Linear Programming

EXERCISE 7.1 [PAGES 232 - 233]

Exercise 7.1 | Q 1.1 | Page 232

Solve graphically : $x \geq 0$

Solution: Consider the line whose equation is $x = 0$. This represents the Y-axis.

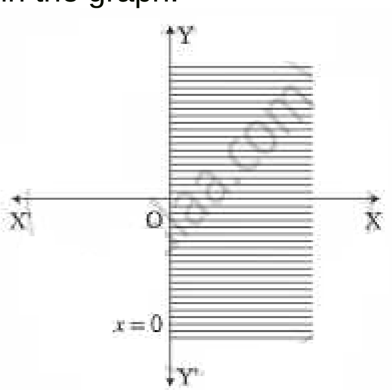
To find the solution set, we have to check any point other than origin.

Let us check the point (1, 1)

When $x = 1, x \geq 0$

$\therefore$ (1, 1) lies in the required region

Therefore, the solution set is the Y-axis and the right side of the Y-axis which is shaded in the graph.



Exercise 7.1 | Q 1.2 | Page 232

Solve graphically : $x \leq 0$

Solution: Consider the line whose equation is $x = 0$.

This represents the Y-axis.

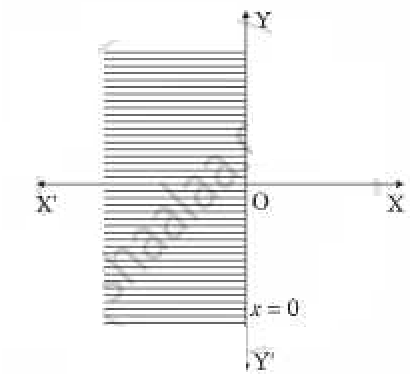
To find the solution set, we have to check any point other than origin.

Let us check the point (1, 1).

When $x = 1, x \not\leq 0$

$\therefore$ (1, 1) does not lie in the required region.

Therefore, the solution set is the Y-axis and the left side of the Y-axis which is shaded in the graph.



Exercise 7.1 | Q 1.3 | Page 232

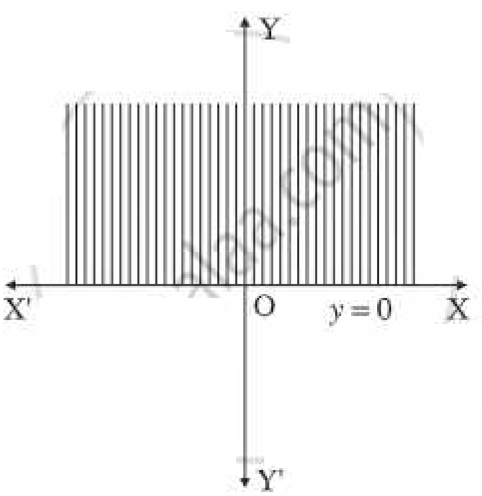
Solve graphically : $y \geq 0$

Solution: Consider the line whose equation is $y = 0$. This represents the X-axis. To find the solution set, we have to check any point other than origin. Let us check the point (1, 1).

When $y = 1, y \geq 0$

∴ (1, 1) lies in the required region.

Therefore, the solution set is the X-axis and above the X-axis which is shaded in the graph.



Exercise 7.1 | Q 1.4 | Page 232

Solve graphically : $y \leq 0$

Solution: Consider the line whose equation is $y = 0$. This represents the X-axis.

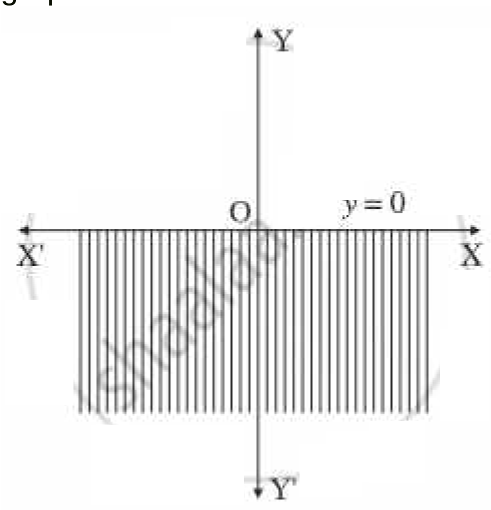
To find the solution set, we have to check any point other than origin.

Let us check the point (1, 1).

When y = 1, y ≰ 0.

∴ (1,1) does not lie in the required region.

Therefore, the solution set is the X-axis and below the X-axis which is shaded in the graph.

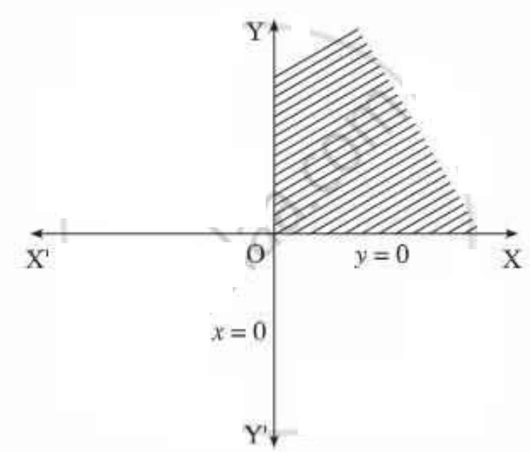


Exercise 7.1 | Q 2.1 | Page 232

Solve graphically : x ≥ 0 and y ≥ 0

Solution: Consider the lines whose equations are x = 0, y = 0.These represents the equations of Y-axis and X-axis respectively, which divide the plane into four parts.

Since x ≥ 0,y ≥ 0, the solution set is in the first quadrant which is shaded in the graph.

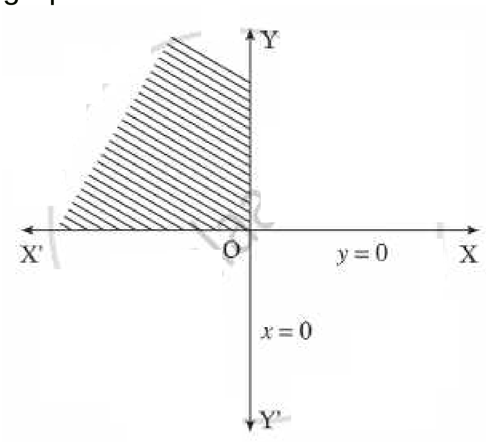


Exercise 7.1 | Q 2.2 | Page 232

Solve graphically : x ≤ 0 and y ≥ 0

Solution: Consider the lines whose equations are x = 0, y = 0.These represents the equations of Y-axis and X-axis respectively, which divide the plane into four parts.

Since x ≤ 0, y ≥ 0, the solution set is in the second quadrant which is shaded in the graph.

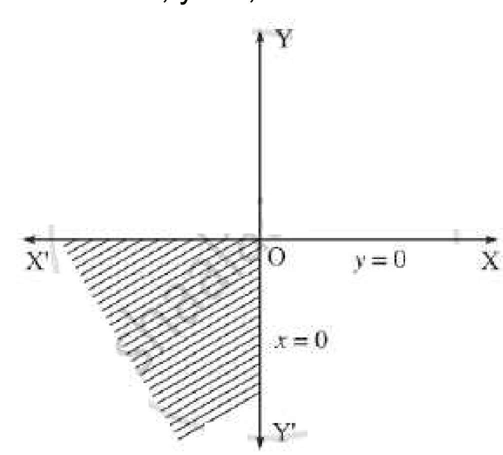


Exercise 7.1 | Q 2.3 | Page 232

Solve graphically : x ≤ 0 and y ≤ 0

Solution: Consider the lines whose equations are x = 0, y = 0.These represents the equations of Y-axis and X-axis respectively, which divide the plane into four parts.

Since x ≤ 0, y ≤ 0, the solution set is in the third quadrant which is shaded in the graph.

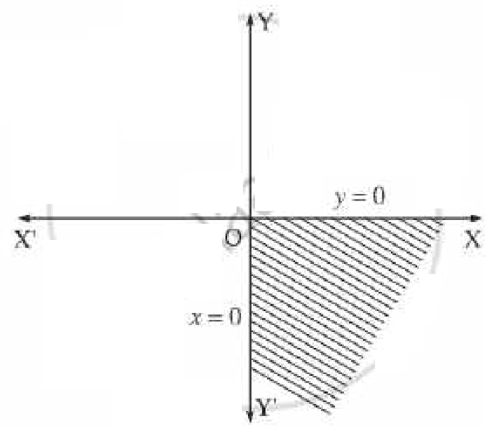


Exercise 7.1 | Q 2.4 | Page 232

Solve graphically : x ≥ 0 and y ≤ 0.

Solution: Consider the lines whose equations are x = 0, y = 0.These represents the equations of Y-axis and X-axis respectively, which divide the plane into four parts.

Since x ≥ 0, y ≤ 0, the solution set is in the fourth quadrant which is shaded in the graph.



Exercise 7.1 | Q 3.1 | Page 232

Solve graphically : 2x – 3 ≥ 0

Solution:

Consider the line whose equation is 2x – 3 ≥ 0, i.e. x = $\frac{3}{2}$

This represents a line parallel to Y-axis passing3through the point $\left(\frac{3}{2}, 0\right)$.

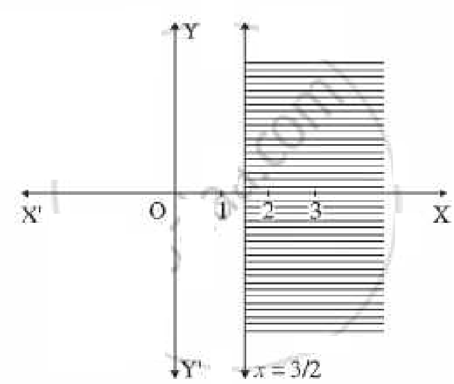
Draw the line x = $\frac{3}{2}$.

To find the solution set, we have to check the position of the origin (0, 0).

When x = 0, 2x – 3 = 2 x 0 – 3 = – 3 $\not\geq$ 0

∴ the coordinates of the origin does not satisfy thegiven inequality.

∴ the solution set consists of the line x = $\frac{3}{2}$ and the2non-origin side of the line which is shaded in thegraph.



Exercise 7.1 | Q 3.2 | Page 232

Solve graphically : 2y – 5 ≥ 0

Solution:

Consider the line whose equation is 2y – 5 = 0, i.e. y = $\frac{5}{2}$

This represents a line parallel to X-axis passing5through the point $\left(0, \frac{5}{2}\right)$

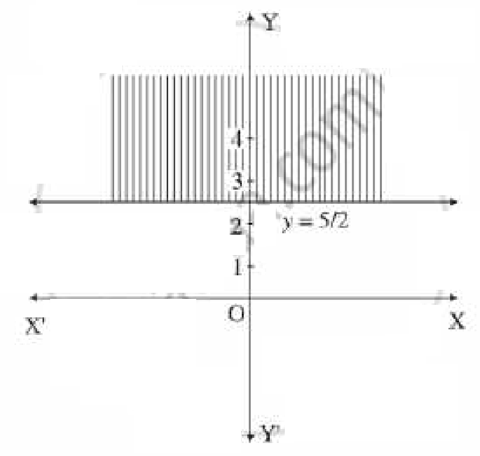
Draw the line y = $\frac{5}{2}$.

To find the solution set, we have to check the position of the origin (0, 0).

When y = 0, 2y – 5 = 2 x – 5 = –5 $\not\geq$ 0

∴ the coordinates of the origin does not satisfy the given inequality.

∴ the solution set consists of the line y == $\frac{5}{2}$ and the non-origin side of the line which is shaded in the graph.



Exercise 7.1 | Q 3.3 | Page 232

Solve graphically : $3x + 4 \le 0$

Solution:

Consider the line whose equation is $3x + 4 = 0$, i.e. $x = -\frac{4}{3}$.

This represents a line parallel to Y-axis passing through the point $\left(-\frac{4}{3}, 0\right)$.

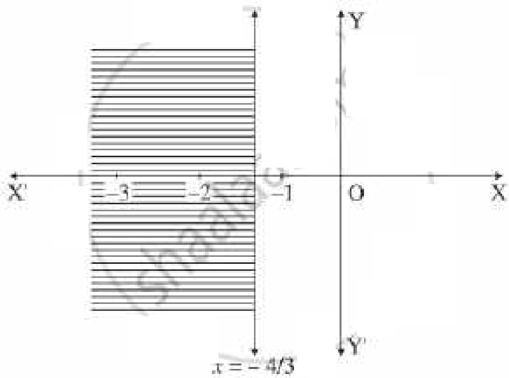
Draw the line $x = -\frac{4}{3}$.

To find the solution set, we have to check the position of the origin (0, 0).

When $x = 0$, $3x + 4 = 3 \times 0 + 4 = 4 \not\le 0$

$\therefore$ the coordinates of the origin does not satisfy thegiven inequality.

$\therefore$ the solution set consists of the line $x = -\frac{4}{3}$ the non-origin side of the line which is shaded in the graph.



Exercise 7.1 | Q 3.4 | Page 232

Solve graphically : 5y + 3 ≤ 0

Solution:

Consider the line whose equation is 5y + 3 ≤ 0, i.e. y = $\frac{-3}{5}$

This represents a line parallel to X-axis passing through the point $\left(0, \frac{-3}{5}\right)$.

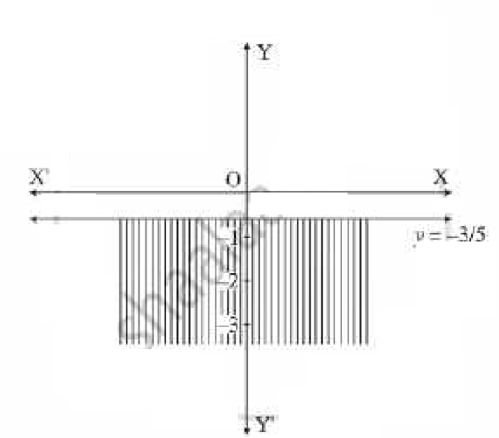
Draw the line y = $\frac{-3}{5}$.

To find the solution set, we have to check the position of the origin (0, 0).

When y = 0, 5y + 3 = 5 x 0 + 3 = 3 $\not\leq$ 0

∴ the coordinates of the origin does not satisfy the given inequality.

∴ the solution set consists of the line y = $\frac{-3}{5}$ and the non-origin side of the line which is shaded in the graph.



Exercise 7.1 | Q 4.1 | Page 232

Solve graphically : x +2y ≤ 6

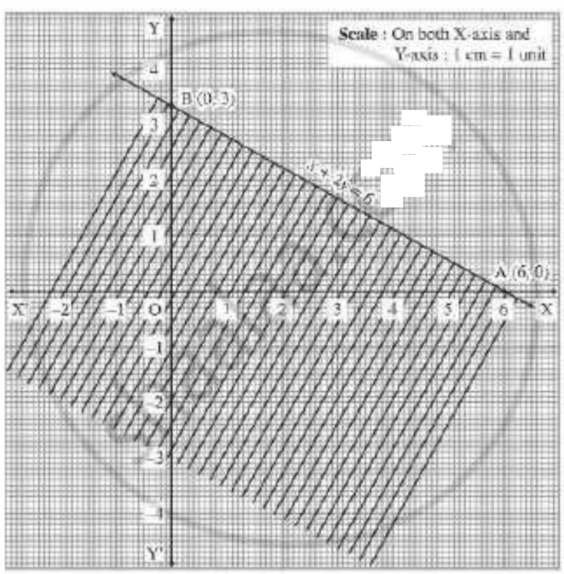
Solution: Consider the line whose equation is x +2y ≤ 6. To find the points of intersection of this line with the coordinate axes.

Put y = 0, we get x = 6.

∴ A = (6, 0) is a point on the line.

Put x = 0, we get 2y = 6, i.e. y = 3

∴ B = (0, 3) is another point on the line.



Draw the line AB joining these points. This line divide the line into two parts.

1. Origin side

2. Non-origin side

To find the solution set, we have to check the position of the origin (0,0) with respect to the line.

When x = 0, y = 0, then x + 2y = 0 which is less than 6.

∴ x + 2y ≤ 6 in this case.

Hence, origin lies in the required region. Therefore,the given inequality is the origin side which is shaded in the graph.

This is the solution set of x + 2y ≤ 6.

Exercise 7.1 | Q 4.2 | Page 232

Solve graphically : 2x – 5y ≥10

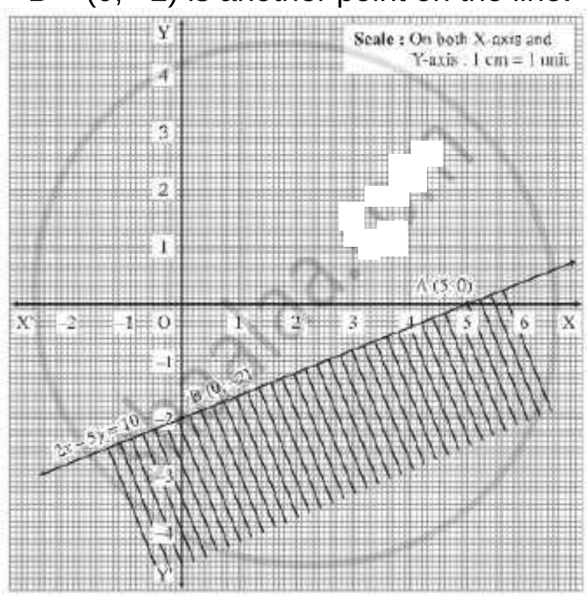
Solution: Consider the line whose equation is 2x – 5y = 10. To find the points of intersection of this line with the coordinate axes.

Put y = 0, we get x = 10, i.e. x = 5,

∴ A = (5, 0) is a point on the line.

Put x = 0, we get –5y = 10, i.e. y = –2

∴ B = (0, –2) is another point on the line.



Draw the line AB joining these points. This line divide the plane in two parts.

1. Origin side

2. Non-origin side

To find the solution set, we have to check the position of the origin (0,0) with respect to the line.

when x = 0, y = 0, then 2x – 5y = 0 which is neither greater non equal to 10.

∴ 2x – 5y $\not\geq$ 10 in the case.

Hence (0,0) will not lie in the required region.

Therefore, the given inequality is the non-origin side, which is shaded in the graph.

This is the solution set of $2x - 5y \geq 10$.

Exercise 7.1 | Q 4.3 | Page 232

Solve graphically : $3x + 2y \geq 0$

Solution: Consider the line whose equation is $3x + 2y = 0$. The constant term is zero, therefore this line is passing through the origin.

$\therefore$one point on the line is $O = (0, 0)$.

To find the another point, we can give any value of x and get the corresponding value of y.

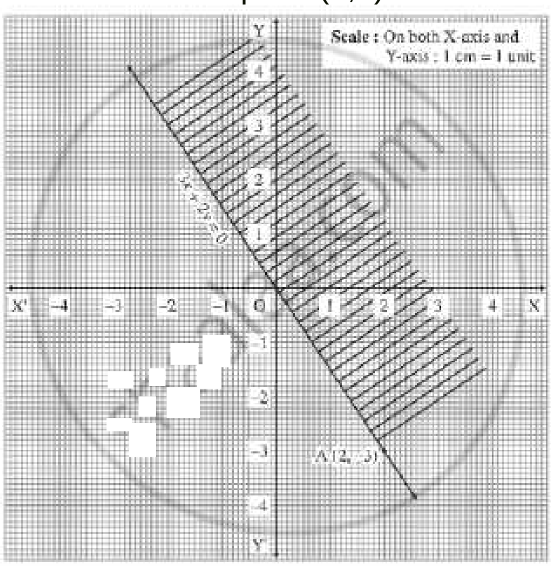
Put $x = 2$, we get $6 + 2y = 0$ i.e. $y = -3$

$\therefore$ $A = (2, -3)$, is another point on the line. Draw the line OA.

To find the solution set, we cannot check (0,0) as it is already on the line.

We can check any other point which is not on the line.

Let us check the point (1,1).



When $x = 1$, $y = 1$, then $3x + 2y = 3 + 2 = 5$ which is grreater than zer.

$\therefore$ $3x + 2y > 0$ in this case.

Hence (1,1) lies in the required region.

Therefore, the required region is the upper side which is shaded in the graph.

This is the solution set of $x + 2y > 0$.

Exercise 7.1 | Q 4.4 | Page 232

Solve graphically : $5x - 3y \leq 0$

Solution: Consider the line whose equation is 5x – 3y = 0. The constant term is zero, therefore this line is passing through the origin.

∴ one point on the line is the origin O = (0, 0).

To find the other point, we can give any value of x and get the corresponding value of y.

