Chapter 6: Determinants

EXERCISE 6.1 [PAGE 83]

Exercise 6.1 | Q 1.1 | Page 83

Evaluate the following determinants: $\begin{vmatrix} 4 & 7 \\ -7 & 0 \end{vmatrix}$

SOLUTION

$\begin{vmatrix} 4 & 7 \\ -7 & 0 \end{vmatrix}$

= 4(0) – (– 7)(7)

= 0 + 49

= 49.

Exercise 6.1 | Q 1.2 | Page 83

Evaluate the following determinants: $\begin{vmatrix} 3 & -5 & 2 \\ 1 & 8 & 9 \\ 3 & 7 & 0 \end{vmatrix}$

SOLUTION

$\begin{vmatrix} 3 & -5 & 2 \\ 1 & 8 & 9 \\ 3 & 7 & 0 \end{vmatrix}$

$= 3\begin{vmatrix} 8 & 9 \\ 7 & 0 \end{vmatrix} - (-5)\begin{vmatrix} 1 & 9 \\ 3 & 0 \end{vmatrix} + 2\begin{vmatrix} 1 & 8 \\ 3 & 7 \end{vmatrix}$

= 3(0 – 63) + 5(0 –27) + 2(7 –24)

= 3(– 63) + 5(– 27) + 2(– 17)

= – 189 – 135 – 34

= –358.

Exercise 6.1 | Q 1.3 | Page 83

Evaluate the following determinants: $\begin{vmatrix} 1 & i & 3 \\ i^3 & 2 & 5 \\ 3 & 2 & i^4 \end{vmatrix}$

SOLUTION

$\begin{vmatrix} 1 & i & 3 \\ i^3 & 2 & 5 \\ 3 & 2 & i^4 \end{vmatrix}$

$= \begin{vmatrix} 1 & i & 3 \\ -i & 2 & 5 \\ 3 & 2 & 1 \end{vmatrix}$ ...[$\because$ $i^2 = -1$, $i^4 = 1$]

$= 1\begin{vmatrix} 2 & 5 \\ 2 & 1 \end{vmatrix} - i\begin{vmatrix} -i & 5 \\ 3 & 1 \end{vmatrix} + 3\begin{vmatrix} -i & 2 \\ 3 & 2 \end{vmatrix}$

$= 1(2 - 10) - i(-i - 15) + 3(-2i - 6)$

$= -8 + i^2 + 15i - 6i - 18$

$= i^2 - 26 + 9i$

$= -1 - 26 + 9i$ ...[$\because$ $i^2 = -1$]

$= -27 + 9i.$

Exercise 6.1 | Q 1.4 | Page 83

Evaluate the following determinants: $\begin{vmatrix} 5 & 5 & 5 \\ 5 & 4 & 4 \\ 5 & 4 & 8 \end{vmatrix}$

SOLUTION

$\begin{vmatrix} 5 & 5 & 5 \\ 5 & 4 & 4 \\ 5 & 4 & 8 \end{vmatrix}$

$= 5\begin{vmatrix} 4 & 4 \\ 4 & 8 \end{vmatrix} - 5\begin{vmatrix} 5 & 4 \\ 5 & 8 \end{vmatrix} + 5\begin{vmatrix} 5 & 4 \\ 5 & 4 \end{vmatrix}$

= 5(32 – 16) –5(40 – 20) + 5(20 –20)

= 5(16) – 5(20) + 5(0)

= 80 –100

= – 20.

Exercise 6.1 | Q 1.5 | Page 83

Evaluate the following determinants: $\begin{vmatrix} 2i & 3 \\ 4 & -i \end{vmatrix}$

SOLUTION

$\begin{vmatrix} 2i & 3 \\ 4 & -i \end{vmatrix}$

= 2i(– i) – 3(4)

= – 2i^2 – 12

= – 2(– 1) –12 ...[∵ i^2 = – 1]

= 2 –12

= – 10.

Exercise 6.1 | Q 1.6 | Page 83

Evaluate the following determinants: $\begin{vmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix}$

SOLUTION

$\begin{vmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix}$

$= 3\begin{vmatrix} 1 & -2 \\ 3 & 1 \end{vmatrix} - (-4)\begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix} + 5\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix}$

$= 3(1 + 6) + 4(1 + 4) + 5(3 - 2)$

$= 3(7) + 4(5) + 5(1)$

$= 21 + 20 + 5$

$= 46.$

Exercise 6.1 | Q 1.7 | Page 83

Evaluate the following determinants: $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & C \end{vmatrix}$

SOLUTION

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & C \end{vmatrix}$$

$$= a\begin{vmatrix} b & f \\ f & c \end{vmatrix} - h\begin{vmatrix} h & f \\ h & c \end{vmatrix} + g\begin{vmatrix} h & b \\ g & f \end{vmatrix}$$

$= a(bc - f^2) - h(hc - gf) + g(hf - gb)$

$= abc - af^2 - h^2c + fgh + fgh - g^2b$

$= abc + 2fgh - af^2 - bg^2 - ch^2.$

Exercise 6.1 | Q 1.8 | Page 83

Evaluate the following determinants: $\begin{vmatrix} 0 & a & -b \\ -a & 0 & -c \\ b & c & 0 \end{vmatrix}$

SOLUTION

$$\begin{vmatrix} 0 & a & -b \\ -a & 0 & -c \\ b & c & 0 \end{vmatrix}$$

$$= 0\begin{vmatrix} 0 & -c \\ c & 0 \end{vmatrix} - a\begin{vmatrix} -a & -c \\ b & 0 \end{vmatrix} - b\begin{vmatrix} -a & 0 \\ b & c \end{vmatrix}$$

= 0 – a (0 + bc) – b(– ac – 0)

= – a(bc) – b (– ac)

= – abc + abc

= 0.

Exercise 6.1 | Q 2.1 | Page 83

Find the value(s) of x, if $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$

SOLUTION

$$\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$$

∴ 10 – 12 = 5x – 6x

∴ –2 = – x

∴ x = 2.

Exercise 6.1 | Q 2.2 | Page 83

Find the value(s) of x, if $\begin{vmatrix} 2 & 1 & x+1 \\ -1 & 3 & -4 \\ 0 & -5 & 3 \end{vmatrix}$ = 0

SOLUTION

$$\begin{vmatrix} 2 & 1 & x+1 \\ -1 & 3 & -4 \\ 0 & -5 & 3 \end{vmatrix} = 0$$

$$\therefore 2\begin{vmatrix} 3 & -4 \\ -5 & 3 \end{vmatrix} - 1\begin{vmatrix} -1 & -4 \\ 0 & 3 \end{vmatrix} + (x+1)\begin{vmatrix} -1 & 3 \\ 0 & -5 \end{vmatrix} = 0$$

∴ 2(9 – 20) – 1 (– 3 – 0) + (x + 1) (5 –0) = 0

∴ 2(– 11) – 1(– 3) + (x + 1)(5) = 0

∴ – 22 + 3 + 5x + 5 = 0

$\therefore$ 5x = 14

$\therefore$ x = $\frac{14}{5}$

Exercise 6.1 | Q 2.3 | Page 83

Evaluate the following determinants: $\begin{vmatrix} x-1 & x & x-2 \\ 0 & x-2 & x-3 \\ 0 & 0 & x-3 \end{vmatrix} = 0$

SOLUTION

$$\begin{vmatrix} x-1 & x & x-2 \\ 0 & x-2 & x-3 \\ 0 & 0 & x-3 \end{vmatrix} = 0$$

$\therefore (x-1)\begin{vmatrix} x-2 & x-3 \\ 0 & x-3 \end{vmatrix} - x\begin{vmatrix} 0 & x-3 \\ 0 & x-3 \end{vmatrix} + (x-2)\begin{vmatrix} 0 & x-2 \\ 0 & 0 \end{vmatrix}$ = 0

$\therefore$ (x – 1)[(x –2)(x – 3) – 0] – x(0 –0) + (x – 2)(0 –0) = 0

$\therefore$ (x –1)(x –2)(x –) = 0

$\therefore$ x – 1 = 0 or x – 2 = 0 or x –3 = 0

$\therefore$ x –1 or x = 2 or x = 3.

Exercise 6.1 | Q 3.1 | Page 83

Solve the following equation : $\begin{vmatrix} x & 2 & 2 \\ 2 & x & 2 \\ 2 & 2 & x \end{vmatrix} = 0$

SOLUTION

$$\begin{vmatrix} x & 2 & 2 \\ 2 & x & 2 \\ 2 & 2 & x \end{vmatrix} = 0$$

$$\therefore x\begin{vmatrix} x & 2 \\ 2 & x \end{vmatrix} - 2\begin{vmatrix} 2 & 2 \\ 2 & x \end{vmatrix} + 2\begin{vmatrix} 2 & x \\ 2 & 2 \end{vmatrix} = 0$$

$\therefore x(x^2 - 4) -2(2x - 4) + 2(4 - 2x) = 0$

$\therefore x(x^2 - 4) - 2(2x - 4) -2(2x - 4) = 0$

$\therefore x(x + 2)(x - 2) - 8(x - 2) = 0$

$\therefore x(x + 2)(x - 2) - 8(x - 2) = 0$

$\therefore (x - 2)[x(x + 2) - 8] = 0$

$\therefore (x - 2)(x^2 + 2x - 8) = 0$

$\therefore (x - 2)(x^2 + 4x - 2x -8) = 0$

$\therefore (x - 2)(x + 4)(x - 2) = 0$

$\therefore (x - 2)^2(x + 4) = 0$

$\therefore (x - 2)^2 = 0$ or $x + 4 = 0$

$\therefore x - 2 = 0$ or $x = -4$

$\therefore x = 2$ or $x = -4$.

