Chapter 7: Wave Optics

EXERCISES [PAGES 184 - 185]

Exercises | Q 1.1 | Page 184
Choose the correct option:
Which of the following phenomenon proves that light is a transverse wave?

1. reflection
2. interference
3. diffraction
4. **polarization**

SOLUTION

polarization

Exercises | Q 1.2 | Page 184
Choose the correct option:
Which property of light does not change when it travels from one medium to another?

1. velocity
2. wavelength
3. amplitude
4. **frequency**

SOLUTION

frequency

Explanation:

Color and energy depend on frequency and frequency does not change. The light photons are the same as the photons at the beginning. Frequency is a parameter of the source of emission of light, not the medium it is traveling through.
Therefore, When light travels from one medium to another medium which is separated by a sharp boundary, the frequency does not change.

Exercises | Q 1.3 | Page 184
Choose the correct option:
When unpolarized light is passed through a polarizer, its intensity

1. increases
2. **decreases**
3. remain unchanged
4. depends on the orientation of the polarize

SOLUTION

When unpolarized light is passed through a polarizer, its intensity **decreases.**

Exercises | Q 1.4 | Page 184

Choose the correct option:
In Young's double-slit experiment, the two coherent sources have different intensities. If the ratio of the maximum intensity to the minimum intensity in the interference pattern produced is 25 : 1, what is the ratio of the intensities of the two sources?

1. 5 : 1
2. 25 : 1
3. 3 : 2
4. **9 : 4**

SOLUTION

9 : 4

Exercises | Q 1.5 | Page 184
Choose the correct option:
In Young's double-slit experiment, a thin uniform sheet of glass is kept in front of the two slits, parallel to the screen having the slits. The resulting interference pattern will satisfy:

1. **The interference pattern will remain unchanged**
2. The fringe width will decrease
3. The fringe width will increase
4. The fringes will shift

SOLUTION

The interference pattern will remain unchanged

Exercises | Q 2.1 | Page 184
Answer in brief:
What are primary and secondary sources of light?

SOLUTION

1. **Primary sources of light:**
   The sources that emit light on their own are called primary sources. This emission of light may be due to

   a. the high temperature of the source, e.g., the Sun, the stars, objects heated to high temperature, a flame, etc.

   b. the effect of current being passed through the source, e.g., tubelight, TV, etc.

   c. chemical or nuclear reactions taking place in the source, e.g., firecrackers, nuclear energy generators, etc.

2. **Secondary sources of light:**
   Some sources are not self-luminous, i.e., they do not emit light on their own, but reflect or scatter the light incident on them. Such sources of light are called secondary sources, e.g. the moon, the planets, objects such as humans,

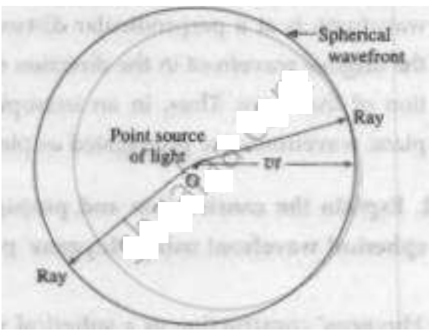
animals, plants, etc. These objects are visible due to reflected light.
Many of the sources that we see around are secondary sources and most of them are extended sources.

Exercises | Q 2.2 | Page 184

Answer in brief:
What is wavefront? How is it related to rays of light? What is the shape of the wavefront at a point far away from the source of light?

SOLUTION

Wavefront or wave surface:
The locus of all points where waves starting simultaneously from a source reach at the same instant of time and hence the particles at the points oscillate with the same phase is called a wavefront or wave surface.

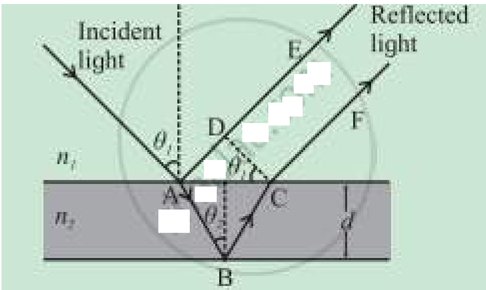


Consider a point source of light 0 in a homogeneous isotropic medium in which the speed of light is v. The source emits light in all directions. In time t, the disturbance (light energy) from the source, covers a distance vt in all directions, i.e., it reaches out to all points which are at a distance vt from the point source. The locus of these points which are in the same phase is the surface of a sphere with centre 0 and radius vt. It is a spherical wavefront. In a given medium, a set of straight lines can be drawn which are perpendicular to the wavefront. According to Huygens, these straight lines are the rays of light. Thus, rays are always normal to the wavefront. In the case of a spherical wavefront, the rays are radial. If a wavefront has travelled a large distance away from the source, a small portion of this wavefront appears to be plane. This part is a plane wavefront.

Exercises | Q 2.3 | Page 184

Answer in brief:
Why are multiple colours observed over a thin film of oil floating on water? Explain with the help of a diagram.

SOLUTION



Interference due to a thin film:
The brilliant colours of soap bubbles and thin films on the surface of water are due to the interference of light waves reflected from the upper and lower surfaces of the film. The two rays have a path difference which depends on the point on the film that is being viewed. This is shown in above figure.

The incident wave gets partially reflected from upper surface as shown by ray AE. The rest of the light gets refracted and travels along AB. At B it again gets partially reflected and travels along BC. At Cit refracts into air and travels along CF. The parallel rays AE and CF have a phase difference due to their different path lengths in different media. As can be seen from the figure, the phase difference depends on the angle of incidence θ_1, i.e., the angle of incidence at the top surface which is the angle of viewing, and also on the wavelength of the light as the refractive index of the material of the thin film depends on it. The two waves propagating along AE and CF interfere producing maxima and minima for different colours at different angles of viewing. One sees different colours when the film is viewed at different angles.

As the reflection is from the denser boundary, there is an additional phase difference of π radians (or an additional path difference λ). This should be taken into account for mathematical analysis.