Put x = 3, we get 15 – 3y = 0, i.e. y = 5

∴ A = (3, 5) is another point on the line. Draw the line OA.

To find the solution set, we cannot check O(0,0), as it is already on the line. We can check any other point which is not on the line.

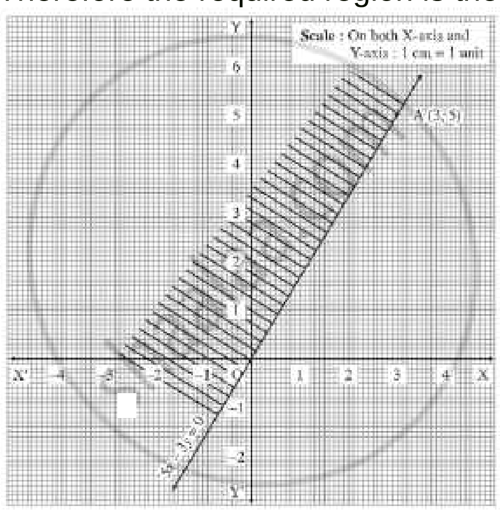
Let us check the point (1, – 1).

When x = 1, y = – 1 then 5x – 3y = 5 + 3 = 8 which is neither less nor equal to zero.

∴ 5x – 3y $\nleq$ 0 in this case.

Hence (1, – 1) will not lie in the required region.

Therefore the required region is the upper side which is shaded in the graph.



This is the solution set of 5x – 3y ≤ 0.

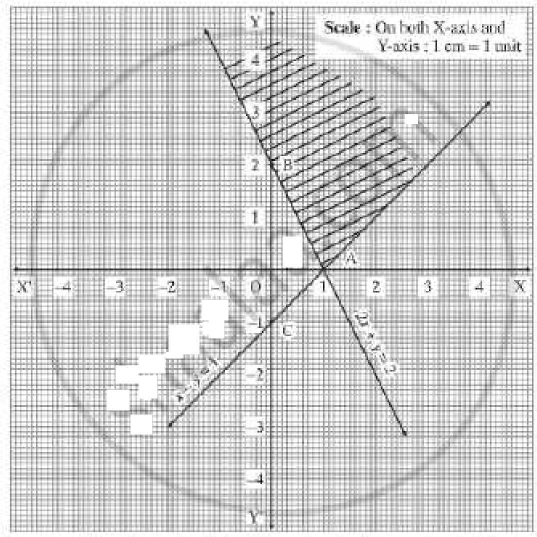
Exercise 7.1 | Q 5.1 | Page 233

Solve graphically : 2x + y ≥ 2 and x – y ≤ 1

Solution: First we draw the lines AB and AC whose equations are 2x + y = 2 and x – y = 1 respectively.

Line	Equation	Points on the X-axis	Points on the Y-axis	Sign	Region
AB	2x + y = 2	A(1, 0)	B(0, 2)	≥	non-origin ssde of line AB

AC	x – y = 1	A(1, 0)	C(0, –1)	≤	origin side of the line AC



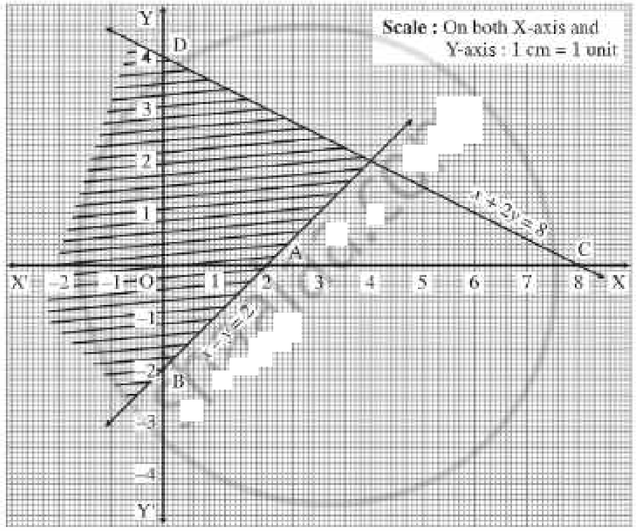
The solution set of the given system of inequalities is shaded in the graph.

Exercise 7.1 | Q 5.2 | Page 233

Solve graphically : x – y ≤ 2 and x + 2y ≤ 8

Solution: First we draw the lines AB and CD whose equations are x – y = 2 and x + 2y = 8 respectively.

Line	Equation	Points on the X-axis	Points on the Y-axis	Sign	Region
AB	x – y ≤ 2	A(2, 0)	B(0, –2)	≤	origin side of line AB
CD	x + 2y ≤ 8	C(8, 0)	D(0, 4)	≤	origin side of line CD



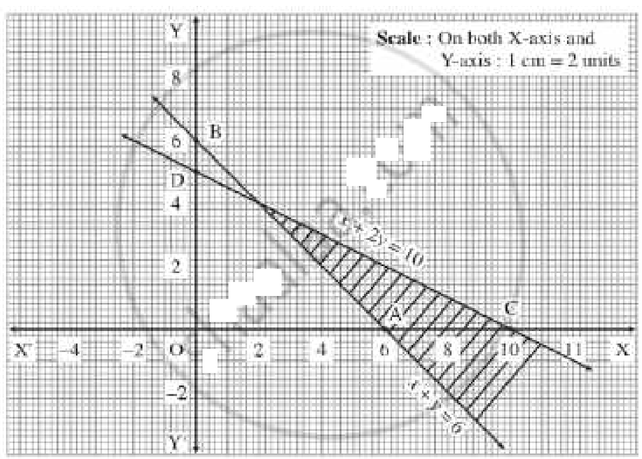
The solution set of the given system of inequalities is shaded in the graph.

Exercise 7.1 | Q 5.3 | Page 233

Solve graphically : x + y ≥ 6 and x + 2y ≤ 10

Solution: First we draw the lines AB and CD whose equations are x + y = 6 and x + 2y = 10 respectively.

Line	Equation	Points on the X-axis	Points on the Y-axis	Sign	Region
AB	x + y = 6	A(6, 0)	B(0, 6)	≥	non- origin side of line AB
CD	x + 2y = 1	D(0, 5)	D(0, 5)	≤	origin side of the line CD



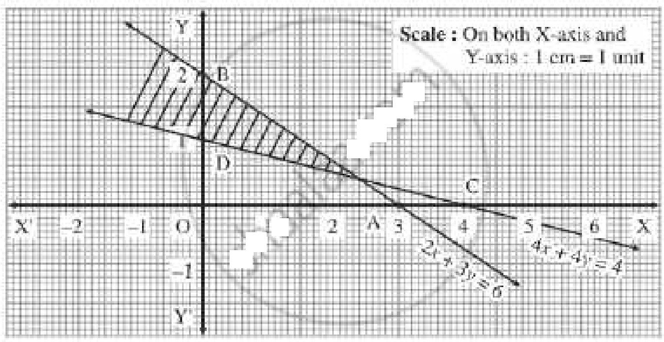
The solution set of the given system of inequalities is shaded in the graph.

Exercise 7.1 | Q 5.4 | Page 233

Solve graphically : 2x + 3y≤ 6 and x + 4y ≥ 4

Solution: First we draw the lines AB and CD whose equations are 2x + 3y = 6 and x + 4y = 4 respectively.

Line	Equation	Points on the X-axis	Points on the Y-axis	Sign	Region
AB	2x + 3y = 6	A(3, 0)	B(0, 2)	≤	origin side of line AB
CD	x + 4y = 4	C(4, 0)	D(0. 1)	≥	non-origin side of line CD



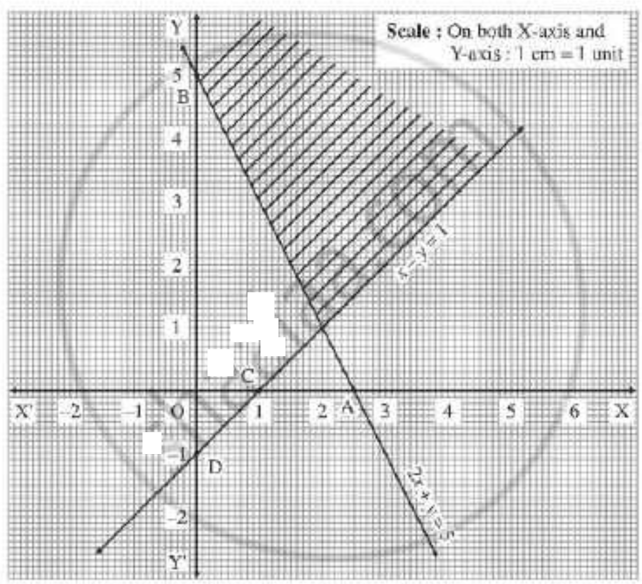
The solution set of the given system of inequalities shaded in the graph.

Exercise 7.1 | Q 5.5 | Page 233

Solve graphically : 2x + y ≥ 5 and x – y ≤ 1

Solution: First we draw the lines AB and CD whose equations are 2x + y = 5 and x – y = 1 respectively.

Line	Equation	Points on the X-axis	Points on the Y-axis	Sign	Region
AB	2x + y = 5	A(2.5, 0)	B(0, 5)	≥	non-origin side of line AB
CD	x – y = 1	C(1, 0)	D(0, –1)	≤	origin side of line CD



The solution set of the given system of inequations is shaded in the graph.

EXERCISE 7.2 [PAGE 234]

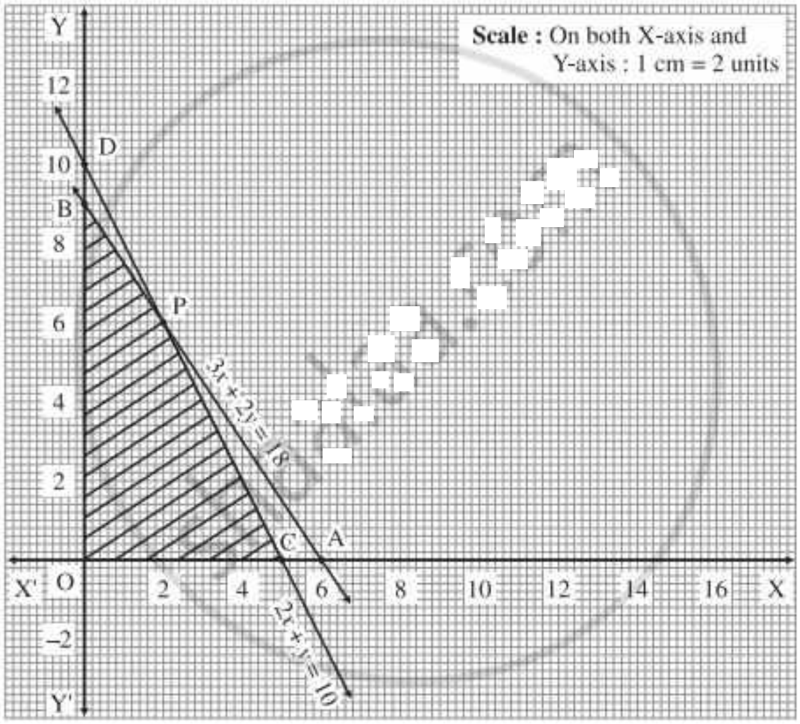
Exercise 7.2 | Q 1 | Page 234

Find the feasible solution of the following inequation:

3x + 2y ≤ 18, 2x + y ≤ 10, x ≥ 0, y ≥ 0

Solution: First we draw the lines AB and CD whose equations are 3x + 2y = 18 and 2x + y = 10 respectively.

Line	Equation	Points on the X-axis	Points on the Y-axis	Sign	Region
AB	3x + 2y = 18	A (6,0)	B (0,9)	≤	origin side of line AB
CD	2x + y = 10	C (5,0)	D(0,10)	≤	origin side of line CD



The feasible solution is OCPBO which is shaded in the graph.

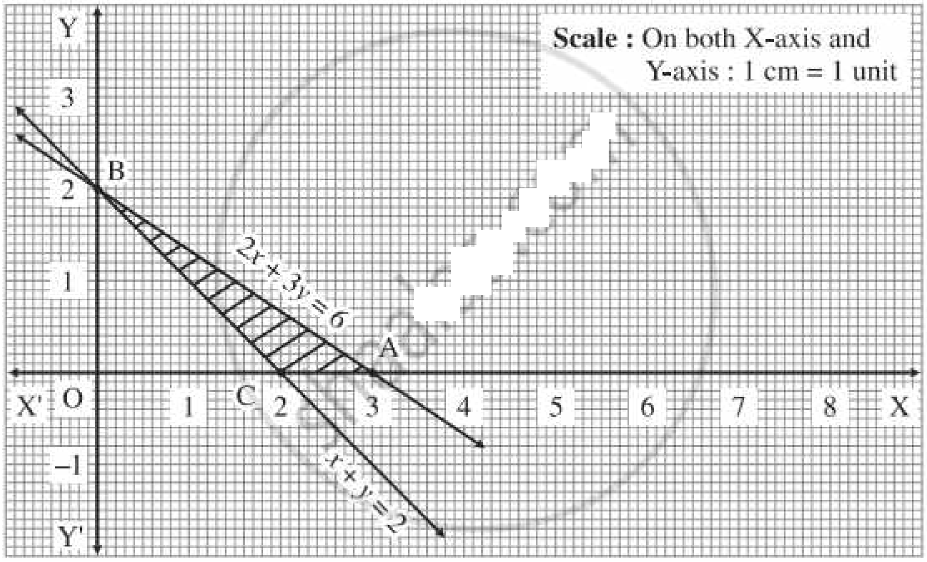
Exercise 7.2 | Q 2 | Page 234

Find the feasible solution of the following inequation:

$2x + 3y \le 6$, $x + y \ge 2$, $x \ge 0$, $y \ge 0$

Solution: First we draw the lines AB and CB whose equations are $2x + 3y = 6$ and $x + y = 2$ respectively.

Line	**Equation**	**Points on the X-axis**	**Points on the Y-axis**	**Sign**	**Region**
AB	$2x + 3y = 6$	A (3,0)	B (0,2)	$\le$	origin side of line AB
CB	$x + y = 2$	C (2,0)	D(0,2)	$\ge$	non -origin side of line CB



The feasible solution is Δ ABC which is shaded in the graph.

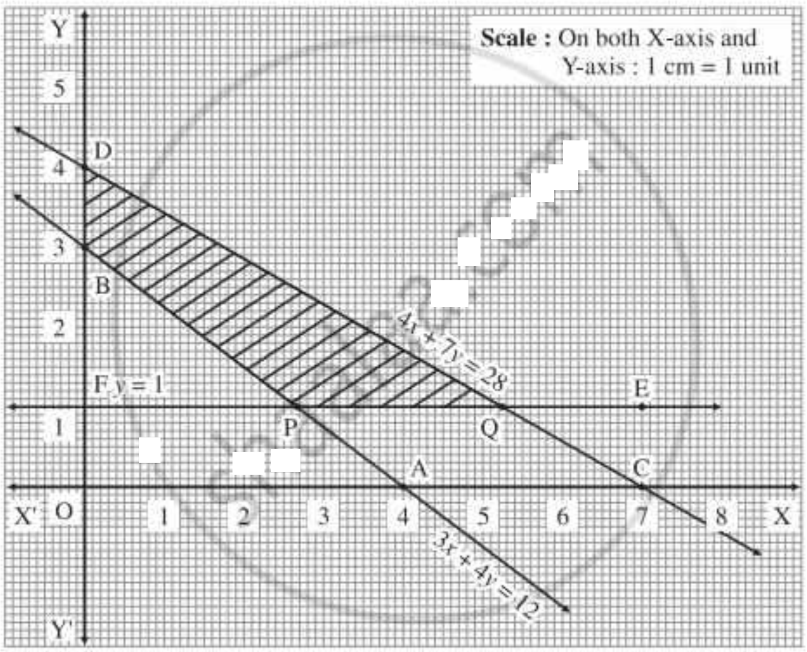
Exercise 7.2 | Q 3 | Page 234

Find the feasible solution of the following inequation:

$3x + 4y \geq 12$, $4x + 7y \leq 28$, $y \geq 1$, $x \geq 0$.

Solution: First we draw the lines AB, CD and EF whose equations are $3x + 4y = 12$ and $4x + 7y = 28$ and $y = 1$ respectively.

Line	**Equation**	**Points on the X-axis**	**Points on the Y-axis**	**Sign**	**Region**
AB	$3x + 4y = 12$	A (4,0)	B (0,3)	≥	non-origin side of line AB
CB	$4x + 7y = 28$	C (7,0)	D(0,4)	≤	origin side of line CD
EF	$y = 1$	-	F(0,1)	≥	non-origin side of line EF



The feasible solution is PQDB which is shaded in the graph.

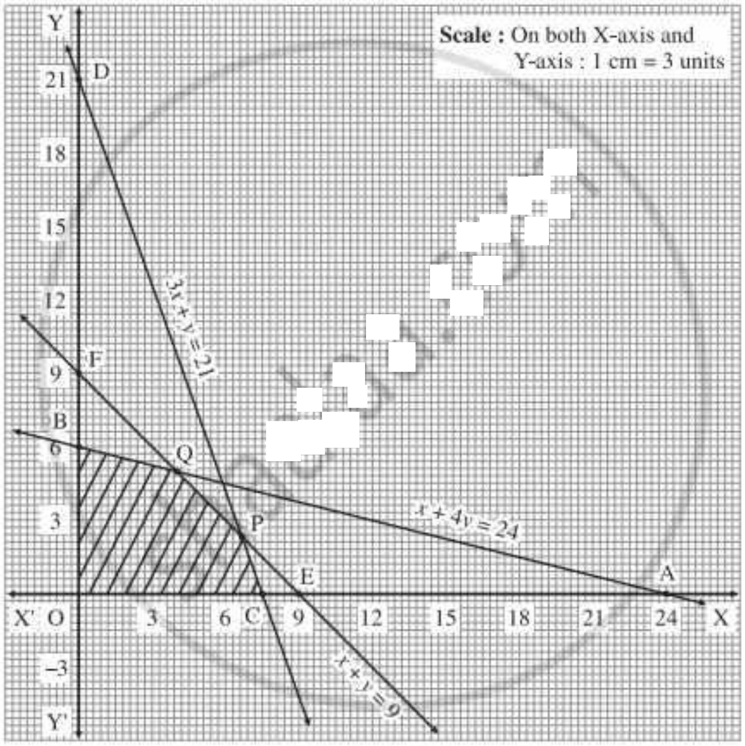
Exercise 7.2 | Q 4 | Page 234

Find the feasible solution of the following inequation:

$x + 4y \le 24$, $3x + y \le 21$, $x + y \le 9$, $x \ge 0$, $y \ge 0$.

Solution: First we draw the lines AB, CD and EF whose equations are $x + 4y = 24$, $3x + y = 21$ and $x + y = 9$ respectively.

Line	Equation	Points on the X-axis	Points on the Y-axis	Sign	Region
AB	$x + 4y = 24$	A (24,0)	B (0,6)	$\le$	origin side of line AB
CB	$3x + y = 21$	C (7,0)	D(0,21)	$\le$	origin side of line CD
EF	$x + y = 9$	E(9,0)	F(0,9)	$\le$	origin side of line EF



The feasible solution is OCPQBO which is shaded in the graph.

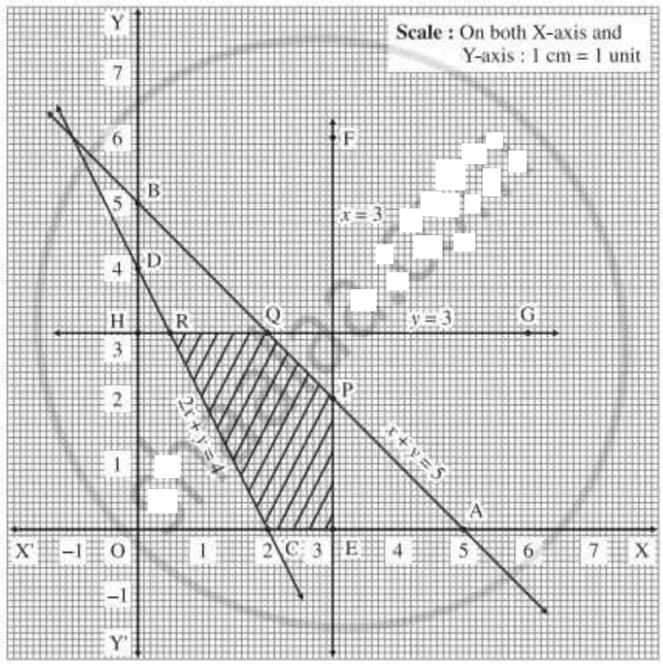
Exercise 7.2 | Q 5 | Page 234

Find the feasible solution for the following system of linear inequations:

$0 \le x \le 3$, $0 \le y \le 3$, $x + y \le 5$, $2x + y \ge 4$

Solution: First we draw the lines AB, CD, EF and GH whose equations are $x + y = 5$, $2x + y = 4$, $x = 3$ and $y = 3$ respectively.

