CHAPTER : 4 INVERSE TRIGONOMETRIC FUNCTIONS

Exercise 4.1

1. Find all the values of x such that

(i) $-10\pi \le x \le 10\pi$ and $\sin x = 0$

Soln:- $\sin x = 0$

$\sin x = \sin 0$

$x = n\pi + (-1)^n (0)$

$= n\pi$, $n = 0, \pm 1, \pm 2 \ldots \pm 10$.

$\sin\theta = \sin\alpha$

$\theta = n\pi + (-1)^n \alpha$

$n \in R$

(ii) $-8\pi \le x \le 8\pi$ and $\sin x = -1$

Soln $\sin x = -1$:

$\sin x = -\sin \pi/2 = \sin(-\pi/2)$

$x = (4n-1)\pi/2$, $n = 0, \pm 1, \pm 2, \pm 3, 4$

நா. காமாட்சி, M.Sc., B.Ed., M.Phil., Ph.D.
முதுகலை ஆசிரியர் (கணிதம்)
அரசு மேல்நிலைப்பள்ளி
கோவிந்தவாடி, காஞ்சிபுரம் (Dt)

2. Find the period and amplitude of -

(i) $y = \sin 7x$.

For amplitude use the form $y = a \sin(bx - c) + d$.

amplitude $= |a|$

$a = 1$ $\therefore |a| = 1$

Period using the formula $\frac{2\pi}{|b|} = \frac{2\pi}{|7|} = \frac{2\pi}{7}$

$\therefore$ amplitude $= 1$

period $= \frac{2\pi}{7}$

(ii) $y = -\sin(\frac{1}{3}x)$

$a = -1$ $b = 1/3$

$\therefore$ amplitude $|a| = |-1| = 1$

period $= \frac{2\pi}{|b|} = \frac{2\pi}{|1/3|} = 3(2\pi) = 6\pi$

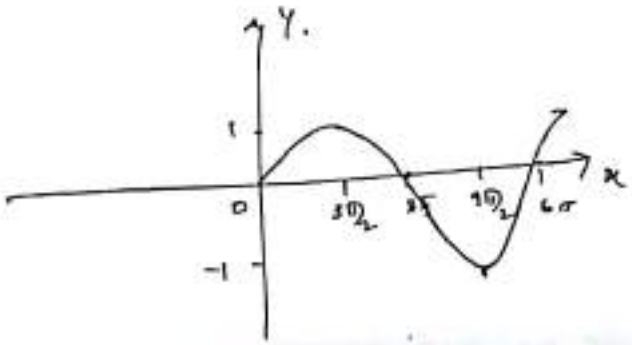
(iii) $y = 4\sin(-2x)$

Here $a = 4$ $b = -2$ $\therefore$ amplitude $= |4| = 4$

period $= \frac{2\pi}{|b|} = \frac{2\pi}{|-2|} = \frac{2\pi}{2} = \pi$

3) Sketch the graph of $y = \sin(\frac{1}{3}x)$ for $0 \le x \le 6\pi$.

x	$f(x)$
0	0
$3\pi/2$	1
3π	0
$9\pi/2$	-1
6π	0



(3)

4. Find the value of (i) $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$

Soln $y = \sin^{-1}(\sin 2\theta_3)$

$= \sin^{-1}[\sin(\pi - \theta_3)] = \sin^{-1}(\sin\theta_3) = \theta_3$

(ii) $\sin^{-1}[\sin(5\theta_4)]$

$y = \sin^{-1}[\sin(5\theta_4)] = \sin^{-1}[\sin(\pi + \theta_4)] = \sin^{-1}(-\sin\theta_4)$

$= \sin^{-1}[\sin(-\theta_4)] = -\theta_4.$

5. For what value of x does $\sin x = \sin^{-1} x$?

Soln $\sin\theta = 0$

$\sin^{-1}(0) = 0$ $\quad \therefore x = 0$

6) Find the domain of the following.

(i) $f(x) = \sin^{-1}\left(\frac{x^2+1}{2x}\right)$

Soln. Range of $\sin^{-1} x$ is $[-1, 1]$.

$-1 \le \frac{x^2+1}{2x} \le 1.$

Multiply by $2x^2 > 0$.

$-2x^2 \le x(x^2+1) \le 2x^2$

$-2x^2 \le x(x^2+1)$

$x(x^2+1) + 2x^2 \ge 0$

$x(x^2+1+2x) \ge 0$

$x(x+1)^2 \ge 0$

$x \ge 0 \quad (x+1)^2 \ge 0$

$x = -1.$

$x(x^2+1) - 2x^2 \le 0$

$x[x^2+1-2x] \le 0$

$x(x-1)^2 \le 0$

$x \le 0 \quad x = 1.$

$x = 0$ doesn't play with us this time.

$\therefore$ Solution is $\{-1, 1\}$

(iii) $g(x) = 2\sin^{-1}(2x-1) - \pi/4.$

Soln Range of $\sin^{-1} x$ is $[-1, 1]$.

$-1 \le 2x-1 \le 1$

$-1 \le 2x-1$

$2x - 1 \ge -1 \quad \Rightarrow \quad 2x \ge -1+1$

$2x \ge 0 \quad \Rightarrow \quad x \ge 0.$

$2x - 1 \leq 1$

$2x \leq 2$

$x \leq 1$

$x \geq 0$ & $x \leq 1$ ∴ solution is $[0, 1]$.

7) Find the value of $\sin^{-1}\left(\sin\frac{5\pi}{9}\cos\theta_9 + \cos\frac{5\pi}{9}\sin\frac{\pi}{9}\right)$

Soln.

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

$A = \frac{5\pi}{9}$ $B = \theta_9$.

$= \sin^{-1}\left(\sin\left[\frac{5\pi}{9} + \theta_9\right]\right)$

$= \sin^{-1}\left[\sin\left[\frac{6\pi}{9}\right]\right] = \sin^{-1}\left(\sin\left[\frac{2\pi}{3}\right]\right)$

$= \sin^{-1}\left(\sin\left(\pi - \theta_3\right)\right]$

$= \sin^{-1}\left(\sin\theta_3\right) = \theta_3$.

ந. ஸாலாட்சி, M.Sc., B.Ed., M.Phil., Ph.D.
முதுகலை ஆசிரியர் (கணிதம்)
அரசு மேல்நிலைப்பள்ளி
கோவிந்தவாடி, காஞ்சிபுரம் (Dt)

Exercise 4.2

1. Find all values of x such that (i) $-6\pi \le x \le 6\pi$ and $\cos x = 0$

Soln $\cos x = 0$

$\cos x = \cos \pi/2$

$x = (2n+1)\pi/2$, $n = 0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5$.

