SETS

Definition: Any well defined collection of objects is called set.

Some standard sets:

1. The set of all natural numbers (or positive integers) = N.

$N = \{1, 2, 3, ...\}$.

2. The set of all whole numbers – W

$W = \{0, 1, 2, 3, 4 ....\}$.

3. Set of Integers – Z (or) I

$Z = \{....-3, -2, -1, 0, 1, 2, 3 ....\}$

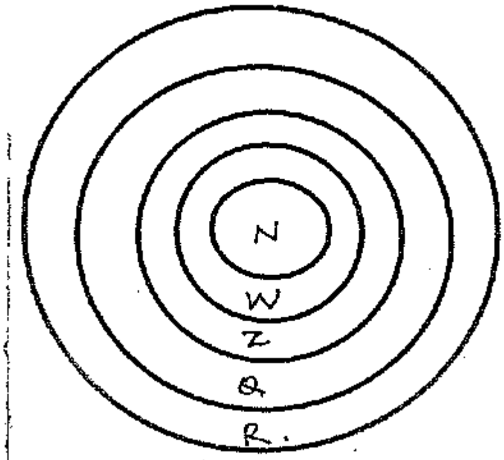
4. Set of rational numbers – Q

$Q = \{x : x = \frac{m}{n}$ where m and n are integers $n \neq 0\}$

5. Set of Real numbers – R.

$R = \{x : x$ is real number$\}$.

$R = \{x : x$ is either rational (or) irrational number$\}$



Null set (or) void set: A set containing no elements is called empty set (ϕ)

Eg: The collection of all integers whose square is <0 is empty set.

Finite and infinite sets. A set is called finite if the processes of counting of its different elements comes to an end. Otherwise it is called infinite. The empty set is always taken as finite.

Order of a finite set: The number of different elements in a finite set S is called order of S. (ie) $n(S)$

Cardinal numbers: The number of distinct elements in a finite set is called cardinal number.

1. The order of the empty set is zero
2. A set whose order is 1 is singleton set. (ie) Singleton set is the set which contains only one distinct element.

Equivalent set: Two finite sets A and B are said to be equivalent $(A \sim B)$ iff they contain the number of distinct elements are same

(ie) $n(A) = n(B)$ (ie) cardinal number are same

Equal set: Two sets A and B are said to be equal $(A = B)$ iff every member of A is a member of B and every member of B is a member of A.

Singleton set: A set which contains only one element is known as singleton set.

Universal set :

<u>Properties of power set:</u> 1) If $A \subseteq B$ then $P(A) \subseteq P(B)$

2) $P(A) \cap P(B) = P(A \cap B)$

3) $P(A \cup B) \neq P(A) \cup P(B)$

1) Natural number: $N = \{1, 2, 3, \cdots\}$.

2) Whole Number : $W = \{0, 1, 2, 3 \cdots\}$. $N \subset W$.

3) Integer $= Z = \{\cdots -4, -3, -2, -1, 0, 1, 2, 3, 4 \cdots\}$. $W \subset Z$.

4) Rational Number : $Q = \{\frac{p}{q}, p, q \in Z \text{ and } q \neq 0\}$

<u>Note:</u> All integers are also rational number $\because n$ can be represented by $\frac{n}{1}$.

$Z \subset Q$. $\therefore N \subset W \subset Z \subset Q$.

5) Irrational number: A number which cannot be written in the form p/q where p and q both are integers and $q \neq 0$ called irrational Number

$$Q' = \{x : x \in R, x \notin Q\}.$$

Note: A non terminating, non repeating decimal number are irrational number.

Set Operations.

Intersection of two sets: Let A and B be any two sets then the set consisting of the elements which belong to both A and B is called the intersection of A and B $(A \cap B)$

Disjoint sets: Two sets A and B are called disjoint iff $A \cap B = \phi$ (i.e) iff they have no elements in common.

Difference of two sets: Let A, B be any two sets then $A - B$ is the set consisting of all the elements which belong to A but donot belong to B.

Symmetric difference of two sets: Let A, B be two sets. Symmetric difference of A and B denoted by $A \Delta B$ is defined as the set $(A-B) \cup (B-A)$.

Complement of a set: Let A be any given set and U be the universal set, Then the set consisting of all the elements of U which donot belong to A is called the Compliment of A. and is denoted by $\bar{A}$ (or) A'.

Basic laws of Set Theory: If A and B are any two sets and U is the universal set then

1) Indempotent laws : $A \cup A = A$ and $A \cap A = A$.
2) Identity laws: $A \cup U = U$ and $A \cap U = A$ and $A \cup \varphi = A$

$$A \cap \varphi = \varphi$$

3) Commutative law $A \cup B = B \cup A$, $A \cap B = B \cap A$.
4). Compliment laws: $A \cup \bar{A} = U$, $A \cap \bar{A} = \varphi$
5) Associative law: If A, B, C are any three sets then
   1) $(A \cup B) \cup C = A \cup (B \cup C)$
   2) $(A \cap B) \cap C = A \cap (B \cap C)$
6) Distributive law: If A, B, C are any three sets then
   1) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
   2) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
7) De Morgan's law: If A and B are any two sets then 1) $(A \cup B)' = A' \cap B'$
   2) $(A \cap B)' = A' \cup B'$ 3) $A - (B \cup C) = (A-B) \cap (A-C)$
   4) $A - (B \cap C) = (A-B) \cup (A-C)$

Sub set: Let A and B are two sets s.t every member of A is a member of B, then A is called the sub set of B. $A \subseteq B$.

If A is the sub set of B we say that A is contained in B (or) B contains A then B is the super set.

$N \subset W \subset Z \subset Q \subset R$.

Note: 1. Two sets A and B are equal iff $A \subset B$, $B \subset A$.

2. Empty set is also a sub set of every set.

Proper Sub set: Let A be the sub set of B, we say that A is a proper sub set of B if $A \neq B$ (ie) if there exists atleast one element in B which does not belong to A. A sub set which is not proper, is called improper sub set.

1. Every set is an improper sub set of itself

2. When A is empty, then the null set is the proper sub set.

Power set: The set formed by all the sub sets of a given set A is called power set of A. denoted by P(A)

Note: If $n(A) = n$, then $n[P(A)] = 2^n$

Working Rule to write the power set

1. First write φ

2. write down the singleton sub sets of each containing only one element.

3. write all the sub sets two elements from the set A.

Continue this way and in the end write A itself

Enclose all these in braces to get the power of A.

Comparable set: Two sets A and B are said to be comparable iff either $A \subset B$ (or) $B \subset A$.

Clearly equal sets are always comparable set

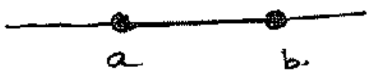
Universal sets: All sets under investigation are regarded as sub sets of a fixed set. We call this set the the universal set 'U'

Venn Diagram: We can illustrat the relationship between the sets with the help of the diagrams known as Venn diagrams. Generally we represent the universal set U by the rectangle enclosing a certain region and its sub sets by closed curve with in the rectangle.

Intervals as Subsets of R.

1) The set $\{x \in R : a < x < b\}$ is called open interval and is denoted by (a, b)

2). The set $\{x \in R : a \leq x \leq b\}$ is called closed interval and denoted by $[a, b]$



3). The set $\{x \in R, a < x \leq b\}$ is an interval left open and right closed.

4) The set $\{x \in R, a \leq x < b\}$ is an interval left closed and right open

5. The set $\{x \in R : x < a\}$ is an interval which is denoted by $(-\infty, a)$ It is open on both sides.

6) The set $\{x \in R : x \leq a\}$ is an interval which is denoted by $(-\infty, a]$ It is closed on the right and open on the left

7. The set $\{x \in R : x > a\}$ is an interval which is denoted by (a, ∞) both open

8. The set $\{x \in R : x \geq a\}$ is an interval which is denoted by $[a, \infty)$ left closed Right open.

Note: 1) The first four is finite interval.

2) The last four is infinite interval.

3) Each interval (finite, or infinite) is an infinite set containing infinitely many rational and infinitely many irrational numbers.

Operation on two sets: 1) **union of two sets:**

Let A, B be any two sets then the set consisting of elements which belong to A or B or both is called union of A and B. $(A \cup B)$

Applications of sets: Relation involving order of sets

1) If A and B are finite sets and $A \cap B = \varphi$ then

$$n(A \cup B) = n(A) + n(B)$$

$$n(\bar{A}) = n(U) - n(A) \Rightarrow n(A) + n(\bar{A}) = n(U)$$

In particular $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

when $A \cap B = \varphi$ $\quad n(A \cup B) = n(A) + n(B)$

2) If A and B are any two sets for which $A \cap B = \varphi$ then $A \cup B$ is the union of mutually disjoint sets $A-B$, $A \cap B$, $B-A$

Hence if both A and B are finite sets

$$n(A \cup B) = n(A-B) + n(B-A) + n(A \cap B)$$

3) If A and B are any two ~~events~~ finite sets for which $A \cap B = \varphi$ then A is union of disjoint sets $A-B$ and $A \cap B$ and B is the union of disjoint sets $B-A$ and $A \cap B$ then

$$\left.\begin{aligned} n(A-B) &= n(A) - n(A \cap B) \\ n(B-A) &= n(B) - n(A \cap B) \end{aligned}\right\} \; X$$

4) If A ~~and~~, B and C are any three finite sets then

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$

Laws of difference of sets: For any two sets A and B we have

1) $A - B = A \cap B'$
2) $B - A = B \cap A'$
3) $A - B \subseteq A$
4) $B - A \subseteq B$
5) $A - B = A \Leftrightarrow A - B = \varphi$
6) $(A-B) \cup B = A - B \neq A \cup B$
7) $(A-B) \cup (B-A) = (A \cup B) - (A \cap B)$

For any three sets A, B, C

1) $A - (B \cap C) = (A-B) \cup (A-C)$
2) $A - (B \cup C) = (A-B) \cap (A-C)$
3) $A - (B-C) = (A \cap B) - (A \cap C)$
4) $A \cap (B-C) = (A \cap B) - (A \cap C)$
5) $A \cap (B \Delta C) = (A \cap B) \Delta (A \cap C)$.

Properties of Compliment sets.

1) $(A')' = A = U - A'$ (law of double complimentation)
2) $A \cup A' = U$
3) $A \cap A' = \varphi$
4) $\varphi' = U$
5) $U' = \varphi$
6) $(A \cup B)' = U - (A \cup B)$

De Morgan's law

1) If A and B are any two sets then 1) $(A \cup B)' = A' \cap B'$

2) $(A \cap B)' = A' \cup B'$

2) $A \Delta B = B \Delta A$
3) $(A \Delta B) \Delta C = A \Delta (B \Delta C)$
4) $A \cap (B \Delta C) = (A \cap B) \Delta (A \cap C)$

Important Results:

1) $n(A\cup B) = n(A-B) + n(B-A) + n(A\cap B)$

2) $n(A) = n(A-B) + n(A\cap B)$

3) $n(B) = n(B-A) + n(A\cap B)$

4) $n(A\Delta B) = n[(A-B)\cup(B-A)]$
$= n(A-B) + n(B-A)$
$= n(A) + n(B) - 2n(A\cap B)$ $\quad \because A-B, B-A$ are disjoint

5) $n(A'\cup B') = n[(A\cap B)'] = n(U) - n(A\cap B)$, $\quad n(A'\cap B') = n(A\cup B)' = n(U) - n(A\cup B)$

6) $n(A-B) = n(A\cap B') = n(A) - n(A\cap B)$

If A, B, C are three finite sets, then

1) $n(A\cup B\cup C) = n(A) + n(B) + n(C) - n(A\cap B) - n(B\cap C) - n(A\cap C) + n(A\cap B\cap C)$

2) $n(A \text{ only}) = n(A) - n(A\cap B) - n(A\cap C) + n(A\cap B\cap C)$

3) $n(\bar{A}\cap\bar{B}\cap\bar{C}) = n(U) - n(A\cup B\cup C)$

Identity of 1) $A\cup\phi = A$, $\quad A\cap U = A$

Idempotent $A\cup A = A$, $\quad A\cap A = A$.

Absorption: $A\cup(A\cap B) = A$

$A\cap(A\cup B) = A$.

4) $n(A\cap B'\cap C') = n\{A\cap(B\cup C)'\} = n(A) - n\{A\cap(B\cup C)\}$.

5) $\{(A\cup B'\cup C)\cap(A\cap B'\cap C')\}\cup\{(A\cup B\cup C')\cap(B'\cap C')\} = B'\cap C'$.

Relations

Ordered pair: An ordered pair consists of two objects or elements in a given fixed order.

Equal of ordered pairs: Two ordered pairs (a_1, b_1) and (a_2, b_2) are equal iff $a_1 = a_2$ and $b_1 = b_2$

Cartesian product of sets: For two nonempty sets A and B the set of all ordered pairs (a, b) s.t $a \in A$ and $b \in B$ is called Cartesian product of the sets A and B denoted by $A \times B$.

(ie) $A \times B = \{(a, b) : a \in A \text{ and } b \in B\}$

If there are three sets A, B and C and $a \in A$, $b \in B$ and $c \in C$ then $A \times B \times C = \{(a, b, c) : a \in A, b \in B, c \in C\}$

Properties of Cartesian product:

1. For three sets A, B and C

1) $n(A \times B) = n(A)\,n(B)$

2. $A \times B = \varphi$. if either A or B is any empty set

3. $A \times (B \cup C) = (A \times B) \cup (A \times C)$

4. $A \times (B \cap C) = (A \times B) \cap (A \times C)$

5. $A \times (B - C) = (A \times B) - (A \times C)$

6. $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$

7. $A \subseteq B$ and $C \subseteq D$ then $(A \times C) \subseteq (B \times D)$

8. $A \subseteq B$ then $A \times A \subseteq (A \times B) \cap (B \times A)$

9. $A \times B = B \times A \Leftrightarrow A = B$

10. If either A or B is an infinite set then $A \times B$ is infinite

11. $A \times (B' \cup C')' = (A \times B) \cap (A \times C)$

12. $A \times (B' \cap C')' = (A \times B) \cup (A \times C)$

13. If A and B be any two non empty sets having n elements in common then $A \times B$ and $B \times A$ have n^2 elements in common.

14. $A \neq B$ then $A \times B \neq B \times A$

15. $A = B$ then $A \times B = B \times A$, 16) $A \subseteq B$ then $A \times C \subseteq B \times C$ for any set C.

Note: $n(A \times B) = n(A) \cdot n(B)$

$n(A \times B \times C) = n(A) \cdot n(B) \cdot n(C)$

$(A \times B) \cap (B \times A) = (A \cap B) \times (B \cap A)$

$A \times (B' \cap C')' = (A \times B) \cup (A \times C)$

Relation: If A and B are two nonempty sets then the relation R from A to B is a subset of $A \times B$.

If $R \subseteq A \times B$ and $(a, b) \in R$, then we say that a is related to b by the relation R.

Domain and Range: Let R be the relation from the set A to B Then the set of all first components or Co-ordinates of the ordered pairs belonging to R is called the domain of R, while the set of all second elements or Co-ordinates of the ordered Pairs belonging to R is the Range of R.

