Chapter 3: Skewness

EXERCISE 3.1 [PAGE 43]

Exercise 3.1 | Q 1 | Page 43

For a distribution, mean = 100, mode = 127 and SD = 60. Find the Pearson coefficient of skewness Sk_p.

SOLUTION

Given, Mean = 100, Mode = 127, S.D. = 60

Pearsonian coefficient of skewness,

$$Sk_p = \frac{\text{Mean} - \text{Mode}}{\text{S.D.}}$$

$$= \frac{100 - 27}{60}$$

$$= \frac{-27}{60}$$

$$= -0.45$$

Exercise 3.1 | Q 2 | Page 43

The mean and variance of the distribution is 60 and 100 respectively. Find the mode and the median of the distribution if Sk_p = – 0.3.

SOLUTION

Given, Mean = 60, Variance = 100, $Sk_p = -0.3$

$\therefore$ S.D. $= \sqrt{\text{Variance}} = \sqrt{100} = 10$

$Sk_p = -0.3$

Pearsonian coefficient of skewness,

$$Sk_p = \frac{\text{Mean} - \text{Mode}}{\text{S.D.}}$$

$$\therefore -0.3 = \frac{60 - \text{Mode}}{10}$$

$\therefore -3 = 60 - \text{Mode}$

$\therefore$ Mode $= 60 + 3 = 63$

Mean – Mode = 3(Mean – Median)

$\therefore 60 - 63 = 3(60 - \text{Median})$

$\therefore -3 = 180 - 3$ Median

$\therefore 3$ Median $= 180 + 3 = 183$

$$\therefore \text{Median} = \frac{183}{3}$$

$\therefore$ Median = 61

Exercise 3.1 | Q 3 | Page 43

For a data set, sum of upper and lower quartiles is 100, difference between upper and lower quartiles is 40 and median is 30. Find the coefficient of skewness.

SOLUTION

Given, $Q_3 + Q_1 = 100$ ....(i)

$Q_3 - Q_1 = 40$ ....(ii)

Median $= Q_2 = 30$

Adding (i) and (ii), we get

$2Q_3 = 140$

$\therefore Q_3 = \dfrac{140}{2} = 70$

Substituting the value of Q_3 in (i), we get

$70 + Q_1 = 100$

$\therefore Q_1 = 100 - 70 = 30$

$$Sk_b = \frac{Q_3 + Q_1 - 2Q_2}{Q_3 - Q_1}$$

$$= \frac{70 + 30 - 2(30)}{40}$$

$$= \frac{70 + 30 - 60}{40}$$

$$\therefore Sk_b = \frac{40}{40}$$

$\therefore Sk_b = 1$

Exercise 3.1 | Q 4 | Page 43

For a data set with upper quartile equal to 55 and median equal to 42. If the distribution is symmetric, find the value of lower quartile.

SOLUTION

Upper quartile = Q_3 = 55

Median = Q_2 = 42

Since the distribution is symmetric.

$\therefore Sk_b = 0$

$$Sk_b = \frac{Q_3 + Q_1 - 2Q_2}{Q_3 - Q_1}$$

$$\therefore 0 = \frac{Q_3 + Q_1 - 2Q_2}{Q_3 - Q_1}$$

$\therefore 0 = Q_3 + Q_1 - 2Q_2$

$\therefore Q_1 = 2Q_2 - Q_3$

$\therefore Q_1 = 2(42) - 55$

$\therefore Q_1 = 84 - 55$

$\therefore Q_1 = 29$

Exercise 3.1 | Q 5 | Page 43

Obtain the coefficient of skewness by formula and comment on nature of the distribution.

Height in inches	No. of females
Less than 60	10
60 – 64	20
64 – 68	40
68 – 72	10
72 – 76	2

SOLUTION

We construct the less than cumulative frequency table as given below.