Exercise 6.1 | Q 3.2 | Page 83

Solve the following equation : $\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 2x & 5x^2 \end{vmatrix} = 0$

SOLUTION

$$\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 2x & 5x^2 \end{vmatrix} = 0$$

$\therefore 1\begin{vmatrix} -2 & 5 \\ 2x & 5x^2 \end{vmatrix} - 4\begin{vmatrix} 1 & 5 \\ 1 & 5x^2 \end{vmatrix} + 20\begin{vmatrix} 1 & -2 \\ 1 & 2x \end{vmatrix}$ = 0

∴ 1(– 10x^2 –10x) – 4(5x^2 – 5) + 20(2x + 2) = 0

∴ – 10x^2 – 10x – 20^2 + 20 + 40x + 40 = 0

∴ – 30x^2 + 30x + 60 = 0

∴ x^2 – x –2 = 0 ...[Dividing throughout by (– 30)]

∴ x^2 – 2x + x – 2 = 0

∴ (x – 2)(x + 1) = 0

∴ x – 2 = 0 or x + 1 = 0

∴ x = 2 or x = – 1.

Exercise 6.1 | Q 4 | Page 83

Find the value of x, if $\begin{vmatrix} x & -1 & 2 \\ 2x & 1 & -3 \\ 3 & -4 & 5 \end{vmatrix}$ = 29

SOLUTION

$\begin{vmatrix} x & -1 & 2 \\ 2x & 1 & -3 \\ 3 & -4 & 5 \end{vmatrix}$ = 29

$\therefore x\begin{vmatrix} 1 & -3 \\ -4 & 5 \end{vmatrix} - (-1)\begin{vmatrix} 2x & -3 \\ 3 & 5 \end{vmatrix} + 2\begin{vmatrix} 2x & 1 \\ 3 & -4 \end{vmatrix}$ = 29

∴ x(5 – 12) + 1(10x + 9) + 2(– 8x –3) = 29

∴ – 7x + 10x + 9 – 16x – 6 = 29

∴ – 13x + 3 = 29

∴ – 13x = 26

∴ x = – 2.

Exercise 6.1 | Q 5 | Page 83

Find x and y if $\begin{vmatrix} 4i & i^3 & 2i \\ 1 & 3i^2 & 4 \\ 5 & -3 & i \end{vmatrix}$ = x + iy, where i = $\sqrt{-1}$.

SOLUTION

$\begin{vmatrix} 4i & i^3 & 2i \\ 1 & 3i^2 & 4 \\ 5 & -3 & i \end{vmatrix}$

= $\begin{vmatrix} 4i & -i & 2i \\ 1 & -3 & 4 \\ 5 & -3 & i \end{vmatrix}$ ...[∵ i^2 = – 1]

= $4i\begin{vmatrix} -3 & 4 \\ -3 & i \end{vmatrix} - (-i)\begin{vmatrix} 1 & 4 \\ 5 & i \end{vmatrix} + 2i\begin{vmatrix} 1 & -3 \\ 5 & -3 \end{vmatrix}$

= 4i(– 3i + 12) + i(i – 20) + 2i(– 3 + 15)

= – 12i^2 + 48i + i^2 – 20i + 24i

= – 11i^2 + 52i

= – 11(– 1) + 52i ...[∵ i^2 = – 1]

= 11 + 52i

Comparing with x + iy, we get

x = 11, y = 52.

EXERCISE 6.2 [PAGE 89]

Exercise 6.2 | Q 1.1 | Page 89

Without expanding evaluate the following determinants : $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$

SOLUTION

Let D = $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$

Applying $C_3 \to C_3 + C_2$, we get

D = $\begin{vmatrix} 1 & a & a+b+c \\ 1 & b & a+b+c \\ 1 & c & a+b+c \end{vmatrix}$

Taking (a + b + c) common from C_3, we get

D = $(a+b+c)\begin{vmatrix} 1 & a & 1 \\ 1 & b & 1 \\ 1 & c & 1 \end{vmatrix}$

∴ D = (a + b + c)(0) ...[∵ C_1 and C_3 are identical]

∴ D = 0.

Exercise 6.2 | Q 1.2 | Page 89

Without expanding evaluate the following determinants : $\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix}$

SOLUTION

Let D = $\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix}$

Taking (3x) common from R_3, we get

D = $3x\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 2 & 3 & 4 \end{vmatrix}$

= (3x)(0) ...[$\because$ R_1 and R_3 are identical]

= 0.

Exercise 6.2 | Q 1.3 | Page 89

Without expanding evaluate the following determinants : $\begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix}$

SOLUTION

Let D = $\begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix}$

Applying $C_3 \to C_3 - 9C_2$, we get

D = $\begin{vmatrix} 2 & 7 & 2 \\ 3 & 8 & 3 \\ 5 & 9 & 5 \end{vmatrix}$

= 0 ...[$\because$ C_1 and C_3 are identical]

Exercise 6.2 | Q 2 | Page 89

Using properties of determinants, show that $\begin{vmatrix} a+b & a & b \\ a & a+c & c \\ b & c & b+c \end{vmatrix}$ = 4abc.

SOLUTION

L.H.S. = $\begin{vmatrix} a+b & a & b \\ a & a+c & c \\ b & c & b+c \end{vmatrix}$

Applying $C_1 \to C_1 - (C_2 + C_3)$, we get

L.H.S. = $\begin{vmatrix} 0 & a & b \\ -2c & a+c & c \\ -3c & c & b+c \end{vmatrix}$

Taking (– 2) common from $C_{1,}$ we get

L.H.S. = $-2\begin{vmatrix} 0 & a & b \\ c & a+c & c \\ c & c & b+c \end{vmatrix}$

Applying $C_2 \to C_2 - C_1$ and $C_3 \to C_2 - C_1$, we get

L.H.S. = $-2\begin{vmatrix} 0 & a & b \\ c & a & 0 \\ c & 0 & b \end{vmatrix}$

= 2[0(ab – 0) – a(bc – 0) + b(0 – ac)]

= – 2(0 – abc – abc)

= – 2(– 2abc)

= 4abc

= R.H.S.

Exercise 6.2 | Q 3 | Page 89

Solve the following equation: $\begin{vmatrix} x+2 & x+6 & x-1 \\ x+6 & x-1 & x+2 \\ x-1 & x+2 & x+6 \end{vmatrix}$ = 0

SOLUTION

$\begin{vmatrix} x+2 & x+6 & x-1 \\ x+6 & x-1 & x+2 \\ x-1 & x+2 & x+6 \end{vmatrix}$ = 0

Applying $R_2 \to R_2 - R_1$ and R2 $\to R_3 - R_3$, we get

$\begin{vmatrix} x+2 & x+6 & x-1 \\ 4 & -7 & 3 \\ -3 & -4 & 7 \end{vmatrix}$ = 0

∴ (x + 2)(– 49 + 12) – (x + 6)(28 + 9) + (x – 1) (– 16 – 21) = 0

∴ (x + 2) (– 37) – (x + 6) (37) + (x – 1) (– 37) = 0

$\therefore$ 3x + 7 = 0

$\therefore$ x = $\dfrac{-7}{3}$.

Exercise 6.2 | Q 4 | Page 89

If $\begin{vmatrix} 4+x & 4-x & 4-x \\ 4-x & 4+x & 4-x \\ 4-x & 4-x & 4+x \end{vmatrix}$ = 0, then find the values of x.

SOLUTION

$\begin{vmatrix} 4+x & 4-x & 4-x \\ 4-x & 4+x & 4-x \\ 4-x & 4-x & 4+x \end{vmatrix}$ = 0

Applying $C_1 \to C_1 + C_2 + C_3$, we get

$\begin{vmatrix} 12- & x & 4-x & 4-x \\ 12- & x & 4+x & 4-x \\ 12- & x & 4-x & 4+x \end{vmatrix}$ = 0

Taking (12 – x) common from C_1, we get

$(12-x)\begin{vmatrix} 1 & 4-x & 4-x \\ 1 & 4+x & 4-x \\ 1 & 4-x & 4+x \end{vmatrix}$ = 0

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, we get

$(12-x)\begin{vmatrix} 1 & 4-x & 4-x \\ 0 & 2x & 0 \\ 0 & 0 & 2x \end{vmatrix}$ = 0

$\therefore$ (12 – x)[1(4x^2 – 0) –(4 – x)(0 – 0) + (4 –x)(0 – 0)] = 0

$\therefore (12 - x)(4x^2) = 0$

$\therefore x^2(12 - x) = 0$

$\therefore x = 0$ or $12 - x = 0$

$\therefore x = 0$ or $x = 12$.

