ELECTROSTATICS

PHYSICS - VOL 1 **UNIT - 1**

NAME :

STANDARD : 12 SECTION :

SCHOOL :

EXAM NO :

கற்க கசடற கற்பவை கற்

றபின் நிற்க அதற்கு தக

கற்பதற்கு தகுதியான நூல்களை பழுதில்லாமல் கற்க வேண்டும். கற்றதற்கு பின்னர் கற்ற அக்கல்விக்கு தகுந்தபடி நடக்கவும் வேண்டும்

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PART - II 2 MARK QUESTIONS AND ANSWERS

1. What is Electrostatics?

- The branch of electricity which deals with stationary charges is called electrostatics.

2. What is called triboelectric charging?

- Charging the objects through rubbing is called triboelectric charging.

3. Like charges repels. Unlike charges attracts. Prove.

- A negatively charged rubber rod is repeled by another negatively charged rubber rod.
- But a negatively charged rubber rod is attracted by a positively charged glass rod.
- This proves like charges repels and unlike charges attracts.

4. State conservation of electric charges.

- The total electric charge in the universe is constant and charge can neither be created nor be destroyed
- In any physical process, the net change in charge will be zero. This is called conservation of charges.

5. State quantisation of electric charge.

- The charge 'q' of any object is equal to an integral multiple of this fundamental unit of charge 'e' (i.e) $q = n\,e$
- where, n → integer and $e = 1.6 \, X\, 10^{-19}\, C$

6. State Coulomb's law in electrostatics.

- According to Coulomb law, the force on the point charge q_2 exerted by another point charge q_1 is

$$\vec{F}_{21} = k\,\frac{q_1\,q_2}{r^2}\,\hat{r}_{12}$$

- where, k → constant
  $\hat{r}_{12}$ → unit vector directed from q_1 to q_2

7. Define one coulomb (1 C)

- The S.I unit of charge is coulomb (C)
- One Coulomb is that charge which when placed in free space or air at a distance 1 m from an equal and similar charge repels with a force of $9 \, X\, 10^9$ N

8. Define relative permittivity.

- From Coulomb's law, the electrostatic force is

$$\vec{F}_{21} = \frac{1}{4\,\pi\,\varepsilon}\,\frac{q_1\,q_2}{r^2}\,\hat{r}_{12} = \frac{1}{4\,\pi\,\varepsilon_o\,\varepsilon_r}\,\frac{q_1\,q_2}{r^2}\,\hat{r}_{12}$$

- Here $\varepsilon = \varepsilon_o\,\varepsilon_r$ is called permittivity of any medium
- ε_0 is called permittivity of free space or vacuum and ε_r is called relative permittivity.
- Thus The ratio of permittivity of the medium to the permittivity of free space is called relative permittivity or dielectric constant. $\left[\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}\right]$.
- It has no unit and for air $\varepsilon_r = 1$ and for other dielectric medium $\varepsilon_r > 1$

9. Give the vector form of Coulomb's law.

- The force on the point charge q_2 exerted by another point charge q_1 is

$$\vec{F}_{21} = \frac{1}{4\,\pi\,\varepsilon_0}\,\frac{q_1\,q_2}{r^2}\,\hat{r}_{12}$$

- Similarly the force on the point charge q_1 exerted by another point charge q_2 is

$$\vec{F}_{12} = \frac{1}{4\,\pi\,\varepsilon_0}\,\frac{q_1\,q_2}{r^2}\,\hat{r}_{21}$$

- **Here,** $\hat{r}_{12}$ → unit vector directed from q_1 to q_2
  $\hat{r}_{21}$ → unit vector directed from q_2 to q_1

10. Distinguish between Coulomb force and Gravitational force.

Coulomb force	Gravitational force
It acts between two charges	It acts between two masses
It can be attractive or repulsive	It is always attractive
It is always greater in magnitude	It is always lesser in magnitude
It depends on the nature of the medium	It is independent of the medium
If charges are in motion, another force called Lorentz force come in to play in addition to Coulomb force	Gravitional force is the same whether two masses are at rest or in motion

11. Define superposition principle.

- According to Superposition principle, the total force acting on a given charge is equal to the vector sum of forces exerted on it by all the other charges.

12. Define electric field.

- The electric field at a point 'P' at a distance 'r' from the point charge 'q' is the force experienced by a unit charge. Its S.I unit is $N\,C^{-1}$

13. What is called electric dipole. Give an example.

- Two equal and opposite charges separated by a small distance constitute an electric dipole.
  (e.g) CO, HCl, NH_4, H_2O

14. Define electric dipole moment. Give its unit.

- The magnitude of the electric dipole moment (p) is equal to the product of the magnitude of one of the charges (q) and the distance ($2a$) between them. (i.e) $|\vec{p}| = q.2a$
- Its unit is $C\,m$

15. Define potential difference. Give its unit.

- The electric potential difference is defined as the workdone by an external force to bring unit positive charge from one point to another point against the electric field.
- Its unit is ***volt (V)***

16. Define electrostatic potential. Give its unit.

- The electric potential at a point is equal to the work done by an external force to bring a unit positive charge with constant velocity from infinity to the point in the region of the external electric field.
- Its unit is ***volt (V)***

17. Define electrostatic potential energy.

- The electric potential energy of two point charges is equal to the amount of workdone to assemble the charges or workdone in bringing a charge from infinite distance. (i.e) $U = W = q\,V$

18. Define electric flux.

- The number of electric field lines crossing a given area kept normal to the electric field lines is called electric flux (Φ_E).
- Its S.I unit is $N\,m^2\,C^{-1}$. It is a scalar quantity.

19. State Gauss law.

Gauss law states that if a charge 'Q' is enclosed by an arbitrary closed surface, then the total electric flux through the closed surface is equal to $\frac{1}{\varepsilon_O}$ times the net charge enclosed by the surface.

$$\Phi_E = \oint \vec{E}\,.\,d\vec{A} = \frac{Q_{inside}}{\varepsilon_0}$$

20. What is called a capacitor?

- Capacitor is a device used to store electric charge and electric energy.
- It consists of two conducting plates or sheets separated by some distance.
- Capacitors are widely used in many electronic circuits and in many area of science and technology.

21. Define capacitance of a capacitor.

- The capacitance of a capacitor is defined as the ratio of the magnitude of charge (Q) on either of the conductor plates to the potential difference (V) existing between the conductors. (i.e) C = Q/V
- Its unit is ***farad (F)*** or ***C V⁻¹***

22. Define energy density of a capacitor.

- The energy stored per unit volume of space is defined as energy density and it is derived as,

$$u_E = \frac{U}{volume} = \frac{1}{2}\, \varepsilon_o E^2$$

23. Define action of point or corona discharge.

- Smaller the radius of curvature, larger the charge density. Hence charges are accumulated at the sharp points.
- Due to this, the electric field near this sharp edge is very high and it ionized the surrounding air.
- The positive ions are repelled and negative ions are attracted towards the sharp edge.
- This reduces the total charge of the conductor near the sharp edge. This is called action of points or corona discharge

PART - III 3 MARK QUESTIONS AND ANSWERS

1. Discuss the basic properties of electric charge.

(i) **Electric charge :**

- ♦ Like mass, the electric charge is also an intrinsic and fundamental property of particles.
- ♦ The unit of electric charge is coulomb

(ii) **Conservation of electric charge :**

- ♦ The total electric charge in the universe is constant and charge can neither be created nor be destroyed.
- ♦ In any physical process, the nte change in charge will be zero. This is called conservation of charges

(iii) **Quanisation of charge :**

- ♦ The chage 'q' of any object is equal to an integral multiple of this fundamental unit of charge 'e' (i.e) $\boldsymbol{q = n\, e}$
- ♦ where n → integer and $\boldsymbol{e = 1.6 \ X\ 10^{-19}\ C}$

2. Define superposition principle. Explain how superposition principle explans the interaction between multiple charges.

__Superposition principle__ :

- According to Superposition principle, the total force acting on a given charge is equal to the vector sum of forces exerted on it by all the other charges.