(ii) $-5\pi \le x \le 5\pi$, and $\cos x = 1$.

Soln $\cos x = 1$

$\cos x = \cos 0$

$x = (2n+1)\pi$, $n = 0, \pm 1, \pm 2, -3$.

2. State the reason for $\cos^{-1}[\cos(-\pi/6)] \neq -\pi/6$

Soln $\cos^{-1}[\cos(-\pi/6)] = \cos^{-1}[\cos \pi/6]$

$= \pi/6 \neq -\pi/6$. $[\because \cos(-\theta) = \cos\theta]$

3. Is $\cos^{-1}(-x) = \pi - \cos^{-1} x$ true? Justify your answer.

Soln. $\cos^{-1}(-x) = \pi - \cos^{-1} x$.

Take $x = \cos\theta \Rightarrow \theta = \cos^{-1} x$

$\cos^{-1}(-x) = \pi - \theta$

$\cos(\pi - \theta) = -\cos\theta = -x \in [-1, 1]$

$\pi - \cos^{-1} x = \pi - \cos^{-1} x = \pi - \theta$.

$\pi - \cos^{-1} x = \cos^{-1}(-x)$ is true.

4) Find the principal value of $\cos^{-1}(1/2)$

Soln $y = \cos^{-1}(\frac{1}{2})$ $\cos^{-1} x$ range is $[0, \pi]$.

$\cos y = 1/2 = \cos \pi/3$.

$y = \pi/3 \in [0, \pi]$

Principal value is $\pi/3$

5) Find the value of (i) $2\cos^{-1}(1/2) + \sin^{-1}(\frac{1}{2})$

Soln $y = \cos^{-1}(\frac{1}{2})$

$\cos y = \frac{1}{2} = \cos \pi/3$

$y = \pi/3 \in [0, \pi]$

$x = \sin^{-1}(\frac{1}{2})$

$\sin x = 1/2 = \sin \pi/6$

$x = \pi/6 \in [0, \pi]$

$2\cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}\left(\frac{1}{2}\right) = 2\left(\frac{\pi}{3}\right) + \frac{\pi}{6} = \frac{2\pi}{3} + \frac{\pi}{6} = \frac{4\pi+\pi}{6}$ (6)

$= 5\pi/6.$

(ii) $\cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}(-1)$

$x = \cos^{-1}\left(\frac{1}{2}\right)$

$\cos x = \frac{1}{2} = \cos\frac{\pi}{3}$

$x = \frac{\pi}{3}$

$y = \sin^{-1}(-1)$

$\sin y = -1 = \sin\left(-\frac{\pi}{2}\right)$

$y = -\frac{\pi}{2}.$

$\cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}(-1) = \frac{\pi}{3} - \frac{\pi}{2} = \frac{2\pi - 3\pi}{6} = -\frac{\pi}{6}.$

(iii) $\cos^{-1}\left[\cos\frac{\pi}{7}\cos\frac{\pi}{17} - \sin\frac{\pi}{7}\sin\frac{\pi}{17}\right]$ $\left[\because \cos(A+B) = \cos A\cos B - \sin A\sin B\right]$

$= \cos^{-1}\left[\cos\left(\frac{\pi}{7} + \frac{\pi}{17}\right)\right]$

$= \cos^{-1}\left[\cos\left(\frac{17\pi+7\pi}{119}\right)\right]$

$= \cos^{-1}\left[\cos\left(\frac{24\pi}{119}\right)\right] = \frac{24\pi}{119}.$

b) Find the domain of $f(x) = \sin^{-1}\left(\frac{|x|-2}{3}\right) + \cos^{-1}\left(\frac{1-|x|}{4}\right)$

Soln

$-1 \le \sin^{-1} \le 1.$

$-1 \le \frac{|x|-2}{3} \le 1$

$-3 \le |x| - 2 \le 3.$

$-3+2 \le |x| \le 3+2$

$-1 \le |x| \le 5$

Since $-1 \le |x|$ is not possible

$|x| \le 5$

$-5 \le x \le 5$

$\therefore x \in [-5, 5].$

நா. காமாட்சி, M.Sc., B.Ed., M.Phil., Ph.D.
முதுகலை ஆசிரியர் (கணிதம்)
அரசு மேல்நிலைப்பள்ளி
கோவிந்தவாடி, காஞ்சிபுரம் (Dt)

(b) $g(x) = \sin^{-1}x + \cos^{-1}x$.

Range of $\sin x$ and $\cos x$ is $[-1, 1]$

$-1 \le x \le 1$.

$\therefore x \in [-1, 1]$.

7) For what value of x the inequality $\pi/2 < \cos^{-1}(3x-1) < \pi$ holds

Soln

$$\pi/2 < \cos^{-1}(3x-1) < \pi$$

$$\cos\pi/2 < (3x-1) < \cos\pi$$

$$0 < 3x-1 < -1$$

$$0+1 < 3x < -1+1$$

$$1 < 3x < 0$$

$$0 < 3x < 1$$

$$0 < x < 1/3 \quad \therefore 0 < x < 1/3$$

திரு. காமராஜ், M.Sc., B.Ed., M.Phil.,
முதுகலை ஆசிரியர் (கணிதம்)
அரசு மேல்நிலைப்பள்ளி
கொடிங்கனாடு, காஞ்சிபுரம் (DT)

8) Find the value of (i) $\cos\left[\cos^{-1}\left(\frac{4}{5}\right) + \sin^{-1}\left(\frac{4}{5}\right)\right]$

We know that $\sin^{-1}x + \cos^{-1}x = \pi/2$.

$$\cos\left[\cos^{-1}\left(\frac{4}{5}\right) + \sin^{-1}\left(\frac{4}{5}\right)\right] = \cos\pi/2 = 0.$$

(ii) $\cos^{-1}\left[\cos\frac{4\pi}{3}\right] + \cos^{-1}\left[\cos\frac{5\pi}{4}\right]$

$$\cos\frac{4\pi}{3} = \cos\left[\pi + \pi/3\right] = -\cos(\pi/3) = \cos(-\pi/3) = \cos\pi/3$$

$$\cos\frac{5\pi}{4} = \cos\left[\pi + \pi/4\right] = -\cos\pi/4 = \cos(-\pi/4) = \cos\pi/4$$

LHS: $\cos^{-1}\left[\cos\frac{4\pi}{3}\right] + \cos^{-1}\left[\cos\frac{5\pi}{4}\right]$

$$= \cos^{-1}\left[\cos(+\pi/3)\right] + \cos^{-1}\left(\cos(+\pi/4)\right) = +\pi/3 + \pi/4$$

$$= +\frac{4\pi + 3\pi}{12} = \frac{7\pi}{12}.$$

Exercise 4.3

1. Find the domain of following functions

(i) $\tan^{-1}\sqrt{9-x^2}$ (ii) $\frac{1}{2}\tan^{-1}(1-x^2) - \pi/4$.

