10. ORDINARY DIFFERENTIAL EQUATIONS

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Differential Equation, order, Degree

Order. – highest order derivative present in the D.E.

Degree – Integral power of the highest order derivative (D.E. is expressible in a polynomial form)

– If not expressible as a highest order derivative as the leading term then the degree of D.E. is not defined.

Exercise 10.1

1) determine the order, degree (if exists) of the D.E.

(i) $\left(\frac{dy}{dx}\right) + xy = \cot x$. order = 1, degree = 1

(ii) $\left(\frac{d^3y}{dx^3}\right)^{2/3} - 3\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 4 = 0$

$\left(\frac{d^3y}{dx^3}\right)^{2/3} = 3\frac{d^2y}{dx^2} - 5\frac{dy}{dx} - 4$ order = 3, degree = 2

cubing both sides.

$\left(\frac{d^3y}{dx^3}\right)^2 = \left(3\frac{d^2y}{dx^2} - 5\frac{dy}{dx} - 4\right)^3$

(iii) $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 = x\sin\left(\frac{d^2y}{dx^2}\right)$ order = 2

This can not be expressed as polynomial with $\frac{d^2y}{dx^2}$ as the leading term.

degree = not defined.

(iv) $\sqrt{\frac{dy}{dx}} - 4\frac{dy}{dx} - 7x = 0$

$\sqrt{\frac{dy}{dx}} = 4\frac{dy}{dx} + 7x$

squaring both sides

$\frac{dy}{dx} = \left(4\left(\frac{dy}{dx}\right) + 7x\right)^2$

$\frac{dy}{dx} = 16\left(\frac{dy}{dx}\right)^2 + 56x\frac{dy}{dx} + 49x^2$ order = 1, degree = 2

$16\left(\frac{dy}{dx}\right)^2 + 56x\frac{dy}{dx} - \frac{dy}{dx} + 49x^2 = 0$

(v) $y\left(\frac{dy}{dx}\right) = \frac{x}{\left(\frac{dy}{dx}\right)+\left(\frac{dy}{dx}\right)^3}$

$y\left(\frac{dy}{dx}\right)^2 + y\left(\frac{dy}{dx}\right)^4 = x$.

order = 1
degree = 4

(vi) $x^2\frac{d^2y}{dx^2} + \left[1+\left(\frac{dy}{dx}\right)^2\right]^{1/2} = 0$

$x^2\frac{d^2y}{dx^2} = -\left[1+\left(\frac{dy}{dx}\right)^2\right]^{1/2}$

squaring both sides

$x^4\left(\frac{d^2y}{dx^2}\right)^2 = 1+\left(\frac{dy}{dx}\right)^2$

order = 2
degree = 2

(vii) $\left(\frac{d^2y}{dx^2}\right)^3 = \sqrt{1+\left(\frac{dy}{dx}\right)}$

Squaring both sides

$\left(\frac{d^2y}{dx^2}\right)^6 = 1+\frac{dy}{dx}$

order = 2
degree = 6

(viii) $\frac{d^2y}{dx^2} = xy + \cos\left(\frac{dy}{dx}\right)$

order = 2
degree = does not exist

(ix) $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + \int y\,dx = x^3$

diff w-r to x

$\frac{d^3y}{dx^3} + 5\frac{d^2y}{dx^2} + y = 3x^2$

order = 3
degree = 1

(x) $x = e^{xy\frac{dy}{dx}}$

taking log

$\log x = xy\frac{dy}{dx}$

order = 1
degree = 1

(book answer wrong)

———x———

classification of D.E.

ordinary D.Equation (ODE)

contains only ordinary derivatives of one or more functions w.r.to. a ~~sig~~ single independent variable

partial D.Equation (PDE)

Involving only partial derivatives of one or more functions of 2 or more independent variable.

Exercise 10.2

1) Express each of the following physical statements in the form of differential equation.

(i) Radium decays at a rate proportional to the amount q present.

given q be the amount of Radium at present any time t

$$\frac{dq}{dt} \propto q$$

$$\frac{dq}{dt} = kq \quad (k \text{ is a constant})$$

(ii) The population p of a city increases at a rate proportional to the product of population and to the difference between 5,00,000 and the population.

p be the population

rate of population $\propto p \cdot (500000 - p)$

$$\frac{dp}{dt} \propto p(5{,}00{,}000 - p)$$

$$\frac{dp}{dt} = kp(500000 - p)$$

(iii) For a certain substance, the rate of change of vapor pressure P w.r.to temperature T is proportional to the vapor pressure and inversely proportional to the square of the temperature.

rate of change of P w.r.to T $\propto \frac{P}{T^2}$

$$\frac{dP}{dT} \propto \frac{P}{T^2}$$

$$\frac{dP}{dT} = \frac{kP}{T^2}$$

book answer $\frac{dP}{dt} = \frac{kP}{T^2}$ wrong

(iv) A saving amount pays 8% interest per year, compounded continuously. In addition, the income from another investment is credited to the amount continuously at the rate of ₹400 per year.

Let x be the amount any time t

$$\frac{dx}{dt} \propto x$$

$$\frac{dx}{dt} = kx \qquad k = 8\% = \frac{8}{100}$$

$$\frac{dx}{dt} = \frac{8}{100}x$$

400 income added $\quad \frac{dx}{dt} = \frac{8}{100}x + 400 \qquad \frac{dx}{dt} = \frac{2x}{25} + 400$

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2) Assume that a spherical rain drop evaporates at a rate proportional to its surface area. Form a D.E. involving the rate of change of the radius of the rain drop.

Let r be the radius V be the volume and A be the surface area of spherical rain drop

$$\frac{dV}{dt} \propto -A$$

volume of sphere $= \frac{4}{3}\pi r^3$

$SA = 4\pi r^2$

$$\frac{dV}{dt} = -kA$$

$$\frac{d}{dt}\left(\frac{4}{3}\pi r^3\right) = -k\, 4\pi r^2$$

$$\frac{4\pi}{3}\frac{d}{dt}(r^3) = -4k\pi r^2$$

$$\frac{1}{3} 3r^2 \frac{dr}{dt} = -kr^2$$

$$\frac{dr}{dt} = -k$$

This is the required D.E.

—x—

Formation of D.E.

Exercise 10.3

1) Find the differential equation of the family of
(i) all non-vertical lines in a plane
(ii) all non-horizondal lines in a plane.

(i) The equation of any non-vertical family of line in a plane is

$ax+by+c=0$ where $a \in R$, $b \neq 0$

a, b two constants diff. w.r to x two times

$a+by'=0 \therefore y' = a/b$

$y''=0$ or $\frac{d^2y}{dx^2}=0$.

$\boxed{\text{when } b=0 \Rightarrow x = \text{con vertical line}}$

(ii) The equation of all non-horizondal family of line in a plane is $ax+by=c$ with $a \neq 0$

diff w.r to y

$$\frac{dx}{dy}a+b = 0$$

$$\frac{dx}{dy} = \frac{-b}{a}$$

$\boxed{\text{when } a=0 \Rightarrow y = \text{constant horizondal}}$

again diff w.r to y

$$\frac{d^2x}{dy^2}=0$$

2) Form the differential equation of all straight lines touching the circle. $x^2+y^2=r^2$

Equation of tangent to the circle $x^2+y^2=r^2$ is

$\boxed{\text{to eliminate m only}}$

(m is a arbitrary constant) $y = mx \pm r\sqrt{1+m^2}$ —①

diff w.r to x

$$\frac{dy}{dx} = m$$

$$① \Rightarrow \quad y = \frac{dy}{dx}(x) \pm r\sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

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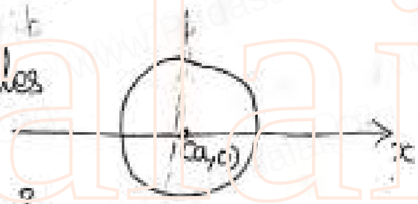
$$y - x\frac{dy}{dx} = \pm r\sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

squaring both sides

$$\left(y - x\frac{dy}{dx}\right)^2 = r^2\left(1+\left(\frac{dy}{dx}\right)^2\right)$$

3) Find the differential equation of the family of circles passing through the origin and having their centres on the x-axis

Equation of family of circles passing through origin and centre $(a,0)$ is



$$(x-a)^2+(y-0)^2 = a^2$$

$$(x-a)^2+y^2 = a^2 \quad —①$$

$\boxed{\text{to eliminate a only}}$

a is only arbitrary constant

diff w.r to x

$$2(x-a)+2y\frac{dy}{dx} = 0$$

$$x-a = -y\frac{dy}{dx} \qquad a = x+y\frac{dy}{dx}$$

$$① \Rightarrow \left(-y\frac{dy}{dx}\right)^2 + y^2 = \left(x+y\frac{dy}{dx}\right)^2$$

$$y^2\left(\frac{dy}{dx}\right)^2 + y^2 = x^2 + y^2\left(\frac{dy}{dx}\right)^2 + 2xy\frac{dy}{dx}$$

$$x^2+2xy\frac{dy}{dx} - y^2 = 0$$

4) Find the differential equation of the family of all the parabolas with the latus rectum $4a$ and whose axes are parallel to the x-axis.

Equation of parabola.

