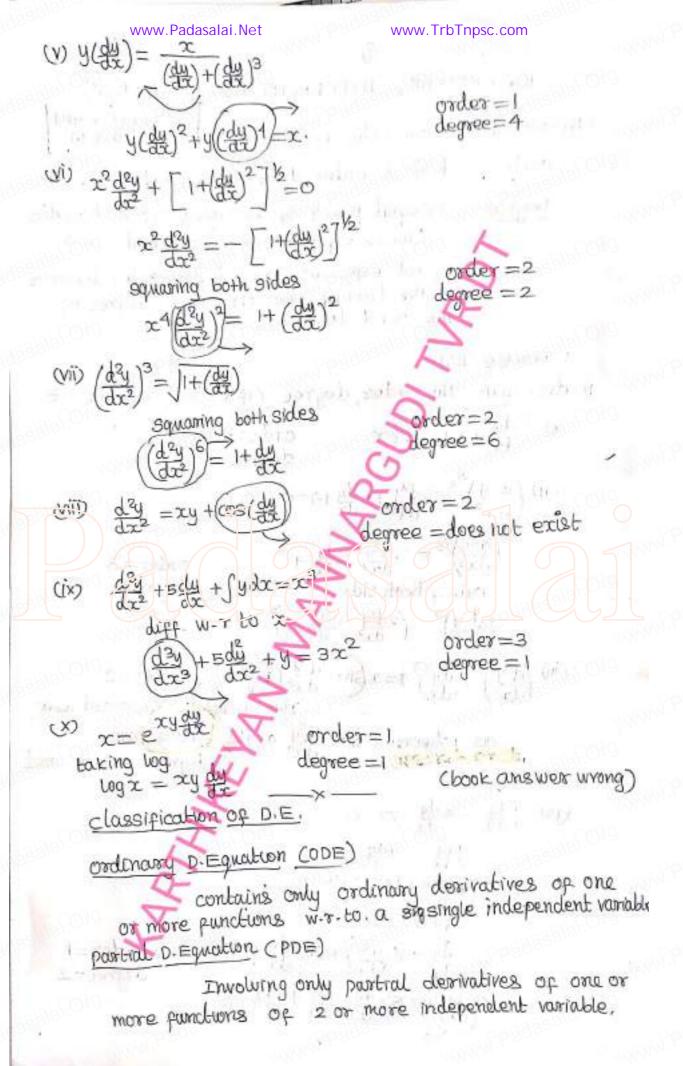
as polynomial with dry as the leading term. degree = not defined,

. and to mytophysection

(14) John -4 88 -7x=0

three transporter squaring both sides +7x 2 and as squaring both sides

dy = 16(dy)2+ 56xdy +49x2 degree=2 16(機)和56次號一般 +40次=0



- 1) Express each of the following physical statements in the form of differential equation.
  - 1) Radium decays at a rate proportional to the amount of given q be the amount of Radium at present any time 💢

de xa

dg=kg. (kis a constant)

(ii) The population popacity increases at a rate proportional to the product of population and to the difference between 5,00,000 and the population.

p be the population rate of population & Pi (500000-P)

df a P(5,00,000-P)

dp = k P ( B 00000-P)

(iii) For a certain substance, the rate of change of vapor pressure P w. sto temperature i is proportional to the verpor pressure and inversely proportional to the square of the temperature,

rate of change of P & P

pook ausmed

(IV) A saving amount pays 87 interest per year, compounded continuously. In addition, the income from another investment is credited +D+he amount continuously at the rate of ≥ 400 per year,

tet is be the amount any time t

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de a re of = KX

K=8%=8

<del>な</del>= 800 x

400 income added

www.Padasalai.Net www.TrbTnpsc.com 2) Assume that a spherical rain drop evaporates at a rate proportional to its surface area, Form a D.E. involving the rate of change of the radius of the rain drop.

Let & be the radius V be the volume and A be the surface area of spherical rain drop

$$\frac{dV}{dt} \propto -A$$

$$\frac{dV}{dt} \propto -A$$

$$\frac{dV}{dt} = -KA$$

This is the required D.E.

the offense and the service of the

volume of

Sphore=47173

3A = 4T1 2

Formation of D.E. named oute will

Exarcise to 3 or with Aud Company

i) First the differential equation of the family of

(i) all non-vertical lines in a plane

i) all non-horizondal lines in a plane;

ci The equation of any non-vertical line in aplane is

axt-by+c=owhere acr, b+0

a, b two constants diff. w. r to x two times

a+by'=0 : 
$$y'=a_{10}$$
  
 $y''=0$  or  $\frac{d^2y}{dx^2}=0$ .

when b=0 >

(i) The equation of all non-horizondal family of line in a plane is outby= C with ato

when a=0=> y=constant hom 20ndal

2) Form the differential equation of all straight lines toeching the circle x2+y2=12

Equation of tangent to the (to eliminate m

(m is a corbitrary constant)  $y=mx\pm\sqrt{1+m^2}$  — ① grist m. 1 to oc

 $\frac{dy}{dx} = m$ 

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A. Karthikeyan

y-x2 = = + V 开键

squaring both sides

(y-xell)2= 22 (1+(ell)2)

3) Find the differential equation of the family of circles passing through the origin and having their centres on the x-axis

> exquation cif framily of circles pussing through might and centre (200) is (x-a) + (y-0)2 = a2



a is only arbitrary constant + y = a? —

to eliminate a only

diff w. rto =0  $\sqrt{x-\alpha} = -y \frac{dy}{dx}$ 

(-ygg)2+y2= (I+ygg

x2+27y # -y2=0

4) Find the differential equation of the family of all the parabolas with the latus redum to and whose axes are parallel to the x-axis,

Equation of parabola.

