Chapter 15: Structure of Atoms and Nuclei

EXERCISES [PAGES 342 – 343]

Exercises | Q 1.1 | Page 342

Choose the correct option.
In which of the following systems will the radius of the first orbit of the electron be smallest?

1. hydrogen
2. singly ionized helium
3. deuteron
4. **Tritium**

SOLUTION

Tritium

Exercises | Q 1.2 | Page 342

Choose the correct option.
The radius of the 4^{th} orbit of the electron will be smaller than its 8^{th} orbit by a factor of

1. 2
2. **4**
3. 8
4. 16

SOLUTION

4

Exercises | Q 1.3 | Page 342

Choose the correct option.
In the spectrum of the hydrogen atom which transition will yield the longest wavelength?

1. n = 2 to n = 1
2. n = 5 to n = 4
3. n = 7 to n = 6
4. **n = 8 to n = 7**

SOLUTION

n = 8 to n = 7

Exercises | Q 1.4 | Page 342

Choose the correct option.
Which of the following properties of a nucleus does not depend on its mass number?

1. Radius
2. Mass
3. Volume
4. **Density**

SOLUTION

Density

Exercises | Q 1.5 | Page 342

Choose the correct option.
If the number of nuclei in a radioactive sample at a given time is N, what will be the number at the end of two half-lives?

1. N/2
2. **N/4**
3. 3N/4
4. N/8

SOLUTION

N/4

Exercises | Q 2.1 | Page 342

Answer in brief.
State the postulates of Bohr's atomic model.

SOLUTION

The postulates of Bohr's atomic model (for the hydrogen atom):

1. The electron revolves with a constant speed in a circular orbit around the nucleus. The necessary centripetal force is the Coulomb force of attraction of the positive nuclear charge on the negatively charged electron.

2. The electron can revolve without radiating energy only in certain orbits, called allowed or stable orbits, in which the angular momentum of the electron is equal to an integral multiple of $h/2\pi$, where h is Planck's constant.

3. Energy is radiated by the electron only when it jumps from one of its orbits to another orbit having lower energy. The energy of the quantum of electromagnetic radiation, i.e., the photon, emitted is equal to the energy difference of the two states.

Exercises | Q 2.2 | Page 342

Answer in brief.
State the difficulties faced by Rutherford's atomic model.

SOLUTION

1. According to Rutherford, the electrons revolve in circular orbits around the atomic nucleus. The circular motion is an accelerated motion. According to the classical electromagnetic theory, an accelerated charge continuously radiates energy. Therefore, an electron during its orbital motion should go on radiating energy. Due to the loss of energy, the radius of its orbit should go on decreasing. Therefore, the electron should move along a spiral path and finally fall into the nucleus in a very short time, of the order of 10^{-16} s in the case of a hydrogen atom. Thus, the atom should be unstable. We exist because atoms are stable.

2. If the electron moves along such a spiral path, the radius of its orbit would continuously decrease. As a result, the speed and frequency of revolution of the electron would go on increasing. The electron, therefore, would emit radiation of continuously changing frequency, and hence give rise to a continuous spectrum. However, the atomic spectrum is a line spectrum.

Exercises | Q 2.3 | Page 342

Answer in brief.
What are alpha, beta and gamma decays?

SOLUTION

1. A radioactive transformation in which an α-particle is emitted is called α-decay.
   In an α-decay, the atomic number of the nucleus decreases by 2 and the mass number decreases by 4.
   Example: ${}^{238}_{92}\mathrm{U} \longrightarrow {}^{234}_{90}\mathrm{Th} + {}^{4}_{2}\alpha$
   $Q = [m_U - m_{Th} - m_\alpha]c^2$
2. A radioactive transformation in which a β-particle is emitted is called β-decay.
   In a β^- -decay, the atomic number of the nucleus increases by 1 and the mass number remains unchanged.
   Example: ${}^{234}_{90}\mathrm{Th} \longrightarrow {}^{234}_{91}\mathrm{Pa} + {}^{0}_{-1}\mathrm{e} + \bar{v}_e$
   where $\bar{\mathbf{v}}_e$ is the antineutrino emitted to conserve the momentum, energy and spin.
   $Q = [m_{Th} - m_{pa} - m_e]c^2$
   In a β^+ -decay, the atomic number of the nucleus decreases by 1 and the mass number remains unchanged.
   Example: ${}^{30}_{15}\mathrm{P} \longrightarrow {}^{30}_{14}\mathrm{Si} + {}^{0}_{+1}\mathrm{e} + \mathrm{v}_e$
   where v_e is the neutrino emitted to conserve the momentum, energy and spin.
   $Q = [m_P - m_{Si} - m_e]c^2$
3. A given nucleus does not emit α- and -β particles simultaneously. However, on emission of α or β-particles, most nuclei are left in an excited state. A nucleus in an excited state emits a γ-ray photon in a transition to the lower energy state. Hence, α- and β-particle emissions are often accompanied by γ-rays.

Exercises | Q 2.4 | Page 342

Define excitation energy.

SOLUTION

The excitation energy of an electron in an atom: The energy required to transfer an electron from the ground state to an excited state (a state of higher energy) is called the excitation energy of the electron in that state.

Exercises | Q 2.5 | Page 342

Define ionization energy of an electron in an atom.

SOLUTION

The ionization energy of an electron in an atom is defined as the minimum energy required to remove the least strongly bound electron from a neutral atom such that its total energy is zero.

Exercises | Q 3 | Page 342

State the postulates of Bohr's atomic model and derive the expression for the energy of an electron in the atom.

SOLUTION

The postulates of Bohr's atomic model (for the hydrogen atom):

1. The electron revolves with a constant speed in a circular orbit around the nucleus. The necessary centripetal force is the Coulomb force of attraction of the positive nuclear charge on the negatively charged electron.

2. The electron can revolve without radiating energy only in certain orbits, called allowed or stable orbits, in which the angular momentum of the electron is equal to an integral multiple of h/27t, where h is Planck's constant.

3. Energy is radiated by the electron only when it jumps from one of its orbits to another orbit having lower energy. The energy of the quantum of electromagnetic radiation, i.e., the photon, emitted is equal to the energy difference of the two states.

