

Chapter-7

Probability Distribution

1. binomial
2. Poisson
3. Normal

Binomial $(n \leq 10)$

$\Rightarrow$ Mean $>$ Variance

Mean $= np$

Variance $= npq$.

$S.D = \sqrt{npq}$

$q \rightarrow$ Failure

$p \rightarrow$ Success

} depends upon Question

$$p + q = 1$$

$$\Rightarrow p = 1 - q \quad ; \quad q = 1 - p$$

$$p, q \leq 1.$$

$$P(x) = {}^{n}C_{x}\, p^{x}\, q^{n-x}, \quad n = 0 \text{ to } x$$

$${}^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$$

$0! = 1$

$1! = 1$

${}^{n}C_{0} = {}^{n}C_{n} = 1$

${}^{n}C_{1} = n$

${}^{n}C_{r} = {}^{n}C_{n-r}$

${}^{5}C_{2} = {}^{5}C_{3}$

$\dfrac{5!}{2!\,3!} = \dfrac{5 \times 4}{2 \times 1} \dfrac{3!}{3!}$

${}^{5}C_{2} = \dfrac{5 \times 4}{2 \times 1}$

${}^{5}C_{3} = \dfrac{5 \times 4}{2 \times 1}$

Exercise 7.1

16. mean = 5   S.D = 2

$np = 5$ $\sqrt{npq} = 2$

$npq = 4$

$\frac{npq}{np} = \frac{4}{5}$

$q = \frac{4}{5}$, $p = \frac{1}{5}$

$np = 5$

$n\frac{1}{5} = 5$

$\boxed{n = 25}$

$P(x) = n_{C_x}\, p^x q^{n-x}$

$= 25_{C_x} \left(\frac{1}{5}\right)^x \left(\frac{4}{5}\right)^{25-x}$

6. $n = 20$, $P = 5\%$

$= \frac{5}{100}$

$P = 0.05$; $q = 0.95$

$P(x) = n_{C_x} (P)^x q^{n-x}$

i) $P(x=3)$:

$= 20_{C_3} (0.05)^3 (0.95)^{20-3}$

$= \frac{20 \times 19 \times \cancel{18}^{3}}{3 \times 2 \times 1} (0.05)^3 (0.95)^{17}$

$= 1140 \times 0.00005226$

$= 0.059$

ii) $P(x \geq 2) = 1 - P(x < 2)$

$= 1 - P(x=0) + P(x=1)$

$= 1 - \left[20_{C_0} (0.05)^0 (0.95)^{20} + 20_{C_1} (0.05)^1 (0.95)^{19} \right]$

Scanned by CamScanner

$= 1 - [1(0.3584) + 20(0.05)(0.373)]$

$= 1 - 0.7337$

$= 0.2643.$

iii) $P(x=4) = 20_{C_4} (0.05)^4 (0.95)^{16}$

$= \frac{20\times19\times18\times17}{4\times3\times2\times1} \times 0.00000625 \times 0.4401$

$= 0.0133.$

iv) mean $= np = 20 \times 0.05$

$= 1$

Variance $= npq = 1 \times 0.95$

$= 0.95.$

15. x = doublet.

$n=4$. $P(\text{doublet}) = \frac{6}{36} = \frac{1}{6} = p$

$q = \frac{5}{6}$

$P(x) = n_{C_x} p^x q^{n-x}$

$P(x=2) = 4_{C_2} \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^2$

$= \frac{\cancel{4}^{2}\times3}{\cancel{2}\times1} \cdot \frac{5^2}{6\,\cancel{4}\,3} = \frac{21}{216}$ (or) 0.115

17 mean = 4 , var = 3

$np = 4$, $npq = 3$

$4q = 3$

$q = 3/4$; $p = 1/4$; $np = 4$

$n \cdot 1/4 = 4$

$n = 16$

$P(x) = {}^{n}C_{x}\, p^{x} q^{n-x}$

$= {}^{16}C_{x} \left(\frac{1}{4}\right)^{x} \left(\frac{3}{4}\right)^{16-x}$

$P(x=15) = {}^{16}C_{15} \left(\frac{1}{4}\right)^{15} \left(\frac{3}{4}\right)^{1}$

$= 16 \times \frac{3}{(4)^{16}}$

$= (4)^{2} \times \frac{3}{(4)^{\cancel{16}\,14}}$

$= \frac{3}{(4)^{14}}$

10 x = defective.

$n = 4$.

$p = 18\% = \frac{18}{100} = \frac{9}{50}$

$q = \frac{41}{50}$

$P(x) = {}^{n}C_{x}\, p^{x} q^{n-x}$

i) $P(x=1) = {}^{4}C_{1} \left(\frac{9}{50}\right)^{1} \left(\frac{41}{50}\right)^{3}$

$= 4 \times 0.18 \times 0.5514$

$= 0.3969$.

ii) $P(x=0) = {}^{4}C_{0} \left(\frac{9}{50}\right)^{0} \left(\frac{41}{50}\right)^{4}$

$= 0.4521$

iii) atmost 2 $P(x \leq 2) = P(x=2) + P(x=1) + P(x=0)$

$= {}^4C_2 \left(\frac{9}{50}\right)^2 \left(\frac{41}{50}\right)^2 + 0.3969 + 0.4521$

$= \frac{4 \times 3}{2 \times 1} \times 0.0324 \times 0.6724$

$= 0.1307 + 0.849$

$= 0.9797$.

Eg: 7.2

coin tossed 6 times

$n = 6, \quad p = \frac{1}{2}, \quad q = \frac{1}{2}$

head:

$P(x=2) : \quad {}^nC_x \, p^x \, q^{n-x}$

$= {}^6C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{6-2}$

$= \frac{6 \times 5}{2 \times 1} \left(\frac{1}{4}\right)\left(\frac{1}{2}\right)^4$

$= 15 \left(\frac{1}{4}\right)\left(\frac{1}{16}\right)$

$P(x=2) = \frac{15}{64}$.

Eg: 7.1 3:2

win: $p = \frac{3}{5}, \quad q = \frac{2}{5}, \quad n = 5.$

$P(x) = {}^nC_x \, p^x \, q^{n-x}$

$P(x \geq 3) = 1 - P(x < 3)$

$= 1 - \left[P(x=0) + P(x=1) + P(x=2)\right]$

$= 1 - \left[{}^5C_0 \left(\frac{3}{5}\right)^0 \left(\frac{2}{5}\right)^5 + {}^5C_1 \left(\frac{3}{5}\right)^1 \left(\frac{2}{5}\right)^4 + {}^5C_2 \left(\frac{3}{5}\right)^2 \left(\frac{2}{5}\right)^3\right]$

$\cancel{[0.01024 + 5(0.6)(0.0256)]} +$

$$= 1 - \left[\frac{2^5}{5^5} + 5 \times \frac{3}{5^5} \times \frac{2^4}{5^4} + \frac{5 \times 4}{2 \times 1} \cdot \frac{3^2}{5^2} \times \frac{2^3}{5^3} \right]$$

$$= 1 - \left[\frac{2^5}{5^5} + 5 \times \frac{3}{5^5} \times 2^4 + \frac{3^2}{5^3} \times 2^3 \right]$$

$$= 1 - \left[\frac{32 + 240 + 720}{3125} \right]$$

$$= 1 - \frac{992}{3125}$$

$$= 1 - 0.3174$$

$$P(x \geq 3) = 0.6826$$

Eg: 7.3

$npq = 16$

$12q = 16$

$q = \frac{16}{12} \not< 1$

$\therefore$ The Gn statement is wrong.