(hikare arbitrary constants) (y-K)2= 4a(x-h) -0
outf. w. rto x

eliminate hik only

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$$2(y-k) y' = 4a(1)$$

$$(y-k)y' = 2a - 2$$

$$again diff w rto x:$$

$$(y-k)y'' + (y')(y') = 0$$

$$(y-k)y'' + (y')(y') = -y^{2}$$

$$(y-k) = -y^{2}$$

$$(y-k) = -y^{2}$$

$$(y-k) = -y^{2}$$

$$(y')^{3} = 2ay''$$

$$-(y')^{3} = 2ay''$$

$$2ay'' + (y')^{3} = 0$$

5) Find the differential equation of the family of parabolog with vertex at (0,-1) and having axis along the y-axis

Equation of parabola

$$(x-0)^2 = 4a(y+1)$$
 $e^2 = 4a(y+1) = 0$ 
 $e^2 = 4a(y+1) = 0$ 

diff wer to 
$$x$$
 and arbitrary con  $x = 4a \, dy$ 

$$x = 2a \, dy$$

$$x = 2a \, dy$$

$$y' = 2a \, (y') \, (y+1)$$

$$y'x^4 = 2y+2$$

$$xy'-2y-2=0$$

6) Find the differential equations of the family of all the ellipses having foci on the y-axis and centre at the origin.

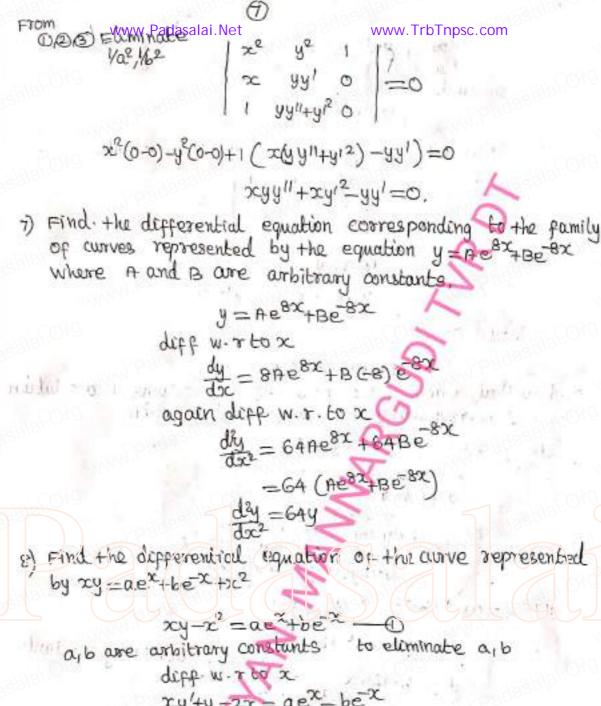
Equation of Ellipse.

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 \quad \text{Ollminate a, b}$$

$$\text{diff w. r. to } x \quad \text{a, b are arbitrary constant}$$

$$\frac{2x}{b^2} + \frac{2yy!}{a^2} = 0 \quad \text{O}$$

$$\frac{3c}{b^2} + \frac{yyl}{a^2} = 0 - 2$$
again diff. w-r to x
$$\frac{1}{b^2} + \frac{(yy'' + y'y')}{a^2} = 0 - 3$$



 $xy-x^2 = ae^x + be^x = 0$ a,b are arbitrary constants to eliminate a,b

diff w. x to x  $xy'+y-2x = ae^x - be^x$ again diff. w. x to x  $xy''+y'+y'-2 = ae^x + be^x$ Using  $0 \Rightarrow xy''+y'+2y'-2 = xy-x^2$   $xy''+2y'+x^2-xy'-2=0$ ,

ellipses having foci on the x-axis and centre at origin.

Equation of ellipse  $\frac{x^2 + y^2}{a^2} = 1 - 0$ a, b are arbitrary constant diff without  $\frac{2x}{a^2} + \frac{2yy!}{b^2} = 0$ 

$$\frac{2c}{a^2} + \frac{yy}{b^2} = 0$$

again diff. w. rto x

$$\frac{1}{a^2} + (\frac{yy'' + y'y'}{b^2}) = 0$$

Eliminate 1/2, to compare its co-efficients

$$\begin{vmatrix} x^2 & y^2 & 1 \\ x & yy' & 0 \\ 1 & yy''+y'^2 & 0 \end{vmatrix} = 0$$

solution of ODE.

Exercise 10,4

1 show that each of the pollowing expressions is a solution of the corresponding given differential equation.

diff white 
$$0.2$$
  
 $y' = 4x^2$   
 $y' = 4x^2$   
 $xy' = 2(2)x^2$   
 $xy' = 2(2)x^2$   
 $xy' = 2(2)x^2$ 

$$y = ae^{x} + be^{x}$$

a, b are arbitrary constants.

or to x

 $y' = ae^{x} - be^{-x}$ 

or dept. w. r. to x.

@ Find value of m so that the function y=emc is a solution of the given differential equation

@> y'-my=0

y'=my

again desp wir to x

$$y'' - my' = 0$$

$$9^{11} = my^{1}$$

$$\frac{g = mg}{g = m^2 g}$$

my-Bmy+6y=0

m=2/3

3 The slope of the tangent to the curve atomy point is the receprocal of 4 times the ordinate at that point, The curve passes through (2,15). Find the equation of the curve.

Let the point be percy) slope = .

aydy = dx.

Integrating 4 (ydy = jax

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24 = x+C -This curve passing through (215)

0=> 2(5)2=2+C

Required equation of curve 2y2=x+48%

4) show that  $y=e^{-sc}+mx+n$  is a solution of the differential ex (dry )-1=0 equation

y=e-x+mx+n two constants min

diff. w.r to x

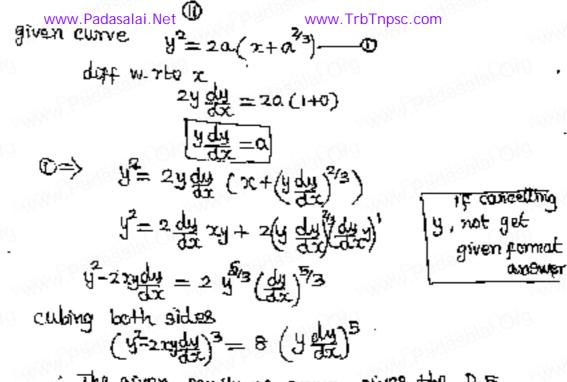
번=excn+m

again diff wirtox.

