1. Applications of Matrices and Determinants.

* If A is a matrix of order $m \times n$
  $\Rightarrow \rho(A) \leq$ minimum of $\{m, n\}$
* If $|A| = 0 \Rightarrow \rho(A) < n$ [A is order of $n \times n$]
* If $|A| \neq 0 \Rightarrow \rho(A) = n$

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Ex 1.1 Find the rank of each matrices

① (i) $\begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$

Let $A = \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$

order of A is 2×2

$\Rightarrow \rho(A) \leq 2$

Consider second order minor $\begin{vmatrix} 5 & 6 \\ 7 & 8 \end{vmatrix} = 40 - 42$
$= -2 \neq 0$

There is a minor of order 2, which is not zero

$\therefore \rho(A) = 2$

(ii) Let $A = \begin{pmatrix} 1 & -1 \\ 3 & -6 \end{pmatrix}$

order of A is 2×2

$\Rightarrow \rho(A) \leq 2$

Consider second order minor

$\begin{vmatrix} 1 & -1 \\ 3 & -6 \end{vmatrix} = -6 + 3$
$= -3 \neq 0$

There is a minor of order 2 which is not zero

$\therefore \rho(A) = 2$

(iii) Let $A = \begin{pmatrix} 1 & 4 \\ 2 & 8 \end{pmatrix}$

order of A is 2×2

$\Rightarrow \rho(A) \leq 2$

consider second order minor

$\begin{vmatrix} 1 & 4 \\ 2 & 8 \end{vmatrix} = 8 - 8$

$= 0$

Since second order minor vanishes,

$\Rightarrow \rho(A) \neq 2$

consider first order minor $|1| \neq 0$

There is a minor of order 1, which is not zero

$\therefore \rho(A) = 1$

(iv) Let $A = \begin{pmatrix} 2 & -1 & 1 \\ 3 & 1 & -5 \\ 1 & 1 & 1 \end{pmatrix}$

order of A is 3×3

$\Rightarrow \rho(A) \leq 3$

consider third order minor

$\begin{vmatrix} 2 & -1 & 1 \\ 3 & 1 & -5 \\ 1 & 1 & 1 \end{vmatrix} = 2\begin{vmatrix} 1 & -5 \\ 1 & 1 \end{vmatrix} + 1\begin{vmatrix} 3 & -5 \\ 1 & 1 \end{vmatrix} + 1\begin{vmatrix} 3 & 1 \\ 1 & 1 \end{vmatrix}$

$= 2(1+5) + 1(3+5) + 1(3-1)$

$= 12 + 8 + 2$

$= 22 \neq 0$

There is a minor of 3 which is not zero.

$\therefore \rho(A) = 3$

Echelon form and finding the rank of matrix

(iv) Let $A = \begin{pmatrix} 2 & -1 & 1 \\ 3 & 1 & -5 \\ 1 & 1 & 1 \end{pmatrix}$

order of A is 3×3

$\Rightarrow \rho(A) \leq 3$

$R_1 \leftrightarrow R_3$

$\sim \begin{pmatrix} 1 & 1 & 1 \\ 3 & 1 & -5 \\ 2 & -1 & 1 \end{pmatrix}$

$R_2 \to R_2 - 3R_1$

$R_3 \to R_3 - 2R_1$

$\sim \begin{pmatrix} 1 & 1 & 1 \\ 0 & -2 & -8 \\ 0 & -3 & -1 \end{pmatrix}$

$R_2 \to R_2/2$

$\sim \begin{pmatrix} 1 & 1 & 1 \\ 0 & -1 & -4 \\ 0 & -3 & -1 \end{pmatrix}$

$R_3 \to R_3 - 3R_2$

$\sim \begin{pmatrix} 1 & 1 & 1 \\ 0 & -1 & -4 \\ 0 & 0 & 11 \end{pmatrix}$

It is in echelon form

The number of non zero rows is 3

$\therefore \rho(A) = 3$

(v) Let $A = \begin{pmatrix} -1 & 2 & -2 \\ 4 & -3 & 4 \\ -2 & 4 & -4 \end{pmatrix}$

The order of A is 3×3

$\Rightarrow \rho(A) \leq 3$

$R_1 \to R_1/(-1)$

$R_3 \to R_3/(-2)$

$\sim \begin{pmatrix} 1 & -2 & 2 \\ 4 & -3 & 4 \\ 1 & -2 & 2 \end{pmatrix}$

$R_2 \to R_2 - 4R_1$

$R_3 \to R_3 - R_1$

$\sim \begin{pmatrix} 1 & -2 & 2 \\ 0 & 5 & -4 \\ 0 & 0 & 0 \end{pmatrix}$

It is in echelon form

The number of non zero rows is 2

$\therefore \rho(A) = 2$

(vi) Let $A = \begin{pmatrix} 1 & 2 & -1 & 3 \\ 2 & 4 & 1 & -2 \\ 3 & 6 & 3 & -7 \end{pmatrix}$

The order of A is 3×4

$\Rightarrow \rho(A) \leq 3$

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 3R_1$

$$\sim \begin{pmatrix} 1 & 2 & -1 & 3 \\ 0 & 0 & 3 & -8 \\ 0 & 0 & 6 & -16 \end{pmatrix}$$

$R_3 \to R_3 - 2R_2$

$$\sim \begin{pmatrix} 1 & 2 & -1 & 3 \\ 0 & 0 & 3 & -8 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

It is in echelon form

The number of non zero rows is 2

$\therefore \rho(A) = 2$

(vii) Let $A = \begin{pmatrix} 3 & 1 & -5 & -1 \\ 1 & -2 & 1 & -5 \\ 1 & 5 & -7 & 2 \end{pmatrix}$

The order of A is 3×4

$\Rightarrow \rho(A) \leq 3$

$R_1 \leftrightarrow R_3$

$$\sim \begin{pmatrix} 1 & 5 & -7 & 2 \\ 1 & -2 & 1 & -5 \\ 3 & 1 & -5 & -1 \end{pmatrix}$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - 3R_1$

$$\sim \begin{pmatrix} 1 & 5 & -7 & 2 \\ 0 & -7 & 8 & -7 \\ 0 & -14 & 16 & -7 \end{pmatrix}$$

$R_3 \to R_3 - 2R_2$

$$\sim \begin{pmatrix} 1 & 5 & -7 & 2 \\ 0 & -7 & 8 & -7 \\ 0 & 0 & 0 & 7 \end{pmatrix}$$

It is in echelon form

The number of non zero rows is 3

$\therefore \rho(A) = 3$

(viii) Let $A = \begin{pmatrix} 1 & -2 & 3 & 4 \\ -2 & 4 & -1 & -3 \\ -1 & 2 & 7 & 6 \end{pmatrix}$

The order of A is 3×4

$\therefore \rho(A) \leq 3$

$R_2 \to R_2 + 2R_1$

$R_3 \to R_3 + R_1$

$$\sim \begin{pmatrix} 1 & -2 & 3 & 4 \\ 0 & 0 & 5 & 5 \\ 0 & 0 & 10 & 10 \end{pmatrix}$$

$R_3 \to R_3 - 2R_2$

$$\sim \begin{pmatrix} 1 & -2 & 3 & 4 \\ 0 & 0 & 5 & 5 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

It is in echelon form

The number of non zero rows is 2

$\therefore \rho(A) = 2$

(2) If $A = \begin{pmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -2 & 3 \\ -2 & 4 & -6 \\ 5 & 1 & -1 \end{pmatrix}$

then find the rank of AB and the rank of BA.

Soln

$$AB = \begin{pmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{pmatrix} \begin{pmatrix} 1 & -2 & 3 \\ -2 & 4 & -6 \\ 5 & 1 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 1-2-5 & -2+4-1 & 3-6+1 \\ 2+6+20 & -4-12+4 & 6+18-4 \\ 3+4+15 & -6-8+3 & 9+12-3 \end{pmatrix}$$

$$AB = \begin{pmatrix} -6 & 1 & -2 \\ 28 & -12 & 20 \\ 22 & -11 & 18 \end{pmatrix}$$

order of AB is 3×3

$\Rightarrow \rho(AB) \leq 3$.