Eg: 7.5

$n = 5$, P(Head)

$p(x) = \frac{1}{2}$, $q = \frac{1}{2}$.

$$P(x) = n_{C_x} \cdot p^x q^{n-x}$$

$$P(x = 3) = 5_{C_3} \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2$$

$$= \frac{5 \times 4}{3 \times 2} \frac{1}{2^3} \times \frac{1}{2^2}$$

$$= \frac{5 \times \cancel{4}^2}{2 \times 1} \times \frac{1}{2^{5}{}_4}$$

$$= \frac{5}{16}.$$

Eg: 7.6 mean $= 20$, $np = 20$

S.D $= (4)^2$ $npq = 16$

$20q = 16$ $q = \frac{16}{20}$

$q = \frac{4}{5}$, $P = \frac{1}{5}$

$np = 20$

$n\frac{1}{5} = 20$ $n = 100$

The Parameters are

$n = 100$, $P = \frac{1}{5}$, $q = \frac{4}{5}$

$P(x) = {}^{100}C_x \left(\frac{1}{5}\right)^x \left(\frac{4}{5}\right)^{100-x}$

Eg: 7.7

$E(x) = 2$ $\quad$ $var(x) = \frac{4}{3}$

$\downarrow$

mean $= 2$ $\quad$ $npq = \frac{4}{3}$

$\downarrow$

$np = 2$

$2q = \frac{4}{3}$

$q = \frac{4}{3 \times 2}$

$q = \frac{\cancel{4}^2}{\cancel{6}3}$

$q = \frac{2}{3}$, $P = \frac{1}{3}$, $np = 2$

$n(\frac{1}{3}) = 2$

$n = 6$

$P(x) = {}^nC_x \, p^x \, q^{n-x}$

$P(x=5) = {}^6C_5 \left(\frac{1}{3}\right)^5 \left(\frac{2}{3}\right)^1$

$= \cancel{6}^2 \times \frac{1}{3^5} \times \frac{2}{\cancel{3}}$

$= \frac{4}{243}$

$= 0.016$.

Eg: 7.8

$n = 7$ (weak)

$P = \frac{\cancel{9}^3}{\cancel{30}_{10}}$ $\quad$ $P = \frac{3}{10}$, $q = \frac{7}{10}$

$P(x) = {}^nC_x \, p^x \, q^{n-x}$

$P(x \geq 2) = 1 - P(x < 2)$

$= 1 - P(x=0) + P(x=1)$

$= 1 - \left[7_{C_0} \left(\frac{3}{10}\right)^0 \left(\frac{7}{10}\right)^7 + 7_{C_1} \left(\frac{3}{10}\right)^1 \left(\frac{7}{10}\right)^6 \right]$

$= 1 - [0.082 + 0.247]$

$= 1 - 0.329$

$= 0.671$

Eg: 7.4

$P = 0.4, \quad q = 0.6, \quad n = 5$

i) $P(x=1) = 5_{C_1} (0.4)^1 (0.6)^4$

$= 5 \times 0.4 \times 0.1296$

$= 0.2592$

ii) $P(x \geq 1) = 1 - P(x < 1)$

$= 1 - P(x=0)$

$= 1 - 5_{C_0} (0.4)^0 (0.6)^5$

$= 1 - 1(0.07776)$

$= 0.9222$

Eg: 7.9

P(Guessing Correctly) $= \frac{1}{2}$

$P = \frac{1}{2}, \; q = \frac{1}{2}, \; n = 10$

$P(x) = n_{C_x} p^x q^{n-x}$

$P(x \geq 6) = 1 - P(x < 6)$

$= 1 - P(x=0) + P(x=1) + P(x=2) + P(x=3) + P(x=4) + P(x=5)$

$$= 1 - {}^{10}c_0 \left(\tfrac{1}{2}\right)^0 \left(\tfrac{1}{2}\right)^{10} + {}^{10}c_1 \left(\tfrac{1}{2}\right)^1 \left(\tfrac{1}{2}\right)^9 + {}^{10}c_2 \left(\tfrac{1}{2}\right)^2 \left(\tfrac{1}{2}\right)^8 + {}^{10}c_3 \left(\tfrac{1}{2}\right)^3 \left(\tfrac{1}{2}\right)^7 +$$

$$ {}^{10}c_4 \left(\tfrac{1}{2}\right)^4 \left(\tfrac{1}{2}\right)^6 + {}^{10}c_5 \left(\tfrac{1}{2}\right)^5 \left(\tfrac{1}{2}\right)^5$$

$$= 1 - \left[(1)\left(\tfrac{1}{2}\right)^{10} + 10\left(\tfrac{1}{2}\right)^{10} + \frac{10 \times 9}{2 \times 1}\left(\tfrac{1}{2}\right)^{10} + \frac{10 \times 9 \times 8}{3 \times 2 \times 1}\left(\tfrac{1}{2}\right)^{10} + \right.$$

$$\left. \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1}\left(\tfrac{1}{2}\right)^{10} + \frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2 \times 1}\left(\tfrac{1}{2}\right)^{10} \right]$$

$$= 1 - \left(\frac{1}{2}\right)^{10} [1 + 10 + 45 + 120 + 210 + 252]$$

$$= 1 - (0.000976)(638)$$

0.000976(638

$$= 1 - 0.62304$$

$$= 0.3769$$

8. x = girls.

$P(G) = \frac{1}{2}$, $q = \frac{1}{2}$, $n = 3$.

$P(x) = {}^nc_x \, p^x q^{n-x}$.

$P(x=2) = {}^3c_2 \left(\tfrac{1}{2}\right)^2 \left(\tfrac{1}{2}\right)^1$

$= 3 \times \frac{1}{4} \times \frac{1}{2}$

$= 0.375$.

9. mean $= 1.2$

$np = 1.2$, $n = 6$

$6p = 1.2$

$p = \frac{1.2}{6}$

$p = 0.2$, $q = 0.8$.

$P(x) = {}^nc_x \, p^x q^{n-x}$

$P(x<2) = P(x=0) + P(x=1)$

$= {}^6c_0 (0.2)^0 (0.8)^6 + {}^6c_1 (0.2)^1 (0.8)^5$

$= 0.262 + 0.3932$

$= 0.6552.$

11.

$P = 0.09, \quad q = 0.91$

$P(x \geq 1) = \frac{1}{3}$

$1 - P(x<1) = \frac{1}{3}$

$P(x<1) = 1 - \frac{1}{3}$

$P(x=0) = 2/3$

$P(x) = {}^nc_x p^x q^{n-x}.$

${}^nc_0 (0.09)^0 (0.91)^n = \frac{2}{3}$

$(0.91)^n = 0.67$

$\therefore n = 5$ or more

7. x : reading news paper

$P = 40\%$
$= 0.4$

$q = 0.6, n = 9.$

$P(x) = {}^nc_x \, p^x q^{n-x}$

i) $P(x=0) = \cancel{9c_0}^{1} (0.\cancel{4})^0 (0.6)^9$

$= 0.01007.$

ii) $P(x=9) = \cancel{9c_9}^{1} (0.4)^9 (0.6)^0$

$= 1(0.000262)$

$= 0.000262$

iii) $P(x \geq \frac{2}{3} \times 9)$

$= P(x \geq 6)$

$= P(x=6) + P(x=7) + P(x=8) + P(x=9)$

$= 9c_6 (0.4)^6 (0.6)^3 + 9c_7 (0.4)^7 (0.6)^2 + 9c_8 (0.4)^8 (0.6)^1$

$+ 9c_9 (0.4)^9 (0.6)^0$

$= \frac{9 \times 8 \times 7}{3 \times 2 \times 1} \times 0.00088 + \frac{9 \times 8}{2 \times 1} \times 0.000589 + 9 \times 0.00039 + 1 \times 0.00026$

