RAY OPTICs

PHYSICS - VOL 2 | **UNIT - 6**

NAME :

STANDARD : 12 SECTION :

SCHOOL :

EXAM NO :

தொட்டனைத் தூறும் மணற்கேணி மாந்தர்க்குக்
கற்றனைத் தூறும் அறிவு

மணலில் உள்ள கேணியில் தோண்டிய அளவிற்கு நீர் ஊறும். அதுபோல
நாம் கற்ற கல்வியின் அளவிற்கு அறிவு வளரும்

webStrake

victory R. SARAVANAN. M.Sc, M.Phil, B.Ed.,
PG ASST (PHYSICS)
GBHSS, PARANGIPETTAI - 608 502
webStrake Recognized Teacher

2 & 3 mark questions and answers

1. Define reflection.

- The bouncing back of light in to the same medium when it encounters a reflecting surface is called reflection of light.

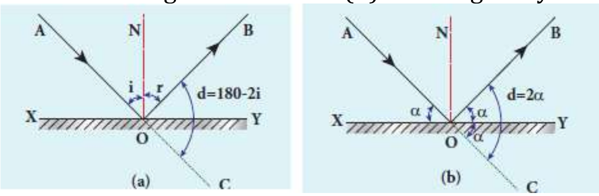
2. State the laws of reflection.

(1) The incident ray, reflected ray and the normal to the surface all are coplanar.

(2) The angle of incidence (i) is equal to angle of reflection (r). That is $\boldsymbol{i = r}$

3. What is the angle of deviation due to reflection?

- The angle between the incident and deviated ray is called angle of deviation (d) of the light ray.



- From figure (a),

$$d = 180° - (i + r) \qquad [i = r]$$
$$\boldsymbol{d = 180° - 2\,i}$$

- The angle between the incident ray and the reflecting surface is called *glancing angle* (α).
- From figure (b),

$$d = \angle BOY + \angle YOC = \alpha + \alpha = \boldsymbol{2\,\alpha}$$

4. What are the characteristics of the image formed by the plane mirror?

<u>Characteristics of the image of the plane mirror</u> :

- Virtual, erect and laterally inverted.
- Size of image is equal to the size of the object.
- The distance of the image behind the mirror is equal to the distance of object in front of it.
- If an object placed between two plane mirrors inclined at an angle θ, then the number (n) of images formed is,

(1) If $\left[\frac{360°}{\theta}\right]$ even, then ; $n = \left[\frac{360°}{\theta} - 1\right]$ for objects placed symmetrically or unsymmetrically.

(2) If $\left[\frac{360°}{\theta}\right]$ odd, then ; $n = \left[\frac{360°}{\theta} - 1\right]$ for objects placed symmetrically

(3) If $\left[\frac{360°}{\theta}\right]$ odd, then ; $n = \left[\frac{360°}{\theta}\right]$ for objects placed unsymmetrically

5. Distinguish convex mirror and concave mirror?

Convex mirror	Concave mirror
It is a spherical mirror in which reflection takes place at the convex surface and other surface is silvered	It is a spherical mirror in which reflection takes place at the concave surface and other surface is silvered

6. Define (1) centre of curvature, (2) Radius of curvature (3) pole, (4) principal axis, (5) focus or focal point, (6) focal length, (7) focal plane

(1) **<u>Centre of curvature</u>** :

- The centre of the sphere of which the mirror is a part is called centre of curvature (C)

(2) **<u>Radius of curvature</u>** :

- The radius of the sphere of which the spherical mirror is a part is called the radius of curvature (R) of the mirror.

(3) **<u>Pole (or) Optic centre</u>** :

- The middle point on the spherical surface of the mirror (or) the geometrical centre of the mirror is called the pole (P) of the mirror.

(4) **<u>Principal axis</u>** :

- The line joining the pole (P) and the centre of curvature (C) is called the principal axis (or) optical axis of the mirror.

(5) **<u>Focus or Focal point</u>** :

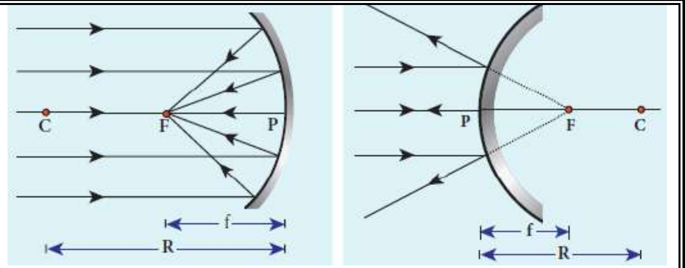
- Light rays travelling parallel and close to the principal axis when incident on a spherical mirror, converge at a point for concave mirror or appears to diverge from a point for convex mirror on the principal axis. This point is called the focus or focal point (F) of the mirror

(6) **<u>Focal length</u>** :

- The distance between the pole (P) and the Focus (F) is called the focal length (f) of the mirror.

(7) **<u>Focal plane</u>** :

- The plane through the focus and perpendicular to the principal axis is called the focal plane of the mirror.



7. Define paraxial rays and marginal rays.

<u>Paraxial rays</u> :

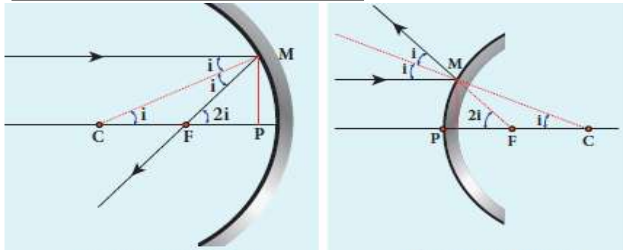
- The rays travelling very close to the principal axis and make small angle with it are called paraxial rays.