Soln

$9 - x^2 \geq 0$

$9 \geq x^2$.

$x^2 \leq 9$

$x \leq \pm 3$.

domain $[-3, 3]$

(ii) Range of $\tan^{-1}x$ is R

$-\infty < 1 - x^2 < \infty$

$-\infty < -x^2 < \infty$

$-\infty < x < \infty$

$x \in R$.

domain $= R$

நா. காமாட்சி, M Sc., B Ed., M Phil., Ph.D.
முதுகலை ஆசிரியர் (கணிதம்)
அரசு மேல்நிலைப்பள்ளி
கோவிந்தவாடி, காஞ்சிபுரம் (Dt)

2) Find the value of (i) $\tan^{-1}(\tan\frac{5\pi}{4})$.

Soln $\tan^{-1}[\tan(\frac{5\pi}{4})] = \tan^{-1}[\tan(\pi + \pi/4)]$

$= \tan^{-1}[\tan\pi/4] = \pi/4$.

(ii) $\tan^{-1}(\tan(-\pi/6)) = -\pi/6$.

3) Find the value of

(i) $\tan[\tan^{-1}(\frac{7\pi}{4})] = 7\pi/4$.

We know that $\tan[\tan^{-1}(x)] = x$.

(ii) $\tan[\tan^{-1}[1947]] = 1947$.

(iii) $\tan[\tan^{-1}(-0.2021)] = -0.2021$.

4) Find the value of

(i) $\tan[\cos^{-1}(\frac{1}{2}) - \sin^{-1}(\frac{-1}{2})]$

(9)

Soln $\tan\left[\cos^{-1}\left(\frac{1}{2}\right) - \sin^{-1}\left(\frac{-1}{2}\right)\right]$

$= \tan\left[\cos^{-1}\frac{1}{2} + \sin^{-1}\frac{1}{2}\right]$ $\left[\because \sin^{-1}(-x) = -\sin^{-1}x\right]$

$= \tan\left(\frac{\pi}{2}\right) = \infty.$ $\left[\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}\right]$

(ii) $\sin\left[\tan^{-1}\left(\frac{1}{2}\right) - \cos^{-1}\left(\frac{4}{5}\right)\right]$

Q: $\tan^{-1}\frac{1}{2}$.

$\tan a = \frac{1}{2} > 0$. (lies in I & III quadrant]

$b = \cos^{-1}\left(\frac{4}{5}\right)$

$\cos b = \frac{4}{5} > 0$. (lies in I & IV quadrant only].

$\sin\left(\tan^{-1}\left(\frac{1}{2}\right) - \cos^{-1}\left(\frac{4}{5}\right)\right] = \sin(a-b]$

$= \sin a\cos b - \cos a \sin b$ [$\because \sin(A-B) = \sin A\cos B - \cos A\sin B$]

$= \frac{1}{\sqrt{5}}\left(\frac{4}{5}\right) - \frac{2}{\sqrt{5}}\cdot\frac{3}{5}$

$= \frac{4}{5\sqrt{5}} - \frac{6}{5\sqrt{5}} = -\frac{2}{5\sqrt{5}}.$

நா. காமாட்சி, M.Sc., B.Ed., M.Phil., P
முதுகலை ஆசிரியர் (கணிதம்)
அரசு மேல்நிலைப்பள்ளி
கோவிந்தவாடி, காஞ்சிபுரம் (Dt)

(iii) $\cos\left[\sin^{-1}\left(\frac{4}{5}\right) - \tan^{-1}\left(\frac{3}{4}\right)\right]$

Soln $a = \sin^{-1}\left(\frac{4}{5}\right)$ [lies in I & II quadrant only]

$\sin a = \frac{4}{5} > 0$ (lies in I & II quadrant only]

$\cos a = \pm\frac{3}{5}$

opp = 4, Hyp = 5, Adj $= \sqrt{5^2-4^2} = \sqrt{25-16} = \sqrt{9} = 3.$

$b = \tan^{-1}\left(\frac{3}{4}\right) \Rightarrow \tan b = \frac{3}{4}.$

$\sin b = \frac{3}{5}$

opp = 3, adj = 4, Hyp $= \sqrt{4^2+3^2} = \sqrt{16+9} = \sqrt{25} = 5$

$\cos\left[\sin^{-1}\left(\frac{4}{5}\right) - \tan^{-1}\left(\frac{3}{4}\right)\right]$

$= \cos(a-b) = \cos a\cos b + \sin a\sin b$

$= \left(\frac{3}{5}\right)\left(\frac{4}{5}\right) + \left(\frac{4}{5}\right)\left(\frac{3}{5}\right) = \frac{12}{25} + \frac{12}{25} = \frac{24}{25}.$

Exercise 4.4

1. Find the principal value of

(i) $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$

Soln $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = \alpha$

$\sec\alpha = \frac{2}{\sqrt{3}} = \sec \pi/6$

$\alpha = \pi/6$.

(ii) $\cot^{-1}(\sqrt{3})$

Soln $\cot^{-1}(\sqrt{3}) = \alpha$.

$\sqrt{3} = \cot\alpha$

$\cot\pi/6 = \cot\alpha$

$\alpha = \pi/6$

நா. காமாட்சி, M.Sc., B.Ed., M.Phil., Ph.D.
முதுகலை ஆசிரியர் (கணிதம்)
அரசு மேல்நிலைப்பள்ளி
கோவிந்தவாடி, காஞ்சிபுரம் (Dt)

(iii) $\operatorname{cosec}^{-1}(-\sqrt{2})$

Soln $\operatorname{cosec}^{-1}(-\sqrt{2}) = \alpha$

$\operatorname{cosec}\alpha = -\sqrt{2} = -\operatorname{cosec}(\pi/4) = \operatorname{cosec}(-\pi/4)$

$\alpha = -\pi/4$.

2) Find the value of

(i) $\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$

Soln $x = \tan^{-1}(\sqrt{3})$

$\tan x = \sqrt{3} = \tan\pi/6$

$x = \pi/6$

$y = \sec^{-1}(-2)$

$\sec y = -2 = -\sec\pi/3$

$\sec y = \sec(-\pi/3)$

$\sec y = \sec(\pi + \pi/3)$

$y = \pi + \pi/3$

[$\because \sec(-\theta) = \sec\theta$]

$\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2) = \pi/6 - \pi/3$

$= \frac{\pi - 2\pi}{6} = -\pi/6$.