__Explanation__ :

- Consider a system of 'n' charges $q_1, q_2, \ \dots\ , q_n$
- By Coulomb's law, force on q_1 by $q_2, \ \dots\ , q_n$ are

$$\vec{F}_{12} = k\, \frac{q_1\, q_2}{r_{21}^2}\ \hat{r}_{21}$$

$$\vec{F}_{13} = k\, \frac{q_1\, q_2}{r_{31}^2}\ \hat{r}_{31}$$

finally. $$\vec{F}_{1n} = k\, \frac{q_1\, q_2}{r_{n1}^2}\ \hat{r}_{n1}$$

- Then total force action on q_1 due to all charges,

$$\vec{F}_1^{tot} = \vec{F}_{12} + \vec{F}_{13} + \dots \dots + \vec{F}_{1n}$$

$$\boldsymbol{\vec{F}_1^{tot} = k\left[\frac{q_1\, q_2}{r_{21}^2}\ \hat{r}_{21} + \frac{q_1\, q_3}{r_{31}^2}\ \hat{r}_{31} + \dots + \frac{q_1\, q_n}{r_{n1}^2}\ \hat{r}_{n1}\right]}$$

3. Explain Electric field at a point dueto system of charges (or) Superposition of electric fields.

__Superposition of electric field__ :

- The electric field at an arbitrary point due to system of point charges is simply equal to the vector sum of the electric fields created by the individual point charges. This is called superposition of electric fields.

__Explanation__ :

- Consider a system of 'n' charges $q_1, q_2, \ \dots\ , q_n$
- The electric field at 'P' due to 'n' charges

$$\vec{E}_1 = \frac{1}{4\,\pi\,\varepsilon_0}\frac{q_1}{r_{1P}^2}\ \hat{r}_{1P}$$

$$\vec{E}_2 = \frac{1}{4\,\pi\,\varepsilon_0}\frac{q_2}{r_{2P}^2}\ \hat{r}_{2P}$$

finally, $$\vec{E}_n = \frac{1}{4\,\pi\,\varepsilon_0}\frac{q_n}{r_{nP}^2}\ \hat{r}_{nP}$$

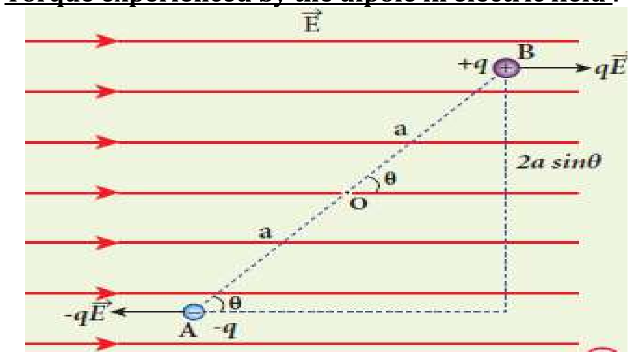
- The total electric field at 'P' due to all these 'n' charges will be,

$$\vec{E}_{tot} = \vec{E}_1 + \vec{E}_2 + \dots \dots + \vec{E}_n$$

$$\boldsymbol{\vec{E}_{tot} = \frac{1}{4\,\pi\,\varepsilon_0}\left[\frac{q_1}{r_{1P}^2}\ \hat{r}_{1P} + \frac{q_2}{r_{2P}^2}\ \hat{r}_{2P} + \dots + \frac{q_n}{r_{nP}^2}\ \hat{r}_{nP}\right]}$$

4. Derive an expression for torque experienced by an electric dipole placed in the uniform electric field.

__Torque experienced by the dipole in electric field__ :



- Let a dipole of moment $\vec{\boldsymbol{p}}$ is placed in an uniform electric field $\vec{\boldsymbol{E}}$
- The force on '+q' = $+\boldsymbol{q\,\vec{E}}$

  The force on '-q' = $-\,\boldsymbol{q\,\vec{E}}$
- Then the total force acts on the dipole is zero.
- But these two forces constitute a ***couple*** and the dipole experience a torque which tend to rotate the dipole along the field.

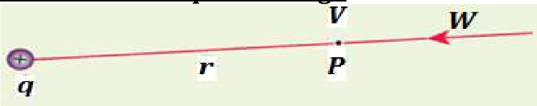
- The total torque on the dipole about the point 'O'

$$\vec{\tau} = \overrightarrow{OA}\ X\left(-\,\boldsymbol{q}\,\vec{\boldsymbol{E}}\right) + \overrightarrow{OB}\ X\left(+\boldsymbol{q}\,\vec{\boldsymbol{E}}\right)$$
$$|\vec{\tau}| = |\overrightarrow{OA}|\,|-\boldsymbol{q}\,\vec{\boldsymbol{E}}|\sin\theta + |\overrightarrow{OB}|\,|\boldsymbol{q}\,\vec{\boldsymbol{E}}|\sin\theta$$
$$\tau = (OA + OB) q\ E \sin\theta$$
$$\tau = 2\,a\,q\,E\sin\theta \qquad \because [OA = OB = a]$$
$$\boldsymbol{\tau = p\,E\sin\theta}$$

- where, $2\,a\,q = p \rightarrow$ dipole moment
- In vector notation, $\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{p}}\ \boldsymbol{X}\ \vec{\boldsymbol{E}}$
- The torque is maximum, when $\theta = 90°$

5. Obtain an expression electric potential at a point due to a point charge.

<u>Potential due to a point charge</u> :



- Consider a point charge $+\boldsymbol{q}$ at origin.
- 'P' be a point at a distance 'r' from origin.
- By definition, the electric field at 'P' is

$$\vec{E} = \frac{1}{4\,\pi\,\varepsilon_0}\frac{q}{r^2}\,\hat{r}$$

- Hence electric potential at 'P' is

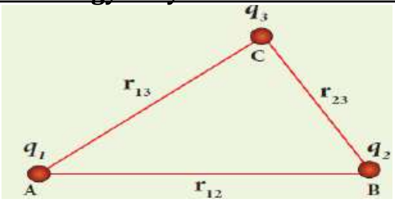
$$V = -\int_{\infty}^{r}\vec{E}\,.\,\overrightarrow{dr} = -\int_{\infty}^{r}\frac{1}{4\,\pi\,\varepsilon_0}\frac{q}{r^2}\,\hat{r}\,.\,\overrightarrow{dr}$$
$$V = -\int_{\infty}^{r}\frac{1}{4\,\pi\,\varepsilon_0}\frac{q}{r^2}\,\hat{r}\,.\,dr\,\hat{r} \qquad [\because \overrightarrow{dr} = dr\,\hat{r}]$$
$$V = -\frac{q}{4\,\pi\,\varepsilon_0}\int_{\infty}^{r}\frac{1}{r^2}\,dr \qquad [\because\ \hat{r}\,.\,\hat{r} = 1]$$
$$V = -\frac{q}{4\,\pi\,\varepsilon_0}\left[-\frac{1}{r}\right]_{\infty}^{r} = \frac{q}{4\,\pi\,\varepsilon_0}\left[\frac{1}{r} - \frac{1}{\infty}\right]$$
$$\boldsymbol{V = \frac{1}{4\,\pi\,\varepsilon_0}\frac{q}{r}}$$

- If the source charge is negative $(-q)$, then the potential also negative and it is given by

$$\boldsymbol{V = -\frac{1}{4\,\pi\,\varepsilon_0}\frac{q}{r}}$$

6. Obtain an expression for potential energy due to a collection of three point charges which are separated by finite distances.