Consider third order minor

$$\begin{vmatrix} -6 & 1 & -2 \\ 28 & -12 & 20 \\ 22 & -11 & 18 \end{vmatrix} = -6 \begin{vmatrix} -12 & 20 \\ -11 & 18 \end{vmatrix} - 1 \begin{vmatrix} 28 & 20 \\ 22 & 18 \end{vmatrix} - 2 \begin{vmatrix} 28 & -12 \\ 22 & -11 \end{vmatrix}$$

$$= -6(-216+220) - 1(504-440) - 2(-308+264)$$

$= -6(4) -1(64) -2(-44)$
$= -24 - 64 + 88$

$|AB| = 0$

Since third order minor vanishes

$\Rightarrow \rho(AB) \neq 3$

consider second order minor

$$\begin{vmatrix} 1 & -2 \\ -12 & 20 \end{vmatrix} = 20-24$$
$$= -4 \neq 0$$

There is a minor of order 2 which is not zero

$\therefore \rho(AB) = 2$

$$BA = \begin{pmatrix} 1 & -2 & 3 \\ -2 & 4 & -6 \\ 5 & 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 1-4+9 & 1+6-6 & -1-8+9 \\ -2+8-18 & -2-12+12 & 2+16-18 \\ 5+2-3 & 5-3+2 & -5+4-3 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & 1 & 0 \\ -12 & -2 & 0 \\ 4 & 4 & -4 \end{pmatrix}$$

order of BA is 3×3

$\Rightarrow \rho(BA) \leq 3$

Consider third order minor

$$\begin{vmatrix} 6 & 1 & 0 \\ -12 & -2 & 0 \\ 4 & 4 & -4 \end{vmatrix} = 6\begin{vmatrix} -2 & 0 \\ 4 & -4 \end{vmatrix} - 1\begin{vmatrix} -12 & 0 \\ 4 & -4 \end{vmatrix} + 0\begin{vmatrix} -12 & -2 \\ 4 & 4 \end{vmatrix}$$

$$= 6(8-0) - 1(48-0) + 0$$

$$= 6(8) - 1(48)$$

$$= 48 - 48$$

$$= 0$$

since third order minor vanishes

$\Rightarrow \rho(BA) \neq 3$

consider second order minor

$$\begin{vmatrix} -2 & 0 \\ 4 & -4 \end{vmatrix} = 8 - 0$$

$$= 8 \neq 0$$

There is a minor of order 2 which is not zero

$\therefore \rho(BA) = 2$

Testing the consistency of non homogeneous linear equations by rank method

* If $\rho(A, B) = \rho(A) = n$

$\Rightarrow$ The given equations are consistent with unique solution.

* If $\rho(A, B) = \rho(A) < n$

$\Rightarrow$ The given equations are consistent with Infinitely many solutions.

* If $\rho(A, B) \neq \rho(A)$

$\Rightarrow$ The given equations are inconsistent with no solution.

(3) Solve the following system of equations by rank method

$x+y+z=9$, $2x+5y+7z=52$,
$2x-y-z=0$

Soln

The matrix equation is

$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & 5 & 7 \\ 2 & -1 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 9 \\ 52 \\ 0 \end{pmatrix}$$

$$A \quad X = B$$

The Augmented matrix is

$$(A,B) = \begin{pmatrix} 1 & 1 & 1 & 9 \\ 2 & 5 & 7 & 52 \\ 2 & -1 & -1 & 0 \end{pmatrix}$$

$R_2 \rightarrow R_2 - 2R_1$
$R_3 \rightarrow R_3 - 2R_1$

$$\sim \begin{pmatrix} 1 & 1 & 1 & 9 \\ 0 & 3 & 5 & 34 \\ 0 & -3 & -3 & -18 \end{pmatrix}$$

$R_3 \rightarrow R_3 + R_2$

$$\sim \begin{pmatrix} 1 & 1 & 1 & 9 \\ 0 & 3 & 5 & 34 \\ 0 & 0 & 2 & 16 \end{pmatrix}$$

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It is in echelon form.

$\rho(A,B) = 3$, $\rho(A) = 3$, $n = 3$

Since $\rho(A,B) = \rho(A) = n$

$\Rightarrow$ The given system is consistent with unique solution.

The given system equivalent to matrix equation.

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 3 & 5 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 9 \\ 34 \\ 16 \end{pmatrix}$$

$x + y + z = 9 \rightarrow (1)$

$3y + 5z = 34 \rightarrow (2)$

$2z = 16$

$z = 8$

(2) $\Rightarrow$ $3y+5(8)=34$

$3y+40=34$

$3y=34-40$

$3y=-6$

$y=-2$

(1) $\Rightarrow$ $x-2+8=9$

$x+6=9$

$x=3$

$\therefore$ $x=3,\ y=-2,\ z=8$

(4) Show that the equations $5x+3y+7z=4$, $3x+26y+2z=9$ and $7x+2y+10z=5$ are consistent and solve them by rank method.

Soln The matrix equation is

$$\begin{pmatrix} 5 & 3 & 7 \\ 3 & 26 & 2 \\ 7 & 2 & 10 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 9 \\ 5 \end{pmatrix}$$

$$A \quad X = B$$

The Augmented matrix is

$$(A|B) = \begin{pmatrix} 5 & 3 & 7 & 4 \\ 3 & 26 & 2 & 9 \\ 7 & 2 & 10 & 5 \end{pmatrix}$$

$R_2 \to 5R_2 - 3R_1$

$R_3 \to 5R_3 - 7R_1$

$$\sim \begin{pmatrix} 5 & 3 & 7 & 4 \\ 0 & 121 & -11 & 33 \\ 0 & -11 & 1 & -3 \end{pmatrix}$$

$R_3 \to 11R_3 + R_2$

$$\sim \begin{pmatrix} 5 & 3 & 7 & 4 \\ 0 & 121 & -11 & 33 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

It is in echelon form

$\rho(A, B) = 2, \ \rho(A) = 2, \ n = 3$

since $\rho(A, B) = \rho(A) < n$

$\Rightarrow$ The given system is consistent with infinitely many solution.

The given system equivalent to matrix equation.

$$\begin{pmatrix} 5 & 3 & 7 \\ 0 & 121 & -11 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 33 \\ 0 \end{pmatrix}$$

$5x + 3y + 7z = 4 \rightarrow (1)$

$121y - 11z = 33 \rightarrow (2)$

put $z = k$

$(2) \Rightarrow 121y - 11k = 33$

$121y = 33 + 11k$

$\div 11$

$11y = 3 + k$

$y = \dfrac{3+k}{11}$

$$(1) \Rightarrow 5x + 3\left(\frac{3+k}{11}\right) + 7k = 4$$

$$5x + \frac{9+3k}{11} + 7k = 4$$

$$5x + \frac{9+3k+77k}{11} = 4$$

$$5x + \frac{9+80k}{11} = 4$$

$$5x = 4 - \frac{9+80k}{11}$$

$$5x = \frac{44-9-80k}{11}$$

$$5x = \frac{35-80k}{11}$$

$$x = \frac{1}{5}\left(\frac{35-80k}{11}\right)$$

$$x = \frac{7-16k}{11}$$

$$(x, y, z) = \left(\frac{7-16k}{11}, \frac{3+k}{11}, k\right), \ k \in R$$

$$(x, y, z) = \left[\frac{1}{11}(7-16k), \frac{1}{11}(3+k), k\right], k \in R$$

(5) Show that the following system of equations have unique solutions.

$x+y+z=3$, $x+2y+3z=4$

$x+4y+9z=6$ by rank method.

Sln

The matrix equation is

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 6 \end{pmatrix}$$

$$A \quad X = B$$

The Augmented matrix is

$$(A, B) = \begin{pmatrix} 1 & 1 & 1 & 3 \\ 1 & 2 & 3 & 4 \\ 1 & 4 & 9 & 6 \end{pmatrix}$$

$R_2 \to R_2 - R_1$
$R_3 \to R_3 - R_1$

$$\sim \begin{pmatrix} 1 & 1 & 1 & 3 \\ 0 & 1 & 2 & 1 \\ 0 & 3 & 8 & 3 \end{pmatrix}$$

$R_3 \to R_3 - 3R_2$

$$\sim \begin{pmatrix} 1 & 1 & 1 & 3 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 2 & 0 \end{pmatrix}$$

It is in echelon form

$\rho(A, B) = 3, \ \rho(A) = 3, \ n = 3.$

Since $\rho(A, B) = \rho(A) = n$

$\Rightarrow$ The given system is consistent with unique solution.

The given system is equivalent to matrix equation.

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}$$

$x + y + z = 3 \to (1)$

$y + 2z = 1 \to (2)$

$2z = 0$

$z = 0$

$(2) \Rightarrow$

$y + 2(0) = 1$

$y = 1$

$(1) \Rightarrow \quad x+1+0 = 3$

$x+1=3$

$x=2$

$\therefore \quad x=2, \; y=1, \; z=0$

(b) For what values of the parameter λ, will the following equations fail to have unique solution: $3x-y+\lambda z=1$, $2x+y+z=2$, $x+2y-\lambda z=-1$ by rank method.

