

GEOMI

NTRODUCTION

Circles are geometric shapes you can see all around you. The significance of the concept of a circle can be well understood from the fact that the wheel is one of the ground-breaking inventions in the history of mankind

Parts of Circle:

A circle, you can describe, is the set of all points in a plane at a constant distance from a fixed point. The fixed point is the centre of the circle; the constant distance corresponds to a radius of the circle. A line that cuts the circle in two parts is called a secant of the circle. A line segment whose end points lie on the circle is called a chord of the circle..

Exercise 4.1

Fill i	n the blanks :	
(i)	Twice of the radius is called of the circle.	Ans. diameter]
(ii)	Longest chord passes through the of the circle.	[Ans. centre]
(iii)	Distance from the centre to any point on the circumference of the circle is called [Ans. radius]	
(iv)	A part of a circle between any two points is called a/an	of the circle. [Ans. arc]
(v)	A circle divides the plane into parts. [An	s. three parts]
Wri	te True or False. Give reasons for your answers.	
(i)	Line segment joining any two points on the circle is called radius of the circle.	
		[Ans. False]
(ii)	Point of concurrency of the diameter is the centre of the circle.	[Ans. True]
(iii)		[Ans. True]
(iv)		[Ans. True]
(v)	Sector is the region between the chord and its corresponding are	. [Ans. False]

and the circle is circumcircu

4.4 Properties of Chords of a Circle:

erties of Chords of a Circle:

In this chapter, already we come across lines, angles, triangles and quadrilateral lines chapter, already we come across lines, angles, triangles and quadrilateral lines of the lines In this chapter, already we come across lines, angles, and quadrilateral lines that the properties of the Recently we have seen a new member circle. Using all the properties of the Recently we have seen a new member circle. Now, we are going to discuss In this chapter, arready

Recently we have seen a new member circle. Now, we are going to discuss so we get some standard results one by one. Now, we are going to discuss so we get some standard results of the circle. properties based on chords of the circle.

Perpendicular from the Centre to a Chord:

Theorem 2:
The perpendicular from the centre of a circle to a chord bisects the chord.

Converse of Theorem 2:
The line joining the centre of the circle and the midpoint of a chord is perpendicular. to the chord.

Angle Subtended by Chord at the Centre: Angle Subtended by Chord at the Celling to two equal chords. Now we are going to 4.4.2

Theorem 3: Equal chords of a circle subtend equal angles at the centre Theorem 3: Equal chords of a chicke and the midpoint of a chord is perpendicular.

The line joining the centre of the circle and the midpoint of a chord is perpendicular. to the chord.

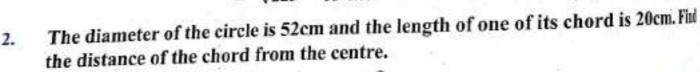
Exercise 4.2

- The radius of the circle is 25cm and the length of one of its chord is 40cm, Find 1. the distance of the chord from the centre.
- Sol. Distance of the chord from the centre

$$= \sqrt{(25\text{cm})^2 - (20\text{ cm})^2}$$

$$= \sqrt{(625 - 400)\text{cm}}$$

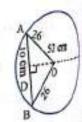
$$= \sqrt{225} = 15\text{ cm}$$



The distance of the chord from the centre O Sol.

OD =
$$\sqrt{26^2 - 10^2}$$

= $\sqrt{676 - 100}$
= $\sqrt{576} = 24$ cm

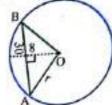


5 in 1 0 Maths - Term II - 9th Std o Chapter 4 0 Geometry

The chord of length 30 cm is drawn at the distance of 8cm from the centre of the circle. Find the radius of the circle.

Radius of the circle =
$$\sqrt{8^2 + 15^2} = \sqrt{64 + 225}$$

= $\sqrt{289} = 17$ cm



Find the length of the chord AC where AB and CD are the two diameters perpendicular to each other of a circle with radius $4\sqrt{2}$ cm and also find $\angle OAC$ and $\angle OCA$.

MOAC is an isoceles triangle with one angle 90°

 al_8

Se

The

lar

to

lar

Find

Find

Sol.