(h, k are arbitrary constants) $(y-k)^2 = 4a(x-h)$ —①

diff. w.r to x

$\boxed{\text{eliminate h, k only}}$

$$2(y-k)\,y' = 4a(1)$$

$$(y-k)y' = 2a \quad \text{---} (2)$$

again diff w.r to x

$$(y-k)y'' + (y')(y') = 0$$

$$(y-k)y'' = -y'^2$$

$$(y-k) = \frac{-y'^2}{y''}$$

$$(2) \Rightarrow \; -\frac{y'^2}{y''}\,y' = 2a$$

$$-(y')^3 = 2ay''$$

$$2ay'' + (y')^3 = 0$$

5) Find the differential equation of the family of parabolas with vertex at $(0,-1)$ and having axis along the y-axis

Equation of parabola

$$(x-0)^2 = 4a(y+1)$$

$\boxed{\text{eliminate } a \text{ only}}$

$$x^2 = 4a(y+1) \quad \text{---} (1)$$

a is arbitrary constant

diff w.r to x

$$\cancel{2}x = \cancel{4}^{2}a\frac{dy}{dx}$$

$$\frac{x}{y'} = 2a$$

$$(1) \Rightarrow \; x^2 = 2\left(\frac{x}{y'}\right)(y+1)$$

$$\frac{y'x^{\cancel{2}}}{\cancel{x}} = 2y+2$$

$$xy' - 2y - 2 = 0$$

6) Find the differential equations of the family of all the ellipses having foci on the y-axis and centre at the origin.

Equation of Ellipse

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 \quad \text{---} (1)$$

$\boxed{\text{eliminate } a, b}$

a, b are arbitrary constant

diff w.r to x

$$\frac{2x}{b^2} + \frac{2yy'}{a^2} = 0 \quad \text{---}$$

$$\frac{x}{b^2} + \frac{yy'}{a^2} = 0 \quad \text{---} (2)$$

again diff. w.r to x

$$\frac{1}{b^2} + \frac{(yy'' + y'y')}{a^2} = 0 \quad \text{---} (3)$$

From ①②③ Eliminate $1/a^2, 1/b^2$

$$\begin{vmatrix} x^2 & y^2 & 1 \\ x & yy' & 0 \\ 1 & yy''+y'^2 & 0 \end{vmatrix} = 0$$

$$x^2(0-0) - y^2(0-0) + 1\left(x(yy''+y'^2) - yy'\right) = 0$$

$$xyy'' + xy'^2 - yy' = 0.$$

7) Find the differential equation corresponding to the family of curves represented by the equation $y = Ae^{8x} + Be^{-8x}$ where A and B are arbitrary constants.

$$y = Ae^{8x} + Be^{-8x}$$

diff w.r.to x

$$\frac{dy}{dx} = 8Ae^{8x} + B(-8)e^{-8x}$$

again diff w.r.to x

$$\frac{d^2y}{dx^2} = 64Ae^{8x} + 64Be^{-8x}$$

$$= 64(Ae^{8x} + Be^{-8x})$$

$$\frac{d^2y}{dx^2} = 64y$$

8) Find the differential equation of the curve represented by $xy = ae^x + be^{-x} + x^2$

$$xy - x^2 = ae^x + be^{-x} \quad \text{---} ①$$

a, b are arbitrary constants to eliminate a, b

diff w.r.to x

$$xy' + y - 2x = ae^x - be^{-x}$$

again diff w.r.to x

$$xy'' + y' + y' - 2 = ae^x + be^{-x}$$

Using ① ⇒ $xy'' + 2y' - 2 = xy - x^2$

$$xy'' + 2y' + x^2 - xy - 2 = 0.$$

———x———

Example 10.6 Find the D.E of the family of all ellipses having foci on the x-axis and centre at origin.

Equation of ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{---} ①$$

a, b are arbitrary constant

diff w.r.to x

$$\frac{2x}{a^2} + \frac{2yy'}{b^2} = 0$$

$$\frac{x}{a^2}+\frac{yy'}{b^2}=0$$

again diff. w.r.to x

$$\frac{1}{a^2}+\frac{(yy''+y'y')}{b^2}=0$$

Eliminate $1/a^2, \frac{1}{b^2}$ compare its co-efficients.

$$\begin{vmatrix} x^2 & y^2 & 1 \\ x & yy' & 0 \\ 1 & yy''+y'^2 & 0 \end{vmatrix}=0$$

$$1(x(yy''+y'^2)-yy'=0$$

$$xyy''+xy'^2-yy'=0.$$

solution of ODE.

Exercise 10.4

① show that each of the following expressions is a solution of the corresponding given differential equation.

(i) $y=2x^2$; $xy'=2y$

$$y=2x^2 \text{ — ①}$$

diff w.r.to x

$$y'=4x.$$

multiply by x $\quad xy'=4x^2$

$$xy'=2(x^2)$$

using ① $\rightarrow$ $xy'=2y$

(ii) $y=ae^x+be^{-x}$; $y''-y=0$

$$y=ae^x+be^{-x}$$

a, b are arbitrary constants.

diff w.r.to x

$$y'=ae^x-be^{-x}$$

again diff. w.r.to x.

$$y''=ae^x+be^{-x}$$

$$y''=y \rightarrow y''-y=0$$

② Find value of m so that the function $y=e^{mx}$ is a solution of the given differential equation

(i) $y'+2y=0$ — ①

$$y=e^{mx}.$$

diff w.r.to x

$$y'=e^{mx}m$$

$$y'=my$$

$$y'-my=0 \text{ — ②}$$

compare ① & ② $\quad -m=2$

$$m=-2$$

(ii) $y''-5y'+6y=0$ —— ①

②⇒ $y'-my=0$ $\boxed{y'=my}$

again diff w.r to x

$y''-my'=0$

$y''=my'$

$=m\,my$

$\boxed{y''=m^2y}$

①⇒ $m^2y-5my+6y=0$

$y(m^2-5m+6)=0$

$y=0$, $m^2-5m+6=0$

$(m-2)(m-3)=0$

$m=2,3$

③ The slope of the tangent to the curve at any point is the reciprocal of 4 times the ordinate at that point. The curve passes through (2,5). Find the equation of the curve.

Let the point be $P(x,y)$

$$\text{slope}=\frac{1}{4y}$$

$$\frac{dy}{dx}=\frac{1}{4y}$$

$$4y\,dy=dx$$

Integrating $4\int y\,dy=\int dx$

$$\cancel{4}^2\frac{y^2}{\cancel{2}}=x+c$$

$$2y^2=x+c \quad \text{——①}$$

This curve passing through (2,5)

①⇒ $2(5)^2=2+c$ $c=48$

$50-2=c$

①⇒ Required equation of curve $2y^2=x+48$

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4) Show that $y=e^{-x}+mx+n$ is a solution of the differential equation $e^x\left(\frac{d^2y}{dx^2}\right)-1=0$

$y=e^{-x}+mx+n$ two constants m,n

diff. w.r to x

$$\frac{dy}{dx}=e^{-x}(-1)+m$$

again diff w.r to x.

$$\frac{d^2y}{dx^2}=e^{-x} \Rightarrow \frac{d^2y}{dx^2}=\frac{1}{e^x}$$

$$e^x\left(\frac{d^2y}{dx^2}\right)=1$$

$$e^x\left(\frac{d^2y}{dx^2}\right)-1=0$$

∴ given function is a solution of the given D.E.

5) Show that $y=ax+\frac{b}{x}$, $x\neq 0$ is a solution of the differential equation $x^2y''+xy'-y=0$

$$y=ax+\frac{b}{x} \quad \text{—①}$$

two constants a, b

diff w-r to x.

$$y'=a(1)+b\left(\frac{-1}{x^2}\right) \quad \text{—②}$$

again diff. w-r to x

$$y''=0+\frac{2b}{x^3}$$

$$2b=x^3y''$$

$$b=\frac{x^3y''}{2}$$

②⇒ $y'=a-\frac{x^3y''}{2}\times\frac{1}{x^2}$

$$y'=a-\frac{xy''}{2}$$

$$y'+\frac{xy''}{2}=a$$

①⇒ $y=\left(y'+\frac{xy''}{2}\right)x+\frac{x^3y''}{2x}$

$$y=xy'+\frac{x^2y''}{2}+\frac{x^2y''}{2}$$

$$y=xy'+\frac{2x^2y''}{2}$$

$$x^2y''+xy'-y=0$$

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$$\begin{vmatrix} y & x & \frac{1}{x} \\ y' & 1 & \frac{-1}{x^2} \\ y'' & 0 & \frac{2}{x^3} \end{vmatrix}=0$$

$$y\left(\frac{2}{x^3}-0\right)-x\left(\frac{y'2}{x^3}+\frac{y''}{x^2}\right)+\frac{1}{x}(0-y'')=0$$

$$\frac{2y}{x^3}-\frac{2y'}{x^2}-\frac{y''}{x}-\frac{y''}{x}=0$$

$$\frac{2y-2y'x-y''x^2-y''x^2}{x^3}=0$$

$$-2y''x^2-2xy'+2y=0$$

$$-2\div \quad x^2y''+xy'-y=0$$

∴ The given function is a solution of given D.E.