<u>Potential energy of system of three charges</u> :



- Electrostatic potential energy of a system of charges is defined as the work done to assemble the charges
- consider a point charge $\boldsymbol{q_1}$ at 'A'
- Electric potential at 'B' due to $\boldsymbol{q_1}$ is,

$$V_{1B} = \frac{1}{4\,\pi\,\varepsilon_0}\frac{q_1}{r_{12}}$$

- To bring second charge $\boldsymbol{q_2}$ to 'B', work has to be done against the electric field created by $\boldsymbol{q_1}$
- The work done on the charge $\boldsymbol{q_2}$ is,

$$W = q_2\,V_{1B} = \frac{1}{4\,\pi\,\varepsilon_0}\frac{q_1\,q_2}{r_{12}}$$

- This work done is stored as electrostatic potential energy of system of two charges $\boldsymbol{q_1}$ and $\boldsymbol{q_2}$

$$\boldsymbol{U = \frac{1}{4\,\pi\,\varepsilon_0}\frac{q_1\,q_2}{r_{12}}} \qquad ----(1)$$

- The potential at 'C' due to charges $\boldsymbol{q_1}$ & $\boldsymbol{q_2}$

$$V_{1C} = \frac{1}{4\,\pi\,\varepsilon_0}\frac{q_1}{r_{13}} \qquad \& \qquad V_{2C} = \frac{1}{4\,\pi\,\varepsilon_0}\frac{q_2}{r_{23}}$$

- To bring third charge $\boldsymbol{q_3}$ to 'C' , work has to be done against the electric field due to $\boldsymbol{q_1}$ & $\boldsymbol{q_2}$.
- Thus work done on charge $\boldsymbol{q_3}$ is,

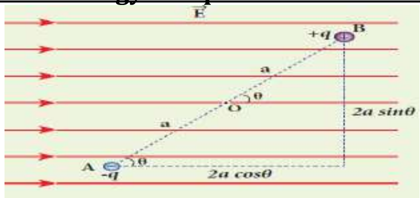
$$W = q_3\,(V_{1C} + V_{2C}) = q_3\,\frac{1}{4\,\pi\,\varepsilon_0}\left[\frac{q_1}{r_{13}} + \frac{q_2}{r_{23}}\right]$$
$$(or)\quad \boldsymbol{U = \frac{1}{4\,\pi\,\varepsilon_0}\left[\frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right]} \qquad ----(2)$$

- Hence the the total electrostatic potential energy of system of three point charges is

$$\boldsymbol{U = \frac{1}{4\,\pi\,\varepsilon_0}\left[\frac{q_1\,q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right]} \qquad ---(3)$$

7. Obtain an expression for electrostatic potential energy of a dipole in a uniform electric field.

<u>Potential energy of dipole in uniform electric field</u>:



- Let a dipole of moment $\vec{\boldsymbol{p}}$ is placed in a uniform electric field $\vec{\boldsymbol{E}}$
- Here the dipole experience a torque, which rotate the dipole along the field.
- To rotate the dipole from θ' to θ against this torque, work has to be done by an external torque (τ_{ext}) and it is given by,

$$W = \int_{\theta'}^{\theta}\tau_{ext}\,d\theta = \int_{\theta'}^{\theta}p\,E\sin\theta\ d\theta$$
$$W = p\,E\,[-\cos\theta]_{\theta'}^{\theta} = -p\,E\,[\cos\ \theta - \cos\ \theta']$$
$$W = p\,E\,[\cos\ \theta' - \cos\ \theta]$$

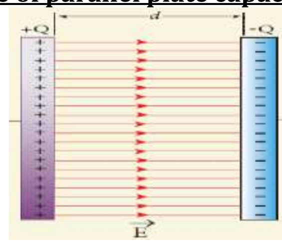
- This work done is stored as electrostatic potential energy of the dipole.
- Let the initial angle be $\theta' = 90°$, then

$$U = W = p\,E\,[\cos\ 90° - \cos\ \theta]$$
$$\boldsymbol{U = -\,p\,E\cos\theta = -\,\vec{p}\,.\,\vec{E}}$$

- If $\theta = 180°$, then potential energy is maximum
- If $\theta = 0°$, then potential energy is mimimum

8. Derive an expression for capacitance of parallel plate capacitor.

<u>Capacitance of parallel plate capacitor</u> :



- Consider a capacitor consists of two parallel plates each of area 'A' separated by a distance 'd'
- Let '$\boldsymbol{\sigma}$' be the surface charge density of the plates.
- The electric field between the plates,

$$\boldsymbol{E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\,\varepsilon_0}} \qquad ------- \quad (1)$$

- Since the field is uniform, the potential difference between the plates,

$$V = E\, d = \left[\frac{Q}{A\, \varepsilon_0}\right] d \quad ------ \quad (2)$$

- Then the capacitance of the capacitor,

$$C = \frac{Q}{V} = \frac{Q}{\left[\frac{Q}{A\,\varepsilon_0}\right] d}$$

$$C = \frac{\varepsilon_0\, A}{d} \quad -------- \quad (3)$$

- Thus capacitance is,
  (i) directly proportional to the Area (A) and
  (ii) inversely proportional to the separation (d)

9. **Derive an expression for energy stored in capacitor**

Energy stored in capacitor:

- Capacitor is a device used to store charges and energy.
- When a battery is connected to the capacitor, electrons of total charge '-Q' are transferred from one plate to other plate. For this work is done by the battery.
- This work done is strored as electrostatic energy in capacitor.
- To transfer $'dQ'$ for a potential difference 'V', the work done is

$$dW = V\, dQ = \frac{Q}{C}\, dQ \qquad \left[\because\ V = \frac{Q}{C}\right]$$

- The total work done to charge a capacitor,

$$W = \int_0^Q \frac{Q}{C}\, dQ = \frac{1}{C}\left[\frac{Q^2}{2}\right]_0^Q = \frac{Q^2}{2\, C}$$

- This work done is stored as electrostatic energy of the capacitor, (i.e)

$$U_E = \frac{Q^2}{2\, C} = \frac{1}{2}\, C\, V^2 \qquad [\because\ Q = C\, V]$$

- We know that, $V = E\, d$ & $C = \frac{\varepsilon_0\, A}{d}$

$$\therefore\ U_E = \frac{1}{2}\frac{\varepsilon_0\, A}{d}(E\, d)^2 = \frac{1}{2}\,\varepsilon_0\,(A\, d)\, E^2$$

- *where,* $(A\, d) \rightarrow\ volume$
- The energy stored per unit volume of space is defined as energy density ((u_E).

$$u_E = \frac{U_E}{volume} = \frac{1}{2}\,\varepsilon_0\, E^2$$

10. **Give the applications and disadvantage of capacitors**

Applications of capacitor:

- Flash capacitors are used in digital camera to take photographs
- During cardiac arrest, a device called heart defibrillator is used to give a sudden surge of a large amount of electrical energy to the patient's chest to retrieve the normal heart function. This defibrillator uses a capacitor of 175 μF charged to a high voltage of around 2000 V
- Capacitors are used in the ignition system of automobile engines to eliminate sparking.
- Capacitors are used to reduce power fluctuations in power supplies and to increase the efficiency of power transmission.

Disadvantages :

- Even after the battery or power supply is removed, the capacitor stores charges and energy for some time. It caused unwanted shock.

11. **Write a note on microwave oven.**

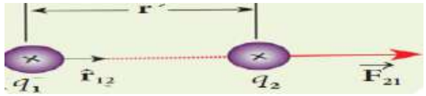
Microwave oven :

- It works on the principle of torque acting on an electric dipole.
- The food we consume has water molecules which are permanent electric dipoles. Oven produce microwaves that are oscillating electromagnetic fields and produce torque on the water molecules.
- Due to this torque on each water molecule, the molecules rotate very fast and produce thermal energy.
- Thus, heat generated is used to heat the food.