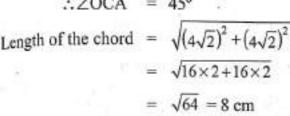
$$\angle OAC + \angle OCA = 180^{\circ} - 90^{\circ}$$

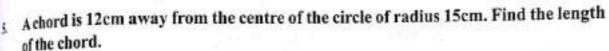
$$2\angle OAC = 90^{\circ} \quad (\because \angle OAC = \angle OCA)$$

$$\angle OAC = 45^{\circ}$$

$$\angle OCA = 45^{\circ}$$

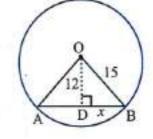
$$Length of the chord = \sqrt{(4\sqrt{2})^2 + (4\sqrt{2})^2}$$





BD =
$$\sqrt{15^2 - 12^2}$$

= $\sqrt{225 - 144}$
= $\sqrt{81}$
= 9 cm



:. length of the chord AB = 9 + 9 = 18 cm

In a circle, AB and CD are two parallel chords with centre O and radius 10 cm such that AB = 16 cm and CD = 12 cm determine the distance between the two chords?

In the distance between the two chord FE

$$OE = \sqrt{10^2 - 6^2} = \sqrt{100 - 36} = \sqrt{64} = 8 \text{ cm}$$

$$OF = \sqrt{10^2 - 8^2}$$

$$= \sqrt{100 - 64}$$

$$= \sqrt{36} = 6 \text{ cm}$$

$$\therefore FE = 8 \text{ cm} + 6 \text{ cm} = 14 \text{ cm}$$

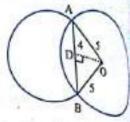
F 8 10 10 6 6 D x D

: Distance between the chords is 14 cm

Two circles of radii 5 cm and 3 cm intersect at two points and the distance bets their centres is 4 cm. Find the length of the common chord. 7.

Sol.

OD = DP =
$$\frac{4\text{cm}}{2}$$
 = 2 cm
AD = BD = $\sqrt{5^2 - 4^2}$
= $\sqrt{25 - 16}$
= $\sqrt{9}$
= 3 cm



... The length of the common chord AB = AD + BD = (3 + 3) cm = 6 cm

Angle Subtended by an Arc of a Circle: 4.4.3

Now we are going to verify the relationship between the angle subtended by at Now we are going to verify the relationship between the angle subtended by at the circumference. arc at the centre and the angle subtended on the circumference.

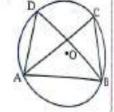
Angle at the Centre and the Circumference: 4.4.4

Theorem 5:
The angle subtended by an arc of the circle at the centre is double the angle sub. tended by it at any point on the remaining part of the circle.

Angles at the Circumference to the same Segment : Angles at the Circumference to the sand AB. C and D are the points on the 4.4.5 circumference of the circle in the same segment. Join the radius OA and OB

$$\frac{1}{2} \angle AOB = \angle ACB$$
 (by theorem 5)

and
$$\frac{1}{2} \angle AOB = \angle ADB$$
 (by theorem 5)

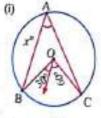


This conclusion leads to the new result.

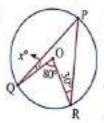
Theorem 6: Angles in the same segment of a circle are equal

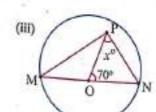
Exercise 4.3

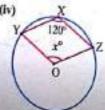
Find the value of x^0 in the following: 1.



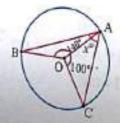






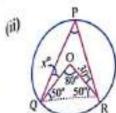


(v)





$$x^{o} = \frac{1}{2} \angle BOC = \frac{1}{2} \times (30^{o} + 60^{o}) = \frac{1}{2} \times 90^{o} = 45^{o}$$

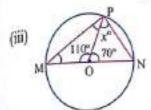


$$\angle QPR = \frac{1}{2} \angle QOR = \frac{1}{2} \times 80^{\circ} = 40^{\circ}$$