 $e^{x}(\frac{dsy}{dsx}) = 1$ 

ex (dry)-1=0

www.Padasalai.Net www.TrbTnpsc.com 5) show that y=ax+ = , x = 0 is a solution of the differential equation x2y11+xy1-y=0 y=ax+b -0 two constants a, b diff w-r to x. y = a(1)+b(-{2}e)-0 again diff. wir tox J11 = 0+ 26 ②> y'=a-型"x= y'=a-型" y(是-の)x(生生型)
y(+xy!)=a +提(0-y=xy+xx2y" -zy"x2-zxy+zy=0 1-2: 2011-xy1-y=0. x24"+xy-4=0 .. The given function is a striumon of given DE. 6) show that y=activity, where a and b are arbitrary constants, is a solution of the Die day +3 th =0 y=ae32+ postery amount  $\frac{dx}{dx} = aG3)e^{-3x} + 0 \Rightarrow \frac{dx}{dx} = -3ae^{3x} - 0$ again diff w. r to x  $\frac{d^{2}y}{dx^{2}} = 9ae^{3x}$   $\frac{d^{2}y}{dx^{2}} = -3(-3ae^{3x})$  $\frac{d^2y}{dx^2} = -3\frac{dy}{dx}$ => 2000 +3 600 =0 . given function is a solution of given D.E. 1 Show that the D.E. representing the family of curves y= 2a (x+a2/3), where a is a tre parameter, is /y²-2xy器)3=8(y器)5,



8) show that y=acostac is a solution of the DE.

\[ \frac{d2y}{dx^2} + b^2y = 0
\] ... The given family of curve gives the D.E.

y=acosbx gitt mix fo or died (zolnies) b agouth disp wir to ic des = - acrosor (8)  $\frac{d^2y}{dx^2} = -b^2y \quad \Rightarrow \quad \frac{d^2y}{dx^2} + b^2y = 0$ .. The given punction is a solution of the given D.E.

variable Separable Method

G. Korthukeyan This works of

expansion 10.2

) If F is the constant force generated by the motor of an automobile of mass M, its velocity V is given by M dy = F-KV where x is a constant, Express V interms of t given that V=0 when t=0.

$$M \frac{dV}{dt} = F - KV$$

$$\frac{dV}{F - KV} = \frac{1}{M} dt$$
Integrating  $\int_{F - KV} \frac{dV}{F - KV} = \int_{F} \frac{dV}{M} dt$ 

Will things to the sides by 
$$-k$$
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$$\int \frac{kdV}{E-kV} = -\frac{k}{M} dt$$

