Chapter : 3 Theory of Equations (2)

Exercise 3.1

1. If the sides of a cubic box are increased by 1, 2, 3 units respectively to form a cuboid, then the volume is increased by 52 cubic units. Find the volume of the cuboid.

soln Let the sides of cubic boxes are x, x, x.

volume of a cube $= x^3$

it is increased by 1, 2, 3.

cuboid sides be $(x+1), (x+2)\ (x+3)$

volume of a cuboid

$$(x+1)(x+2)(x+3) = x^3+52.$$

$$(x^2+3x+2)(x+3) = x^3+52$$

$$\cancel{x^3}+3x^2+2x+3x^2+9x+6 = \cancel{x^3}+52$$

$$6x^2+11x+6-52=0$$

$$6x^2+11x-46=0$$

$$6x^2-12x+23x-46=0$$

$$6x(x-2)+23(x-2)=0$$

$x-2=0$ $6x+23=0$

$x=2$ $6x=-23$

$x=-\frac{23}{6}$ (is not possible)

$\therefore x=2.$

volume of a cube $= x^3 = 2^3 = 8.$

2. Construct a cubic equation with roots.

(i) 1, 2 and 3

$\alpha=1,\ \beta=2\ \ \gamma=3$

$\alpha+\beta+\gamma = 6$

$\alpha\beta+\beta\gamma+\gamma\alpha = 2+6+3=11$

$\alpha\beta\gamma = 6.$

$$x^3-(\alpha+\beta+\gamma)x^2+(\alpha\beta+\beta\gamma+\gamma\alpha)x-\alpha\beta\gamma=0$$

$$x^3-6x^2+11x-6=0$$

(ii) 1, 1 and -2.

$\alpha=1\quad \beta=1\quad \gamma=-2.$

நா. காமாட்சி, M.Sc., B.Ed., M.Phil., Ph.D
முதுகலை ஆசிரியர் (கணிதம்)
அரசு மேல்நிலைப்பள்ளி
கோவிந்தவாடி, காஞ்சிபுரம் (Dt)

Scanned by CamScanner

3

$\alpha+\beta+\gamma = 1+1-2 = 0$

$\alpha\beta+\beta\gamma+\gamma\alpha = 1-2-2 = -3$

$\alpha\beta\gamma = 1(1)(-2) = -2$

$x^3 - 0x^2 - 3x + 2 = 0$ $\boxed{x^3 - 3x + 2 = 0}$

(ii) $2, -2$ and 4

$\alpha = 2 \quad \beta = -2 \quad \gamma = 4$

$\alpha+\beta+\gamma = 2-2+4 = 4$

$\alpha\beta+\beta\gamma+\gamma\alpha = -4-8+8 = -4.$

$\alpha\beta\gamma = 2(-2)(4) = -16.$

$\boxed{x^3 - 4x^2 - 4x + 16 = 0}$

3. If α, β and γ are the roots of the cubic equation $x^3+2x^2+3x+4=0$ form a cubic equation whose roots are

(i) $2\alpha, 2\beta, 2\gamma.$

$x^3+2x^2+3x+4=0$

$\alpha+\beta+\gamma = \frac{-b}{a} = \frac{-2}{1} = -2$

$\alpha\beta+\beta\gamma+\gamma\alpha = c/a = \frac{3}{1} = 3$

$\alpha\beta\gamma = -d/a = -4.$

(i) $2\alpha+2\beta+2\gamma = 2(\alpha+\beta+\gamma) = 2(-2) = -4$

$(2\alpha)(2\beta)+(2\beta)(2\gamma)+(2\gamma)(2\alpha) = 4\alpha\beta+4\beta\gamma+4\alpha\gamma$

$= 4(\alpha\beta+\beta\gamma+\alpha\gamma) = 4(3) = 12.$

$2\alpha(2\beta)(2\gamma) = 8(\alpha\beta\gamma) = 8(-4) = -32.$

$x^3-(-4)x^2+12x-(-32) = 0$

$x^3+4x^2+12x+32 = 0.$

4. Solve the equation $3x^3-16x^2+23x-6=0$ if the product of two roots is 1

Soln Let the roots be $\alpha, \beta, \gamma.$

given $\alpha\beta = 1$

$\beta = 1/\alpha.$

நா. காமாட்சி, M.Sc., B.Ed., M.Phil., Ph.D.
முதுகலை ஆசிரியர் (கணிதம்)
அரசு மேல்நிலைப்பள்ளி
கோவிந்தவாடி, காஞ்சிபுரம் (Dt)

(4)

Sum of the roots $\alpha + \frac{1}{\alpha} + \gamma = \frac{16}{3}$ — (1)

Product of the roots $\alpha \cdot \frac{1}{\alpha} \cdot \gamma = \frac{6}{3} = 2$

$\gamma = 2.$

Put in (1)

$\alpha + 1/\alpha + 2 = \frac{16}{3} \Rightarrow \alpha + \frac{1}{\alpha} = \frac{16}{3} - 2 = \frac{16-6}{3} = \frac{10}{3}$

$\frac{\alpha^2+1}{\alpha} = \frac{10}{3} \Rightarrow 3\alpha^2 + 3 = 10\alpha \Rightarrow 3\alpha^2 - 10\alpha + 3 = 0$

$(3\alpha - 1)(\alpha - 3) = 0$

$\alpha = 3 \quad \alpha = 1/3$

If $\alpha = 3 \quad \beta = \frac{1}{3}, \gamma = 2$

(or) If $\alpha = \frac{1}{3} \quad \beta = 3 \quad \gamma = 2.$

5. Find the Sum of squares of roots of the equation $2x^4 - 8x^3 + 6x^2 - 3 = 0$

Soln

$\alpha + \beta + \gamma + \delta = 8/2 = 4$

$\alpha\beta + \beta\gamma + \gamma\alpha + \delta\alpha + \beta\delta + \gamma\delta = \frac{6}{2} = 3.$

$(a+b+c+d)^2 = a^2 + b^2 + c^2 + d^2 + 2(ab + bc + ac + ad + bc + bd + cd)$

$\alpha^2 + \beta^2 + \gamma^2 + \delta^2 = (\alpha + \beta + \gamma + \delta)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha + \delta\alpha + \beta\delta + \gamma\delta)$

$= 4^2 - 2(3) = 16 - 6 = 10.$

6. Solve the equation $x^3 - 9x^2 + 14x + 24 = 0$ if it is given that two of its roots are in the ratio 3 : 2.

Soln. Let the roots be $3\alpha, 2\alpha, \beta$.

Sum of the roots : $3\alpha + 2\alpha + \beta = +9$

$5\alpha + \beta = 9$ — (1).

Product of two roots : $(3\alpha)(2\alpha) + 2\alpha(\beta) + 3\alpha(\beta) = 14$

$6\alpha^2 + 5\alpha\beta = 14$ — (2)

Product of 3 roots : $(3\alpha)(2\alpha)\beta = -24$

$\alpha^2\beta = -4$ (3)

(1) $\Rightarrow \quad \beta = 9 - 5\alpha.$

(2) $\Rightarrow \quad 6\alpha^2 + 5\alpha(9 - 5\alpha) = 14$

$6\alpha^2 + 45\alpha - 25\alpha^2 = 14$

(5)

$-19\alpha^2+45\alpha-14=0$

$14\alpha^2-45\alpha+14=0$

$(\alpha-2)(\alpha-\frac{7}{19})=0$

$\alpha=2 \quad \alpha=\frac{7}{19}$

266, $-\frac{38}{19}$, $-\frac{7}{19}$

If $\alpha=2$. $\beta=9-5(\alpha)=9-5(2)=9-10=-1$

roots are $3\alpha, 2\alpha, \beta$

$3(2), 2(2), -1$ (ie) $6, 4, -1$

If $\alpha=\frac{2}{19}$ $\beta=9-5(\frac{2}{19})=\frac{136}{19}$.

roots are $3\alpha, 2\alpha, \beta$ (ie) $\frac{21}{19}, \frac{14}{19}, \frac{136}{19}$.