(ii) $\sin^{-1}(-1) + \cos^{-1}\left(\frac{1}{2}\right) + \cot^{-1}(2)$.

$x = \sin^{-1}(-1)$

$\sin x = -1 = \sin(-\pi/2)$

$x = -\pi/2$.

(11)

$y = \cos^{-1}\left(\frac{1}{2}\right)$

$\cos y = \frac{1}{2} = \cos \frac{\pi}{3}$

$y = \frac{\pi}{3}$.

$z = \cot^{-1}(2)$

$\cot z = 2$

$z = \cot^{-1}(2)$. is constant.

$\sin^{-1}(-1) + \cos^{-1}\left(\frac{1}{2}\right) + \cot^{-1}(2)$

$= -\frac{\pi}{2} + \frac{\pi}{3} + \cot^{-1}(2)$

$= \frac{-3\pi + 2\pi}{6} + \cot^{-1}(2) = -\frac{\pi}{6} + \cot^{-1}(2)$

3) $\cot^{-1}(1) + \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) - \sec^{-1}(-\sqrt{2})$

Soln

$x = \cot^{-1}(1)$

$\cot x = 1 = \cot \frac{\pi}{4}$

$x = \frac{\pi}{4}$

$y = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

$\sin y = -\frac{\sqrt{3}}{2} = \sin\left(-\frac{\pi}{3}\right)$

$y = -\frac{\pi}{3}$

$z = \sec^{-1}(-\sqrt{2})$

$\sec z = -\sqrt{2}$

$\sec z = -\sec\frac{\pi}{4} = \sec\left(-\frac{\pi}{4}\right) = \sec\frac{\pi}{4}$

$(\sec(-\theta) = \sec\theta)$

$z = \pm\frac{\pi}{4}$

$\cot^{-1}(1) + \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) - \sec^{-1}(-\sqrt{2})$

$= \frac{\pi}{4} - \frac{\pi}{3} - \frac{\pi}{4} = -\frac{\pi}{3}$.

நா. காமாட்சி, M.Sc., B.Ed., M.Phil., Ph.D.
முதுகலை ஆசிரியர் (கணிதம்)
அரசு மேல்நிலைப்பள்ளி
கோவிந்தவாடி, காஞ்சிபுரம் (Dt)

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Exercise 4.5

1. Find the value if it exists if not, give the reason for non existence

(i) $\sin^{-1}(\cos\pi)$ (ii) $\tan^{-1}(\sin(-5\pi/2))$ (iii) $\sin^{-1}(\sin 5)$

Soln

(i) $\sin^{-1}(\cos\pi) = \sin^{-1}(-1) = -\sin^{-1}(1) = -\pi/2$

(ii) $\tan^{-1}(\sin(-5\pi/2)) = \tan^{-1}[-\sin 5\pi/2]$

$= \tan^{-1}[-\sin(2\pi + \pi/2)] = \tan^{-1}(-\sin\pi/2)$

$= \tan^{-1}(-1) = -\tan^{-1}(1) = -\pi/4.$

(iii) $\sin^{-1}(\sin 5)$

$-\pi/2 \le \sin^{-1}x \le \pi/2$ ~~ex $\sin(\pi/2) \le$~~

$-3\pi/2 \le 5 \le 2\pi$

$-\pi/2 \le 5 - 2\pi \le 0 \le \pi/2$

$\sin(5-2\pi) = \sin 5$

$\sin^{-1}(\sin 5) = 5 - 2\pi$

நா. காமாட்சி, M.Sc., B.Ed., M.Phil., Ph.D.
முதுகலை ஆசிரியர் (கணிதம்)
அரசு மேல்நிலைப்பள்ளி
கோவிந்தவாடி, காஞ்சிபுரம் (Dt)

2) Find the value of the expression in terms of x with the help of a reference triangle

(i) $\sin(\cos^{-1}(1-x))$

$= \sin[\cos^{-1}(\text{side of adj} / \text{side of Hyp})]$ [$\because \cos(\text{adj}/\text{Hyp}) = \frac{1-x}{1}$, adj $= 1-x$, Hyp $= 1$, opp $= \sqrt{1^2-(1-x)^2}$ $= \sqrt{1-(1+x^2-2x)}$ $= \sqrt{1-1-x^2+2x}$ $= \sqrt{2x-x^2}$]

$= \text{opp}/\text{Hyp} = \sqrt{2x-x^2}/1 = \sqrt{2x-x^2}.$

(ii) $\cos(\tan^{-1}(3x-1))$

$= \cos[\text{side of opp} / \text{side of adj}]$

$= \text{Adj}/\text{Hyp} = 1/\sqrt{9x^2-6x+2}$

[$\because$ opp $= 3x-1$, adj $= 1$, Hyp $= \sqrt{(3x-1)^2+1^2} = \sqrt{9x^2+1-6x+1} = \sqrt{9x^2-6x+2}$]

(iii) $\tan(\sin^{-1}[x+1/2)] = \tan(\sin^{-1}(\frac{2x+1}{2}))$

$\tan[\text{side of opp} / \text{side of Hyp}]$

$= \text{opp}/\text{adj} = \frac{2x+1}{\sqrt{3-4x-4x^2}}$

[$\because$ opp $= 2x+1$, Hyp $= 2$, adj $= \sqrt{2^2-(2x+1)^2} = \sqrt{4-(4x^2+1+4x)} = \sqrt{4-4x^2-1-4x} = \sqrt{3-4x-4x^2}$]

(iii) $2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$.

Soln W.K.T $2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2}$

$$2\tan^{-1}(\cos x) = \tan^{-1}\left(\frac{2\cos x}{1-\cos^2 x}\right) = \tan^{-1}\left(\frac{2\cos x}{\sin^2 x}\right)$$

$$\Rightarrow \tan^{-1}\left(2\frac{\cos x}{\sin^2 x}\right) = \tan^{-1}(2\operatorname{cosec} x)$$

$$2\frac{\cos x}{\sin^2 x} = 2\operatorname{cosec} x$$

$$\frac{\cos x}{\sin^2 x} = \frac{1}{\sin x}$$

$$\cot x = 1 = \cot\frac{\pi}{4},$$

$$x = \frac{\pi}{4}$$

(iv) $\cot^{-1}x - \cot^{-1}(x+2) = \frac{\pi}{12}$, $x > 0$

Soln $\tan^{-1}\frac{1}{x} - \tan^{-1}\frac{1}{x+2} = \frac{\pi}{12} = \frac{180}{12} = 15^\circ$

$$\tan^{-1}\left(\frac{\frac{1}{x}-\frac{1}{x+2}}{1+\frac{1}{x}\cdot\frac{1}{x+2}}\right) = 15^\circ$$

$$\tan^{-1}\frac{\frac{(x+2)-(x)}{x(x+2)}}{\frac{x(x+2)+1}{x(x+2)}} = 15^\circ$$

$$\frac{2}{x^2+2x+1} = \tan 15^\circ = \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$$\frac{2}{(x+1)^2} = \frac{\sqrt{3}-1}{\sqrt{3}+1}\left[\frac{\sqrt{3}+1}{\sqrt{3}+1}\right] = \frac{(\sqrt{3}-1)^2}{(\sqrt{3}^2-1^2)^2}$$

$$\frac{2}{(x+1)^2} = \frac{2}{(\sqrt{3}+1)^2}.$$

Comparing both sides

$$x = \sqrt{3}$$

தா. காமாட்சி, M.Sc., B.Ed., M.Phil., Ph.D.
முதுகலை ஆசிரியர் (கணிதம்)
அரசு மேல்நிலைப்பள்ளி
கேவிந்தவாடி, கள்ளக்குறிச்சி (Dt)