$$\log(F-kV) = -\frac{k}{M} + C \longrightarrow 0$$

when  $V=0$ ,  $V=0$ 

$$0 \Rightarrow \log F = 0 + C$$

$$C = \log F$$

$$0 \Rightarrow \log(F-kV) = -\frac{kt}{M} + \log F$$

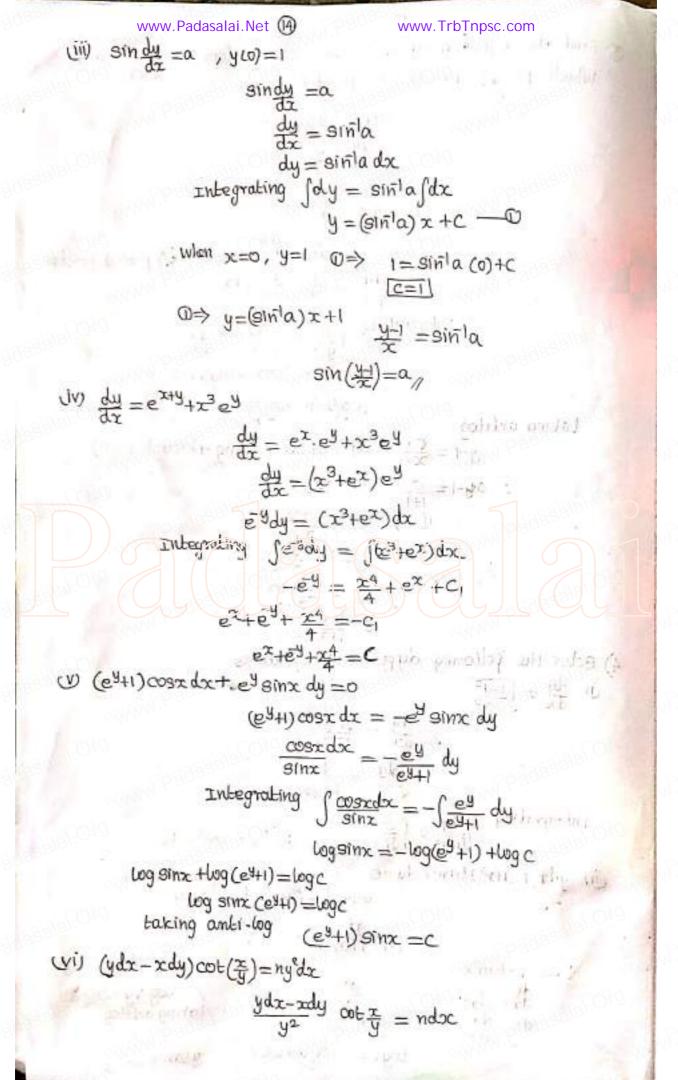
$$\frac{kt}{M} = \log F - \log(F-kV)$$

$$\frac{kt}{M} = \log F - \log(F-kV)$$

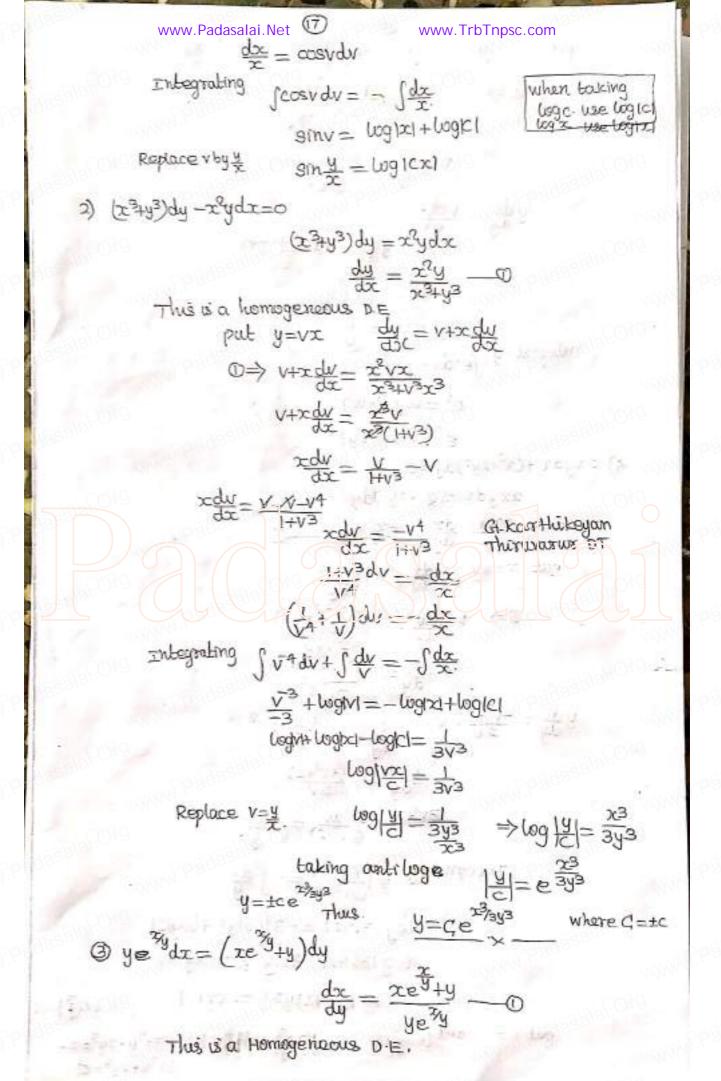
$$\frac{kt}{M} = \log F - \log(F-kV)$$

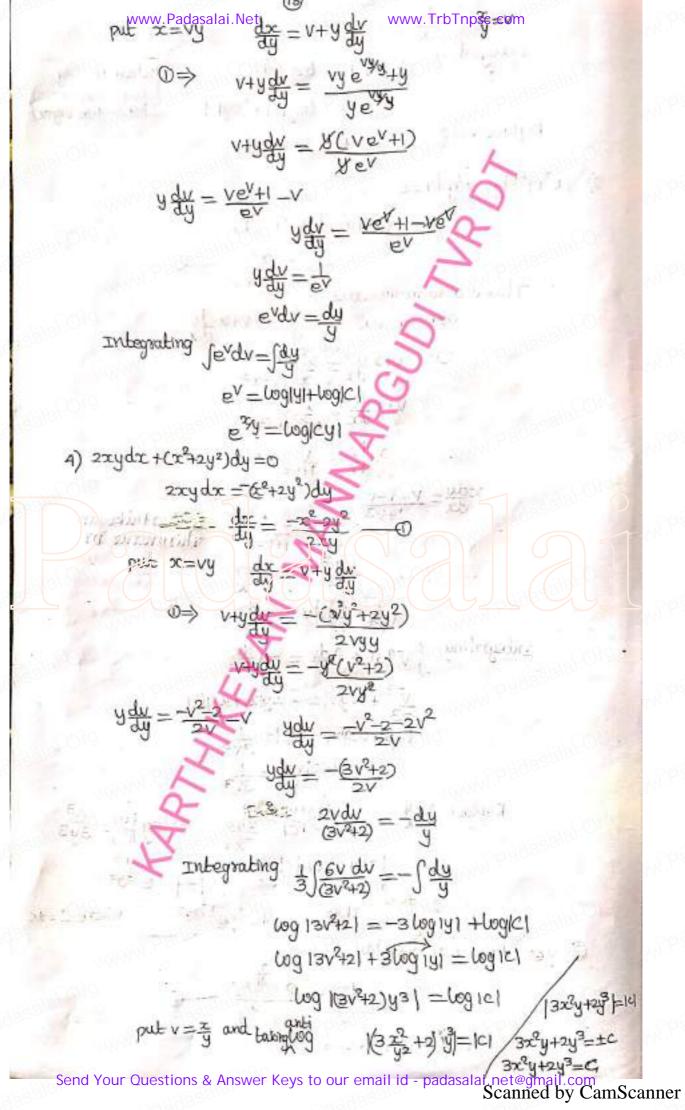
$$\frac{kt}{M} = \log F - \log F$$

www.Padasalai.Ne www.Padasalai.Net www.TrbTnpsc.com www.TrbTnpsc.com of the curve whose slope is  $\frac{y-1}{x^2+x}$  and which passes through the point (1,0) slope = y-1 on = 4-1 = x(x+1) doc (using postial praction)  $\frac{dy}{y-1} = \left(\frac{1}{x} - \frac{1}{x+1}\right) dx$ Integrating  $\int \frac{dy}{y-1} = \int \frac{dx}{x} - \int \frac{dx}{x+1}$ lug y+ = lugx - lug(x+1) + lug c (109(y-1)= 109xc taking antilog y-1 = Cx - 0 This passing through (1,0)  $0 \Rightarrow y_1 = \frac{-2x}{x+1}$ y = 1-2x  $=\frac{|x+|-2x}{|x+1|}$ y= 1-12 4) solve the following differential equations 山 器 司提 (11 (11 (11 ) (1) (1) die = 11-52 Integrating  $\int \frac{dy}{\sqrt{1-y^2}} = \int \frac{dx}{\sqrt{1-x^2}}$  sixiy =  $\sin^4 x + c$ (ii) ydx + (1+x2) tarilx dy=0 ydx=-(Hx2) tanizdy (1+2) tank = -dy no pro dy = - dy / logv+logy=logc take v=tanilx  $\frac{dv}{v} = -\frac{dy}{y} / \log vy = \log c$ Integrating  $\int \frac{dv}{v} = -\int \frac{dy}{y} / \frac{\log vy}{vy} = c$ 张-tz  $dv = \frac{dx}{1+x^2}$ wgv = - logy + log c / y tanta = c



