

UNIT – 09 KINETIC THEORY OF GASES

TWO MARKS AND THREE MARKS:

01. What is the microscopic origin of pressure?

With the help of kinetic theory of gases, the pressure is linked to the velocity of molecules. $P = \frac{1}{3} \frac{N}{V} m \overline{v^2}$ m – mass of a molecule;

N - Avogadro Number ; V – Volume ; $\overline{v^2}$ – Average velocity molecules.

02. What is the microscopic origin of temperature?

$$\text{Average Kinetic Energy / Molecule : } KE = \epsilon = \frac{3}{2} NkT$$

03. Why moon has no atmosphere?

The escape speed of gases on the surface of Moon is much less than the root mean square speeds of gases due to low gravity. Due to this all the gases escape from the surface of the Moon.

04. Write the expression for rms speed, average speed and most probable speed of a gas molecule.

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}} ; v_{\text{ave}} = \sqrt{\frac{8RT}{\pi M}} ; v_{\text{mp}} = \sqrt{\frac{2RT}{M}}$$

05. What is the relation between the average kinetic energy and pressure?

The internal energy of the gas is given by $U = \frac{3}{2} NkT$

The above equation can also be written as $U = \frac{3}{2} PV$ Since $PV = NkT$

$$P = \frac{2}{3} \frac{U}{V} = \frac{2}{3} u \quad \text{-----1}$$

From the equation (1), we can state that the pressure of the gas is equal to two thirds of internal energy per unit volume or internal energy density. $u = \frac{U}{V}$

Writing pressure in terms of mean kinetic energy density using equation.

$$P = \frac{1}{3} n m \overline{v^2} = \frac{1}{3} \rho \overline{v^2} \quad \text{-----2}$$

where $\rho = nm$ = mass density (Note n is number density)

Multiply and divide R.H.S of equation (2) by 2, we get $P = \frac{2}{3} \left(\frac{\rho}{2} \overline{v^2} \right)$

$$P = \frac{2}{3} \overline{KE} \quad \text{-----3}$$

From the equation (3), pressure is equal to 2/3 of mean kinetic energy per unit volume.

06. Define the term degrees of freedom.

The minimum number of independent coordinates needed to specify the position and configuration of a thermo-dynamical system in space is called the degree of freedom of the system.

07. State the law of equipartition of energy.

According to kinetic theory, the average kinetic energy of system of molecules in thermal equilibrium at temperature T is uniformly distributed to all degrees of freedom (x or y or freedom will get $\frac{1}{2} kT$ of energy. This is called law of equipartition of energy.

08. Define mean free path and write down its expression.

Average distance travelled by the molecule between collisions is called mean free path (λ). We can calculate the mean free path based on kinetic theory.

09. Deduce Charles' law based on kinetic theory.

Charles' law: From the equation $P = \frac{2}{3} \frac{U}{V} = \frac{2}{3} u$ we get $PV = \frac{2}{3} U$

For a fixed pressure, the volume of the gas is proportional to internal energy of the gas or average kinetic energy of the gas and the average kinetic energy is directly proportional to absolute temperature.

It implies that $V \propto T$ or $\frac{V}{T} = \text{Constant}$.

10. Deduce Boyle's law based on kinetic theory.

Boyle's law: From the equation $P = \frac{2}{3} \frac{U}{V} = \frac{2}{3} u$ we get $PV = \frac{2}{3} U$

But the internal energy of an ideal gas is equal to N times the average kinetic energy (ϵ) of each molecule. $U = N\epsilon$

For a fixed temperature, the average translational kinetic energy ϵ will remain constant. It implies that $PV = \frac{2}{3} N\epsilon$ Thus $PV = \text{constant}$

Therefore, pressure of a given gas is inversely proportional to its volume provided the temperature remains constant. This is Boyle's law.