3) Find the value of (i) $\sin^{-1}\left[\cos\left(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)\right]$ (4)

Soln $\sin^{-1}\left[\cos\left(\sin^{-1}\frac{\sqrt{3}}{2}\right)\right] = \sin^{-1}\left[\cos\frac{\pi}{3}\right] = \sin^{-1}\left[\frac{1}{2}\right]$

$= \frac{\pi}{6}$

(ii) $\cot\left[\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{4}{5}\right]$

$\cot\left[\sin^{-1}\frac{3}{5} + \cos^{-1}\frac{3}{5}\right]$

$\cot\left(\frac{\pi}{2}\right) = 0 \quad \left[\because \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}\right]$

$\sin^{-1}\frac{3}{5} = \frac{\text{opp}}{\text{hyp}}$

$\text{adj} = \sqrt{5^2-3^2} = \sqrt{25-9} = \sqrt{16} = 4$

$\cos^{-1}\frac{\text{adj}}{\text{hyp}} = \cos^{-1}\frac{3}{5}$

(iii) $\sin^{-1}\frac{3}{5} = a$ & $\cot^{-1}\frac{3}{2} = b$

$\sin a = 3/5$ $\quad \cot b = 3/2 \Rightarrow \tan b = \frac{2}{3}$

$\cos a = \sqrt{1-\sin^2 a} = \sqrt{1-9/25} = \sqrt{\frac{16}{25}} = \frac{4}{5}$.

(iv) $\tan\left(\sin^{-1}\frac{3}{5} + \cot^{-1}\frac{3}{2}\right) = \tan(a+b)$

$= \frac{\tan a + \tan b}{1 - \tan a \tan b} = \frac{3/4 + 2/3}{1 - \frac{3}{4}\times\frac{2}{3}}$

$= \frac{\frac{9+8}{12}}{\frac{12-6}{12}} = \frac{17}{6}$.

$\left[\because \tan a = \frac{\sin a}{\cos a} = \frac{3/5}{4/5} = \frac{3}{4}\right]$

ந. சாமாட்சி, M.Sc., B.Ed., M.Phil., Ph.D.
முதுகலை ஆசிரியர் (கணிதம்)
அரசு மேல்நிலைப்பள்ளி
கோவிந்தவாடி, காஞ்சிபுரம் (Dt)

4) Prove that (i) $\tan^{-1}\frac{2}{11} + \tan^{-1}\frac{7}{24} = \tan^{-1}\frac{1}{2}$

Soln $\tan^{-1}A + \tan^{-1}B = \tan^{-1}\left(\frac{A+B}{1-AB}\right)$

$\tan^{-1}\left(\frac{2}{11}\right) + \tan^{-1}\left[\frac{7}{24}\right) = \tan^{-1}\left(\frac{\frac{2}{11}+\frac{7}{24}}{1-\frac{2}{11}\cdot\frac{7}{24}}\right)$

$= \tan^{-1}\left(\frac{\frac{48+77}{11\times 24}}{\frac{264-14}{11\times 24}}\right) = \tan^{-1}\left(\frac{125}{250}\right)$

$= \tan^{-1}\left(\frac{1}{2}\right)$.

(ii) $\sin^{-1}\frac{3}{5} - \cos^{-1}\frac{12}{13} = \sin^{-1}\frac{16}{65}$

Soln

$x = \sin^{-1}\frac{3}{5}$ | $y = \cos^{-1}\frac{12}{13}$

$\sin x = \frac{3}{5}$ | $\cos y = \frac{12}{13}$

$\cos x = \frac{4}{5}$ | $\sin y = \frac{5}{13}$

$\sin x = \frac{3}{5} = \frac{opp}{hyp}$

$adj = \sqrt{5^2-3^2} = \sqrt{5-3} = \sqrt{16} = 4$

$\cos y = \frac{12}{13} = \frac{adj}{hyp}$

$opp = \sqrt{13^2-12^2} = \sqrt{169-144} = \sqrt{25} = 5$

$\sin^{-1}\left(\frac{3}{5}\right) - \cos^{-1}\left(\frac{12}{13}\right) =$

Let $\sin(x-y) = \sin x \cos y - \cos x \sin y$

$= \frac{3}{5}\left(\frac{12}{13}\right) - \frac{4}{5}\left(\frac{5}{13}\right) = \frac{36}{65} - \frac{20}{65} = \frac{16}{65}$

$\sin(x-y) = \frac{16}{65}$

$(x-y) = \sin^{-1}\left(\frac{16}{65}\right)$

5) Prove that $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \tan^{-1}\left(\frac{x+y+z-xyz}{1-xy-yz-zx}\right)$

Soln

$\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \tan^{-1}\left(\frac{x+y}{1-xy}\right) + \tan^{-1}z$

$= \tan^{-1}\left(\frac{\frac{x+y}{1-xy} + z}{1 - \left(\frac{x+y}{1-xy}\right)z}\right)$

$= \tan^{-1}\left(\frac{\frac{x+y+z(1-xy)}{1-xy}}{\frac{(1-xy)-(xz+yz)}{1-xy}}\right) = \tan^{-1}\left(\frac{x+y+z-xyz}{1-xy-yz-xz}\right)$

நா. காமாட்சி, M.Sc., B.Ed., M.Phil., Ph.D.
முதுகலை ஆசிரியர் (கணிதம்)
அரசு மேல்நிலைப்பள்ளி
கோவிந்தவாடி, காஞ்சிபுரம் (Dt)

6) If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$ show that $x+y+z = xyz$.