Soln

The matrix equation is,

$$\begin{pmatrix} 3 & -1 & \lambda \\ 2 & 1 & 1 \\ 1 & 2 & -\lambda \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$$

$$A \qquad X = B$$

The Augmented matrix is

$$(A|B) = \begin{pmatrix} 3 & -1 & \lambda & 1 \\ 2 & 1 & 1 & 2 \\ 1 & 2 & -\lambda & -1 \end{pmatrix}$$

$R_1 \leftrightarrow R_3$

$$\sim \begin{pmatrix} 1 & 2 & -\lambda & -1 \\ 2 & 1 & 1 & 2 \\ 3 & -1 & \lambda & 1 \end{pmatrix}$$

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 3R_1$

$$\sim \begin{pmatrix} 1 & 2 & -\lambda & -1 \\ 0 & -3 & 1+2\lambda & 4 \\ 0 & -7 & 4\lambda & 4 \end{pmatrix}$$

$$R_3 \to 3R_3 - 7R_2$$

$$\sim \begin{pmatrix} 1 & 2 & -\lambda & -1 \\ 0 & -3 & 1+2\lambda & 4 \\ 0 & 0 & -2\lambda-7 & -16 \end{pmatrix}.$$

It is in echelon form

Since given system fail to have unique solution

$\Rightarrow \rho(A|B) = \rho(A) < n$

(or)

$\rho(A|B) \neq \rho(A)$

From echelon form $\rho(A|B) = 3$

but $\rho(A) \neq 3$

$\Rightarrow -2\lambda - 7 = 0$

$-2\lambda = 7$

$\lambda = -7/2$

(7). The price of three commodities X, Y and Z are x, y and z respectively. Mr. Anand purchases 6 units of Z and sells 2 units of X and 3 units of Y. Mr. Aman purchases a unit of Y and sells 3 units of X and 2 units of Z. Mr. Amit purchases a unit of X and sells 3 units of Y and a unit of Z. In the process they earn ₹ 5000 ₹ 2000 and ₹ 5500 respectively. Find the prices per unit of three commodities by rank method.

Soln

Anand $2x+3y-6z=5000$

Amar $3x-y+2z=2000$

Amit $-x+3y+z=5500$

The matrix equation is

$$\begin{pmatrix} 2 & 3 & -6 \\ 3 & -1 & 2 \\ -1 & 3 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{pmatrix} 5000 \\ 2000 \\ 5500 \end{pmatrix}$$

$$A \quad X = B$$

The Augmented matrix is

$$(A,B) = \begin{pmatrix} 2 & 3 & -6 & 5000 \\ 3 & -1 & 2 & 2000 \\ -1 & 3 & 1 & 5500 \end{pmatrix}$$

$R_3 \to R_3(-1)$

$$\sim \begin{pmatrix} 2 & 3 & -6 & 5000 \\ 3 & -1 & 2 & 2000 \\ 1 & -3 & -1 & -5500 \end{pmatrix}$$

$R_1 \leftrightarrow R_3$

$$\sim \begin{pmatrix} 1 & -3 & -1 & -5500 \\ 3 & -1 & 2 & 2000 \\ 2 & 3 & -6 & 5000 \end{pmatrix}$$

$R_2 \to R_2 - 3R_1$

$R_3 \to R_3 - 2R_1$

$$\sim \begin{pmatrix} 1 & -3 & -1 & -5500 \\ 0 & 8 & 5 & 18500 \\ 0 & 9 & -4 & 16000 \end{pmatrix}$$

$R_3 \to 8R_3 - 9R_2$

$$\sim \begin{pmatrix} 1 & -3 & -1 & -5500 \\ 0 & 8 & 5 & 18500 \\ 0 & 0 & -77 & -38500 \end{pmatrix}$$

It is in echelon form

$\rho(A|B) = 3$, $\rho(A) = 3$, $n = 3$

Since $\rho(A|B) = \rho(A) = n$

$\Rightarrow$ The given system is consistent with unique solution.

$$\begin{pmatrix} 1 & -3 & -1 \\ 0 & 8 & 5 \\ 0 & 0 & -77 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -5500 \\ 18500 \\ -38500 \end{pmatrix}$$

$x - 3y - z = -5500 \to (1)$

$8y + 5z = 18500 \to (2)$

$-77z = -38500$

$z = \frac{38500}{77}$

$z = 500$

$(2) \Rightarrow 8y + 5(500) = 18500$

$8y + 2500 = 18500$

$8y = 18500 - 2500$

$8y = 16000$

$y = \frac{16000}{8}$

$y = 2000$

$$(1) \Rightarrow x - 3(2000) - 500 = -5500$$

$$x - 6000 - 500 = -5500$$

$$x - 6500 = -5500$$

$$x = -5500 + 6500$$

$$x = 1000$$

$\therefore x = 1000,\ y = 2000,\ z = 500$

(8). An Amount of ₹ 5000 is ~~to~~ be deposited in three different bonds bearing 6%, 7% and 8% per year respectively. Total annual income is ₹ 358. If the income from first two investments is ₹ 70 more than the income from the third then find the amount of investment in each bond by rank method.

Sln

Let $x, y, \& z$ be three amount which deposited in bonds bearing 6%, 7% & 8%.

$$x + y + z = 5000 \rightarrow (I)$$

$$6\% x + 7\% y + 8\% z = 358$$

$$\Rightarrow \frac{6}{100}x + \frac{7}{100}y + \frac{8}{100}z = 358$$

$\times(100)$

$$6x + 7y + 8z = 35800 \rightarrow (II)$$

$$\frac{6}{100}x + \frac{7}{100}y = \frac{8}{100}z + 70$$

$$\frac{6}{100}x + \frac{7}{100}y - \frac{8}{100}z = 70$$

$$6x + 7y - 8z = 7000 \rightarrow \text{(iii)}$$

The matrix equation is

$$\begin{pmatrix} 1 & 1 & 1 \\ 6 & 7 & 8 \\ 6 & 7 & -8 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5000 \\ 35800 \\ 7000 \end{pmatrix}$$

$$A \quad X = B$$

The Augmented matrix is

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$$(A, B) = \begin{pmatrix} 1 & 1 & 1 & 5000 \\ 6 & 7 & 8 & 35800 \\ 6 & 7 & -8 & 7000 \end{pmatrix}$$

$R_2 \rightarrow R_2 - 6R_1$

$R_3 \rightarrow R_3 - 6R_1$

$$\sim \begin{pmatrix} 1 & 1 & 1 & 5000 \\ 0 & 1 & 2 & 5800 \\ 0 & 1 & -14 & -23000 \end{pmatrix}$$

$R_3 \rightarrow R_3 - R_2$

$$\sim \begin{pmatrix} 1 & 1 & 1 & 5000 \\ 0 & 1 & 2 & 5800 \\ 0 & 0 & -16 & -28800 \end{pmatrix}$$

It is in echelon form

$\rho(A, B) = 3, \; \rho(A) = 3, \; n = 3$

Since $\rho(A, B) = \rho(A) = n$

$\Rightarrow$ The given system is consistent with unique solution.

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & -16 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5000 \\ 5800 \\ -28800 \end{pmatrix}$$

$x+y+z = 5000 \rightarrow (1)$

$y+2z = 5800 \rightarrow (2)$

$-16z = -28800$

$z = \frac{28800}{16}$

$z = 1800$

$(2) \Rightarrow y + 2(1800) = 5800$

$y + 3600 = 5800$

$y = 5800 - 3600$

$y = 2200$

$(1) \Rightarrow x + 2200 + 1800 = 5000$

$x + 4000 = 5000$

$x = 5000 - 4000$

$x = 1000$

$\therefore x = 1000, \quad y = 2200, \quad z = 1800$

Cramer's Rule

* If $\Delta \neq 0 \Rightarrow$ Cramer's Rule applicaple

* $x = \frac{\Delta x}{\Delta}, \ y = \frac{\Delta y}{\Delta}, \ z = \frac{\Delta z}{\Delta}$

Ex 1.2 Solve by cramer's rule

1. (i) $2x + 3y = 7, \ 3x + 5y = 9$

$$\Delta = \begin{vmatrix} 2 & 3 \\ 3 & 5 \end{vmatrix}$$

$= 10 - 9$

$= 1 \neq 0$

$\therefore$ we can apply cramer's Rule.

$$\Delta x = \begin{vmatrix} 7 & 3 \\ 9 & 5 \end{vmatrix}$$

$= 35 - 27$

$\Delta x = 8$

$$\Delta y = \begin{vmatrix} 2 & 7 \\ 3 & 9 \end{vmatrix}$$

$= 18 - 21$

$= -3$

$x = \frac{\Delta x}{\Delta} = \frac{8}{1} = 8$

$y = \frac{\Delta y}{\Delta} = \frac{-3}{1} = -3$

$\therefore x = 8, y = -3$

(ii) $5x + 3y = 17 \ ; \ 3x + 7y = 31$

$$\Delta = \begin{vmatrix} 5 & 3 \\ 3 & 7 \end{vmatrix}$$

$= 35 - 9$

$= 26 \neq 0$

$\therefore$ we can apply Cramer's Rule.

