

CLASS -XII MATHEMATICS NCERT SOLUTIONS

Determinants

Exercise 4.2

Answers

$$1. \begin{vmatrix} 2 & 4 \\ -5 & -1 \end{vmatrix} = 2(-1) - 4(-5) = -2 + 20 = 18$$

$$2. (i) \begin{vmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{vmatrix} = \cos \theta \cdot \cos \theta - (-\sin \theta) \cdot \sin \theta = \cos^2 \theta + \sin^2 \theta = 1$$

$$(ii) \begin{vmatrix} x^2 - x + 1 & x - 1 \\ x + 1 & x + 1 \end{vmatrix} = (x^2 - x + 1)(x + 1) - (x + 1)(x - 1) \\ = (x^3 + 1) - (x^2 - 1) = x^3 - x^2 + 2$$

$$3. \text{ Given: } A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}, \quad \text{then } 2A = 2 \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 2 \times 1 & 2 \times 2 \\ 2 \times 4 & 2 \times 2 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 8 & 4 \end{bmatrix}$$

$$\text{L.H.S.} = |2A| = \begin{vmatrix} 2 & 4 \\ 8 & 4 \end{vmatrix} = 2 \times 4 - 4 \times 8 = 8 - 32 = -24$$

$$\text{R.H.S.} = 4|A| = 4 \begin{vmatrix} 1 & 2 \\ 4 & 2 \end{vmatrix} = 4(1 \times 2 - 4 \times 8) = 4(-6) = -24$$

Since L.H.S. = R.H.S.

Hence, proved.

$$4. \text{ Given: } A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}, \quad \text{then } 3A = 3 \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 3 \\ 0 & 3 & 6 \\ 0 & 0 & 12 \end{bmatrix}$$

$$\text{L.H.S.} = |3A| = \begin{vmatrix} 3 & 0 & 3 \\ 0 & 3 & 6 \\ 0 & 0 & 12 \end{vmatrix} = 3(36 - 0) = 3 \times 36 = 108$$

$$\text{R.H.S.} = 27|A| = 27 \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{vmatrix} = 27 [1(4 - 0)] = 27 \times 4 = 108$$

Since L.H.S. = R.H.S.

Hence, proved.

5. Evaluate the determinants:

$$(i) \text{ Given: } \begin{vmatrix} 3 & -1 & -2 \\ 0 & 0 & -1 \\ 3 & -5 & 0 \end{vmatrix}$$

$$\text{Expanding along first row, } 3 \begin{vmatrix} 0 & -1 \\ -5 & 0 \end{vmatrix} - (-1) \begin{vmatrix} 0 & -1 \\ 3 & 0 \end{vmatrix} + (-2) \begin{vmatrix} 0 & 0 \\ 3 & -5 \end{vmatrix}$$

$$= 3(0-5)+1\{0-(-3)\}-2(0-0)$$

$$= -15+3-0 = -12$$

(ii) Given: $\begin{vmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix}$

Expanding along first row, $3\begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix} - 4\begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix} + 5\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix}$

$$= 3(1+6)+4\{1-(-4)\}+5(3-2)$$

$$= 3\times 7+4\times 5+5\times 1$$

$$= 21+20+5 = 46$$

(iii) Given: $\begin{vmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{vmatrix}$

Expanding along first row, $0\begin{vmatrix} -1 & -3 \\ -2 & 0 \end{vmatrix} - 3\begin{vmatrix} -1 & -3 \\ -2 & 0 \end{vmatrix} + 2\begin{vmatrix} -1 & 0 \\ -2 & 3 \end{vmatrix}$

$$= 0(0+9)-(0-6)+2(-3-0)$$

$$= 0+6-6 = 0$$

(iv) Given: $\begin{vmatrix} 2 & -1 & -2 \\ 0 & 0 & -1 \\ 3 & -5 & 0 \end{vmatrix}$

Expanding along first row, $2\begin{vmatrix} 0 & -1 \\ 3 & 0 \end{vmatrix} - 1\begin{vmatrix} 0 & -1 \\ 3 & 0 \end{vmatrix} - 2\begin{vmatrix} 0 & 0 \\ 3 & -5 \end{vmatrix}$

$$= 2(0-5)+(0+3)-2(0-6)$$

$$= -10+3+12 = 5$$

6. Given: $A = \begin{bmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{bmatrix}, \quad \Rightarrow \quad |A| = \begin{vmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{vmatrix}$

Expanding along first row, $1\begin{vmatrix} 1 & -3 \\ 5 & -9 \end{vmatrix} - 1\begin{vmatrix} 2 & -3 \\ 5 & -9 \end{vmatrix} + (-2)\begin{vmatrix} 2 & 1 \\ 5 & 4 \end{vmatrix}$

$$= \{-9-(-12)\}-\{-18-(-15)\}-2(8-5)$$

$$= -9+12-(-18+15)-2(3)$$

$$= 3-(-3)-6$$

$$= 3+3-6 = 0$$

7. (i) Given: $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$

$$\Rightarrow 2 - 20 = 2x^2 - 24$$

$$\Rightarrow -18 = 2x^2 - 24$$

$$\Rightarrow 2x^2 = -18 + 24$$

$$\Rightarrow 2x^2 = 6$$

$$\Rightarrow x^2 = 3$$

$$\Rightarrow x = \pm\sqrt{3}$$

(ii) $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$

$$\Rightarrow 10 - 12 = 5x - 6x$$

$$\Rightarrow -2 = -x$$

$$\Rightarrow x = 2$$

8. Given: $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$

$$\Rightarrow x^2 - 36 = 36 - 36$$

$$\Rightarrow x^2 - 36 = 0$$

$$\Rightarrow x^2 = 36$$

$$\Rightarrow x = \pm 6$$

Therefore, option (B) is correct.

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Answers

1. Given: $\begin{vmatrix} x & a & x+a \\ y & b & y+b \\ z & c & z+c \end{vmatrix} = 0$

Operating $C_1 \rightarrow C_1 + C_2$ $\begin{vmatrix} x+a & a & x+a \\ y+b & b & y+b \\ z+c & c & z+c \end{vmatrix} = 0 \Rightarrow 0 = 0$ [$\because C_1$ and C_2 are identical]

\Rightarrow L.H.S. = R.H.S.

2. On $\begin{vmatrix} a-b & b-c & a-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$ Operating $C_1 \rightarrow C_1 + C_2 + C_3$

$= \begin{vmatrix} a-b+b-c+c-a & b-c & a-a \\ b-c+c-a+a-b & c-a & a-b \\ c-a+a-b+b-c & a-b & b-c \end{vmatrix} = \begin{vmatrix} 0 & b-c & a-a \\ 0 & c-a & a-b \\ 0 & a-b & b-c \end{vmatrix} = 0 = \text{R.H.S.}$

[\because All entries of one column here first are zero]

3. On $\begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix}$, operating $C_3 \rightarrow C_3 - C_1$

$= \begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 72 \\ 5 & 9 & 81 \end{vmatrix} = 9 \begin{vmatrix} 2 & 7 & 7 \\ 3 & 8 & 8 \\ 5 & 9 & 9 \end{vmatrix} = 9 \times 0 = 0$ [\because two columns are identical] Proved.

4. Given: $\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix} = \begin{vmatrix} 1 & bc & ab+ac \\ 1 & ca & bc+ba \\ 1 & ab & ca+cb \end{vmatrix}$

Operating $C_3 \rightarrow C_3 + C_2$ $\begin{vmatrix} 1 & bc & ab+ab+ac \\ 1 & ca & ab+ab+ac \\ 1 & ab & ab+ab+ac \end{vmatrix} = (ab+ab+ac) \begin{vmatrix} 1 & bc & 1 \\ 1 & ca & 1 \\ 1 & ab & 1 \end{vmatrix}$

$= (ab+ab+ac)(0) = 0$ [\because two columns are identical] Proved.

$$\begin{aligned}
5. \quad \text{L.H.S.} &= \begin{vmatrix} (b+c) & q+r & y+z \\ (c+a) & r+p & z+x \\ (a+b) & p+q & x+y \end{vmatrix} \quad \text{operating } R_1 \rightarrow R_1 + R_2 + R_3 \\
&= \begin{vmatrix} b+c+c+a+a+b & q+r+r+p+p+q & y+z+z+x+x+y \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix} \\
&= \begin{vmatrix} 2(a+b+c) & 2(p+q+r) & 2(x+y+z) \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix} = 2 \begin{vmatrix} (a+b+c) & (p+q+r) & (x+y+z) \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix} \\
&= 2 \begin{vmatrix} b & q & y \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix} \quad [\text{operating } R_1 \rightarrow R_1 - R_2] \\
&= 2 \begin{vmatrix} b & q & y \\ c+a & r+p & z+x \\ a & p & x \end{vmatrix} \quad [\text{operating } R_3 \rightarrow R_3 - R_1] \\
&= 2 \begin{vmatrix} b & q & y \\ c & r & z \\ a & p & x \end{vmatrix} \quad [\text{operating } R_2 \rightarrow R_2 - R_3] \\
&= -2 \begin{vmatrix} b & q & y \\ a & p & x \\ c & r & z \end{vmatrix} \quad [\text{Interchanging } R_2 \text{ and } R_3] \\
&= -(-2) \begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix} \quad [\text{Interchanging } R_2 \text{ and } R_3] = 2 \begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix} = \text{R.H.S.}
\end{aligned}$$

$$6. \quad \text{Let } \Delta = \begin{vmatrix} 0 & a & -b \\ -a & 0 & -c \\ b & c & 0 \end{vmatrix} = \Delta = (-1)^3 \begin{vmatrix} 0 & -a & b \\ a & 0 & c \\ -b & -c & 0 \end{vmatrix} \quad [\text{Taking } (-1) \text{ common from each row}]$$

Interchanging rows and columns in the determinants on R.H.S.,

$$\begin{aligned}
\Delta &= - \begin{vmatrix} 0 & a & -b \\ -a & 0 & -c \\ b & c & 0 \end{vmatrix} & \Rightarrow \quad \Delta = -\Delta & \Rightarrow \quad \Delta + \Delta = 0 \\
\Rightarrow \quad 2\Delta &= 0 & \Rightarrow \quad \Delta = 0 & \text{Proved.}
\end{aligned}$$

$$7. \quad \text{L.H.S.} = \begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix}$$

Taking common a, b, c from R_1, R_2, R_3 respectively,

$$= abc \begin{vmatrix} -a & a & a \\ a & -b & b \\ a & b & -c \end{vmatrix} = abc \begin{vmatrix} 0 & 0 & 2c \\ a & -b & c \\ a & b & -c \end{vmatrix} \quad [\text{operating } R_1 \rightarrow R_1 + R_2]$$

$$= abc.2c \begin{vmatrix} a & -b \\ a & b \end{vmatrix} = abc.2c(ab + ab) = abc.2c.2ab = 4a^2b^2c^2 = \text{R.H.S.}$$

$$8. \quad (i) \quad \text{L.H.S.} = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \quad \text{operating } R_2 \rightarrow R_2 - R_1 \text{ and } R_3 \rightarrow R_3 - R_1,$$

$$= \begin{vmatrix} 1 & a & a^2 \\ 0 & b-a & b^2-a^2 \\ 0 & c-a & c^2-a^2 \end{vmatrix} = 1 \begin{vmatrix} b-a & b^2-a^2 \\ c-a & c^2-a^2 \end{vmatrix} \quad [\text{Expanding along 1st column}]$$

$$= (b-a)(c-a) \begin{vmatrix} 1 & b+a \\ 1 & c+a \end{vmatrix} = (b-a)(c-a)(c+a-b-a)$$

$$= (b-a)(c-a)(c-b) = (a-b)(b-c)(c-a) = \text{R.H.S.} \quad \text{Proved.}$$

$$(ii) \quad \text{L.H.S.} = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} \quad \text{operating } C_2 \rightarrow C_2 - C_1 \text{ and } C_3 \rightarrow C_3 - C_1$$

$$= \begin{vmatrix} 1 & 0 & 0 \\ a & b-a & c-a \\ a^3 & b^3-a^3 & c^3-a^3 \end{vmatrix} = 1 \begin{vmatrix} b-a & c-a \\ b^3-a^3 & c^3-a^3 \end{vmatrix}$$

$$= 1 \begin{vmatrix} b-a & c-a \\ (b-a)(b^2+a^2+ab) & (c-a)(c^2+a^2+ac) \end{vmatrix}$$

$$= (b-a)(c-a) \begin{vmatrix} 1 & 1 \\ (b^2+a^2+ab) & (c^2+a^2+ac) \end{vmatrix}$$

$$= (b-a)(c-a)(c^2+a^2+ac-b^2-a^2-ab) = (b-a)(c-a)(c^2-b^2+ac-ab)$$

$$= (b-a)(c-a)[(c-b)(c+b)a(c-b)] = (b-a)(c-a)(c-b)(c+b+a)$$

$$= -(a-b)(c-a)[-(c-b)(c+b+a)] = (a-b)(b-c)(c-a)(a+b+c) = \text{R.H.S.}$$

$$9. \begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx)$$

$$\text{L.H.S.} = \begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix} = \begin{vmatrix} x^2 & x^3 & xyz \\ y^2 & y^3 & xyz \\ z^2 & z^3 & xyz \end{vmatrix} \quad [\text{Multiplying } R_1, R_2, R_3 \text{ by } x, y, z \text{ respectively}]$$