Soln

$\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$

$\tan^{-1}\left(\frac{x+y}{1-xy}\right) + \tan^{-1}z$

$\tan^{-1}\left(\frac{\frac{x+y}{1-xy} + z}{1 - \left(\frac{x+y}{1-xy}\right)\cdot z}\right) = \pi$

(14)

$$\tan^{-1}\left(\frac{\frac{x+y+z(1-xy)}{1-xy}}{\frac{1-xy-(xz+yz)}{1-xy}}\right)=\pi$$

$$\frac{x+y+z-xyz}{1-xy-xz-yz}=\tan\pi=0.$$

$$x+y+z-xyz=0$$

$$\boxed{x+y+z=xyz}$$

7) Prove that $\tan^{-1}x+\tan^{-1}\frac{2x}{1-x^2}=\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)\ |x|<\frac{1}{\sqrt{3}}$.

Soln

$$\tan^{-1}x+\tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

$$=\tan^{-1}\left[\frac{x+\frac{2x}{1-x^2}}{1-x\left(\frac{2x}{1-x^2}\right)}\right]=\tan^{-1}\left(\frac{\frac{x(1-x^2)+2x}{1-x^2}}{\frac{1-x^2-2x^2}{1-x^2}}\right)$$

$$=\tan^{-1}\left(\frac{x-x^3+2x}{1-3x^2}\right)$$

$$=\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$$

if $3x^2<1$

$x^2<\frac{1}{3}$

$|x|<\frac{1}{\sqrt{3}}$

8) Simplify $\tan^{-1}\frac{x}{y}-\tan^{-1}\left(\frac{x-y}{x+y}\right)$

Soln

$$\tan^{-1}\left(\frac{x}{y}\right)-\tan^{-1}\left(\frac{x-y}{x+y}\right)$$

$$=\tan^{-1}\left(\frac{\frac{x}{y}-\frac{x-y}{x+y}}{1+\frac{x}{y}\cdot\left(\frac{x-y}{x+y}\right)}\right)=\tan^{-1}\left(\frac{\frac{x(x+y)-y(x-y)}{y(x+y)}}{\frac{y(x+y)+x(x-y)}{y(x+y)}}\right)$$

$$=\tan^{-1}\left(\frac{x^2+xy-xy+y^2}{xy+y^2+x^2-xy}\right)=\tan^{-1}\left(\frac{x^2+y^2}{x^2+y^2}\right)=\tan^{-1}(1)$$

$$=\frac{\pi}{4}.$$

நா. காமாட்சி, M.Sc., B.Ed., M.Phil., Ph.D.
முதுகலை ஆசிரியர் (கணிதம்)
அரசு மேல்நிலைப்பள்ளி
செய்யூர், காஞ்சிபுரம் (Dt)

9) Solve: (i) $\sin^{-1}\frac{5}{x} + \sin^{-1}\frac{12}{x} = \frac{\pi}{2}$.

Soln $\sin^{-1}\frac{5}{x} = \frac{\pi}{2} - \sin^{-1}\frac{12}{x}$

$\frac{\pi}{2} - (\cos^{-1}\frac{5}{x}) = \frac{\pi}{2} - \sin^{-1}\frac{12}{x}$

$\cos^{-1}\frac{5}{x} = \sin^{-1}\frac{12}{x}$

$\cos\theta = \frac{5}{x}$ $\quad \sin\theta = \frac{12}{x}$

$\sin^2\theta + \cos^2\theta = 1$

$\left(\frac{5}{x}\right)^2 + \left(\frac{12}{x}\right)^2 = 1$

$\frac{25}{x^2} + \frac{144}{x^2} = 1$

$\frac{169}{x^2} = 1$

$x^2 = 169$

$x = \pm 13$.

(ii) $2\tan^{-1}x = \cos^{-1}\left(\frac{1-a^2}{1+a^2}\right) - \cos^{-1}\left(\frac{1-b^2}{1+b^2}\right)$ $a > 0$.

Soln

W.K.T $2\tan^{-1}x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

$\cos^{-1}\left(\frac{1-a^2}{1+a^2}\right) = 2\tan^{-1}a$

$\cos^{-1}\left(\frac{1-b^2}{1+b^2}\right) = 2\tan^{-1}b$

RHS $\cos^{-1}\left(\frac{1-a^2}{1+a^2}\right) - \cos^{-1}\left(\frac{1-b^2}{1+b^2}\right) = 2\tan^{-1}a - 2\tan^{-1}b$

$= 2[\tan^{-1}a - \tan^{-1}b]$

$= 2\left(\tan^{-1}\frac{a-b}{1+ab}\right)$

LHS = RHS

$2\tan^{-1}\left(\frac{a-b}{1+ab}\right) = 2\tan^{-1}x$

$x = \frac{a-b}{1+ab}$. $\quad a > 0$.

நா. காமாட்சி, M.Sc., B.Ed., M.Phil., Ph.D.
முதுகலை ஆசிரியர் (கணிதம்)
அரசு மேல்நிலைப்பள்ளி
கோவிந்தவாடி, காஞ்சிபுரம் (Dt)

16) Find the number of solution of the equation

$\tan^{-1}(x-1) + \tan^{-1}x + \tan^{-1}(x+1) = \tan^{-1}3x$.

Soln $\tan^{-1}(x-1) + \tan^{-1}x + \tan^{-1}(x+1)$

$= \tan^{-1}(x-1) + \tan^{-1}(x+1) + \tan^{-1}x$

$= \tan^{-1}\left(\frac{x-1+x+1}{1-(x-1)(x+1)}\right) + \tan^{-1}x$

$= \tan^{-1}\left(\frac{2x}{1-(x^2-1)}\right) + \tan^{-1}x$

$= \tan^{-1}\left(\frac{2x}{1-x^2+1}\right) + \tan^{-1}x = \tan^{-1}\left(\frac{2x}{2-x^2}\right) + \tan^{-1}x$

$= \tan^{-1}\left(\frac{\frac{2x}{2-x^2} + x}{1 - \frac{2x}{2-x^2}\cdot x}\right) = \tan^{-1}\left(\frac{\frac{2x + 2x - x^3}{2-x^2}}{\frac{2-x^2-2x^2}{2-x^2}}\right)$

$= \tan^{-1}\left(\frac{4x-x^3}{2-3x^2}\right)$

Given LHS = RHS

$\tan^{-1}\left(\frac{4x-x^3}{2-3x^2}\right) = \tan^{-1}3x$

$\frac{4x-x^3}{2-3x^2} = 3x$

$4x - x^3 = 6x - 9x^3$

$8x^3 = 2x$

$8x^3 - 2x = 0$

$2x(x^2-1) = 0$

$x = 0 \qquad x^2 = 1$

$x = \pm 1$.

number of solutions are $0, 1, -1$.