Exercises | Q 2.4 | Page 184
Answer in brief:
In Young's double-slit experiment what will we observe on the screen when white light is incident on the slits but one slit is covered with a red filter and the other with a violet filter? Give reasons for your answer.

SOLUTION

In Young's double-slit experiment, when white light is incident on the slits and one of the slit is covered with a red filter, the light passing through this slit will emerge as the light having red colour. The other slit which is covered with a violet filter will give light having violet colour as emergent light. The interference fringes will involve mixing of red and violet colours. At some points, fringes will be red if constructive interference occurs for red colour and destructive interference occurs for violet colour. At some points, fringes will be violet if constructive interference occurs for violet colour and destructive interference occurs for red colour. The central fringe will be bright with the mixing of red and violet colours.

Exercises | Q 2.5 | Page 184

Answer in brief:

Explain what is the optical path length. How is it different from actual path length?

SOLUTION

Consider, a light wave of angular frequency w and wave vector k travelling through vacuum along the x-direction. The phase of this wave is (kx - ωt). The speed of light in vacuum is c and that in the medium is v.

$$k = \frac{2\pi}{\lambda} = \frac{2\pi v}{v\lambda} = \frac{\omega}{v} \text{ as } \omega = 2\pi v \text{ and } v = v\lambda\text{, where } v \text{ is the frequency of light.}$$

If the wave travels a distance Δ x, its phase changes by Δ Φ = kΔx = ω Δx/v.

Similarly, if the wave is travelling in vacuum,

k = ω/c and Δ Φ = ω Δ x/c

Now, consider a wave travelling a distance Δ x in the medium, the phase difference generated is,

Δ Φ' = k' Δ x = ωn Δ x/c = ω Δ x'/c ...(1)

where Δ x' = n Δ x .....(2)

The distance nΔ x is called the optical path length of the light in the medium; it is the distance the light would have travelled in the same time t in vacuum (with the speed c).

The optical path length in a medium is the corresponding path in a vacuum that the light travels at the same time as it takes in the given medium.

$$\text{Now, speed} = \frac{\text{distance}}{\text{time}}$$

$$\therefore \text{time} = \frac{\text{distance}}{\text{speed}}$$

$$\therefore t = \frac{d_{medium}}{v_{medium}} = \frac{d_{vaccum}}{v_{vaccum}}$$

$$\text{Hence, the optical path} = d_{vacuum}$$

$$= \frac{v_{vaccum}}{v_{medium}} \times d_{medium}$$

$$= n \times d_{medium}$$

Thus, a distance d travelled in a medium of refractive index n introduces a path difference = nd - d = d (n - 1) over a ray travelling equal distance through vacuum.

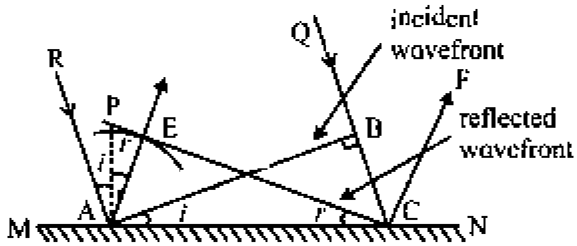
Exercises | Q 3 | Page 184

Answer in brief:

Derive the laws of reflection of light using Huygens' principle.

SOLUTION

Reflection of a plane wavefront of light at a plane surface



Where MN: Plane mirror,
RA and QC: Incident rays,
AP: Normal to MN,
AB: Incident wavefront,
i: Angle of incident,
CE: Reflected wavefront,
r: Angle of reflection

When wavefront AB is incident on the mirror, at first, point A becomes a secondary source and emits secondary waves in the same medium. If T is the time taken by the incident wavefront to travel from B to C, then BC = vT. During this time, the secondary wave originating at A covers the same distance, so that the secondary spherical wavelet has a radius vT at time T.

To construct the reflected wavefront, a hemisphere of radius vT is drawn from point A. Draw a tangent EC to the secondary wavelet.

The arrow AE shows the direction of propagation of the reflected wave.

AP is the normal to MN at A,

∠RAP = i = angle of incidence and

∠PAE = r = angle of reflection

In Δ ABC and Δ AEC,

AE = BC and ∠ABC = ∠AEC = 90°

∴ Δ ABC and Δ AEC are congruent.

∴ ∠ACE = ∠BAC = i .....(1)

Also, as AE is perpendicular to CE and AP is perpendicular to AC,

∠ACE = ∠PAE = r .....(2)

∴ From Eqs (1) and (2),

i = r

Thus, the angle of incidence is equal to the angle of reflection. This is the first law of reflection. Also, it can be seen from the figure that the incident ray and reflected ray lie on the opposite sides of the normal to the reflecting surface at the point of incidence and all of them lie in the same plane. This is the second law of reflection. Thus, the laws of reflection of light can be deduced by Huygens' construction of a plane wavefront.

Exercises | Q 4 | Page 184

Answer in brief:

Explain what is meant by polarization and derive Malus' law.

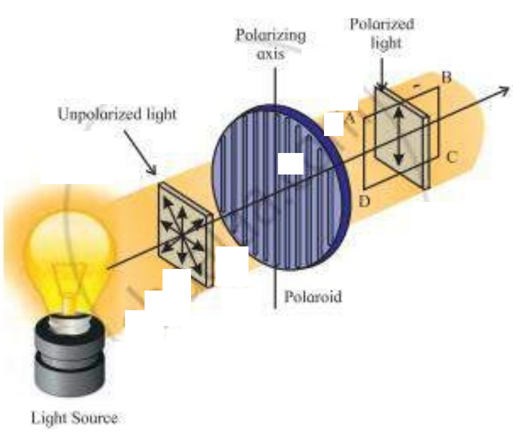
SOLUTION

According to the electromagnetic theory of light, a light wave consists of electric and magnetic fields vibrating at right angles to each other and to the direction of propagation of the wave. If the vibrations of $\vec{E}$ in a light wave are in all directions perpendicular to the direction of propagation of light, the wave is said to be unpolarized.

