3/01/2020

Chapter - 6

Random variable & Mathematical Expectation.

R.V.
X

Pmf: $\boxed{\sum P(x) = 1}$

Pdf: $\boxed{\int_{-\infty}^{\infty} f(x)\,dx = 1}$

Cumulative distribution function:

$$F(x) = P(X \le x) = \int_{-\infty}^{x} f(t)\,dt.$$

$$F'(x) = \frac{dF(x)}{d(x)} = f(x).$$

$P(a < x < b) = F(b) - F(a).$

$P(X < a) = 1 - P(X \ge a)$

0 —— a —— $= 1$

$P(X > a) = 1 - P(X \le a)$

$P(X \le a) = 1 - P(X > a)$

$P(X \ge a) = 1 - P(X < a)$

eg 6.1

Car	0	1	2	3	4	
House hold	30	320	380	190	80.	$= 1000$

x	0	1	2	3	4	
$P(x)$	0.03	0.32	0.38	0.19	0.08	$= 1$

$\Rightarrow P(x) > 0$,

$\sum P(x) = 1 \Rightarrow$ It is pmf.

eg 6.2

x	0	1	2	3	4	5	6	7
$P(x)$	0	a	$2a$	$2a$	$3a$	a^2	$2a^2$	$7a^2+a$

Qn:

(i) pf $\Rightarrow \sum P(x) = 1$

$0 + a + 2a + 2a + 3a + a^2 + 2a^2 + 7a^2 + a = 1$

$9a + 10a^2 = 1$

$10a^2 + 9a - 1 = 0$

$\boxed{a = 1/10}$ $\quad a = -1$

-10; $\frac{-1}{10}$, $\frac{10}{10}$; 9

(ii) $P(X < 3) = 0 + 1 + 2$

$= 3a = \boxed{\frac{3}{10}}$

(iii) $P(X > 2) = 1 - P(X \le 2)$

$= 1 - (0 + 1 + 2)$

$= 1 - \frac{3}{10} = \boxed{7/10}$

(iv) $P(2 < x \le 5) = 3 + 4 + 5$

$= 5a + a^2$

$= \frac{5}{10} + \frac{1}{100}$

$= \frac{50 + 1}{100}$

$= \boxed{\frac{51}{100}}$

eg 6.3.

$$P(x) = \begin{cases} \frac{x}{20}, & x = 0, 1, 2, 3, 4, 5 \\ 0, & \text{otherwise.} \end{cases}$$

(i) $P(X < 3) = 0 + 1 + 2$

$= \frac{0}{20} + \frac{1}{20} + \frac{2}{20}$

$= \boxed{\frac{3}{20}}$

(ii) $P(2 < x \le 4) = 3 + 4 = \frac{3}{20} + \frac{4}{20}$

$= \boxed{\frac{7}{20}}$

eg 6.4 :

3 coins :

$n(S) = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

$= 2^3 = 8$

X = head $= 0, 1, 2, 3$

x	0	1	2	3
$p(x)$	$1/8$	$3/8$	$3/8$	$1/8$

$= 1$

$P(x) \ge 0$

$\Sigma P(x) = 1$ It is pmf.

eg 6.5

X = sum of upturn $(d_1 + d_2)$

x	2	3	4	5	6	7	8	9	10	11	12
$p(x)$	$1/36$	$2/36$	$3/36$	$4/36$	$5/36$	$6/36$	$5/36$	$4/36$	$3/36$	$2/36$	$1/36$

eg 6 = (1,5) (5,1) (2,4) (4,2) (3,3) $= \frac{5}{36}$

(1)

x	0	1	2	3
$p(x)$	0.3	0.2	0.4	0.1

$F(x) = P(X \le x)$

$F(0) = P(X \le 0) = P(X = 0) = 0.3$

$F(1) = P(X \le 1) = 0 + 1 = 0.3 + 0.2 = 0.5$

$F(2) = P(X \le 2) = 0 + 1 + 2 = 0.9$

$F(3) = P(X \le 3) = 0 + 1 + 2 + 3 = 1.0 = \boxed{1}$

x	0	1	2	3
$P(X \le x)$	0.3	0.5	0.9	1

(2). $P(x) = \begin{cases} 0.3 & x=3 \\ 0.2 & x=5 \\ 0.3 & x=8 \\ 0.2 & x=10 \\ 0 & \text{otherwise.} \end{cases}$