$$= \frac{xyz}{xyz} \begin{vmatrix} x^2 & x^3 & 1 \\ y^2 & y^3 & 1 \\ z^2 & z^3 & 1 \end{vmatrix} = \begin{vmatrix} x^2 & x^3 & 1 \\ y^2 & y^3 & 1 \\ z^2 & z^3 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} x^2 & x^3 & 1 \\ y^2 - x^2 & y^3 - x^3 & 0 \\ z^2 - x^2 & z^3 - x^3 & 0 \end{vmatrix} \quad [\text{operating } R_2 \rightarrow R_2 - R_1, R_3 \rightarrow R_3 - R_1]$$

$$= 1 \begin{vmatrix} y^2 - x^2 & y^3 - x^3 \\ z^2 - x^2 & z^3 - x^3 \end{vmatrix} = \begin{vmatrix} (y-x)(y+x) & (y-x)(y^2+x^2+yx) \\ (z-x)(z+x) & (z-x)(z^2+x^2+zx) \end{vmatrix}$$

$$= (y-x)(z-x) \begin{vmatrix} y+x & y^2+x^2+yx \\ z+x & z^2+x^2+zx \end{vmatrix}$$

$$= (y-x)(z-x) [(y+x)(z^2+x^2+zx) - (z+x)(y^2+x^2+xy)]$$

$$= (y-x)(z-x) [yz^2 + yx^2 + xyz + xz^2 + x^3 + x^2z - zy^2 - zx^2 - xyz - xy^2 - x^3 - x^2y]$$

$$= (y-x)(z-x) [yz^2 - zy^2 + xz^2 - xy^2] = (y-x)(z-x) [yz(z-y) + x(z^2 - y^2)]$$

$$= (y-x)(z-x) [yz(z-y) + x(z-y)(z+y)] = (y-x)(z-x)(z-y) [yz + x(z+y)]$$

$$= -(x-y)(z-x)[- (y-z)] [yz + xz + xy] = (x-y)(y-z)(z-x)(xy + yz + zx) = \text{R.H.S.}$$

$$10. (i) \quad \text{L.H.S.} = \begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = \begin{vmatrix} 5x+4 & 5x+4 & 5x+4 \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} \quad [R_1 \rightarrow R_1 + R_2 + R_3]$$

$$= (5x+4) \begin{vmatrix} 1 & 1 & 1 \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$$

$$= (5x+4) \begin{vmatrix} 1 & 0 & 0 \\ 2x & 4-x & 0 \\ 2x & 0 & 4-x \end{vmatrix} \quad [\text{operating } C_2 \rightarrow C_2 - C_1 \text{ and } C_3 \rightarrow C_3 - C_1]$$

$$= (5x+4).1 \begin{vmatrix} 4-x & 0 \\ 0 & 4-x \end{vmatrix} = (5x+4)(4-x)^2 = \text{R.H.S.}$$

$$\begin{aligned} \text{(ii) L.H.S.} &= \begin{vmatrix} y+k & y & y \\ y & y+k & y \\ y & 2x & y+k \end{vmatrix} = \begin{vmatrix} 3y+k & y & y \\ 3y+k & y+k & y \\ 3y+k & y & y+k \end{vmatrix} \quad [C_1 \rightarrow C_1 + C_2 + C_3] \\ &= (3y+k) \begin{vmatrix} 1 & y & y \\ 1 & y+k & y \\ 1 & y & y+k \end{vmatrix} \\ &= (3y+k) \begin{vmatrix} 1 & y & y \\ 0 & k & 0 \\ 0 & 0 & k \end{vmatrix} \quad [\text{operating } C_2 \rightarrow C_2 - C_1 \text{ and } C_3 \rightarrow C_3 - C_1] \\ &= (3y+k).1 \begin{vmatrix} k & 0 \\ 0 & k \end{vmatrix} = (3y+k)k^2 = k^2(3y+k) = \text{R.H.S.} \quad \text{Proved.} \end{aligned}$$

$$\begin{aligned} 11. \text{ (i) L.H.S.} &= \begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} \\ &= \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} \quad [R_1 \rightarrow R_1 + R_2 + R_3] \\ &= (a+b+c) \begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} \\ &= (a+b+c) \begin{vmatrix} 1 & 0 & 0 \\ 2b & -b-c-a & 0 \\ 2c & 0 & -c-a-b \end{vmatrix} \quad [C_2 \rightarrow C_2 - C_1 \text{ and } C_3 \rightarrow C_3 - C_1] \\ &= (a+b+c).1 \begin{vmatrix} -b-c-a & 0 \\ 0 & -c-a-b \end{vmatrix} = (a+b+c)\{-(b+c+a)\}\{-(c+a+b)\} \\ &= (a+b+c)^3 = \text{R.H.S.} \quad \text{Proved.} \end{aligned}$$

$$\begin{aligned} \text{(ii) L.H.S.} &= \begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix} \\ &= \begin{vmatrix} 2(x+y+z) & x & y \\ 2(x+y+z) & y+z+2x & y \\ 2(x+y+z) & x & z+x+2y \end{vmatrix} \quad [C_1 \rightarrow C_1 + C_2 + C_3] \end{aligned}$$

$$\begin{aligned}
&= 2(x+y+z) \begin{vmatrix} 1 & x & y \\ 1 & y+z+2x & y \\ 1 & x & z+x+2y \end{vmatrix} \\
&= 2(x+y+z) \begin{vmatrix} 1 & x & y \\ 0 & x+y+z & 0 \\ 0 & 0 & x+y+z \end{vmatrix} \quad [R_2 \rightarrow R_2 - R_1 \text{ and } R_3 \rightarrow R_3 - R_1] \\
&= 2(x+y+z) \cdot 1 \cdot \begin{vmatrix} x+y+z & 0 \\ 0 & x+y+z \end{vmatrix} = 2(x+y+z) [(x+y+z)^2 - 0] \\
&= 2(x+y+z)^3 = \text{R.H.S.} \quad \text{Proved.}
\end{aligned}$$

$$\begin{aligned}
12. \text{ L.H.S.} &= \begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = \begin{vmatrix} 1+x+x^2 & 1+x+x^2 & 1+x+x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} \quad [R_1 \rightarrow R_1 + R_2 + R_3] \\
&= (1+x+x^2) \begin{vmatrix} 1 & 1 & 1 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (1+x+x^2) \begin{vmatrix} 1 & 0 & 0 \\ x^2 & 1-x^2 & x-x^2 \\ x & x^2-x & 1-x \end{vmatrix} \quad [C_2 \rightarrow C_2 - C_1 \text{ and } C_3 \rightarrow C_3 - C_1] \\
&= (1+x+x^2) \cdot 1 \cdot \begin{vmatrix} 1-x^2 & x-x^2 \\ x^2-x & 1-x \end{vmatrix} = (1+x+x^2) \cdot 1 \cdot \begin{vmatrix} (1-x)(1+x) & x(1-x) \\ -x(1-x) & 1-x \end{vmatrix} \\
&= (1+x+x^2) [(1-x)^2(1+x) + x^2(1-x)^2] = (1+x+x^2)(1-x)^2(1+x+x^2) \\
&= (1+x+x^2)^2(1-x)^2 = [(1+x+x^2)(1-x)]^2 = (1-x+x-x^2+x^2-x^3)^2 \\
&= (1-x^3)^2 = \text{R.H.S.} \quad \text{Proved.}
\end{aligned}$$

$$\begin{aligned}
13. \text{ L.H.S.} &= \begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} \\
&= \begin{vmatrix} 1+a^2+b^2 & 0 & -2b \\ 0 & 1+a^2+b^2 & 2a \\ b(1+a^2+b^2) & -a(1+a^2+b^2) & 1-a^2-b^2 \end{vmatrix} \quad [C_1 \rightarrow C_1 - b C_3 \text{ and } C_2 \rightarrow C_2 + a C_3] \\
&= (1+a^2+b^2)^2 \begin{vmatrix} 1 & 0 & -2b \\ 0 & 1 & 2a \\ b & -a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^2 \begin{vmatrix} 1 & 0 & -2b \\ 0 & 1 & 2a \\ 0 & -a & 1-a^2+b^2 \end{vmatrix} \quad [R_3 \rightarrow R_3 - b R_1] \\
&= (1+a^2+b^2)^2 \begin{vmatrix} 1 & 2a \\ -a & 1-a^2+b^2 \end{vmatrix} = (1+a^2+b^2)^2 (1-a^2+b^2+2a^2) = (1+a^2+b^2)^3 = \text{R.H.S.}
\end{aligned}$$

$$14. \text{ L.H.S.} = \begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix}$$

Multiplying C_1, C_2, C_3 by a, b, c respectively and then dividing the determinant by abc ,

$$= \frac{1}{abc} \begin{vmatrix} a(a^2+1) & ab^2 & ac^2 \\ a^2b & b(b^2+1) & bc^2 \\ a^2c & b^2c & c(c^2+1) \end{vmatrix} = \frac{abc}{abc} \begin{vmatrix} a^2+1 & b^2 & c^2 \\ a^2 & b^2+1 & c^2 \\ a^2 & b^2 & c^2+1 \end{vmatrix}$$

$$= \frac{abc}{abc} \begin{vmatrix} 1+a^2+b^2+c^2 & b^2 & c^2 \\ 1+a^2+b^2+c^2 & b^2+1 & c^2 \\ 1+a^2+b^2+c^2 & b^2 & c^2+1 \end{vmatrix} \quad [C_1 \rightarrow C_1 + C_2 + C_3]$$

$$= (1+a^2+b^2+c^2) \begin{vmatrix} 1 & b^2 & c^2 \\ 1 & b^2+1 & c^2 \\ 1 & b^2 & c^2+1 \end{vmatrix}$$

$$= (1+a^2+b^2+c^2) \begin{vmatrix} 1 & b^2 & c^2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \quad [R_2 \rightarrow R_2 - R_1 \text{ and } R_3 \rightarrow R_3 - R_1]$$

$$= (1+a^2+b^2+c^2)(1)(1-0) = 1+a^2+b^2+c^2 = \text{R.H.S.} \quad \text{Proved.}$$

$$15. \text{ Let } A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \text{ be a square matrix of order } 3 \times 3. \quad \dots\dots\dots(i)$$

$$\therefore kA = \begin{bmatrix} ka_{11} & ka_{12} & ka_{13} \\ ka_{21} & ka_{22} & ka_{23} \\ ka_{31} & ka_{32} & ka_{33} \end{bmatrix} \Rightarrow |kA| = \begin{vmatrix} ka_{11} & ka_{12} & ka_{13} \\ ka_{21} & ka_{22} & ka_{23} \\ ka_{31} & ka_{32} & ka_{33} \end{vmatrix}$$

$$\Rightarrow |kA| = k^3 \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = k^3 |A| \quad [\text{From eq. (i)}]$$

Therefore, option (C) is correct.

16. Since, Determinant is a number associated to a square matrix.

Therefore, option (C) is correct.

CLASS -XII MATHEMATICS NCERT SOLUTIONS**Determinants****Exercise 4.3**

Answers

$$\begin{aligned} 1. \quad (i) \quad \text{Area of triangle} &= \text{Modulus of } \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \left| \frac{1}{2} \begin{bmatrix} 1 & 0 & 1 \\ 6 & 0 & 1 \\ 4 & 3 & 1 \end{bmatrix} \right| \\ &= \left| \frac{1}{2} [1(0-3) - 0(6-4) + 1(18-0)] \right| = \left| \frac{1}{2} (-3+18) \right| = \left| \frac{15}{2} \right| = \frac{15}{2} \text{ sq. units} \end{aligned}$$

$$\begin{aligned} (ii) \quad \text{Area of triangle} &= \text{Modulus of } \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \left| \frac{1}{2} \begin{bmatrix} 2 & 7 & 1 \\ 1 & 1 & 1 \\ 10 & 8 & 1 \end{bmatrix} \right| \\ &= \left| \frac{1}{2} [2(1-8) - 7(1-10) + 1(8-10)] \right| = \left| \frac{1}{2} [2(-7) - 7(-9) - 2] \right| \\ &= \left| \frac{1}{2} (-14 + 63 - 2) \right| = \left| \frac{1}{2} (63 - 16) \right| = \left| \frac{47}{2} \right| = \frac{47}{2} \text{ sq. units} \end{aligned}$$

$$\begin{aligned} (iii) \quad \text{Area of triangle} &= \text{Modulus of } \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \left| \frac{1}{2} \begin{bmatrix} -2 & -3 & 1 \\ 3 & 2 & 1 \\ -1 & -8 & 1 \end{bmatrix} \right| \\ &= \left| \frac{1}{2} [-2(2+8) - (-3)(3+1) + 1(-24+2)] \right| = \left| \frac{1}{2} [-2(10) + 3(4) - 22] \right| \\ &= \left| \frac{1}{2} (-20 + 12 - 22) \right| = \left| \frac{1}{2} \times (-30) \right| = \left| \frac{1}{2} \times 30 \right| = 15 \text{ sq. units} \end{aligned}$$

$$\begin{aligned} 2. \quad \text{Area of triangle ABC} &= \text{Modulus of } \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \left| \frac{1}{2} \begin{bmatrix} a & b+c & 1 \\ b & c+a & 1 \\ c & a+b & 1 \end{bmatrix} \right| \\ &= \left| \frac{1}{2} [a(c+a-a-b) - (b+c)(b-c) + 1\{b(a+b) - c(c+a)\}] \right| \\ &= \left| \frac{1}{2} [a(c-b) - (b^2 - c^2) + (ab + b^2 - c^2 - ac)] \right| = \left| \frac{1}{2} (ac - ab - b^2 + c^2 + ab + b^2 - c^2 - ac) \right| \\ &= \left| \frac{1}{2} \times 0 \right| = 0 \end{aligned}$$

Therefore, points A, B and C are collinear.