6) Show that $y=ae^{-3x}+b$, where a and b are arbitrary constants, is a solution of the D.E $\frac{d^2y}{dx^2}+3\frac{dy}{dx}=0$

$$y=ae^{-3x}+b$$

diff w. r to x

$$\frac{dy}{dx}=a(-3)e^{-3x}+0 \Rightarrow \frac{dy}{dx}=-3ae^{-3x} \quad \text{—①}$$

again diff w. r to x

$$\frac{d^2y}{dx^2}=9ae^{-3x}$$

$$\frac{d^2y}{dx^2}=-3\left(-3ae^{-3x}\right)$$

$$\frac{d^2y}{dx^2}=-3\frac{dy}{dx}$$

$$\Rightarrow \frac{d^2y}{dx^2}+3\frac{dy}{dx}=0$$

∴ given function is a solution of given D.E

⑦ Show that the D.E. representing the family of curves $y^2=2a\left(x+a^{2/3}\right)$, where a is a +ve parameter, is $\left(y^2-2xy\frac{dy}{dx}\right)^3=8\left(y\frac{dy}{dx}\right)^5$.

given curve $y^2 = 2a(x + a^{2/3})$ ——①

diff w.r.to x

$$2y\frac{dy}{dx} = 2a(1+0)$$

$$\boxed{y\frac{dy}{dx} = a}$$

① ⇒ $y^2 = 2y\frac{dy}{dx}\left(x + \left(y\frac{dy}{dx}\right)^{2/3}\right)$

$$y^2 = 2\frac{dy}{dx}xy + 2\left(y\frac{dy}{dx}\right)^{2/3}\left(\frac{dy}{dx}y\right)'$$

$$y^2 - 2xy\frac{dy}{dx} = 2y^{5/3}\left(\frac{dy}{dx}\right)^{5/3}$$

If cancelling y, not get given format answer

cubing both sides

$$\left(y^2 - 2xy\frac{dy}{dx}\right)^3 = 8\left(y\frac{dy}{dx}\right)^5$$

∴ The given family of curve gives the D.E.

8) show that $y = a\cos bx$ is a solution of the D.E. $\frac{d^2y}{dx^2} + b^2y = 0$

$y = a\cos bx$

diff w.r to x

$$\frac{dy}{dx} = a(-\sin bx)b$$

again diff w.r to x

$$\frac{d^2y}{dx^2} = -a\cos bx\,(b^2)$$

$$\frac{d^2y}{dx^2} = -b^2y \Rightarrow \frac{d^2y}{dx^2} + b^2y = 0$$

∴ The given function is a solution of the given D.E.

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Variable separable method

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Exercise 10.5

1) If F is the constant force generated by the motor of an automobile of mass M, its velocity V is given by $M\frac{dV}{dt} = F - KV$ where K is a constant, Express V interms of t given that V=0 when t=0.

$$M\frac{dV}{dt} = F - KV$$

$$\frac{dV}{F-KV} = \frac{1}{M}dt$$

Integrating $\int\frac{dV}{F-KV} = \int\frac{1}{M}dt$

multiply both sides by $-k$

$$\int \frac{-k\,dv}{F-kv} = \frac{-k}{M}\int dt$$

$$\log(F-kv) = -\frac{k}{M}t + c \quad \text{--- (1)}$$

when $t=0$, $v=0$

$$(1) \Rightarrow \log F = 0 + c$$

$$c = \log F$$

$$(1) \Rightarrow \log(F-kv) = -\frac{kt}{M} + \log F$$

$$\frac{kt}{M} = \log F - \log(F-kv)$$

$$\frac{kt}{M} = \log\left(\frac{F}{F-kv}\right)$$

taking antilog $\quad e^{kt/M} = \frac{F}{F-kv}$

$$(F-kv)e^{kt/M} = F$$

(2) The velocity v of a parachute falling vertically satisfies the equation $v\frac{dv}{dx} = g\left(1-\frac{v^2}{k^2}\right)$ where g and k are constants. If v and x are both initially zero, find v interms of x.

$$v\frac{dv}{dx} = g\left(1-\frac{v^2}{k^2}\right)$$

$$\frac{v\,dv}{\left(1-\frac{v^2}{k^2}\right)} = g\,dx$$

multiply both sides by $\frac{-2}{k^2}$

$$\frac{-\frac{2v}{k^2}\,dv}{\left(1-\frac{v^2}{k^2}\right)} = \frac{-2g}{k^2}\,dx$$

Integrating $\quad \int \frac{-\frac{2v}{k^2}\,dv}{(1-v^2/k^2)} = \frac{-2g}{k^2}\int dx$

$$\log\left(1-\frac{v^2}{k^2}\right) = \frac{-2gx}{k^2} + c \quad \text{--- (1)}$$

given $v=0$, $x=0$ $\quad (1) \Rightarrow \log(1-0) = 0 + c$

$$\boxed{c=0}$$

$$(1) \Rightarrow \log\left(1-\frac{v^2}{k^2}\right) = \frac{-2gx}{k^2} + 0$$

taking antilog $\quad 1-\frac{v^2}{k^2} = e^{\frac{-2gx}{k^2}}$

$$1-e^{\frac{-2gx}{k^2}} = \frac{v^2}{k^2} \qquad k^2\left(1-e^{\frac{-2gx}{k^2}}\right) = v^2$$

3) Find the equation of the curve whose slope is $\frac{y-1}{x^2+x}$ and which passes through the point (1,0)

$$\text{slope} = \frac{y-1}{x^2+x}$$

$$\frac{dy}{dx} = \frac{y-1}{x^2+x}$$

$$\frac{dy}{y-1} = \frac{dx}{x^2+x}$$

$$= \frac{1}{x(x+1)}\,dx \quad \text{(using partial fraction)}$$

$$\frac{dy}{y-1} = \left(\frac{1}{x} - \frac{1}{x+1}\right)dx$$

Integrating $\int \frac{dy}{y-1} = \int \frac{dx}{x} - \int \frac{dx}{x+1}$

$$\log y-1 = \log x - \log(x+1) + \log c$$

$$\log(y-1) = \log \frac{xc}{x+1}$$

taking antilog

$$y-1 = \frac{cx}{x+1} \quad \text{---(1)} \quad \text{This passing through } (1,0)$$

$$\therefore 0-1 = \frac{c}{1+1} \qquad \boxed{-2=c}$$

$$(1) \Rightarrow y-1 = \frac{-2x}{x+1}$$

$$y = 1 - \frac{2x}{x+1}$$

$$= \frac{x+1-2x}{x+1}$$

$$y = \frac{1-x}{1+x}$$

4) solve the following differential equations

(i) $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$

$$\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$$

$$\frac{dy}{\sqrt{1-y^2}} = \frac{dx}{\sqrt{1-x^2}}$$

Integrating $\int \frac{dy}{\sqrt{1-y^2}} = \int \frac{dx}{\sqrt{1-x^2}}$ $\quad \sin^{-1} y = \sin^{-1} x + c$

(ii) $y\,dx + (1+x^2)\tan^{-1}x\,dy = 0$

$$y\,dx = -(1+x^2)\tan^{-1}x\,dy$$

$$\frac{dx}{(1+x^2)\tan^{-1}x} = \frac{-dy}{y}$$

take $v = \tan^{-1}x$

$\frac{dv}{dx} = \frac{1}{1+x^2}$

$dv = \frac{dx}{1+x^2}$

$$\frac{dv}{v} = -\frac{dy}{y}$$

Integrating $\int \frac{dv}{v} = -\int \frac{dy}{y}$

$$\log v = -\log y + \log c$$

$\log v + \log y = \log c$

$\log vy = \log c$

taking antilog

$vy = c$

$y \tan^{-1}x = c$

(iii) $\sin\frac{dy}{dx}=a$, $y(0)=1$

$$\sin\frac{dy}{dx}=a$$
$$\frac{dy}{dx}=\sin^{-1}a$$
$$dy=\sin^{-1}a\,dx$$

Integrating $\int dy = \sin^{-1}a\int dx$

$$y=(\sin^{-1}a)x+C \quad \text{--- (1)}$$

$\therefore$ when $x=0$, $y=1$ (1) $\Rightarrow$ $1=\sin^{-1}a\,(0)+C$

$$\boxed{C=1}$$

(1) $\Rightarrow$ $y=(\sin^{-1}a)x+1$

$$\frac{y-1}{x}=\sin^{-1}a$$
$$\sin\left(\frac{y-1}{x}\right)=a$$

(iv) $\frac{dy}{dx}=e^{x+y}+x^3e^y$

$$\frac{dy}{dx}=e^x.e^y+x^3e^y$$
$$\frac{dy}{dx}=(x^3+e^x)e^y$$
$$e^{-y}dy=(x^3+e^x)dx$$

Integrating $\int e^{-y}dy = \int(x^3+e^x)dx$

$$-e^{-y}=\frac{x^4}{4}+e^x+C_1$$
$$e^x+e^{-y}+\frac{x^4}{4}=-C_1$$
$$e^x+e^{-y}+\frac{x^4}{4}=C$$

(v) $(e^y+1)\cos x\,dx+e^y\sin x\,dy=0$

$$(e^y+1)\cos x\,dx=-e^y\sin x\,dy$$
$$\frac{\cos x\,dx}{\sin x}=-\frac{e^y}{e^y+1}dy$$