Exercise 6.2 | Q 5 | Page 89

Without expanding determinants, show that $\begin{vmatrix} 1 & 3 & 6 \\ 6 & 1 & 4 \\ 3 & 7 & 12 \end{vmatrix} + \begin{vmatrix} 2 & 3 & 3 \\ 2 & 1 & 2 \\ 1 & 7 & 6 \end{vmatrix} = 10 \begin{vmatrix} 1 & 2 & 1 \\ 3 & 1 & 7 \\ 3 & 2 & 6 \end{vmatrix}$

SOLUTION

$$\text{L.H.S.} = \begin{vmatrix} 1 & 3 & 6 \\ 6 & 1 & 4 \\ 3 & 7 & 12 \end{vmatrix} + 4 \begin{vmatrix} 2 & 3 & 3 \\ 2 & 1 & 2 \\ 1 & 7 & 6 \end{vmatrix}$$

$$= \begin{vmatrix} 2 & 3 & 3 \\ 12 & 1 & 2 \\ 6 & 7 & 6 \end{vmatrix} + \begin{vmatrix} 8 & 3 & 3 \\ 8 & 1 & 2 \\ 4 & 7 & 6 \end{vmatrix}$$

$$= \begin{vmatrix} 2+8 & 3 & 3 \\ 12+8 & 1 & 2 \\ 6+4 & 7 & 6 \end{vmatrix}$$

$$= \begin{vmatrix} 10 & 3 & 3 \\ 20 & 1 & 2 \\ 10 & 7 & 6 \end{vmatrix}$$

Interchanging rows and columns, we get

$$\text{L.H.S.} = \begin{vmatrix} 10 & 20 & 10 \\ 3 & 1 & 7 \\ 3 & 2 & 6 \end{vmatrix}$$

Taking 10 common from R_1, we get

L.H.S. = $10\begin{vmatrix} 1 & 2 & 1 \\ 3 & 1 & 7 \\ 3 & 2 & 6 \end{vmatrix}$

= R.H.S.

Exercise 6.2 | Q 6.1 | Page 89

Without expanding determinants, find the value of $\begin{vmatrix} 10 & 57 & 107 \\ 12 & 64 & 124 \\ 15 & 78 & 153 \end{vmatrix}$

SOLUTION

Let D = $\begin{vmatrix} 10 & 57 & 107 \\ 12 & 64 & 124 \\ 15 & 78 & 153 \end{vmatrix}$

Applying $C_3 \to C_3 - C_2$, we get

D = $\begin{vmatrix} 10 & 57 & 50 \\ 12 & 64 & 60 \\ 15 & 78 & 75 \end{vmatrix}$

Taking (5) common from C_3, we get

D = $5\begin{vmatrix} 10 & 57 & 10 \\ 12 & 64 & 12 \\ 15 & 78 & 15 \end{vmatrix}$

= 5(0) ...[$\because$ C_1 and C_3 are identical]

= 0.

Exercise 6.2 | Q 6.2 | Page 89

Without expanding determinants, find the value of $\begin{vmatrix} 2014 & 2017 & 1 \\ 2020 & 2023 & 1 \\ 2023 & 2026 & 1 \end{vmatrix}$

SOLUTION

Let D = $\begin{vmatrix} 2014 & 2017 & 1 \\ 2020 & 2023 & 1 \\ 2023 & 2026 & 1 \end{vmatrix}$

Applying $C_2 \to C_2 - C_1$, we get

D = $\begin{vmatrix} 2014 & 3 & 1 \\ 2020 & 3 & 1 \\ 2023 & 3 & 1 \end{vmatrix}$

Taking (3) common from C_2, we get

D = 3$\begin{vmatrix} 2014 & 1 & 1 \\ 2020 & 1 & 1 \\ 2023 & 1 & 1 \end{vmatrix}$

= 3(0) ...[$\because$ C_2 and C_3 are identical]

= 0.

Exercise 6.2 | Q 7.1 | Page 89

Without expanding determinants, prove that $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} b_1 & c_1 & a_1 \\ b_2 & c_2 & a_2 \\ b_3 & c_3 & a_3 \end{vmatrix} = \begin{vmatrix} c_1 & a_1 & b_1 \\ c_2 & a_2 & b_2 \\ c_3 & a_3 & b_3 \end{vmatrix}$

SOLUTION

Let D = $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ ...(i)

Let E = $\begin{vmatrix} b_1 & c_1 & a_1 \\ b_2 & c_2 & a_2 \\ b_3 & c_3 & a_3 \end{vmatrix}$

Applying $C_1 \leftrightarrow C_2$, we get

E = $-\begin{vmatrix} c_1 & a_1 & b_1 \\ c_2 & a_2 & b_2 \\ c_3 & a_3 & b_3 \end{vmatrix}$

$$E = -\begin{vmatrix} c_1 & a_1 & b_1 \\ c_2 & a_2 & b_2 \\ c_3 & a_3 & b_3 \end{vmatrix}$$

Applying $C_1 \leftrightarrow C_3$, we get

$$E = -(-1)\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$\therefore E = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \qquad \text{...(ii)}$$

Let $F = \begin{vmatrix} c_1 & a_1 & b_1 \\ c_2 & a_2 & b_2 \\ c_3 & a_3 & b_3 \end{vmatrix}$

Applying $C_1 \leftrightarrow C_2$, we get

$$F = -\begin{vmatrix} a_1 & c_1 & b_1 \\ a_2 & c_2 & b_2 \\ a_3 & c_3 & b_3 \end{vmatrix}$$

Applying $C_2 \leftrightarrow C_3$, we get

$$F = -(-1)\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$\therefore F = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \qquad \text{...(iii)}$$

From (i), (ii) and (iii), we get

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} b_1 & c_1 & a_1 \\ b_2 & c_2 & a_2 \\ b_3 & c_3 & a_3 \end{vmatrix} = \begin{vmatrix} c_1 & a_1 & b_1 \\ c_2 & a_2 & b_2 \\ c_3 & a_3 & b_3 \end{vmatrix}$$

Exercise 6.2 | Q 7.2 | Page 89

Without expanding determinants, prove that $\begin{vmatrix} 1 & yz & y+z \\ 1 & zx & z+x \\ 1 & xy & x+y \end{vmatrix} = \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$

SOLUTION

L.H.S. = $\begin{vmatrix} 1 & yz & y+z \\ 1 & zx & z+x \\ 1 & xy & x+y \end{vmatrix}$

$= \begin{vmatrix} 1 & yz & x+y+z-x \\ 1 & zx & y+z+x-y \\ 1 & xy & z+x+y-z \end{vmatrix}$

$= \begin{vmatrix} 1 & yz & x+y+z \\ 1 & zx & x+y+z \\ 1 & xy & x+y+z \end{vmatrix} + \begin{vmatrix} 1 & yz & -x \\ 1 & zx & -y \\ 1 & xy & -z \end{vmatrix}$

$= \begin{vmatrix} 1 & yz & x+y+z \\ 1 & zx & x+y+z \\ 1 & xy & x+y+z \end{vmatrix} + \begin{vmatrix} 1 & yz & x \\ 1 & zx & y \\ 1 & xy & z \end{vmatrix}$

In 1st determinant, taking (x + y + z) common from C_3 and in 2nd determinant, taking $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ common from R_1, R_2, R_3 respectively, we get

L.H.S. = $(x+y+z)\begin{vmatrix} 1 & yz & 1 \\ 1 & zx & 1 \\ 1 & xy & 1 \end{vmatrix} - \dfrac{1}{xyz}\begin{vmatrix} x & xyz & x^2 \\ y & xyz & y^2 \\ z & xyz & z^2 \end{vmatrix}$

In 2nd determinant, taking xyz common from C_2, we get

L.H.S. = $(x+y+z)(0) - \dfrac{xyz}{xyz}\begin{vmatrix} x & 1 & x^2 \\ y & 1 & y^2 \\ z & 1 & z^2 \end{vmatrix}$ ...[$\because$ C_1 and C_2 are identical in 1st determinant]

$= -\begin{vmatrix} x & 1 & x^2 \\ y & 1 & y^2 \\ z & 1 & z^2 \end{vmatrix}$

Applying $C_1 \leftrightarrow C_2$, we get

L.H.S. = $\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$ = R.H.S.