PART - IV 5 MARK QUESTIONS & ANSWERS

1. **Explain in detail Coulomb's law and its various aspects.**

Coulomb's law :



- Consider two point charges q_1 and q_2 separated by a distance $'r'$
- According to Coulomb law, the force on the point charge q_2 exerted by q_1 is

$$\vec{F}_{21} = k\, \frac{q_1\, q_2}{r^2}\ \hat{r}_{12}$$

- where, k → constant
  $\hat{r}_{12}$ → unit vector directed from q_1 to q_2

Important aspects :

- Coulomb law states that the electrostatic force is
  1) directly proportional to the product of the magnitude of two point charges
  2) inversely proportional to the square of the distance between them
- The force always lie along the line joining the two charges.
- In S.I units, $k = \frac{1}{4\,\pi\varepsilon_0} = 9\ X\ 10^9\ N\ m^2 C^{-2}$
- Here is the permittivity of free space or vacuum and its value is

$$\varepsilon_0 = \frac{1}{4\,\pi k} = 8.85\ X\ 10^{-12}\ C^2\ N^{-1} m^{-2}$$

- The magnitude of electrostatic force between two charges each of 1 C separated by a distance of 1 m is $9\ X\ 10^9\ N$
- The Coulomb law in vacuum and in medium are,

$$\vec{F}_{21} = \frac{1}{4\,\pi\varepsilon_0}\frac{q_1\, q_2}{r^2}\,\hat{r}_{12} \quad \& \quad \vec{F}_{21} = \frac{1}{4\,\pi\varepsilon}\frac{q_1\, q_2}{r^2}\,\hat{r}_{12}$$

where, $\varepsilon = \varepsilon_o\varepsilon_r$ ⟶ permittivity of the medium

Thus the relative permittivity of the given medium is defined as , $\varepsilon_r = \frac{\varepsilon}{\varepsilon_o}$. For air or vacuum, $\varepsilon_r = 1$ and for all other media $\varepsilon_r > 1$

- Coulomb's law has same structure as Newton's law of gravitation. (i.e)

$$F_{Coulomb} = k\,\frac{q_1 q_2}{r^2} \quad \& \quad F_{Newton} = G\,\frac{m_1 m_2}{r^2}$$

- Here $k = 9\ X\ 10^9\ N\ m^2 C^{-2}$ and
  $G = 6.626\ X\ 10^{-11}\ N\ m^2 kg^{-2}$

Since 'k' is much more greater than 'G', the electrostatic force is always greater than gravitational force for smaller size objects

- Electrostatic force between two point charges depends on the nature of the medium in which two charges are kept at rest.
- Depending upon the nature of the charges, it may either be attractive or repulsive
- If the charges are in motion, another force called Lorentz force come in to play in addition with Coulomb force.
- Electrostatic force obeys Newton's third law. (i.e) $\vec{F}_{21} = -\vec{F}_{12}$

2. Define electric field. Explain its various aspects.

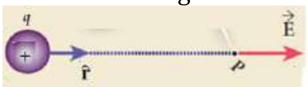
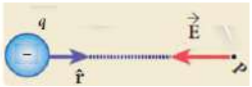
Electric field :

- The electric field at the point 'P' at a distance 'r' from the point charge 'q' is the force experienced by a unit charge and is given by

$$\vec{E} = \frac{\vec{F}}{q_o} = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}\hat{r}$$

Important aspects :

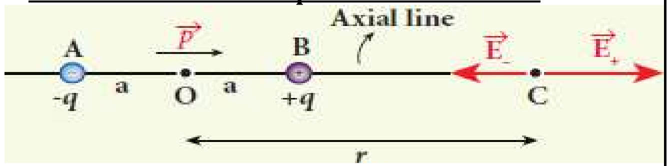
- If 'q' is positive, the electric field points away and if 'q' is negative the electric field points towards the source charge.

- The force experienced by the test charge q_o placed in electric field $\vec{E}$ is , $\vec{F} = q_o\vec{E}$
- The electric field is independent of test charge q_o and it depends only on souce charge q
- Electric field is a vector quantity. So it has unique direction and magnitude at every point.
- Since electric field is inversely proportional to the distance, as distance increases the field decreases.
- The test charge is made sufficiently small such that it will not modify the electric field of the source charge.
- For continuous and finite size charge distributions, integration techniques must bt used
- There are two kinds of electric field. They are
  (1) Uniform or constant field
  (2) Non uniform field

3. Calculate the electric field due to a dipole on its axial line.

Electric field due to dipole on its axial line :



- Consider a dipole AB along X - axis. Its diplole moment be $\boldsymbol{p = 2qa}$ and its direction be along $-q$ to $+q$.
- Let 'C' be the point at a distance 'r' from the mid point 'O' on its axial line.
- Electric field at C due to $+q$

$$\vec{E}_+ = \frac{1}{4\pi\varepsilon_o}\frac{q}{(r-a)^2}\hat{p}$$

- Electric field at C due to $-q$

$$\vec{E}_- = -\frac{1}{4\pi\varepsilon_o}\frac{q}{(r+a)^2}\hat{p}$$

- Since $+q$ is located closer to pont 'C' than $-q$, $\vec{E}_+ > \vec{E}_-$
- By superposition principle, the total electric field at 'C' due to dipole is,

$$\vec{E}_{tot} = \vec{E}_+ + \vec{E}_-$$

$$\vec{E}_{tot} = \frac{1}{4\pi\varepsilon_o}\frac{q}{(r-a)^2}\hat{p} - \frac{1}{4\pi\varepsilon_o}\frac{q}{(r+a)^2}\hat{p}$$

$$\vec{E}_{tot} = \frac{1}{4\pi\varepsilon_o}q\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right]\hat{p}$$

$$\vec{E}_{tot} = \frac{1}{4\pi\varepsilon_o}q\left[\frac{(r+a)^2-(r-a)^2}{(r-a)^2(r+a)^2}\right]\hat{p}$$

$$\vec{E}_{tot} = \frac{1}{4\pi\varepsilon_o}q\left[\frac{r^2+a^2+2ra-r^2-a^2+2ra}{\langle(r-a)(r+a)\rangle^2}\right]\hat{p}$$

$$\vec{E}_{tot} = \frac{1}{4\pi\varepsilon_o}q\left[\frac{4ra}{(r^2-a^2)^2}\right]\hat{p}$$

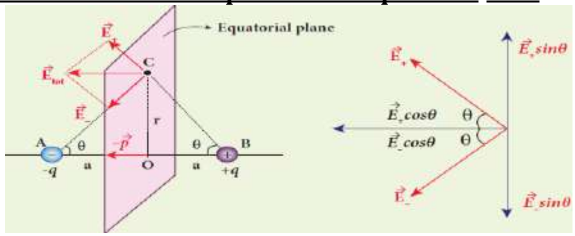
- Here the direction of total electric field is the dipole moment $\vec{p}$.
- If $r \gg a$, then neglecting a^2. We get

$$\vec{E}_{tot} = \frac{1}{4\pi\varepsilon_o}q\left[\frac{4ra}{r^4}\right]\hat{p} = \frac{1}{4\pi\varepsilon_o}q\left[\frac{4a}{r^3}\right]\hat{p}$$

$$\vec{E}_{tot} = \frac{1}{4\pi\varepsilon_o}\frac{2\vec{p}}{r^3} \qquad [\,q\,2a\,\hat{p} = \vec{p}\,]$$

4. Calculate the electric field due to a dipole on its equatorial line.

Electric field due to dipole on its equatorial line :