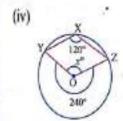
In AQPR

$$\angle R + \angle P + \angle Q = 180^{\circ}$$

 $80^{\circ} + 40^{\circ} + \angle Q = 180^{\circ}$;
 $\angle Q = 180^{\circ} - 120^{\circ} = 60^{\circ}$
 $\angle Q = x^{\circ} + 50^{\circ} = 60^{\circ}$
 $x^{\circ} = 60^{\circ} - 50^{\circ} = 10^{\circ}$



$$\angle$$
MPN = 90° (Angle subtended by the diameter is 90°)
 \angle OMP + \angle OPM = 180° - 110° = 70°
 \angle MPO = $\frac{70}{2}$ = 35°
But \angle MPN = 90°
 $\therefore x = \angle$ OPN = 90° - 35° = 55°



Central angle is twice that of angle subtended on the circumference

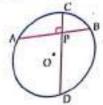
$$\angle YOZ = 120^{\circ} \times 2 = 240^{\circ}$$

 $\therefore x + \angle YOZ = 360^{\circ}$
 $x^{\circ} = 360^{\circ} - 240^{\circ} = 120^{\circ}$

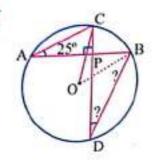
∠BOC =
$$360^{\circ} - 240^{\circ} = 120^{\circ}$$

∴∠BAC = $x^{\circ} = \frac{120^{\circ}}{2} = 60^{\circ}$

In the given figure, $\angle CAB = 25^{\circ}$, find $\angle BDC$, $\angle DBA$ and $\angle COB$



Sol.



(ii)
$$\angle DBA = \angle DCA = 180 - (90 + 25^{\circ})$$

= $180^{\circ} - 115^{\circ}$
= 65

(ii)
$$\angle COB = 2 \angle CAB = 2 \times 25^{\circ} = 50^{\circ}$$

Cyclic Quadrilaterals: 4.5

Now we see a special quadrilateral with its properties called "Cyclic Quadrilateral". A quadrilateral is called cyclic quadrilateral if all its four vertices lie on the circumference of the circle. Now we are going to learn the special property of cyclic quadrilateral.

Theorem 7:

Opposite angles of a cyclic quadrilateral are supplementary.

Let us see the converse of theorem 7, which is very useful in solving problems.

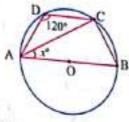
Converse of Theorem 7:

If a pair of opposite angles of a quadrilateral is supplementary, then the quadrilateral is supplementary. eral is cyclic

Exterior Angle of a Cyclic Quadrilateral:

Theorem 8: If one side of a cyclic quadrilateral is produced then the exterior angle is equal to the interior opposite angle..

find the value of x in the given figure.



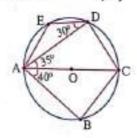
In the cyclic quadrilateral ABCD

∠ABC =
$$180^{\circ}$$
 – 120° = 60°
∠BCA = 90°
∴ $x = \angle BAC = 180^{\circ}$ – $(90^{\circ} + 60^{\circ}) = 30^{\circ}$

In the given figure, AC is the diameter of the circle with centre O. If $\angle ADE = 30^\circ$; $\angle DAC = 35^\circ$ and $\angle CAB = 40^\circ$.

(ii) ∠ACB

(iii) ∠DAE

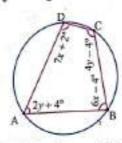


$$= 180^{\circ} - 125^{\circ} = 55^{\circ}$$

$$= 180^{\circ} - 130^{\circ} = 50^{\circ}$$

$$\angle CAE = 180^{\circ} - 120^{\circ} = 60^{\circ}$$

Find all the angles of the given cyclic quadrilateral ABCD in the figure.



In the cyclic quadrilateral
$$\angle A + \angle C = 180^{\circ}$$

$$2y + 4 + 4y - 4' = 180^{\circ}$$

$$6v = 180^{\circ}$$

$$y = \frac{180^{\circ}}{6} = 30^{\circ}$$

ŊΓ

-

e

f

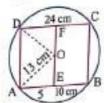
4. AB and CD are two parallel sides of a cyclic quadrilateral ABCD such the AB = 10cm, CD = 24cm and the radius of the circle is 13cm. Find the shape distance between the two sides AB and CD.