7) If α, β and γ are the roots of the polynomial equation $ax^3+bx^2+cx+d=0$ find the value of $\sum\frac{\alpha}{\beta\gamma}$ in terms of the coefficients

Soln

$$\sum\frac{\alpha}{\beta\gamma}=\frac{\alpha}{\beta\gamma}+\frac{\beta}{\alpha\gamma}+\frac{\gamma}{\alpha\beta}=\frac{\alpha^2+\beta^2+\gamma^2}{\alpha\beta\gamma}$$

$\alpha+\beta+\gamma=\frac{-b}{a}$, $\alpha\beta\gamma=-\frac{d}{a}$, $\alpha\beta+\beta\gamma+\gamma\alpha=c/a$

$$\alpha^2+\beta^2+\gamma^2=(\alpha+\beta+\gamma)^2-2(\alpha\beta+\beta\gamma+\gamma\alpha)$$

$$=\left(\frac{-b}{a}\right)^2-2\left(\frac{c}{a}\right)=\frac{b^2}{a^2}-\frac{2c}{a}$$

$$\sum\frac{\alpha}{\beta\gamma}=\frac{b^2/a^2-\frac{2c}{a}}{-d/a}=\frac{b^2-2ac}{a^2}\times\frac{a}{-d}=\frac{b^2-2ac}{-ad}$$

$$=\frac{2ac-b^2}{ad}.$$

8) If α, β, γ and δ are the roots of the polynomial equation $2x^4+5x^3-7x^2+8=0$. find a quadratic equation with integer coefficients whose roots are $\alpha+\beta+\gamma+\delta$ and $\alpha\beta\gamma\delta$.

Soln

p: $\alpha+\beta+\gamma+\delta=-5/2$

q: $\alpha\beta\gamma\delta=-8/2=-4$

நா. காமாட்சி, M.Sc., B.Ed., M.Phil., Ph.D.
முதுகலை ஆசிரியர் (கணிதம்)
அரசு மேல்நிலைப்பள்ளி
கோவிந்தவாடி, காஞ்சிபுரம் (Dt)

Scanned by CamScanner

$p+q = -5/2 - 4 = \frac{-5-8}{2} = \frac{-13}{2}$ (6)

$pq = -5/2(-4) = +10$

$x^2 - (p+q)x + pq = 0$

$x^2 + \frac{13}{2}x + 10 = 0$

$2x^2 + 13x + 20 = 0.$

9) If p and q are the roots of the equation $lx^2+nx+n=0$ show that $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}} = 0.$

Soln. p & q are the roots of the roots

$p+q = -\frac{n}{l}, \quad pq = \frac{n}{l}.$

LHS $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}} = \frac{\sqrt{p}}{\sqrt{q}} + \frac{\sqrt{q}}{\sqrt{p}} + \frac{\sqrt{n}}{\sqrt{l}} = \frac{p+q}{\sqrt{p}\sqrt{q}} + \frac{\sqrt{n}}{\sqrt{l}}$

$= \frac{-\frac{n}{l}}{\sqrt{pq}} + \frac{\sqrt{n}}{\sqrt{l}} = \frac{-\frac{n}{l}}{\frac{\sqrt{n}}{\sqrt{l}}} + \frac{\sqrt{n}}{\sqrt{l}}$

$= \frac{-\frac{n}{l} + \frac{n}{l}}{\sqrt{\frac{n}{l}}} = 0 = $ RHS.

10) If the equations $x^2+px+q=0$ and $x^2+p'x+q'=0$ have a common root, show that it must be equal to $\frac{pq'-p'q}{q-q'}$ or $\frac{q-q'}{p'-p}$

Soln let α be common roots of $x^2+px+q=0$; $x^2+p'x+q'=0$

(ie) $\alpha^2+p\alpha+q=0$ and $\alpha^2+p'\alpha+q'=0$

$\frac{\alpha^2}{pq'-p'q} = \frac{\alpha}{q-q'} = \frac{1}{p'-p}$

$$\begin{array}{cccc} \alpha^2 & \alpha & 1 & \\ 1 & p & q & 1 \\ 1 & p' & q' & 1 \end{array}$$

$\alpha^2 = \frac{pq'-p'q}{p'-p}$ $\boxed{\alpha = \frac{q-q'}{p'-p}}$

$\alpha = \frac{\alpha^2}{\alpha} = \frac{\frac{pq'-p'q}{p'-p}}{\frac{q-q'}{p'-p}} = \frac{pq'-p'q}{q-q'}$ Hence proved.

11) Formulate into a mathematical problem to find a number such that when its cube root is added to it, the result is 6.

Soln. Let the number be x

Its cube root be $\sqrt[3]{x}$.

$$x + \sqrt[3]{x} = 6$$

$$\sqrt[3]{x} = 6 - x$$

$$x^{1/3} = 6 - x \Rightarrow x = (6-x)^3$$

$$x = 6^3 - x^3 + 3(x^2)6 - 3(6^2)x$$

$$216 - x^3 + 18x^2 - 108x - x = 0$$

$$-x^3 + 18x^2 - 109x + 216 = 0$$

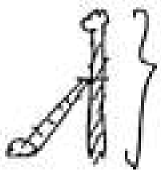
$$x^3 - 18x^2 + 109x - 216 = 0$$

12) A 12 metre tall tree was broken into two parts. It was found that the height of the part which was left standing was the cube root of the length of the part that was cut away. Formulate this into a mathematical problem to find the height of the part which was cut away

Soln Let the height of tree $= 12$.

length of cut part $= x^3$

Length of left out part $= \sqrt[3]{x^3} = x$.



12m.

$$x + x^3 = 12.$$

$$\boxed{x^3 + x - 12 = 0}$$

நா. காமாட்சி, M.Sc., B.Ed., M.Phil., Ph.D,
முதுகலை ஆசிரியர் (கணிதம்)
அரசு மேல்நிலைப்பள்ளி
கோவிந்தவாடி, காஞ்சிபுரம் (Dt)

Scanned by CamScanner

(8)

Exercise 3.2

1. If k is real, discuss the nature of the roots of the polynomial equation $2x^2+kx+k=0$ in terms of k

Soln

$\Delta = b^2-4ac$

$a=2 \quad b=k \quad c=k$

$\Delta = k^2-4.2(k)$

$\Delta = k^2-8k$.

If $k<0$, The polynomial has real roots $(\Delta>0)$

if $\Delta=0$ $\quad k^2-8k=0$

$k(k-8)=0$

$k=0$ (or) $k=8$

$\therefore k=0$ or $k=8$ The roots are real & equal

if $0<k<8$ $\quad \Delta<0$ The roots are imaginary

if $k>8$ The roots are real and distinct.

2) Find a polynomial equation of minimum degree with rational coefficients having $2+i\sqrt{3}$ as a root.