$= 0.0739 + 0.0212 + 0.00351 + 0.00026$

$= 0.~~0~~99.$

12. $~~N~~ = 28,$

$n = 5, \quad P(\text{foreign}) = \frac{18}{28} = 0.64$

$q = 0.36.$

$P(x) = n_{c_x} \, p^x \, q^{n-x}.$

$P(x \geq 3) = 1 - P(x < 3)$

$= 1 - P(x=0) + P(x=1) + P(x=2)$

$= 1 - [5c_0 (0.64)^0 (0.36)^5 + 5c_1 (0.64)^1 (0.36)^4 + 5c_2 (0.64)^2 (0.36)^3]$

$= 1 - [0.006047 + 5(0.64)(0.0167) + 10(0.4096)(0.0467)]$

$= 1 - [0.006047 + 0.053728 + 0.19128]$

$= 1 - 0.251048$

$= 0.748952$

13. $p = \frac{1}{2}$, $q = \frac{1}{2}$, $n = 4$.

$$P(x) = n_{C_x}\, p^x\, q^{n-x}$$

i) $P(x \geq 1) = 1 - P(x < 1)$

$= 1 - P(x = 0)$

$= 1 - 4_{C_0} \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^4$

$= 1 - 0.0625$

$= 0.9375$

out of 750 $= 0.9375 \times 750$

$= 703$ boys.

ii) P(atmost 2) $= P(x \leq 2)$

$= 1 - P(x > 2)$

$= 1 - [P(x=3) + P(x=4)]$

$= 1 - \left[4_{C_3} \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^1 + 4_{C_4} \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^0\right]$

$= 1 - \left(4 \times \frac{1}{16} + \frac{1}{16}\right)$

$= 1 - 0.3125$

$= 0.6875$. out of 750 $= 0.6875 \times 750$

$= 516$. girls.

iii) P(both sexes).

$= 1 - [P(x=0) + P(x=4)]$

$= 1 - \left[4_{C_0} \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^4 + 4_{C_4} \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^4\right]$

$= 1 - [0.0625 + 0.0625]$

$= 1 - 0.1250$

$= 0.8750$

out of 750 $= 0.8750 \times 750$

$= 656$ G & Boys.

20 $p = 2q$, $n = 5$

$p = 2(1-p)$

$p = 2 - 2p$

$3p = 2$ $\boxed{p = 2/3}$ $q = \frac{1}{3}$

$P(x) = {}^nC_x \, p^x \, q^{n-x}$.

i) $P(x=3) = {}^5C_3 \left(\frac{2}{3}\right)^3 \left(\frac{1}{3}\right)^2$

$= 10 \times \frac{8}{243}$

$= 0.3292$.

ii) $P(x \geq 3) = 1 - P(x=0) + P(x=1) + P(x=2)$

$= 1 - \left[{}^5C_0 \left(\frac{2}{3}\right)^0 \left(\frac{1}{3}\right)^5 + {}^5C_1 \left(\frac{2}{3}\right)^1 \left(\frac{1}{3}\right)^4 + {}^5C_2 \left(\frac{2}{3}\right)^2 \left(\frac{1}{3}\right)^3 \right]$

$= 1 - (0.0041 + 0.0411 + 0.1646)$

$= 1 - 0.2098$

$= 0.7902$

14. $n = 15$, $p = 40\%$

$p = 0.4$, $q = 0.6$.

$P(x) = {}^nC_x \, p^x \, q^{n-x}$.

i) $P(x=3) = {}^{15}C_3 \, (0.4)^3 (0.6)^{12}$

$= \frac{\cancel{15}^{5} \times \cancel{14}^{7} \times 13}{\cancel{3} \times \cancel{2} \times 1} (0.064)(0.00217)$

$= 0.0631$.

ii) $P(x=12)$

$= P(x=3)$

$= {}^{15}C_3 (0.4)^3 (0.6)^{12}$

$= 0.0631$

iii) $P(x \geq 3) = 1 - P(x<3)$

$= 1 - P(x=0) + P(x=1) + P(x=2)$

$= 1 - \left[{}^{15}C_0 (0.4)^0 (0.6)^{15} + {}^{15}C_1 (0.4)^1 (0.6)^{14} + {}^{15}C_2 (0.4)^2 (0.6)^{13} \right]$

$= 1 - \left[0.0004687 + 15(0.4)(0.0007812) + \frac{15 \times 14}{2 \times 1}(0.16))(0.001302) \right]$

$= 1 - [0.0004687 + 0.0046872 + 0.021874]$

$= 1 - 0.0270299$

$= 0.9729$

18. $P = \frac{3}{100}$ $P = 0.03$, $q = 0.97$, $n = 10$

$p(x) = {}^{n}C_{x} \; p^{x} q^{n-x}$

$P(x \geq 1) = 1 - P(x<1)$

$= 1 - P(x=0)$

$= 1 - \left[{}^{10}C_0 (0.03)^0 (0.97)^{10} \right]$

$= 1 - 0.73742$

$= 0.2625$

19. $P = 0.73$, $q = 0.27$, $n = 5$

$p(x) = {}^{n}C_{x} \; p^{x} q^{n-x}$

$P(x \geq 3) = 1 - [P(x < 3)]$

$= 1 - [P(x=0) + P(x=1) + P(x=2)]$

$= 1 - [5c_0 (0.73)^0 (0.27)^5 + 5c_1 (0.73)^1 (0.27)^4 + 5c_2 (0.73)^2 (0.27)^3]$

$= 1 - [0.00143 + 5(0.73)(0.00531) + \frac{5 \times 4}{2 \times 1} (0.5329)(0.0196)]$

$= 1 - [0.00143 + 0.01938 + 0.10444]$

$= 1 - 0.12525$

$= 0.874$

Eg: 7.11

mean $= np$

variance $= npq$.

Sum $\Rightarrow np + npq = 24$ ... (1)

Product $\Rightarrow np \times npq = 128$

$npq = 24 - np$

$np(24 - np) = 128$

$24np - (np)^2 = 128$

$np^2 - 24np + 128 = 0$

128: −16, −8; 24

$np = 16$, $np = 8$ → Its value is above 1 so it is omitted.

$(1) \Rightarrow np = 16$.

$np + npq = 24$

$16 + 16q = 24$

$16q = 24 - 16$

$16q = 8$ $\quad q = \frac{8}{16}$

$q = \frac{1}{2}$, $P = \frac{1}{2}$

$np = 16$

$n(\frac{1}{2}) = 16$

$n = 32$

$(1) \Rightarrow np = 8$

$np + npq = \cancel{128}\ 24$

$8 + 8q = 24$

$8q = 24 - 8$

$8q = 16$

$q = 2 \cancel{\nless} 1 \qquad \therefore np \neq 8.$

$P(x) = {}^{n}C_{x}\, p^{x} q^{n-x}.$

$= {}^{32}C_{x} \left(\frac{1}{2}\right)^{x} \left(\frac{1}{2}\right)^{32-x}$

$x = 0 \text{ to } 32.$

Eg: 7.10

$P = 0.2,\ q = 0.8,\ n = 10.$

$P(x) = {}^{n}C_{x}\, p^{x} q^{n-x}.$

ii) $P(\text{atleast } 1) = P(x \geq 1)$

$= 1 - P(x < 1)$

$= 1 - P(x = 0)$

$= 1 - \left[{}^{10}C_{0}\, (0.2)^{0} (0.8)^{10}\right]$

$= 1 - [0.10737]$

$= 0.8926.$

i) $P(x = 1) = {}^{10}C_{1} (0.2)^{1} (0.8)^{9}$

$= 10\,(0.2)(0.1342)$

$= 0.2684.$

Eg: 7.12 A, B $P = \frac{1}{2}$, $q = \frac{1}{2}$.

a) A beats B exactly 3 games out of 4

$n = 4$, $x = 3$.