If the vibrations of the electric field $\vec{E}$ in a light wave are confined to a single plane containing the direction of propagation of the wave so that its electric field is restricted along one particular direction at right angles to the direction of propagation of the wave, the wave is said to be plane-polarized or linearly polarized.

This phenomenon of restricting the vibrations of light, i.e., of the electric field vector in a particular direction, which is perpendicular to the direction of the propagation of the wave is called polarization of light.

Polarization of light



Consider an unpolarized light wave travelling along the x-direction. Let c, v and λ be the speed, frequency and wavelength, respectively, of the wave. The magnitude of its electric field $\left(\vec{E}\right)$ is,

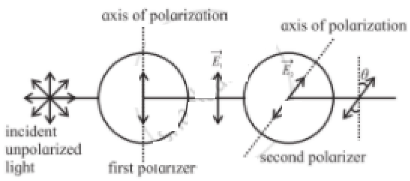
$E = E_0 \sin (kx - \omega t)$, where $E_0 = E_{max}$ = amplitude

of the wave, $\omega = 2\pi v$ = angular frequency of the wave and $k = 2\pi/\lambda$ = magnitude of the wave vector or propagation vector.

The intensity of the wave is proportional to $|\mathbf{E}_0|^2$.

The direction of the electric field can be anywhere in the y-z plane. This wave is passed through two identical polarizers as shown in the following figure.

Unpolarized light passing through two identical polarizers



When a wave with its electric field inclined at an angle Φ to the axis of the first polarizer is passed through the polarizer, the component $E_0 \cos \Phi$ will pass through it. The other component $E_0 \sin \Phi$ which is perpendicular to it will be blocked.

Now, after passing through this polarizer, the intensity of this wave will be proportional to the square of its amplitude, i.e., proportional to $|\mathbf{E}_0 \cos \phi|^2$.

The intensity of the plane-polarized wave emerging from the first polarizer can be obtained by averaging $|\mathbf{E}_0 \cos \phi|^2$ over all values of Φ between 0 and 180°. The intensity of the wave will be proportional to $\frac{1}{2}|\mathbf{E}_0|^2$ as the average value of $\cos^2 \Phi$ over this range is $\frac{1}{2}$. Thus the intensity of an unpolarized wave reduces by half after passing through a polarizer.

When the plane-polarized wave emerges from the first polarizer, let us assume that its electric field $\left(\vec{\mathbf{E}}_1\right)$ is along they-direction. In the above figure Thus, this electric field is

$\vec{\mathbf{E}}_1 = \hat{\mathbf{j}}\mathbf{E}_{10} \sin (kx - \omega t)$ ....(1)

where E_{10} is the amplitude of this polarized wave. The intensity of the polarized wave,

$I_1 \propto |E_{10}|^2$ .....(2)

Now, this wave passes through the second polarizer whose polarization axis (transmission axis) makes an angle θ with the y-direction. This allows only the component $E_{10} \cos \theta$ to pass through it. Thus, the amplitude of the wave which passes through the second polarizer is $E_{10} = E_{10} \cos \theta$ and its intensity,

$I_2 \propto |E_{20}|^2$

$\therefore I_2 \propto |E_{10}|^2 \cos^2 \theta$

$\therefore I_2 = I_1 \cos^2 \theta$ ......(3)

Thus, when plane-polarized light of intensity I_1 is incident on the second identical polarizer, the intensity of light transmitted by the second polarizer varies as $\cos^2 \theta$, i.e., $I_2 = I_1 \cos^2 \theta$, where θ is the angle between the transmission axes of the two polarizers. This is known as Malus' law.

Exercises | Q 5 | Page 184

What is Brewster's law? Derive the formula for Brewster angle.

SOLUTION

Brewster's law: The tangent of the polarizing angle is equal to the refractive index of the reflecting medium with respect to the surrounding (${}_1n_2$).

$$\text{If } \theta_B = {}_1n_2 = \frac{n_2}{n_1}$$

Here n_1 is the absolute refractive index of the surrounding and n_2 is that of the reflecting medium.

The angle θ_B is called the Brewster angle.

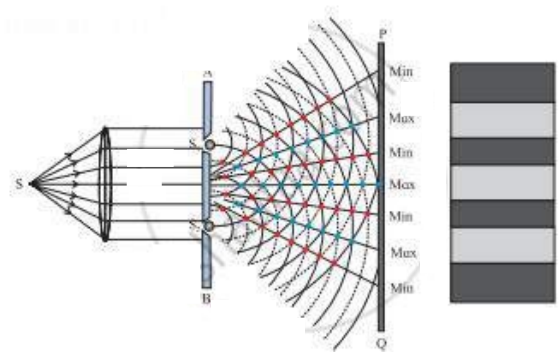
Exercises | Q 6 | Page 184

Describe Young's double-slit interference experiment and derive conditions for occurrence of dark and bright fringes on the screen. Define fringe width and derive a formula for it.

SOLUTION

Description of Young's double-slit interference experiment:

1. A plane wavefront is obtained by placing a linear source S of monochromatic light at the focus of a convex lens. It is then made to pass through an opaque screen AB having two narrow and similar slits S_1 and S_2. S_1 and S_2 are equidistant from S so that the wavefronts starting simultaneously from S and reaching S_1 and S_2 at the same time are in phase. A screen PQ is placed at some distance from screen AB as shown in following figure.



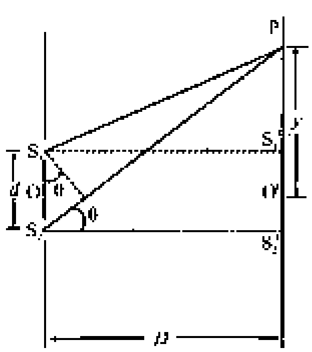
Young's double-slit experiment

2. S_1 and S_2 act as secondary sources. The crests/- troughs of the secondary wavelets superpose and interfere constructively along straight lines joining the black dots shown in the above figure. The point where these lines meet the screen have high intensity and is bright.

3. Similarly, there are points shown with red dots where the crest of one wave coincides with the trough of the other. The corresponding points on the screen are dark due to destructive interference. These dark and bright regions are called fringes or bands and the whole pattern is called an interference pattern.