$$\Delta x = \begin{vmatrix} 17 & 3 \\ 31 & 7 \end{vmatrix}$$
$$= 119 - 93$$
$$\Delta x = 26$$

$$\Delta y = \begin{vmatrix} 5 & 17 \\ 3 & 31 \end{vmatrix}$$
$$= 155 - 51$$
$$= 104$$

$$x = \frac{\Delta x}{\Delta} = \frac{26}{26} = 1$$

$$y = \frac{\Delta y}{\Delta} = \frac{104}{26} = 4$$

$\therefore x = 1, y = 4$

(iii) $2x + y - z = 3$, $x + y + z = 1$, $x - 2y - 3z = 4$

$$\Delta = \begin{vmatrix} 2 & 1 & -1 \\ 1 & 1 & 1 \\ 1 & -2 & -3 \end{vmatrix}$$

$$= 2\begin{vmatrix} 1 & 1 \\ -2 & -3 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 1 & -3 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix}$$

$$= 2(-3+2) - 1(-3-1) - 1(-2-1)$$

$$= -2 + 4 + 3$$

$$\Delta = 5 \neq 0$$

we can apply Cramer's rule

$$\Delta x = \begin{vmatrix} 3 & 1 & -1 \\ 1 & 1 & 1 \\ 4 & -2 & -3 \end{vmatrix}$$

$$= 3\begin{vmatrix} 1 & 1 \\ -2 & -3 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 4 & -3 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 4 & -2 \end{vmatrix}$$

$$= 3(-3+2) - 1(-3-4) - 1(-2-4)$$

$$= -3 + 7 + 6$$

$$\Delta x = 10$$

$$\Delta y = \begin{vmatrix} 2 & 3 & -1 \\ 1 & 1 & 1 \\ 1 & 4 & -3 \end{vmatrix}$$

$$= 2\begin{vmatrix} 1 & 1 \\ 4 & -3 \end{vmatrix} - 3\begin{vmatrix} 1 & 1 \\ 1 & -3 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 1 & 4 \end{vmatrix}$$

$$= 2(-3-4) - 3(-3-1) - 1(4-1)$$

$$= \cancel{-8 + 6 - 3}$$

$$= -14 + 12 - 3$$

$$\Delta y = -5$$

$$\Delta z = \begin{vmatrix} 2 & 1 & 3 \\ 1 & 1 & 1 \\ 1 & -2 & 4 \end{vmatrix}$$

$$= 2\begin{vmatrix} 1 & 1 \\ -2 & 4 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 1 & 4 \end{vmatrix} + 3\begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix}$$

$$= 2(4+2) - 1(4-1) + 3(-2-1)$$

$$= 12 - 3 - 9$$

$$\Delta z = 0$$

$$x = \frac{\Delta x}{\Delta} = \frac{10}{5} = 2$$

$$y = \frac{\Delta y}{\Delta} = \frac{-5}{5} = -1$$

$$z = \frac{\Delta z}{\Delta} = \frac{0}{5} = 0$$

$$\therefore x = 2,\ y = -1,\ z = 0$$

(iv) $x+y+z=6$, $2x+3y-z=5$

$6x-2y-3z=-7$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & -1 \\ 6 & -2 & -3 \end{vmatrix}$$

$$= 1\begin{vmatrix} 3 & -1 \\ -2 & -3 \end{vmatrix} - 1\begin{vmatrix} 2 & -1 \\ 6 & -3 \end{vmatrix} + 1\begin{vmatrix} 2 & 3 \\ 6 & -2 \end{vmatrix}$$

$$= 1(-9-2) - 1(-6+6) + 1(-4-18)$$

$$= -11 - 0 - 22$$

$\Delta = -33 \neq 0$ $\therefore$ we can apply Cramer's rule

$$\Delta x = \begin{vmatrix} 6 & 1 & 1 \\ 5 & 3 & -1 \\ -7 & -2 & -3 \end{vmatrix}$$

$$= 6\begin{vmatrix} 3 & -1 \\ -2 & -3 \end{vmatrix} - 1\begin{vmatrix} 5 & -1 \\ -7 & -3 \end{vmatrix} + 1\begin{vmatrix} 5 & 3 \\ -7 & -2 \end{vmatrix}$$

$$= 6(-9-2) - 1(-15-7) + 1(-10+21)$$

$$= -66 + 22 + 11$$

$\Delta x = -33$

$$\Delta y = \begin{vmatrix} 1 & 6 & 1 \\ 2 & 5 & -1 \\ 6 & -7 & -3 \end{vmatrix}$$

$$= 1\begin{vmatrix} 5 & -1 \\ -7 & -3 \end{vmatrix} - 6\begin{vmatrix} 2 & -1 \\ 6 & -3 \end{vmatrix} + 1\begin{vmatrix} 2 & 5 \\ 6 & -7 \end{vmatrix}$$

$$= 1(-15-7) - 6(-6+6) + 1(-14-30)$$

$$= -22 - 0 - 44$$

$\Delta y = -66$

$$\Delta z = \begin{vmatrix} 1 & 1 & 6 \\ 2 & 3 & 5 \\ 6 & -2 & -7 \end{vmatrix}$$

$$= 1\begin{vmatrix} 3 & 5 \\ -2 & -7 \end{vmatrix} - 1\begin{vmatrix} 2 & 5 \\ 6 & -7 \end{vmatrix} + 6\begin{vmatrix} 2 & 3 \\ 6 & -2 \end{vmatrix}$$

$$= 1(-21+10) - 1(-14-30) + 6(-4-18)$$

$$= -11 + 44 - 132$$

$$\Delta z = -99$$

$$x = \frac{\Delta x}{\Delta} = \frac{-33}{-33} = 1$$

$$y = \frac{\Delta y}{\Delta} = \frac{-66}{-33} = 2$$

$$z = \frac{\Delta z}{\Delta} = \frac{-99}{-33} = 3$$

$\therefore x = 1, y = 2, z = 3$

(v) $x + 4y + 3z = 2, \; 2x - 6y + 6z = -3$

$5x - 2y + 3z = -5$

$$\Delta = \begin{vmatrix} 1 & 4 & 3 \\ 2 & -6 & 6 \\ 5 & -2 & 3 \end{vmatrix}$$

$$= 1\begin{vmatrix} -6 & 6 \\ -2 & 3 \end{vmatrix} - 4\begin{vmatrix} 2 & 6 \\ 5 & 3 \end{vmatrix} + 3\begin{vmatrix} 2 & -6 \\ 5 & -2 \end{vmatrix}$$

$$= 1(-18+12) - 4(6-30) + 3(-4+30)$$

$$= -6 + 96 + 78$$

$$\Delta = 168 \neq 0$$

$\therefore$ we can apply Cramer's rule.

$$\Delta x = \begin{vmatrix} 2 & 4 & 3 \\ -3 & -6 & 6 \\ -5 & -2 & 3 \end{vmatrix}$$

$$= 2\begin{vmatrix} -6 & 6 \\ -2 & 3 \end{vmatrix} - 4\begin{vmatrix} -3 & 6 \\ -5 & 3 \end{vmatrix} + 3\begin{vmatrix} -3 & -6 \\ -5 & -2 \end{vmatrix}$$

$$= 2(-18+12) - 4(-9+30) + 3(6-30)$$

$$= -12 - 84 - 72$$

$$\Delta x = -168$$

$$\Delta y = \begin{vmatrix} 1 & 2 & 3 \\ 2 & -3 & 6 \\ 5 & -5 & 3 \end{vmatrix}$$

$$= 1\begin{vmatrix} -3 & 6 \\ -5 & 3 \end{vmatrix} - 2\begin{vmatrix} 2 & 6 \\ 5 & 3 \end{vmatrix} + 3\begin{vmatrix} 2 & -3 \\ 5 & -5 \end{vmatrix}$$

$$= 1(-9+30) - 2(6-30) + 3(-10+15)$$

$$= 21 + 48 + 15$$

$$\Delta y = 84$$

$$\Delta z = \begin{vmatrix} 1 & 4 & 2 \\ 2 & -6 & -3 \\ 5 & -2 & -5 \end{vmatrix}$$

$$= 1\begin{vmatrix} -6 & -3 \\ -2 & -5 \end{vmatrix} - 4\begin{vmatrix} 2 & -3 \\ 5 & -5 \end{vmatrix} + 2\begin{vmatrix} 2 & -6 \\ 5 & -2 \end{vmatrix}$$

$$= 1(30-6) - 4(-10+15) + 2(-4+30)$$

$$= 24 - 20 + 52$$

$$\Delta z = 56$$

$$x = \frac{\Delta x}{\Delta} = \frac{-168}{168} = -1$$

$$y = \frac{\Delta y}{\Delta} = \frac{84}{168} = \frac{1}{2}$$

$$z = \frac{\Delta z}{\Delta} = \frac{56}{168} = \frac{1}{3}$$

$\therefore x = -1,\ y = \frac{1}{2},\ z = \frac{1}{3}$

(2) A Commodity was produced by using 3 units of labour and 2 units of capital, the total cost is ₹ 62. If the commodity had been produced by using 4 units of labour and one unit of capital, the cost is ₹ 56. what is the cost per unit of labour and capital? (Use determinant method).