Soln Let the roots be $2+i\sqrt{3}$

another roots be $2-i\sqrt{3}$.

Sum of the roots $= 2+i\sqrt{3}+2-i\sqrt{3} = 4$

Product of the roots $= (2+i\sqrt{3})(2-i\sqrt{3}) = 2^2+\sqrt{3}^2$

$=4+3=7$

$x^2-(SR)x+PR=0$

$\boxed{x^2-4x+7=0}$

3) Find a polynomial equation of minimum degree with rational coefficients having $2i+3$ as a root

Soln Let the roots be $3+2i$

another root be $3-2i$

$SR = 3+2i+3-2i = 6$

$PR = (3+2i)(3-2i) = 3^2+2^2 = 9+4 = 13$

$x^2-(SR)x+PR$

$\boxed{x^2-6x+13=0}$

நா. காமாட்சி, M.Sc., B.Ed., M.Phil
முதுகலை ஆசிரியர் (கணிதம்)
அரசு மேல்நிலைப்பள்ளி
கோவிந்தவாடி, காஞ்சிபுரம் (

4. Find a polynomial equation of minimum degree with rational coefficients having $\sqrt{5}-\sqrt{3}$ as a root. (9)

Soln Let the root be $\sqrt{5}-\sqrt{3}$
another root is $\sqrt{5}+\sqrt{3}$

$SR = \sqrt{5}-\cancel{\sqrt{3}}+\sqrt{5}+\cancel{\sqrt{3}} = 2\sqrt{5}$

$PR = (\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3}) = \sqrt{5}^2-\sqrt{3}^2 = 5-3=2$

$x^2-(SR)x+PR=0$

$x^2-2\sqrt{5}x+2=0$ which is not rational coefficient.

To make rational coefficient.

$(x^2-2\sqrt{5}x+2)(x^2+2+2\sqrt{5}x)=0$

$(x^2+2)^2-(2\sqrt{5}x)^2=0$ $\quad (2\sqrt{5})^2 = 4\times5=20$

$x^4+4+4x^2-20x^2=0$

$\boxed{x^4-16x^2+4=0}$ is a rational coefficients polynomial equation.

5) Prove that a straight line and parabola cannot intersect at more than two points.

Soln. Let the equation of St. line be $y=mx+c$

Let the equation of parabola $y^2=4ax$

$(mx+c)^2=4ax$

$m^2x^2+c^2+2mcx-4ax=0$

$m^2x^2+(2x)(mc-2)+c^2=0$

which is a quadratic equation.

∴ This equation can't have more than two solution.

நா. காமாட்சி, M.Sc., B.Ed., M.Phil., Ph.D.
முதுகலை ஆசிரியர் (கணிதம்)
அரசு மேல்நிலைப்பள்ளி
கோவிந்தவாடி, காஞ்சிபுரம் (Dt)

Scanned by CamScanner

Exercise 3.3

(10)

1. Solve the cubic equation $2x^3-x^2-18x+9=0$ if sum of two of its roots vanishes

Soln $2x^3-x^2-18x+9=0$

Let the roots be α, β, γ.

Given $\alpha+\beta=0$.

$\alpha+\beta+\gamma=\frac{1}{2}$

$\gamma=\frac{1}{2}$.

$\alpha+\beta=0$ $\quad x^2-(\alpha+\beta)x+\alpha\beta=0$

$x^2-0x+P=0$

$(x^2+P)=0$

$\gamma=1/2$.

$(x-1/2)=0$

$2x-1=0$.

$\therefore (x^2+P)(2x-1/2)=2x^3-x^2-18x+9$.

$2x^3-x^2+2px-p=2x^3-x^2-18x+9$.

$-P=9$

$P=-9$

$x^2-9=0$

$x^2=9$

$x=\pm 3$ $\quad$ Roots are $3, -3, 1/2$.

2. Solve the equation $9x^3-36x^2+44x-16=0$ if the roots form an arithmetic progression.

Soln. Let the roots are in A.P

Let us take $a-d, a, a+d$.

Sum of the roots $= a-\cancel{d}+a+a+\cancel{d} = \frac{36}{9} = 4$

$3a=4$

$a=\frac{4}{3}$

Product of the roots $= (a-d)a(a+d) = (a^2-d^2)a = \frac{16}{9}$

$\left(\left(\frac{4}{3}\right)^2-d^2\right)\left(\frac{4}{3}\right)=\frac{16}{9}$ $\frac{4}{3}$

$\frac{16}{9}-d^2=\frac{4}{3}$

$\frac{16}{9}-\frac{4}{3}=d^2$

$$d^2 = \frac{16-12}{9} = \frac{4}{9}$$

$$d = \frac{2}{3}.$$

$\therefore a-d = \frac{4}{3} - \frac{2}{3} = \frac{2}{3}$

$a+d = \frac{4}{3} + \frac{2}{3} = \frac{6}{3} = 2$

Roots are $a-d, a, a+d$ be $\frac{2}{3}, \frac{4}{3}, 2$.

5) Solve the equation $3x^3 - 26x^2 + 52x - 24 = 0$ if its roots form a geometric progression.

Soln. Let the roots form a G.P. be $\frac{a}{\lambda}, a, a\lambda$

Product of the roots $\frac{a}{\cancel{\lambda}} \times a \times a \cancel{\lambda} = \frac{24}{3} = 8$

$$a^3 = 8$$

$$\boxed{a = 2}$$

Sum of the roots

$$\frac{a}{\lambda} + a + a\lambda = \frac{26}{3}$$

$$a\left(\frac{1}{\lambda} + 1 + \lambda\right) = \frac{26}{3}$$

$$\cancel{2}\left(\frac{1+\lambda+\lambda^2}{\lambda}\right) = \frac{\cancel{26}^{13}}{3}$$

$$3 + 3\lambda + 3\lambda^2 = 13\lambda.$$

$$3\lambda^2 + 3\lambda - 13\lambda + 3 = 0$$

$$3\lambda^2 - 10\lambda + 3 = 0$$

$$(\lambda - 3)(3\lambda - 1) = 0$$

$$\lambda = 3 \text{ (or) } \lambda = 1/3.$$

If $\lambda = 3$

$\frac{a}{\lambda} = \frac{2}{3}$, $a\lambda = 2(3) = 6$

If $\lambda = 1/3$

$\frac{a}{\lambda} = \frac{2}{1/3} = 6$ $a\lambda = 2\left(\frac{1}{3}\right) = \frac{2}{3}$

Roots are $a/\lambda, a, a\lambda$

if $\lambda = 3$ roots are $2/3, 2, 6$.

if $\lambda = 1/3$ roots are $6, 2, 2/3$

Scanned by CamScanner

4) Determine k and solve the equation $2x^3-6x^2+3x+k=0$ if one of its roots is twice the sum of the other two roots.

Soln. $2x^3-6x^2+3x+k=0$ Let the roots be α, β, γ

Given $\alpha = 2(\beta+\gamma)$.

$\alpha+\beta+\gamma = 3$ — (1) $\alpha\beta+\beta\gamma+\gamma\alpha = 3/2$ — (2) $\alpha\beta\gamma = -k/2$ — (3)

(1) ⇒ $2(\beta+\gamma)+(\beta+\gamma) = 3$

$3(\beta+\gamma) = 3$

$(\beta+\gamma) = 1$. ⇒ $\gamma = 1-\beta = 1-\left(\frac{1+\sqrt{3}}{2}\right) = \frac{2-1-\sqrt{3}}{2} = \frac{1-\sqrt{3}}{2}$.