Consider the electron revolving in the nth orbit around the nucleus of an atom with the atomic number Z. Let m and -e be the mass and the charge of the electron, r the radius of the orbit and v the linear speed of the electron.

According to Bohr's first postulate,

centripetal force on the electron = electrostatic force of attraction exerted on the electron by the nucleus

$$\therefore \frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0} = \frac{Ze^2}{r^2} \quad ...(1)$$

where ε_0 is the permittivity of free space.

$\therefore$ Kinetic energy (KE) of the electron

$$= \frac{1}{2}mv^2 = \frac{Ze^2}{8\pi\varepsilon_0 r} \quad .....(2)$$

The electric potential due to the nucleus of charge + Ze at a point at a distance r from it is

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Ze}{r}$$

$\therefore$ Potential energy (PE) of the electron

= charge on the electron x electric potential

$$= -e \times \frac{1}{4\pi\varepsilon_0}\frac{Ze}{r} = -\frac{Ze^2}{4\pi\varepsilon_0 r} \quad ....(3)$$

Hence, the total energy of the electron in the nth orbit is

$$E = KE + PE = \frac{-Ze^2}{4\pi\varepsilon_0 r} + \frac{Ze^2}{8\pi\varepsilon_0 r}$$

$$\therefore E = \frac{-Ze^2}{8\pi\varepsilon_0 r} \quad ......(4)$$

This shows that the total energy of the electron in the nth orbit of the atom is inversely proportional to the radius of the orbit as Z, ε_0 and e are constants. The radius of the nth orbit of the electron is

$$r = \frac{\varepsilon_0 h^2 n^2}{\pi m Z e^2} \quad ....(5)$$

where his Planck's constant.

From Eqs. (4) and (5), we get,

$$E_n = -\frac{Ze^2}{8\pi\varepsilon_0}\left(\frac{\pi m Z e^2}{\varepsilon_0 h^n \hat{} \; 2}\right) = -\frac{mZ^2e^4}{8\varepsilon_0^2 h^2 n^2} \quad ....(6)$$

This gives the expression for the energy of the electron in the nth Bohr orbit. The minus sign in the expression shows that the electron is bound to the nucleus by the electrostatic force of attraction.

As m, Z, e, ε_0 and h are constant, we get

$$E_n \propto \frac{1}{n^2}$$

i.e., the energy of the electron in a stationary energy state is discrete and is inversely proportional to the square of the principal quantum number.

Exercises | Q 4 | Page 342

Starting from the formula for the energy of an electron in the nth orbit of the hydrogen atom, derive the formula for the wavelengths of Lyman and Balmer series spectral lines and determine the shortest wavelengths of lines in both these series.

SOLUTION

According to Bohr's third postulate for the model of the hydrogen atom, an atom radiates energy only when an electron jumps from a higher energy state to a lower energy state and the energy of the quantum of electromagnetic radiation emitted in this process is equal to the energy difference between the two states of the electron.

This emission of radiation gives rise to a spectral line. The energy of the electron in a hydrogen atom, when it is in an orbit with the principal quantum number n, is

$$E_n = -\frac{me^4}{8\varepsilon_0^2 h^2 n^2}$$

where m = mass of electron, e = electronic charge, h = Planck's constant and ε_0 = permittivity of free space.

Let E_m be the energy of the electron in a hydrogen atom when it is in an orbit with the principal quantum number m and E_n, its energy in an orbit with the principal quantum number n, n < m. Then

$$E_m = -\frac{me^4}{8\varepsilon_0^2 h^2 m^2} \text{ and } E_n = -\frac{me^4}{8\varepsilon_0^2 h^2 n^2}$$

Therefore, the energy radiated when the electron jumps from the higher energy state to the lower energy state is

$$E_m - E_n = \frac{-me^4}{8\varepsilon_0^2 h^2 m^2} - \left(\frac{-me^4}{8\varepsilon_0^2 h^2 n^2}\right)$$

$$= \frac{me^4}{8\varepsilon_0^2 h^2}\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$$

This energy is emitted in the form of a quantum of radiation (photon) with energy hv, where v is the frequency of the radiation.

$\therefore E_m - E_n = hv$

$\therefore v = \frac{E_m - E_n}{h} = \frac{me^4}{8\varepsilon_0^2 h^3}\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$

The wavelength of the radiation is $\lambda = \frac{c}{v}$, where c is the speed of radiation in free space.

The wave number, $\bar{v} = \frac{1}{\lambda} = \frac{v}{c}$

$\therefore \bar{v}\frac{1}{\lambda} = \frac{me^4}{8\varepsilon_0^2 h^3 c}\left(\frac{1}{n^2} - \frac{1}{m^2}\right) = R\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$ where R $\left(\frac{me^4}{8\varepsilon_0^2 h^3 c}\right)$ is a constant called the Rydberg constant.

This expression gives the wave number of the radiation emitted and hence that of a line in the hydrogen spectrum.

For the Lyman series, n = 1, m = 2, 3, 4, ...∞

$\therefore \frac{1}{\lambda_L} = R\left(\frac{1}{1^2} - \frac{1}{m^2}\right)$ and for the shortest wavelength line in this series, $\frac{1}{\lambda_{Ls}} = R\left(\frac{1}{1^2}\right)$ as m = ∞.

For the Balmer series, n = 2, m = 3, 4, 5, ...∞.

$\therefore \frac{1}{\lambda_B} = R\left(\frac{1}{4} - \frac{1}{m^2}\right)$ and for the shortest wavelength line in this series,

$$\frac{1}{\lambda_{Bs}} = R\left(\frac{1}{4}\right) \text{ as } m = \infty.$$

Exercises | Q 5 | Page 342

Determine the maximum angular speed of an electron moving in a stable orbit around the nucleus of the hydrogen atom.

SOLUTION

The radius of the nth Bohr orbit is

$$r = \frac{\varepsilon_0 h^2 n^2}{\pi m Z e^2} \quad ....(1)$$

and the linear speed of an electron in this orbit is

$$v = \frac{Ze^2}{2\varepsilon_0 nh} \quad ....(2)$$

where ε_0 ≡ permittivity of free space, h =Planck's constant, n ≡ principal quantum number, m ≡ electron mass, e ≡ electronic charge and Z = the atomic number of the atom.