11. Deduce Avogadro's law based on kinetic theory.

This law states that at constant temperature and pressure, equal volumes of all gases contain the same number of molecules. For two different gases at the same temperature and pressure, according to kinetic theory of gases,

From equation $P = \frac{1}{3} \frac{N_1}{V} m_1 v_1^2 = \frac{1}{3} \frac{N_2}{V} m_2 v_2^2 \text{ ----- } 1$

Where \bar{v}_1^2 and \bar{v}_2^2 are the mean square speed for two gases and N_1 and N_2 are the number of gas molecules in two different gases.

At the same temperature, average kinetic energy per molecule is the same for

two gases. $\frac{1}{2} m_1 \bar{v}_1^2 = \frac{1}{2} m_2 \bar{v}_2^2$ ----- 2

Dividing the equation (1) by (2) we get $N_1 = N_2$

This is Avogadro's law. It is sometimes referred to as Avogadro's hypothesis or Avogadro's Principle.

12. List the factors affecting the mean free path.

- 1) Mean free path increases with increasing temperature. As the temperature increases, the average speed of each molecule will increase. It is the reason why the smell of hot sizzling food reaches several meter away than smell of cold food.
- 2) Mean free path increases with decreasing pressure of the gas and diameter of the gas molecules.

13. What is the reason for Brownian motion?

According to kinetic theory, any particle suspended in a liquid or gas is continuously bombarded from all the directions so that the mean free path is almost negligible. This leads to the motion of the particles in a random and zig-zag manner.

14. What are the factors which affect Brownian motion?

- 1) Brownian motion increases with increasing temperature.
- 2) Brownian motion decreases with bigger particle size, high viscosity and density of the liquid (or) gas.

15. What is meant by rms speed of the molecules of a gas? Is rms speed same as the average speed?

The rms speed of the molecule of a gas is defined as the square root of the mean of the square of speeds of all molecules.

No, rms speed is different from the average speed. $V_{\text{rms}} = \sqrt{\frac{v_1^2 + v_2^2 + v_3^2}{3}}$

\bar{V} = Average speed = $\frac{V_1 + V_2 + V_3}{3}$

16. Why No hydrogen in Earth's atmosphere?

As the root mean square speed of hydrogen is much less than that of nitrogen, it easily escapes from the earth's atmosphere. In fact, the presence of nonreactive nitrogen instead of highly combustible hydrogen deters many disastrous consequences.

17. Mention the different ways of increasing the number of molecular collisions per unit time in a gas.

The numbers of collisions per unit time can be increased by increasing the temperature of the gas, increasing the number of molecules, and decreasing the volume of the gas.

18. On which factors does the average kinetic energy of gas molecular depend?

The average kinetic energy of a gas molecular depends only on the absolute temperature of the gas and is directly proportional to it.

19. When a gas is heated, its temperature increases. Explain it on the basis of kinetic temperature of gases.

When a gas is heated, the rms velocity of its molecule increases. As $V_{rms} \propto \sqrt{T}$. So the temperature of the gas increases.

20. What is an ideal gas? (or) What is perfect gas?

An ideal gas is that gas which obeys the gas laws. i.e. Charle's law, Boyle's law etc, at all values of temperature and pressure. Molecules of such a gas should be free from intermolecular attraction.

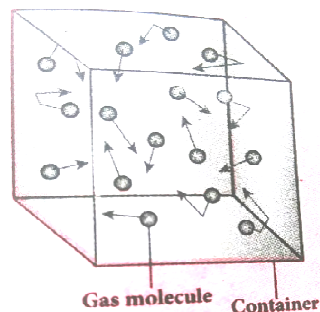
FIVE MARKS

01. Write down the postulates of kinetic theory of gases.

- 1) All the molecules of a gas are identical, elastic spheres.
- 2) The molecules of different gases are different.
- 3) The number of molecules in a gas is very large and the average separation between them is larger than size of the gas molecules.
- 4) The molecules of a gas are in a state of continuous random motion.
- 5) The molecules collide with one another and also with the walls of the container.
- 6) These collisions are perfectly elastic so that there is no loss of kinetic energy during collisions.
- 7) Between two successive collisions, a molecule moves with uniform velocity.
- 8) The molecules do not exert any force of attraction or repulsion on each other except during collision. The molecules do not possess any potential energy and the energy is wholly kinetic.
- 9) The collisions are instantaneous. The time spent by a molecule in each collision is very small compared to the time elapsed between two consecutive collisions.
- 10) These molecules obey Newton's laws of motion even though they move randomly.