$F(x) = \begin{cases} 0 & \text{if } x < 3 \\ 0.3 & \text{if } 3 \le x < 5 \\ 0.5, & \text{if } 5 \le x < 8 \\ 0.8, & \text{if } 8 \le x < 10 \\ & \text{if } x \le 10. \end{cases}$

(3) $P(X=x) = \begin{cases} kx & x=2,4,6 \\ k(x-2) & x=8 \\ 0 & \text{otherwise} \end{cases}$

Gm pf

x	2	4	6	8
$P(x)$	$2k$	$4k$	$6k$	$6k$

$\sum P(x) = 1$

$2k + 4k + 6k + 6k = 1$

$18k = 1$

$\boxed{k = \frac{1}{18}}$

(6).

value of $X = x$	0	1	2	3	4	5	6	7
$P(x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

(i) find k.

$\sum P(x) = 1$

$\Rightarrow 0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$

$9k + 10k^2 = 1$

$10k^2 + 9k - 1 = 0$

$(k+1)(k - 1/10) = 0$

$\boxed{k = -1}$ $\boxed{k = 1/10}$

(+ve) values

$\therefore \boxed{k = 1/10}$

(ii) $P(x < 6) = 0 + 1 + 2 + 3 + 4 + 5$

$= 0 + k + 2k + 2k + 3k + k^2.$

$= 8k + k^2$

$= 8\left(\frac{1}{10}\right) + \left(\frac{1}{100}\right)$

$= \frac{80+1}{100} = \boxed{\frac{81}{100}}$

$P(x \ge 6) = 6 + 7$

$= 2k^2 + 7k^2 + k$

$= 9k^2 + k = 90$

$= \frac{9}{100} + \frac{1}{10}$

$= \frac{9+10}{100}$

$= \boxed{\frac{19}{100}}$

$P(0 < x < 5) = 1+2+3+4$

$= k+2k+2k+3k$

$= 8k$

$= \boxed{\frac{8}{10}}$

(iii) $P(X \le x) > 1/2$

$P(X \le 0) = 0 \ngtr 1/2$

$P(X \le 1) = k = \frac{1}{10} \ngtr 1/2$

$P(X \le 2) = 3k = \frac{3}{10} \ngtr 1/2$

$P(X \le 3) = 5k = \frac{5}{10} \ngtr 1/2$

$P(X \le 4) = 8k = \frac{8}{10} > 1/2$

$\boxed{x = 4}$

(7) $\int_{-3}^{3} f(x)\,dx = 1$

Gn. $f(x) = \begin{cases} \frac{1}{16}(9+x^2+6x), & -3 \le x < -1 \\ \frac{1}{16}(6-2x^2), & -1 \le x < 1 \\ \frac{1}{16}(9+x^2-6x), & 1 \le x \le 3. \end{cases}$

L.H.S:

$\int_{-3}^{3} f(x)\,dx = \int_{-3}^{-1} f(x)\,dx + \int_{-1}^{1} f(x)\,dx + \int_{1}^{3} f(x)\,dx$

$= \frac{1}{16}\left[\int_{-3}^{-1}(9+x^2+6x)\,dx + \int_{-1}^{1}(6-2x^2)\,dx + \int_{1}^{3}(9+x^2-6x)\,dx\right]$