$P(x) = {}^nC_x \, p^x q^{n-x}$.

$= {}^4C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^1$

$= 4 \left(\frac{1}{2^4}\right)$

$= 4 \left(\frac{1}{16}\right)$

$= 0.25 = 25\%$.

b) A beats B exactly in 5 games out of 8.

$n = 8$, $x = 5$

$P(x) = {}^nC_x \, p^x q^{n-x}$

$= {}^8C_5 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^3$

$= \frac{8 \times 7 \times \cancel{6}}{\cancel{3} \times 2 \times 1} (0.03125)(0.125)$

$= 0.21875$

$= 21.875\%$

$\therefore$ first event is more probable.

Exercise 7.2

Poisson Distribution

$(e^{-\lambda})$

mean = var = $np = npq = \lambda$

$$P(x) = \frac{e^{-\lambda} . \lambda^x}{x!}$$

9. $\lambda = 2.8 = np$

x = death.

$$P(x) = \frac{e^{-\lambda} . \lambda^x}{x!}$$

$$P(x=2) = \frac{e^{-2.8} . (2.8)^2}{2!}$$

$$= \frac{0.06 \times 7.84}{2}$$

$$= 0.2352.$$

10. mean = $4 = \lambda$.

$$P(x) = \frac{e^{-\lambda} . \lambda^x}{x!}$$

$$P(x=0) = \frac{e^{-4} \times 4^0}{0!}$$

$$= \frac{0.0183 \times 1}{1}$$

$$= 0.0183.$$

out of 100 = 0.0183×100

= 1.83

= 2 days.

ii) $P(x \geq 2) = 1 - P(x<2)$

$= 1 - P(x=0) + P(x=1)$

$= 1 - \left(\frac{e^{-4} 4^0}{0!} + \frac{e^{-4} \cdot (4)^1}{1!} \right)$

$= 1 - (0.0183 + 0.0183 \times 4)$

$= 1 - 0.0915$

$= 0.9085$

out of $100 = 0.9085 \times 100$

$= 90.8 = 91$ days.

iii) $P(x \leq 3) = \underbrace{P(x=0) + P(x=1)} + P(x=2) + P(x=3)$.

$= 0.0915 + \frac{e^{-4} \cdot (4)^2}{2!} + \frac{e^{-4} (4)^3}{3!}$

$= 0.0915 + \frac{0.0183 \times 16}{2} + \frac{0.0183 (64)}{3 \times 2}$

$= 0.0915 + 0.1464 + 0.1952$

$= 0.4331$.

out of $100 = 0.4331 \times 100$

$= 43$ days.

7. $P = \frac{5}{100}$, $n = 120$

$\lambda = np = \frac{5}{100} \times 120$

$\lambda = 6$.

$P(x) = \frac{e^{-\lambda} \lambda^x}{x!}$

$P(x=0) = \frac{e^{-6} (6)^0}{0!}$

$= 0.0025$

mean = 1.5 = λ

$\lambda = 1.5$

$P(x) = \frac{e^{-\lambda} \cdot \lambda^x}{x!}$

i) $P(x=0) = \frac{e^{-1.5} (1.5)^0}{0!}$

$= 0.2231$

ii) P(some demand is refused).

$P(x>2) = 1 - P(x \leq 2)$

$= 1 - P(x=0) + P(x=1) + P(x=2)$

$= 1 - \left[\frac{e^{-1.5} \cdot (1.5)^0}{0!} + \frac{e^{-1.5}(1.5)^1}{1!} + \frac{e^{-1.5}(1.5)^2}{2!}\right]$

$= 1 - e^{-1.5}\left[(1.5)^0 + (1.5)^1 + \frac{(1.5)^2}{2}\right]$

$= 1 - 0.2231 [1 + 1.5 + 1.125]$

$= 1 - [0.2231 \times 3.625]$

$= 1 - 0.8087$

$= 0.1912.$

C.Q:

mean = 3

$\lambda = 3.$

$P(x) = {}^{n}c_{x} \frac{e^{-\lambda} \lambda^x}{x!}$

out of 1000 drivers.

i) $P(x=0) = \frac{e^{-3} \cdot (3)^0}{0!}$

$= 0.0498.$

out of 1000 = $0.0498 \times 1000 = 49.8$

ii) $P(x \geq 3) = 1 - P(x < 3)$

$= 1 - [P(x=0) + P(x=1) + P(x=2)]$

$= 1 - \left[\frac{e^{-3}(3)^0}{0!} + \frac{e^{-3}(3)^1}{1!} + \frac{e^{-3}(3)^2}{2!} \right]$

$= 1 - e^{-3}\left[1 + 3 + \frac{9}{2} \right]$

$= 1 - 0.0498(8.5)$

$= 1 - 0.4233$

$= 0.5767$

out of 1000 $= 0.5767 \times 1000$

$= 576.7 = 577$

C.Q:

$P(x=2) = P(x=3)$ find $P(x=5)$

$P(x) = \frac{e^{-\lambda} \cdot \lambda^x}{x!}$

$\frac{\cancel{e^{-\lambda}} \cdot \cancel{\lambda^2}}{2!} = \frac{\cancel{e^{-\lambda}} \cdot (\lambda)^{\cancel{3}}}{3! \, 3}$

$1 = \frac{\lambda}{3}$

$\lambda = 3$

$P(x=5) = \frac{e^{-3} \cdot (3)^5}{5!}$

$= \frac{e^{-3} \times 243}{5 \times 4 \times 3 \times 2 \times 1}$

$= \frac{0.0497 \times 243}{120}$

$= 0.100645$ (or)

0.101

CQ: In a busstand the buses are comes inside the busstand per minute are is given by Poisson distribution is

$\lambda = 0.9$ is 9 buss.

i) exactly 9 busses comes inside within 5 minutes.

ii) Less than 10 busses comes inside within 8 minutes.

iii) atleast 14 busses comes inside within 11 minutes

i) $P(x=9)$ $\lambda = 0.9 \times 5$

$\lambda = 4.5$

$$P(x) = \frac{e^{-\lambda} \cdot \lambda^{x}}{x!}$$

$$= \frac{e^{-4.5} \cdot (4.5)^{9}}{9!}$$

ii) $P(x < 10) = P(x = 0 \text{ to } 9)$

$\lambda = 8$ min.

$= 8 \times 0.9$

$\lambda = 7.2$

$$= \sum_{x=0}^{9} \frac{e^{-7.2} \cdot (7.2)^{x}}{x!}$$

iii) $P(x \geq 14) = P(x = 0 \text{ to } 13)$

$\lambda = 11$ min

$= 11 \times 0.9$

$\lambda = 9.9$

$= 1 - P(x < 14)$

$= 1 - P(x = 0 \text{ to } 13)$

$$= 1 - \sum_{x=0}^{13} \frac{e^{-9.9} (9.9)^{x}}{x!}$$

9. $\lambda = 2.5$

$$P(x) = \frac{e^{-\lambda} \lambda^{x}}{x!}$$

i) $P(x=0) = \frac{e^{-2.5} \cdot (2.5)^{0}}{0!}$

$= 0.0821$

ii) $P(x=3) = \frac{e^{-2.5} (2.5)^{3}}{3!}$

$= \frac{0.0821 \times 15.625}{3 \times 2 \times 1}$

$= 0.2138$

iii) $P(x \geq 5) = 1 - P(x < 5)$

$= 1 - [P(x=0) + P(x=1) + P(x=2) + P(x=3) + P(x=4)]$

$= 1 - e^{-2.5}\left[\frac{(2.5)^0}{0!} + \frac{(2.5)^1}{1!} + \frac{(2.5)^2}{2!} + \frac{(2.5)^3}{3!} + \frac{(2.5)^4}{4!}\right]$