Soln Let x be cost per unit of labour
y be cost per unit of capital.

$$3x + 2y = 62$$
$$4x + y = 56$$

$$\Delta = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$$
$$= 3 - 8$$
$$= -5 \neq 0$$

$\therefore$ we can apply Cramer's rule

$$\Delta x = \begin{vmatrix} 62 & 2 \\ 56 & 1 \end{vmatrix}$$
$$= 62 - 112$$
$$\Delta x = -50$$

$$\Delta y = \begin{vmatrix} 3 & 62 \\ 4 & 56 \end{vmatrix}$$

$$= 168 - 248$$

$$\Delta y = -80$$

$$x = \frac{\Delta x}{\Delta} = \frac{-50}{-5} = 10$$

$$y = \frac{\Delta y}{\Delta} = \frac{-80}{-5} = 16$$

$\therefore$ cost of one unit of labour = ₹10

cost of one unit of capital = ₹16

(3). A total of ₹ 8600 was invested in two accounts. One account earned $4\frac{3}{4}$% annual interest and the other earned $6\frac{1}{2}$% annual interest. If the total interest for one year was ₹ 431.25, how much was invested in each ~~other~~ account? (Use determinant method)

Soln

Let x & y be amount invested in two accounts.

$$x + y = 8600$$

$$4\frac{3}{4}\% \cdot x + 6\frac{1}{2}\% \cdot y = 431.25$$

$$\frac{19}{4}\% \cdot x + \frac{13}{2}\% \cdot y = 431.25$$

$$\frac{19}{400}x + \frac{13}{200}y = 431.25$$

$$\frac{19x + 26y}{400} = 431.25$$

$$19x + 26y = 172500$$

$$\Delta = \begin{vmatrix} 1 & 1 \\ 19 & 26 \end{vmatrix}$$

$$= 26 - 19$$

$$\Delta = 7 \neq 0$$

$\therefore$ we can apply cramer's rule

$$\Delta_x = \begin{vmatrix} 8600 & 1 \\ 172500 & 26 \end{vmatrix}$$

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$$= 223600 - 172500$$

$$\Delta_x = 51100$$

$$\Delta_y = \begin{vmatrix} 1 & 8600 \\ 19 & 172500 \end{vmatrix}$$

$$= 172500 - 163400$$

$$\Delta_y = 9100$$

$$x = \frac{\Delta_x}{\Delta} = \frac{51100}{7} = 7300$$

$$y = \frac{\Delta_y}{\Delta} = \frac{9100}{7} = 1300$$

$\therefore$ Amount invested at $4\frac{3}{4}\% = ₹\ 7300$

Amount invested at $6\frac{1}{2}\% = ₹\ 1300$

(4) At marina two types of games viz., Horse riding and Quad Bikes riding are available on hourly rent. Keren and Benita spent ₹ 780 and ₹ 560 during the month of may.

Name	Number of hours		Total amount spent (in ₹)
	Horse Riding	Quad Bike Riding	
Keren	3	4	780
Benita	2	3	560

Find the hourly charges for the two games (rides) (use determinant method)

Soln

Let x be Horse Riding charge
y be Quad Bike Riding charge

$3x+4y = 780$

$2x+3y = 560$

$$\Delta = \begin{vmatrix} 3 & 4 \\ 2 & 3 \end{vmatrix}$$

$= 9-8$

$= 1 \neq 0$

$\therefore$ we can apply cramer's rule

$$\Delta x = \begin{vmatrix} 780 & 4 \\ 560 & 3 \end{vmatrix}$$

$= 2340 - 2240$

$\Delta x = 100$

$$\Delta y = \begin{vmatrix} 3 & 780 \\ 2 & 560 \end{vmatrix}$$

$$= 1680 - 1560$$

$$\Delta y = 120$$

$$x = \frac{\Delta x}{\Delta} = \frac{100}{1} = 100$$

$$y = \frac{\Delta y}{\Delta} = \frac{120}{1} = 120$$

∴ Horse Riding charge per hour = ₹ 100

Quad Bike Riding charge per hour = ₹ 120

(5) In a market survey three Commodities A, B and C were considered. In finding out the index number some fixed weights were assigned to the three varieties in each of the commodities. The table below provides the information regarding the consumption of three commodities according to the three varities and also the total weight received by the commodity.

Commodity Variety	Variety I	Variety II	Variety III	Total weight
A	1	2	3	11
B	2	4	5	21
C	3	5	6	27

Find the weights assigned to the three varieties by using Cramer's Rule.

Let x, y & z be weights assigned to variety I, II & III

$x+2y+3z=11$

$2x+4y+5z=21$

$3x+5y+6z=27$

$$\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{vmatrix}$$

$$= 1\begin{vmatrix} 4 & 5 \\ 5 & 6 \end{vmatrix} - 2\begin{vmatrix} 2 & 5 \\ 3 & 6 \end{vmatrix} + 3\begin{vmatrix} 2 & 4 \\ 3 & 5 \end{vmatrix}$$

$$= 1(24-25) - 2(12-15) + 3(10-12)$$

$$= -1 + 6 - 6$$

$\Delta = -1 \neq 0$

$\therefore$ we can apply cramer's rule

$$\Delta x = \begin{vmatrix} 11 & 2 & 3 \\ 21 & 4 & 5 \\ 27 & 5 & 6 \end{vmatrix}$$

$$= 11\begin{vmatrix} 4 & 5 \\ 5 & 6 \end{vmatrix} - 2\begin{vmatrix} 21 & 5 \\ 27 & 6 \end{vmatrix} + 3\begin{vmatrix} 21 & 4 \\ 27 & 5 \end{vmatrix}$$

$$= 11(24-25) - 2(126-135) + 3(105-108)$$

$$= -11 + 18 - 9$$

$\Delta x = -2$

$$\Delta y = \begin{vmatrix} 1 & 11 & 3 \\ 2 & 21 & 5 \\ 3 & 27 & 6 \end{vmatrix}$$

$$= 1\begin{vmatrix} 21 & 5 \\ 27 & 6 \end{vmatrix} - 11\begin{vmatrix} 2 & 5 \\ 3 & 6 \end{vmatrix} + 3\begin{vmatrix} 2 & 21 \\ 3 & 27 \end{vmatrix}$$

$= 1(126-135) - 11(12-15) + 3(54-63)$

$= -9 + 33 - 27$

$\Delta y = -3$

$$\Delta z = \begin{vmatrix} 1 & 2 & 11 \\ 2 & 4 & 21 \\ 3 & 5 & 27 \end{vmatrix}$$

$$= 1\begin{vmatrix} 4 & 21 \\ 5 & 27 \end{vmatrix} - 2\begin{vmatrix} 2 & 21 \\ 3 & 27 \end{vmatrix} + 11\begin{vmatrix} 2 & 4 \\ 3 & 5 \end{vmatrix}$$

$= 1(108-105) - 2(54-63) + 11(10-12)$

$= 3 + 18 - 22$

$\Delta z = -1$

$x = \frac{\Delta x}{\Delta} = \frac{-2}{-1} = 2$

$y = \frac{\Delta y}{\Delta} = \frac{-3}{-1} = 3$

$z = \frac{\Delta z}{\Delta} = \frac{-1}{-1} = 1$

$x = 2, y = 3, z = 1$

(6) A total of ₹ 8500 was invested in three interest earning accounts. The interest rates were 2%, 3% and 6%. If the total simple interest for one year was ₹ 380 and the amount invested at 6% was equal to the sum of the amounts in the other two accounts, then how much was invested in each account? (use Cramer's rule).

Soln

Let x, y & z be amounts invested at 2%, 3% & 6% respectively.

$$x+y+z = 8500 \rightarrow \text{(I)}$$

$$2\% \cdot x + 3\% \cdot y + 6\% \cdot z = 380$$

$$\frac{2}{100}x + \frac{3}{100}y + \frac{6}{100}z = 380$$

$$2x + 3y + 6z = 38000 \rightarrow \text{(II)}$$

$$z = x+y$$

$$x+y-z=0 \rightarrow \text{(III)}$$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 6 \\ 1 & 1 & -1 \end{vmatrix}$$

$$= 1\begin{vmatrix} 3 & 6 \\ 1 & -1 \end{vmatrix} - 1\begin{vmatrix} 2 & 6 \\ 1 & -1 \end{vmatrix} + 1\begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix}$$

$$= 1(-3-6) - 1(-2-6) + 1(2-3)$$

$$= -9+8-1$$

$$\Delta = -2 \neq 0$$

$\therefore$ we can apply cramer's rule

$$\Delta x = \begin{vmatrix} 8500 & 1 & 1 \\ 38000 & 3 & 6 \\ 0 & 1 & -1 \end{vmatrix}$$

$$= 8500\begin{vmatrix} 3 & 6 \\ 1 & -1 \end{vmatrix} - 1\begin{vmatrix} 38000 & 6 \\ 0 & -1 \end{vmatrix} + 1\begin{vmatrix} 38000 & 3 \\ 0 & 1 \end{vmatrix}$$

$$= 8500(-3-6) - 1(-38000-0) + 1(38000-0)$$

$$= -76500 + 38000 + 38000$$

$$\Delta x = -500$$

$$\Delta y = \begin{vmatrix} 1 & 8500 & 1 \\ 2 & 38000 & 6 \\ 1 & 0 & -1 \end{vmatrix}$$

$$= 1\begin{vmatrix} 38000 & 6 \\ 0 & -1 \end{vmatrix} - 8500\begin{vmatrix} 2 & 6 \\ 1 & -1 \end{vmatrix} + 1\begin{vmatrix} 2 & 38000 \\ 1 & 0 \end{vmatrix}$$

$$= 1(-38000-0) - 8500(-2-6) + 1(0-38000)$$

$$= -38000 + 68000 - 38000$$

$$\Delta y = -8000$$

$$\Delta z = \begin{vmatrix} 1 & 1 & 8500 \\ 2 & 3 & 38000 \\ 1 & 1 & 0 \end{vmatrix}$$