EXERCISE 6.3 [PAGES 93 - 94]

Exercise 6.3 | Q 1.1 | Page 93

Solve the following equations using Cramer's Rule: x + 2y – z = 5, 2x – y + z = 1, 3x + 3y = 8

SOLUTION

Given equations are
x + 2y – z = 5
2x – y + z = 1
3x + 3y = 8
i.e. 3x + 3y + 0z = 8

$\therefore$ D = $\begin{vmatrix} 1 & 2 & -1 \\ 2 & -1 & 1 \\ 3 & 3 & 0 \end{vmatrix}$

= 1(0 – 3) – 2(0 – 3) – 1(6 + 3)

= – 3 + 6 – 9

= – 6

$D_x = \begin{vmatrix} 5 & 2 & -1 \\ 1 & -1 & 1 \\ 8 & 3 & 0 \end{vmatrix}$

= 5(0 – 3) – 2(0 – 8) + (– 1)(3 + 8)

= – 15 + 16 – 11

= – 10

$D_y = \begin{vmatrix} 1 & 5 & -1 \\ 2 & 1 & 1 \\ 3 & 8 & 0 \end{vmatrix}$

= 1(0 – 8) – 5(0 – 3) + 1(6 – 3)

= – 8 + 15 – 13

= – 6

$D_z = \begin{vmatrix} 1 & 2 & 5 \\ 2 & -1 & 1 \\ 3 & 3 & 8 \end{vmatrix}$

= 1(– 8 – 3) – 2(16 – 3) + 5(6 + 3)

= – 11 – 26 + 45

= 8

By Cramer's Rule,

x = $\dfrac{D_y}{D} = \dfrac{-10}{-6} = \dfrac{5}{3}$

y = $\dfrac{D_y}{D} = \dfrac{-6}{-6}$ = 1

z = $\dfrac{D_z}{D} = \dfrac{8}{-6} = \dfrac{-4}{3}$

∴ x = $\dfrac{5}{3}, y = 1 \text{ and } z = \dfrac{-4}{3}$ are the solution of the given equations.

Exercise 6.3 | Q 1.2 | Page 93

Solve the following equations using Cramer's Rule: 2x – y + 6z = 10, 3x + 4y – 5z = 11, 8x – 7y – 9z = 12

SOLUTION

Given equations are

$2x - y + 6z = 10$

$3x + 4y - 5z = 11$

$8x - 7y - 9z = 12$

$$D = \begin{vmatrix} 2 & -1 & 6 \\ 3 & 4 & -5 \\ 8 & -7 & -9 \end{vmatrix}$$

$= 2(-36 - 35) - (-1)(-27 + 40) + 6(-21 - 32)$

$= -142 + 13 - 318$

$= -447$

$$D_x = \begin{vmatrix} 10 & -1 & 6 \\ 11 & 4 & -5 \\ 12 & -7 & -9 \end{vmatrix}$$

$= 10(-36 - 35) - (-1)(-99 + 60) + 6(-77 - 48)$

$= -710 - 39 - 750$

$= -1499$

$$D_y = \begin{vmatrix} 2 & 10 & 6 \\ 3 & 11 & -5 \\ 8 & 12 & -9 \end{vmatrix}$$

$= 2(-99 + 60) - 10(-27 + 40) + 6(36 - 88)$

$= -78 - 130 - 312$

$= -520$

$$D_x = \begin{vmatrix} 2 & -1 & 10 \\ 3 & 4 & 11 \\ 8 & -7 & 12 \end{vmatrix}$$

= 2(48 + 77) – (– 1)(36 – 88) + 10(– 21 – 32)

= 250 – 52 – 530

= – 332

By Cramer's Rule,

x = $\frac{D_x}{D} = \frac{-1499}{-447} = \frac{1499}{447}$

y = $\frac{D_y}{D} = \frac{-520}{-447} = \frac{520}{447}$

z = $\frac{D_z}{D} = \frac{-332}{-447} = \frac{332}{447}$

∴ x = $\frac{1449}{447}, y = \frac{520}{447}$ and $z = \frac{332}{447}$ are the solutions of the given equations.

Exercise 6.3 | Q 1.4 | Page 93

Solve the following equations using Cramer's Rule:

$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = -2,\ \frac{1}{x} - \frac{2}{y} + \frac{1}{z} = 3,\ \frac{2}{x} - \frac{1}{y} + \frac{3}{z}$ = -1

SOLUTION

Let $\frac{1}{x}$ p, $\frac{1}{y}$ = q, $\frac{1}{z}$ = r

∴ The given equations become

p + q + r = – 2

p – 2q + r = 3

2p – q + 3r = – 1

D = $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -2 & 1 \\ 2 & -1 & 3 \end{vmatrix}$

= 1(– 6 + 1) – 1(3 – 2) + 1(– 1 + 4)

= 5 – 1 + 3

= – 3

$D_p = \begin{vmatrix} -2 & 1 & 1 \\ 3 & -2 & 1 \\ -1 & -1 & 3 \end{vmatrix}$

= – 2(– 6 + 1) – 1(9 + 1) + 1(– 3 – 2)

= 10 – 10 – 5

= – 5

$D_q = \begin{vmatrix} 1 & -2 & 1 \\ 1 & 3 & 1 \\ 2 & -1 & 3 \end{vmatrix}$

= 1(9 + 1) + 2(3 – 2) + 1(– 1 –6)

= 10 + 2 – 7

= 5

$D_r = \begin{vmatrix} 1 & 1 & -2 \\ 1 & -2 & 3 \\ 2 & -1 & -1 \end{vmatrix}$

= 1(2 + 3) – 1(– 1 –6) – 2(– 1 + 4)

= 5 + 7 – 6

= 6

By Cramer's Rule,

p = $\frac{D_p}{D} = \frac{-5}{-3} = \frac{5}{3}$

q = $D - \frac{q}{q} = \frac{-5}{3}$,

r = $\frac{D_r}{D} = \frac{6}{-3}$ = – 2

$\therefore \frac{1}{x} = \frac{5}{3}, \frac{1}{y} = \frac{-5}{3}, \frac{1}{z}$ = – 2

$\therefore$ x = $\frac{3}{5}, y = \frac{-3}{5}, z = \frac{-1}{2}$ are the solution of the given equatios.

Exercise 6.3 | Q 1.5 | Page 93

Solve the following equations using Cramer's Rule:

$\frac{2}{x} - \frac{1}{y} + \frac{3}{z} = 4, \frac{1}{x} - \frac{1}{y} + \frac{1}{z} = 2, \frac{3}{x} + \frac{1}{y} - \frac{1}{z}$ = 2

SOLUTION

Let $\frac{1}{x} = \text{p}, \frac{1}{y} = \text{q}, \frac{1}{z}$ = r

$\therefore$ The given equations become

2p – q + 3r = 4

p – q – r = 2

3p + q – r = 2

D = $\begin{vmatrix} 2 & -1 & 3 \\ 1 & -1 & 1 \\ 3 & 1 & -1 \end{vmatrix}$

= 2(1 – 1) – (– 1)(– 1 – 3) + 3(1 + 3)

= 0 – 4 + 12

= 8

D_p = $\begin{vmatrix} 4 & -1 & 3 \\ 2 & -1 & 1 \\ 2 & 1 & -1 \end{vmatrix}$

= 4(1 – 1) – (– 1)(– 2 –2) + 3(2 + 2)
= 0 – 4 + 12
= 8

$D_q = \begin{vmatrix} 2 & 4 & 3 \\ 1 & 2 & 1 \\ 3 & 2 & -1 \end{vmatrix}$

= 2(–2 – 2) –4(– 1 – 3) + 3(2 – 6)

= – 8 + 16 – 12

= – 4

$D_r = \begin{vmatrix} 2 & -1 & 4 \\ 1 & -1 & 2 \\ 3 & 1 & 2 \end{vmatrix}$

= 2(– 2 – 2) – (– 1)(2 – 6) + 4(1 + 3)

= – 8 – 4 + 16

= 4

By Cramer's Rule,

p = $\dfrac{D_p}{D} = \dfrac{8}{8}$ = 1

q = $\dfrac{D_q}{D} = \dfrac{-4}{8} = \dfrac{-1}{2}$

r = $\dfrac{D_r}{D} = \dfrac{4}{8} = \dfrac{1}{2}$

$\therefore \dfrac{1}{x} = 1, \dfrac{1}{y} = \dfrac{-1}{2}, \dfrac{1}{z} = \dfrac{1}{2}$

∴ x = 1, y = – 2 and z = 2 are the solutions of the given equations.

Exercise 6.3 | Q 2 | Page 93

An amount of ₹ 5,000 is invested in three plans at rates 6%, 7% and 8% per annum respectively. The total annual income from these investments is ₹ 350. If the total annual income from first two investments is ₹ 70 more than the income from the third, find the amount invested in each plan by using Cramer's Rule.