$= 1 - 0.0821 [1 + 2.5 + 3.125 + 2.604 + 1.627]$

$= 1 - 0.0821 \times 10.856$

$= 1 - 0.8912$

$= 0.1087$

10. $\lambda = 0.25$

$$P(x) = \frac{e^{-\lambda} \cdot \lambda^{x}}{x!}$$

$P(x \geq 2) = 1 - P(x < 2)$

$= 1 - [P(x=0) + P(x=1)]$

$= 1 - e^{-0.25}\left[\frac{(2.5)^0}{0!} + \frac{(0.25)^1}{1}\right]$

$= 1 - 0.7788 [1 + 0.25]$

$= 1 - 0.7788 (1.25)$ $= 0.0265$

12. $\lambda = 2$.

$$P(x) = \frac{e^{-\lambda} \lambda^{x}}{x!}$$

i) $P(x=0) = \dfrac{e^{-2} \times (2)^0}{0!}$

$= 0.1353$.

ii) $P(x \geq 3) = 1 - P(x < 3)$

$= 1 - [P(x=0) + P(x=1) + P(x=2)]$

$= 1 - e^{-2}\left[\dfrac{(2)^0}{0!} + \dfrac{(2)^1}{1!} + \dfrac{(2)^2}{2!}\right]$

$= 1 - 0.1353\ [1 + 2 + 2]$

$= 1 - 0.6765$

$= 0.3235$.

Eg : 7.14

$$P(x) = \frac{e^{-\lambda} \lambda^{x}}{x!}$$

first $P(x=0) = 0.2725$

$\dfrac{e^{-\lambda} \cdot \lambda^0}{0!} = 0.2725$

$e^{-\lambda} = 0.2725$.

$e^{\lambda} = \dfrac{1}{0.2725} = 3.6697$

$\lambda = 1.30$

$= 1.3$

Next $P(x=1) = \dfrac{e^{-1.3} (1.3)^1}{1!}$

$= 0.2725 \times 1.3$

$= 0.3543$.

Eg : 7.15

λ = mean = $\frac{390}{520}$ = 0.75.

$P(x) = \frac{e^{-\lambda} \lambda^{x}}{x!}$

P(x = 0) no error = $\frac{e^{-0.75} \times (0.75)^{0}}{0!}$

= 0.4723.

for 5 page = $(0.4723)^5$

= 0.0235.

Eg : 7.16

$\lambda = 10$

$P(x) = \frac{e^{-\lambda} \cdot \lambda^{x}}{x!}$

$P(x \leq 5) = P(x=0) + P(x=1) + P(x=2) + P(x=3) + P(x=4) + P(x=5)$

$= e^{-10}\left[\frac{(10)^0}{0!} + \frac{(10)^1}{1!} + \frac{(10)^2}{2!} + \frac{(10)^3}{3!} + \frac{(10)^4}{4!} + \frac{(10)^5}{5!}\right]$

$= 0.000045\left[1 + 10 + \frac{100}{2} + \frac{1000}{3 \times 2 \times 1} + \frac{10000}{4 \times 3 \times 2 \times 1} + \frac{100000}{5 \times 4 \times 3 \times 2 \times 1}\right]$

$= 0.000045\left[1 + 10 + 50 + 166.67 + 416.67 + 833.33\right]$

$= 0.000045\left[1477.64\right]$

= 0.0665.

Eg 7.17

$$\lambda = \frac{\frac{1}{5}}{100}$$

$$\lambda = \frac{1}{500} = 0.002 \times 10$$

$$\lambda = 0.02$$

$$P(x) = \frac{e^{-\lambda} \lambda^{x}}{x!}$$

i) $P(x=0) = \frac{e^{-0.02} (0.02)^{0}}{0!}$

$= 0.9802$

In 10000 packets $= 0.9802 \times 10$

$= 98020.$

ii) $P(x=1) = \frac{e^{-0.02} \cdot (0.02)^{1}}{1!}$

$= 0.9802 \times 0.02$

$= 0.0196$

In 10000 packets $= 1960.$

iii) $P(x=2) = \frac{e^{-0.02} (0.02)^{2}}{2!}$

$= \frac{0.9802 \times 0.0004}{2}$

Eg: 7.18

$\lambda = 2000 \times 0.001$

$\lambda = 2$

$$P(x) = \frac{e^{-\lambda} \cdot \lambda^{x}}{x!}$$

i) $P(x=3) = \dfrac{e^{-2}(2)^3}{3!}$

$= \dfrac{0.1353 \times 8}{3\times2\times1}$

$= 0.1804$

ii) $P(x \geq 2) = 1 - P(x \leq 2)$

$= 1 - [P(x=0) + P(x=1)] + P(x=2)$

$= 1 - e^{-2}\left[\dfrac{(2)^0}{0!} + \dfrac{(2)^1}{1!}\right] + \dfrac{(2)^2}{2!}]$

$= 1 - 0.1353\,[1+2]2]$

$= 1 - 0.1353(5)$

$= 0.3235.$

Eg: 7.19

$\lambda = 8$ $\quad P(x) = \dfrac{e^{-\lambda} \cdot \lambda^x}{x!}$

i) $P(x=5) = \dfrac{e^{-8}(8)^5}{5!}$

$= \dfrac{0.000335 \times 32768}{5\times4\times3\times2\times1}$

$= 0.0915 \times 100$

$= 9.16\%$

Eg: 7.20

$P = \dfrac{1}{80}$ $\quad \lambda = \dfrac{1}{80} \times 30$

$\lambda = 0.375.$

$P(x \geq 2) = 1 - P(x < 2)$

$= 1 - [P(x=0) + P(x=1)].$

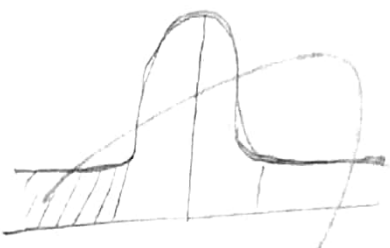
$= 1 - e^{-0.375}\left[\dfrac{(0.375)^0}{0!} + \dfrac{(0.375)^1}{1!}\right]$

$0.6873\,(1 + 0.375) \quad = 0.0556$

Exercise 7.3

Normal Distribution

$P(Z < 2) = 0.11$



$P(Z < 2) = 0.5 - P(Z < 2)$

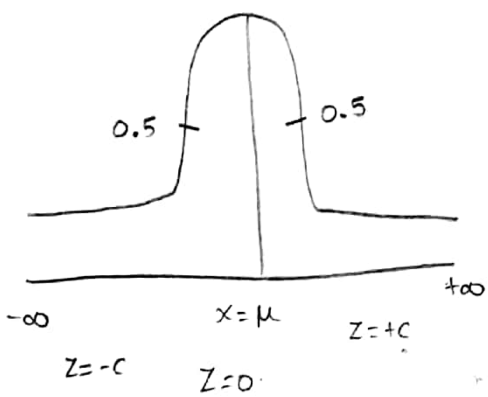
$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

μ = Mean = np = npq = λ.