- Consider a dipole AB along X - axis. Its diplole moment be $\boldsymbol{p = 2qa}$ and its direction be along $-q$ to $+q$.
- Let 'C' be the point at a distance 'r' from the mid point 'O' on its equatorial plane.
- Electric field at C due to $+q$ (along BC)

$$|\vec{E}_+| = \frac{1}{4\pi\varepsilon_o}\frac{q}{(r^2+a^2)}$$

- Electric field at C due to $-q$ (along CA)

$$|\vec{E}_-| = \frac{1}{4\pi\varepsilon_o}\frac{q}{(r^2+a^2)}$$

- Here $|\vec{E}_+| = |\vec{E}_-|$
- Resolve $\vec{E}_+$ and $\vec{E}_-$ in to two components.
- Here the perpendicular components $|\vec{E}_+|\sin\theta$ and $|\vec{E}_-|\sin\theta$ are equal and opposite will cancel each other
- But the horizontal components $|\vec{E}_+|\cos\theta$ and $|\vec{E}_-|\cos\theta$ are equal and in same direction $(-\hat{p})$ will added up to give total electric field. Hence

$$\vec{E}_{tot} = |\vec{E}_+|\cos\theta\,(-\hat{p}) + |\vec{E}_-|\cos\theta\,(-\hat{p})$$

(or) $$\vec{E}_{tot} = -2\,|\vec{E}_+|\cos\theta\,\hat{p}$$

$$\vec{E}_{tot} = -2\left[\frac{1}{4\pi\varepsilon_o}\frac{q}{(r^2+a^2)}\right]\cos\theta\,\hat{p}$$

$$\vec{E}_{tot} = -\left[\frac{1}{4\pi\varepsilon_o}\frac{2q}{(r^2+a^2)}\right]\frac{a}{(r^2+a^2)^{\frac{1}{2}}}\hat{p}$$

$$\vec{E}_{tot} = -\frac{1}{4\pi\varepsilon_o}\frac{2qa}{(r^2+a^2)^{\frac{3}{2}}}\hat{p}$$

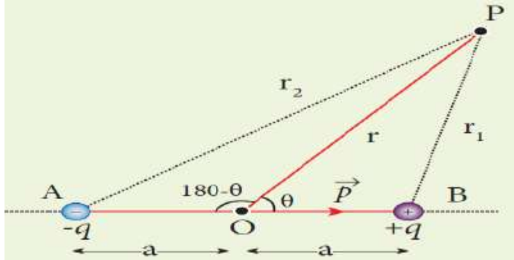
$$\vec{E}_{tot} = -\frac{1}{4\pi\varepsilon_o}\frac{p\,\hat{p}}{(r^2+a^2)^{\frac{3}{2}}} = -\frac{1}{4\pi\varepsilon_o}\frac{\vec{p}}{(r^2+a^2)^{\frac{3}{2}}}$$

- If $r \gg a$ then neglecting a^2

$$\vec{E}_{tot} = -\frac{1}{4\pi\varepsilon_o}\frac{\vec{p}}{r^3} \qquad [\,q\,2a\,\hat{p} = p\,\hat{p} = \vec{p}\,]$$

5. Derive an expression for electro static potential due to electric dipole.

Electrostatic potential due to dipole :



- Consider a dipole AB along X - axis. Its diplole moment be $\boldsymbol{p = 2qa}$ and its direction be along $\boldsymbol{-q}$ to $\boldsymbol{+q}$
- Let 'P' be the point at a distance 'r' from the mid point 'O'
- Let $\angle POA = \theta$, $BP = r_1$ and $AP = r_2$
- Electric potential at P due to $+\boldsymbol{q}$

$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_1}$$

- Electric potential at P due to $-\boldsymbol{q}$

$$V_2 = -\frac{1}{4\pi\varepsilon_0}\frac{q}{r_2}$$

- Then total potential at 'P' due to dipole is

$$V = V_1 + V_2 = \frac{1}{4\pi\varepsilon_0}\, q \left[\frac{1}{r_1} - \frac{1}{r_2}\right] \qquad ---(1)$$

- Apply cosine law in Δ BOP

$$r_1^2 = r^2 + a^2 - 2\,r\,a\cos\theta$$

$$r_1^2 = r^2\left[1 + \frac{a^2}{r^2} - \frac{2\,a}{r}\cos\theta\right]$$

- If $a \ll r$ then neglecting $\frac{a^2}{r^2}$

$$r_1{}^2 = r^2\left[1 - \frac{2\,a}{r}\cos\theta\right]$$

$$r_1 = r\left[1 - \frac{2d}{r}\cos\theta\right]^{\frac{1}{2}}$$

$$\frac{1}{r_1} = \frac{1}{r}\left[1 - \frac{2\,a}{r}\cos\theta\right]^{-\frac{1}{2}}$$

$$\frac{1}{r_1} = \frac{1}{r}\left[1 + \frac{a}{r}\cos\theta\right] \qquad ----(2)$$

- Apply cosine law in Δ AOP

$$r_2^2 = r^2 + a^2 - 2\,r\,a\cos(180° - \theta)$$

$$r_2^2 = r^2\left[1 + \frac{a^2}{r^2} + \frac{2\,a}{r}\cos\theta\right]$$

- If $a \ll r$ then neglecting $\frac{a^2}{r^2}$

$$r_2{}^2 = r^2\left[1 + \frac{2\,a}{r}\cos\theta\right]$$

$$r_2 = r\left[1 + \frac{2\,a}{r}\cos\theta\right]^{\frac{1}{2}}$$

$$\frac{1}{r_2} = \frac{1}{r}\left[1 + \frac{2\,a}{r}\cos\theta\right]^{-\frac{1}{2}}$$

$$\frac{1}{r_2} = \frac{1}{r}\left[1 - \frac{a}{r}\cos\theta\right] \qquad ----(3)$$

- Put equation (2) and (3) in (1)

$$V = \frac{1}{4\pi\varepsilon_0}q\left\{\frac{1}{r}\left[1 + \frac{a}{r}\cos\theta\right] - \frac{1}{r}\left[1 - \frac{a}{r}\cos\theta\right]\right\}$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}\left[1 + \frac{a}{r}\cos\theta - 1 + \frac{a}{r}\cos\theta\right]$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}\frac{2\,a}{r}\cos\theta = \frac{1}{4\pi\varepsilon_0}\frac{2\,q\,a}{r^2}\cos\theta$$

$$\boldsymbol{V = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^2}\cos\theta} \qquad [p = 2qa]$$

$$(or)\;\boldsymbol{V = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\cdot\hat{r}}{r^2}} \qquad [p\cos\theta = \vec{p}\cdot\hat{r}]$$

- Here $\hat{r}$ is the unit vector along OP

case -1 : If $\theta = 0°$; $\cos\theta = 1$ then,

$$\boldsymbol{V = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^2}}$$

Case -2 : If $\theta = 180°$; $\cos\theta = -1$ then,

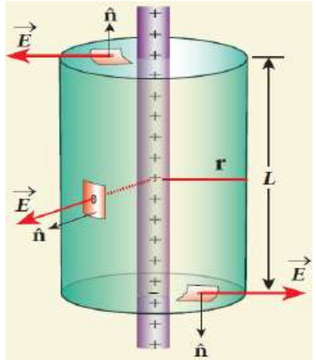
$$\boldsymbol{V = -\frac{1}{4\pi\varepsilon_0}\frac{p}{r^2}}$$

Case -3 : If $\theta = 90°$; $\cos\theta = 0$ then,

$$\boldsymbol{V = 0}$$

6. Obtain an expression for electric field due to an infinitely long charged wire.

Electric field due to infinitely long charged wire :