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                       cot写)d(多)=ndx
                Integrating foot($) d($) = nfdx
                       \log \sin x = nx + c
taking antilog \sin (x) = e^{nx + c}
 (VII) & ->125-x2=0
                         dy = x\sqrt{25-x^2} dx put 25-x^2 = t
                      Integrating dy = \int t^{1/2} dt xdx = \frac{dt}{2}
                        +2y = -\frac{L^{3/2}}{3/2} + 2C
 multiply both sides by 36
                      = - F3/2+3 (ac)
                                3y=-(25-x2)3/2 +3C
(mi) starsy dy = ex(xlogx+1)dx
                                                        G. Korthi keyari
                 cosydy = ex (xlogz+1) dx
                                                       The source of
                  \cos y \, dy = e^{x} \left( \log x + \frac{1}{x} \right) dx
                         = (e^{x \log x} + e^{x} \perp dx
\Rightarrow \cos y dy = e^{x \log x} dx
+ e^{x} \perp dx
                          = \log x d(e^x) + e^x d(\log x)
                                                   (douv)=udv+vdu)
               cosydy = d(exlogx)
            Integrating Scosydy = Id (exlogx)
                              siny = e^{x}\log x + c.
   tany \frac{dy}{dx} = cos(x+y) + cos(x-y)
              tany dy = coexcesy-sinxsiny + coexcesy+sinxsiny
               tany dy = 2009x cosy
                                                /Integrating
                                               Isecy tany dy=szcoxdx
           tany dy = 2 cos x doc /
                                                 Secy = 29 inx+c
               secy tamy dy = 2 cos xdx
  Send Your Questions & Answer Keys to our email id - padasalai.net@gmail.com
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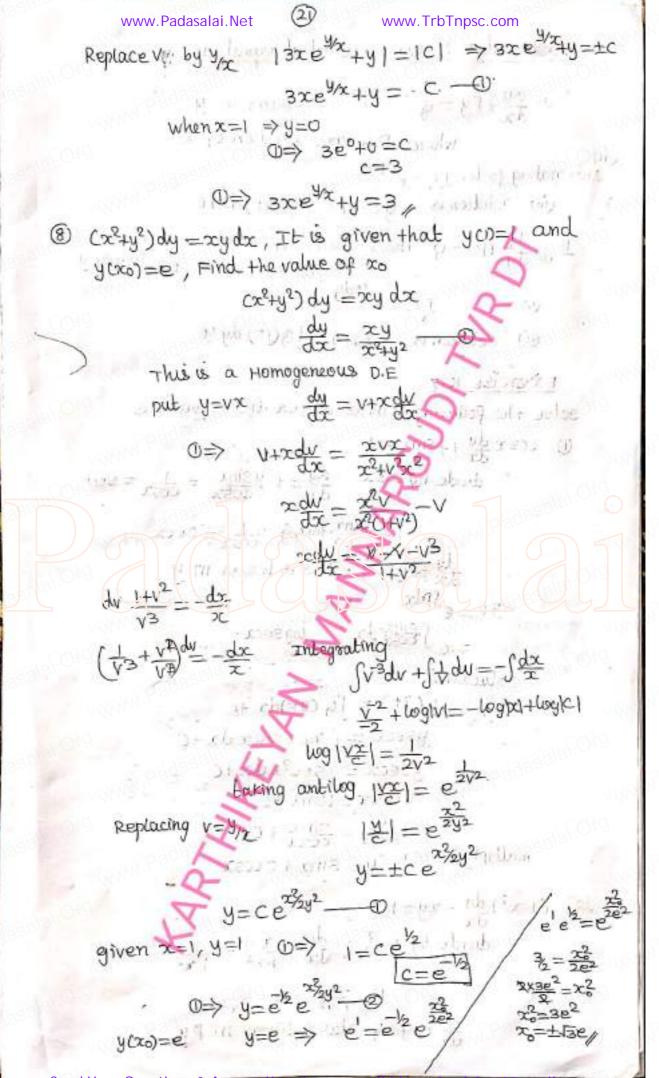




www.TrbTnpsc.com This is a homogeneous DE.  $\frac{dy}{dx} = \frac{y^2 - 2xy}{x^2 - 2xy} - 0$ put y=vx dy=v+xdy  $0 \Rightarrow v + x \frac{dv}{dx} = \frac{v^2 x^2 - 2xvx}{x^2 - 2xvx}$  $V^{+}x\frac{dv}{dx} = \frac{\chi^{0}(\sqrt{2}-2v)}{\chi^{0}(\sqrt{-2}v)}$  $2\frac{dv}{dx} = \frac{\sqrt{2-2V}}{1-2V} - V \implies \frac{\sqrt{2-2V} - V + 2V^2}{dx}$  $2 \frac{dv}{dx} = \frac{3\sqrt{-3}V}{1-2V}$ 3(v2V) dv = & 1-2V dv = 3 dz Using pointral praction (-+) dv=3 dx Integrating S铁+J於=-3J安 lisgiv-1+-lingiv = -3 lingin-1 +- lingic! log (vev-01 + alog 12) == log 12.1 اعا وفعا ــ الخروسا+ زربيكا وصا taking and leg  $|x^3(v^2v)| = \log |c|$   $|x^3(v^2v)| = |c|$ pulting v=其 129(些-美) = 1c1 G.kartkikegan Therwaster of 소급하 - \* = +c xy2-x2y=+c where q=+c  $xy^2-x^2y=C$ ©  $x\frac{dy}{dx} = y - x\cos^2(\frac{y}{2}) - 0$   $y = v + x\frac{dy}{2} = v + x\frac{dy}{2}$ ##= V+~# · 0 今 x (v+xxx) = vx-x·cos² yx 1/2K+2gH = 1/2K-2 cogV

Find Your Orientions & V

www.Padasalai Net www.TrbTnpsc.com Integrating Jeers du = - Jax tanv = - logiz | + log ic | banv = lug|ş| tany - wolf taking antily etan = | \frac{1}{2} = | \frac{1}{2} | Telany = +C was etany = 4  $\Im (1+3e^{4x})dy +3e^{4x}(1-4x)dx=0$ ; given that y=0 when x=1(1+3e4x) dy = -3e4x (1-4x) dx  $\frac{dy}{dx} = \frac{-3e^{3/x}(1-3/x)}{1+3e^{3/x}}$ This is a Homogeneous DE by hax #= N+xdx  $0 \Rightarrow y \neq dy \leq -3 e^{14/2} (1 - \sqrt{2})$  $x \frac{dx}{dx} = -3e^{y} + 3ve^{y} = v$   $v + x \frac{dx}{dx} = -3e^{y} (1-v)$  $\frac{3e^{V+1}}{3e^{V+1}} dV = -\frac{3}{3e^{V+1}} dx$   $\frac{3e^{V+1}}{3e^{V+1}} dV = -\frac{3e^{V+1}}{3e^{V+1}} dx$   $\frac{3e^{V+1}}{3e^{V+1}} dV = -\frac{3e^{V+1}}{3e^{V+1}} dx$ Integrating JevH du= -Jdx logicaev+v) = -logizi+logici log1=(zev+v) = log1c1 