$= \frac{1}{16}\left[\left(9x+\frac{x^3}{3}+\frac{6x^2}{2}\right)_{-3}^{-1} + \left(6x-\frac{2x^3}{3}\right)_{-1}^{1} + \left(9x+\frac{x^3}{3}-\frac{6x^2}{2}\right)_{1}^{3}\right]$

$= \frac{1}{16}\left[\left(-9-\frac{1}{3}+\frac{6}{2}\right) - \left(-27-\frac{27}{3}+\frac{54}{2}\right) + \left(6-\frac{2}{3}\right) - \left(-6+\frac{2}{3}\right) + \left(27+\frac{27}{3}-\frac{54}{2}\right) - \left(9+\frac{1}{3}-\frac{6}{2}\right)\right]$

$= \frac{1}{16}\left[-9-\frac{1}{3}+3+27+9-27+6-\frac{2}{3}+6-\frac{2}{3}+27+9-27-9-\frac{1}{3}+3\right]$

$= \frac{1}{16}\left[-\frac{6}{3}+18\right]$

$= \frac{1}{16}(16)$

$= \boxed{1}$ $\therefore$ RHS

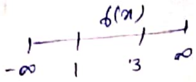
(8). $F(x) = \begin{cases} 0 & , x \le 1 \\ k(x-1)^4 & , 1 < x \le 3 \\ 0 & , x > 3 \end{cases}$

$F'(x) = f(x)$

$f(x) = d(F(x)) = \begin{cases} 0 & , x \le 1 \\ 4k(x-1)^3 & , 1 < x \le 3 \\ 0 & , x > 3 \end{cases}$

(i) k

$\int_{-\infty}^{\infty} f(x)\,dx = 1$



$\int_1^3 4k(x-1)^3\,dx = 1$

$4k \left[\frac{(x-1)^4}{4(1)} \right]_1^3 = 1$

$k(16-0) = 1$

$\boxed{k = 1/16}$

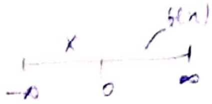
(ii) Pdf.

$f(x) = \begin{cases} \frac{1}{4}(x-1)^3 & , 1 < x \le 3 \\ 0 & , \text{otherwise.} \end{cases}$

(9). $f(x) = \begin{cases} Ae^{-x/5} & x \ge 0 \\ 0 & \text{otherwise.} \end{cases}$

(i) A:

w.k.t. Pdf $\int_{-\infty}^{\infty} f(x)\,dx = 1$



$\Rightarrow \int_0^{\infty} Ae^{-x/5}\,dx = 1$

$A \left(\frac{e^{-x/5}}{-1/5} \right)_0^{\infty} = 1$

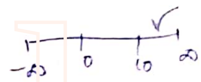
$-5A(e^{-\infty} - e^{-0}) = 1$

$-5A(0-1) = 1$

$\boxed{A = 1/5}$

$e^{\infty} = \infty$
$e^{-\infty} = 0$
$e^{-0} = e^0 = 1$

(ii) (a) More than 10 min



$P(X > 10) = \int_{10}^{\infty} \frac{1}{5} e^{-x/5}\,dx$

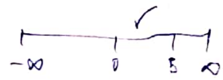
$= \frac{1}{5} \left[\frac{e^{-x/5}}{-1/5} \right]_{10}^{\infty}$