$\therefore \alpha = 2(\beta+\gamma) = 2(1) = 2$ $\boxed{\therefore \alpha = 2}$

(3) ⇒ $\alpha\beta\gamma = -\frac{k}{2}$

$2\beta\left(\frac{1}{\beta}\right) = -k/2$

$4 = -k$

$\boxed{k = -4}$

(2) ⇒ $2\beta+\beta\left(\frac{1}{\beta}\right)+\frac{1}{\beta}(2) = \frac{3}{2}$

$2\beta+1+\frac{2}{\beta} = \frac{3}{2}$

(2) ⇒ $2\beta+\beta(1-\beta)+(1-\beta)2 = \frac{3}{2}$

$\cancel{2\beta}+\beta-\beta^2+2-\cancel{2\beta} = 3/2$

$-\beta^2+\beta+2 = \frac{3}{2}$

$-2\beta^2+2\beta+4-3=0$

$2\beta^2-2\beta-1=0$

$\beta = \frac{2\pm\sqrt{4+8}}{4} = \frac{2\pm\sqrt{12}}{4} = \frac{2\pm2\sqrt{3}}{4}$

$\boxed{\beta = \frac{1\pm\sqrt{3}}{2}}$ $\boxed{\therefore \gamma = \frac{1\pm\sqrt{3}}{2}}$

(3) ⇒ $\alpha\beta\gamma = -k/2$

$2\left(\frac{1+\sqrt{3}}{2}\right)\left(\frac{1-\sqrt{3}}{2}\right) = -k/2$ ⇒ $\boxed{k=2}$

$-k = 1-3$ ⇒ $-k = -2$

5) Find all zeros of the polynomial $x^6-3x^5-5x^4+22x^3-39x^2-39x+135$, if it is known that $1+2i$ and $\sqrt{3}$ are two of its zeros.

Soln Given roots $1+2i$, $\sqrt{3}$

Another roots be $1-2i$, $-\sqrt{3}$.

(i) SR of roots $= 1+\cancel{2i}+1-\cancel{2i} = 2$

PR of roots $= (1+2i)(1-2i) = 1^2+2^2 = 1+4 = 5$

$\boxed{x^2-2x+5=0}$

(ii) SR of roots $= \sqrt{3}-\sqrt{3} = 0$

PR of roots $= (\sqrt{3})(-\sqrt{3}) = -3$

$x^2-0x-3=0$ $\boxed{x^2-3=0.}$

$(x^2-2x+5)(x^2-3) = x^4-2x^3+2x^2+6x-15$

$x^6-3x^5-5x^4+22x^3-39x^2-39x+135$

$= (x^4-2x^3+2x^2+6x-15)(x^2+px-9)$ on both sides.

Equate coefficient of x

$-39 = -54-15p$

$-39+54 = -15p$

$15 = -15p$

$\boxed{p=-1}$

$\therefore x^2-x-9=0$

$x = \dfrac{1 \pm \sqrt{1-4\cdot1\cdot(-9)}}{2\cdot 1} = \dfrac{1\pm\sqrt{1+36}}{2} = \dfrac{1\pm\sqrt{37}}{2}.$

Roots are $\dfrac{1+\sqrt{37}}{2}, \dfrac{1-\sqrt{37}}{2}, 1+2i, 1-2i, \sqrt{3}, -\sqrt{3}.$

6) Solve the cubic equations

(i) $2x^3-9x^2+10x=3$

$2x^3-9x^2+10x-3=0.$

sum of the coefficients $2-9+10-3=0$

$\therefore x=1$ is one of the roots

$$\begin{array}{c|cccc} 1 & 2 & -9 & 10 & -3 \\ & 0 & 2 & -7 & 3 \\ \hline & 2 & -7 & 3 & \boxed{0} \end{array}$$

$2x^2-7x+3=0$

$(x-3)(2x-1)=0$

$x=3$ (or) $x=1/2.$

6; -3, -1; $-\frac{6}{2}$, $-\frac{1}{2}$

Roots are $3, 1/2, 1.$

7) Solve the equation $8x^3-2x^2-7x+3=0.$

Sum of alternative coefficients are equal

$8-7 = -2+3$

$1=1$

$\therefore x+1$ is a root.

நா. காமாட்சி, M.Sc., B.Ed., M.Phil., Ph.D
முதுகலை ஆசிரியர் (கணிதம்)
அரசு மேல்நிலைப்பள்ளி
கோவிந்தவாடி, காஞ்சிபுரம் (Dt)

Scanned by CamScanner

$$\begin{array}{r|rrrr} -1 & 8 & -2 & -7 & 3 \\ & 0 & -8 & 10 & -3 \\ \hline & 8 & -10 & 3 & \boxed{0} \end{array}$$

24: $-\frac{\cancel{6}\,3}{\cancel{8}\,4}$, $-\frac{\cancel{4}\,1}{\cancel{8}\,2}$

$8x^2 - 10x + 3 = 0$

$(4x-3)(2x-1) = 0$

$4x = 3$ $\quad 2x = 1$

$x = 3/4$ $\quad x = 1/2$

Roots are $3/4, 1/2, -1$.

7) Solve the equation: $x^4 - 14x^2 + 45 = 0$

Soln: put $t = x^2$

45: -9, -5

$t^2 - 14t + 45 = 0$

$(t-9)(t-5) = 0$

$t - 9 = 0$ $\quad t - 5 = 0$

$t = 9$ $\quad t = 5$

$x^2 = 9$ $\quad x^2 = 5$

$x = \pm 3$ $\quad x = \pm\sqrt{5}$

Roots are $3, -3, \sqrt{5}, -\sqrt{5}$.

நா. காமாட்சி, M.Sc., B.Ed., M.Phil., Ph.D
முதுகலை ஆசிரியர் (கணிதம்)
அரசு மேல்நிலைப்பள்ளி
கோவிந்தவாடி, காஞ்சிபுரம் (Dt)

Scanned by CamScanner

Exercise 3.4

1. Solve (i) $(x-5)(x-7)(x+6)(x+4) = 504$

Soln

$(x-5)(x+4)(x-7)(x+6) - 504 = 0$

$(x^2-x-20)(x^2-x-42) - 504 = 0$

put $y = x^2 - x$.

$(y-20)(y-42) - 504 = 0$

$y^2 - 42y - 20y + 840 - 504 = 0$

$y^2 - 62y + 336 = 0$

$(y-56)(y-6) = 0$

$y = 56 \quad y = 6$

336: $-56, -6$.

$x^2 - x = 56$ or $x^2 - x = 6$.

$x^2 - x - 56 = 0 \qquad x^2 - x - 6 = 0$

$(x-8)(x+7) = 0 \qquad (x+2)(x-3) = 0$

$x = 8, -7 \qquad x = 3, x = -2$.

6: $+2, -3$

Roots are $-2, 3, 8$ and -7.

2) $(x-4)(x-7)(x-2)(x+1) = 16$.