Line	**Equation**	**Points on the X-axis**	**Points on the Y-axis**	**Sign**	**Region**
AB	$x + y = 5$	A(5, 0)	B(0, 5)	$\le$	origin side of line AB
CD	$2x + y = 4$	C(2, 0)	D(0, 4)	$\ge$	non-origin side of line CD
EF	$x = 3$	E(3, 0)	-	$\le$	origin side of line EF
GH	$y = 3$	-	H(0, 3)	$\le$	origin side of line GH



The feasible solution is CEPQRC which is shaded in the graph.

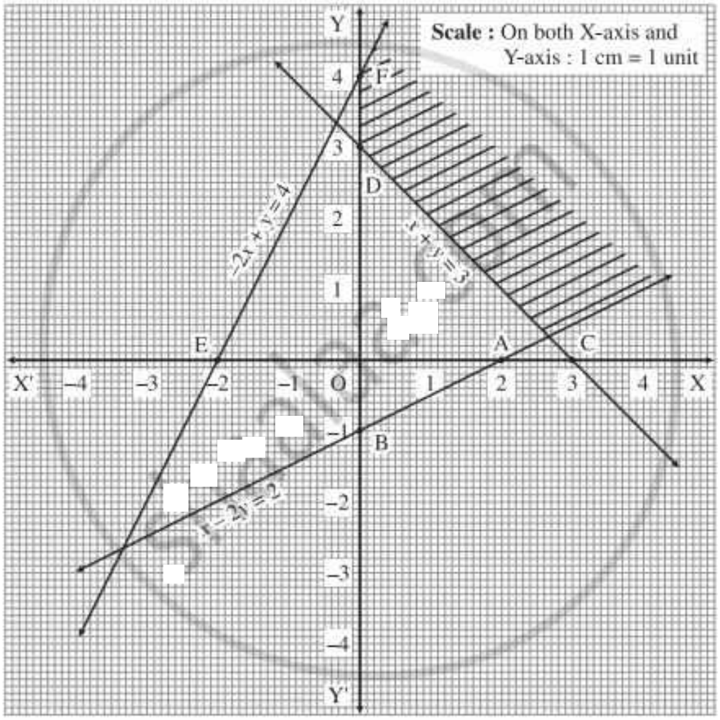
Exercise 7.2 | Q 6 | Page 234

Find the feasible solution of the following inequations:

x - 2y ≤ 2, x + y ≥ 3, - 2x + y ≤ 4, x ≥ 0, y ≥ 0

Solution: First we draw the lines AB, CD and EF whose equations are x - 2y = 2, x + y = 3 and - 2x + y = 4 respectively.

Line	**Equation**	**Points on the X-axis**	**Points on the Y-axis**	**Sign**	**Region**
AB	x - 2y = 2	A(2, 0)	B(0,-1)	≤	origin side of line AB
CD	x + y = 3	C(3, 0)	D(0,3)	≥	non-origin side of line AB
EF	- 2x + y = 4	E(-2,0)	F(0,4)	≤	origin side of line EF



The feasible solution is shaded in the graph.

Exercise 7.2 | Q 7 | Page 234

A company produces two types of articles A and B which require silver and gold. Each unit of A requires 3 gm of silver and 1 gm of gold, while each unit of B requires 2 gm of silver and 2 gm of gold. The company has 6 gm of silver and 4 gm of gold. Construct the inequations and find feasible solution graphically

Solution: Let the company produces x units of article A and y units of article B.
The given data can be tabulated as:

	Article A (x)	**Article B (y)**	**Availability**
Gold	1	2	4
Silver	3	2	6

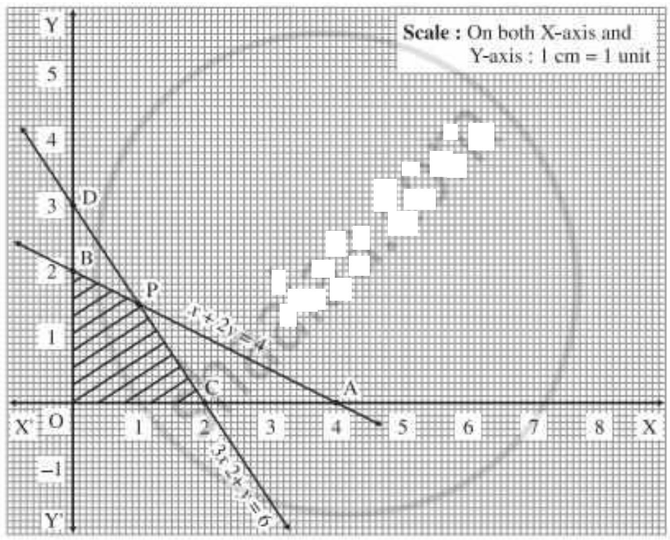
Inequations are:

$x + 2y \leq 4$ and $3x + 2y \leq 6$

x and y are number of items, $x \geq 0$, $y \geq 0$

First we draw the lines AB and CD whose equations are $x + 2y = 4$ and $3x + 2y = 6$ respectively.

Line	Equation	Points on the X-axis	Points on the Y-axis	Sign	Region
AB	x +2y = 4	A(4, 0)	B(0, 2)	≤	origin side of line AB
CD	3x + 2y = 6	C(2, 0)	D(0, 3)	≤	origin side of line CD



The feasible solution is OCPBO which is shaded in the graph.

Exercise 7.2 | Q 8 | Page 234

A furniture dealer deals in tables and chairs. He has ₹ 15,000 to invest and a space to store at most 60 pieces. A table costs him ₹ 150 and a chair ₹ 750. Construct the inequations and find the feasible solution.

Solution: Let x be the number of tables and y be the number of chairs. Then x ≥ 0, y ≥ 0.

The dealer has a space to store at most 60 pieces. ∴ x + y ≤ 60

Since, the cost of each table is ₹ 150 and that of each chair is ₹ 750, the total cost of x tables and y chairs is 150x + 750y. Since the dealer has ₹ 15,000 to invest, 150x + 750y ≤ 15000

Hence the system of inequations are

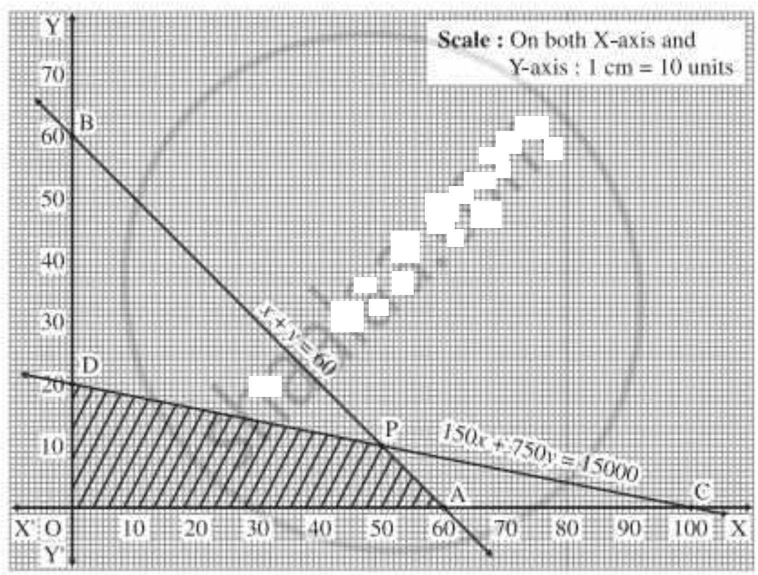
x + y ≤ 60, 150x + 750y ≤ 15000, x ≥ 0, y ≥ 0.

First we draw the lines AB and CD whose equations are

x + y = 60 and 150x + 750y = 15000 respectively.

Line	Equation	Points on the X-axis	Points on the Y-axis	Sign	Region

AB	x + y =60	A(60,0)	B(0,60)	≤	origin side of line AB
CD	150x + 750y = 15000	C(100,0)	D(0,20)	≤	origin side of line CD



The feasible solution is OAPDO which is shaded in the graph.

EXERCISE 7.3 [PAGES 237 - 378]

Exercise 7.3 | Q 1 | Page 237

A manufacturing firm produces two types of gadgets A and B, which are first processed in the foundry and then sent to the machine shop for finishing. The number of man-hours of labour required in each shop for production of A and B and the number of man-hours available for the firm is as follows :

Gadgets	Foundry	Machine shop
A	10	5
B	6	4
Time available (hour)	60	35

Profit on the sale of A is ₹ 30 and B is ₹ 20 per units. Formulate the LPP to have maximum profit.

Solution: Let the number of gadgets A produced by the firm be x and the number of gadgets B produced by the firm be y.
The profit on the sale of A is ₹ 30 per unit and on the sale of B is ₹ 20 per unit.

∴ total profit is $z = 30x + 20y$

This is a linear function that is to be maximized. Hence it is the objective function. The constraints are as per the following table:

Gadgets	**Foundry**	**Machine shop**	**Total available Time (in hour)**
A	10	5	60
B	6	4	35

From the table total man-hours of labour required for x units of gadget A and y units of gadget B in foundry is (10x + 6y) hours and total man-hours of labour required in machine shop is (5x + 4y) hours.

Since the maximum time available in foundry and machine shops are 60 hours and 35 hours respectively. Therefore, the constraints are $10x + 6y \le 60$, $5x + 4y \le 35$.

Since, x and y cannot be negative, we have $x \ge 0$, $y \ge 0$. Hence, the given LPP can be formulated as:

Maximize $z = 30x + 20y$, subject to

$10x + 6y \le 60$,
$5x + 4y \le 35$,
$x \ge 0$, $y \ge 0$

Exercise 7.3 | Q 2 | Page 237

Fodder → **Nutrient ↓**	Fodder 1	Fodder 2
Nutrients A	2	1
Nutrients B	2	3
Nutrients C	1	1

The cost of fodder 1 is ₹ 3 per unit and that of fodder 2 ₹ 2. Formulate the LPP to minimize the cost.

Solution: Let x units of fodder 1 and y units of fodder 2 be prescribed. The cost of fodder 1 is ₹ 3 per unit and cost of fodder 2 is ₹ 2 per unit.

∴ total cost is $z = 3x + 2y$

This is the linear function which is to be minimized. Hence it is the objective function. The constraints are as per the following table:

Fodder → **Nutrient ↓**	Fodder 1	Fodder 2	Minimum requirements
Nutrients A	2	1	14
Nutrients B	2	3	22
Nutrients C	1	1	1

From table fodder contains $(2x + y)$ units of nutrients A,$(2x + 3y)$ units of nutrients B and $(x + y)$ units of nutrients C. The minimum requirements of these nutrients are 14 units, 22 units, and 1 unit respectively.

Therefore, the constraints are

$2x + y \geq 14$, $2x + 3y \geq 22$, $x + y \geq 1$

Since, number of units (i.e. x and y) cannot be negative, we have, $x \geq 0$, $y \geq 0$.

Hence, the given LPP can be formulated as

Minimize $z = 3x + 2y$, subject to

$2x + y \geq 14$, $2x + 3y \geq 22$, $x + y \geq 1$, $x \geq 0$, $y \geq 0$.

Exercise 7.3 | Q 3 | Page 237

A company manufactures two types of chemicals Aand B. Each chemical requires two types of raw material P and Q. The table below shows number of units of P and Q required to manufacture one unit of A and one unit of B and the total availability of P and Q.

Chemical→ Raw Material ↓	**A**	**B**	**Availability**
P	3	2	120
Q	2	5	160

The company gets profits of ₹ 350 and ₹ 400 by selling one unit of A and one unit of B respectively. (Assume that the entire production of A and B can be sold). How many units of the chemicals A and B should be manufactured so that the company gets a maximum profit? Formulate the problem as LPP to maximize profit.

Solution: Let the company manufactures x units of chemical A and y units of chemical B. Then the total profit to the company is p = ₹(350x + 400y).

This is a linear function that is to be maximized. Hence, it is an objective function.

The constraints are as per the following table:

Chemical→ Raw Material ↓	**A** **(x)**	**B** **(y)**	**Availability**
P	3	2	120
Q	2	5	160

The raw material P required for x units of chemical A and y units of chemical B is 3x + 2y. Since the maximum availability of P is 120, we have the first constraint as $3x + 2y \le 120$.

Similarly, considering the raw material Q, we have $2x + 5y \le 160$.

Since, x and y cannot be negative, we have, $x \ge 0, y \ge 0$.
Hence, the given LPP can be formulated as:

Maximize p = 350x + 400y, subject to

$3x + 2y \le 120, 2x + 5y \le 160, x \ge 0, y \ge 0$

Exercise 7.3 | Q 4 | Page 237

A printing company prints two types of magazines A and B. The company earns ₹ 10 and ₹ 15 in magazines A and B per copy. These are processed on three Machines I, II, III. Magazine A requires 2 hours on Machine I, 5 hours on Machine II, and 2 hours on machine III. Magazine B requires 3 hours on machine I, 2 hours on machine II and 6 hours on Machine III. Machines I, II, III are available for 36, 50, and 60 hours per week respectively. Formulate the LPP to determine weekly production of magazines A and B, so that the total profit is maximum.

Solution: Let the company prints x magazine of type A and y magazine of type B.

Profit on sale of magazine A is ₹ 10 per copy and magazine B is ₹ 15 per copy.
Therefore, the total earning z of the company is z = ₹(10x + 15y).

This is a linear function that is to be maximized. Hence, it is an objective function.

The constraints are as per the following table:

Magazine type →	Time required per unit		Available time per week (in hours)
Machine type ↓	Magazine A (x)	Magazine B (y)	
Machine I	2	3	36
Machine II	5	2	50

Machine III	2	6	60

From the table, the total time required for Machine I is $(2x + 3y)$ hours, for Machine II is $(5x + 2y)$ hours and Machine III is $(2x + 6y)$ hours.

The machines I, II, III are available for 36, 50, and 60 hours per week. Therefore, the constraints are $2x + 3y \leq 36$, $5x + 2y \leq 50$, $2x + 6y \leq 60$.

Since x and y cannot be negative. We have, $x \geq 0$, $y \geq 0$. Hence, the given LPP can be formulated as:

Maximize $z = 10x + 15y$, subject to

$2x + 3y \leq 36$, $5x + 2y \leq 50$, $2x + 6y \leq 60$, $x \geq 0$, $y \geq 0$.

Exercise 7.3 | Q 5 | Page 237

A manufacturer produces bulbs and tubes. Each of these must be processed through two machines M_1 and M_2. A package of bulbs requires 1 hour of work on Machine M_1 and 3 hours of work on Machine M_2. A package of tubes requires 2 hours on Machine M_1 and 4 hours on Machine M_2. He earns a profit of ₹ 13.5 per package of bulbs and ₹ 55 per package of tubes. Formulate the LPP to maximize the profit, if he operates the machine M_1, for almost 10 hours a day and machine M_2 for almost 12 hours a day.

Solution: Let the number of packages of bulbs produced by manufacturer be x and packages of tubes be y. The manufacturer earns a profit of ₹ 13.5 per package of bulbs and ₹ 55 per package of tubes.
Therefore, his total profit is p = ₹ $(13.5x + 55y)$

This is a linear function that is to be maximized. Hence, it is an objective function.

The constraints are as per the following table:

	Bulbs (x)	**Tubes (y)**	**Available Time**
Machine M_1	1	2	10
Machine M_2	3	4	12

From the table, the total time required for Machine M_1 is $(x + 2y)$ hours and for Machine M_2 is $(3x + 4y)$ hours. Given Machine M_1 and M_2 are available for at most 10 hours and 12 hours a day respectively.

Therefore, the constraints are $x + 2y \leq 10$, $3x + 4y \leq 12$. Since, x and y cannot be negative, we have, $x \geq 0$, $y \geq 0$. Hence, the given LPP can be formulated as:

Maximize $p = 13.5x + 55y$, subject to

$x + 2y \leq 10$, $3x + 4y \leq 12$, $x \geq 0$, $y \geq 0$

Exercise 7.3 | Q 6 | Page 238

A company manufactures two types of fertilizers F_1 and F_2. Each type of fertilizer requires two raw materials A and B. The number of units of A and B required to manufacture one unit of fertilizer F_1 and F_2 and availability of the raw materials A and B per day are given in the table below:

Fertilizers→ Raw Material ↓	**F_1**	**F_2**	**Availability**
A	2	3	40
B	1	4	70

By selling one unit of F_1 and one unit of F_2, the company gets a profit of ₹ 500 and ₹ 750 respectively. Formulate the problem as LPP to maximize the profit.

Solution: Let the company manufactures x units of fertilizers F_1 and y units of fertilizers F_2. Then the total profit to the company is $z = ₹(500x + 750y)$.

This is a linear function that is to be maximized. Hence, it is an objective function.

Fertilizers→ Raw Material ↓	**F_1**	**F_2**	**Availability**
A	2	3	40
B	1	4	70

The raw material A required for x units of Fertilizers F_1 and y units of Fertilizers F_2 is $2x + 3y$. Since the maximum availability of A is 40, we have the first constraint as $2x + 3y \leq 40$.

Similarly, considering the raw material B, we have $x + 4y \leq 70$.

Since, x and y cannot be negative, we have, $x \geq 0$, $y \geq 0$.
Hence, the given LPP can be formulated as:

Maximize $z = 500x + 750y$, subject to

$2x + 3y \leq 40$, $x + 4y \leq 70$, $x \geq 0$, $y \geq 0$

Exercise 7.3 | Q 7 | Page 237

A doctor has prescribed two different units of foods A and B to form a weekly diet for a sick person. The minimum requirements of fats, carbohydrates and proteins are 18, 28, 14 units respectively. One unit of food A has 4 units of fat, 14 units of carbohydrates and 8 units of protein. One unit of food B has 6 units of fat, 12 units of carbohydrates and 8 units of protein. The price of food A is ₹ 4.5 per unit and that of food B is ₹ 3.5 per unit. Form the LPP, so that the sick person's diet meets the requirements at a minimum cost.

Solution: Let x units of food A and y units of food B be prescribed in the weekly diet of a sick person.
The price for food A is ₹ 4.5 per unit and for food B is ₹ 3.5 per unit.

∴ Total cost is z = ₹ (4.5x + 3.5y)
We construct a table with constraints of fats, carbohydrates and proteins as follows:

Nutrients\Foods	**A (x)**	**B (y)**	**Minimum requirement**
Fats	**4**	**6**	**18**
Carbohydrates	**14**	**12**	**28**
Protein	**8**	**8**	**14**

From the table, diet of sick person must include (4x + 6y) units of fats, (14x + 12y) units of carbohydrates and (8x + 8y) units of proteins
∴ The constraints are
$4x + 6y \geq 18$,
$14x + 12y \geq 28$,
$8x + 8y \geq 14$.
Since x and y cannot be negative, we have $x \geq 0$, $y \geq 0$
∴ Given problem can be formulated as follows:
Minimize $z = 4.5x + 3.5y$
Subject to $4x + 6y \geq 18$, $14x + 12y \geq 28$, $8x + 8y \geq 14$, $x \geq 0$, $y \geq 0$.

Exercise 7.3 | Q 8 | Page 238

If John drives a car at a speed of 60 km/hour, he has to spend ₹ 5 per km on petrol. If he drives at a faster speed of 90 km/hour, the cost of petrol increases ₹ 8 per km. He has ₹ 600 to spend on petrol and wishes to travel the maximum distance within an hour. Formulate the above problem as L.P.P.

Solution: Let John travel x_1 km at speed of 60 km/hr and x_2 km at a speed of 90 km/hr.

∴ Total distance = $(x_1 + x_2)$ km

Time = $\frac{\text{Distance}}{\text{Speed}}$

Time to travel x1 km = $\left(\frac{x_1}{60}\right)$ hours and time to travel x_2 km = $\left(\frac{x_2}{90}\right)$ hours.

∴ Total time = $\left(\frac{x_1}{60} + \frac{x_2}{90}\right)$hours

But John wishes to travel maximum distance within an hour.

$\therefore \frac{x_1}{60} + \frac{x_2}{90} \le 1$

John has to spend ₹ 5 per km at 60 km/hr and ₹ 8 per km at 90 km/hr.

∴ Total cost = ₹ $(5x_1 + 8x_2)$

But John has ₹ 600 to spend on petrol

∴ $5x_1 + 8x_2 \le 600$

Since x_1 and x_2 cannot be negative, we have $x_1 \ge 0$, $x_2 \ge 0$

∴ Given problem can be formulated as follows:

Maximize Z = x1 + x2,

Subject to $\frac{x_1}{60} + \frac{x_2}{90} \le 1, 5x_1 + 8x_2 \le 600$, $x_1 \ge 0$, $x_2 \ge 0$.

