Chapter 7: Limits

EXERCISE 7.1 [PAGE 100]

Exercise 7.1 | Q 1.1 | Page 100

Evaluate the following limits: $\lim_{x \to 3}\left[\frac{\sqrt{x+6}}{x}\right]$

SOLUTION

$$\lim_{x \to 3}\left[\frac{\sqrt{x+6}}{x}\right]$$

$$= \frac{\lim_{x \to 3}\sqrt{x+6}}{\lim_{x \to 3} x}$$

$$= \frac{\sqrt{3+6}}{3}$$

$$= \frac{\sqrt{9}}{3}$$

$$= \frac{3}{3}$$

= 1.

Exercise 7.1 | Q 1.2 | Page 100

Evaluate the following limits: $\lim_{x \to 2}\left[\frac{x^{-3} - 2^{-3}}{x - 2}\right]$

SOLUTION

$$\lim_{x \to 2}\left[\frac{x^{-3} - 2^{-3}}{x - 2}\right]$$

$$= (-3).(2)^{-4} \quad \ldots\left[\lim_{x \to \mathrm{a}}\frac{x^{\mathrm{n}} - \mathrm{a}^{\mathrm{n}}}{x - \mathrm{a}} = \mathrm{na}^{\mathrm{n}-1}\right]$$

$= -3 \times \frac{1}{2^4}$

$= \frac{-3}{16}$

Exercise 7.1 | Q 1.3 | Page 100

Evaluate the following limits: $\lim_{x \to 5} \left[\frac{x^3 - 125}{x^5 - 3125}\right]$

SOLUTION

$$\lim_{x \to 5} \left[\frac{x^3 - 125}{x^5 - 3125}\right]$$

$$= \lim_{x \to 5} \frac{\left(\frac{x^3 - 5^3}{x - 5}\right)}{\left(\frac{x^5 - 5^5}{x - 5}\right)} \quad \ldots \left[\begin{array}{c}\because x \to 5 \therefore x \neq 5 \\ \therefore x - 5 \neq 0\end{array}\right]$$

$$= \frac{\lim_{x \to 5} \frac{x^3 - 5^3}{x - 5}}{\lim_{x \to 5} \frac{x^5 - 5^5}{x - 5}}$$

$$= \frac{3(5)^2}{5(5)^4} \quad \ldots \left[\because \lim_{x \to \mathrm{a}} \frac{x^{\mathrm{n}} - \mathrm{a}^{\mathrm{n}}}{x - \mathrm{a}} = \mathrm{n} \cdot \mathrm{a}^{\mathrm{n}-1}\right]$$

$$= \frac{3}{(5)^3}$$

$$= \frac{3}{125}.$$

Exercise 7.1 | Q 1.4 | Page 100

Evaluate the following limits: if $\lim_{x \to 1} \left[\frac{x^4 - 1}{x - 1}\right] = \lim_{x \to \mathrm{a}} \left[\frac{x^3 - \mathrm{a}^3}{x - \mathrm{a}}\right]$, find all the value of a.

SOLUTION

$$\lim_{x\to 1}\left[\frac{x^4-1}{x-1}\right] = \lim_{x\to a}\left[\frac{x^3-a^3}{x-a}\right]$$

$$\lim_{x\to 1}\frac{x^4-(1)^4}{x-1} = \lim_{x\to a}\frac{x^3-a^3}{x-a}$$

$$\therefore 4(1)^3 = 3a^2 \quad \ldots\left[\because \lim_{x\to a}\frac{x^n-a^n}{x-a} = na^{n-1}\right]$$

$\therefore 3a^2 = 4$

$\therefore a^2 = \dfrac{4}{3}$

$\therefore a = \pm\dfrac{2}{\sqrt{3}}$.

Exercise 7.1 | Q 2.1 | Page 100

Evaluate the following limits: $\lim_{x\to 7}\left[\dfrac{\left(\sqrt[3]{x}-\sqrt[3]{7}\right)\left(\sqrt[3]{x}+\sqrt[3]{7}\right)}{x-7}\right]$

SOLUTION

$$\lim_{x\to 7}\left[\frac{\left(\sqrt[3]{x}-\sqrt[3]{7}\right)\left(\sqrt[3]{x}+\sqrt[3]{7}\right)}{x-7}\right]$$

$$= \lim_{x\to 7}\left[\frac{\left(x^{\frac{1}{3}}-7^{\frac{1}{3}}\right)\left(x^{\frac{1}{3}}+7^{\frac{1}{3}}\right)}{x-7}\right]$$

$$= \lim_{x\to 7}\left[\frac{x^{\frac{2}{3}}7^{\frac{2}{3}}}{x-7}\right] \quad \ldots[\because (a-b)(a+b) = a^2-b^2]$$

$$= \frac{2}{7}(7)^{\frac{-1}{3}} \quad \ldots\left[\therefore \lim_{x\to a}\frac{x^n-a^n}{x-a} = na^{n-1}\right]$$

$= \frac{2}{3} \cdot \frac{1}{7^{\frac{1}{3}}}$

$= \frac{2}{3\sqrt[3]{7}}.$

Exercise 7.1 | Q 2.2 | Page 100

Evaluate the following limits: if $\lim_{x \to 5} \left[\frac{x^{\mathrm{k}} - 5^{\mathrm{k}}}{x - 5}\right]$ = 500, find all possible values of k.

SOLUTION

$\lim_{x \to 5} \left[\frac{x^{\mathrm{k}} - 5^{\mathrm{k}}}{x - 5}\right]$ = 500

$\therefore \mathrm{k}(5)^{\mathrm{k}-1} = 500 \quad \ldots \left[\because \lim_{x \to \mathrm{a}} \frac{x^{\mathrm{n}} - \mathrm{a}^{\mathrm{n}}}{x - \mathrm{a}} = \mathrm{na}^{\mathrm{n}-1}\right]$

$\therefore$ k$(5)^{k-1}$ = 4 x 125

$\therefore$ k$(5)^{k-1}$ = 4 x $(5)^3$

$\therefore$ k$(5)^{k-1}$ = 4 x $(5)^{4-1}$

Comparing both sides, we get

k = 4.

Exercise 7.1 | Q 2.3 | Page 100

Evaluate the following limits: $\lim_{x \to 0} \left[\frac{(1 - x)^8 - 1}{(1 - x)^2 - 1}\right]$

SOLUTION

$\lim_{x \to 0} \frac{(1 - x)^8 - 1}{(1 - x)^2 - 1}$

Put 1 – x = y

As x → 0, y → 1

$\therefore \lim_{x \to 0} \frac{(1-x)^8 - 1}{(1-x)^2 - 1}$

$= \lim_{y \to 1} \frac{y^8 - 1^8}{y^2 - 1^2}$

$= \lim_{y \to 1} \frac{\frac{y^8 - 1^8}{y-1}}{\frac{y^2 - 1^2}{y-1}} \quad \ldots \begin{bmatrix} \because y \to 1 \therefore y \neq 1 \\ \therefore y - 1 \neq 0 \end{bmatrix}$

$= \frac{\lim_{y \to 1} \frac{y^8 - 1^8}{y-1}}{\lim_{y \to 1} \frac{y^2 - 1^2}{y-1}}$

$= \frac{8(1)^7}{2(1)^1} \quad \ldots \left[\because \lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}\right]$

= 4

Alternative method:

$\lim_{x \to 0} \frac{(1-x)^8 - 1}{(1-x)^2 - 1}$

Put 1 – x = y

As x → 0, y → 1

$\therefore \lim_{x \to 0} \frac{(1-x)^8 - 1}{(1-x)^2 - 1}$

$= \lim_{y \to 1} \frac{y^8 - 1}{y^2 - 1}$

$= \lim_{y \to 1} \frac{(y^4 - 1)(y^4 + 1)}{y^2 - 1}$

$= \lim_{y \to 1} \frac{(y^2 - 1)(y^2 + 1)(y^4 + 1)}{y^2 - 1}$

$= \lim_{y\to1} \left(y^2+1\right)\left(y^4+1\right) \quad \ldots \begin{bmatrix} \because y\to1 \therefore y\neq1 \\ \therefore y^2\neq1 \\ \therefore y^2-1\neq0 \end{bmatrix}$

= (2) (2)

= 4.

Exercise 7.1 | Q 3.1 | Page 100

Evaluate the following limits: $\lim_{x\to0}\left[\frac{\sqrt[3]{1+x}-\sqrt{1+x}}{x}\right]$

SOLUTION

$$\lim_{x\to0}\frac{\sqrt[3]{1+x}-\sqrt{1+x}}{x}$$

$$= \lim_{x\to0}\frac{(1+x)^{\frac{1}{3}}-(1+x)^{\frac{1}{2}}}{x}$$

Put 1 + x = y

As x → 0, y → 1

$$\therefore \lim_{x\to0}\frac{(1+x)^{\frac{1}{3}}-(1+x)^{\frac{1}{2}}}{x}$$

$$= \lim_{y\to1}\frac{y^{\frac{1}{3}}-y^{\frac{1}{2}}}{y-1}$$

$$= \lim_{y\to1}\frac{\left(y^{\frac{1}{3}}-1\right)-\left(y^{\frac{1}{2}}-1\right)}{y-1}$$

$$= \lim_{y\to1}\left(\frac{y^{\frac{1}{3}}-1}{y-1}-\frac{y^{\frac{1}{2}}-1}{y-1}\right)$$

$$= \lim_{y\to1}\frac{y^{\frac{1}{3}}-1^{\frac{1}{3}}}{y-1}-\lim_{y\to1}\frac{y^{\frac{1}{2}}-1^{\frac{1}{2}}}{y-1}$$

$= \lim_{y\to1} \frac{y^{\frac{1}{3}} - 1^{\frac{1}{3}}}{y-1} - \lim_{y\to1} \frac{y^{\frac{1}{2}} - 1^{\frac{1}{2}}}{y-1}$

$= \frac{1}{3}(1)^{\frac{-2}{3}} - \frac{1}{2}(1)^{\frac{-1}{2}}$ ... $\left[\because \lim_{x\to a} \frac{x^n - a^n}{x-a} = na^{n-1}\right]$

$= \frac{1}{3} - \frac{1}{2}$

$= \frac{2-3}{6}$

$= -\frac{1}{6}.$

Exercise 7.1 | Q 3.2 | Page 100

Evaluate the following limits: $\lim_{y\to1}\left[\frac{2y-2}{\sqrt[3]{7+y}-2}\right]$

SOLUTION

$\lim_{y\to1} \frac{2y-2}{\sqrt[3]{7+y}-2}$

$= \lim_{y\to1} \frac{2(y-1)}{(7+y)^{\frac{1}{3}} - 8^{\frac{1}{3}}}$ ... $\left[\because 2 = \left(2^3\right)^{\frac{1}{3}} = 8^{\frac{1}{3}}\right]$

$= \lim_{y\to1} \frac{2}{\frac{(y+7)^{\frac{1}{3}} - 8^{\frac{1}{3}}}{y-1}}$

$= \frac{\lim_{y\to1} 2}{\lim_{y\to1} \frac{(y+7)^{\frac{1}{3}} - 8^{\frac{1}{3}}}{(y+7)-8}}$

Let y + 7 = x

As y → 1, x → 8

$= \dfrac{2}{\lim\limits_{x \to 8} \frac{x^{\frac{1}{3}} - 8^{\frac{1}{3}}}{x - 8}}$

$= \dfrac{2}{\frac{1}{3}(8)^{\frac{-2}{2}}}$ $\quad \ldots \left[\lim\limits_{x \to a} \dfrac{x^n - a^n}{x - a} = na^{n-1}\right]$

$= 2(3) \cdot (8)^{\frac{2}{3}}$

$= 6\left(2^3\right)^{\frac{2}{3}}$

$= 6 \times (2)^2$

= 24.

