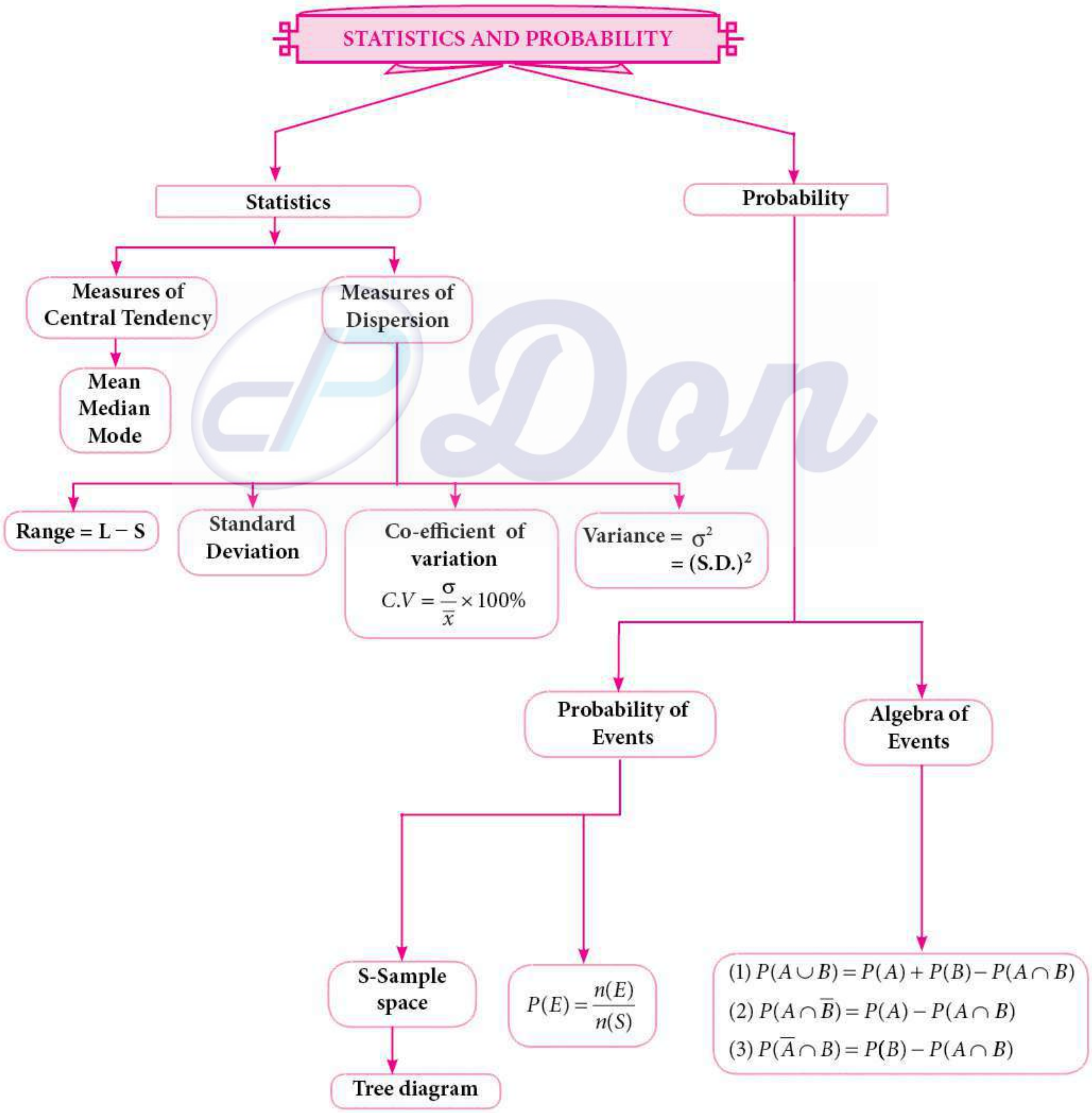
UNIT 8 STATISTICS AND PROBABILITY

MIND MAP



MEASURES OF DISPERSION

Key Points

Prasanta Chandra Mahalanobis is the "Father of Indian Statistics".

- He was awarded the Padma Vibhushan in 1968 for introducing innovative techniques for conducting large scale sample surveys.
- The Government of India has designated 29th June every year, coinciding with his birth anniversary as "National Statistics Day".
- STATISTICS is derived from the Latin word status which means a Political State.
- The study of statistics is concerned with scientific methods for collecting, organising, summarising, presenting, analysing data and making meaningful decisions.

Recalling Measures of Central Tendency

- A number that represents the whole data is called a Measures of Central Tendency or an average.
- The Measures of Central Tendency usually will be near to the middle value of the data.
- The Most Common Measures of Central Tendency are

  (i) Arithmetic mean — Arithmetic mean $\bar{x} = \dfrac{\text{Sum of all observations}}{\text{Number of observations}}$

  (ii) Median

  (iii) Mode

Methods of Finding Mean

- **Ungrouped data**
  - **Direct method**: $\bar{x} = \dfrac{\sum_{i=1}^{n} x_i}{n}$
- **Grouped data**
  - **Direct method**: $\bar{x} = \dfrac{\sum_{i=1}^{n} x_i f_i}{\sum_{i=1}^{n} f_i}$
  - **Assumed Mean Method**: $\bar{x} = A + \dfrac{\sum_{i=1}^{n} f_i d_i}{\sum_{i=1}^{n} f_i}$, Where $d_i = x_i - A$
  - **Step Deviation Method**: $\bar{x} = A + C \times \dfrac{\sum_{i=1}^{n} f_i d_i}{\sum_{i=1}^{n} f_i}$, Where $d_i = \dfrac{x_i - A}{C}$

- The representation of facts numerically is called Data.
- Each entry in the data is called an Observation.
- The quantities which are being considered in a survey are called Variables. Variables are generally denoted by $x_1, x_2, x_3, \ldots$
- The number of times a variable occurs in a given data is called the Frequency of that variable. It is generally denoted as $f_1, f_2, f_3, f_4 \ldots$
- Dispersion is a measure which gives an idea about the scatteredness of the values.
- Measures of Variation (or) Dispersion of a data provide an idea of how observations spread out (or) Scattered throughout the data.
- Different Measures of Dispersion are
  - Range
  - Quartile Deviation
  - Variance
  - Mean Deviation
  - Standard Deviation
  - Co-efficient of Variation

Range:

- The difference between the largest value and the smallest value is called Range.

$$\text{Range R} = L - S$$

- Co-efficient of Range $= \dfrac{L-S}{L+S}$

  where L – Largest Value
  S – Smallest value

- If the frequency of the initial class is zero, then the next class will be considered for the calculation of range.
- We know that the squares of deviations from the mean $(\bar{x})$, i.e., $(x_i - \bar{x})^2 \geq 0$ for all observations x_i, i = 1, 2, 3,..., n.
- If the deviations from the mean $x_i - \bar{x}$ are small, then the squares of the deviations will be very small.

Variance:

- The mean of the squares of the deviations from the mean is called Variance. It is denoted by σ^2.
- Variance = Mean of squares of deviations.
- Variance $\sigma^2 = \dfrac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

Standard Deviation:

- The positive square root of the variance is called Standard Deviation.
- Standard deviation is the positive square root of the mean of the squares of deviations of the given values from their mean.
- Standard deviation gives the clear idea about how far the values are spreading or deviating from their mean.
- Standard deviation $\sigma = \dfrac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$
- Karl Pearson was the first person to use the word standard deviation.
- German mathematician Gauss used the word mean error.
- The standard deviation and mean have same units in which the data are given.

CALCULATION OF STANDARD DEVIATION

Ungrouped data

- **Direct Method**: $\sigma = \sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2}$
- **Mean Method**: $\sigma = \sqrt{\frac{\Sigma d_i^2}{n}}$ where $d_i = x_i - \bar{x}$
- **Assumed Mean Method**: $\sigma = \sqrt{\frac{\Sigma d_i^2}{n} - \left(\frac{\Sigma d_i}{n}\right)^2}$ where $d_i = x_i - A$, $A -$ assumed mean
- **Step Deviation Method**: $\sigma = C \times \sqrt{\frac{\Sigma d_i^2}{n} - \left(\frac{\Sigma d_i}{n}\right)^2}$ where $d_i = \frac{x_i - A}{C}$
  C – Common divisor
  A – Assumed mean

Grouped data (Both discrete and continuous)

- **Mean Method**: $\sigma = \sqrt{\frac{\Sigma f_i d_i^2}{N}}$ where $N = \sum_{i=1}^{n} f_i$, $d_i = x_i - \bar{x}$
- **Assumed Mean Method**: $\sigma = \sqrt{\frac{\Sigma f_i d_i^2}{N} - \left(\frac{\Sigma f_i d_i}{N}\right)^2}$ where $d_i = x_i - A$, $A -$ Assumed mean
  $\sigma = \sqrt{\frac{\Sigma f_i (x_i - \bar{x})^2}{N}}$ *if continuous*
- **Step Deviation Method**: $\sigma = C \times \sqrt{\frac{\Sigma f_i d_i^2}{N} - \left(\frac{\Sigma f_i d_i}{N}\right)^2}$, $d_i = \frac{x_i - A}{C}$

- If the data values are given directly then to find standard deviation we use

$$\sigma = \sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2}$$

- If the squares of the deviations from the mean of each observation is given then to find the standard deviation we use $\sigma = \sqrt{\frac{\Sigma (x_i - \bar{x})^2}{n}}$
- When the mean value is not an integer, then we use Assumed mean method to find standard deviation.
- The standard deviation will not change when we add or subtract some fixed constant to all the values.
- When we multiply or divide each data by a fixed constant then the standard deviation is also get multiplied or divided by the constant.

Worked Examples

8.1 **Find the range and co-efficient of range of the following data: 25, 67, 48, 53, 18, 39, 44.**

Sol :

Largest value L = 67; Smallest value S = 18

Range R = L – S = 67 – 18 = 49

Co-efficient of range= $\frac{L-S}{L+S}$

Co-efficient of range= $\frac{67-18}{67+18} = \frac{49}{85} = 0.576$

8.2 **Find the range of the following distribution.**

Age (in years)	Number of students
16-18	0
18-20	4
20-22	6
22-24	8
24-26	2
26-28	2

Sol :

Here Largest value L = 28

Smallest value S = 18

Range R = L – S = 28 –18

= 10 Years

8.3 **The range of a set of data is 13.67 and the largest value is 70.08. Find the smallest value.**

Sol :

Range R = 13.67

Largest value L = 70.08

Range R = L – S

13.67 = 70.08 – S

S = 70.08 – 13.67 = 56.41

Therefore, the smallest value is 56.41.

8.4 **The number of televisions sold in each day of a week are 13, 8, 4, 9, 7, 12, 10. Find its standard deviation.**

Sol :

x_i	x_i^2
13	169
8	64
4	16
9	81
7	49
12	144
10	100
$\Sigma x_i = 63$	$\Sigma x_i^2 = 623$

Standard deviation

$$\sigma = \sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2}$$

$$= \sqrt{\frac{623}{7} - \left(\frac{63}{7}\right)^2}$$

$$= \sqrt{89-81} = \sqrt{8}$$

Hence $\sigma \simeq 2.83$

8.5 **The amount of rainfall in a particular season for 6 days are given as 17.8 cm, 19.2 cm, 16.3 cm, 12.5 cm, 12.8 cm and 11.4 cm. Find its standard deviation.**

Sol :

Arranging the numbers in ascending order we get 11.4, 12.5, 12.8, 16.3, 17.8, 19.2.

Number of observations n = 6

$$\text{Mean} = \frac{11.4+12.5+12.8+16.3+17.8+19.2}{6}$$

$$= \frac{90}{6} = 15$$

x_i	$d_i = x_i - \bar{x}$ $= x - 15$	d_i^2
11.4	– 3.6	12.96
12.5	– 2.5	6.25
12.8	– 2.2	4.84
16.3	1.3	1.69
17.8	2.8	7.84
19.2	4.2	17.64
		$\Sigma d_i^2 = 51.22$

$$\text{Standard deviation } \sigma = \sqrt{\frac{\Sigma d_i^2}{n}}$$

$$= \sqrt{\frac{51.22}{6}} = \sqrt{8.53}$$

Hence $\sigma \simeq 2.9$

8.6 **The marks scored by 10 students in a class test are 25, 29, 30, 33, 35, 37, 38,40, 44, 48. Find the standard deviation.**

Sol :

The mean of marks is 35.9 which is not an integer. Hence we take assumed mean A = 35, n = 10.

x_i	$d_i = x_i - A$ $d_i = x_i - 35$	d_i^2
25	– 10	100
29	– 6	36
30	– 5	25
33	– 2	4
35	0	0
37	2	4
38	3	9
40	5	25
44	9	81
48	13	169
	$\Sigma d_i = 9$	$\Sigma d_i^2 = 453$

Standard deviation

$$\sigma = \sqrt{\frac{\Sigma d_i^2}{n} - \left(\frac{\Sigma d_i}{n}\right)^2}$$

$$= \sqrt{\frac{453}{10} - \left(\frac{9}{10}\right)^2}$$

$$= \sqrt{45.3 - 0.81}$$

$$= \sqrt{44.49}$$

Henec $\sigma \simeq 6.67$

8.7 The amount that the children have spent for purchasing some eatables in one day trip of a school are 5, 10, 15, 20, 25, 30, 35, 40. Using step deviation method, find the standard deviation of the amount they have spent.

Sol :

We note that all the observations are divisible by 5. Hence we can use the step deviation method. Let Assumed mean A = 20, n = 8.

x_i	$d_i = x_i - A$ $d_i = x_i - 20$	$d_i = \frac{x_i - A}{c}$ c = 5	d_i^2
5	– 15	– 3	9
10	– 10	– 2	4
15	– 5	– 1	1
20	0	0	0
25	5	1	1
30	10	2	4
35	15	3	9
40	20	4	16
		$\Sigma d_i = 4$	$\Sigma d_i^2 = 44$

Standard deviation

$$\sigma = \sqrt{\frac{\Sigma d_i^2}{n} - \left(\frac{\Sigma d_i}{n}\right)^2} \times c$$

$$= \sqrt{\frac{44}{8} - \left(\frac{4}{8}\right)^2} \times 5 = \sqrt{\frac{11}{2} - \frac{1}{4}} \times 5$$

$$= \sqrt{5.5 - 0.25} \times 5 = 2.29 \times 5$$

$$\sigma \simeq 11.45$$

8.8. Find the standard deviation of the following data 7, 4, 8, 10, 11. Add 3 to all the values then find the standard deviation for the new values.

Sol :

Arranging the values in ascending order we get 4, 7, 8, 10, 11 and n = 5

x_i	x_i^2
4	16
7	49
8	64
10	100
11	121
$\Sigma x_i = 40$	$\Sigma x_i^2 = 350$

Standard deviation

$$\sigma = \sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2}$$

$$= \sqrt{\frac{350}{5} - \left(\frac{40}{5}\right)^2}$$

$$\sigma = \sqrt{6} \simeq 2.45$$

When we add 3 to all the values, we get the new values as 7, 10, 11, 13, 14.

x_i	x_i^2
7	9
10	100
11	121
13	169
14	196
$\Sigma x_i = 55$	$\Sigma x_i^2 = 635$

Standard deviation

$$\sigma = \sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2}$$

$$= \sqrt{\frac{635}{5} - \left(\frac{55}{5}\right)^2}$$

$$\sigma = \sqrt{6} \simeq 2.45$$

From the above, we see that the standard deviation will not change when we add some fixed constant to all the values.

8.9 **Find the standard deviation of the data 2, 3, 5, 7, 8. Multiply each data by 4. Find the standard deviation of the new values.**

Sol :

Given, n = 5

x_i	x_i^2
2	4
3	9
5	25
7	49
8	64
$\Sigma x_i = 25$	$\Sigma x_i^2 = 151$

Standard deviation

$$\sigma = \sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2}$$

$$\sigma = \sqrt{\frac{151}{5} - \left(\frac{25}{5}\right)^2}$$

$$= \sqrt{30.2 - 25}$$

$$= \sqrt{5.2} \simeq 2.28$$

When we multiply each data by 4, we get the new values as 8, 12, 20, 28, 32.

x_i	x_i^2
8	64
12	144
20	400
28	784
32	1024
$\Sigma x_i = 100$	$\Sigma x_i^2 = 2416$

Standard deviation

$$\sigma = \sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2}$$

$$= \sqrt{\frac{2416}{5} - \left(\frac{100}{5}\right)^2}$$

$$= \sqrt{483.2 - 400} = \sqrt{83.2}$$

$$\sigma = \sqrt{16 \times 5.2} = 4\sqrt{5.2} \simeq 9.12$$

From the above, we see that when we multiply each data by 4 the standard deviation is also get multiplied by 4.

8.10 **Find the mean and variance of the first n natural numbers.**

Sol :

$$\text{Mean } \bar{x} = \frac{\textit{Sum of all the observations}}{\textit{Number of observations}}$$

$$= \frac{\Sigma x_i}{n} = \frac{1+2+3+\dots\dots+n}{n} = \frac{n(n+1)}{2 \times n}$$

$$\text{Mean } \bar{x} = \frac{n+1}{2}$$

$$\text{Variance } \sigma^2 = \frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2$$

$$\begin{bmatrix} \Sigma x_i^2 = 1^2 + 2^2 + 3^2 + \dots + n^2 \\ (\Sigma x_i)^2 = (1+2+3+\dots\dots+n)^2 \end{bmatrix}$$

$$= \frac{n(n+1)(2n+1)}{6 \times n} - \left[\frac{n(n+1)}{2 \times n}\right]^2$$

$$= \frac{2n^2+3n+1}{6} - \frac{n^2+2n+1}{4}$$

$$= \frac{4n^2+6n+2-3n^2-6n-3}{12}$$

$$\text{Variance } \sigma^2 = \frac{n^2-1}{12}.$$

8.11 **48 students were asked to write the total number of hours per week they spent for watching television. With this information find the standard deviation of hours spent for watching television.**

x	6	7	8	9	10	11	12
f	3	6	9	13	8	5	4

Sol :

x_i	f_i	x_if_i	$d_i = x_i - \bar{x}$	d_i^2	$f_i d_i^2$
6	3	18	– 3	9	27
7	6	42	– 2	4	24
8	9	72	– 1	1	9
9	13	117	0	0	0
10	8	80	1	1	8
11	5	55	2	4	20
12	4	48	3	9	36
	N = 48	$\Sigma x_i f_i = 432$	$\Sigma d_i = 0$		$\Sigma f_i d_i^2 = 124$

Mean $\bar{x} = \dfrac{\Sigma x_i f_i}{N} = \dfrac{432}{48} = 9$ (Since $N = \Sigma f_i$)

Standard deviation

$$\sigma = \sqrt{\frac{\Sigma f_i d_i^2}{N}} = \sqrt{\frac{124}{48}} = \sqrt{2.58}$$

$$\sigma \simeq 1.6$$

8.12 **The marks scored by the students in a slip test are given below.**

x	4	6	8	10	12
f	7	3	5	9	5

Find the standard deviation of their marks.

Sol :

Let the assumed mean, A = 8

x_i	f_i	$d_i = x_i - A$	$f_i d_i$	$f_i d_i^2$
4	7	– 4	– 28	112
6	3	– 2	– 6	12
8	5	0	0	0
10	9	2	18	36
12	5	4	20	80
	N = 29		$\Sigma f_i d_i = 4$	$\Sigma f_i d_i^2 = 240$

Standard deviation

$$\sigma = \sqrt{\frac{\Sigma f_i d_i^2}{N} - \left(\frac{\Sigma f_i d_i}{N}\right)^2}$$

$$= \sqrt{\frac{240}{29} - \left(\frac{4}{29}\right)^2}$$

$$= \sqrt{\frac{240 \times 29 - 16}{29 \times 29}}$$

$$\sigma = \sqrt{\frac{6944}{29 \times 29}} \simeq 2.87$$

8.13 **Marks of the students in a particular subject of a class are given below.**

Marks	Number of students
0-10	8
10–20	12
20–30	17
30–40	14
40–50	9
50–60	7
60–70	4

Find its standard deviation.

Sol :

Let Assumed mean, A = 35, c = 10

Marks	Mid value(x_i)	f_i	$d_i = x_i - A$	$d_i = \frac{x_i - A}{C}$	$f_i d_i$	$f_i d_i^2$
0-10	5	8	– 30	– 3	– 24	72
10-20	15	12	– 20	– 2	– 24	48
20-30	25	17	– 10	– 1	– 17	17
30-40	35	14	0	0	0	0
40-50	45	9	10	1	9	9
50-60	55	7	20	2	14	28
60-70	65	4	30	3	12	36
		N = 71			$\Sigma f_i d_i = -30$	$\Sigma f_i d_i^2 = 210$

Standard deviation

$$\sigma = c \times \sqrt{\frac{\Sigma f_i d_i^2}{N} - \left(\frac{\Sigma f_i d_i}{N}\right)^2}$$

$$\sigma = 10 \times \sqrt{\frac{210}{71} - \left(-\frac{30}{71}\right)^2}$$

$$\sigma = 10 \times \sqrt{\frac{210}{71} - \frac{900}{5041}}$$

$$\sigma = 10 \times \sqrt{2.779} \;;\; \simeq 16.67$$

8.14 **The mean and standard deviation of 15 observations are found to be 10 and 5 respectively. On rechecking it was found that one of the observation with value 8 was incorrect. Calculate the correct mean and standard deviation if the correct observation value was 23.**

Sol :

$n = 15$, $\bar{x} = 10$, $\sigma = 5$; $\bar{x} = \frac{\Sigma x}{n}$;

$\Sigma x = 15 \times 10 = 150$

Wrong observation value = 8.

Correct observation value = 23.

Correct total = 150 – 8 + 23 = 165

Correct mean $\bar{x} = \frac{165}{15} = 11$

Standard deviation $\sigma = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}$

Incorrect value of $\sigma = 5 = \sqrt{\frac{\Sigma x^2}{15} - (10)^2}$

$25 = \frac{\Sigma x^2}{15} - 100 \Rightarrow \frac{\Sigma x^2}{15} = 125$

Incorrect value of $\Sigma x^2 = 1875$

Correct value of $\Sigma x^2 = 1875 - 8^2 + 23^2 = 2340$

Correct standard deviation $\sigma = \sqrt{\frac{2340}{15} - (11)^2}$

$\sigma = \sqrt{156 - 121} = \sqrt{35} \simeq 5.9$

Progress Check

1. **_____ is a value that represent a set of data**

   Ans : An observation

2. **The sum of all the observations divided by number of observations is _____**

   Ans : Arithmetic Mean

3. **If the sum of 10 data values is 265 then their mean is ______**

   Ans : 26.5

4. **If the sum and mean of a data are 407 and 11 respectively, then the number of observations in the data are ______**

   Ans : 37

5. **The range of first 10 prime numbers is ______**

   Ans : 27

   Range = L – S = 29 – 2 = 27

6. **If the variance is 0.49, then the standard deviation is ______**

   Ans : 0.7

Thinking Corner

1. **Does the mean, median and mode are same for a given data?**

   Ans : In a perfectly symmetrical distribution, the mean and median are the same. If the distribution has one mode, the mode is same as the mean and median.

   That is a symmetrical distribution which is unimodal has its mean, median and mode the same.

2. **What is the difference between the arithmetic mean and average?**

   Ans : The measures of Central Tendency Mean, median and mode are called averages. Arithmetic mean is the sum of all observation divided by total number of observations. So, Arithmetic mean is one type of averages.

3. **The mean of n observation is $\bar{x}$, if first term is increased by 1 second term is increased by 2 and so on. What will be the new mean?**

   Ans : Given the mean of n observations is $\bar{x}$. Let x_1, x_2, x_3, ..., x_n be the n observations.

   Then $\frac{x_1 + x_2 + x_3 + ... + x_n}{n} = \bar{x}$.

   If 1 is added to the first term, 2 to the second term and so on.