Exercise 7.3 | Q 9 | Page 378

The company makes concrete bricks made up of cement and sand. The weight of a concrete brick has to be at least 5 kg. Cement costs ₹ 20 per kg and sand costs of ₹ 6 per kg. Strength consideration dictates that a concrete brick should contain minimum 4 kg of cement and not more than 2 kg of sand. Form the L.P.P. for the cost to be minimum.

Solution1: Let the company use x_1 kg of cement and x_2 kg of sand to make concrete bricks.

Cement costs ₹ 20 per kg and sand costs ₹ 6 per kg.

∴ the total cost c = ₹ $(20x_1 + 6x_2)$

This is a linear function which is to be minimized.

Hence, it is an objective function.

Total weight of brick = (x_1 + x_2) kg

Since the weight of concrete brick has to be at least 5 kg,

∴ $x_1 + x_2 \geq 5$

Since concrete brick should contain minimum 4 kg of cement and not more than 2 kg of sand,

$x_1 \geq 4$ and $0 \leq x_2 \leq 2$

Hence, the given LPP can be formulated as:

Minimize c = $20x_1 + 6x_2$, subject to

$x_1 + x_2 \geq 5$, $x_1 \geq 4$, $0 \leq x_2 \leq 2$.

SOLUTION 2

Let the concrete brick contain x_1 kg of cement and x_2 kg of sand Cement costs ₹ 20 per kg and sand costs ₹ 6 per kg.

∴ Total cost = ₹ ($20x_1 + 6x_2$)

Weight of a concrete brick has to be at least 5 kg.

∴ $x_1 + x_2 \geq 5$

The brick should contain minimum 4 kg of cement.

∴ $x_1 \geq 4$

The brick should contain not more than 2 kg of sand.

∴ $x_2 \leq 2$

Since x_1 and x_2 cannot be negative, we have $x_1 \geq 0$, $x_2 \geq 0$

∴ Given problem can be formulated as follows:

Minimize Z = $20x_1 + 6x_2$

Subject to $x_1 + x_2 \geq 5$, x1 ≥ 4, $x_2 \leq 2$, $x_1 \geq 0$, $x_2 \geq 0$.

EXERCISE 7.4 [PAGE 241]

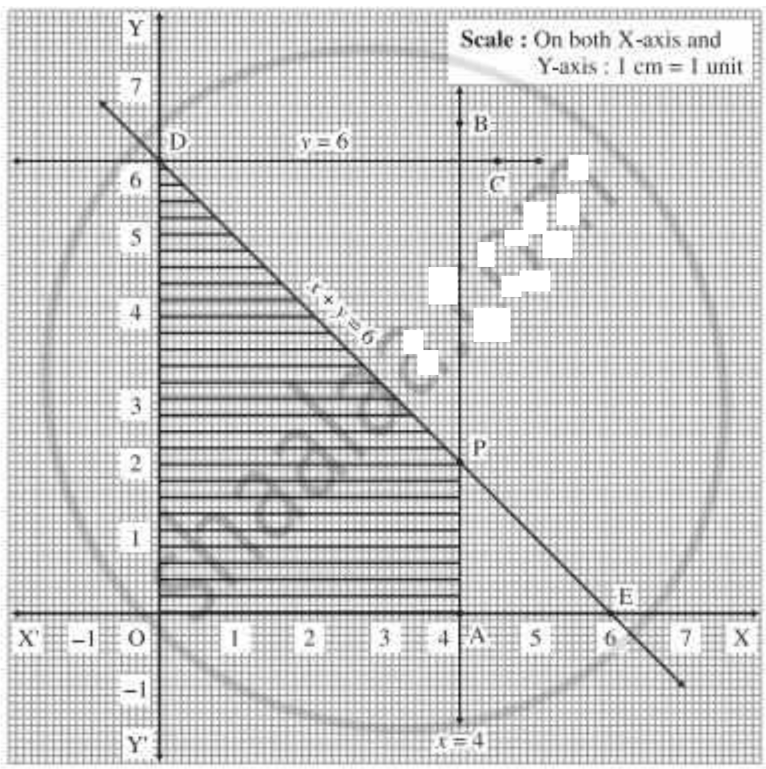
Exercise 7.4 | Q 1 | Page 241

Solve the following LPP by graphical method:

Maximize z = 11x + 8y, subject to x ≤ 4, y ≤ 6, x + y ≤ 6, x ≥ 0, y ≥ 0,

Solution: First we draw the lines AB, CD and ED whose equations are x = 4, y = 6 and x + y = 6 respectively.

Line	Equation	Points on the X-axis	Points on the Y-axis	Sign	Region
AB	x = 4	A(4, 0)	-	≤	origin side of the line AB
CD	y = 6	-	D(0, 6)	≤	origin side of the line CD
EF	x + y = 6	E(6, 0)	D(0, 6)	≤	origin side of the line ED



The feasible region is shaded portion OAPDO in the graph.

The vertices of the feasible region are O (0, 0), A (4, 0), P and D (0, 6)

P is point of intersection of lines x + y = 6 and x = 4.

Substituting x = 4 in x + y = 6, we get

4 + x = 6 ∴ y = 2 ∴ P is (4, 2)

∴ the corner points of feasible region are O (0, 0), A (4, 0), P (4, 2) and D (0, 6).

The values of the objective function z = 11x + 8y at these vertices are

$z(O) = 11(0) + 8(0) = 0 + 0 = 0$

$z(a) = 11(4) + 8(8) = 44 + 0 = 44$

$z(P) = 11(4) + 8(2) = 44 + 16 = 60$

$z(D) = 11(0) + 8(2) = 0 + 16 = 16$

$\therefore$ z has maximum value 60, when $x = 4$ and $y = 2$.

Exercise 7.4 | Q 2 | Page 241

Solve the following LPP by graphical method:

Maximize $z = 4x + 6y$, subject to $3x + 2y \leq 12$, $x + y \geq 4$, $x, y \geq 0$.

Solution: First we draw the lines AB, AD whose equations are $3x + 2y = 12$ and $x + y = 4$ respectively.

Line	Equation	Points on the X-axis	Points on the Y-axis	Sign	Region
AB	$3x + 2y = 12$	A(4, 0)	B(0, 6)	$\leq$	origin side of the line AB
AC	$x + y = 4$	A(4, 0)	C(0, 4)	$\geq$	non-origin side of line AC

The feasible region is the Δ ABC which is shaded in the graph.

The vertices of the feasible region (i.e. corner points) are A(4, 0), B (0, 6) and C (0, 4).

The values of the objective function z = 4x + 6y at these vertices are

z(a) = 4(4) + 6(0) = 16 + 0 = 16

z(B) = 4(0) + 6(6) = 0 + 36 = 36

z(C) = 4(0) + 6(4) = 0 + 24 = 24

∴ z has maximum value 36, when x = 0 and y = 6.

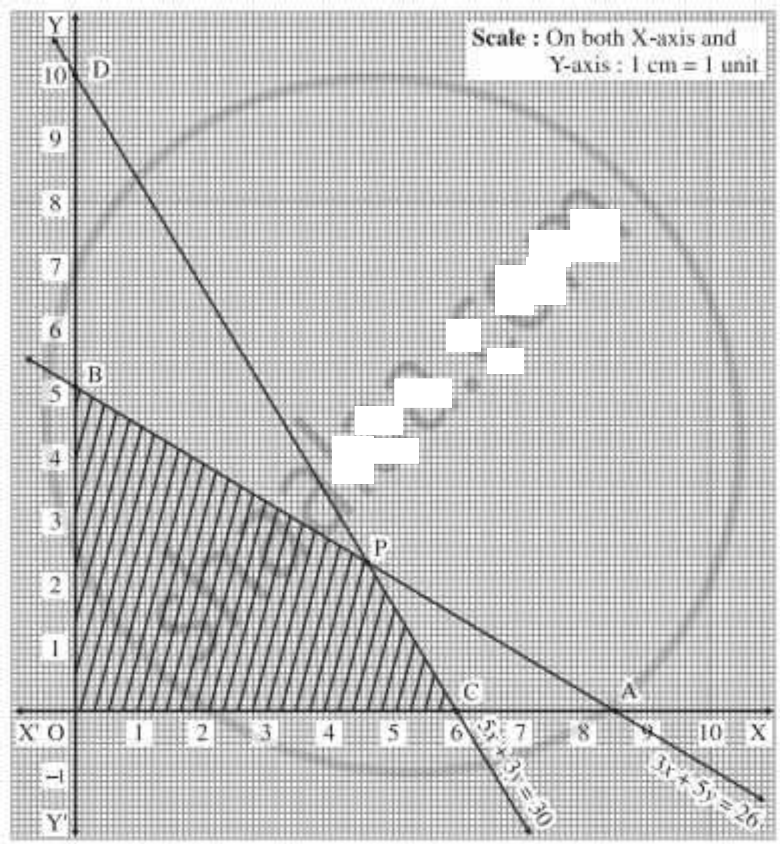
Exercise 7.4 | Q 3 | Page 241

Solve the following LPP by graphical method:

Maximize z = 7x + 11y, subject to 3x + 5y ≤ 26, 5x + 3y ≤ 30, x ≥ 0, y ≥ 0.

Solution: First we draw the lines AB and CD whose equations are 3x + 5y = 26 and 5x + 3y = 30 respectively.

Line	Equation	Points on the X-axis	Points on the Y-axis	Sign	Region
AB	3x + 5y = 26	A$\left(\frac{26}{3}, 0\right)$	B $\left(0, \frac{26}{5}\right)$	≤	origin side of line AB
CD	5x + 3y = 30	C(6, 0)	D(0, 10)	≤	origin side of line CD



The feasible region is OCPBO which is shaded in the graph.

The vertices of the feasible region are O (0, 0), C (6, 0), P and B

$$\left(0, \frac{26}{5}\right).$$

The vertex P is the point of intersection of the lines 3x + 5y = 26 .....(1)

and 5x + 3y = 30 .....(2)

Multiplying equation (1) by 3 and equation (2) by 5, we get

9x + 15y = 78

and 25x + 15y = 150

On subtracting, we get

16x = 72

∴ x = 72/16 = 9/2 = 4.5

Substituting x = 4.5 in equation (2), we get

5(4.5) + 3y = 30

$22.5 + 3y = 30$

$\therefore 3y = 7.5$

$\therefore y = 2.5$

$\therefore$ P is (4.5, 2.5)

The values of the objective function $z = 7x + 11y$ at these corner points are

$z(O) = 7(0) + 11(0) = 0 + 0 = 0$

$z(C) = 7(6) + 11(0) = 42 + 0 = 42$

$z(P) = 7(4.5) + 11(2.5) = 31.5 + 27.5 = 59.0 = 59$

$$z(B) = 7(0) + 11\left(\frac{26}{5}\right) = \frac{286}{5} = 57.2$$

$\therefore$ z has maximum value 59, when $x = 4.5$ and $y = 2.5$.

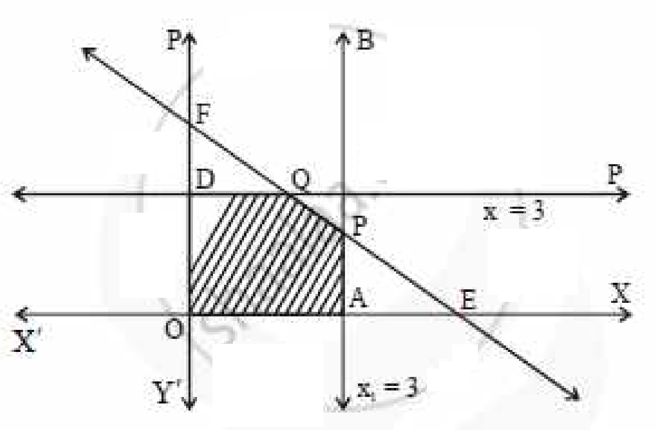
Exercise 7.4 | Q 4 | Page 241

Solve the following L.P.P graphically:

Maximize: $Z = 10x + 25y$
Subject to: $x \leq 3$, $y \leq 3$, $x + y \leq 5$, $x \geq 0$, $y \geq 0$

Solution: First we draw the lines AB, CD and EF whose equations are $x = 3$, $y = 3$ and $x + y = 5$ respectively.

Line	Equation	Point on the X-axis	Point on the Y-axis	Sign	Region
AB	$x = 3$	A(3,0)	-	$\leq$	origin side of line AB
CD	$y = 3$	-	D(0,3)	$\leq$	origin side of line CD
EF	$x + y = 5$	E(5,0)	F(0,5)	$\leq$	origin side of line EF



The feasible region is OAPQDO which is shaded in the figure.
The vertices of the feasible region are O (0,0), A (3, 0), P, Q and D (0, 3)
P is the point of intersection of the lines x + y = 5 and x = 3
Substituting x = 3 in x + y = 5, we get,

3+ y=5

y = 2
P≡ (3, 2)
Q is the point of intersection of the lines x + y = 5 and y = 3
Substituting y = 3 in x + y = 5, we get,
x + 3 = 5

x = 2
Q ≡ (2,3)
The values of the objective function z = 10x + 25y at these vertices are

Z(O) =10(0)+ 25(0)= 0
Z(A) =10(3) + 25(0) = 30
Z(P) =10(3) + 25(2) = 30 + 50 = 80
Z(Q) =10(2) + 25(3) = 20 + 75 = 95
Z(D) =10(0) + 25(3) =75

Z has max imumvalue 95, when x = 2 and y = 3.

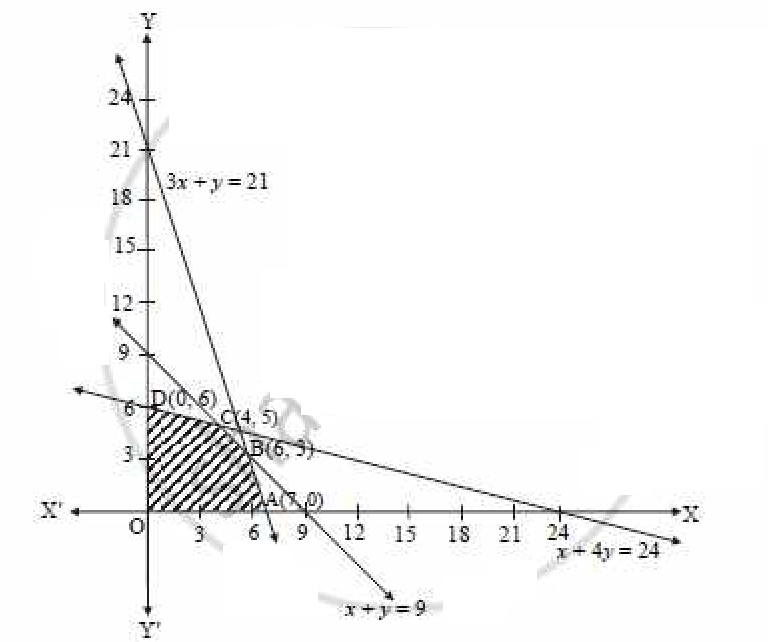
Exercise 7.4 | Q 5 | Page 241

Solve the following LPP by graphical method:

Maximize: z = 3x + 5y
Subject to: x + 4y ≤ 24
3x + y ≤ 21
x + y ≤ 9
x ≥ 0, y ≥ 0

Solution: To draw the feasible region, construct table as follows :

Inequality	x + 4y ≤ 24	3x + y ≤ 21	x + y ≤ 9
Corresponding equation (of line)	x + 4y = 24	3x + y = 21	x + y = 9
Intersection of line with X-axis	(24, 0)	(7, 0)	(9, 0)
Intersection of line with Y-axis	(0, 6)	(0, 21)	(0, 9)
Region	Origin side	Origin side	Origin side



Shaded portion OABCD is the feasible region,
whose vertices are O(0, 0), A(7, 0), B, C and (0, 6)
B is the point of intersection of the lines 3x + y = 21 and x+y = 9.
Solving the above equations, we get x = 6, y = 3
∴ B ≡ (6, 3)
C is the point of intersection of the lines
x + 4y = 24
and x + y = 9.
Solving the above equations, we get
x = 4, y = 5
∴ C ≡ (4, 5)
Here, the objective function is Z = 3x + 5y,

Z at O(0, 0) = 3(0) + 5(0) = 0
Z at A(7, 0) = 3(7) + 5(0) = 21
Z at B(6, 3) = 3(6) + 5(3) = 18 + 15 = 33
Z at C(4, 5) = 3(4) + 5(5) = 12 + 25 = 37
Z at D(0, 6) = 3(0) + 5(6) = 30

∴ Z has maximum value 37 at C(4, 5).

∴ Z is maximum, when x = 4, y = 5

Exercise 7.4 | Q 6 | Page 241

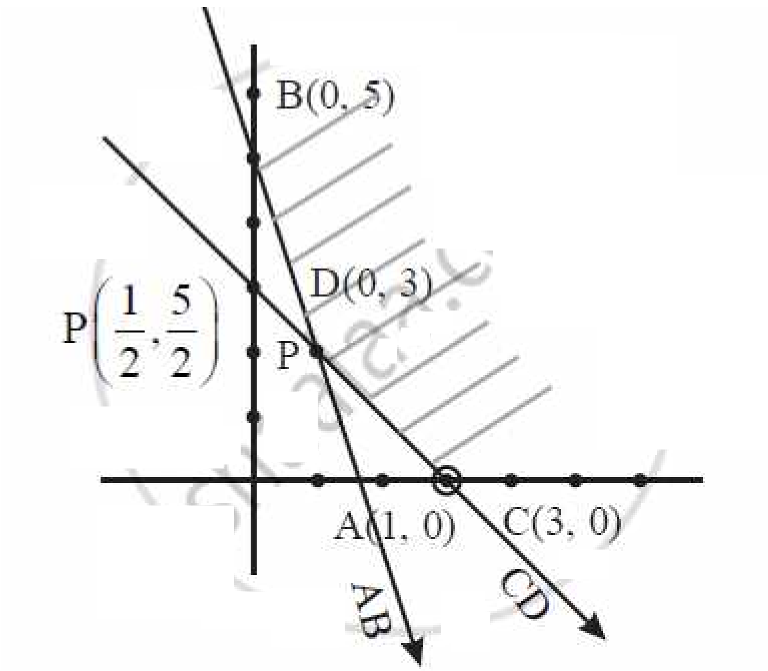
Solve the following LPP by graphical method:

Minimize Z = 7x + y subject to 5x + y ≥ 5, x + y ≥ 3, x ≥ 0, y ≥ 0

Solution: First we draw the lines AB and CD whose equations are 5x + y = 5 and x + y = 3 respectively.

Line	Inequation	Points on x	Points on y	Sign	Feasible region
AB	5x + y = 5	A(1,0)	B(0,5)	≥	Non - origin side AB
CD	x + y = 3	C(3,0)	D(0,3)	≥	Non - origin side of line CD

1 unit = 1 cm both axis



common feasible region BPC

Points	Minimize z = 7x + y
B(0,5)	Z(B) = 7(0) + 5 = 5
$P\left(\frac{1}{2}, \frac{5}{2}\right)$	$Z(P) = 7 \times \frac{1}{2} + \frac{5}{2} = 6$
C(3,0)	Z(C) = 7x(3) + 0 = 21

Z is minimum at x = 0, y = 5 and min (z) = 5

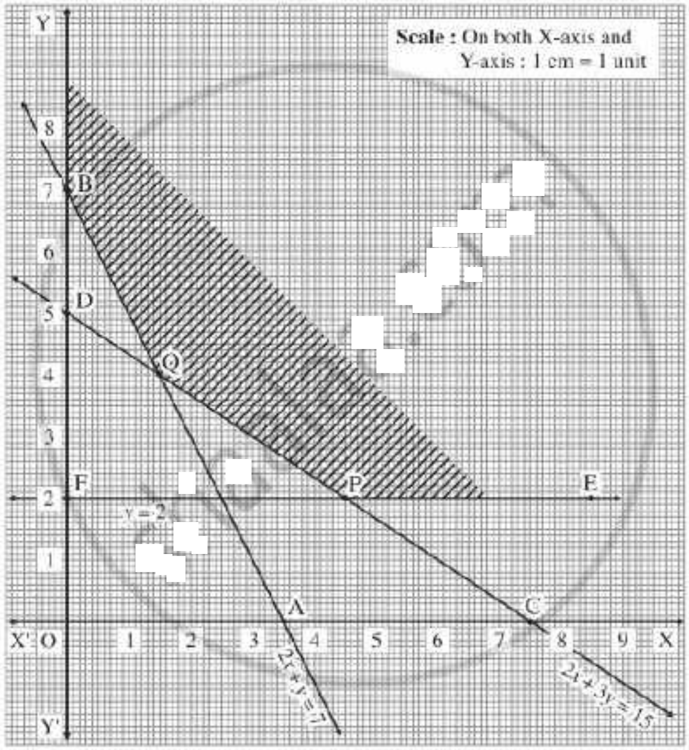
Exercise 7.4 | Q 7 | Page 241

Solve the following LPP by graphical method:

Minimize z = 8x + 10y, subject to 2x + y ≥ 7, 2x + 3y ≥ 15, y ≥ 2, x ≥ 0, y ≥ 0.