Conditions for occurrence of dark and bright lunges on the screen:

Consider Young's double-slit experimental set up. Two narrow coherent light sources are obtained by wavefront splitting as monochromatic light of wavelength λ emerges out of two narrow and closely spaced, parallel slits S_1 and S_2 of equal widths. The separation $S_1 S_2 = d$ is very small. The interference pattern is observed on a screen placed parallel to the plane of S_1S_2 and at considerable distance.D (D » d) from the slits. OO' is the perpendicular bisector of a segment S_1S_2,



Geometry of the double-slit experiment

Consider, a point P on the screen at a distance y from O' (y « 0). The two light waves from S_1 and S_2 reach P along paths S_1P and S_2P, respectively. If the path difference (Δl) between S_1P and S_2P is an integral multiple of λ, the two waves arriving there will interfere constructively producing a bright fringe at P. On the contrary, if the path difference between S_1P and S_2P is a half-integral multiple of λ, there will be destructive interference and a dark fringe will be produced at P.

From above figure,

$(S_2P)^2 = (S_2S_2')^2 + (PS_2')^2$

$= (S_2S_2')^2 + (PO' + O'S_2')^2$

$$= D^2 + \left(y + \frac{d}{2}\right)^2 \qquad ....(1)$$

and $(S_1P)^2 = (S_1S_1')^2 + (PS_1')^2$

$= (S_1S_1')^2 + (PQ' - Q'S_1)^2$

$$= D^2 + \left(y - \frac{d}{2}\right)^2 \qquad .....(2)$$

$$(S_2P)^2 - (S_1P)^2 = \left\{D^2 + \left(y + \frac{d}{2}\right)^2\right\} - \left\{D^2 + \left(y - \frac{d}{2}\right)^2\right\}$$

$\therefore (S_2P + S_1P)(S_2P - S_1P)$

$$= \left[D^2 + y^2 + \frac{d^2}{4} + yd\right] - \left[D^2 + y^2 + \frac{d^2}{4} - yd\right] = 2yd$$

$\therefore S_2P + S_1P = \Delta l = 2yd/S_2P + S_1P$

In practice, D » y and D » d,

$\therefore S_2P + S_1P \cong 2D$

$\therefore$ Path difference,

Δ l = $S_2P + S_1P \cong 2\,\frac{yd}{2D} = y\frac{d}{D}$ ....(3)

The expression for the fringe width (or band width) : The distance between consecutive bright (or dark) fringes is called the fringe width (or bandwidth) W. Point P will be bright (maximum intensity), if the

path difference, Δ l = $y_n\frac{d}{D} = n\lambda$ where n = 0, 1, 2, 3, ...

Point P will be dark (minimum intensity equal to zero), if $y_m\frac{d}{D} = (2m-1)\frac{\lambda}{2}$, where, m = 1,2,3...,

Thus, for bright fringes (or bands),

$$y_n = 0, \lambda\frac{D}{d}, \frac{2\lambda D}{d} \ldots$$

and for dark fringes (or bands),

$$y_n = \frac{\lambda}{2}\frac{D}{d}, 3\frac{\lambda}{2}\frac{D}{d}, 5\frac{\lambda}{2}\frac{D}{d} \ldots.$$

These conditions show that the bright and dark fringes (or bands) occur alternately and are equally spaced. For Point O', the path difference (S_2O' - S_1O') = 0. Hence, point O' will be bright. It corresponds to the centre of the central bright fringe (or band). On both sides of O', the interference pattern consists of alternate dark and bright fringes (or band) parallel to the slit.

Let y_n and y_{n+1}, be the distances of the nth and (m + 1)th bright fringes from the central bright fringe.

$$\therefore \frac{y_n d}{D} = n\lambda$$

$$\therefore y_n = \frac{n\lambda D}{d} \quad .....(4)$$

$$\text{and } \frac{y_{n+1}d}{D} = (n+1)\lambda$$

$$\therefore (y_{n+1}) = \frac{(n+1)\lambda D}{d} \quad .....(5)$$

The distance between consecutive bright fringes

$$= y_{n+1} - y_n = \frac{\lambda D}{d}[(n+1) - n] = \frac{\lambda D}{d} \quad ....(6)$$

Hence, the fringe width,

$$\therefore W = \triangle y = y_{n+1} - y_n = \frac{\lambda D}{d} \text{ (for bright fringes) ... (7)}$$

Alternately, let y_m and y_{m+1} be the distances of the m th and (m + 1)th dark fringes respectively from the central bright fringe.

$$\therefore \frac{y_m d}{D} = (2m-1)\frac{\lambda}{2} \text{ and}$$

$$\frac{y_{m+1} d}{D} = [2(m+1)-1]\frac{\lambda}{2} = (2m+1)\frac{\lambda}{2} \quad \text{....(8)}$$

$$\therefore y_m = (2m - 1)\frac{\lambda D}{2d} \text{ and}$$

$$y_{m+1} = (2m+1)\frac{\lambda D}{2d} \quad \text{.....(9)}$$

∴ The distance between consecutive dark fringes,

$$y_{m+1} - y_m = \frac{\lambda D}{2d}[(2m+1)-(2m-1)] = \frac{\lambda D}{d} \quad \text{....(10)}$$

$$\therefore W = y_{m+1} - y_m$$

$$= \frac{\lambda D}{d} \text{ (for dark fringes)} \quad \text{.....(11)}$$

Eqs. (7) and (11) show that the fringe width is the same for bright and dark fringes.

Exercises | Q 7 | Page 184

What are the conditions for obtaining a good interference pattern? Give reasons.

SOLUTION

The conditions necessary for obtaining well defined and steady interference pattern:

1. **The two sources of light should be coherent:**
   The two sources must maintain their phase relation during the time required for observation. If the phases and phase differences vary with time, the positions of maxima and minima will also change with time and consequently the interference pattern will change randomly and rapidly, and a steady interference pattern would not be observed. For coherence, the two secondary sources must be derived from a single original source.