(ie) The set of elements of A is domain and the set of image elements is range.

The set of elements of B is Co-domain

Range $\subseteq$ of codomain.

Types of Relation:

1. **Void Relation:** As $\phi \subset A \times A$ for any set A, ϕ is the relation (or) empty relation on A Called empty relation or void relation.
2. **Universal Relation:** $\because A \times A \subseteq A \times A$, so $A \times A$ is a relation on A called the universal relation.
3. **Identity relation:** The relation $I = \{(a, a) : a \in A\}$ is called identity relation.
4. **Reflexive relation:** A relation R is said to be reflexive relation if every element of A related it itself;

   $(a, a) \in R \ \forall a \in A \Rightarrow R$ is reflexive.
5. **Symmetric Relation:** A relation R is said to be symmetric relation iff $(a, b) \in R \Rightarrow (b, a) \in R \ \forall a, b \in R$.
6. **Anti symmetric relation:** A relation R is said to be anti symmetric relation iff $(a, b) \in R$ and $(b, a) \in R \Rightarrow a = b, \forall a, b \in A$.
7. **Transitive Relation:** A relation R is said to be transitive relation iff $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$

   $\forall a, b, c \in A$.

8) Equivalance Relation: A relation R is said to be an equivalance relation if it is reflexive, symmetric and transitive on A.

9. Partial order relation: A relation R is said to be partial order relation, if it is reflexive, symmetric and antisymmetric.

10). Total order Relation: A relation R on a set A is said to be a total order relation on A if R is partial order relation on A.

Inverse Relation: If A and B are two non empty sets and R be a relation from A to B s.t $R = \{(a,b) : a \in A, b \in B\}$ then the inverse of R denoted by R^{-1} is the relation from B to A and is defined by $R^{-1} = \{(b,a) : (a,b) \in R\}$.

Equivalence classes of an Equivalance Relation:

Let R be equivalence ration in A $(A \neq \phi)$ Let $a \in A$. Then the equivalance class of a equivalence relation denoted by $[a]$ is defined as the set of all those points of A which are related to a under the relation R.

Composition of Relation: Let R and S be two relations from the set A to B and B to C respectively, Then we can define the relation SoR from A to C s.t $(a,c) \in SoR \Leftrightarrow \exists\, b \in B$ s.t $(a,b) \in R$ and $(b,c) \in S$.

Congruence Modulo n: Let n be an arbitrary but fixed integer. Two integers a and b are said to be congruence modulo n if a−b is divisible by n

$$a \equiv b \pmod{n} \Leftrightarrow a-b \text{ is divisible by } n.$$

1. If R and S are two equivalence relation on a set A then $R \cap S$ is also equivalence relation on A.
2. The union of two equivalence relation on a set is not necessarily an equivalence relation on a set
3. If R is an equivalence relation on a set A then R^{-1} is also an equivalance relation on A.
4. If a set A has n elements, then the number of reflexive relation from A to A is 2^{n^2-n}

Examples: Empty relation: 1) Consider a relation R in the set $A = \{1,2,3,4\}$ given by $R = \{(a,b), a \in A, b \in A, a \neq b \text{ and } a = b\}$. This is the empty set as no pair (a, b) satisfies the condition $a \neq b$, $a = b$.

2) Consider the relation R in the set $A = \{1,3,5,7\}$ given by

$$R = \{(a,b) : a-b = 9\}.$$ This is empty set as no pair satisfies the condition $a - b = 9$ in A.

Universal Relation: 1) Let $A = \{1,2,3\}$ then

$R = \{(1,1)\ (1,2)\ (1,3),\ (2,1)\ (2,2)\ (2,3)\ (3,1)\ (3,2)\ (3,3)\}$ is universal relation in A.

2) Let us consider the relation R in the set $A = \{1,3,5,7\}$ given by

$$R = \{(a,b) : |a-b| \geq 0\}.$$

As all pairs (a, b) in $A \times A$ satisfies $|a-b| \geq 0$ $\therefore R = \{(a,b) : |a-b| \geq 0\}$ is the whole set $A \times A$. $\therefore$ R is universal set.

Identity relation: If $A = \{1,2,3,4,5,6\}$ then $I = \{(1,1)\ (2,2)\ (3,3)\ (4,4)\ (5,5)\ (6,6)\}$

In any identity relation on A, every element of A should be related to itself only.

Inverse relation: If $A = \{1,2,3,4\}$ $B = \{a,b,c\}$

$R = (1,a)\ (2,b)\ (3,a)\ (4,b)\}$ then

$R^{-1} = (a,1)\ (b,2)\ (a,3)\ (b,4)$

We conclude that the domain of R is identical with Range of R^{-1} and the range of R is identical with the domain of R^{-1}.

(ie) Domain of R = Range of R^{-1}, Range of R = Domain of R^{-1}.

Reflexive relation: 1. Let A be the set of triangles in a plane. The relation R in A 'is similar to' is reflexive because every Δle is similar to itself.

2. Let A be the set of all lines in a plane and R means 'is parallel to'. Now every line $a \in A$ is parallel to itself.

3. The relation '>' on the set R is not reflexive because no real number can be not greater than to itself.

Symmetric relation: i) Let A be the set of lines in a plane and let R means 'is perpendicular to' then R is symmetric. (ie) if the line a is $\perp r$ to the line b then it implies that the line b is $\perp r$ to a.

$\therefore$ R is symmetric.

2. Let N be the set of natural numbers and R means a divides b Then the relation R is anti symmetric. $\because$ a divides b and b divides a only when $a = b$.

Transitive relation: 1. Let A be the set of all lines in a plane and R be the relation in A defined by 'is parallel to'. Then if the line a is Parallel to b and b is parallel to c then a is parallel c Hence R is transitive.

2) Let A be the set of members of a family and R means 'is the brother of' Now if a is a brother of b and b is the brother of c then a is the brother of c $\therefore$ R is transitive.

3) If The R is taken as a subset of $A \times A$ Then $(a,b) \in R$ and $(b,c) \in R \Rightarrow (a,c) \in R$. R is called transitive relation.

Equivalance relation:- A relation R on a set A is called an equivalance relation on A when R is reflexive, Symmetric, and transitive

(ie) if 1) $(a,a) \in R$, $a \in A$

2) $(a,b) \in R, \Rightarrow (b,a) \in R$, where ever $a,b \in A$.

3) $(a,b) \in R$, and $(b,c) \in R \Rightarrow (a,c) \in R \ \forall a,b,c \in A$

Example: Let A be the set of all triangles in a plane and let R be defined by 'is Congruent to'

1. Since every triangle is congruent to itself R is reflexive

2. Since if triangle a is congruent to $\triangle$le b then the triangle b is congruent to triangle a Then R is symmetric.

3. If the triangle a is congruent to triangle b and triangle b is congruents to the $\triangle$le c then the triangle a is congruent to c. Then R is transitive.

$\therefore$ The relation defined above is reflexive, symmetric, transitive Hence R is an equivalance relation.

Horizontal test to find the function through its Graphs -

Let the function be given as a curve in the plane. If the horizontal through a point y in a co-domain meets the curve at some point then the x-coordinate of all the points give pre image of y

1. If the horizontal line through a point y in the co-domain does not meet the curve, then there will be no-pre image for y and hence the function is not onto
2. If the horizontal line through at least one point meets the curve at more than one point, Then the function is not one-one
3. If for all y in the range the horizontal line through y meets the curve at only one point then the function is one-one.

Method to solve problems Based on types of relation-

For Reflexive: we have to show that $\forall a \in A$, $(a,a) \in R$. For this we take an arbitrary element of set A in form of variable (say x) and then Check (x,x) satisfies the given condition of R or not (ie $(x,x) \in R$ or not) If it satisfies this then R is reflexive.

For Symmetric: For $a, b \in A$ if $(a, b) \in R$ then $(b,a) \in R$ for this take two arbitrary elements of A in the form of two variables (x, and y) s.t $(x, y) \in R$ and then check (y,x) satisfies the given condition of R or not ((ie) if $(x,y) \in R \Rightarrow (y,x) \in R$) Then R is symmetric otherwise not.

For Transitive. $\forall a, b, c \in A$, $(a,b) \in R, (b,c) \in R$ then $(a, c) \in R$. For we take three arbitrary elements of A. in the form of x, y, z s.t $(x,y) \in R$ and $(y,z) \in z$ then check whether (x,z) satisfies the condition of R or not. (ie) $(x, z) \in R$. If the condition satisfies then R is transitive otherwise not.

Functions

Definition: A relation R from a set A to the set B is said to the function if every element of set A has one and only one image in set B.

(or) A function f from $A \to B$ is a relation s.t the domain of f is A and no two distinct ordered pairs in f have the same first element.

Eg: The relation 'son of' is a function but the relation 'father of' is not a function.

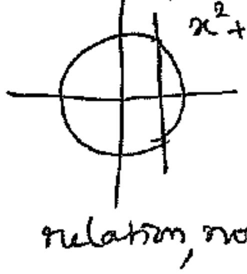
Note: Function is also a mapping.

Characteristics of a function $f: A \to B$:

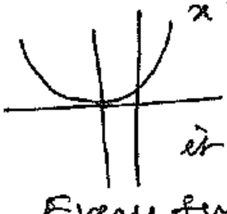
1) For each element $x \in A$, there is a unique element image y in B.
2) $y \in B$ is the value of the function.
3. $f: A \to B$ is not a function, if there is an element in A which has more than one image in B but more than one element of A may be associated with same elements of B.
4. $f: A \to B$ is not a function if an element of A does not have an image in B.

Identification of a function with its Graphs:

If we draw a vertical line (ie) $\parallel^{el}$ to y axis, then if this line intersects the graph of the expression in more than one point then it is a relation. If it intersects at only one point then it is a function.



relation, not a function



it is a function.

Every function is a relation.

Relation need not be a function.

Classification: Real functions are generally classified under two categories algebraic function and transcendental function.

I Algebraic function: 1) **polynomial function:** If the function $y=f(x)$ is given by $f(x) = a_0x^n + a_1x^{n-1} + \dots + a_{n-1}x + a_n$

$$= \sum_{i=0}^{n} a_i x^{n-i}$$

where $a_0, a_1, a_2, \dots a_n$ are real numbers and n is any non-negative integer then $f(x)$ is called a polynomial function in x. then the degree of the polynomial is n.

Eg: $y = f(x) = 3x^5 - 4x^2 - 2x + 1$ is a polynomial fn. of degree 5.

2) Rational function: If the function $y = f(x)$ is given by $f(x) = \frac{\varphi(x)}{\psi(x)}$ where $\varphi(x)$ and $\psi(x)$ are polynomial function then $f(x)$ is called rational fn in x.

3) Irrational function: The algebraic function containing two or more terms having non integral rational power of x is called irrational function Eg: $y = f(x) = 2\sqrt{x} - \sqrt[3]{x} + b$

In General Algebraic fn, is in the form $y = f(x)$

II Transcendental function: A function which is not algebraic fn. is known as Transcendental function

Eg: Trigonometric, Inverse Trigometric, Exponential, logarithmic function.

Explicit Function: A function is said to be explicit function if it is expressed as $y = f(x)$

Implicit function: A function is said to be implicit function if it is expressed $f(x, y) = c$. where c is constant.

Eg $\cos(x+y) - (\cos x - y) = 2$

Intervals of the function:

1) closed interval: The set of real numbers x s.t $a \le x \le b$ is called closed interval and is denoted by $[a, b]$

2) open interval: The set of all real numbers s.t $a < x < b$ is called open interval and is denoted by (a, b)

3) Semi open $= [a, b) = a \le x < b$.

semi closed $(a, b] = a < x \le b$.

Periodic Function: A function $f(x)$ is said to be a periodic fn. of x provided there exist a real number $T > 0$ s.t

$$F(T + x) = f(x) \ \forall x \in R.$$

The smallest positive real number T, satisfying the above condition is known as period or the fundamental period of $f(x)$.

Important points 1) Constant function is a periodic function with no fundamental period.

2. If $f(x)$ is a periodic function with period T then $\frac{1}{f(x)}$, $\sqrt{f(x)}$ is also periodic with same period T.

3. If $f(x)$ is periodic with T_1 and $g(x)$ is periodic with T_2 then $f(x)+g(x)$ is periodic with period equal to LCM of T_1 and T_2 Provided there is positive k. s.t $f(k+x) = g(x)$, $g(k+x) = f(x)$.

4. $\sin x$, $\cos x$, $\operatorname{cosec} x$, $\sec x$ are periodic functions with period 2π

5. $\tan x$, $\cot x$ is periodic f_n with period π.

6. $|\sin x|$, $|\cos x|$, $|\tan x|$, $|\cot x|$, $|\sec x|$ and $|\operatorname{cosec} x|$ is also a Periodic f_n with period π.

7. $\sin^n x$, $\cos^n x$, $\sec^n x$, $\operatorname{cosec}^n x$ are periodic function with period 2π when n is odd (or) even.

8. $\tan^n x$, $\cot^n x$ are periodic with period π

9. $|\sin x|+|\cos x|$, $|\tan x + \cot x|$, $|\sec x + \operatorname{cosec} x|$ are periodic with Period $\pi/2$

Even Function: A real function $f(x)$ is an even function if

$f(-x) = f(x)$ Eg 1) $f(x) = x^2$, $f(-x) = x^2$ $f(x) = \cos x$, $f(-x) = \cos(-x) = \cos x$.

Odd Function: A real function $f(x)$ is an odd function if $f(-x) = -f(x)$ Eg $f(x) = x^3$, $f(-x) = -x^3$ | $f(x) = \sin x$, $f(-x) = \sin(-x) = -\sin x$

Properties: 1 Even $\pm$ Even = Even

2 odd $\pm$ odd = odd.

3 Even * odd = odd f_n.

4 Even × even = Even

5 odd × odd = even.

6 If $f \circ g$, $g \circ f$ is even if any one of f and g or both are even.

7. $f \circ g$, $g \circ f$ is odd if f and g are both are odd.

8. The graph of the even f_n is symmetrical to Y axis
The graph of the odd f_n is symmetrical about origin (or) opposite quadrants.

9. An even f_n can never be the one to one, odd function may or may not be one-one.

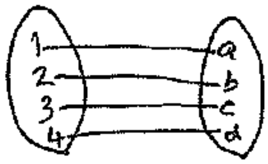
Types of function: (Mappings)

one-to-one or injective function: A function $f: A \to B$ is defined to be one to one if different elements in A have different images in B. (ie) if no two different elements in A have same image in B. in other words every element in A has unique image in B.

For every $x_1, x_2 \in A$, $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

if $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$

Eg: 1) $f: R \to R$ defined by $f(x) = 2x+3$



$f(x_1) = 2x_1+3$, $f(x_2) = 2x_2+3$.

If $f(x_1) = f(x_2)$

$2x_1+3 = 2x_2+3$

$2x_1 = 2x_2$

$x_1 = x_2$ then f is one to one.

Eg: 2 $f: R \to R$ defined by $f(x) = |x| + x$.