Solution: First we draw the lines AB, CD and EF whose equations are 2x + y = 7, 2x + 3y = 15 and y = 2 respectively.

Line	Equation	Points on the X-axis	Points on the Y-axis	Sign	Region
AB	2x + y = 7	A(3.5, 0)	B(0, 7)	≥	non-origin side of line AB
CD	2x + 3y = 15	C(7.5, 0)	D(0, 5)	≥	non-origin side of line CD
EF	y = 2	-	F(0,2)	≥	non-origin side of line EF



The feasible region is EPQBY which is shaded in the graph. The vertices of the feasible region are P, Q and B (0, 7). P is the point of intersection of the lines 2x + 3y = 15 and y = 2.

Substituting y = 2 in 2x + 3y = 15, we get

2x + 3(2) = 15

∴ 2x = 9

∴ x = 4.5

∴ P = (4.5, 2)

Q is the point of intersection of the lines

2x + 3y = 15 .....(1)

and 2x + y = 7

On subtracting, we get

2y = 8

∴ y = 4

∴ from (2), 2x + 4 = 7

∴ 2x = 3

∴ x = 1.5

∴ Q = (1.5, 4)

The values of the objective function z = 8x + 10y at these vertices are

z(P) = 8(4.5) + 10(2) = 36 + 20 = 56

z(Q) = 8(1.5) + 10(4) = 12 + 40 = 52

z(B) = 8(0) + 10(7) = 70

∴ z has minimum value 52, when x = 1.5 and y = 4.

Exercise 7.4 | Q 8 | Page 241

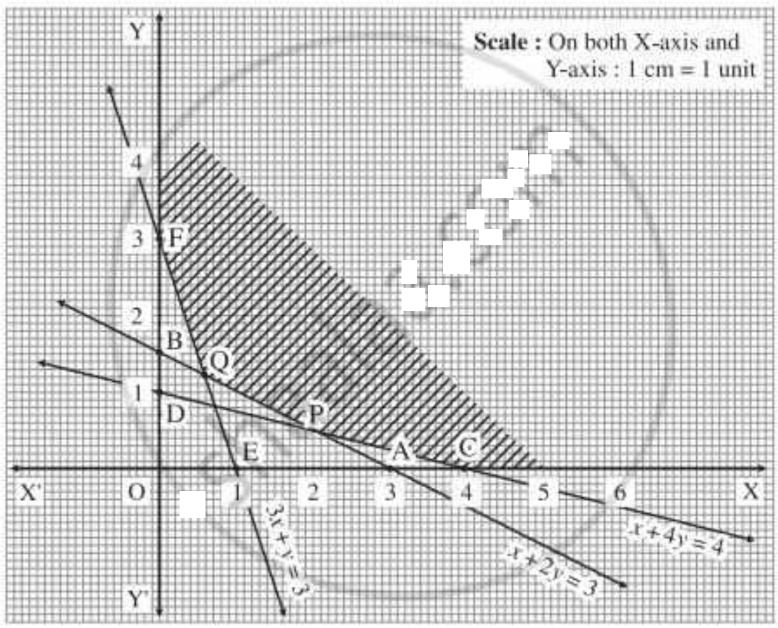
Solve the following LPP by graphical method:

Minimize z = 6x + 21y, subject to x + 2y ≥ 3, x + 4y ≥ 4, 3x + y ≥ 3, x ≥ 0, y ≥ 0.

Solution: First we draw the lines AB, CD and EF whose equations are x + 2y = 3, x + 4y = 4 and 3x + y = 3 respectively.

First we draw the lines AB, CD and EF whose equations are x + 2y = 3, x + 4y = 4 and 3x + y = 3 respectively.

Line	Equation	Points on the X-axis	Points on the Y-axis	Sign	Region
AB	x + 2y = 3	A(3, 0)	B(0,3/2)	≥	non-origin side of line AB
CD	x + 4y = 4	C(4, 0)	D(0, 1)	≥	non-origin side of line CD
EF	3x + y = 3	E(1, 0)	F(0, 3)	≥	non-origin side of line EF



The feasible region is XCPQFY which is shaded in the graph.

The vertices of the feasible region are C(4, 0), P, Q and F (0, 3).

P is the point of intersection of the lines x + 4y = 4 and x + 2y = 3

On subtracting, we get

2y = 1

∴ y = 1/2

Substituting y = 1/2 in x + 2y = 3, we get

x + 2, (1/2) = 3

∴ x = 2

∴ P ≡ (2,1/2)

Q is the point of intersection of the lines

x + 2y = 3 ....(1)

and 3x + y = 3 ...(2)

Multiplying equation (1) by 3, we get

3x + 6y = 9

Subtracting equation (2) from this equation, we get

5y = 6

$\therefore y = \frac{6}{5}$

$\therefore$ from (1), $x + 2\left(\frac{6}{5}\right) = 3$

$\therefore x = 3 - \frac{12}{5} = \frac{3}{5}$

$\therefore Q \equiv \left(\frac{3}{5}, \frac{6}{5}\right)$

The values of the objective function z = 6x + 21y at these vertices are

z(C) = 6(4) + 21(0) = 24

$z(P) = 6(2) + 21\left(\frac{1}{2}\right)$

= 12 + 10.5 = 22.5

$z(Q) = 6\left(\frac{3}{5}\right) + 21\left(\frac{6}{5}\right)$

$= \frac{18}{5} + \frac{126}{5} = \frac{144}{5} = 28.8$

z(F) = 6(0) + 21(3) = 63

$\therefore$ z has minimum value 22.5, when x = 2 and $y = \frac{1}{2}$.

MISCELLANEOUS EXERCISE 7 [PAGES 242 - 243]

Miscellaneous exercise 7 | Q 1 | Page 242

Select the appropriate alternatives for each of the following question:

The value of objective function is maximum under linear constraints

1. at the centre of feasible region
2. at (0, 0)

3. **at a vertex of feasible region**
4. the vertex which is of maximum distance from (0, 0).

Solution: at a vertex of feasible region

Miscellaneous exercise 7 | Q 2 | Page 242

Select the appropriate alternatives for each of the following question:

Which of the following is correct?

1. Every LPP has an optimal solution
2. A LPP has unique optimal solution
3. **If LPP has two optimal solutions, then it has infinite number of optimal solutions**
4. The set of all feasible solution of LPP may not be convex set

Solution: If LPP has two optimal solutions, then it has infinite number of optimal solutions

Miscellaneous exercise 7 | Q 3 | Page 242

Select the appropriate alternatives for each of the following question:

Objective function of LPP is

1. a constraint
2. **a function to be maximized or minimized**
3. a relation between the decision variables
4. equation of a straight line

Solution: a function to be maximized or minimized

Miscellaneous exercise 7 | Q 4 | Page 242

Select the appropriate alternatives for each of the following question:

The maximum value of $z = 5x + 3y$ subject to the constraints $3x + 5y \leq 15$, $5x + 2y \leq 10$, $x, y \geq 0$ is

1. 235
2. 235/9
3. **235/19**
4. 235/3

Solution: 235/19

Miscellaneous exercise 7 | Q 5 | Page 242

Select the appropriate alternatives for each of the following question:

The maximum value of $z = 10x + 6y$ subject to the constraints $3x + y \le 12$, $2x + 5y \le 34$, $x, \ge 0$, $y \ge 0$ is

1. **56**
2. 65
3. 55
4. 66

Solution: 56

Miscellaneous exercise 7 | Q 6 | Page 242

Select the appropriate alternatives for each of the following question:

The point of which the maximum value of $x + y$ subject to the constraints $x + 2y \le 70$, $2x + y \le 95$, $x, \ge 0$, $y \ge 0$ is is obtained at

1. (30, 25)
2. (20, 35)
3. (35, 20)
4. **(40, 15)**

Solution: (40, 15)

Miscellaneous exercise 7 | Q 7 | Page 242

Select the appropriate alternatives for each of the following question:

Of all the points of the feasible region, the optimal value of z obtained at the point lies

1. inside the feasible region
2. at the boundary of the feasible region
3. **at vertex of feasible region**
4. outside the feasible region

Solution: at vertex of feasible region

Miscellaneous exercise 7 | Q 8 | Page 242

Select the appropriate alternatives for each of the following question:

Feasible region is the set of points which satisfy

1. the objective function
2. **all the given constraints**
3. some of the given constraints

4. only one constraint

Solution: all the given constraints

Miscellaneous exercise 7 | Q 9 | Page 243

Select the appropriate alternatives for each of the following question:

Solution of LPP to minimize z = 2x + 3y, such that x ≥ 0, y ≥ 0, 1 ≤ x + 2y ≤ 10 is

1. **x = 0, y = 1/2**
2. x = 1/2, y = 0
3. x = 1, y = 2
4. x = 1/2, y = 1/2

Solution: x = 0, y = 1/2

Miscellaneous exercise 7 | Q 10 | Page 243

Select the appropriate alternatives for each of the following question:

The corner points of the feasible solution given by the inequation x + y ≤ 4, 2x + y ≤ 7, x ≥ 0, y ≥ 0 are

1. (0, 0), (4, 0), (7, 1), (0, 4)
2. **(0, 0), (7/2,0), (3, 1), (0, 4)**
3. (0, 0), (7/2,0), (3, 1), (0, 7)
4. (0, 0), (4, 0), (3, 1), (0, 7)

Solution: (0, 0), (7/2,0), (3, 1), (0, 4)

Miscellaneous exercise 7 | Q 11 | Page 243

Select the appropriate alternatives for each of the following question:

The corner points of the feasible solution are (0, 0), (2, 0), (12/7,3/7), (0, 1). Then z = 7x + y is maximum at

1. (0, 0)
2. **(2, 0)**
3. (12/7,3/7)
4. (0, 1)

Solution: (2, 0)

Miscellaneous exercise 7 | Q 12 | Page 243

Select the appropriate alternatives for each of the following question:

If the corner points of the feasible solution are (0, 0), (3, 0), (2, 1), (0, 7/3) the maximum value of z = 4x + 5y is

1. 12
2. **13**
3. 35/3
4. 0

Solution: 13

Miscellaneous exercise 7 | Q 13 | Page 243

Select the appropriate alternatives for each of the following question:

If the corner points of the feasible solution are (0, 10), (2, 2) and (4, 0), then the point of minimum z = 3x + 2y

1. **(2, 2)**
2. (0, 10)
3. (4, 0)
4. (3, 4)

Solution: (2, 2)

Miscellaneous exercise 7 | Q 14 | Page 243

Select the appropriate alternatives for each of the following question:

The half-plane represented by 3x + 2y < 8 contains the point

1. (1, 5/2)
2. (2, 1)
3. **(0, 0)**
4. (5, 1)

Solution: (0, 0)

Miscellaneous exercise 7 | Q 15 | Page 243

Select the appropriate alternatives for each of the following question:

The half-plane represented by 4x + 3y >14 contains the point

1. (0, 0)
2. (2, 2)
3. **(3, 4)**
4. (1, 1)

Solution: (3, 4)

MISCELLANEOUS EXERCISE 7 [PAGES 243 - 245]

Miscellaneous exercise 7 | Q 1.1 | Page 243

Solve each of the following inequations graphically using XY-plane:

$4x - 18 \geq 0$

Solution: Consider the line whose equation is $4x - 18 \geq 0$ i.e. $x = 18/4 = 9/2 = 4.5$

This represents a line parallel to Y-axis passing3through the point (4.5,0)

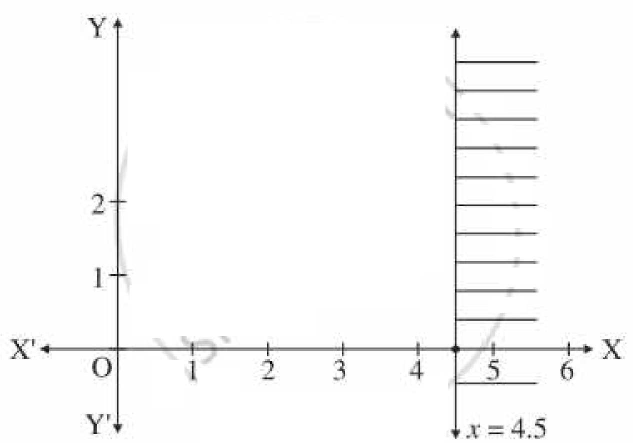
Draw the line $x = 4.5$

To find the solution set, we have to check the position of the origin (0, 0).

When $x = 0$, $4x – 18 = 4 \times 0 – 18 = – 18 > 0$

∴ the coordinates of the origin does not satisfy thegiven inequality.

∴ the solution set consists of the line $x = 4.5$ and the non-origin side of the line which is shaded in the graph.



Miscellaneous exercise 7 | Q 1.2 | Page 243

Solve each of the following inequations graphically using XY-plane:

$- 11x - 55 \leq 0$

Solution: Consider the line whose equation is $- 11x - 55 \leq 0$ i.e. x = -5`

This represents a line parallel to Y-axis passing3through the point (-5,0)

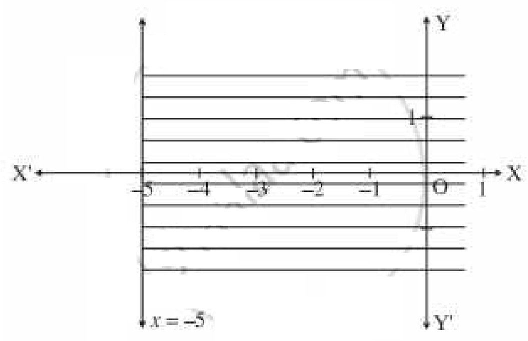
Draw the line $x = - 5$

To find the solution set, we have to check the position of the origin (0, 0).

When x = 0, - 11x - 55 = - 11(0) - 55 = - 55 > 0

∴ the coordinates of the origin does not satisfy thegiven inequality.

∴ the solution set consists of the line x = -5 and the non-origin side of the line which is shaded in the graph.



Miscellaneous exercise 7 | Q 1.3 | Page 243

Solve each of the following inequations graphically using XY-plane:

5y - 12 ≥ 0

Solution: Consider the line whose equation is 5y - 12 ≥ 0 i.e. y = 12/5

This represents a line parallel to X-axis passing3through the point (0,12/5)

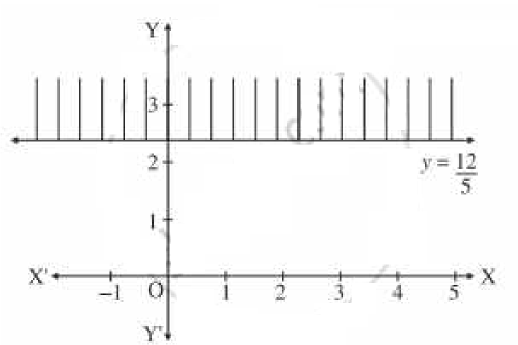
Draw the line y = 12/5

To find the solution set, we have to check the position of the origin (0, 0).

When y = 0, 5y - 12 = 5(0) - 12 = - 12 > 0

∴ the coordinates of the origin does not satisfy the given inequality.

∴ the solution set consists of the line y = 12/5 and the non-origin side of the line which is shaded in the graph.



Miscellaneous exercise 7 | Q 1.4 | Page 243

Solve each of the following inequations graphically using XY-plane:

y ≤ - 3.5

Solution: Consider the line whose equation is y ≤ - 3.5 i.e. y = - 3.5

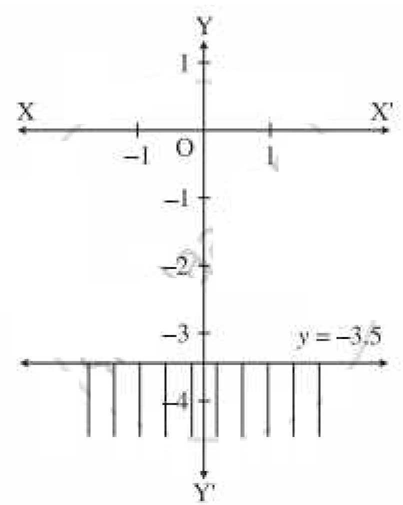
This represents a line parallel to X-axis passing3through the point (0,-3.5)

Draw the line y = - 3.5

To find the solution set, we have to check the position of the origin (0, 0).

∴ the coordinates of the origin does not satisfy the given inequality.

∴ the solution set consists of the line y = - 3.5 and the non-origin side of the line which is shaded in the graph.



Miscellaneous exercise 7 | Q 2.4 | Page 243

Sketch the graph of the following inequations in XOY-coordinate system:

|x + 5| ≤ y

Solution: |x + 5| ≤ y

∴ - y ≤ x + 5 ≤ y

∴ - y ≤ x + 5   and x + 5 ≤ y

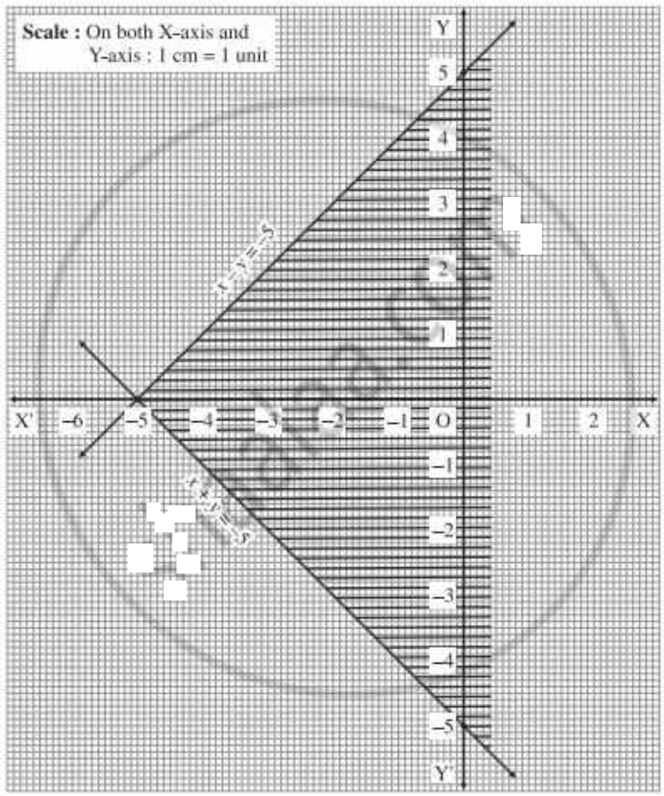
∴ x + y ≥ - 5   and x - y ≤ - 5

First we draw the lines AB and AC whose equations are x + y = - 5 and x - y = - 5 respectively.

Line	Equation	Points on the X-axis	Points on the Y-axis	Sign	region
AB	x + y = - 5	A(- 5, 0)	B(0, -5)	≥	origin side of line AB

AC	x - y = - 5	A(-5, 0)	C(0, 5)	≤	non-origin side of line AB

The graph of $|x + 5| \le y$ is as below:

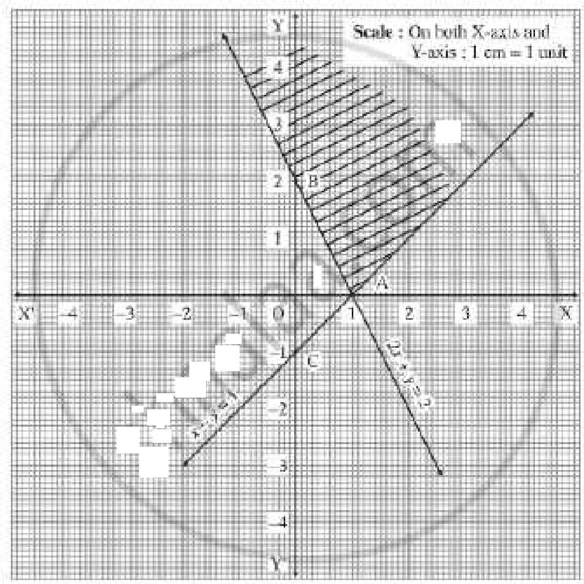


Miscellaneous exercise 7 | Q 3.1 | Page 243

Solve graphically : $2x + y \ge 2$ and $x – y \le 1$

Solution: First we draw the lines AB and AC whose equations are $2x + y = 2$ and $x – y = 1$ respectively.