3. (i) Given: Area of triangle = Modulus of $\frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 4$

$$\Rightarrow \text{Modulus of } \frac{1}{2} \begin{vmatrix} k & 0 & 1 \\ 4 & 0 & 1 \\ 0 & 2 & 1 \end{vmatrix} = 4$$

$$\Rightarrow \left| \frac{1}{2} [k(0-2) - 0 + 1(8-0)] \right| = 4 \quad \Rightarrow \quad \left| \frac{1}{2} (-2k + 8) \right| = 4$$

$$\Rightarrow |-k + 4| = 4 \quad \Rightarrow \quad -k + 4 = \pm 4$$

$$\text{Taking positive sign, } -k + 4 = 4 \quad \Rightarrow \quad k = 0$$

$$\text{Taking negative sign, } -k + 4 = -4 \quad \Rightarrow \quad k = 8$$

(ii) Given: Area of triangle = Modulus of $\frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 4$

$$\Rightarrow \frac{1}{2} \begin{vmatrix} -2 & 0 & 1 \\ 0 & 4 & 1 \\ 0 & k & 1 \end{vmatrix} = 4 \quad \Rightarrow \quad \left| \frac{1}{2} [-2(4-k) - 0 + 1(0-0)] \right| = 4$$

$$\Rightarrow \left| \frac{1}{2} (-8 + 2k) \right| = 4 \quad \Rightarrow \quad |-k + 4| = 4 \quad \Rightarrow \quad -k + 4 = \pm 4$$

$$\text{Taking positive sign, } -k + 4 = 4 \quad \Rightarrow \quad k = 0$$

$$\text{Taking negative sign, } -k + 4 = -4 \quad \Rightarrow \quad k = 8$$

4. (i) Let P(x, y) be any point on the line joining the points (1, 2) and (3, 6).

Then, Area of triangle that could be formed by these points is zero.

$$\therefore \text{Area of triangle} = \text{Modulus of } \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \text{Modulus of } \frac{1}{2} \begin{vmatrix} x & y & 1 \\ 1 & 2 & 1 \\ 3 & 6 & 1 \end{vmatrix} = 0 \quad \Rightarrow \quad \frac{1}{2} [x(2-6) - y(1-3) + 1(6-6)] = 0$$

$$\Rightarrow -4x + 2y = 0 \quad \Rightarrow \quad -2x + y = 0$$

$$\Rightarrow y = 2x \quad \text{which is required line.}$$

(ii) Let P(x, y) be any point on the line joining the points (3, 1) and (9, 3).

Then, Area of triangle that could be formed by these points is zero.

$$\therefore \text{Area of triangle} = \text{Modulus of } \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \text{Modulus of } \frac{1}{2} \begin{vmatrix} x & y & 1 \\ 3 & 1 & 1 \\ 9 & 3 & 1 \end{vmatrix} = 0 \quad \Rightarrow \quad \frac{1}{2} [x(1-3) - y(3-9) + 1(9-9)] = 0$$

$$\Rightarrow -2x + 6y = 0 \quad \Rightarrow \quad -x + 3y = 0$$

$$\Rightarrow x - 3y = 0 \quad \text{which is required line.}$$

5. Given: Area of triangle = Modulus of $\frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 35$

$$\Rightarrow \text{Modulus of } \frac{1}{2} \begin{vmatrix} 2 & -6 & 1 \\ 5 & 4 & 1 \\ k & 4 & 1 \end{vmatrix} = 35 \quad \Rightarrow \quad \left| \frac{1}{2} [2(4-4) - (-6)(5-k) + 1(20-4k)] \right| = 35$$

$$\Rightarrow \left| \frac{1}{2} [0 + 30 - 6k + 20 - 4k] \right| = 35 \quad \Rightarrow \quad \left| \frac{1}{2} [50 - 10k] \right| = 35$$

$$\Rightarrow |25 - 5k| = 35 \quad \Rightarrow \quad 25 - 5k = \pm 35$$

$$\text{Taking positive sign, } 25 - 5k = 35 \quad \Rightarrow \quad k = -2$$

$$\text{Taking negative sign, } 25 - 5k = -35 \quad \Rightarrow \quad k = 12$$

Therefore, option (D) is correct.

CLASS -XII MATHEMATICS NCERT SOLUTIONS**Determinants****Exercise 4.4**

Answers

1. (i) Let $\Delta = \begin{vmatrix} 2 & -4 \\ 0 & 3 \end{vmatrix}$

$$M_{11} = \text{Minor of } a_{11} = |3| = 3$$

$$\text{and } A_{11} = (-1)^{1+1} M_{11} = (-1)^2 (3) = 3$$

$$M_{12} = \text{Minor of } a_{12} = |0| = 0$$

$$\text{and } A_{12} = (-1)^{1+2} M_{12} = (-1)^3 (0) = 0$$

$$M_{21} = \text{Minor of } a_{21} = |-4| = 4$$

$$\text{and } A_{21} = (-1)^{2+1} M_{21} = (-1)^3 (-4) = 4$$

$$M_{22} = \text{Minor of } a_{22} = |2| = 2$$

$$\text{and } A_{22} = (-1)^{2+2} M_{22} = (-1)^4 (2) = 2$$

(ii) Let $\Delta = \begin{vmatrix} a & c \\ b & d \end{vmatrix}$

$$M_{11} = \text{Minor of } a_{11} = |d| = d$$

$$\text{and } A_{11} = (-1)^{1+1} M_{11} = (-1)^2 (d) = d$$

$$M_{12} = \text{Minor of } a_{12} = |b| = b$$

$$\text{and } A_{12} = (-1)^{1+2} M_{12} = (-1)^3 (b) = -b$$

$$M_{21} = \text{Minor of } a_{21} = |c| = c$$

$$\text{and } A_{21} = (-1)^{2+1} M_{21} = (-1)^3 (c) = -c$$

$$M_{22} = \text{Minor of } a_{22} = |a| = a$$

$$\text{and } A_{22} = (-1)^{2+2} M_{22} = (-1)^4 (a) = a$$

2. (i) Let $\Delta = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

$$M_{11} = \text{Minor of } a_{11} = \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0 - 0 = 0$$

$$\text{and } A_{11} = (-1)^{1+1} M_{11} = (-1)^2 (0) = 0$$

$$M_{12} = \text{Minor of } a_{12} = \begin{vmatrix} 0 & 0 \\ 0 & 0 \end{vmatrix} = 0 - 0 = 0$$

$$\text{and } A_{12} = (-1)^{1+2} M_{12} = (-1)^3 (0) = 0$$

$$M_{13} = \text{Minor of } a_{13} = \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} = 0 - 0 = 0$$

$$\text{and } A_{13} = (-1)^{1+3} M_{13} = (-1)^4 (0) = 0$$

$$M_{21} = \text{Minor of } a_{21} = \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0 - 0 = 0$$

$$\text{and } A_{21} = (-1)^{2+1} M_{21} = (-1)^3 (0) = 0$$

$$M_{22} = \text{Minor of } a_{22} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1$$

$$\text{and } A_{22} = (-1)^{2+2} M_{22} = (-1)^4 (1) = 1$$

$$M_{23} = \text{Minor of } a_{23} = \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 0 - 0 = 0$$

$$\text{and } A_{23} = (-1)^{2+3} M_{23} = (-1)^5 (0) = 0$$

$$M_{31} = \text{Minor of } a_{31} = \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} = 0 - 0 = 0 \quad \text{and} \quad A_{31} = (-1)^{3+1} M_{31} = (-1)^4 (0) = 0$$

$$M_{32} = \text{Minor of } a_{32} = \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 0 - 0 = 0 \quad \text{and} \quad A_{32} = (-1)^{3+2} M_{32} = (-1)^5 (0) = 0$$

$$M_{33} = \text{Minor of } a_{33} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1 \quad \text{and} \quad A_{31} = (-1)^{3+3} M_{33} = (-1)^6 (1) = 1$$

(ii) Let $\Delta = \begin{vmatrix} 1 & 0 & 4 \\ 3 & 5 & -5 \\ 0 & 2 & 2 \end{vmatrix}$

$$M_{11} = \text{Minor of } a_{11} = \begin{vmatrix} 5 & -1 \\ 1 & 2 \end{vmatrix} = 10 - (-1) = 11 \quad \text{and} \quad A_{11} = (-1)^{1+1} M_{11} = (-1)^2 (11) = 11$$

$$M_{12} = \text{Minor of } a_{12} = \begin{vmatrix} 3 & -1 \\ 0 & 2 \end{vmatrix} = 6 - 0 = 6 \quad \text{and} \quad A_{12} = (-1)^{1+2} M_{12} = (-1)^3 (6) = -6$$

$$M_{13} = \text{Minor of } a_{13} = \begin{vmatrix} 3 & 5 \\ 0 & 1 \end{vmatrix} = 3 - 0 = 3 \quad \text{and} \quad A_{13} = (-1)^{1+3} M_{13} = (-1)^4 (3) = 3$$

$$M_{21} = \text{Minor of } a_{21} = \begin{vmatrix} 0 & 4 \\ 1 & 2 \end{vmatrix} = 0 - 4 = -4 \quad \text{and} \quad A_{21} = (-1)^{2+1} M_{21} = (-1)^3 (-4) = 4$$

$$M_{22} = \text{Minor of } a_{22} = \begin{vmatrix} 1 & 4 \\ 0 & 2 \end{vmatrix} = 2 - 0 = 2 \quad \text{and} \quad A_{22} = (-1)^{2+2} M_{22} = (-1)^4 (2) = 2$$

$$M_{23} = \text{Minor of } a_{23} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1 \quad \text{and} \quad A_{23} = (-1)^{2+3} M_{23} = (-1)^5 (1) = -1$$

$$M_{31} = \text{Minor of } a_{31} = \begin{vmatrix} 0 & 4 \\ 5 & -1 \end{vmatrix} = 0 - 20 = -20 \quad \text{and} \quad A_{31} = (-1)^{3+1} M_{31} = (-1)^4 (-20) = -20$$

$$M_{32} = \text{Minor of } a_{32} = \begin{vmatrix} 1 & 4 \\ 3 & -1 \end{vmatrix} = -1 - 12 = -13 \quad \text{and} \quad A_{32} = (-1)^{3+2} M_{32} = (-1)^5 (-13) = 13$$

$$M_{33} = \text{Minor of } a_{33} = \begin{vmatrix} 1 & 0 \\ 3 & 5 \end{vmatrix} = 5 - 0 = 5 \quad \text{and} \quad A_{31} = (-1)^{3+3} M_{33} = (-1)^6 (5) = 5$$

3. Elements of second row of Δ are $a_{21} = 2, a_{22} = 0, a_{23} = 1$

$$A_{21} = \text{Cofactor of } a_{21} = (-1)^{2+1} \begin{vmatrix} 3 & 8 \\ 2 & 3 \end{vmatrix} = (-1)^3 (9 - 16) = -(-7) = 7$$

$$A_{22} = \text{Cofactor of } a_{22} = (-1)^{2+2} \begin{vmatrix} 5 & 8 \\ 1 & 3 \end{vmatrix} = (-1)^4 (15 - 8) = 7$$

$$A_{23} = \text{Cofactor of } a_{23} = (-1)^{2+3} \begin{vmatrix} 5 & 3 \\ 1 & 2 \end{vmatrix} = (-1)^5 (10-3) = -7$$

$$\therefore \Delta = a_{21} + A_{21} + a_{22}A_{22} + a_{23}A_{23} = 2(7) + 0(7) + 1(-7) = 14 - 7 = 7$$

4. Elements of third column of Δ are $a_{13} = yz, a_{23} = zx, a_{33} = xy$

$$A_{13} = \text{Cofactor of } a_{13} = (-1)^{1+3} \begin{vmatrix} 1 & y \\ 1 & z \end{vmatrix} = (-1)^4 (z-y) = z-y$$

$$A_{23} = \text{Cofactor of } a_{23} = (-1)^{2+3} \begin{vmatrix} 1 & x \\ 1 & z \end{vmatrix} = (-1)^5 (z-x) = x-z$$

$$A_{33} = \text{Cofactor of } a_{33} = (-1)^{3+3} \begin{vmatrix} 1 & x \\ 1 & y \end{vmatrix} = (-1)^6 (y-x) = y-x$$

$$\begin{aligned} \therefore \Delta &= a_{13} + A_{13} + a_{23}A_{23} + a_{33}A_{33} = yz(z-y) + zx(x-z) + xy(y-x) \\ &= yz^2 - y^2z + zx^2 - xz^2 + xy^2 - x^2y = (yz^2 - y^2z) + (xy^2 - xz^2) + (xz^2 - x^2y) \\ &= yz(z-y) + x(y^2 - z^2) - x^2(y-z) = (y-z)[-yz + x(y+z) - x^2] \\ &= (y-z)(-yz + xy + xz - x^2) = (y-z)[-y(z-x) + x(z-x)] \\ &= (y-z)(z-x)(-y+x) = (x-y)(y-x)(z-x) \end{aligned}$$

5. Option (D) is correct.

CLASS -XII MATHEMATICS NCERT SOLUTIONS**Determinants****Exercise 4.5**

Answers

1. Here $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \Rightarrow |A| = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$

$\therefore A_{11} = \text{Cofactor of } a_{11} = (-1)^2 (4) = 4$

$A_{12} = \text{Cofactor of } a_{12} = (-1)^3 (3) = -3$

$A_{21} = \text{Cofactor of } a_{21} = (-1)^3 (2) = -2$

$A_{22} = \text{Cofactor of } a_{22} = (-1)^4 (1) = 1$

$\therefore \text{adj. } A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

2. Here $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 5 \\ -2 & 0 & 1 \end{bmatrix} \Rightarrow |A| = \begin{vmatrix} 1 & -1 & 2 \\ 2 & 3 & 5 \\ -2 & 0 & 1 \end{vmatrix}$