First order Linear Differential equation 工的dy+py=q. This is Linear in y where P.g are function of x Integrating factor IF = e Spdx. uin solution is y (IF) = fg (IF) dx+c I.F. = e Spoly whome Pig are function of y 正的 dx+px=g This is linear in 2 (ii) (ii) Solution is z (IF) = [G(IF) dy+c. exercise 10.7 solve the following Linear differential equations O cosx提+yamc=1 divide by cosx dy =+ y sinx = tosx p= ginx tonz, q = 1 = Seloc the py= 0 , This is linear in y IF= e SPdx =e secx = e secx solutionis y (IF) = Jg (IF)dx +c Asecx = lescx secx gx+C ysecx = sec2xdx+C y secx = tanx +c multiply by  $\cos x$   $y = \sin x + c$ multiply by  $\cos x$   $y = \sin x + c \cos x$ ② (1-x2)# -xy=1 divide by  $1-x^2$   $\frac{dy}{dx} - \frac{xy}{1-x^2} = \frac{1}{1-x^2}$  $p = \frac{-x}{1-x^2} \quad Q = \frac{1}{1-x^2}$ # +Py=9 This is linear in Py

$$IF. = e^{\int_{-\infty}^{\infty} dx}$$

$$= e^{\int_{-\infty}^{\infty} dx}$$

e nartuloe

$$3(IE) = \int \frac{1}{4\pi^2} dx + C$$

$$3(IE) = \int \frac{1}{4\pi^2} \sqrt{1-x^2} dx + C$$

$$3(I-x^2) = \int \frac{1}{4\pi^2} dx + C$$

③ 岩+茅=shix

$$p=k_{x}$$
,  $q=9$  in  $x$ 

$$\frac{dy}{dx}+py=q$$
 This is linear in  $y$ 

$$IF=e^{\int pdx}$$

$$=e^{\int \frac{p}{dx}}=e^{\int p}$$

Bolution is

IF = 
$$x$$
 $y(IF) = \int f(IF) dx + C$ 
 $y(x) = \int f(x) dx + C$ 
 $f(x) = \int f(x) dx + C$ 

wex duestimate december v=-cosse

= xccosx)-Jecosxodx+c Judu=uv-Judu

 $xy = -x\cos xx + \sin x + C$ 

$$x(y+\cos x) = \sin x + C$$
.

€ (Eg+1) \$\frac{4}{12} + 5\frac{1}{12} = \frac{1}{12} \frac{1}{12}

$$P = \frac{2\pi}{241}$$
  $Q = \sqrt{\frac{24}{241}}$ 

solution is 
$$y(IF) = \int g(IF) dx + C$$

$$g(x)H) = \int \frac{dx}{dx} dx + C$$

y (x2+1) = = 12/12/14 + 2 log |x+12/14| +C 5) (2x-10y3) dy +ydx=0  $y dx = -(2x - 10y^3) dy$  $y\frac{dx}{dy} = -2x + 10y^3$  $\frac{dx}{dy} = -\frac{2x}{y} + \frac{10y^3^2}{y}$ \$\frac{4x}{5y} + \frac{2x}{y} = 10y^2 dx+Px=9 This is linear in x IF=eSpay=eSigdy=elogy=elogy=y2 solution is xCIF)= [qCIF) dy+00 xy2= Sloy2(y2)dy+C =10 g 4 dy + c = 11 xy2=2454c/ 6 xsmz ill + Cxcos:c+sinx ill = sinx. divide by Esinx  $\frac{dy}{dx} + \frac{x\cos x + \sin x}{x \sin x} y = \frac{\sin x}{x \sin x}$  $p = \frac{x \cos x + \sin x}{x \sin x}$   $q = \frac{1}{x}$ dy +py=9 This is linear in y  $I = e^{\int p dx} = e^{\int \frac{xaosx + sinx}{x sinx}} dx$   $= e^{\int g(csinx)} = xsinx.$ solution is y(IF)= g(IF) dx+C y(xsim)=[+xxxsinxdx+( xysinx = -cosx+c@ (y-esin'x) dx +11-x2 =0 (y-esint2) dx =- J1-x2

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$$y = e^{\sin^2 x} = -\frac{dy}{dx} \sqrt{1-x^2}$$

$$\frac{dy}{dx} + \frac{y}{1-x^2} = e^{\sin^2 x}$$

$$\frac{dx}{dx} + \frac{dx}{1-x^2} = e^{\sin^2 x}$$

$$\frac{dx}{dx} + \frac{dx}{1-x^2} + \frac{dx}{1-x^2}$$

$$\frac{dx}{dx} + \frac{dx}{1-x^2} = e^{\sin^2 x}$$

$$\frac{dx}{1-x^2} + \frac{dx}{1-x^2}$$

$$\frac{dx}{1-x^2} + \frac{dx}{$$

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$$(1+x+xy^2)$$
  $\frac{1}{2}$   $\frac{1$ 

IF = e Spdy = e Sydy = e logy = y xCIF)= JQ(IF) dy+c es nortulos

xy = S= Lynny & dy \* xy = - tanily +c

xy+tamily=c 3

10) 
$$\frac{dy}{dx} + \frac{y}{x \log x} = \frac{\sin 2x}{\log x}$$

 $f = \frac{1}{2\log x}$   $G = \frac{\sin 2x}{\log x} \Rightarrow \frac{dy}{dx} + fy = G$ (irecasin u This is lirenor in y pode Stogx da wg log z

IF=Wg>C solution is y(IF)= [9 IFdx+C / glogx = J sin2x logx dx+C → ylog x = -cos2x + C.

ywgx+cosex = c

① (x+a) dy -2y = (x+a)4 divide by x+a dy - 24 = (x+a)3

This is linear in y IF= e Spax - 12 adx.