Integrating $\int\frac{\cos x\,dx}{\sin x}=-\int\frac{e^y}{e^y+1}dy$

$$\log\sin x=-\log(e^y+1)+\log C$$

$$\log\sin x+\log(e^y+1)=\log C$$
$$\log\sin x\,(e^y+1)=\log C$$

taking anti-log $(e^y+1)\sin x=C$

(vi) $(ydx-xdy)\cot\left(\frac{x}{y}\right)=ny^2dx$

$$\frac{ydx-xdy}{y^2}\cot\frac{x}{y}=n\,dx$$

$$\cot\left(\frac{x}{y}\right) d\left(\frac{x}{y}\right) = n\,dx$$

Integrating $\int \cot\left(\frac{x}{y}\right) d\left(\frac{x}{y}\right) = n\int dx$

$$\log \sin\frac{x}{y} = nx+c$$

taking antilog $\sin\left(\frac{x}{y}\right) = e^{nx+c}$

(vii) $\frac{dy}{dx} - x\sqrt{25-x^2} = 0$

$$\frac{dy}{dx} = x\sqrt{25-x^2}$$

$$dy = x\sqrt{25-x^2}\,dx$$

put $25-x^2 = t$

$-2x = \frac{dt}{dx}$

$x\,dx = \frac{dt}{-2}$

$$\therefore dy = \sqrt{t}\left(\frac{dt}{-2}\right)$$

Integrating $+2\int dy = -\int t^{1/2}\,dt$

$$+2y = -\frac{t^{3/2}}{3/2} + 2c$$

multiply bothsides by $\frac{3}{2}$

$$\therefore \frac{3}{2}(2y) = -t^{3/2} + \frac{3}{2}(2c)$$

$$3y = -(25-x^2)^{3/2} + 3c$$

(viii) $x\cos y\,dy = e^x(x\log x+1)\,dx$

$$\cos y\,dy = e^x\,\frac{(x\log x+1)}{x}\,dx$$

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$$\cos y\,dy = e^x\left(\log x+\frac{1}{x}\right)dx$$

$$= \left(e^x\log x + e^x\frac{1}{x}\right)dx$$

$\Rightarrow \cos y\,dy = e^x\log x\,dx + e^x\frac{1}{x}\,dx$

$$= \log x\, d(e^x) + e^x\, d(\log x)$$

$$\cos y\,dy = d(e^x\log x)$$

$\boxed{d(uv) = u\,dv + v\,du}$

Integrating $\int \cos y\,dy = \int d(e^x\log x)$

$$\sin y = e^x\log x + c.$$

(ix) $\tan y\frac{dy}{dx} = \cos(x+y) + \cos(x-y)$

$$\tan y\frac{dy}{dx} = \cos x\cos y - \cancel{\sin x\sin y} + \cos x\cos y + \cancel{\sin x\sin y}$$

$$\tan y\frac{dy}{dx} = 2\cos x\cos y$$

$$\frac{\tan y\,dy}{\cos y} = 2\cos x\,dx$$

$$\sec y\tan y\,dy = 2\cos x\,dx$$

Integrating

$\int \sec y\tan y\,dy = \int 2\cos x\,dx$

$\sec y = 2\sin x + c$

(x) $\frac{dy}{dx} = \tan^2(x+y)$ —— ①

Let $x+y=z$

diff w.r.to x

$$1+\frac{dy}{dx} = \frac{dz}{dx}$$

$$\frac{dy}{dx} = \frac{dz}{dx}-1$$

$$① \Rightarrow \frac{dz}{dx}-1 = \tan^2 z$$

$$\frac{dz}{dx} = 1+\tan^2 z$$

$$\frac{dz}{dx} = \sec^2 z \Rightarrow \frac{dz}{\sec^2 z} = dx.$$

$$\cos^2 z\, dz = dx$$

$$\left(\frac{1+\cos 2z}{2}\right)dz = dx.$$

integrating $\frac{1}{2}\int(1+\cos 2z)\,dz = \int dx$

$$\frac{1}{2}\left[z+\frac{1}{2}\sin 2z\right] = x+C.$$

Replace $z=x+y$

$$\frac{1}{2}\left[(x+y)+\frac{1}{2}\sin 2(x+y)\right] = x+C$$

$$\frac{1}{2}\left[(x+y)+\frac{1}{2}\cancel{2}\sin(x+y)\cos(x+y)\right] = x+C$$

$$\frac{1}{2}\left[(x+y)+\sin(x+y)\cos(x+y)\right] = x+C.$$

Homogeneous Differential Equation

Exercise 10.6

① Solve the following differential equations.

1) $\left[x+y\cos\left(\frac{y}{x}\right)\right]dx = x\cos\left(\frac{y}{x}\right)dy$ $\qquad \frac{x+y\cos\frac{y}{x}}{x\cos\frac{y}{x}} = \frac{dy}{dx}$ —— ①

This is a homogeneous D.E.

put $y=vx$

$\therefore \frac{dy}{dx} = v+x\frac{dv}{dx}$ $\qquad ① \Rightarrow \frac{x+vx\cos\frac{vx}{x}}{x\cos\frac{vx}{x}} = v+x\frac{dv}{dx}.$

$$\frac{\cancel{x}(1+v\cos v)}{\cancel{x}\cos v} = v+x\frac{dv}{dx}$$

$$\frac{1+v\cos v}{\cos v} - v = x\frac{dv}{dx}$$

$$\frac{1+\cancel{v\cos v}-\cancel{v\cos v}}{\cos v} = x\frac{dv}{dx}$$

$$\frac{1}{\cos v} = x\frac{dv}{dx}$$

$$\frac{dx}{x} = \cos v\, dv$$

Integrating $\int \cos v\, dv = -\int \frac{dx}{x}$

when taking $\log c$ use $\log|c|$
~~$\log x$ use $\log|x|$~~

$$\sin v = \log|x| + \log|c|$$

Replace v by $\frac{y}{x}$ $\quad \sin\frac{y}{x} = \log|cx|$

2) $(x^3+y^3)dy - x^2y\,dx = 0$

$$(x^3+y^3)dy = x^2y\,dx$$

$$\frac{dy}{dx} = \frac{x^2y}{x^3+y^3} \quad \text{---} (1)$$

This is a homogeneous D.E

put $y = vx$ $\quad \frac{dy}{dx} = v + x\frac{dv}{dx}$

$$(1) \Rightarrow v + x\frac{dv}{dx} = \frac{x^2vx}{x^3+v^3x^3}$$

$$v + x\frac{dv}{dx} = \frac{\cancel{x^3}v}{\cancel{x^3}(1+v^3)}$$

$$x\frac{dv}{dx} = \frac{v}{1+v^3} - v$$

$$x\frac{dv}{dx} = \frac{\cancel{v} - \cancel{v} - v^4}{1+v^3}$$

$$x\frac{dv}{dx} = \frac{-v^4}{1+v^3}$$

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$$\frac{1+v^3}{v^4}dv = -\frac{dx}{x}$$

$$\left(\frac{1}{v^4} + \frac{1}{v}\right)dv = -\frac{dx}{x}$$

Integrating $\int v^{-4}dv + \int \frac{dv}{v} = -\int\frac{dx}{x}$

$$\frac{v^{-3}}{-3} + \log|v| = -\log|x| + \log|c|$$

$$\log|v| + \log|x| - \log|c| = \frac{1}{3v^3}$$

$$\log\left|\frac{vx}{c}\right| = \frac{1}{3v^3}$$

Replace $v = \frac{y}{x}$. $\quad \log\left|\frac{y}{c}\right| = \frac{1}{\frac{3y^3}{x^3}} \Rightarrow \log\left|\frac{y}{c}\right| = \frac{x^3}{3y^3}$

taking anti loge $\quad \left|\frac{y}{c}\right| = e^{\frac{x^3}{3y^3}}$

$y = \pm c e^{x^3/3y^3}$ thus $\quad \underline{y = Ce^{x^3/3y^3}}$ where $C = \pm c$

(3) $ye^{x/y}dx = (xe^{x/y} + y)dy$

$$\frac{dx}{dy} = \frac{xe^{\frac{x}{y}} + y}{ye^{x/y}} \quad \text{---} (1)$$

This is a Homogeneous D.E.

put $x = vy$ $\quad \frac{dx}{dy} = v + y\frac{dv}{dy}$

$$① \Rightarrow \quad v + y\frac{dv}{dy} = \frac{vy\,e^{vy/y} + y}{y\,e^{vy/y}}$$

$$v + y\frac{dv}{dy} = \frac{\cancel{y}(ve^v + 1)}{\cancel{y}e^v}$$

$$y\frac{dv}{dy} = \frac{ve^v + 1}{e^v} - v$$

$$y\frac{dv}{dy} = \frac{ve^v + 1 - ve^v}{e^v}$$

$$y\frac{dv}{dy} = \frac{1}{e^v}$$

$$e^v dv = \frac{dy}{y}$$

Integrating $\int e^v dv = \int \frac{dy}{y}$

$$e^v = \log|y| + \log|C|$$

$$e^{x/y} = \log|Cy|$$

4) $2xy\,dx + (x^2 + 2y^2)dy = 0$

$$2xy\,dx = -(x^2 + 2y^2)dy$$

$$\frac{dx}{dy} = \frac{-x^2 - 2y^2}{2xy} \quad —①$$

put $x = vy$ $\quad \frac{dx}{dy} = v + y\frac{dv}{dy}$

$$① \Rightarrow \quad v + y\frac{dv}{dy} = \frac{-(v^2y^2 + 2y^2)}{2vy\,y}$$

$$v + y\frac{dv}{dy} = -\frac{\cancel{y^2}(v^2+2)}{2v\cancel{y^2}}$$

$$y\frac{dv}{dy} = \frac{-v^2 - 2}{2v} - v \qquad y\frac{dv}{dy} = \frac{-v^2 - 2 - 2v^2}{2v}$$

$$y\frac{dv}{dy} = \frac{-(3v^2+2)}{2v}$$

$$\frac{2v\,dv}{(3v^2+2)} = -\frac{dy}{y}$$

Integrating $\frac{1}{3}\int \frac{6v\,dv}{(3v^2+2)} = -\int \frac{dy}{y}$

$$\log|3v^2+2| = -3\log|y| + \log|C|$$

$$\log|3v^2+2| + 3\log|y| = \log|C|$$

$$\log|(3v^2+2)y^3| = \log|C|$$

put $v = \frac{x}{y}$ and taking anti log $\quad \left|\left(3\frac{x^2}{y^2} + 2\right)y^3\right| = |C|$

$$|3x^2y + 2y^3| = |C|$$

$$3x^2y + 2y^3 = \pm C$$

$$3x^2y + 2y^3 = C_1$$

(5) $(y^2-2xy)dx = (x^2-2xy)dy$

This is a homogeneous D.E.