Height in inches	No. of females (f)	Less than cumulative frequency (c.f.)
Less than 60	10	10
60 – 64	20	30 ← Q_1
64 – 68	40	70 ← Q_2, Q_3
68 – 72	10	80
72 – 76	2	82
Total	N = 82	

Q_1 class = class containing $\left(\frac{N}{4}\right)^{th}$ observation

$\therefore \frac{N}{4} = \frac{82}{4} = 20.5$

Cumulative frequency which is just greater than (or equal) to 20.5 is 30.

$\therefore Q_1$ lies in the class 60 – 64.

L = 60, f = 20, c.f. = 10, h = 4

$\therefore Q_1 = L + \frac{h}{f}\left(\frac{N}{4} - c.f.\right)$

$= 60 + \frac{4}{20}(20.5 - 10)$

$= 60 + \frac{1}{5} \times 10.5$

= 60 + 2.1

$\therefore Q_1 = 62.1$

Q_2 class = class containing $\left(\frac{N}{2}\right)^{th}$ observation

$\therefore \frac{\mathbf{N}}{\mathbf{2}} = \frac{\mathbf{82}}{\mathbf{4}}$ = 41

Cumulative frequency which is just greater than (or equal) to 41 is 70.

$\therefore$ Q_2 lies in the class 64 - 68

$\therefore$ L = 64, h = 4, f = 40, c.f. = 30

$\therefore$ Q_2 = $\mathbf{L} + \frac{\mathbf{h}}{\mathbf{f}}\left(\frac{\mathbf{N}}{\mathbf{2}} - \mathbf{c.f.}\right)$

= $\mathbf{64} + \frac{\mathbf{4}}{\mathbf{40}}(\mathbf{41} - \mathbf{30})$

= $\mathbf{64} + \frac{\mathbf{1}}{\mathbf{10}}(\mathbf{11})$

= 64 + 1.1

$\therefore$ Q_2 = 65.1

Q_3 class = class containing $\left(\frac{\mathbf{3N}}{\mathbf{4}}\right)^{\mathbf{th}}$ observation

$\therefore \frac{3N}{4} = \frac{3 \times 82}{4} = 61.5$

Cumulative frequency which is just greater than (or equal) to 61.5 is 70.

$\therefore Q_3$ lies in the class 64 – 68

L = 64, f = 40, c.f. = 30, h = 4

$\therefore Q_3 = L + \frac{h}{f}\left(\frac{3N}{4} - c.f.\right)$

$= 64 + \frac{4}{40}(61.5 - 30)$

$= 64 + \frac{1}{10} \times 31.5$

= 64 + 3.15

$\therefore Q_3 = 67.15$

$Sk_b = \frac{Q_3 + Q_1 - 2Q_2}{Q_3 - Q_1}$

$= \frac{67.15 + 62.1 - 2(65.1)}{67.15 - 62.1}$

$= \frac{129.25 - 130.2}{5.05}$

$= \frac{-0.95}{5.05}$

$\therefore Sk_b = -0.1881$

Since, $Sk_b < 0$, the distribution is negatively skewed.

Exercise 3.1 | Q 6 | Page 43

Find Sk_p for the following set of observations.
17, 17, 21, 14, 15, 20, 19, 16, 13, 17, 18

SOLUTION

$\sum x_i$ = 17 + 17 + 21 + 14 + 15 + 20 + 19 + 16 + 13 + 17 + 18 = 187

Here, n = 11

Mean = $\frac{\sum x_i}{n}$

= $\frac{187}{11}$

= 17

Mode = Observation that occurs most frequently in the data

= 17

Sk_p = $\frac{\text{Mean} - \text{Mode}}{\text{S.D.}}$

= $\frac{17 - 17}{\text{S.D.}}$

= $\frac{0}{\text{S.D.}}$

= 0

Exercise 3.1 | Q 7 | Page 43

Calculate Sk_b for the following set of observations of yield of wheat in kg from 13 plots:
4.6, 3.5, 4.8, 5.1, 4.7, 5.5, 4.7, 3.6, 4.2, 3.5, 3.6, 5.2