Sol. In this figure,

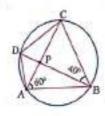
oe,
OE =
$$\sqrt{13^3 - 5^2} = \sqrt{169 - 25} = \sqrt{144} = 12 \text{ cm}$$

OF = $\sqrt{13^2 - 12^2} = \sqrt{169 - 144} = \sqrt{25} = 5 \text{ cm}$

The shortest distance between = 12 + 5 = 17 cm



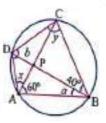
5. In the given figure, ABCD is a cyclic quadrilateral where diagonals intersectate such that ∠DBC = 40° and ∠BAC = 60° find (i) ∠CAD (ii) ∠BCD



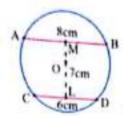
Sol.

$$\angle CAD = \angle CBD = 40^{\circ}$$

(:. In cyclic quadrilateral opposite angles are supplemental



In the given figure, AB and CD are the parallel chords of a circle with centre O. such that AB = 8cm and CD = 6cm. If OM \perp AB and OL \perp CD distance between LM is 7cm. Find the radius of the circle?



In the figure

$$LM = 7 cm$$

Let OM =
$$(7-x)$$
 cm

$$MB = \frac{8}{2} = 4 \text{ cm}$$

OB =
$$\sqrt{4^2 + (7-x)^2}$$

OD =
$$\sqrt{3^2 + x^2}$$

$$\sqrt{16+(7-x)^2} = \sqrt{3^2+x^2}$$

4 M 4 7 cm 0 D

Squaring both sides

$$16 + (7 - x)^2 = 9 + x^2$$

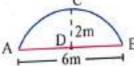
$$16 + 49 - 14x + x^2 = 9 + x^2$$

$$14x = 65 - 9$$

$$14x = 56$$
; $x = \frac{54}{14} = 4$

:. Radius OD =
$$\sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5 \text{ cm}$$

The arch of a bridge has dimensions as shown, where the arch measure 2m at its highest point and its width is 6m. What is the radius of the circle that contains the arch?



If CD = 2 cm and R is the radius, i.e.
$$OC = OA = OB = R$$

$$:: OD = OC - DC = R - 2 \text{ cm}$$

AD = DB =
$$\frac{6}{2}$$
 = 3 cm

(By Pythogoras theore

$$OB^2 = OD^2 + BD^2$$

 $OB^2 = (R-2)^2 + 3^2$

$$OB^{2} = OD^{2} + BD^{2}$$
 $R^{2} = (R-2)^{2} + 3^{2}$
 $R^{e'} = R - 4R + 4 + 9$
 $= R^{e'} - 4R + 13$

$$\Rightarrow$$

$$\Rightarrow$$

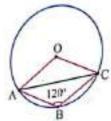
$$4R = 13$$

$$R = \frac{13}{4} = 3.25 \text{ cm}$$



In figure ∠ABC = 120°, where A,B and C are points on the circle with centre

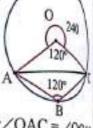
8. Find ∠OAC?



Sol

reflex
$$\angle AOC = 2 \times \angle ABC = 2 \times 120^{\circ} = 240^{\circ}$$

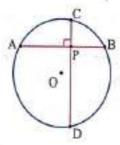
 $\therefore \angle AOC = 360^{\circ} - 240^{\circ} = 120^{\circ}$
Hence $\angle OAC + \angle OCA = 180^{\circ} - 120^{\circ} = 60^{\circ}$
 $2\angle AOC = 60^{\circ}$



$$\angle OAC = \frac{60^{\circ}}{2} = 30^{\circ}$$

[:: <OAC = <000

A school wants to conduct tree plantation programme. For this a teacher allottel 9. a circle of radius 6m ground to nineth standard students for planting sappling Four students plant trees at the points A,B,C and D as shown in figure. Has AB = 8m, CD = 10m and AB \(\perp \) CD. If another student places a flower pot at the point P, the intersection of AB and CD, then find the distance from the centre to !.



Sol. In the figure

$$OA = OD = 6 cm$$

AB = 8 cm (Chord)

OM =
$$\sqrt{6^2 - 4^2}$$
 (: OM bisects the chord and \perp^r to the chord = $\sqrt{36-16} = \sqrt{20}$ cm

ON =
$$\sqrt{6^2 - 5^2} = \sqrt{36 - 25} = \sqrt{11}$$
 cm

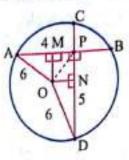
of MPM is a rectangle with all the angles 90° and with length $\sqrt{20}$ cm, breadth $\sqrt{11}$ cm.

need to find OP which is the diagonal of the rectangle ONPM.