$$\frac{dy}{dx} = \frac{y^2-2xy}{x^2-2xy} \quad \text{---(1)}$$

put $y=vx$ $\quad \frac{dy}{dx} = v + x\frac{dv}{dx}$

$$(1) \Rightarrow v + x\frac{dv}{dx} = \frac{v^2x^2-2xvx}{x^2-2xvx}$$

$$v+x\frac{dv}{dx} = \frac{x^2(v^2-2v)}{x^2(1-2v)}$$

$$x\frac{dv}{dx} = \frac{v^2-2v}{1-2v} - v \Rightarrow x\frac{dv}{dx} = \frac{v^2-2v-v+2v^2}{1-2v}$$

$$x\frac{dv}{dx} = \frac{3v^2-3v}{1-2v}$$

$$\frac{1-2v}{3(v^2-v)}dv = \frac{dx}{x}$$

$$\frac{1-2v}{v(v-1)}dv = 3\frac{dx}{x}$$

Using partial fraction

$$\left(-\frac{1}{v-1} - \frac{1}{v}\right)dv = 3\frac{dx}{x}$$

Integrating $\int \frac{dv}{v-1} + \int \frac{dv}{v} = -3\int \frac{dx}{x}$

$$\log|v-1| + \log|v| = -3\log|x| + \log|c|$$

$$\log|v(v-1)| + 3\log|x| = \log|c|$$

$$\log|(v^2-v)| + \log|x^3| = \log|c|$$

$$\log|x^3(v^2-v)| = \log|c|$$

taking antilog $\quad |x^3(v^2-v)| = |c|$

putting $v=\frac{y}{x}$ $\quad \left|x^3\left(\frac{y^2}{x^2}-\frac{y}{x}\right)\right| = |c|$

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$$\frac{x^3y^2}{x^2} - \frac{x^3y}{x} = \pm c$$

$$xy^2 - x^2y = \pm c$$

$$xy^2 - x^2y = C_1 \quad \text{where } C_1 = \pm c$$

(6) $x\frac{dy}{dx} = y - x\cos^2\left(\frac{y}{x}\right)$ ---(1)

This is a Homogeneous D.E

put $y=vx$ $\quad \frac{dy}{dx} = v + x\frac{dv}{dx}$

$$(1) \Rightarrow x\left(v + x\frac{dv}{dx}\right) = vx - x\cdot\cos^2\frac{vx}{x}$$

$$\cancel{vx} + x^2\frac{dv}{dx} = \cancel{vx} - x\cos^2 v$$

$$x^{\cancel{2}}\frac{dv}{dx} = -\cancel{x}\cos^2 v$$

$$\frac{dv}{\cos^2 v} = -\frac{dx}{x}$$

Integrating $\int \sec^2 v \, dv = -\int \frac{dx}{x}$

$$\tan v = -\log|x| + \log|c|$$

$$\tan v = \log\left|\frac{c}{x}\right|$$

$$\tan\frac{y}{x} = \log\left|\frac{c}{x}\right|$$

taking antilog $e^{\tan\frac{y}{x}} = \left|\frac{c}{x}\right|$

$$x e^{\tan\frac{y}{x}} = \pm c$$

$$\therefore x e^{\tan y/x} = c$$

⑦ $(1+3e^{y/x})dy + 3e^{y/x}(1-y/x)dx = 0$, given that $y=0$ when $x=1$

$$(1+3e^{y/x})\,dy = -3e^{y/x}(1-y/x)\,dx$$

$$\frac{dy}{dx} = \frac{-3e^{y/x}(1-y/x)}{1+3e^{y/x}} \quad \text{——} ①$$

This is a Homogeneous D.E.

put $y = vx$ $\quad \frac{dy}{dx} = v + x\frac{dv}{dx}$

$$① \Rightarrow v + x\frac{dv}{dx} = \frac{-3e^{vx/x}(1-vx/x)}{1+3e^{vx/x}}$$

$$v + x\frac{dv}{dx} = \frac{-3e^v(1-v)}{1+3e^v}$$

$$x\frac{dv}{dx} = \frac{-3e^v+3ve^v}{1+3e^v} - v$$

$$= \frac{-3e^v+3ve^v-v-3ve^v}{1+3e^v}$$

$$x\frac{dv}{dx} = \frac{-(3e^v+v)}{(3e^v+1)} \qquad x\,dv = \frac{-(3e^v+v)}{(3e^v+1)}\,dx$$

$$\left(\frac{3e^v+1}{3e^v+v}\right)dv = -\frac{dx}{x}$$

Integrating $\int \frac{3e^v+1}{3e^v+v}\,dv = -\int \frac{dx}{x}$

$$\log|(3e^v+v)| = -\log|x| + \log|c|$$

$$\log|x(3e^v+v)| = \log|c|$$

taking antilog $|3xe^v + vx| = |c|$

Replace v by y/x $\quad |3xe^{y/x}+y|=|c| \Rightarrow 3xe^{y/x}+y=\pm c$

$$3xe^{y/x}+y=c \quad \text{---} (1)$$

when $x=1 \Rightarrow y=0$

$$(1) \Rightarrow 3e^0+0=c$$
$$c=3$$

$$(1) \Rightarrow 3xe^{y/x}+y=3$$

(8) $(x^2+y^2)dy=xy\,dx$, It is given that $y(1)=1$ and $y(x_0)=e$, Find the value of x_0

$$(x^2+y^2)\,dy=xy\,dx$$

$$\frac{dy}{dx}=\frac{xy}{x^2+y^2} \quad \text{---} (1)$$

This is a Homogeneous D.E

put $y=vx \quad \frac{dy}{dx}=v+x\frac{dv}{dx}$

$$(1) \Rightarrow v+x\frac{dv}{dx}=\frac{xvx}{x^2+v^2x^2}$$

$$x\frac{dv}{dx}=\frac{x^2v}{x^2(1+v^2)}-v$$

$$x\frac{dv}{dx}=\frac{\cancel{v}-\cancel{v}-v^3}{1+v^2}$$

$dv\,\frac{1+v^2}{v^3}=-\frac{dx}{x}$

$\left(\frac{1}{v^3}+\frac{v^2}{v^3}\right)dv=-\frac{dx}{x}$ $\quad$ Integrating

$$\int v^{-3}dv+\int\frac{1}{v}dv=-\int\frac{dx}{x}$$

$$\frac{v^{-2}}{-2}+\log|v|=-\log|x|+\log|c|$$

$$\log\left|\frac{vx}{c}\right|=\frac{1}{2v^2}$$

taking antilog, $\left|\frac{vx}{c}\right|=e^{\frac{1}{2v^2}}$

Replacing $v=y/x$ $\quad \left|\frac{y}{c}\right|=e^{\frac{x^2}{2y^2}}$

$$y=\pm c\,e^{x^2/2y^2}$$

$$y=ce^{x^2/2y^2} \quad \text{---} (1)$$

given $x=1, y=1$ $\quad (1) \Rightarrow 1=ce^{1/2}$

$$\boxed{c=e^{-1/2}}$$

$$(1) \Rightarrow y=e^{-1/2}e^{x^2/2y^2} \quad \text{---} (2)$$

$y(x_0)=e \quad y=e \Rightarrow e^1=e^{-1/2}e^{\frac{x_0^2}{2e^2}}$

$e^1e^{1/2}=e^{\frac{x_0^2}{2e^2}}$

$\frac{3}{2}=\frac{x_0^2}{2e^2}$

$\frac{2\times3e^2}{2}=x_0^2$

$x_0^2=3e^2$

$x_0=\pm\sqrt{3}e$

First order Linear Differential equation

I (i) $\frac{dy}{dx}+Py=Q$ This is Linear in y

where P, Q are function of x

(ii) Integrating factor $I.F = e^{\int P dx}$.

(iii) solution is $y(IF)=\int Q(IF)dx+c$

II (i) $\frac{dx}{dy}+Px=Q$ This is linear in x

where P, Q are function of y

(ii) $I.F. = e^{\int P dy}$

(iii) Solution is $x(IF)=\int Q(IF)\,dy+c$.

Exercise 10.7

solve the following Linear differential equations

① $\cos x\frac{dy}{dx}+y\sin x=1$

divide by $\cos x$ $\quad \frac{dy}{dx}+y\frac{\sin x}{\cos x}=\frac{1}{\cos x}$

$P=\frac{\sin x}{\cos x}=\tan x, Q=\frac{1}{\cos x}=\sec x$

$\frac{dy}{dx}+Py=Q$, This is linear in y

$IF = e^{\int P dx}$

$= e^{\int \tan x dx} = e^{\log \sec x} = \sec x$

solution is

$y(IF)=\int Q(IF)dx+c$

$y\sec x=\int \sec x\sec x\,dx+C$

$y\sec x=\int \sec^2 x\,dx+C$

$y\sec x=\tan x+C$

$\frac{y}{\cos x}=\frac{\sin x}{\cos x}+C$

multiply by $\cos x$ $\quad y=\sin x+C\cos x$.