SOLUTION

The given data can be arranged in ascending order as follows:

3.5, 3.5, 3.5, 3.6, 3.6, 4.2, 4.6, 4.7, 4.7, 4.8, 5.1, 5.2, 5.5

Here, n = 13

Q_1 = value of $\left(\frac{n+1}{4}\right)^{th}$ observation

= value of $\left(\frac{13\ +1}{4}\right)^{th}$ observation

= value of $(3.50)^{th}$ observation

= value pf 3^{rd} observation + 0.5 (value of 4^{th} observation – value of 3^{rd} observation)

= 3.5 + 0.50 (3.6 – 3.5)

= 3.5 + 0.50 × 0.1

= 3.5 + 0.05

∴ Q_1 = 3.55

Q_2 = value of $2\left(\frac{n+1}{4}\right)^{th}$ observation

$= \text{value of } 2\left(\frac{13+1}{4}\right)^{\text{th}}$ observation

= value of $(2 \times 3.50)^{\text{th}}$ observation

= value of 7^{th} observation

$\therefore Q^2 = 4.6$

Q_3 = value of $3\left(\frac{n+1}{4}\right)^{\text{th}}$ observation

= value of $3\left(\frac{13+1}{4}\right)^{\text{th}}$ observation

= value of $(3 \times 3.50)^{\text{th}}$ observation

= value of $(10.50)^{\text{th}}$ observation

= value of 10^{th} observation + 0.5 (value of 11^{th} observation – value of 10^{th} observation)

$= 4.8 + 0.50\ (5.1 - 4.8)$

$= 4.8 + 0.50 \times 0.3$

$= 4.8 + 0.15$

$\therefore Q_3 = 4.95$

$\therefore Sk_b = \frac{Q_3 + Q_1 - 2Q_2}{Q_3 - Q_1}$

$= \frac{4.95 + 3.55 - 2(4.6)}{4.95 + 3.55}$

$= \frac{8.5 - 9.2}{1.4}$

$= \frac{-0.7}{1.4}$

$\therefore Sk_b = -0.5$

Exercise 3.1 | Q 8 | Page 43

For a frequency distribution $Q_3 - Q_2 = 90$ And $Q_2 - Q_1 = 120$, find Sk_b.

SOLUTION

Given, $Q_2 - Q_1 = 120$, $Q_3 - Q_2 = 90$

Bowley's co-efficient of skewness,

$$\therefore Sk_b = \frac{Q_3 + Q_1 - 2Q_2}{Q_3 - Q_1}$$

$$= \frac{Q_3 - Q_2 - Q_2 + Q_1}{Q_3 - Q_2 + Q_2 - Q_1}$$

$$= \frac{(Q_3 - Q_2) - (Q_2 - Q_1)}{(Q_3 - Q_2) + (Q_2 - Q_1)}$$

$$= \frac{90 - 120}{90 + 120}$$

$$= \frac{-30}{210}$$

$$= \frac{-1}{7}$$

$$\therefore Sk_b = -0.1429$$

MISCELLANEOUS EXERCISE 3 [PAGE 44]

Miscellaneous Exercise 3 | Q 1 | Page 44

For a distribution, mean = 100, mode = 80 and S.D. = 20. Find Pearsonian coefficient of skewness Sk_p.

SOLUTION

Given, Mean = 100, Mode = 80, S.D. = 20

$Sk_p = \dfrac{\text{Mean} - \text{Mode}}{\text{S.D.}}$

$= \dfrac{100 - 80}{20}$

$= \dfrac{20}{20}$

$= 1$

$\therefore Sk_p = 1$

Miscellaneous Exercise 3 | Q 2 | Page 44

For a distribution, mean = 60, median = 75 and variance = 900. Find Pearsonian coefficient of skewness Sk_p.