$\therefore A_{11} = + \begin{vmatrix} 3 & 5 \\ 0 & 1 \end{vmatrix} = 3$ $A_{12} = - \begin{vmatrix} 2 & 5 \\ -2 & 1 \end{vmatrix} = -(2+10) = -12$

$A_{13} = + \begin{vmatrix} 2 & 3 \\ -2 & 0 \end{vmatrix} = 6$ $A_{21} = - \begin{vmatrix} -1 & 2 \\ 0 & 1 \end{vmatrix} = -(-1) = 1$

$A_{22} = + \begin{vmatrix} 1 & 2 \\ -2 & 1 \end{vmatrix} = 1+4 = 5$ $A_{23} = - \begin{vmatrix} 1 & -1 \\ -2 & 0 \end{vmatrix} = -(-2) = 2$

$A_{31} = + \begin{vmatrix} -1 & 2 \\ 3 & 5 \end{vmatrix} = -5-6 = -11$ $A_{32} = - \begin{vmatrix} 1 & 2 \\ 2 & 5 \end{vmatrix} = -(5-4) = -1$

$A_{33} = + \begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix} = 3+2 = 5$

$\therefore \text{adj. } A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} = \begin{bmatrix} 3 & -12 & 5 \\ 1 & 5 & 2 \\ -11 & -1 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 1 & -11 \\ -12 & 5 & -1 \\ 6 & 2 & 5 \end{bmatrix}$

3. Let $A = \begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix} \Rightarrow \text{adj. } A = \begin{bmatrix} -6 & -3 \\ 4 & 2 \end{bmatrix}$

$\Rightarrow A(\text{adj. } A) = \begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix} \begin{bmatrix} -6 & -3 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} -12+12 & -6+6 \\ 24-24 & 12-12 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \dots\dots(i)$

$$\text{Again } (\text{adj. } A) \cdot A = \begin{bmatrix} -6 & -3 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix} = \begin{bmatrix} -12+12 & -18+18 \\ 8-8 & 12-12 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \dots(ii)$$

$$\text{And } |A| = \begin{vmatrix} 2 & 3 \\ -4 & -6 \end{vmatrix} = 2(-6) - 3(-4) = -12 + 12 = 0$$

$$\text{Again } |A|I = |A|I_2 = (0) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \dots(iii)$$

\therefore From eq. (i), (ii) and (iii) $A \cdot (\text{adj. } A) = (\text{adj. } A) \cdot A = |A|I$

$$4. \text{ Let } A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix} \Rightarrow |A| = \begin{vmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{vmatrix}$$

$$\therefore A_{11} = + \begin{vmatrix} 0 & -2 \\ 0 & 3 \end{vmatrix} = +0+0=0 \quad A_{12} = - \begin{vmatrix} 3 & -2 \\ 1 & 3 \end{vmatrix} = -(9+2)=-11$$

$$A_{13} = + \begin{vmatrix} 3 & 0 \\ 1 & 0 \end{vmatrix} = +(0-0)=0 \quad A_{21} = - \begin{vmatrix} -1 & 2 \\ 0 & 3 \end{vmatrix} = -(-3-0)=3$$

$$A_{22} = + \begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix} = 3-2=1 \quad A_{23} = - \begin{vmatrix} 1 & -1 \\ 1 & 0 \end{vmatrix} = -(0+1)=-1$$

$$A_{31} = + \begin{vmatrix} -1 & 2 \\ 0 & -2 \end{vmatrix} = 2-0=2 \quad A_{32} = - \begin{vmatrix} 1 & 2 \\ 3 & -2 \end{vmatrix} = -(-2-6)=8$$

$$A_{33} = + \begin{vmatrix} 1 & -1 \\ 3 & 0 \end{vmatrix} = 3+0=3$$

$$\therefore \text{adj. } A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} = \begin{bmatrix} 0 & -11 & 0 \\ 3 & 1 & -1 \\ 2 & 8 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix}$$

$$\therefore A \cdot (\text{adj. } A) = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix} \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 0+11+0 & 3-1-2 & 2-8+6 \\ 0-0-0 & 9+0+2 & 6+0-6 \\ 0+0+0 & 3+0-3 & 2+0+9 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix} \dots(i)$$

$$\text{Again } (\text{adj. } A) \cdot A = \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 0+9+2 & 0+0+0 & 0-6+6 \\ -11+3+8 & 11+0+0 & -22-2+24 \\ 0-3+3 & 0-0+0 & 0+2+9 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix} \quad \text{.....(ii)}$$

And $|A| = \begin{vmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{vmatrix} = 1(0-0) - (-1)(9+2) + 2(0-0) = 0+11+0 = 11$

Also $|A|I = |A|I_3 = 11 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$
(iii)

\therefore From eq. (i), (ii) and (iii) $A \cdot (\text{adj. } A) = (\text{adj. } A) \cdot A = |A|I$

5. Let $A = \begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix} \quad \therefore \quad |A| = \begin{vmatrix} 2 & -2 \\ 4 & 3 \end{vmatrix} = 6 - (-8) = 6+8 = 14 \neq 0$

\therefore Matrix A is non-singular and hence A^{-1} exist.

Now $\text{adj. } A = \begin{bmatrix} 3 & 2 \\ -4 & 2 \end{bmatrix} \quad \text{And} \quad A^{-1} = \frac{1}{|A|} \text{adj. } A = \frac{1}{14} \begin{bmatrix} 3 & 2 \\ -4 & 2 \end{bmatrix}$

6. Let $A = \begin{bmatrix} -1 & 5 \\ -3 & 2 \end{bmatrix} \quad \therefore \quad |A| = \begin{vmatrix} -1 & 5 \\ -3 & 2 \end{vmatrix} = -2 - (-15) = -2+15 = 13 \neq 0$

\therefore Matrix A is non-singular and hence A^{-1} exist.

Now $\text{adj. } A = \begin{bmatrix} 2 & -5 \\ 3 & -1 \end{bmatrix} \quad \text{And} \quad A^{-1} = \frac{1}{|A|} \text{adj. } A = \frac{1}{13} \begin{bmatrix} 2 & -5 \\ 3 & -1 \end{bmatrix}$

7. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix} \quad \therefore \quad |A| = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{vmatrix} = 1(10-0) - 2(0-0) + 3(-0) = 10 \neq 0$

\therefore A^{-1} exists.

$$A_{11} = + \begin{vmatrix} 2 & 4 \\ 0 & 5 \end{vmatrix} = +(10-0) = 10,$$

$$A_{12} = - \begin{vmatrix} 0 & 4 \\ 0 & 5 \end{vmatrix} = -(0-0) = 0,$$

$$A_{13} = + \begin{vmatrix} 0 & 2 \\ 0 & 0 \end{vmatrix} = +(0-0) = 0,$$

$$A_{21} = - \begin{vmatrix} 2 & 3 \\ 0 & 5 \end{vmatrix} = -(10-0) = -10,$$

$$A_{22} = + \begin{vmatrix} 1 & 3 \\ 0 & 5 \end{vmatrix} = +(5-0) = 5,$$

$$A_{23} = - \begin{vmatrix} 1 & 2 \\ 0 & 0 \end{vmatrix} = -(0-0) = 0,$$

$$A_{31} = + \begin{vmatrix} 2 & 3 \\ 2 & 4 \end{vmatrix} = +(8-6) = 2,$$

$$A_{32} = - \begin{vmatrix} 1 & 3 \\ 0 & 4 \end{vmatrix} = -(4-0) = -4,$$

$$A_{33} = + \begin{vmatrix} 1 & 2 \\ 0 & 2 \end{vmatrix} = +(2-0) = 2$$

$$\therefore \text{adj. } A = \begin{bmatrix} 10 & 0 & 0 \\ -10 & 5 & 0 \\ 2 & -4 & 2 \end{bmatrix} = \begin{bmatrix} 10 & -10 & 2 \\ 0 & 5 & -4 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} \text{adj. } A = \frac{1}{10} \begin{bmatrix} 10 & -10 & 2 \\ 0 & 5 & -4 \\ 0 & 0 & 2 \end{bmatrix}$$

$$8. \text{ Let } A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{bmatrix} \quad \therefore |A| = \begin{vmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{vmatrix} = 1(-3-0) - 0 + 0 = -3 \neq 0$$

$\therefore A^{-1}$ exists.

$$A_{11} = + \begin{vmatrix} 3 & 0 \\ 2 & -1 \end{vmatrix} = +(-3-0) = -3,$$

$$A_{12} = - \begin{vmatrix} 3 & 0 \\ 5 & -1 \end{vmatrix} = -(-3-0) = 3,$$

$$A_{13} = + \begin{vmatrix} 3 & 3 \\ 5 & 2 \end{vmatrix} = +(6-15) = -9,$$

$$A_{21} = - \begin{vmatrix} 0 & 0 \\ 2 & -1 \end{vmatrix} = -(0-0) = 0,$$

$$A_{22} = + \begin{vmatrix} 1 & 0 \\ 5 & -1 \end{vmatrix} = +(-1-0) = -1,$$

$$A_{23} = - \begin{vmatrix} 1 & 0 \\ 5 & 2 \end{vmatrix} = -(2-0) = -2,$$

$$A_{31} = + \begin{vmatrix} 0 & 0 \\ 3 & 0 \end{vmatrix} = +(0-0) = 0,$$

$$A_{32} = - \begin{vmatrix} 1 & 0 \\ 3 & 0 \end{vmatrix} = -(0-0) = 0,$$

$$A_{33} = + \begin{vmatrix} 1 & 0 \\ 3 & 3 \end{vmatrix} = +(3-0) = 3$$

$$\therefore \text{adj. } A = \begin{bmatrix} -3 & 3 & -9 \\ 0 & -1 & -2 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} -3 & 0 & 0 \\ 3 & -1 & 0 \\ -9 & -2 & 3 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} \text{adj. } A = \frac{-1}{3} = \begin{bmatrix} -3 & 0 & 0 \\ 3 & -1 & 0 \\ -9 & -2 & 3 \end{bmatrix}$$

$$9. \text{ Let } A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ -7 & 2 & 1 \end{bmatrix} \quad \therefore |A| = \begin{vmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ -7 & 2 & 1 \end{vmatrix} = 2\{(-1)-(4)\} + 3(8-7) = -3 \neq 0$$

∴ A^{-1} exists.

$$A_{11} = + \begin{vmatrix} -1 & 0 \\ 2 & 1 \end{vmatrix} = +(-1-0) = -1,$$

$$A_{12} = - \begin{vmatrix} 4 & 0 \\ -7 & 1 \end{vmatrix} = -(4-0) = -4,$$

$$A_{13} = + \begin{vmatrix} 4 & -1 \\ -7 & 2 \end{vmatrix} = +(8-7) = 1,$$

$$A_{21} = - \begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix} = -(1-6) = 5,$$

$$A_{22} = + \begin{vmatrix} 2 & 3 \\ -7 & 1 \end{vmatrix} = +(2+21) = 23,$$

$$A_{23} = - \begin{vmatrix} 2 & 1 \\ -7 & 2 \end{vmatrix} = -(4+7) = -11,$$

$$A_{31} = + \begin{vmatrix} 1 & 3 \\ -1 & 0 \end{vmatrix} = +(0+3) = 3,$$

$$A_{32} = - \begin{vmatrix} 2 & 3 \\ 4 & 0 \end{vmatrix} = -(0-12) = 12,$$

$$A_{33} = + \begin{vmatrix} 2 & 1 \\ 4 & -1 \end{vmatrix} = +(-2-4) = -6$$

$$\therefore \text{adj. } A = \begin{bmatrix} -1 & 4 & 1 \\ 5 & 23 & -11 \\ 3 & 12 & -6 \end{bmatrix} = \begin{bmatrix} -1 & 5 & 3 \\ -4 & 23 & 12 \\ 1 & -11 & -6 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} \text{adj. } A = \frac{-1}{3} = \begin{bmatrix} -1 & 5 & 3 \\ -4 & 23 & 12 \\ 1 & -11 & -6 \end{bmatrix}$$

$$10. \text{ Let } A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \quad \therefore |A| = \begin{vmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{vmatrix} = 1(8-6) - (-1)(0+9) + 2(0-6) = -1 \neq 0$$

∴ A^{-1} exists.

$$A_{11} = + \begin{vmatrix} 2 & -3 \\ -2 & 4 \end{vmatrix} = +(8-6) = 2,$$

$$A_{12} = - \begin{vmatrix} 0 & -3 \\ 3 & 4 \end{vmatrix} = -(0+9) = -9,$$

$$A_{13} = + \begin{vmatrix} 0 & 2 \\ 3 & -2 \end{vmatrix} = +(0-6) = -6,$$

$$A_{21} = - \begin{vmatrix} -1 & 2 \\ -2 & 4 \end{vmatrix} = -(-4+4) = 0,$$

$$A_{22} = + \begin{vmatrix} 2 & 2 \\ 3 & 4 \end{vmatrix} = +(4-6) = -2,$$

$$A_{23} = - \begin{vmatrix} 1 & -1 \\ 3 & -2 \end{vmatrix} = -(-2+3) = -1,$$

$$A_{31} = + \begin{vmatrix} -1 & 2 \\ 2 & -3 \end{vmatrix} = +(3-4) = -1,$$

$$A_{32} = - \begin{vmatrix} 1 & 2 \\ 0 & -3 \end{vmatrix} = -(-3-0) = 3,$$

$$A_{33} = + \begin{vmatrix} 1 & -1 \\ 0 & 2 \end{vmatrix} = +(2-0) = 2$$

$$\therefore \text{adj. } A = \begin{bmatrix} 2 & -9 & -6 \\ 0 & -2 & -1 \\ -1 & 3 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 0 & -1 \\ -9 & -2 & 3 \\ -6 & -1 & 2 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} \text{adj.} A = \frac{-1}{1} = \begin{bmatrix} 2 & 0 & -1 \\ -9 & -2 & 3 \\ -6 & -1 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$$

$$11. \text{ Let } A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & \sin \alpha \\ 0 & \sin \alpha & -\cos \alpha \end{bmatrix} \quad \therefore |A| = \begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & \sin \alpha \\ 0 & \sin \alpha & -\cos \alpha \end{vmatrix}$$

$$= 1(-\cos^2 \alpha - \sin^2 \alpha) - 0 + 0 = -(\cos^2 \alpha + \sin^2 \alpha) = -1 \neq 0$$