② $(1-x^2)\frac{dy}{dx}-xy=1$

divide by $1-x^2$ $\quad \frac{dy}{dx}-\frac{xy}{1-x^2}=\frac{1}{1-x^2}$

$P=\frac{-x}{1-x^2}\quad Q=\frac{1}{1-x^2}$

$\frac{dy}{dx}+Py=Q$ This is linear in y

$$IF = e^{\int p\,dx}$$
$$= e^{\frac{1}{2}\int \frac{2x}{1-x^2}dx}$$
$$= e^{\frac{1}{2}\log(1-x^2)} = e^{\log(1-x^2)^{1/2}}$$
$$IF = \sqrt{1-x^2}$$

solution is

$$y(IF) = \int Q(IF)\,dx + C$$
$$y\sqrt{1-x^2} = \int \frac{1}{1-x^2}\sqrt{1-x^2}\,dx + C$$
$$y\sqrt{1-x^2} = \int \frac{1}{\sqrt{1-x^2}}\,dx + C$$
$$y\sqrt{1-x^2} = \sin^{-1}x + C$$

③ $\frac{dy}{dx} + \frac{y}{x} = \sin x$

$$P = 1/x \quad , \quad Q = \sin x$$

$\frac{dy}{dx} + Py = Q$ This is linear in y

$$IF = e^{\int p\,dx}$$
$$= e^{\int \frac{1}{x}dx} = e^{\log x}$$
$$IF = x$$

solution is

$$y(IF) = \int Q(IF)\,dx + C$$
$$yx = \int \sin x\,(x)\,dx + C$$
$$xy = \int \underset{u}{x}\, \underset{dv}{\sin x\,dx} + C \qquad u = x \quad dv = \sin x\,dx$$
$$du = dx \quad v = -\cos x$$
$$= x(-\cos x) - \int(-\cos x)\,dx + C \qquad \int u\,dv = uv - \int v\,du$$
$$xy = -x\cos x + \sin x + C.$$
$$xy + x\cos x = \sin x + C$$
$$x(y + \cos x) = \sin x + C.$$

④ $(x^2+1)\frac{dy}{dx} + 2xy = \sqrt{x^2+4}$

divide by x^2+1 $\qquad \frac{dy}{dx} + \frac{2x}{x^2+1}y = \frac{\sqrt{x^2+4}}{x^2+1}$

$$P = \frac{2x}{x^2+1} \qquad Q = \frac{\sqrt{x^2+4}}{x^2+1}$$

$\frac{dy}{dx} + Py = Q$, This is linear in y

$$IF = e^{\int p\,dx} = e^{\int \frac{2x}{x^2+1}dx} = e^{\log(x^2+1)} = x^2+1$$

solution is

$$y(IF) = \int Q(IF)\,dx + C$$
$$y(x^2+1) = \int (x^2+1)\frac{\sqrt{x^2+4}}{x^2+1}\,dx + C$$
$$= \int \sqrt{2^2+x^2}\,dx + C$$

$$y(x^2+1) = \frac{x}{2}\sqrt{2^2+x^2} + \frac{2^4}{2}\log|x+\sqrt{x^2+4}|+C$$

$$y(x^2+1) = \frac{x}{2}\sqrt{x^2+4} + 2\log|x+\sqrt{x^2+4}|+C$$

5) $(2x-10y^3)dy + ydx = 0$

$$ydx = -(2x-10y^3)dy$$

$$y\frac{dx}{dy} = -2x+10y^3$$

$$\frac{dx}{dy} = -\frac{2x}{y} + \frac{10y^{\cancel{3}2}}{\cancel{y}}$$

$$\frac{dx}{dy} + \frac{2x}{y} = 10y^2$$

$$P=\frac{2}{y} \quad Q=10y^2$$

$\frac{dx}{dy}+Px=Q$ This is linear in x

$$IF = e^{\int Pdy} = e^{\int \frac{2}{y}dy} = e^{2\log y} = e^{\log y^2} = y^2$$

solution is $x(IF) = \int Q(IF)dy + C$

$$xy^2 = \int 10y^2(y^2)dy + C$$

$$= 10\int y^4 dy + c$$

$$= \cancel{10}^{2}\frac{y^5}{\cancel{5}} + C$$

$$xy^2 = 2y^5 + C$$

⑥ $x\sin x\frac{dy}{dx} + (x\cos x+\sin x)y = \sin x$

divide by $x\sin x$

$$\frac{dy}{dx} + \frac{x\cos x+\sin x}{x\sin x}y = \frac{\cancel{\sin x}}{x\cancel{\sin x}}$$

$$P = \frac{x\cos x+\sin x}{x\sin x} \quad Q=\frac{1}{x}$$

$\frac{dy}{dx}+Py=Q$ This is linear in y

$$IF = e^{\int Pdx} = e^{\int \frac{x\cos x+\sin x}{x\sin x}dx}$$

$$= e^{\log(x\sin x)} = x\sin x$$

solution is $y(IF) = \int Q(IF)dx + C$

$$y(x\sin x) = \int \frac{1}{\cancel{x}}(\cancel{x})\sin x\,dx + C$$

$$xy\sin x = -\cos x + C$$

⑦ $(y-e^{\sin^{-1}x})\frac{dx}{dy} + \sqrt{1-x^2} = 0$

$$(y-e^{\sin^{-1}x})\frac{dx}{dy} = -\sqrt{1-x^2}$$

$y - e^{\sin^{-1}x} = -\frac{dy}{dx}\sqrt{1-x^2}$

$$\frac{dy}{dx}\sqrt{1-x^2} + y = e^{\sin^{-1}x}$$

divide by $\sqrt{1-x^2}$ $\quad \frac{dy}{dx} + \frac{y}{\sqrt{1-x^2}} = \frac{e^{\sin^{-1}x}}{\sqrt{1-x^2}}$

$$P = \frac{1}{\sqrt{1-x^2}} \qquad Q = \frac{e^{\sin^{-1}x}}{\sqrt{1-x^2}} \qquad \Rightarrow \frac{dy}{dx} + Py = Q$$

This is linear in y

$$IF = e^{\int P dx} = e^{\int \frac{1}{\sqrt{1-x^2}} dx} = e^{\sin^{-1}x}$$

solution is $\quad y(IF) = \int Q(IF)dx + C$

$$y\,e^{\sin^{-1}x} = \int \frac{e^{\sin^{-1}x}}{\sqrt{1-x^2}} e^{\sin^{-1}x} dx + C$$

put $\sin^{-1}x = t$

$\frac{1}{\sqrt{1-x^2}} dx = dt$

$$= \int e^t e^t dt + C$$

$$= \int e^{2t} dt + C$$

$$y\,e^{\sin^{-1}x} = \frac{e^{2t}}{2} + C$$

$$y\,e^{\sin^{-1}x} = \frac{e^{2\sin^{-1}x}}{2} + C$$

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(8) $\frac{dy}{dx} + \frac{y}{(1-x)\sqrt{x}} = 1-\sqrt{x}$

$$P = \frac{1}{(1-x)\sqrt{x}} \qquad Q = 1-\sqrt{x} \qquad \Rightarrow \frac{dy}{dx} + Py = Q$$

This is linear in y

$$IF = e^{\int P dx} = e^{\int \frac{1}{(1-x)\sqrt{x}} dx}$$

put
$\sqrt{x} = t$
$\frac{1}{2\sqrt{x}} dx = dt$
$\frac{dx}{\sqrt{x}} = 2dt$

$$= e^{\int \frac{2dt}{1-t^2}} = e^{\int \frac{2dt}{(1+t)(1-t)}}$$

$$= e^{\int \left(\frac{1}{1+t} + \frac{1}{1-t}\right)dt} = e^{\log(1+t) - \log(1-t)}$$

$$e^{\log\left(\frac{1+t}{1-t}\right)} = e^{\log\left(\frac{1+\sqrt{x}}{1-\sqrt{x}}\right)} = \frac{1+\sqrt{x}}{1-\sqrt{x}}$$

$$IF = \frac{1+\sqrt{x}}{1-\sqrt{x}}$$

solution is $\quad y(IF) = \int Q(IF)dx + C$

$$y\left(\frac{1+\sqrt{x}}{1-\sqrt{x}}\right) = \int \frac{1+\sqrt{x}}{(1-\sqrt{x})}(1-\sqrt{x})dx + C$$

$$= x + \frac{x^{3/2}}{3/2} + C$$

$$y\left(\frac{1+\sqrt{x}}{1-\sqrt{x}}\right) = x + \frac{2}{3}x\sqrt{x} + C$$

9) $(1+x+xy^2)\frac{dy}{dx}+(y+y^3)=0$

$$1+x+xy^2\frac{dy}{dx}=-(y+y^3)$$

$$1+x(1+y^2)=-y(1+y^2)\frac{dx}{dy}$$

$$y(1+y^2)\frac{dx}{dy}+x(1+y^2)=-1$$

divide by $(1+y^2)y$ $\quad \frac{dx}{dy}+\frac{x}{y}=\frac{-1}{y(1+y^2)}$

This is linear in x

$$P=\frac{1}{y} \quad Q=\frac{-1}{y(1+y^2)}$$

$$IF=e^{\int Pdy}=e^{\int \frac{1}{y}dy}=e^{\log y}=y$$

solution is $\quad x(IF)=\int Q(IF)dy+c$

$$xy=\int \frac{-1}{y(1+y^2)}y\,dy+c$$

$$xy=-\tan^{-1}y+c$$

$$xy+\tan^{-1}y=c$$

10) $\frac{dy}{dx}+\frac{y}{x\log x}=\frac{\sin 2x}{\log x}$

$$P=\frac{1}{x\log x} \quad Q=\frac{\sin 2x}{\log x} \Rightarrow \frac{dy}{dx}+Py=Q$$