Exercise 7.1 | Q 3.3 | Page 100

Evaluate the following limits: $\lim\limits_{x \to a} \left[\dfrac{(z+2)^{\frac{3}{2}} - (a+2)^{\frac{3}{2}}}{z - a}\right]$

SOLUTION

$\lim\limits_{x \to a} \dfrac{(z+2)^{\frac{3}{2}} - (a+2)^{\frac{3}{2}}}{z - a}$

Put z + 2 = y and a + 2 = b

As z → a, z + 2 → a + 2

i.e. y → b

$\therefore \lim\limits_{z \to a} \dfrac{(z+2)^{\frac{3}{2}} - (a+2)^{\frac{3}{2}}}{z - a}$

$= \lim\limits_{y \to b} \dfrac{y^{\frac{3}{2}} - b^{\frac{3}{2}}}{(y-2) - (b-2)}$

$= \lim\limits_{y \to b} \dfrac{y^{\frac{3}{2}} - b^{\frac{3}{2}}}{y - b}$

$$= \frac{3}{2} \cdot b^{\frac{1}{2}} \quad \ldots\left[\because \lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}\right]$$

$$= \frac{3}{2}(a+2)^{\frac{1}{2}} \qquad \ldots[\because b = a + 2]$$

Exercise 7.1 | Q 3.4 | Page 100

Evaluate the following limits: $\lim_{x \to 5}\left[\frac{x^3 - 125}{x^2 - 25}\right]$

SOLUTION

$$\lim_{x \to 5} \frac{x^3 - 125}{x^2 - 25}$$

$$= \lim_{x \to 5} \frac{\frac{x^3 - 125}{x^2 - 25}}{\frac{x^2 - 25}{x - 5}} \quad \ldots\left[\begin{matrix}\text{As } x \to 5 x \neq 5 \\ \therefore x - 5 \neq 0 \\ \text{Divide Numerator and} \\ \text{Denominator by} x - 5.\end{matrix}\right]$$

$$= \lim_{x \to 5} \frac{\left(\frac{x^3 - 5^3}{x - 5}\right)}{\left(\frac{x^2 - 5^2}{x - 5}\right)}$$

$$= \frac{3(5)^2}{2(5)^1} \quad \ldots\left[\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}\right]$$

$$= \frac{15}{2}.$$

EXERCISE 7.2 [PAGE 102]

Exercise 7.2 | Q 1.1 | Page 102

Evaluate the following limits: $\lim_{x \to 2}\left[\frac{z^2 - 5z + 6}{z^2 - 4}\right]$

SOLUTION

$$\lim_{x \to 2} \frac{z^2 - 5z + 6}{z^2 - 4}$$

$$= \lim_{z \to 2} \frac{(z-3)(z-2)}{(z+2)(z-2)}$$

$$= \lim_{x \to 2} \frac{z-3}{z-2} \quad \ldots \begin{bmatrix} \text{As } z \to 2\ z \neq 2 \\ \therefore z - 2 \neq 0 \end{bmatrix}$$

$$= \frac{2-3}{2+2}$$

$$= -\frac{1}{4}.$$

Exercise 7.2 | Q 1.2 | Page 102

Evaluate the following limits: $\lim_{x \to -3} \left[\frac{x+3}{x^2+4x+3}\right]$

SOLUTION

$$\lim_{x \to -3} \left[\frac{x+3}{x^2+4x+3}\right]$$

$$= \lim_{x \to -3} \frac{x+3}{(x+3)(x+1)}$$

$$= \lim_{x \to -3} \frac{1}{x+1} \quad \ldots \begin{bmatrix} \text{As } x \to -3\ x \neq -3 \\ \therefore x + 3 \neq 0 \end{bmatrix}$$

$$= \frac{1}{-3+1}$$

$$= -\frac{1}{2}.$$

Exercise 7.2 | Q 1.3 | Page 102

Evaluate the following limits: $\lim_{y \to 0} \left[\frac{5y^3+8y^2}{3y^4-16y^2}\right]$

SOLUTION

$$\lim_{y \to 0} \left[\frac{5y^3 + 8y^2}{3y^4 - 16y^2}\right]$$

$$= \lim_{y \to 0} \frac{y^2(5y + 8)}{y^2(3y^2 - 16)}$$

$$= \lim_{y \to 0} \frac{5y + 8}{3y^2 - 16} \quad \ldots \begin{bmatrix} \text{As } y \to 0 \ \ y \neq 0 \\ \therefore y^2 \neq 0 \end{bmatrix}$$

$$= \frac{5(0) + 8}{3(0)^2 - 16}$$

$$= \frac{8}{-16}$$

$$= -\frac{1}{2}.$$

Exercise 7.2 | Q 1.4 | Page 102

Evaluate the following limits: $\lim_{x \to -2} \left[\frac{-2x - 4}{x^3 + 2x^2}\right]$

SOLUTION

$$\lim_{x \to -2} \left[\frac{-2x - 4}{x^3 + 2x^2}\right]$$

$$= \lim_{x \to -2} \frac{-2(x + 2)}{x^2(x + 2)}$$

$$= \lim_{x \to -2} \frac{-2}{x^2} \quad \ldots \begin{bmatrix} \text{As } x \to -2 \ x \neq -2 \\ \therefore x + 2 \neq 0 \end{bmatrix}$$

$$= \frac{(-2)}{(-2)^2}$$

$$= \frac{-2}{4}$$

$$= \frac{-1}{2}.$$

Exercise 7.2 | Q 2.1 | Page 102

Evaluate the following limits: $\lim_{u \to 1}\left[\frac{u^4 - 1}{u^3 - 1}\right]$

SOLUTION

$$\lim_{u \to 1}\left[\frac{u^4 - 1}{u^3 - 1}\right]$$

$$= \lim_{u \to 1}\frac{\left[\frac{u^4 - 1^4}{u - 1}\right]}{\left[\frac{u^3 - 1^3}{u - 1}\right]} \quad \ldots \left[\begin{array}{c}\because u \to 1;\ u \neq 1 \\ \therefore u - 1 \neq 0\end{array}\right]$$

$$= \frac{4(1)^2}{3(1)^2} \quad \ldots \left[\because \lim_{x \to \mathrm{a}}\frac{x^{\mathrm{n}} - \mathrm{a}^{\mathrm{n}}}{z - \mathrm{a}} = \mathrm{na}^{\mathrm{n}-1}\right]$$

$$= \frac{4}{3}.$$

Exercise 7.2 | Q 2.2 | Page 102

Evaluate the following limits: $\lim_{x \to 3}\left[\frac{1}{x - 3} - \frac{9x}{x^3 - 27}\right]$

SOLUTION

$$\lim_{x \to 3}\left[\frac{1}{x - 3} - \frac{9x}{x^3 - 27}\right]$$

$$= \lim_{x \to 3}\left[\frac{1}{x - 3} - \frac{9x}{x^3 - 3^3}\right]$$

$$= \lim_{x \to 3}\left[\frac{1}{x - 3} - \frac{9x}{x^3 - 3^3}\right]$$

$$= \lim_{x \to 3}\left[\frac{1}{x - 3} - \frac{9x}{(x - 3)(x^2 + 3x + 9)}\right]$$

$$= \lim_{x \to 3}\left[\frac{x^2 + 3x + 9 - 9x}{(x - 3)(x^2 + 3x + 9)}\right]$$

$= \lim_{x \to 3} \left[\frac{x^2 + 3x + 9 - 9x}{(x-3)(x^2+3x+9)} \right]$

$= \lim_{x \to 3} \left[\frac{x^2 - 6x + 9}{(x-3)(x^2+3x+9)} \right]$

$= \lim_{x \to 3} \left[\frac{(x-3)^2}{(x-3)(x^2+3x+9)} \right]$

$= \lim_{x \to 3} \left[\frac{x-3}{x^2+3x+9} \right] \quad \ldots \begin{bmatrix} \because x \to 3,\, x \neq 3 \\ \therefore x - 3 \neq 0 \end{bmatrix}$

$= \frac{3-3}{(3)^2 + 3(3) + 9}$

$= \frac{0}{27}$

= 0.

Exercise 7.2 | Q 2.3 | Page 102

Evaluate the following limits: $\lim_{x \to 2} \left[\frac{x^3 - 4x^2 + 4x}{x^2 - 1} \right]$

SOLUTION

$\lim_{x \to 2} \left[\frac{x^3 - 4x^2 + 4x}{x^2 - 1} \right]$

$= \lim_{x \to 2} \frac{x(x^2 - 4x + 4)}{(x^2 - 1)}$

$= \lim_{x \to 2} \frac{x(x-2)^2}{x^2 - 1}$

$= \frac{2(0)}{(2)^2 - 1}$

$= \dfrac{2 \times 0}{3}$

= 0.

Exercise 7.2 | Q 3.1 | Page 102

Evaluate the following limits: $\lim_{x \to -2} \left[\dfrac{x^7 + x^5 + 160}{x^3 + 8}\right]$

SOLUTION

$$\lim_{x \to -2} \left[\frac{x^7 + x^5 + 160}{x^3 + 8}\right]$$

$$= \lim_{x \to -2} \frac{\left(x^7 + 128\right) + \left(x^5 + 32\right)}{x^3 + 8}$$

$$= \lim_{x \to -2} \frac{\frac{\left(x^7+128\right)+\left(x^5+32\right)}{x+2}}{\frac{x^3+8}{x+2}} \quad \ldots \begin{bmatrix} \text{As } x \to -2, x \neq -2 \\ \therefore x + 2 \neq 0 \\ \text{Divide Numerator and} \\ \text{Denominator by} x + 2 \end{bmatrix}$$

$$= \frac{\lim_{x \to -2} \left(\frac{x^7+2^7}{x+2} + \frac{x^5+2^5}{x+2}\right)}{\lim_{x \to -2} \left(\frac{x^3+2^3}{x+2}\right)}$$

$$= \frac{\lim_{x \to -2} \frac{x^7-(-2)^7}{x-(-2)} + \lim_{x \to -2} \frac{x^5-(-2)^5}{x-(-2)}}{\lim_{x \to -2} \frac{x^3-(-2)^3}{x-(-2)}}$$

$= \dfrac{7(-2)^6 + 5(-2)^4}{3(-2)^2} \quad \ldots \left[\lim_{x \to \mathrm{a}} \dfrac{x^{\mathrm{n}} - \mathrm{a}^{\mathrm{n}}}{x - \mathrm{a}} = \mathrm{na}^{\mathrm{n}-1}\right]$

$= \dfrac{7(64) + 5(16)}{3(4)}$

$= \dfrac{448 + 80}{12}$

$= \dfrac{528}{12}$

= 44.

Exercise 7.2 | Q 3.2 | Page 102

Evaluate the following limits: $\lim_{y \to \frac{1}{2}} \left[\dfrac{1 - 8y^3}{y - 4y^3}\right]$

SOLUTION

$\lim_{y \to \frac{1}{2}} \left[\dfrac{1 - 8y^3}{y - 4y^3}\right]$

$= \lim_{y \to \frac{1}{2}} \dfrac{1 - 8y^3}{y(1 - 4y^2)}$

$= \lim_{y \to \frac{1}{2}} \dfrac{(1)^3 - (2y)^3}{y\left[(1)^2 - (2y)^2\right]}$

$= \lim_{y \to \frac{1}{2}} \dfrac{(1 - 2y)\left(1 + 2y + 4y^2\right)}{y(1 - 2y)(1 + 2y)}$

$= \lim_{y \to \frac{1}{2}} \dfrac{1 + 2y + 4y^2}{y(1 + 2y)} \ldots. \left[\begin{array}{c} \because y \to \frac{1}{2}, \therefore y \neq \frac{1}{2} \\ \therefore 2y \neq 1, \therefore 2y - 1 \neq 0 \\ \therefore 1 - 2y \neq 0 \end{array}\right]$

$= \dfrac{1 + 2\left(\frac{1}{2}\right) + 4\left(\frac{1}{2}\right)^2}{\frac{1}{2}\left[1 + 2\left(\frac{1}{2}\right)\right]}$

$= \dfrac{1+1+1}{\frac{1}{2}(2)}$

= 3.

Exercise 7.2 | Q 3.3 | Page 102

Evaluate the following limits: $\lim_{v\to\sqrt{2}}\left[\dfrac{v^2+v\sqrt{2}-4}{v^2-3v\sqrt{2}+4}\right]$

SOLUTION

$$\lim_{v\to\sqrt{2}}\left[\frac{v^2+v\sqrt{2}-4}{v^2-3v\sqrt{2}+4}\right]$$

Consider, $v^2+v\sqrt{2}-4 = v^2+\sqrt{2}v-4$

$= v^2+2\sqrt{2}v-\sqrt{2}v-4$

$= v\left(v+2\sqrt{2}\right)-\sqrt{2}\left(v+2\sqrt{2}\right)$

$= \left(v+2\sqrt{2}\right)\left(v-\sqrt{2}\right)$

$v^2-3v\sqrt{2}+4 = v^2-3\sqrt{2}v+4$

$= v^2-2\sqrt{2}v-\sqrt{2}v+4$

$= v\left(v-2\sqrt{2}\right)-\sqrt{2}\left(v-2\sqrt{2}\right)$

$= \left(v-2\sqrt{2}\right)\left(v-\sqrt{2}\right)$

$\therefore \lim_{v\to\sqrt{2}}\left[\dfrac{v^2+v\sqrt{2}-4}{v^2-3v\sqrt{2}+4}\right]$

$$= \lim_{v \to \sqrt{2}} \frac{\left(v + 2\sqrt{2}\right)\left(v - \sqrt{2}\right)}{\left(v - 2\sqrt{2}\right)\left(v - \sqrt{2}\right)}$$

$$= \lim_{v \to \sqrt{2}} \frac{v + 2\sqrt{2}}{v - 2\sqrt{2}} \quad \ldots \begin{bmatrix} \text{As } v \to \sqrt{2}, v \neq \sqrt{2} \\ \therefore v - \sqrt{2} \neq 0 \end{bmatrix}$$

$$= \frac{\sqrt{2} + 2\sqrt{2}}{\sqrt{2} - 2\sqrt{2}}$$

$$= \frac{3\sqrt{2}}{-\sqrt{2}}$$

= − 3.