SOLUTION

Let the amount of each investment be ₹ x, ₹ y and ₹ z.
According to the given conditions,

x + y + z = 5000
6%x + 7y + 8z = 350

$\therefore \frac{6}{100}x + \frac{7}{100}y + \frac{8}{100}z$ = 350

∴ 6x + 7y + 8z = 35000

6%x + 7%y = 8%z + 70

$\therefore \frac{6}{100}x + \frac{7}{100}y = \frac{8}{100}z + 70$

∴ 6x + 7y = 8z + 7000

∴ 6x + 7y – 8z = 7000

$\therefore D = \begin{vmatrix} 1 & 1 & 1 \\ 6 & 7 & 8 \\ 6 & 7 & -8 \end{vmatrix}$

= 1(– 56 – 56) – 19– 48 – 48) + 1(42 –42)

= – 112 + 96 + 0

= – 16

$D_x = \begin{vmatrix} 5000 & 1 & 1 \\ 35000 & 7 & 8 \\ 7000 & 7 & -8 \end{vmatrix}$

Taking 1000 common from C_1, we get

$D_x = \begin{vmatrix} 5 & 1 & 1 \\ 35 & 7 & 8 \\ 7 & 7 & -8 \end{vmatrix}$

Applying $C_1 \to C_1 - {}^5C_3$ and $C_2 \to C_2 - C_3$, we get

$D_x = 1000\begin{vmatrix} 0 & 0 & 1 \\ -5 & -1 & 8 \\ 47 & 15 & -8 \end{vmatrix}$

= 1000 [0 – 0 + 1(– 75 + 47)

= 1000 x (– 28) = – 28000

$$D_y = \begin{vmatrix} 1 & 5000 & 1 \\ 6 & 35000 & 8 \\ 6 & 7000 & -8 \end{vmatrix}$$

Taking 1000 common from C_2, we get

$$D_y = 1000\begin{vmatrix} 1 & 5 & 1 \\ 6 & 35 & 8 \\ 6 & 7 & -8 \end{vmatrix}$$

Applying $C_1 \rightarrow C_1 - C_3$ and $C_2 \rightarrow C_2 - {}^5C_3$, we get

$$D_y = 1000\begin{vmatrix} 0 & 0 & 1 \\ -2 & -5 & 8 \\ 14 & 47 & -8 \end{vmatrix}$$

= 1000 [0 – 0 + 1(– 94 + 70)

= 1000(– 24)

= – 24000

$$D_z = \begin{vmatrix} 1 & 1 & 5000 \\ 6 & 7 & 35000 \\ 6 & 7 & 7000 \end{vmatrix}$$

Taking 1000 common from C_3, we get

$$D_z = 1000\begin{vmatrix} 0 & 1 & 5 \\ 6 & 7 & 35 \\ 6 & 7 & 7 \end{vmatrix}$$

Applying $C_1 \rightarrow C_1 - C_2$ and $C_3 \rightarrow C_3 - {}^5C_2$, we get

$$D_z = 1000\begin{vmatrix} 0 & 1 & 0 \\ -1 & & 0 \\ -1 & 7 & -28 \end{vmatrix}$$

$= 1000[0 - 1(28 - 0) + 0]$

$= 1000 \times (-28)$

$= -28000$

By Cramer's Rule,

$$x = \frac{D_x}{D} = \frac{-28000}{-16} = 1750$$

$$y = D\frac{y}{D} = \frac{-24000}{-16} = 1500$$

$$z = D\frac{z}{D} = \frac{-28000}{-16} = 1750$$

∴ Amounts of investments are ₹ 1750, ₹ 1500 and ₹ 1750.

Exercise 6.3 | Q 3 | Page 93

Show that the following equations are consistent: $2x + 3y + 4 = 0$, $x + 2y + 3 = 0$, $3x + 4y + 5 = 0$

SOLUTION

Given equations are

$2x + 3y + 4 = 0$

$x + 2y + 3 = 0$

$3x + 4y + 5 = 0$

$$\therefore \begin{vmatrix} 2 & 3 & 4 \\ 1 & 2 & 3 \\ 3 & 4 & 5 \end{vmatrix}$$

$= 2(10 - 12) - 3(5 - 9) + 4(4 - 6)$

$= 2(-2) - 3(-4) + 4(-2)$

$= -4 + 12 - 8$

$= 0$

$\therefore$ The given equations are consistent.

Exercise 6.3 | Q 4.1 | Page 93

Find k, if the following equations are consistent: x + 3y + 2 = 0, 2x + 4y – k = 0, x – 2y – 3k = 0

SOLUTION

Given equations are

$x + 3y + 2 = 0$

$2x + 4y - k = 0$

$x - 2y - 3k = 0$

Since, these equations are consistent.

$$\therefore \begin{vmatrix} \mathbf{1} & \mathbf{3} & \mathbf{2} \\ \mathbf{2} & \mathbf{4} & -\mathbf{k} \\ \mathbf{1} & -\mathbf{2} & -\mathbf{3k} \end{vmatrix} = 0$$

$\therefore 1(-12k - 2k) - 3(-6k + k) + 2(-4 - 4) = 0$

$\therefore -14k + 15k - 16 = 0$

$\therefore k - 16 = 0$

$\therefore k = 16$

Exercise 6.3 | Q 4.2 | Page 93

Find k, if the following equations are consistent: (k – 2)x + (k – 1)y = 17, (k – 1)x + (k – 2)y = 18, x + y = 5

SOLUTION

Given equations are
$(k - 2)x + (k - 1)y = 17$
$(k - 1)x + (k - 2)y = 18$
$x + y = 5$
Since, these equations are consistent.

$$\therefore \begin{vmatrix} k-2 & k-1 & -17 \\ k-1 & k-2 & -18 \\ 1 & 1 & -5 \end{vmatrix} = 0$$

Applying $R_1 \to R_1 - R_2$, we get

$$\begin{vmatrix} -1 & 1 & 1 \\ k-1 & k-21 & -18 \\ 1 & 1 & -5 \end{vmatrix} = 0$$

$\therefore -1(-5k + 10 + 18) - 1(-5k + 5 + 18) + 1(k - 1 - k + 2) = 0$

$\therefore -1(-5k + 28) - 1(-5k + 23) + 1(1) = 0$

$\therefore 5k - 28 + 5k - 23 + 1 = 0$

$\therefore 10k - 50 = 0$

$\therefore k = 5.$

Exercise 6.3 | Q 5.1 | Page 93

Find the area of the triangle whose vertices are: (4, 5), (0, 7), (–1, 1)

SOLUTION

Here, $A(x_1, y_1) \equiv A(4, 5)$, $B(x_2, y_2) \equiv B(0, 7)$, $C(x_3, y_3) \equiv C(-1, 1)$

$$\text{Area of a triangle} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$\therefore A(\Delta\, ABC) = \frac{1}{2}\begin{vmatrix} 4 & 5 & 1 \\ 0 & 7 & 1 \\ -1 & 1 & 1 \end{vmatrix}$$

$$= \frac{1}{2}[4(7-1) - 5(0+1) + 1(0+7)]$$

$$\therefore A(\Delta\, ABC) = \frac{1}{2}(24 - 5 + 7)$$

= 13 sq.units.

Exercise 6.3 | Q 5.2 | Page 93

Find the area of the triangle whose vertices are: (3, 2), (–1, 5), (–2, –3)

SOLUTION

Here, $A(x_1, y_1) \equiv A(3, 2)$, $B(x_2, y_2) \equiv B(-1, 5)$, $C(x_3, y_3) \equiv C(-2, -3)$

$$\text{Area of a triangle} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$\therefore A(\Delta ABC) = \frac{1}{2}\begin{vmatrix} 3 & 2 & 1 \\ -1 & 5 & 1 \\ -2 & -3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}[3(5+3) - 2(-1+2) + 1(3+10)]$$

$$= \frac{1}{2}(24 - 2 + 13)$$

$$\therefore A(\Delta ABC) = \frac{35}{2} \text{ sq.unitts.}$$

Exercise 6.3 | Q 5.3 | Page 93

Find the area of the triangle whose vertices are: (0, 5), (0, – 5), (5, 0)

SOLUTION

Here, $A(x_1, y_1) \equiv A(0, 5)$, $B(x_2, y_2) \equiv B(0, -5)$, $C(x_3, y_3) \equiv C(5, 0)$

$$\text{Area of a triangle} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$\therefore A(\Delta ABC) = \frac{1}{2}\begin{vmatrix} 0 & 5 & 1 \\ 0 & -5 & 1 \\ 5 & 0 & 1 \end{vmatrix}$$

$$= \frac{1}{2}[0(-5-0) - 5(0-5) + 1(0+25)]$$

$= \frac{1}{2}(0 + 25 + 25)$

$= \frac{50}{2}$

$\therefore$ A(ΔABC) = 25 sq. units

Exercise 6.3 | Q 6 | Page 93

Find the value of k, if the area of the triangle with vertices at A(k, 3), B(–5, 7), C(–1, 4) is 4 square units.

SOLUTION

Here, $A(x_1, y_1) \equiv A(k, 3)$, $B(x_2, y_2) \equiv B(-5, 7)$, $C(x_3, y_3) \equiv C(-1, 4)$
A(ΔABC) = 4 sq. units

$$\text{Area of a triangle} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$\therefore \frac{1}{2}\begin{vmatrix} k & 3 & 1 \\ -5 & 7 & 1 \\ -1 & 4 & 1 \end{vmatrix} = \pm 4$$

$\therefore k(7 - 4) - 3(-5 + 1) + 1(-20 + 7) = \pm 8$

$\therefore 3k + 12 - 13 = \pm 8$

$\therefore 3k - 1 = \pm 8$

$\therefore 3k - 1 = 8$ or $3k - 1 = -8$

$\therefore 3k = 9$ or $3k = -7$

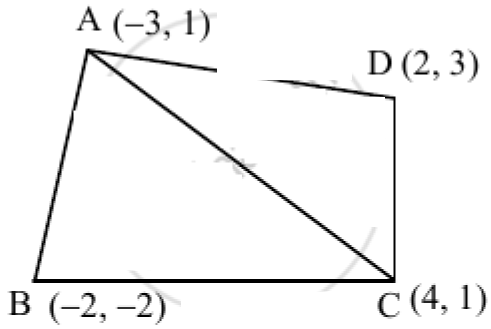
$\therefore k = 3$ or $k = \frac{-7}{3}$

Exercise 6.3 | Q 7 | Page 93

Find the area of the quadrilateral whose vertices are A(–3, 1), B(–2, –2), C(4, 1), D(2, 3).