$\therefore A^{-1}$ exists.

$$A_{11} = + \begin{vmatrix} \cos \alpha & \sin \alpha \\ \sin \alpha & -\cos \alpha \end{vmatrix} = +(-\cos^2 \alpha - \sin^2 \alpha) = -(\cos^2 \alpha + \sin^2 \alpha) = -1,$$

$$A_{12} = - \begin{vmatrix} 0 & \sin \alpha \\ 0 & -\cos \alpha \end{vmatrix} = -(0 - 0) = 0, \quad A_{13} = + \begin{vmatrix} 0 & \cos \alpha \\ 0 & \sin \alpha \end{vmatrix} = +(0 - 0) = 0,$$

$$A_{21} = - \begin{vmatrix} 0 & 0 \\ \sin \alpha & -\cos \alpha \end{vmatrix} = -(0 - 0) = 0, \quad A_{22} = + \begin{vmatrix} 1 & 0 \\ 0 & -\cos \alpha \end{vmatrix} = +(-\cos \alpha - 0) = -\cos \alpha,$$

$$A_{23} = - \begin{vmatrix} 1 & 0 \\ 0 & \sin \alpha \end{vmatrix} = -(\sin \alpha - 0) = \sin \alpha, \quad A_{31} = + \begin{vmatrix} 0 & 0 \\ \cos \alpha & \sin \alpha \end{vmatrix} = (0 - 0) = 0,$$

$$A_{32} = - \begin{vmatrix} 1 & 0 \\ 0 & \sin \alpha \end{vmatrix} = -(\sin \alpha - 0) = -\sin \alpha, \quad A_{33} = + \begin{vmatrix} 1 & 0 \\ 0 & \cos \alpha \end{vmatrix} = +(\cos \alpha - 0) = \cos \alpha$$

$$\therefore \text{adj. } A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos \alpha & -\sin \alpha \\ 0 & -\sin \alpha & \cos \alpha \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos \alpha & -\sin \alpha \\ 0 & -\sin \alpha & \cos \alpha \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} \text{adj.} A = - \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos \alpha & -\sin \alpha \\ 0 & -\sin \alpha & \cos \alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & \sin \alpha \\ 0 & \sin \alpha & -\cos \alpha \end{bmatrix}$$

$$12. \text{ Given: Matrix } A = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix} \quad \therefore |A| = \begin{vmatrix} 3 & 7 \\ 2 & 5 \end{vmatrix} = 15 - 14 = 1 \neq 0$$

$$\therefore A^{-1} = \frac{1}{|A|} \text{adj. } A = \frac{1}{1} \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$$

$$\text{Matrix } B = \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix} \quad \therefore |B| = \begin{vmatrix} 6 & 8 \\ 7 & 9 \end{vmatrix} = 54 - 56 = -2 \neq 0$$

$$\therefore B^{-1} = \frac{1}{|B|} \text{adj. } B = \frac{1}{-2} \begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix}$$

$$\text{Now } AB = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix} = \begin{bmatrix} 18+49 & 24+63 \\ 12+35 & 16+45 \end{bmatrix} = \begin{bmatrix} 67 & 87 \\ 47 & 61 \end{bmatrix}$$

$$\therefore |AB| = \begin{vmatrix} 67 & 87 \\ 47 & 61 \end{vmatrix} = 67(61) - 87(47) = 4087 - 4089 = -2 \neq 0$$

$$\text{Now L.H.S.} = (AB)^{-1} = \frac{1}{|AB|} \text{adj. } (AB) = \frac{1}{-2} \begin{bmatrix} 61 & -87 \\ -47 & 67 \end{bmatrix} \quad \text{.....(i)}$$

$$\begin{aligned} \text{R.H.S.} = B^{-1}A^{-1} &= \frac{1}{-2} \begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix} \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix} = \frac{1}{-2} \begin{bmatrix} 45+16 & -63-24 \\ -35-12 & 49+18 \end{bmatrix} \\ &= \frac{1}{-2} \begin{bmatrix} 61 & -87 \\ -47 & 67 \end{bmatrix} \quad \text{.....(ii)} \end{aligned}$$

$$\therefore \text{ From eq. (i) and (ii), we get } \quad \text{L.H.S.} = \text{R.H.S.} \quad \Rightarrow \quad (AB)^{-1} = B^{-1}A^{-1}$$

$$13. \text{ Given: } A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \quad \therefore A^2 = A.A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

$$\begin{aligned} \text{L.H.S.} = A^2 - 5A + 7I &= A^2 - 5A + 7I_2 = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 5 \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} 8-15 & 5-5 \\ -5+5 & 3-10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} \\ &= \begin{bmatrix} -7+7 & 0+0 \\ 0+0 & -7+7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0 = \text{R.H.S.} \quad \Rightarrow \quad A^2 - 5A + 7I_2 = 0 \quad \text{.....(i)} \end{aligned}$$

To find: A^{-1} , multiplying eq. (i) by A^{-1} .

$$\Rightarrow A^2 A^{-1} - 5A.A^{-1} + 7I_2 A^{-1} = 0.A^{-1} \quad \Rightarrow \quad A - 5I_2 + 7A^{-1} = 0$$

$$\Rightarrow 7A^{-1} = -A + 5I_2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 5 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 8 & 1 \\ -1 & 7 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{7} \begin{bmatrix} 8 & 1 \\ -1 & 7 \end{bmatrix}$$

$$14. \text{ Given: } A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} \quad \therefore A^2 = A.A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 9+2 & 6+2 \\ 3+1 & 2+1 \end{bmatrix} = \begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix}$$

$$\therefore A^2 + aA + bI_2 = 0 \quad \Rightarrow \quad \begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix} + a \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} + b \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix} + \begin{bmatrix} 3a & 2a \\ a & a \end{bmatrix} + \begin{bmatrix} b & 0 \\ 0 & b \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 11+3a+b & 8+2a+0 \\ 4+a+0 & 3+a+b \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\therefore \text{ We have } 11+3a+b=0 \quad \dots\dots\dots(i)$$

$$8+2a+0=0 \quad \Rightarrow \quad 2a=-8 \quad \Rightarrow \quad a=-4$$

Here $a=-4$ satisfies $4+a+0=0$ also, therefore $a=-4$

$$\text{Putting } a=-4 \text{ in eq. (i), } 11-12+b=0 \Rightarrow b-1=0 \Rightarrow b=1$$

Here also $b=1$ satisfies $3+a+b=0$, therefore $b=1$

Therefore, $a=-4$ and $b=1$

$$15. \text{ Given: } A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} \quad \therefore \quad A^2 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 1+1+1 & 1+2-1 & 1-3+3 \\ 1+2-6 & 1+4+3 & 1-6-9 \\ 2-1+6 & 2-2-3 & 2+3+9 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}$$

$$\text{Now } A^3 = A^2 A = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 4+2+2 & 4+4-1 & 4-6+3 \\ -3+8-28 & -3+16+14 & -3-24-42 \\ 7-3+28 & 7-6-14 & 7+9+42 \end{bmatrix} = \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix}$$

$$\text{L.H.S.} = A^3 - 6A^2 + 5A + 11I$$

$$= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix} - 6 \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix} + 5 \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} + 11 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix} - \begin{bmatrix} 24 & 12 & 6 \\ -18 & 48 & -84 \\ 42 & -18 & 84 \end{bmatrix} + \begin{bmatrix} 5 & 5 & 5 \\ 5 & 10 & -15 \\ 10 & -5 & 15 \end{bmatrix} + \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

$$= \begin{bmatrix} 8-24+5 & 7-12+5 & 1-6+5 \\ -23+18+5 & 27-48+10 & -69+84-15 \\ 32-42+10 & -13+18-5 & 58-84+15 \end{bmatrix} + \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

$$= \begin{bmatrix} -11 & 0 & 0 \\ 0 & -11 & 0 \\ 0 & 0 & -11 \end{bmatrix} + \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0 = \text{R.H.S.}$$

Now, to find A^{-1} , multiplying $A^3 - 6A^2 + 5A + 11I = 0$ by A^{-1}

$$\Rightarrow A^3 A^{-1} - 6A^2 A^{-1} + 5A A^{-1} + 11I A^{-1} = 0 A^{-1}$$

$$\Rightarrow A^2 - 6A + 5I + 11A^{-1} = 0 \quad \Rightarrow \quad 11A^{-1} = 6A - 5I - A^2$$

$$\Rightarrow 11A^{-1} = 6 \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} - 5 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}$$

$$\Rightarrow 11A^{-1} = \begin{bmatrix} 6 & 6 & 6 \\ 6 & 12 & -18 \\ 12 & -6 & 18 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} - \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}$$

$$\Rightarrow 11A^{-1} = \begin{bmatrix} 6-5-4 & 6-2 & 6-1 \\ 6+3 & 12-5-8 & -18+14 \\ 12-7 & -6+3 & 18-5-14 \end{bmatrix} = \begin{bmatrix} -3 & 4 & 5 \\ 9 & -1 & -4 \\ 5 & -3 & -1 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{11} \begin{bmatrix} -3 & 4 & 5 \\ 9 & -1 & -4 \\ 5 & -3 & -1 \end{bmatrix}$$

16. Given: $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \therefore A^2 = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

$$\Rightarrow A^2 = \begin{bmatrix} 4+1+1 & -2-2-1 & 1+1+2 \\ -2-2-1 & 1+4+1 & -1-2-2 \\ 2+1+2 & -1-2-2 & 1+1+4 \end{bmatrix} = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}$$

Now $A^3 = A^2 A = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

$$= \begin{bmatrix} 12+5+5 & -6-10-5 & 6+5+10 \\ -10-6-5 & 5+12+5 & -5-6-10 \\ 10+5+6 & -5-10-6 & 5+5+12 \end{bmatrix} = \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix}$$

L.H.S. = $A^3 - 6A^2 + 9A - 4I$

$$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} - 6 \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} + 9 \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} - 4 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} - \begin{bmatrix} 36 & -30 & 30 \\ -30 & 36 & -30 \\ 30 & -30 & 36 \end{bmatrix} + \begin{bmatrix} 18 & -9 & 9 \\ -9 & 18 & -9 \\ 9 & -9 & 18 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$\begin{aligned}
&= \begin{bmatrix} 22-36 & -21+30 & 21-30 \\ -21+30 & 22-36 & -21+30 \\ 21-30 & -21+30 & 22-36 \end{bmatrix} + \begin{bmatrix} 18-4 & -9-0 & 9-0 \\ -9-0 & 18-4 & -9-0 \\ 9-0 & -9-0 & 18-4 \end{bmatrix} \\
&= \begin{bmatrix} -14 & 9 & -9 \\ 9 & -14 & 9 \\ -9 & 9 & -14 \end{bmatrix} + \begin{bmatrix} -14 & 9 & -9 \\ 9 & -14 & 9 \\ -9 & 9 & -14 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0 = \text{R.H.S.}
\end{aligned}$$

Now, to find A^{-1} , multiplying $A^3 - 6A^2 + 9A - 4I$ by A^{-1}

$$\Rightarrow A^3 A^{-1} - 6A^2 A^{-1} + 9A A^{-1} - 4I A^{-1} = 0 \cdot A^{-1}$$

$$\Rightarrow A^2 - 6A + 9I - 4A^{-1} = 0 \quad \Rightarrow \quad 4A^{-1} = A^2 - 6A + 9I$$

$$\Rightarrow 4A^{-1} = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} - 6 \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} + 9 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow 4A^{-1} = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} - \begin{bmatrix} 12 & -6 & 6 \\ -6 & 12 & -6 \\ 6 & -6 & 12 \end{bmatrix} + \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$\Rightarrow 4A^{-1} = \begin{bmatrix} 6-12+9 & -5+6+0 & 5-6+0 \\ -5+6+0 & 6-12+9 & -5+6+0 \\ 5-6+0 & -5+6+0 & 6-12+9 \end{bmatrix} = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{4} \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

17. If A is a non-singular matrix of order $n \times n$, then $|\text{adj. } A| = |A|^{n-1}$

$$\therefore \text{Putting } n = 3, \quad |\text{adj. } A| = |A|^2$$

Therefore, option (B) is correct.

18. Since $AA^{-1} = I$

$$\therefore |AA^{-1}| = |I| \quad \Rightarrow \quad |A| |A^{-1}| = 1 \quad \Rightarrow \quad |A^{-1}| = \frac{1}{|A|}$$

Therefore, option (B) is correct.

CLASS -XII MATHEMATICS NCERT SOLUTIONS**Determinants****Exercise 4.6**

Answers

1. Matrix form of given equations is $AX = B \Rightarrow \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

$$\therefore A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$\therefore |A| = \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = 3 - 4 = -1 \neq 0$$

Therefore, Unique solution and hence equations are consistent.

2. Matrix form of given equations is $AX = B \Rightarrow \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$

$$\therefore A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$

$$\therefore |A| = \begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} = 2 - (-1) = 3 \neq 0$$

Therefore, Unique solution and hence equations are consistent.