Soln

$(x-4)(x-2)(x-7)(x+1) - 16 = 0$

$(x^2-6x+8)(x^2-6x-7) - 16 = 0$

put $y = x^2 - 6x$

$(y+8)(y-7) - 16 = 0$

$y^2 - 7y + 8y - 56 - 16 = 0$

$y^2 + y - 72 = 0$

$(y+9)(y-8) = 0$

$y = -9 \qquad y = 8$

$x^2 - 6x = -9 \qquad x^2 - 6x = 8$

$x^2 - 6x + 9 = 0 \qquad x^2 - 6x - 8 = 0$

$(x-3)^2 = 0$

$x = 3, 3$

$x = \frac{6 \pm \sqrt{36 - 4 \cdot 1 \cdot 8}}{2 \cdot 1} = \frac{6 \pm \sqrt{36+32}}{2}$

$= \frac{6 \pm \sqrt{68}}{2} = \frac{6 \pm 2\sqrt{17}}{2}$

$= \frac{\cancel{2}(3 \pm \sqrt{17})}{\cancel{2}} = 3 \pm \sqrt{17}$.

Roots are $3, 3, 3 \pm \sqrt{17}$.

நா. காமாட்சி, M.Sc., B.Ed., M.Phil., Ph.D.
முதுகலை ஆசிரியர் (கணிதம்)
அரசு மேல்நிலைப்பள்ளி
கோவிந்தவாடி, காஞ்சிபுரம் (Dt)

Scanned by CamScanner

2. Solve: $(2x-1)(x+3)(x-2)(2x+3)+20=0$. (15)

Solve

$(x+3)(x-2)(2x-1)(2x+3)+20=0$

$(x^2+x-6)(4x^2+6x-2x-3)+20=0$

$(x^2+x-6)(4x^2+4x-3)+20=0$

$4(x^2+x-6)(x^2+x-3/4)+20=0$

$(\div 4)\quad (x^2+x-6)(x^2+x-3/4)+\frac{20}{4}=0.$

Put $y=x^2+x$.

$(y-6)(y-3/4)+\frac{20}{4}=0$

$y^2-\frac{3}{4}y-6y+\frac{9}{2}+\frac{20}{4}=0$

$y^2-y\left(\frac{27}{4}\right)+\frac{38}{4}=0$

$(\times 4)\quad 4y^2-27y+38=0$

$4y^2-8y-19y+38=0$

$(4y-19)(y-2)=0$

$y=2 \qquad y=\frac{19}{4}$

152, -19, -8, -11

$x^2+x=2$	$x^2+x=\frac{19}{4}$
$x^2+x-2=0$	$4x^2+4x=19$
$(x+2)(x-1)=0$	$4x^2+4x-19=0$
$x=-2 \quad x=1$	$x=\frac{-4\pm\sqrt{16-4.1.(-19)}}{2.8}$

$2|92$, $2|46$, 23

$=\frac{-4\pm\sqrt{16+76}}{16}$

$=\frac{-4\pm\sqrt{92}}{8}=\frac{-4\pm 2\sqrt{23}}{8}$

$=\boxed{\frac{-2\pm\sqrt{23}}{4}}$

நா. காமாட்சி, M.Sc., B.Ed., M.Phil., Ph.D.
முதுகலை ஆசிரியர் (கணிதம்)
அரசு மேல்நிலைப்பள்ளி
கோவிந்தவாடி, காஞ்சிபுரம் (Dt)

Scanned by CamScanner

Exercise 3.5

1. Solve the following equation.

(i) $\sin^2 x - 5\sin x + 4 = 0$

put $\sin x = t$

$t^2 - 5t + 4 = 0$

$(t-4)(t-1) = 0$

$t = 4$ or $t = 1$

$\sin x = 4$ (is not possible)

$\sin x = 1$

$\sin x = \sin \pi/2$

$x = 2n\pi + \pi/2 \ \forall n \in \mathbb{Z}.$

2) $12x^3 + 8x = 29x^2 - 4$

$12x^3 - 29x^2 + 8x - 4 = 0.$

2	12	−29	8	−4
	0	24	−10	−4
	12	−5	−2	0

$12x^2 - 5x - 2 = 0$

$12x^2 - 8x + 3x - 2 = 0$

$4x(3x-2) + 1(3x-2) = 0$

$(3x-2)(4x+1) = 0$

$3x = 2$, $x = \frac{2}{3}$

$4x = -1$, $x = \frac{-1}{4}$

Roots are $\frac{2}{3}, 2, \frac{-1}{4}.$

நா. காமாட்சி, M.Sc., B.Ed., M.Phil., Ph.D.
முதுகலை ஆசிரியர் (கணிதம்)
அரசு மேல்நிலைப்பள்ளி
கோவிந்தவாடி, காஞ்சிபுரம் (Dt)

3) Examine for the rational roots.

$2x^3 - x^2 - 1 = 0.$

soln: Sum of the coeffts $2 - 1 - 1 = 0$

$x = 1$ is a root.

$$\begin{array}{r|rrrr} 1 & 2 & -1 & 0 & -1 \\ & 0 & 2 & 1 & 1 \\ \hline & 2 & 1 & 1 & \boxed{0} \end{array}$$

$2x^2+x+1=0$

$x = \frac{-1 \pm \sqrt{1-4 \cdot 2 \cdot 1}}{2 \cdot 1} = \frac{-1 \pm \sqrt{1-8}}{2} = \frac{-1 \pm i\sqrt{7}}{2}$

which is imaginary roots

$\therefore x=1$ is rational roots.

(ii) $x^8-3x+1=0$.

Soln $a_n=1 \quad a_0=1.$

If $\frac{p}{q}$ is a root of the polynomial. $(p,q)=1$

by rational root theorem.

it have ~~no~~ no rational roots.

நா. காமாட்சி, M.Sc., B.Ed., M.Phil., Ph.D
முதுகலை ஆசிரியர் (கணிதம்)
அரசு மேல்நிலைப்பள்ளி
கோவிந்தவாடி, காஞ்சிபுரம் (Dt)

3) Solve $8x^{3/2n} - 8x^{-3/2n} = 63.$

put $k = \frac{3}{2n}$. $\therefore 8x^k - 8x^{-k} = 63$

$8x^k - 8/x^k = 63$

$8x^{2k} - 8 = 63x^k$

$8x^{2k} - 63x^k - 8 = 0$

$(x^k-8)(8x^k+1)=0.$

$x^k-8=0$

$x^k=8$

$x^{3/2n}=8$

$x = 8^{2n/3}$

$= (2^3)^{2n/3} = (2^2)^n$

$x=4^n$.

$\therefore x=4^n$ is a root of the equation.