SOLUTION

Given, Mean = 60, Median = 75, Variance = 900

$\therefore$ S.D. $= \sqrt{\text{Variance}} = \sqrt{900} = 30$

$Sk_p = \dfrac{3(\text{Mean} - \text{Median})}{\text{S.D.}}$

$= \dfrac{3(60 - 75)}{30}$

$= \dfrac{3(-15)}{30}$

$= \dfrac{-15}{10}$

$\therefore Sk_p = -1.5$

Miscellaneous Exercise 3 | Q 3 | Page 44

For a distribution, $Q_1 = 25$, $Q_2 = 35$ and $Q_3 = 50$. Find Bowley's coefficient of skewness Sk_b.

SOLUTION

Given, $Q_1 = 25$, $Q_2 = 35$ and $Q_3 = 50$

$$Sk_b = \frac{Q_3 + Q_1 - 2Q_2}{Q_3 - Q_1}$$

$$= \frac{50 + 25 - 2(35)}{50 - 25}$$

$$= \frac{75 - 70}{25}$$

$$= \frac{5}{25}$$

$$= \frac{1}{5}$$

$\therefore Sk_b = 0.2$

Miscellaneous Exercise 3 | Q 4 | Page 44

For a distribution $Q_3 - Q_2 = 40$, $Q_2 - Q_1 = 60$. Find Bowley's coefficient of skewness Sk_b.

SOLUTION

Given, $Q_3 - Q_2 = 40$, $Q_2 - Q_1 = 60$

$$Sk_b = \frac{Q_3 + Q_1 - 2Q_2}{Q_3 - Q_1}$$

$$= \frac{Q_3 - Q_2 - Q_2 + Q_1}{Q_3 - Q_2 + Q_2 - Q_1}$$

$$= \frac{(Q_3 - Q_2) - (Q_2 - Q_1)}{(Q_3 - Q_2) + (Q_2 - Q_1)}$$

$$= \frac{40 - 60}{40 + 60}$$

$$= -\frac{20}{100}$$

$$= -\frac{1}{5}$$

$\therefore Sk_b = -0.2$

Miscellaneous Exercise 3 | Q 5 | Page 44

For a distribution, Bowley's coefficient of skewness is 0.6. The sum of upper and lower quartiles is 100 and median is 38. Find the upper and lower quartiles.

SOLUTION

Given, $Sk_b = 0.6$, $Q_3 + Q_1 = 100$,

Median = $Q_2 = 38$

$$Sk_b = \frac{Q_3 + Q_1 - 2Q_2}{Q_3 - Q_1}$$

$$\therefore 0.6 = \frac{100 - 2(38)}{Q_3 - Q_1}$$

$\therefore 0.6(Q_3 - Q_1) = 100 - 76 = 24$

$$\therefore Q_3 - Q_1 = \frac{24}{0.6}$$

$\therefore Q_3 - Q_1 = 40$ ....(i)

$Q_3 + Q_1 = 100$ ....(ii) (given)

Adding (i) and (ii), we get

$2Q_3 = 140$

$$\therefore Q_3 = \frac{140}{2} = 70$$

Substituting the value of Q_3 in (ii), we get

$70 + Q_1 = 100$

$\therefore Q_1 = 100 - 70 = 30$

$\therefore$ upper quartile = 70 and lower quartile = 30.

Miscellaneous Exercise 3 | Q 6 | Page 44

For a frequency distribution, the mean is 200, the coefficient of variation is 8% and Karl Pearsonian's coefficient of skewness is 0.3. Find the mode and median of the distribution.