<u>Marginal rays</u> :

- The rays travelling far away from the principal axis and fall on the mirror far away from the pole are called as marginal rays.

8. Obtain the relation between focal length (f) and radius of curvature (R) of the spherical mirror.

<u>Relation between f and R</u> :



(a) Concave mirror (b) Convex Mirror

- Let 'C' be the centre of curvature of the mirror.
- Consider a light ray parallel to the principal axis and incident at 'M' on the mirror.
- After reflection, it will passes through principal focus 'F'
- The line 'CM' is the normal to the mirror at 'M'
- From the figure (a),
  angle of incidence ; $i = \angle AMC$
  angle of reflection ; $r = \angle CMF$
- By the law of reflection. we have, $\boldsymbol{i = r}$
- Thus, , $\angle MCP = i$ & $\angle MFP = 2\,i$
- From ΔMCP and ΔMFP

$$\tan i = \frac{PM}{PC}$$
$$\tan 2\,i = \frac{PM}{PF}$$

- As the angles are small, we have $\tan i \approx i$ and $\tan 2i \approx 2i$. So

$$i = \frac{PM}{PC} \quad ------ \quad (1)$$
$$2i = \frac{PM}{PF} \quad ------ \quad (2)$$

- Put eqn (1) in eqn (2)

$$2\frac{PM}{PC} = \frac{PM}{PF}$$
$$(or) \quad 2\ PF = PC$$
$$(or) \quad 2\ f = R$$
$$(or) \quad \boldsymbol{f = \frac{R}{2}} \quad ------ \quad (3)$$

9. Define refractive index.

- Refractive index (n) of a transparent medium is defined as the ratio of speed of light in vacuum (or air) to the speed of light on that medium.

$$\boldsymbol{n = \frac{c}{v}}$$

10. Define optical path.

- Optical path of a medium is defined as the distance (d′) light travels in vacuum in the same time it travels a distance (d) in the medium.
- If 'n' is the refractive index of the medium. then optical path is ; **d′ = n d**

11. What is called refraction?

- Refraction is passing through of light from one optical medium to another optical medium through a boundary.

12. State the laws of refraction (Snell's law).

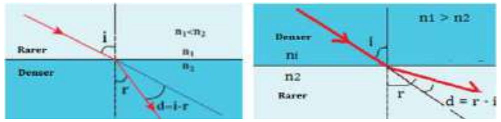
- The incident ray, refracted ray and normal are all coplanar.
- The ratio of angle of incident 'i' in the first medium to the angle of reflection 'r' in the second medium is equal to the ratio of refractive index of the second medium 'n_2' to that of the refractive index of the first medium 'n_1'

$$\frac{\sin i}{\sin r} = \frac{n_2}{n_1}$$
$$(or) \quad \boldsymbol{n_1 \sin i = n_2 \sin r}$$

13. What is the angle of deviation due to refraction?

Angle of deviation due to refraction :

- The angle between the incident and deviated ray is called angle of deviation.
- When light travels from rarer to denser medium it deviates towards normal. Hence the angle of deviation ; $\boldsymbol{d = i - r}$
- When light travels from denser to rarer medium it deviates away normal. Hence the angle of deviation ; $\boldsymbol{d = r - i}$



14. What is the principle of reversiability?

- The principle of reversibility states that, light will be follow exactly the same path if its direction of travel is reversed.
- This is true for both reflection and refraction.

15. Define relative refractive index.

- From Snell's law, $\frac{\sin i}{\sin r} = \frac{n_2}{n_1}$
- Here the term $\left[\frac{n_2}{n_1}\right]$ is called relative refractive index of second medium with respect to the first medium and it is denoted by n_{21} (i,e.) $\boldsymbol{n_{21} = \frac{n_2}{n_1}}$

16. Give the useful relations obtained from the concept of relative refractive index.

(1) <u>Inverse rule</u> :

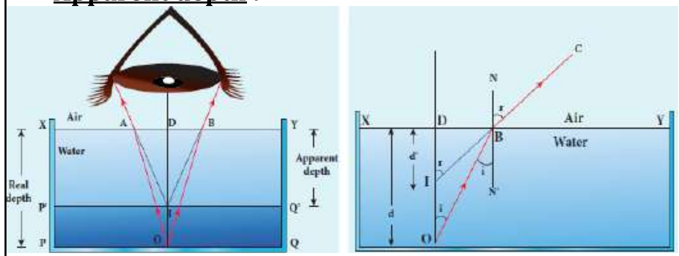
$$\boldsymbol{n_{12} = \frac{1}{n_{21}} \quad (or) \quad \frac{n_1}{n_2} = \frac{1}{\left[\frac{n_2}{n_1}\right]}}$$

(2) <u>Chain rule</u> :

$$\boldsymbol{n_{32} = n_{31}\ X\ n_{12} \quad (or) \quad \frac{n_3}{n_2} = \frac{n_3}{n_1}\ X\ \frac{n_1}{n_2}}$$

17. Obtain the equation for apparent depth.

<u>Apparent depth</u> :