Exercise 7.2 | Q 3.4 | Page 102

Evaluate the following limits: $\lim_{x \to 3}\left[\frac{x^2 + 2x - 15}{x^2 - 5x + 6}\right]$

SOLUTION

$$\lim_{x \to 3}\left[\frac{x^2 + 2x - 15}{x^2 - 5x + 6}\right]$$

$$= \lim_{x \to 3} \frac{(x+5)(x-3)}{(x-2)(x-3)}$$

$$= \lim_{x \to 3} \frac{x+5}{x-2} \quad \ldots \begin{bmatrix} \text{as } x \to 3, x \neq 3 \\ \therefore x - 3 \neq 0 \end{bmatrix}$$

$$= \frac{3+5}{3-2}$$

= 8.

EXERCISE 7.3 [PAGE 103]

Exercise 7.3 | Q 1.1 | Page 103

Evaluate the following limits: $\lim_{x \to 0} \left[\frac{\sqrt{6+x+x^2}-\sqrt{6}}{x}\right]$

SOLUTION

$$\lim_{x \to 0} \left[\frac{\sqrt{6+x+x^2}-\sqrt{6}}{x}\right]$$

$$= \lim_{x \to 0} \left[\frac{\sqrt{6+x+x^2}-\sqrt{6}}{x} \times \frac{\sqrt{6+x+x^2}+\sqrt{6}}{\sqrt{6+x+x^2}+\sqrt{6}}\right]$$

$$= \lim_{x \to 0} \frac{\left(6+x+x^2\right)-6}{x\left(\sqrt{6+x+x^2}+\sqrt{6}\right)}$$

$$= \lim_{x \to 0} \frac{x+x^2}{x\left(\sqrt{6+x+x^2}+\sqrt{6}\right)}$$

$$= \lim_{x \to 0} \frac{x(1+x)}{x\left(\sqrt{6+x+x^2}+\sqrt{6}\right)}$$

$$= \lim_{x \to 0} \frac{1+x}{\sqrt{6+x+x^2}+\sqrt{6}} \quad ...[\because x \to 0, \therefore x \neq 0]$$

$$= \frac{(1+0)}{\sqrt{6}+\sqrt{6}}$$

$$= \frac{1}{2\sqrt{6}}.$$

Exercise 7.3 | Q 1.2 | Page 103

Evaluate the following limits: $\lim_{y \to 0} \left[\frac{\sqrt{1-y^2}-\sqrt{1+y^2}}{y^2}\right]$

SOLUTION

$$\lim_{y \to 0}\left[\frac{\sqrt{1-y^2}-\sqrt{1+y^2}}{y^2}\right]$$

$$= \lim_{y \to 0}\left[\frac{\sqrt{1-y^2}-\sqrt{1+y^2}}{y^2}\right]$$

$$= \lim_{y \to 0}\left[\frac{\sqrt{1-y^2}-\sqrt{1+y^2}}{y^2} \times \frac{\sqrt{1-y^2}+\sqrt{1+y^2}}{\sqrt{1-y^2}+\sqrt{1+y^2}}\right]$$

$$= \lim_{y \to 0}\frac{\left(1-y^2\right)-\left(1+y^2\right)}{\left(\sqrt{1-y^2}+\sqrt{1+y^2}\right)}$$

$$= \lim_{y \to 0}\frac{1-y^2-1-y^2}{y^2\left(\sqrt{1-y^2}+\sqrt{1+y^2}\right)}$$

$$= \lim_{y \to 0}\frac{-2y^2}{y^2\left(\sqrt{1-y^2}+\sqrt{1+y^2}\right)}$$

$$= \lim_{y \to 0}\frac{-2}{\sqrt{1-y^2}+\sqrt{1+y^2}} \quad \ldots \begin{bmatrix} \because y \to 0, \ \therefore y \neq 0, \\ \therefore y^2 \neq 0 \end{bmatrix}$$

$$= \frac{-2}{\sqrt{1-0^2}+\sqrt{1+0^2}}$$

$$= \frac{-2}{1+1}$$

= − 1.

Exercise 7.3 | Q 1.3 | Page 103

Evaluate the following limits: $\lim_{x \to 2}\left[\frac{\sqrt{2+x}-\sqrt{6-x}}{\sqrt{x}-\sqrt{2}}\right]$

SOLUTION

$$\lim_{x\to 2}\left[\frac{\sqrt{2+x}-\sqrt{6-x}}{\sqrt{x}-\sqrt{2}}\right]$$

$$= \lim_{x\to 2}\left[\frac{\sqrt{2+x}-\sqrt{6-x}}{\sqrt{x}-\sqrt{2}}\times\frac{\sqrt{2+x}+\sqrt{6-x}}{\sqrt{2+x}+\sqrt{6-x}}\times\frac{\sqrt{x}+\sqrt{2}}{\sqrt{x}+\sqrt{2}}\right]\ \ldots\left[\begin{array}{c}\text{By taking conjuagates}\\ \text{of both, the numerator}\\ \text{as well as the}\\ \text{Denomerator}\end{array}\right]$$

$$= \lim_{x\to 2}\left[\frac{(2+x)-(6-x)}{(x-2)}\times\frac{\sqrt{x}+\sqrt{2}}{\sqrt{2+x}+\sqrt{6\ -x}}\right]$$

$$= \lim_{x\to 2}\left[\frac{-4+2x}{x-2}\times\frac{\sqrt{x}+\sqrt{2}}{\sqrt{2+x}+\sqrt{6-x}}\right]$$

$$= \lim_{x\to 2}\left[\frac{2(x-2)}{x-2}\times\frac{\sqrt{x}+\sqrt{2}}{\sqrt{2+x}+\sqrt{6-x}}\right]$$

$$= \lim_{x\to 2}\left[\frac{2\left(\sqrt{x}+\sqrt{2}\right)}{\sqrt{2+x}+\sqrt{6\ -x}}\right]\ \ldots\left[\begin{array}{c}\because x\to 2;\, x\neq 2\\ \therefore x-2\neq 0\end{array}\right]$$

$$= \frac{2\left(\sqrt{2}+\sqrt{2}\right)}{\sqrt{2+2}+\sqrt{6-2}}$$

$$= \frac{2\left(2\sqrt{2}\right)}{2+2}$$

$$= \frac{4\sqrt{2}}{4}$$

$$= \sqrt{2}.$$

Exercise 7.3 | Q 2.1 | Page 103

Evaluate the following limits: $\lim_{x\to \mathrm{a}}\left[\frac{\sqrt{\mathrm{a}+2x}-\sqrt{3x}}{\sqrt{3\mathrm{a}+x}-2\sqrt{x}}\right]$

SOLUTION

$$\lim_{x \to a}\left[\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}\right]$$

$$= \lim_{x \to a}\left[\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x-2\sqrt{x}}} \times \frac{\sqrt{a+2x}+\sqrt{3x}}{\sqrt{a+2x}+\sqrt{3x}} \times \frac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{3a+x}+2\sqrt{x}}\right]$$

$$= \lim_{x \to a}\left[\frac{(a+2x)-3x}{(3a+x)-4x} \times \frac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{a+2x}+\sqrt{3x}}\right]$$

$$= \lim_{x \to a}\left[\frac{a-x}{3a-3x} \times \frac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{a+2x}+\sqrt{3x}}\right]$$

$$= \lim_{x \to a}\left[\frac{-(x-a)}{-3(x-a)} \times \frac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{a+2x}+\sqrt{3x}}\right]$$

$$= \lim_{x \to a}\left[\frac{\sqrt{3a+x}+2\sqrt{x}}{3\left(\sqrt{a+2x}+\sqrt{3x}\right)}\right] \quad \ldots \left[\begin{matrix}\because x \to a,\, x \neq a \\ \therefore x-a \neq 0\end{matrix}\right]$$

$$= \frac{\sqrt{3a+x}+2\sqrt{x}}{3\left(\sqrt{a+2x}+\sqrt{3x}\right)}$$

$$= \frac{\sqrt{4a}+2\sqrt{a}}{3\left(\sqrt{3a}+\sqrt{3a}\right)}$$

$$= \frac{2\sqrt{a}+2\sqrt{a}}{3\left(2\sqrt{3a}\right)}$$

$$= \frac{4\sqrt{a}}{6\sqrt{3}\sqrt{a}}$$

$= \dfrac{2}{3\sqrt{3}}.$

Exercise 7.3 | Q 2.2 | Page 103

Evaluate the following limits: $\lim_{x \to 2} \left[\dfrac{x^2 - 4}{\sqrt{x+2} - \sqrt{3x-2}}\right]$

SOLUTION

$$\lim_{x \to 2} \left[\frac{x^2 - 4}{\sqrt{x+2} - \sqrt{3x-2}}\right]$$

$$= \lim_{x \to 2} \left[\frac{x^2 - 4}{\sqrt{x+2} - \sqrt{3x-2}} \times \frac{\sqrt{x+2} + \sqrt{3x-2}}{\sqrt{x+2} + \sqrt{3x-2}}\right]$$

$$= \lim_{x \to 2} \frac{(x^2 - 4)\left(\sqrt{x+2} + \sqrt{3x-2}\right)}{(x+2) - (3x-2)}$$

$$= \lim_{x \to 2} \frac{(x^2 - 4)\left(\sqrt{x+2} + \sqrt{3x-2}\right)}{-2x + 4}$$

$$= \lim_{x \to 2} \frac{(x+2)(x-2)\left(\sqrt{x+2} + \sqrt{3x-2}\right)}{-2(x-2)}$$

$$= \lim_{x \to 2} \frac{(x+2)\left(\sqrt{x+2} + \sqrt{3x-2}\right)}{-2} \quad \ldots \begin{bmatrix} \text{As } x \to 2, x \neq 2 \\ \therefore x - 2 \neq 0 \end{bmatrix}$$

$$= \frac{(2+2)\left(\sqrt{2+2} + \sqrt{3(2)-2}\right)}{-2}$$

$$= \frac{4(2+2)}{-2}$$

$$= \frac{16}{-2}$$

$= -8.$

Exercise 7.3 | Q 3.1 | Page 103

Evaluate the following limits: $\lim_{x\to1}\left[\frac{x^2+x\sqrt{x}-2}{x-1}\right]$

SOLUTION

$$\lim_{x\to1}\left[\frac{x^2+x\sqrt{x}-2}{x-1}\right]$$

$$= \lim_{x\to1}\left[\frac{\left(x^2-1\right)+\left(x\sqrt{x}-1\right)}{x-1}\right]$$

$$= \lim_{x\to1}\left[\frac{x^2-1}{x-1}+\frac{x^{\frac{1}{2}}-1}{x-1}\right] \quad \ldots\left[\because x\sqrt{x}=x^1\cdot x^{\frac{1}{2}}=x^{1+\frac{1}{2}}=x^{\frac{3}{2}}\right]$$

$$= \lim_{x\to1}\left(\frac{x^2-1^2}{x-1}\right)+\lim_{x\to1}\left(\frac{x^{\frac{3}{2}}-1^{\frac{3}{2}}}{x-1}\right)$$

$$= 2(1)1\ \frac{3}{2}(1)^{\frac{1}{2}} \quad \ldots\left[\because\ \lim_{x\to\mathrm{a}}\frac{x^{\mathrm{n}}-\mathrm{a}^{\mathrm{n}}}{x-\mathrm{a}}=\mathrm{na}^{\mathrm{n}-1}\right]$$

$$= 2+\frac{3}{2}$$

$$= \frac{7}{2}.$$

Exercise 7.3 | Q 3.2 | Page 103

Evaluate the following limits: $\lim_{x\to0}\left[\frac{\sqrt{1+x^2}-\sqrt{1+x}}{\sqrt{1+x^3}-\sqrt{1+x}}\right]$