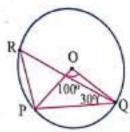
:.OP =
$$\sqrt{ON^2 + NP^2} = \sqrt{\sqrt{11^2 + \sqrt{20^2}}}$$

(: OM = NP, opposite sides of the rectangle)

$$= \sqrt{11+20} = \sqrt{31} = 5.56 \text{ cm} = 5.6 \text{ cm}$$



In the given figure, $\angle POQ=100^{\circ}$ and $\angle PQR=30^{\circ}$, then find $\angle RPO$.



$$\angle PQR = 30^{\circ}$$

$$\angle PQR = \frac{1}{2} \angle POQ = \frac{1}{2} \times 100 = 50^{\circ}$$
 ... (1)

In AOPQ,

$$\angle OPQ = \angle OQP = \angle POQ = 180^{\circ}$$

$$2\angle OPQ + 100 = 180^{\circ}$$

$$(180^{\circ} - 80^{\circ} = 100)$$

∴∠OPQ = 40°

in ΔPRQ,

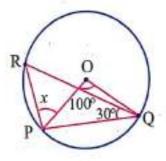
$$\angle R + \angle P + \angle Q = 180^{\circ}$$

$$50^{\circ} + (40 + x) + 30^{\circ} = 180^{\circ}$$

$$(40 + x)^{\circ} = 180^{\circ} - 80 = 100^{\circ}$$

 $(40 + x)^{\circ} = 180^{\circ} - 80 = 100^{\circ}$

$$x^0 = 100^{\circ} - 40^{\circ} = 60^{\circ}$$



4.6

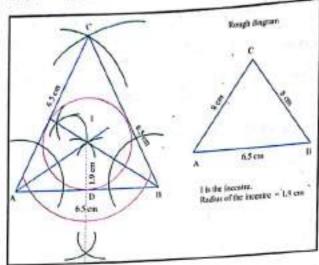
In the first term we have learnt to locate circumcentre and ortho centre of a triangle. Now we are ready to locate in centre and centroid of a triangle. For this we use (i) the construction of perpendicular bisector of a line segment (ii) the construction of angle bisector of a given angle.

Cake Incircle of a Triangle Incentre

Exercise 4.5

Draw an equilateral triangle of side 6.5 cm and locate its incentre. Also draw the incircle.

Sol. Side = 6.5 cm



Construction:

Step 1 : Draw △ABC with AB ≈ BC 3

Step 2: Construct angle bisectors of any two angles (A and B) and let then meet at I. I is the incentre of ΔABC.

Step 3: Draw perpendicular from | to any one of the side (AB) to meet AB at D

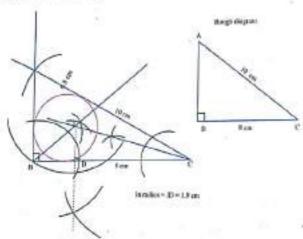
Step 4: With I as centre, ID as radius draw the circle. This circle touches at the sides of triangle internally.

Step 5: Measure in radius. In radius = 1.9 cm.

Draw a right triangle whose hypotenuse is 10 cm and one of the legs is 8 cm.
 Locate its incentre and also draw the incircle.

Sol. hypotenuse = 10 cm

One of the legs = 8 cm



Construction:

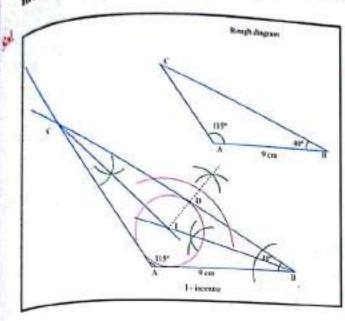
Step 1: Draw \triangle ABC with BC = 8 cm. AC = 10 cm with right angle at B.

Step 2: Construct angle bisectors of any two angles (B and C) and let them meet at I. I is the incentre.