SOLUTION

A(–3, 1), B(–2, –2), C(4, 1), D(2, 3)



A(ΔABCD) = A(ΔABC) + A(ΔACD)

Area of triangle = $\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

A(ΔABC) = $\frac{1}{2}\begin{vmatrix} -3 & 1 & 1 \\ -2 & -2 & 1 \\ 4 & 1 & 1 \end{vmatrix}$

= $\frac{1}{2}[-3(-2-1)-1(-2-4)+1(-2+8)]$

= $\frac{1}{2}(9+6+6)$

∴ A(ΔABC) = $\frac{21}{2}$ sq. units'.

∴ A(ΔACD) = $\frac{1}{2}\begin{vmatrix} -3 & 1 & 1 \\ 4 & 1 & 1 \\ 2 & 3 & 1 \end{vmatrix}$

= $\frac{1}{2}[-3(1-3)-1(4-2)+1(12-2)]$

= $\frac{1}{2}(6-2+10)$

$\therefore$ A(ΔACD) = 7 sq. units

$\therefore$ A(ΔABCD) = A(ΔABC) + A(ΔACD)

$= \frac{21}{2} + 7$

$= \frac{35}{2}$ sq. units.

Exercise 6.3 | Q 8 | Page 93
By using determinant, show that the following points are collinear: P(5, 0), Q(10, –3), R(–5, 6)

SOLUTION

Here, $P(x_1, y_1) \equiv P(5, 0)$, $Q(x_2, y_2) \equiv Q(10, -3)$, $R(x_3, y_3) \equiv R(-5, 6)$

If A(ΔPQR) = 0, then the points P, Q, R are collinear.

$$\therefore A(\Delta PQR) = \frac{1}{2}\begin{vmatrix} 5 & 0 & 1 \\ 10 & -3 & 1 \\ -5 & 6 & 1 \end{vmatrix}$$

$$= \frac{1}{2}[5(-3-6) - 0(10+5) + 1(60-15)]$$

$$= \frac{1}{2}(-45+0+45) = 0$$

$\therefore$ A(ΔPQR) = 0

$\therefore$ Points P, Q and R are collinear.

Exercise 6.3 | Q 9 | Page 94
The sum of three numbers is 15. If the second number is subtracted from the sum of first and third numbers, then we get 5. When the third number is subtracted from the sum of twice the first number and the second number, we get 4. Find the three numbers.

SOLUTION

Let the three numbers be x, y and z.
According to the given conditions,
$x + y + z = 15$
$x + z - y = 5$ i.e. $x - y + z = 5$
$2x + y - z = 4$

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 2 & 1 & -1 \end{vmatrix}$$

$= 1(1 - 1) - 1(-1 - 2) + 1(1 + 2)$

$= 1(0) - 1(-3) + 1(3)$

$= 0 + 3 + 3$

$= 6 \neq 0$

$$D_x = \begin{vmatrix} 15 & 1 & 1 \\ 5 & -1 & 1 \\ 4 & 1 & -1 \end{vmatrix}$$

$= 15(1 - 1) - 1(-5 - 4) + 1(5 + 4)$

$= 15(0) - 1(-9) + 1(9)$

$= 0 + 9 + 9$

$= 18$

$$D_y = \begin{vmatrix} 1 & 15 & -1 \\ 1 & 5 & 1 \\ 2 & 4 & -1 \end{vmatrix}$$

$= 1(-5 - 4) - 15(-1 - 2) + 1(4 - 10)$

$= 1(-9) - 15(-3) + 1(-6)$

$= -9 + 45 - 6$

$= 30$

$$D_z = \begin{vmatrix} 1 & 1 & 15 \\ 1 & -1 & 5 \\ 2 & 1 & 4 \end{vmatrix}$$

$= 1(-4 - 5) - 1(4 - 10) + 15(1 + 2)$

$= 1(-9) - 1(-6) + 15(3)$

$= -9 + 6 + 45$

$= 42$

By Cramer's Rule,

x = $\frac{D_x}{D} = \frac{18}{6}$ = 3,

y = $\frac{D_y}{D} = \frac{30}{6}$ = 5,

z = $\frac{D_x}{D} = \frac{42}{6}$ = 7

∴ The three numbers are 3, 5 and 7.

MISCELLANEOUS EXERCISE 6 [PAGES 94 - 95]

Miscellaneous Exercise 6 | Q 1.1 | Page 94

Evaluate: $\begin{vmatrix} 2 & -5 & 7 \\ 5 & 2 & 1 \\ 9 & 0 & 2 \end{vmatrix}$

SOLUTION

$\begin{vmatrix} 2 & -5 & 7 \\ 5 & 2 & 1 \\ 9 & 0 & 2 \end{vmatrix}$

= $2\begin{vmatrix} 2 & 1 \\ 9 & 2 \end{vmatrix} - (-5)\begin{vmatrix} 5 & 1 \\ 9 & 2 \end{vmatrix} + 7\begin{vmatrix} 5 & 2 \\ 9 & 0 \end{vmatrix}$

= = 2(4 – 0) + 5(10 – 9) + 7(0 – 18)

= 2(4) + 5(1) + 7(– 18)

= 8 + 5 – 126

= – 113.

Miscellaneous Exercise 6 | Q 1.2 | Page 94

Evaluate: $\begin{vmatrix} 1 & -3 & 12 \\ 0 & 2 & -4 \\ 9 & 7 & 2 \end{vmatrix}$

SOLUTION

$$\begin{vmatrix} 1 & -3 & 12 \\ 0 & 2 & -4 \\ 9 & 7 & 2 \end{vmatrix}$$

$= 1\begin{vmatrix} 2 & -4 \\ 7 & 2 \end{vmatrix} - (-3)\begin{vmatrix} 0 & 4 \\ 9 & 2 \end{vmatrix} + 12\begin{vmatrix} 0 & 2 \\ 9 & 7 \end{vmatrix}$

= 1(4 + 28) + 3(0 + 36) + 12(0 – 18)

= 1(32) + 3(36) + 12(– 18)

= 32 + 108 – 216

= – 76.

Miscellaneous Exercise 6 | Q 2.1 | Page 94

Find the value (s) of x, if $\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & -5 \\ 1 & 2x & 5x^2 \end{vmatrix}$ = 0

SOLUTION

$\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & -5 \\ 1 & 2x & 5x^2 \end{vmatrix}$ = 0

$\therefore 1(-10x^2 + 10x) - 4(5x^2 + 5) + 20(2x + 2) = 0$

$\therefore -10x^2 + 10x - 20x^2 - 20 + 40x + 40 = 0$

$\therefore -30x^2 + 50x + 20 = 0$

$\therefore 3x^2 - 5x - 2 = 0$ ...[Dividding throughtout by (– 10)]

$\therefore 3x^2 - 6x + x - 2 = 0$

$\therefore 3x(x - 2) + 1(x - 2) = 0$

$\therefore (x - 2)(3x + 1) = 0$

$\therefore$ x – 2 = 0 or 3x + 1 = 0

$\therefore$ x = 2 or x = $-\frac{1}{3}$.

Miscellaneous Exercise 6 | Q 2.2 | Page 94

Find the value (s) of x, if $\begin{vmatrix} 1 & 2x & 4x \\ 1 & 4 & 16 \\ 1 & 1 & 1 \end{vmatrix}$ = 0

SOLUTION

$\begin{vmatrix} 1 & 2x & 4x \\ 1 & 4 & 16 \\ 1 & 1 & 1 \end{vmatrix}$ = 0

$\therefore$ 1(4 – 16) – 2x(1 – 16) + 4x(1 – 4) = 0

$\therefore$ 1(– 12) – 2x(–15) + 4x(– 3) = 0

$\therefore$ – 12 + 30x – 12x = 0

$\therefore$ 18x = 12

$\therefore$ x = $\frac{12}{18} = \frac{2}{3}$.

Miscellaneous Exercise 6 | Q 3 | Page 95

By using properties of determinants, prove that $\begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$ = 0.

SOLUTION

L.H.S. = $\begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$

Applying $R_1 \to R_1 + R_2$, we get

L.H.S. = $\begin{vmatrix} x+y+z & z+y+z & x+y+z \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$

Taking (x + y + z) common from R_1, we get

L.H.S. = $(x+y+z)\begin{vmatrix} 1 & 1 & 1 \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$

= (x + y + z) (0) ...[∵ R_1 and R_3 are identical]

= 0

= R.H.S.