SOLUTION

$$\lim_{x \to 0} \left[\frac{\sqrt{1+x^2} - \sqrt{1+x}}{\sqrt{1+x^3} - \sqrt{1+x}} \right]$$

$$= \lim_{x \to 0} \frac{\left(\sqrt{1+x^2} - \sqrt{1+x}\right)\left(\sqrt{1+x^3} + \sqrt{1+x}\right)\left(\sqrt{1+x^2} + \sqrt{1+x}\right)}{\left(\sqrt{1+x^3} - \sqrt{1+x}\right)\left(\sqrt{1+x^3} + \sqrt{1+x}\right)\left(\sqrt{1+x^2} + \sqrt{1+x}\right)}$$

$$= \lim_{x \to 0} \frac{\left[1+x^2-(1+x)\right]\left(\sqrt{1+x^3} + \sqrt{1+x}\right)}{\left[1+x^3-(1+x)\right]\left(\sqrt{1+x^2} + \sqrt{1+x}\right)}$$

$$= \lim_{x \to 0} \frac{x(x-1)\left(\sqrt{1+x^3} + \sqrt{1+x}\right)}{x(x^2-1)\left(\sqrt{1-x^2} + \sqrt{1+x}\right)}$$

$$= \lim_{x \to 0} \frac{\sqrt{1+x^3} + \sqrt{1+x}}{(x+1)\left(\sqrt{1+x^2} + \sqrt{1+x}\right)}$$ ...[$\because$ x → 0, $\therefore$ x ≠ 0, $\therefore$ x − 1 ≠ 0]

$$= \frac{\sqrt{1+0^3} + \sqrt{1+0}}{(0+1)\left(\sqrt{1+0^2} + \sqrt{1+0}\right)}$$

$$= \frac{1+1}{1(1+1)}$$

= 1

Exercise 7.3 | Q 3.3 | Page 103

Evaluate the following limits: $\lim_{x \to 4} \left[\frac{x^2+x-20}{\sqrt{3x+4}-4} \right]$

SOLUTION

$$\lim_{x \to 4}\left[\frac{x^2+x-20}{\sqrt{3x+4}-4}\right]$$

$$= \lim_{x \to 4}\left[\frac{(x+5)(x-4)}{\sqrt{3x+4}-4} \times \frac{\sqrt{3x+4}+4}{\sqrt{3x+4}+4}\right]$$

$$= \lim_{x \to 4}\frac{(x+5)(x-4)\left(\sqrt{3x+4}+4\right)}{3x+4-16}$$

$$= \lim_{x \to 4}\frac{(x+5)(x-4)\left(\sqrt{3x+4}+4\right)}{3x-12}$$

$$= \lim_{x \to 4}\frac{(x+5)(x-4)\left(\sqrt{3x+4}+4\right)}{3(x-4)}$$

$$= \lim_{x \to 4}\frac{(x-5)\left(\sqrt{3x+4}+4\right)}{3} \quad \ldots\begin{bmatrix}\because x \to 4, x \neq 4 \\ \therefore x-4 \neq 0\end{bmatrix}$$

$$= \frac{(4+5)\left(\sqrt{3(4)+4}+4\right)}{3}$$

$$= \frac{9(4+4)}{3}$$

= 3(8)

= 24.

Exercise 7.3 | Q 3.4 | Page 103

Evaluate the following limits: $\lim_{x \to 2}\left[\frac{x^3-8}{\sqrt{x+2}-\sqrt{3x-2}}\right]$

SOLUTION

$$\lim_{x\to 2}\left[\frac{x^3-8}{\sqrt{x+2}-\sqrt{3x-2}}\right]$$

$$= \lim_{x\to 2}\left[\frac{x^3-8}{\sqrt{x+2}-\sqrt{3x-2}}\times\frac{\sqrt{x+2}+\sqrt{3x-2}}{\sqrt{x+2}+\sqrt{3x-2}}\right]$$

$$= \lim_{x\to 2}\frac{(x^3-8)\left(\sqrt{x+2}+\sqrt{3x-2}\right)}{(x+2)-(3x-2)}$$

$$= \lim_{x\to 2}\frac{(x^3-8)\left(\sqrt{x+2}+\sqrt{3x-2}\right)}{-2x+4}$$

$$= \lim_{x\to 2}\frac{(x-2)(x^2+2x+4)\left(\sqrt{x+2}+\sqrt{3x-2}\right)}{-2(x-2)}$$

$$= \lim_{x\to 2}\frac{(x^2+2x+4)\left(\sqrt{x+2}+\sqrt{3x-2}\right)}{-2} \quad \ldots\begin{bmatrix}\text{As } x\to 2,\ x\neq 2\\ \therefore x-2\neq\end{bmatrix}$$

$$= -\frac{1}{2}\lim_{x\to 2}(x^2+2x+4)\times\lim_{x\to 2}\left(\sqrt{x+2}+\sqrt{3x-2}\right)$$

$$= -\frac{1}{2}\times\left[(2)^2+2(2)+4\right]\times\left(\sqrt{2+2}+\sqrt{3(2)-2}\right)$$

$$= -\frac{1}{2}\times 12\times(2+2)$$

= – 6 x 4

= – 24.

Exercise 7.3 | Q 4.1 | Page 103

Evaluate the following limits: $\lim_{y\to 2}\left[\frac{2-y}{\sqrt{3-y}-1}\right]$

SOLUTION

$$\lim_{y \to 2}\left[\frac{2-y}{\sqrt{3-y}-1}\right]$$

$$= \lim_{y \to 2}\left[\frac{2-y}{\sqrt{3-y}-1} \times \frac{\sqrt{3-y}+1}{\sqrt{3-y}+1}\right]$$

$$= \lim_{y \to 2}\frac{(2-y)\left(\sqrt{3-y}+1\right)}{3-y-1}$$

$$= \lim_{y \to 2}\frac{(2-y)\left(\sqrt{3-y}+1\right)}{2-y}$$

$$= \lim_{y \to 2}\left(\sqrt{3-y}+1\right) \quad \ldots\left[\begin{array}{c}\text{As } y \to 2, y \neq 2\\ \therefore y-2 \neq 0 \therefore 2-y \neq 0\end{array}\right]$$

$$= \sqrt{3-2}+1$$

= 1 + 1

= 2.

Exercise 7.3 | Q 4.2 | Page 103

Evaluate the following limits: $\lim_{z \to 4}\left[\frac{3-\sqrt{5+z}}{1-\sqrt{5-z}}\right]$

SOLUTION

$$\lim_{z \to 4}\left[\frac{3-\sqrt{5+z}}{1-\sqrt{5-z}}\right]$$

$$= \lim_{z \to 4}\left[\frac{3-\sqrt{5+z}}{1-\sqrt{5-z}} \times \frac{3+\sqrt{5+z}}{1+\sqrt{5-z}} \times \frac{1+\sqrt{5-z}}{3+\sqrt{5+z}}\right]$$

$$= \lim_{z \to 4}\left[\frac{9-(5+z)}{1-(5-z)} \times \frac{1+\sqrt{5-z}}{3+\sqrt{5+z}}\right]$$

$$= \lim_{z\to 4}\left[\frac{4-z}{-4+z}\times\frac{1+\sqrt{5-z}}{3+\sqrt{5+z}}\right]$$

$$= \lim_{z\to 4}\left[\frac{-(z-4)}{z-4}\times\frac{1+\sqrt{5-z}}{3+\sqrt{5+z}}\right]$$

$$= \lim_{z\to 4}\left[\frac{-1+\sqrt{5-z}}{3+\sqrt{5+z}}\right] \quad \ldots\left[\begin{array}{c}\because z\to 4, \therefore z\neq 4,\\ \therefore z-4\neq 0\end{array}\right]$$

$$= \frac{-\left(1+\sqrt{5-4}\right)}{3+\sqrt{5+4}}$$

$$= \frac{-(1+1)}{3+3}$$

$$= \frac{-2}{6}$$

$$= -\frac{1}{3}.$$

EXERCISE 7.4 [PAGE 105]

Exercise 7.4 | Q 1.1 | Page 105

Evaluate the following: $\lim_{x\to 0}\left[\frac{9^x-5^x}{4^x-1}\right]$

SOLUTION

$$\lim_{x\to 0}\frac{9^x-5^x}{4^x-1}$$

$$= \lim_{x\to 0}\frac{9x-1+1-5x}{4^x-1}$$

$$= \lim_{x\to 0}\frac{(9^x-1)-(5^x-1)}{4^x-1}$$

$= \lim_{x \to 0} \frac{\frac{9^x - 1 - 5^x - 1}{x}}{\frac{4^x - 1}{x}}$ ...[∵ x → 0, ∴ x ≠ 0]

$= \lim_{x \to 0} \frac{\left(\frac{9^x - 1}{x}\right) - \left(\frac{5^x - 1}{x}\right)}{\left(\frac{4^x - 1}{x}\right)}$

$= \frac{\lim_{x \to 0} \frac{9^x - 1}{x} - \lim_{x \to 0} \frac{5^x - 1}{x}}{\lim_{x \to 0} \frac{4^x - 1}{x}}$

$= \frac{\log 9 - \log 5}{\log 4}$ $\ldots\left[\because \lim_{x \to 0} \frac{a^x - 1}{x} = \log a\right]$

$= \frac{1}{\log 4} \log\left(\frac{9}{5}\right).$

Exercise 7.4 | Q 1.2 | Page 105

Evaluate the following: $\lim_{x \to 0}\left[\frac{5^x + 3^x - 2^x - 1}{x}\right]$

SOLUTION

$\lim_{x \to 0} \frac{5^x + 3^x - 2^x - 1}{x}$

$\lim_{x \to 0} \frac{(5^x - 1) + (3^x - 2^x)}{x}$

$= \lim_{x \to 0} \frac{(5^x - 1) + (3^x - 1) - (2^x - 1)}{x}$

$= \lim_{x \to 0}\left(\frac{5^x - 1}{x} + \frac{3^x - 1}{x} - \frac{2^x - 1}{x}\right)$

$= \lim_{x \to 0}\left(\frac{5^x - 1}{x}\right) + \lim_{x \to 0}\left(\frac{3^x - 1}{x}\right) - \lim_{x \to 0}\left(\frac{2^x - 1}{x}\right)$

$= \log 5 + \log 3 - \log 2$ $\ldots\left[\lim_{x \to 0} \frac{a^x - 1}{x} - \log a\right]$

$= \log \frac{5 \times 3}{2}$

$= \log \frac{15}{2}.$

Exercise 7.4 | Q 1.3 | Page 105

Evaluate the following: $\lim_{x \to 0} \left[\frac{\log(2+x) - \log(2-x)}{x} \right]$

SOLUTION

$$\lim_{x \to 0} \left[\frac{\log(2+x) - \log(2-x)}{x} \right]$$

$$= \lim_{x \to 0} \frac{\log\left[2\left(1+\frac{x}{2}\right)\right] - \log\left[2\left(1-\frac{x}{2}\right)\right]}{x}$$

$$= \lim_{x \to 0} \frac{\log 2 + \log\left(1+\frac{x}{2}\right) - \left[\log 2 + \log\left(1-\frac{x}{2}\right)\right]}{x}$$

$$= \lim_{x \to 0} \frac{\log\left(1+\frac{x}{2}\right) - \log\left(1-\frac{x}{2}\right)}{x}$$

$$= \lim_{x \to 0} \left[\frac{\log\left(1+\frac{x}{2}\right)}{x} - \frac{\log\left(1-\frac{x}{2}\right)}{x} \right]$$

$$= \lim_{x \to 0} \left[\frac{\log\left(1+\frac{x}{2}\right)}{2\left(\frac{x}{2}\right)} - \frac{\log\left(1-\frac{x}{2}\right)}{(-2)\left(-\frac{x}{2}\right)} \right]$$

$$= \frac{1}{2} \lim_{x \to 0} \frac{\log\left(1+\frac{x}{2}\right)}{\frac{x}{2}} + \frac{1}{2} \lim_{x \to 0} \frac{\log\left(1-\frac{x}{2}\right)}{-\frac{x}{2}}$$

$$= \frac{1}{2}(1) + \frac{1}{2}(1) \quad \ldots. \left[\begin{array}{c} \because x \to 0, \frac{x}{2} \to 0 \ \text{ and} \\ \lim_{x \to 0} \frac{\log(1+x)}{x} = 1 \end{array} \right]$$

= 1.

Exercise 7.4 | Q 2.1 | Page 105

Evaluate the following: $\lim_{x\to0} \frac{3^x + 3^{-x} - 2}{x^2}$

SOLUTION

$\lim_{x\to0} \frac{3^x + 3^{-x} - 2}{x^2}$

$= \lim_{x\to0} \frac{3^x + \frac{1}{3^x} - 2}{x^2}$

$= \lim_{x\to0} \frac{(3^x)^2 + 1 - 2(3^x)}{3^x \cdot x^2}$

$= \lim_{x\to0} \frac{(3^x - 1)^2}{x^2 \cdot (3^x)}$ ...[∵ $a^2 - 2ab + b^2 = (a - b)^2$]

$= \lim_{x\to0} \left(\frac{3^x - 1}{x}\right)^2 \times \frac{1}{3^x}$

$= \lim_{x\to0} \left(\frac{3^x - 1}{x}\right)^2 \times \frac{1}{\lim_{x\to0} (3^x)}$

$= (\log 3)^2 \times \frac{1}{3^0}$

$= (\log 3)^2 \times \frac{1}{1}$ .... $\left[\lim_{x\to0} \frac{a^x - 1}{x} = \log a\right]$

$= (\log 3)^2$.