நா. காமாட்சி, M.Sc., B.Ed., M.Phil., Ph.D.
முதுகலை ஆசிரியர் (கணிதம்)
அரசு மேல்நிலைப்பள்ளி
கோவிந்தவாடி, காஞ்சிபுரம் (Dt)

Exercise 4.6

Choose the correct or the most suitable answer from the given four alternatives.

1. The value of $\sin^{-1}(\cos x)$ if $0 \le x \le \pi$ is

Soln $\sin^{-1}(\cos x) = \sin^{-1}\left(\sin\left(\frac{\pi}{2} - x\right)\right) = \frac{\pi}{2} - x$

Ans (3) $\frac{\pi}{2} - x$.

2. If $\sin^{-1}x + \sin^{-1}y = \frac{2\pi}{3}$ Then $\cos^{-1}x + \cos^{-1}y$ is equal to.

Soln $\sin^{-1}x + \cos^{-1}x + \cos^{-1}y + \sin^{-1}y = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

$= \frac{2\pi}{3} + \cos^{-1}x + \cos^{-1}y = \pi$

$\cos^{-1}x + \cos^{-1}y = \pi - \frac{2\pi}{3} = \frac{3\pi - 2\pi}{3} = \frac{\pi}{3}$

Ans (2) $\frac{\pi}{3}$

3. $\sin^{-1}\frac{3}{5} - \cos^{-1}\frac{12}{13} + \sec^{-1}\frac{5}{3} - \operatorname{cosec}^{-1}\frac{13}{12}$ is equal to.

$= \sin^{-1}\frac{3}{5} - \sec^{-1}\frac{13}{12} + \cos^{-1}\frac{3}{5} - \operatorname{cosec}^{-1}\frac{13}{12}$

$= \left(\sin^{-1}\frac{3}{5} + \cos^{-1}\frac{3}{5}\right) - \left(\sec^{-1}\frac{13}{12} + \operatorname{cosec}^{-1}\frac{13}{12}\right)$

$= \frac{\pi}{2} - \frac{\pi}{2} = 0$

Ans (3) 0.

4) If $\sin^{-1}x = 2\sin^{-1}\alpha$ has a solution then.

$-\frac{\pi}{2} \le 2\sin^{-1}\alpha \le \frac{\pi}{2}$

$-\frac{\pi}{4} \le \sin^{-1}\alpha \le \frac{\pi}{4}$

$\sin\left(-\frac{\pi}{4}\right) \le \alpha \le \sin\frac{\pi}{4}$

$-\frac{1}{\sqrt{2}} \le \alpha \le \frac{1}{\sqrt{2}}$

$|\alpha| \le \frac{1}{\sqrt{2}}$.

Ans (1) $|\alpha| \le \frac{1}{\sqrt{2}}$

5) $\sin^{-1}(\cos x) = \frac{\pi}{2} - x$ is valid for

$\cos x = \sin\frac{\pi}{2} - x$.

$\cos x \in [0, \pi]$ $\therefore$ $0 \le x \le \pi$

Ans (2) $0 \le x \le \pi$

நா. காமாட்சி, M.Sc., B.Ed., M.Phil., Ph.D.
முதுகலை ஆசிரியர் (கணிதம்)
அரசு மேல்நிலைப்பள்ளி
கோவிந்தவாடி, காஞ்சிபுரம் (Dt)

(2)

6) If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{3\pi}{2}$. The value of

$x^{2017} + y^{2018} + z^{2019} - \frac{9}{x^{101}+y^{101}+z^{101}}$ is

Given $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = 3\frac{\pi}{2}$ $\left(\because \sin^{-1}(1) = \frac{\pi}{2}\right.$

$\left.\frac{\pi}{2}+\frac{\pi}{2}+\frac{\pi}{2} = 3\frac{\pi}{2}\right)$

$\therefore x = y = z = 1$

$x^{2017} + y^{2018} + z^{2019} - \frac{9}{x^{101}+y^{101}+z^{101}}$

$= 1+1+1 - \frac{9}{1+1+1} = 3 - \frac{9}{3} = 3-3 = 0$

Ans (1) 0

7) If $\cot^{-1}x = \frac{2\pi}{5}$ for some $x \in R$ The value of $\tan^{-1}x$ is

Soln $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$.

$\tan^{-1}x = \frac{\pi}{2} - \cot^{-1}x = \frac{\pi}{2} - \frac{2\pi}{5} = \frac{5\pi-4\pi}{10} = \frac{\pi}{10}$

Ans (3) $\frac{\pi}{10}$

8) The domain of the function defined by $f(x) = \sin^{-1}\sqrt{x-1}$ is

Soln $f(x) = \sin^{-1}\sqrt{x-1}$

Soln $\sqrt{x-1} \geq 0$ $-1 \leq \sqrt{x-1} \leq 1$

$\therefore 0 \leq \sqrt{x-1} \leq 1$

$0 \leq x-1 \leq 1$

$1 \leq x \leq 2$

$x \in [1,2]$.

Ans (1) $[1,2]$

நா. காமாட்சி, M.Sc., B.Ed., M.Phil., Ph.D.
முதுகலை ஆசிரியர் (கணிதம்)
அரசு மேல்நிலைப்பள்ளி
கோவிந்தவாடி, காஞ்சிபுரம் (Dt)

9) If $x = 1/5$ The value of $\cos(\cos^{-1}x + 2\sin^{-1}x)$ is

Soln $\cos[\cos^{-1}x + \sin^{-1}x + \sin^{-1}x] = \cos\left(\frac{\pi}{2} + \sin^{-1}x\right)$

$\left[\because \cos(90+\theta) = -\sin\theta\right)$

$= -\sin(\sin^{-1}x)$

$= -x = -\frac{1}{5}$

Ans (4) $-\frac{1}{5} = x$.

10) $\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{2}{9}$ is equal to.

Soln $\tan^{-1}\left(\frac{\frac{1}{4}+\frac{2}{9}}{1-\frac{1}{4}\cdot\frac{2}{9}}\right) = \tan^{-1}\left(\frac{\frac{9+8}{36}}{\frac{36-2}{36}}\right) = \tan^{-1}\left(\frac{17}{34}\right)$

$= \tan^{-1}\left(\frac{1}{2}\right)$ Ans (4) $\tan^{-1}\frac{1}{2}$. (21)

11) If the function $y(x) = \sin^{-1}(x^2-3)$ then x belongs to

Soln $-1 \le x^2-3 \le 1$

$-1 \le x^2-3 \le 1$

$-1+3 \le x^2 \le 1+3 \Rightarrow 2 \le x^2 \le 4$

$\pm\sqrt{2} \le x \le \pm 2$.