$= -\left[e^{-\infty} - e^{-10/5} \right]$

$= -[-e^{-2}]$

$= \boxed{1/e^2}$

(b) less than 5



$P(X < 5) = \int_0^5 \frac{1}{5} e^{-x/5}\,dx$

$= \frac{1}{5}\left(\frac{e^{-x/5}}{-1/5}\right)_0^5$

$= -(e^{-5/5} - e^0)$

$= -(e^{-1} - 1) = -(1/e - 1)$

$= (1 - 1/e)$

$= \boxed{\frac{e-1}{e}}$

(c) 5 to 10

$-\infty \quad 0 \quad 5 \quad 10 \quad \infty$

$P(5 < x < 10) = \int_5^{10} \frac{1}{5} e^{-x/5}\, dx$

$= \frac{1}{5}\left[\frac{e^{-x/5}}{-1/5}\right]_5^{10}$

$= -[e^{-10/5} - e^{-5/5}]$

$= -(e^{-2} - e^{-1})$

$= -\left(\frac{1}{e^2} - \frac{1}{e}\right)$

$= 1/e - 1/e^2$

$= \boxed{\frac{e-1}{e^2}}$

(10). $F(x) = \begin{cases} 0 & x \le 0 \\ x/2, & 0 \le x < 1 \\ 1/2, & 1 \le x < 2 \\ x/4, & 2 \le x < 4 \\ 1, & x \ge 4. \end{cases}$

(i) Yes,

Pdf $\Rightarrow f(x) = \begin{cases} 0, & x \le 0 \\ 1/2, & 0 \le x < 1 \\ 0, & 1 \le x < 2 \\ 1/4, & 2 \le x < 4 \\ 0, & x \ge 4. \end{cases}$

(ii) (a) More than 3

$0 \quad 3 \quad 4$

$P(x > 3) = \int_3^4 \frac{1}{4}\, dx$

$= \frac{1}{4}(x)_3^4 = \frac{1}{4}(4-3) = \boxed{1/4}$

(b) less than 3.

$0 \quad 3 \quad 4$

$P(x < 3) = \int_0^3 f(x)\, dx$

$1/2 \quad 1/4$
$0 \quad 1 \quad 2 \quad 3$

$= \int_0^1 \frac{1}{2}\, dx + \int_2^3 \frac{1}{4}\, dx$

$= \frac{1}{2}(x)_0^1 + \frac{1}{4}(x)_2^3$

$= \frac{1}{2}(1-0) + \frac{1}{4}(3-2)$

$= \frac{1}{2} + \frac{1}{4} = 3/4$

eg: 6.11

$$f(x) = \begin{cases} Ax, & \text{for } 0 \le x < 10 \\ A(20-x), & \text{for } 10 \le x < 20 \\ 0, & \text{otherwise} \end{cases}$$

a) find the A.

$$\int_{-\infty}^{\infty} f(x)\,dx = 1.$$

$$= \int_0^{10} Ax\,dx + \int_{10}^{20} A(20-x)\,dx = 1$$

$$= A\left\{(x)_0^{10} + (20-x)_{10}^{20}\right\} dx = 1$$

$$= A\left(\left(\frac{x^2}{2}\right)_0^{10} + \left(20x - \frac{x^2}{2}\right)_{10}^{20}\right) dx = 1$$

$$= A\left(\frac{100}{2} - 0\right) + \left(\left(20(20) - \frac{400}{2}\right) - \left(200 - \frac{100}{2}\right)\right) dx = 1$$