For $x_1 \neq x_2$ (ie) $-2 \neq +3$

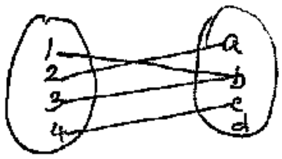
$f(-2) = -x + x = 2-2 = 0$

$f(-3) = -x+3 = 0 \Rightarrow f(-2) = f(-3)$ But $-2 \neq -2$

$\therefore f$ is not one-one.

Many to one: A function $f: A \to B$ is said to be many to one function if at least two elements in A have same image in B.

For $x_1 \neq x_2 \Rightarrow f(x_1) = f(x_2)$



Eg: $f(x) = x^2$

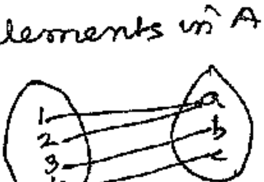
Let $-1, 1 \in R$ and $-1 \neq +1$ (ie) $x_1 \neq x_2$

$f(-1) = 1$ (ie) $f(x_1) = f(x_2)$ $\therefore f$ is many to one

$f(1) = 1$

On to function: A function $f: A \to B$ is said to be onto (Surjective) if every element of B is the image of some elements in A under f

(ie) Range = Codomain.

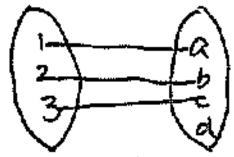


(Note) It may be one to one (or) many to one

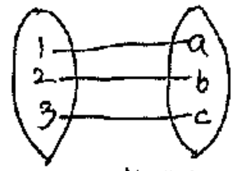
Into function : A function $f: A \to B$ is called into function if there is atleast one element of B which has no pre image in A.

Eg: $A = \{1, 2, 3\}$ $B = \{a\ b\ c\ d\}$

$f: (1, a)\ (2, b)\ (3, c)$



Bijective function : A function $f: A \to B$ is called bijective if f is both one to one and on.



Composition of functions :

Let $f: A \to B$ and $g: B \to C$ be two functions. Then the composition of function f and g denoted by $f \circ g$ is defined by $f \circ g : A \to C$. 15

Note: 1) In General $f \circ g \neq g \circ f$. Composition of function is not commutative.

2) $f \circ g = g \circ f = I$ iff g and f are inverses to each other.

3) If f and g are two bijections then composition of function is also bijection

4. Composition of any function with the identity function is the function itself.

Invertable function: A function $f: A \to B$ is said to invertable if there exists a function $g: B \to A$ s.t $g \circ f = f \circ g = I$.

$\therefore$ Function g is called the inverse of f and is denoted by f^{-1}

$\therefore g = f^{-1}$.

Note: $f: A \to B$, $g: B \to A$ be two functions s.t $g \circ f = f \circ g = I$ then f must be both one to one and onto.

Otherwise f^{-1} does not exist.

Inverse function (if exist) is unique.

Procedure to find out the inverse of a function.

Let $f: A \to B$ be bijection. In order to find f^{-1}

Put $y = f(x)$, solve $y = f(x)$ and find the x value in terms of y. Now replace x by $f^{-1}(y)$.

1) Find the inverse fn Let $f : R \rightarrow R$ be a fn defined by

Ex: $f(x) = 2x+1$. Find f^{-1}.

Sol: Let $g = f^{-1}$ $\qquad g \circ f = I$

$$(g \circ f)(x) = x$$

$$g \circ (f(x)) = x$$

$$g(2x+1) = x. \qquad \text{Let } 2x+1 = y$$

$$x = \frac{1}{2}(y-1)$$

$$\therefore g(y) = \frac{y-1}{2}$$

$$f^{-1}(y) = \frac{y-1}{2} \Rightarrow f^{-1}(x) = \frac{x-1}{2}.$$

2) Let $f : R \rightarrow R$, $f(x) = x+1$ find f^{-1}.

Let $g = f^{-1}$.

Ex:

$$(g \circ f)(x) = I = x.$$

$$g \circ f(x) = x.$$

$$g(x+1) = x. \qquad \text{let } x+1 = y \Rightarrow x = y-1.$$

$$g(y) = y-1.$$

$$f^{-1}(y) = y-1.$$

$$f^{-1}(x) = x-1.$$

3) Find the domain of the rational fn $f(x) = \frac{x^2+x+2}{x^2-x}$

$$q(x) = x^2 - x = x(x-1) = 0$$

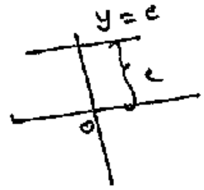
$$x = 0, 1$$

$\therefore$ Domain of fn $S = R - \{0, 1\}$.

Types of function:

1. Constant function:

Let c be a fixed real number. The function that associates to each real number x, this fixed number c is called a constant function. (ie) $y = f(x) = c \ \forall R$.



⊗ Here range set is singleton set. ⁂

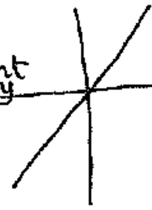
Domain = R

range = {c}

2) Identity fn: The function that associates to each real number x for the same is called the identity fn

(ie) $y = f(x) = x \ \forall x \in R$.

For each element of x there is a unique element in y at the same time all Real value of x has image and Range = Codomain



Domain - R

range - R.

Hence Constant fn is both one-one and onto. ∴ f^{-1} exists.

3) Linear function: A function $f: R \to R$ is defined in the form $f(x) = ax + b$ where a and b constant is called linear fn

Domain = R
range = R

Note 1. Graph of the linear fn is st. line

2. Inverse of linear fn always exist and f^{-1} is also linear.

∴ inverse fn of Linear fn is one to one and onto

4) Polynomial fn: Let $f: R \to R$, $f(x) = a_n x^n + a_{n-1}x^{n-1} + \dots + a_1 x + a_0$ where $a_0, a_1, \dots a_n$ are real numbers, $a_n \neq 0$. then f is the polynomial function.

1) ∴ $f: R \to R$ defined by $f(x) = x^3 + 5x^2 + 3$. is a cubic polynomial fun or polynomial fn of degree two.

2) $f: R \to R$ defined by $f(x) = ax^2 + bx + c$ $\quad a, b, c \neq 0$ is quadratic polynomial (or) polynomial fn of degree two.

Rational f^n Let $p(x)$ and $q(x)$ be any two polynomial f^n. Let S be the sub set of R obtaining after removing all values of x for which $q(x)=0$ from R.

The ~~set~~ function $f: S \to R$ defined by $f(x) = \frac{p(x)}{q(x)}$, $q(x) \neq 0$ is called the rational f^n

Exponential f^n: For any number $a>0$, $a \neq 1$. the function $f: R \to R$ defined by $f(x) = a^x$ is called an exponential function.

For exponential function the range is always R^+. (set of all +ve real numbers) Domain R, range $(0, \infty)$

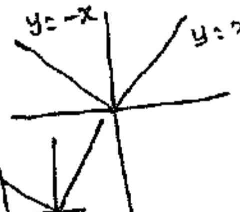
Logarithmic functions: The general form is $f(x) = \log_a x$, $a \neq 1$ and any +ve numb.

Domain $(0, \infty)$ of a log f^n becomes codomain of exponential f^n,

Codomain $(-\infty, \infty)$ of log f^n. becomes the domain of exponential f^n

The inverse of exponential f^n is logarithmic f^n

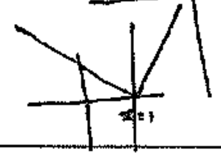
Modulus f^n $y = f(x) = |x|$ where $|x|$ denotes the absolute value of x that is $|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$



Domain R,

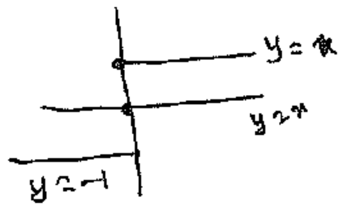
Range $= [0, \infty)$

For $f(x) = |x-1|$ =



Signum f^n: $f(x) = \begin{cases} \frac{|x|}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ (or) $f(x) = \begin{cases} \frac{x}{|x|} & x \neq 0 \\ 0 & x = 0 \end{cases}$

$y = f(x) = \text{sgn}(x) = \begin{matrix} -1 & x<0 \\ 0 & x=0 \\ 1 & > \end{matrix}$



Domain $= R$

Range $\{-1, 0, 1\}$

Maths - one Mark Tips.

Functions and Graphs.

1. Number line

2. Interval : Subset of real line is called an interval if it contains atleast two numbers and contains all the real numbers lying between any two of its elements.

Eg: the set of all real numbers x s.t $x > 6$

" x s.t $-2 \le x \le 5$

" x s.t $x < 5$.

3) Set of all natural numbers is not an interval.

Set of all non zero real numbers is also not an interval

Since: 1) Between any two rational number there are infinitely many real numbers which are not included in the given set.

2. Here 0 is absent. It fails to contain every real number between any two real numbers say -1 and 1.

4) A finite interval is said to be closed if it contains both of its end points and open if it contains neither of its end points and it is denoted by [ ], ( ) respectively.

$[2, 5] = 2 \le x \le 5$, $(2, 5) = 2 < x < 5$.

5. We cannot write closed interval by $-\infty$ or ∞. They are not representives of Real numbers.

6) Neighbourhood : 1) In a number line the neighbourhood of a point (real number) is defined by an open interval of small lengths

2) In a plane neighbourhood of a point is defined is an open circle with small radius.

3) In a space neighbourhood of a point is defined as an open sphere with very small radius.

7) A variable is an independent variable when it has any arbitrary value. (radius)

A variable is said to dependent when its value depends on the other variable. (V, A, S)

8) Cartesian product:

Let $A = \{a_1, a_2, a_3\}$, $B = \{b_1, b_2\}$. The Cartesian product of these two sets A and B is denoted by $A \times B$ is

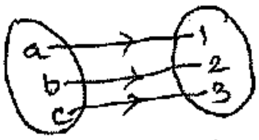
$$A \times B = \{(a_1 b_1)(a_1 b_2)(a_2 b_1)(a_2 b_2)(a_3 b_1)(a_3 b_2)\}$$

$$A \times B \neq B \times A$$

9) Relation: Let A and B be any two sets. A relation from $A \to B$ is the subset of the cartesian product of $A \times B$.

10) Function: Relation may be function (or) may not be function

In the ordered pair set If the first elements are different and the second element are also different then the relation said to be function.



In the ordered pair set If the first element are same and the second element are different it is not a function.

Function must be relation

11) Domain: Co-domain, Range

The range may or may not be equal to Co-domain.

Domain is the set A

Codomain is the set B.

The image set of the elements A is Range.

12) If the domain is not stated explicitly for the function $y = f(x)$ The domain is assumed to be the largest set of x values which the formula gives real y values.

$$f(x) = \sqrt{4 - x^2} \qquad [-2, 2].$$

In this function for $-2 \leq x \leq 2$ $f(x)$ is gives real value otherwise imaginary.

$\therefore [-2, 2]$ is the domain.

13) Similarly for the suitable value of x the value of $f(x)$ (ie) image set is the range.

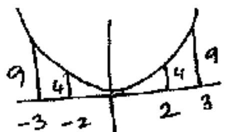
$$f(x) = \sqrt{9 - x^2} \qquad [0, 3] \quad \text{(ie) images}$$

For any suitable value of x the rest will be in $[0, 3]$

$\therefore [0, 3]$ is the range.

Function	Domain (x)	Range (y or $f(x)$)
$y = x^2$	$(-\infty, \infty)$	$[0, \infty)$
$y = \sqrt{x}$	$[0, \infty)$	$[0, \infty)$
$y = \frac{1}{x}$	R − {0} Non zero Real numbers	R − {0}
$y = \sqrt{1 - x^2}$	$[-1, 1]$	$[0, 1]$
$y = \sin x$	$(-\infty, \infty)$ $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ principal domain	$[-1, 1]$
$y = \cos x$	$(-\infty, \infty)$ $[0, \pi]$ principal domain	$[-1, 1]$
$y = \tan x$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ principal domain	$(-\infty, \infty)$
$y = e^x$	$(-\infty, \infty)$	$(0, \infty)$
$y = \log_e x$	$(0, \infty)$	$(-\infty, \infty)$

14) Graph of the function: The graph of a function f is the graph of the equation $y = f(x)$



1) $f(x) = x^2$

for every x there is a unique y

If we draw a vertical line it cuts the curve at only one place

$\therefore$ The function $y = x^2$ is a graph.

(Note: For the function, in the ordered pair first value are different, & second elements may be same)

$y^2 = x$.

2) $y^2 = x$ or $y = \pm\sqrt{x}$.

If we draw a vertical line it cuts the curve at two points

(ie) For unique value of x two different value of y. Hence it is not a graph. at $x=2$, $y = \pm\sqrt{2}$ $(2, \sqrt{2})$ $(2, -\sqrt{2})$

$x^2 + y^2 = 4$

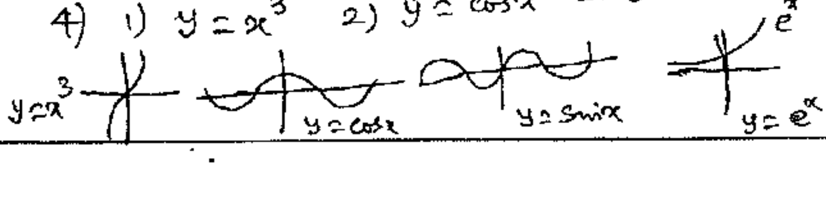
3) $x^2 + y^2 = 4$.

$x^2 + y^2 = 4$ is a circle with radius $2r$.

If we draw a vertical line at $x = 1$ it cuts the curve at two places (ie) For $x = 1$ $y = \pm\sqrt{3}$ $(1, \sqrt{3})$ $(1, -\sqrt{3})$.

Hence it is not a function.

4) 1) $y = x^3$ 2) $y = \cos x$ 3) $y = \sin x$ 4) $y = e^x$



are graph of the functions.

15) Types of functions

1. ~~on to functions~~: If the range and co-domain are equal then the function is on to. otherwise it is called into fn.

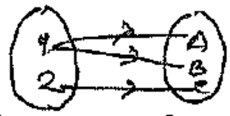
(ie) the image set is equal to co-domain then it is called onto.

(ie) every element in the co-domain has pre image.

(ie) If any ~~one~~ ~~any~~ single element in the co-domain is not ~~connected~~ ~~with~~ ~~to the~~ Connected with the element of A. (not onto)

2) In the ordered pair. first elements are different second element are same.

3). any ~~two~~ element in the set A has ~~same~~ Two images in B. But range and co-domain should be same



4) onto function is also known as surjective function.

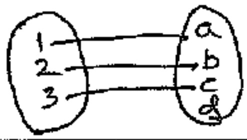
2) one to one function :

A function is said to be one to one if each element of the domain. range is associated with exactly one element of the

(ie) two different elements in the domain A have different images in the Co-domain B. But in co-domain there may be some element which has no pre image.

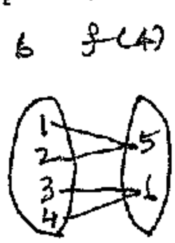
2) in ordered pair set the first elements are different and the second elements are also different.

(ie) $a_1 \neq a_2 \Rightarrow f(a_1) \neq f(a_2)$



Example 1) Let $A = \{1, 2, 3, 4\}$ $B = \{5, 6\}$.