- We observe that the bottom of a tank filled with water with water appears raised as shown.
- Light OB from the object 'O' passes through water get refracted in air
- The refracted ray BC appers to come from 'I' which is just above 'O' (i.e) the object is appears to be at 'I'
- Refractive index of water $= \boldsymbol{n_1}$
  Refractive index of air $= \boldsymbol{n_2}$
  Angle of incidence in water $= \boldsymbol{i}$
  Angle of refraction in air $= \boldsymbol{r}$
  Original depth of tank $= \boldsymbol{DO = d}$
  Apparent depth of tank $= \boldsymbol{DI = d'}$
- Here $\boldsymbol{n_1 > n_2}$. Hence , $\boldsymbol{i < r}$
- By Snell's law in product form,

$$n_1 \sin i = n_2 \sin r$$

- As the angles aresmall, we can write

$$\sin i \approx tan\ i \quad \& \quad \sin r \approx tanr$$

Hence, $\quad n_1 \tan i = n_2 \tan r \quad ---- \quad (1)$

- In $\Delta DOB\ and\ \Delta DIB$,

$$\tan i = \frac{DB}{DO} = \frac{DB}{d}$$
$$\tan r = \frac{DB}{DI} = \frac{DB}{d'}$$

- Put this in eqn (1)

$$n_1\left[\frac{DB}{d}\right] = n_2\left[\frac{DB}{d'}\right]$$
$$n_1\frac{1}{d} = n_2\frac{1}{d'}$$
$$\therefore \quad \boldsymbol{d' = \frac{n_2}{n_1}\ d}$$

- For air ; $\boldsymbol{n_2 = 1}$ and let $\boldsymbol{n_2 = n}$, then apparent depth

$$\boldsymbol{d' = \frac{d}{n}}$$

- Thus the bottom appears to be elevated by $(d - d')$

$$\boldsymbol{d - d' = d - \frac{d}{n} = d\left(1 - \frac{1}{n}\right)}$$

18. Define critical angle.

- The angle of incidence in the denser medium for which the refracted ray graces the boundary is called critical angle $\boldsymbol{i_C}$

19. Define total internal reflection.

- If the angle of incidence in the denser medium is greater than the critical angle, there is no refraction possible in the rarer medium.
- The entire light is reflected back in to the denser medium itself, This phenomenon is called total internal reflection.

20. What are the conditions to achieve total internal reflection?

- Light must travel from denser to rarer medium
- Angle of incidence must be greater than critical angle ($i > i_C$)

21. Obtain an expression for critical angle.

Critical angle:

- When light ray passes from denser medium to rarer medium, it bends away from normal. So $i < r$
- As i increases, r also increases rapidly and at a certain stage it just gracing the boundary ($r = 90°$). The corresponding anle of incidence is called critical angle (i_C)
- From Snell's law of product form

$$\boldsymbol{n_1 \sin i = n_2 \sin r}$$

- When $i = i_C$, then $r = 90°$

$$\boldsymbol{n_1 \sin i_C = n_2 \sin 90°}$$

$$\boldsymbol{n_1 \sin i_C = n_2}$$

$$\boldsymbol{\sin i_C = \frac{n_2}{n_1}}$$

- If the rarer medium is air, then $\boldsymbol{n_2 = 1}$ and let $\boldsymbol{n_1 = n}$, then

$$\boldsymbol{\sin i_C = \frac{1}{n}}$$

$$\text{(or)} \quad \boldsymbol{i_C = \sin^{-1}\left(\frac{1}{n}\right)}$$

22. Define silvered lenses.

- If one of the surfaces of a lens is silvered from outside, then such a lens is said to be a silvered lens. It is a combination of a lens and a mirror.
- A silvered lens is basically a modified mirror and its power is given by

$$\boldsymbol{P = 2\, P_{lens} + P_{mirror}}$$

$$(\boldsymbol{or}) \quad \left[\frac{1}{-f}\right] = \left[\frac{2}{f_{lens}}\right] + \left[\frac{1}{-f_{mirror}}\right]$$

23. Write a note on prism.

- A prism is a triangular block of glass or plastic which is bounded by the three plane faces not parallel to each other.
- Its one face is grounded which is called base.
- The other two faces are polished which are called refracing faces of the prism.
- The angle between the two refracting faces is called angle of prism (A)

24. Define angle of minimum deviation.

- The angle between incident ray and emergent ray is called angle of deviation (d).
- When the angle of incidence increases, the angle of deviation decreases, reaches a minimum value and then continues to increase.
- The minimum value of angle of deviation is called angle of minimum deviation (D).

25. What is called dispersion of light?

- The splitting of white light in to its constituent colours is called dispersion of light.
- This band of colours of light is called its spectrum.
- The spectrum consists seven colours in the order VIBGYOR

26. Define dispersive power.

- Dispersive power (ω) is the ability of the material of the prism to cause prism.
- It is defined as the ratio of the angular dispersion for the extreme colours to the deviation for any mean colour.

27. What is Rayleigh's scattering?

- The scattering of light by atoms and molecules which have size ($\boldsymbol{a}$) very less than that of the wavelength (λ) of light is called Rayleigh's scattering.
  (i.e) condition for Rayleigh's scattering is $\boldsymbol{a << \lambda}$

28. State Rayleigh's scattering law.

- The intensity (I) of Rayleigh's scattering is inversely proportional to fourth power of wavelength (λ) $(i.e.)$ $\boldsymbol{I \propto \frac{1}{\lambda^4}}$

29. Why does sky appears blue colour?

- According to Rayleigh's scattering, shorter wavelenths (violet) scattered much more than longer wavelengths (Red)
- As our eyes are more sensitive to blue colour than violet, the sky appears blue during day time.