- Consider an infinitely long straight wire of uniform linear charge density 'λ'
- Let 'P' be a point at a distance 'r' from the wire. Let 'E' be the electric field at 'P'
- Consider a cylindrical Gaussian surface of length 'L' and radius 'r'
- The electric flux through the top surface,

$$\Phi_{top} = \int \vec{E}\cdot\vec{dA} = \int E\,dA\cos 90° = 0$$

- The electric flux through the bottom surface,

$$\Phi_{bottom} = \int \vec{E}\cdot\vec{dA} = \int E\,dA\cos 90° = 0$$

- The electric flux through the curved surface,

$$\Phi_{curve} = \int \vec{E}\cdot\vec{dA} = \int E\,dA\cos 0° = E\int dA$$

$$\Phi_{curve} = E\;2\,\pi\,r\,L$$

- Then the total electric flux through the Gaussian surface,

$$\Phi_E = \Phi_{top} + \Phi_{bottom} + \Phi_{curve}$$

$$\boldsymbol{\Phi_E = E\;(2\,\pi\,r\,L)}$$

- By Gauss law,

$$\Phi_E = \frac{Q_{in}}{\varepsilon_o}$$

$$E\;(2\,\pi\,r\,L) = \frac{\lambda\,L}{\varepsilon_o}$$

$$\boldsymbol{E = \frac{\lambda}{2\,\pi\,\varepsilon_o\,r}}$$

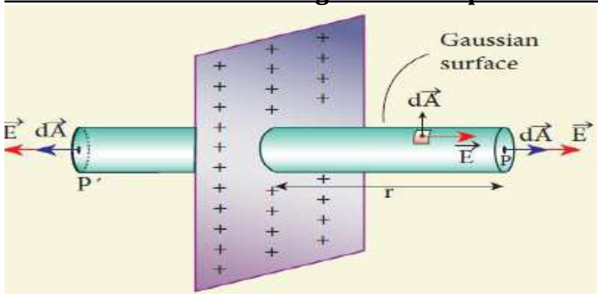
- In Vector notation,

$$\boldsymbol{\vec{E} = \frac{\lambda}{2\,\pi\,\varepsilon_o\,r}\,\hat{r}}$$

- Here $\hat{\boldsymbol{r}} \rightarrow$ unit vector perpendicular to the curved surface outwards.
- If $\lambda > 0$, then $\vec{E}$ points perpendicular outward $(\hat{r})$ from the wire and if $\lambda < 0$, then $\vec{E}$ points perpendicular inward $(-\hat{r})$

7. Obtain an expression for electric field due to an charged infinite plane sheet.

<u>Electric field due to charged infinite plane sheet</u> :



- Consider an infinite plane sheet of uniform surface charge density 'σ'
- Let 'P' be a point at a distance 'r' from the sheet. Let 'E' be the electric field at 'P'
- Here the direction of electric field is perpendicularly outward from the sheet.
- Consider a cylindrical Gaussian surface of length '2r' and area of cross section 'A'
- The electric flux through plane surface 'P'

$$\Phi_P = \int \vec{E}\,.\,\vec{dA} = \int E\,dA \cos 0° = \int E\,dA$$

- The electric flux through plane surface 'P''

$$\boldsymbol{\Phi_{P'}} = \int \vec{E}\,.\,\vec{dA} = \int E\,dA \cos\ 0° = \int E\,dA$$

- The electric flux through the curved surface,

$$\Phi_{curve} = \int \vec{E}\,.\,\vec{dA} = \int E\,dA \cos\ 90° = 0$$

- The total electric flux through through the Gaussian surface,

$$\Phi_E = \Phi_P + \boldsymbol{\Phi_{P'}} + \Phi_{curve}$$

$$\boldsymbol{\Phi_E} = \int E\,dA + \int E\,dA + 0 = 2\ E \int dA$$

$$\boldsymbol{\Phi_E = 2\,E\,A}$$

- By Gauss law,

$$\Phi_E = \frac{Q_{in}}{\varepsilon_o}$$

$$2\,\mathrm{E\,A} = \frac{\sigma\,A}{\varepsilon_o}$$

$$\mathbf{E} = \frac{\boldsymbol{\sigma}}{\boldsymbol{2\,\varepsilon_o}}$$

- In vector notation,

$$\vec{\boldsymbol{E}} = \frac{\boldsymbol{\sigma}}{\boldsymbol{2\,\varepsilon_o}}\,\boldsymbol{\hat{n}}$$

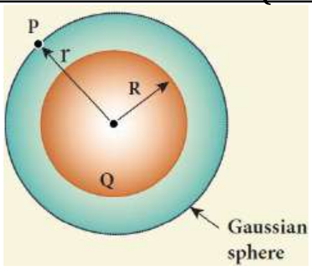
- Here $\hat{n} \rightarrow$ unit vector perpendicular to the plane sheet outwards.
- If $\sigma > 0$, then $\vec{E}$ points perpendicular outward $(\hat{n})$ from the plane sheet and if $\sigma < 0$, then $\vec{E}$ points perpendicular inward $(-\,\hat{n})$

8. Obtain an expression for electric field due to an uniformly charged spherical shell.

<u>Electric field due to charged spherical shell</u> :

- Consider an uniformly charged spherical shell of radius 'R' and charge 'Q'

1) **<u>At a point outside the shell ($r > R$)</u>** :



- Let P be the point outside the shell at a distance 'r' from its centre.
- Here electric field points radially outwards if Q >0 and radially inward if Q < 0.
- Consider a spherical Gaussian surface of radius 'r' which encloses the total charge 'Q'
- Since $\vec{E}$ and $\vec{dA}$ are along radially outwards, we have $\theta = 0°$
- The electric flux through the Gaussian surface,

$$\Phi_E = \oint \vec{E}\,.\,\vec{dA} = \oint E\,dA\,\cos\ 0°$$

$$\boldsymbol{\Phi_E = E \oint dA = E\,(4\,\pi\,r^2)}$$

- By Gauss law,

$$\Phi_E = \frac{Q_{in}}{\varepsilon_o}$$

$$E\ (4\,\pi\,r^2) = \frac{Q}{\varepsilon_o}$$

$$\boldsymbol{E} = \frac{\boldsymbol{1}}{\boldsymbol{4\,\pi\,\varepsilon_o}}\,\frac{\boldsymbol{Q}}{\boldsymbol{r^2}}$$

- In vector notation,

$$\vec{\boldsymbol{E}} = \frac{\boldsymbol{1}}{\boldsymbol{4\,\pi\,\varepsilon_o}}\,\frac{\boldsymbol{Q}}{\boldsymbol{r^2}}\,\boldsymbol{\hat{r}}$$

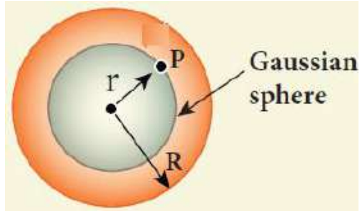
- Here $\hat{r} \rightarrow$ unit vector acting radiallyh outward from the spherical surface.

2) **<u>At a point on the surface of the shell ($r = R$):</u>**

- If the point lies on the surface of the charged shell, then $= \boldsymbol{R}$. Then the electric field,

$$\vec{\boldsymbol{E}} = \frac{\boldsymbol{1}}{\boldsymbol{4\,\pi\,\varepsilon_o}}\,\frac{\boldsymbol{Q}}{\boldsymbol{R^2}}\,\boldsymbol{\hat{r}}$$

3) **<u>At a point inside the shell ($r < R$)</u>** :



- Let 'P' be the point inside the charged shell at a distance 'r' from its centre.
- Consider the spherical Gaussian surface of radius 'r'
- Since there is no charge inside the Gaussian surface, Q = 0
- Then from Gauss law,

$$\Phi_E = \oint \vec{E}\,.\,\vec{dA} = \frac{Q_{in}}{\varepsilon_o}$$

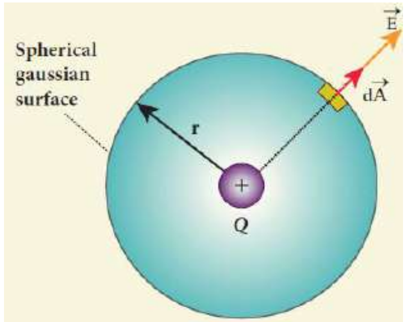
$$E\ (4\,\pi\,r^2) = 0$$

$$\boldsymbol{E = 0}$$

- Thus the electric field due to the uniform charged spherical shell is zero at all points inside the shell.