Since angular speed $\omega = \frac{v}{r}$, then from Eqs. (1) and (2), we get,

$$\omega = \frac{v}{r} = \frac{Ze^2}{2\varepsilon_0 nh} \cdot \frac{\pi m Z e^2}{\varepsilon_0 h^2 n^2} = \frac{\pi m Z^2 e^4}{2\varepsilon_0^2 h^3 n^3} \quad ...(3)$$

which gives the required expression for the angular speed of an electron in the nth Bohr orbit. From Eq. (3), the frequency of revolution of the electron,

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \times \frac{\pi m Z^2 e^4}{2\varepsilon_0^2 h^3 n^3} = \frac{m Z^2 e^4}{4\varepsilon_0^2 h^3 n^3} \quad .....(4)$$

Obtain the formula for w and continue as follows:

$\omega(\text{maximum}) = \frac{\pi me^4}{2\varepsilon_0^2 h^3}$ (for Z = 1 and n = 1)

$$= \frac{(3.142)\left(9.1\times10^{-31}\mathrm{kg}\right)\left(1.6\times10^{-19}\mathrm{C}\right)^4}{(2)\left(8.85\times10^{-12}\mathrm{C^2/N.m^2}\right)^2\left(6.63\times10^{-34}\mathrm{J.s}\right)^3}\ \text{rad/s}$$

$$= \frac{(3.142)(9.1)(1.6)^4\left(10^{-107}\right)}{(2)(8.85)^2(6.63)^3\left(10^{-126}\right)}$$

$= 4.105 \times 10^{16}$ rad/s

Exercises | Q 6 | Page 342

Determine the series limit of Balmer, Paschen, and Pfund series, given the limit for Lyman series is 912 Å.

SOLUTION

Data: $\lambda_{L\infty}$ = 912 Å

For hydrogen spectrum, $\frac{1}{\lambda} = R_H\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$

$$\therefore \frac{1}{\lambda_{L\infty}} = R_H\left(\frac{1}{1^2} - \frac{1}{\infty}\right) = R_H \quad ...(1)$$

as n = 1 and m = ∞

$$\frac{1}{\lambda_{B\infty}} = R_H\left(\frac{1}{4} - \frac{1}{\infty}\right) = \frac{R_H}{4} \quad .....(2)$$

as n = 2 and m = ∞

$$\frac{1}{\lambda_{Pa\infty}} = R_H\left(\frac{1}{9} - \frac{1}{\infty}\right) = \frac{R_H}{9} \quad .....(3)$$

as n = 3 and m = ∞

$$\frac{1}{\lambda_{Pf\infty}} = R_H\left(\frac{1}{25} - \frac{1}{\infty}\right) = \frac{R_H}{25} \quad ......(4)$$

as n = S and m = ∞

From Eqs. (1) and (2), we get,

$$\frac{\lambda_{B\infty}}{\lambda_{L\infty}} = \frac{R_H}{R_H/4} = 4$$

$\therefore\ \lambda_{B\infty} = 4\lambda_{L\infty} = (4)(912)$ = 3648 Å

This is the series limit of the Balmer series.

rom Eqs. (1) and (3), we get,

$$\frac{\lambda_{Pa\infty}}{\lambda_{L\infty}} = \frac{R_H}{R_H/9} = 9$$

$\therefore\ \lambda_{Pa\infty} = 9\lambda_{L\infty} = (9)(912)$ = 8208 Å

This is the series limit of the Paschen series.

From Eqs. (1) and (4), we get,

$$\frac{\lambda_{Pf\infty}}{\lambda_{L\infty}} = \frac{R_H}{R_H/25} = 25$$

$\therefore\ \lambda_{Pf\infty} = 25\ \lambda_{L\infty} = (25)(912)$ = 22800 Å

Exercises | Q 7 | Page 342

Describe alpha, beta and gamma decays and write down the formulae for the energies generated in each of these decays.

SOLUTION

1. A radioactive transformation in which an α-particle is emitted is called α-decay.

   In an α-decay, the atomic number of the nucleus decreases by 2 and the mass number decreases by 4.

   Example: ${}^{238}_{92}\mathrm{U} \longrightarrow {}^{234}_{90}\mathrm{Th} + {}^{4}_{2}\alpha$

   $Q = [m_U - m_{Th} - m_\alpha]c^2$

2. A radioactive transformation in which a β-particle is emitted is called β-decay.

   In a β^- -decay, the atomic number of the nucleus increases by 1 and the mass number remains unchanged.

   Example: ${}^{234}_{90}\mathrm{Th} \longrightarrow {}^{234}_{91}\mathrm{Pa} + {}_{-1}^{0}\mathrm{e} + \bar{v}_e$

   where $\mathbf{v}_e$ is the antineutrino emitted to conserve the momentum, energy and spin.

   $Q = [m_{Th} - m_{pa} - m_e]c^2$

   In a β^+ -decay, the atomic number of the nucleus decreases by 1 and the mass number remains unchanged.

   Example: ${}^{30}_{15}\mathrm{P} \longrightarrow {}^{30}_{14}\mathrm{Si} + {}_{+1}^{0}\mathrm{e} + \mathrm{v}_e$

   where v_e is the neutrino emitted to conserve the momentum, energy and spin.

   $Q = [m_P - m_{Si} - m_e]c^2$

3. A given nucleus does not emit α- and -β particles simultaneously. However, on emission of α or β-particles, most nuclei are left in an excited state. A nucleus in an excited state emits a γ-ray photon in a transition to the lower energy state. Hence, α- and β-particle emissions are often accompanied by γ-rays.

$${}^{A}_{Z}\mathrm{X} \longrightarrow {}^{4}_{2}\alpha + {}^{A-4}_{Z-2}\mathrm{Y} + \text{energy released}$$

$${}^{A}_{Z}\mathrm{X} \longrightarrow {}_{-1}^{0}\beta + {}_{Z+1}^{A}\mathrm{Y} + \text{energy released}$$

$${}^{A}_{Z}\mathrm{X} \longrightarrow {}^{0}_{0}\gamma + {}^{A}_{Z}\mathrm{X}$$ (Energy released is carried by the γ-ray photon).