30. Why does sky and Sun looks reddish during sunset and sunrise?

- During sunset or sunrise, the light from Sun travels a greater distance through atmosphere.
- Hence the blue light which has shorter wavelength is scattered away and less scattered red light of longer wavelength reaches observer
- This is the reason for reddish appearance of sky and Sun during sunrise and sunset.

31. Why does cloud appears as white colour?

- When size of particles or water drops are greater than the wavelength of light ($a \gg \lambda$), the intensity of scattering is equal for all the wavelength.
- Since clouds contains large amount of dust and water droplets, all the colours get equally scattered irrespective of wavelength. This is the reason for the whitish appearance of cloud.
- But the rain clouds appear dark because of the condensation of water droplets on dust paricles that make the cloud become opque.

32. How are rainbows formed?

Formation of rainbows :

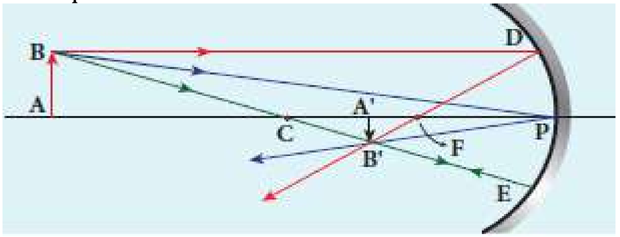
- Rainbows are formed due to dispersion of sunlight through droplets of water during rainy days.
- Rainbow is observed during rainfall or after rainfall or looking water fountain provided the Sun is at the back of the observer.
- When sun light falls on the water drop suspended air, it splits in to its constituent seven colours. Here waterdrops acts as a glass prism.
- Primary rainbow is formed when ***one total internal reflection*** takes place inside the drop. The angle of view for violet to red in primary rainbow is $\boldsymbol{40°}$ to $\boldsymbol{42°}$
- Secondary rainbow is formed when ***two total internal reflection*** takes place inside the drop. The angle of view for violet to red in primary rainbow is $\boldsymbol{52°}$ to $\boldsymbol{54°}$

5 - Mark Question & Answer

1. Derive the mirror equation and the equation for lateral magnification.

Mirror equation :

- The equation which gives the relation between object distance (u), image distance (v) and focal length (f) is of spherical mirror is called mirror equation.



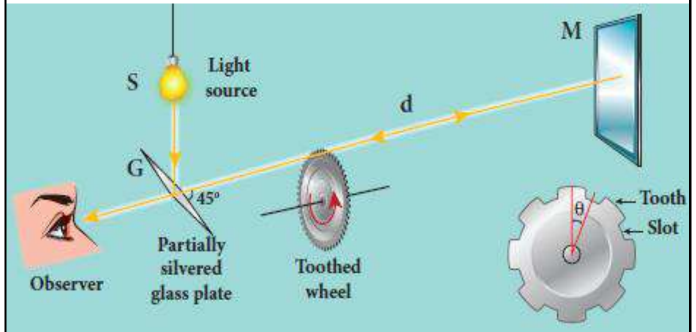
- Let an object AB is placed on the principle axis of a concave mirror beyond the centre of curvature 'C'
- The real and inverted image A^1B^1 is formed between C and F
- By the laws of reflection,
 angle of incidence (i) = angle of reflection (r)
$$\angle BPA = \angle B^1PA^1$$
- From figure, $\Delta\, BPA$ and $\Delta\, B^1PA^1$ are similar triangles. So
$$\frac{A^1B^1}{AB} = \frac{PA^1}{PA} \qquad ------ \quad (1)$$
- Also $\Delta\, DPF$ and $\Delta\, B^1A^1F$ are similar triangles. So
$$\frac{A^1B^1}{PD} = \frac{A^1F}{PF} \qquad [PD = AB]$$
$$\frac{A^1B^1}{AB} = \frac{A^1F}{PF} \qquad ------ \quad (2)$$
- From eqn (1) and (2),
$$\frac{PA^1}{PA} = \frac{A^1F}{PF}$$
$$\frac{PA^1}{PA} = \frac{PA^1 - PF}{PF} \qquad ---- \quad (3)$$
- By applying sign conventions,
$$PA = -u \quad ; \quad PA^1 = -v \quad ; \quad PF = -f$$
$$\frac{-v}{-u} = \frac{-v-(-f)}{-f}$$
$$(or) \qquad \frac{v}{u} = \frac{v-f}{f}$$
$$(or) \qquad \frac{v}{u} = \frac{v}{f} - 1$$
- Dividing both sides by v
$$\frac{1}{u} = \frac{1}{f} - \frac{1}{v}$$
$$\boldsymbol{\frac{1}{v} + \frac{1}{u} = \frac{1}{f}} \qquad ------ \quad \mathbf{(4)}$$
- This is called mirror equation. It is also valid for convex mirror.