2. **The light should be monochromatic:**
   Otherwise, interference will result in complex coloured bands (fringes) because the separation of successive bright bands (fringes) is different for different colours. It also may produce overlapping bands.

3. The two light sources should be of equal brightness, i.e., the waves must have the same amplitude. The interfering light waves should have the same amplitude. Then, the points where the waves meet in opposite phase will be completely dark

(zero intensity). This will increase the contrast of the interference pattern and make it more distinct.

4. **The two light sources should be narrow:**
   If the source apertures are wide in comparison with the light wavelength, each source will be equivalent to multiple narrow sources and the superimposed pattern will consist of bright and less bright fringes. That is, the interference pattern will not be well defined.

5. **The interfering light waves should be in the same state of polarization:**
   Otherwise, the points where the waves meet in opposite phase will not be completely dark and the interference pattern will not be distinct.

6. **The two light sources should be closely spaced and the distance between the screen and the sources should be large:**
   Both these conditions are desirable for appreciable fringe separation. The separation of successive bright or dark fringes is inversely proportional to the closeness of the slits and directly proportional to the screen distance.

Exercises | Q 8 | Page 184

Answer in brief:
What is meant by coherent sources?

SOLUTION

Coherent sources: Two sources of light are said to be coherent if the phase difference between the emitted waves remains constant.

Exercises | Q 9 | Page 184
What is the diffraction of light? How does it differ from interference? What are Fraunhofer and Fresnel diffractions?

SOLUTION

- **The phenomenon of diffraction of light:**

When light passes by the edge of an obstacle or through a small opening or a narrow slit and falls on a screen, the principle of rectilinear propagation of light from geometrical optics predicts a sharp shadow. However, it is found that some of the light deviates from its rectilinear path and penetrates into the region of the geometrical shadow. This is a general characteristic of wave phenomena, which occurs whenever a portion of the wavefront is obstructed in some way. This bending of light waves at an edge into the region of the geometrical shadow is called diffraction of light.

- **Differences between interference and diffraction:**

1. The term interference is used to characterize the superposition of a few coherent waves (say, two). But when the superposition at a point involves a large number of waves coming from different parts of the same wavefront, the effect is referred to as diffraction.

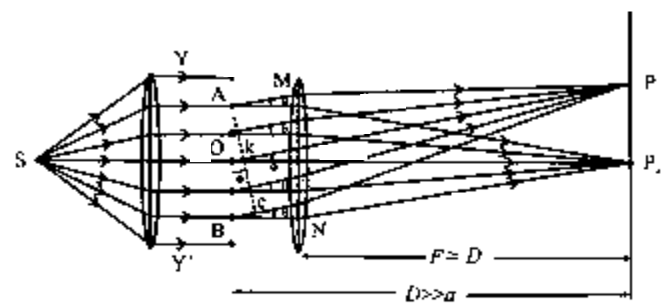
2. Double-slit interference fringes are all of equal width. In single-slit diffraction pattern, only the non-central maxima are of equal width which is half of that of the central maximum.

3. In double-slit interference, the bright and dark fringes are equally spaced. In diffraction, only the non-central maxima lie approximately halfway between the minima.

4. In double-slit interference, bright fringes are of equal intensity. In diffraction, successive noncentral maxima decrease rapidly in intensity.

- **Diffraction can be classified into two types depending on the distances involved in the experimental setup:**

1. **Fraunhofer diffraction:**
   In this class of diffraction, both the source and the screen are at infinite distances from the aperture. This is achieved by placing the source at the focus of a convex lens and the screen at the focal plane of another convex lens.



   Set up for Fraunhofer diffraction

2. **Fresnel diffraction:**
   In this class of diffraction, either the source of light or the screen or both are at finite distances from the diffracting aperture. The incident wavefront is either cylindrical or spherical depending on the source. A lens is not needed to observe the diffraction pattern on the screen.

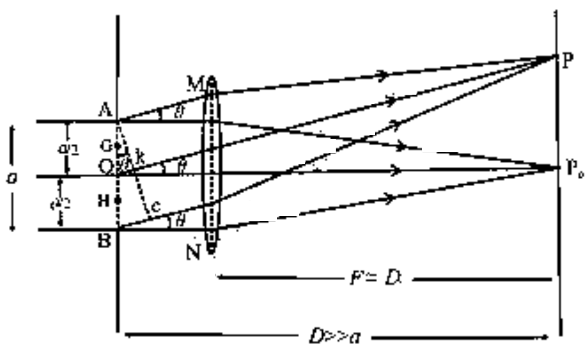
Exercises | Q 10 | Page 184

Derive the conditions for bright and dark fringes produced due to diffraction by a single slit.

SOLUTION

When a parallel beam of monochromatic light of wavelength λ. illuminates a single slit of finite width a, we observe on a screen some distance from the slit, a broad pattern of alternate dark and bright fringes. The pattern consists of a central bright fringe, with successive dark and bright fringes of diminishing intensity on both sides. This is called the diffraction pattern of a single slit.

Consider a single slit illuminated with a parallel beam of monochromatic light perpendicular to the plane of the slit. The diffraction pattern is obtained on a screen at a distance D (» a) from the slit and at the focal plane of the convex lens,



Fraunhofer diffraction due to a single slit

We can imagine the single slit as being made up of a large number of Huygens' sources evenly distributed over the width of the slit. Then the maxima and minima of the pattern arise from the interference of the various Huygens' wavelets.

Now, imagine the single slit as made up of two adjacent slits, each of width a/2. Since the incident plane wavefronts are parallel to the plane of the slit, all the Huygens sources at the slit will be in phase. They will therefore also in phase at the point P_0 on the screen, where P_0 is equidistant from all the Huygens sources. At P_0 then, we get the central maximum.

For the first minimum of intensity on the screen, the path difference between th e waves from the Huygens sources A and O (or 0 and B) is λ/2, which is the condition for destructive interference. Suppose, the nodal line OP for the first minimum subtends an angle θ at the slit; θ is very small. With P as the centre and PA as radius, strike an arc intersecting PB at C. Since, D » a, the arc AC can be considered a straight line at right angles to PB.