 $= e^{2\log(\alpha+\alpha)} = e^{\log(\alpha+\alpha)^2}$  $= \cot 0$ 

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$$y(IF) = \int G(IF) dx + C$$
 $y(\overline{x}_{0})^{2} = \int \frac{1}{\overline{x}_{0}} (x + \alpha)^{\frac{3}{2}} dx + C$ 
 $y(\overline{x}_{0})^{2} = Cx + \alpha)^{2} + C$ 
 $y(\overline{x}_{0})^{2} = Cx + \alpha)^{4} + 2Cx + \alpha)^{2}$ 

(12) 
$$\frac{dy}{dx} = \frac{3in^2x}{1+x^3} - \frac{3x^2}{1+x^3}y$$
$$\frac{dy}{dx} + \frac{3x^2}{1+x^3}y = \frac{3in^2x}{1+x^3}$$

This is linear in y

$$IF = e^{\int \frac{3x^2}{1+x^3} dy} = e^{\int \frac{3x^2}{1+x$$

$$y(IF) = \int g(IF) dx + C$$
  
 $yx = \int x \log x dx + C$ 

u=logx dv=xdx du=for v=== Judy=uv-Judu

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$$p=\frac{2}{2}$$
  $G=x\log x \Rightarrow \frac{1}{2}+Py=9$   
This is linear in y  $\int_{-\infty}^{\infty} dx = e^{2\log x} = e^{\log x^2}$ 

$$IF = x^{2}$$
Solution is 
$$V(IF) = \{0, (2, 1)\}$$

solution is 
$$y(IF) = \int Q(IF) dx + C$$
  
 $yx^2 = \int x^2 wgx x^2 dx + C$ 

$$= \int \log x \, \frac{3}{2} \, dx \, dx$$

$$= \frac{x^4}{4} \log x - \int \frac{x^3}{4} \, dx \, dx$$

$$= \frac{x^4 (00x - 1) x^4}{4} + C$$

$$yx^2 = x \frac{1\log x}{4} + C$$

Solution is 
$$y(IF) = \int g(IF) dx + C$$

given 
$$g=2$$
 when  $x=1$   $0 \Rightarrow 203=\frac{12}{2}+C$ 

$$0 \Rightarrow yx^3 = x^2 + \frac{3}{2}$$

## Applications of First order ODE

Exercise 10.8

1) The rate of increase in the number of bacteria in a certain culture is proportional to the number present Given that the number triples in 5 hours, find how many bacteria will be present after 10 hours.

Let A be the number of bactoria at any time t

a kanthikeyan hiruvarur DT

wwhen to A=A0

$$A = A_0$$

$$0 \Rightarrow A_0 = Ce^0$$

$$0 \Rightarrow A_0 = Ce^0$$

ii) when t=5 A=3A0

in when t=10, A=?

apton 10 hours in = 0 times the (initial value)

@ Find the population of a city at any time to given that the rate of increase of population is proportional to the population at that instant and that in a period of 40 years the population increased from 3,00,000 to 4,00,000

Let A bethe population of city at any time t

when t=0, A=3,00,000  $= Ce^{0}$  C=3,00,000

0 = 3,00,000 eKE \_ @ SHURS

when t=40 A=4,00,000

€ > 4,00,000 = 3,00,000 € \( \frac{4}{3} = e^{40 \text{ k}} \)

3 The equation of electromotive force for an electric circuit containing resistance and self-inductance is ==Ri+Ldi where E is the Electromotive force is given to the circuit R is the Resistance and L-coeppicient op induction. Find the current i at time t when E=0