   Then the terms will be $x_1 + 1, x_2 + 2, x_3 + 3, ... x_n + n$. Now the mean

   $$= \frac{x_1 + 1 + x_2 + 2 + x_3 + 3 + ... + x_n + n}{n}$$

   $$= \frac{x_1 + x_2 + x_3 + ... + x_n}{n} + \frac{1 + 2 + 3 + ... + n}{n}$$

   $$= \bar{x} + \frac{\frac{n(n+1)}{2}}{n}$$

$$= \bar{x} + \frac{n(n+1)}{2} \times \frac{1}{n}$$

$$= \bar{x} + \frac{(n+1)}{2}$$

4. Can variance be negative?

Ans : Variance cannot be negative, because it is the mean of the squares of the deviations from the mean. Any squared value never be negative.

5. Can the standard deviation be more than the variance?

Ans : The variance is the square of the standard deviation.

So if the variance less than 1, then the standard deviation be more than the variance, otherwise variance has the greater value.

For example if variance = 0.001, then the standard deviation = $\sqrt{0.01}$ = 0.1

6. For any collection of n values.

(i) $\Sigma(x_i - \bar{x}) =$ _____

(ii) $(\Sigma x_i) - \bar{x} =$ _____

(i) $\Sigma(x_i - \bar{x}) = \Sigma d_i$, the sum of deviations from the mean of each observations.

(ii) $\Sigma x_i - \bar{x} = \Sigma x_i - \frac{\Sigma x_i}{n}$ $\left[\because \bar{x} = \frac{\Sigma x_i}{n}\right]$

$$= \frac{(n-1)}{n}\Sigma x_i$$

7. (i) The standard deviation of a data is 2.8, if 5 is added to all the data values then the new standard deviation is _____

Ans : 2.8

(ii) If S is the standard deviation p, q, r then standard deviation of p – 3, q – 3, r – 3 is _____

Ans : S

Exercise 8.1

1. Find the range and coefficient of range of the following data.

(i) 63, 89, 98, 125, 79, 108, 117, 68

(ii) 43.5, 13.6, 18.9, 38.4, 61.4, 29.8

Sol :

(i) 63, 89, 98, 125, 79, 108, 117, 68

Largest value L = 125

Smallest value S = 63

Range R = L – S = 125 – 63 = 62

Coefficient of range $= \frac{L-S}{L+S} = \frac{125-63}{125+63}$

$= \frac{62}{188} = 0.329 = 0.33$

∴ Range = 62 ; coefficient of range = 0.33

(ii) 43.5, 13.6, 18.9, 38.4, 61.4, 29.8

Largest value L = 61.4

Smallest value S = 13.6

Range = L – S

= 61.4 – 13.6 = 47.8

Coefficient of range $= \frac{61.4-13.6}{61.4+13.6}$

$= \frac{47.8}{75} = 0.637 = 0.64$

∴ Range = 47.8 ; co-efficient of range = 0.64.

2. If the range and the smallest value of a set of data are 36.8 and 13.4 respectively, then find the largest value.

Sol :

Given range = 36.8

Smallest value = 13.4

Range R = L – S

36.8 = L – 13.4

36.8 + 13.4 = L

L = 50.2

∴ The largest value L = 50.2

3. Calculate the range of the following data.

Income	400-450	450-500	500-550	550-600	600-650
Number of workers	8	12	30	21	6

Sol :

Here the Largest value L = 650

Smallest value S = 400

∴ Range R = L – S

R = 650 – 400 = 250

∴ Range R = 250

4. A teacher asked the students to complete 60 pages of a record note book. Eight students have completed only 32, 35, 37, 30, 33, 36, 35 and 37 pages. Find the standard deviation of the pages yet to be completed by them.

Sol :

Total pages = 60

Completed pages by the students are

32, 35, 37, 30, 33, 36, 35, 37

Let the pages yet to be completed by 8 students be x_i

Completed Pages	Pages yet to be completed (x_i) x_i = 60 – completed pages	x_i^2
32	28	784
35	25	625
37	23	529
30	30	900
33	27	729
36	24	576
35	25	625
37	23	529
	$\Sigma x_i^2 = 205$	$\Sigma x_i^2 = 5297$

$$\text{Standard deviation } \sigma = \sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2}$$

$$= \sqrt{\frac{9497}{8} - \left(\frac{275}{8}\right)^2}$$

$$= \sqrt{1187.125 - (34.375)^2} = \sqrt{1187.125 - 1181.64063}$$

$$= \sqrt{5.48437} = 2.34 \qquad \therefore \sigma \simeq 2.34$$

5. Find the variance and standard deviation of the wages of 9 workers given below: ₹ 310, ₹ 290, ₹ 320, ₹ 280, ₹ 300, ₹ 290, ₹ 320, ₹ 310, ₹ 280.

Sol :

Arranging the numbers in ascending order

₹ 280, ₹ 280, ₹ 290, ₹ 290, ₹ 300, ₹ 310, ₹ 310, ₹ 320, ₹ 320

$$\text{Mean} = \frac{280+280+290+290+300+310+310+320+320}{9}$$

$$\bar{x} = \frac{2700}{9} = 300$$

x_i	$d_i = x_i - \bar{x} = x_i - 300$	d_i^2
280	– 20	400
280	– 20	400
290	– 10	100
290	– 10	100
300	0	0
310	10	100
310	10	100
320	20	400
320	20	400
		$\sum d_i^2 = 2000$

Standard Deviation

$$\sigma = \sqrt{\frac{\Sigma d_i^2}{n}}$$

$$\sigma = \sqrt{\frac{2000}{9}}$$

$$\sigma = \sqrt{222.222...} = 14.907 = 14.91$$

We have $\sigma = \sqrt{222.222}$

Variance $\sigma^2 = 222.22$

$\therefore$ Variance = 222.22 ;

Standard deviation = 14.91

6. A wall clock strikes the bell once at 1 O'clock, 2 times at 2 O'clock, 3 times at 3 O'clock and so on. How many times will it strike in a particular day? Find the standard deviation of the number of strikes the bell make a day.

Sol :

Clock strikes once at 1, twice at 2, thrice at 3 and so on.

But it only strikes 12 times at most and then it repeats. So, Number of times clock strikes = 2 (1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12)

=78 × 2 = 156 times.

$\therefore$ In a particular day the clock strikes 156 times.

Also standard deviation for first n natural numbers is $\sqrt{\frac{n^2-1}{12}}$

$\therefore$ Standard deviation of the number of strikes

$$= 2\sqrt{\frac{n^2-1}{12}}$$

$$= 2\sqrt{\frac{12^2-1}{12}} = 2\sqrt{\frac{144-1}{12}}$$

$$= 2\sqrt{\frac{143}{12}} = 2\sqrt{11.92}$$

$$= 2 \times 3.45 = 6.9$$

$\therefore$ Standard deviation of the number of strikes the bell make a day = 6.9

7. Find the standard deviation of first 21 natural numbers.

Sol : Standard deviation of first n natural numbers

$$= \sqrt{\frac{n^2-1}{12}}$$

$\therefore$ SD of first 21 natural numbers

$$= \sqrt{\frac{21^2-1}{12}}$$

$$= \sqrt{\frac{441-1}{12}} = \sqrt{\frac{440}{12}}$$

$$= \sqrt{36.6666} = 6.05$$

Standard deviation of first 21 natural numbers = 6.05

8. If the standard deviation of a data is 4.5 and if each value of the data is decreased by 5, then find the new standard deviation.

Sol : The standard deviation of a given data is 4.5. If we subtract some fixed constant from all the data, the standard deviation will not change.

$\therefore$ Each value of the data decreased by 5, the new standard deviation will not change.

$\therefore$ New standard deviation = 4.5

9. If the standard deviation of a data is 3.6 and each value of the data is divided by 3, then find the new variance and standard deviation.

Sol :

Standard deviation of a data = 3.6.

When each value of the data is divided by a fixed constant, then the new standard deviation is also get divided by the constant.

$\therefore$ If each value is divided by 3, then

New standard deviation = $\frac{3.6}{3} = 1.2$

New variance = $\sigma^2 = (1.2)^2$

$\therefore$ New variance = 1.44

New standard deviation= 1.2

10. The rainfall recorded in various places of five districts in a week are given below.

Rainfall (in mm)	45	50	55	60	65	70
Number of places	5	13	4	9	5	4

Find its standard deviation

Sol : Let the assumed mean A = 55

x_i	f_i	$d_i = x_i - A$ $d_i = x_i - 55$	$f_i d_i$	d_i^2	$f_i d_i^2$
45	5	– 10	– 50	100	500
50	13	– 5	– 65	25	325
55	4	0	0	0	0
60	9	5	45	25	225
65	5	10	50	100	500
70	4	15	60	225	900
	$\Sigma f_i = 40$		$\Sigma f_i d_i = 40$		$\Sigma f_i d_i^2 = 2450$

$\Sigma f_i = N = 40$

$$\text{Standard deviation } \sigma = \sqrt{\frac{\Sigma f_i d_i^2}{N} - \left(\frac{\Sigma f_i d_i}{N}\right)^2}$$

$$= \sqrt{\frac{2450}{40} - \left(\frac{40}{40}\right)^2} = \sqrt{61.25 - 1^2}$$

$$= \sqrt{61.25 - 1} = \sqrt{60.25}$$

Standard deviation σ = 7.76

11. In a study about viral fever, the number of people affected in a town were noted as

Age in years	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Number of people affected	3	5	16	18	12	7	4

Find its standard deviation.

Sol :

Let us take the assumed mean A = 20 and C = 5

Ages	Mid value x_i	f_i	$d_i = x_i - A$ $d_i = x_i - 20$	$d_i = \frac{x_i - A}{C}$ $d_i = \frac{x_i - A}{5}$	$f_i d_i$	$f_i d_i^2$
0-10	5	3	– 15	– 3	– 9	27
10-20	15	5	– 5	– 1	– 5	5
20-30	25	16	5	1	16	16
30-40	35	18	15	3	54	162
40-50	45	12	25	5	60	300
50-60	55	7	35	7	49	343
60-70	65	4	45	9	36	324
		$\Sigma f_i = N = 65$			$\Sigma f_i d_i = 201$	$\Sigma f_i d_i^2 = 1177$

Standard deviation $\sigma = C \times \sqrt{\frac{\Sigma f_i d_i^2}{N} - \left(\frac{\Sigma f_i d_i}{N}\right)^2}$

$$\sigma = 5 \times \sqrt{\frac{1177}{65} - \left(\frac{201}{65}\right)^2}$$

$$\sigma = 5 \times \sqrt{\frac{1177}{65} - \frac{40401}{4225}}$$

$$\sigma = 5 \times \sqrt{\frac{76505 - 40401}{4225}}$$

$$\sigma = 5 \times \sqrt{8.54}$$

$$\sigma = 5 \times 2.92 = 14.6$$

Standard deviation $\sigma = 14.6$

12. The measurements of the diameters (in cms) of the plates prepared in a factory are given below. Find its standard deviation.

Diameter (cm)	21-24	25-28	29-32	33-36	37-40	41-44
Number of plates	15	18	20	16	8	7

Sol : Let the assumed mean A = 34.5

Diameters (cm)	Mid value x_i	f_i	$d_i = \frac{x_i - A}{2}$, $d_i = \frac{x_i - 34.5}{2}$	$f_i d_i$	d_i^2	$f_i d_i^2$
20.5–24.5	22.5	15	– 6	– 90	36	540
24.5–28.5	26.5	18	– 4	– 72	16	288
28.5–32.5	30.5	20	– 2	– 40	4	80
32.5–36.5	34.5	16	0	0	0	0
36.5–40.5	38.5	8	2	16	4	32
40.5–44.5	42.5	7	4	28	16	112
		Σf_i = N = 84		$\Sigma f_i d_i$ = – 158		$\Sigma f_i d_i^2$ = 1052

Standard deviation $\sigma = C \times \sqrt{\frac{\Sigma f_i d_i^2}{N} - \left(\frac{\Sigma f_i d_i}{N}\right)^2}$

$$\sigma = 2 \times \sqrt{\frac{1052}{84} - \left(\frac{-158}{84}\right)^2}$$

$$\sigma = 2 \times \sqrt{\frac{22092}{1764} - \frac{6241}{1764}}$$

$$\sigma = 2 \times \sqrt{\frac{15851}{1764}} = 2 \times \sqrt{8.98}$$

$$\sigma = 2 \times 2.99 \simeq 5.99$$

$\therefore$ Standard deviation $\sigma \simeq 5.99 \simeq 6$

13. The time taken by 50 students to complete a 100 meter race are given below. Find its standard deviation

Time taken (sec)	8.5 - 9.5	9.5 - 10.5	10.5 - 11.5	11.5 - 12.5	12.5 - 13.5
Number of students	6	8	17	10	9

Sol :

Let x_i are the midvalues of the given set

Assumed mean A = 11

Time taken (sec)	x_i	f_i	$d_i = x_i - 11$	d^2	fd	fd^2
8.5–9.5	9	6	– 2	4	– 12	24
9.5–10.5	10	8	– 1	1	– 8	8
10.5–11.5	11	17	0	0	0	0
11.5–12.5	12	10	1	1	10	10
12.5–13.5	13	9	2	4	18	36
		$\Sigma f_i = N$ = 50			$\Sigma f_i d_i$ = 8	$\Sigma f_i d_i^2$ = 78

Standard deviation

$$\sigma = \sqrt{\frac{\Sigma f_i d_i^2}{N} - \left(\frac{\Sigma f_i d_i}{N}\right)^2}$$

$$= \sqrt{\frac{78}{50} - \left(\frac{8}{50}\right)^2}$$

$$= \sqrt{\frac{78}{50} - \frac{64}{2500}}$$

$$= \sqrt{\frac{3900-64}{2500}} = \sqrt{\frac{3836}{2500}}$$

$$= \sqrt{1.5344} \simeq 1.238 \simeq 1.24$$

$\therefore$ Standard deviation $\sigma \simeq 1.24$

14. For a group of 100 candidates the mean and standard deviation of their marks were found to be 60 and 15 respectively. Later on it was found that the scores 45 and 72 were wrongly entered as 40 and 27. Find the correct mean and standard deviation.

Sol :

Mean $\bar{x}$ = 60

Standard deviation σ = 15

Wrong scores = 40 and 27

Correct scores = 45 and 72.

$$\text{Old Mean} = \frac{\Sigma x_i}{n} = 60$$

$$\frac{\Sigma x_i}{100} = 60$$

$$\text{Old } \Sigma x_i = 60 \times 100 = 6000$$

$$\therefore \text{Correct } \Sigma x_i = 6000 - \text{wrong scores} + \text{correct scores}$$

$$= 6000 - (40 + 27) + (45 + 72)$$

$$= 6000 - 67 + 117$$

$$\therefore \text{Correct } \Sigma x_i = 6050$$

$$\text{Correct Mean} = \text{Correct } \frac{\Sigma x_i}{n} = \frac{6050}{100}$$

$$\text{Correct Mean} = 60.5$$

Old Standard deviation $\sigma = 15$

$$\sigma = \sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2}$$

$$15 = \sqrt{\frac{\Sigma x_i^2}{n} - (60)^2}$$

Squaring on both sides

$$225 = \frac{\Sigma x_i^2}{n} - 3600$$

$$225 + 3600 = \frac{\Sigma x_i^2}{100}$$

$$\frac{old\ \Sigma x_i^2}{100} = 3825$$

$$\text{Old } \Sigma x_i^2 = 3825 \times 100$$

$$\text{Old } \Sigma x_i^2 = 382500$$

$$\text{Correct } \Sigma x_i^2 = 382500 - (\text{wrong scores})^2 + (\text{correct scores})^2$$

$$= 382500 - 40^2 - 27^2 + 45^2 + 72^2$$

$$= 382500 - 1600 - 729 + 2025 + 5184$$

$$= 382500 - 2329 + 2025 + 5184$$

$$= 389709 - 2329$$

$$\text{Correct } \Sigma x_i^2 = 387380$$

$$\text{Correct } \sigma = \sqrt{\frac{387380}{100} - (60.5)^2}$$

$$= \sqrt{3873.80 - 3660.25}$$

$$= \sqrt{213.55}$$

$$\sigma = 14.61$$

$\therefore$ Correct $\sigma = 14.61$; Correct Mean $= 60.5$

15. The mean and variance of seven observations are 8 and 16 respectively. If five of these are 2 , 4, 10, 12 and 14, then find the remaining two observations.

Sol :

Mean $= 8$

Variance $= 16.$

Let a and b be the missing observations

$$\frac{\textit{Sum of observations}}{\textit{Number of observations}} = \text{Mean}$$

$$\frac{2+4+10+12+14+a+b}{7} = 8$$

$$42 + a + b = 56$$

$$a + b = 56 - 42 = 14 \quad \text{......(1)}$$

$$b = 14 - a$$

$$\text{Variance} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$$

$$= \frac{\sum_{i=1}^{7}(x_i - \bar{x})^2}{7}$$

$$= \frac{(2-8)^2 + (4-8)^2 + (10-8)^2 + (12-8)^2 + (14-8)^2 + (a-8)^2 + (b-8)^2}{7}$$

$$16 = \frac{36 + 16 + 4 + 16 + 36 + (a-8)^2 + (b-8)^2}{7}$$

$$108 + (a-8)^2 + (b-8)^2 = 112$$

$$(a-8)^2 + (b-8)^2 = 4 \quad \text{... (2)}$$

Put $b = 14 - a$ in (2)

$$(a-8)^2 + (b-8)^2 = 4$$

$$(a-8)^2 + ((14-a)-8)^2 = 4$$

$$(a-8)^2 + (6-a)^2 = 4$$

$$a^2 + 64 - 16a + 36 + a^2 - 12a - 4 = 0$$

$$2a^2 - 28a + 96 = 0$$

Divided by 2,

$$a^2 - 14a + 48 = 0$$

$$(a-6)(a-8) = 0$$

$$a = 6 \text{ or } a = 8$$

$$a = 6 \Rightarrow b = 14 - a = 14 - 6 = 8$$

$$a = 6 \text{ and } b = 8$$

$\therefore$ Remaining numbers are 6 and 8.

CO-EFFICIENT OF VARIATION

Key Points

Co-efficient of variation is used for comparing two or more data for corresponding changes even if they are in different units.

- It is expressed in percentage
- Co-efficient of variation C.V $= \frac{\sigma}{\overline{x}} \times 100\%$
- The set of data showing less co-efficient of variation is more stable than the other set which shows higher co-efficient of variation.

Worked Examples

8.15 The mean of a data is 25.6 and its co-efficient of variation is 18.75. Find the standard deviation.

Sol :

Mean $\overline{x} = 25.6$, Co-efficient of variation, C.V. $= 18.75$

Co-efficient of variation, C.V. $= \frac{\sigma}{\overline{x}} \times 100\%$

$$18.75 = \frac{\sigma}{25.6} \times 100\,; \Rightarrow \sigma = 4.8$$

8.16 The following table gives the values of mean and variance of heights and weights of the 10th standard students of a school.

	Height	Weight
Mean	155 cm	46.50 kg
Variance	72.25 cm^2	28.09 kg^2

Which is more varying than the other?

Sol :

For comparing the two data, first we have to find their Co-efficient of Variations

Mean $\overline{x}_1 = 155$ cm, variance $\sigma_1^2 = 72.25\ cm^2$

Therefore standard deviation $\sigma_1 = 8.5$

Co-efficient of variation $C.V_1 = \frac{\sigma_1}{\overline{x}_1} \times 100\%$

$C.V_1 = \frac{8.5}{155} \times 100\% = 5.48\%$ (for heights)

Mean $\overline{x}_2 = 46.50$ kg, Variance $\sigma_2^2 = 28.09\ kg^2$

Standard deviation $\sigma_2 = 5.3$ kg

Coefficient of variation $C.V_2 = \frac{\sigma_2}{\overline{x}_2} \times 100\%$

$C.V_2 = \frac{5.3}{46.50} \times 100\% = 11.40\%$ (for weights)

$C.V_1 = 5.48\%$ and $C.V_2 = 11.40\%$

Since $C.V_2 > C.V_1$, the weight of the students is more varying than the height.

8.17 The consumption of number of guava and orange on a particular week by a family are given below.

Number of Guavas	3	5	6	4	3	5	4
Number of Oranges	1	3	7	9	2	6	2

Which fruit is consistently consumed by the family?

Sol : First we find the Coefficient of variation for guavas and oranges separately.

Number of guavas n = 7

x_i	x_i^2
3	9
5	25
6	36
4	16
3	9
5	25
4	16
$\Sigma x_i = 30$	$\Sigma x_i^2 = 136$

$$\text{Mean } \bar{x}_1 = \frac{30}{7} = 4.29$$

$$\text{Standard deviation } \sigma_1 = \sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2}$$

$$\sigma_1 = \sqrt{\frac{136}{7} - \left(\frac{30}{7}\right)^2}$$

$$= \sqrt{19.43 - 18.40} \simeq 1.01$$

Co-efficient of Variation for guavas

$$C.V_1 = \frac{\sigma_1}{\bar{x}_1} \times 100\% = \frac{1.01}{4.29} \times 100\% = 23.54\%$$

Number of oranges n = 7

x_i	x_i^2
1	1
3	9
7	49
9	81
2	4
6	36
2	4
$\Sigma x_i = 30$	$\Sigma x_i^2 = 184$

$$\text{Mean } \bar{x}_2 = \frac{30}{7} = 4.29$$

$$\text{Standard deviation } \sigma_2 = \sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2}$$

$$\sigma_2 = \sqrt{\frac{184}{7} - \left(\frac{30}{7}\right)^2}$$

$$= \sqrt{26.29 - 18.40} = 2.81$$

Coefficient of Variation for Oranges

$$C.V_2 = \frac{\sigma_2}{\bar{x}_2} \times 100\% = \frac{2.81}{4.29} \times 100\%$$

$$= 65.50\%$$

$C.V_1 = 23.54\%$, $C.V_2 = 65.50\%$. Since, $C.V_1 < C.V_2$, we can conclude that the consumption of guavas is more consistent than oranges.

Progress Check

1. Co-efficient of variation is a relative measure of _____.

Ans : Standard deviation

2. When the standard deviation is divided by the mean we get _____.

Ans : Co-efficient of variation

3. The coefficient of variation depends upon _____and _____.

Ans : Mean and Standard deviation

4. If the mean and standard deviation of a data are 8 and 2 respectively then the coefficient of variation is _____.

Ans : 25%

5. When comparing two data, the data with _____ coefficient of variation is inconsistent.

Ans : More

Exercise 8. 2

1. The standard deviation and mean of a data are 6.5 and 12.5 respectively. Find the coefficient of variation.

Sol :

Standard deviation $\sigma = 6.5$

Mean $\bar{x} = 12.5$

$$\text{Coefficient of variation C.V} = \frac{\sigma}{\bar{x}} \times 100\%$$

$$= \frac{6.5}{12.5} \times 100\%$$

$$= \frac{65}{125} \times 100\%$$

$$= \frac{13}{25} \times 100\%$$

$$= 52\%$$

$\therefore$ Co-efficient of variation is 52%

2. The standard deviation and coefficient of variation of a data are 1.2 and 25.6 respectively. Find the value of mean.

Sol :

Standard deviation = 1.2

Coefficient of variation = 25.6

$$\text{Coefficient of variation C.V} = \frac{\sigma}{\bar{x}} \times 100\%$$

$$25.6 = \frac{1.2}{\bar{x}} \times 100\%$$

$$\bar{x} = \frac{1.2}{25.6} \times 100 = 4.687$$

$\therefore$ Mean $\bar{x} = 4.69$

3. If the mean and coefficient of variation of a data are 15 and 48 respectively, then find the value of standard deviation.