$f(1) = 5$, $f(2) = 5$, $f(3) = 6$ $f(4) = 6$. then S.T f is on to function.



ordered Pair
$(1, 5)$ $(2, 5)$ $(3, 6)$ $(4, 6)$.

range $= \{5, 6\}$

co-domain $= \{5, 6\}$. $\therefore$ The given function is onto.

Example 2) Let $A = \{1, 2, 3, 4\}$ $B = \{5, 6, 7\}$, $f(1) = 5$, $f(2) = 5$

$f(3) = 6$ $f(4) = 6$ then S.T it is not onto (or) into

Range $= \{5, 6\}$, co domain $= \{5, 6, 7\}$ Range $\neq$ Codomain
$\therefore$ It is not onto

Example for not ~~onto~~: one to one

S.T the function $y = x^2$ is not onto.

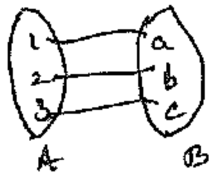
For $x = 1$ $y = 1$ $(1,1)$ $(-1,1)$

$x = -1$ $y = 1$ In the ordered pair the first element are different but second elements are same.

(i.e) different element has same image

Example for one to one

$A = \{1, 2, 3\}$ $B = \{a, b, c\}$ S.T the function is defined by $\{(1,a)\ (2,b)\ (3,c)\}$ is one to one fn.



different elements in A has different images in B. ∴ it is one to one.

16) A function is said to be injective if it is one-to-one

17) A function is said to be bijective if it is both one to one and onto. and surjective if it is onto.

Example: S.T the function $f : R \to R$ defined by $f(x) = x+1$ is bijective

For the set of all R, for the different real numbers have different images under $f(x) = x+1$ and no same element has the different images ∴ it is onto

Further $f(a_1) = f(a_2) \Rightarrow a_1 + 1 = a_2 + 1 \Rightarrow a_1 = a_2$ Hence it is one to one. ∴ it is bijective

18) $f : A \to A$ is defined by $f(x) = x$ $\forall x \in A$ then the function is said to be identity function and is denoted by I.

If every element has the same image then it is identity fn.

19) For the function f, if f^{-1} exist it must be one to one and onto (ie) f^{-1} exists iff f is one to one and onto.

20) For the inverse function f^{-1} the co domain of f becomes domain of f^{-1}.

21) Since all the functions are relation, the inverse function is also a relation.

22) If the function is not one to one and onto f^{-1} does not exist.

23) In general $f \circ g \neq g \circ f$ (ie) the composition of function need not be commutative. There may be some special cases have commutative.

24) By the definition of identity fn $f \circ g = g \circ f = I$.

25) Let $f : A \to B$ and be a function. If there exists a fun $g : B \to A$ s.t $f \circ g = I_B$ and $g \circ f = I_A$ then g is called Called the inverse of f. and is denoted by f^{-1}

26) The domain and the co-domain of both f and g are same $f \circ g = g \circ f = I$.

27) If f^{-1} exists then f is said to be invertable

28. $f \circ f^{-1} = f^{-1} \circ f = I$. (commutative admissable for I)

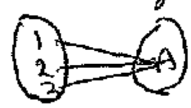
29) $(f \pm g)(x) = f(x) \pm g(x)$

30) $f \cdot g(x) = f(x) \cdot g(x)$

31) $\frac{f}{g}(x) = f(x) / g(x)$

32. $(cf)(x) = c \cdot f(x)$

33. Product of two functions is different from composition of function

34. <u>Constant fn</u>: If the range of the function is singleton set then the function is called constant fn 

Eg: 'is a son of' is a constant fn between sons and father here father is a range

35) Linear function: A fn $f: R \to R$ is defined in the form $f(x) = ax+b$ then the function is called linear functions.

The graph of the linear fn is a st. line

Inverse of linear fn always exists and also linear.

36) Polynomial function:

If $f: R \to R$ is defined by $f(x) = a_n x^n + a_{n-1} x^{n-1} \cdots + a_1 x + a_0$ where $a_0, a_1 \ldots a_n$ are all real numbers $a_n \neq 0$ then f is a polynomial fn. of degree n

37) Rational fn: Let $p(x)$ and $q(x)$ are any two polynomial fn. Let S be the subset of R obtained after removing all values of x for which $q(x) = 0$ from R.

The function $f: S \to R$ defined by $f(x) = \frac{p(x)}{q(x)}$ $q(x) \neq 0$ is called a rational fn

Note: To find the domain of a rational fn put $q(x) = 0$ and find x value. Except these x value the remaining value in R is the domomain of the rational fn.

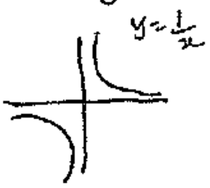
38) Exponential Function: For any number $a \neq 1$ and $a > 0$ the fn $f: R \to R$ is defined by $f(x) = a^x$ is called the exponential fn.

For Exponential fn. the range is always R^+.

Note: The curve of $f(x) = e^x$ lies between the curves ~~between~~ 2^x and 3^x.

39) Logarithmic fn: The inverse of exponential fn is a logarithmic fn. The general form of logarithmic fn is $f(x) = \log_a x$ $a \neq 1$ and $a > 0$

40) Reciprocal of a fn: The fn $f: S \to R$ defined by $g(x) = \frac{1}{f(x)}$ is called reciprocal fn of $f(x)$



a) The graph of reciprocal fn $g(x) = \frac{1}{x}$ does not meet neither axes for finite real number.

b) Reciprocal fn is associated with product of two fns

(ie) if f and g are reciprocals of each other then $f(x) \cdot g(x) = 1$

c) Inverse fn. associated with composition of fn.

(ie) if f and g are two fns $f \circ g = g \circ f = I$.

41) Absolute value fn (or) modulus fn

If $f: R \rightarrow R$ defined by $f(x) = |x|$ then the function is called absolute fn. of x

$$\text{where } |x| = \begin{cases} x & x \geq 0 \\ -x & < 0 \end{cases}$$

42) Step function: a) Greatest integer fn

The fun whose value at any real number x is the greatest integer less than or equal to x is called Greatest integer fn and is denoted by $\lfloor x \rfloor$. $f: R \rightarrow R$ defined by $f(x) = \lfloor x \rfloor$

$\lfloor 2.5 \rfloor = 2$. The domain of the fn is R and the range of the fn is Z

b) Least integer fn: The fn whose value at any real number is x is smallest integer greater than or equal to x is called the least integer fn and is denoted by $f(x) = \lceil x \rceil$

$\lceil 2.5 \rceil = 3$.

43) Signum fn: If $f: R \rightarrow R$ is defined by

$$f(x) = \begin{cases} \frac{|x|}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

Then f is called the signum fn

Domain is R and the range $\{-1, 0, 1\}$

44) If $f(-x) = f(x)$ for all x in the domain then the fn is called even fn

If $f(-x) = -f(x)$ for all x in the domain then the fn is called odd fn.

The graph of even fn is symmetric about y axis

The graph of odd fn is symmetric about origin.

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T.G.Venkatesan

Properties of Equivalence relation: :

1) The inverse of an equivalence relation is also an equivalence relation.

2. The intersection of two equivalence relation on a set A is also an equivalence relation

3. The union of two equivalence relation on a set A not necessarily equivalence relation on A.

Note: The relation 'congruence modulo n' is an equivalence relation in the set Z of all integers (Proof important)

Function as a relation.

1) A relation f from non empty set A to non empty set B is said to be a function if every element of set A has only one image in set B.

2) and no element in A does not have image in B.

Note: Every function is relation. But the converse need not be true (ie) every relation need not be function.

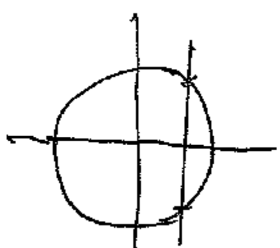
$f : \{(1,2)\ (2,3)\ (3,4)\}$ is a function $A = \{1,2,3\}$ $B = \{2,3,4\}$.

$f : \{(1,2)\ (1,3)\ (2,4)\}$ is not a function. Since $3 \in A$ does not have image and $1 \in A$ has two images $2,3 \in B$.

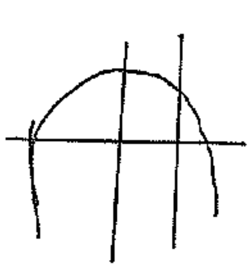
Identification of a function with its Graph.

Draw a vertical line (ie) any line parallel to y axis. This line intersect the graph of the curve at more than one point (ie) for one value of x it has two value of y (ie) for unique element in A has two images in y Hence it is a relation but not function.

If the vertical line cuts the curve at only one point (ie) for unique value of x has unique image in y. Hence this relation is a function. This is known as vertical line test.



not function



function.

Working Rule for finding the Domain of Real function -

For Rational function.

1. Put the denominator equal to zero and find the values of x
2. The Value of Value of x are the values at which rational function is not defined. Then omit the set of these values from R to get required domain.

For square root function.

We know that expression under the square root should not be negative. So for finding the domain of square root function put the expression without root greater than or equal to 0 and find then the value of x for which it is positive.

Working rule for finding Range of real functions.

1. Find the domain of the function $y = f(x)$
2. Transform the equation $y = f(x)$ as $x = g(y)$ (ie) convert x in terms of y.
3) Find the values of y from $x = g(y)$ s.t The values of x are real and lying in the domain of f.
4) The value of y obtained be the range of f

Properties of logarithmic function

1. $\log_a 1 = 0$ $a > 0$ and $a \neq 1$
2. $\log_a a = 1$ "
3. $\log_a(xy) = \log_a x + \log_a y$
4. $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$
5. $\log_a x^m = m \log_a x$
6. $\log_{a^n} x^m = \frac{m}{n} \log_a(x)$
7. $\log_a m = \frac{1}{\log_m a}$
8. $\log_a b \cdot \log_b x = \log_a x$.

Note! $\log_a x$ and a^x are inverses to each other. So their graphs are mirror image each other in the $y = x$.

Algebra of real function-

1. Addition of two real function:

The sum of the two functions $(f+g)$ defined by

$(f+g)x = f(x) + g(x) \quad \forall x \in D_1 \cap D_2$ D-Domain

$\therefore$ The domain of $f+g = D_1 \cap D_2$

2) Subtraction of two real function.

The difference of two real functions $(f-g)$ is defined by

$(f-g)(x) = f(x) - g(x) \quad \forall x \in D_1 \cap D_2$

$\therefore$ The domain of $f-g$ is $D_1 \cap D_2$

3) Multiplication of two real functions.

The product of the function (fg) is defined by $(fg)(x) = f(x).g(x)$
$\forall x \in D_1 \cap D_2$

$\therefore$ The domain of $fg = D_1 \cap D_2$

4. Quotient of two real function.

The quotient of the function is defied by $\frac{f}{g}(x) = \frac{f(x)}{g(x)}$
$\forall x \in D_1 \cap D_2 - \{x \mid g(x) \neq 0\}$.

Multiplication of real function by a scalar.

Multiplication of real function by a constand is defined as $(cf)(x) = c.f(x) \quad \forall x \in D_1$

The domain is D_1

Methods to check whether a function is one-one or many to one.

1. Consider any two arbitrary elements say $a_1, a_2 \in$ (domain)
2) Put $f(a_1) = f(a_2)$ and simplify the equation
3) If we get $a_1 = a_2$ f is one-one and if we get $a_1 \neq a_2$ f many to one.

Note! In order to prove f is not one-one it is sufficient to show $f(1) = 1$ and $f(2) = 1$

Methods to check whether the function is onto or into

1) Let $f: A \rightarrow B$ be the given function. Consider y be any arbitrary element in B. 2) Put $y = f(x)$ and simply it to obtain x in terms of y then let $x = g(y)$ under the given condition simplify then we get then f is onto or f is not onto

(or) If range = codomain (ie every element in the codomain has pre image in domain) then f is onto.

Note: If a function is one-one and onto (bejective) then only f^{-1} exists.

Composition of function:

Let $f : A \to B$ and $g : B \to C$ be any two functions then the composition of f and g denoted by gof is defined as $gof : A \to C$ given by

$$gof(x) = g[f(x)] \in A.$$

Clearly domain of gof = domain of f.

Note: 1) $gof(x) = g[f(x)]$ First f rule applied and then g rule is applied

2) Generally $gof \neq fog$

3) If $f : R \to R$ and $g : R \to R$ are real function then gof, fog exists.

4) Two functions f and g having the same domain D are said to be equal if $f(x) = g(x)$ $\forall x \in D$.

Composition of function is associtive.

If for any three function f, g and h are three functions s.t $(fog)oh = fo(goh)$ exist and $(fog)oh = fo(goh)$

Properties 1. If $f : A \to B$ and $g : B \to C$ are one-one then $gof : A \to C$ is also one-one

2) If $f : A \to B$, $g : B \to C$ are onto then $gof : A \to C$ is also onto.

3) If $f : A \to B$ and $g : B \to C$ are two functions then $gof : A \to C$ is onto $\Rightarrow g : B \to C$ is onto

$gof : A \to C$ is one-one $\Rightarrow A \to B$ is one-one.

Note: If gof is one-one, then it is not necessary that both f and g are one-one

If gof is onto then it is not necessary that both g and f are onto.

Method to show that the given function is invertable -

1. Show that f is one-one
2. Show that f is onto Then f is invertable.

To find f^{-1} let us take $y = f(x)$ and find x in terms of y - then

$f^{-1}(y) =$ Put the x value in terms y.

Replace y by x

$\therefore f^{-1}(x) :-$ — - -

Properties of f^{-1}.

1) The inverse is unique
2) The inverse of a bijective function is also bijective
3) If $(f^{-1})^{-1} = f$.

Graphing functions - The following type of transformation used in graphing

1) Reflection : a) The graph $y = -f(x)$ is the reflection of a graph of f about the x axis

b) The graph $y = f(-x)$ is the reflection of a graph f about y axis.

c) The graph $f^{-1}(x)$ is the reflection of the graph f about the line $y = x$.

Translation: $y = f(x+c)$ $c>0$ causes the shift to the left

$y = f(x-c)$ $c>0$ causes the shift to the right

$y = f(x)+d$ $d>0$ causes to shift to the upward

$y = f(x)-d$ $d>0$ causes to shift to the downward.

Dialation 1) If we multiplying a function by a constant >1 the graph moves away from the x axis

2) If we multiply a function by a constant <1 then the graph moves towards the x axis.

1) The number of elements in set A is m and in set B is n then the number of relations are 2^{mn}.
2. The number of elements in set A is n The number of relation on the set A itself $= 2^{n^2}$
3) If the number of elements in a set A is n the number of reflexive relation is $2^{n(n-1)}$
4) If the number of elements in a set A is n then the number of symmetric relation is $2^{\frac{n(n+1)}{2}}$

1) If A and B are two non empty finite sets containing m and n elements respectively.