9. Obtain Gauss law from Coulomb's law.

<u>Gauss law from Coulomb's law</u> :



- Consider a charged particle of charge '+q'
- Draw a Gaussian spherical surface of radius 'r' around this charge.
- Due to symmentry, the electric field $\vec{E}$ at all the points on the spherical surface have same magnitude and radially outward in direction.

- If a test charge 'q_o' is placed on the Gaussian surface, by Coulomb law the force acting it is,

$$|\vec{F}| = \frac{1}{4\pi\varepsilon_o}\frac{Q\,q_o}{r^2}$$

- By definition, the electric field,

$$|\vec{E}| = \frac{|\vec{F}|}{q_o} = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r^2} \qquad ----(1)$$

- Since the area element $\overrightarrow{dA}$ is along the electric field $\vec{E}$, we have $\theta = 0°$. Hence the electric flux through the Gaussian surface is,

$$\Phi_E = \oint \vec{E}\,.\overrightarrow{dA} = \oint E\,dA\cos 0° = E\oint dA$$

- Here $\oint dA = 4\pi r^2 \rightarrow$ area of Gaussian sphere
- Put in equation (1)

$$\Phi_E = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r^2}\;X\;4\pi r^2$$

$$\therefore \qquad \boldsymbol{\Phi_E = \frac{Q}{\varepsilon_o}}$$

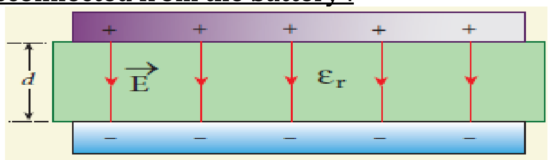
- This is known as Gauss law.

Result :

- The total electric flux through the closed surface depends only on the charges enclosed by the surface and independent of charges outside the surface.
- The total electric flux is independent of the location of charges inside the closed surface and shape on the closed surface.
- Gauss law is another form of Coulomb law and also applicable to charges in motion.

10. Explain in detail the effect of dielectric placed in a parallel plate capacitor when the capacitor is disconnected from the battery.

Effect of dielectrics when the capacitor is disconnected from the battery :



- Consider a parallel plate capacitor.
- Area of each plates = A
  Distance between the plates = d
  Voltage of battery = V_o
  Total charge on the capacitor = Q_o
- So the capacitance of capacitor without dielectric,

$$C_o = \frac{Q_o}{V_o}$$

- The battery is then disconnected from the capacitor and the dielectric is inserted between the plates. This decreases the electric field.
- Electric field without dielectric = E_o
  Electric field with dielectric = E
  Relative permittivity or dielectric constant = ε_r

$$\therefore \qquad E = \frac{E_o}{\varepsilon_r}$$

- Since $\varepsilon_r > 1$, we have $E < E_o$
- Hence electrostatic potential between the plates is reduced and at the same time the charge Q_o remains constant.

$$V = E\,d = \frac{E_o}{\varepsilon_r}\,d = \frac{V_o}{\varepsilon_r}$$

- Then the capacitance of a capacitor with dielectric,

$$C = \frac{Q_o}{V} = \frac{Q_o}{\left[\frac{V_o}{\varepsilon_r}\right]} = \varepsilon_r\frac{Q_o}{V_o} = \varepsilon_r\,C_o$$

- Since $\varepsilon_r > 1$, we have $C > C_o$.
- Thus insertion of dielectric slab increases the capacitance.
- We have, $\boldsymbol{C_o} = \frac{\varepsilon_0 A}{d}$

$$\therefore \quad \boldsymbol{C = \frac{\varepsilon_r\,\varepsilon_0\,A}{d} = \frac{\varepsilon\,A}{d}}$$

Where, $\boldsymbol{\varepsilon_r\,\varepsilon_0 = \varepsilon} \rightarrow$ permitivity of the dielectric medium

- The energy stored in the capacitor without dielectric,

$$U_o = \frac{1}{2}\frac{{Q_o}^2}{C_o}$$

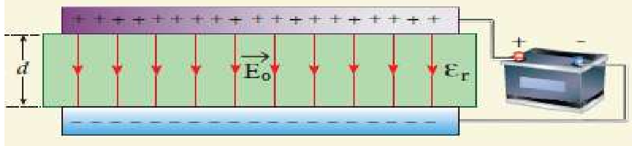
- After the dielectric is inserted,

$$\boldsymbol{U = \frac{1}{2}\frac{Q_o^2}{C} = \frac{1}{2}\frac{Q_o^2}{\varepsilon_r\,C_o} = \frac{U_o}{\varepsilon_r}}$$

- Since $\varepsilon_r > 1$, we have $U < U_o$
- There is a decrease in energy because, when the dielectric is inserted, the capacitor spend some energy to pulling the dielectric slab inside.

11. Explain in detail the effect of dielectric placed in a parallel plate capacitor when the battery remains connected to the capacitor.

Effect of dielectrics when the battery remains connected to the capacitor:



- Consider a parallel plate capacitor.
- Area of each plates = A
  Distance between the plates = d
  Voltage of battery = V_o
  Total charge on the capacitor = Q_o
- So the capacitance of capacitor without dielectric,

$$C_o = \frac{Q_o}{V_o}$$

- Dielectric is inserted between the plates and the battery is remains in connected with the capacitor.
- So the charges stored in the capacitor is increased.
- Total charge without dielectric = Q_o
  Total charge with dielectric = Q
  Relative permittivity (dielectric constat) = ε_r

$$\therefore \qquad \boldsymbol{Q = \varepsilon_r\,Q_o}$$

- Since $\varepsilon_r > 1$, we have $Q < Q_o$
- Here the potential difference between the plates remains constant. But the charges increases and the new capacitance will be

$$C = \frac{Q}{V_o} = \frac{\varepsilon_r\,Q_o}{V_o} = \varepsilon_r\,C_o$$

- Since $\varepsilon_r > 1$, we have $C > C_o$
- Hence capacitance increases with the insertion of dielectric slab.
- We know that, $\boldsymbol{C_o} = \frac{\varepsilon_0 A}{d}$

$$\therefore \quad \boldsymbol{C = \frac{\varepsilon_r\,\varepsilon_0\,A}{d} = \frac{\varepsilon\,A}{d}}$$

Where, $\boldsymbol{\varepsilon_r\,\varepsilon_0 = \varepsilon} \rightarrow$ permitivity of the dielectric medium

- The energy stored in the capacitor without dielectric,

$$U_o = \frac{1}{2}\,C_o\,{V_o}^2$$

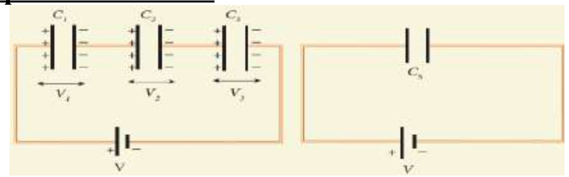
- After the dielectric is inserted,

$$\boldsymbol{U = \frac{1}{2}\,C\,V_o^2 = \frac{1}{2}\,\varepsilon_r\,C_oV_o^2 = \varepsilon_r\,U_o}$$

- Since $\varepsilon_r > 1$, we have $U > U_o$
- So there is increase in energy when the dielectric is inserted

12. Derive the expression for resultant capacitance, when capacitors are connected in series and in parallel.

Capacitors in series :



- Consider three capacitors of capacitance C_1, C_2 and C_3 connected in series with a battery of voltage V . In series connection,
  1) Each capacitor has same amount of charge (Q)
  2) But potential difference across each capacitor will be different.
- Let V_1, V_2, V_3 be the potential difference across C_1, C_2, C_3 respectively, then

$$V = V_1 + V_2 + V_3$$
$$V = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} \qquad [\because\ Q = C\,V]$$
$$V = Q\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right] \qquad ------(1)$$