$$= A(50 + \cancel{400} - \cancel{200} - 200 + 50) = 1$$

$$= A(100) = 1$$

$$\boxed{A = 1/100}$$

(b) (i) $P(10 \le X \le 20)$

$$= \int_{10}^{20} A(20-x)\,dx.$$

$$= \int_{10}^{20} \frac{1}{100}(20-x)\,dx$$

$$= 1/100 \left[20x - \frac{x^2}{2}\right]_{10}^{20}.$$

$$= 1/100 \left(400 - \frac{400}{2} - \left(200 - \frac{100}{2}\right)\right)$$

$$= \frac{1}{100}(400 - 200 - 200 + 50)$$

$$= 1/100\,(50)$$

$$= \boxed{0.5}$$

(ii) $P(0 \le X < 10) = \int_0^{10} Ax\,dx.$

$$= \int_0^{10} 1/100\; x\,dx$$

$$= 1/100 \int_0^{10} x\,dx$$

$$= 1/100 \left[\frac{x^2}{2}\right]_0^{10}$$

$$= \frac{1}{100}\left[\frac{100}{2} - 0\right]$$

$$= 1/100\,(50)$$

$$= \boxed{0.5}$$

$S = \{(HHH), (HHT), (HTH), (HTT), (THH), (THT), (TTH), (TTT)\}$

eg. 6.6

X

$P(x)$ Cumulative

x	0	1	2	3
$P(x)$	$1/8$	$3/8$	$3/8$	$1/8$
$f(x)$	$1/8$	$4/8$	$7/8$	1

$$f(x) = \begin{cases} 0, & \text{for } x < 0 \\ 1/8, & 0 \le x < 1 \\ 4/8, & 1 \le x < 2 \\ 7/8, & 2 \le x < 3 \\ 1, & x \le 3 \end{cases}$$

eg 6.7

$f(x) = 5x^4 \quad 0 \le x \le 1$

(i) $P(X \le a_1) = P(x > a_1)$

$P(X \le a_1) = 1 - P(X \le a_1)$

$= \boxed{P(X > a) = 1 - P(X \le a)}$ formula

$2 P(X \le a_1) = 1$

$P(X \le a_1) = 1/2$

$\int_0^{a_1} f(x)\,dx = 1/2$

$\int_0^{a_1} 5x^4\,dx = 1/2$

$5\left(\frac{x^5}{5}\right)_0^{a_1} = 1/2$

$a_1^5 - 0 = 1/2$

$\boxed{a_1 = \left(\frac{1}{2}\right)^{1/5}}$

(ii) $P(X > a_2) = 0.05 = \frac{5}{100} = 1/20$

$\int_{a_2}^{1} f(x)\,dx = 1/20$

$5\left(\frac{x^5}{5}\right)_{a_2}^{1} = 1/20$

$1 - (a_2)^5 = 1/20$

$(a_2)^5 = 1 - 1/20 = 19/20$

$a_2 = \left(\frac{19}{20}\right)^{1/5}$

$\boxed{a_2 = (0.95)^{1/5}}$ //

ex. 6.2

discrete

$E(X) = \Sigma x\, p(x)$

Mean $= \mu_1'$

$E(X^2) = \Sigma x^2\, p(x)$

μ_2'

continuous.

$E(X) = \int_{-\infty}^{\infty} x f(x).dx$

Mean

$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)dx$

$Var(X) = E(X^2) - (E(X))^2$

$\mu_2 = \mu_2' - (\mu_1')^2$.

(i) $E(a) = a$, $E(aX) = a E(X)$

$Var(a) = 0$

$Var(aX) = a^2 Var(X)$

$S.D = \sqrt{Var(X)}$.

① $X = 1, 2, 3, 4, 5, 6$

x	1	2	3	4	5	6
$p(x)$	1/6	1/6	1/6	1/6	1/6	6/6

$E(X) = \Sigma(x)p(x)$

$= 1/6 + 2/6 + 3/6 + 4/6 + 5/6 + 6/6$

$= \frac{21}{6}$

$\boxed{E(X) = 3.5}$

②.

x	0	1	2	3
$p(x)$	0.2	0.1	0.4	0.3

$E(X) = \Sigma x\, p(x)$

$= 0 + 0.1 + 0.8 + 0.9$

$\boxed{E(X) = 1.8}$

③

x	3	4	5
$p(x)$	0.1	0.1	0.2.

$E(X) = \Sigma x\, P(x) = 0.3 + 0.4 + 1.0$

$= \boxed{1.7}$

$E(x^2) = \Sigma x^2 P(x)$

$= 9\times0.1 + 16\times0.1 + 25\times0.2$

$= 0.9 + 1.6 + 5.0$

$\boxed{E(x^2) = 7.5}$

$Var(x) = E(x^2) - (E(x))^2$

$= 7.5 - (1.7)^2$

$= 7.5 - 2.89$

$\boxed{Var(x) = 4.61}$

$S.D = \sqrt{Var(x)}$

$= \sqrt{4.61}$

$= 2.147$

$\boxed{S.D = 2.15}$

x	5000	-8000
$P(x)$	0.62	0.38

(6). $E(X) = 5000\times0.62 - 8000\times0.38$

$= 3100 - 3040$

$= \boxed{₹\ 60}$

(14) $E(x^2) = \ 5$

(14).