The number of functions from $A \to B = n^m$

The number of one-one function (injective) $= \begin{cases} {}^nC_m \cdot m! & n \geq m \\ 0 & n < m \end{cases}$

The number of onto function (surjective) $= \begin{cases} \sum_{r=1}^{n} (-1)^{n-r} \cdot {}^nC_r \, r^m & m \geq n \\ 0 & m = n \end{cases}$

The number of one-one and onto bijective $\Big\} = \begin{cases} n! & m = n \\ 0 & m \neq n. \end{cases}$

1) If f is injective $n(A) \leq n(B)$
2) If f is surjective $n(A) \geq n(B)$
3) If f is bijective $n(A) = n(B)$

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DOMAIN, RANGE, CONTUNITY, DIFFERENTIABLITY AND GRAPH OF SOME DIFFERENT FUNCTIONS. (TABLE)

Continuity and Differentiability of Different Functions

Type of Functions	Curve	Domain and Range	Continuity and Differentiability
Identity function	$f(x) = x$,	Domain $= R$, Range $=]-\infty, \infty[$	Continuous and differentiable in their domain
Exponential function	$f(x) = a^x$, $a > 0, a \neq 1$	Domain $= R$, Range $=]0, \infty[$	
Logarithmic function	$f(x) = \log_a x$; $x, a > 0$ and $a \neq 1$	Domain $= (0, \infty)$, Range $= R$	
Root function	$f(x) = \sqrt{x}$	Domain $= [0, \infty)$, Range $= [0, \infty)$	Continuous and differentiable in $(0, \infty)$
Greatest integer function	$f(x) = [x]$	Domain $= R$, Range $= I$	Other than integral value it is continuous and differentiable
Least integer function	$f(x) = (x)$	Domain $= R$, Range $= I$	
Fractional part function	$f(x) = \{x\} = x - [x]$	Domain $= R$, Range $= [0, 1)$	
Signum function	$f(x) = \frac{\lvert x \rvert}{x}$ $= \begin{cases} -1, & x < 0 \\ 0, & x = 0 \\ 1, & x > 0 \end{cases}$	Domain $= R$, Range $= \{-1, 0, 1\}$	Continuous and differentiable everywhere except at $x = 0$
Constant function	$f(x) = c$	Domain $= R$, Range $= \{c\}$, where $c \to$ constant	
Polynomial function		Domain $= R$	Continuous and differentiable in their domain

Type of Functions	Curve	Domain and Range	Continuity and Differentiability
Sine function	$y = \sin x$	Domain $= R$, Range $= [-1, 1]$	Continuous and differentiable in their domain
Cosine function	$y = \cos x$	Domain $= R$, Range $= [-1, 1]$	
Tangent function	$y = \tan x$	Domain $= R - \left\{(2n+1)\frac{\pi}{2}\right\}$, Range $= R$	
Cosecant function	$y = \operatorname{cosec} x$	Domain $= R - n\pi$, Range $= (-\infty, -1] \cup [1, \infty)$	
Secant function	$y = \sec x$	Domain $= R - \left\{(2n+1)\frac{\pi}{2}\right\}$, Range $= (-\infty, -1] \cup [1, \infty)$	
Cotangent function	$y = \cot x$	Domain $= R - \{n\pi\}$, Range $= R$	
Arc sine function	$y = \sin^{-1} x$	Domain $= [-1, 1]$, Range $= \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	Continuous and differentiable in their domain
Arc cosine function	$y = \cos^{-1} x$	Domain $= [-1, 1]$, Range $= [0, \pi]$	
Arc tangent function	$y = \tan^{-1} x$	Domain $= R$, Range $= \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$	
Arc cosecant function	$y = \operatorname{cosec}^{-1} x$	Domain $= (-\infty, 1] \cup [1, \infty)$, Range $= \left(\frac{\pi}{2}, \frac{\pi}{2}\right) - \{0\}$	
Arc secant function	$y = \sec^{-1} x$	Domain $= (-\infty, -1] \cup [1, \infty)$, Range $= [0, \pi] - \left\{\frac{\pi}{2}\right\}$	
Arc cotangent function	$y = \cot^{-1} x$	Domain $= R$, Range $= (0, \pi)$	

11) $y = \sin x$.

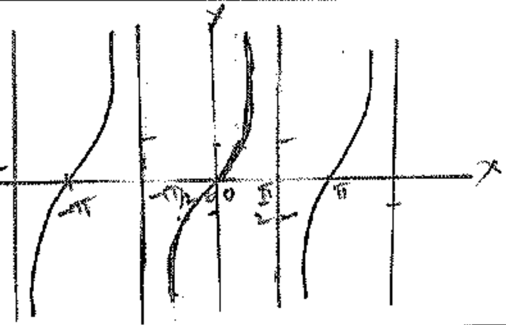
Domain $(-\infty, \infty)$
Range $[-1, 1]$ Principle Domain $[-\pi/2, \pi/2]$

12) $y = \cos x$

Domain $(-\infty, \infty)$
Range $[-1, 1]$ Principle Domain $[0, \pi]$

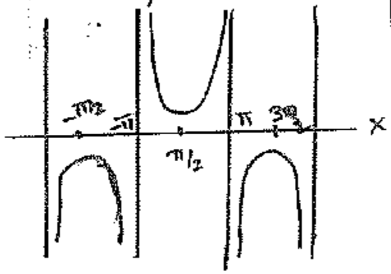
13) $y = \tan x$.

Domain $R - \{(2k+1)\frac{\pi}{2}\}$ $k \in Z$
Range $(-\infty, \infty)$



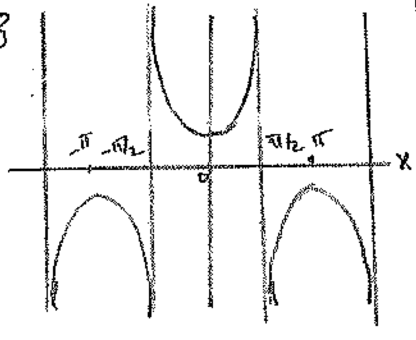
14) $y = \operatorname{cosec} x$

Domain $R - \{(2k+1)\pi/2\}$ $k \in Z$.
Range
$(-\infty, -1] \cup [1, \infty)$



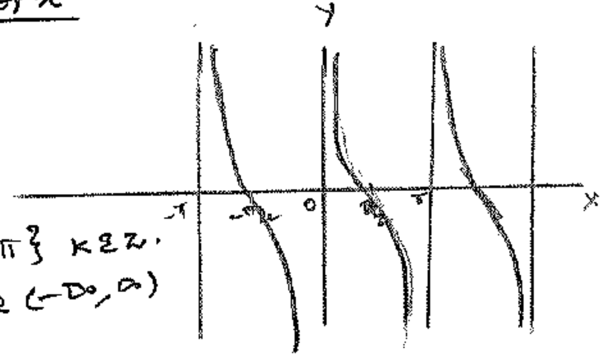
15) $y = \sec x$.

Domain $R - \{k\pi\}$
Range
$(-\infty, -1] \cup [1, \infty)$



16) $y = \cot x$.

Domain
$R - \{k\pi\}$ $k \in Z$.
Range $(-\infty, \infty)$



1. **Relation**: Let A and B be any two sets. A relation from $A \to B$ is a subset of a Cartesian product $A \times B$

2) a) **Function**: one to one, many to one But not one to many and every element of A should have image

b) The relation 'son of' is a function (many to one)

c) The relation 'father of' is not a function (one to many)

d) Relation need not be function or it may function

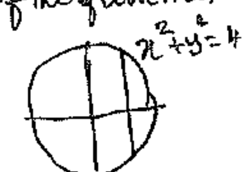
e) But Function should be relation.

3) Graph of the function.

1) For every x there is unique image. But any element in A does not posses two images

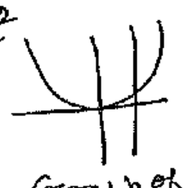
2) (ie) If the vertical line cuts the curve in only one point then it is a graph of a fn. If the vertical line cuts the curve in two point it is not the Graph of the function.

$x^2+y^2=4$



not Graph.

$y = x^2$



Graph of the function.

4) **One to One**: A function is said to be one-one if each element of the range is associated with exactly one element of the domain. (ie) all the element in B has preimage Codomain = Range [Injective].

5) **on to**: If the range of the fn is equal to Codomain then the fn is called an onto fn. It may be one to one (or) many to one [Surjective]

If both one-one and onto it is bejective.

Note: If the fun is one-one and onto then f^{-1} (fn) exist s.t $f \circ f^{-1} = f^{-1} \circ f = I(x)$

note: composition of fn need not be commutative always. $f \circ g \neq g \circ f$

Roster form (or) Tabulated form. 1, 2, 3 Marks.

1. Represent the following sets in the roster form.

1. $\{x : x$ is a vowel in English alphabets$\}$.
2. $\{x : x$ is an integer where $-1 \le x \le 7\}$
3. $\{x : x$ is two digit number s.t the sum of the digits is $10\}$
4. $\{x : x$ is a natural number, x is a perfect square, $x \le 100\}$
5. $\{x : x$ is a ~~word~~ letter in the word MAHARASHTRA$\}$
6. $\{x : x$ is a positive integer and a multiple of $5\}$

Sol: 1) $\{a, e, i, o, u\}$

2) $\{-1, 0, 1, 2, 3, 4, 5, 6, 7\}$.

3) $\{19, 91, 28, 82, 37, 73, 46, 64, 55\}$

4. $\{1, 4, 9, 16, 25, 36, 49, 64, 81, 100\}$

5. $\{M, A, H, R, S, T\}$

6. $\{5, 10, 15, 20, 25 \ldots\}$.

2) Exhibit in tabulation form.

1) $A = \{x : x$ is a natural number$\} \{3 < x < 7\}$
2. $B = \{x : x$ is a prime number, $x < 20\}$
3. $C = \{x : x$ is a prime number which is a divisor of $60\}$.

1) $A = \{4, 5, 6\}$

2. $B = \{2, 3, 5, 7, 11, 13, 17, 19\}$

3. C : Divisor of 60 are $1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60\}$.

$= \{$prime number$\}$

$= \{2, 3, 5\}$.

3) Represent the fowing sets in the set Builder form.

1. $\{2, 3, 5\}$ 2) $\{M, A, T, H, E, I, C, S\}$ 3) $\{2, 4, 6, 8, 10\}$ 4) $\{3, 9, 27, 81\}$

5) $\{5, 10, 15, 20\}$ 6) $\{0, 3, 6, 9, 12, 15, 18, \ldots\}$

1. $\{x : x$ is a prime divisor of $30\}$
2. $\{x : x$ is the letter of the word MATHEMATICS$\}$
3. $\{x : x = 3^n$ where n is a natural number $< 4\}$

4) $\{x : x = 5n \quad n \in N, n \le 4$

5) $\{x : x = 3n, n \in N, n$ is a whole number.

4) Represents the following sets in the set builder form.

$A = \{7, 8, 9, 10, 11\}$ $B = 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4} \cdots \}$ $C = \{1, 4, 9 \cdots 100\}$.

$A = \{x : x$ is the natural number between 6 and 12 $\}$

$B = \{x : x = \frac{1}{n} \mid n$ is a natural number $\}$

$C = \{x : x = n^2 \mid n$ is a natural number $\}$.

5) write the roster form as well as set builder form of set containing the elements 0, 2, 4, 6, 8

Roster form: $A = \{0, 2, 4, 6, 8\}$.

Set Builder form $B = \{x : x = 2(n-1) \mid n \leq 5\}$

6) write the set of all vowels in English alphabet which precedes s

$A = \{a, e, i, o\}$.

7) write the set in the Roster form $\{x : x$ is a positive integer and divisor of 9 $\}$.

$A = \{1, 3, 9\}$.

8) write the set of all natural numbers x. s.t $4x + 9 < 50$ in the roster form.

when $x = 1$ $4 + 9 = 13$

$x = 2$ $8 + 9 = 17$

$x = 3$ $12 + 9 = 21$

$\vdots$

$x = 10$ $40 + 9 = 49 < 50$

$\therefore A = \{1, 2, 3, \cdots 10\}$.

9) write the following in Roster form.

1. $\{x : x \in Z$ and $|x| \leq 2\}$ 2) $x : x = \frac{n}{1+n^2}$ and $1 \leq n \leq 3$ n, N.

1) $x : x \in Z$ and $|x| \leq 2$

$|x| \leq 2 \Rightarrow -2 \leq x \leq 2$

$\therefore A = \{-2, -1, 0, 1, 2\}$.

2) $x : x = \frac{n}{1+n^2}$ $1 \leq n \leq 3$

$x = 1$ $= \frac{1}{2}$

2 $= \frac{2}{5}$

3 $= \frac{3}{10}$

$\therefore A = \{\frac{1}{2}, \frac{2}{5}, \frac{3}{10}\}$.

(10) write in the Rosterform.

1) $A = \{a_n : n \in N, a_{n+1} = 3a_n, a_1 = 1$

2) $B = \{a_n = n \in N, a_{n+2} = a_{n+1} + a_n$ and $a_1 = a_2 = 1$.

1) $a_1 = 1$ and $a_{n+1} = 3a_n$.

$$a_2 = a_{1+1} = 3a_1 = 3$$
$$a_3 = a_{2+1} = 3a_2 = 3 \cdot 3 = 3^2$$
$$a_4 = a_{3+1} = 3a_3 = 3 \cdot a^2 = 3^3 \text{ and so on.}$$

$\therefore A = \{1, 3, 3^2, 3^3 \ldots\}$.

2) $a_1 = a_2 = 1$

$$a_3 = a_{1+2} = a_2 + a_1 = 2$$
$$a_4 = a_{2+2} = a_3 + a_2 = 2 + 1 = 3$$
$$a_5 = a_{3+2} = a_4 + a_3 = 3 + 2 = 5 \text{ and so on.}$$

$\therefore B = \{1, 1, 2, 3, 5 \ldots\}$.

11) write the set $A = \{14, 21, 28, 35 \ldots 98\}$ in the set builder form.

The givers numbers are natural number >13 and <99. and multiply of 7.

$\{x : x$ is a natural number multiply of 7 and $13 < x < 99$

$\{x = 7n \mid n \in N$ and $2 \leq x \leq 14\}$

12) Describe the set in the set Builder form.

$B = 53, 59, 61, 67, 71, 73, 79, 83, 89, 97\}$.

$\{x : x =$ prime numbers between 50 and 100$\}$

13) write the set $D = \{\frac{1}{2}, \frac{2}{5}, \frac{3}{10}, \frac{4}{17}, \frac{5}{26} \text{———}\}$ in the set builder form.

$D = \{x : x = \frac{n}{n^2+1} \mid n \in N \quad n \leq 5\}$

14) using Roster (or) listing method to express the set

$A = \{x : x = n^3, n \in N$ and $x < 80\}$

$A = \{1, 8, 27, 64\}$.

15) List elements of the set $A = \{x : x$ is an integer $-\frac{1}{2} \leq x < \frac{9}{2}$

$A = \{0, 1, 2, 3, 4\}$

16) Express the set $D = \{x : x = \frac{n^2-1}{n^2+1}, n \in N$ and $x < 4\}$ in roster form.

$x=1, \quad x = 0$

$x=2, \quad x = \frac{3}{5}$ $\quad \therefore A = \{0, 3/5, 8/10\}$

$x=3 \quad x = \frac{8}{10}$

17) Describe the following set in Roster form $\{x : x$ is a letter of the word PROPORTION$\}$.