02. Derive the expression of pressure exerted by the gas on the walls of the container.

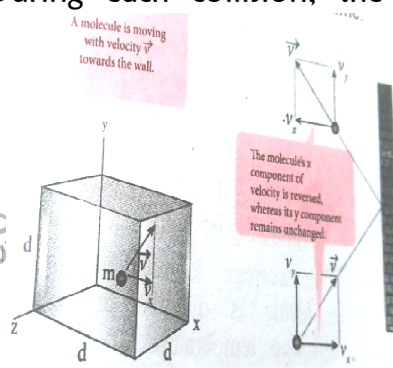
1) Consider a monatomic gas of N molecules each having a mass m inside a cubical container of side l as shown in the Figure (a).



2) The molecules of the gas are in random motion. They collide with each other and also with the walls of the container. As the collisions are elastic in nature, there is no loss of energy, but a change in momentum occurs.

3) The molecules of the gas exert pressure on the walls of the container due to collision on it. During each collision, the molecules impart certain momentum to the wall. Due to transfer of momentum, the walls experience a continuous force.

4) The force experienced per unit area of the walls of the container determines the pressure exerted by the gas. It is essential to determine the total momentum transferred by the molecules in a short interval of time.



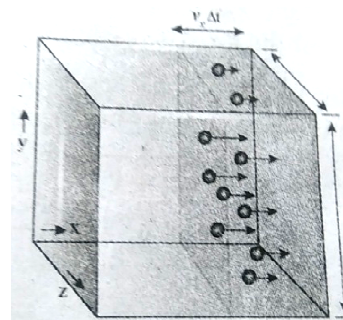
5) A molecule of mass m moving with a velocity \vec{v} having components (v_x, v_y, v_z) hits the right side wall. Since we have assumed that the collision is elastic, the particle rebounds with same speed and its x -component is reversed. This is shown in the Figure (b). The components of velocity of the molecule after collision are $(-v_x, v_y, v_z)$.

The x -component of momentum of the molecule before collision = mv_x

The x -component of momentum of the molecule after collision = $-mv_x$

6) The change in momentum of the molecule in x direction = Final momentum – initial momentum = $-mv_x - mv_x = -2mv_x$ According to law of conservation of linear momentum, the change in momentum of the wall = $2mv_x$

7) The number of molecules hitting the right side wall in a small interval of time Δt is calculated as follows. The molecules within the distance of $v_x \Delta t$ from the right side wall and moving towards the right will hit the wall in the time interval Δt . This is shown in the Figure. The number of molecules that will hit the right side wall in a time interval Δt is equal to the product of volume ($Av_x \Delta t$) and number density of the molecules (n). Here A is area of the wall and n is number of molecules per unit volume $\left(\frac{N}{V}\right)$. We have assumed that the number density is the same throughout the cube.



8) Not all the n molecules will move to the right, therefore on an average only half of the n molecules move to the right and the other half moves towards left side. The number of molecules that hit the right side wall in a time interval $\Delta t = \frac{n}{2} Av_x \Delta t$ ----- 1

In the same interval of time Δt , the total momentum transferred by the molecules $\Delta p = \frac{n}{2} Av_x \Delta t \times 2mv_x = Av_x^2 nm\Delta t$ -----2

9) From Newton's second law, the change in momentum in a small interval of time gives rise to force. The force exerted by the molecules on the wall (in magnitude) $F = \frac{\Delta p}{\Delta t} = nm Av_x^2$ ----- 3

Pressure, P = force divided by the area of the wall.