Miscellaneous Exercise 6 | Q 4.1 | Page 95

Without expanding the determinants, show that $\begin{vmatrix} \mathrm{b+c} & \mathrm{bc} & \mathrm{b^2c^2} \\ \mathrm{c+a} & \mathrm{ca} & \mathrm{c^2a^2} \\ \mathrm{a+b} & \mathrm{ab} & \mathrm{a^2b^2} \end{vmatrix}$ = 0

SOLUTION

L.H.S. = $\begin{vmatrix} \mathrm{b+c} & \mathrm{bc} & \mathrm{b^2c^2} \\ \mathrm{c+a} & \mathrm{ca} & \mathrm{c^2a^2} \\ \mathrm{a+b} & \mathrm{ab} & \mathrm{a^2b^2} \end{vmatrix}$

Taking bc, ca, ab common from R_1, R_2, R_3 respectively, we get

L.H.S. = $(\mathrm{bc})(\mathrm{ca})(\mathrm{ab})\begin{vmatrix} \frac{\mathrm{b-c}}{\mathrm{bc}} & 1 & \mathrm{bc} \\ \frac{\mathrm{c+a}}{\mathrm{ca}} & 1 & \mathrm{ca} \\ \frac{\mathrm{a+b}}{\mathrm{ab}} & 1 & \mathrm{ab} \end{vmatrix}$

Taking abc common from C_3, we get

L.H.S. = $\left(a^2b^2c^2\right)(abc)\begin{vmatrix} \frac{1}{c}+\frac{1}{b} & 1 & \frac{1}{a} \\ \frac{1}{a}+\frac{1}{c} & 1 & \frac{1}{b} \\ \frac{1}{b}+\frac{1}{a} & 1 & \frac{1}{c} \end{vmatrix}$

Applying $C_1 \to C_1 + C_3$, we get

L.H.S. = $a^3b^3c^3\begin{vmatrix} \frac{1}{a}+\frac{1}{b}+\frac{1}{c} & 1 & \frac{1}{a} \\ \frac{1}{a}+\frac{1}{b}+\frac{1}{c} & 1 & \frac{1}{b} \\ \frac{1}{a}+\frac{1}{b}+\frac{1}{c} & 1 & \frac{1}{c} \end{vmatrix}$

Taking $\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$ common from C_1, we get

L.H.S. = $a^3b^3c^3\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\begin{vmatrix} 1 & 1 & \frac{1}{a} \\ 1 & 1 & \frac{1}{b} \\ 1 & 1 & \frac{1}{c} \end{vmatrix}$

= $a^3b^3c^3\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)(0)$ ...[C_1 and C_2 are identical]

= 0

= R.H.S.

Miscellaneous Exercise 6 | Q 4.2 | Page 95

Without expanding the determinants, show that $\begin{vmatrix} xa & yb & zc \\ a^2 & b^2 & c^2 \\ 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} x & y & z \\ a & b & c \\ bc & ca & ab \end{vmatrix}$

SOLUTION

L.H.S. = $\begin{vmatrix} xa & yb & zc \\ a^2 & b^2 & c^2 \\ 1 & 1 & 1 \end{vmatrix}$

= Taking a, b, c common from C_1, C_2, C_3 respectively, we get

$$\text{L.H.S.} = abc\begin{vmatrix} x & y & z \\ a & b & c \\ \frac{1}{a} & \frac{1}{b} & \frac{1}{c} \end{vmatrix}$$

$$= \begin{vmatrix} x & y & z \\ a & b & c \\ \frac{abc}{a} & \frac{abc}{b} & \frac{abc}{c} \end{vmatrix}$$

$$= \begin{vmatrix} x & y & z \\ a & b & c \\ bc & ca & ab \end{vmatrix}$$

= R.H.S.

Miscellaneous Exercise 6 | Q 4.3 | Page 95

Without expanding the determinants, show that $\begin{vmatrix} l & m & n \\ e & d & f \\ u & v & w \end{vmatrix} = \begin{vmatrix} n & f & w \\ l & e & u \\ m & d & v \end{vmatrix}$

SOLUTION

$$\text{L.H.S.} = \begin{vmatrix} l & m & n \\ e & d & f \\ u & v & w \end{vmatrix}$$

Interchanging rows and columns, we get

$$\text{L.H.S.} = \begin{vmatrix} l & e & u \\ m & d & v \\ n & f & w \end{vmatrix}$$

Applying $R_2 \leftrightarrow R_3$, we get

L.H.S. = $\begin{vmatrix} l & e & u \\ m & f & w \\ m & d & v \end{vmatrix}$

Applying $R_1 \leftrightarrow R_2$, we get

L.H.S. = $\begin{vmatrix} n & f & w \\ l & e & u \\ m & d & v \end{vmatrix}$

= R.H.S.

Miscellaneous Exercise 6 | Q 4.4 | Page 95

Without expanding the determinants, show that $\begin{vmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{vmatrix}$ = 0

SOLUTION

Let D = $\begin{vmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{vmatrix}$

Taking (– 1) common from R_1, R_2, R_3, we get

D = $(-1)^3 \begin{vmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{vmatrix}$

Interchanging rows and columns, we get

D = $-1 \begin{vmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{vmatrix}$

∴ D = – 1(D)

$\therefore$ 2D = 0

$\therefore$ D = 0

$\therefore \begin{vmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{vmatrix} = 0.$

Miscellaneous Exercise 6 | Q 5.1 | Page 95

Solve the following linear equations by Cramer's Rule: 2x – y + z = 1, x + 2y + 3z = 8, 3x + y – 4z = 1

SOLUTION

Given equations are

2x – y + z = 1

x + 2y + 3z = 8

3x + y – 4z = 1

$D = \begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & -4 \end{vmatrix}$

= 2(– 8 – 3) – (– 1)(– 4 – 9) + 1(1 – 6)

= 2(– 11) + 1(– 13) + 1(– 5)

= – 22 – 13 – 5

= – 40 ≠ 0

$D_x = \begin{vmatrix} 1 & -1 & 1 \\ 8 & 2 & 3 \\ 1 & 1 & -4 \end{vmatrix}$

= 1(– 8 – 3) – (– 1)(– 32 – 30 + 1(8 – 2)

= 1(– 11) + 1(– 35) + 1(6)

= – 11 – 35 + 6

= – 40

$D_y = \begin{vmatrix} 2 & 1 & 1 \\ 1 & 8 & 3 \\ 3 & 1 & -4 \end{vmatrix}$

= 2(– 32 – 3) – 1(– 4 – 9) + 1(1 – 24)

= 2(- 35) – 1(– 13) + 1(– 23)

= – 70 + 13 – 23

= – 80

$D_z = \begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & 8 \\ 3 & 1 & 1 \end{vmatrix}$

= 2(2 – 8) – (– 1)(1 – 24) + 1(1 – 6)

= 2(– 6) + 1(– 23) + 1(– 5)

= – 12 – 23– 5

= – 40

By Cramer's Rule,

x = $\frac{D_x}{D} = \frac{-40}{-40}$ = 1

y = $\frac{D_y}{D} = \frac{-80}{-40}$ = 2,

z = $\frac{D_z}{D} = \frac{-40}{-40}$ = 1

∴ x = 1, y = 2 and z = 1 are the solutions of the given equations.

Miscellaneous Exercise 6 | Q 5.2 | Page 95

Solve the following equations using Cramer's Rule:

$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = -2, \ \frac{1}{x} - \frac{2}{y} + \frac{1}{z} = 3, \ \frac{2}{x} - \frac{1}{y} + \frac{3}{z}$ = -1

SOLUTION

Let $\frac{1}{x}$ p, $\frac{1}{y}$ = q, $\frac{1}{z}$ = r

∴ The given equations become

p + q + r = − 2

p − 2q + r = 3

2p − q + 3r = − 1

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -2 & 1 \\ 2 & -1 & 3 \end{vmatrix}$$

= 1(− 6 + 1) − 1(3 − 2) + 1(− 1 + 4)

= 5 − 1 + 3

= − 3

$$D_p = \begin{vmatrix} -2 & 1 & 1 \\ 3 & -2 & 1 \\ -1 & -1 & 3 \end{vmatrix}$$

= − 2(− 6 + 1) − 1(9 + 1) + 1(− 3 − 2)

= 10 − 10 − 5

= − 5

$$D_q = \begin{vmatrix} 1 & -2 & 1 \\ 1 & 3 & 1 \\ 2 & -1 & 3 \end{vmatrix}$$

= 1(9 + 1) + 2(3 − 2) + 1(− 1 −6)

= 10 + 2 − 7

= 5

$$D_r = \begin{vmatrix} 1 & 1 & -2 \\ 1 & -2 & 3 \\ 2 & -1 & -1 \end{vmatrix}$$

= 1(2 + 3) – 1(– 1 –6) – 2(– 1 + 4)

= 5 + 7 – 6

= 6

By Cramer's Rule,

p = $\frac{\mathrm{D_p}}{\mathrm{D}} = \frac{-5}{-3} = \frac{5}{3}$

q = $\mathrm{D} - \frac{\mathrm{q}}{\mathrm{q}} = \frac{-5}{3}$,

r = $\frac{\mathrm{D_r}}{\mathrm{D}} = \frac{6}{-3}$ = – 2

∴ $\frac{1}{x} = \frac{5}{3}, \frac{1}{y} = \frac{-5}{3}, \frac{1}{z}$ = – 2

∴ x = $\frac{3}{5}, y = \frac{-3}{5}, z = \frac{-1}{2}$ are the solution of the given equatios.