$$= 1\begin{vmatrix} 3 & 38000 \\ 1 & 0 \end{vmatrix} - 1\begin{vmatrix} 2 & 38000 \\ 1 & 0 \end{vmatrix} + 8500\begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix}$$

$$= 1(0-38000) - 1(0-38000) + 8500(2-3)$$

$$= -38000 + 38000 - 8500$$

$$\Delta z = -8500$$

$$x = \frac{\Delta x}{\Delta} = \frac{-500}{-2} = 250$$

$$y = \frac{\Delta y}{\Delta} = \frac{-8000}{-2} = 4000$$

$$z = \frac{\Delta z}{\Delta} = \frac{-8500}{-2} = 4250$$

∴ ~~x = 250~~ Amount Invested at 2% = ₹250

Amount Invested at 3% = ₹4000

Amount Invested at 6% = ₹4250

Ex 1·3

① The subscribtion department of a magazine sends out a letter to a large mailing list inviting subscriptions for the magazine. Some of ~~the~~ people recieving this letter already subscribe to the magazine while others do not. From ~~the~~ this mailing list, 45% of those who already subscribe will subscribe again while 30% of those who do not now subscribe will subscribe. On the last letter, it was found that 40% of those receiving it ordered a subscription. What percent of those receiving the current letter can be expected to order a subscription?

Sln

Transistion probability matrix

$$T = \begin{array}{c c} & \begin{array}{cc} S & N \end{array} \\ \begin{array}{c} S \\ N \end{array} & \begin{pmatrix} 0.45 & 0.55 \\ 0.30 & 0.70 \end{pmatrix} \end{array} \qquad \begin{array}{cc} S & N \\ (0.40 & 0.60) \end{array}$$

$$\begin{array}{cc} S & N \\ (0.40 & 0.60) \end{array} \begin{array}{c c} & \begin{array}{cc} S & N \end{array} \\ \begin{array}{c} S \\ N \end{array} & \begin{pmatrix} 0.45 & 0.55 \\ 0.30 & 0.70 \end{pmatrix} \end{array} =$$

$$= (0.18 + 0.18 \quad 0.22 + 0.42)$$

$$= \begin{array}{cc} S & N \\ (0.36 & 0.64) \end{array}$$

∴ 36% of those recieving the current letter can be expected to order a subscribtion.

(2) A new transit system has just gone into operation in chennai. Of those who use the transit system this year, 30% will switch over to using metro train next year and 70% will continue to use the transit system. Of those who use metro train this year, 70% will continue to use metro train next year and 30% will switch over to the transit system. Suppose the population of chennai city remains constant and that 60% of the commuters use the transit system and 40% of the commuters use metro train this year.
(i) what percent of commuters will be using the transit system after one year?
(ii) what percent of commuters will be using the transit system in the long run?

Soln

Transition probability matrix

$$T = \begin{array}{c c} & \begin{array}{cc} S & M \end{array} \\ \begin{array}{c} S \\ M \end{array} & \begin{pmatrix} 0.70 & 0.30 \\ 0.30 & 0.70 \end{pmatrix} \end{array} \qquad \begin{array}{cc} S & M \end{array} \\ \begin{pmatrix} 0.60 & 0.40 \end{pmatrix}$$

After one year

$$\begin{array}{cc} S & M \end{array} \\ \begin{pmatrix} 0.60 & 0.40 \end{pmatrix} \begin{array}{c c} & \begin{array}{cc} S & M \end{array} \\ \begin{array}{c} S \\ M \end{array} & \begin{pmatrix} 0.70 & 0.30 \\ 0.30 & 0.70 \end{pmatrix} \end{array} = \begin{pmatrix} 0.42+0.12 & 0.18+0.28 \end{pmatrix}$$

$$= \begin{array}{cc} S & M \end{array} \\ \begin{pmatrix} 0.54 & 0.46 \end{pmatrix}$$

$\therefore$ 54% of commuters will be using the transit system after one year.

(ii) At equilibrium

$$(S \quad M)\begin{pmatrix} 0.70 & 0.30 \\ 0.30 & 0.70 \end{pmatrix} = (S \quad M) \qquad \begin{array}{l} S+M=1 \\ M=1-S \end{array}$$

$$0.70S + 0.30M = S$$

$$0.70S + 0.30(1-S) = S$$

$$0.70S + 0.30 - 0.30S = S$$

$$0.70S - 0.30S - S = -0.30$$

$$S(0.70 - 0.30 - 1) = -0.30$$

$$-0.60S = -0.30$$

$$S = \frac{0.30}{0.60}$$

$$S = 0.50$$

$\therefore$ 50% of commuters will be using the transit system in the long run.

(3) Two types of soaps A and B are in the market. Their present market shares are 15% for A and 85% for B. Of those who bought A the previous year, 65% continue to buy it again while 35% switch over to B. Of those who bought B the previous year, 55% buy it again and 45% switch over to A. Find their market shares after one year and when is the equilibrium reached?

Soln

Transition probability matrix

$$T = \begin{array}{c c} & \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{pmatrix} 0.65 & 0.35 \\ 0.45 & 0.55 \end{pmatrix} \end{array} \qquad \begin{array}{cc} A & B \\ \end{array} \begin{pmatrix} 0.15 & 0.85 \end{pmatrix}$$

After one year

$$\begin{array}{cc} A & B \end{array} \begin{pmatrix} 0.15 & 0.85 \end{pmatrix} \begin{array}{c c} & \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{pmatrix} 0.65 & 0.35 \\ 0.45 & 0.55 \end{pmatrix} \end{array} =$$

$$= \begin{pmatrix} 0.0975 + 0.3825 & 0.0525 + 0.4675 \end{pmatrix}$$

$$= \begin{array}{cc} A & B \end{array} \begin{pmatrix} 0.48 & 0.52 \end{pmatrix}$$

$A = 48\%$ $\quad B = 52\%$

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At equilibrium

$$\begin{pmatrix} A & B \end{pmatrix} \begin{pmatrix} 0.65 & 0.35 \\ 0.45 & 0.55 \end{pmatrix} = \begin{pmatrix} A & B \end{pmatrix}$$

$A + B = 1$
$\Rightarrow B = 1 - A$

$0.65A + 0.45B = A$

$0.65A + 0.45(1-A) = A$

$0.65A + 0.45 - 0.45A = A$

$0.65A - 0.45A - A = -0.45$

$A(0.65 - 0.45 - 1) = -0.45$

$-0.80A = -0.45$

$A = \dfrac{0.45}{0.80} = 0.5625$

$A = 56.25\%$

$B = 43.75\%$

(4) Two products A and B currently share the market with shares 50% and 50% each respectively. Each week some brand switching takes place. Of those who bought A the previous week, 60% buy it again whereas 40% switch over to B. Of those who bought B the previous week, 80% buy it again where as 20% switch over to A. Find their shares after one week and after two weeks. If the price war continues, when is the equilibrium reached?

Soln

Transition probability matrix

$$T = \begin{array}{c c} & \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{pmatrix} 0.60 & 0.40 \\ 0.20 & 0.80 \end{pmatrix} \end{array} \qquad \begin{array}{cc} A & B \\ \end{array} \begin{pmatrix} 0.50 & 0.50 \end{pmatrix}$$

After one week

$$\begin{pmatrix} 0.50 & 0.50 \end{pmatrix} \begin{array}{c c} & \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{pmatrix} 0.60 & 0.40 \\ 0.20 & 0.80 \end{pmatrix} \end{array} = \begin{pmatrix} 0.30+0.10 & 0.20+0.40 \end{pmatrix}$$

$$= \begin{pmatrix} 0.40 & 0.60 \end{pmatrix}$$

$A = 40\%$ $B = 60\%$

After two week

$$\begin{pmatrix} 0.40 & 0.60 \end{pmatrix} \begin{array}{c c} & \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{pmatrix} 0.60 & 0.40 \\ 0.20 & 0.80 \end{pmatrix} \end{array}$$

$$= \begin{pmatrix} 0.24+0.12 & 0.16+0.48 \end{pmatrix}$$

$$= \begin{pmatrix} 0.36 & 0.64 \end{pmatrix}$$

$A = 36\%$ $B = 64\%$

At equilibrium

$A + B = 1$
$\Rightarrow B = 1 - A$

$$\begin{pmatrix} A & B \end{pmatrix} \begin{pmatrix} 0.60 & 0.40 \\ 0.20 & 0.80 \end{pmatrix} = \begin{pmatrix} A & B \end{pmatrix}$$