Sol :

Mean $\bar{x} = 15$

Coefficient of variation (C.V) = 48

Coefficient of variation C.V $= \frac{\sigma}{\bar{x}} \times 100\%$

$$48 = \frac{\sigma}{15} \times 100\%$$

$$\frac{48 \times 15}{100} = \sigma$$

$$\sigma = \frac{36}{5} = 7.2$$

$\therefore$ Standard deviation $\sigma = 7.2$

4. If n = 5, $\bar{x} = 6$, $\Sigma x^2 = 765$, then calculate the coefficient of variation.

Sol :

To find the coefficient of variation we need standard deviation (σ)

$$\sigma = \sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2}$$

$$\frac{\Sigma x^2}{n} = \frac{765}{5} = 153$$

$$\left(\frac{\Sigma x}{n}\right)^2 = (\bar{x})^2 = 6^2 = 36$$

$$\therefore \quad \sigma = \sqrt{(153) - 36} = \sqrt{117}$$

$$= \sqrt{3 \times 3 \times 13}$$

$$\sigma = 3\sqrt{13}$$

Coefficient of variation $= \frac{\sigma}{\bar{x}} \times 100\%$

$$= \frac{3\sqrt{13}}{6} \times 100\% = \frac{\sqrt{13}}{2} \times 100\% = \frac{3.60555}{2} \times 100\%$$

$= 1.80277 \times 100\% = 180.277$ %

$\therefore$ Coefficient of variation = 180.28 %

5. Find the coefficient of variation of 24, 26, 33, 37, 29, 31.

Sol :

Ascending order : 24, 26, 29, 31, 33, 37

Standard deviation $\sigma = \sqrt{\frac{\Sigma d_i^2}{N} - \left(\frac{\Sigma d_i}{N}\right)^2}$

Let the assumed mean A = 31

x_i	$d_i = x_i - A$ $d_i = x_i - 31$	d_i^2
24	– 7	49
26	– 5	25
29	– 2	4
31	0	0
33	2	4
37	6	36
	$\Sigma d_i = -6$	$\Sigma d_i^2 = 118$

$$\therefore \quad \sigma = \sqrt{\frac{118}{6} - \left(\frac{-6}{6}\right)^2}$$

$$\sigma = \sqrt{19.67 - (-1)^2} = \sqrt{19.7 - 1}$$

$$\sigma = \sqrt{18.67} = 4.3$$

Mean $\bar{x} = \frac{\Sigma x_i}{N}$

Mean $\bar{x} = \frac{24 + 26 + 29 + 31 + 33 + 37}{6}$

$$\bar{x} = \frac{180}{6} = 30$$

Now co-efficient of variation C.V $= \frac{\sigma}{\bar{x}} \times 100\%$

$$C.V = \frac{4.3}{30} \times 100\% = 0.1433 \times 100\%$$

$= 14.33\%$

Co-efficient of variation of the given data $= 14.33\%$

6. The time taken (in minutes) to complete a homework by 8 students in a day are given by 38, 40, 47, 44, 46, 43, 49, 53. Find the coefficient of variation.

Sol :

Given data are 38, 40, 43, 44, 46, 47, 49, 53.

Standard deviation $\sigma = \sqrt{\frac{\Sigma d_i^2}{N} - \left(\frac{\Sigma d_i}{N}\right)^2}$

Let us take the assumed mean A = 44

x_i	$d_i = x_i - A$ $d_i = x_i - 44$	d_i^2
38	– 6	36
40	– 4	16
43	– 1	1
44	0	0
46	2	4
47	3	9
49	5	25
53	9	81
	$\Sigma d_i = 8$	$\Sigma d_i^2 = 172$

$$\sigma = \sqrt{\frac{\Sigma d_i^2}{N} - \left(\frac{\Sigma d_i}{N}\right)^2}$$

$$\sigma = \sqrt{\frac{172}{8} - \left(\frac{8}{8}\right)^2} = \sqrt{21.5 - (1)^2}$$

$$\sigma = \sqrt{21.5 - 1} = \sqrt{20.5} = 4.53$$

Mean $\bar{x} = \frac{\textit{Sum of the given observations}}{\textit{Number of observations}}$

$$\bar{x} = \frac{38 + 40 + 43 + 44 + 46 + 47 + 49 + 53}{8}$$

$$\bar{x} = \frac{360}{8} = 45$$

Co-efficient of variation C.V = $\frac{\sigma}{\bar{x}} \times 100\%$

$$= \frac{4.53}{45} \times 100\% = \frac{453}{45}$$

$\therefore$ Co-efficient of variation = 10.07%

7. The total marks scored by two students Sathya and Vidhya in 5 subjects are 460 and 480 with standard deviation 4.6 and 2.4 respectively. Who is more consistent in performance?

Sol :

(i) Sathya

Sum of marks in 5 subjects = 460

$\therefore$ Mean marks of sathya $\bar{x} = \frac{\sum x_i}{N} = \frac{460}{5} = 92$

Standard deviation σ = 4.6

$\therefore$ Co-efficient of variation C.V = $\frac{\sigma}{\bar{x}} \times 100\%$

$$= \frac{4.6}{92} \times 100\% = \frac{460}{92}\%$$

$$= \frac{10}{2}\% = 5\%$$

C.V of Sathya = 5%.

(ii) Vidhya

Sum of the marks in 5 subjects = 480

$\therefore$ Mean marks of Vidhya $\bar{x} = \frac{\sum x_i}{N} = \frac{480}{5} = 96$

Standard deviation σ = 2.4

Co-efficient of variation C.V. = $\frac{\sigma}{\bar{x}} \times 100\%$

$$= \frac{2.4}{96} \times 100\% = \frac{240}{96}\%$$

$$= \frac{10}{4}\% = 2.5\%$$

$\therefore$ C. V. of Vidhya = 2.5%

C. V. of Vidhya < C. V. of Sathya

$\therefore$ Vidhya is more consistent in her performance.

8. The mean and standard deviation of marks obtained by 40 students of a class in three subjects Mathematics, Science and Social Science are given below.

Subjects	Mean	SD
Mathematics	56	12
Science	65	14
Social Science	60	10

Which of the three subjects shows highest variation and which shows lowest variation in marks?

Sol :

Mathematics:

Mean $\bar{x}$ = 56; SD σ = 12

Co-efficient of variation C. V = $\frac{\sigma}{\bar{x}} \times 100\%$

$$= \frac{12}{56} \times 100\% = \frac{1200}{56}\% = 21.43\%$$

Science:

Mean $\bar{x} = 65$; SD $\sigma = 14$

Co-efficient of variation C. V = $\frac{\sigma}{\bar{x}} \times 100\%$

$$= \frac{14}{65} \times 100\% = \frac{1400}{65}\%$$

$$= 21.538\% = 21.54\%$$

Social Science:

Mean $\bar{x} = 60$; SD $\sigma = 10$

Co-efficient of variation C. V = $\frac{\sigma}{\bar{x}} \times 100\%$

$$= \frac{10}{60} \times 100\% = \frac{100}{6}\%$$

$$= 16.666\%$$

$$= 16.67\%$$

The highest variation is in the subject science and the lowest variation is in the subject social science.

9. The temperature of two cities A and B in a winter season are given below.

Temperature of city A (in degree Celsius)	18	20	22	24	26
Temperature of city B (in degree Celsius)	11	14	15	17	18

Find which city is more consistent in temperature changes?

Sol :

Standard deviation $\sigma = \sqrt{\frac{\sum d_i^2}{N} - \left(\frac{\sum d_i}{N}\right)^2}$

For the city A

Let the assumed mean A = 22

x_i	$d_i = x_i - A$ $d_i = x_i - 22$	d_i^2
18	– 4	16
20	– 2	4
22	0	0
24	2	4
26	4	16
	$\sum d_i = 0$	$\sum d_i^2 = 40$

$$\sigma = \sqrt{\frac{40}{5} - (0)^2} = \sqrt{8}$$

$$\sigma = 2.828 = 2.83$$

Mean $\bar{x} = \frac{\sum x_i}{N} = \frac{18+20+22+24+26}{5}$

$$= \frac{110}{5} = 22$$

Co-efficient of variation C. V. = $\frac{\sigma}{\bar{x}} \times 100\%$

$$= \frac{2.83}{22} \times 100\% = 12.86\%$$

For the city B

Let the Assumed mean B = 15

x_i	$d_i = x_i - B$ $d_i = x_i - 15$	d_i^2
11	– 4	16
14	– 1	1
15	0	0
17	2	4
18	3	9
	$\sum d_i = 0$	$\sum d_i^2 = 30$

Standard deviation $\sigma = \sqrt{\frac{30}{5} - \left(\frac{0}{5}\right)^2}$

$$= \sqrt{6-0} = \sqrt{6} = 2.449$$

Mean $\bar{x} = \frac{\sum x_i}{N}$

$$= \frac{11+14+15+17+18}{5}$$

$$\bar{x} = \frac{75}{5} = 15.$$

$\therefore$ Co-efficient of variation C. V = $\frac{\sigma}{\bar{x}} \times 100\%$

$$= \frac{2.4495}{15} \times 100\%$$

$$= \frac{244.95}{15}\% = 16.33\%$$

C. V. of temperature of city A < C. V of temperature of city B.

$\therefore$ City A is more consistent in temperature change.

PROBABILITY OF AN EVENT

Key Points

1. Random Experiment:

A random experiment is an experiment in which

(i) The set of all possible outcomes are known.

(ii) Exact outcome is not known.

Example: Tossing a coin, rolling a die.

2. **Sample Space:**

The set of all possible outcomes in a random experiment is called a sample space. It is generally denoted by S.

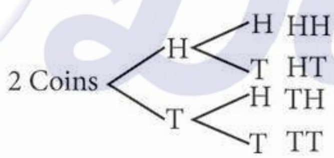
Example: When tossing a coin the possible outcomes are {H,T}.

$\therefore$ Sample space S = {H, T}

3. **Tree Diagram:**

Tree diagram allow us to see visually all the possible outcomes of an experiment. Each branch of a tree diagram represent a possible outcome.

Example: When tossing two coins.



$\therefore$ Sample space S = {HH, HT, TH, TT}

4. **Event:**

In a random experiment, each possible outcome is called an event.

Thus an event will be a subset of the sample space.

Example: Getting two heads on tossing two coins.

5. **Trial:**

Performing an experiment once is called a trial.

6. **Equally likely events:**

Two or more events are said to be equally likely if each one of them has an equal chance of occurring.

Example: Head and tail are equally likely events in tossing a coin.

7. **Certain events:**

In an experiment, the event which surely occur is called certain event.

Example: When we roll a die, the event of getting any natural number from 1 to 6 is a certain event.

8. **Impossible event:**

In an experiment if an event has no scope to occur then it is called an impossible event.

Example: When we toss two coins, the event of getting three heads is an impossible event.

9. **Mutually exclusive events:**

Events A and B are said to be mutually exclusive if $A \cap B = \phi$

Example: On rolling a die the events of getting odd numbers and even numbers are mutually exclusive events.

10. **Exhaustive Events:**

The collection of events whose union is the whole sample space are called exhaustive events.

Example: On rolling a die the events of getting odd numbers and even numbers are exhaustive events.

11. **Complementary events:**

The complement of an event A is the event representing collection of sample points not in A.

It is denoted by A' or A^C or $\overline{A}$.

The event A and its complement A' are mutually exclusive and exhaustive.

12. **Elementary event:**

If an event E consists of only one outcome then it is called an elementary event.

Example: In tossing a coin, event of getting a head is an elementary event.

13. **Probability of an event:**

In a random experiment, let S be the sample space $E \subseteq S$. Then E is an event.

The probability of occurance of E is defined as

$$P(E) = \frac{\textit{Number of outcomes favourable to occurance of E}}{\textit{Number of all possible outcomes}}$$

$$P(E) = \frac{n(E)}{n(S)}$$

14. **Results:**

(i) $P(E) = \frac{n(E)}{n(S)}$ (ii) $P(S) = \frac{n(S)}{n(S)} = 1$

i.e., The probability of sure event is 1.

(iii) $P(\phi) = \frac{n(\phi)}{n(S)} = \frac{0}{n(S)} = 0$

The probability of impossible event is 0

(iv) $0 \le P(E) \le 1$

The probability value always lies from 0 to 1

(v) $P(\overline{E}) = 1 - P(E)$

(vi) $P(E) + P(\overline{E}) = 1$

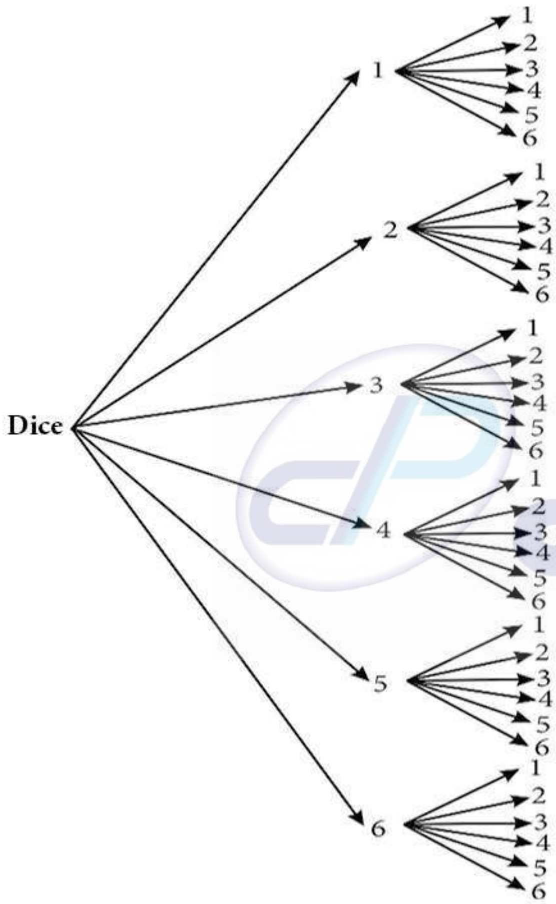
Worked Examples

8.18 Express the sample space for rolling two dice using tree diagram.

Sol :

When we roll two dice, since each die contain 6 faces marked with 1, 2, 3, 4, 5, 6 the tree diagram will look like

Hence, the sample space can be written as Figure



S = {(1, 1) , (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)
(2, 1) , (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)
(3, 1) , (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)
(4, 1) , (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)
(5, 1) , (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)
(6, 1) , (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

8.19 A bag contains 5 blue balls and 4 green balls. A ball is drawn at random from the bag. Find the probability that the ball drawn is (i) blue (ii) not blue.

Sol : Total number of possible outcomes

n(S) = 5 + 4 = 9

(i) Let A be the event of getting a blue ball. Number of favourable outcomes for the event A. Therefore, n(A) = 5

Probability that the ball drawn is blue.

Therefore, $P(A) = \frac{n(A)}{n(S)} = \frac{5}{9}$

(ii) $\overline{A}$ will be the event of not getting a blue ball.

So, $P(\overline{A}) = 1 - P(A) = 1 - \frac{5}{9} = \frac{4}{9}$

8.20 Two dice are rolled. Find the probability that the sum of outcomes is

(i) equal to 4 **(ii) greater than 10**

(iii) less than 13

Sol :

When we roll two dice, the sample space is given by

S = {(1, 1) , (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)
(2, 1) , (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)
(3, 1) , (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)
(4, 1) , (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)
(5, 1) , (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)
(6, 1) , (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)};

n(S) = 36

(i) Let A be the event of getting the sum of outcome values equal to 4.

Then A = {(1, 3), (2, 2), (3, 1)} ; n(A) = 3.

Probability of getting the sum of outcomes values equal to 4 is

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{36} = \frac{1}{12}$$

(ii) Let B be the event of getting the sum of outcome values greater than 10.

B = {(5, 6), (6, 5), (6, 6); n(B) = 3

Probability of getting the sum of outcomes greater than 10 is

$$P(B) = \frac{n(B)}{n(S)} = \frac{3}{36} = \frac{1}{12}$$

(iii) Let C be the event of getting the sum of outcomes less than 13. Here all the outcomes have the sum value less than 13. Hence C = S.

Therefore, n(C) = n(S) = 36

Probability of getting the total value less than 13 is

$$P(C) = \frac{n(C)}{n(S)} = \frac{36}{36} = 1.$$

8.21 Two coins are tossed together. What is the probability of getting different faces on the coins?

Sol :

When two coins are tossed together, the sample space is $S = \{HH, HT, TH, TT\}$; $n(S) = 4$

Let A be the event of getting different faces on the coins.

$A = \{HT, TH\}$; $n(A) = 2$

Probability of getting different faces on the coins is

$$P(A) = \frac{n(A)}{n(S)} = \frac{2}{4} = \frac{1}{2}$$

8.22 From a well shuffled pack of 52 cards, one card is drawn at random. Find the probability of getting (i) red card (ii) heart card (iii) red king (iv) face card (v) number card

Sol :

$n(S) = 52$

(i) Let A be the event of getting a red card.

$n(A) = 26$

Probability of getting a red card is

$$P(A) = \frac{n(C)}{n(S)} = \frac{26}{52} = \frac{1}{2}$$

(ii) Let B be the event of getting a heart card.

$n(B) = 13$

Probability of getting a heart card is

$$P(B) = \frac{n(B)}{n(S)} = \frac{13}{52} = \frac{1}{4}$$

(iii) Let C be the event of getting a red king card. A red king card can be either a diamond king or a heart king.

$n(C) = 2$

Probability of getting a red king card is

$$P(C) = \frac{n(C)}{n(S)} = \frac{2}{52} = \frac{1}{26}$$

(iv) Let D be the event of getting a face card. The face cards are Jack (J), Queen (Q), and King (K).

$n(D) = 4 \times 3 = 12$

Probability of getting a face card is

$$P(D) = \frac{n(D)}{n(S)} = \frac{12}{52} = \frac{3}{13}$$

(v) Let E be the event of getting a number card. The number cards are 2, 3, 4, 5, 6, 7, 8, 9 and 10.

$n(E) = 4 \times 9 = 36$

Probability of getting a number card is

$$P(E) = \frac{n(E)}{n(S)} = \frac{36}{52} = \frac{9}{13}$$

8.23 What is the probability that a leap year selected at random will contain 53 Saturdays. (Hint: $366 = 52 \times 7 + 2$)

Sol :

A leap year has 366 days. So it has 52 full weeks and 2 days. 52 Saturdays must be in 52 full weeks.

The possible chances for the remaining two days will be the sample space.

S = {(Sun-Mon, Mon-Tue, Tue-Wed, Wed-Thu, Thu-Fri, Fri-Sat, Sat-Sun)}

$n(S) = 7$

Let A be the event of getting 53rd Saturday.

Then A = {Fri-Sat, Sat-Sun}

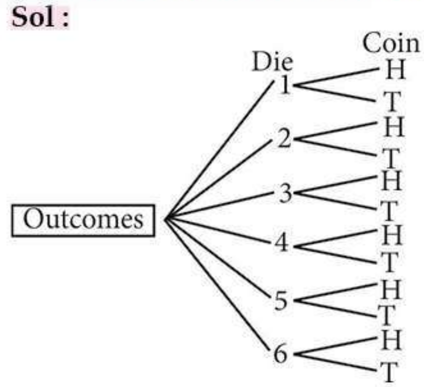
$n(A) = 2$

Probability of getting 53 Saturdays in a leap year is

$$P(A) = \frac{n(A)}{n(S)} = \frac{2}{7}$$

8.24 A die is rolled and a coin is tossed simultaneously. Find the probability that the die shows an odd number and the coin shows a head.

Sol :



Sample space

$S = \{1H, 1T, 2H, 2T, 3H, 3T, 4H, 4T, 5H, 5T, 6H, 6T\}$;

$n(S) = 12$

Let A be the event of getting an odd number and a head.

$A = \{1H, 3H, 5H\}$; $n(A) = 3$

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{12} = \frac{1}{4}$$

8.25 A bag contains 6 green balls, some black and red balls. Number of black balls is as twice as number of red ball. Probability of getting a green ball is thrice the probability of getting a red ball. Find (i) number of black balls (ii) total number of balls.

Sol : Number of Green balls is n(G) = 6

Let number of red balls is n(R) = x

Therefore, number of black balls is n(B) = 2x

Total number of balls n(S) = 6 + x+ 2x = 6 + 3x

It is given that, P(G) = 3 × P(R)

$$\frac{6}{6+3x} = 3 \times \frac{x}{6+3x}$$

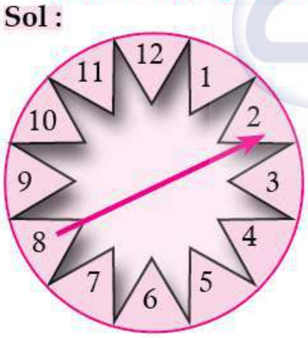
$$3x = 6 \Rightarrow x = 2$$

(i) Number of black balls = 2 × 2 = 4

(ii) Total number of balls = 6 + (3 × 2) = 12

8.26 A game of chance consists of spinning an arrow which is equally likely to come to rest pointing to one of the numbers 1, 2, 3, ... 12. What is the probability that it will point to (i) 7 (ii) a prime number (iii) a composite number?

Sol :



Sample space S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}

n (S) = 12

(i) Let A be the event of resting in 7. n(A) = 1

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{12}$$

(ii) Let B be the event that the arrow will come to rest in a prime number.

B = {2, 3, 5, 7, 11}; n(B) = 5

$$P(B) = \frac{n(B)}{n(S)} = \frac{5}{12}$$

(iii) Let C be the event that arrow will come to rest in composite number.

C = {4, 6, 8, 9, 10, 12}; n(C) = 6

$$P(C) = \frac{n(C)}{n(S)} = \frac{6}{12} = \frac{1}{2}$$

Progress Check

1. An experiment in which a particular outcome cannot be predicted is called _____.

Ans : Random Experiment

2. The set of all possible outcomes is called _____.

Ans : Sample space

3. Which of the following values cannot be a probability of an event? (a) –0.0001 (b) 0.5 (c) 1.001 (d) 1 (e) 20% (f) 0.253 (g) $\frac{1-\sqrt{5}}{2}$ (h) $\frac{\sqrt{3}+1}{4}$

Ans : Probability of an event lies between 0 and 1

$0 \le P(E) \le 1$

∴ (a) – 0.0001 (< 0)

(c) 1.001 (> 1)

(g) $\frac{1-\sqrt{5}}{2}$ (< 0)

are the numbers cannot be a probability.

Thinking Corner

1. What will be the probability that a non leap year will have 53 Saturdays?

Ans : In a non leap year will be 52 saturdays and 1 day will be left.

This one day can be any of the 7 days.

∴ Sample space S = {Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}

n (S) = 7

Let A be the event of getting saturday

∴ n (A) = 1

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A) = \frac{1}{7}$$

∴ Probability that a non leap year will have 53 saturdays is $\frac{1}{7}$.

2. What is the complement event of an impossible event?

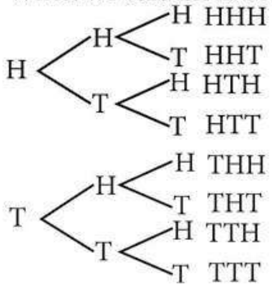
Ans : Complement event of an impossible event is the sample space.