Miscellaneous Exercise 6 | Q 5.3 | Page 95

Solve the following linear equations by Cramer’s Rule: x – y + 2z = 7, 3x + 4y – 5z = 5, 2x – y + 3z = 12

SOLUTION

Given equations are

x – y + 2z = 7

3x + 4y – 5z = 5

2x – y + 3z = 12

D = $\begin{vmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{vmatrix}$

= 1(12 – 5) – (– 1)(9 + 1) + (–3– 8)

= 1(7) + 1(19) + 2(– 11)

= 7 + 19 – 22

= 4 ≠ 0

$$D_x = \begin{vmatrix} 7 & -1 & 2 \\ 5 & 4 & -5 \\ 12 & -1 & 3 \end{vmatrix}$$

= 7(12 ≠ 5) – (– 1)(15 + 60) + 2(–5 – 48)

= 7(7) + 1(75) + 2(– 53)

= 49 + 75 – 106

= 18

$$D_y = \begin{vmatrix} 1 & 7 & 2 \\ 3 & 5 & -5 \\ 2 & 12 & 3 \end{vmatrix}$$

= 1(15 + 60) –7(9 + 10) + 2(36 – 10)

= 1(75) – 7(9) + 2(26)

= 75 – 133 + 52

= – 6

$$D_z = \begin{vmatrix} 1 & -1 & 7 \\ 3 & 4 & 5 \\ 2 & -1 & 12 \end{vmatrix}$$

= 1(48 + 5) – (– 1)(36 – 1) + 7(– 3 – 8)

= 1(53) (26) 7(– 1)

= 3 + 26 – 77

= 2

By Cramer's Rule,

$$x = \frac{D_x}{D} = \frac{18}{4} = \frac{9}{2},$$

$$y = \frac{D_y}{D} = \frac{-}{4} = \frac{-3}{2},$$

$z = \frac{D_z}{D} = \frac{2}{4} = \frac{1}{2}$

$\therefore x = \frac{9}{2}, y = \frac{-3}{2}$ and $z = \frac{1}{2}$ are the solutions of the given equations.

Miscellaneous Exercise 6 | Q 6.1 | Page 95

Find the value (s) of k, if the following equations are consistent: 3x + y – 2 = 0, kx + 2y – 3 = 0 and 2x – y = 3

SOLUTION

Given equations are

3x + y – 2 = 0

kx + 2y – 3 = 0

2x – y = 3 i.e. 2x – y – 3 = 0

Since, these equations are consistent.

$$\begin{vmatrix} 3 & 1 & -2 \\ k & 2 & -3 \\ 2 & -1 & -3 \end{vmatrix} = 0$$

$\therefore 3(-6 - 3) - 1(-3k + 6) - 2(-k - 4) = 0$

$\therefore 3(-9) - 1(-3k + 6) - 2(-k - 4) = 0$

$\therefore -27 + 3k - 6 + 2k + 8 = 0$

$\therefore 5k - 25 = 0$

$\therefore k = \frac{25}{5}$

$= 5.$

Miscellaneous Exercise 6 | Q 6.2 | Page 95

Find the value (s) of k, if the following equations are consistent: kx + 3y + 4 = 0, x + ky + 3 = 0, 3x + 4y + 5 = 0

SOLUTION

Given equations are

kx + 3y + 4 = 0

x + ky + 3 = 0

3x + 4y + 5 = 0
Since, these equations are consistent.

$$\therefore \begin{vmatrix} k & 3 & 4 \\ 1 & k & 3 \\ 3 & 4 & 5 \end{vmatrix} = 0$$

$\therefore$ k(5k – 12) – 3(5 –9) + 4(4 – 3k) = 0

$\therefore$ 5k^2 – 12k + 12 + 16 – 12k = 0

$\therefore$ 5k^2 – 24k + 28 = 0

$\therefore$ 5k^2 – 10k – 14k + 28 = 0

$\therefore$ 5k(k – 2) –14(k – 2) = 0

$\therefore$ (k – 2) (5k – 14) = 0

$\therefore$ k – 2 = 0 or 5k – 14 = 0

$\therefore$ k = 2 or k = $\frac{14}{5}$

Miscellaneous Exercise 6 | Q 7.1 | Page 95

Find the area of triangles whose vertices are A(− 1, 2), B(2, 4), C(0, 0)

SOLUTION

Here, A(x_1, y_1) ≡ A(–1, 2), B(x_2, y_2) ≡ B(2, 4), C(x_3, y_3) ≡ C(0, 0)

$$\text{Area of a triangle} = \frac{1}{2}\begin{vmatrix} x1 & y1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$\therefore A(\Delta ABC) = \frac{1}{2}\begin{vmatrix} -1 & 2 & 1 \\ 2 & 4 & 1 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= \frac{1}{2}[-1(4-0) - 2(2-0) + 1(0-0)]$$

$$= \frac{1}{2}(-4-4)$$

$= \frac{1}{2}(-8)$

$= -4$

Since, area cannot be negative.

$\therefore$ A(ΔABC) = 4 sq. units

Miscellaneous Exercise 6 | Q 7.2 | Page 95

Find the area of triangles whose vertices are P(3, 6), Q(−1, 3), R(2, −1)

SOLUTION

Here, P(x_1, y_1) ≡ P(3, 6), Q(x_2, y_2) ≡ Q(−1, 3), R(x_3, y_3) □ R(2, −1)

$$\text{Area of triangle} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$\therefore A(\Delta PQR) = \frac{1}{2}\begin{vmatrix} 3 & 6 & 1 \\ -1 & 3 & 1 \\ 2 & -1 & 1 \end{vmatrix}$$

$$= \frac{1}{2}[3(3+1) - 6(-1-2) + 1(1-6)]$$

$$= \frac{1}{2}[3(4) - 6(-3) + 1(-5)]$$

$$= \frac{1}{2}(12 + 18 - 5)$$

$$\therefore A(\Delta PQR) = \frac{25}{2} \text{ sq. units}$$

Miscellaneous Exercise 6 | Q 7.3 | Page 95

Find the area of triangles whose vertices are L(1, 1), M(−2, 2), N(5, 4)

SOLUTION

Here, L(x_1, y_1) ≡ L(1, 1), M(x_2, y_2) ≡ M(–2, 2), N(x_3, y_3) ≡ N(5, 4)

Area of a triangle = $\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

∴ A(ΔLMN) = $\frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ -2 & 2 & 1 \\ 5 & 4 & 1 \end{vmatrix}$

= $\frac{1}{2}[1(2-4) - 1(-2-5) + 1(-8-10)]$

= $\frac{1}{2}[1(-2) - 1(-7) + 1(-18)]$

= $\frac{1}{2}(-2+7-18)$

= $-\frac{13}{2}$

Since, area cannot be negative.

∴ A(ΔLMN) = $\frac{13}{2}$ sq. units

Miscellaneous Exercise 6 | Q 8.1 | Page 95

Find the value of k, if area of ΔPQR is 4 square units and vertices are P(k, 0), Q(4, 0), R(0, 2).

SOLUTION

Here, P(x_1, y_1) ≡ P(k, 0), Q(x_2, y_2) ≡ Q(4, 0), R(x_3, y_3) ≡ R(0, 2)
A(ΔPQR) = 4 sq. units

Area of a triangle = $\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y3 & 1 \end{vmatrix}$

∴ ± 4 = $\frac{1}{2}\begin{vmatrix} \mathrm{k} & 0 & 1 \\ 4 & 0 & 1 \\ 0 & 2 & 1 \end{vmatrix}$

$\therefore \pm 4 = \frac{1}{2}[k(0-2) - 0 + 1(8-0)]$

$\therefore \pm 8 = -2k + 8$

$\therefore 8 = -2k + 8$ or $-8 = -2k + 8$

$\therefore -2k = 0$ or $2k = 16$

$\therefore k = 0$ or $k = 8$

Miscellaneous Exercise 6 | Q 8.2 | Page 95

Find the value of k, if area of ΔLMN is 33/2 square units and vertices are L(3, − 5), M(− 2, k), N(1, 4).

SOLUTION

Here, $L(x_1, y_1) \equiv L(3, -5)$, $M(x_2, y_2) \equiv M(-2, k)$, $N(x_3, y_3) \equiv N(1, 4)$

$A(\Delta LMN) = \frac{33}{2}$ q. units

Area of a triangle $= \frac{1}{2}\begin{vmatrix} x_1 & y1 & 1 \\ x2 & y_2 & 1 \\ x3 & y_3 & 1 \end{vmatrix}$

$$\therefore \pm\frac{33}{2} = \frac{1}{2}\begin{vmatrix} 3 & -5 & 1 \\ -2 & k & 1 \\ 1 & 4 & 1 \end{vmatrix}$$

$$\therefore \pm\frac{33}{2} = \frac{1}{2}[3(k-4) - (-5)(-2-1) + 1(-8-k)]$$

$\therefore \pm 33 = 3k - 12 - 5 - 8 - k$

$\therefore \pm 33 = 2k - 35$

$\therefore 2k - 35 = 33$ or $2k - 35 = -33$

$\therefore 2k = 68$ or $2k = 2$

$\therefore k = 34$ or $k = 1$