$0.60A + 0.20B = A$

$0.60A + 0.20(1-A) = A$

$0.60A + 0.20 - 0.20A = A$

$0.60A - 0.20A - A = -0.20$

$-0.60A = -0.20$

$A = \frac{0.20}{0.60}$

$A = 0.8333$

$A = 33.33\%$

$\therefore$ Equilibrium reached when $A = 33.33\%$ $B = 66.67\%$

Miscellaneous problems

(1) Find the rank of the matrix

$$A = \begin{pmatrix} 1 & -3 & 4 & 7 \\ 9 & 1 & 2 & 0 \end{pmatrix}$$

order of A is 2×4

$\Rightarrow \rho(A) \leq 2$

consider second order minor

$$\begin{vmatrix} 1 & -3 \\ 9 & 1 \end{vmatrix} = 1+27 = 28 \neq 0$$

There is a minor of order 2 which is not zero

$\therefore \rho(A) = 2.$

(2) Find the rank of the matrix

$$A = \begin{pmatrix} -2 & 1 & 3 & 4 \\ 0 & 1 & 1 & 2 \\ 1 & 3 & 4 & 7 \end{pmatrix}$$

order of A is 3×4

$\rightarrow \rho(A) \leq 3$

$R_1 \leftrightarrow R_3$

$$\sim \begin{pmatrix} 1 & 3 & 4 & 7 \\ 0 & 1 & 1 & 2 \\ -2 & 1 & 3 & 4 \end{pmatrix}$$

$R_3 \rightarrow R_3 + 2R_1$

$$\sim \begin{pmatrix} 1 & 3 & 4 & 7 \\ 0 & 1 & 1 & 2 \\ 0 & 7 & 11 & 18 \end{pmatrix}$$

$R_3 \rightarrow R_3 - 7R_2$

$$\sim \begin{pmatrix} 1 & 3 & 4 & 7 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 4 & 4 \end{pmatrix}$$

It is in echelon form

The number of non zero rows is 3

$\therefore \rho(A) = 3$

(3) Find the rank of the matrix

$$A = \begin{pmatrix} 4 & 5 & 2 & 2 \\ 3 & 2 & 1 & 6 \\ 4 & 4 & 8 & 0 \end{pmatrix}$$

order of A is 3×4

$\Rightarrow \rho(A) \leq 3$

$R_3 \to R_3/4$

$$\sim \begin{pmatrix} 4 & 5 & 2 & 2 \\ 3 & 2 & 1 & 6 \\ 1 & 1 & 2 & 0 \end{pmatrix}$$

$R_1 \leftrightarrow R_3$

$$\sim \begin{pmatrix} 1 & 1 & 2 & 0 \\ 3 & 2 & 1 & 6 \\ 4 & 5 & 2 & 2 \end{pmatrix}$$

$R_2 \to R_2 - 3R_1$

$R_3 \to R_3 - 4R_1$

$$\sim \begin{pmatrix} 1 & 1 & 2 & 0 \\ 0 & -1 & -5 & 6 \\ 0 & 1 & -6 & 2 \end{pmatrix}$$

$R_3 \to R_3 + R_2$

$$\sim \begin{pmatrix} 1 & 1 & 2 & 0 \\ 0 & -1 & -5 & 6 \\ 0 & 0 & -11 & 8 \end{pmatrix}$$

It is in echelon form

The number of non zero rows is 3

$\therefore \rho(A) \leq 3$

4. Examine the consistency of the system of equations.

$x+y+z=7$, $x+2y+3z=18$, $y+2z=6$

Soln

The matrix equation is

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 0 & 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 7 \\ 18 \\ 6 \end{pmatrix}$$

$$A \quad X = B$$

The Augmented matrix is

$$(A, B) = \begin{pmatrix} 1 & 1 & 1 & 7 \\ 1 & 2 & 3 & 18 \\ 0 & 1 & 2 & 6 \end{pmatrix}$$

$R_2 \to R_2 - R_1$

$$\sim \begin{pmatrix} 1 & 1 & 1 & 7 \\ 0 & 1 & 2 & 11 \\ 0 & 1 & 2 & 6 \end{pmatrix}$$

$R_3 \to R_3 - R_2$

$$\sim \begin{pmatrix} 1 & 1 & 1 & 7 \\ 0 & 1 & 2 & 11 \\ 0 & 0 & 0 & -5 \end{pmatrix}$$

It is in echelon form

$\rho(A) = 2, \ \rho(A|B) = 3, \ n = 3$

Since $\rho(A) \neq \rho(A|B)$

$\Rightarrow$ The given system ~~has~~ is inconsistent with no solution.

(5) Find k if the equations $2x+3y-z=5$, $3x-y+4z=2$, $x+7y-6z=k$ are consistent

Soln

The matrix equation is

$$\underset{A}{\begin{pmatrix} 2 & 3 & -1 \\ 3 & -1 & 4 \\ 1 & 7 & -6 \end{pmatrix}} \underset{X}{\begin{pmatrix} x \\ y \\ z \end{pmatrix}} = \underset{B}{\begin{pmatrix} 5 \\ 2 \\ k \end{pmatrix}}$$

The Augmented matrix is

$$(A|B) = \begin{pmatrix} 2 & 3 & -1 & 5 \\ 3 & -1 & 4 & 2 \\ 1 & 7 & -6 & k \end{pmatrix}$$

$R_1 \leftrightarrow R_3$

$$\sim \begin{pmatrix} 1 & 7 & -6 & k \\ 3 & -1 & 4 & 2 \\ 2 & 3 & -1 & 5 \end{pmatrix}$$

$R_2 \rightarrow R_2 - 3R_1$

$R_3 \rightarrow R_3 - 2R_1$

$$\sim \begin{pmatrix} 1 & 7 & -6 & k \\ 0 & -22 & 22 & 2-3k \\ 0 & -11 & 11 & 5-2k \end{pmatrix}$$

$R_3 \to 2R_3 - R_2$

$$\sim \begin{pmatrix} 1 & 7 & -6 & k \\ 0 & -22 & 22 & 2-3k \\ 0 & 0 & 0 & 8-k \end{pmatrix}$$

It is in echelon form

since equations are consistent

$\Rightarrow \rho(A) = \rho(A, B)$

From echelon form

$\rho(A) = 2 \Rightarrow \rho(A, B) = 2$

$\Rightarrow 8 - k = 0$

$k = 8$

(6) Find k if the equations

$x+y+z=1$, $3x-y-z=4$, $x+5y+5z=k$

are inconsistent.

Soln

The matrix equation is

$$\begin{pmatrix} 1 & 1 & 1 \\ 3 & -1 & -1 \\ 1 & 5 & 5 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \\ k \end{pmatrix}$$

$A \quad X = B$

The Augmented matrix is

$$(A, B) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 3 & -1 & -1 & 4 \\ 1 & 5 & 5 & k \end{pmatrix}$$

$R_2 \to R_2 - 3R_1$

$R_3 \to R_3 - R_1$

$$\sim \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & -4 & -4 & 1 \\ 0 & 4 & 4 & k-1 \end{pmatrix}$$

$R_3 \to R_3 + R_2$

$$\sim \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & -4 & -4 & 1 \\ 0 & 0 & 0 & k \end{pmatrix}$$

It is in echelon form

Since equations are inconsistent

$\rho(A) \neq \rho(A, B)$

from echelon form

$\rho(A) = 2 \Rightarrow \rho(A, B) \neq 2$

$\Rightarrow \rho(A, B) = 3$

$\Rightarrow k \neq 0$

(or)

k is any real value except 0.

(7) Solve the equations

$x+2y+z=7$, $2x-y+2z=4$, $x+y-2z=-1$

by using Cramer's rule.

Sln

$$\Delta = \begin{vmatrix} 1 & 2 & 1 \\ 2 & -1 & 2 \\ 1 & 1 & -2 \end{vmatrix}$$

$$= 1\begin{vmatrix} -1 & 2 \\ 1 & -2 \end{vmatrix} - 2\begin{vmatrix} 2 & 2 \\ 1 & -2 \end{vmatrix} + 1\begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix}$$

$$= 1(2-2) - 2(-4-2) + 1(2+1)$$

$$= 0 + 12 + 3$$

$\Delta = 15 \neq 0$

$\therefore$ we can apply cramer's rule.

$$\Delta x = \begin{vmatrix} 7 & 2 & 1 \\ 4 & -1 & 2 \\ -1 & 1 & -2 \end{vmatrix}$$

$$= 7\begin{vmatrix} -1 & 2 \\ 1 & -2 \end{vmatrix} - 2\begin{vmatrix} 4 & 2 \\ -1 & -2 \end{vmatrix} + 1\begin{vmatrix} 4 & -1 \\ -1 & 1 \end{vmatrix}$$

$$= 7(2-2) - 2(-8+2) + 1(4-1)$$

$$= 0 + 12 + 3$$

$\Delta x = 15$

$$\Delta y = \begin{vmatrix} 1 & 7 & 1 \\ 2 & 4 & 2 \\ 1 & -1 & -2 \end{vmatrix}$$

$$= 1\begin{vmatrix} 4 & 2 \\ -1 & -2 \end{vmatrix} - 7\begin{vmatrix} 2 & 2 \\ 1 & -2 \end{vmatrix} + 1\begin{vmatrix} 2 & 4 \\ 1 & -1 \end{vmatrix}$$

$$= 1(-8+2) - 7(-4-2) + 1(-2-4)$$

$$= -6 + 42 - 6$$

$\Delta y = 30$

$$\Delta z = \begin{vmatrix} 1 & 2 & 7 \\ 2 & -1 & 4 \\ 1 & 1 & -1 \end{vmatrix}$$

$$= 1\begin{vmatrix} -1 & 4 \\ 1 & -1 \end{vmatrix} - 2\begin{vmatrix} 2 & 4 \\ 1 & -1 \end{vmatrix} + 7\begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix}$$

$= 1(1-4) - 2(-2-4) + 7(2+1)$

$= -3 + 12 + 21$

$\Delta z = 30$

$x = \frac{\Delta x}{\Delta} = \frac{15}{15} = 1$

$y = \frac{\Delta y}{\Delta} = \frac{30}{15} = 2$

$z = \frac{\Delta z}{\Delta} = \frac{30}{15} = 2$

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$x = 1, y = 2, z = 2$

(8) The cost of 2kg of wheat and 1kg of sugar is ₹100. The cost of 1kg of wheat and 1kg of rice is ₹80. The cost of 3kg of wheat, 2kg of sugar and 1 kg of rice is ₹220. Find the cost of each per kg, using Cramer's rule.