- Let C_S be the equivalent capacitance of capacitor in series connection, then

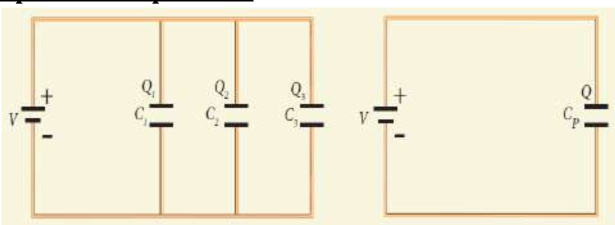
$$V = \frac{Q}{C_S} \qquad ------(2)$$

- From (1) and (2) , we have

$$\frac{Q}{C_S} = Q\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right]$$
$$\boldsymbol{\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

- Thus the inverse of the equivalent capacitance of capacitors connected in series is equal to the sum of the inverses of each capacitance.
- This equivalent capacitance $\boldsymbol{C_S}$ is always less than the smallest individual capacitance in the series

Capacitors in parallel :



- Consider three capacitors of capacitance C_1, C_2 and C_3 connected in parallel with a battery of voltage V .In parallel connection,
  1) Each capacitor has same potential difference (V)
  2) But charges on each capacitor will be different
- Let Q_1, Q_2, Q_3 be the charge on C_1, C_2, C_3 respectively, then

$$Q = Q_1 + Q_2 + Q_3$$
$$Q = C_1 V + C_2 V + C_3 V \qquad [\because\ Q = C\,V]$$
$$Q = V\,[C_1 + C_2 + C_3] \qquad -----(1)$$

- Let C_P be the equivalent capacitance of capacitor in parallel connection, then

$$Q = C_P\,V \qquad ------(2)$$

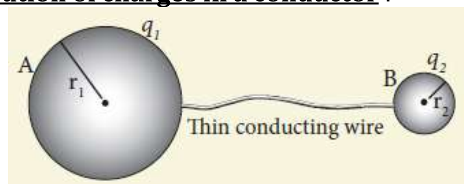
- From (1) and (2),

$$C_P\,V = V\,[C_1 + C_2 + C_3]$$
$$\boldsymbol{C_P = C_1 + C_2 + C_3}$$

- Thus the equivalent capacitance of capacitors connected in parallel is equal to the sum of the individual capacitances.
- The equivalent capacitance $\boldsymbol{C_P}$ in a parallel connection is always greater than the largest individual capacitance.

13. Explain in detail how charges are distributed in a conductor and the principle behind the lightning conductor.

Distribution of charges in a conductor :



- Consider two conducting spheres 'A' and 'B' of radii $\boldsymbol{r_1}$ and $\boldsymbol{r_2}$. Let $\boldsymbol{r_1 > r_2}$
- Let the two spheres are connected by a thin conducting wire.
- If a charge 'Q' is given to either A or B, this charge is redistributed in both the spheres until their potential becomes same.
- Now they are uniformly charged and attain electrostatic equilibrium.
- At this stage, let the surface charge densities of A and B are σ_1 and σ_2 respectively, then
  Charge residing on suface of A $= q_1 = \sigma_1 4\,\pi\,r_1^2$
  Charge residing on suface of B $= q_2 = \sigma_2 4\,\pi\,r_2^2$
- Then the total charge ; $Q = q_1 + q_2$
- There is no net charge inside the conductors.
- Electrostic potential on the surface of A and B is

$$V_A = \frac{1}{4\,\pi\,\varepsilon_0}\frac{q_1}{r_1} \qquad \& \qquad V_B = \frac{1}{4\,\pi\,\varepsilon_0}\frac{q_2}{r_2}$$

- Under elecrostic equilibrium. $V_A = V_B$

$$\therefore \quad \frac{1}{4\,\pi\,\varepsilon_0}\frac{q_1}{r_1} = \frac{1}{4\,\pi\,\varepsilon_0}\frac{q_2}{r_2}$$
$$\frac{q_1}{r_1} = \frac{q_2}{r_2}$$
$$\frac{\sigma_1 4\,\pi\,r_1^2}{r_1} = \frac{\sigma_2 4\,\pi\,r_2^2}{r_2}$$
$$\sigma_1\,r_1 = \sigma_2\,r_2$$
$$(or) \qquad \boldsymbol{\sigma\,r = constant}$$

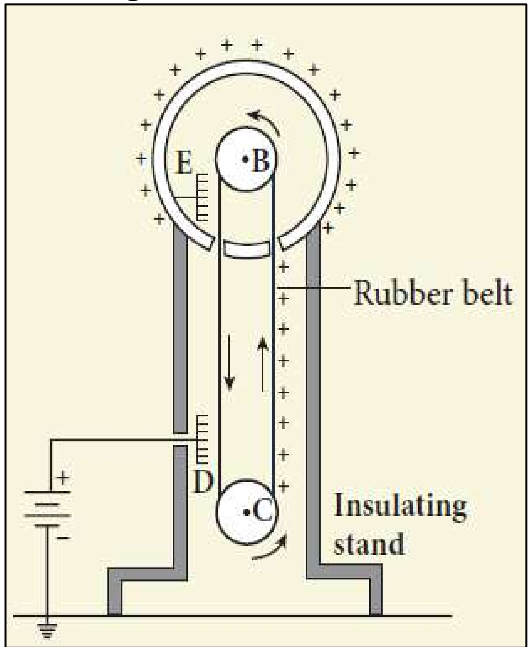
- Thus the surface charge density is inversely proportional to the radius of the sphere.
- Hence for smaller radius , the charge density will be larger and vice versa

Principle of lightning conductor (Action of point) :

- Action of point is the principle behind the lightning conductor.
- We know that smaller the radius of curvature, the larger is the charge density.
- If the conductor has sharp end which has larger curvature (smaller radius), it has a large charge accumulation.
- As a result, the electric field near this edge is very high and it ionizes the surrounding air.
- The positive ions are repelled at the sharp edge and negative ions are attracted towards the sharper edge.
- This reduces the total charge of the conductor near the sharp edge. This is called action of points or corona discharge.

14. Explain in detail the construction and working of Van de Graff generator.

Van de Gralff generator :



- It is designed by Robert Van de Graff.
- It produce large electro static potential difference of about $10^7\ V$

Principle :

- Electro static induction
- Action of points

Construction :

- It consists of large hollow spherical conductor 'A' fixed on the insulating stand.
- Pulley 'B' is mounted at the centre of the sphere and another pulley 'C' is fixed at the bottom.
- A belt made up of insulating material like silk or rubber runs over the pulleys.
- The pulley 'C' is driven continuously by the electric motor.
- Two comb shaped metallic conductor D and E are fixed near the pulleys.
- The comb 'D' is maintained at a positive potential of $10^4\ V$ by a power supply.
- The upper comb 'E' is connected to the inner side of the hollow metal sphere.

Working :

- Due to the high electgric field near comb 'D', air between the belt and comb 'D' gets ionized.
- The positive charges are pushed towards the belt and negative charges are attracted towards the comb 'D'
- The positive charges stick to the belt and move up.
- When the positive charges reach the comb 'E' a large amount of negative and positive charges are induced on either side of comb 'E' due to electrostatic induction.
- As a result. the positive charges are pushed away from the comb 'E' and they reach the outer surface of the sphere.
- These positive charges are distributed uniformly on the outer surface of the hollow sphere.
- At the same time, the negative charges neutralize the positive charges in the belt due to corona discharge before it passes over the pulley.
- When the belt descends, it has almost no net charge.
- This process continues until the outer surface produces the potential difference of the order of $10^7\ V$ which is the limiting value.
- Beyond this, the charges starts leaking to the surroundings due to ionization of air.
- It is prevented by enclosing the machine in a gas filled steel chamber at very high pressure.

Applications :

- The high voltage produced in this Van de Graff generator is used to accelerate positive ions (protons and deuterons) for nuclear disintegrations and other applications.