x	4	-2
$P(x)$	1/2	1/2

$E(x) = \Sigma x P(x)$

$= 4/2 - 2/2 = 2/2 = 1$

$E(x^2) = \Sigma x^2 P(x)$

$= \overset{8}{\cancel{16}}\times\frac{1}{\cancel{2}} + \overset{2}{\cancel{4}}\times\frac{1}{\cancel{2}}$

$= 8+2 = 10$

$Var(x) = E(x^2) - (E(x))^2$

$= 10 - 1 = \boxed{9}$

(9). $f(x) = \begin{cases} 2x, & 0 \le x \le 1 \\ 0, & \text{otherwise.} \end{cases}$

$E(x) = \int_{-\infty}^{\infty} x f(x)\, dx$

$-\infty \quad 0 \quad 1 \quad \infty$

$= \int_0^1 x(2x)\,dx$

$= 2\left(\frac{x^3}{3}\right)_0^1$

$= 2/3\,[1-0]$

$\boxed{E(x) = 2/3}$

(5) $f(x) = \begin{cases} \frac{3}{x^4}, & x \geq 1 \\ 0, & \text{otherwise} \end{cases}$

$E(x) = \int_{-\infty}^{\infty} x f(x) \, dx$

$= \int_{1}^{\infty} x \times \frac{3}{x^4} \, dx$

$= 3 \int_{1}^{\infty} \frac{1}{x^3} \, dx$

$= 3 \left[-\frac{1}{2x^2} \right]_{1}^{\infty}$

$= -\frac{3}{2} \left(\frac{1}{\infty} - \frac{1}{1} \right)$

$= -\frac{3}{2} (0 - 1)$

$\boxed{E(x) = \frac{3}{2}}$ → Mean.

$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x) \, dx$

$= \int_{1}^{\infty} x^2 \left(\frac{3}{x^4} \right) dx$

$= 3 \int_{1}^{\infty} \frac{1}{x^2} \, dx$

$= 3 \left(-\frac{1}{x} \right)_{1}^{\infty}$

$= -3 \left[\frac{1}{\infty} - \frac{1}{1} \right]$

$= -3(0 - 1)$

$\boxed{E(x^2) = 3}$

$\text{var}(x) = E(x^2) - (E(x))^2$

$= 3 - \frac{9}{4}$

$= \boxed{\frac{3}{4}}$

(15) $Y = 2x + 1$

Take var

$\text{var } Y = \text{var}(2x + 1)$

$= \text{var}(2x) + \text{var}(1)$

$= 2^2 \text{ var}(x) + 0$

$= 4(5)$

$\boxed{\text{Var } Y = 20}$

eg 6.24

$f(x) = ke^{-|x|}, \quad -\infty < x < \infty$

(i) Q1:

Pdf: $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$|x| = +x$
$= -x$

$\int_{-\infty}^{\infty} ke^{-|x|}\,dx = 1$

$2\int_0^{\infty} ke^{-x}\,dx = 1$

$\left[\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx \quad f \text{ is even}, \quad f(x) = e^{-|x|} = \text{is even}\right]$

$2\int_0^{\infty} ke^{-x}dx = 1$

$-|-x| = -x$

$2k\left(\frac{e^{-x}}{-1}\right)_0^{\infty} = 1$

$-2k\,(e^{-\infty} - e^0) = 1$

$= 2k\,(0-1) = 1$

$\boxed{k = 1/2}$

$E(x) = \int_{-\infty}^{\infty} x f(x)\,dx \qquad k\int_{-\infty}^{\infty} x e^{-|x|}dx = 0$