Exercises | Q 8 | Page 342

Explain what are nuclear fission and fusion giving an example of each. Write down the formulae for energy generated in each of these processes.

SOLUTION

Nuclear fission is a nuclear reaction in which a heavy nucleus of an atom, such as that of uranium, splits into two or more fragments of comparable size, either spontaneously or as a result of bombardment of a neutron on the nucleus (induced fission). It is followed by the emission of two or three neutrons. The mass of the original nucleus is more than the sum of the masses of the fragments. This mass difference is released as energy, which can be enormous as in the fission of ^{235}U.

Nuclear fission was discovered by Lise Meitner, Otto Frisch, Otto Hahn and Fritz Strass mann in 1938.

The products of the fission of ^{235}U by thermal neutrons are not unique. A variety of fission fragments are produced with mass number A ranging from about 72 to about 138, subject to the conservation of mass-energy, momentum, number of protons (Z) and number of neutrons (N}. A few typical fission equations are

(1) $^{235}_{92}\mathrm{U} + {}^{1}_{0}\mathrm{n} \longrightarrow {}^{236}_{92}\mathrm{U} \longrightarrow {}^{140}_{54}\mathrm{Xe} + {}^{94}_{38}\mathrm{Sr} + 2\,{}^{1}_{0}\mathrm{n} + 200\,\mathrm{MeV}$

(235 + 1 = 236 = 140 + 94 + 2)

(2) $^{235}_{92}\mathrm{U} + {}^{1}_{0}\mathrm{n} \longrightarrow {}^{236}_{92}\mathrm{U} \longrightarrow {}^{144}_{56}\mathrm{Ba} + {}^{90}_{36}\mathrm{Kr} + 2\,{}^{1}_{0}\mathrm{n} + 200\,\mathrm{MeV}$

(235 + 1 = 236 = 144 + 90 + 2)

(3) $^{235}_{92}\mathrm{U} + {}^{1}_{0}\mathrm{n} \longrightarrow {}^{236}_{92}\mathrm{U} \longrightarrow {}^{148}_{57}\mathrm{La} + {}^{85}_{35}\mathrm{Br} + 3\,{}^{1}_{0}\mathrm{n} + 200\,\mathrm{MeV}$

(235 + 1 = 236 = 148 + 85 + 3)

A type of nuclear reaction in which lighter ato c nuclei (of low atomic number) fuse to form a heavier nucleus (of higher atomic number) with the release of enormous amount of energy is called nuclear fusion.

Very high temperatures, of about 107 K to 108 K, are required to carry out nuclear fusion. Hence, such a reaction is also called a thermonuclear reaction.

Example: The D-T reaction, being used in experimental fusion reactors, fuses a deuteron and triton nuclei at temperatures of about 10^8 K.

$$\underset{\text{(deuteron)}}{{}^{2}_{1}\mathrm{D}} + \underset{\text{(triton)}}{{}^{3}_{1}\mathrm{T}} \longrightarrow \underset{\text{(helium nucleus)}}{{}^{4}_{2}\mathrm{He}} + \underset{\text{(neutron)}}{{}^{1}_{0}\mathrm{n}} + 17.6\,\mathrm{MeV}$$

Exercises | Q 9.1 | Page 342

Describe the principles of a nuclear reactor.

SOLUTION

In a nuclear reactor fuel rods are used to provide a suitable fissionable material such as $^{236}_{92}\mathrm{U}$. Control rods are used to start or stop the reactor. Moderators are used to slowing down the fast neutrons ejected in nuclear fission to the appropriate lower speeds. The material used as a coolant removes the energy released in the nuclear reaction by converting it into thermal energy for the production of electricity.

Exercises | Q 9.2 | Page 342

What is the difference between a nuclear reactor and a nuclear bomb?

SOLUTION 1

Nuclear Reactor
A nuclear reactor is a machine where electricity and heat energy is generated by utilizing the power of atoms. In this mechanism, nuclear chain reactions are produced, controlled, and contained releasing a tremendous amount of energy. This controlled energy is used in electricity generation and radioactive isotopes production. These isotopes are used in the treatment and research of cancer in the medical field. All operating nuclear reactors are “critical.” When reactors are running at a constant power level, they are said to be in a “critical condition.”

These reactors use heavy atoms as fuel instead of fossil fuels. Fast-moving electrons strike a radioactive nucleus such as Plutonium-239 or Uranium-235 causing the nucleus to split. This splitting process is known as fission. In the process of fission, a tremendous amount of energy, radiation, and free electrons are released. These free electrons that are released are guided to strike other nuclei and so on causing a chain reaction.

Neutron moderators and neutron poisons control these fast-moving electrons and slow them down while becoming absorbed in other nuclei, thus managing the output of electricity from a reactor. The moderators are heavy water, water, and solid graphite.

Nuclear Bomb

In a nuclear bomb, there is a nuclear device having massive destructive power coming from uncontrolled fusion and fission reactions. The fusion and fission processes generate a tremendous amount of energy with a small amount of matter. This matter is usually the unstable nuclei of Plutonium-239 and Uranium-235. An atom bomb is categorized as a fission bomb and a hydrogen bomb as a fusion bomb are both weapons of mass destruction.

In World War II, Hiroshima and Nagasaki are recent examples of such mass destruction. In fusion bombs, nuclear fusion is the result of a huge amount of released energy while in the case of fission bombs the released energy is the result of fission reactions.

SOLUTION 2

In a nuclear reactor, a nuclear fission chain reaction is used in a controlled manner, while in a nuclear bomb, the nuclear fission chain reaction is not controlled, releasing tremendous energy in a very short time interval.

Exercises | Q 10 | Page 342

Calculate the binding energy of an alpha particle given its mass to be 4.00151 u.