This is Linear in i But SRIE RE

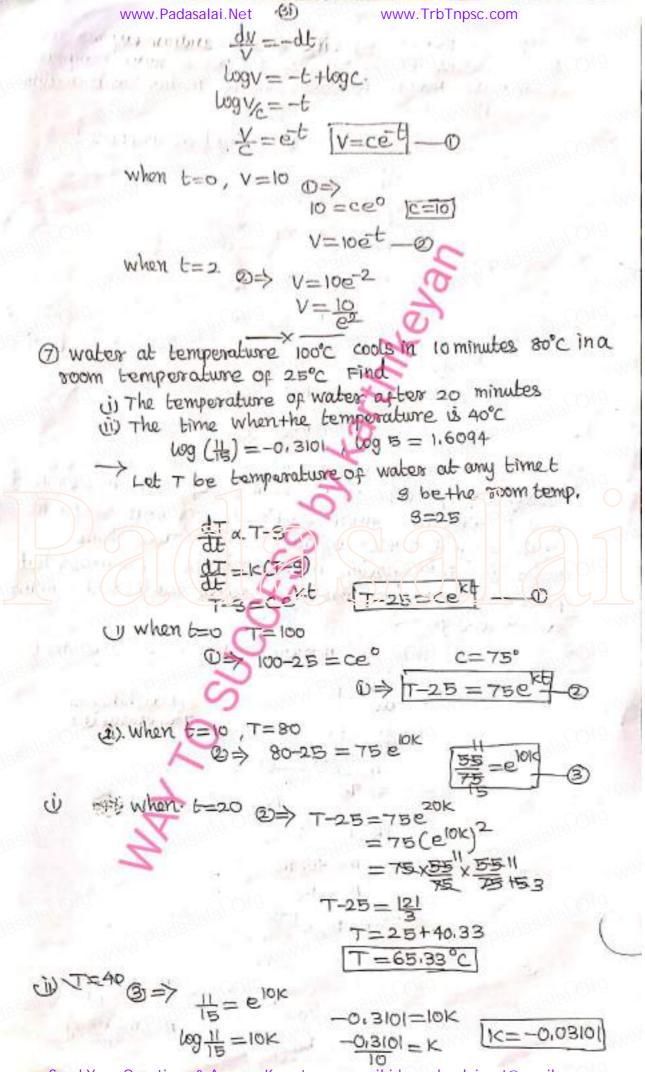
solution is

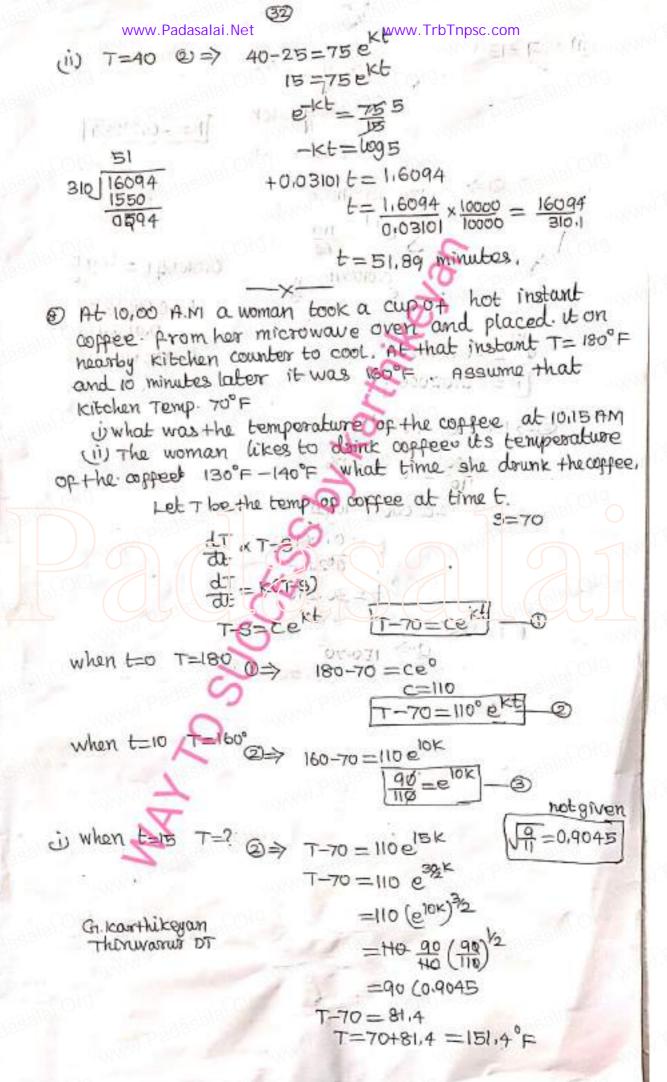
when 
$$E=0$$
  $\Rightarrow 0$ 

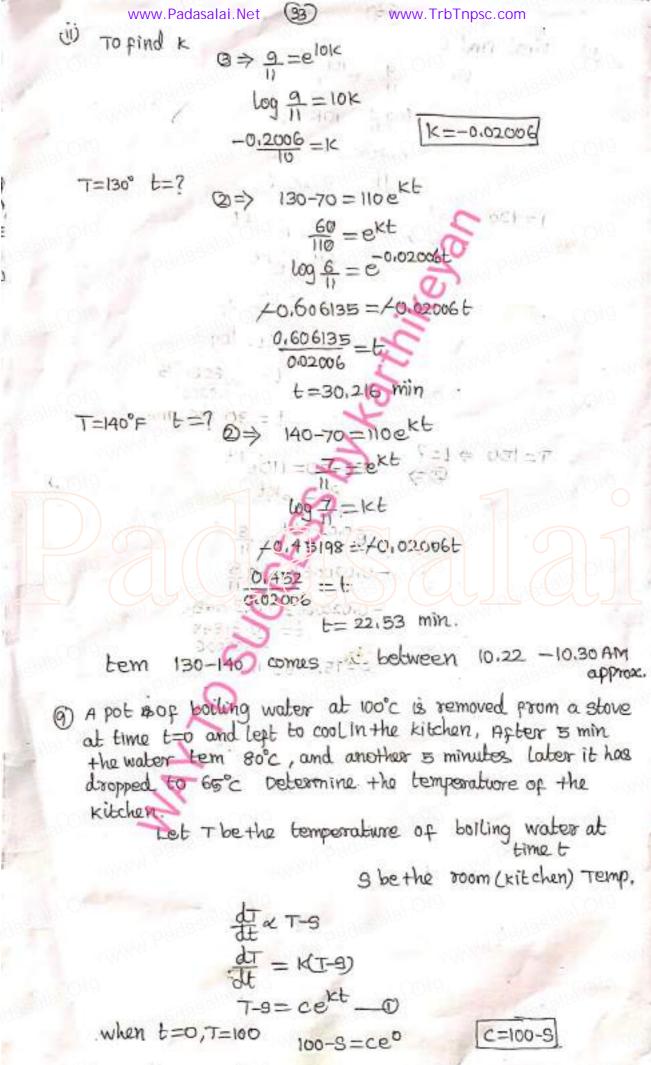
when E=0+C ieRt=c

1 The engine of a motor boat moving at 10 m/s is shut off. Given that the retardation at any subsequent time (after shutting off the engine) equal to the velocity at that time. Find the velocity after 2 seconds of switching off the engine,

Let v be the velocity at any time t given retardation = velocity







33

when t=5 T=80°

when t=10 T=65

$$0 \Rightarrow 65-9 = (100-9)(65K)^2$$

35=100

Kitchen Temperature=20°C. 05=20°C.

10 n tank initially contains 50 litres of pure water, starting at time t=0 a brine containing with 2 grams of dissolved salt por litre plows into the tank at the rate of 3 litres per minute . The mixture is kept uniform by stirring and the well stirred michane simultaneously flows out of the toric at the scome rate Find the amount of salt present in the tark at any time (100,

Let act) denote the amount of sall in the fank ac time t.

inflow rate = 2×3=6

out flow 7 30 x

$$\frac{dx}{dx} = 6 - \frac{2}{3}x$$

$$= \frac{2}{3}(x - 6x\frac{2}{3})$$

$$\frac{dx}{dx} = -\frac{2}{3}(x - 100)$$

$$\frac{dx}{dx} = \frac{2}{3}(x - 100)$$

when . 6=0 x=0

GriveHour Suggestions 9715634957

3 supposeur Radasalai Net deposits 10,000 Indum rupees ina bank account at the rate of By per annum compounded continuously. How much money will be in his bank account 18 months later? Let A be the Amount at any time t. dA = 0.05A t=0, A=10,000 @=> 10,000 = ce0 C=10,000 antiquings of all a fact of the t=1/2=3/2 1/11 0000 € 0.05x3 92.1121 110000 A =10,000 e @ Assume that the rate at which radioactive nuclei decay is proportional to the number of such nuclei that are present in a given sample. In a certain sample lox of the original number of redivactive nuclei have undergone disintegration in a period of 100 years. What percentage of the anginal madicialtive nuclei will remain outter 1000 years. Let A be the Amount of nuclei at any time t Give your suggestions 3-2/ = 92-11 gt ⊗ U G. Karthikeyam Thirwarur (DT) 9715634957 A=cekt\_O when be A=A0 => A0=CeO [C=Ad] A= AoeKt @ (ii) when t=100 A=907. Ao 2)=> 2 A= Aa e 100K (ii) When t=1000 A=  $= A_0 \left( e^{100K} \right)^{10} = A_0 \frac{9^{10} \times 100}{10^{10} \times 100} \times .$  $= Ao \left(\frac{9}{10}\right) IO \qquad A = Ao \left(\frac{9}{108}\right)$ ADIED/CI---