Then, Δ ABC is a right-angled triangle similar to Δ OP_0P.

This means that, ∠ BAC = θ

∴ BC = a sin θ

∴ Difference in path length,

BC = PB - PA = (PB - PO) + (PO - PA)

$$= \frac{\lambda}{2} + \frac{\lambda}{2} = \lambda$$

$\therefore$ a sin θ = λ

$$\therefore \sin\theta \cong \theta = \frac{\lambda}{a} \quad .....(1)$$

($\because$ θ is very small and in radian)

The other nodal lines of intensity minima can be understood in a similar way. In general, then, for the mth minimum (m = ± 1, ± 2, ± 3,....)

$$\theta_m = \frac{m\lambda}{a} \text{ (mth minimum) ...(2)}$$

as θ_m is very small and in radian.

Between the successive minima, the intensity rises to secondary maxima when the path difference is an odd-integral multiple of λ/2:

$$a \sin\theta_m = (2m + 1)\frac{\lambda}{2} = \left(m + \frac{1}{2}\right)\lambda$$

i.e., at angles given by,

$$\theta_m \cong \sin\theta_m = \left(m + \frac{1}{2}\right)\frac{\lambda}{a}$$

(mth secondary maximum) ......(3)

Exercises | Q 11 | Page 184

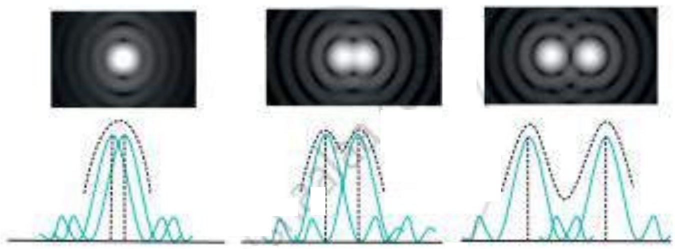
Describe Rayleigh's criterion for resolution. Explain it for a telescope and a microscope.

SOLUTION

Rayleigh's criterion for minimum resolution:

Two overlapping diffraction patterns due to two point sources are acceptably or just resolved if the center of the central peak of one diffraction pattern is as far as the first minimum of the other pattern.

The 'sharpness' of the central maximum of a diffraction pattern is measured by the angular separation between the center of the peak and the first minimum. It gives the limit of resolution.



(a) Unresolved (b) Just resolved (c) well resolved

Two overlapping diffraction patterns due to two point sources are not resolved if the angular separation between the central peaks is less than the limit of resolution in the first figure. They are said to be just separate or resolved if the angular separation between the central peaks is equal to the limit of resolution in the second figure. They are said to be well resolved if the angular separation between the central peaks is more than the limit of resolution in the third figure.

Resolving power of an optical instrument:

The primary aim of using an optical instrument is to see fine details, whether observing a star system through a telescope or a living cell through a microscope. After passing through an optical system, light from two adjacent parts of the object should produce sharp, distinct (separate) images of those parts. The objective lens or mirror of a telescope or microscope acts like a circular aperture. The diffraction pattern of a circular aperture consists of a central bright spot (called the Airy disc and corresponds to the central maximum) and concentric dark and bright rings.

Light from two close objects or parts of an object after passing through the aperture of an optical system produces overlapping diffraction patterns that tend to obscure the image. If these diffraction patterns are so broad that their central maxima overlap substantially, it is difficult to decide if the intensity distribution is produced by two separate objects or by one.

The resolving power of an optical instrument, e.g., a telescope or microscope, is a measure of its ability to produce detectably separate images of objects that are dose together.

Definition: The smallest linear or angular separation between two point objects which appear just resolved when viewed through an optical instrument is called the limit of resolution of the instrument and its reciprocal is called the resolving power of the instrument.

Exercises | Q 12 | Page 184

White light consists of wavelengths from 400 nm to 700 nm. What will be the wavelength range seen when white light is passed through a glass of refractive index 1.55?

SOLUTION

Let λ_1 and λ_2 be the wavelengths of light in water for 400 nm and 700 nm (wavelengths in a vacuum) respectively.

Let λ_a be the wavelength of light in vacuum.

$$\lambda_2 = \frac{\lambda_a}{n} = \frac{700 \times 10^{-9}\text{m}}{1.55} = 451.61 \times 10^{-9}\text{ m}$$

The wavelength range seen when white light is passed through the glass would be 258.06 nm to 451.61 run.

Exercises | Q 13 | Page 184

The optical path of a ray of light of a given wavelength travelling a distance of 3 cm in flint glass having refractive index 1.6 is the same as that on travelling a distance x cm through a medium having a refractive index 1.25. Determine the value of x.

SOLUTION

Let d_{fg} and d_m m be the distances by the ray of light in the flint glass and the medium respectively. Also, let n_{fg} and n_m be the refractive indices of the flint glass and the medium respectively.

Data: d_{fg} = 3 cm, n_{fg} = 1.6, n_m = 1.25,

Data: d_{fg} = 3 cm, n_{fg} = 1.6, n_m = 1.25,

Optical path = $\mathbf{n_m \times d_m = n_{fg} \times d_{fg}}$

$$\therefore \mathbf{d_m = \frac{n_{fg} \times d_{fg}}{n_m} = \frac{1.6 \times 3}{1.25} = 3.84} \text{ cm}$$

Thus, x cm = 3.84 cm

∴ x = 3.84

Exercises | Q 14 | Page 185

A double-slit arrangement produces interference fringes for sodium light (λ = 589 nm) that are 0.20° apart. What is the angular fringe separation if the entire arrangement is immersed in water (n = 1.33)?