Lateral magnification:

- It is defined as the ratio of the height of the image (h^1) to the height of the object (h)..
- From eqn (1)
$$\frac{A^1B^1}{AB} = \frac{PA^1}{PA}$$
$$\frac{-h^1}{h} = \frac{-v}{-u}$$
- Hence magnification,
$$\boldsymbol{m = \frac{h^1}{h} = -\frac{v}{u}} \qquad ---- \quad \mathbf{(5)}$$
- Using eqn (4)
$$\boldsymbol{m = \frac{h^1}{h} = \frac{f-v}{f} = \frac{f}{f-u}} \qquad -- \mathbf{(6)}$$

2. Describe the Fizeau's method to determine speed of light.

Fizeau's method :



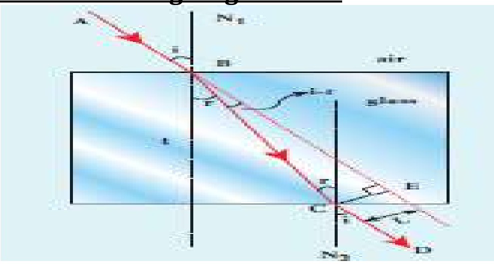
- The light from the source S was first allowed to fall on a partially silvered glass plate G kept at an angle of 45° to the vertical.
- The light then allowed to pass through a rotating toothed-wheel with N -teeth and N -cuts.
- The speed of rotation of the wheel could be varied through an external mechanism.
- The light passing through one cut in the wheel get reflected by a mirror M kept at a long distance 'd' (about 8 km) from the toothed wheel.
- If the toothed wheel was not rotating, the reflected light from the mirror would again pass through the same cut and reach the observer through G.

Working :

- The angular speed of the rotation of the toothed wheel was increased until light passing through one cut would completely be blocked by the adjacent tooth. Let that angular speed be ω
- The total distance traveled by the light from the toothed wheel to the mirror and back to the wheel is '2d' and the time taken be 't'.
- Then the speed of light in air,
$$v = \frac{2\,d}{t}$$
- But the angular speed is,
$$\omega = \frac{\theta}{t}$$
- Here θ is the angle between the tooth and the slot which is rotated by the toothed wheel within that time 't' . Then,
$$\theta = \frac{total\ angle\ of\ the\ circle\ in\ radian}{number\ of\ teeth\ + number\ of\ cuts}$$
$$\theta = \frac{2\,\pi}{2\,N} = \frac{\pi}{N}$$
- Hence angular speed,
$$\omega = \frac{\left(\frac{\pi}{N}\right)}{t} = \frac{\pi}{N\,t}$$
$$(or) \quad t = \frac{\pi}{N\,\omega}$$
- Therefore the speed of light in air,
$$v = \frac{2\,d}{t} = \frac{2\,d}{\left(\frac{\pi}{N\,\omega}\right)}$$
$$\boldsymbol{v = \frac{2\,d\,N\,\omega}{\pi}}$$
- The speed of light in air was determined as,
$$\boldsymbol{v = 2.99792\ X\ 10^{8}\ m\,s^{-1}}$$

3. Derive the equation for lateal displacement of light passing through a glass slab.

Refraction through a glass slab :



- Thickness of glass slab = t
  Refractive index of glass = n
- The perpendicular distance 'CE'between refracted ray and incident ray at C gives the lateral displacement (L).
- *In* ΔBCE , $\sin(i-r) = \frac{L}{BC}$

$$BC = \frac{L}{\sin(i-r)}$$

- *In* ΔBCF, $\cos r = \frac{t}{BC}$

$$BC = \frac{t}{\cos r}$$

- Hence,

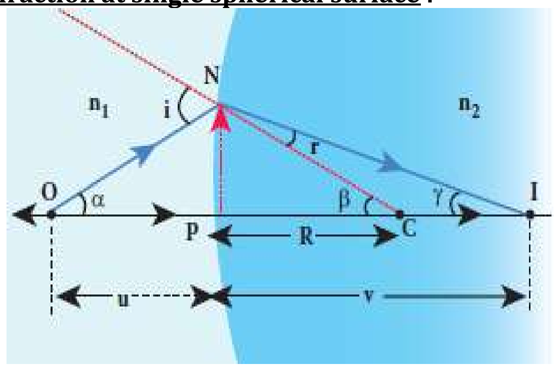
$$\frac{L}{\sin(i-r)} = \frac{t}{\cos r}$$

$$\boldsymbol{L = t\left[\frac{\sin(i-r)}{\cos r}\right]}$$

- Therfore lateral displacement depends on,
  (1) thickness of the glass slab
  (2) angle of incidence

4. Derive equation for refraction at single spherical surface.

Refraction at single spherical surface :



- Refractive index of rarer medium $= n_1$
  Refractive index of spherical medium $= n_2$
  Centre of curvature of spherical surface $= C$
  Point object in rarer medium $= O$
  Point image formed in denser medium $= I$
- Apply Snell's law of product form at the point N

$$n_1 \sin i = n_2 \sin r$$

- Since the angles are small, we have,
  $\sin i \approx i$ மற்றும் $\sin r \approx r$

$$\therefore \quad n_1 i = n_2 r \quad ---- (1)$$

- *Let*, $\angle NOP = \alpha$, $\angle NCP = \beta$, $\angle NIP = \gamma$, then

$$\tan\alpha = \frac{PN}{PO} \quad (or) \quad \alpha = \frac{PN}{PO}$$

$$\tan\beta = \frac{PN}{PC} \quad (or) \quad \beta = \frac{PN}{PC}$$

$$\tan\gamma = \frac{PN}{PI} \quad (or) \quad \gamma = \frac{PN}{PI}$$

- From figure , $i = \alpha + \beta$ and

$$\beta = r + \gamma \quad (or) \quad r = \beta - \gamma$$

- Put the values of i and r in eqn (1)