$$P = \frac{F}{A} = nmv_x^2 \text{ ----- 4}$$

Since all the molecules are moving completely in random manner, they do not have same speed. So we can replace the term v_x^2 by the average $\overline{v_x^2}$ in equation. $P = nm \overline{v_x^2}$ ----- 5

10) Since the gas is assumed to move in random direction, it has no preferred direction of motion (the effect of gravity on the molecules is neglected). It implies that the molecule has same average speed in all the three direction. So, $\overline{v_x^2} = \overline{v_y^2} = \overline{v_z^2}$. The mean square speed is written as $\overline{v^2} = \overline{v_x^2} = \overline{v_y^2} = \overline{v_z^2}$

$$\overline{v_x^2} = \frac{1}{3} \overline{v^2}$$

Using this in equation (5), we get, $\overline{v_x^2} = \frac{1}{3} nm \overline{v^2}$ or

$$P = \frac{1}{3} \frac{N}{V} m \overline{v^2} \text{ as } \left[n = \frac{N}{V} \right]$$

03. Explain in detail the kinetic interpretation of temperature.

1) To understand the microscopic origin of temperature in the same way,

Rewrite the equation $P = nm \overline{v_x^2}$

$$P = \frac{1}{3} \frac{N}{V} m \overline{v^2} ; PV = \frac{1}{3} m \overline{v^2} \text{ -----1}$$

Comparing the equation (1) with ideal gas equation $PV = NkT$,

$$NkT = \frac{1}{3} Nm \overline{v^2} ;$$

$$KT = \frac{1}{3} m \overline{v^2} \text{ -----2}$$

Multiply the above equation by 3/2 on both sides,

$$\frac{3}{2} KT = \frac{1}{2} m \overline{v^2} \text{ ----- 3}$$

R.H.S of the equation (3) is called average kinetic energy of a single molecule

(\overline{KE}) . The average kinetic energy per molecule $\overline{KE} = \epsilon = \frac{3}{2} KT$ ----- 4

2) Equation (3) implies that the temperature of a gas is a measure of the average translational kinetic energy per molecule of the gas.

Equation 4 is a very important result from kinetic theory of gas. We can infer the following from this equation.

- 3) The average kinetic energy of the molecule is directly proportional to absolute temperature of the gas. The equation (3) gives the connection between the macroscopic world (temperature) to microscopic world (motion of molecules).
- 4) The average kinetic energy of each molecule depends only on temperature of the gas not on mass of the molecule. In other words, if the temperature of an ideal gas is measured using thermometer, the average kinetic energy of each molecule can be calculated without seeing the molecule through naked eye.
- 5) By multiplying the total number of gas molecules with average kinetic energy of each molecule, the internal energy of the gas is obtained.

Internal energy of ideal gas $U = N\left(\frac{1}{2}m\overline{v^2}\right)$

By using equation (3) $U = \frac{3}{2}NKT$ ----- 5

From equation (5), we understand that the internal energy of an ideal gas depends only on absolute temperature and is independent of pressure and volume.

04. Describe the total degrees of freedom for mono-atomic molecule, diatomic molecule and tri-atomic molecule.

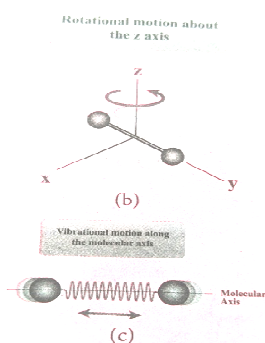
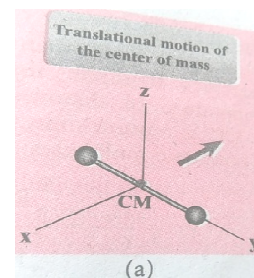
Mono-atomic molecule: A mono-atomic molecule by virtue of its nature has only three translational degrees of freedom. Therefore $f = 3$

Example: Helium, Neon, Argon

Diatomic molecule: There are two cases.