SOLUTION

Data: M = 4.00151 u, m_p = 1.00728 u, m_n = 1.00866 u, 1 u = 931.5 MeV/c^2

The binding energy of an alpha particle =

$(Zm_p + Nm_n - M)c^2$

$= (2m_p + 2m_n - M)c^2$

$= [(2)(1.00728\ u) + 2(1.00866\ u) - 4.00151\ u]c^2$

$= (2.01456 + 2.01732 - 4.00151)(931.5)$ MeV

= 28.289655 MeV

$= 28.289655 \times 10^6$ eV $\times 1.602 \times 10^{-10}$ J

$= 4.532002731 \times 10^{-12}$ J

Exercises | Q 11 | Page 342

An electron in hydrogen atom stays in its second orbit for 10^{-8} s. How many revolutions will it make around the nucleus at that time?

SOLUTION

Data: $z = 1$, $m = 9.1 \times 10^{-31}$ kg, $e = 1.6 \times 10^{-19}$ C, $\varepsilon_0 = 8.85 \times 10^{-12}$ $C^2/N{\cdot}m^2$, $h = 6.63 \times 10^{-34}$ J.s, $n = 2$, $t = 10^{-8}$ s

The periodic time of the electron in a hydrogen atom,

$$T = \frac{4\varepsilon_0^2 h^3 n^3}{\pi m e^4}$$

$$= \frac{(4)\left(8.85 \times 10^{-12}\right)^2\left(6.63 \times 10^{-34}\right)^3(8)}{(3.142)\left(9.1 \times 10^{-31}\right)\left(1.6 \times 10^{-19}\right)^4}$$

$$= \frac{(4)(8.85)^2(6.63)^3(8)}{(3.142)(9.1)(1.6)^4} \times 10^{-19}\text{s}$$

$= 3.898 \times 10^{-16}$ s

Let N be the number of revolutions made by the electron in time t. Then, t =NT.

$$\therefore N = \frac{t}{T} = \frac{10^{-8}}{3.898 \times 10^{-16}} = 2.565 \times 10^7$$

Exercises | Q 12 | Page 342

Determine the binding energy per nucleon of the amenctum isotope $^{244}_{95}\text{Am}$, given the mass of $^{244}_{95}\text{Am}$ to be 244.06428 u.

SOLUTION

Data: Z = 95, N = 244 - 95 = 149, m_p = 1.00728 u, m_n = 1.00866 u, M = 244.06428 u, 1 u = 931.5 MeV/c^2

The binding energy per nucleon,

$$\frac{E_B}{A} = \frac{(Zm_p + Nm_n - M)c^2}{A}$$

$$= \frac{[95(1.00728) + 149(1.00866) - 244.06428]uc^2}{244}$$

$$= \left(\frac{95.6916 + 150.29034 - 244.06428}{244}\right)(931.5) \text{ MeV nucleon}$$

= 7.3209 MeV/nucleon

Exercises | Q 13 | Page 342

Calculate the energy released in the nuclear reaction $^7_3\text{Li} + \text{p} \longrightarrow 2\,\alpha$ given mass of ^7_3Li atom and of helium atom to be 7.016 u and 4.0026 u respectively.

SOLUTION

Data: M_1 ($^7_3\text{Li atom}$) = 7.016 u, M_2 = (He atom)

= 4.0026 u, m_p = 1.00728 u, 1 u = 931.5 MeV/c^2

$\Delta M = M_1 + m_p - 2M_2$

= (7.016 + 1.00728- 2(4.0026)]u

= 0.01808 u = (0.01808)(931.5) MeV/c^2

= 16.84152 MeV/c^2

Therefore, the energy released in the nuclear reaction= $(\Delta M)c^2$ = 16.84152 MeV

Exercises | Q 14.1 | Page 343

Complete the following equation describing nuclear decay.

$^{226}_{88}\text{Ra} \longrightarrow {}^{4}_{2}\alpha + \_\_\_\_$

SOLUTION

$$^{226}_{88}\text{Ra} \longrightarrow {}^{4}_{2}\alpha + {}^{222}_{86}\text{Em}$$

Em (Emanation) ≡ Rn (Radon)

Here, α particle is emitted and radon is formed.

Exercises | Q 14.2 | Page 343

Complete the following equation describing nuclear decay.

$^{19}_{8}\text{O} \longrightarrow \text{e}^{-} + \_\_\_\_$

SOLUTION

$$^{19}_{8}\text{O} \longrightarrow \text{e}^{-} + {}^{19}_{9}\text{F}$$

Here, e$^-$ ≡ $^{0}_{-1}\beta$ is emitted and fluorine is formed.

Exercises | Q 14.3 | Page 343

Complete the following equation describing nuclear decay.

$^{228}_{90}\text{Th} \longrightarrow \alpha + \_\_\_\_$

SOLUTION

$$^{228}_{90}\text{Th} \longrightarrow {}^{4}_{2}\alpha + {}^{224}_{88}\text{Ra}$$

Here, α particles is emitted and radium is formed.

Exercises | Q 14.4 | Page 343

Complete the following equation describing nuclear decay.

$^{12}_{7}\text{N} \longrightarrow {}^{12}_{6}\text{C} + \_\_\_\_$

SOLUTION

$$ {}_{7}^{12}\mathrm{N} \longrightarrow {}_{6}^{12}\mathrm{C} + + {}_{1}^{0}\beta $$

${}_{1}^{0}\beta$ is e^+ (positron)

Here, β^+ is emitted and carbon is formed.