Scanned by CamScanner

4. Solve $2\sqrt{\frac{x}{a}}+3\sqrt{\frac{a}{x}}=\frac{b}{a}+\frac{6a}{b}$

$$\left(2\frac{\sqrt{x}}{\sqrt{a}}+3\frac{\sqrt{a}}{\sqrt{x}}\right)^2=\left(\frac{b}{a}+\frac{6a}{b}\right)^2$$

$$4\cdot\frac{x}{a}+9\frac{a}{x}+\cancel{12}=\frac{b^2}{a^2}+\frac{6^2a^2}{b^2}+\cancel{12}$$

multiply this eqn by $a/4\,x$

$$\left(\frac{a}{4}x\right)\left(\frac{4}{a}\right)x+\left(\frac{a}{4}x\right)\left(\frac{9a}{x}\right)=\left(\frac{a}{4}x\right)\frac{b^2}{a^2}+\left(\frac{a}{4}\right)x\;\frac{6^2a^2}{b^2}$$

$$x^2+\frac{9a^2}{4}=\frac{b^2x}{4a}+\frac{\overset{9}{\cancel{36}}a^3x}{\cancel{4}b^2}$$

$$x^2-\frac{b^2x}{4a}+\frac{9a^2}{4}-\frac{9a^3x}{b^2}=0$$

$$x\left(x-\frac{b^2}{4a}\right)-\frac{9a^3}{b^2}\left(x-\frac{b^2}{4a}\right)=0$$

$$\left(x-\frac{b^2}{4a}\right)\left(x-\frac{9a^3}{b^2}\right)=0$$

$$x=\frac{b^2}{4a}\quad\text{or}\quad x=\frac{9a^3}{b^2}.\qquad\text{Ans}\quad\frac{b^2}{4a},\ \frac{9a^3}{b^2}.$$

5) solve the equations

(i) $6x^4-35x^3+62x^2-35x+6=0$

Soln This equation is Type I even degree reciprocal equation. Hence it can be rewritten as

$$6\left(x^2+\frac{1}{x^2}\right)-35\left(x+\frac{1}{x}\right)+62=0\quad (1)$$

$$y=x+\frac{1}{x}.$$

$$y^2=x^2+\frac{1}{x^2}+2$$

$$y^2-2=x^2+\frac{1}{x^2}$$

நா. காமாட்சி, M.Sc., B.Ed., M.Phil., Ph.D.
முதுகலை ஆசிரியர் (கணிதம்)
அரசு மேல்நிலைப்பள்ளி
கோவிந்தவாடி, காஞ்சிபுரம் (Dt)

150, $-30, -50$

$$(1)\Rightarrow\ 6(y^2-2)-35y+62=0$$

$$6y^2-12-35y+62=0$$

$$6y^2-35y+50=0$$

$$\left(y-\frac{5}{2}\right)\left(y-\frac{10}{3}\right)=0$$

150; -15, -20; $\frac{-15}{6}=\frac{5}{2}$, $\frac{-20}{6}=\frac{10}{3}$

20

$y = 5/2$ $\quad y = \frac{10}{3}$

$x + \frac{1}{x} = 5/2$ $\quad x + 1/x = \frac{10}{3}$

$\frac{x^2+1}{x} = 5/2$ $\quad \frac{x^2+1}{x} = \frac{10}{3}$

$2x^2 + 2 - 5x = 0$ $\quad 3x^2 + 3 = 10x$

$2x^2 - 5x + 2 = 0$ $\quad 3x^2 - 10x + 3 = 0$

$x = 2, 1/2$ $\quad x = 3, 1/3$

roots are $2, 1/2, 3, \frac{1}{3}$.

(11) $x^4 + 3x^3 - 3x - 1 = 0$

Soln This equation is Type I even degree reciprocal equation. Hence it can be rewritten as

$(x^2 + \frac{1}{x^2}) + 3(x - \frac{1}{x}) + 0 = 0$. — ①

$y = x - \frac{1}{x}$ $\Rightarrow y^2 = x^2 + \frac{1}{x^2} - 2$

$y^2 + 2 = x^2 + \frac{1}{x^2}$.

① ⇒ $(y^2+2) + 3y = 0$

$y^2 + 3y + 2 = 0$

$(y+2)(y+1) = 0$

$y = -2 \quad y = -1$.

$+2 = +2 \times +1$

$x - \frac{1}{x} = -2$ $\quad x - \frac{1}{x} = -1$

$\frac{x^2-1}{x} = -2$ $\quad \frac{x^2-1}{x} = -1$

$x^2 - 1 = -2x$ $\quad x^2 - 1 = -x$

$x^2 + 2x + 1 = 0$ $\quad x^2 + x - 1 = 0$

$(x+1)^2 = 0$ $\quad x = \frac{-1 \pm \sqrt{1 - 4.1.(-1)}}{2 \cdot 1} = \frac{-1 \pm \sqrt{1+4}}{2}$

$x = -1, -1$ $\quad = \frac{-1 \pm \sqrt{5}}{2}$.

Roots are $-1, -1, \frac{-1 \pm \sqrt{5}}{2}$.

ந.காமாட்சி, M.Sc., B.Ed., M.Phil
முதுகலை ஆசிரியர் (கணித)
அரசு மேல்நிலைப்பள்ளி
கோவிந்தவாடி, காஞ்சிபுரம்

7) Solve the equation $6x^4-5x^3-38x^2-5x+6=0$ if it is Known that $\frac{1}{3}$ is a solution.

(21)

Soln $6x^4-5x^3-38x^2-5x+6=0$

$6(x^2+\frac{1}{x^2})-5(x+1/x)-38=0$

$x+1/x=y$; $y^2-2=x^2+1/x^2$

$6(y^2-2)-5y-38=0$

$6y^2-12-5y-38=0$

$6y^2-5y-50=0$

$6y^2-20y+15y-50=0$

$2y(3y-10)+5(3y-10)=0$

$(3y-10)(2y+5)=0$

$3y=10$ $\quad 2y=-5$

$y=\frac{10}{3}$ $\quad y=-5/2$

$x+\frac{1}{x}=\frac{10}{3}$

$\frac{x^2+1}{x}=\frac{10}{3}$

$3x^2+3=10x$

$3x^2-10x+3=0$

$(x-3)(3x-1)=0$

$x=3$ $\quad x=1/3$

$x+\frac{1}{x}=-\frac{5}{2}$

$\frac{x^2+1}{x}=-\frac{5}{2}$

$2x^2+2=-5x$

$2x^2-5x+2=0$

$(x-2)(x-\frac{1}{2})=0$

$x=2,\quad x=1/2$.

-5, 4, -4, -1, $\frac{-4}{2}$, $\frac{-1}{2}$

roots are $3, 1/3, 2, 1/2$

b) Find all real numbers satisfying

$4^x-3(2^{x+2})+2^5=0$.

$(2^2)^x-3(2^x.2^2)+2^5=0$

$(2^x)^2-12(2^x)+2^5=0$.

32, -8, -4

put $2^x=t$ $\quad t^2-12t+32=0$

$(t-8)(t-4)=0$

$t=8$ $\quad t=4$

$2^x=8=2^3$ $\quad 2^x=4=(\pm 2)^2$

$x=3$ $\quad x=\pm 2$.

Scanned by CamScanner

Exercise 3.6

1. Discuss the maximum possible number of positive and negative roots of the polynomial equation $9x^9-4x^8+4x^7-3x^6+2x^5+x^3+7x^2+7x+2=0$.

Soln. Clearly there are 4 sign changes for the given polynomial $p(x)$ and hence number of positive roots of $p(x)$ can't be more than four.

$p(-x) = -9x^9-4x^8-4x^7-3x^6-2x^5-x^3+7x^2-7x+2=0$

There is two sign changes. hence the number of negative roots can't be more than two.

$\therefore$ it has at most 4 positive roots and at most two negative roots.

(2) Discuss the maximum possible number of positive and negative roots of the polynomial equations x^2-5x+6 and $x^2-5x+16$. Also draw rough sketch of the graphs.

$p(x) = (x^2-5x+6)(x^2-5x+16)$

$= x^4-5x^3+16x^2-5x^3+25x^2-80x+6x^2-30x+96=0$

$x^4-10x^3+57x^2-110x+96=0$

Ans. it has two sign changes

$\therefore$ it has two +ve real roots

$P(-x) = x^4+110x^3+57x^2+110x+96$

it has no sign change.

no –ve real roots.