1) **At Normal temperature** A molecule of a diatomic

gas consists of two atoms bound to each other by a force of attraction. Physically the molecule can be regarded as a system of two point masses fixed at the ends of a mass less elastic spring. The center of mass lies in the center of the diatomic molecule. So, the motion of the center of mass requires three translational degrees of freedom (figure a). In addition, the diatomic molecule can rotate about three mutually perpendicular axes (figure b). But the moment of inertia about its own axis of rotation is negligible (about y



axis in the figure). Therefore, it has only two rotational degrees of freedom (one rotation is about Z axis and another rotation is about Y axis). Therefore totally there are five degrees of freedom.

$$f = 5$$

2) At High Temperature At a very high temperature such as 5000 K, the diatomic molecules possess additional two degrees of freedom due to vibrational motion [one due to kinetic energy of vibration and the other is due to potential energy] (Figure c). So totally there are seven degrees of freedom.

$$f = 7.$$

Examples: Hydrogen, Nitrogen, Oxygen.

3) Tri-atomic molecules There are two cases.

Linear tri-atomic molecule In this type, two atoms lie on either side of the central atom as shown in the Figure. Linear tri-atomic molecule has three translational degrees of freedom. It has two rotational degrees of freedom because it is similar to diatomic molecule except there is an additional atom at the center. At normal temperature, linear tri-atomic molecule will have five degrees of freedom. At high temperature it has two additional vibrational degrees of freedom. So a linear tri-atomic molecule has seven degrees of freedom. **Example:** Carbon dioxide

Non-linear tri-atomic molecule In this case, the three atoms lie at the vertices of a triangle as shown in the Figure. It has three translational degrees of freedom and three rotational degrees of freedom about three mutually orthogonal axes. The total degrees of freedom, $f = 6$

Example: Water, Sulphurdioxide.

05. Derive the ratio of two specific heat capacities of mono-atomic, diatomic and Tri-atomic molecules.

Application of law of equipartition energy in specific heat of a gas

Meyer's relation $C_p - C_v = R$ connects the two specific heats for one mole of an ideal gas.

Equipartition law of energy is used to calculate the value of $C_p - C_v$ and the ratio between them $\gamma = \frac{C_p}{C_v}$. Here γ is called adiabatic exponent.

i) Monatomic molecule:

$$\text{Average kinetic energy of a molecule} = \left[\frac{3}{2} kT \right]$$

$$\text{Total energy of a mole of gas} = \frac{3}{2} kT \times N_A = \frac{3}{2} RT$$

$$\text{For one mole, the molar specific heat at constant volume } C_v = \frac{dU}{dT} = \frac{d}{dT} \left[\frac{3}{2} RT \right]$$

$$C_v = \left[\frac{3}{2} R \right] ; C_p = C_v + R$$

$$= \frac{3}{2} R + R = \frac{5}{2} R$$

The ratio of specific heats, $\gamma = \frac{C_P}{C_V}$;

$$= \frac{\frac{5}{2} R}{\frac{3}{2} R} = \frac{5}{3} \quad \gamma = 1.67$$

ii) Diatomic molecule:

Average kinetic energy of a diatomic molecule at low temperature = $\frac{5}{2} kT$

Total energy of one mole of gas = $\frac{5}{2} kT \times N_A$; = $\frac{5}{2} RT$

(Here, the total energy is purely kinetic) For one mole Specific heat at

constant volume. $C_V = \frac{dU}{dT}$; = $\left[\frac{5}{2} RT \right]$; $C_V = \frac{5}{2} R$

But, $C_P = C_V + R$

$$= \frac{5}{2} R + R = \frac{7}{2} R$$

The ratio of specific heats, $\gamma = \frac{C_P}{C_V}$;

$$= \frac{\frac{7}{2} R}{\frac{5}{2} R} = \frac{7}{5} \quad \gamma = 1.40$$

Energy of a diatomic molecule at high temperature is equal to $\frac{7}{2} RT$

$$C_V = \frac{dU}{dT} ; = \left[\frac{7}{2} RT \right] ; C_V = \frac{7}{2} R$$

But, $C_P = C_V + R$

$$= \frac{7}{2} R + R = \frac{9}{2} R$$

Note that the C_V and C_P are higher for diatomic molecules than the mono atomic molecules. It implies that to increase the temperature of diatomic gas molecules by 1°C it require more heat energy than mono-atomic molecules.