3. Matrix form of given equations is $AX = B \Rightarrow \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$

$$\therefore A = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$$

$$\therefore |A| = \begin{vmatrix} 1 & 3 \\ 2 & 6 \end{vmatrix} = 6 - 6 = 0$$

$$\text{Now } (\text{adj. } A) B = \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 8 \end{bmatrix} = \begin{bmatrix} 30 - 24 \\ -10 + 8 \end{bmatrix} = \begin{bmatrix} 6 \\ -2 \end{bmatrix} \neq 0$$

Therefore, given equations are inconsistent, i.e., have no common solution.

4. Matrix form of given equations is $AX = B \Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ a & a & 2a \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$

$$\text{Here } A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ a & a & 2a \end{bmatrix} \quad \therefore |A| = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ a & a & 2a \end{vmatrix}$$

$$\Rightarrow |A| = 1(6a - 2a) - 1(4a - 2a) + 1(2a - 3a) = 4a - 2a - a = a \neq 0$$

Therefore, Unique solution and hence equations are consistent.

$$5. \text{ Matrix form of given equations is } AX = B \Rightarrow \begin{bmatrix} 3 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$$

$$\text{Here } A = \begin{bmatrix} 3 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{bmatrix} \therefore |A| = \begin{vmatrix} 3 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{vmatrix}$$

$$\Rightarrow |A| = 3(0 - 5) - (-1)(0 + 3) + (-2)(0 - 6) = 3(-5) + 3 + 12 = -15 + 15 = 0$$

$$\text{Now } (\text{adj. } A) = \begin{bmatrix} -5 & 10 & 5 \\ -3 & 6 & 3 \\ -6 & 12 & 6 \end{bmatrix}$$

$$\text{And } (\text{adj. } A) B = \begin{bmatrix} -5 & 10 & 5 \\ -3 & 6 & 3 \\ -6 & 12 & 6 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} -10 - 10 + 15 \\ -6 - 6 + 9 \\ -12 - 12 + 18 \end{bmatrix} = \begin{bmatrix} -5 \\ -3 \\ -6 \end{bmatrix} \neq 0$$

Therefore, given equations are inconsistent.

$$6. \text{ Matrix form of given equations is } AX = B \Rightarrow \begin{bmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix}$$

$$\text{Here } A = \begin{bmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{bmatrix} \therefore |A| = \begin{vmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{vmatrix}$$

$$\Rightarrow |A| = 5(18 + 10) - (-1)(12 - 25) + 4(-4 - 15) = 140 - 13 - 76 = 140 - 89 = 51 \neq 0$$

Therefore, Unique solution and hence equations are consistent.

$$7. \text{ Matrix form of given equations is } AX = B \Rightarrow \begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

$$\text{Here } A = \begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

$$\therefore |A| = \begin{vmatrix} 5 & 2 \\ 7 & 3 \end{vmatrix} = 15 - 14 = 1 \neq 0$$

Therefore, solution is unique and $X = A^{-1}B = \frac{1}{|A|}(\text{adj. } A)B$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{1} \begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 12-10 \\ -28+25 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$

Therefore, $x=2$ and $y=-3$

8. Matrix form of given equations is $AX = B \Rightarrow \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

Here $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

$$\therefore |A| = \begin{vmatrix} 2 & -1 \\ 3 & 4 \end{vmatrix} = 8 - (-3) = 8 + 3 = 11 \neq 0$$

Therefore, solution is unique and $X = A^{-1}B = \frac{1}{|A|}(\text{adj. } A)B$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{11} \begin{bmatrix} 4 & 1 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} -2 \\ 3 \end{bmatrix} = \frac{1}{11} \begin{bmatrix} -8+3 \\ 6+6 \end{bmatrix} = \frac{1}{11} \begin{bmatrix} -5 \\ 12 \end{bmatrix} = \begin{bmatrix} -5/11 \\ 12/11 \end{bmatrix}$$

Therefore, $x = \frac{-5}{11}$ and $y = \frac{12}{11}$

9. Matrix form of given equations is $AX = B \Rightarrow \begin{bmatrix} 4 & -3 \\ 3 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$

Here $A = \begin{bmatrix} 4 & -3 \\ 3 & -5 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$

$$\therefore |A| = \begin{vmatrix} 4 & -3 \\ 3 & -5 \end{vmatrix} = 8 - (-3) = -20 - (-9) = -20 + 9 = -11 \neq 0$$

Therefore, solution is unique and $X = A^{-1}B = \frac{1}{|A|}(\text{adj. } A)B$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-11} \begin{bmatrix} -5 & 3 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} 3 \\ 7 \end{bmatrix} = \frac{1}{-11} \begin{bmatrix} -15+21 \\ -9+28 \end{bmatrix} = \frac{1}{-11} \begin{bmatrix} 6 \\ 19 \end{bmatrix} = \begin{bmatrix} -6/11 \\ -19/11 \end{bmatrix}$$

Therefore, $x = \frac{-6}{11}$ and $y = \frac{-19}{11}$

10. Matrix form of given equations is $AX = B \Rightarrow \begin{bmatrix} 5 & 2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$

Here $A = \begin{bmatrix} 5 & 2 \\ 3 & 2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$

$$\therefore |A| = \begin{vmatrix} 5 & 2 \\ 3 & 2 \end{vmatrix} = 10 - 6 = 4 \neq 0$$

Therefore, solution is unique and $X = A^{-1}B = \frac{1}{|A|}(\text{adj. } A)B$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 2 & -2 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 6-10 \\ -9+25 \end{bmatrix} = \frac{1}{4} \begin{bmatrix} -4 \\ 16 \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$$

Therefore, $x = -1$ and $y = 4$

11. Matrix form of given equations is $AX = B \Rightarrow \begin{bmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 0 & 3 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 3/2 \\ 9 \end{bmatrix}$

Here $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 0 & 3 & -5 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 1 \\ 3/2 \\ 9 \end{bmatrix}$

$$\therefore |A| = \begin{vmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 0 & 3 & -5 \end{vmatrix} = 2(10+3) - 1(-5-0) + 1(3-0) = 26 + 5 + 3 = 34 \neq 0$$

Therefore, solution is unique and $X = A^{-1}B = \frac{1}{|A|}(\text{adj. } A)B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{34} \begin{bmatrix} 13 & 8 & 1 \\ 5 & -10 & 3 \\ 3 & -6 & -5 \end{bmatrix} \begin{bmatrix} 1 \\ 3/2 \\ 9 \end{bmatrix} = \frac{1}{34} \begin{bmatrix} 13+12+9 \\ 5-15+27 \\ 3-9-45 \end{bmatrix} = \frac{1}{34} \begin{bmatrix} 34 \\ 17 \\ -51 \end{bmatrix} = \begin{bmatrix} 1 \\ 1/2 \\ -3/2 \end{bmatrix}$$

Therefore, $x = 1$, $y = \frac{1}{2}$ and $z = \frac{3}{2}$

12. Matrix form of given equations is $AX = B \Rightarrow \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$

Here $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$

$$\therefore |A| = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{vmatrix} = 1(1+3) - (-1)(2+3) + 1(2-1) = 4 + 5 + 1 = 10 \neq 0$$

Therefore, solution is unique and $X = A^{-1}B = \frac{1}{|A|}(\text{adj. } A)B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{10} \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix} = \frac{1}{10} \begin{bmatrix} 16+0+4 \\ -20+0+10 \\ 4-0+6 \end{bmatrix} = \frac{1}{10} \begin{bmatrix} 20 \\ -10 \\ 10 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

Therefore, $x = 2, y = -1$ and $z = 1$

13. Matrix form of given equations is $AX = B \Rightarrow \begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$

Here $A = \begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$

$$\therefore |A| = \begin{vmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{vmatrix} = 2(4+1) - 3(-2-3) + 3(-1+6) = 10 + 15 + 15 = 40 \neq 0$$

Therefore, solution is unique and $X = A^{-1}B = \frac{1}{|A|}(\text{adj. } A)B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{40} \begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix} \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix} = \frac{1}{40} \begin{bmatrix} 25-12+27 \\ 25+52+3 \\ 25-44-21 \end{bmatrix} = \frac{1}{40} \begin{bmatrix} 40 \\ 80 \\ -40 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

Therefore, $x = 1, y = 2$ and $z = -1$

14. Matrix form of given equations is $AX = B \Rightarrow \begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$

Here $A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$

$$\therefore |A| = \begin{vmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{vmatrix} = 1(12-5) - (-1)(9+10) + 2(-3-8) = 7 + 9 - 22 = 4 \neq 0$$

Therefore, solution is unique and $X = A^{-1}B = \frac{1}{|A|}(\text{adj. } A)B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix} \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 49-5-36 \\ -133+5+132 \\ -77+5+84 \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 8 \\ 4 \\ 12 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

Therefore, $x = 2, y = 1$ and $z = 3$

15. Given: Matrix $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$ $\therefore |A| = \begin{vmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{vmatrix}$

$$\Rightarrow |A| = 2(-4+4) - (-3)(-6+4) + 5(3-2) = 0 - 6 + 5 = -1 \neq 0$$

$$\therefore A^{-1} \text{ exists and } A^{-1} = \frac{1}{|A|} (\text{adj. } A) \quad \dots\dots\dots(i)$$

Now, $A_{11} = 0, A_{12} = 2, A_{13} = 1$ and $A_{21} = -1, A_{22} = -9, A_{23} = -5$ and $A_{31} = 2, A_{32} = 23, A_{33} = 13$

$$\therefore \text{adj. } A = \begin{bmatrix} 0 & 2 & 1 \\ -1 & -9 & -5 \\ 2 & 23 & 13 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

$$\therefore \text{From eq. (i), } A^{-1} = \frac{1}{-1} \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$

Now, Matrix form of given equations is $AX = B \Rightarrow \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$

Here $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$

Therefore, solution is unique and $X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix} = \begin{bmatrix} 0-5+6 \\ -22-45+69 \\ -11-25+39 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Therefore, $x = 1, y = 2$ and $z = 3$

16. Let ₹ x , ₹ y , ₹ z per kg be the prices of onion, wheat and rice respectively.

\therefore According to given data, we have three equations,

$$4x + 3y + 2z = 60$$

$$2x + 4y + 6z = 90$$

$$6x + 2y + 3z = 70$$

Matrix form of given equations is $AX = B \Rightarrow \begin{bmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$

Here $A = \begin{bmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$

$$\therefore |A| = \begin{vmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{vmatrix} = 4(12-12) - 3(6-36) + 2(4-24) = 0 + 90 - 40 = 50 \neq 0$$

Therefore, solution is unique and $X = A^{-1}B = \frac{1}{|A|}(\text{adj. } A)B$ (i)

Now, $A_{11} = 0, A_{12} = 30, A_{13} = -20$

$A_{21} = -5, A_{22} = 0, A_{23} = 10$

$A_{31} = 10, A_{32} = -20, A_{33} = 10$

$$\therefore \text{adj. } A = \begin{bmatrix} 0 & 30 & -20 \\ -5 & 0 & 10 \\ 10 & -20 & 10 \end{bmatrix} = \begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}$$

$$\Rightarrow \text{From eq. (i), } \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{50} \begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix} \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$$

$$= \frac{1}{50} \begin{bmatrix} -450 + 700 \\ 1800 - 1400 \\ -1200 + 900 + 700 \end{bmatrix} = \frac{1}{50} \begin{bmatrix} 250 \\ 400 \\ 400 \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix}$$

Therefore, $x = 5, y = 8$ and $z = 8$

Hence, the cost of onion, wheat and rice are ₹ 5, ₹ 8 and ₹ 8 per kg.

CLASS -XII MATHEMATICS NCERT SOLUTIONS

Determinants

Miscellaneous Exercise

Answers

1. Let $\Delta = \begin{vmatrix} x & \sin \theta & \cos \theta \\ -\sin \theta & -x & 1 \\ \cos \theta & 1 & x \end{vmatrix}$

Expanding along first row,

$$\begin{aligned} \Delta &= x \begin{vmatrix} -x & 1 \\ 1 & x \end{vmatrix} - \sin \theta \begin{vmatrix} -\sin \theta & 1 \\ \cos \theta & x \end{vmatrix} + \cos \theta \begin{vmatrix} -\sin \theta & -x \\ \cos \theta & 1 \end{vmatrix} \\ \Rightarrow \Delta &= x(-x^2 - 1) - \sin \theta(-x \sin \theta - \cos \theta) + \cos \theta(-\sin \theta + x \cos \theta) \\ \Rightarrow \Delta &= -x^3 - x + x \sin^2 \theta + \sin \theta \cos \theta - \sin \theta \cos \theta + x \cos^2 \theta \\ \Rightarrow \Delta &= -x^3 - x + x(\sin^2 \theta + \cos^2 \theta) = -x^3 - x + x = -x^3 \text{ which is independent of } \theta. \end{aligned}$$

2. L.H.S. = $\begin{vmatrix} a & a^2 & bc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix}$

Multiplying R_1 by a , R_2 by b and R_3 by c ,

$$\begin{vmatrix} a^2 & a^3 & abc \\ b^2 & b^3 & abc \\ c^2 & c^3 & abc \end{vmatrix} = \frac{abc}{abc} \begin{vmatrix} a^2 & a^3 & 1 \\ b^2 & b^3 & 1 \\ c^2 & c^3 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} a^2 & a^3 & 1 \\ b^2 & b^3 & 1 \\ c^2 & c^3 & 1 \end{vmatrix} = - \begin{vmatrix} 1 & a^3 & a^2 \\ 1 & b^3 & b^2 \\ 1 & c^3 & c^2 \end{vmatrix} \quad [\text{Interchanging } C_1 \text{ and } C_3]$$

$$= (-)(-)\begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} = \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} \quad [\text{Interchanging } C_2 \text{ and } C_3] \quad \text{Proved.}$$