Exercises | Q 15.1 | Page 343

Calculate the energy released in the following reaction, given the masses to be
${}_{88}^{223}\mathrm{Ra}$: 223.0185 u, ${}_{82}^{209}\mathrm{Pb}$: 208.9811 u, ${}_{6}^{14}\mathrm{C}$: 14.00324 u, ${}_{92}^{236}\mathrm{U}$: 236.0456 u, ${}_{56}^{140}\mathrm{Ba}$: 139.9106 u, ${}_{92}^{36}\mathrm{Kr}$: 93.9341 u, ${}_{11}^{6}\mathrm{C}$: 11.01143 u, ${}_{11}^{5}\mathrm{B}$: 11.0093 u. Ignore neutrino energy.
${}_{88}^{223}\mathrm{Ra} \longrightarrow {}_{82}^{209}\mathrm{Pb} + {}_{6}^{14}\mathrm{C}$

SOLUTION

$$ {}_{88}^{223}\mathrm{Ra} \longrightarrow {}_{82}^{209}\mathrm{Pb} + {}_{6}^{14}\mathrm{C} $$

The energy released in this reaction = $(\Delta M)c^2$

= [223.0185 - (208.9811+14.00324)](931.5) MeV

= 31.820004 Me V

Exercises | Q 15.2 | Page 343

Calculate the energy released in the following reaction, given the masses to be
${}_{88}^{223}\mathrm{Ra}$: 223.0185 u, ${}_{82}^{209}\mathrm{Pb}$: 208.9811 u, ${}_{6}^{14}\mathrm{C}$: 14.00324 u, ${}_{92}^{236}\mathrm{U}$: 236.0456 u, ${}_{56}^{140}\mathrm{Ba}$: 139.9106 u, ${}_{36}^{94}\mathrm{Kr}$: 93.9341 u, ${}_{6}^{11}\mathrm{C}$: 11.01143 u, ${}_{5}^{11}\mathrm{B}$: 11.0093 u. Ignore neutrino energy.
${}_{92}^{236}\mathrm{U} \longrightarrow {}_{56}^{140}\mathrm{Ba} + {}_{36}^{94}\mathrm{Kr} + 2\,\mathrm{n}$

SOLUTION

$$ {}_{92}^{236}\mathrm{U} \longrightarrow {}_{56}^{140}\mathrm{Ba} + {}_{36}^{94}\mathrm{Kr} + 2\,\mathrm{n} $$

The energy released in this reaction =

$(\Delta M)c^2$ = (236.0456 - (139.9106 + 93.9341 + (2)(1.00866)](931.S)MeV

= 171.00477 MeV

Exercises | Q 15.3 | Page 343

Calculate the energy released in the following reaction, given the masses to be

$^{223}_{88}\mathrm{Ra}$: 223.0185 u, $^{209}_{82}\mathrm{Pb}$: 208.9811 u, $^{14}_{6}\mathrm{C}$: 14.00324 u, $^{236}_{92}\mathrm{U}$: 236.0456 u, $^{140}_{56}\mathrm{Ba}$: 139.9106 u, $^{94}_{36}\mathrm{Kr}$: 93.9341 u, $^{11}_{6}\mathrm{C}$: 11.01143 u, $^{11}_{5}\mathrm{B}$: 11.0093 u. Ignore neutrino energy.

$^{11}_{6}\mathrm{C} \longrightarrow {}^{11}_{5}\mathrm{B} + \mathrm{e}^{+} + \mathrm{neutrino}$

SOLUTION

$$^{11}_{6}\mathrm{C} \longrightarrow {}^{11}_{5}\mathrm{B} + \mathrm{e}^{+} + \mathrm{neutrino}$$

The energy released in this reaction = $(\Delta M)c^2$

= [11.01143 - (11.0093 + 0.00055)](931.5} MeV

=1.47177 MeV

Exercises | Q 16 | Page 343

Sample of carbon obtained from any living organism has a decay rate of 15.3 decays per gram per minute. A sample of carbon obtained from very old charcoal shows a disintegration rate of 12.3 disintegrations per gram per minute. Determine the age of the old sample given the decay constant of carbon to be 3.839×10^{-12}per second.

SOLUTION

Data: 15.3 decays per gram per minute (living organism), 12.3 disintegrations per gram per minute (very old charcoal). Hence, we have,

$$\frac{\mathrm{A(t)}}{\mathrm{A_0}} = \frac{12.3}{15.3}, \ \lambda = 3.839 \times 10^{-12} \text{ per second}$$

$$\mathrm{A(t)} = \mathrm{A_0}e^{-\lambda t} \ \therefore e^{\lambda t} = \frac{\mathrm{A_0}}{\mathrm{A}}$$

$$\therefore \lambda t = \log_e\left(\frac{\mathrm{A_0}}{\mathrm{A}}\right)$$

$$\therefore t = \frac{2.303}{\lambda}\log_{10}\left(\frac{\mathrm{A_0}}{\mathrm{A}}\right)$$

$$= \frac{2.303}{3.839 \times 10^{-12}}\log_{10}\left(\frac{15.3}{12.3}\right)$$

$$= \frac{2.303 \times 10^{12}}{3.839}(\log 15.3 - \log 12.3)$$

$$= \frac{2.303 \times 10^{12}}{3.839}(1.1847 - 1.0899)$$

$$= \frac{(2.303)(0.0948)}{3.839} \times 10^{12}\text{s}$$

= 5.687×10^{10} s

$$= \frac{5.687 \times 10^{10}\text{s}}{3.156 \times 10^{7}\text{s per year}}$$

= 1802 years

Exercises | Q 17 | Page 343

The half-life of $^{90}_{38}\text{Sr}$ is 28 years. Determine the disintegration rate of its 5 mg sample.

SOLUTION

Data: $T_{1/2}$ = 28 years = $28 \times 3.156 \times 10^7$ s

= 8.837×10^8 s, M = 5mg = 5×10^{-3} g

90 grams of $^{90}_{38}\text{Sr}$ contain 6.02×10^{23} atoms

Hence, here, N = $\frac{(6.02 \times 10^{23})(5 \times 10^{-3})}{90}$

= 3.344×10^{19} atoms

∴ The disintegration rate = $N\lambda = N\frac{0.693}{T_{1/2}}$

$$= \frac{(3.344 \times 10^{19})(0.693)}{8.837 \times 10^{8}}$$

= 2.622×10^{10} disintegrations per second

Exercises | Q 18 | Page 343

What is the amount of $^{60}_{27}\mathrm{Co}$ necessary to provide a radioactive source of strength 10.0 mCi, its half-life being 5.3 years?