Soln

Let x be cost of wheat per kg

y be cost of sugar per kg

z be cost of rice per kg

$2x + y = 100$

$x + z = 80$

$3x + 2y + z = 220$

$$\Delta = \begin{vmatrix} 2 & 1 & 0 \\ 1 & 0 & 1 \\ 3 & 2 & 1 \end{vmatrix}$$

$$= 2\begin{vmatrix} 0 & 1 \\ 2 & 1 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} + 0\begin{vmatrix} 1 & 0 \\ 3 & 2 \end{vmatrix}$$

$$= 2(0-2) - 1(1-3) + 0$$

$$= -4+2$$

$\Delta = -2 \neq 0$

$\therefore$ we can apply Cramer's rule

$$\Delta x = \begin{vmatrix} 100 & 1 & 0 \\ 80 & 0 & 1 \\ 220 & 2 & 1 \end{vmatrix}$$

$$= 100\begin{vmatrix} 0 & 1 \\ 2 & 1 \end{vmatrix} - 1\begin{vmatrix} 80 & 1 \\ 220 & 1 \end{vmatrix} + 0\begin{vmatrix} 80 & 0 \\ 220 & 2 \end{vmatrix}$$

$$= 100(0-2) - 1(80-220) + 0$$

$$= -200 + 140$$

$\Delta x = -60$

$$\Delta y = \begin{vmatrix} 2 & 100 & 0 \\ 1 & 80 & 1 \\ 3 & 220 & 1 \end{vmatrix}$$

$$= 2\begin{vmatrix} 80 & 1 \\ 220 & 1 \end{vmatrix} - 100\begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} + 0\begin{vmatrix} 1 & 80 \\ 3 & 220 \end{vmatrix}$$

$$= 2(80-220) - 100(1-3) + 0$$

$$= -280 + 200$$

$\Delta y = -80$

$$\Delta_z = \begin{vmatrix} 2 & 1 & 100 \\ 1 & 0 & 80 \\ 3 & 2 & 220 \end{vmatrix}$$

$$= 2\begin{vmatrix} 0 & 80 \\ 2 & 220 \end{vmatrix} - 1\begin{vmatrix} 1 & 80 \\ 3 & 220 \end{vmatrix} + 100\begin{vmatrix} 1 & 0 \\ 3 & 2 \end{vmatrix}$$

$$= 2(0-160) - 1(220-240) + 100(2-0)$$

$$= -320 + 20 + 200$$

$$\Delta_z = -100$$

$$x = \frac{\Delta x}{\Delta} = \frac{-60}{-2} = 30$$

$$y = \frac{\Delta y}{\Delta} = \frac{-80}{-2} = 40$$

$$z = \frac{\Delta z}{\Delta} = \frac{-100}{-2} = 50$$

$\therefore$ cost of wheat per kg = ₹ 30

Cost of sugar per kg = ₹ 40

Cost of rice per kg = ₹ 50

(9) A Salesman has the following record of sales during three months for three items A, B and C which have different rates of commission.

Months	Sales of units			Total commission drawn (in ₹)
	A	B	C	
January	90	100	20	800
February	130	50	40	900
March	60	100	30	850

Find out the rate of commission on the items. A, B and C by using Cramer's rule.

Soln
Let x, y & z be commission on the items A, B & C.

$$90x + 100y + 20z = 800$$
$$\div 10 \quad 9x + 10y + 2z = 80 \rightarrow (1)$$
$$130x + 50y + 40z = 900$$
$$\div 10$$
$$13x + 5y + 4z = 90 \rightarrow (2)$$
$$60x + 100y + 30z = 850$$
$$\div 10 \quad 6x + 10y + 3z = 85 \rightarrow (3)$$

$$\Delta = \begin{vmatrix} 9 & 10 & 2 \\ 13 & 5 & 4 \\ 6 & 10 & 3 \end{vmatrix}$$

$$= 9\begin{vmatrix} 5 & 4 \\ 10 & 3 \end{vmatrix} - 10\begin{vmatrix} 13 & 4 \\ 6 & 3 \end{vmatrix} + 2\begin{vmatrix} 13 & 5 \\ 6 & 10 \end{vmatrix}$$

$$= 9(15-40) - 10(39-24) + 2(130-30)$$

$$= -225 - 150 + 200$$

$$\Delta = -175 \neq 0$$

$\therefore$ we can apply cramer's rule.

$$\Delta x = \begin{vmatrix} 80 & 10 & 2 \\ 90 & 5 & 4 \\ 85 & 10 & 3 \end{vmatrix}$$

$$= 80\begin{vmatrix} 5 & 4 \\ 10 & 3 \end{vmatrix} - 10\begin{vmatrix} 90 & 4 \\ 85 & 3 \end{vmatrix} + 2\begin{vmatrix} 90 & 5 \\ 85 & 10 \end{vmatrix}$$

$$= 80(15-40) - 10(270-340) + 2(900-425)$$

$$= -2000 + 700 + 950$$

$$\Delta x = -350$$

$$\Delta y = \begin{vmatrix} 9 & 80 & 2 \\ 13 & 90 & 4 \\ 6 & 85 & 3 \end{vmatrix}$$

$$\cancel{= 9 \begin{vmatrix} 90 & 4 \\ 85 & 3 \end{vmatrix} - 80(39-24) + 2(}$$

$$= 9 \begin{vmatrix} 90 & 4 \\ 85 & 3 \end{vmatrix} - 80 \begin{vmatrix} 13 & 4 \\ 6 & 3 \end{vmatrix} + 2 \begin{vmatrix} 13 & 90 \\ 6 & 85 \end{vmatrix}$$

$$= 9(270 - 340) - 80(39 - 24) + 2(1105 - 540)$$

$$= -630 - 1200 + 1130$$

$$\Delta y = -700$$

$$\Delta z = \begin{vmatrix} 9 & 10 & 80 \\ 13 & 5 & 90 \\ 6 & 10 & 85 \end{vmatrix}$$

$$= 9 \begin{vmatrix} 5 & 90 \\ 10 & 85 \end{vmatrix} - 10 \begin{vmatrix} 13 & 90 \\ 6 & 85 \end{vmatrix} + 80 \begin{vmatrix} 13 & 5 \\ 6 & 10 \end{vmatrix}$$

$$= 9(425 - 900) - 10(1105 - 540) + 80(130 - 30)$$

$$= -4275 - 5650 + 8000$$

$$\Delta z = -1925$$

$$x = \frac{\Delta x}{\Delta} = \frac{-350}{-175} = 2$$

$$y = \frac{\Delta y}{\Delta} = \frac{-700}{-175} = 4$$

$$z = \frac{\Delta z}{\Delta} = \frac{-1925}{-175} = 11$$

$\therefore$ Commission on A = ₹ 2

Commission on B = ₹ 4

Commission on C = ₹ 11

(10) The subscription department of a magazine sends out a letter to a large mailing list inviting subscriptions for the magazine. Some of people recieving this letter already subscribe to the magazine while others do not. From this mailing list, 60% of those who already subscribe will subscribe again while 25% of those who do not now subscribe will subscribe. On the last letter it was found that 40% of those receiving it ordered subscription. what percent of those receiving the current letter can be expected to order a subscription.

Soln.

Transition probability matrix

$$T = \begin{array}{c c} & \begin{array}{cc} S & N \end{array} \\ \begin{array}{c} S \\ N \end{array} & \begin{pmatrix} 0.60 & 0.40 \\ 0.25 & 0.75 \end{pmatrix} \end{array}$$

$$\begin{array}{cc} S & N \\ (0.40 & 0.60) \end{array}$$

$$\begin{array}{cc} S & N \\ (0.40 & 0.60) \end{array} \begin{array}{c c} & \begin{array}{cc} S & N \end{array} \\ \begin{array}{c} S \\ N \end{array} & \begin{pmatrix} 0.60 & 0.40 \\ 0.25 & 0.75 \end{pmatrix} \end{array}$$

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$$= (0.24 + 0.15 \quad 0.16 + 0.45)$$

$$= \begin{array}{cc} S & N \\ (0.39 & 0.61) \end{array}$$

$\therefore$ 39% of those receiving the current letter can be expected to order a subscription.