3. Let $\Delta = \begin{vmatrix} \cos \alpha \cos \beta & \cos \alpha \sin \beta & -\sin \alpha \\ -\sin \beta & \cos \beta & 0 \\ \sin \alpha \cos \beta & \sin \alpha \sin \beta & \cos \alpha \end{vmatrix}$

Expanding along first row,

$$\begin{aligned} &= \cos \alpha \cos \beta (\cos \alpha \cos \beta - 0) - \cos \alpha \sin \beta (-\cos \alpha \sin \beta - 0) - \sin \alpha (-\sin \alpha \sin^2 \beta - \sin \alpha \cos^2 \beta) \\ &= \cos^2 \alpha \cos^2 \beta + \cos^2 \alpha \sin^2 \beta + \sin^2 \alpha (\sin^2 \beta + \cos^2 \beta) \end{aligned}$$

$$\begin{aligned}
&= \cos^2 \alpha (\cos^2 \beta + \sin^2 \beta) + \sin^2 \alpha (\sin^2 \beta + \cos^2 \beta) \\
&= \cos^2 \alpha + \sin^2 \alpha \\
&= 1
\end{aligned}$$

4. Given: $\Delta = \begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 0$

$$\Rightarrow \Delta = \begin{vmatrix} 2(a+b+c) & 2(a+b+c) & 2(a+b+c) \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 0 \quad [R_1 \rightarrow R_1 + R_2 + R_3]$$

$$\Rightarrow \Delta = 2(a+b+c) \begin{vmatrix} 1 & 1 & 1 \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 0$$

Here, Either $2(a+b+c) = 0 \Rightarrow a+b+c = 0$ (i)

Or $\begin{vmatrix} 1 & 1 & 1 \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 0$

$$\Rightarrow \begin{vmatrix} 1 & 0 & 0 \\ c+a & a+b-c-a & b+c-c-a \\ a+b & b+c-a-b & c+a-a-b \end{vmatrix} = 0 \quad [C_2 \rightarrow C_2 - C_1 \text{ and } C_3 \rightarrow C_3 - C_1]$$

$$\Rightarrow \begin{vmatrix} b-c & b-a \\ c-a & c-b \end{vmatrix} = 0 \quad [\text{Expanding along first row}]$$

$$\Rightarrow (b-c)(c-b) - (b-a)(c-a) = 0$$

$$\Rightarrow bc - b^2 - c^2 + bc - bc + ab + ac - a^2 = 0$$

$$\Rightarrow -a^2 - b^2 - c^2 + ab + bc + ca = 0$$

$$\Rightarrow 2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca = 0$$

$$\Rightarrow a^2 + a^2 + b^2 + b^2 + c^2 + c^2 - 2ab - 2bc - 2ca = 0$$

$$\Rightarrow (a^2 + b^2 - 2ab) + (b^2 + c^2 - 2bc) + (a^2 + c^2 - 2ca) = 0$$

$$\Rightarrow (a-b)^2 + (b-c)^2 + (c-a)^2 = 0$$

$$\Rightarrow a-b=0 \text{ and } b-c=0 \text{ and } c-a=0 \quad [x^2 + y^2 + z^2 = 0, \text{ then } x=0, y=0, z=0]$$

$$\Rightarrow a=b \text{ and } b=c \text{ and } c=a \quad \text{.....(ii)}$$

Therefore, from eq. (i) and (ii), either $a+b+c=0$ or $a=b=c$

$$5. \text{ Given: } \begin{vmatrix} x+a & x & x \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 3x+a & 3x+a & 3x+a \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0 \quad [R_1 \rightarrow R_1 + R_2 + R_3]$$

$$\Rightarrow (3x+a) \begin{vmatrix} 1 & 1 & 1 \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0$$

$$\text{Either } 3x+a=0 \Rightarrow x = \frac{-a}{3} \quad \dots\dots\dots(i)$$

$$\text{Or } \begin{vmatrix} 1 & 1 & 1 \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1 & 0 & 0 \\ x & a & 0 \\ x & 0 & a \end{vmatrix} = 0 \quad [C_2 \rightarrow C_2 - C_1 \text{ and } C_3 \rightarrow C_3 - C_1]$$

$$\Rightarrow 1(a^2 - 0) = 0 \Rightarrow a^2 = 0 \Rightarrow a = 0$$

But this is contrary as given that $a \neq 0$.

Therefore, from eq. (i), $x = \frac{-a}{3}$ is only the solution.

$$6. \text{ L.H.S.} = \begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = \begin{vmatrix} a^2 & bc & c(a+c) \\ a(a+b) & b^2 & ac \\ ab & b(b+c) & c^2 \end{vmatrix}$$

$$= abc \begin{vmatrix} a & c & (a+c) \\ (a+b) & b & a \\ b & (b+c) & c \end{vmatrix}$$

$$= abc \begin{vmatrix} a-a-b-b & c-b-b-c & a+c-a-c \\ a+b & b & a \\ b & b+c & c \end{vmatrix} \quad [R_1 \rightarrow R_1 - R_2 - R_3]$$

$$= abc \begin{vmatrix} -2b & -2b & 0 \\ a+b & b & a \\ b & b+c & c \end{vmatrix} = abc \begin{vmatrix} -2b & 0 & 0 \\ a+b & -a & a \\ b & c & c \end{vmatrix} \quad [C_2 \rightarrow C_2 - C_1]$$

$$= abc(-2b)(-ac-ac) = 4a^2b^2c^2 = \text{R.H.S.} \quad \text{Proved.}$$

7. Given: $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$

Since, $(AB)^{-1} = B^{-1}A^{-1}$ [Reversal law](i)

Now $|B| = \begin{vmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{vmatrix} = 1(3-0) - 2(-1-0) + (-2)(2-0) = 3+2-4=1 \neq 0$

Therefore, B^{-1} exists.

$\therefore B_{11}=3, B_{12}=1, B_{13}=2$ and $B_{21}=2, B_{22}=1, B_{23}=2$ and $B_{31}=6, B_{32}=2, B_{33}=5$

$\therefore \text{adj. } B = \begin{bmatrix} 3 & 1 & 2 \\ 2 & 1 & 2 \\ 6 & 2 & 5 \end{bmatrix}' = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$

$\therefore B^{-1} = \frac{1}{|B|}(\text{adj. } B) = \frac{1}{1} \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$

From eq. (i), $(AB)^{-1} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$

$\Rightarrow (AB)^{-1} = \begin{bmatrix} 9-30+30 & -3+12-12 & 3-10+12 \\ 3-15+10 & -1+6-4 & 1-5+4 \\ 6-30+25 & -2+12-10 & 2-10+10 \end{bmatrix} = \begin{bmatrix} 9 & -3 & 5 \\ -2 & 3 & 1 \\ 1 & 0 & 2 \end{bmatrix}$

8. Given: Matrix $A = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix}$ $\therefore |A| = \begin{vmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{vmatrix}$

$\Rightarrow |A| = 1(15-1) - (-2)(-10-1) + 1(-2-3) = 14-22-5 = -13 \neq 0$

Therefore, A^{-1} exists.

$\therefore A_{11}=14, A_{12}=11, A_{13}=-5$ and $A_{21}=11, A_{22}=4, A_{23}=-3$

and $A_{31}=-5, A_{32}=-3, A_{33}=-1$

$\therefore \text{adj. } A = \begin{bmatrix} 14 & 11 & -5 \\ 11 & 4 & -3 \\ -5 & -3 & -1 \end{bmatrix} = B \text{ (say)}$

$$\therefore A^{-1} = \frac{1}{|A|}(\text{adj. } A) = \frac{-1}{13} \begin{bmatrix} 14 & 11 & -5 \\ 11 & 4 & -3 \\ -5 & -3 & -1 \end{bmatrix} \quad \text{.....(i)}$$

$$\Rightarrow |B| = \begin{vmatrix} 14 & 11 & -5 \\ 11 & 4 & -3 \\ -5 & -3 & -1 \end{vmatrix} = 14(-4-9) - 11(-11-15) - 5(-33+20) = 169 \neq 0$$

Therefore, B^{-1} exists.

$$\therefore B_{11} = -13, B_{12} = 26, B_{13} = -13 \text{ and } B_{21} = 26, B_{22} = -39, B_{23} = -13 \\ \text{and } B_{31} = -13, B_{32} = -13, B_{33} = -65$$

$$\therefore \text{adj. } B = \begin{bmatrix} -13 & 26 & -13 \\ 26 & -39 & -13 \\ -13 & -13 & -65 \end{bmatrix} = -13 \begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix}$$

$$\Rightarrow B^{-1} = (\text{adj. } A)^{-1} = \frac{1}{|B|}(\text{adj. } B) = \frac{1}{169}(-13) \begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix} = \frac{-1}{13} \begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix} \quad \text{....(ii)}$$

Now to find $\text{adj. } A^{-1} = \text{adj. } C$ (say), where

$$C = A^{-1} = \frac{-1}{13} \begin{bmatrix} 14 & 11 & -5 \\ 11 & 4 & -3 \\ -5 & -3 & -1 \end{bmatrix} = \begin{bmatrix} -14/13 & -11/13 & 5/13 \\ -11/13 & -4/11 & 3/13 \\ 5/13 & 3/11 & 1/13 \end{bmatrix}$$

$$C = A^{-1} = \frac{-14}{13} \left(\frac{-4}{169} - \frac{9}{169} \right) - \left(\frac{-11}{13} \right) \left(\frac{-11}{169} - \frac{15}{169} \right) + \frac{5}{13} \left(\frac{-33}{169} + \frac{20}{169} \right)$$

$$C = A^{-1} = \frac{-14}{13} \left(\frac{-13}{169} \right) + \frac{11}{13} \left(\frac{-26}{169} \right) + \frac{5}{13} \left(\frac{-13}{169} \right) = \frac{14}{169} - \frac{22}{169} - \frac{5}{169} = \frac{-13}{169} = \frac{-1}{13} \neq 0$$

Therefore, C^{-1} exists.

$$\therefore C_{11} = \frac{-1}{13}, C_{12} = \frac{2}{13}, C_{13} = \frac{-1}{13} \text{ and } C_{21} = \frac{2}{13}, C_{22} = \frac{-3}{13}, C_{23} = \frac{-1}{13} \\ \text{and } C_{31} = \frac{-1}{13}, C_{32} = \frac{-1}{13}, C_{33} = \frac{-5}{13}$$

$$\therefore \text{adj. } A = \text{adj. } (A^{-1}) = \begin{vmatrix} -1/13 & 2/13 & -1/13 \\ 2/13 & -3/13 & -1/13 \\ -1/13 & -1/13 & -5/13 \end{vmatrix} = \frac{-1}{13} \begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix} \quad \text{.....(iii)}$$

Again $(A^{-1})^{-1} = C^{-1} = \frac{1}{|C|}(\text{adj. } C) = \frac{1}{-1/13} \left(\frac{-1}{13} \right) \begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix} = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix} = A \text{ (given)}$

(i) $(\text{adj. } A)^{-1} = \text{adj. } (A^{-1})$

$$\Rightarrow \frac{-1}{13} \begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix} = \frac{-1}{13} \begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix} \quad [\text{From eq. (ii) and (iii)}]$$

(ii) $(A^{-1})^{-1} = A$

$$\Rightarrow \frac{-1}{13} \begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix} = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix}$$

9. Let $\Delta = \begin{vmatrix} x & y & x+y \\ y & x+y & x \\ x+y & x & y \end{vmatrix} = \begin{vmatrix} 2(x+y) & 2(x+y) & 2(x+y) \\ y & x+y & x \\ x+y & x & y \end{vmatrix} \quad [R_1 \rightarrow R_1 + R_2 + R_3]$

$$= 2(x+y) \begin{vmatrix} 1 & 1 & 1 \\ y & x+y & x \\ x+y & x & y \end{vmatrix}$$

$$= 2(x+y) \begin{vmatrix} 1 & 0 & 0 \\ y & x+y-y & x-y \\ x+y & x-x-y & y-x-y \end{vmatrix} \quad [C_2 \rightarrow C_2 - C_1 \text{ and } C_3 \rightarrow C_3 - C_1]$$

$$= 2(x+y) \begin{vmatrix} 1 & 0 & 0 \\ y & x & x-y \\ x+y & -y & -x \end{vmatrix} = 2(x+y) \cdot 1 \begin{vmatrix} x & x-y \\ -y & -x \end{vmatrix}$$

$$= 2(x+y) \{-x^2 + y(x-y)\} = 2(x+y)(-x^2 + xy - y^2)$$

$$= -2(x+y)(x^2 - xy + y^2) = -2(x^3 + y^3)$$

10. Let $\Delta = \begin{vmatrix} 1 & x & y \\ 1 & x+y & y \\ 1 & x & x+y \end{vmatrix}$

$$= \begin{vmatrix} 1 & x & y \\ 0 & x+y-x & 0 \\ 0 & 0 & x+y-y \end{vmatrix}$$

$$[R_2 \rightarrow R_2 - R_1 \text{ and } R_3 \rightarrow R_3 - R_1]$$

$$= \begin{vmatrix} 1 & x & y \\ 0 & y & 0 \\ 0 & 0 & x \end{vmatrix} = 1 \begin{vmatrix} y & 0 \\ 0 & x \end{vmatrix} = xy$$