Exercise 8. 3

1. Write the sample space for tossing three coins using tree diagram.

Sol :

When we toss three coins the outcome will be

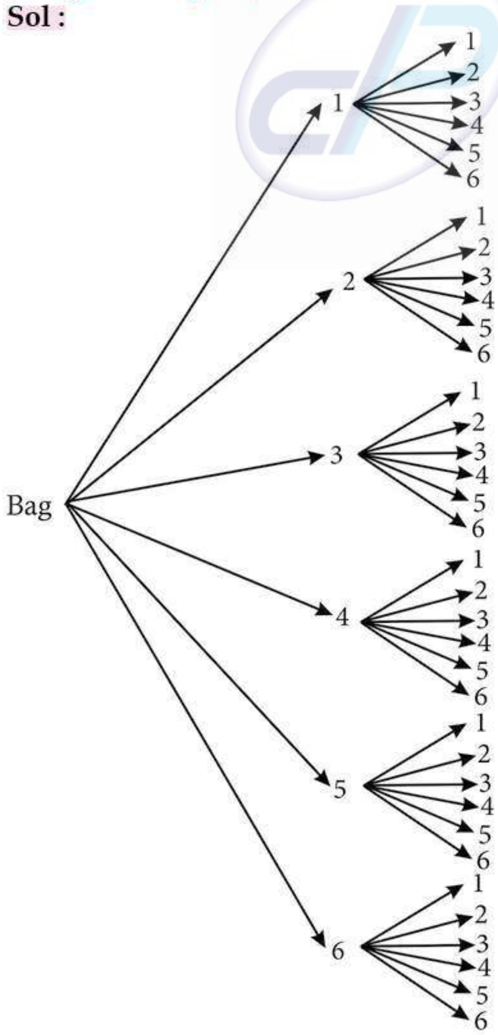


∴ The sample space = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

Total number of outcomes = 8

2. Write the sample space for selecting two balls from a bag containing 6 balls numbered 1 to 6 (using tree diagram).

Sol :



Sample Space

S = {(1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6)
(2, 1) (2, 2) (2, 3) (2, 4) (2, 5) (2, 6)
(3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (3, 6)
(4, 1) (4, 2) (4, 3) (4, 4) (4, 5) (4, 6)
(5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (5, 6)
(6, 1) (6, 2) (6, 3) (6, 4) (6, 5) (6, 6)}

∴ Total number of outcomes = 36

3. If A is an event of a random experiment such that P(A) : P$(\bar{A})$ = 17 : 15 and n(S) = 640 then find (i) P$(\bar{A})$ (ii) n(A)

Sol :

Given P(A) : P$(\bar{A})$ = 17 : 15

(i) $$\frac{P(A)}{P(\bar{A})} = \frac{17}{15}$$

$$\frac{P(A)}{1-P(A)} = \frac{17}{15} \quad [\because P(\bar{A}) = 1 - P(A)]$$

$$15\,P(A) = 17\,[1 - P(A)]$$

$$15\,P(A) = 17 - 17\,P(A)$$

$$32\,P(A) = 17; \quad P(A) = \frac{17}{32}$$

$$P(\bar{A}) = 1 - P(A) = 1 - \frac{17}{32} = \frac{32-17}{32} = \frac{15}{32}$$

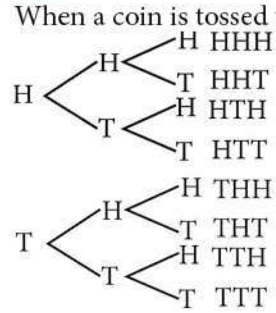
(ii) $$P(A) = \frac{n(A)}{n(S)} = \frac{17}{32}$$

n(A) = 17

4. A coin is tossed thrice. What is the probability of getting two consecutive tails?

Sol :

When a coin is tossed thrice, the outcome will be



The sample space S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

n(S) = 8

Let A be the event of getting two consecutive tails

A = {HTT, TTH, TTT}

$$n(A) = 3$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{8}$$

Probability of getting two consecutive tails $= \frac{3}{8}$

5. At a fete, cards bearing numbers 1 to 1000, one number on one card are put in a box. Each player selects one card at random and that card is not replaced. If the selected card has a perfect square number greater than 500, the player wins a prize. What is the probability that (i) the first player wins a prize (ii) the second player wins a prize, if the first has won?

Sol :

Sample space $S = \{1, 2, 3, 4, ... 1000\}$

$$n(S) = 1000$$

Let 'A' be the event of selecting a card that is perfect square greater than 500

$$A = \{23^2, 24^2, 25^2, 26^2, 27^2, 28^2, 29^2, 30^2, 31^2\}$$

$$n(A) = 9$$

(i) Probability of the first player wins a prize
By picking one of the cards from A he may win a prize.

$$\therefore \quad P(A) = \frac{n(A)}{n(S)} = \frac{9}{1000}$$

(ii) Probability of the second player winning a prize if the first has won
Let 'B' be the event of second player wins a prize.
Since the card picked in **(i)** is not replaced

$$n(B) = 8$$

$$n(S) = 1000 - 1 = 999$$

$$\therefore \quad P(B) = \frac{n(B)}{n(S)} = \frac{8}{999}$$

6. A bag contains 12 blue balls and x red balls. If one ball is drawn at random (i) what is the probability that it will be a red ball? (ii) If 8 more red balls are put in the bag and if the probability of drawing a red ball will be twice that of the probability in (i) then find x.

Sol :

Total number of balls = blue balls + red balls

$$n(S) = 12 + x$$

(i) The probability that it will be a red ball:
Let A be the event of selecting a red ball

$$\therefore \quad n(A) = x$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A) = \frac{x}{12+x}$$

(ii) If 8 more red balls are put in the bag then
the number of red balls = x + 8
Total number of balls = 12 + x + 8
Probability of drawing a red ball

$$P(B) = \frac{n(B)}{n(S)}$$

$$= \frac{x+8}{12+x+8} = \frac{x+8}{x+20}$$

If the probability of drawing a red ball will be twice that of the probability **(i)**, then

$$\frac{x+8}{x+20} = 2\times\left[\frac{x}{12+x}\right]$$

$$(x+8)(12+x) = 2x(x+20)$$

$$12x + 96 + x^2 + 8x = 2x^2 + 40x$$

$$2x^2 - x^2 + 40x - 12x - 8x - 96 = 0$$

$$x^2 + 20x - 96 = 0$$

$$(x-4)(x+24) = 0$$

$x = 4$ and $x = -24$ (not possible)

$\therefore$ By applying the value of x in P(A)

we get $P(A) = \frac{4}{12+4} = \frac{4}{16}$

$$P(A) = \frac{1}{4}$$

$$x = 4$$

7. Two unbiased dice are rolled once. Find the probability of getting

(i) a doublet (equal numbers on both dice)

(ii) the product as a prime number

(iii) the sum as a prime number

(iv) the sum as 1

Sol : When two unbiased dice are rolled, the Sample Space

S = {(1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6)
(2, 1) (2, 2) (2, 3) (2, 4) (2, 5) (6, 6)
(3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (3, 6)
(4, 1) (4, 2) (4, 3) (4, 4) (4, 5) (4, 6)
(5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (6, 6)
(6, 1) (6, 2) (6, 3) (6, 4) (6, 5) (6, 6)}

$$n(S) = 36$$

(i) Let A be the event of getting a doublet

A = {(1, 1) (2, 2) (3, 3) (4, 4) (5, 5) (6, 6)}

$n(A) = 6$

$$P(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

(ii) Let B be the event of getting the product as a prime number.

B = {(1, 2) (1, 3) (1, 5) (2, 1) (3, 1) (5, 1)}

$n(B) = 6$

$$P(B) = \frac{6}{36} = \frac{1}{6}$$

(iii) Let C be the event of getting the sum as a prime number.

C = {(1, 1) (1, 2) (1, 4) (1, 6) (2, 1) (2, 3) (2, 5) (3, 2) (3, 4) (4, 1) (4, 3) (5, 2) (5, 6) (6, 1) (6, 5)}

$n(C) = 15$

$$P(C) = \frac{n(C)}{n(S)} = \frac{15}{36} = \frac{5}{12}$$

(iv) Let D be the event of getting the sum as 1.

Since it is an impossible event.

$n(D) = 0$ and $P(D) = 0$

8. Three fair coins are tossed together. Find the probability of getting

(i) all heads **(ii) atleast one tail**

(iii) atmost one head

(iv) atmost two tails

Sol :

When three fair coins are tossed together the sample space

S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

$n(S) = 8$

(i) Let A be the event of getting all heads

A = {HHH}

$n(A) = 1$

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{8}$$

(ii) Let B be the event of getting atleast one tail

B = {HHT, HTH, HTT, THH, THT, TTH, TTT}

$n(B) = 7$

$$P(B) = \frac{n(B)}{n(S)} = \frac{7}{8}$$

(iii) Let C be the event of getting atmost one head

C = {HTT, THT, TTH, TTT}

$n(C) = 4$

$$P(C) = \frac{n(C)}{n(S)} = \frac{4}{8} = \frac{1}{2}$$

(iv) Let D be the event of getting atmost two tails

D = {HHH, HHT, HTH, HTT, THH, THT, TTH}

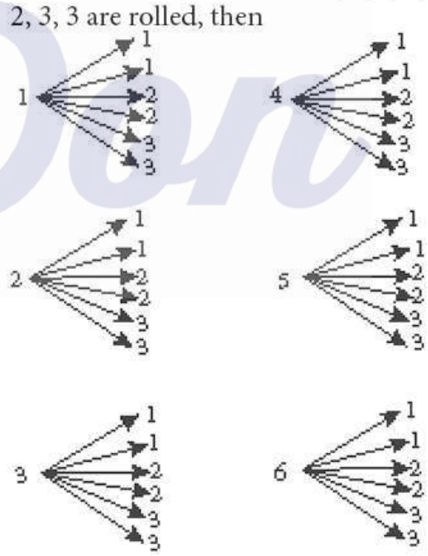
$n(D) = 7$

$$P(D) = \frac{n(D)}{n(S)} = \frac{7}{8}$$

9. Two dice are numbered 1, 2, 3, 4, 5, 6 and 1, 1, 2, 2, 3,3 respectively. They are rolled and the sum of the numbers on them is noted. Find the probability of getting each sum from 2 to 9 separately.

Sol :

When two dice numbered 1, 2, 3, 4, 5, 6 and 1, 1, 2, 2, 3, 3 are rolled, then



Sample Space

S = {(1, 1) (2, 1) (3, 1) (4, 1) (5, 1) (6, 1)
(1, 1) (2, 1) (3, 1) (4, 1) (5, 1) (6, 1)
(1, 2) (2, 2) (3, 2) (4, 2) (5, 2) (6, 2)
(1, 2) (2, 2) (3, 2) (4, 2) (5, 2) (6, 2)
(1, 3) (2, 3) (3, 3) (4, 3) (5, 3) (6, 3)
(1, 3) (2, 3) (3, 3) (4, 3) (5, 3) (6, 3)}

$n(S) = 36$

Let A, B, C, D, E, F, G, H be the event of getting the sum 2, 3, 4, 5, 6, 7, 8 and 9 respectively.

(i) A = {(1, 1) (1, 1)}

$n(A) = 2$

$$P(A) = \frac{n(A)}{n(S)} = \frac{2}{36}$$

(ii) Sum 3

$$B = \{(2, 1)\ (2, 1)\ (1, 2)\ (1, 2)\}$$
$$n(B) = 4$$
$$P(B) = \frac{n(B)}{n(S)} = \frac{4}{36}$$

(iii) Sum 4

$$C = \{(3, 1)\ (3, 1)\ (2, 2)\ (2, 2)\ (1, 3)\ (1, 3)\}$$
$$n(C) = 6$$
$$P(C) = \frac{n(C)}{n(S)} = \frac{6}{36}$$

(iv) Sum 5

$$D = \{(4, 1)\ (4, 1)\ (3, 2)\ (3, 2)\ (2, 3)\ (2, 3)\}$$
$$n(D) = 6$$
$$P(D) = \frac{n(D)}{n(S)} = \frac{6}{36}$$

(v) Sum 6

$$E = \{(5, 1)\ (5, 1)\ (4, 2)\ (4, 2)\ (3, 3)\ (3, 3)\}$$
$$n(E) = 6$$
$$P(E) = \frac{n(E)}{n(S)} = \frac{6}{36}$$

(vi) Sum 7

$$F = \{(6, 1)\ (6, 1)\ (5, 2)\ (5, 2)\ (4, 3)\ (4, 3)\}$$
$$n(F) = 6$$
$$P(F) = \frac{n(F)}{n(S)} = \frac{6}{36}$$

(vii) Sum 8

$$G = \{(6, 2)\ (6, 2)\ (5, 3)\ (5, 2)\ (5, 3)\}$$
$$n(G) = 4$$
$$P(G) = \frac{n(G)}{n(S)} = \frac{4}{36}$$

(viii) Sum 9

$$H = \{(6, 3)\ (6, 3)\}$$
$$n(H) = 2$$
$$P(H) = \frac{n(H)}{n(S)} = \frac{2}{36}$$

10. A bag contains 5 red balls, 6 white balls, 7 green balls, 8 black balls. One ball is drawn at random from the bag. Find the probability that the ball drawn is

(i) white **(ii) black or red**

(iii) not white

(iv) neither white nor black

Sol :

Total number of balls = 5 red + 6 white + 7 green + 8 black

$$n(S) = 26$$

(i) Let A be the event of getting white ball

$$n(A) = 6$$
$$P(A) = \frac{n(A)}{n(S)} = \frac{6}{26} = \frac{3}{13}$$

(ii) Let B be the event of getting black or red

$$n(B) = 5 + 8 = 13$$
$$P(B) = \frac{n(B)}{n(S)} = \frac{13}{26} = \frac{1}{2}$$

(iii) Since A is the event of getting white ball $\overline{A}$ is the event of not getting white ball

$$P(\overline{A}) = 1 - P(A)$$
$$= 1 - \left(\frac{3}{13}\right) = \frac{13-3}{13}$$
$$P(\overline{A}) = \frac{10}{13}$$

(iv) Let 'C' be the event of getting neither white nor black.

$$C = 5 \text{ red} + 7 \text{ green}$$
$$n(C) = 12$$
$$P(C) = \frac{n(C)}{n(S)} = \frac{12}{26} = \frac{6}{13}$$

11. In a box there are 20 non-defective and some defective bulbs. If the probability that a bulb selected at random from the box found to be defective is $\frac{3}{8}$ then, find the number of defective bulbs.

Sol :

Number of non defective bulbs = 20.

Let x be the number of defective bulbs

Then total number of bulbs n(S) = 20 + x

Let 'A' be the event of getting defective bulbs

$$P(A) = \frac{x}{20+x} = \frac{3}{8}$$

$$8x = 3(20 + x)$$
$$8x = 60 + 3x$$
$$8x - 3x = 60$$
$$5x = 60$$
$$x = \frac{60}{5}$$
$$x = 12$$

$\therefore$ Number of defective bulbs = 12

12. The king and queen of diamonds, queen and jack of hearts, jack and king of spades are removed from a deck of 52 playing cards and then well shuffled. Now one card is drawn at random from the remaining cards. Determine the probability that the card is (i) a clavor (ii) a queen of red card (iii) a king of black card

Sol :

Suits of playing cards		Spade ♠	Heart ♥	Clavor ♣	Diamond ♦
Existing cards		A	A	A	A
		2	2	2	2
		3	3	3	3
		4	4	4	4
		5	5	5	5
		6	6	6	6
		7	7	7	7
		8	8	8	8
		9	9	9	9
		10	10	10	10
				J	J
		Q		Q	
			K	K	
Set of playing cards in each suit		13	13	13	13
Remaining cards after removing some of them		11	11	13	11

Total number of cards remaining
$= 11 + 11 + 13 + 11 = 46$
$n(S) = 46$

(i) Let A be the event of getting a clavor card

$$n(A) = 13$$
$$P(A) = \frac{n(A)}{n(S)} = \frac{13}{46}$$

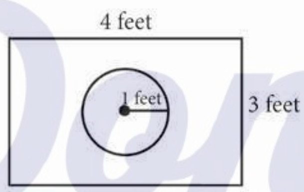
(ii) Let B be the event of getting queen of red card

$$n(B) = 0 \text{ (Removed red queens)}$$
$$P(B) = \frac{n(B)}{n(S)} = 0$$

(iii) Let C be the event of getting a king of black card

$$n(C) = 1$$
$$P(C) = \frac{n(C)}{n(S)} = \frac{1}{46}$$

13. Some boys are playing a game, in which the stone thrown by them landing in a circular region (given in the figure) is considered as win and landing other than the circular region is considered as loss. What is the probability to win the game?



Sol :

Area of the rectangle = (length × breadth) square units

$$= 4 \times 3 \text{ square feet}$$
$$n(S) = 12 \text{ square feet}$$

Area of the circle $= \pi r^2$ square units

$$= \pi (1)^2 \text{ square feet}$$
$$= \pi \text{ square feet}$$

Let 'A' be the event of winning the game,

then $n(A) = \pi$

Probability of winning the game $= \frac{n(A)}{n(S)} = \frac{\pi}{12}$

$$= \frac{3.14}{12} = \frac{1.57}{6} = \frac{157}{600}$$

14. Two customers Priya and Amuthan are visiting a particular shop in the same week (Monday to Saturday). Each is equally likely to visit the shop on any one day as on another day. What is the probability that both will visit the shop on

(i) the same day **(ii) different days**

(iii) consecutive days?

Sol :

If Priya and Vidhya are visiting the shop in the same week, the sample space

S = {(Mon, Mon) (Mon, Tue) (Mon, Wed)
(Mon, Thur) (Mon, Fri) (Mon, Sat)
(Tue, Mon) (Tue, Tue) (Tue, Wed)
(Tue, Thur) (Tue, Fri) (Tue, Sat)
(Wed, Mon) (Wed, Tue) (Wed, Wed)
(Wed, Thur) (Wed, Fri) (Wed, Sat)
(Thur, Mon) (Thur, Tue) (Thur, Wed)
(Thur, Thur) (Thur, Fri) (Thur, Sat)
(Fri, Mon) (Fri, Tue) (Fri, Wed) (Fri, Thur)
(Fri, Fri) (Fri, Sat) (Sat, Mon) (Sat, Tue)
(Sat, Wed) (Sat, Thur) (Sat, Fri) (Sat, Sat)}

$n(S) = 36$

(i) Let 'A' be the event that both will visit the shop on the same day.

A = {(Mon, Mon) (Tue, Tue) (Wed, Wed) (Thur, Thur) (Fri, Fri) (Sat, Sat)}

$n(A) = 6$

$$P(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

(ii) Both will visit the shop on different days

$$P(\overline{A}) = 1 - P(A) = 1 - \frac{1}{6}$$

$$P(\overline{A}) = \frac{6-1}{6} = \frac{5}{6}$$

i.e., probability that both will visit the shop on different days $= \frac{5}{6}$

(iii) Let 'B' be the event that both will visit the shop on consecutive days

B = {(Mon, Tue) (Tue, Wed) (Wed, Thur) (Thur, Fri) (Fri, Sat)}

$n(B) = 5$

$$P(B) = \frac{n(B)}{n(S)} = \frac{5}{36}$$

15. In a game, the entry fee is ₹ 150. The game consists of tossing a coin 3 times. Dhana bought a ticket for entry. If one or two heads show, she gets her entry fee back. If she throws 3 heads, she receives double the entry fees. Otherwise she will lose. Find the probability that she (i) gets double entry fee (ii) just gets her entry fee (iii) loses the entry fee.

Sol :

In tossing a coin 3 times, the sample space

S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

$n(S) = 8$

(i) Let 'A' be the event of getting double entry fee. She received double entry fee when she throws 3 heads.

$\therefore$ A = {HHH}

$n(A) = 1$

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{8}$$

Probability of getting double entry fee $= \frac{1}{8}$

(ii) Let 'B' be the event of getting the entry fee back. She receives her entry fee back when she throws one or two heads.

$\therefore$ B = {HHT, HTH, HTT, THH, THT, TTH}

$n(B) = 6$

$$P(B) = \frac{n(B)}{n(S)} = \frac{6}{8} = \frac{3}{4}$$

Probability of getting back entry fee $= \frac{3}{4}$

(iii) Let 'C' be the event of losing the entry fee

C = {TTT}

$n(C) = 1$

$$P(C) = \frac{n(C)}{n(S)} = \frac{1}{8}$$

Probability of losing the entry fee $= \frac{1}{8}$

ALGEBRA OF EVENTS

Key Points

In a random experiment, let S be the sample space. Let $A \subseteq S$ and $B \subseteq S$. Then A and B are events.

- $A \cap B$ is an event that occurs only when both A and B occurs.
- $A \cup B$ is an event that occurs only when atleast one of A or B occurs.
- $\overline{A}$ is an event that occurs only when A doesn't occur.
- $A \cap \overline{A} = \phi$
- $A \cup \overline{A} = S$
- If A and B are mutually exclusive events, the n $P(A \cup B) = P(A) + P(B)$
- P(Union of events) = Σ (Probability of events)
- $P(\overline{A \cup B}) = P(\overline{A} \cap \overline{B})$

Theorem 1

If A and B are two events associated with a random experiment, then

1. $P(A \cap \overline{B}) = P(\text{only } A) = P(A) - P(A \cap B)$
2. $P(\overline{A} \cap B) = P(\text{only } B) = P(B) - P(A \cap B)$

Addition Theorem of Probability

1. If A and B are any two non mutually exclusive events then
   $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
2. If A, B and C are any three non mutually exclusive events then

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$

Worked Examples

8.27 If $P(A) = 0.37$, $P(B) = 0.42$, $P(A \cap B) = 0.09$ then find $P(A \cup B)$.

Sol :

$P(A) = 0.37$, $P(B) = 0.42$, $P(A \cap B) = 0.09$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = 0.37 + 0.42 - 0.09 = 0.7$

8.28 What is the probability of drawing either a king or a queen in a single draw from a well shuffled pack of 52 cards?

Sol :

Total number of cards = 52

Number of king cards = 4

Probability of drawing a king card = $\frac{4}{52}$

Number of queen cards = 4

Probability of drawing a queen card = $\frac{4}{52}$

Both the events of drawing a king and queen are mutually exclusive

$\Rightarrow P(A \cup B) = P(A) + P(B)$

Therefore, probability of drawing either a king or a queen is = $\frac{4}{52} + \frac{4}{52} = \frac{2}{13}$

8.29 Two dice are rolled together. Find the probability of getting a doublet or sum of faces as 4.

Sol :

When two dice are rolled together, there will be $6 \times 6 = 36$ outcomes.

Let S be the sample space. Then n(S) = 36

Let A be the event of getting a doublet and B be the event of getting face sum 4.

Then $A = \{(1, 1), (2, 2) (3, 3), (4, 4), (5, 5) (6, 6)\}$

$B = \{(1, 3), (2, 2), (3, 1)\}$

Therefore, $A \cap B = \{(2, 2)\}$

Then, $n(A) = 6, n(B) = 3, n(A \cap B) = 1$

$$P(A) = \frac{n(A)}{n(S)} = \frac{6}{36}$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{3}{36}$$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{36}$$

Therefore, P (getting a doublet or a total of 4)

$$= P(A \cup B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{6}{36} + \frac{3}{36} - \frac{1}{36} = \frac{8}{36} = \frac{2}{9}$$

Hence, the required probability is $\frac{2}{9}$.

8.30 If A and B are two events such that $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$ and P (A and B) $= \frac{1}{8}$, find (i) P(A or B) (ii) P(not A and not B).

Sol :

(i) $P(A \text{ or } B) = P(A \cup B)$
$= P(A) + P(B) - P(A \cap B)$

$$P(A \text{ or } B) = \frac{1}{4} + \frac{1}{2} - \frac{1}{8} = \frac{5}{8}$$

(ii) $P(\text{not } A \text{ and not } B) = P(\overline{A} \cap \overline{B})$

$= P(\overline{A \cup B})$

$= 1 - P(A \cup B)$

$$P(\text{not } A \text{ and not } B) = 1 - \frac{5}{8} = \frac{3}{8}$$

8.31 A card is drawn from a pack of 52 cards. Find the probability of getting a king or a heart or a red card.

Sol :

Total number of cards = 52; n(S) = 52

Let A be the event of getting a king card,

n(A) = 4

$$P(B) = \frac{n(A)}{n(S)} = \frac{4}{52}$$

Let B be the event of getting a heart card,

n(B) = 13

$$P(B) = \frac{n(B)}{n(S)} = \frac{13}{52}$$

Let C be the event of getting a red card,

n(C) = 26

$$P(C) = \frac{n(C)}{n(S)} = \frac{26}{52}$$

$P(A \cap B)$ = P(getting heart king) $= \frac{1}{52}$

$P(B \cap C)$ = P(getting red and heart) $= \frac{13}{52}$

$P(A \cap C)$ = P(getting red king) $= \frac{2}{52}$

$P(A \cap B \cap C)$ = P(getting heart, king which is red) $= \frac{1}{52}$

Therefore, required probability is

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$

$$= \frac{4}{52} + \frac{13}{52} + \frac{26}{52} - \frac{1}{52} - \frac{13}{52} - \frac{2}{52} + \frac{1}{52} = \frac{28}{52} = \frac{7}{13}.$$

8.32 In a class of 50 students, 28 opted for NCC, 30 opted for NSS and 18 opted both NCC and NSS. One of the students is selected at random. Find the probability that

(i) The student opted for NCC but not NSS.