$$n_1(\alpha + \beta) = n_2(\beta - \gamma)$$

$$n_1\alpha + n_1\beta = n_2\beta - n_2\gamma$$

$$(or) \quad n_1\alpha + n_2\gamma = n_2\beta - n_1\beta$$

$$(or) \quad n_1\alpha + n_2\gamma = (n_2 - n_1)\beta$$

- Put α , β and γ , we have

$$n_1\left[\frac{PN}{PO}\right] + n_2\left[\frac{PN}{PI}\right] = (n_2 - n_1)\left[\frac{PN}{PC}\right]$$

$$(or) \quad \frac{n_1}{PO} + \frac{n_2}{PI} = \frac{n_2 - n_1}{PC}$$

- Using Cartesian sign convension, we get

$$PO = -u \; ; \quad PI = +v \; ; \quad PC = +R$$

$$\therefore \quad \frac{n_1}{-u} + \frac{n_2}{v} = \frac{n_2 - n_1}{R}$$

$$\boldsymbol{(or) \quad \frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}} \quad --- \quad (2)$$

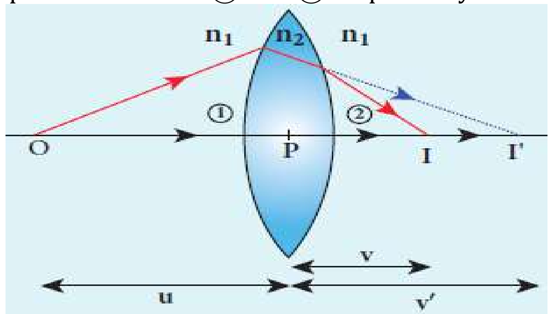
- Here rarer medium is air and hence $\boldsymbol{n_1 = 1}$ and let the refractive index of second medium be $\boldsymbol{n_2 = n}$. Therefore

$$\boldsymbol{\frac{n}{v} - \frac{1}{u} = \frac{n-1}{R}} \quad --- \quad \textbf{(3)}$$

5. Obtain Lens maker formula and metion its significance.

Lens maker's formula :

- A thin lens of refractive index n_2 is placed in a medium of refractive index n_1
- Let R_1 and R_2 be radii of curvature of two spherical surfaces 1 and 2 respectively



- Let P be pole of the lens and O be the Point object.
- Here I^1 be the image to be formed due the refracton at the surface 1 and I be the final image obtanined due the refracton at the surface 2
- We know that, equation for single spherical surface

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

- For refracting surface 1, the light goes from n_1 *to* n_2 .
- Hence

$$\frac{n_2}{v^1} - \frac{n_1}{u} = \frac{n_2 - n_1}{R_1} \quad --- \quad (1)$$

- For refracting surface 2, the light goes from n_2 *to* 1 . Hence

$$\frac{n_1}{v} - \frac{n_2}{v^1} = \frac{n_1 - n_2}{R_2} \quad --- \quad (2)$$

- Adding equation (1) and (2), we get,

$$\frac{n_2}{v^1} - \frac{n_1}{u} + \frac{n_1}{v} - \frac{n_2}{v^1} = \frac{n_2 - n_1}{R_1} + \frac{n_1 - n_2}{R_2}$$

$$\frac{n_1}{v} - \frac{n_1}{u} = (n_2 - n_1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\frac{1}{v} - \frac{1}{u} = \frac{(n_2 - n_1)}{n_1}\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\boldsymbol{\frac{1}{v} - \frac{1}{u} = \left(\frac{n_2}{n_1} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]} \quad --- \textbf{(2)}$$

- If the object is at infinity, the image is formed at the forcus of the lens. Thus, for $u = \infty$, $v = f$
  Then equation becomes,

$$\frac{1}{f} - \frac{1}{\infty} = \left(\frac{n_2}{n_1} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\boldsymbol{\frac{1}{f} = \left(\frac{n_2}{n_1} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]} \quad --- \textbf{(3)}$$

- Here first medium is air and hence $n_1 = 1$ and let the refractive index of second medium be $n_2 = n$. Therefore

$$\frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \quad --(4)$$

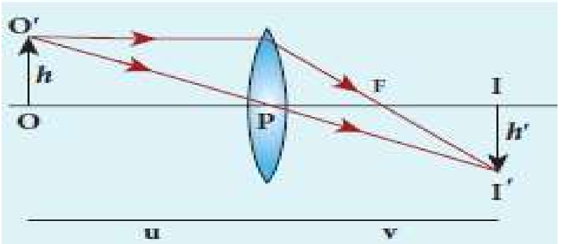
- The above equation is called ***lens maker's formula***.
- By comparing eqn (2) and (3)

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad ------- (5)$$

- This equation is known as ***lens equation***..

6. Derive the equation for thin lens and obtain its magnification.

<u>Magnification of thin lens</u> :



- Let an object OO^1 is placed on the principal axis with its height perpendicular to the principal axis.
- The ray O^1P passing through the pole of the lens goes undeviated.
- But the ray parallel to principal axis, after refraction it passes through secondary focus 'F'
- At the point of intersection of these two rays, an inverted, real image II^1 is formed.
- Height of object ; $OO^1 = h$
  Height of image ; $II^1 = \boldsymbol{h^1}$
- The lateral magnification (m) is defined as the ration of the heiht of the image to that of the object.