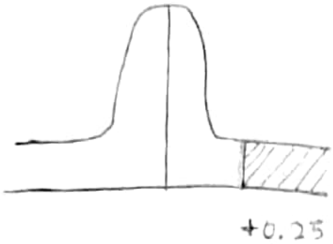
σ = Standard deviation

$\Rightarrow$ $z = \frac{x-\mu}{\sigma}$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}z}.$$

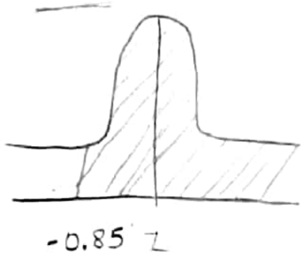


$(z > c) = 0.25$



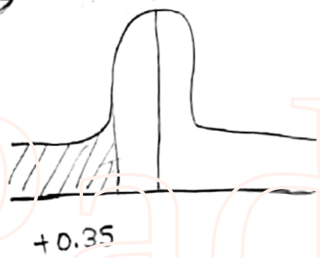
$P(z > c) = 0.5 - P(z = c)$

$(z > c) = \underline{0.85}$



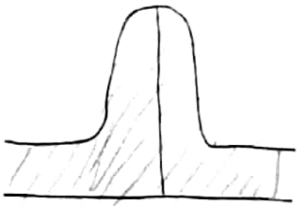
$P(z > c) = 0.5 + P(z = c).$

$(z < c) = 0.35$

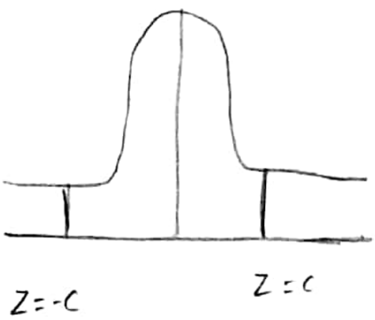


$P(z < c) = 0.5 - P(z = c)$

$(z < c) = 0.99$



$P(z < c) = 0.5 + P(z = c).$



$P(-c < z < c)$

$= 2P(0 < z < c)$

(or)

$= 2P(z = c).$

Eg : 7.21

i) $P(z > 1.09)$

$P(z > 1.09)$

$= 0.5 - P(z = 1.09)$

$= 0.5 - 0.3621$

$= 0.1379$

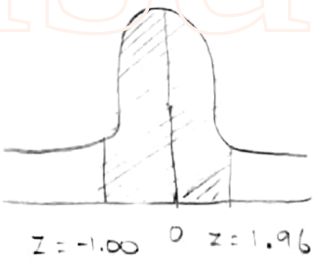
ii) $P(z < -1.65)$

$P(z < -1.65) = 0.5 - P(z = 1.65)$

$= 0.5 - 0.4505$

$= 0.0495.$

iii) $P(-1.00 < z < 1.96)$



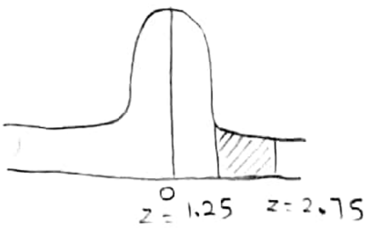
$P(-1.00 < z < 1.96)$

$= P(z = 1.00) + P(z = 1.96)$

$= 0.3413 + 0.4750$

$= 0.8163.$

iv) $P(1.25 < z < 2.75)$



$P(1.25 < z < 2.75)$

$= P(2.75) - P(1.25)$

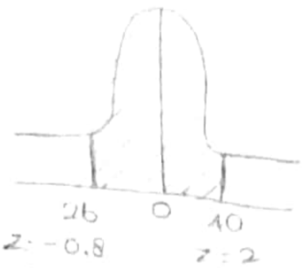
$= 0.4970 - 0.3944$

$= 0.1026.$

Eg : 7.22

$\mu = 30, \sigma = 5.$

i) $26 \le x \le 40$



$x = 26$

$z = \dfrac{\cancel{x} - \mu}{\sigma}$

$= \dfrac{26-30}{5}$

$= \dfrac{-4}{5}$

$z = -0.8$

$x = 40$

$= \dfrac{40-30}{5}$

$= \dfrac{10}{5}$

$z = 2.$

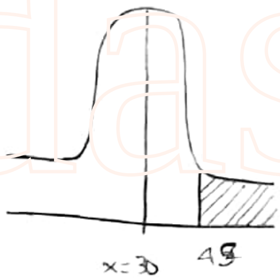
$P(26 \le x \le 40) = P(-0.8 \le z \le 2)$

$= P(z = 0.8) + P(z = 2)$

$= 0.2881 + 0.4772$

$= 0.7653.$

ii) $x > 45$



$x = 45.$

$z = \dfrac{45-30}{5}$

$= \dfrac{15}{5}$

$z = 3$

$P(x > 45) = P(z > 3)$

$= 0.5 - P(z = 3)$

$= 0.5 - 0.4987$

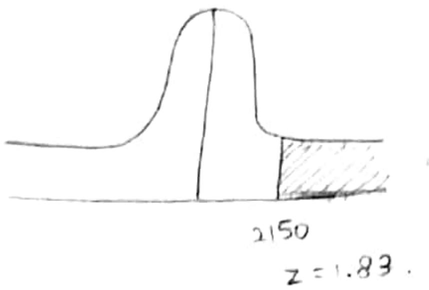
$= 0.0013.$

5. /

5. $\mu = 2040$, $\sigma = 60$

x = bulb

i) $P(x > 2150)$



$x = 2150$

$$z = \frac{x - \mu}{\sigma}$$

$$= \frac{2150 - 2040}{60}$$

$$= \frac{110}{60}$$

$z = 1.83$

$$P(x > 1.83) = 0.5 - P(z = 1.83)$$

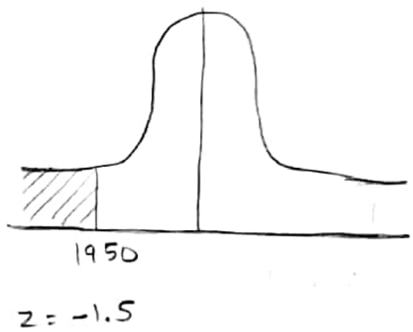
$$= 0.5 - 0.4664.$$

$$= 0.0336$$

out of 2000 $= 0.0336 \times 2000$

$= 67.2$. (or) 67 bulbs

ii) $P(x < 1950)$.



$x = 1950$.

$$Z = \frac{1950 - 2040}{60}$$

$$= \frac{-90}{60}$$

$Z = -1.5$

$$P(x < 1950) = 0.5 - \cancel{1950}\ P(z = 1.5)$$

$$= 0.5 - 0.4332$$

$$= 0.0668.$$

out of 2000 $= 0.6688 \times 2000$

$= 133.6$. (or) 134 bulbs

iii) $P(1920 < x < 2100)$

$x = 1920$ $\quad$ $x = 2100$

$z = \frac{1920 - 2040}{60}$, $z = \frac{2100 - 2040}{60}$

$= \frac{-120}{60}$ $\quad$ $= \frac{60}{60}$

$z = -2$ $\quad$ $z = 1.$

$P(1920 < x < 2100) = P(z=2) + P(z=1)$

$= 0.4772 + 0.3413$

$= 0.8185.$

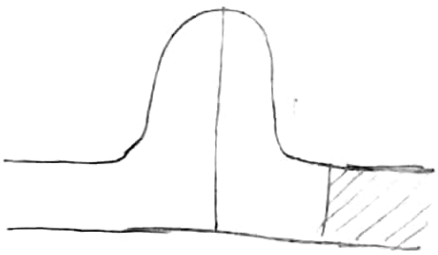
out of 2000 $= 0.8185 \times 2000$

$= 1637$

8. $\mu = 68.$, $\sigma = 3.$

i) $P(x > 72).$

$x = 72$



$z = \frac{x - \mu}{\sigma}$

$= \frac{72 - 68}{3}$

$z = 1.33$

$P(x > 72) = 0.5 - P(z = 1.33)$

$= 0.5 - 0.4082.$

$= 0.0918$

out of 500 $= 0.0918 \times 500$

$= 45.9$ $\quad$ or 46

$= 46.$

ii) $P(x \le 64)$.