Exercise 7.4 | Q 2.2 | Page 105

Evaluate the following: $\lim_{x\to0} \left[\frac{3 + x}{3 - x}\right]^{\frac{1}{x}}$

SOLUTION

$$\lim_{x\to 0}\left(\frac{3+x}{3-x}\right)^{\frac{1}{x}}$$

$$= \lim_{x\to 0}\left(\frac{1+\frac{x}{3}}{1-\frac{x}{3}}\right)^{\frac{1}{x}} \quad \ldots\left[\begin{array}{c}\text{Divide numerator and}\\ \text{denominato by 3}\end{array}\right]$$

$$= \lim_{x\to 0}\frac{\left(1+\frac{x}{3}\right)^{\frac{1}{x}}}{\left(1-\frac{x}{3}\right)^{\frac{1}{x}}}$$

$$= \lim_{x\to 0}\frac{\left(1+\frac{x}{3}\right)^{\frac{3}{x}\times\frac{1}{3}}}{\left(1-\frac{x}{3}\right)^{\frac{-3}{x}\times\frac{1}{-3}}}$$

$$= \frac{\lim_{x\to 0}\left[\left(1+\frac{x}{3}\right)^{\frac{3}{x}}\right]^{\frac{1}{3}}}{\lim_{x\to 0}\left[\left(1-\frac{x}{3}\right)^{\frac{-3}{x}}\right]^{-\frac{1}{3}}}$$

$$= \frac{e^{\frac{1}{3}}}{e^{\frac{-1}{3}}} \quad \ldots\left[\begin{array}{c}\because x\to 0, \frac{x}{3}\to 0, \frac{-x}{3}\to 0 \text{ and}\\ \lim_{x\to 0}(1+x)^{\frac{1}{x}} = e\end{array}\right]$$

$$= e^{\frac{1}{3}+\frac{1}{3}}$$

$$= e^{\frac{2}{3}}.$$

Exercise 7.4 | Q 2.3 | Page 105

Evaluate the following: $\lim_{x\to 0}\left[\frac{\log(3-x)-\log(3+x)}{x}\right]$

SOLUTION

$$\lim_{x\to 0}\frac{\log(3-x)-\log(3+x)}{x}$$

$$= \lim_{x\to 0}\frac{1}{x}\log\left(\frac{3-x}{3+x}\right)$$

$$= \lim_{x\to 0} \log\left(\frac{3-x}{3+x}\right)^{\frac{1}{x}}$$

$$= \lim_{x\to 0} \log\left(\frac{1-\frac{x}{3}}{1+\frac{x}{3}}\right)^{\frac{1}{x}}$$

$$= \log\left[\lim_{x\to 0} \frac{\left(1-\frac{x}{3}\right)^{\frac{1}{x}}}{\left(1+\frac{x}{3}\right)^{\frac{1}{x}}}\right]$$

$$= \log\left[\frac{\left\{\lim_{x\to 0} \left(1-\frac{x}{3}\right)^{\frac{-3}{x}}\right\}^{\frac{-1}{3}}}{\left\{\lim_{x\to 0} \left(1+\frac{x}{3}\right)^{\frac{3}{x}}\right\}^{\frac{1}{3}}}\right]$$

$$= \log\left(\frac{e^{-\frac{1}{3}}}{e^{\frac{1}{3}}}\right) \quad \ldots\left[\because x \to 0, \pm\frac{x}{3} \to 0 \text{ and } \lim_{x\to 0}(1+x)^{\frac{1}{x}} = e\right]$$

$$= \log e^{\frac{-2}{3}}$$

$$= -\frac{2}{3}\cdot \log e$$

$$= -\frac{2}{3}(1)$$

$$= -\frac{2}{3}.$$

Exercise 7.4 | Q 3.1 | Page 105

Evaluate the following: $\lim_{x\to 0}\left[\frac{a^{3x}-b^{2x}}{\log 1+4x}\right]$

SOLUTION

$$\lim_{x\to 0}\left[\frac{\mathrm{a}^{3x}-\mathrm{b}^{2x}}{\log 1+4x}\right]$$

$$= \lim_{x\to 0}\ \frac{a^{3x}-1-\mathrm{b}^{2x}-1}{\log 1+4x}$$

$$= \lim_{x\to 0}\ \frac{\frac{a^{3x}-1-\mathrm{b}^{2x}-1}{x}}{\frac{\log 1+4x}{x}}$$

$$= \frac{\lim_{x\to 0}\left[\frac{a^{3x}-1}{x}-\frac{\mathrm{b}^{2x}-1}{x}\right]}{\lim_{x\to 0}\frac{\log 1+4x}{4x}}$$

$$= \frac{\lim_{x\to 0}\left[\frac{a^{3x}-1}{3x}\right]\times 3-\lim_{x\to 0}\left[\frac{\mathrm{b}^{2x}-1}{2x}\right]\times 2}{\lim_{x\to 0}\frac{\log 1+4x}{4x}\times 4}$$

$$= \frac{3\log \mathrm{a}-2\log \mathrm{b}}{1\times 4}\quad \ldots\left[\begin{array}{c}\because x\to 0{,}2x\to 0{,}3x\to 0\\ 4x\to 0\ \text{ and }\ \lim_{x\to 0}\frac{\mathrm{a}^{x}-1}{x}=\log \mathrm{a}\\ \text{and }\ \lim_{x\to 0}\frac{\log(1+x)}{x}=1\end{array}\right]$$

$$= \frac{1}{4}\left(\log \mathrm{a}3-\log \mathrm{b}^{2}\right)$$

$$= \frac{1}{4}\log\left(\frac{\mathrm{a}^{3}}{\mathrm{b}^{2}}\right).$$

Exercise 7.4 | Q 3.2 | Page 105

Evaluate the following: $\lim_{x\to 0}\left[\frac{(2^{x}-1)^{2}}{(3^{x}-1)\times\log(1+x)}\right]$

SOLUTION

$$\lim_{x\to 0}\left[\frac{(2^x-1)^2}{(3^x-1)\times\log(1+x)}\right]$$

$$= \lim_{x\to 0}\frac{\frac{(2^x-1)^2}{x^2}}{\frac{3^x-1\cdot\log 1+x}{x^2}} \quad \ldots \begin{bmatrix}\text{Divide Numerator and}\\ \text{Denominator by} x^2\\ \text{As } x\to 0, x\neq 0\\ \therefore x^2\neq 0\end{bmatrix}$$

$$= \frac{\lim_{x\to 0}\left(\frac{2^x-1}{x}\right)^2}{\lim_{x\to 0}\left[\left(\frac{3^x-1}{x}\right)\times\frac{\log 1+x}{x}\right]}$$

$$= \frac{\lim_{x\to 0}\left(\frac{2^x-1}{x}\right)^2}{\lim_{x\to 0}\left(\frac{3^x-1}{x}\right)\times\lim_{x\to 0}\frac{\log 1+x}{x}}$$

$$= \frac{(\log 2)^2}{\log 3\times 1} \quad \ldots \begin{bmatrix}\lim_{x\to 0}\frac{a^x-1}{x}=\log a,\\ \lim_{x\to 0}\frac{\log(1+x)}{x}=1\end{bmatrix}$$

$$= \frac{(\log 2)^2}{\log 3}.$$

Exercise 7.4 | Q 3.3 | Page 105

Evaluate the following: $\lim_{x\to 0}\left[\frac{15^x-5^x-3^x+1}{x^2}\right]$

SOLUTION

$$\lim_{x \to 0}\left[\frac{15^x - 5^x - 3^x + 1}{x^2}\right]$$

$$= \lim_{x \to 0} \frac{5^x \cdot 3^x - 5^x - 3^x + 1}{x^2}$$

$$= \lim_{x \to 0} \frac{5^x(3^x - 1) - 1(3^x - 1)}{x^2}$$

$$= \lim_{x \to 0} \frac{(3^x - 1)(5^x - 1)}{x^2}$$

$$= \lim_{x \to 0}\left(\frac{3^x - 1}{x} \times \frac{5^x - 1}{x}\right)$$

$$= \lim_{x \to 0} \frac{3^x - 1}{x} \times \lim_{x \to 0} \frac{5^x - 1}{x}$$

$$= \log 3 \cdot \log 5 \ \ldots \left[\lim_{x \to 0} \frac{\mathrm{a}^x - 1}{x} = \log \mathrm{a}\right]$$

Exercise 7.4 | Q 3.4 | Page 105

Evaluate the following: $\lim_{x \to 2}\left[\frac{3^{\frac{x}{2}} - 3}{3^x - 9}\right]$

SOLUTION

$$\lim_{x \to 2}\left[\frac{3^{\frac{x}{2}} - 3}{3^x - 9}\right]$$

$$= \lim_{x \to 2}\left[\frac{3^{\frac{x}{2}} - 3}{\left(3^{\frac{x}{2}}\right)^2 - (3)^2}\right]$$

$$= \lim_{x \to 2} \frac{3^{\frac{x}{2}} - 3}{\left(3^{\frac{x}{2}} - 3\right)\left(3^{\frac{x}{2}} + 3\right)}$$

$$= \lim_{x \to 2} \frac{1}{3^{\frac{x}{2}} + 3} \ \ldots \left[\begin{array}{c} \text{As } x \to 2, \frac{x}{2} \to 1 \\ \therefore 3^{\frac{x}{2}} \to 3^1 \ \therefore 3^{\frac{x}{2}} \neq 3 \\ \therefore\ 3^{\frac{x}{2}} - 3 \neq 0 \end{array}\right]$$

$= \dfrac{1}{3^{\frac{2}{2}} + 3}$

$= \dfrac{1}{3^1 + 3}$

$= \dfrac{1}{6}.$

Exercise 7.4 | Q 4.1 | Page 105

Evaluate the following: $\lim_{x\to 0}\left[\dfrac{(25)^x - 2(5)^x + 1}{x^2}\right]$

SOLUTION

$$\lim_{x\to 0}\left[\frac{(25)^x - 2(5)^x + 1}{x^2}\right]$$

$$= \lim_{x\to 0}\left[\frac{(5)^{2x} - 2(5)^x + 1}{x^2}\right]$$

$$= \lim_{x\to 0}\left[\frac{(5^x)^2 - 2(5)^x + 1}{x^2}\right]$$

$$= \lim_{x\to 0}\frac{(5^x - 1)^2}{x^2}$$

$$= \lim_{x\to 0}\left(\frac{5^x - 1}{x}\right)^2$$

$$= \log 5^2 \quad \ldots\left[\lim_{x\to 0}\frac{\mathrm{a}^x - 1}{x} = \log \mathrm{a}\right]$$

Exercise 7.4 | Q 4.2 | Page 105

Evaluate the following: $\lim_{x\to 0}\left[\dfrac{(49)^x - 2(35)^x + (25)^x}{x^2}\right]$

SOLUTION

$$\lim_{x \to 0}\left[\frac{(49)^x - 2(35)^x + (25)^x}{x^2}\right]$$

$$= \lim_{x \to 0}\left[\frac{\left(7^2\right)^x - 2(7 \times 5)^x + \left(5^2\right)^x}{x^2}\right]$$

$$= \lim_{x \to 0}\left[\frac{\left(7^x\right)^2 - 2\left(7^x - 5^x\right)^x + \left(5^x\right)^2}{x^2}\right]$$

$$= \lim_{x \to 0}\frac{\left(7^x - 5^x\right)^2}{x^2}$$

$$= \lim_{x \to 0}\left[\frac{7^x - 1 - 5^x - 1}{x}\right]^2$$

$$= \lim_{x \to 0}\left[\frac{7^x - 1}{x} - \frac{5^x - 1}{x}\right]^2$$

$$= \left[\lim_{x \to 0}\frac{7^x - 1}{x} - \lim_{x \to 0}\frac{5^x - 1}{x}\right]^2$$

$$= (\log 7 - \log 5)^2 \quad \ldots\left[\lim_{x \to 0}\frac{\mathrm{a}^x - 1}{x} = \log \mathrm{a}\right]$$

$$= \left(\log \frac{7}{5}\right)^2.$$

MISCELLANEOUS EXERCISE 7 [PAGES 105 - 106]

Exercise 7.4 | Q 1.1 | Page 105

Evaluate the following: $\lim_{x \to 0}\left[\frac{9^x - 5^x}{4^x - 1}\right]$

SOLUTION

$$\lim_{x \to 0} \frac{9^x - 5^x}{4^x - 1}$$

$$= \lim_{x \to 0} \frac{9x - 1 + 1 - 5x}{4^x - 1}$$

$$= \lim_{x \to 0} \frac{(9^x - 1) - (5^x - 1)}{4^x - 1}$$

$$= \lim_{x \to 0} \frac{\frac{9^x - 1 - 5^x - 1}{x}}{\frac{4^x - 1}{x}} \quad ...[\because x \to 0, \therefore x \neq 0]$$

$$= \lim_{x \to 0} \frac{\left(\frac{9^x - 1}{x}\right) - \left(\frac{5^x - 1}{x}\right)}{\left(\frac{4^x - 1}{x}\right)}$$

$$= \frac{\lim_{x \to 0} \frac{9^x - 1}{x} - \lim_{x \to 0} \frac{5^x - 1}{x}}{\lim_{x \to 0} \frac{4^x - 1}{x}}$$

$$= \frac{\log 9 - \log 5}{\log 4} \quad \ldots\left[\because \lim_{x \to 0} \frac{a^x - 1}{x} = \log a\right]$$

$$= \frac{1}{\log 4} \log\left(\frac{9}{5}\right).$$

Exercise 7.4 | Q 1.2 | Page 105

Evaluate the following: $\lim_{x \to 0} \left[\frac{5^x + 3^x - 2^x - 1}{x}\right]$