This is linear in y

$$IF=e^{\int Pdx}=e^{\int \frac{1/x}{\log x}dx}=e^{\log\log x}$$

$$IF=\log x$$

solution is $\quad y(IF)=\int Q\, IF\,dx+c$

$$y\log x=\int \frac{\sin 2x}{\log x}\log x\,dx+c$$

$$y\log x=-\frac{\cos 2x}{2}+c$$

$$y\log x+\frac{\cos 2x}{2}=c$$

11) $(x+a)\frac{dy}{dx}-2y=(x+a)^4$

divide by $x+a$ $\quad \frac{dy}{dx}-\frac{2y}{x+a}=(x+a)^3$

$$P=\frac{-2}{x+a} \quad Q=(x+a)^3 \Rightarrow \frac{dy}{dx}+Py=Q$$

This is linear in y $\quad IF=e^{\int Pdx}=e^{-\int \frac{2}{x+a}dx}$

$$=e^{-2\log(x+a)}=e^{\log(x+a)^{-2}}$$

$$=(x+a)^{-2}$$

$IF = \frac{1}{(x+a)^2}$

solution is $y(IF) = \int Q(IF)dx + C$

$y\left(\frac{1}{(x+a)^2}\right) = \int \frac{1}{(x+a)^2}(x+a)^4 dx + C$

$y\frac{1}{(x+a)^2} = \frac{(x+a)^2}{2} + C$

$2y = (x+a)^4 + 2C(x+a)^2$

(12) $\frac{dy}{dx} = \frac{\sin^2 x}{1+x^3} - \frac{3x^2}{1+x^3}y$

$\frac{dy}{dx} + \frac{3x^2}{1+x^3}y = \frac{\sin^2 x}{1+x^3}$

$P = \frac{3x^2}{1+x^3}$, $Q = \frac{\sin^2 x}{1+x^3}$ $\Rightarrow \frac{dy}{dx} + Py = Q$

This is linear in y

$IF = e^{\int pdx} = e^{\int \frac{3x^2}{1+x^3}dy} = e^{\log(1+x^3)} = (1+x^3)$

solution is

$y(IF) = \int Q(IF)dx + C$

$y(1+x^3) = \int \frac{\sin^2 x}{1+x^3}(1+x^3)dx + C$

$y(1+x^3) = \int\left(\frac{1}{2} - \frac{\cos 2x}{2}\right)dx + C$

$y(1+x^3) = \frac{x}{2} - \frac{1}{4}\sin 2x + C$

13) $x\frac{dy}{dx} + y = x\log x$

divide by x $\quad \frac{dy}{dx} + \frac{y}{x} = \log x$

$P = \frac{1}{x}$ $\quad Q = \log x \Rightarrow \frac{dy}{dx} + Py = Q$

This is linear in y

$IF = e^{\int pdx} = e^{\int \frac{1}{x}dx} = e^{\log x} = x$

$y(IF) = \int Q(IF)dx + C$

$yx = \int x\log x\, dx + C$ $\qquad u = \log x \quad dv = xdx$

$xy = \int \underset{u}{\log x}\, \underset{dv}{xdx} + C$ $\qquad du = \frac{1}{x}dx \quad v = \frac{x^2}{2}$

$\int udv = uv - \int vdu$

$xy = \frac{x^2}{2}\log x - \int \frac{x^2}{2}\frac{1}{x}dx + C$

$xy = \frac{x^2\log x}{2} - \frac{x^2}{4} + C$

$4xy = 2x^2\log x - x^2 + 4C$

14) $x\frac{dy}{dx}+2y-x^2\log x=0$

$$x\frac{dy}{dx}+2y=x^2\log x$$

divide by x $\quad \frac{dy}{dx}+\frac{2y}{x}=x\log x$

$$P=\frac{2}{x} \quad Q=x\log x \Rightarrow \frac{dy}{dx}+Py=Q$$

This is linear in y

$$IF=e^{\int Pdx}=e^{\int \frac{2}{x}dx}=e^{2\log x}=e^{\log x^2}$$

$$IF=x^2$$

solution is $\quad y(IF)=\int Q(IF)dx+C$

$$yx^2=\int x\log x\, x^2dx+C$$

$$=\int \underbrace{\log x}_{u}\, \underbrace{x^3dx}_{dv}+C$$

$u=\log x \quad | \quad dv=x^3dx$

$du=\frac{1}{x}dx \quad v=\frac{x^4}{4}$

$\int udv=uv-\int vdu$

$$=\frac{x^4}{4}\log x-\int \frac{x^{4}}{4}\cdot\frac{1}{x}dx+C$$

$$=\frac{x^4\log x}{4}-\frac{1}{4}\frac{x^4}{4}+C$$

$$yx^2=\frac{x^4\log x}{4}-\frac{x^4}{16}+C.$$

15) $\frac{dy}{dx}+\frac{3y}{x}=\frac{1}{x^2}$, given that $y=2$ when $x=1$

$$P=\frac{3}{x} \quad Q=\frac{1}{x^2}$$

$\frac{dy}{dx}+Py=Q$, This is linear in y

$$IF=e^{\int Pdx}=e^{\int \frac{3}{x}dx}=e^{3\log x}=e^{\log x^3}$$

$$IF=x^3$$

solution is $\quad y(IF)=\int Q(IF)dx+C$

$$yx^3=\int \frac{1}{x^2}x^3dx+C$$

$$yx^3=\int x dx+C$$

$$yx^3=\frac{x^2}{2}+C \quad \text{——} (1)$$

given $y=2$ when $x=1$ $\quad (1)\Rightarrow 2(1)^3=\frac{1^2}{2}+C$

$$2-\frac{1}{2}=C$$

$$C=3/2$$

$$(1)\Rightarrow yx^3=\frac{x^2}{2}+\frac{3}{2}$$

$$2yx^3=x^2+3$$

Applications of First order ODE

Exercise 10.8

① The rate of increase in the number of bacteria in a certain culture is proportional to the number present. Given that the number triples in 5 hours, find how many bacteria will be present after 10 hours.

Let A be the number of bacteria at any time t

$$\frac{dA}{dt} \propto A$$

$$\frac{dA}{dt} = kA$$

$$A = ce^{kt} \quad \text{——} ①$$

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(i) when $t=0$ $A = A_0$

$① \Rightarrow \quad A_0 = ce^0 \qquad \boxed{c = A_0}$

$① \Rightarrow \quad A = A_0 e^{kt} \text{——} ②$

(ii) when $t=5$ $A = 3A_0$

$② \Rightarrow \quad 3A_0 = e^{5k}(A_0) \qquad \boxed{e^{5k} = 3}$

(iii) when $t=10$, $A = ?$

$② \Rightarrow \quad A = A_0 e^{10k}$

$= A_0 (e^{5k})^2 = A_0 (3)^2$

$= 9A_0$

After 10 hours $A = 9$ times the (initial value) A_0

② Find the population of a city at any time t, given that the rate of increase of population is proportional to the population at that instant and that in a period of 40 years the population increased from 3,00,000 to 4,00,000

Let A be the population of city at any time t

$$\frac{dA}{dt} \propto A$$

$$\frac{dA}{dt} = kA$$

$$A = ce^{kt} \text{——} ①$$

when $t=0$, $A = 3,00,000$

$$3,00,000 = ce^0$$

$$\boxed{c = 3,00,000}$$

$① \Rightarrow \quad A = 3,00,000 e^{kt} \text{——} ②$

when $t=40$ $A = 4,00,000$

$② \Rightarrow \quad 4,00,000 = 3,00,000 e^{40k}$

$$\frac{4}{3} = e^{40k}$$

$$e^{40k} = \frac{4}{3}$$

$$(e^k)^{40} = \frac{4}{3} \qquad e^k = \left(\frac{4}{3}\right)^{\frac{1}{40}}$$

$$② \Rightarrow \quad A = 3{,}00{,}000(e^k)^t$$

$$A = 3{,}00{,}000\left(\frac{4}{3}\right)^{\frac{t}{40}}$$

③ The equation of electromotive force for an electric circuit containing resistance and self-inductance is $E = Ri + L\frac{di}{dt}$ where E is the Electromotive force is given to the circuit R is the Resistance and L-coefficient of induction. Find the current i at time t when $E = 0$.

given $\quad L\frac{di}{dt} + Ri = E$

divide by L $\quad \frac{di}{dt} + \frac{Ri}{L} = \frac{E}{L}$

$$P = \frac{R}{L}, \quad Q = \frac{E}{L} \Rightarrow \frac{di}{dt} + Pi = Q$$

This is Linear in i

$$IF = e^{\int P dt} = e^{\int \frac{R}{L} dt} = e^{\frac{Rt}{L}}$$

solution is

$$i(IF) = \int (Q \cdot IF)\, dt + C$$

$$i e^{\frac{Rt}{L}} = \int \frac{E}{L} e^{\frac{Rt}{L}} dt + C$$

$$i e^{\frac{Rt}{L}} = \frac{E}{L} \frac{e^{\frac{Rt}{L}}}{\frac{R}{L}} + C$$

$$i e^{\frac{Rt}{L}} = \frac{E}{R} e^{\frac{Rt}{L}} + C \quad \text{——} ①$$

when $E = 0 \Rightarrow$

$$i e^{\frac{Rt}{L}} = 0 + C$$

$$i e^{Rt/L} = C$$

$$i = C e^{-\frac{Rt}{L}}$$

④ The engine of a motor boat moving at 10 m/s is shut off. Given that the retardation at any subsequent time (after shutting off the engine) equal to the velocity at that time. Find the velocity after 2 seconds of switching off the engine.