$\therefore [-2,-\sqrt{2}] \cup [\sqrt{2}, 2]$ Ans (3)

12) If $\cot^{-1}2$ and $\cot^{-1}3$ are two angles of a triangle Then the third angle is

Soln $A+B+C = \pi$ (triangle.

$\cot^{-1}2 + \cot^{-1}3 + C = \pi$

$\cot^{-1}\left(\frac{1-2(3)}{2+3}\right) + C = \pi$

$\cot^{-1}\left(\frac{1-6}{5}\right) + C = \pi$

$\cot^{-1}\left(\frac{-5}{5}\right) + C = \pi \Rightarrow \cot^{-1}(-1) + C = \pi$

$\frac{\pi}{4} + C = \pi$

$C = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$ Ans (2)

13) $\sin^{-1}(\tan\frac{\pi}{4}) - \sin^{-1}\left(\sqrt{\frac{3}{x}}\right) = \frac{\pi}{6}$ Then x is a root of the equation.

Soln $\sin^{-1}(1) - \sin^{-1}\left(\sqrt{\frac{3}{x}}\right) = \frac{\pi}{6}$

$\frac{\pi}{2} - \sin^{-1}\sqrt{\frac{3}{x}} = \frac{\pi}{6}$

$\cos^{-1}\sqrt{\frac{3}{x}} = \frac{\pi}{6}$.

$\sqrt{\frac{3}{x}} = \cos\frac{\pi}{6} = \frac{1}{2}$

$\frac{\sqrt{3}}{\sqrt{x}} = \frac{1}{2}$

$\sqrt{x} = 2\sqrt{3}$

$x = 4(3) = 12$

$x = 12$.

ந. காமாட்சி, M.Sc., B.Ed., M.Phil., PhD
முதுகலை ஆசிரியர் (கணிதம்)
அரசு மேல்நிலைப்பள்ளி
கொளத்தூர், காஞ்சிபுரம் (Dt)

14) $\sin^{-1}(2\cos^2 x - 1) + \cos^{-1}(1 - 2\sin^2 x) = \sin^{-1}(2\cos^2 x - 1) + \cos^{-1}(1 - \sin^2 x - \sin^2 x)$

$= \sin^{-1}(2\cos^2 x - 1) + \cos^{-1}[\cos^2 x - (1 - \cos^2 x)]$ (22)

$= \sin^{-1}(2\cos^2 x - 1) + \cos^{-1}(\cos^2 x - 1 + \cos^2 x)$

$= \sin^{-1}(2\cos^2 x - 1) + \cos^{-1}(2\cos^2 x - 1)$

$= \pi/2$ $\quad \because \sin^{-1}x + \cos^{-1}x = \pi/2$

Ans (3)

15) If $\cot^{-1}\sqrt{\sin\alpha} + \tan^{-1}\sqrt{\sin\alpha} = 4$ then $\cos 2\alpha$ is equal to

Soln $\cot^{-1}x + \tan^{-1}x = \pi/2$

$\therefore 4 = \pi/2$

$\cos 2\alpha = \cos 2(\pi/2) = \cos\pi = -1$ Ans (1)

16) If $|x| \le 1$ Then $2\tan^{-1}x - \sin^{-1}\frac{2x}{1+x^2}$ is equal to

Soln $\sin^{-1}\frac{2x}{1+x^2} = 2\tan^{-1}x$

Ans (3)

$\therefore 2\tan^{-1}x - 2\tan^{-1}x = 0$

17) The eqn $\tan^{-1}x - \cot^{-1}x = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \pi/6$

Soln $\tan^{-1}x + \cot^{-1}x = \pi/2$.

Add (2) & (1) $\quad 2\tan^{-1}x = \pi/6 + \pi/2 = 2\pi/3$.

$\tan^{-1}x = \pi/3$

$x = \sqrt{3}$ Ans (2)

which is unique solution.

18) If $\sin^{-1}x + \cot^{-1}\left(\frac{1}{2}\right) = \pi/2$ Then x is equal to.

$\sin^{-1}x + \cos^{-1}\left(\frac{1}{\sqrt{5}}\right) = \pi/2$

w.k.t $\sin^{-1}x + \cos^{-1}x = \pi/2$

$\therefore x = \frac{1}{\sqrt{5}}$

$\cot^{-1}\frac{1}{2} = \frac{\text{adj}}{\text{opp}}$

$\text{Hyp} = \sqrt{2^2+1^2} = \sqrt{5}$

$\cos^{-1}\frac{\text{adj}}{\text{Hyp}} = \frac{1}{\sqrt{5}}$

Ans. $\frac{1}{\sqrt{5}}$.

நா. காமாட்சி, M.Sc., B.Ed., M.Phil., Ph.D.
முதுகலை ஆசிரியர் (கணிதம்)
அரசு மேல்நிலைப்பள்ளி
..., காஞ்சிபுரம் (Dt)

19) If $\sin^{-1}\frac{x}{5} + \operatorname{cosec}^{-1}\frac{5}{4} = \frac{\pi}{2}$ then the values of x.

Soln $\sin^{-1}\frac{x}{5} + \cos^{-1}\frac{3}{5} = \frac{\pi}{2}$

$= \sin^{-1}\frac{x}{5} + \frac{\pi}{2} - \sin^{-1}\frac{3}{5} = \frac{\pi}{2}$

$\sin^{-1}\frac{x}{5} = \sin^{-1}\frac{3}{5}$

$\frac{x}{5} = \frac{3}{5} \Rightarrow x = 3$ Ans (4)

$\operatorname{cosec}\theta = \frac{\text{Hyp}}{\text{opp}} = \frac{5}{4}$

$\text{adj} = \sqrt{5^2-4^2}$

$= \sqrt{25-16} = \sqrt{9} = 3$

20) $\sin(\tan^{-1}x)$, $|x| < 1$ is equal to.

Soln ~~$\sin(\tan^{-1}x)$~~ Let $a = \tan^{-1}x$

$\tan a = x$

W.K.T $1 + \tan^2 a = \sec^2 a$

$1 + x^2 = \sec^2 a$

$\sec a = \sqrt{1+x^2}$

$\frac{1}{\cos a} = \sqrt{1+x^2}$

$\cos a = \frac{1}{\sqrt{1+x^2}}$

$\sin a = \sqrt{1-\cos^2 a} = \sqrt{1 - \frac{1}{1+x^2}} = \sqrt{\frac{1+x^2-1}{1+x^2}}$

$= \frac{x}{\sqrt{1+x^2}}$ Ans (4)

நா. காமாட்சி, M.Sc., B.Ed., M.Phil., Ph.D.
முதுகலை ஆசிரியர் (கணிதம்)
அரசு மேல்நிலைப்பள்ளி
கோவிந்தவாடி, காஞ்சிபுரம் (Dt)

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