$A = \{P, R, O, T, I, N\}$.

Symmetric difference of two sets. $A \Delta B = (A-B) \cup (B-A)$

(or) $= (A-B) - (A \cap B)$

18) P.T $A - (B \cap C) = (A-B) \cup (A-C)$

$\because$ Suppose the symbol $\cap$, (or) $\cup$ are in brackets LHS Change the symbol in RHS. Suppose $-$ (or) Δ is in bracket in LHS. No change the symbol in RHS.

Proof: Let $x \in A - (B \cap C)$

$\Rightarrow x \in A$, and $x \notin B \cap C$.

$\Rightarrow x \in A$ and $x \notin B$ (or) $x \notin C$.

$\Rightarrow x \in A$ and $x \notin B$ (or) $x \in A$ and $x \notin C$

$\Rightarrow x \in A-B$ (or) $x \in A-C$

$\Rightarrow x \in (A-B) \cup (A-C)$

$\Rightarrow A - (B \cup C) \subseteq (A-B) \cup (A-C)$ — ①

Let $x \in (A-B) \cup (A-C)$

$\Rightarrow x \in A-B$ (or) $x \in A-C$

$\Rightarrow x \in A, x \notin B$ (or) $x \in A, x \notin C$

$\Rightarrow x \in A, (x \notin B$ or $x \notin C)$

$\Rightarrow x \in A, x \notin (B \cap C)$

$\Rightarrow x \in A - (B \cap C)$

$\therefore (A-B) \cup (A-C) \subseteq A - (B \cap C)$ — ②

$\therefore$ From ① and ② $A - (B \cap C) = (A-B) \cup (A-C)$

19) P.T $A \cap (B - C) = (A \cap B) - (A \cap C)$

Proof: Let $x \in A \cap (B-C)$

$\Rightarrow x \in A, x \in B - C$

$\Rightarrow x \in A, (x \in B, x \notin C)$

$\Rightarrow (x \in A, x \in B)$ and $x \in A, x \notin C$.

$\Rightarrow x \in A \cap B$ and $x \notin A \cap C$

$\Rightarrow x \in (A \cap B) - (A \cap C)$

$\therefore A \cap (B-C) \subseteq (A \cap B) - (A \cap C)$ — ①

Let $x \in (A \cap B) - (A \cap C)$

$\Rightarrow x \in A \cap B$ and $x \notin (A \cap C)$

$\Rightarrow x \in (A$ and $x \in B)$ and $(x \in A$ and $x \notin C)$

$\Rightarrow x \in A$ and $(x \in B$ and $x \notin C)$

$\Rightarrow x \in A$ and $x \in B - C$.

$\Rightarrow x \in A \cap (B - C)$

$\Rightarrow (A \cap B) - (A \cap C) \subseteq A \cap (B-C)$ — ②

From ① and ② $A \cap (B-C) = (A \cap B) - (A \cap C)$

20) P.T $A \cap (B \Delta C) = (A \cap B) \Delta (A \cap C)$

Proof: LHS: $A \cap (B \Delta C) = A \cap \{(B-C) \cup (C-B)\}$

$= A \cap (B-C) \cup A \cap (C-B)$

$= \{(A \cap B) - (A \cap C)\} \cup \{(A \cap C) - (A \cap B)\}$

$= \cancel{(A \cap B)} \cancel{(A \cap B) \cup (A \cap C)} - \cancel{(A}$

$(A \cap B) \Delta (A \cap C) =$ RHS.

21) If $A = \{x : x \text{ is a factor of } 20\}$ and $B = \{5, 10, 15, 20\}$ find $A - B$ and $B - A$.

Sol: $A = \{1, 2, 4, 5, 10, 20\}$, $B = \{5, 10, 15, 20\}$

$A - B = \{1, 2, 4\}$

$B - A = \{15\}$.

22) If S is the set of all prime numbers and $M = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ then evaluate 1) $M \cup (S \cap M)$ 2) $(S - M) \cap (M - S)$

Sol: 1) $S = \{2, 3, 5, 7, 11, 13, 17, 19, \ldots\}$

$M = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$.

$S \cap M = \{2, 3, 5, 7\}$

$M \cup (S \cap M) = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\} = M$.

2) $S - M = \{11, 13, 17, 19 \ldots\}$

$M - S = \{0, 1, 4, 6, 8, 9\}$

$(S - M) \cap (M - S) = \phi$

23) Verify $A - (B \cup C) = (A - B) \cap (A - C)$ with sets $A = \{1, 2, 3\}$ $B = \{3, 4, 5\}$ and $C = \{1, 3, 5\}$.

Sol: $A = \{1, 2, 3\}$

$B = \{3\ 4\ 5\}$

$C = \{1, 3, 5\}$.

LHS $B \cup C = \{1, 3, 4, 5\}$ $A - (B \cup C) = \{2\}$

RHS $=$ $A - B = \{1, 2\}$

$A - C = \{2\}$ $\therefore (A - B) \cap (A - C) = \{2\}$

$\therefore$ LHS $=$ RHS.

24 If $U = \{1, 2, 3, 4, 6, 7, 8, 9, 10\}$, $A = \{1, 3, 5, 7, 9\}$ $B = \{2, 4, 6, 8, 10\}$ and $C = \{1, 2, 3, 4\}$ Then

Find) 1) U', 2) $A \cup A'$ 3) $A \cap (B - C)$ 4) $A - (B \cap C)$ 5) $A' \cap (B \cup C)'$

6) A' 7) $A \cap A'$ 8) $A - (B \cup C)$ 9) $A' \cup (B \cup C)$

10) $A' \cup (B' \cap C')$.

26) For any two sub sets A, B of the universal set U then S.T
$A \cup (A \cap B) = A$

Sol: Let x be any arbitrary element of $A \cup (A \cap B)$

$x \in A \cup (A \cap B) = \{x : x \in A \text{ (or) } x \in A \cap B\}$
$= \{x : x \in A \text{ or } x \in A, x \in B\}$
$= \{x : x \in A\}$
$= A.$

27) For any two sets A and B P.T $A \cap (A \cup B) = A$.

Sol: Let $x \in A \cap (A \cup B) = \{x : x \in A, x \in A \cup B\}$
$= \{x : x \in A, x \in A \text{ or } x \in B\}$
$= \{x \in A\}$
$= A.$

28) For any two sets A and B P.T $A-B \cap B-A = \varphi$.

Sol: $x \in A - B \Rightarrow x \in A$ and $x \notin B$.
$\Rightarrow x \notin B-A$.
any $x \in B-A \Rightarrow x \in B$ and $x \notin A$
$\Rightarrow x \notin A-B$.
$\therefore (A-B) \cap (B-A) = \varphi$.

$A \Delta B = (A-B) \cup (B-A)$
(or) $(A \cup B) - (A \cap B)$

29) State which of the sets given below are finite or infinite.
1) $\{x : x$ is a prime number, x is even$\}$ ⟶ infinite set
2) Set of all rivers in India ⟶ finite set
3.) Set of all concentric circles ⟶ infinite set
4) $\{x : x$ is a multiple of 2, x is an integer$\}$ ⟶ infinite set
5) The set of months in a year ⟶ finite set
6) $\{1, 2, 3, \ldots\}$ ⟶ infinite set
7) $\{1, 2, 3, \ldots 100\}$ ⟶ finite set
8) The set of prime number less than 99 ⟶ finite set
9) The set of lines which are parallel to x axis is ⟶ infinite set
10) The set of circles in a plane passing through the origin ⟶ infinite set

11) The set of lines which are parallel to y-axis ⟶ infinite set
12) The set of letters in English alphabets ⟶ finite set
13) The set of numbers which multiplied by 5 ⟶ infinite set
14) The set of animals living on the earth ⟶ infinite set
15) The set of all historical monuments in India ⟶ finite set.
16) $\{x : x$ is an integer $x < 5\}$ ⟶ finite set
17) $\{x : x$ is a real number $1 < x < 2\}$ ⟶ infinite set
18) $\{x : x$ is a natural number $> 500\}$ ⟶ infinite set
19) $\{x : x$ is an integer, x is a factor of $1000\}$ ⟶ finite set.

30) In a survey, it was found that 21 people liked product A, 26 liked Product B and 29 liked product C. If 14 liked products A and B, 14 liked B and C, 12 liked C and A and 8 liked all the three products. Find how many liked C only.

$n(A) = 21$, $n(B) = 26$ $n(C) = 29$, $n(A\cap B) = 14$ $n(B\cap C) = 14$, $n(C\cap A) = 12$

$n(A\cap B\cap C) = 8$ $\qquad \because n(A\cap B') = n(A) - n(A\cap B)$

Now $n(A'\cap B'\cap C) = n\{(A\cup B)'\cap C\} = n(C) - n\{(A\cup B)\cap C\}$

$= n(C) - n\{(A\cap C) \cup (B\cap C)\}$

$= n(C) - \{n(A\cap C) + n(B\cap C) - n(A\cap B\cap C)\}$

$= 29 - (12 + 14 - 8) = 29 - 18 = 11.$

31) Let A and B are two sets s.t $A\times B$ consists of 6 elements. If the three elements of $A\times B$ are $(1,4)$ $(2,6)$ $(3,6)$. Find $A\times B$ and $B\times A$

Sol: Let $A = \{1, 2, 3\}$
$B = \{4, 6\}$

$A\times B = \{(1,4)\ (2,4)\ (3,4)\ (1,6)\ (2,6)\ (3,6)\}$

$B\times A = (4,1)\ (4,2)\ (4,3)\ (6,1)\ (6,2)\ (6,3)$

32) Let A and B be two sets s.t $n(A) = 3$, $n(B) = 2$, If $(x,1)$ $(y,2)$ $(z,1)$ are in $A\times B$. find A and B. where x, y, z are distinct. (Already done)

33) Let $A = \{1, 2\}$ $B = \{3, 4\}$ write $A\times B$. How many subsets will $A\times B$ have.

$n(A) = 2$, $n(B) = 2$. $\qquad n(A\times B) = 4$

$n[P(A\times B)] = 2^4 = 16.$

34) If $A = \{x : x^2 - 5x + 6 = 0\}$ $B = \{2, 4\}$ $C = \{4, 5\}$ Then find $A\times(B\cap C)$

$x^2 - 5x + 6 = 0 \Rightarrow (x-2)(x-3) = 0$ $x = \{2, 3\}$

$\therefore A = \{2, 3\}$ $B = \{2, 4\}$ $C = \{4, 5\}$.

$B\cap C = \{4\}$ $\quad A\times(B\cap C) = \{(2,4)\ (3,4)\}$

35) If $A = \{a, b, c, d, e\}$ $B = \{a, c, e, g\}$ $C = \{b, e, f, g\}$. Then verify that

i) $A \cap (B - C) = (A \cap B) - (A \cap C)$ | 2) $A - (B \cap C) = (A - B) \cup (A - C)$

1) $A \cap (B - C) = (A \cap B) - (A \cap C)$

LHS $B - C = \{a, c, f\}$

$A \cap (B - C) = \{a, c\}$ — ①

RHS $A \cap B = \{a, c, e\}$

$A \cap C = \{b, e\}$

$\therefore (A - B) - (A \cap C) = \{a, c\}$ — ②

$\therefore$ LHS = RHS

LHS: $B \cap C = \{e, g\}$

$A - (B \cap C) = \{a, b, c, d\}$ — ①

RHS $A - B = \{b, d\}$

$A - C = \{a, c, d\}$

$(A - B) \cup (A - C) = \{a, b, c, d\}$ — ②

$A - (B \cap C) = (A - B) \cup (A - C)$

LHS = RHS.

36) Let $A = \{1, 2, 4, 5\}$ $B = \{2, 3, 5, 6\}$ $C = \{4, 5, 6, 7\}$ verify the following identities. 1) $A \cap (B - C) = (A \cap B) - (A \cap C)$

2) $A - (B \cup C) = (A - B) \cap (A - C)$

3) $A - (B \cap C) = (A - B) \cup (A - C)$

Solution: 1) $A \cap (B - C) = (A \cap B) - (A \cap C)$

LHS: $B - C = \{2, 3, \}$

$A \cap (B - C) = \{2\}$ — ①

RHS $A \cap B = \{2, 5\}$

$(A \cap C) = \{4, 5\}$

$(A \cap B) - (A \cap C) = \{2\}$ — ②

From ① and ② $A \cap (B - C) = (A \cap B) - (A \cap C)$

2) $A - (B \cup C) = (A - B) \cap (A - C)$

LHS: $B \cup C = \{2, 3, 4, 5, 6, 7\}$

$A - (B \cup C) = \{1\}$ — ②

RHS ~~$A \cap B = \{2, 5\}$~~ $A - B = \{1, 4\}$

~~$A \cap C = \{4, 5\}$~~ $A - C = \{1, 2, 5\}$

$(A - B) \cap (A - C) = \{1\}$ — ②

From ① and ② $A - (B \cup C) = (A - B) \cap (A - C)$

37) If $A = \{1, 2, 3\}$ $B = \{2, 3, 4, 5\}$ $C = \{2, 4, 6, 8\}$ verify that

1) $A - (A - B) = A \cap B$

2) $A \cap (B - C) = (A \cap B) - (A \cap C)$

1) LHS: $A - B = \{1\}$

$A - (A - B) = \{2, 3\}$

RHS : $A \cap B = \{2, 3\}$.

$\therefore$ LHS = RHS.

2) LHS $B - C = \{3, 5\}$

$A \cap (B - C) = \{3\}$ — ①

$A \cap B = \{2, 3\}$ $(A \cap B) - (A \cap C) = \{3\}$

$A \cap C = \{2, \}$ — ②

$\therefore A \cap (B - C) = (A \cap B) - (A \cap C)$

38) For any two sets A and B $A \Delta B = (A \cup B) - (A \cap B)$

(or) $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$

38) Let $S = \{x \mid x \text{ is a positive multiple of 3 less than } 100\}$.
$P = \{x \mid x \text{ is a prime number less than } 20\}$ then $n(S) + n(P) = ?$

$S = \{3, 6, 9, 12 \ldots 99\} \Rightarrow n(S) = 33$

$P = \{2, 3, 5, 7, 11, 13, 17, 19\} \Rightarrow n(P) = 8$

$\therefore n(S) + n(P) = 33 + 8 = 41.$

39) A set contains n elements. Its power set contains 2^n elements.

40) If $n(A) = 10$, $n(B) = 6$, $n(C) = 5$ for three disjoint sets A, B, C then $n(A \cup B \cup C) = ?$

$n(A \cup B \cup C) = n(A) + n(B) + n(C)$
$= 10 + 6 + 5 = 21.$

Book Problems

1. Find the number of subsets of $A = \{x : x = 4n+1 \;\; 2 \le n \le 5, n \in N\}$.

Sol: $A = \{x : x = 4n+1 \quad n = 2, 3, 4, 5\}$

TBP $= \{9, 13, 17, 21\} \Rightarrow n(A) = 4.$

Number of subsets $= 2^4 = 16.$

2. In a survey of 5000 persons in a town, it was found that TBP 45% of the persons know language A, 25% know language B, 10% knows language C, 5% language A and B, 4% knows language B and C and 4% knows C and A. If 3% of the person know all the three languages, find the number of persons who knows only language A.