Step 3: Draw perpendicular from I to any side of the triangle to meet BC at D.

Step 4: With I as centre, ID as radius draw the incircle, which touches all the three sides of the triangle internally. In radius = 1.9 cm.

praw AABC given the incircle. (Note: You can check from the above examples that the incentre of any triangle is always in its interior).



Construction:

Step 1: Draw $\triangle ABC$ with AB = 9 cm. $\angle A = 115^{\circ}, \angle B = 40^{\circ}$.

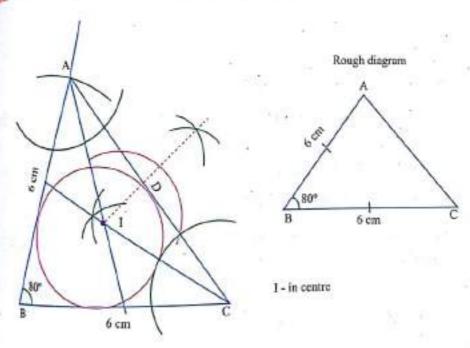
Step 2: Construct angle bisectors of any two angles (B and C). Let them meet at I. I is the incentre of ΔABC.

Step 3: Draw perpendicular from I to any side (BC) to meet BC at D.

Step 4: Draw incircle, with I as centre and ID a radius. Measures the in radius.

Construct $\triangle ABC$ in which AB = BC = 6cm and $B = 80^{\circ}$. Locate its in centre and draw the incircle.

In $\triangle ABC$, AB = BC = 6 cm, $\angle B = 80^{\circ}$.



Construction:

Step 1: Draw $\triangle ABC$ with BC = 6 cm. AB = 6 cm, AB = 6 cm, and $\angle B = 80^{\circ}$.

Step 2: Construct the incentre I and ID is the in radius, as in the previous sums.

Step 3: Draw incircle with I as centre and ID as radius. It touches all the three sides internally.

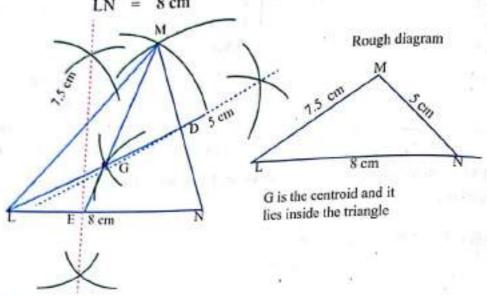
Step 4. Massure in radius In radius = 1.7 cm

Construct the ALMN such that LM = 7.5cm, MN = 5cm and LN = 8cm. its centroid.

Sol. In ALMN

$$MN = 5 cm$$

$$LN = 8 cm$$



Construction:

Step 1: Draw ALMN with LNM = 8 cm, MN = 5 cm, LM = 7.5 cm

Step 2: Construct perpendicular bisectors for any two sides (LN and MN) to fe the mid points of LM and MN.

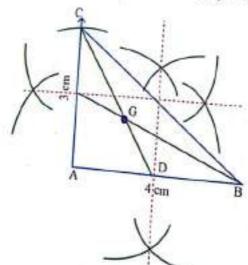
Step 3: Draw the medians LD, ME. Let them meet at G.

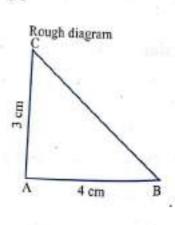
Step 4: G is the centroid of the triangle LMN.

Draw and locate the centroid of the triangle ABC where right angle at a 2. AB = 4cm and AC = 3cm.

Sol. In AABC.

$$AB = 4 \text{ cm}, AC = 3 \text{ cm}, \angle A = 90^{\circ}$$





Construction :

Draw ΔABC with AB = 4 cm, AC = 3 cm, ∠A = 90°

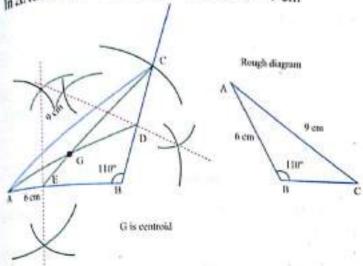
2: Draw perpendicular bisectors of any two sides (AB and AC) to find the mid

3: Draw the medians CD and BE. Let them meet at G.