Line	Equation	Points on the X-axis	Points on the Y-axis	Sign	Region
AB	2x + y = 2	A(1, 0)	B(0, 2)	≥	non-origin ssde of line AB
AC	x – y = 1	A(1, 0)	C(0, –1)	≤	origin side of the line AC



The solution set of the given system of inequalities is shaded in the graph.

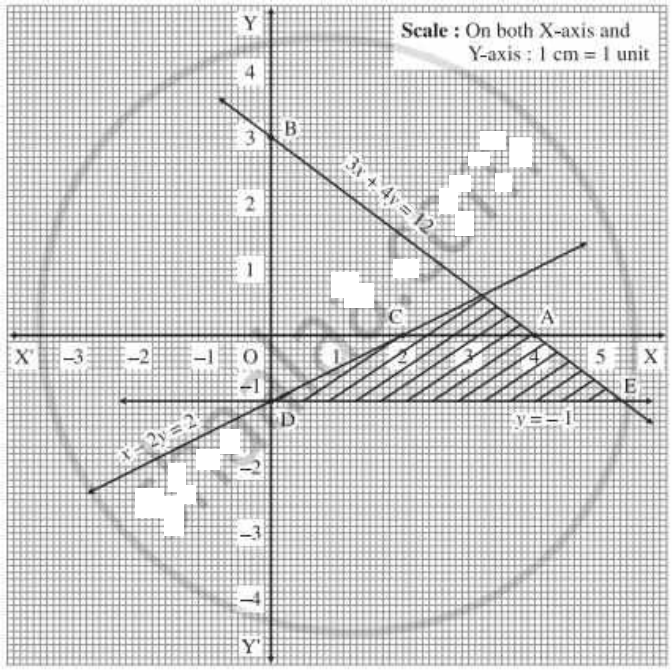
Miscellaneous exercise 7 | Q 3.3 | Page 243

Find graphical solution for the following system of linear in equation:

$3x + 4y \leq 12$, $x - 2y \geq 2$, $y \geq -1$

Solution: First we draw the lines AB, CD and ED whose equations are $3x + 4y = 12$, $x - 2y = 2$ and $y = -1$ respectively.

Line	Equation	Points on the X-axis	Points on the Y-axis	Sign	Region
AB	$3x + 4y = 12$	A(4, 0)	B(0, 3)	≤	origin side of line AB
CD	$x - 2y = 2$	C(2, 0)	D(0, -1)	≥	non-origin side of line CD
ED	$y = -1$	-	D(0, -1)	≥	origin side of line ED



The solution set of given system of inequation is shaded in the graph.

Miscellaneous exercise 7 | Q 5.1 | Page 244

Solve the following LPP:

Maximize z = $5x_1 + 6x_2$ subject to $2x_1 + 3x_2 \le 18$, $2x_1 + x_2 \le 12$, $x_1 \ge 0$, $x_2 \ge 0$.

Solution: First we draw the lines AB and CD whose equations are $2x_1 + 3x_2 = 18$ and $2x_1 + x_2 = 12$ respectively.

Line	Equation	Points on the X-axis	Points on the Y-axis	Sign	Region
AB	$2x_1 + 3x_2 = 18$	A(9, 0)	B(0, 6)	≤	origin side of line AB
CD	$2x_1 + x_2 = 12$	C(6, 0)	Q(0, 12)	≤	origin side of line CD

The feasible region is OCPBO which is shaded in the graph. The vertices of the feasible region are O (0, 0), C (6, 0), P and B (0, 6).

P is the point of intersection of the lines

$2x_1 + 3x_2 = 18$ ....(1)

and $2x_1 + x_2 = 12$ ....(2)

On subtracting, we get

$2x_2 = 6$

∴ $x_2 = 3$

Substituting $x_2 = 3$ in (2), we get

$2x_1 + 3 = 12$

$\therefore x_1 = 9$

$\therefore$ P is (9/2, 3)

The values of objective function $z = 5x_1 + 6x_2$ at these vertices are

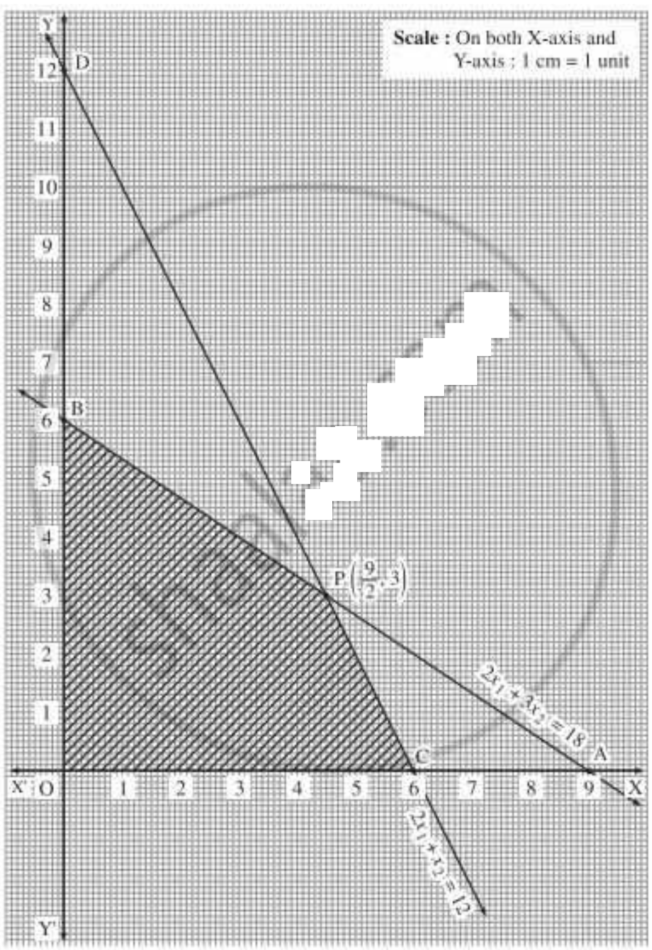
$z(O) = 5(0) + 6(0) = 0 + 0 = 0$

$z(C) = 5(6) + 6(0) = 30 + 0 = 30$

$$z(P) = 5\left(\frac{9}{2}\right) + 6(3) = \frac{45}{2} + 18 = \frac{45 + 36}{2} = \frac{81}{2} = 40.5$$

$z(B) = 5(0) + 6(3) = 0 + 18 = 18$

Maximum value of z is 40.5 when $x_1 = 9/2$ $y_1 = 3$.



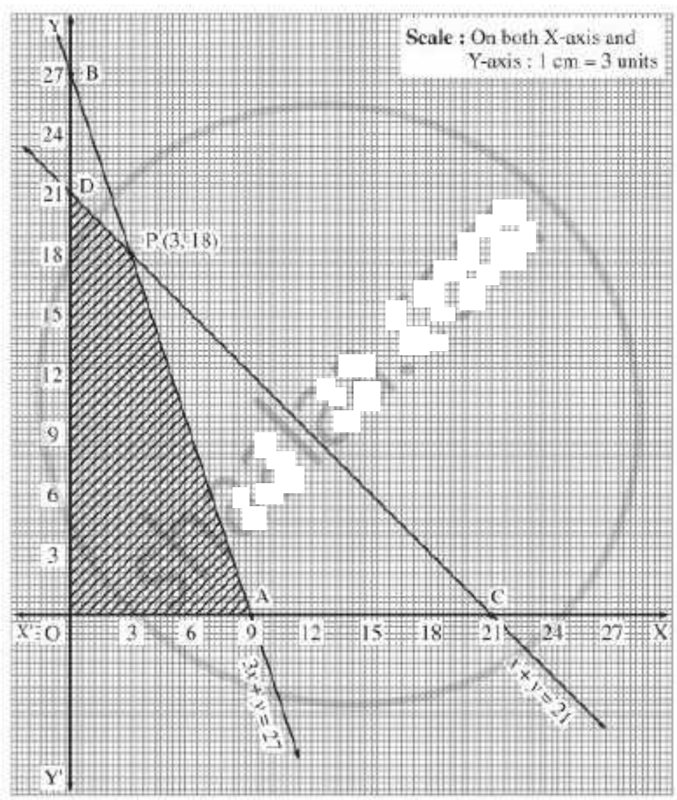
Miscellaneous exercise 7 | Q 5.2 | Page 244

Solve the following LPP:

Maximize z = 4x + 2y subject to 3x + y ≤ 27, x + y ≤ 21, x ≥ 0, y ≥ 0.

Solution: First we draw the lines AB and CD whose equations are 3x + y = 27 and x + y = 21 respectively.

Line	Equation	Points on the X-axis	Points on the Y-axis	Sign	Region
AB	3x + y = 27	A(9, 0)	B(0, 27)	≤	origin side of line AB
CD	x + y = 21	C(21, 0)	O(0, 21)	≤	origin side of line CD



The feasible region is OAPDO which is shaded region in the graph. The vertices of the feasible region are O(0, 0), A (9, 0), P and D (0, 21). P is the point of intersection of lines

3x + y = 27 ....(1)

and x + y = 21 ....(2)

On substracting, we get 2x = 6

∴ $x = 3$

Substituting $x = 3$ in equation (1), we get

$9 + y = 27$

∴ $y = 18$

∴ P ≡ (3, 18)

The values of the objective function $z = 4x + 2y$ at these vertices are

$z(O) = 4(0) + 2(0) = 0 + 0 = 0$

$z(a) = 4(9) + 2(0) = 36 + 0 = 36$

$z(P) = 4(3) + 2(18) = 12 + 36 = 48$

$z(D) = 4(0) + 2(21) = 0 + 42 = 42$

∴ 2 has minimum value 48 when $x = 3$, $y = 18$.

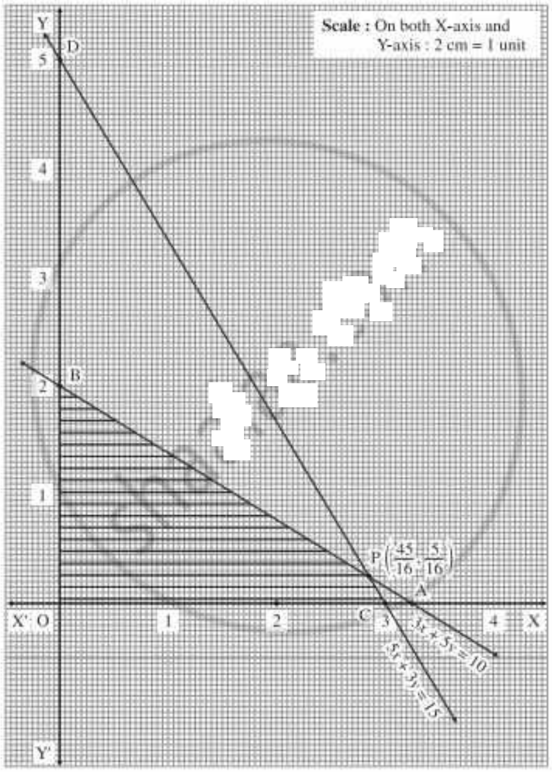
Miscellaneous exercise 7 | Q 5.3 | Page 244

Solve the following LPP:

Maximize $z = 6x + 10y$ subject to $3x + 5y \leq 10$, $5x + 3y \leq 15$, $x \geq 0$, $y \geq 0$.

Solution: First we draw the lines AB and CD whose equations are $3x + 5y = 10$ and $5x + 3y = 15$ respectively.

Line	Equation	Points on the X-axis	Points on the Y-axis	Sign	Region
AB	$3x + 5y = 10$	A (10/3, 0)	B(0, 2)	≤	origin side of line AB
CD	$5x + 3y = 15$	C(3, 0)	O(0, 5)	≤	origin side of line CD



The feasible region is OCPBD which is shaded region in the graph.
The vertices of the feasible region are O(0, 0), C (3, 0), P and B (0, 2).
P is the point of intersection of lines

3x + 5y = 10 ....(1)

and 5x + 3y = 15 ....(2)

Multiplying equation (1) by 5 and equation (2) by 3, we get

15x + 25y = 50

15x + 9y = 45

On subtracting, we get

16y = 5

$\therefore$ y = $\frac{5}{16}$

Substituting y = $\frac{5}{16}$ in equation (1), we get

$3x + \frac{25}{16} = 10$

$\therefore$ 3x = $10 - \frac{25}{16} = \frac{135}{16}$

$\therefore$ x = $\frac{45}{16}$

$\therefore$ P ≡ $\left(\frac{45}{16}, \frac{5}{16}\right)$

The values of objective function z = 6x + 10y at these vertices are

z(O) = 6(0) + 10(0) = 0 + 0 = 0

z(C) = 6(3) + 10(0) = 18 + 0 = 18

z(P) = $6\left(\frac{45}{16}\right) + 10\left(\frac{5}{10}\right) = \frac{270}{16} + \frac{50}{16} = \frac{320}{16} = 20$

z(B) = 6(0) + 10(2) = 0 + 20 = 20

The maximum value of z is 20 at P$\left(\frac{45}{16}, \frac{5}{16}\right)$ and B(0, 2) two consecutive vertices.

$\therefore$ z has maximum value 20 at each point of line segment

PB where B is (0, 2) and P is $\left(\frac{45}{16}, \frac{5}{16}\right)$

Hence, there are infinite number of optimum solutions.

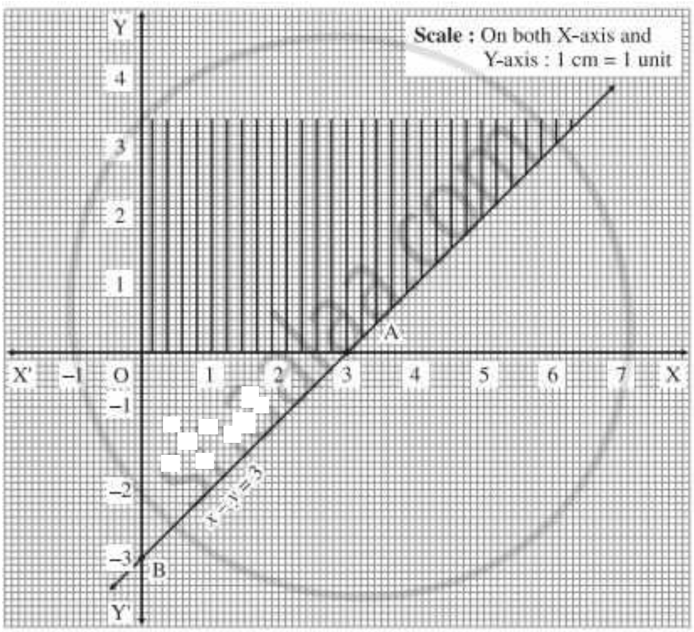
Miscellaneous exercise 7 | Q 5.4 | Page 244

Solve the following LPP:

Maximize z = 2x + 3y subject to x - y ≥ 3, x ≥ 0, y ≥ 0.

Solution: First we draw the lines AB whose equations are x - y = 3.

Line	**Equation**	**Points on the X-axis**	**Points on the Y-axis**	**Sign**	**Region**
AB	x - y = 3	A(3, 0)	B(0, -3)	≥	non-origin side of line AB



The feasible region is shaded which is unbounded. Therefore, the value of objective function can be increased indefinitely. Hence, this LPP has unbounded solution.

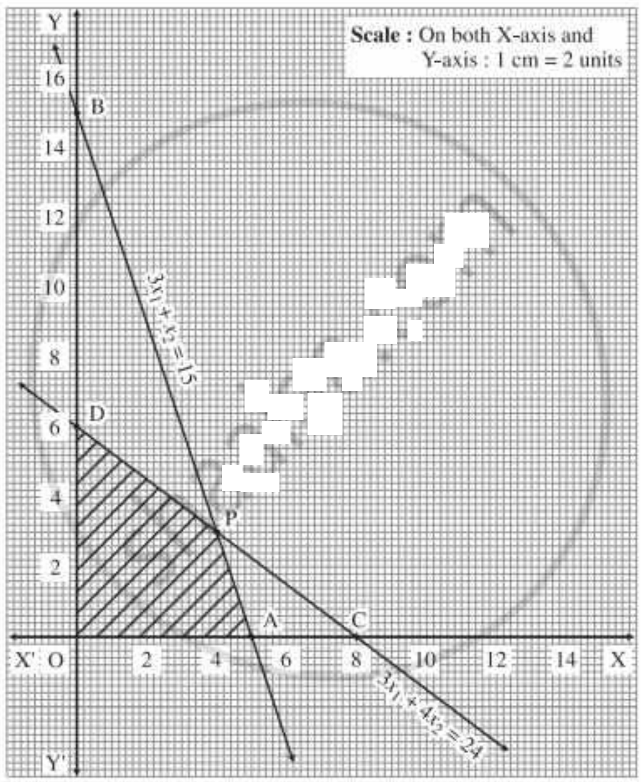
Miscellaneous exercise 7 | Q 6.1 | Page 244

Solve the following LPP:

Maximize $z = 4x_1 + 3x_2$ subject to
$3x_1 + x_2 \leq 15$, $3x_1 + 4x_2 \leq 24$, $x_1 \geq 0$, $x_2 \geq 0$.

Solution: We first draw the lines AB and CD whose equations are $3x_1 + x_2 = 15$ and $3x_1 + 4x_2 = 24$ respectively.

Line	Equation	Points on the X-axis	Points on the Y-axis	Sign	Region
AB	$3x_1 + x_2 = 15$	A(5, 0)	B(0,15)	≤	origin side of the line AB
CD	$3x_1 + 4x_2 = 24$	C(8, 0)	D(0, 6)	≤	origin side of the line CD



The feasible region is OAPDO which is shaded in the graph.

The Vertices of the feasible region are O(0, 0), A(5, 0), P and D(0, 6).

P is the point of intersection of lines.

$3x_1 + 4x_2 = 24$ ....(1)

and $3x_1 + x_2 = 15$ ....(2)

On subtracting, we get

$3x_2 = 9 \quad \therefore x_2 = 3$

Substituting $x_2 = 3$ in (2), we get

$3x_1 + 3 = 15$

$\therefore 3x_1 = 12$

$\therefore x_1 = 4$

$\therefore$ P is (4, 3)

The values of objective function $z = 4x_1 + 3x_2$ at these vertices are

$z(O) = 4(0) + 3(0) = 0 + 0 = 0$

$z(a) = 4(5) + 3(0) = 20 + 0 = 20$

$z(P) = 4(4) + 3(3) = 16 + 9 = 25$

$z(D) = 4(0) + 3(6) = 0 + 18 = 18$

$\therefore$ z has maximum value 25 when x = 4 and y = 3.

Miscellaneous exercise 7 | Q 6.2 | Page 244

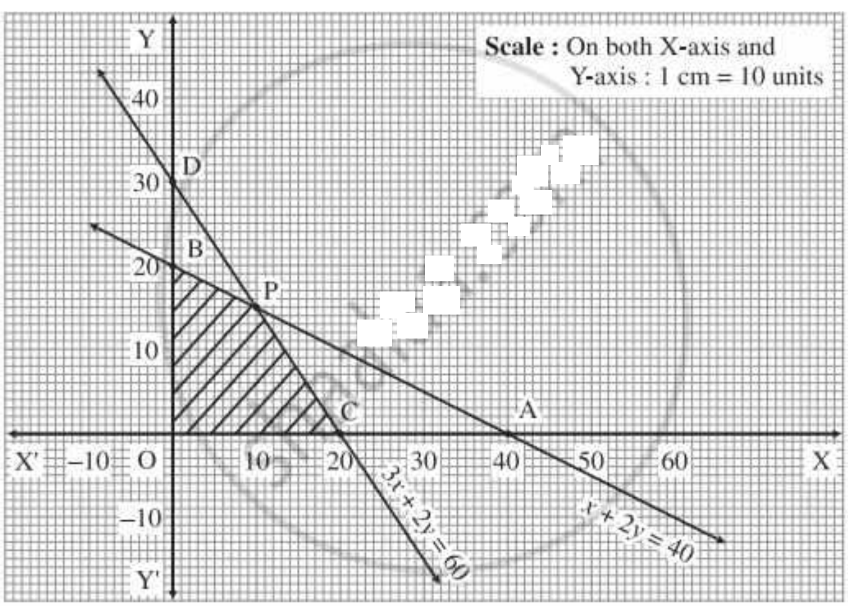
Solve the following LPP:

Maximize $z = 60x + 50y$ subject to

$x + 2y \le 40$, $3x + 2y \le 60$, $x \ge 0$, $y \ge 0$.

Solution: We first draw the lines AB and CD whose equations are $x + 2y = 40$ and $3x + 2y = 60$ respectively.