Sol. $45\% \text{ of } 5000 = 2250$, $25\%\ 5000 = 1250$, $10\%\ 5000 = 500$

$4\% \text{ of } 5000 = 200 \quad 3\%\ 5000 = 150.$

(ie) $n(A) = 2250$, $n(B) = 1250$ $n(C) = 500$, $n(A \cap B) = 250$

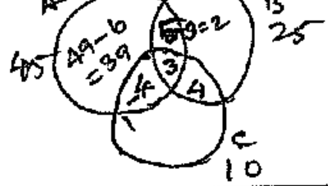
$n(B \cap C) = 200 \quad n(A \cap C) = 200 \quad n(A \cap B \cap C) = 150.$

Number of persons who knows A only

$n(A \cap B' \cap C') = n\{A \cap (B \cup C)'\}.$
$= n(A) - n(A \cap B) - n(A \cap C) + n(A \cap B \cap C)$
$= 2250 - 250 - 200 + 150$
$= 1950.$

(or) 39% of 5000 people speaks A only

$= \cancel{5000} \times \frac{39}{\cancel{100}} = 1950.$



3) If $X = \{1, 2, 3, 4 \ldots 10\}$ and $A = \{1, 2, 3, 4, 5\}$ Find the number of subsets of $B \subseteq X$ s.t $A - B = \{4\}$

Sol: $X = \{1, 2, 3, 4, \ldots 10\}$, $A = \{1, 2, 3, 4, 5\}$

Let $B = C \cup A = C \cup \{1, 2, 3, 4, 5\} = \{1, 2, 3, 5, 6, 7, 8, 9, 10\}$

$\therefore B = \{6, 7, 8, 9, 10\}$

$\therefore A - B = \{4\}$ $\therefore$ The number of subsets of $\{6, 7, 8, 9, 10\}$.
$n = 5$ $\therefore$ subsets $= 2^5 = 32.$

4. If A and B are two sets so that $n(B-A) = 2n(A-B) = 4n(A\cap B)$ and if $n(A\cup B) = 14$. Then find $n[P(A)]$.

Sol: Let $n(A\cap B) = k$.

$\therefore n(B-A) = \cancel{4}\overset{2}{}k$

$\cancel{2}n(A-B) = \cancel{4}k$

$\therefore n(A\cup B) = n(A-B) + n(B-A) + n(A\cap B)$

$= 4k + 2k + k$

$14 = 7k$

$\therefore k = 2$

$\therefore n(B-A) = 8$

$n(A-B) = 4$

$n(A) = n(A-B) + n(A\cap B)$

$= 4 + 2 = 6$

$\therefore n[P(A)] = 2^6 = 64.$

$n(A\cup B) = n(A-B) + n(B-A) + n(A\cap B)$.

$\underline{\underline{n(A) = n(A-B) + n(A\cap B)}}$

5) Two sets have m and k elements. If the total number of subsets of the first set is 112 more than that of the second set. Find the value of k and m.

Sol: Let A, B be two sets s.t $n(A) = m$ and $n(B) = n$. $m > n$.

$n[P(A)] = 2^m,\ n[P(B)] = 2^k.$

$2^m - 2^k = 112$

$2^k[2^{m-k} - 1] = 112$

$= 2^4 \times 7.$

$$\begin{array}{r|l} 2 & 112 \\ \hline 2 & 56 \\ \hline 2 & 28 \\ \hline 2 & 14 \\ \hline & 7 \end{array}$$

$\Rightarrow 2^k = 2^4$ and $2^{m-k} - 1 = 7$

$k = 4$

$2^{m-4} = 8 = 2^3$

2

$m - 4 = 3$

$m = 7.$

6) If $n(A) = 10$ and $n(A\cap B) = 3$, find $n[(A\cap B)' \cap A]$

Sol: $(A\cap B)' = A'\cup B'$

$(A\cap B)'\cap A = (A'\cup B')\cap A$

$= (A'\cap A)\cup(B'\cap A)$

$= \phi \cup (B'\cap A)$

$= B'\cap A = A - B.$

$\therefore n[(A\cap B)'\cap A] = n(A-B)$

$= n(A) - n(A\cap B)$

$= 10 - 3$

$= \underline{\underline{7}}.$

12) If A and B be two sets s.t $n(A)=3$, and $n(B)=2$, If $(x,1)$ $(y,2)$ $(z,1)$ are in $A\times B$. find A and B where x, y, z are distinct elements.

TBP

Sol: $A=\{x,y,z\}$ $n(A)=3$.

$B=\{1,2\}$ $n(B)=2$ and $A\times B = \{(x,1)(y,1)(z,1)(x,2)(y,2)(z,2)\}$

13) If $A\times A$ has 16 elements $S=\{(a,b)\in A\times A : a<b\}$; $(-1,2)$ $(0,1)$ are two elements of S, then find the remaining elements of S.

TBP

Solution let $A=\{-1,0,1,2\}$

$S=(-1,0)\ (-1,1)\ (-1,2)\ (0,1)\ (0,2)\ (1,2)$

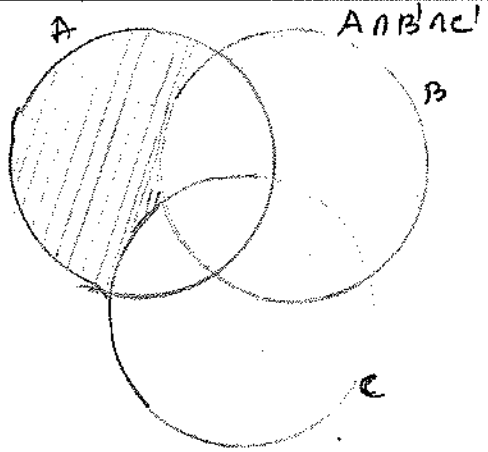
14) P.T $((A\cup B'\cup C)\cap(A\cap B'\cap C'))\cup((A\cup B\cup C')\cap(B'\cap C')) = B'\cap C'$.

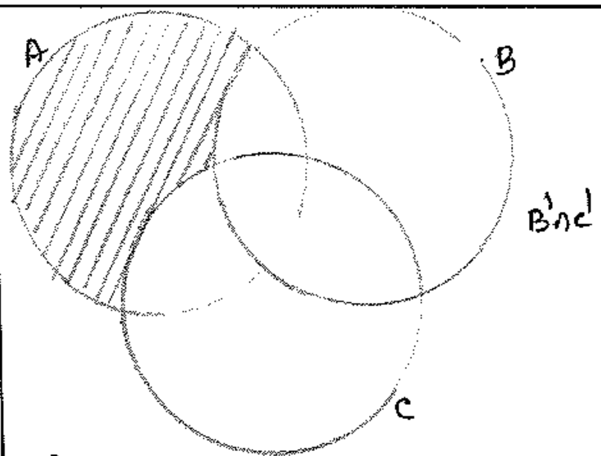
Sol: $A\cap B'\cap C' \subseteq A \subseteq A\cup B'\cup C \Rightarrow (A\cap B'\cap C')\cap(A\cup B'\cup C) = A\cap B'\cap C'$

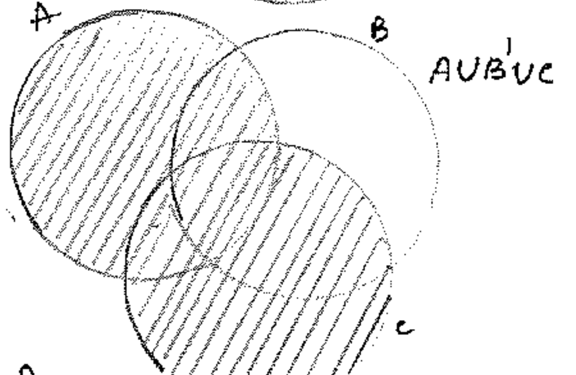
Also $B'\cap C' \subseteq C' \subseteq A\cup B\cup C' \Rightarrow (A\cup B\cup C')\cap(B'\cap C') = B'\cap C'$

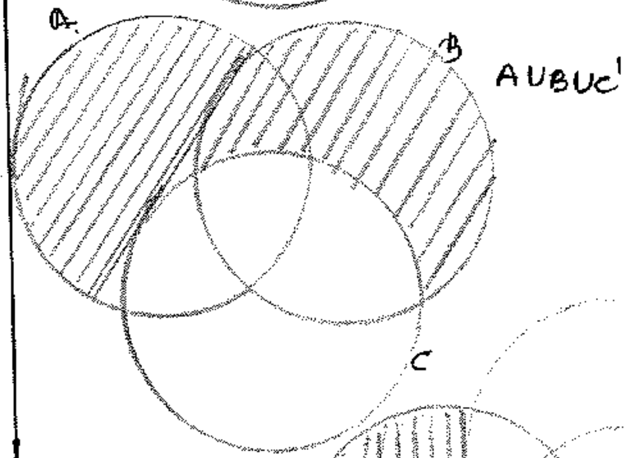
But $A'\cap B'\cap C' \subseteq B'\cap C'$

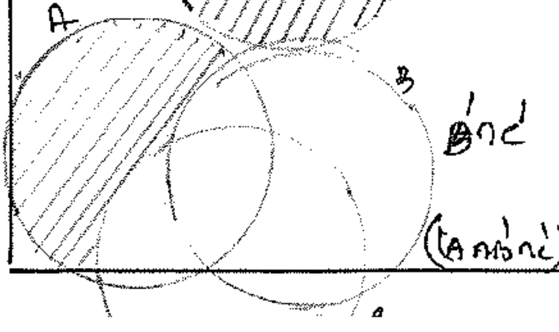
$\therefore ((A\cap B'\cap C')\cap(A\cup B'\cup C))\cup((A\cup B\cup C')\cap(B'\cap C')) = B'\cap C'$.

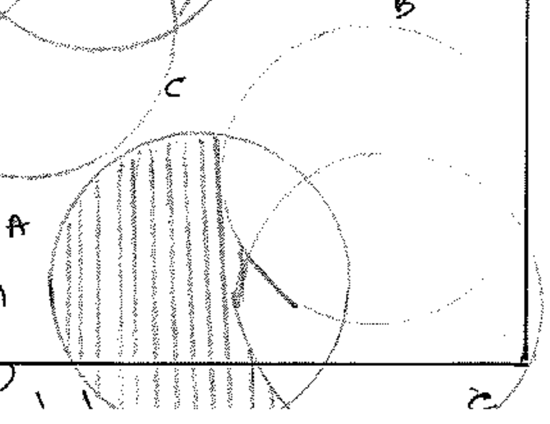












$((A\cap B'\cap C')\cap(A\cup B'\cup C))\cup$
$(A\cup B\cup C')\cap(B'\cap C')$

7) If $A = \{1,2,3,4\}$ $B = \{3,4,5,6\}$ find $n\{(A \cup B) \times (A \cap B) \times (A \Delta B)\}$

Sol: $A \cup B = \{1,2,3,4,5,6\}$ $A \cap B = \{3,4\}$ $n(A \Delta B) = 4$

TBP $n(A \cup B) = 6$ $n(A \cap B) = 2$

$A \Delta B$ = is A symmetric difference B.

$A \Delta B = (A-B) \cup (B-A)$

$\therefore n[(A \cup B) \times (A \cap B) \times (A \Delta B)] = 6 \times 2 \times 4 = 48$.

8) If $P(A)$ denotes the power set of A then find $n[P(P(P(\phi)]$

Sol: $P(\phi)$ contains 1 element

TBP

$P(P(\phi)) = 2^1$

$P(P(P(\phi)) = 2^2$

$= 4$

$\therefore n[P(P(P(\phi)] = 4$.

9) If $n(P(A)) = 1024$ $n(A \cup B) = 15$ $n(P(B)) = 32$ find $n(A \cap B)$

Sol: If $n[P(A)] = 1024 = 2^{10}$ $\therefore n(A) = 10$

$n[P(B)] = 32 = 2^5 = n(B) = 5$

$$\begin{array}{r|l} 2 & 1024 \\ 2 & 512 \\ 16 & 256 \\ & 16 \end{array}$$

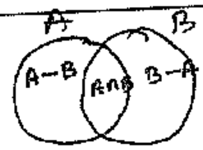
TBP

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$n(A \cap B) = 10 + 5 - n(A \cup B)$

$= 15 - 15 = 0$

10) If $n(A \cap B) = 3$ and $n(A \cup B) = 10$ then find $n(P(A \Delta B))$



$A \Delta B = (A-B) \cup (B-A)$

$= (A \cup B) - (A \cap B)$

TBP $n(A \Delta B) = n(A \cup B) - n(A \cap B)$

$= 10 - 3 = 7$

$n[P(A \Delta B)] = 2^7 = 128$

11) For a set A, $A \times A$ contains 16 elements and two of its elements are $(1,3)$ and $(0,2)$ Find the element A.

Let $A = \{1,3,0,2\}$

TBP

$A = \{1,3,0,2\}$

$A \times A$ = contains 16 elements also contains $(1,3), (0,2) \ldots$

$\therefore A = \{1,3,0,2\}$.

4) If $A = \{1,4\}$, $B = \{2,3,6\}$ $C = \{2,3,7\}$ Verify

1) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

IBP LHS: $B \cup C = \{2,3,6,7\}$

$A \times (B \cup C) = \{(1,2)\ (1,3)\ (1,6)\ (1,7)\ (4,2)\ (4,3)\ (4,6)\ (4,7)\}$ —— ①

RHS: $A \times B = \{(1,2), (1,3), (1,6), (4,2)\ (4,3)\ (4,6)\}$

$A \times C = \{(1,2)\ (1,3)\ (1,7)\ (4,2)\ (4,3)\ (4,7)\}$

$(A \times B) \cup (A \times C) = \{(1,2)\ (1,3)\ (1,6), (1,7)\ (4,2)\ (4,3)\ (4,6)\ (4,7)\}$ —— ②

From ① and ② $A \times (B \cup C) = (A \times B) \cup (A \times C)$

2) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

LHS $B \cap C = \{2,3\}$

$A \times (B \cap C) = \{(1,2)\ (1,3)\ (4,2)\ (4,3)\}$ —— ①

$A \times B = (1,2)\ (1,3)\ (1,6), (4,2), (4,3), (4,6)\}$

$A \times C = \{(1,2)\ (1,3)\ (1,7)\ (4,2)\ (4,3)\ (4,7)\}$

$(A \times B) \cap (A \cap C) = \{(1,2)\ (1,3)\ (4,2)\ (4,3)\}$ —— ②

From ① and ② $A \times (B \cap C) = (A \times B) \cap (A \times C)$

3) $(A \times B) \cap (B \times A) = (A \cap B) \times (B \cap A)$

LHS $A \times B = \{(1,2)\ (1,3)\ (1,6), (4,2), (4,3), (4,6)\}$

$B \times A = \{(21)\ (24)\ (31)\ (34)\ (61)\ (64)\}$

$(A \times B) \cap (B \times A) = \{\cdot\}$ —— ①

RHS $A \cap B = \{\cdot\}$

$B \cap A = \{\cdot\}$

$(A \cap B) \times (A \cap C) = \{\cdot\}$ —— ②

From ① and ② $(A \times B) \cap (B \times A) = (A \cap B) \times (B \cap A)$

4) $C - (B - A) = (C \cap A) \cup (C \cap B')$

LHS $B - A = \{2,3,6\}$

$C - (B - A) = \{7\}$ —— ①

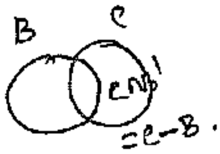
RHS: $(C \cap A) = \{\cdot\}$.