(ii) The student opted for NSS but not NCC.

(iii) The student opted for exactly one of them.

Sol :

Total number of students n(S) = 50

Let A and B be the events of students opted for NCC and NSS respectively.

$n(A) = 28, n(B) = 30, n(A \cap B) = 18$

$$P(A) = \frac{n(A)}{n(S)} = \frac{28}{50}$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{30}{50}$$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{18}{50}$$

(i) Probability of the students opted for NCC but not NSS

$$P(A \cap \overline{B}) = P(A) - P(A \cap B)$$
$$= \frac{28}{50} - \frac{18}{50} = \frac{1}{5}$$

(ii) Probability of the students opted for NSS but not NCC.

$$P(\overline{A} \cap B) = P(B) - P(A \cap B)$$
$$= \frac{30}{50} - \frac{18}{50} = \frac{6}{25}$$

(iii) Probability of the students opted for exactly one of them

$$= P[(A \cap \overline{B}) \cup (\overline{A} \cap B)]$$
$$= P(A \cap \overline{B}) + P(\overline{A} \cap B)$$
$$= \frac{1}{5} + \frac{6}{25} = \frac{11}{25}$$

(Note that $(A \cap \overline{B}), (\overline{A} \cap B)$ are exclusive events).

8.33 **A and B are two candidates seeking admission to IIT, the probability that A getting selected is 0.5 and the probability that both A and B getting selected is 0.3. Prove that the probability of B being selected is at most 0.8.**

Sol :

$$P(A) = 0.5, P(A \cap B) = 0.3$$

We have $P(A \cup B) \leq 1$

$$P(A) + P(B) - P(A \cap B) \leq 1$$
$$0.5 + P(B) - 0.3 \leq 1$$
$$P(B) \leq 1 - 0.2$$
$$P(B) \leq 1 - 0.2$$
$$P(B) \leq 0.8$$

Therefore, probability of B getting selected is atmost 0.8.

Progress Check

1. P(only A) = _____

Ans : $P(A) - P(A \cap B)$

2. $P(\overline{A} \cap B)$ = _____

Ans : $P(B) - P(A \cap B)$

3. $A \cap B$ and $\overline{A} \cap B$ are _____ events.

Ans : Mutually exclusive

4. $P(\overline{A} \cap \overline{B})$ = _____

Ans : $P(\overline{A \cup B})$

5. If A and B are mutually exclusive events then $P(A \cap B)$ = _____

Ans : 0

6. If $P(A \cap B) = 0.3$, $P(\overline{A} \cap B) = 0.45$, then P(B) = _____

Ans : 0.75

Exercise 8.4

1. If $P(A) = \frac{2}{3}$, $P(B) = \frac{2}{5}$, $P(A \cup B) = \frac{1}{3}$ then find $P(A \cap B)$.

Sol :

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$\frac{1}{3} = \frac{2}{3} + \frac{2}{5} - P(A \cap B)$$
$$P(A \cap B) = \frac{2}{3} + \frac{2}{5} - \frac{1}{3}$$
$$P(A \cap B) = \frac{10+6-5}{15} = \frac{16-5}{15} = \frac{11}{15}$$

2. A and B are two events such that, P(A) = 0.42, P(B) = 0.48 and $P(A \cap B)$ = 0.16. Find (i) P(not A) (ii) P(not B) (iii) P(A or B)

Sol :

(i) Given $P(A) = 0.42$

$$\therefore P(\text{not } A) = 1 - P(A)$$
$$P(\overline{A}) = 1 - 0.42 = 0.58$$

(ii) Given $P(B) = 0.48$

$P(\text{not } B) = 1 - P(B)$

$P(\bar{B}) = 1 - 0.48 = 0.52$

(iii) $P(A \text{ or } B) = P(A \cup B)$

$= P(A) + P(B) - P(A \cap B)$

$= 0.42 + 0.48 - 0.16$

$= 0.90 - 0.16$

$P(A \text{ or } B) = 0.74$

3. If A and B are two mutually exclusive events of a random experiment and P(not A) = 0.45, $P(A \cup B)$ = 0.65, then find P(B).

Sol :

Since A and B are mutually exclusive events

$P(A \cap B) = 0$

$P(\text{not } A) = 0.45$

$\therefore \quad P(A) = 1 - P(\text{not } A)$

$P(A) = 1 - 0.45 = 0.55$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.65 = 0.55 + P(B) - 0$

$P(B) = 0.65 - 0.55 = 0.1$

4. The probability that atleast one of A and B occur is 0.6. If A and B occur simultaneously with probability 0.2, then find $P(\bar{A}) + P(\bar{B})$.

Sol :

$P(A \text{ or } B) = P(A \cup B) = 0.6$

$P(A \cap B) = 0.2$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.6 = P(A) + P(B) - 0.2$

$P(A) + P(B) = 0.6 + 0.2 = 0.8$

$[\therefore P(A) = 1 - P(\bar{A});\ P(B) = 1 - P(\bar{B})]$

$[1 - P(\bar{A})] + [1 - P(\bar{B})] = 0.8$

$1 - P(\bar{A}) + 1 - P(\bar{B}) = 0.8$

$2 - [P(\bar{A}) + P(\bar{B})] = 0.8$

$P(\bar{A}) + P(\bar{B}) = 2 - 0.8 = 1.2$

5. The probability of happening of an event A is 0.5 and that of B is 0.3. If A and B are mutually exclusive events, then find the probability that neither A nor B happen.

Sol : $P(A) = 0.5$

$P(B) = 0.3$

Since A and B are mutually exclusive events

$P(A \cap B) = 0$

$P(\text{either } A \text{ or } B) = P(A \cup B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = 0.5 + 0.3 - 0 = 0.8$

$P(\text{neither } A \text{ nor } B) = P(\overline{A \cup B})$

$P(\overline{A \cup B}) = 1 - P(A \cup B) = 1 - 0.8 = 0.2$

Probability of neither A nor B happen = 0.2

6. Two dice are rolled once. Find the probability of getting an even number on the first die or a total of face sum 8.

Sol :

When two dice are rolled once, the sample space

$S = \{(1, 1)\ (1, 2)\ (1, 3)\ (1, 4)\ (1, 5)\ (1, 6)$
$(2, 1)\ (2, 2)\ (2, 3)\ (2, 4)\ (2, 5)\ (2, 6)$
$(3, 1)\ (3, 2)\ (3, 3)\ (3, 4)\ (3, 5)\ (3, 6)$
$(4, 1)\ (4, 2)\ (4, 3)\ (4, 4)\ (4, 5)\ (4, 6)$
$(5, 1)\ (5, 2)\ (5, 3)\ (5, 4)\ (5,5)\ (5, 6)$
$(6, 1)\ (6, 2)\ (6, 3)\ (6, 4)\ (6, 5)\ (6, 6)\}$

$n(S) = 36$

Let 'A' be the event of getting an even number on the first die.

$A = \{(2, 1)\ (2, 2)\ (2, 3)\ (2, 4)\ (2, 5)\ (2, 6)$
$(4, 1)\ (4, 2)\ (4, 3)\ (4, 4)\ (4, 5)\ (4, 6)$
$(6, 1)\ (6, 2)\ (6, 3)\ (6, 4)\ (6, 5)\ (6, 6)\}$

$n(A) = 18$

$$P(A) = \frac{n(A)}{n(S)} = \frac{18}{36}$$

Let B be the event of getting total face sum 8.

$B = \{(2, 6)\ (3, 5)\ (4, 4)\ (5, 3)\ (6, 2)\}$

$n(B) = 5$

$$P(B) = \frac{n(B)}{n(S)} = \frac{5}{36}$$

$A \cap B = \{(4, 4)\ (6, 2)\ (2, 6)\}$

$n(A \cap B) = 3$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{3}{36}$$

Now $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{18}{36} + \frac{5}{36} - \frac{3}{36}$$

$$P(A \cup B) = \frac{18 + 5 - 3}{36} = \frac{23 - 3}{36} = \frac{20}{36} = \frac{5}{9}$$

Probability of getting even number in the first die or a total face sum 8 is $\frac{5}{9}$.

7. From a well-shuffled pack of 52 cards, a card is drawn at random. Find the probability of it being either a red king or a black queen.

Sol :

Total number of cards $= 52$

$n(S) = 52$

Let 'A' be the event of getting a red king card

$n(A) = 2$

$$P(A) = \frac{n(A)}{n(S)} = \frac{2}{52}$$

Let 'B' be the event of getting black queen card

$n(B) = 2$

$$P(B) = \frac{n(B)}{n(S)} = \frac{2}{52}$$

Since A and B are mutually exclusive events

$A \cap B = 0$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = \frac{2}{52} + \frac{2}{52} - 0$$

$$P(A \cup B) = \frac{4}{52} = \frac{1}{13}$$

Probability of getting either a red king or a black queen $= \frac{1}{13}$.

8. A box contains cards numbered 3, 5, 7, 9, …. 35, 37. A card is drawn at random from the box. Find the probability that the drawn card have either multiples of 7 or a prime number.

Sol :

Cards in the box $= \{3, 5, 7, 9, \ldots, 37\}$

$$\text{Number of cards} = \frac{l-a}{d} + 1$$

$$= \frac{37-3}{2} + 1 = \frac{34}{2} + 1 = 17 + 1$$

$\therefore$ Number of cards $= 18$

$n(S) = 18$

Let A be the event of selecting a number which is multiple of 7.

$A = \{7, 21, 35\}$

$n(A) = 3$

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{18}$$

Let B be the event of selecting a prime number

$B = \{3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37\}$

$n(B) = 11$

$$P(B) = \frac{n(B)}{n(S)} = \frac{11}{18}$$

$A \cap B = \{7\}$

$n(A \cap B) = 1$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)}$$

$$P(A \cap B) = \frac{1}{18}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = \frac{3}{18} + \frac{11}{18} - \frac{1}{18}$$

$$= \frac{3+11-1}{18} = \frac{13}{18}$$

Probability of drawing a card either multiple of 7 or a prime number is $\frac{13}{18}$.

9. Three unbiased coins are tossed once. Find the probability of getting atmost 2 tails or at least 2 heads.

Sol :

When three coins are tossed the sample space

S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

$n(S) = 8$

Let A be the event of getting atmost 2 tails (i.e., 0 tail, 1 tail, 2 tails}

A = {HHH, HHT, HTH, HTT, THH, THT, TTH}

$n(A) = 7$

$$P(A) = \frac{n(A)}{n(S)} = \frac{7}{8}$$

Let B be the event of getting atleast 2 heads. (i.e., 2 heads, 3 heads)

B = {HHT, HTH, THH, HHH}

$n(B) = 4$

$$P(B) = \frac{n(B)}{n(S)} = \frac{4}{8}$$

$A \cap B$ = {HHT, HTH, THH, HHH}

$$n(A\cap B) = 4$$

$$P(A\cap B) = \frac{n(A\cap B)}{n(S)} = \frac{4}{8}$$

$$P(A\cup B) = P(A) + P(B) - P(A\cap B)$$

$$P(A\cup B) = \frac{7}{8}+\frac{4}{8}-\frac{4}{8} = \frac{7+4-4}{8} = \frac{7}{8}$$

Probability of getting atmost two tails or atleast 2 heads = $\frac{7}{8}$.

10. The probability that a person will get an electrification contract is $\frac{3}{5}$ and the probability that he will not get plumbing contract is $\frac{5}{8}$. The probability of getting atleast one contract is $\frac{5}{7}$. What is the probability that he will get both?

Sol :

Let A be the event of getting electrification contract.

$$P(A) = \frac{3}{5}$$

Let B be the event of getting plumbing contract

$$P(\bar{B}) = \frac{5}{8}$$

$$1 - P(B) = \frac{5}{8}$$

$$P(B) = 1 - \frac{5}{8} = \frac{3}{8}$$

Also $P(A\cup B) = \frac{5}{7}$

$$P(A\cup B) = P(A) + P(B) - P(A\cap B)$$

$$\frac{5}{7} = \frac{3}{5}+\frac{3}{8} - P(A\cap B)$$

$$P(A\cap B) = \frac{3}{5}+\frac{3}{8}-\frac{5}{7}$$

$$= \frac{168+105-200}{280} = \frac{73}{280}$$

Probability of getting both contracts = $\frac{73}{280}$.

11. In a town of 8000 people, 1300 are over 50 years and 3000 are females. It is known that 30% of the females are over 50 years. What is the probability that a chosen individual from the town is either a female or over 50 years?

Sol :

Total number of people = 8000

$$n(S) = 8000$$

Let A be the event of selecting a female

Number of females n(A) = 3000

$$P(A) = \frac{n(A)}{n(S)} = \frac{3000}{8000}$$

Let 'B' be the event of selecting an individual as over 50 years.

Number of people who are over 50 years = 1300

n(B) = 1300

$$P(B) = \frac{n(B)}{n(S)} = \frac{1300}{8000}$$

$$n(A\cap B) = 30\% \text{ of } 3000$$

$$n(A\cap B) = \frac{30}{100}\times 3000 = 900$$

$$P(A\cap B) = \frac{n(A\cap B)}{n(S)} = \frac{900}{8000}$$

$$P(A\cup B) = P(A) + P(B) - P(A\cap B)$$

$$P(A\cup B) = \frac{3000}{8000}+\frac{1300}{8000}-\frac{900}{8000}$$

$$P(A\cup B) = \frac{3000+1300-900}{8000}$$

$$= \frac{3400}{8000} = \frac{17}{40}$$

$\therefore$ Probability that a chosen individual is either female or over 50 years is $\frac{17}{40}$.

12. A coin is tossed thrice. Find the probability of getting exactly two heads or atleast one tail or two consecutive heads.

Sol :

A coin is tossed thrice by the sample space

S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

n(S) = 8

Let A be the event of getting exactly two heads

$\therefore$ A = {HHT, HTH, THH}

n(A) = 3

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{8}$$

Let B be the event of getting atleast one tail.

B = {HHT, HTH, HTT, THH, THT, TTH, TTT}

$$n(B) = 7$$
$$P(B) = \frac{n(B)}{n(S)} = \frac{7}{8}$$

Let C be the event of getting consecutively two heads.

$$C = \{HHT, THH, HHH\}$$
$$n(C) = 3$$
$$P(C) = \frac{n(C)}{n(S)} = \frac{3}{8}$$
$$A \cap B = \{HHT, HTH, THH\}$$
$$n(A \cap B) = 3$$
$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{3}{8}$$
$$B \cap C = \{HHT, THH\}$$
$$n(A \cap C) = 2$$
$$P(B \cap C) = \frac{n(B \cap C)}{n(S)} = \frac{2}{8}$$
$$A \cap C = \{HHT, THH\}$$
$$n(A \cap C) = 2$$
$$P(A \cap C) = \frac{n(A \cap C)}{n(S)} = \frac{2}{8}$$
$$A \cap B \cap C = \{HHT, THH\}$$
$$n(A \cap B \cap C) = 2$$
$$P(A \cap B \cap C) = \frac{n(A \cap B \cap C)}{n(S)} = \frac{2}{8}$$
$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$
$$P(A \cup B \cup C) = \frac{3}{8} + \frac{7}{8} + \frac{3}{8} - \frac{3}{8} - \frac{2}{8} - \frac{2}{8} + \frac{2}{8}$$
$$= \frac{3+7+3-3-2-2+2}{8}$$
$$= \frac{8}{8} = 1$$

$\therefore$ The required probability is 1.

13. If A, B, C are any three events such that probability of B is twice as that of probability of A and probability of C is thrice as that of probability of A and if $P(A \cap B) = \frac{1}{6}$, $P(B \cap C) = \frac{1}{4}$, $P(A \cap C) = \frac{1}{8}$, $P(A \cup B \cup C) = \frac{9}{10}$ and $P(A \cap B \cap C) = \frac{1}{15}$, then find P(A), P(B) and P(C)?

Sol :

Given $P(A \cap B) = \frac{1}{6}$

$$P(B \cap C) = \frac{1}{4}$$
$$P(A \cap C) = \frac{1}{8}$$
$$P(A \cup B \cup C) = \frac{9}{10}$$
$$P(A \cap B \cap C) = \frac{1}{15}$$

Also given that $P(B) = 2\,P(A)$

$$P(C) = 3\,P(A)$$

Now $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

$$\frac{9}{10} = P(A) + 2\,P(A) + 3\,(P(A)) - \frac{1}{6} - \frac{1}{4} - \frac{1}{8} + \frac{1}{15}$$
$$\frac{9}{10} = 6\,P(A) - \frac{1}{6} - \frac{1}{4} - \frac{1}{8} + \frac{1}{15}$$
$$6\,P(A) = \frac{9}{10} + \frac{1}{6} + \frac{1}{4} + \frac{1}{8} - \frac{1}{15}$$
$$6\,P(A) = \frac{108+20+30+15-8}{120}$$
$$6\,P(A) = \frac{165}{120}$$
$$P(A) = \frac{165}{120 \times 6} = \frac{11}{48}$$
$$P(B) = 2 \times \frac{11}{48} = \frac{11}{24}$$
$$P(C) = 3 \times \frac{11}{48} = \frac{11}{16}$$
$$\therefore \quad P(A) = \frac{11}{48};\ P(B) = \frac{11}{24};\ P(C) = \frac{11}{16}.$$

14. In a class of 35, students are numbered from 1 to 35. The ratio of boys and girls in 4 : 3. The roll numbers of students begin with boys and end with girls. Find the probability that a student selected is either a boy with prime roll number or a girl with composite roll number or an even roll number.

Sol :

$$\begin{aligned}
\text{Total students} &= 35 \\
n(S) &= 35 \\
\text{Boys : girls} &= 4:3 \\
\text{Let the number of boys} &= 4x \\
\text{and number of girls} &= 3x \\
4x + 3x &= 35 \\
7x &= 35 \\
x &= \frac{35}{7} = 5
\end{aligned}$$

$\therefore$ Number of boys $= 4 \times 5 = 20$

Number of girls $= 3 \times 5 = 15$

Boys are numbered $= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$

Girls are numbered $= \{21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35\}$

Let A be the event of getting a boy with prime roll number.

$$A = \{2, 3, 5, 7, 11, 13, 17, 19\}$$

$$n(A) = 8$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{8}{35}$$

Let B be the event of getting a girl with composite roll number.

$$B = \{21, 22, 24, 25, 26, 27, 28, 30, 32, 33, 34, 35\}$$

$$n(B) = 12$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{12}{35}$$

Let C be the event of getting an even roll number.

$$C = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34\}$$

$$n(C) = 17$$

$$P(C) = \frac{n(C)}{n(S)} = \frac{17}{35}$$

Since A and B are mutually exclusive events

$$P(A \cap B) = 0$$

$$B \cap C = \{22, 24, 26, 28, 30, 32, 34\}$$

$$n(B \cap C) = 7$$

$$P(B \cap C) = \frac{n(B \cap C)}{n(S)} = \frac{7}{35}$$

$$A \cap C = \{2\}$$

$$n(A \cap C) = 1$$

$$P(A \cap C) = \frac{n(A \cap C)}{n(S)} = \frac{1}{35}$$

$$P(A \cap B \cap C) = 0 \quad [\because n(A \cap B) = 0]$$

$$\begin{aligned}
P(A \cup B \cup C) &= P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C) \\
&= \frac{8}{35} + \frac{12}{35} + \frac{17}{35} - 0 - \frac{7}{35} - \frac{1}{35} + 0 \\
&= \frac{8 + 12 + 17 - 7 - 1}{35} = \frac{29}{35}
\end{aligned}$$

$\therefore$ Required probability $= \dfrac{29}{35}$

Exercise 8.5

Multiple Choice Questions:

1. Which of the following is not a measure of dispersion?

(1) Range
(2) Standard deviation
(3) Arithmetic mean
(4) Variance [Ans: (3)]

2. The range of the data 8, 8, 8, 8, 8 … 8 is

(1) 0 (2) 1
(3) 8 (4) 3 [Ans: (1)]

Sol :

$$\begin{aligned}
\text{Range} &= L - S \\
&= 8 - 8 \\
&= 0
\end{aligned}$$

3. The sum of all deviations of the data from its mean is

(1) Always positive (2) Always negative
(3) Zero (4) non-zero integer
[Ans: (3)]

4. The mean of 100 observations is 40 and their standard deviation is 3. The sum of squares of all deviations is

(1) 40000 (2) 160900
(3) 160000 (4) 30000 [Ans: (2)]

Sol :

$$N = 100$$
$$\bar{x} = 40$$
$$\sigma = 3$$
$$\sigma = \sqrt{\frac{\sum d_i^2}{N} - \left(\frac{\sum d_i}{N}\right)^2}$$
$$3^2 = \frac{\sum d_i^2}{100} - 40^2$$
$$9 = \frac{\sum d_i^2}{100} - 1600$$
$$9 + 1600 = \frac{\sum d_i^2}{100}$$
$$1609 \times 100 = \sum d_i^2$$
$$\sum d_i^2 = 1{,}60{,}900$$

5. Variance of first 20 natural numbers is

(1) 32.25 (2) 44.25
(3) 33.25 (4) 30 [Ans: (3)]

Sol :

Sum of first 20 natural numbers

$$= \frac{n(n+1)}{2} = \frac{20 \times 21}{2}$$
$$\sum x_i = 210$$
$$\sigma = \sqrt{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2}$$
$$\sum x_i^2 = 1^2 + 2^2 + 3^2 + \ldots + 20^2$$
$$= \frac{n(n+1)(2n+1)}{6}$$
$$= \frac{20 \times 21 \times 41}{6} = 10 \times 7 \times 41$$
$$= 2870$$
$$\sigma = \sqrt{\frac{2870}{20} - \left(\frac{210}{20}\right)^2}$$
$$\text{Variance } \sigma^2 = \frac{2870}{20} - \left(\frac{210}{20}\right)^2$$
$$= 143.5 - \frac{441}{4}$$
$$= 143.5 - 110.25 = 33.25$$

6. The standard deviation of a data is 3. If each value is multiplied by 5 then the new variance is

(1) 3 (2) 15
(3) 5 (4) 225 [Ans: (4)]

Sol : standard deviation $\sigma = 3$

Each value is multiplied by 5, then standard deviation also gets multiplied by 5.

$\therefore$ New standard deviation $\sigma = 3 \times 5 = 15$

Variance $\sigma^2 = 15^2 = 225.$

7. If the standard deviation of x, y, z is p then the standard deviation of 3x + 5, 3y + 5, 3z + 5 is

(1) 3p + 5 (2) 3p
(3) p + 5 (4) 9p + 15 [Ans: (2)]

Sol : standard deviation $\sigma = p$

If each value is multiplied by 3, the

New standard deviation $\sigma = 3p$

If 5 is added to each value the standard deviation does not change.