The ratio of specific heats, $\gamma = \frac{C_P}{C_V}$;

$$= \frac{\frac{9}{2} R}{\frac{7}{2} R} = \frac{9}{7} \quad \gamma = 1.28$$

iii) Tri-atomic molecule:

a) Linear molecule:

Energy of one mole = $\frac{7}{2} kT \times N_A$; = $\frac{7}{2} RT$

$$C_V = \frac{dU}{dT} ; = \frac{d}{dT} \left[\frac{7}{2} RT \right] ; C_V = \frac{7}{2} R$$

But, $C_P = C_V + R$

$$= \frac{7}{2} R + R = \frac{9}{2} R$$

The ratio of specific heats, $\gamma = \frac{C_P}{C_V}$;

$$= \frac{\frac{9}{2} R}{\frac{7}{2} R} = \frac{9}{7} \quad \gamma = 1.28$$

b) Non-linear molecule:

$$\text{Energy of a mole} = \frac{6}{2} kT \times N_A = \frac{6}{2} RT = 3RT$$

$$C_V = \frac{dU}{dT} ; = 3R ;$$

$$\text{But, } C_P = C_V + R ;$$

$$= 3R + R = 4R$$

The ratio of specific heats, $\gamma = \frac{C_P}{C_V}$

$$= \frac{4R}{3R} = \frac{4}{3} \quad \gamma = 1.33$$

Note that according to kinetic theory model of gases the specific heat capacity at constant volume and constant pressure are independent of temperature. But in reality it is not sure. The specific heat capacity varies with the temperature.

6. Explain in detail the Maxwell Boltzmann distribution function.

1) The air molecules are moving in random directions. The speed of each molecule is not the same even though macroscopic parameters like temperature and pressure are fixed.

2) Each molecule collides with every other molecule and they exchange their speed. Section we calculated the rms speed of each molecule and not the speed of each molecule which is rather difficult.

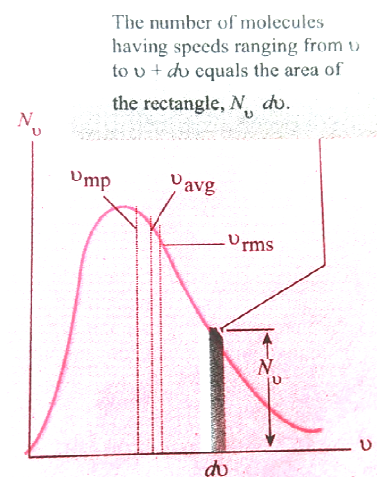
3) In this scenario we can find the number of gas molecules that move with the speed of 5 m s^{-1} to 10 m s^{-1} or 10 m s^{-1} to 15 m s^{-1} etc.

4) In general our interest is to find how many gas molecules have the range of speed from v to $v + dv$. This is given by Maxwell's speed distribution

$$\text{function. } N_v = 4_p N \left(\frac{m}{2\pi kT} \right)^{\frac{3}{2}} v^2 \frac{mv^2}{e^{2kT}}$$

The above expression is graphically shown as follows

5) for a given temperature the number of molecules having lower speed increases parabolically but decreases exponentially after reaching most probable speed. The rms speed, average



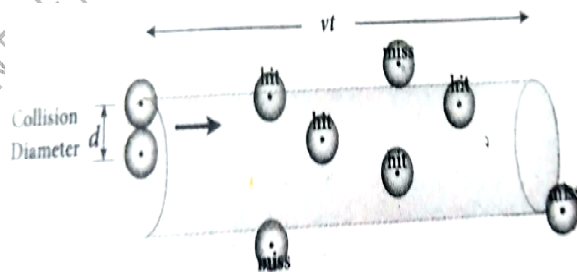
speed and most probable speed are indicated in the Figure. It can be seen that the rms speed is greatest among the three.