Line	Equation	Points on the X-axis	Points on the Y-axis	Sign	Region
AB	$x + 2y = 40$	A(40,0)	B(0,20)	$\le$	origin side of line AB
CD	$3x + 2y = 60$	C(20,0)	D(0,30)	$\le$	origin side of line CD



The feasible region is OCPBO which is shaded in the graph.
The vertices of the feasible region are O (0, 0), C (20, 0), P and B (0, 20).
P is the point of intersection of the lines.

$3x + 2y = 60$ ....(1)

and $x + 2y = 40$ .....(2)

On subtracting, we get

$2x = 20$ $\therefore x = 10$

Substituting x = 10 in (2), we get

$10 + 2y = 40$

$\therefore 2y = 30$

$\therefore y = 15$

$\therefore$ P is (10, 15)

The values of the objective function $z = 60x + 50y$ at these vertices are

$z(O) = 60(0) + 50(0) = 0 + 0 = 0$

$z(C) = 60(20) + 50(0) = 1200 + 0 = 1200$

$z(P) = 60(10) + 50(15) = 600 + 750 = 1350$

$z(B) = 60(0) + 50(20) = 0 + 1000 = 1000$

$\therefore$ z has maximum value 1350 at x = 10, y = 15.

Miscellaneous exercise 7 | Q 6.3 | Page 244

Solve the following LPP:

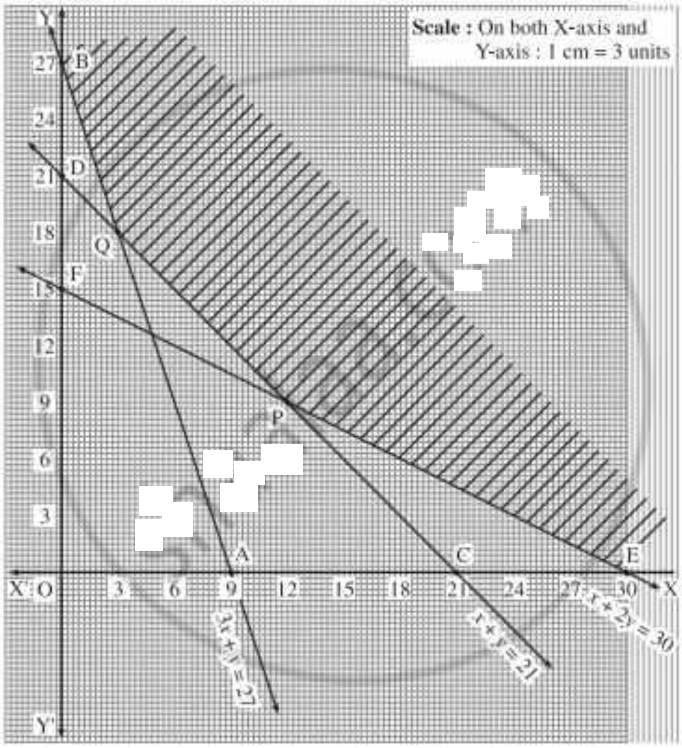
Minimize $z = 4x + 2y$ subject to

$3x + y \geq 27$, $x + y \geq 21$, $x + 2y \geq 30$, $x \geq 0$, $y \geq 0$.

Solution: We first draw the lines AB, CD and EF whose equations are $3x + y = 27$, $x + y = 21$, $x + 2y = 30$ respectively.

Line	Equation	Points on the X-axis	Points on the Y-axis	Sign	Region
AB	$3x + y = 27$	A(9,0)	B(0,27)	≥	non-origin side of line AB
CD	$x + y = 21$	C(21,0)	D(0,21)	≥	non-origin side of line CD

EF	x + 2y = 30	E(30,0)	F(0,15)	≥	non-origin side of line EF



The feasible region is XEPQBY which is shaded in the graph.

The vertices of the feasible region are E (30, 0), P, Q and B (0, 27).

P is the point of intersection of the lines

x + 2y = 30 ....(1)

and x + y = 21 ......(2)

On subtracting, we get

y = 9

Substituting y = 9 in (2), we get

x + 9 = 21

∴ x = 12

∴ P is (12, 9)

Q is the point of intersection of the lines

x + y = 21 ....(2)

and 3x + y = 2 ....(3)

On subtracting, we get

$2x = 6 \quad \therefore x = 3$

Substituting $x = 3$ in (2), we get

$3 + y = 21 \quad \therefore y = 18$

$\therefore$ Q is (3, 18)

The values of the objective function $z = 4x + 2y$ at these vertices are

$z(E) = 4(30) + 2(0) = 120 + 0 = 120$

$z(P) = 4(12) + 2(9) = 48 + 18 = 66$

$z(Q) = 4(3) + 2(18) = 12 + 36 = 48$

$z(B) = 4(0) + 2(27) = 0 + 54 = 54$

$\therefore$ z has minimum value 48, when $x = 3$ and $y = 18$.

Miscellaneous exercise 7 | Q 7 | Page 244

A carpenter makes chairs and tables. Profits are ₹ 140 per chair and ₹ 210 per table. Both products are processed on three machines: Assembling, Finishing and Polishing. The time required for each product in hours and availability of each machine is given by the following table:

Product → / Machine ↓	**Chair (x)**	**Table (y)**	**Available time (hours)**
Assembling	3	3	36
Finishing	5	2	50
Polishing	2	6	60

Formulate the above problem as LPP. Solve it graphically

Solution: Let the number of chairs and tables made by the carpenter be x and y respectively.

The profits are ₹ 140 per chair and ₹ 210 per table.

$\therefore$ total profit z = ₹ $(140x + 210y)$

This is the objective function which is to be maximized. The constraints are as per the following table:

	Chair (x)	**Table (y)**	**Available time (hours)**

Assembling	3	3	36
Finishing	5	2	50
Polishing	2	6	60

From the table, the constraints are

$3x + 3y \leq 36$, $5x + 2y \leq 50$, $2x + 6y \leq 60$.

The number of chairs and tables cannot be negative.

$\therefore x \geq 0, y \geq 0$

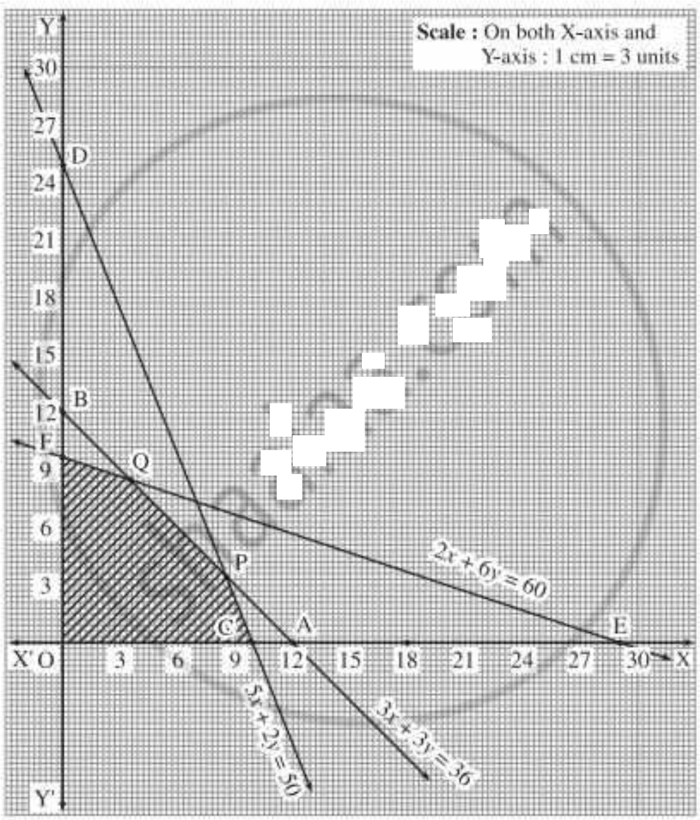
Hence, the mathematical formulation of given LPP is:

Maximize $z = 140x + 210y$, subject to

$3x + 3y \leq 36$, $5x + 2y \leq 50$, $2x + 6y \leq 60$, $x \geq 0$, $y \geq 0$

We first draw the lines AB, CD and EF whose equations are $3x + 3y = 36$, $5x + 2y = 50$ and $2x + 6y = 60$ respectively.

Line	**Equation**	**Points on the X-axis**	**Points on the Y-axis**	**Sign**	**Region**
AB	$3x + 3y = 36$	A(12,0)	B(0,12)	$\leq$	origin side of line AB
CD	$5x + 2y = 50$	C(10,0)	D(0,25)	$\leq$	origin side of line CD
EF	$2x + 6y = 60$	E(30,0)	F(0,10)	$\leq$	origin side of line EF



The feasible region is OCPQFO which is shaded in the graph.
The vertices of the feasible region are O (0, 0), C (10, 0), P, Q and F (0, 10).
P is the point of intersection of the lines

$5x + 2y = 50$ … (1)

and $3x + 3y = 36$ … (2)

Multiplying equation (1) by 3 and equation (2) by 2, we get

$15x + 6y = 150$

$6x + 6y = 72$

On subtracting, we get

$9x = 78 \quad \therefore x = \frac{26}{3}$

Substituting $x = \frac{26}{3}$ in (2), we get

$$3\left(\frac{26}{3}\right) + 3y = 36$$

$\therefore 3y = 10$

$\therefore y = \frac{10}{3}$

$\therefore$ P is $\left(\frac{26}{3}, \frac{10}{3}\right)$

Q is the point of intersection of the lines

$3x + 3y = 36$ ....(2)

and $2x + 6y = 60$ ......(3)

Multiplying equation (2) by 2, we get

$6x + 6y = 72$

Subtracting equation (3) from this equation, we get

$4x = 12 \quad \therefore x = 3$

Substituting x = 3 in (2), we get

$3(3) + 3y = 36$

$\therefore 3y = 27 \quad \therefore y = 9$

$\therefore$ Q is (3, 9).

Hence, the vertices of the feasible region are O (0, 0),

C(10, 0), P(26/3, 10/3), Q(3,9) and F(0,10)

The values of the objective function z = 140x + 210y at these vertices are

z(O) = 140(0) + 210(0) = 0 + 0 = 0

z(C) = 140(10) + 210(0) = 1400 + 0 = 1400

$$z(P) = 140\left(\frac{26}{3}\right) + 210\left(\frac{10}{3}\right) = \frac{360 + 2100}{3} = \frac{5740}{3} = 1913.33$$

z(Q) = 140(3) + 210(9) = 420 + 1890 = 2310

z (F) = 140(0) + 210(10) = 0 + 2100 = 2100

∴ z has maximum value 2310 when x = 3 and y = 9.

Hence, the carpenter should make 3 chairs and 9 tables to get the maximum profit of ₹ 2310.

Miscellaneous exercise 7 | Q 8 | Page 244

A company manufactures bicycles and tricycles each of which must be processed through machines A and B. Machine A has maximum of 120 hours available and machine B has maximum of 180 hours available. Manufacturing a bicycle requires 6 hours on machine A and 3 hours on machine B. Manufacturing a tricycle requires 4 hours on machine A and 10 hours on machine B.

If profits are Rs. 180 for a bicycle and Rs. 220 for a tricycle, formulate and solve the L.P.P. to determine the number of bicycles and tricycles that should be manufactured in order to maximize the profit.

Solution: Let x number of bicycles and y number of tricycles be manufactured by the company.
Total profit Z = 180x + 220y
This is the objective function to be maximized.
The given information can be tabulated as shown below:

	Bicycles (x)	Tricycles (y)	Maximum availability of time (hrs)
Machine A	6	4	120
Machine B	3	10	180

The constraints are 6x + 4y ≤ 120, 3x + 10y ≤ 180, x ≥ 0, y ≥ 0

Given problem can be formulated as

Maximize Z = 180x + 220y

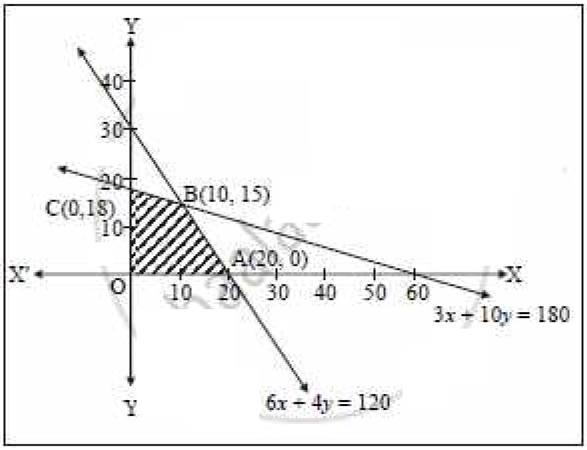
Subject to, 6x + 4y ≤ 120, 3x + 10y ≤ 180 , x ≥ 0, y ≥ 0.

To draw the feasible region, construct the table as follows:

Inequality	6x + 4y ≤ 120	3x + 10y ≤ 180
Corresponding equation (of line)	6x + 4y = 120	3x + 10y = 180

Intersection of line with X-axis	(20, 0)	(60, 0)
Intersection of line with Y-axis	(0, 30)	(0, 18)
Region	Origin side	Origin side

Shaded portion OABC is the feasible region, whose vertices are O=(0, 0), A =(20, 0), B and C = (0, 18)



B is the point of intersection of the lines 3x + 10y = 180 and 6x + 4y = 120.

Solving the above equations, we get
B = (10, 15) Here the objective function is,

Z = 180x + 220y

Z at O(0, 0) = 180(0) + 220(0) = 0
Z at A(20, 0) = 180(20) + 220(0) = 3600
Z at B(10, 15) = 180(10) + 220(15) = 5100
Z at C(0, 18) = 180(0) + 220(18) = 3960

Z has maximum value 5100 at B(10, 15)
Z is maximum when x = 10, y = 15

Thus, the company should manufacture 10 bicycles and 15 tricycles to gain maximum profit of Rs.5100.

Miscellaneous exercise 7 | Q 9 | Page 244

A chemical company produces two compounds, A and B. The following table gives the units of ingredients, C and D per kg of compounds A and B as well as minimum requirements of C and D and costs per kg of A and B. Find the quantities of A and B which would give a supply of C and D at a minimum cost.

	Compound	**Minimum requirement**

	A	B	
Ingredient C Ingredient D	1 3	2 1	80 75
Cost (in Rs) per kg	4	6	-

Solution: Let x kg of compound A and y kg of compound B were produced.
Quantity cannot be negative.
Therefore, $x,y \geq 0$

	Compound		Minimum requirement
	A	B	
Ingredient C Ingredient D	1 3	2 1	80 75
Cost (in Rs) per kg	4	6	-

According to question, the constraints are

$x+2y \geq 80$

$3x+y \geq 75$
Cost (in Rs) per kg of
compound A and compound B is Rs 4 and Rs 6 respectively.Therefore, cost of x kg of compound A and y kg of compound B is 4x and 6y respectively.
Total cost = Z = $4x+6y$

which is to be minimised.
Thus, the mathematical formulation of the given linear programming problem is
Min Z = $4x+6y$
subject to

$x+2y \geq 80$
$3x+y \geq 75$

$x,y \geq 0$
First we will convert inequations into equations as follows:
$x + 2y = 80$, $3x + y = 75$, $x = 0$ and $y = 0$

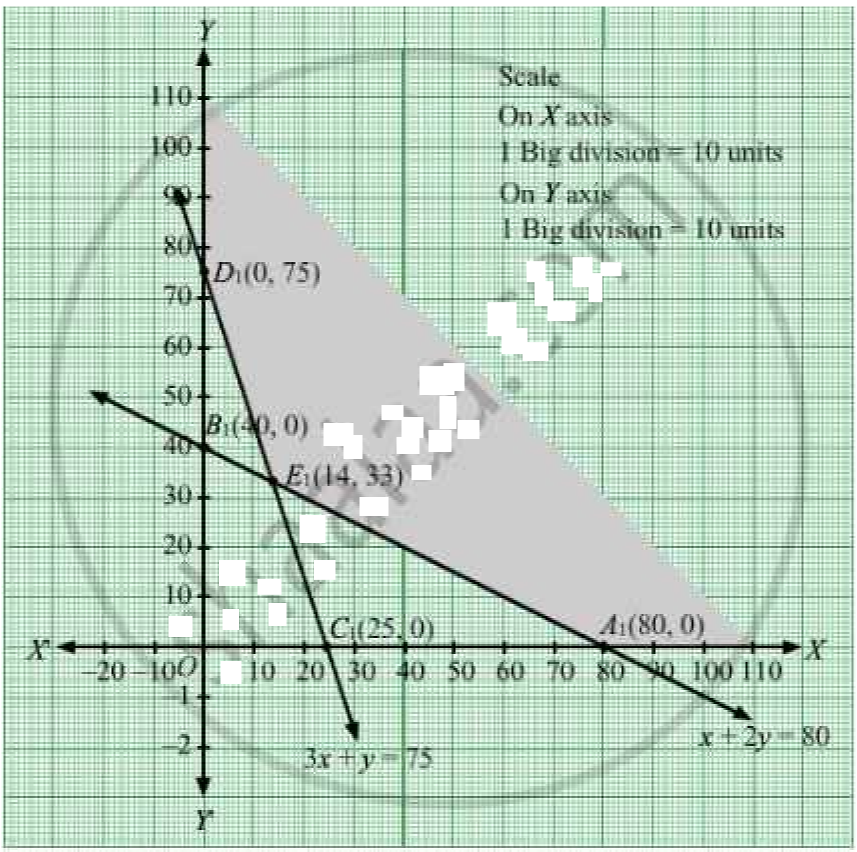
Region represented by $x + 2y \geq 80$:
The line $x + 2y = 80$ meets the coordinate axes at $A_1(80, 0)$ and $B_1(0, 40)$ respectively. By joining these points we obtain the line $x + 2y = 80$. Clearly (0,0) does not satisfies the $x + 2y = 80$. So, the region which does not contain the origin represents the solution set of the inequation $x + 2y \geq 80$.

Region represented by $3x + y \geq 75$:
The line $3x + y = 75$ meets the coordinate axes at $C_1(25, 0)$ and $D_1(0, 75)$ respectively.

By joining these points we obtain the line 3x + y =75. Clearly (0,0) does not satisfies the inequation 3x + y ≥ 75. So,the region which does not contain the origin represents the solution set of the inequation 3x + y ≥ 75.

Region represented by x ≥ 0 and y ≥ 0:
Since, every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the inequations x ≥ 0, and y ≥ 0.

The feasible region determined by the system of constraints x + 2y ≥ 80, 3x + y ≥ 75, x ≥ 0, and y ≥ 0 are as follows.



The corner points are $D_1(0, 75)$, $E_1(14, 33)$ and $A_1(80, 0)$.

The values of Z at these corner points are as follows

Corner point	Z= 4x + 6y
D_1	450
E_1	254
A_1	320

The minimum value of Z is 254 which is attained at E_1 (14,33)
Thus, the minimum cost is Rs 254 obtained when 14 units of compound A and 33 units of compound B were produced.

Miscellaneous exercise 7 | Q 10 | Page 244

A company produces mixers and food processors. Profit on selling one mixer and one food processor is Rs 2,000 and Rs 3,000 respectively. Both the products are processed through three machines A, B, C. The time required in hours for each product and total time available in hours per week on each machine arc as follows:

Machine	Mixer	Food Processor	Available time
A	3	3	36
B	5	2	50
C	2	6	60

How many mixers and food processors should be produced in order to maximize the profit?

Solution: Let x = number of mixers are sold
y = number of food processors are sold

Profit function z = 2000x + 3000y

This is the objective function which is to be maximized. From the given table in the problem, the constraints are

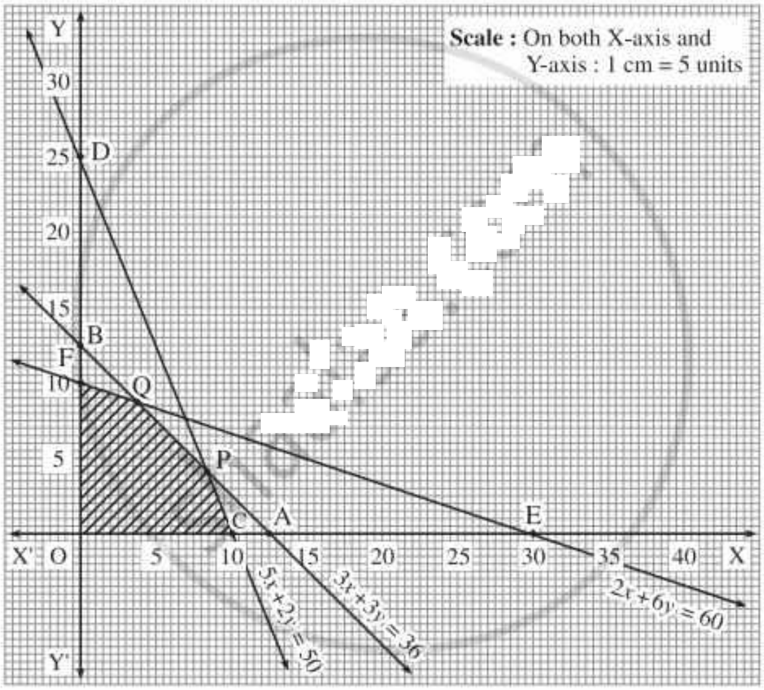
$3x + 3y \leq 36$ (above machine A)
$5x\ 2y \leq 50$ (about machine B)
$2x + 6y \leq 60$ (about machine C)

As the number of mixers and food processors are non-negative.