$\therefore$ New standard deviation $\sigma = 3p$

8. If the mean and coefficient of variation of a data are 4 and 87.5% then the standard deviation is

(1) 3.5 (2) 3
(3) 4.5 (4) 2.5 [Ans: (1)]

Sol :

$$\text{C. V} = \frac{\sigma}{\bar{x}} \times 100\%$$
$$87.5\% = \frac{\sigma}{4} \times 100\%$$
$$87.5 \times 4 = \sigma \times 100$$
$$350.0 = \sigma \times 100$$
$$\sigma = \frac{350}{100} = 3.5$$

9. Which of the following is incorrect?

(1) P (A) > 1 (2) $0 \le P(A) \le 1$
(3) $P(\phi) = 0$ (4) $P(A) + P(\bar{A}) = 1$

[Ans: (1)]

10. The probability a red marble selected at random from a jar containing p red, q blue and r green marbles is

(1) $\frac{q}{p+q+r}$ (2) $\frac{p}{p+q+r}$

(3) $\frac{p+q}{p+q+r}$ (4) $\frac{p+r}{p+q+r}$ [Ans: (2)]

Sol : Total marbles = p red + q blue + r green

n (S) = p + q + r

Let A be the event of selecting a red marbles

n (A) = p

$$P(A) = \frac{n(A)}{n(S)} = \frac{p}{p+q+r}$$

11. A page is selected at random from a book. The probability that the digit at units place of the page number chosen is less than 7 is

(1) $\frac{3}{10}$ (2) $\frac{7}{10}$

(3) $\frac{3}{9}$ (4) $\frac{7}{9}$ [Ans: (2)]

Sol : The digits that may accur in units place

$$S = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$$
$$n(S) = 10.$$

Let A be the event of selecting page with unit place less than 7.

$$A = \{0, 1, 2, 3, 4. 5, 6\}$$
$$n(A) = 7$$
$$P(A) = \frac{n(A)}{n(S)} = \frac{7}{10}$$

12. The probability of getting a job for a person is $\frac{x}{3}$. If the probability of not getting the job is $\frac{2}{3}$ then the value of x is

(1) 2 (2) 1

(3) 3 (4) 1.5 [Ans: (2)]

Sol : Let A be the event of getting a job.

$$P(A) + P(\overline{A}) = 1$$
$$\frac{x}{3} + \frac{2}{3} = 1$$
$$\frac{x+2}{3} = 1$$
$$x + 2 = 3$$
$$x = 3 - 2$$
$$x = 1.$$

13. Kamalam went to play a lucky draw contest. 135 tickets of the lucky draw were sold. If the probability of Kamalam winning is $\frac{1}{9}$, then the number of tickets bought by Kamalam is

(1) 5 (2) 10

(3) 15 (4) 20 [Ans: (3)]

Sol :

Let A be the event of winning the contest.

$$P(A) = \frac{1}{9}$$
$$\frac{n(A)}{n(S)} = \frac{1}{9}$$
$$\frac{n(A)}{n(S)} = \frac{1 \times 15}{9 \times 15} = \frac{15}{135}$$
$$n(A) = 15.$$

14. If a letter is chosen at random from the English alphabets {a, b, ..., z}, then the probability that the letter chosen precedes x

(1) $\frac{12}{13}$ (2) $\frac{1}{13}$

(3) $\frac{23}{26}$ (4) $\frac{3}{26}$ [Ans: (3)]

Sol : Let $S = \{a, b, ..., z\}$

$$n(S) = 26$$

Let A be the event of choosing a letter that precedes x.

$$A = \{a, b, c, d, ..., w\}$$
$$n(A) = 23$$
$$P(A) = \frac{n(A)}{n(S)} = \frac{23}{26}$$

15. A purse contains 10 notes of ₹ 2000, 15 notes of ₹ 500 and 25 notes of ₹ 200. One note is drawn at random. What is the probability that the note is either a ₹ 500 note or ₹ 200 note?

(1) $\frac{1}{5}$ (2) $\frac{3}{10}$

(3) $\frac{2}{3}$ (4) $\frac{4}{5}$ [Ans: (4)]

Sol :

Total notes $n(S) = 10 + 15 + 25$

$$n(S) = 50$$

Let A be the event of drawing ₹ 500 note

$$n(A) = 15$$
$$P(A) = \frac{n(A)}{n(S)}$$
$$P(A) = \frac{15}{50}$$

Let B be the event of drawing ₹ 200 note

$$n(B) = 25$$
$$P(B) = \frac{n(B)}{n(S)} = \frac{25}{50}$$

Since A and B are mutually exclusive events.

$$P(A \cap B) = 0$$
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$P(A \cup B) = \frac{15}{50} + \frac{25}{50} - 0 = \frac{40}{50} = \frac{4}{5}$$

UNIT EXERCISE - 8

1. The mean of the following frequency distribution is 62.8 and the sum of all frequencies is 50. Compute the missing frequencies f_1 and f_2.

Class Interval	0-20	20-40	40-60	60-80	80-100	100-120
Frequency	5	f_1	10	f_2	7	8

Sol :

Let A = 50 and C = 20.

Class Interval	f_i	x_i	$d_i = x_i - A$, $d_i = x_i - 50$	$d_i = \frac{x_i - 50}{C}$, $d_i = \frac{x_i - 50}{20}$	$f_i d_i$
0-20	5	10	– 40	– 2	– 10
20-40	f_1	30	– 20	– 1	$-f_1$
40-60	10	50	0	0	0
60-80	f_2	70	20	1	f_2
80-100	7	90	40	2	14
100-120	8	110	60	3	24
	$\sum f_i = 30 + f_1 + f_2$, $\sum f_i = 50$				$\sum f_i d_i = 28 + f_2 - f_1$

$$\text{Given mean } \bar{x} = 62.8$$

$$\sum f_i = 50$$

$$30 + f_1 + f_2 = 50$$

$$f_1 + f_2 = 50 - 30$$

$$f_1 + f_2 = 20 \quad \ldots (1)$$

$$\text{Mean } \bar{x} = A + C \times \frac{\sum_{i=1}^{n} f_i d_i}{\sum_{i=1}^{n} f_i}$$

$$62.8 = 50 + 20 \times \left[\frac{28 + f_2 - f_1}{50}\right]$$

$$62.8 - 50 = \frac{2}{5}\left(28 + f_2 - f_1\right)$$

$$12.8 \times 5 = 56 + 2f_2 - 2f_1$$

$$64.0 - 56 = 2(f_2 - f_1)$$

$$\frac{8}{2} = f_2 - f_1$$

$$f_2 - f_1 = 4$$

$$f_1 - f_2 = -4 \quad \ldots (2)$$

$$(1) + (2) \Rightarrow 2f_1 = 16$$

$$f_1 = \frac{16}{2} = 8$$

$$(2) \Rightarrow f_1 - f_2 = -4$$

$$8 - f_2 = -4$$

$$-f_2 = -4 - 8$$

$$-f_2 = -12$$

$$f_2 = 12$$

$\therefore$ $f_1 = 8$ and $f_2 = 12$.

2. The diameter of circles (in mm) drawn in a design are given below.

Diameters	33-36	37-40	41-44	45-48	49-52
Number of circles	15	17	21	22	25

Calculate the standard deviation.

Sol :

Let A = 42.5 and C = 4

Diameters	Number of circles f_i	x_i	$d_i = \frac{x_i - A}{C}$, $d_i = \frac{x_i - 42.5}{4}$	$f_i d_i$	d_i^2	$f_i d_i^2$
33-36	15	34.5	$\frac{34.5 - 42.5}{4} = -2$	– 30	4	60
37-40	17	38.5	$\frac{38.5 - 42.5}{4} = -1$	– 17	1	17
41-44	21	42.5	$\frac{42.5 - 42.5}{4} = 0$	0	0	0
45-48	22	46.5	$\frac{46.5 - 42.5}{4} = 1$	22	1	22
49-52	25	50.5	$\frac{50.5 - 42.5}{4} = 2$	50	4	100

$$\text{Mean } \bar{x} = A + \frac{\sum f_i d_i}{\sum f_i} \times C$$

$$\bar{x} = 42.5 + \frac{25}{100} \times 4$$

$$\bar{x} = 42.5 + 1 = 43.5$$

Standard Deviation

$$\sigma = C \times \sqrt{\frac{\sum f_i d_i^2}{N} - \left(\frac{\sum f_i d_i}{N}\right)^2}$$

$$\sigma = 4 \times \sqrt{\frac{199}{100} - \left(\frac{25}{100}\right)^2}$$

$$\sigma = 4 \times \sqrt{\frac{199}{100} - \frac{625}{10000}}$$

$$\sigma = 4 \times \sqrt{\frac{19900 - 625}{10000}}$$

$$\sigma = 4\sqrt{\frac{19275}{10000}} = 4\sqrt{1.9275}$$

$$= 4 \times 1.388 = 5.55$$

$\therefore$ Standard deviation $\sigma = 5.55$.

3. The frequency distribution is given below

x	k	2k	3k	4k	5k	6k
f	2	1	1	1	1	1

In the table k is a positive integer, has a variance of 160. Determine the value of k.

Sol :

Standard deviation

$$\sigma = \sqrt{\frac{\sum f_i x_i^2}{N} - \left(\frac{\sum f_i x_i}{N}\right)^2}$$

$$\text{Variance } \sigma^2 = \frac{\sum f_i x_i^2}{N} - \left(\frac{\sum f_i x_i}{N}\right)^2$$

x_i	f_i	x_i^2	$f_i x_i$	$f_i x_i^2$
k	2	k^2	2k	$2k^2$
2k	1	$4k^2$	2k	$4k^2$
3k	1	$9k^2$	3k	$9k^2$
4k	1	$16k^2$	4k	$16k^2$
5k	1	$25k^2$	5k	$25k^2$
6k	1	$36k^2$	6k	$36k^2$
	$N = \sum f_i = 7$		$\sum f_i x_i = 22\text{ k}$	$\sum f_i x_i^2 = 92k^2$

$$\therefore \text{ Variance } \sigma^2 = \frac{92k^2}{7} - \left(\frac{22k}{7}\right)^2$$

$$160 = \frac{92k^2}{7} - \frac{484k^2}{49}$$

$$160 = \frac{644k^2 - 484k^2}{49}$$

$$160 = \frac{160k^2}{49} \Rightarrow k^2 = \frac{160 \times 49}{160}$$

$$k^2 = 49 \Rightarrow k = \pm 7$$

Given that k is a positive integer

$\therefore \quad k = 7$.

4. The standard deviation of some temperature data in degree Celsius (°C) is 5. If the data were converted into degree Fahrenheit (°F) then what is the variance?

Sol :

Given the standard deviation of some temperature data in degree Celsius is 5.

$$\sigma_C = 5$$

Let the temperature data be

$x_1^o C,\ x_2^o C,\ x_3^o C,\ x_4^o C,\ x_5^o C, \ldots$

To convert Celsius to Fahrenheit, we have

$$F = \frac{9}{5} \times \text{Celsius temperature} + 32$$

If the temperature data are converted into Fahrenheit (°F), then the data will be

$\frac{9}{5}x_1 + 32,\ \frac{9}{5}x_2 + 32,\ \frac{9}{5}x_3 + 32,\ \frac{9}{5}x_4 + 32,\ \frac{9}{5}x_5 + 32, \ldots$

i.e., Every temperature in Celsius is multiplied by $\frac{9}{5}$ and 32 is added to it.

When we add a constant to each data, the S.D of the given data will not change

Also, when we multiply each data by a constant, the S.D of the new data also get multiplied by the constant.

$\therefore$ Standard deviation of the new data in Fahrenheit will be $\sigma_F = \frac{9}{5} \times \sigma_C$ [$\because + 32$ will not affect new S.D]

$$= \frac{9}{5} \times 5 = 9^o$$

$\therefore$ New variance $\sigma_F^2 = 9^2 = 81$

$\therefore$ Variance $= 81$°F.

5. If for a distribution $\sum(x-5) = 3$, $\sum(x-5)^2 = 43$ and total number of observations is 18. Find the mean and standard deviation.

Sol :

$$\text{Given } \sum(x-5) = 3$$

$$N = 18$$

$$\sum(x-5) = 3$$

$$\sum x - \sum 5 = 3$$

$$\Rightarrow \quad \sum x - 18 \times 5 = 3$$

$$\sum x - 90 = 3$$

$$\sum x = 3 + 90 = 93$$

$$\sum(x-5)^2 = 43$$

$$\sum(x^2 + 25 - 10x) = 43$$

$$\sum x^2 + \sum 25 - 10\sum x = 43$$

$$\sum x^2 + 25 \times 18 - 10 \times 93 = 43$$

$$\sum x^2 = 43 + 930 - 450 = 523$$

Now standard deviation $\sigma = \sqrt{\dfrac{\sum x^2}{N} - \left(\dfrac{\sum x}{N}\right)^2}$

$$= \sqrt{\frac{523}{18} - \left(\frac{93}{18}\right)^2}$$

$$= \sqrt{\frac{523 \times 18 - 93 \times 93}{18 \times 18}}$$

$$= \frac{1}{18}\sqrt{9414 - 8649}$$

$$= \frac{27.66}{18} = 1.53$$

$\therefore$ Standard deviation = 1.53

Also mean $= \dfrac{\sum x}{N}$

$$\bar{x} = \frac{93}{18}$$

$$= 5.166$$

Mean = 5.17

6. Prices of peanut packets in various places of two cities are given below. In which city, prices were more stable?

Prices in city A	20	22	19	23	16
Prices in city B	10	20	18	12	15

Sol :

Prices of city A

n = 5

x_i	20	22	19	23	16	$\sum x_i = 100$
x_i^2	400	484	361	529	256	$\sum x_i^2 = 2030$

Mean $\bar{x} = \dfrac{\sum x_i}{n}$

$$\bar{x} = \frac{100}{5} = 20$$

Standard deviation $\sigma_A = \sqrt{\dfrac{\sum x_i^2}{n} - \left(\dfrac{\sum x_i}{n}\right)^2}$

$$= \sqrt{\frac{2030}{5} - \left(\frac{100}{5}\right)^2}$$

$$= \sqrt{406 - 400} = \sqrt{6} = 2.45$$

Co-efficient of variation C.V. $= \dfrac{\sigma}{\bar{x}} \times 100\%$

$$= \frac{2.45}{20} \times 100\% = \frac{245}{20}\%$$

$$= \frac{122.5}{10}\% = 12.25\%$$

For prices in city A, Co efficient of variation = 12.25%

For prices in city B.

n = 5

x_i	10	20	18	12	15	$\sum x_i = 75$
x_i^2	100	400	324	144	225	$\sum x_i^2 = 1193$

$$\bar{x} = \frac{75}{5} = 15$$

$$\sigma_B = \sqrt{\frac{1193}{5} - \left(\frac{75}{5}\right)^2}$$

$$= \sqrt{238.6 - 15^2}$$

$$= \sqrt{238.6 - 225} = \sqrt{13.6}$$

$$= 3.69$$

For prices in city B, co-efficient of variation

$$\text{C.V.} = \frac{3.69}{15} \times 100\%$$

$$= \frac{369}{15}\% = 24.6\%$$

C.V. of city A < C.V. of city B.

$\therefore$ The prices were more stable in city A.

7. If the range and coefficient of range of the data are 20 and 0.2 respectively, then find the largest and smallest values of the data.

Sol :

Given range = 20

$$L - S = 20 \quad \text{... (1)}$$

Co-efficient of range = 0.2

$$\frac{L-S}{L+S} = 0.2$$

$$L - S = 0.2\,(L + S)$$

$$20 = 0.2\,(L + S)$$

$$\frac{20}{0.2} = L + S$$

$$L + S = \frac{200}{2}$$

$$L + S = 100 \quad \text{... (2)}$$

$$L - S = 20$$

$$(1) + (2) \Rightarrow \quad 2L = 120$$

$$L = \frac{120}{2} = 60$$

Put L = 60 in (2)

$$L + S = 100$$
$$60 + S = 100$$
$$S = 100 - 60 = 40$$

$\therefore$ L = 60 and S = 40.

8. If two dice are rolled, then find the probability of getting the product of face value 6 or the difference of face values 5.

Sol : If two dice are rolled, then the sample space

S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

n (S) = 36

Let A be the event of getting the product of the face value 6.

A = {(1, 6), (2, 3), (3, 2), (6, 1)}

n (A) = 4

$$P(A) = \frac{n(A)}{n(S)} = \frac{4}{36}$$

Let 'B' be the event of getting the difference of the face value 5.

B = {(1, 6), (6, 1)}

n (B) = 2

$$P(B) = \frac{n(B)}{n(S)} = \frac{2}{36}$$

$A \cap B$ = {(1, 6), (6, 1)}

n $(A \cap B)$ = 2

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{2}{36}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = \frac{4}{36} + \frac{2}{36} - \frac{2}{36} = \frac{4}{36} = \frac{1}{9}$$

$\therefore$ The required probability = $\frac{1}{9}$.

9. In a two children family, find the probability that there is atleast one girl in a family.

Sol : A family has two children

Let girl is denoted by 'g' and boy be denoted by 'b'.

Then the sample space

S = {(g, g), (g, b), (b, g), (b, b)}

n (S) = 4

Let A be the event of having atleast one girl baby.

A = {(g, g), (g, b), (b, g)}

n (A) = 3

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{4}$$

$\therefore$ Probability of getting atleast one girl is $\frac{3}{4}$.

10. A bag contains 5 white and some black balls. If the probability of drawing a black ball from the bag is twice the probability of drawing a white ball then find the number of black balls.

Sol :

Let A be the event of drawing black ball.

Let the number of black balls = x

n (A) = x

Total number of balls = 5 + x

n (S) = 5 + x

$$P(A) = \frac{n(A)}{n(S)} = \frac{x}{5+x}$$

Let 'B' be the event of drawing a white ball

n (B) = 5

$$P(B) = \frac{n(B)}{n(S)} = \frac{5}{5+x}$$

Given that $\quad \frac{x}{5+x} = 2\left(\frac{5}{5+x}\right)$

$$\frac{x(5+x)}{5+x} = 2 \times 5$$

$$x = 10$$

$\therefore$ Number of black balls = 10.

11. The probability that a student will pass the final examination in both English and Tamil is 0.5 and the probability of passing neither is 0.1. If the probability of passing the English examination is 0.75, what is the probability of passing the Tamil examination?

Sol :

Let T denote Tamil and E denote English

$$P(T \cap E) = 0.5$$

$$\text{P (Passing neither)} = P(T' \cap E')$$
$$= P(T \cup E)' = 0.1$$

$$P(T \cup E) = 1 - P(T \cup E)'$$

$$= 1 - 0.1 = 0.9$$

Also given $P(E) = 0.75$

$$P(T \cup E) = P(T) + P(E) - P(T \cap E)$$

$$0.9 = P(T) + 0.75 - 0.5$$

$$P(T) = 0.9 - 0.75 + 0.5 = 0.65$$

∴ Probability of passing Tamil exam = 0.65

$$P(T) = \frac{65}{100} = \frac{13}{20}$$

12. The King, Queen and Jack of the suit spade are removed from a deck of 52 cards. One card is selected from the remaining cards. Find the probability of getting (i) a diamond (ii) a queen (iii) a spade (iv) a heart card bearing the number 5.

Sol :

Suits of playing cards	Spade ♠	Heart ♥	Clavor ♣	Diamond ♦
Existing Cards	A	A	A	A
	2	2	2	2
	3	3	3	3
	4	4	4	4
	5	5	5	5
	6	6	6	6
	7	7	7	7
	8	8	8	8
	9	9	9	9
	10	10	10	10
		J	J	J
		Q	Q	Q
		K	K	K
Total Number of cards in each suit	13	13	13	13
Remaining cards after removing some of them.	10	13	13	13

After removing King, Queen and Jack of spade the remaining number of cards

$$n(S) = 10 + 13 + 13 + 13$$

$$n(S) = 49$$

(i) Let A be the event of selecting a card from diamond.

$$n(A) = 13$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{13}{49}$$

∴ Probability of getting a diamond card = $\frac{13}{49}$.

(ii) Let 'B' be the event of getting a queen.

$$n(B) = 3$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{3}{49}$$

∴ Probability of getting a queen = $\frac{3}{49}$.

(iii) Let 'C' be the event of getting a spade

$$n(C) = 10$$

$$P(C) = \frac{n(C)}{n(S)} = \frac{10}{49}$$

∴ Probability of getting a spade = $\frac{10}{49}$.

(iv) Let D be the event of getting 5 of heart.

$$n(D) = 1$$

$$P(D) = \frac{n(D)}{n(S)} = \frac{1}{49}$$

∴ Probability of getting 5 of heart = $\frac{1}{49}$.

CREATIVE QUESTIONS

I. Multiple Choice Questions

Measures of Dispersion:

1. Statistically, spread or scatterness of observations in a data is called _____

(1) Discriminant (2) Dispersion
(3) Range (4) Standard deviation. [Ans: (2)]

2. Mean of squared deviations of some observations from their arithmetic mean is called _____

(1) Standard deviation
(2) Variation
(3) Median (4) Mode [Ans: (2)]

3. Positive square root of mean of squared deviations of some observations from the arithmetic mean is called _____

(1) Standard deviation
(2) Variation
(3) Median (4) Mode [Ans: (1)]

4. Sum of deviations of a variable from its mean is always

(1) 0 (2) 1
(3) 2 (4) 5 [Ans: (1)]

5. Standard deviation of first 50 natural numbers is _____

(1) 45.43 (2) 14.43
(3) 20.43 (4) 16.43 [Ans: (2)]

Sol :

Standard deviation of first n natural numbers

$$= \sqrt{\frac{n^2-1}{12}}$$

Standard deviation of first 50 natural numbers

$$= \sqrt{\frac{50^2-1}{12}} = \sqrt{\frac{2500-1}{12}}$$

$$= \sqrt{\frac{2499}{12}} = \sqrt{208.25} = 14.43$$

6. Standard deviation of population is denoted by _____

(1) Ω (2) ω
(3) σ (4) Δ [Ans: (3)]

7. Price of apple per kg for three days are as 98, 97, 100, then the value of standard deviation with assumed mean method is.

(1) 15 (2) 10
(3) 1 (4) 11 [Ans: (3)]

Sol :

x_i	f_i	$d_i = x_i - A = x_i - 98$	d_i^2	f_id_i	$f_id_i^2$
97	1	– 1	1	– 1	1
98	1	0	0	0	0
100	1	2	4	2	4
				$\sum f_id_i = 1$	$\sum f_id_i^2 = 5$

$$\sigma = \sqrt{\frac{\sum f_id_i^2}{N} - \left(\frac{\sum f_id_i}{N}\right)^2}$$

$$= \sqrt{\frac{5}{3} - \left(\frac{1}{3}\right)^2}$$

$$= \sqrt{1.666 - 0.333} = \sqrt{1.333}$$

$$= 1.15 \simeq 1$$

8. In statistics, distance or dispersion from central value is classified as

(1) Standard variance
(2) Sample variance
(3) Standard root
(4) Standard deviation [Ans: (4)]

9. Range of the scores 80, 90, 90, 85, 60, 70, 75, 85, 90, 60, 80 is _____

(1) 30 (2) 70
(3) 90 (4) 40 [Ans: (1)]

Sol :

Range $= L - S = 90 - 60 = 30$

10. Coefficient of range of 5, 6, 7, 8, 9, 54 is _____

(1) $\frac{39}{49}$ (2) $\frac{49}{59}$

(3) $\frac{59}{69}$ (4) $\frac{69}{79}$ [Ans: (2)]

Sol : *Co-efficient of range* $= \frac{L-S}{L+S} = \frac{54-5}{54+5} = \frac{49}{59}$

11. If the total sum of squares is 20 and sample variance is 5, then total number of observations are

(1) 15 (2) 25

(3) 4 (4) 35 [Ans: (3)]

Sol : $\sum(x_i - \bar{x})^2 = 20$

Variance = 5

$$\sigma = \sqrt{\frac{\sum(x_i - \bar{x})^2}{n}} = \sqrt{\frac{20}{n}}$$

$$\text{Variance } \sigma^2 = \frac{20}{n}$$

$$5 = \frac{20}{n}$$

$$n = \frac{20}{5} = 4$$

Coefficient of Variation:

12. If mean is 25 and standard deviation is 5 then co-efficient of variation is

(1) 100% (2) 25%

(3) 20% (4) None of these

[Ans: (3)]

Sol : $C.V. = \frac{\sigma}{\bar{x}} \times 100\% = \frac{5}{25} \times 100\% = 20\%$

13. _____ is used to compare the variation or dispersion in two or more sets of data even though they are measured in different units.

(1) Range

(2) Standard deviation

(3) Co-efficient of variation

(4) Mean deviation. [Ans: (3)]

14. _____ is used to criterion of consistence is for consistence performance.

(1) Range

(2) Standard deviation

(3) Co-efficient of variation

(4) Mean deviation [Ans: (3)]

15. If the co-efficient of variation of marks of Brinda is 25% and that of Buvana is 40%. Who is more stable in scoring?

(1) Brinda (2) Buvana

(3) Both (4) None [Ans: (1)]

Sol :

Less co-efficient of variation means the data are more stable.

∴ Brinda is more stable.

Probability of an Event:

16. If a digit is chosen at random from the digits 1, 2, 3, 4, 5, 6, 7, 8, 9, then the probability that it is odd is

(1) $\frac{4}{9}$ (2) $\frac{5}{9}$

(3) $\frac{1}{9}$ (4) $\frac{2}{3}$ [Ans: (2)]

Sol : S = {1, 2, 3, 4, 5, 6, 7, 8, 9}

n (S) = 9

A be the event of selecting odd number.