$$m = \frac{II^1}{OO^1} \quad ---- (1)$$

- ΔPOO^1 and ΔPII^1 are similar triangles . So ,

$$\frac{II^1}{OO^1} = \frac{PI}{PO}$$

- Using Cartesian sign convension,

$$m = \frac{-h^1}{h} = \frac{v}{-u}$$

$$(or) \qquad \boldsymbol{m = \frac{h^1}{h} = \frac{v}{u}} \quad ---- (2)$$

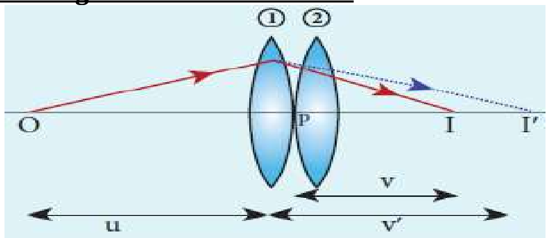
- The magnification is negative for real image and positive for virtural image.
- Thus for convex lens, the magnification is negative, and for concave lens, the magnification is positive.
- Combining the lens equation and magnification equation, we get

$$\boldsymbol{m = \frac{h^1}{h} = \frac{f}{f+u}}$$

$$(or) \qquad \boldsymbol{m = \frac{h^1}{h} = \frac{f-v}{f}}$$

7. Derive the equation for effective forcal length for lenses in contact.

<u>Focal length of lenses in contact</u> :



- Let us consider two lenses 1 and 2 of focal lengths f_1 and f_2 placed co-axially in contact with each other.
- Let the object is placed at 'O' beyond the principal focus of 1 on the principal axis.
- It forms an image at I^1
- This image I^1 acts as an object for lens 2 and hence the final image is formed at 'I'
- Writing the lens equation for lens 1

$$\frac{1}{v^1} - \frac{1}{u} = \frac{1}{f_1} \quad ---- (1)$$

- Writing the lens equation for lens 2

$$\frac{1}{v} - \frac{1}{v^1} = \frac{1}{f_2} \quad ---- (2)$$

- Adding equation (1) and (2)

$$\frac{1}{v^1} - \frac{1}{u} + \frac{1}{v} - \frac{1}{v^1} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\boldsymbol{\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2}} \quad --- (3)$$

- If this combination acts as a single lens of focal lenth 'F', then, ,

$$\boldsymbol{\frac{1}{v} - \frac{1}{u} = \frac{1}{F}} \quad --- (4)$$

- Compare eqn (3) and (4)

$$\boldsymbol{\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}} \quad --- (5)$$

- For any number of lenses,

$$\boldsymbol{\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} + \frac{1}{f_4} + \cdots}$$

- Let P_1, P_2, P_3, P_4 ... be the power of each lens, then the net power of the lens combination,

$$\boldsymbol{P = P_1 + P_2 + P_3 + P_4 + \cdots}$$

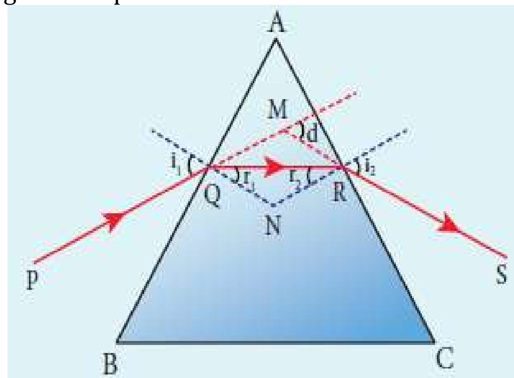
- Let m_1, m_2, m_3, m_4 ... be the magnification of each lens, then the net magnification of the lens combination,

$$\boldsymbol{m = m_1 \, X \, m_2 \, X \, m_3 \, X \, m_4 \, X \ldots}$$

8. Derive the equation for angle of deviation produced by af prism and thus obtain the equation for refractive index of material of the prism.

<u>Angle of deviation (d)</u> :

- Let 'ABC' be the section of triangular prism.
- Here face 'BC' is grounded and it is called base of the prism.
- The other two faces 'AB' and 'AC' are polished which are called refracting faces.
- The angle between two refraction faces is called angle of the prism 'A'



- Here, 'PQ' be incident ray, 'QR' be refracted ray and 'RS' be emergent ray.
- The angle between incident ray and emergent ray is called ***angle of deviation (d)***
- Let QN and RN be the normal drawn at the points Q and R
- The incident and emergent ray meet at a point M
- From figure, $\angle MQR = d_1 = i_1 - r_1$
  and $\angle MRQ = d_2 = i_2 - r_2$

- Then total angle of deviation,

$$d = d_1 + d_2$$
$$d = (i_1 - r_1) + (i_2 - r_2)$$
$$d = (i_1 + i_2) - (r_1 + r_2) \qquad --- \quad (1)$$

- In the quadrilateral AQNR, $\angle Q = \angle R = 90°$. Hence

$$A + \angle QNR = 180°$$
$$(or) \qquad A = 180° - \angle QNR \qquad --- \quad (2)$$

- In ΔQNR,

$$r_1 + r_2 + \angle QNR = 180°$$
$$r_1 + r_2 = 180° - \angle QNR \quad --- (3)$$

- From eqn (2) and (3)

$$A = r_1 + r_2 \qquad ---- \ (4)$$

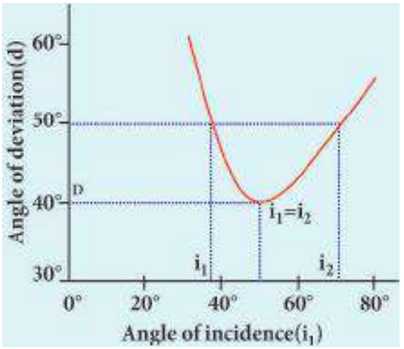
- Put eqn (4) in eqn (1),

$$\boldsymbol{d = (i_1 + i_2) - A} \qquad ----(5)$$

- Thus the angle of deviation depends on,
  (1) the angle of incidence (i_1)
  (2) the angle of the prism (A)
  (3) the material of the prism (n)
  (4) the wavelength of the light (λ)

Angle of minimum deviation (D) :

- A graph is plotted between the angle of incidence along x-axis and angle of deviation along y-axis.
- From the graph, as angle of incidence increases, the angle of deviation decreases, reaches a minimum value and then continues to increase.