SOLUTION

Mean = $\bar{x}$ = 200, Coefficient of variation,

C.V. = 8%, $Sk_p = 0.3$

C.V. = $\frac{\sigma}{\bar{x}} \times 100$, where σ = standard deviation

$\therefore 8 = \frac{\sigma}{200} \times 100$

$\therefore \sigma = \frac{8 \times 200}{100} = 16$

Now, $Sk_p = \frac{\text{Mean} - \text{Mode}}{\text{S.D.}}$

$\therefore 0.3 = \frac{200 - \text{Mode}}{16}$

$\therefore 0.3 \times 16 = 200 - \text{Mode}$

$\therefore$ Mode = 200 – 4.8 = 195.2

Since, Mean – Mode = 3(mean – Median)

$\therefore$ 200 – 195.2 = 3(200 – Median)

$\therefore$ 4.8 = 600 – 3 Median

$\therefore$ 3 Median = 600 – 4.8 = 595.2

$\therefore \text{Median} = \frac{595.2}{3} = 198.4$

Miscellaneous Exercise 3 | Q 7 | Page 44

Calculate Karl Pearsonian's coefficient of skewness Sk_p from the following data:

Marks above	0	10	20	30	40	50	60	70	80
No. of students	120	115	108	98	85	60	18	5	0

SOLUTION

The given table is the cumulative frequency table of more than type. From this table, we have to prepare the frequency distribution table and then calculate the value of Skp. Construct the following table:

Mark above	**No. of students 'more than' (c.f.)**	**Class-interval**	**Frequency f_i**	**Mid value x_i**	**$f_i x_i$**	**$f_i x_i^2$**
0	120	0 – 10	5	5	25	125
10	115	10 – 20	7	15	105	1575
20	108	20 – 30	10	25	250	6250
30	98	30 – 40	13	35	455	15925
40	85	40 – 50	25	45	1125	50625
50	60	50 – 60	42	55	2310	127050
60	18	60 – 70	13	65	845	54925
70	5	70 – 80	5	75	375	28125
80	0	80 – 90	0	85	0	0
		Total	**120**	**–**	**5490**	**284600**

From the table, N = 120, $\sum f_i x_i = 5490$ and $\sum f_i x_i^2 = 284600$

Mean = $\bar{x} - \dfrac{\sum f_i x_i}{N} = \dfrac{5490}{120}$ = 45.75

Maximum frequency 42 is of the class 50 – 60.

∴ Mode lies in the class 50 – 60.

∴ L = 50, f1 = 42, f0 = 25, f2 = 13, h = 10

∴ Mode = $L + \dfrac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$

= $50 + \dfrac{42 - 25}{2(42) - 25 - 13} \times 10$

= $50 + \dfrac{17}{84 - 38} \times 10$

= $50 + \dfrac{17}{46} \times 10$

= 50 + 3.6957

= 53.6957

S.D. = $\sqrt{\dfrac{\sum f_i x_i^2}{N} - (\bar{x})^2}$

= $\sqrt{\dfrac{284600}{120} - (45.75)^2}$

= $\sqrt{2371.6667 - 2093.0625}$

= $\sqrt{278.6042}$

= 16.6914

Pearsonian's coefficient of skewness:

Sk_p = $\dfrac{\text{Mean} - \text{Mode}}{\text{S.D.}}$

$= \dfrac{45.75 - 53.6957}{16.6914}$

$= -\dfrac{7.9457}{16.6914}$

$\therefore Sk_p = -0.4760$

Alternate Method:

Let u = $\dfrac{x - 45}{10}$

Mark above	No. of students 'more than' (c.f.)	Class	Frequency (f_i)	Mid value x_i	u_i	f_iu_i	$f_iu_i^2$
0	120	0 – 10	5	5	− 4	− 20	80
10	115	10 – 20	7	15	− 3	− 21	63
20	108	20 – 30	10	25	− 2	− 20	40
30	98	30 – 40	13	35	− 1	− 13	13
40	85	40 – 50	25	45	0	0	0
50	60	50 – 60	42	55	1	42	42
60	18	60 – 70	13	65	2	26	52
70	5	70 – 80	5	75	3	15	45
80	0	80 – 90	0	85	4	0	0
		Total	**120**			**9**	**335**