$x = 64$

$z = \frac{64-68}{3}$

$z = -1.33$

$P(x \le 64) = 0.5 - P(z = 1.33)$

$= 0.5 - 0.4082$

$= 0.0918$

out of $500 = 0.0918 \times 500$

$= 45.9$

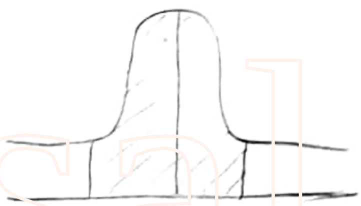
or (46)

iii) $P(65 < x < 71)$.

$x = 65$ $\qquad$ $x = 71$

$z = \frac{65-68}{3}$ $\qquad$ $z = \frac{71-68}{3}$

$z = -1$ $\qquad$ $z = 1$



$P(65 < x < 71) = P(z = 1) + P(z = 1)$

$= 0.3413 + 0.3413$

$= 0.6826$

out of $500 = 0.6826 \times 500$

$= 341.3$

$= 341$

$= P(-1 < z < 1)$

$= 2P(z = 1)$

$= 2 \times 0.3413$

$= 0.6826$

out $500 = 0.6826 \times 500$

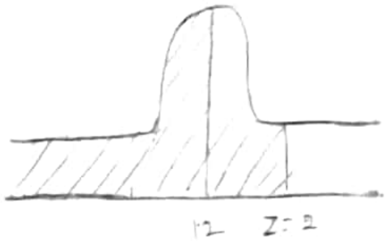
$= 341.3$

$= 341$

d. / μ =

7. $\mu = 12$, $\sigma = 4$

i) $P(x \le 20)$



$x = 20$

$$z = \frac{x-\mu}{\sigma}$$

$$= \frac{20-12}{4}$$

$z = 2$

$P(x \le 20) = 0.5 + P(z=2)$

$= 0.5 + 0.4772$

$= \cancel{0.028}$

$= 0.9772$

ii) $P(0 \le x \le 12)$

$x = 0$ $\quad$ $x = 12$.

$z = \frac{0-12}{4}$ $\quad$ $z = \frac{12-12}{4}$

$z = -3$ $\quad$ $z = 0$

$P(0 \le x \le 12) = P(x=3) + P(x=0)$

$= 0.4987 + 0.000$

$= 0.4987$.

9. $\mu = 16.28$, $\sigma = 0.12$.

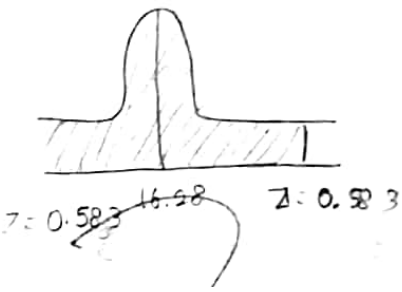
i) $P(x \le 16.35)$

$x = 16.35$

$$z = \frac{x-\mu}{\sigma}$$

$$= \frac{16.35-16.28}{12}$$

$= \frac{0.07}{0.12}$ $\quad$ $z = 0.\cancel{00}583$

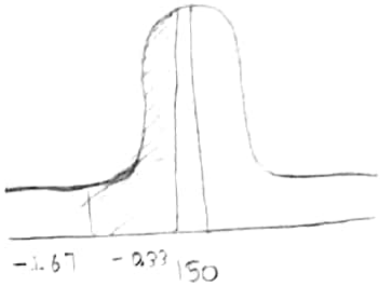


$P(x < 16.35) = 0.5 + P(z < 0.0058)$

$= 0.5 + 0.2190$

$= 0.719$

Eg: 7.23 $\mu = 150000$ $\sigma = 15000$



i) $P(1,25,000 < x < 1,45,000)$

$x = 1,25,000$ $x = 1,45,000$

$z = \frac{x-\mu}{\sigma}$ $z = \frac{x-\mu}{\sigma}$

$= \frac{1,25,000 - 1,50,000}{15000}$ $= \frac{1,45,000 - 1,50,000}{15000}$

$z = -1.67$ $z = -0.33$

$P(1,25,000 < x < 1,45,000) = \cancel{P(z = -0.3)}$

$= P(z = 1.67) - P(z = 0.33)$

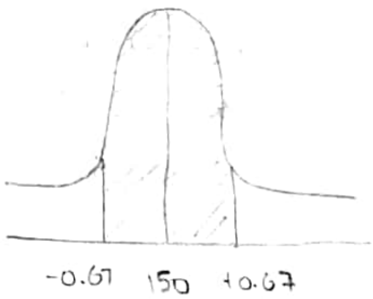
$= 0.4525 - 0.1293$

$= 0.3232.$

out of 550 =

0.3232×550

$= 178.$



ii) $P(1,40,000 < x < 1,60,000)$

$x = 1,40,000$ $x = 1,60,000$

$z = \frac{x-\mu}{\sigma}$ $z = \frac{x-\mu}{\sigma}$

$= \frac{1,40,000 - 1,50,000}{15000}$ $= \frac{1,60,000 - 1,50,000}{15000}$

$z = -0.67$ $z = 0.67$

$P(1,40,000 < x < 1,60,000) = P(z = 0.67) + P(z = 0.67)$

$= 0.2486 + 0.2486$

$= 0.4972.$

out of 550 = $0.4972 \times 550 = 273.$

Eg: 7.24

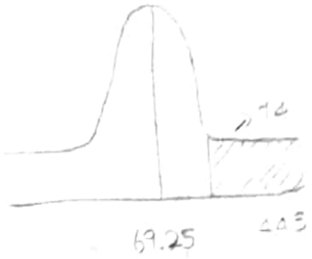
$\mu = 69.25$, Variance $= 10.8$

$S.D = \sqrt{\text{variance}}$

$= \sqrt{10.8}$

$\sigma = 3.286$

i) $P(x > 74)$



$x = 74$.

$z = \frac{x - \mu}{\sigma}$

$= \frac{74 - 69.25}{3.286}$

$z = 1.445$.

$P(x > 74) = 0.5 - P(z = 1.445)$.

$= 0.5 - 0.4251$

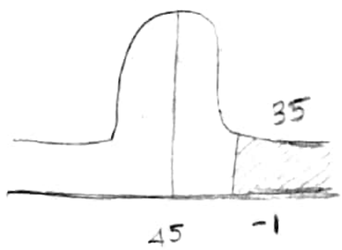
$= 0.0749$.

out of 1200 children $= 0.0749 \times 1200 = 90$.

Eg: 7.25.

$\mu = 45$, $\sigma = 10$.

i) $P(x < 35)$



$x = 35$.

$z = \frac{x - \mu}{\sigma}$

(or)

$= \frac{35 - 45}{10}$

$z = -1$

$P(x < 35) = 0.5 - P(z = 1)$

$= 0.5 - 0.3413$

$= 0.1587$

out of $1300 = 0.1587 \times 1300 = 206$

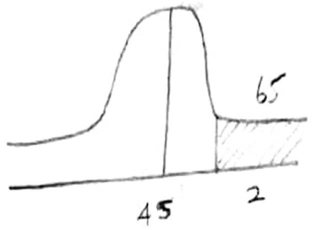
ii) $P(x>65)$

$x = 65$

$z = \frac{x-\mu}{\sigma}$

$= \frac{65-45}{10}$

$z = 2.$



$P(x>65) = 0.5 - P(z=2)$

$= 0.5 - 0.4772$

$= 0.0228$

out of 1300 students = 30.