$$\begin{aligned} 11. \text{ L.H.S.} &= \begin{vmatrix} \alpha & \alpha^2 & \beta+\gamma \\ \beta & \beta^2 & \gamma+\alpha \\ \gamma & \gamma^2 & \alpha+\beta \end{vmatrix} = \begin{vmatrix} \alpha & \alpha^2 & \alpha+\beta+\gamma \\ \beta & \beta^2 & \alpha+\beta+\gamma \\ \gamma & \gamma^2 & \alpha+\beta+\gamma \end{vmatrix} & [C_3 \rightarrow C_3 + C_1] \\ &= (\alpha+\beta+\gamma) \begin{vmatrix} \alpha & \alpha^2 & 1 \\ \beta & \beta^2 & 1 \\ \gamma & \gamma^2 & 1 \end{vmatrix} = (\alpha+\beta+\gamma) \begin{vmatrix} \alpha & \alpha^2 & 1 \\ \beta-\alpha & \beta^2-\alpha^2 & 0 \\ \gamma-\alpha & \gamma^2-\alpha^2 & 0 \end{vmatrix} & [R_2 \rightarrow R_2 - R_1 \text{ and } R_3 \rightarrow R_3 - R_1] \end{aligned}$$

$$\begin{aligned} &\text{Expanding along third column,} & (\alpha+\beta+\gamma) \begin{vmatrix} \beta-\alpha & \beta^2-\alpha^2 \\ \gamma-\alpha & \gamma^2-\alpha^2 \end{vmatrix} \\ &= (\alpha+\beta+\gamma) \begin{vmatrix} \beta-\alpha & (\beta-\alpha)(\beta+\alpha) \\ \gamma-\alpha & (\gamma-\alpha)(\gamma+\alpha) \end{vmatrix} = (\alpha+\beta+\gamma)(\beta-\alpha)(\gamma-\alpha) \begin{vmatrix} 1 & (\beta+\alpha) \\ 1 & (\gamma+\alpha) \end{vmatrix} \\ &= (\alpha+\beta+\gamma)(\beta-\alpha)(\gamma-\alpha)(\gamma+\alpha-\beta-\alpha) = (\alpha+\beta+\gamma)(\beta-\alpha)(\gamma-\alpha)(\gamma-\beta) \\ &= (\alpha+\beta+\gamma)[-(\alpha-\beta)](\gamma-\alpha)[-(\beta-\gamma)] = (\alpha+\beta+\gamma)(\alpha-\beta)(\beta-\gamma)(\gamma-\alpha) = \text{R.H.S.} \end{aligned}$$

$$12. \text{ L.H.S.} = \begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+py^3 \\ z & z^2 & 1+pz^3 \end{vmatrix} = \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + \begin{vmatrix} x & x^2 & px^3 \\ y & y^2 & py^3 \\ z & z^2 & pz^3 \end{vmatrix} = \Delta_1 + \Delta_2 \text{ (say)} \quad \dots\dots\dots(i)$$

$$\text{Now } \Delta_2 = \begin{vmatrix} x & x^2 & px^3 \\ y & y^2 & py^3 \\ z & z^2 & pz^3 \end{vmatrix} = pxyz\Delta_2 = \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = -pxyz \begin{vmatrix} x^2 & x & 1 \\ y^2 & y & 1 \\ z^2 & z & 1 \end{vmatrix} = pxyz \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix}$$

$$\Rightarrow \text{From eq. (i), L.H.S.} = \Delta_1 + pxyz\Delta_1 \quad \dots\dots\dots(ii)$$

$$\text{Now } \Delta_1 = \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} = \begin{vmatrix} x & x^2 & 1 \\ y-x & y^2-x^2 & 0 \\ z-x & z^2-x^2 & 1 \end{vmatrix} \quad [R_2 \rightarrow R_2 - R_1 \text{ and } R_3 \rightarrow R_3 - R_1]$$

$$\begin{aligned} &\text{Expanding along third column,} & \Delta_1 = \begin{vmatrix} y-x & y^2-x^2 \\ z-x & z^2-x^2 \end{vmatrix} = \begin{vmatrix} y-x & (y-x)(y+x) \\ z-x & (z-x)(z+x) \end{vmatrix} \\ &= (y-x)(z-x) \begin{vmatrix} 1 & y+x \\ 1 & z+x \end{vmatrix} = (y-x)(z-x)(z+x-y-x) = (y-x)(z-x)(z-y) \\ &= (x-y)(y-z)(z-x) \end{aligned}$$

$$\therefore \quad \text{From eq. (i), L.H.S.} = (y-x)(z-x)(z-y) + pxyz(y-x)(z-x)(z-y) \\ = (1+pxyz)(y-x)(z-x)(z-y) = \text{R.H.S.}$$

$$\begin{aligned} 13. \text{ L.H.S.} &= \begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix} = \begin{vmatrix} a+b+c & -a+b & -a+c \\ a+b+c & 3b & -b+c \\ a+b+c & -c+b & 3c \end{vmatrix} [C_1 \rightarrow C_1 + C_2 + C_3] \\ &= (a+b+c) \begin{vmatrix} 1 & -a+b & -a+c \\ 1 & 3b & -b+c \\ 1 & -c+b & 3c \end{vmatrix} \\ &= (a+b+c) \begin{vmatrix} 1 & -a+b & -a+c \\ 0 & 2b+a & -b+a \\ 1 & -c+a & 2c+a \end{vmatrix} [R_2 \rightarrow R_2 - R_1 \text{ and } R_3 \rightarrow R_3 - R_1] \\ &= (a+b+c) \cdot 1 \begin{vmatrix} 2b+a & a-b \\ a-c & 2c+a \end{vmatrix} = (a+b+c) [(2b+a)(2c+a) - (a-b)(a-c)] \\ &= (a+b+c) [4bc + 2ab + a^2 - a^2 + ac + ab - bc] = (a+b+c)(3ab + 3bc + 3ac) \\ &= 3(a+b+c)(ab + bc + ac) = \text{R.H.S.} \end{aligned}$$

$$\begin{aligned} 14. \text{ L.H.S.} &= \begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 4+3p+2q \\ 3 & 6+3p & 10+6p+3q \end{vmatrix} \\ &= \begin{vmatrix} 1 & 1+p & 1+p+q \\ 0 & 1 & 2+p \\ 0 & 3 & 7+3p \end{vmatrix} [R_2 \rightarrow R_2 - 2R_1 \text{ and } R_3 \rightarrow R_3 - 3R_1] \\ &= 1 \begin{vmatrix} 1 & 2+p \\ 3 & 7+3p \end{vmatrix} - 0 + 0 = 7 + 3p - 3(2+p) = 7 + 3p - 6 - 3p = 1 = \text{R.H.S.} \end{aligned}$$

$$\begin{aligned} 15. \text{ L.H.S.} &= \begin{vmatrix} \sin \alpha & \cos \alpha & \cos(\alpha + \delta) \\ \sin \beta & \cos \beta & \cos(\beta + \delta) \\ \sin \gamma & \cos \gamma & \cos(\gamma + \delta) \end{vmatrix} = \begin{vmatrix} \sin \alpha & \cos \alpha & \cos \alpha \cos \delta - \sin \alpha \sin \delta \\ \sin \beta & \cos \beta & \cos \beta \cos \delta - \sin \beta \sin \delta \\ \sin \gamma & \cos \gamma & \cos \gamma \cos \delta - \sin \gamma \sin \delta \end{vmatrix} \\ &= \begin{vmatrix} \sin \alpha & \cos \alpha & \cos \alpha \cos \delta - \sin \alpha \sin \delta + \sin \alpha \sin \delta \\ \sin \beta & \cos \beta & \cos \beta \cos \delta - \sin \beta \sin \delta + \sin \beta \sin \delta \\ \sin \gamma & \cos \gamma & \cos \gamma \cos \delta - \sin \gamma \sin \delta + \sin \gamma \sin \delta \end{vmatrix} [C_3 \rightarrow C_3 + (\sin \delta)C_1] \end{aligned}$$

$$\begin{aligned}
&= \begin{vmatrix} \sin \alpha & \cos \alpha & \cos \alpha \cos \delta \\ \sin \beta & \cos \beta & \cos \beta \cos \delta \\ \sin \gamma & \cos \gamma & \cos \gamma \cos \delta \end{vmatrix} = \cos \delta \begin{vmatrix} \sin \alpha & \cos \alpha & \cos \alpha \\ \sin \beta & \cos \beta & \cos \beta \\ \sin \gamma & \cos \gamma & \cos \gamma \end{vmatrix} \\
&= \cos \delta(0) \quad [\because C_2 \text{ and } C_3 \text{ have become identical}] \\
&= 0 = \text{R.H.S.}
\end{aligned}$$

16. Putting $\frac{1}{x} = u, \frac{1}{y} = v$ and $\frac{1}{z} = w$ in the given equations,

$$2u + 3v + 10w = 4; \quad 4u - 6v + 5w = 1; \quad 6u + 9v - 20w = 2$$

$$\therefore \text{ the matrix form of given equations is } \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix} \quad [AX = B]$$

$$\text{Here, } A = \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}, X = \begin{bmatrix} u \\ v \\ w \end{bmatrix} \text{ and } B = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$\therefore |A| = \begin{vmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{vmatrix} = 2(120 - 45) - 3(-80 - 30) + 10(36 + 36) = 150 + 330 + 750 = 1200 \neq 0$$

$$\therefore A^{-1} \text{ exists and unique solution is } X = A^{-1}B \quad \dots\dots\dots(i)$$

$$\text{Now } A_{11} = 75, A_{12} = 110, A_{13} = 72 \text{ and } A_{21} = 150, A_{22} = -100, A_{23} = 0 \\ \text{and } A_{31} = 75, A_{32} = 30, A_{33} = -24$$

$$\therefore \text{adj. } A = \begin{bmatrix} 75 & 110 & 72 \\ 150 & -100 & 0 \\ 75 & 30 & -24 \end{bmatrix} = \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

$$\text{And } A^{-1} = \frac{\text{adj. } A}{|A|} = \frac{1}{1200} \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

$$\therefore \text{ From eq. (i), } \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \frac{1}{1200} \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \frac{1}{1200} \begin{bmatrix} 300 + 150 + 150 \\ 440 - 100 + 60 \\ 288 + 0 - 48 \end{bmatrix} = \frac{1}{1200} \begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5} \end{bmatrix}$$

$$\therefore u = \frac{1}{2}, v = \frac{1}{3}, w = \frac{1}{5} \Rightarrow x = \frac{1}{u} = 2, y = \frac{1}{v} = 3, z = \frac{1}{w} = 5$$

17. According to question, $b - a = c - b$ (i)

$$\text{Let } \Delta = \begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix} = \begin{vmatrix} x+2 & x+3 & x+2a \\ 1 & 1 & 2(b-a) \\ 1 & 1 & 2(c-b) \end{vmatrix} [R_2 \rightarrow R_2 - R_1 \text{ and } R_3 \rightarrow R_3 - R_1]$$

$$= \begin{vmatrix} x+2 & x+3 & x+2a \\ 1 & 1 & 2(b-a) \\ 1 & 1 & 2(b-a) \end{vmatrix} [\text{From eq. (i)}] = 0 [\because R_2 \text{ and } R_3 \text{ have become identical}]$$

Therefore, option (A) is correct.

18. Given: Matrix $A = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix} \therefore |A| = \begin{vmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{vmatrix}$

$$\Rightarrow |A| = x(yz - 0) - 0 + 0 = xyz \neq 0$$

$\therefore A^{-1}$ exists and unique solution is $X = A^{-1}B$ (i)

Now $A_{11} = yz, A_{12} = 0, A_{13} = 0$ and $A_{21} = 0, A_{22} = xz, A_{23} = 0$ and $A_{31} = 0, A_{32} = 0, A_{33} = xy$

$$\therefore \text{adj. } A = \begin{vmatrix} yz & 0 & 0 \\ 0 & xz & 0 \\ 0 & 0 & xy \end{vmatrix} = \begin{vmatrix} yz & 0 & 0 \\ 0 & xz & 0 \\ 0 & 0 & xy \end{vmatrix}$$

$$\text{And } A^{-1} = \frac{\text{adj. } A}{|A|} = \frac{1}{xyz} \begin{vmatrix} yz & 0 & 0 \\ 0 & xz & 0 \\ 0 & 0 & xy \end{vmatrix} = \begin{vmatrix} \frac{yz}{xyz} & 0 & 0 \\ 0 & \frac{xz}{xyz} & 0 \\ 0 & 0 & \frac{xy}{xyz} \end{vmatrix} = \begin{vmatrix} \frac{1}{x} & 0 & 0 \\ 0 & \frac{1}{y} & 0 \\ 0 & 0 & \frac{1}{z} \end{vmatrix} = \begin{vmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{vmatrix}$$

Therefore, option (A) is correct.

19. Given: Matrix $A = \begin{bmatrix} 1 & \sin \theta & 1 \\ -\sin \theta & 1 & \sin \theta \\ -1 & -\sin \theta & 1 \end{bmatrix} \therefore |A| = \begin{vmatrix} 1 & \sin \theta & 1 \\ -\sin \theta & 1 & \sin \theta \\ -1 & -\sin \theta & 1 \end{vmatrix}$

$$\Rightarrow |A| = 1(1 + \sin^2 \theta) - \sin \theta(-\sin \theta + \sin \theta) + 1(\sin^2 \theta + 1)$$

$$\Rightarrow |A| = 1 + \sin^2 \theta + 1 + \sin^2 \theta = 2 + 2\sin^2 \theta \text{(i)}$$

Since $-1 \leq \sin \theta \leq 1 \Rightarrow 0 \leq \sin^2 \theta \leq 1$ [$\because \sin^2 \theta$ cannot be negative]

$$\Rightarrow 0 \leq 2\sin^2 \theta \leq 2 \Rightarrow 2 \leq 2 + 2\sin^2 \theta \leq 4 \Rightarrow 2 \leq \text{Det. } A \leq 4$$

Therefore, option (D) is correct.