SOLUTION

$$\lim_{x \to 0} \frac{5^x + 3^x - 2^x - 1}{x}$$

$$\lim_{x \to 0} \frac{(5^x - 1) + (3^x - 2^x)}{x}$$

$$= \lim_{x \to 0} \frac{(5^x - 1) + (3^x - 1) - (2^x - 1)}{x}$$

$$= \lim_{x \to 0} \left(\frac{5^x - 1}{x} + \frac{3^x - 1}{x} - \frac{2^x - 1}{x} \right)$$

$$= \lim_{x \to 0} \left(\frac{5^x - 1}{x} \right) + \lim_{x \to 0} \left(\frac{3^x - 1}{x} \right) - \lim_{x \to 0} \left(\frac{2^x - 1}{x} \right)$$

$$= \log 5 + \log 3 - \log 2 \quad \ldots \left[\lim_{x \to 0} \frac{\mathrm{a}^x - 1}{x} - \log \mathrm{a} \right]$$

$$= \log \frac{5 \times 3}{2}$$

$$= \log \frac{15}{2}.$$

Exercise 7.4 | Q 1.3 | Page 105

Evaluate the following: $\lim_{x \to 0} \left[\frac{\log(2 + x) - \log(2 - x)}{x} \right]$

SOLUTION

$$\lim_{x \to 0} \left[\frac{\log(2 + x) - \log(2 - x)}{x} \right]$$

$$= \lim_{x \to 0} \frac{\log\left[2\left(1 + \frac{x}{2}\right)\right] - \log\left[2\left(1 - \frac{x}{2}\right)\right]}{x}$$

$$= \lim_{x \to 0} \frac{\log 2 + \log\left(1 + \frac{x}{2}\right) - \left[\log 2 + \log\left(1 - \frac{x}{2}\right)\right]}{x}$$

$$= \lim_{x \to 0} \frac{\log\left(1 + \frac{x}{2}\right) - \log\left(1 - \frac{x}{2}\right)}{x}$$

$$= \lim_{x \to 0} \left[\frac{\log\left(1 + \frac{x}{2}\right)}{x} - \frac{\log\left(1 - \frac{x}{2}\right)}{x} \right]$$

$$= \lim_{x \to 0} \left[\frac{\log\left(1 + \frac{x}{2}\right)}{2\left(\frac{x}{2}\right)} - \frac{\log\left(1 - \frac{x}{2}\right)}{(-2)\left(-\frac{x}{2}\right)} \right]$$

$$= \frac{1}{2} \lim_{x \to 0} \frac{\log\left(1 + \frac{x}{2}\right)}{\frac{x}{2}} + \frac{1}{2} \lim_{x \to 0} \frac{\log\left(1 - \frac{x}{2}\right)}{-\frac{x}{2}}$$

$= \frac{1}{2}(1) + \frac{1}{2}(1)$ .... $\begin{bmatrix} \because x \to 0, \frac{x}{2} \to 0 \text{ and} \\ \lim\limits_{x \to 0} \frac{\log(1+x)}{x} = 1 \end{bmatrix}$

= 1.

Exercise 7.4 | Q 2.1 | Page 105

Evaluate the following: $\lim\limits_{x \to 0} \frac{3^x + 3^{-x} - 2}{x^2}$

SOLUTION

$$\lim_{x \to 0} \frac{3^x + 3^{-x} - 2}{x^2}$$

$$= \lim_{x \to 0} \frac{3^x + \frac{1}{3^x} - 2}{x^2}$$

$$= \lim_{x \to 0} \frac{(3^x)^2 + 1 - 2(3^x)}{3^x \cdot x^2}$$

$$= \lim_{x \to 0} \frac{(3^x - 1)^2}{x^2 \cdot (3^x)}$$ ...[$\because a^2 - 2ab + b^2 = (a - b)^2$]

$$= \lim_{x \to 0} \left(\frac{3^x - 1}{x}\right)^2 \times \frac{1}{3^x}$$

$$= \lim_{x \to 0} \left(\frac{3^x - 1}{x}\right)^2 \times \frac{1}{\lim\limits_{x \to 0} (3^x)}$$

$$= (\log 3)^2 \times \frac{1}{3^0}$$

$$= (\log 3)^2 \times \frac{1}{1} \quad .... \left[\lim_{x \to 0} \frac{a^x - 1}{x} = \log a\right]$$

$= (\log 3)^2$.

Exercise 7.4 | Q 2.2 | Page 105

Evaluate the following: $\lim_{x \to 0} \left[\frac{3+x}{3-x}\right]^{\frac{1}{x}}$

SOLUTION

$$\lim_{x \to 0} \left(\frac{3+x}{3-x}\right)^{\frac{1}{x}}$$

$$= \lim_{x \to 0} \left(\frac{1+\frac{x}{3}}{1-\frac{x}{3}}\right)^{\frac{1}{x}} \quad \ldots \begin{bmatrix}\text{Divide numerator and} \\ \text{denominato by 3}\end{bmatrix}$$

$$= \lim_{x \to 0} \frac{\left(1+\frac{x}{3}\right)^{\frac{1}{x}}}{\left(1-\frac{x}{3}\right)^{\frac{1}{x}}}$$

$$= \lim_{x \to 0} \frac{\left(1+\frac{x}{3}\right)^{\frac{3}{x} \times \frac{1}{3}}}{\left(1-\frac{x}{3}\right)^{\frac{-3}{x} \times \frac{1}{-3}}}$$

$$= \frac{\lim_{x \to 0} \left[\left(1+\frac{x}{3}\right)^{\frac{3}{x}}\right]^{\frac{1}{3}}}{\lim_{x \to 0} \left[\left(1-\frac{x}{3}\right)^{\frac{-3}{x}}\right]^{-\frac{1}{3}}}$$

$$= \frac{e^{\frac{1}{3}}}{e^{\frac{-1}{3}}} \quad \ldots \begin{bmatrix}\because x \to 0, \frac{x}{3} \to 0, \frac{-x}{3} \to 0 \text{ and} \\ \lim_{x \to 0} (1+x)^{\frac{1}{x}} = e\end{bmatrix}$$

$$= e^{\frac{1}{3}+\frac{1}{3}}$$

$$= e^{\frac{2}{3}}.$$

Exercise 7.4 | Q 2.3 | Page 105

Evaluate the following: $\lim_{x \to 0} \left[\frac{\log(3-x) - \log(3+x)}{x}\right]$

SOLUTION

$$\lim_{x\to 0} \frac{\log(3-x)-\log(3+x)}{x}$$

$$= \lim_{x\to 0} \frac{1}{x}\log\left(\frac{3-x}{3+x}\right)$$

$$= \lim_{x\to 0} \log\left(\frac{3-x}{3+x}\right)^{\frac{1}{x}}$$

$$= \lim_{x\to 0} \log\left(\frac{1-\frac{x}{3}}{1+\frac{x}{3}}\right)^{\frac{1}{x}}$$

$$= \log\left[\lim_{x\to 0} \frac{\left(1-\frac{x}{3}\right)^{\frac{1}{x}}}{\left(1+\frac{x}{3}\right)^{\frac{1}{x}}}\right]$$

$$= \log\left[\frac{\left\{\lim_{x\to 0}\left(1-\frac{x}{3}\right)^{\frac{-3}{x}}\right\}^{\frac{-1}{3}}}{\left\{\lim_{x\to 0}\left(1+\frac{x}{3}\right)^{\frac{3}{x}}\right\}^{\frac{1}{3}}}\right]$$

$$= \log\left(\frac{e^{-\frac{1}{3}}}{e^{\frac{1}{3}}}\right) \quad \ldots\left[\because x\to 0, \pm\frac{x}{3}\to 0 \text{ and } \lim_{x\to 0}(1+x)^{\frac{1}{x}} = e\right]$$

$$= \log e^{\frac{-2}{3}}$$

$$= -\frac{2}{3}\cdot \log e$$

$$= -\frac{2}{3}(1)$$

$$= -\frac{2}{3}.$$

Exercise 7.4 | Q 3.1 | Page 105

Evaluate the following: $\lim_{x\to 0}\left[\frac{a^{3x}-b^{2x}}{\log 1+4x}\right]$

SOLUTION

$$\lim_{x \to 0}\left[\frac{\mathrm{a}^{3x}-\mathrm{b}^{2x}}{\log 1+4x}\right]$$

$$= \lim_{x \to 0} \frac{a^{3x}-1-\mathrm{b}^{2x}-1}{\log 1+4x}$$

$$= \lim_{x \to 0} \frac{\frac{a^{3x}-1-\mathrm{b}^{2x}-1}{x}}{\frac{\log 1+4x}{x}}$$

$$= \frac{\lim_{x \to 0}\left[\frac{a^{3x}-1}{x}-\frac{\mathrm{b}^{2x}-1}{x}\right]}{\lim_{x \to 0}\frac{\log 1+4x}{4x}}$$

$$= \frac{\lim_{x \to 0}\left[\frac{a^{3x}-1}{3x}\right]\times 3-\lim_{x \to 0}\left[\frac{\mathrm{b}^{2x}-1}{2x}\right]\times 2}{\lim_{x \to 0}\frac{\log 1+4x}{4x}\times 4}$$

$$= \frac{3\log \mathrm{a}-2\log \mathrm{b}}{1\times 4} \quad \ldots\left[\begin{array}{c}\because x \to 0, 2x \to 0, 3x \to 0 \\ 4x \to 0 \text{ and } \lim_{x \to 0}\frac{\mathrm{a}^{x}-1}{x}=\log \mathrm{a} \\ \text{and } \lim_{x \to 0}\frac{\log(1+x)}{x}=1\end{array}\right]$$

$$= \frac{1}{4}\left(\log \mathrm{a}3-\log \mathrm{b}^{2}\right)$$

$$= \frac{1}{4}\log\left(\frac{\mathrm{a}^{3}}{\mathrm{b}^{2}}\right).$$

Exercise 7.4 | Q 3.2 | Page 105

Evaluate the following: $\lim_{x \to 0}\left[\frac{\left(2^{x}-1\right)^{2}}{\left(3^{x}-1\right)\times\log(1+x)}\right]$

SOLUTION

$$\lim_{x \to 0}\left[\frac{\left(2^{x}-1\right)^{2}}{\left(3^{x}-1\right)\times\log(1+x)}\right]$$

$$= \lim_{x \to 0} \frac{\frac{(2^x-1)^2}{x^2}}{\frac{3^x-1 \cdot \log 1+x}{x^2}} \quad \ldots \begin{bmatrix} \text{Divide Numerator and} \\ \text{Denominator by} x^2 \\ \text{As } x \to 0, x \neq 0 \\ \therefore x^2 \neq 0 \end{bmatrix}$$

$$= \frac{\lim_{x \to 0} \left(\frac{2^x-1}{x}\right)^2}{\lim_{x \to 0} \left[\left(\frac{3^x-1}{x}\right) \times \frac{\log 1+x}{x}\right]}$$

$$= \frac{\lim_{x \to 0} \left(\frac{2^x-1}{x}\right)^2}{\lim_{x \to 0} \left(\frac{3^x-1}{x}\right) \times \lim_{x \to 0} \frac{\log 1+x}{x}}$$

$$= \frac{(\log 2)^2}{\log 3 \times 1} \quad \ldots \begin{bmatrix} \lim_{x \to 0} \frac{\mathrm{a}^x-1}{x} = \log \mathrm{a}, \\ \lim_{x \to 0} \frac{\log(1+x)}{x} = 1 \end{bmatrix}$$

$$= \frac{(\log 2)^2}{\log 3}.$$

Exercise 7.4 | Q 3.3 | Page 105

Evaluate the following: $\lim_{x \to 0} \left[\frac{15^x - 5^x - 3^x + 1}{x^2}\right]$

SOLUTION

$$\lim_{x \to 0} \left[\frac{15^x - 5^x - 3^x + 1}{x^2}\right]$$

$$= \lim_{x \to 0} \frac{5^x \cdot 3^x - 5^x - 3^x + 1}{x^2}$$

$$= \lim_{x \to 0} \frac{5^x(3^x - 1) - 1(3^x - 1)}{x^2}$$

$$= \lim_{x \to 0} \frac{(3^x - 1)(5^x - 1)}{x^2}$$

$$= \lim_{x \to 0} \left(\frac{3^x - 1}{x} \times \frac{5^x - 1}{x} \right)$$

$$= \lim_{x \to 0} \frac{3^x - 1}{x} \times \lim_{x \to 0} \frac{5^x - 1}{x}$$

$$= \log 3 \cdot \log 5 \quad \ldots \left[\lim_{x \to 0} \frac{a^x - 1}{x} = \log a \right]$$