SOLUTION

Data: θ_1 = 0.20°, n_w = 1.33

In the first approximation,

D sin θ_1 = y_1 and D sin θ_2 = y_2

$$\therefore \frac{\sin\theta_2}{\sin\theta_1} = \frac{\mathbf{y_2}}{\mathbf{y_1}} \quad ....(1)$$

Now, $\mathbf{y} \propto \frac{\lambda \mathbf{D}}{\mathbf{d}}$

For given d and D,

$y \propto \lambda$

$$\therefore \frac{\mathbf{y_2}}{\mathbf{y_1}} = \frac{\lambda_2}{\lambda_1} \quad ....(2)$$

Now, $\mathbf{n_w} = \frac{\lambda_1}{\lambda_2}$ ....(3)

From Eqs. (1), (2) and (3), we get,

$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{y_2}{y_1} = \frac{1}{n_w}$$

$$\therefore \sin\theta_2 = \frac{\sin\theta_1}{n_w}$$

$$= \frac{\sin 0.2}{1.33} = \frac{0.0035}{1.33}$$

= 0.0026

$$\therefore \theta_2 = \sin^{-1} 0.0026 = 9\prime = 0.15^\circ$$

This is the required angular fringe separation

Exercises | Q 15 | Page 185

Answer in brief:
In a double-slit arrangement, the slits are separated by a distance equal to 100 times the wavelength of the light passing through the slits.
(a) What is the angular separation in radians between the central maximum and an adjacent maximum?
(b) What is the distance between these maxima on a screen 50.0 cm from the slits?

SOLUTION

Data: d = 100λ, D = 50.0 cm

(a) The conditions for maximum in Young's experiment is given by:

d sin (θ) = n λ, n = 0, 1, 2...

the angle between the central maximum and it's adjacent can be determined by setting m equals to 1, so:

d sin (θ) = λ

$$\theta = \sin^{-1}\left(\frac{\lambda}{d}\right)$$

$$\theta = \sin^{-1}\left(\frac{\lambda}{100\lambda}\right)$$

$$\theta = \sin^{-1}\left(\frac{1}{100}\right)$$

= 0.9' = **0.01571 rad**

(b) The distance between the central axis and the first maximum is given by:

$$D \sin \theta = D\left(\frac{\lambda}{d}\right)$$

$$= (50.0 \text{ cm})\left(\frac{\lambda}{100\lambda}\right)$$

$$= \mathbf{0.50 \text{ cm}}$$

Exercises | Q 16 | Page 185

Unpolarized light with intensity 10 is incident on two polaroids. The axis of the first polaroid makes on the angle of 50° with the vertical, and the axis of the second polaroid is horizontal. What is the intensity of the light after it has passed through the second polaroid?

SOLUTION

According to Malus' law, when the unpolarized light with intensity I_0 is incident on the first polarizer, the polarizer polarizes this incident light. The intensity of light becomes $I_1 = I_0/2$.

Now, $I_2 = I_1 \cos^2\theta$

$$\therefore I_2 = \left(\frac{I_0}{2}\right)\cos^2(\theta_2 - \theta_1)$$

$$= \left(\frac{I_0}{2}\right)\cos^2(90^\circ - 50^\circ)$$

$$\therefore I_2 = \left(\frac{I_0}{2}\right)\cos^2 40^\circ$$

The intensity of light after it has passed through the second polaroid $\left(\frac{I_0}{2}\right)\cos^2 40^\circ = \frac{I_0}{2}(0.76660)^2$

$$= \mathbf{0.2934\ I_0}$$

Exercises | Q 17 | Page 185

In a biprism experiment, the fringes are observed in the focal plane of the eyepiece at a distance of 1.2 m from the slits. The distance between the central bright band and the 20th bright band is 0.4 cm. When a convex lens is placed between the biprism and the eyepiece, 90 cm from the eyepiece, the distance between the two virtual magnified images is found to be 0.9 cm. Determine the wavelength of light used.

SOLUTION

Data: D = 1.2 m

The distance between the central bright band and the 20th bright band is 0.4 cm.

$\therefore y_{20} = 0.4 \text{ cm} = 0.4 \times 10^{-2}$ m

$$W = \frac{y_{20}}{20} = \frac{0.4}{20} \times 10^{-2}\text{m} = 2 \times 10^{-4}\text{m},$$

$d_1 = 0.9 \text{ cm} = 0.9 \times 10^{-2}$ m, $v_1 = 90 \text{ cm} = 0.9$ m

$\therefore u_1 = D - v_1 = 1.2 \text{ m} - 0.9 \text{ m} = 0.3$ m

Now, $\frac{d_1}{d} = \frac{v_1}{u_1}$

$$\therefore d = \frac{d_1 u_1}{v_1} = \frac{(0.9 \times 10^{-2})(0.3)}{0.9} \text{ m}$$

$= 3 \times 10^{-3}$ m

$\therefore$ The wavelength of light,

$$\lambda = \left(\frac{Wd}{D}\right) = \frac{2 \times 10^{-4} \times 3 \times 10^{-3}}{1.2} \text{ m}$$

$= 5 \times 10^{-7}$ m

$= 5 \times 10^{-7} \times 10^{10}$ Å

= 5000 Å

Exercises | Q 18 | Page 185

In Fraunh offer diffraction by a narrow slit, a screen is placed at a distance of 2 m from the lens to obtain the diffraction pattern. If the slit width is 0.2 mm and the first minimum is 5 mm on either side of the central maximum, find the wavelength of light.

SOLUTION

Data: D = 2 m, y_{1d} = 5 mm = 5×10^{-3} m,

a = 0.2 mm = 0.2×10^{-3} m = 2×10^{-4} m

$$y_{md} = m\frac{\lambda D}{a}$$

$$\therefore \lambda = \frac{y_{1d}a}{D} \qquad ....(\because m = 1)$$

$$\lambda = \frac{5 \times 10^{-3} \times 2 \times 10^{-4}}{2}$$

$\lambda = 5 \times 10^{-7}$ m = $5 \times 10^{-7} \times 10^{-10}$ Å = **5000 Å**

Exercises | Q 19 | Page 185

The intensity of the light coming from one of the slits in Young's experiment is twice the intensity of the light coming from the other slit. What will be the approximate ratio of the intensities of the bright and dark fringes in the resulting interference pattern?