$y = x^2-5x+6$

$x=1 \quad y = 1-5+6 = 2$

$x=2 \quad y = 4-10+6 = 0$

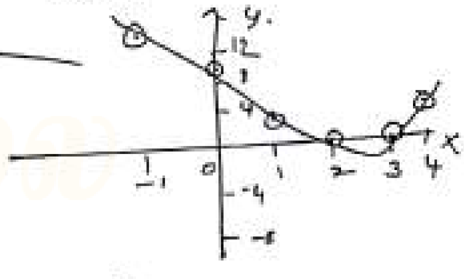
$x=0 \quad y = 6$

$x=3 \quad y = 9-15+6=0$

$x=-1 \quad y = 1+5+6=12$

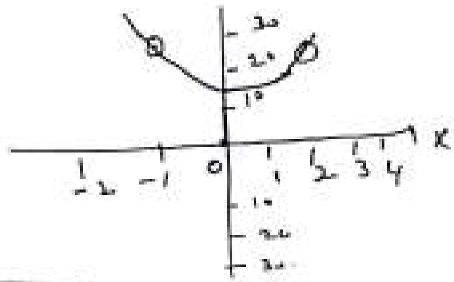
$x=4 \quad y = 16-20+6 = 22$

$(1,2) \; (0,6) \; (-1,12)$



$y = x^2-5x+16$

x	0	1	-1	2	4
y	16	12	22	10	.



நா. காமாட்சி, M.Sc., B.Ed., M.Phil., Ph.D.
முதுகலை ஆசிரியர் (கணிதம்)
அரசு மேல்நிலைப்பள்ளி
கோவிந்தவாடி, காஞ்சிபுரம் (Dt)

3. Show that the equation $x^9-5x^5+4x^4+2x^2+1=0$ has atleast 6 imaginary solution.

Soln $P(x)$ has only one sign change
it has at most one +ve roots
$P(-x) = -x^9+5x^5+4x^4+2x^2+1=0$
it has only one sign changes
it has at most one -ve roots
clearly 0 is not a roots.
So maximum number of real roots is 3
and hence there are atleast six imaginary solutions

4. Determine the number of positive & negative roots of the equation $x^9-5x^8-14x^7=0$

Soln $P(x) = x^9-5x^8-14x^7$. it has only one sign change
∴ it has one +ve roots.
$P(-x) = -x^9-5x^8+14x^7$. it has only one sign change
it has one -ve roots.
∴ It has one +ve & one -ve roots.

5) Find the exact number of real roots & imaginary of the equation $x^9+9x^7+7x^5+5x^3+3x$.

Soln $P(x) = x^9+9x^7+7x^5+5x^3+3x$. So it has no +ve
it has no sign change.
real roots.
$P(-x) = -x^9-9x^7-7x^5-5x^3-3x$.
It has no sign changes.
So it has no -ve real roots.
it has no positive real roots and no negative real roots.

நா. காமாட்சி, M.Sc., B.Ed., M.Phil., Ph.D.
முதுகலை ஆசிரியர் (கணிதம்)
அரசு மேல்நிலைப்பள்ளி
கோவிந்தவாடி, காஞ்சிபுரம் (Dt)

Exercise 3.7

(24)

1. Choose the most suitable answer

1. A zero of x^3+64 is

$x^3 = -64$

$x^3 = -4\times-4\times-4 = (-4)^3$

$x = -4$

Ans (4) -4

2. If f and g are polynomials of degree m and n respectively and if $h(x) = (f\circ g)(x)$ then the degree of h is

soln For example $f(x) = x^m$ $g(x) = x^n$

degree $= m$ degree $= n$

$(f\circ g)(x) = f(g(x)) = f(x^n) = (x^n)^m = x^{mn}$.

degree $= mn$.

Ans (1) mn.

3. A polynomial equation in x of degree n always has

Ans (3) n imaginary roots.

4) If α, β and γ are the roots x^3+px^2+qx+r then $\sum\frac{1}{\alpha}$ is

$\alpha+\beta+\gamma = -p$, $\alpha\beta+\beta\gamma+\gamma\alpha = q$,

$\alpha\beta\gamma = -r$.

$\sum\frac{1}{\alpha} = \frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma} = \frac{\beta\gamma+\alpha\gamma+\alpha\beta}{\alpha\beta\gamma} = \frac{q}{-r}$

Ans (1) $-\frac{q}{r}$.

(5) According to the rational root theorem which number is not possible rational root of $4x^7+2x^4-10x^3-5$?

By rational root theorm.

$\frac{p}{q}$ is a root of a polynomial $a_0 = -5$ $a_n = 4$

$(4, -5)$ p must divide 5 & q must divid 4.

Possible values of p are $+1, -1, +5, -5$.

Posible values of q are $+1, -1, -5/4, 4/5$

$5/4$ is not possible rational roots

Ans (2) $5/4$.

நா. காமாட்சி, M.Sc., B.Ed., M.Phil., Ph.D.,
முதுகலை ஆசிரியர் (கணிதம்)
அரசு மேல்நிலைப்பள்ளி
கோவிந்தவாடி, காஞ்சிபுரம் (Dt)

Scanned by CamScanner

6) The polynomial $x^3 - kx^2 + 9x$ has three real roots iff k satisfies

$\Delta \geq 0$ $\quad k^2 - 4.1.9 \geq 0$

$k^2 \geq 36$

$|k| \geq 6.$

நா. காமாட்சி, M.Sc., B.Ed., M.Phil., Ph.D.,
முதுகலை ஆசிரியர் (கணிதம்)
அரசு மேல்நிலைப்பள்ளி
கோவிந்தவாடி, காஞ்சிபுரம் (Dt)

Ans. (4) $|k| \geq 6$

7) The number of real number of $[0, 2\pi]$ satisfying $\sin^4 x - 2\sin^2 x + 1$ is

put $\sin^2 x = t$ $\quad t^2 - 2t + 1 = 0 \Rightarrow (t-1)^2 = 0$

$t = 1, \quad t = 1$

$\sin^2 x = 1$

$\sin x = \pm 1$

$\therefore \sin x = 1$ $\quad \sin x = -1$

$\sin x = \sin \pi/2$ $\quad \sin x = \sin(\pi + \pi/2)$

$\boxed{x = \pi/2}$ $\quad \boxed{x = 3\pi/2}$.

number of real numbers in $[0, 2\pi]$ is 2.

9) If $x^3 + 12x^2 + 10ax + 1999$ definitely has a +ve roots if need to minimum one sign change
if it is a +ve root then only we can get one sign chang
$\therefore a \leq 0$

Ans (3) $a < 0$

10) The polynomial $x^3 + 2x + 3$ has.

$P(x) = x^3 + 2x + 3$, no sign changes $\Rightarrow$ no +ve real roots

$P(-x) = -x^3 - 2x + 3$, one sign changes $\Rightarrow$ one -ve real root

$\therefore$ it has one negative & two imaginary roots

11) The number of +ve roots of the polynomials $\sum_{j=0}^{n} nC_r (-1)^r x^r$ is

$\sum_{j=0}^{n} nCr (-1)^r x^r = x^n - nC_1 x^{n-1} + nC_2 x^{n-2} + \dots + (-1)^n$.

$P(x)$ has 'n' changes $\therefore$ it has n +ve roots

Ans (2) n

$P(-x)$ has no changes

$\therefore$ No -ve roots.