A = {1, 3, 5, 7, 9}

n (A) = 5

$$P(A) = \frac{n(A)}{n(S)} = \frac{5}{9}$$

17. In a single throw of die, the probability of getting a multiple of 3 is

(1) $\frac{1}{2}$ (2) $\frac{1}{3}$

(3) $\frac{1}{6}$ (4) $\frac{2}{3}$ [Ans: (2)]

Sol :

For a single through of a die S = {1, 2, 3, 4, 5, 6}

n(S) = 6.

Let E be the event of getting a multiple of 3.

E = {3, 6}

n (A) = 2

$$P(E) = \frac{n(E)}{n(S)} = \frac{2}{6} = \frac{1}{3}$$

18. The probability of throwing a number greater than 2 with a fair dice is

(1) $\frac{3}{5}$ (2) $\frac{2}{5}$

(3) $\frac{2}{3}$ (4) $\frac{1}{3}$ [Ans: (3)]

Sol : For a die n(S) = 6

Let E be the event of throwing a number greater then 2.

E = {3, 4, 5, 6}

n(E) = 4

$$P(E) = \frac{n(E)}{n(S)} = \frac{4}{6} = \frac{2}{3}$$

19. A card is dropped from a pack of 52 playing cards. The probability that it is an ace is

(1) $\frac{1}{4}$ (2) $\frac{1}{13}$

(3) $\frac{1}{52}$ (4) $\frac{12}{13}$ [Ans: (2)]

Sol : n (S) = 52.

Number of ace n(A) = 4

$$P(A) = \frac{n(A)}{n(S)} = \frac{4}{52} = \frac{1}{13}$$

20. The probability of a certain event is

(1) 0 (2) 1

(3) $\frac{1}{2}$ (4) Not exists. [Ans: (2)]

21. The probability of an impossible event is

(1) 0 (2) 1

(3) $\frac{1}{2}$ (4) Not exists. [Ans: (1)]

22. Which of the following is not the probability of accurance of an event

(1) 0.2 (2) 0.4

(3) 0.8 (4) 1.6 [Ans: (4)]

Algebra of Events:

23. If P (E) = 0.05, then P (not E) =

(1) – 0.05 (2) 0.5

(3) 0.9 (4) 0.95 [Ans: (4)]

Sol :

$$P(E') = 1 - P(E) = 1 - 0.05 = 0.95$$

24. Which of the following statement is wrong.

(1) $A \cap B$ is an event that occurs only when both A and B occurs.

(2) $A \cup B$ is an event that occurs only when at least one of A or B occurs.

(3) $\bar{A}$ is an event that occurs only when A does not occur.

(4) $\bar{B}$ is an event that occurs when B occur. [Ans: (4)]

25. $A \cup \bar{A}$ = _____

(1) 0 (2) 1

(3) ϕ (4) S [Ans: (4)]

26. $A \cap \bar{A}$ = _____

(1) 0 (2) 1

(3) ϕ (4) S [Ans: (3)]

27. $P(\overline{A \cup B})$ = _____

(1) $P(\bar{A} \cup \bar{B})$ (2) $P(\bar{A} \cap \bar{B})$

(3) $P(A \cup B)$ (4) $P(A \cap B)$ [Ans: (2)]

II. Very Short Answer Questions

1. Find the co-efficient of range of the following data 13, 15, 21, 25, 18, 27, 43, 31

Sol :

Largest value L = 43

Smallest value S = 13

Coefficient of range= $\frac{L-S}{L+S} = \frac{43-13}{43+13} = \frac{30}{56}$

Coefficient of range= $\frac{15}{28} = 0.54$

2. Find the range of the following data 70.3, 43.2, 80.5, 93.4, 100, 13.7

Sol :

Largest value L = 100

Smallest value S = 13.7

Range = L – S = 100 – 13.7

Range = 86.3

3. Calculate the range of the following data.

Marks	40-50	50-60	60-70	70-80	80-90	90-100
Students	20	13	27	17	40	8

Sol : Largest value L = 100

Smallest value S = 40

Range = L – S = 100 – 40

Range = 60

4. Find the standard deviation of first 50 natural numbers

Sol : Standard deviation of first n natural numbers

$$= \sqrt{\frac{n^2-1}{12}}$$

$\therefore$ S.D. of first 50 natural numbers = $\sqrt{\frac{50^2-1}{12}}$

$$= \sqrt{\frac{2500-1}{12}} = \sqrt{\frac{2499}{12}}$$

$$= \sqrt{208.25} = 14.43$$

Standard deviation of first 50 natural numbers = 14.43

5. If the standard deviation of a data is 20.5 and each value of the data is increased by 12, then find the new standard deviation:

Sol :

Given the standard deviation = 20.5.

If we add a fixed constant to each data, the standard deviation never change.

∴ If we add 12 to each data the new standard deviation will not change.

∴ New standard deviation = 20.5

6. The standard deviation and mean of a data is given by 60 and 120. Find the coefficient of variation.

Sol :

Standard deviation $\sigma = 60$

Mean $\bar{x} = 120$

Co-efficient of variation

$$\text{C. V} = \frac{\sigma}{\bar{x}} \times 100\%$$

$$= \frac{60}{120} \times 100\% = \frac{6000}{120}$$

$$= 50\%$$

Co-efficient of variation is 50%.

7. The standard deviation and coefficient of variation of some data are 9 and 72. Find the mean.

Sol :

Given the co-efficient of variation C. V = 72

Standard deviation $\sigma = 9$

$$\text{C. V} = \frac{\sigma}{\bar{x}} \times 100\%$$

$$72 = \frac{9}{\bar{x}} \times 100\%$$

$$\bar{x} = \frac{9}{72} \times 100 = \frac{100}{8}$$

∴ Mean $\bar{x} = 12.5$

8. If the mean and coefficient of variation of a data are 30 and 96 respectively, then find the value of standard deviation.

Sol :

Mean $\bar{x} = 30$

Co-efficient of variation C. V = $\frac{\sigma}{\bar{x}} \times 100\%$

$$96 = \frac{\sigma}{30} \times 100\%$$

$$\sigma = \frac{96 \times 30}{100} = \frac{288}{10}$$

∴ Standard deviation $\sigma = 28.8$

9. The probability that it will rain tomorrow is 0.85. What is the probability that it will not rain tomorrow?

Sol :

P (will rain) + P (will not rain) = 1

∴ P (will not rain) = 1 – P (will rain)

= 1 – 0.85 = 0.15

Probability that will not rain tomorrow = 0.15.

10. A die is thrown. Find the probability of getting an even prime number?

Sol :

When a die is thrown, the sample space

S = {1, 2, 3, 4, 5, 6}

n(S) = 6

Let A be the event of getting an even prime

A = {2}

n(A) = 1

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{6}$$

Probability of getting an even prime is $\frac{1}{6}$

11. An urn contains 10 red and 8 white balls. One ball is drawn at random. Find the probability that the ball drawn is white.

Sol :

Total number of balls = 10 red + 8 white

n(S) = 18

Let A be the event of drawing white ball

n(A) = 8

$$P(A) = \frac{8}{18} = \frac{4}{9}$$

12. A bag contains 3 red balls, 5 black balls and 4 white balls. A ball is drawn at random from the bag. What is the probability that the ball drawn is not red?

Sol :

Total number of balls = 3 red + 5 black + 4 white

n(S) = 12

Let A be the event of drawing red ball

n(A) = 3

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A) = \frac{3}{12} = \frac{1}{4}$$

$$\therefore \quad P(\overline{A}) = 1 - P(A) = 1 - \frac{1}{4}$$

$$P(\overline{A}) = \frac{4-1}{4} = \frac{3}{4}$$

$\therefore$ Probability of the ball drawn is not red is $\frac{3}{4}$

13. What is the probability that a number selected from the numbers 1, 2, 3, …15 is a multiple of 4?

Sol : Let $S = \{1, 2, 3, ..., 15\}$

$$n(S) = 15$$

Let A be the event of selecting a multiple of 4

$$A = \{4, 8, 12\}$$

$$n(A) = 3$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{15} = \frac{1}{15}$$

$\therefore$ Probability of selecting a multiple of 4 is $\frac{1}{15}$.

14. In a lottery there are 10 prizes and 25 blanks. What is the probability of getting prize?

Sol :

Total results in the lottery = 10 prizes + 25 blanks

$$n(S) = 35$$

Let A be the event of getting prize

$$n(A) = 10$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{10}{35} = \frac{2}{7}$$

Probability of getting price is $\frac{2}{7}$

15. The probability of winning a game is 0.3. What is the probability of loosing it?

Sol :

P(winning) + P(loosing) = 1

0.3 + P(loosing) = 1

P(loosing) = 1 – 0.3 = 0.7

$\therefore$ Probability of loosing the game is 0.7

16. A bag contains cards which are numbered from 2 to 90. A card is drawn at random from the bag. Find the probability that it bears a number which is a perfect square?

Sol :

$$S = \{2, 3, 4, 5, ..., 90\}$$

$$n(S) = 89$$

Let A be the event of drawing a perfect square

$$A = \{4, 9, 16, 25, 36, 49, 64, 81\}$$

$$n(A) = 8$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{8}{89}$$

$\therefore$ Probability of drawing a perfect square = $\frac{8}{89}$

17. If $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{2}$, $P(A \cap B) = \frac{1}{4}$, then find $P(A' \cap B')$.

Sol : Given $P(A) = \frac{1}{3}$

$$P(B) = \frac{1}{2}$$

$$P(A \cap B) = \frac{1}{4}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = \frac{1}{3} + \frac{1}{2} - \frac{1}{4}$$

$$= \frac{4+6-3}{12} = \frac{7}{12}$$

$$P(\overline{A \cup B}) = 1 - P(A \cup B)$$

$$P(\overline{A \cup B}) = 1 - \frac{7}{12} = \frac{12-7}{12} = \frac{5}{12}$$

$$\therefore \quad P(A' \cap B') = \frac{5}{12} \left[\because P(\overline{A} \cap \overline{B}) = P(\overline{A \cup B})\right]$$

18. A and B are events such that P (not A or not B) = 0.25. Whether A and B are mutually exclusive or not.

Sol :

We have $P(A \cap B)' = P(A' \cup B')$

Given $P(A' \cup B') = 0.25$

$$P(A \cap B)' = 0.25$$

$$P(A \cap B) = 1 - P(A \cap B)'$$

$$P(A \cap B) = 1 - 0.25 = 0.75$$

$$P(A \cap B) \neq 0.$$

$\therefore$ A and B are not mutually exclusive.

19. Given $P(A) = \frac{3}{5}$ and $P(B) = \frac{1}{5}$. Find P(A or B) if A and B are mutually exclusive events.

Sol : Given A and B are mutually exclusive events.

$$\therefore \quad P(A \cap B) = 0$$

$$P(A \cup B) = P(A) + P(B)$$

$$= \frac{3}{5} + \frac{1}{5}$$

$$P(\text{'A' or 'B'}) = \frac{4}{5}$$

20. If A and B are two events associated with a random experiment such that $P(A) = 0.3$, $P(B) = 0.2$ and $P(A \cap B) = 0.1$. Then find the value of $P(A \cap \overline{B})$.

Sol :

$$P(A \cap \overline{B}) = P\text{ (A only)}$$
$$= P(A) - P(A \cap B) = 0.3 - 0.1$$
$$\therefore \quad P(A \cap \overline{B}) = 0.2$$

III. Short Answer Questions:

1. The standard deviation of 50 data is 12.6 and each value of the data is divided by 6, then find the new variance and standard deviation.

Sol :

Standard deviation = 12.6

When each value of the data is divided by a fixed constant, the new standard deviation also get divided by the fixed constant.

$\therefore$ New standard deviation $= \dfrac{12.6}{6} = 2.1$

New variance $= \sigma^2 = (2.1)^2 = 4.41$

New Standard deviation = 2.1

New variance = 4.41

2. If $n = 10$, $\overline{x} = 1.8$, $\sum x^2 = 1500$. Calculate the coefficient of variation.

Sol :

First we calculate the standard deviation.

$$\sigma = \sqrt{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2}$$

$$\sigma = \sqrt{\frac{1500}{10} - (1.8)^2} = \sqrt{150 - 3.24} = \sqrt{146.76}$$

$$\sigma = 12.11$$

Now, co-efficient of variation $= \dfrac{\sigma}{\overline{x}} \times 100\%$

$$= \frac{12.11}{1.8} \times 100\%$$

$$\text{C. V} = 672.78\%$$

3. Coefficient of variation of two distributions are 60 and 70 and their standard deviation are 21 and 16 respectively. What are their arithmetic means?

Sol : Given:

For the I distribution	For the II distribution.
$C. V_1 = 60$	$C. V_2 = 70$
$\sigma_1 = 21$	$\sigma_2 = 16$

$$\text{C. V} = \frac{\sigma}{\overline{x}} \times 100\%$$

$$\text{C. V}_1 = \frac{\sigma_1}{\overline{x}_1} \times 100\%$$

$$60 = \frac{21}{\overline{x}_1} \times 100$$

$$\overline{x}_1 = \frac{21}{60} \times 100 = 35$$

$$\text{CV}_2 = \frac{\sigma_2}{\overline{x}_2} \times 100\%$$

$$70 = \frac{16}{\overline{x}_2} \times 100$$

$$\overline{x}_2 = \frac{16}{70} \times 100 = 22.85$$

$$\overline{x}_1 = 35;\ \overline{x}_2 = 22.85$$

4. In a lottery of 50 tickets numbered 1 to 50, one ticket is drawn. Find the probability that the drawn ticket bears a prime number.

Sol :

Let $S = \{1, 2, 3, ...,50\}$

$n(S) = 50$

Let A be the event of drawing a ticket bearing prime number.

$A = \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47\}$

$n(A) = 15$

$$P(A) = \frac{n(A)}{n(S)} = \frac{15}{50} = \frac{3}{10}$$

$\therefore$ Probability of drawing a ticket bearing prime number is $\dfrac{3}{10}$

5. A bag contains 6 red, 8 black and 4 white balls. A ball is drawn at random. What is the probability that ball drawn is not black?

Sol :

Total number of balls = 6 red + 8 black + 4 white

$n(S) = 18$

Let A be the event of drawing a black ball

$n(A) = 8$

$$P(A) = \frac{n(A)}{n(S)} = \frac{8}{18} = \frac{4}{9}$$

P(not black) = 1 – P(A)

$$P(\overline{A}) = 1-\frac{4}{9}$$

$$P(\overline{A}) = \frac{9-4}{9} = \frac{5}{9}$$

$\therefore$ Probability that the drawn ball is not black is $\frac{5}{9}$.

6. Two customers are visiting a particular shop in the same week (Monday to Saturday). Each is equally likely to visit the shop on any one day as on another. What is the probability that both will visit the shop on different days?

Sol :

Two customers can visit the shop on any of the 6 days.

S = {(Mon, Mon) (Mon, Tue) (Mon, Wed) (Mon, Thur) (Mon, Fri) (Mon, Sat) (Tue, Mon) (Tue, Tue) (Tue, Wed) (Tue, Thur) (Tue, Fri) (Tue, Sat) (Wed, Mon) (Wed, Tue) (Wed, Wed) (Wed, Thur) (Wed, Fri) (Wed, Sat) (Thur, Mon) (Thur, Tue) (Thur, Wed) (Thur, Thur) (Thur, Fri) (Thur, Sat) (Fri, Mon) (Fri, Tue) (Fri, Wed) (Fri, Thur) (Fri, Fri) (Fri, Sat) (Sat, Mon) (Sat, Tue) (Sat, Wed) (Sat, Thur) (Sat, Fri) (Sat, Sat)}

n(S) = 36

Let A be the event of the customers visiting the shop on the same day.

A = {(Mon, Mon) (Tue, Tue) (Wed, Wed) (Thur, Thur) (Fri, Fri) (Sat, Sat)}

n(A) = 6

$$P(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

$$P(\overline{A}) = 1 - P(A) = 1 - \frac{1}{6}$$

$$P(\overline{A}) = \frac{6-1}{6} = \frac{5}{6}$$

$\therefore$ Probability that they visit on different days

$= \frac{5}{6}$.

7. Find the probability that a number selected from the numbers 1, 2, 3, ….35 is a multiple of 7.

Sol :

S = {1, 2, 3, 4, ...35}

n(S) = 35

Let A be the event of selecting multiple of 7

A = {7, 14, 21, 28, 35}

n(A) = 5

$$P(A) = \frac{n(A)}{n(S)} = \frac{5}{35} = \frac{1}{7}$$

$\therefore$ Probability of selecting multiple of 7 is $\frac{1}{7}$.

8. A lot consists of 144 ball pens of which 20 are defective and others good. Leena will buy a pen if it is good, but will not buy if it is defective. The shopkeeper draws one pen at random and gives it to her. What is the probability that she will buy it?

Sol : Total pens = 144

n(S) = 144

Number of good pens = 144 – defective pens

= 144 – 20 = 124

Let A be the event of getting good pen

$\therefore$ n(A) = 124

$$P(A) = \frac{n(A)}{n(S)} = \frac{124}{144} = \frac{31}{36}$$

If she gets good pen, she will buy it.

$\therefore$ Probability of buying the pen = $\frac{31}{36}$.

9. A bag contains 6 red balls and some blue balls. If the probability of drawing a blue ball from the bag is twice that of a red ball, find the number of blue balls in the bag.

Sol : Let the number of blue balls = x

$\therefore$ Total balls n(S) = 6 + x

Let A be the event of drawing a blue ball.

n(A) = x

$$P(A) = \frac{x}{6+x}$$

Let B be the event of drawing red ball

n(B) = 6

$$P(B) = \frac{6}{6+x}$$

given P(A) = 2 [P(B)]

$$P(A) = 2\times\frac{6}{6+x}$$

$$\frac{x}{6+x} = 2\times\frac{6}{6+x}$$

$$\frac{x(6+x)}{6+x} = 2\times 6$$

x = 12

$\therefore$ Number of blue balls = 12.

10. A number x is selected from the number 1, 2, 3 and then a second number y is randomly selected from the numbers 1, 4, 9. What is the probability that the product xy of two number will be less than 9?

Sol :

First number is selected from 1, 2, 3 and second from 1, 4, 9.

$S = \{(1, 1)\ (1, 4)\ (1, 9)\ (2, 1)\ (2, 4)\ (2, 9)\ (3, 1)\ (3, 4)\ (3, 9)\}$

$n(S) = 9$

Let A be the event of getting the product of two numbers less than 9.

$A = \{(1, 1)\ (1, 4)\ (2, 1)\ (2, 4)\ (3, 1)\}$

$n(A) = 5$

$$P(A) = \frac{n(A)}{n(S)} = \frac{5}{9}$$

$\therefore$ Required probability $= \frac{5}{9}$.

11. The probability that atleast one of the two events A and B occurs is 0.6. If A and B occur simultaneously with probability 0.3. Evaluate $P(\overline{A}) + P(\overline{B})$

Sol :

Given $P(A \cup B) = 0.6$

$P(A \cap B) = 0.3$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$0.6 = P(A) + P(B) - 0.3$$
$$P(A) + P(B) = 0.6 + 0.3 = 0.9$$
$$[1 - P(\overline{A})] + [1 - P(\overline{B})] = 0.9$$
$$1 - P(\overline{A}) + 1 - P(\overline{B}) = 0.9$$
$$2 - [P(\overline{A}) + P(\overline{B})] = 0.9$$
$$P(\overline{A}) + P(\overline{B}) = 2 - 0.9 = 1.1$$

12. If A and B are mutually exclusive events P(A) = 0.35 and P(B) = 0.45. Find $P(A' \cap B')$.

Sol : $P(A \cup B) = P(A) + P(B)$

$= 0.35 + 0.45 = 0.80$

$P(A' \cap B') = P(A \cup B)'$

$= 1 - P(A \cup B) = 1 - 0.80$

$\therefore P(A' \cap B') = 0.20.$

13. If A and B are two events such that $P(A) = \frac{1}{5}$, $P(B) = \frac{1}{3}$ and P (A and B) = $\frac{1}{7}$ find (i) P (A or B), (ii) P (not A and not B).

Sol :

(i) P(A or B) $= P(A \cup B)$

$= P(A) + P(B) - P(A \cap B)$

$$P(A \text{ or } B) = \frac{1}{5} + \frac{1}{3} - \frac{1}{7} = \frac{21 + 35 - 15}{105}$$

$$P(A \text{ or } B) = \frac{41}{105}$$

(ii) P (not A and not B) $= P(\overline{A} \cap \overline{B})$

$$= P(\overline{A \cup B}) = 1 - P(A \cup B)$$
$$= 1 - \frac{41}{105} = \frac{105 - 41}{105}$$

P (not A and not B) $= \frac{64}{105}$.

IV. Long Answer Questions

1. The mean and standard deviation is the marks of a group of 50 students were 60 and 15 respectively. Later if was found to be the scores 40 and 70 were wrongly entered as 30 and 60. Find the correct mean and standard deviation.

Sol :

Mean $\overline{x} = 60$

Standard deviation $\sigma = 15$.

Wrong scores = 30 and 60.