$= \int_{-\infty}^{\infty} x k e^{-|x|}\,dx$

$= \frac{1}{2}\int_{-\infty}^{\infty} x e^{-|x|}\,dx$ OF EF = OF

$= \frac{1}{2} \times 0 \quad [f(x) = xe^{-|x|}$ = odd function$]$

$\boxed{\text{Mean} = 0}$

$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx$

$= \int_{-\infty}^{\infty} x^2 ke^{-|x|}\,dx$

E×E = E

$= 2k\int_0^{\infty} x^2 e^{-x}\,dx$

$= \cancel{2}\times\frac{1}{\cancel{2}} \int_0^{\infty} x^2 e^{-1x}\,dx$

$\left[\int_0^{\infty} x^n e^{-ax}\,dx = \frac{n!}{a^{n+1}}\right]$

$a = 1 \quad n = 2$

$= \frac{2!}{(1)^{2+1}} = \boxed{2}$

$\text{Var}(x) = E(x^2) - (E(x))^2$
$= 2 - 0$
$= \boxed{2}$

(13) $f(x) = \begin{cases} \frac{1}{30} e^{-x/30}, & x > 0 \\ 0, & x \le 0 \end{cases}$

$-\infty \quad 0 \quad \infty$

$E(x) = \int_{-\infty}^{\infty} x f(x)\,dx$

$-\infty \quad 0 \quad \infty$

$\frac{1}{30}\int x e^{-\frac{1}{30}x}dx \qquad = \int_0^{\infty} x \frac{1}{30} e^{-x/30}\,dx \qquad -\frac{1}{30}x$

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$a = 1/30 \quad n = 1$

$= 1/30 \dfrac{1!}{(1/30)^2}$

$= 1/30 (30)^2$

$= 30 \times 1000$

$= \boxed{30{,}000 \text{ miles}}$

eg 6.23

$f(x) = \begin{cases} e^{-x/100}, & x \geq 100 \\ 0 & x < 100 \end{cases}$

$E(x) = \int_{-\infty}^{\infty} x f(x)\,dx$

$= \int_{100}^{\infty} x e^{-x/100}\,dx$

$u = x$ | $dv = e^{-x/100}\,dx$

$u' = 1$ | $v = \dfrac{e^{-x/100}}{-1/100} = -100 e^{-x/100}$

$u'' = 0$ | $v_1 = \dfrac{-100 e^{-x/100}}{-1/100}$

$v_1 = 10000 e^{-x/100}$

$\int u\,dv = \cancel{uv}$

$uv - u'v_1 + \ldots$

$\int_{100}^{\infty} x e^{-x/100}\,dx = \left[x \times -100 e^{-x/100} - 1 \times 10000 e^{-x/100}\right]_{100}^{\infty}$