Scanned by CamScanner

CHAPTER : 4 INVERSE TRIGONOMETRIC FUNCTIONS

1.

Inverse function	$\sin^{-1}x$	$\cos^{-1}x$	$\tan^{-1}x$	$\csc^{-1}x$	$\sec^{-1}x$	$\tan^{-1}x$
Domain	$[-1,1]$	$[-1,1]$	R	$(-\infty,-1]\cup[1,\infty)$	$(-\infty,-1]\cup[1,\infty)$	R
Range	$[-\frac{\pi}{2},\frac{\pi}{2}]$	$[0,\pi]$	$(-\frac{\pi}{2},\frac{\pi}{2})$	$[-\frac{\pi}{2},\frac{\pi}{2}]-\{0\}$	$[0,\pi]-\{\frac{\pi}{2}\}$	$(0,\pi)$

2. **Property I**

(i) $\sin^{-1}(\sin\theta)=\theta$ if $[-\frac{\pi}{2},\frac{\pi}{2}]$ (ii) $\cos^{-1}(\cos\theta)=\theta$ if $\theta\in[0,\pi]$

(iii) $\tan^{-1}(\tan\theta)=\theta$ if $\theta\in(-\frac{\pi}{2},\frac{\pi}{2})$ (iv) $\csc^{-1}(\csc\theta)=\theta$ if $\theta\in[-\frac{\pi}{2},\frac{\pi}{2}]-\{0\}$

(v) $\sec^{-1}(\sec\theta)=\theta$ if $\theta\in[0,\pi]\setminus\{\frac{\pi}{2}\}$

(vi) $\cot^{-1}(\cot\theta)=\theta$ if $\theta\in(0,\pi)$

3. **Property II**

(i) $\sin(\sin^{-1}x)=x$ if $x\in[-1,1]$ (ii) $\cos(\cos^{-1}x)=x$ if $x\in[-1,1]$

(iii) $\tan(\tan^{-1}x)=x$ if $x\in R$ (iv) $\csc(\csc^{-1}x)=x$ if $x\in R\setminus(-1,1)$

(iv) $\sec(\sec^{-1}x)=x$ if $x\in R\setminus(-1,1)$ (vi) $\cot(\cot^{-1}x)=x$ if $x\in R$.

4. **Property III**

1\. $\sin^{-1}(\frac{1}{x})=\csc x$ if $x\in R\setminus(-1,1)$ (ii) $\cos^{-1}(\frac{1}{x})=\sec x$ if $x\in R\setminus(-1,1)$

(ii) $\tan^{-1}\frac{1}{x}=\begin{cases}\cot^{-1}x & \text{if } x>0\\ -\pi+\cot^{-1}x & \text{if } x<0\end{cases}$

5) **Property IV** (Reflection identities)

(i) $\sin^{-1}(-x)=-\sin^{-1}x$ if $x\in[-1,1]$

(ii) $\tan^{-1}(-x)=-\tan^{-1}x$ if $x\in R$

(iii) $\csc^{-1}(-x)=-\csc^{-1}x$ if $|x|\geq1$ or $x\in R\setminus(-1,1)$

(iv) $\cos^{-1}(-x)=\pi-\cos^{-1}x$ if $x\in[-1,1]$

(v) $\sec^{-1}(-x)=\pi-\sec^{-1}x$ if $|x|\geq1$ or $x\in R\setminus(-1,1)$

(vi) $\cot^{-1}(-x)=\pi-\cot^{-1}x$ if $x\in R$

6. **Property V** (Cofunction inverse identities)

(i) $\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}$, $x\in[-1,1]$ (ii) $\tan^{-1}x+\cot^{-1}x=\frac{\pi}{2}$, $x\in R$

(iii) $\csc^{-1}x+\sec^{-1}x=\frac{\pi}{2}$, $x\in R\setminus(-1,1)$ or $|x|\geq1$

7. **Property VI**

(i) $\sin^{-1}x+\sin^{-1}y=\sin^{-1}(x\sqrt{1-y^2}+y\sqrt{1-x^2})$

where either $x^2+y^2\leq0$ or $xy<0$.

(ii) $\sin^{-1}x + \sin^{-1}y = \sin^{-1}[x\sqrt{1-y^2} - y\sqrt{1-x^2}]$ where either $x^2+y^2 \le 1$ or $xy > 0$

(iii) $\cos^{-1}x + \cos^{-1}y = \cos[xy - \sqrt{1-x^2}\sqrt{1-y^2}]$ if $x+y \geq 0$

(iv) $\cos^{-1}x - \cos^{-1}y = \cos^{-1}[xy + \sqrt{1-x^2}\sqrt{1-y^2}]$ if $x \le y$

(v) $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$ if $xy < 1$

(vi) $\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$ if $xy > -1$

Property VII

(i) $2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right), |x| < 1$

(ii) $2\tan^{-1}x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), x \geq 0$

(iii) $2\tan^{-1}x = \sin^{-1}\left(\frac{2x}{1+x^2}\right), |x| \le 1,$

Property VIII

(i) $\sin^{-1}(2x\sqrt{1-x^2}) = 2\sin^{-1}x$ if $|x| \le \frac{1}{\sqrt{2}}$ or $\frac{-1}{\sqrt{2}} \le x \le \frac{1}{\sqrt{2}}$

(ii) $\sin^{-1}(2x\sqrt{1-x^2}) = 2\cos^{-1}x$ if $\frac{1}{\sqrt{2}} \le x \le 1$.

Property IX

(i) $\sin^{-1}x = \cos^{-1}\sqrt{1-x^2}$ if $0 \le x \le 1$ (ii) $\sin^{-1}x = -\cos^{-1}\sqrt{1-x^2}$ if $-1 \le x \le 0$

(iii) $\sin^{-1}x = \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$ if $-1 < x < 1$ (iv) $\cos^{-1}x = \sin^{-1}\sqrt{1-x^2}$ if $0 \le x \le 1$

(v) $\cos^{-1}x = \pi - \sin^{-1}\sqrt{1-x^2}$ if $-1 \le x < 0$ (vi) $\tan^{-1}x = \sin^{-1}\frac{x}{\sqrt{1+x^2}} = \cos^{-1}\frac{1}{\sqrt{1+x^2}}$ if $x > 0$

Property X

(i) $3\sin^{-1}x = \sin^{-1}(3x - 4x^3)$ $x \in [-\frac{1}{2}, \frac{1}{2}]$

(ii) $3\cos^{-1}x = \cos^{-1}(4x^3 - 3x)$, $x \in [\frac{1}{2}, 1]$.

Trigonometric function	Sine	Cosine	tangent	Cosecant	secant	Cotangent
Domain	$[-\frac{\pi}{2}, \frac{\pi}{2}]$	$[0, \pi]$	$(-\frac{\pi}{2}, \frac{\pi}{2})$	$[-\frac{\pi}{2}, \frac{\pi}{2}] \setminus \{0\}$	$[0, \pi] \setminus \{\frac{\pi}{2}\}$	$(0, \pi)$
Range	$[-1, 1]$	$[-1, 1]$	R	$R \setminus (-1, 1)$	$R \setminus (-1, 1)$	R

நா. காமாட்சி, M.Sc., B.Ed., M.Phil., Ph
முதுகலை ஆசிரியர் (கணிதம்)
அரசு மேல்நிலைப்பள்ளி
கோவிந்தவாடி, காஞ்சிபுரம் (DI)