$\bar{u} = \frac{\sum f_i u_i}{N} = \frac{9}{120}$ = 0.075

$\therefore \bar{x} = 45 + 10(\bar{u})$

= 45 + 10(0.075)

= 45 + 0.75

= 45.75

Var(u) = $\sigma_u^2 = \frac{\sum f_i u_i^2}{N} - (\bar{u})^2$

= $\frac{335}{120} - (0.075)^2$

= 2.7917 – 0.0056

= 2.7861

Var(X) = h^2 x Var(u) = 100 × 2.7861 = 278.61

S.D. = $\sqrt{278.61}$

= 16.6916

Maximum frequency 42 is of the class 50 – 60.

∴ Mode lies in the class 50 – 60.

∴ L = 50, f1 = 42, f0 = 25, f2 = 13, h = 10

∴ Mode = $L + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$

= $50 + \frac{42 - 25}{2(42) - 25 - 13} \times 10$

= $50 + \frac{17}{84 - 38} \times 10$

= $50 + \frac{17}{46} \times 10$

= 50 + 3.6957

= 53.6957

$$\therefore Sk_p = \frac{\text{Mean} - \text{Mode}}{\text{S.D.}}$$

$$= \frac{45.75 - 53.6957}{16.6916}$$

$$= \frac{-7.9457}{16.6916}$$

$= -0.4760$

Miscellaneous Exercise 3 | Q 8 | Page 44

Calculate Bowley's coefficient of skewness Sk_b from the following data:

Marks above	0	10	20	30	40	50	60	70	80
No. of students	120	115	108	98	85	60	18	5	0

SOLUTION

To calculate Bowley's coefficient of skewness Skb, we construct the following table:

Marks above	No. of students 'more than' (c.f.)	Marks	Frequency (fi)	Less than cumulative frequency (c.f.)
0	120	0 – 10	5	5
10	115	10 – 20	7	12
20	108	20 – 30	10	22
30	98	30 – 40	13	35 ← Q_1
40	85	40 – 50	25	60 ← Q_2
50	60	50 – 60	42	102 ← Q_3
60	18	60 – 70	13	115
70	5	70 – 80	5	120

80	0	80 – 90	0	120
		Total	**120**	**–**

Here, N = 120

Q_1 class = class containing the $\left(\frac{N}{4}\right)^{th}$ observation

$\therefore \frac{N}{4} = \frac{120}{4} = 30$

Cumulative frequency which is just greater than (or equal to) 30 is 35.

$\therefore Q_1$ lies in the class 30 – 40.

$\therefore$ L = 30, h = 10, f = 13, c.f. = 22

$\therefore Q_1 = L + \frac{h}{f}\left(\frac{N}{4} - c.f.\right)$

$= 30 + \frac{10}{13}(30 - 22)$

$= 30 + \frac{10}{13}(8)$

= 30 + 6.1538

$\therefore Q_1$ = 36.1538

Q_2 class = class containing the $\left(\frac{N}{2}\right)^{th}$ observation

$\therefore \frac{N}{2} = \frac{120}{2}$ = 60

Cumulative frequency which is just greater than (or equal to) 60 is 60.

$\therefore$ Q_2 lies in the class 40 – 50.

$\therefore$ L = 40, h = 10, f = 25, c.f. = 35

$\therefore$ $Q_2 = L + \frac{h}{f}\left(\frac{N}{2} - c.f.\right)$

$= 40 + \frac{10}{25}(60 - 35)$

$= 40 + \frac{10}{25}(25)$

$\therefore$ Q_2 = 50

Q_3 class = class containing the $\left(\frac{3N}{4}\right)^{th}$ observation

$\therefore \frac{3N}{4} = \frac{3 \times 120}{4}$ = 90

Cumulative frequency which is just greater than (or equal to) 90 is 102.

$\therefore$ Q_3 lies in the class 50 – 60.