Exercise 7.4 | Q 3.4 | Page 105

Evaluate the following: $\lim_{x \to 2} \left[\frac{3^{\frac{x}{2}} - 3}{3^x - 9} \right]$

SOLUTION

$$\lim_{x \to 2} \left[\frac{3^{\frac{x}{2}} - 3}{3^x - 9} \right]$$

$$= \lim_{x \to 2} \left[\frac{3^{\frac{x}{2}} - 3}{\left(3^{\frac{x}{2}}\right)^2 - (3)^2} \right]$$

$$= \lim_{x \to 2} \frac{3^{\frac{x}{2}} - 3}{\left(3^{\frac{x}{2}} - 3\right)\left(3^{\frac{x}{2}} + 3\right)}$$

$$= \lim_{x \to 2} \frac{1}{3^{\frac{x}{2}} + 3} \quad \ldots \left[\begin{array}{c} \text{As } x \to 2, \frac{x}{2} \to 1 \\ \therefore 3^{\frac{x}{2}} \to 3^1 \quad \therefore 3^{\frac{x}{2}} \neq 3 \\ \therefore\ 3^{\frac{x}{2}} - 3 \neq 0 \end{array} \right]$$

$$= \frac{1}{3^{\frac{2}{2}} + 3}$$

$$= \frac{1}{3^1 + 3}$$

$$= \frac{1}{6}.$$

Exercise 7.4 | Q 4.1 | Page 105

Evaluate the following: $\lim_{x \to 0} \left[\frac{(25)^x - 2(5)^x + 1}{x^2} \right]$

SOLUTION

$$\lim_{x\to 0}\left[\frac{(25)^x-2(5)^x+1}{x^2}\right]$$

$$=\lim_{x\to 0}\left[\frac{(5)^{2x}-2(5)^x+1}{x^2}\right]$$

$$=\lim_{x\to 0}\left[\frac{\left(5^x\right)^2-2(5)^x+1}{x^2}\right]$$

$$=\lim_{x\to 0}\frac{\left(5^x-1\right)^2}{x^2}$$

$$=\lim_{x\to 0}\left(\frac{5^x-1}{x}\right)^2$$

$$=\log 5^2 \quad \ldots\left[\lim_{x\to 0}\frac{\mathrm{a}^x-1}{x}=\log \mathrm{a}\right]$$

Exercise 7.4 | Q 4.2 | Page 105

Evaluate the following: $\lim_{x\to 0}\left[\frac{(49)^x-2(35)^x+(25)^x}{x^2}\right]$

SOLUTION

$$\lim_{x\to 0}\left[\frac{(49)^x-2(35)^x+(25)^x}{x^2}\right]$$

$$=\lim_{x\to 0}\left[\frac{\left(7^2\right)^x-2(7\times 5)^x+\left(5^2\right)^x}{x^2}\right]$$

$$=\lim_{x\to 0}\left[\frac{\left(7^x\right)^2-2\left(7^x-5^x\right)^x+\left(5^x\right)^2}{x^2}\right]$$

$$=\lim_{x\to 0}\frac{\left(7^x-5^x\right)^2}{x^2}$$

$$= \lim_{x \to 0} \left[\frac{7^x - 1 - 5^x - 1}{x}\right]^2$$

$$= \lim_{x \to 0} \left[\frac{7^x - 1}{x} - \frac{5^x - 1}{x}\right]^2$$

$$= \left[\lim_{x \to 0} \frac{7^x - 1}{x} - \lim_{x \to 0} \frac{5^x - 1}{x}\right]^2$$

$$= (\log 7 - \log 5)^2 \quad \ldots \left[\lim_{x \to 0} \frac{a^x - 1}{x} = \log a\right]$$

$$= \left(\log \frac{7}{5}\right)^2.$$

MISCELLANEOUS EXERCISE 7 [PAGES 105 - 106]

Miscellaneous Exercise 7 | Q 1 | Page 105

if $\lim_{x \to 2} \frac{x^n - 2}{x - 2}$ = 80 then find the value of n.

SOLUTION

$\lim_{x \to 2} \frac{x^n - 2}{x - 2}$ = 80

∴ $n(2)^{n-1} = 80$ $\quad \ldots \left[\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}\right]$

∴ $n(2)^{n-1} = 5 \times 16$

$= 5 \times (2)^4$

∴ $n(2)^{n-1} = 5 \times (2)^{5-1}$

∴ n = 5.

Miscellaneous Exercise 7 | Q 2.01 | Page 105

Evaluate the following Limits: $\lim_{x \to a} \frac{(x+2)^{\frac{5}{3}} - (a+2)^{\frac{5}{3}}}{x - a}$

SOLUTION

$$\lim_{x \to a} \frac{(x+2)^{\frac{5}{3}} - (a+2)^{\frac{5}{3}}}{x - a}$$

Put x + 2 = y and a + 2 = b

As x → a, x + 2 → a + 2

i.e. y → b.

$$\therefore \lim_{x \to a} \frac{(x+2)^{\frac{5}{3}} - (a+2)^{\frac{5}{3}}}{x - a}$$

$$= \lim_{x \to a} \frac{y^{\frac{5}{3}} - b^{\frac{5}{3}}}{(y-2) - (b-2)}$$

$$= \lim_{y \to b} \frac{y^{\frac{5}{3}} - b^{\frac{5}{3}}}{y - b}$$

$$= \frac{5}{3} b^{\frac{2}{3}} \quad \ldots \left[\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}\right]$$

$$= \frac{5}{3}(a+2)^{\frac{2}{3}} \quad \ldots [\because b = a + 2]$$

Miscellaneous Exercise 7 | Q 2.02 | Page 106

Evaluate the following Limits: $\lim_{x \to 2} \frac{(1+x)^n - 1}{x}$

SOLUTION

$$\lim_{x \to 2} \frac{(1+x)^n - 1}{x}$$

Put 1 + x = y $\quad\quad$ ∴ x = y – 1

As x → 2, y → 1

$$\therefore \lim_{x \to 2} \frac{(1+x)^n - 1}{x}$$

$= \lim_{y \to 1} \frac{y^{n} - 1}{y - 1}$

$= \lim_{y \to 1} \frac{y^{n} - 1^{n}}{y - 1}$

= n(1)n−1 ...$\left[\lim_{x \to a} \frac{x^{n} - a^{n}}{x - a} = na^{n-1}\right]$

= n.

Miscellaneous Exercise 7 | Q 2.03 | Page 106

Evaluate the following Limits: $\lim_{x \to 2} \left[\frac{(x - 2)}{2x^{2} - 7x + 6}\right]$

SOLUTION

$\lim_{x \to 2} \left[\frac{(x - 2)}{2x^{2} - 7x + 6}\right]$

$= \lim_{x \to 2} \frac{(x - 2)}{(x - 2)(2x - 3)}$

$= \lim_{x \to 2} \frac{1}{2x - 3}$ ...$\begin{bmatrix} \text{As } x \to 2, x \neq 2 \\ \therefore x - 2 \neq 0 \end{bmatrix}$

$= \frac{1}{2(2) - 3}$

= 1.

Miscellaneous Exercise 7 | Q 2.04 | Page 106

Evaluate the following Limits: $\lim_{x \to 1} \left[\frac{x^{3} - 1}{x^{2} + 5x - 6}\right]$

SOLUTION

$$\lim_{x\to1}\left[\frac{x^3-1}{x^2+5x-6}\right]$$

$$= \lim_{x\to1}\frac{(x-1)\left(x^2+x+1\right)}{(x-1)(x+6)}$$

$$= \lim_{x\to1}\frac{x^2+x+1}{x+6} \quad \ldots\begin{bmatrix}\text{As } x\to1,\ x\neq1\\ \therefore x-1\neq0\end{bmatrix}$$

$$= \frac{(1)^2+1+1}{1+6}$$

$$= \frac{3}{7}.$$

Miscellaneous Exercise 7 | Q 2.05 | Page 106

Evaluate the following Limits: $\lim_{x\to3}\left[\frac{x-3}{\sqrt{x-2}-\sqrt{4-x}}\right]$

SOLUTION

$$\lim_{x\to3}\left[\frac{x-3}{\sqrt{x-2}-\sqrt{4-x}}\right]$$

$$= \lim_{x\to3}\left[\frac{x-3}{\sqrt{x-2}-\sqrt{4-x}}\times\frac{\sqrt{x-2}+\sqrt{4-x}}{\sqrt{x-2}+\sqrt{4-x}}\right]$$

$$= \lim_{x\to3}\frac{(x-3)\left(\sqrt{x-2}+\sqrt{4-x}\right)}{(x-2)-(4-x)}$$

$$= \lim_{x\to3}\frac{(x-3)\left(\sqrt{x-2}+\sqrt{4-x}\right)}{2x-6}$$

$$= \lim_{x\to3}\frac{(x-3)\left(\sqrt{x-2}+\sqrt{4-x}\right)}{2(x-3)}$$

= $\lim_{x \to 3} \frac{\sqrt{x-2}+\sqrt{4-x}}{2}$ $\quad \ldots \begin{bmatrix} \text{As } x \to 3, x \neq 3 \\ \therefore x-3 \neq 0 \end{bmatrix}$

= $\frac{1}{2} \lim_{x \to 3} \left(\sqrt{x-2}+\sqrt{4-x}\right)$

= $\frac{1}{2}\left(\sqrt{3-2}+\sqrt{4-3}\right)$

= $\frac{1}{2}(1+1)$

= 1.

Miscellaneous Exercise 7 | Q 2.06 | Page 106

Evaluate the following Limits: $\lim_{x \to 4} \left[\frac{3-\sqrt{5+x}}{1-\sqrt{5-x}}\right]$

SOLUTION

$\lim_{x \to 4} \left[\frac{3-\sqrt{5+x}}{1-\sqrt{5-x}}\right]$

= $\lim_{x \to 4} \left[\frac{3-\sqrt{5+x}}{1-\sqrt{5-x}} \times \frac{3+\sqrt{5+x}}{1+\sqrt{5-x}} \times \frac{1+\sqrt{5-x}}{3+\sqrt{5+x}}\right]$

= $\lim_{x \to 4} \left[\frac{9-(5+x)}{1-(5-x)} \times \frac{1+\sqrt{5-x}}{3+\sqrt{5+x}}\right]$

= $\lim_{x \to 4} \left[\frac{4-x}{-4+x} \times \frac{1+\sqrt{5-x}}{3+\sqrt{5+x}}\right]$

= $\lim_{x \to 4} \left[\frac{-(x-4)}{x-4} \times \frac{1+\sqrt{5-x}}{3+\sqrt{5+x}}\right]$

$$= \lim_{x \to 4} \left[\frac{-\left(1+\sqrt{5-x}\right)}{3+\sqrt{5+x}}\right] \quad \ldots \begin{bmatrix}\text{As } x \to 4, x \neq 4 \\ \therefore x-4 \neq 0\end{bmatrix}$$

$$= \frac{-\left(1+\sqrt{5-4}\right)}{3+\sqrt{5+4}}$$

$$= \frac{-\left(1+\sqrt{5-4}\right)}{3+\sqrt{5+4}}$$

$$= \frac{-(1+1)}{3+3}$$

$$= \frac{-2}{6}$$

$$= -\frac{1}{3}.$$

Miscellaneous Exercise 7 | Q 2.07 | Page 106

Evaluate the following Limits: $\lim_{x \to 0} \left[\frac{5^x - 1}{x}\right]$

SOLUTION

$$\lim_{x \to 0} \left[\frac{5^x - 1}{x}\right]$$

$$= \log 5 \quad \ldots \left[\lim_{x \to 0} \frac{\mathrm{a}^x - 1}{x} = \log \mathrm{a}\right]$$

Miscellaneous Exercise 7 | Q 2.08 | Page 106

Evaluate the following Limits: $\lim_{x \to 0} \left(1 + \frac{x}{5}\right)^{\frac{1}{x}}$

SOLUTION

$$\lim_{x \to 0} \left(1 + \frac{x}{5}\right)^{\frac{1}{x}}$$

$$= \lim_{x \to 0} \left[\left(1 + \frac{x}{5}\right)^{\frac{5}{x}}\right]^{\frac{1}{5}}$$

$$= e^{\frac{1}{5}} \quad \ldots \left[\lim_{x \to 0} (1 + x)^{\frac{1}{x}} = e\right]$$

Miscellaneous Exercise 7 | Q 2.09 | Page 106

Evaluate the following Limits: $\lim_{x \to 0} \left[\frac{\log(1 + 9x)}{x}\right]$

SOLUTION

$$\lim_{x \to 0} \left[\frac{\log(1 + 9x)}{x}\right]$$

$$= \lim_{x \to 0} \left[\frac{\log(1 + 9x)}{9x}\right] \times 9$$

$$= 1 \times 9 \quad \ldots \left[\lim_{x \to 0} \frac{\log(1 + x)}{x} = 1\right]$$

= 9.