Eg: 7.26

$\mu = 125, \ \sigma = 18.$

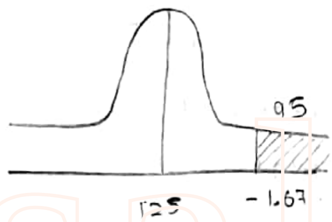
i) $P(x<95)$

$x = 95.$

$z = \frac{x-\mu}{\sigma}$

$= \frac{95-125}{18}$

$z = -1.67.$



$P(x<95) = 0.5 - P(z=1.67)$

$= 0.5 - 0.4525$

$= 0.0475$

out of 900 = $0.0475 \times 900 = 43.$

Eg: 7.27

$\mu = 69.25.$ variance = 9.8

S.D = $\sqrt{\text{variance}}$

$\sigma = 3.13.$

Soldier (6×12)

$x = 72$

i) $P(x>72).$

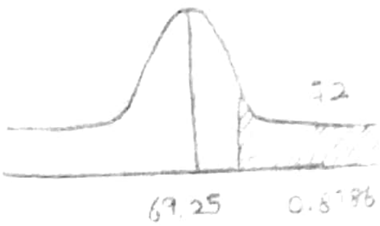
$\cancel{x} \neq 6.$

$x = 72.$

$= \frac{72 - 69.25}{3.13}$

$= 20.21$ ~~2.75~~

$z = 0.8786$



$P(x > 72) = 0.5 - P(z = 0.8786)$

$= 0.5 - 0.3106$

$= 0.1894$

out of 6000 $= 0.1894 \times 6000$

$= 1136.$

Eg: 7.28

$\mu = 5, \quad \sigma = 0.6$

i) $P(x < 6)$.

$x = 6$.

$Z = \frac{X - \mu}{\sigma}$

$= \frac{6-5}{0.6}$

$= 1.67$

$P(x < 6) = 0.5 - P(z = 1.67)$

$= 0.5 - 0.4525$

$= 0.0475$.

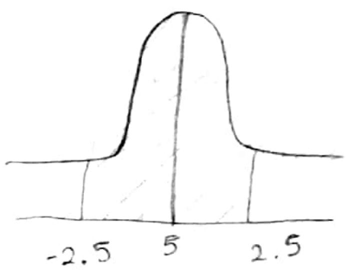
ii) $P(3.5 < x < 6.5)$

$x = 3.5 \qquad x = 6.5$

$z = \frac{x - \mu}{\sigma} \qquad Z = \frac{X - \mu}{\sigma}$

$= \frac{3.5 - 5}{0.6} \qquad = \frac{6.5 - 5}{0.6}$

$z = -2.5 \qquad z = 2.5$



$P(8.5 < x < 6.5) = P(z=2.5) + P(z=2.5)$

$= 0.4918 + 0.4918$

$= 0.9836$

Eg: 7.29

$\mu = 12, \sigma = 3$

i) $P(x > 13)$

$x = 13$.

$z = \frac{x-\mu}{\sigma}$

$= \frac{13-12}{3}$

$= 0.33$.

$P(x > 13) = 0.5 - P(z = 0.33)$

$= 0.5 - 0.1293$

$= 0.3707$.

out of 125 = 0.3707 × 125 = 46.337

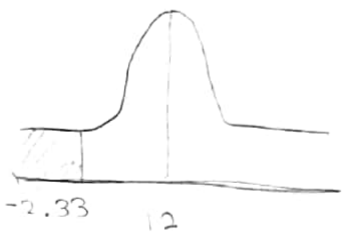
ii) $P(x < 5)$.

$x = 5$

$z = \frac{x-\mu}{\sigma}$

$= \frac{5-12}{3}$

$z = -2.33$.



$P(x>13) = 0.5 - P(z = 2.33)$

$= 0.5 - 0.4901$

$= 0.0099$

out of 125 = 0.0099 × 125 = 1.237.

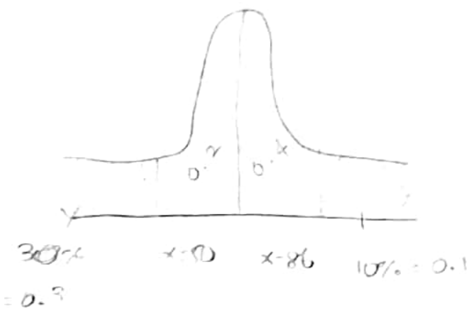
iii) $P(9 < x < 14)$.

$x = 9, \quad x = 14$.

$$Z = \frac{x-\mu}{\sigma} = \frac{9-12}{3} = -1 \qquad Z = \frac{x-\mu}{\sigma} = \frac{14-12}{3} = 0.667$$

$$P(9 < x < 14) = P(z = 1) + P(z = 0.667)$$
$$= 0.3413 + 0.2486$$
$$= 0.5899.$$

6.



$$P(x = 50) = 0.2$$
$$P(z = c) = 0.2$$
$$z = -c = -0.52$$

$$\frac{x-\mu}{\sigma} = -0.52.$$

$$\frac{50-\mu}{\sigma} = -0.52.$$

$$50-\mu = -0.52\sigma.$$

$$\mu - 0.52\sigma = 50 \quad \ldots (1).$$

$$P(x = 86) = 0.4.$$
$$P(z = c) = 0.4$$
$$z = C = 1.28$$

$$\frac{x-\mu}{\sigma} = 1.28$$

$$\frac{86-\mu}{\sigma} = 1.28.$$

$86 - \mu = 1.28\sigma$

$\mu + 1.28\sigma = 86$ (2)

(2) $\rightarrow$ $\mu + 1.28\sigma = 86$

(1) $\rightarrow$ $\mu - 0.5\sigma = 50$

(−) (+) (−)

$1.8\sigma = 36$

$\sigma = \frac{36}{1.8}$

$\sigma = 20$

(1) $\Rightarrow$ $\mu - 0.52\sigma = 50$

$\mu - 0.52(20) = 50$

$\mu - 10.4 = 50$

$\mu = 50 + 10.4$

$\mu = 60.4$

10 $\mu = 400$, $\sigma = 100$

$x = 450 + P$.

i) Penalty $= \frac{200000}{10000} = 20$

$P(x \geq 450 + 20)$

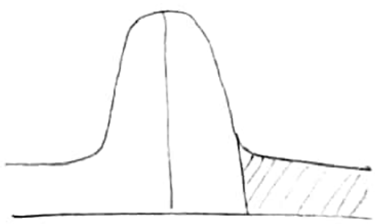
$= P(x \geq 470)$.

$x = 470$

$Z = \frac{x - \mu}{\sigma}$

$= \frac{470 - 400}{100}$

$= \frac{70}{100}$



$Z = 0.7$

$P(x \geq 470) = P(z \geq 0.7)$.

$= 0.5 - P(z = 0.7)$

$= 0.5 - 0.2580$

$= 0.242$

ii) $P(x \leq 500)$

$x = 50$

$z = \frac{500-400}{100}$

$= \frac{100}{100}$

$z = 1$

$P(x \leq 500) = 0.5 + P(z = 1)$

$= 0.5 + 0.3413$

$= 0.8413$

Eg: 7.30

$\mu = 2.25$, $\sigma = 0.25$.

$x = 2.0$

$z = \frac{x - \mu}{\sigma}$

$= \frac{2.0 - 2.25}{0.25}$

$= -1.$

$P(z < 1.0) = 0.5 - P(z = 1.0)$

$= 0.5 - 0.3413$

$= 0.1587$.

Eg: 7.31

$\sigma = 100$, $\mu = 800$.

$P(800 < x < 1000)$

$x = 800$. $x = 1000$.

$z = \frac{x - \mu}{\sigma}$ $\qquad$ $z = \frac{x - \mu}{\sigma}$

$= \frac{800 - 800}{100}$ $\qquad$ $= \frac{1000 - 800}{100}$

$z = 0$ $\qquad$ $z = 2.$

$P(800 < x < 1000) = 0.5 -$

$= P(z=2) - P(z=0)$

$= 0.4772 - 0$

$= 0.4772$