$\therefore$ $Q_3 = L + \frac{h}{f}\left(\frac{3N}{4} - c.f.\right)$

$= 50 + \frac{10}{42}(90 - 60)$

$= 50 + \frac{10}{42}(30)$

= 50 + 7.1429

$\therefore$ Q_3 = 57.1429

Bowley's coefficient of skewness:

$Sk_b = \frac{Q_3 + Q_1 - 2Q_2}{Q_3 - Q_1}$

$$= \frac{57.1429 + 36.1538 - 2(50)}{57.1429 - 36.1538}$$

$$= \frac{93.2967 - 100}{20.9891}$$

$$= \frac{-6.7033}{20.9891}$$

$\therefore Sk_b = -0.3194$

Miscellaneous Exercise 3 | Q 9 | Page 44

Find Sk_p for the following set of observations:
18, 27, 10, 25, 31, 13, 28.

SOLUTION

The given data can be arranged in ascending order as follows:

10, 13, 18, 25, 27, 28, 31.

Here, n = 7

$\therefore$ Median = value of $\left(\frac{n+1}{2}\right)^{th}$ observation

= value of $\left(\frac{7+1}{2}\right)^{th}$ observation

= value of 4^{th} observation

= 25

For finding standard deviation, we construct the following table:

x_i	x_i^2
10	100
13	169
18	324
25	625
27	729
28	784
31	961
152	**3692**

From the table, $\sum x_i = 152, \sum x_i^2 = 3692$

Mean = $\bar{x} = \frac{\sum x_i}{n} = \frac{152}{7}$ = 21.7143

∴ S.D. = $\sqrt{\frac{\sum x_i^2}{n} - (\bar{x})^2}$

= $\sqrt{\frac{3692}{7} - (21.7143)^2}$

= $\sqrt{527.4286 - 471.5108}$

= $\sqrt{55.9178}$

= 7.4778

Coefficient of skewness,

Sk_p = $\frac{3(\text{Mean} - \text{Median})}{\text{S.D.}}$

$$= \frac{3(21.7143 - 25)}{7.4778}$$

$$= \frac{3(-3.2857)}{7.4778}$$

$$= \frac{-9.8571}{7.4778}$$

$\therefore Sk_p = -1.3182$

Miscellaneous Exercise 3 | Q 10 | Page 44

Find Sk_b for the following set of observations:
18, 27, 10, 25, 31, 13, 28.

SOLUTION

The given data can be arranged in ascending order as follows:
10, 13, 18, 25, 27, 28, 31.
Here, n = 7

$$\therefore Q_1 = \text{value of } \left(\frac{n+1}{4}\right)^{th} \text{ observation}$$

$$= \text{value of } \left(\frac{7+1}{4}\right)^{th} \text{ observation}$$

$= \text{value of } 2^{nd}$ observation

$\therefore Q_1 = 13$

$$Q_2 = \text{value of } 2\left(\frac{n+1}{4}\right)^{th} \text{ observation}$$

$$= \text{value of } 2\left(\frac{7+1}{4}\right)^{th} \text{ observation}$$

$= \text{value of } (2 \times 2)^{th}$ observation

$= \text{value of } 4^{th}$ observation

$\therefore Q_2 = 25$

Q_3 = value of $\mathbf{3\left(\frac{n+1}{4}\right)^{th}}$ observation

= value of $\mathbf{3\left(\frac{7+1}{4}\right)^{th}}$ observation

= value of $(3 \times 2)^{th}$ observation

= value of 6^{th} observation

$\therefore Q_3 = 28$

Coefficient of skewness,

$$Sk_b = \frac{Q_3 + Q_1 - 2Q_2}{Q_3 - Q_1}$$

$$= \frac{28 + 13 - 2(25)}{28 - 13}$$

$$= \frac{41 - 50}{15}$$

$$= -\frac{9}{15}$$

$\therefore Sk_b = -0.6$