- The minimum value of angled of deviation is called ***angle of minimum deviation*** (D).
- At minimum deviation,
  (1) $i_1 = i_2$
  (2) $r_1 = r_2$
  (3) Refracted ray 'QR' is parallel to the base 'BC' of the prism.

Refractive index of the material of the prism (n) :

- At angle of minimum deviation,

$$i_1 = i_2 = i$$
$$r_1 = r_2 = r$$

- Put this in equations (4) and (5)

$$A = r + r = 2r$$
$$(or) \qquad \boldsymbol{r = \frac{A}{2}} \qquad ------- \ (6)$$
$$\text{and} \qquad D = (i + i) - A = 2i - A$$
$$(or) \qquad 2i = A + D$$
$$\boldsymbol{i = \frac{A + D}{2}} \qquad ------- \ (7)$$

- Then by Snell's law ,

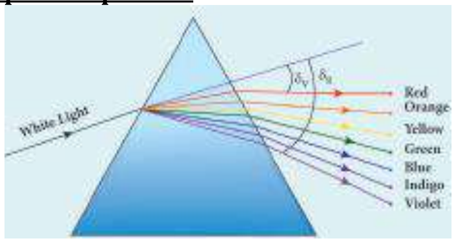
$$n = \frac{\sin i}{\sin r}$$
$$\mathbf{n} = \frac{\mathbf{sin}\left[\frac{\boldsymbol{A+D}}{\mathbf{2}}\right]}{\mathbf{sin}\left[\frac{\boldsymbol{A}}{\mathbf{2}}\right]} \qquad ---- \ (\mathbf{8})$$

9. What is dispersion? Obtain the equation for dispersive power of a medium.

Dispersion :

- Splitting of white light into its constituent colours is called dispersion.
- The coloured band obtained due to dispersion is called spectrum.

Dispersive power :



- Dispersive power ($\boldsymbol{\omega}$) is the ability of the material of the prism to cause dispersion.
- ***It is defined as the ration of the angular dispersion for the extreme colours to the deviation for any mean colour.***
- Let A be the angle of prism and D be the angle of minimum deviation, then the refractive index of the material of the prism is

$$n = \frac{\sin\left[\frac{A+D}{2}\right]}{\sin\left[\frac{A}{2}\right]}$$

- If the angle of the prism is small in the order of 10° then it is called small angle prism. In this prism, the angle of deviation also become small.
- Let A be the angle of prism and δ be the angle of minimum deviation, then the refractive index

$$n = \frac{\sin\left[\frac{A+\delta}{2}\right]}{\sin\left[\frac{A}{2}\right]} \qquad ---- \ (1)$$

- Since A and δ are small, we may write,

$$\sin\left[\frac{A+\delta}{2}\right] \approx \left[\frac{A+\delta}{2}\right]$$
$$\sin\left[\frac{A}{2}\right] \approx \left[\frac{A}{2}\right]$$

- Put this in eqn (1),

$$n = \frac{\left[\frac{A+\delta}{2}\right]}{\left[\frac{A}{2}\right]} = \frac{A+\delta}{A}$$
$$n A = A + \delta$$
$$(or) \qquad \delta = n A - A$$
$$\therefore \qquad \boldsymbol{\delta = (\mathbf{n} - 1) A} \qquad ------- \ (2)$$

- Thus, angle of deviation for violet and red light,

$$\boldsymbol{\delta_V = (\mathbf{n}_V - 1) A} \qquad ------- \ (3)$$
$$\boldsymbol{\delta_R = (\mathbf{n}_R - 1) A} \qquad ------- \ (4)$$

- The angular dispersion is given by,

$$\delta_V - \delta_R = (n_V - 1) A - (n_R - 1) A$$
$$\delta_V - \delta_R = n_V A - A - n_V A + A$$
$$\boldsymbol{\delta_V - \delta_R = (\mathbf{n}_V - \mathbf{n}_R) A} \qquad ------ \ (5)$$

- Let $\boldsymbol{\delta}$ be the angle of deviation for mean ray (yellow) and n be the corresponding refractive index, then

$$\boldsymbol{\delta = (\mathbf{n} - 1) A} \qquad ------- \ (6)$$

- By definition, dispersive power

$$\omega = \frac{angular\ dispersion}{mean\ deviation} = \frac{\boldsymbol{\delta_V - \delta_R}}{\boldsymbol{\delta}}$$
$$\omega = \frac{(n_V - n_R) A}{(n - 1) A}$$
$$\boldsymbol{\omega = \frac{(\mathbf{n}_V - \mathbf{n}_R)}{(\mathbf{n} - 1)}} \qquad -------- \ (\mathbf{7})$$

- Dispersive power is a dimensionless quantity. It has no unit. It is always positive.