6) The area under the graph will give the total number of gas molecules in the system. (ii), Figure shows the speed distribution graph for two different temperatures. As temperature increases, the peak of the curve is shifted to the right. It implies that the average speed of each molecule will increase. But the area under each graph is same since it represents the total number of gas molecules.

07. Derive the expression for mean free path of the gas.

Expression for mean free path

- 1) We know from postulates of kinetic theory that the molecules of a gas are in random motion and they collide with each other. Between two successive collisions, a molecule moves along a straight path with uniform velocity.
- 2) This path is called mean free path. Consider a system of molecules each with diameter d . Let n be the number of molecules per unit volume. Assume that only one molecule is in motion and all others are at rest as shown in the Figure.
- 3) If a molecule moves with average speed v in a time t , the distance travelled is vt . In this time t , consider the molecule to move in an imaginary cylinder of volume $\pi d^2 vt$.
- 4) It collides with any molecule whose center is within this cylinder. Therefore, the number of collisions is equal to the number of molecules in the volume of the imaginary cylinder. It is equal to $\pi d^2 vtn$. The total path length divided by the number of collisions in time t is the mean free path.



$$\text{Mean free path} = \frac{\text{Distance travelled}}{\text{Number of collisions}} ; \lambda = \frac{vt}{n\pi d^2 vt} = \frac{1}{n\pi d^2} \text{----- 1}$$

- 5) Though we have assumed that only one molecule is moving at a time and other molecules are at rest, in actual practice all the molecules are in random motion. So the average relative speed of one molecule with respect to other molecules has to be taken into account. After some detailed calculations (you will learn in higher classes) the correct expression for mean free path. $\lambda = \frac{1}{\sqrt{2}n\pi d^2} \text{----- 2}$
- 6) The equation (1) implies that the mean free path is inversely proportional to number density. When the number density increases the molecular collisions increases so it decreases the distance travelled by the molecule before collisions.

Case1: Rearranging the equation (2) using 'm' (mass of the molecule)

$$\lambda = \frac{m}{\sqrt{2}\pi d^2 mn}$$

But mn = mass per unit volume = ρ (density of the gas)

$$\lambda = \frac{m}{\sqrt{2}\pi d^2 \rho} \text{ Also we know that } PV = NkT$$

$$P = \frac{N}{V} kT = nkT ; n = \frac{P}{kT}$$

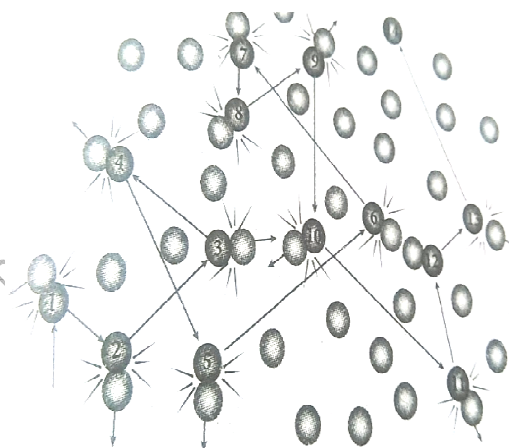
Substituting $n = \frac{P}{kT}$ in equation (2), we get

$$\lambda = \frac{kT}{\sqrt{2}\pi d^2 P}$$

08. Describe the Brownian motion.

1) Brownian motion is due to the bombardment of suspended particles by molecules of the surrounding fluid.

2) According to kinetic theory, any particle suspended in a liquid or gas is continuously bombarded from all the directions so that the mean free path is almost negligible. This leads to the motion of the particles in a random and zig-zag manner



Factors affecting Brownian motion

- 1) Brownian motion increases with increasing temperature.
- 2) Brownian motion decreases with bigger particle size, high viscosity and density of the liquid (or) gas.