SOLUTION

Data: $I_1 : I_2 = 2 : 1$

If E_{10} and E_{20} are the amplitudes of the interfering waves, the ratio of the maximum intensity to the minimum intensity in the fringe system is

$$\frac{I_{max}}{I_{min}} = \left(\frac{E_{10} + E_{20}}{E_{10} - E_{20}}\right)^2 = \left(\frac{r + 1}{r - 1}\right)^2$$

where $r = \dfrac{E_{10}}{E_{20}}$

$$\therefore \frac{I_1}{I_2} = \left(\frac{E_{10}}{E_{20}}\right)^2 = r^2$$

$$\therefore r = \sqrt{\frac{I_1}{I_2}} = \sqrt{2}$$

$$\therefore \frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)^2 = \left(\frac{2.414}{0.414}\right)^2 = (5.83)^2$$

= 33.99 ≈ 34.

∴ The ratio of the intensities of the bright and dark fringes in the resulting interference pattern is 34: 1.

Exercises | Q 20 | Page 185

A parallel beam of green light of wavelength 546 nm passes through a slit of width 0.4 mm. The intensity pattern of the transmitted light is seen on a screen that is 40 cm away. What is the distance between the two first-order minima?

SOLUTION

Data: $\lambda = 546 \text{ nm} = 546 \times 10^{-9}$ m,

$a = 0.4 \text{ mm} = 4 \times 10^{-4}$ m

$D = 40 \text{ cm} = 40 \times 10^{-2}$ m

$$y_{md} = m\frac{\lambda D}{a}$$

$$\therefore y_{1d} = 1\frac{\lambda D}{a}$$ and

$$2y_{1d} = \frac{2\lambda D}{a}$$

$$= \frac{2 \times 546 \times 10^{-9} \times 40 \times 10^{-2}}{4 \times 10^{-4}} \text{ m}$$

$= 2 \times 546 \times 10^{-6} = 1092 \times 10^{-6}$

$= 1.092 \times 10^{-3}$ m = **1.092 mm**

Exercises | Q 21 | Page 185

What must be the ratio of the slit width to the wavelength of light for a single slit to have the first diffraction minimum at 45.0°?

SOLUTION

Data: $\theta = 45°$, m = 1

a sin θ = mλ for (m = 1, 2, 3... minima)

Here, m = 1 (First minimum)

$\therefore$ a sin 45° = (1)λ

$$\therefore \frac{a}{\lambda} = \frac{1}{\sin 45^\circ} = \mathbf{1.414}$$

Exercises | Q 22 | Page 185

Monochromatic electromagnetic radiation from a distant source passes through a slit. The diffraction pattern is observed on a screen 2.50 m from the slit. If the width of the central maximum is 6.00 mm, what is the slit width if the wavelength is
(a) 500 nm (visible light)
(b) 50 µm (infrared radiation)
(c) 0.500 nm (X rays)?

SOLUTION

Data: 2W = 6 mm $\therefore$ W = 3 mm = 3×10^{-3} m, y = 2.5 m,

(a) λ_1 = 500 nm = 5×10^{-7} m

(b) λ_2 = 50 µm = 5×10^{-5} m

(c) λ_3 = 0.500 nm = 5×10^{-10} m

Let a be the slit width.

(a) $W = \frac{y\lambda_1}{a}$

$$\therefore a = \frac{y\lambda_1}{W} = \frac{(2.5)\left(5 \times 10^{-7}\right)}{3 \times 10^{-3}}$$

= 4.167×10^{-4} m

= 0.4167 mm

(b) W = $\frac{y\lambda_2}{a}$

$\therefore$ a = $\frac{y\lambda_2}{W} = \frac{(2.5)(5 \times 10^{-5})}{3 \times 10^{-3}}$

= 4.167×10^{-2} m

= 41.67 mm

(c) W = $\frac{y\lambda_3}{a}$

$\therefore$ a = $\frac{y\lambda_3}{W} = \frac{(2.5)(5 \times 10^{-10})}{3 \times 10^{-3}}$

= 4.167×10^{-7} m

= 4.167×10^{-4} mm

Exercises | Q 23 | Page 185

A star is emitting light at the wavelength of 5000 A. Determine the limit of resolution of a telescope having an objective of a diameter of 200 inch.

SOLUTION

Data: λ = 5000 Å = 5×10^{-7} m

D = 200 × 2.54 cm = 5.08 m

θ = $\frac{1.22\lambda}{D}$

$= \frac{1.22 \times 5 \times 10^{-7}}{5.08}$

= 1.2×10^{-7} rad

Exercises | Q 24 | Page 185

Answer in brief:

The distance between two consecutive bright fringes in a biprism experiment using the light of wavelength 6000 Å is 0.32 mm by how much will the distance change if light of wavelength 4800 Å is used?

SOLUTION

Data: λ_1 = 6000 Å = 6×10^{-7} m, λ_2 = 4800 Å = 4.8×10^{-7} m, W_1 = 0.32 mm = 3.2×10^{-4} m

Distance between consecutive bright fringes,

$$W = \frac{\lambda D}{d}$$

For $\lambda_1, W_1 = \frac{\lambda_1 D}{d}$ and ...(1)

For $\lambda_2, W_2 = \frac{\lambda_2 D}{d}$ and ...(2)

$$\frac{W_2}{W_1} = \frac{\lambda_2 D/d}{\lambda_1 D/d} = \frac{\lambda_2}{\lambda_1}$$

$$\therefore W_2 = \left(\frac{\lambda_2}{\lambda_1}\right) W_1 = \left(\frac{4.8 \times 10^{-7}}{6 \times 10^{-7}}\right)\left(3.2 \times 10^{-4}\right)$$

$$= (0.8)\left(3.2 \times 10^{-4}\right) \text{m}$$

$$= 2.56 \times 10^{-4} \text{ m}$$

$$\therefore \triangle W = W_1 - W_2$$

$= 3.2 \times 10^{-4}$ m - 2.56×10^{-4} m

$= 0.64 \times 10^{-4}$ m

$= 6.4 \times 10^{-5}$ m

= 0.064 mm