G is the centroid of the given triangle.

praw the $\triangle ABC$, where AB=6cm, $B=110^{\circ}$ and AC=9cm and construct the satroid.

 $\ln \Delta ABC$, AB = 6 cm, $\angle B$ = 110°, AC = 9 cm



Construction:

Step 1: Draw ΔABC with AB = 6 cm, ∠B =110°, AC = 9 cm

Step 2: Draw perpendicular bisectors of any two sides (BC and AB) to find the mid points of BC and AB.

Step 3: Construct medians AD and CE. Let them meet at G.

Step 4: G is the centroid of the given $\triangle ABC$.

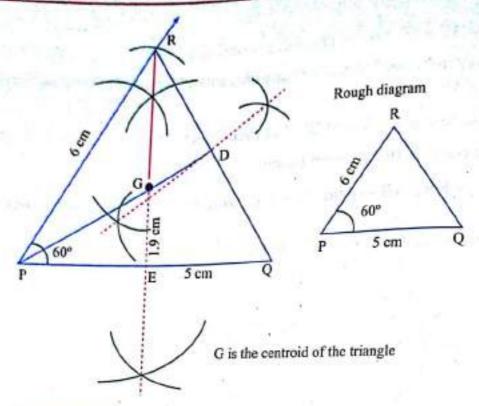
Construct the $\triangle PQR$ such that PQ = 5cm, PR = 6cm and $\angle QPR = 60^{\circ}$ and locate its centroid.

ln ΔPQR.

PQ = 5 cm,

PR = 6 cm

∠OPR = 60°



Construction:

Step 1: Draw APQR with the given measurement

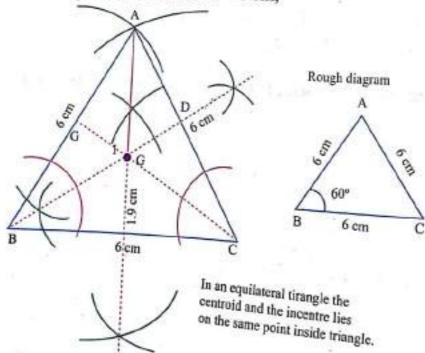
Step 2: Draw perpendicular bisectors of any two sides (PQ and QR) to find the points of PQ and QR.

Step 3: Draw medians PD and RE. Let them meet at G.

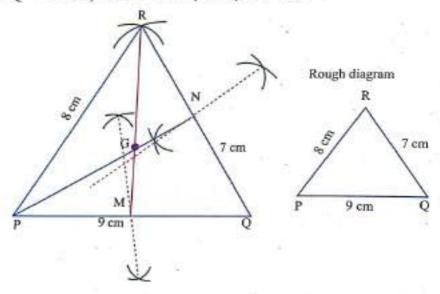
Step 4: G is the centroid of the given $\triangle PQR$.

5. Construct an equilateral triangle of side 6cm and locate its centroid and also incentre. What do you observe from this?

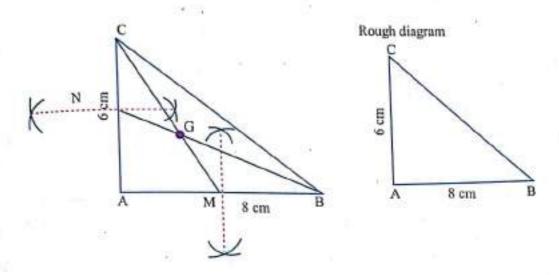
Sol. In an equilateral Δ on side = 6 cm,



Construct the centroid of ΔPQR such that PQ = 9 cm, PQ = 7cm, RP ≈ 8 cm.
 In ΔPQR, PQ = 5 cm, PR = 6 cm, ∠QPR = 60°



priv and locate the centroid of the triangle ABC where right angle at A, ab = 8 cm and AC = 6 cm.



Construction :

- Step 1: Draw ΔABC with the given measurements AB = 8 cm, ∠A = 90° and AC = 6 cm and construct the perpendicular bisector of any two sides (AB and AC) to find the mid points M and N of AB and BC respectively.
- Step 2: Draw the medians (C and BN and let them meet at G. The point G is the centroid of the given ΔABC.