$x \geq 0, y \geq 0$
Mathematical model of L.P.P. is
Maximize Z = 2000x + 3000y Subject to
$3x + 3y \leq 36$, $5x\ 2y \leq 50$, $2x + 6y \leq 60$
and $x \geq 0, y \geq 0$

First we draw the lines AB, CD and EF whose equations are $3x + 3y = 36$, $5x + 2y = 50$ and $2x + 6y = 60$ respectively.

Line	Equation	Points on the X-axis	Points on the Y-axis	Sign	Region
AB	$3x + 3y = 36$	A(12,0)	B(0,12)	$\leq$	origin side of line AB
CD	$5x + 2y = 50$	C(10,0)	D(0,25)	$\leq$	origin side of line CD
EF	$2x + 6y = 60$	E(30,0)	F(0,10)	$\leq$	origin side of line EF



The feasible region is OCPQFO which is shaded in the graph.

The vertices of the feasible region are O (0, 0), C (10, 0), P, Q and F (0, 10).

P is the point of intersection of the lines

3x + 3y = 36 ....(1)

and 5x + 2y = 50 ....(2)

Multiplying equation (1) by 2 and equation (2) by 3, we get,

6x + 6y = 72

15x + 6y = 150

On subtracting, we get

9x = 78

$\therefore x = \frac{26}{3}$

$\therefore$ from (1), $3\left(\frac{26}{3}\right) + 3y = 36$

$\therefore 3y = 10$

$\therefore y = \frac{10}{3}$

$\therefore P = \left(\frac{26}{3}, \frac{10}{3}\right)$

Q is the point of intersection of the lines

$3x + 3y = 36$ . ...(1)

and $2x + 6y = 60$ ...(2)

Multiplying equation (1) by 2, we get

$6x + 6y = 72$

Subtracting equation (3), from this equation, we get

$4x = 12$

$\therefore x = 3$

$\therefore$ from (1), $3(3) + 3y = 36$

$\therefore 3y = 27$

$\therefore y = 9$

$\therefore Q = (3, 9)$

The values of the objective function $z = 2000x + 3000y$ at these vertices are

$z(O) = 2000(0) + 3000(0) = 0 + 0 = 0$

$z(C) = 2000(10) + 3000(0) = 20000 + 0 = 20000$

$$z(P) = 2000\left(\frac{26}{3}\right) + 3000\left(\frac{10}{3}\right) = \frac{52000}{3} + \frac{30000}{3} = \frac{82000}{3}$$

$z(Q) = 2000(3) + 3000(9) = 6000 + 27000 = 33000$

z(F) = 2000(0) + 3000(10) = 30000 + 0 = 30000

∴ the maximum value of z is 33000 at the point (3, 9).

Hence, 3 mixers and 9 food processors should be produced in order to get the maximum profit of ₹ 33,000.

Miscellaneous exercise 7 | Q 11 | Page 245

A chemical company produces a chemical containing three basic elements A, B, C, so that it has at least 16 litres of A, 24 litres of B and 18 litres of C. This chemical is made by mixing two compounds I and II. Each unit of compound I has 4 litres of A, 12 litres of B and 2 litres of C. Each unit of compound II has 2 litres of A, 2 litres of B and 6 litres of C. The cost per unit of compound I is ‘ 800 and that of compound II is ‘ 640. Formulate the problems as LPP and solve it to minimize the cost.

Solution: Let the company buy x units of compound I and y units of compound II.

Then the total cost is z = ₹ (800x + 640y)

This is the objective function that is to be minimized. The constraints are as per the following table:

	Compound I (x)	Compound II (y)	Compound III (z)
Element A	4	2	16
Element B	12	2	24
Element C	2	6	18

From the table, the constraints are

$4x + 2y \geq 16$, $12x + 2y \geq 24$, $2x + 6y \geq 18$

Also, the number of units of compound I and compound II cannot be negative.

∴ $x \geq 0$, $y \geq 0$

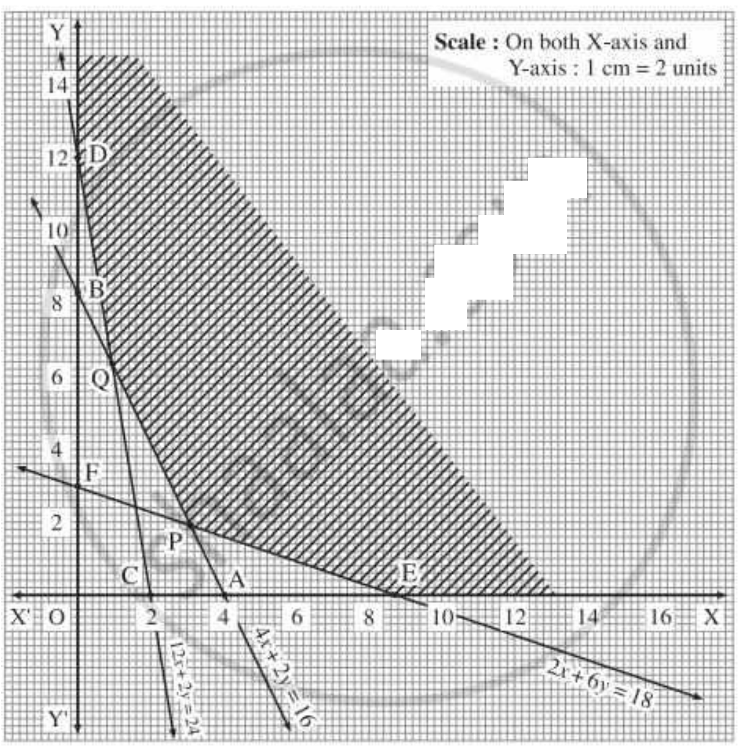
∴ the mathematical formulation of given LPP is

Minimize z = 800x + 640y, subject to

$4x + 2y \geq 16$, $12x + 2y \geq 24$, $2x + 6y \geq 18$, $x \geq 0$, $y \geq 0$.

First we draw the lines AB, CD and EF whose equations are 4x + 2y = 16, 12x + 2y = 24 and 2x + 6y = 18 respectively.

Line	**Equation**	**Points on the X-axis**	**Points on the Y-axis**	**Sign**	**Region**
AB	4x + 2y = 16	A(4, 0)	B(0,8)	≥	non-origin side of line AB
CD	12x + 2y = 24	C(2, 0)	D(0,12)	≥	non-origin side of line CD
EF	2x + 6y = 18	E(9, 0)	F(0,3)	≥	non-origin side of line EF



The feasible region is shaded in the graph.

The vertices of the feasible region are E (9, 0), P, Q and D (0, 12).

P is the point of intersection of the lines

2x + 6y = 18   ...(1)

and $4x + 2y = 16$ ...(2)

Multiplying equation (1) by 2, we get

$4x + 12y = 36$

Subtracting equation (2) from this equation, we get

$10y = 20$

$\therefore y = 2$

$\therefore$ from(1), $2x + 6(2) = 18$

$\therefore 2x = 6$

$\therefore x = 3$

$\therefore P = (3, 2)$

Q is the point of intersection of the lines

$12x + 2y = 24$ ...(3)

and $4x + 2y = 16$ ...(2)

On subtracting, we get

$8x = 8 \quad \therefore x = 1$

$\therefore$ from(2), $4(1) + 2y = 16$

$\therefore 2y = 12$

$\therefore y = 6$

$\therefore Q = (1, 6)$

The values of the objective function $z = 800x + 640y$ at these vertices are

$z(E) = 800(9) + 640(0) = 7200 + 0 = 7200$

$z(P) = 800(3) + 640(2) = 2400 + 1280 = 3680$

$z(Q) = 800(1) + 640(6) = 800 + 3840 = 4640$

z(D) = 800(0) + 640(12) = 0 + 7680 = 7680

∴ the minimum value of z is 3680 at the point (3, 2).

Hence, the company should buy 3 units of compound I and 2 units of compound II to have the minimum cost of ₹ 3680.

Miscellaneous exercise 7 | Q 12 | Page 245

A person makes two types of gift items A and B requiring the services of a cutter and a finisher. Gift item A requires 4 hours of the cutter's time and 2 hours of finisher's time. Fifth item B requires 2 hours of the cutter's time and 4 hours of finisher's time. The cutter and finisher have 208 hours and 152 hours available time respectively every month. The profit on one gift item of type A is ₹ 75 and on one gift item of type, B is ₹ 125. Assuming that the person can sell all the gift items produced, determine how many gift items of each type should he make every month to obtain the best returns?

Solution: Let x: number of gift item A

y: number of gift item B

As numbers of the items are never negative

X ≥ 0; y ≥ 0

	A (x)	**B (y)**
Cutter	4	2
Finisher	2	4
Profit	75	125

Total time required for the cutter = 4x + 2y

Maximum available time 208 hours

∴ 4x+ 2y ≤ 208

Total time required for the finisher 2x +4y

Maximum available time 152 hours

∴ 2x+ 4y ≤ 152

Total Profit is 75x + 125y

∴ L.P.P. of the above problem is

Minimize Z = 75x + 125y

Subject to 4x+ 2y ≤ 208

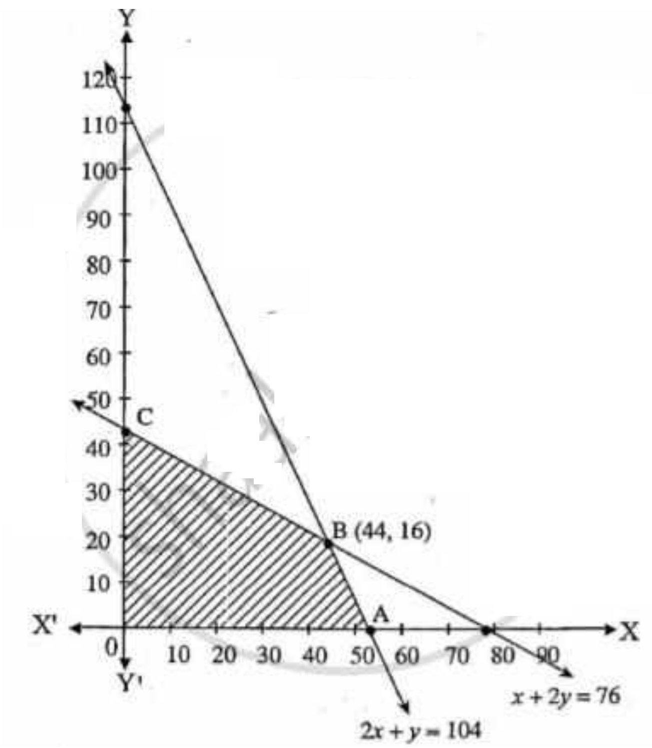
2x + 4y ≤ 152

x ≥ 0 ; y ≥ 0

Graphical solution

2x + y = 104		
x	0	52
y	104	0
(0 , 104) (52 , 0)		

x + 2y = 76		
x	0	0
y	38	76
(0 , 38) (76 , 0)		



Corner points

Now, Z at

Z = (75x + 125y)

0(0, 0) = 75 x 0 + 125 x 0 = 0

A(52, 0) = 75 x 52 + 125 x 0 = 3900

B(44, 16) = 75 x 44 + 125 x 16 = 5300

C(O, 38) = 75 x 0 + 125 x 38 = 4750

∴ A person should make 44 items of type A and 16 Uems of type Band his returns are ₹ 5,300.

Miscellaneous exercise 7 | Q 13 | Page 245

A firm manufactures two products A and B on which profit earned per unit ₹ 3 and ₹ 4 respectively. Each product is processed on two machines M_1 and M_2. The product A requires one minute of processing time on M_1 and two minutes of processing time on M_2, B requires one minute of processing time on M_1 and one minute of processing time on M_2. Machine M_1 is available for use for 450 minutes while M_2 is available for 600 minutes during any working day. Find the number of units of product A and B to be manufactured to get the maximum profit.

Solution: Let the firm manufactures x units of product A and y units of product B.

The profit earned per unit of A is ₹ 3 and B is ₹ 4.

Hence, the total profit is z = ₹ (3x + 4y)

This is the linear function that is to be maximized. Hence, it is an objective function.

The constraints are as per the following table:

Machine	Product A (x)	Product A (y)	Total availability of time (minutes)
M_1	1	1	450
M_2	2	1	600

From the table, the constraints are

$x + y \le 450$, $2x + y \le 600$

Since, the number of gift items cannot be negative, $x \ge 0$, $y \ge 0$.

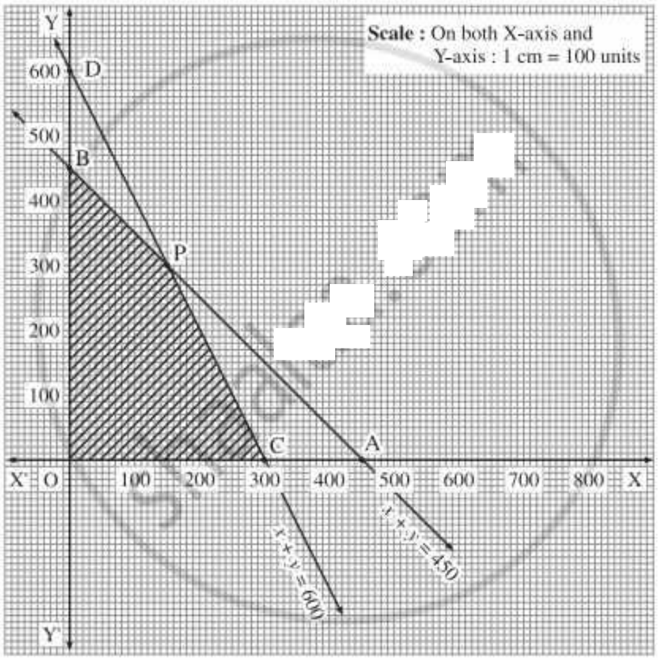
∴ the mathematical formulation of LPP is,

Maximize z = 3x + 4y, subject to

$x + y \le 450$, $2x + y \le 600$, $x \ge 0$, $y \ge 0$

Now, we draw the lines AB and CD whose equations are x + y = 450, 2x + y = 600 respectively.

Line	Equation	Points on the X-axis	Points on the Y-axis	Sign	Region
AB	x + y = 450	A(450,0)	B(0,450)	≤	origin side of line AB
CD	2x + y = 600	C(300,0)	D(0,600)	≤	origin side of line CD



The feasible region is OCPBO which is shaded in the graph.
The vertices of the feasible region are O (0, 0), C (300, 0), P and B (0, 450).

P is the point of intersection of the lines

2x + y = 600 .....(1)

and x + y = 450 .....(2)

On subtracting, we get

∴ x = 150

Substituting x = 150 in equation (2), we get

150 + y = 450

∴ y = 300

∴ P ≡ (150, 300)

The values of the objective function z = 3x + 4y at these vertices are

z(O) = 3(0) + 4(0) = 0 + 0 = 0

$z(C) = 3(300) + 4(0) = 900 + 0 = 900$

$z(P) = 3(150) + 4(300) = 450 + 1200 = 1650$

$z(B) = 3(0) + 4(450) = 0 + 1800 = 1800$

$\therefore$ z has the maximum value 1800 when x =0 and y = 450

Hence, the firm gets maximum profit of ₹ 1800 if it manufactures 450 units of product B and no unit product A.

Miscellaneous exercise 7 | Q 14 | Page 245

A firm manufacturing two types of electrical items A and B, can make a profit of ₹ 20 per unit of A and ₹ 30 per unit of B. Both A and B make use of two essential components a motor and a transformer. Each unit of A requires 3 motors and 2 transformers and each units of B requires 2 motors and 4 transformers. The total supply of components per month is restricted to 210 motors and 300 transformers. How many units of A and B should be manufactured per month to maximize profit? How much is the maximum profit?

Solution: Let the firm manufactures x units of item A and y units of item B.

Firm can make profit of ₹ 20 per unit of A and ₹ 30 per unit of B.

Hence, the total profit is z = ₹ $(20x + 30y)$

This is the objective function which is to be maximized. The constraints are as per the following table:

	Item A (x)	**Item A (y)**	**Total supply**
Motor	3	2	210
Transformer	2	4	300

From the table, the constraints are

$3x + 2y \leq 210,\ 2x + 4y \leq 300$

Since, number of items cannot be negative, $x \geq 0,\ y \geq 0$.

Hence, the mathematical formulation of given LPP is :

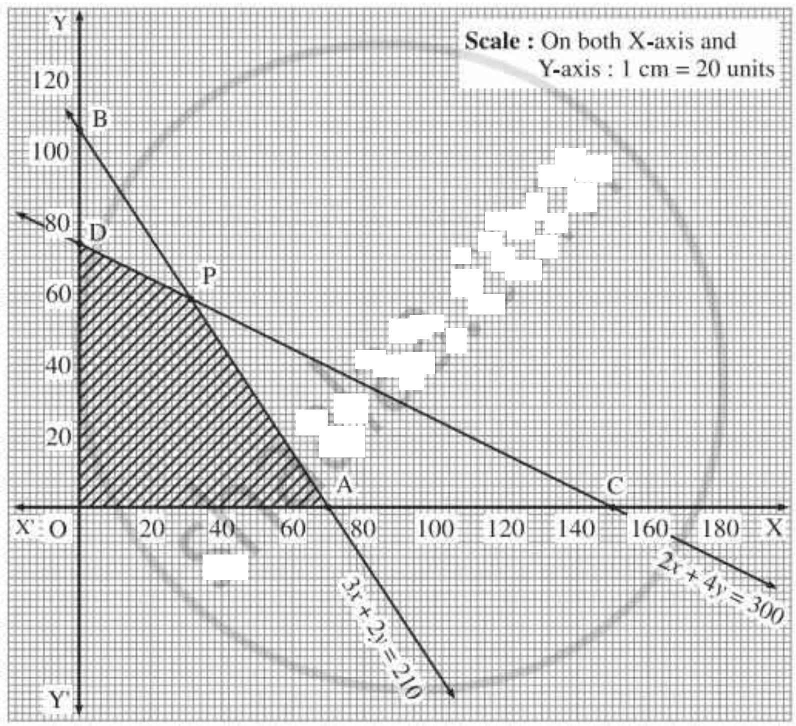
Maximize $z = 20x + 30y$, subject to

$3x + 2y \leq 210,\ 2x + 4y \leq 300,\ x \geq 0,\ y \geq 0$

We draw the lines AB and CD whose equations are

$3x + 2y = 210$ and $2x + 4y = 300$ respectively.

Line	Equation	Points on the X-axis	Points on the Y-axis	Sign	Region
AB	3x + 2y = 210	A(70,0)	B(0,150)	≤	origin side of line AB
CD	2x + 4y = 300	C(150,0)	D(0,75)	≤	origin side of line CD



The feasible region is OAPDO which is shaded in the graph.
The vertices of the feasible region are O (0, 0), A (70, 0), P and D (0, 75).
P is the point of intersection of the lines

2x + 4y = 300 ....(1)

and 3x + 2y = 210 ....(2)

Multiplying equation (2) by 2, we get

6x + 4y = 420

Subtracting equation (1) from this equation, we get

∴ 4x = 120

∴ x = 30

Substituting x = 30 in (1), we get

2(30) + 4y = 300

∴ 4y = 240

$\therefore$ $y = 60$

$\therefore$ P is (30, 60)

The values of the objective function $z = 20x + 30y$ at these vertices are

$z(O) = 20(0) + 30(0) = 0 + 0 = 0$

$z(A) = 20(70) + 30(0) = 1400 + 0 = 1400$

$z(P) = 20(30) + 30(60) = 600 + 1800 = 2400$

$z(D) = 20(0) + 30(75) = 0 + 2250 = 2250$

$\therefore$ z has the maximum value 2400 when $x = 30$ and $y = 60$

Hence, the firm should manufactured 30 units of item A and 60 units of item B to get the maximum profit of ₹ 2400.