Correct scores = 40 and 70

$$\text{Old mean} = \frac{\Sigma x_i}{n} = 60$$
$$\frac{\Sigma x_i}{50} = 60$$
$$\text{Old } \Sigma x_i = 60 \times 50 = 3000$$

Correct Σx_i = 3000 – (wrong scores) + (correct scores)

$= 3000 - [(30 + 60)] + [(40 + 70)]$

$= 3000 - [90] + [110]$

$= 2910 + 110 = 3020$

$$\text{Correct mean} = \frac{\text{correct } \Sigma x_i}{n} = \frac{3020}{50} = 60.4$$

Old standard deviation $\sigma = 15$

$$\sigma = \sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2}$$

$$15 = \sqrt{\frac{\Sigma x_i^2}{n} - 60^2}$$

Squaring on both sides

$$225 = \frac{\Sigma x_i^2}{n} - 3600$$

$$225 + 3600 = \frac{\Sigma x_i^2}{50}$$

$$\text{Old } \frac{\Sigma x_i^2}{50} = 3825$$

$$\text{Old } \Sigma x_i^2 = 3825 \times 50 = 191250$$

Correct $\sum x_i^2$ = 191250 – (wrong scores)^2 + (correct scores)^2

$$= 191250 - 30^2 - 60^2 + 40^2 + 70^2$$

$$= 191250 - 900 - 3600 + 1600 + 4900$$

$$= 191250 - 4500 + 6500$$

$$= 193250$$

$$\text{Correct } \sigma = \sqrt{\frac{193250}{50} - (60.4)^2}$$

$$= \sqrt{3865 - 3648.16} = \sqrt{216.84}$$

Correct σ = 14.7

2. Calculate standard deviation by direct method. 25, 27, 31, 32, 35

Sol :

x_i	x_i^2
25	625
27	729
31	961
32	1024
35	1225
$\Sigma x_i = 150$	$\Sigma x_i^2 = 4564$

$$\text{Standard deviation } \sigma = \sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{N}\right)^2}$$

$$= \sqrt{\frac{4564}{5} - \left(\frac{150}{5}\right)^2} = \sqrt{912.8 - 900} = \sqrt{12.8}$$

$$\therefore \quad \sigma = 3.578$$

3. If N = 10, Σx = 120, Σx^2 = 1530, find the variance.

Sol : $\bar{x} = \frac{\Sigma x}{n} = \frac{120}{10} = 12$

$$\text{Standard deviation } \sigma = \sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{N}\right)^2}$$

$$\sigma = \sqrt{\frac{1530}{10} - \left(\frac{120}{10}\right)^2}$$

$$\sigma = \sqrt{153 - 144} = \sqrt{9} = 3$$

Variance $\sigma^2 = 3^2 = 9$

4. Calculate standard deviation by assumed mean method 40, 44, 54, 60, 62, 64, 70, 80, 90, 96.

Sol : Let A = 60

x_i	$d_i = x_i - A$ $d_i = x_i - 60$	d_i^2
40	– 20	400
44	– 16	256
54	– 6	36
60	0	0
62	2	4
64	4	16
70	10	100
80	20	400
90	30	900
96	36	1296
N = 10	$\Sigma d_i = 60$	$\Sigma d_i^2 = 3408$

$$\text{Standard deviation } \sigma = \sqrt{\frac{\Sigma d_i^2}{n} - \left(\frac{\Sigma d_i}{n}\right)^2}$$

$$= \sqrt{\frac{3408}{10} - \left(\frac{60}{10}\right)^2}$$

$$= \sqrt{340.8 - 36} = \sqrt{304.8}$$

$\therefore$ Standard deviation σ = 17.46

5. Calculate the standard deviation.

x	10	12	14	16	18	20	22
f	3	5	9	16	8	7	2

Sol :

Let A = 14

x_i	f_i	x_if_i	$d_i = x_i - A$ $d_i = x_i - 14$	f_id_i	d_i^2	$f_id_i^2$
10	3	30	– 4	– 12	16	48
12	5	60	– 2	– 10	4	20
14	9	126	0	0	0	0
16	16	256	2	32	4	64
18	8	144	4	32	16	128
20	7	140	6	42	36	252
22	2	44	8	16	64	128
	$\Sigma f_i = 50$	$\Sigma x_i f_i = 800$		$\Sigma f_i d_i = 100$	$\Sigma d_i^2 = 140$	$\Sigma f_i d_i^2 = 640$

Standard deviation $\sigma = \sqrt{\frac{\Sigma f_i d_i^2}{N} - \left(\frac{\Sigma f_i d_i}{N}\right)^2}$

$$= \sqrt{\frac{640}{50} - \left(\frac{100}{50}\right)^2} = \sqrt{\frac{64}{5} - 4}$$

$$= \sqrt{\frac{64-20}{5}} = \sqrt{\frac{44}{5}} = \sqrt{8.8}$$

$\therefore$ Standard deviation $= 2.97$

6. Compute the standard deviation for the following data.

x	0-10	10-20	20-30	30-40	40-50	50-60	60-70
f	1	4	17	45	26	5	2

Sol :

Let A = 35 and C = 10

x_i	Mid value x_i	f_i	$d_i = x_i - A$ $d_i = x_i - 35$	$d_i = \frac{x_i - A}{C}$	f_id_i	$f_id_i^2$
0-10	5	1	– 30	– 3	– 3	9
10-20	15	4	– 20	– 2	– 8	16
20-30	25	17	– 10	– 1	– 17	17
30-40	35	45	0	0	0	0
40-50	45	26	10	1	26	26
50-60	55	5	20	2	10	20
60 - 70	65	2	30	3	6	18
		$\Sigma f_i = 100$			$\Sigma f_i d_i = 14$	$\Sigma f_i d_i^2 = 106$

Standard deviation $\sigma = \sqrt{\frac{\Sigma f_i d_i^2}{N} - \left(\frac{\Sigma f_i d_i}{N}\right)^2} \times C$

$$= \sqrt{\frac{106}{100} - \left(\frac{14}{100}\right)^2} \times 10$$

$$= \sqrt{1.06 - 0.0196} \times 10$$

$$= \sqrt{1.0404} \times 10$$

$$= 1.02 \times 10$$

Standard deviation $\sigma = 10.2$

7. The following values are calculated in respect of heights and weights of the students of class X.

	Height	Weight
Mean	162.6 cm	52.36 kg
Variance	127.69 cm²	23.1361 kg²

Can we say that the weight show greater variation than the heights

Sol :

To compare the variability, we have to calculate their co-efficient of variation.

Given variance of Height = 127.69 cm²

$\therefore$ Standard deviation of Height

$$= \sqrt{127.69\ cm} = 11.3 \text{ cm}$$

Variance of weight = 23.1361 kg

$\therefore$ Standard deviation of weight

$$= \sqrt{23.1361\ kg} = 4.81 \text{ kg.}$$

$\therefore$ Co-efficient of variation C. V $= \frac{\sigma}{x} \times 100\%$

C. V of Height $= \frac{11.3}{162.6} \times 100 = 6.95\%$

C. V of weights $= \frac{4.89}{52.36} \times 100\% = 9.18\%$

C. V of weight > C. V of Height.

$\therefore$ We can say that weights show more variability than heights.

8. The mean and standard deviation of marks obtained by 50 students of a class in three subjects mathematics, physics and chemistry are given below. Which of these three subjects shows the highest variability in marks and shows the lowest?

Subject	Mathematics	Physics	Chemistry
Mean	42	32	40.9
Standard deviation	12	15	20

Sol :

(i) For mathematics

Mean $\bar{x} = 42$; Standard deviation $\sigma = 12$

Co-efficient of variation C. V $= \frac{\sigma}{\bar{x}} \times 100\%$

$= \frac{12}{42} \times 100\% = \frac{1200}{42}\% = 28.57\%$

(ii) For Physics

Mean $\bar{x} = 32$; SD = 15

C. V $= \frac{\sigma}{\bar{x}} \times 100\%$

$= \frac{15}{32} \times 100\% = \frac{1500}{32}\% = 46.87\%$

(iii) For Chemistry:

Mean $\bar{x} = 40.9$; SD $\sigma = 20$

C. V $= \frac{\sigma}{\bar{x}} \times 100\%$

$= \frac{20}{40.9} \times 100\% = \frac{2000}{40.9} = 48.89\%$

As higher the co-efficient of variation, higher in the variability.

$\therefore$ C. V of chemistry is highest and C. V of mathematics is the least.

$\therefore$ Chemistry shows the highest variability and mathematics shows the least variability.

9. From the prices of shares A and B given below. Find which is more stable value.

A	35	54	52	53	56	58	52	50	51	49
B	108	107	105	105	106	107	104	103	104	101

Given that $\sigma_A = 5.92$ and $\sigma_B = 2$ also mean (A) = 51 and mean (B) = 105.

Sol :

Co-efficient of variation C .V $= \frac{\sigma}{\bar{x}} \times 100\%$

$$\text{C. V}_A = \frac{5.92}{51} \times 100\% = \frac{592}{51}$$

$$\text{C. V}_A = 11.60$$

$$\text{C. V of B} = \frac{2}{105} \times 100 = \frac{200}{105}$$

$$\text{C. V}_B = 1.90$$

Shares, whose co-efficient of variation is lesser is considered to be more stable.

$\therefore$ CV of share B is lesser as compared to C. V of share A.

$\therefore$ Share B is more stable.

10. A jar contains 24 marbles some are green and the others are blue. If a marble is drawn at random from the jar, the probability that it is green is $\frac{2}{3}$. Find the number of blue marbles in the jar.

Sol : Let the number of green marbles be x and the number of blue marbles be y.

Total marbles n(S) = x + y = 24 ... (1)

Let A be the event of getting a green marble

$$P(A) = \frac{n(A)}{n(S)} = \frac{x}{24}$$

but given $P(A) = \frac{2}{3}$

$$\therefore \frac{x}{24} = \frac{2}{3}$$

$$x = \frac{2 \times 24}{3} = 16$$

Put x = 16 in (1)

$$x + y = 24$$
$$16 + y = 24$$
$$y = 24 - 16 = 8$$

$\therefore$ Number of green marbles = 16

Number of blue marbles = 8

11. A jar contains 54 marbles each of which is blue, green or white. The probability of selecting a blue marble at random from the jar is $\frac{1}{3}$ and the probability of selecting a green marble at random is $\frac{4}{9}$. How many white marbles does the jar contain?

Sol :

Let the number of blue balls be b, number of green balls be g and number of white balls be w.

$$b + g + w = 54 \quad \dots (1)$$
$$n(S) = 54$$

Let A be the event of selecting blue marble.

$$n(A) = b$$
$$P(A) = \frac{n(A)}{n(S)} = \frac{b}{54}$$

Given $P(A) = \frac{1}{3}$

$$\therefore \quad \frac{1}{3} = \frac{b}{54}$$
$$\Rightarrow \quad b = \frac{54}{3} = 18$$

Let B be the event of selecting a green marble

$$n(B) = g$$
$$P(B) = \frac{g}{54}$$

But $P(B) = \frac{4}{9}$ (given)

$$\therefore \quad \frac{g}{54} = \frac{4}{9}$$
$$g = \frac{4 \times 54}{9} = 24$$

Put $b = 18, g = 24$ in (1)

$$b + g + w = 54$$
$$18 + 24 + w = 54$$
$$42 + w = 54$$
$$w = 54 - 42 = 12$$

$\therefore$ The jar contains 12 white balls.

12. Two dice are thrown simultaneously. Find the probability of getting
(i) an even number as sum
(ii) the total of atleast 10
(iii) a multiple of 3 as the sum.

Sol :

Two dice are thrown, the sample space

S = {(1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6) (2, 1) (2, 2) (2, 3) (2, 4) (2, 5) (2, 6) (3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (3, 6) (4, 1) (4, 2) (4, 3) (4, 4) (4, 5) (4, 6) (5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (5, 6) (6, 1) (6, 2) (6, 3) (6, 4) (6, 5) (6, 6)}

$$n(S) = 36$$

(i) Let A be the event of getting an even number as sum

A = {(1, 1) (1, 3) (3, 1) (2, 2) (1, 5) (5, 1) (2, 4) (4, 2) (3, 3) (2, 6) (6, 2) (4, 4) (5, 3) (3, 5) (5, 5) (6, 4) (4, 6) (6, 6)}

$$n(A) = 18$$
$$P(A) = \frac{n(A)}{n(S)} = \frac{18}{36} = \frac{1}{2}$$

$\therefore$ Required probability = $\frac{1}{2}$.

(ii) Let B be the event of getting total of atleast 10

B = {(6, 4) (4, 6) (5, 5) (6, 5) (5, 6) (6, 6)}

$$n(B) = 6$$
$$P(B) = \frac{n(B)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

$\therefore$ Probability of getting the sum of atleast 10 is $\frac{1}{6}$.

(iii) Let C be the probability getting a multiple of 3 as sum.

C = { (1, 2) (2, 1) (1, 5) (5, 1) (2, 4) (4, 2) (3, 3) (3, 6) (6, 3) (5, 4) (4, 5) (6, 6)}

$$n(C) = 12$$
$$P(C) = \frac{n(C)}{n(S)} = \frac{12}{36} = \frac{1}{3}$$

$\therefore$ Probability of getting a multiple of 3 as sum = $\frac{1}{3}$.

13. Two dice are thrown. The events A, B and C are as follow: A $\rightarrow$ getting an even number in first die, $B \rightarrow$ getting an odd number on the first die, $C \rightarrow$ getting the sum of the numbers on the dice ≤ 5. Check whether A and C are mutually exclusive.

Sol : Given two dice are thrown

Possible outcomes = $6 \times 6 = 36$

Possible outcomes

S = {(1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6) (2, 1) (2, 2) (2, 3) (2, 4) (2, 5) (2, 6) (3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (3, 6) (4, 1) (4, 2) (4, 3) (4, 4) (4, 5) (4, 6) (5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (5, 6) (6, 1) (6, 2) (6, 3) (6, 4) (6, 5) (6, 6)}

$$n(S) = 36$$

A is the event of getting even number in the first die

$$A = \{(2, 1)\ (2, 2)\ (2, 3)\ (2, 4)\ (2, 5)\ (2, 6)\ (4, 1)\ (4, 2)\ (4, 3)\ (4, 4)\ (4, 5)\ (4, 6)\ (6, 1)\ (6, 2)\ (6, 3)\ (6, 4)\ (6, 5)\ (6, 6)\}$$

$$n(A) = 18$$

C is the event of getting the sum of the number less then or equal to 5

$$C = \{(1, 1)\ (1, 2)\ (1, 3)\ (1, 4)\ (2, 1)\ (2, 2)\ (3, 1)\ (3, 2)\ (4, 1)\}$$

$$n(C) = 10$$

$$A \cap C = \{(2, 1)\ (2, 2)\ (2, 3)\ (4, 1)\}$$

$$n(A \cap C) = 4$$

$$n(A \cap C) \neq 0,$$

$\therefore$ A and C are not mutually exclusive.

14. A number is selected from the first 50 natural numbers. What is the probability that it is a multiple of 5 or 11?

Sol : Let S be the sample space

$$S = \{1, 2, 3, 4, 5, \dots 50\}$$

$$n(S) = 50$$

Let A be the set of all multiples of 5.

$$A = \{5, 10, 15, 20, 25, 30, 35, 40, 45, 50\}$$

$$n(A) = 10$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{10}{50}$$

Let B be the event of getting multiple of 11

$$B = \{11, 22, 33, 44\}$$

$$n(B) = 4$$

$$P(B) = \frac{4}{50}$$

$$A \cap B = \{\ \}$$

$$P(A \cap B) = 0$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = \frac{10}{50} + \frac{4}{50} - 0$$

$$P(A \cup B) = \frac{14}{50} = \frac{7}{25}$$

Probability of getting a multiple of 5 or 11 = $\frac{7}{25}$.

15. A number is selected from the first 25 natural numbers. What is the probability that it would be divisible by 4 or 7?

Sol : Let sample space $S = \{1, 2, 3, 4, \dots, 25\}$

$$n(S) = 25$$

Let A be the event of selecting a number divisible by 4.

$$A = \{4, 8, 12, 16, 20, 24\}$$

$$n(A) = 6$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{6}{25}$$

Let B be the event of selecting a number divisible by 7.

$$B = \{7, 14, 21\}$$

$$n(B) = 3$$

$$P(B) = \frac{3}{25}$$

$$A \cap B = 0$$

$$n(A \cap B) = 0$$

$$P(A \cap B) = 0$$

$$P(A \cup B) = P(A) + P(B) = \frac{6}{25} + \frac{3}{25}$$

$$P(A \cup B) = \frac{9}{25}$$

$\therefore$ Probability of selecting a number divisible by 4 or 7 is $\frac{9}{25}$.

16. A number is selected from first 30 natural numbers. What is the probability that if would be divisible by 2 or it is a prime.

Sol :

Sample space $S = \{1, 2, 3, \dots, 30\}$

$$n(S) = 30$$

Let A be the event of selecting a number divisible by 2.

$$A = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30\}$$

$$n(A) = 15$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{15}{30}$$

Let B be the event of getting a prime

$$B = \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29\}$$

$$n(B) = 10$$

$$P(B) = \frac{10}{30}$$

$$A \cap B = \{2\}$$

$$n(A \cap B) = 1$$

$$P(A \cap B) = \frac{1}{30}$$

$$\therefore\ P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = \frac{15}{30} + \frac{10}{30} - \frac{1}{30}$$

$$P(A \cup B) = \frac{24}{30} = \frac{4}{5}$$

$\therefore$ Required probability = $\frac{4}{5}$.

17. There are three persons A, B and C having different ages. The probability that A survives another 5 years is 0.80. 'B' survives another 5 years is 0.60 and C survives for another 5 years is 0.50. The probabilities that A and B survives another 5 years is 0.46. B and C another 5 years is 0.32 and A and C survives another 5 years is 0.48. The probability that all these three survive another 5 years is 0.26. Find the probability that at least one of them survive another 5 years.

Sol :

From the information given

$$P(A) = 0.80$$
$$P(B) = 0.60$$
$$P(C) = 0.50$$
$$P(A \cap B) = 0.46$$
$$P(B \cap C) = 0.32$$
$$P(A \cap C) = 0.48$$
$$P(A \cap B \cap C) = 0.26$$

The probability that at least one of them survives another 5 years is $P(A \cup B \cup C)$.

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) \; P(A \cap C) + P(A \cap B \cap C)$$

$$P(A \cup B \cup C) = 0.80 + 0.60 + 0.50 - 0.46 - 0.32 - 0.48 + 0.26$$

$$\therefore \; P(A \cup B \cup C) = 0.90$$

18. The probability that a student will pass his examination is 0.73. The probability of the student getting a compartment is 0.13 and check whether the probability that the student will either pass or get compartment is 0.96.

Sol :

Let A be the event of passing B get compartment.

Given $P(A \cup B) = 0.96$.

$$P(A) = 0.73$$
$$P(B) = 0.13$$
$$P(A \cup B) = 0.96$$
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$0.96 = 0.73 + 0.13 - P(A \cap B)$$
$$P(A \cap B) = 0.73 + 0.13 - 0.96$$
$$= 0.86 - 0.96 = -0.10$$
$$P(A \cap B) = -0.10$$

The probability cannot be negative.

$\therefore$ The probability that the student will either pass or get compartment is not 0.96.

X Mathematics

Govt. Model Question Paper 2020

Time : 2.30 hrs **Maximum Marks : 100**

Instructions: i) Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.

ii) Use Black or Blue ink to write and underline and pencil to draw diagrams.

Note : This question paper contains four parts.

PART - I

(Marks : 14)

(i) Answer all the **15** questions. **14 × 1 =14**

(ii) Choose the correct answer from the given four alternatives and write the option code and the corresponding answer.

1. If n (A×B) = 6 and A = {1, 3} then n (B) is
 (1) 1 (2) 2 (3) 3 (4) 6

2. Given $F_1 = 1$, $F_2 = 3$ and $F_n = F_{n-1} + F_{n-2}$ then F_5 is
 (1) 3 (2) 5 (3) 8 (4) 11

3. In an A.P, the first term is 1 and the common difference is 4. How many terms of the A.P must be taken for their sum to be equal to 120?
 (1) 6 (2) 7 (3) 8 (4) 9

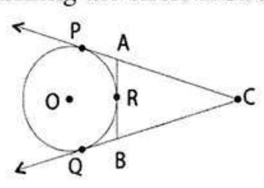
4. f= {(2,a), (3,b), (4,b), (5,c)} is a _____.
 (1) identity function (2) one-one function (3) many-one function (4) constant function

5. The number of points of intersection of quadratic polynomial $x^2 + 4x + 4$ with the x axis is
 (1) 0 (2) 1 (3) 0 (or) 1 (4) 2

6. The non- diagonal elements in any unit matrix are ______
 (1) 0 (2) 1 (3) m (4) n

7. If A is a 2×3 matrix and B is a 3×4 matrix, how many columns does AB have?
 (1) 3 (2) 4 (3) 2 (4) 5

8. In figure CP and CQ are tangents to a circle with centre at O. ARB is another tangent touching the circle at R. If CP= 11 cm and BC= 7 cm then the length of BR is
 (1) 6 cm (2) 5 cm
 (3) 8 cm (4) 4 cm



9. The slope of the line joining (12, 3), (4, a) is $\frac{1}{8}$. The value of 'a' is _____
 (1) 1 (2) 4 (3) –5 (4) 2

10. If $x = a\tan\theta$ and $y = b\sec\theta$ then
 (1) $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$ (2) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ (3) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (4) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$

11. A letter is chosen at random from the letter of the word "PROBABILITY". Find the probability that it is not a vowel.

(1) $\frac{1}{5}$ (2) $\frac{2}{3}$ (3) $\frac{1}{3}$ (4) $\frac{3}{5}$

12. The height of a right circular cone whose radius is 5 cm and slant height is 13 cm will be

(1) 12 cm (2) 10 cm (3) 13 cm (4) 5 cm

13. If the mean and co-efficient of variation of a data are 4 and 87.5% then the standard deviation is

(1) 3.5 (2) 3 (3) 4.5 (4) 2.5

14. Variance of first 20 natural numbers is

(1) 32.25 (2) 44.25 (3) 33.25 (4) 30

PART - II

(Marks :20)

Note : Answer 10 questions.

(Question No. 28 is compulsory) **10 × 2 = 20**

15. Define a function.

16. Compute x such that $10^4 \equiv x \pmod{19}$

17. Simplify $\frac{4x^2y}{2z^2} \times \frac{6xz^3}{20y^4}$

18. Pari needs 4 hours to complete the work. His friend Yuvan needs 6 hours to complete the work. How long will it take to complete if they work together?

19. Find the values of x, y and z from the following equation $\begin{pmatrix} 12 & 3 \\ x & \frac{3}{2} \end{pmatrix} = \begin{pmatrix} y & z \\ 3 & 5 \end{pmatrix}$

20. What length of ladder is needed to reach a height of 7 ft along the wall when the base of the ladder is 4 ft from the wall?

21. Prove that , $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \cos ec\theta + \cot\theta$

22. The radius of a sphere increases by 25%. Find the percentage increase in its surface area.

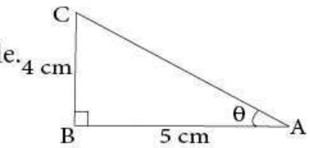
23. The Standard Deviation and Mean of a data are 6.5 and 12.5 respectively. Find the coefficient of variation.

24. If $f(x) = 3 + x$, $g(x) = x - 4$, then check whether $f \circ g = g \circ f$

25. An organization plans to plant saplings in 25 streets in a town in such a way that one sapling for the first street, three for the second, nine for the third and so on. How many saplings are needed to complete the work?

26. Find the 19^{th} term of an A.P $-11, -15, -19, \ldots$

27. Find the value of $\angle BAC$ in the given triangle.



28. The vertices of a triangle are A(–1,3), B(1,–1) and C(5,1). Find the length of the median through the vertex C.

PART - III

(Marks :50)

Note : Answer 10 questions
(Question No. 42 is compulsory) **10 × 5 = 50**

29. Let f be a function $f: N \to N$ be defined by $f(x) = 3x + 2, x \in N$

   i) Find the images of 1, 2, 3 ii) Find the pre-images of 29, 53 iii) Identify the type of function.

30. Let f: A→B be a function defined by $f(x) = \frac{x}{2} - 1$, where A= {2,4,6,10,12}, B = { 0,1,2,4,5,9}. Represent f by

   i) set of ordered pairs ii) a table iii) an arrow diagram iv) a graph

31. The ratio of 6^{th} and 8^{th} terms of an A.P is 7:9.Find the ratio of 9^{th} term to 13^{th} terms.

32. The sum of first n, 2n and 3n terms of an A.P are S_1, S_2 and S_3 respectively. Prove that S_3, = $3(S_2 - S_1)$

33. Find the values of m and n if the expression $\frac{1}{x^4} - \frac{6}{x^3} + \frac{13}{x^2} + \frac{m}{x} + n$ is a perfect square.

34. If α, β are the roots of the equation $2x^2 - x - 1 = 0$ then form the equation whose roots are $\alpha^2 \beta$, $\beta^2 \alpha$.

35. P and Q are the mid-points of the sides CA and CB respectively of a DABC , right angled at C. Prove that $4(AQ^2 + BP^2) = 5AB^2$.

36. Find the equation of a straight line passing through (1,–4) and has intercepts which are in the ratio 2:5.

37. From the top of the tower 60 m high the angles of depression of the top and bottom of a vertical lamp post are observed to be 38^0 and 60^0 respectively. Find the height of the lamp post ($\tan 38^0 = 0.7813$, $\sqrt{3} = 1.732$).

38. Calculate the weight of a hollow brass sphere if the inner diameter is 14 cm and thickness is 1mm, and whose density is 17.3 g/cm^3.

39. Find the Co-efficient of variation of 24, 26, 33, 37, 29, 31.

40. Two dice, one blue and one grey, are thrown at the same time. Write down all the possible outcomes. What is the probability that the sum of the two numbers appearing on the top of the dice is

   (i) 8 (ii) 13 (iii) less than or equal to 12

41. Find two consecutive positive integers, sum of whose squares is 365.

42. A cylindrical bucket of 32 cm high and with radius of base 18 cm, is filled with sand completely. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm, find the radius and slant height of the heap.

PART - IV

(Marks : 16)

Note : Answer both questions. **$2 \times 8 = 16$**

43. (a) PQ is a chord of length 8 cm to a circle of radius 5 cm. The tangents at P and Q intersect at a point T. Find the length of the tangent TP.

(OR)

(b) Draw a triangle ABC of base BC = 8 cm, $\angle A = 60°$ and the bisector of $\angle A$ meets BC at D such that BD = 6 cm.

44. (a) Draw the graph of $y = x^2 + 3x - 4$ and hence use it to solve $x^2 + 3x - 4 = 0$.

(OR)

(b) A motor boat whose speed is 18 km/hr in still water takes 1 hour more to go to 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

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