Let v be the velocity at any time t

given retardation = velocity

$$-\frac{dv}{dt} = v$$

$$\frac{dv}{v} = -dt$$
$$\log v = -t + \log c$$
$$\log v/c = -t$$
$$\frac{v}{c} = e^{-t} \quad \boxed{v = ce^{-t}} \text{ --- (1)}$$

when $t=0$, $v=10$ (1) ⇒

$$10 = ce^{0} \quad \boxed{c=10}$$
$$v = 10e^{-t} \text{ --- (2)}$$

when $t=2$ (2) ⇒ $v = 10e^{-2}$

$$v = \frac{10}{e^2}$$

(7) water at temperature 100°C cools in 10 minutes 80°C in a room temperature of 25°C Find
(i) The temperature of water after 20 minutes
(ii) The time when the temperature is 40°C

$\log\left(\frac{11}{15}\right) = -0.3101$, $\log 5 = 1.6094$

→ Let T be temperature of water at any time t
S be the room temp.
S=25

$$\frac{dT}{dt} \propto T - S$$
$$\frac{dT}{dt} = k(T-S)$$
$$T - S = ce^{kt} \quad \boxed{T - 25 = ce^{kt}} \text{ --- (1)}$$

(i) when $t=0$ $T=100$

(1) ⇒ $100 - 25 = ce^{0}$ $c = 75°$

(1) ⇒ $\boxed{T - 25 = 75e^{kt}}$ --- (2)

(2). when $t=10$, $T=80$

(2) ⇒ $80 - 25 = 75e^{10k}$ $\boxed{\frac{55}{75} = e^{10k}}$ --- (3)

(i) when $t=20$ (2) ⇒

$$T - 25 = 75e^{20k}$$
$$= 75(e^{10k})^2$$
$$= 75 \times \frac{55}{75} \times \frac{55}{75}$$
$$T - 25 = \frac{121}{3}$$
$$T = 25 + 40.33$$
$$\boxed{T = 65.33°C}$$

(ii) $T=40$ (3) ⇒

$$\frac{11}{15} = e^{10k}$$
$$\log\frac{11}{15} = 10k \qquad -0.3101 = 10k$$
$$\frac{-0.3101}{10} = k \qquad \boxed{k = -0.03101}$$

(ii) $T=40$ ② ⇒ $40-25=75e^{kt}$

$15=75e^{kt}$

$e^{-kt}=\frac{75}{15}=5$

$-kt=\log 5$

$+0.03101\,t=1.6094$

$t=\frac{1.6094}{0.03101}\times\frac{10000}{10000}=\frac{16094}{310.1}$

$t=51.89$ minutes.

$310)\overline{16094}(51$
1550
0594

—x—

⑧ At 10.00 A.M a woman took a cup of hot instant coffee from her microwave oven and placed it on nearby kitchen counter to cool. At that instant $T=180°F$ and 10 minutes later it was 160°F. Assume that kitchen Temp 70°F

(i) what was the temperature of the coffee at 10.15 AM

(ii) The woman likes to drink coffee its temperature of the coffee 130°F – 140°F what time she drunk the coffee.

Let T be the temp of coffee at time t.

$S=70$

$\frac{dT}{dt}\propto T-S$

$\frac{dT}{dt}=k(T-S)$

$T-S=Ce^{kt}$ $\quad \boxed{T-70=Ce^{kt}}$ —①

when $t=0$ $T=180$ ① ⇒ $180-70=Ce^{0}$

$C=110$

$\boxed{T-70=110\,e^{kt}}$ —②

when $t=10$ $T=160°$ ② ⇒ $160-70=110e^{10k}$

$\boxed{\frac{9\cancel{0}}{11\cancel{0}}=e^{10k}}$ —③

not given

$\boxed{\sqrt{\frac{9}{11}}=0.9045}$

i) when $t=15$ $T=?$ ② ⇒ $T-70=110e^{15k}$

$T-70=110\,e^{3/2\,k}$

$=110\,(e^{10k})^{3/2}$

$=110\cdot\frac{90}{110}\left(\frac{90}{110}\right)^{1/2}$

$=90\,(0.9045)$

$T-70=81.4$

$T=70+81.4=151.4°F$

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(ii) To find k

$$③ \Rightarrow \frac{9}{11} = e^{10k}$$

$$\log \frac{9}{11} = 10k$$

$$\frac{-0.2006}{10} = k \qquad \boxed{k = -0.02006}$$

$T = 130^\circ \quad t = ?$

$$② \Rightarrow \quad 130 - 70 = 110e^{kt}$$

$$\frac{60}{110} = e^{kt}$$

$$\log \frac{6}{11} = e^{-0.02006t}$$

$$\cancel{-}0.606135 = \cancel{-}0.02006t$$

$$\frac{0.606135}{0.02006} = t$$

$$t = 30.216 \text{ min}$$

$T = 140^\circ F \quad t = ?$

$$② \Rightarrow \quad 140 - 70 = 110e^{kt}$$

$$\frac{7}{11} = e^{kt}$$

$$\log \frac{7}{11} = kt$$

$$\cancel{-}0.45198 = \cancel{-}0.02006t$$

$$\frac{0.452}{0.02006} = t$$

$$t = 22.53 \text{ min.}$$

tem 130-140 comes between 10.22 - 10.30 AM approx.

(9) A pot of boiling water at 100°c is removed from a stove at time t=0 and left to cool in the kitchen, After 5 min the water tem 80°c, and another 5 minutes later it has dropped to 65°c Determine the temperature of the kitchen.

Let T be the temperature of boiling water at time t

S be the room (kitchen) Temp.

$$\frac{dT}{dt} \propto T - S$$

$$\frac{dT}{dt} = k(T - S)$$

$$T - S = ce^{kt} \quad \text{——} ①$$

when $t=0, T=100$ $\qquad 100 - S = ce^{0} \qquad \boxed{c = 100 - S}$

$$T-S=(100-S)e^{kt} \quad \text{——} ①$$

when $t=5$ $T=80^\circ$

$$① \Rightarrow \quad 80-S=(100-S)e^{5k}$$

$$\boxed{e^{5k}=\frac{80-S}{100-S}}$$

when $t=10$ $T=65$

$$① \Rightarrow \quad 65-S=(100-S)e^{10k}$$

$$=(100-S)(e^{5k})^2$$

$$65-S=\cancel{(100-S)}\frac{(80-S)\times(80-S)}{\cancel{(100-S)}(100-S)}$$

$$6500-165S+\cancel{S^2}=6400-160S+\cancel{S^2}$$

$$6500-6400=165S-160S$$

$$5S=100$$

$$S=20^\circ C.$$

Kitchen Temperature $=20^\circ C.$

(10) A tank initially contains 50 litres of pure water. Starting at time $t=0$ a brine containing with 2 grams of dissolved salt per litre flows into the tank at the rate of 3 litres per minute. The mixture is kept uniform by stirring and the well stirred mixture simultaneously flows out of the tank at the same rate. Find the amount of salt present in the tank at any time $t>0$.

Let $x(t)$ denote the amount of salt in the tank at time t.

$$\frac{dx}{dt}=\text{inflow rate}-\text{out flow}$$

inflow rate $= 2\times3=6$

out flow $=\frac{3}{50}x$

$$\frac{dx}{dt}=6-\frac{3}{50}x$$

$$=\frac{-3}{50}\left(x-\cancel{6}^{2}\times\frac{50}{\cancel{3}}\right)$$

$$\frac{dx}{dt}=\frac{-3}{50}(x-100)$$

$$\frac{dx}{x-100}=\frac{-3}{50}dt \qquad x-100=Ce^{-\frac{3t}{50}} \quad \text{——} ①$$

when $t=0$ $x=0$

$$① \Rightarrow \quad 0-100=Ce^{0} \qquad C=-100$$

$$① \Rightarrow \quad x-100=-100e^{-3t/50}$$

$$x=100-100e^{-\frac{3t}{50}}$$

$$x=100(1-e^{-3t/50})$$

Give your suggestions
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—— X —————— X ——

(5) Suppose a person deposits 10,000 Indian rupees in a bank account at the rate of 5% per annum compounded continuously. How much money will be in his bank account 18 months later?

Let A be the Amount at any time t.

$\frac{dA}{dt} \propto A$

$\frac{dA}{dt} = 0.05A$

$A = ce^{+0.05t}$ —— (1)

$t=0,\ A=10,000$ (1) $\Rightarrow$ $10,000 = ce^{0}$

$\boxed{c=10,000}$ $\boxed{A=10,000e^{0.05t}}$ —— (2)

$t = 1\frac{1}{2} = \frac{3}{2}$

(2) $\Rightarrow$ $A = 10,000e^{0.05\times\frac{3}{2}}$

$\boxed{A = 10,000e^{0.075}}$

(6) Assume that the rate at which radioactive nuclei decay is proportional to the number of such nuclei that are present in a given sample. In a certain sample 10% of the original number of radioactive nuclei have undergone disintegration in a period of 100 years. What percentage of the original radioactive nuclei will remain after 1000 years.

Let A be the Amount of nuclei at any time t

$\frac{dA}{dt} \propto A$

$\frac{dA}{dt} = kA$

$A = ce^{kt}$ —— (1)

Give your suggestions
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(i) when $t=0$, $A=A_0$ $\Rightarrow$ $A_0 = ce^{0}$ $\boxed{C=A_0}$

$\boxed{A = A_0e^{kt}}$ —— (2)

(ii) when $t=100$ $A = 90\% A_0$

$A = \frac{9}{10}A_0$

(2) $\Rightarrow$ $\frac{9}{10}A_0 = A_0e^{100k}$ $\boxed{e^{100k} = \frac{9}{10}}$

(iii) when $t=1000$

(2) $\Rightarrow$ $A = A_0e^{1000k}$

$= A_0(e^{100k})^{10}$

$= A_0\left(\frac{9}{10}\right)^{10}$

$A = A_0\frac{9^{10}}{10^{10}}\times 100\%$

$A = A_0\left(\frac{9^{10}}{10^{8}}\%\right)$