Miscellaneous Exercise 7 | Q 2.1 | Page 106

Evaluate the following Limits: $\lim_{x \to 0} \frac{(1 - x)^5 - 1}{(1 - x)^3 - 1}$

SOLUTION

$$\lim_{x \to 0} \left[\frac{(1 - x)^5 - 1}{(1 - x)^3 - 1}\right]$$

Put 1 – x = y

As x → 0, y → 1

$$\therefore \lim_{x\to 0}\left[\frac{(1-x)^5-1}{(1-x)^3-1}\right]$$

$$= \lim_{y\to 1}\frac{y^5-1}{y^3-1}$$

$$= \lim_{y\to 1}\left(\frac{\frac{y^5-1}{y-1}}{\frac{y^3-1}{y-1}}\right) \quad \ldots\left[\begin{array}{c}\text{As } y\to 1, y\neq 1\\ \therefore y-1\neq 0\\ \text{Divide Numerator and}\\ \text{Denominator by } \ y-1\end{array}\right]$$

$$= \frac{\lim\limits_{y\to 1}\frac{y^5-1^5}{y-1}}{\lim\limits_{y>1}\frac{y^3-1^3}{y-1}}$$

$$= \frac{5(1)^4}{3(1)^2} \quad \ldots\left[\lim_{x\to a}\frac{x^n-a^n}{x-a} = na^{n-1}\right]$$

$$= \frac{5}{3}.$$

Miscellaneous Exercise 7 | Q 2.11 | Page 106

Evaluate the following Limits: $\lim\limits_{x\to 0}\left[\frac{a^x+b^x+c^x-3}{x}\right]$

SOLUTION

$$\lim_{x\to 0}\left[\frac{a^x+b^x+c^x-3}{x}\right]$$

$$= \lim_{x\to 0}\frac{(a^x-1)+(b^x-1)+(c^x-1)}{x}$$

$$= \lim_{x\to 0}\left(\frac{a^x-1}{x}+\frac{b^x-1}{x}+\frac{c^x-1}{x}\right)$$

$$= \lim_{x\to 0}\left(\frac{a^x-1}{x}\right)+\lim_{x\to 0}\left(\frac{b^x-1}{x}\right)+\lim_{x\to 0}\left(\frac{c^x-1}{x}\right)$$

= log a + log b + log c ... $\left[\lim_{x \to 0} \frac{a^x - 1}{x} = \log a\right]$

= log (abc).

Miscellaneous Exercise 7 | Q 2.12 | Page 106

Evaluate the following Limits: $\lim_{x \to 0} \frac{e^x + e^{-x} - 2}{x^2}$

SOLUTION

$$\lim_{x \to 0} \frac{e^x + e^{-x} - 2}{x^2}$$

$$= \lim_{x \to 0} \frac{e^x + \frac{1}{e^x} - 2}{x^2}$$

$$= \lim_{x \to 0} \frac{(e^x)^2 + 1 - 2e^x}{x^2 \cdot e^x}$$

$$= \lim_{x \to 0} \frac{(e^x - 1)^2}{x^2 \cdot e^x}$$

$$= \lim_{x \to 0} \left[\left(\frac{e^x - 1}{x}\right)^2 \times \frac{1}{e^x}\right]$$

$$= \lim_{x \to 0} \left(\frac{e^x - 1}{x}\right)^2 \times \frac{1}{\lim_{x \to 0} e^x}$$

$$= (1)^2 \times \frac{1}{e^0} \quad \ldots \left[\lim_{x \to 0} \frac{e^x - 1}{x} = 1\right]$$

$$= 1 \times \frac{1}{1}$$

= 1.

Miscellaneous Exercise 7 | Q 2.13 | Page 106

Evaluate the following Limits: $\lim_{x \to 0} \left[\frac{x(6^x - 3^x)}{(2^x - 1) \cdot \log(1 + x)}\right]$

SOLUTION

$$\lim_{x\to0}\frac{x(6^x-3^x)}{(2^x-1)\cdot\log(1+x)}$$

$$=\lim_{x\to0}\frac{x(3^x\cdot2^x-3^x)}{(2^x-1)\cdot\log(1+x)}$$

$$=\lim_{x\to0}\frac{x\cdot3^x(2^x-1)}{(2^x-1)\cdot\log(1+x)}$$

$$=\lim_{x\to0}\frac{x\cdot3^x}{\log(1+x)}\quad\ldots\begin{bmatrix}\text{As } x\to0,2^x\to2^0\\ \text{i.e.}2^x\to1\ \therefore 2^x\ne1\\ \therefore 2^x-1\ne0\end{bmatrix}$$

$$=\lim_{x\to0}\frac{3^x}{\frac{\log(1+x)}{x}}$$

$$=\frac{\lim\limits_{x\to0}3^x}{\lim\limits_{x\to0}\frac{\log(1+x)}{x}}$$

$$=\frac{3^0}{1}\quad\ldots\left[\lim_{x\to0}\frac{\log(1+x)}{x}=1\right]$$

= 1.

Miscellaneous Exercise 7 | Q 2.14 | Page 106

Evaluate the following Limits: $\lim\limits_{x\to0}\left[\frac{a^{3x}-a^{2x}-a^x+1}{x^2}\right]$

SOLUTION

$$\lim_{x\to0}\frac{a^{3x}-a^{2x}-a^x+1}{x^2}$$

$$=\lim_{x\to0}\frac{a^{2x}\cdot a^x-a^{2x}-a^x+1}{x^2}$$

$$= \lim_{x \to 0} \frac{a^{2x}(a^x - 1) - 1(a^x - 1)}{x^2}$$

$$= \lim_{x \to 0} \frac{(a^x - 1) \cdot (a^{2x} - 1)}{x^2}$$

$$= \lim_{x \to 0} \left(\frac{a^x - 1}{x} \times \frac{a^{2x} - 1}{x} \right)$$

$$= \lim_{x \to 0} \left(\frac{a^x - 1}{x} \right) \times \lim_{x \to 0} \left(\frac{a^{2x} - 1}{2x} \right) \times 2$$

$$= \log a \cdot (2 \log a) \quad \ldots \begin{bmatrix} \text{As } x \to 0,\ 2x \to 0 \ \text{ and} \\ \lim_{x \to 0} \frac{a^x - 1}{x} = \log a \end{bmatrix}$$

$= 2(\log a)^2$.

Miscellaneous Exercise 7 | Q 2.15 | Page 106

Evaluate the following Limits: $\lim_{x \to 0} \left[\frac{(5^x - 1)^2}{x \cdot \log(1 + x)} \right]$

SOLUTION

$$\lim_{x \to 0} \frac{(5^x - 1)^2}{x \cdot \log(1 + x)}$$

$$= \lim_{x \to 0} \frac{\frac{(5^x - 1)^2}{x^2}}{\frac{x \cdot \log(1 + x)}{x^2}} \quad \ldots \begin{bmatrix} \text{As } x \to 0,\ x \neq 0 \therefore x^2 \neq 0 \\ \text{Divide Numerator and} \\ \text{Denominator by } \ x^2 \end{bmatrix}$$

$$= \frac{\lim_{x \to 0} \left(\frac{5^x - 1}{x} \right)^2}{\lim_{x \to 0} \frac{\log(1 + x)}{x}}$$

$= \dfrac{(\log 5)^2}{1} \quad \ldots \left[\begin{array}{c} \lim\limits_{x \to 0} \frac{a^x - 1}{x} = \log a, \\ \lim\limits_{x \to 0} \frac{\log(1+x)}{x} = 1 \end{array}\right]$

$= (\log 5)^2.$

Miscellaneous Exercise 7 | Q 2.16 | Page 106

Evaluate the following Limits: $\lim\limits_{x \to 0} \left[\dfrac{a^{4x} - 1}{b^{2x} - 1}\right]$

SOLUTION

$\lim\limits_{x \to 0} \dfrac{a^{4x} - 1}{b^{2x} - 1}$

$= \lim\limits_{x \to 0} \dfrac{\frac{a^{4x}-1}{x}}{\frac{b^{2x}-1}{x}}$

$= \dfrac{\lim\limits_{x \to 0} \left(\frac{a^{4x}-1}{4x}\right) \times 4}{\lim\limits_{x \to 0} \left(\frac{b^{2x}-1}{2x}\right) \times 2}$

$= \dfrac{4 \log a}{2 \log b} \quad \ldots \left[\begin{array}{c} \text{As } x \to 0,\ 2x \to 0,\ 4x \to 0 \\ \text{and } \lim\limits_{x \to 0} \frac{a^x - 1}{x} = \log a \end{array}\right]$

$= \dfrac{2 \log a}{\log b}.$

Miscellaneous Exercise 7 | Q 2.17 | Page 106

Evaluate the following Limits: $\lim\limits_{x \to 0} \left[\dfrac{\log 100 + \log(0.01 + x)}{x}\right]$

SOLUTION

$$\lim_{x\to 0}\left[\frac{\log 100+\log(0.01+x)}{x}\right]$$

$$= \lim_{x\to 0}\frac{\log[100(0.001+x)]}{x}$$

$$= \lim_{x\to 0}\frac{\log(1+100x)}{x}$$

$$= \lim_{x\to 0}\left[\frac{\log(1+100x)}{100x}\right]\times 100$$

= 1 x 100 ... $\left[\begin{array}{c}\text{As } x\to 0,\ 100\ x\to 0\ \text{ and}\\ \lim_{x\to 0}\frac{\log(1+x)}{x}=1\end{array}\right]$

= 100.

Miscellaneous Exercise 7 | Q 2.18 | Page 106

Evaluate the following Limits: $\lim_{x\to 0}\left[\frac{\log(4-x)-\log(4+x)}{x}\right]$

SOLUTION

$$\lim_{x\to 0}\frac{\log(4-x)-\log(4+x)}{x}$$

$$= \lim_{x\to 0}\frac{\log\left[4\left(1-\frac{x}{4}\right)\right]-\log\left[4\left(1+\frac{x}{4}\right)\right]}{x}$$

$$= \lim_{x\to 0}\frac{\log 4+\log\left(1-\frac{x}{4}\right)-\left[\log 4\ \log\left(1+\frac{x}{4}\right)\right]}{x}$$

$$= \lim_{x\to 0}\frac{\log\left(1-\frac{x}{4}\right)-\log\left(1+\frac{x}{4}\right)}{x}$$

$$= \lim_{x\to 0}\left[\frac{\log\left(1-\frac{x}{4}\right)}{x}-\frac{\log\left(1+\frac{x}{4}\right)}{x}\right]$$

$$= \lim_{x\to 0} \frac{\log\left(1-\frac{x}{4}\right)}{(-4)\left(-\frac{x}{4}\right)} - \lim_{x\to 0} \frac{\log\left(1+\frac{x}{4}\right)}{4\left(\frac{x}{4}\right)}$$

$$= -\frac{1}{4}\lim_{x\to 0} \frac{\log\left(1-\frac{x}{4}\right)}{-\frac{x}{4}} - \frac{1}{4}\lim_{x\to 0} \frac{\log\left(1+\frac{x}{4}\right)}{\frac{x}{4}}$$

$$= -\frac{1}{4}(1) - \frac{1}{4}(1) \quad \ldots \begin{bmatrix} \text{As } x \to 0, \frac{x}{4} \to 0, \frac{-x}{4_>} 0 \\ \text{and } \lim_{x\to 0} \frac{\log(1+x)}{x} = 1 \end{bmatrix}$$

$$= -\frac{1}{2}.$$

Miscellaneous Exercise 7 | Q 2.19 | Page 106

Evaluate the limit of the function if exist at x = 1 where, f(x) = $\begin{cases} 7-4x & x<1 \\ x^2+2 & x \geq 1 \end{cases}$

SOLUTION

f(x) = 7 – 4x : x < 1

= x^2 + 2 : x ≥ 1

$$\lim_{x\to 1^-} \mathrm{f}(x) = \lim_{x\to 1^-} (7-4x)$$

= 7 – 4(1)

= 3

$$\lim_{x\to 1^+} \mathrm{f}(x) = \lim_{x\to 1^+} \left(x^2+2\right)$$

= $(1)^2$ + 2

= 3

$$\therefore \lim_{x\to 1^-} \mathrm{f}(x) = \lim_{x\to 1^+} \mathrm{f}(x)$$

$\therefore \lim_{x\to 1} \mathrm{f}(x)$ exists.

$\therefore \lim_{x\to 1} \mathrm{f}(x)$ = 3.