SOLUTION

Data: Activity= 10.0 mCi = 10.0 x 10^{-3} Ci = (10.0 x 10^{-3})(3.7 x 10^{10}) dis/s = 3.7 x 10^8 dis/s

$T_{1/2}$ = 5.3 years = (5.3)(3.156 × 10^7)s = 1.673 × 10^8 s

$$\text{Decay constant, } \lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{1.673 \times 10^8} s^{-1}$$

= 4.142 x 10^{-9} s^{-1}

$$\therefore N = \frac{\text{activity}}{\lambda} = \frac{3.7 \times 10^8}{4.142 \times 10^{-9}} \text{ atoms}$$

= 8.933 × 10^{16} atoms

= 60 grams of $^{60}_{27}\mathrm{Co}$ contain 6.02 x 10^{23} atoms

∴ Mass of 8.933 x 10^{16} atoms of $^{60}_{27}\mathrm{Co}$

$$= \frac{8.933 \times 10^{16}}{6.02 \times 10^{23}} \times 60g$$

= 8.903 x 10^{-6} g = 8.903 μg

Exercises | Q 19 | Page 343

Disintegration rate of a sample is 10^{10} per hour at 20 hours from the start. It reduces to 6.3 x 10^9 per hour after 30 hours. Calculate its half-life and the initial number of radioactive atoms in the sample.

SOLUTION

Data: $A(t_1)$ = 10^{10} per hour, where t_1 = 20 h,

$A(t_1)$ = 6.3 × 10^{10} per hour, where t_2 = 30 h

$A(t) = A_0e^{-\lambda t}$ $\therefore$ $A(t_1) = \mathbf{A_0e^{-\lambda t_1}}$ and

$A(t_2) = \mathbf{A_0e^{-\lambda t_2}}$

$$\therefore \frac{\mathbf{A(t_1)}}{\mathbf{A(t_2)}} = \left(\frac{\mathbf{e^{-\lambda t_1}}}{\mathbf{e^{-\lambda t_2}}}\right) = \mathbf{e^{\lambda(t_2 - t_1)}}$$

$$\therefore \frac{\mathbf{10^{10}}}{\mathbf{6.3 \times 10^9}} = \mathbf{e^{\lambda(30-20)}} = \mathbf{e^{10\lambda}}$$

$\therefore 1.587 = e^{10\lambda}$

$\therefore 10\lambda = 2.303 \log_{10}(1.587)$

$\therefore \lambda = (0.2303)(0.2007) = 0.04622$ per hour

The half life of the material, $T_{1/2} = \frac{\mathbf{0.693}}{\boldsymbol{\lambda}} = \frac{\mathbf{0.693}}{\mathbf{0.04622}}$

$= 14.99$ hours

Now, $\mathbf{A_0 = A(t_1)e^{\lambda t_1} = 10^{10}e^{(0.04622)(20)}}$

$= \mathbf{10^{10}e^{0.9244}}$

Let $x = \mathbf{e^{0.9244}}$

$\therefore 2.303 \log_{10}x = 0.9244$

$\therefore \log_{10}x = \frac{\mathbf{0.9244}}{\mathbf{2.303}} = \mathbf{0.4014}$

$\therefore x = \text{antilog } 0.4014 = 2.52$

$\therefore A_0 = 2.52 \times 10^{10}$ per hour

Now $A_0 = N_0\lambda$

$$\therefore \mathbf{N_0} = \frac{\mathbf{A_0}}{\boldsymbol{\lambda}} = \frac{\mathbf{2.52 \times 10^{10}}}{\mathbf{0.04622}}$$

$= 5.452 \times 10^{11}$

Exercises | Q 20 | Page 343

The isotope ^{57}Co decays by electron capture to ^{57}Fe with a half-life of 272 d. The ^{57}Fe nucleus is produced in an excited state, and it almost instantaneously emits gamma rays.

(a) Find the mean lifetime and decay constant for ^{57}Co.

(b) If the activity of a radiation source ^{57}Co is 2.0 μCi now, how many ^{57}Co nuclei does the source contain?

c) What will be the activity after one year?

SOLUTION

Data: $T_{1/2}$ = 272 d = 272 × 24 × 60 × 60s = 2.35 × 10^7 s, A_0 = 2.0 μCi= 2.0 × 10^{-6} × 3.7 × 10^{10} = 7.4 × 10^4 dis/s

t = 1 year = 3.156 × 10^7 s

(a) $\mathbf{T_{1/2} = \frac{0.693}{\lambda} = 0.693\tau}$

∴ The mean lifetime for ^{57}Co =

$$\tau = \frac{T_{1/2}}{0.693} = \frac{2.35 \times 10^7}{0.693} = 3.391 \times 10^7 \text{s}$$

The decay constant for ^{57}Co = $\lambda = \frac{1}{\tau}$

$$= \frac{1}{3.391 \times 10^7 \text{s}}$$

= 2.949 × 10^{-8} s^{-1}

(b) $\mathbf{A_0 = N_0\lambda}$

$$\therefore N_0 = \frac{A_0}{\lambda} = A_0\tau$$

$$= (7.4 \times 10^4)(3.391 \times 10^7)$$

= 2.509 × 10^{12} nuclei

(c) A(t) = $A_0 e^{-\lambda t} = 2e^{-(2.949 \times 10^{-8})(3.156 \times 10^7)}$

$= 2e^{-0.9307} = 2/e^{0.9307}$

Let x = $e^{0.9307}$

∴ $\log_e x$ = 0.9307

∴ 2.303 $\log_{10} x$ = 0.9307

$$\therefore \log_{10} x = \frac{0.9307}{2.303} = 0.4041$$

∴ x = antilog 0.4041=2.536

$$\therefore A(t) = \frac{2}{2.536}\mu Ci = 0.7886\mu Ci$$

Exercises | Q 21 | Page 343

A source contains two species of phosphorous nuclei, ${}^{32}_{15}P$ ($T_{1/2}$ = 14.3 d) and ${}^{33}_{15}P$ ($T_{1/2}$ = 25.3 d). At time t = 0, 90% of the decays are from ${}^{32}_{15}P$. How much time has to elapse for only 15% of the decays to be from ${}^{32}_{15}P$?