$= (\infty \times -100 \times 0 - 10000 \times 0)$

$- \left(100 \times -100 e^{-100/100} - 10000 e^{-100/100}\right)$

$= +10000 e^{-1} + 10000 e^{-1}$

$= 2 \times 10000 \times e^{-1}$

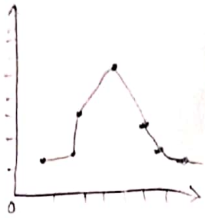
$= 20000 \times \dfrac{1}{2.718}$

$= 20{,}000 \times 0.3679$

$= \boxed{7358 \text{ hours}}$

Eg 6.7

$X=x$	1	2	3	4	5	6	7
$P(x)$	0.10	0.12	0.20	0.30	0.15	0.08	0.05



$P(1) = P(x \le 1) = P(1) = 0.10.$

$P(2) = 0.10 + 0.12 = 0.22$

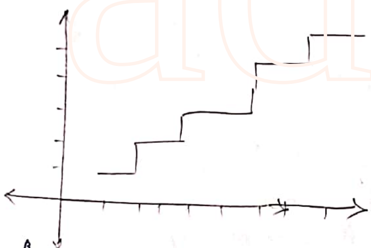
$P(3) = 0.22 + 0.20 = 0.42$

$P(4) = 0.42 + 0.30 = 0.72$

$P(5) = 0.72 + 0.15 = 0.87$

$P(6) = 0.87 + 0.08 = 0.95$

$P(7) = 0.95 + 0.05 = 1.00$



$$f(x) = \begin{cases} 0, & \text{if } x < 1 \\ 0.10, & x \le 1 \\ 0.22, & x \le 2 \\ 0.42 & x \le 3 \\ 0.72 & x \le 4 \\ 0.87 & x \le 5 \\ 0.95 & x \le 6 \\ 1, & x \le 7. \end{cases}$$

Eg 6.8

gn pdf.

$f(x) = ax. \quad 0 \le x \le 1$

$\int_{-\infty}^{\infty} f(x)\,dx = 1$

$-\infty \quad 0 \quad 1 \quad \infty$

$\int_0^1 ax\,dx = 1$

$a\int_0^1 x\,dx = 1$

$a\left(\frac{x^2}{2}\right)_0^1 = 1$

$\frac{a}{2}(1-0) = 1$

$\boxed{a=2}$

$0 \quad 1/2 \quad 1$

$P\left[x \le 1/2\right] =$

$= \int_0^{1/2} ax\,dx$

$= \int_0^{1/2} 2x\,dx$

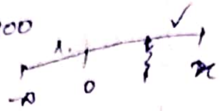
$= \left[\cancel{2}\frac{x^2}{\cancel{2}}\right]_0^{1/2}$

$= 1/4 - 0$

$= \boxed{1/4}$

eg 6.10

$$f(x) = \begin{cases} 100/x^2 , & x \geq 100 \\ 0 , & x < 100 \end{cases}$$



$F'(x) = f(x)$

$$F(x) = \int_{100}^{x} \frac{100}{x^2} \, dx$$

$$= \left[\frac{-100}{x}\right]_{100}^{x}$$

$$= \frac{-100}{x} - \left(\frac{-100}{100}\right)$$

$$= \frac{-100}{x} + 1$$

$$= \boxed{1 - \frac{100}{x}} \rightarrow x \geq 100$$

eg 6.12

$X = x$	1	2	3	4	5	6	7	8	9	10
$P(x)$	0.15	0.10	0.10	0.01	0.08	0.01	0.05	0.02	0.28	0.20

$E(x) = \Sigma x P(x)$

$= (1 \times 0.15) + (2 \times 0.10) + (3 \times 0.10) + (4 \times 0.01) + (5 \times 0.08) + (6 \times 0.01) + (7 \times 0.05) + (8 \times 0.02) + (9 \times 0.28) + (10 \times 0.20$

$\boxed{E(x) = 6.56}$

$E(x^2) = \Sigma x^2 P(x)$

$= (1^2 \times 0.15) + (2^2 \times 0.10) + (3^2 \times 0.10) + (4^2 \times 0.01) + (5^2 \times 0.08) + (6^2 \times 0.01) + (7^2 \times 0.05) + (8^2 \times 0.02) + (9^2 \times 0.28) + (10^2 \times 0.20)$

$= (1 \times 0.15) + (4 \times 0.10) + (9 \times 0.10) + (16 \times 0.01) + (25 \times 0.08) + (36 \times 0.01) + (49 \times 0.05) + (64 \times 0.02) + (81 \times 0.28) + (100 \times 0.20)$

$= \boxed{50.38}$

$Var(x) = E(x^2) - (E(x))^2$

$= 50.38 - (6.56)^2$

$= \boxed{7.35}$

Eg. 6.13

$E(X) = \Sigma x P(x)$

$= \left[\left(0 \times \frac{2}{11}\right) + \left(1 \times \frac{5}{11}\right) + \left(2 \times \frac{4}{11}\right)\right]$

$= \boxed{\frac{13}{11}}$