SOLUTION

Data: ${}^{32}_{15}P$: $T_{1/2}$ = 14.3 d

$$\therefore \lambda_1 = \frac{0.693}{14.3d} = 0.04846\ d^{-1}$$

${}^{32}_{15}P$: $T_{1/2}$ = 25.3 d

$$\therefore \lambda_2 = \frac{0.693}{25.3d} = 0.02739\ d^{-1}$$

At time t = 0, $\frac{N_{O1}\lambda_1}{N_{O2}\lambda_2} = \frac{90\%}{10\%} = 9$ ...(1) and

at time t, $\frac{N_{O1}\lambda_1 e^{-\lambda_1 t}}{N_{O2}\lambda_2 e^{-\lambda_2 t}} = \frac{15\%}{85\%} = \frac{3}{17}$ ...(2)

Dividing Eq. (1) by Eq. (2), we get,

$$\frac{N_{O1}\lambda_1}{N_{O2}\lambda_2} \cdot \frac{N_{O1}\lambda_1 e^{-\lambda_1 t}}{N_{O2}\lambda_2 e^{-\lambda_2 t}} = \frac{9}{3/17} = \frac{153}{3}$$

$$\therefore e^{(\lambda_1-\lambda_2)t} = \frac{153}{3}$$

$$\therefore (\lambda_1 - \lambda_2)t = 2.303\log_{10}\left(\frac{153}{3}\right) = 2.303(\log_{10}153 - \log_{10}3)$$

$\therefore$ (0.04846 - 0.02739) t = 2.303 (2.1847 - 0.4771)

$\therefore$ t = $\frac{(2.303)(1.7076)}{0.02107}$ = 186.6 days

Exercises | Q 22 | Page 343

Before the year 1900 the activity per unit mass of atmospheric carbon due to the presence of ^{14}C averaged about 0.255 Bq per gram of carbon.
(a) What fraction of carbon atoms were ^{14}C?
(b) An archaeological specimen containing 500 mg of carbon, shows 174 decays in one hour. What is the age of the specimen, assuming that its activity per unit mass of carbon when the specimen died was equal to the average value of the air? The half-life of ^{14}C is 5730 years.

SOLUTION

Data: $T_{1/2}$ = 5730 y

$$\therefore \lambda = \frac{0.693}{5730 \times 3.156 \times 10^7} s^{-1}$$

= 3.832×10^{-12} s^{-1}, A = 0.255 Bq per gram of carbon in part (a); M = 500 mg = 500×10^{-3} g,

174 decays in one hour = $\frac{174}{3600}$ dis/s = 0.04833 dis/s in part (b) [per 500 mg]

(a) A = Nλ

$\therefore$ N = $\frac{A}{\lambda} = \frac{0.255}{3.832 \times 10^{-12}}$

= 6.654×10^{10}

Number of atoms in 1 g of carbon = $\frac{6.02 \times 10^{23}}{12} = 5.017 \times 10^{22}$

$$\frac{5.017 \times 10^{22}}{6.654 \times 10^{10}} = 0.7539 \times 10^{12}$$

$\therefore$ 1 ^{14}C atom per 0.7539 x 10^{12} atoms of carbon

$\therefore$ 4 ^{14}C atoms per 3 x 10^{12} atoms of carbon

(b) Present activity per gram = $\frac{0.04833}{500 \times 10^{-3}}$

= 0.09666 dis/s per gram

A_0 = 0.255 dis/s per gram

Now, A(t) = $A_0 e^{-\lambda t}$

$$\therefore \lambda t = 2.303 \frac{\log_{10} A_0}{A} = 2.303 \log_{10}\left(\frac{0.255}{0.09666}\right)$$

$$\therefore t = \frac{2.303 \log 2.638}{3.832 \times 10^{-12}} = \frac{(2.303)(0.4213)}{3.832 \times 10^{-12}}$$

= 25.31 × 10^{10} s

$$= \frac{25.32 \times 10^{10}}{3.156 \times 10^{7}} = 8023 \text{ years}$$

Exercises | Q 23 | Page 343

How much mass of ^{235}U is required to undergo fission each day to provide 3000 MW of thermal power? Average energy per fission is 202.79 MeV.

SOLUTION

Data: Power = 3000 MW = 3 x 10^9 J/s

$\therefore$ Energy to be produced each day

= 3 x 10^9 x 86400 J each day

= 2.592 x 10^{14} J each day

Energy per fission = 202.79 MeV

= 202.79 x 10^6 x 1.6 x 10^{-19} J = 3.245 x 10^{-11} J

∴ Number of fissions each day

$$= \frac{2.592 \times 10^{14}}{3.245 \times 10^{-11}} = 7.988 \times 10^{24} \text{ each day}$$

0.235 kg of ^{235}U contains 6.02 x 10^{23} atoms

$$\therefore \text{M} = \left(\frac{7.988 \times 10^{24}}{6.02 \times 10^{23}}\right)(0.235) = 3.118\text{kg}$$

Exercises | Q 24 | Page 343

In a periodic table the average atomic mass of magnesium is given as 24.312 u. The average value is based on their relative natural abundance on earth. The three isotopes and their masses are $^{24}_{12}\text{Mg}$ (23.98504 u), $^{25}_{12}\text{Mg}$ (24.98584 u), and $^{26}_{12}\text{Mg}$ (25.98259 u). The natural abundance of $^{24}_{12}\text{Mg}$ is 78.99% by mass. Calculate the abundances of other two isotopes.

SOLUTION

Data: Average atomic mass of magnesium = 24.312 u, $^{24}_{12}\text{Mg}$: 23.98504 u, $^{25}_{12}\text{Mg}$: 24.98584 u, $^{26}_{12}\text{Mg}$: 25.98259 u, $^{24}_{12}\text{Mg}$: 78.99% by mass

$$\therefore 24.312 = \frac{(23.98504)(78.99) + (24.98584)\text{x} + (25.98259)(100 - 78.99 - \text{x})}{100}$$

$$\therefore 24.312 = 18.9457831 + \frac{24.98584\text{x}}{100} + 25.98259 - 20.52364784 - \frac{25.98259\text{x}}{100}$$

$$\therefore \frac{0.99675}{100}\text{x} = 0.09272526$$

∴ x = 9.30 %

100 - 78.99 - 9.30 = 11.71

∴ $^{25}_{12}\text{Mg}$: 9.30 % be mass and $^{26}_{12}\text{Mg}$: 11.71 % by mass