$C \cap B' = C - B = \{7\}$

$\therefore (C \cap A) \cup (C \cap B') = \{7\}$ —— ②

$\therefore$ From ① and ②

$C - (B - A) = (C \cap A) \cup (C \cap B')$



5) $(B - A) \cap C = (B \cap C) - A = B \cap (C - A)$

$B - A = \{2,3,6\}$

$(B - A) \cap C = \{2,3\}$ —— ①

$B \cap C = \{2,3\}$

$(B \cap C) - A = \{2,3\}$ —— ②

$C - A = \{2,3,7\}$

$B \cap (C - A) = \{2,3\}$ —— ③

From ①, ②, ③

$(B - A) \cap C = (B \cap C) - A = B \cap (C - A)$

6) $(B - A) \cup C = (B \cup C) - (A - C)$

LHS: $B - A = \{2,3,6\}$

$(B - A) \cup C = \{2,3,6,7\}$ —— ①

RHS $B \cup C = \{2,3,6,7\}$

$A - C = \{1,4\}$

$(B \cup C) - (A - C) = \{2,3,6,7\}$ —— ②

$\therefore$ From ① and ②

$(B - A) \cup C = (B \cup C) - (A - C)$

1) Write the following in Roster form

IBP 1) $x \in N : x^2 < 121$ and x is prime

$A = \{\cancel{1}, 2, 3, 5, 7\}$

2) The set of all positive roots of the equation $(x-1)(x+1)(x^2-1) = 0$

The roots are ± 1

$A = \{1\}$.

3) $\{x \in N : 4x + 9 < 52\}$

$x = 1$, $4x + 9 = 13$
$x = 2$ $= 17$
$x = 3$ $= 21$
$x = 4$ $= 25$
$x = 5$ $= 29$
$x = 6$ $= 33$
$x = 7$ $= 37$
$x = 8$ $= 41$
$x = 9$ $= 45$
$x = 10$ $= 49$

$\therefore x : A = \{1, 2, 3, \ldots 10\}$.

4.

Example 1.3 Prove that

$$((A \cup B' \cup C) \cap (A \cap B' \cap C')) \cup ((A \cup B \cup C') \cap (B' \cap C')) = B' \cap C'.$$

Solution:
We have $A \cap B' \cap C' \subseteq A \subseteq A \cup B' \cup C$ and hence $(A \cup B' \cup C) \cap (A \cap B' \cap C') = A \cap B' \cap C'$.
Also, $B' \cap C' \subseteq C' \subseteq A \cup B \cup C'$ and hence $(A \cup B \cup C') \cap B' \cap C' = B' \cap C'$.
Now as $A \cap B' \cap C' \subseteq B' \cap C'$, we have

$$((A \cup B' \cup C) \cap (A \cap B' \cap C')) \cup ((A \cup B \cup C') \cap (B' \cap C')) = B' \cap C'.$$

Note: Try to simplify the above expression using Venn diagram.

Example 1.4 If $X = \{1, 2, 3, \ldots 10\}$ and $A = \{1, 2, 3, 4, 5\}$, find the number of sets $B \subseteq X$ such that $A - B = \{4\}$

Solution:
For every subset C of $\{6, 7, 8, 9, 10\}$, let $B = C \cup \{1, 2, 3, 5\}$. Then $A - B = \{4\}$. In other words, for every subset C of $\{6, 7, 8, 9, 10\}$, we have a unique set B so that $A - B = \{4\}$. So number of sets $B \subseteq X$ such that $A - B = \{4\}$ and the number of subsets of $\{6, 7, 8, 9, 10\}$ are the same. So the number of sets $B \subseteq X$ such that $A - B = \{4\}$ is $2^5 = 32$.

Example 1.5 If A and B are two sets so that $n(B - A) = 2n(A - B) = 4n(A \cap B)$ and if $n(A \cup B) = 14$, then find $n(\mathscr{P}(A))$.

Solution:
To find $n(\mathscr{P}(A))$, we need $n(A)$.
Let $n(A \cap B) = k$. Then $n(A - B) = 2k$ and $n(B - A) = 4k$.
Now $n(A \cup B) = n(A - B) + n(B - A) + n(A \cap B) = 7k$.
It is given that $n(A \cup B) = 14$. Thus $7k = 14$ and hence $k = 2$.
So $n(A - B) = 4$ and $n(B - A) = 8$. As $n(A) = n(A - B) + n(A \cap B)$, we get $n(A) = 6$ and hence $n(\mathscr{P}(A)) = 2^6 = 64$.

Example 1.6 Two sets have m and k elements. If the total number of subsets of the first set is 112 more than that of the second set, find the values of m and k.

Solution:
Let A and B be the two sets with $n(A) = m$ and $n(B) = k$. Since A contains more elements than B, we have $m > k$. From the given conditions we see that $2^m - 2^k = 112$. Thus we get, $2^k(2^{m-k} - 1) = 2^4 \times 7$.

Then the only possibility is $k = 4$ and $2^{m-k} - 1 = 7$. So $m - k = 3$ and hence $m = 7$.

Relations.

1.10) check the relation $R = \{(1,1)\ (2,2)\ (3,3) \cdots (n,n)\}$ defined on the set $S = \{1,2,3, \ldots n\}$ for the three basic relation.
TBP

Solution: 1) As $(a,a) \in R$ for all $a \in S$, R is reflexive.

2) For $\because (a,b) \notin R$, $(b,a) \notin R$.
Also for every pair $(a,b) \in R$, $(b,a) \in R$ Hence R is symmetric.

3) No pairs $(a,b)\ (b,c)$ are in R $\therefore (a,c) \notin R$.
$\therefore$ R is not transitive is not true $\therefore$ R is transitive
$\because$ R is reflexive, symmetric and transitive this relation is an equivalance relation.

1.11) Let $S = \{1,2,3\}$ $\rho = \{(1,1)\ (\cancel{1},2)\ {}^{(2,2)}\ (\cancel{1},3)\ (3,1)\}$
TBP 1. Is ρ is reflexive? If not state the reason and write the minimum set of ordered pairs ~~of~~ to be included to ρ so as to make it reflexive.

Ans: ρ is not reflexive $\because (3,3) \notin \rho$.
If we include $(3,3)$ in ρ then it is reflexive.

2. Is ρ is symmetric? If not, state the reason, write minimum number of ordered pairs to be included to ρ so as to make it symmetric and write minimum number of ordered pairs to be deleted from ρ so as to make it symmetric.

Ans: ρ is not symmetric $\because$ For $(1,2) \in \rho$, $(2,1) \notin \rho$
Suppose if we include $(2,1)$ in ρ is symmetric (or)
if we delete $(1,2)$ from ρ then it is symmetric.

3) Is ρ is transitive? State the reason, write the minimum number of ordered pair to be included to ρ so as to make it transitive and write minimum number of ordered pairs to be deleted from ρ so as to make it transitive.

Ans. For $(1,3), (3,1) \in \rho$ $(3,3) \notin \rho$ $\therefore$ it is not transitive.
if we include $(3,3)$ $(3,2)$ it is transitive (or) if we delete $(1,3)$ it is transitive.

4) Is ρ is an equivalence relation? if not, write the minimum ordered pair to be included to ρ so that to make ρ as an equivalence relation.

Ans: ρ is not equivalence relation.
If we include $(3,3)$ $(3,2)$ $(2,3)$ and $(2,1)$ in ρ then only it is equivalence relation.

1.12) Let $A = \{0,1,2,3\}$ Construct relations on A of the following types.

1) not reflexive, not symmetric, not transitive

TBP Ans: Constructs the relation $\{(1,2)(2,3)\}$. is not reflexive not symmetric and not transitive.

2) not reflexive, not symmetric and transitive.

Ans: $\{(1,2)\}$ is transitive but not reflexive and not symmetric.

3) not reflexive, symmetric, not transitive

Ans: $\{(1,2)(2,1)\}$ is symmetric, not reflexive, not transitive.

4) not reflexive, symmetric, transitive.

Ans: $\{(1,2)(2,1)(1,1)(2,2)\}$ not reflexive, but symmetric and transitive.

5) reflexive, not symmetric, not transitive.

Ans. $\{(0,0)(1,1)(2,2)(3,3)(1,2)(1,3)\}$ reflexive, not symmetric. not transitive.

6) reflexive, symmetric, not transitive.

Ans: $\{(0,0)(1,1)(2,2)(3,3)(1,2)(2,3)(2,1)(3,2)\}$

reflexive, symmetric, not transitive.

7) reflexive, symmetric, transitive.

Ans: $\{(0,0)(1,1)(2,2)(3,3)\}$.

reflexive, symmetric and transitive.

1.13) In the set z of integers define mR_n if $m-n$ is multiple of 12

TBP P.T R is an equivalence relation.

1) Reflexive: $mR_m = m-m = 0$ and $0 \times 12 = 0$ is multiple of 12 hence mRm is reflexive

2) Symmetric: $mR_n = m-n = 12k$

$nRm \quad n-m = 12(-k)$ Both are multiple of 12

Hence $mR_n = nRm$ $\therefore$ symmetric.

3) transitive: let $mR_n = m-n = 12K$.

$nRp = n-p = 12l$. → multiple of 12

$= m-p = 12(K+l) = mRp$

$\therefore$ transitive.

Hence R is an equivalence relation.

1. Let R be a relation defined on the set of natural number N as follows $R = \{(x,y), x \in N, y \in N, 2x+y = 41\}$ find the domain and range of the relation R. Also verify whether R is reflexive, symmetric and transitive.

Sol: $R = \{(x,y)\ x \in N, y \in N\ \ 2x+y = 41\}$.

Domain $\{1, 2, 3, 4, \ldots 20\}$

Range $\{39, 37, 35, 33, 31 \ldots 1\}$

$\therefore R : \{(1,39)\ (2,37)\ (3,35)\ (4,33) - (19,3)(20,1)\}$.

1) R is not reflexive $\because (2,2) \notin R$.

2) R is not symmetric $\because$ For $(1,39) \in R\ (39,1) \notin R$.

3) R is not transitive $\because (11,19)$ and $(19,3) \in R$.
$(11,3) \notin R$.

Hence R is neither reflexive, nor symmetric, nor transitive.

2) On the set of natural numbers let R be the relation defined by aRb if $2a+3b = 30$. Write down the relation by listing all the pairs. Check whether it is
1) reflexive 2) symmetric, 3) transitive 4) equivalence.

$R : \{(a,b)\ a \in N,\ b \in N\ \ 2a+3b = 30\}$.

Domain $\{6, 9, 12\}$

Range $= \{6, 4, 2\}$

$R = \{(6,6)\ (6,4)\ (6,2)\ (9,6)\ (9,4)\ (9,2)\ (12,6)\ (12,4)\ (12,2)\}$

1) Not reflexive $\because (9,9) \notin R$.

2) Not Symmetric $\because (6,4) \in R$ But $(4,6) \notin R$.

3) It is transitive : $(6,6) \in R\ (6,4) \in R \Rightarrow (6,4) \in R$.
$(9,6) \in R\ (6,4) \in R \Rightarrow (9,4) \in R$.
$(12,6) \in R\ (6,4) \in R \Rightarrow (12,4) \in R$.

$\therefore$ R is not reflexive, not symmetric, and transitive it is an not equivalance relation.

3) On the set of natural numbers let R be the relation by aRb if $a+b \leq 6$ write down the relation by listing all the pairs. Check whether it is 1) reflexive 2) symmetric 3) transitive 4) Equivalence.

Sol: $R : \{(a,b)\ a \in N,\ b \in N,\ a+b \leq 6\}$.

Domain: $\{1, 2\ 3\ 4\ 5\}$

Range: $\{5\ 4\ 3\ 2\ 1\}$

$R = \{(1,5)\ (1,4)\ (1,3)\ (1,2)\ (1,1)\ (2,5)\ (2,4)\ (2,3)\ (2,2)\ (2,1)\ (4,5)\ (4,4)\ (4,3)\ (4,2)\ (4,1)$
$(3,5)\ (3,4)\ (3,3)\ (3,2)\ (3,1)$
$(5,5)\ (5,4)\ (5,3)\ (5,2)\ (5,1)\}$.

1. Reflexive: $\because \{(1,1)\ (2,2)\ (3,3)\ (4,4)\ (5\ 5)\} \in R$. it is reflexive

2) Symmetric: $(5,3) \in R$ and $(3,5) \in R$. $\therefore$ it is symmetric.
(or) $\forall (a,b) \in R\ (b,a) \in R$.

3) transitive: $\forall (a,b) \in R$ and $(b,c) \in R \Rightarrow (a,c) \in R$.
(or) $(3,4),(4,2) \in R \Rightarrow (3,2) \in R$.
Hence transitive.

$\therefore$ it is an equivalence relation.

4) In the set z of integers define xRy if $x-y$ is divisible by 7. P.T R is an equivalence relation. $R = \{(x,y), x \in z, y \in z, x-y$ is divisible by 7$\}$

Sol: Let $z = \{0, \pm 1, \pm 2, \pm 3 \ldots\}$.

1) Reflexive
$x \in z$, $x - x = 0$ which is divisible by 7.
$\therefore$ R is reflexive

2) Symmetric: If $x \in z, y \in z$ and $x-y$ is divisible by 7
then $-(x-y) = y-x$ is also divisible by 7.
$\therefore$ it is symmetric.

3) Transitive: If $x \in z, y \in z$ and $z \in z$.
and $x-y$ is divisible by 7
$y-z$ is divisible by 7
$\Rightarrow x - \cancel{y} + \cancel{y} - z$ is also divisible by 7
$\Rightarrow x-z$ is divisible by 7. Hence it is transitive
$\therefore$ R is an equivalence relation.

5) In a set of all natural number N, let a relation R be defined by $R = \{(a, b) \mid a-b$ is divisible by 3 $a \in N, b \in N\}$ P.T R is an equivalence relation.

Solution: $R = \{(a,b) \mid a \in N, b \in N$ $a-b$ is divisible by 3.

1) <u>Reflexive</u>: $\forall a \in N$ $a-a=0$ which is divisible by 3.

Hence R is Reflexive.

2) <u>Symmetric</u>: $\forall a, b \in N$ $a-b$ is divisible by 3

hence $-(a-b) = b-a$ is also divisible by 3

Hence R is Symmetric.

3) <u>Transitive</u>: $\forall a, b, c \in N$ $a-b$ is divisible by 3

$b-c$ is divisible by 3

Hence $(a-b)+(b-c)$ is divisible by 3.

$a-c$ is divisible by 3

$\therefore$ R is transitive.

Hence R is an equivalence relation.

6) S.T relation R in the set $A = \{1, 2, 3, 4, 5\}$ given by $R: (a,b) : |a-b|$ is even is an equivalence relation.

Solution: Let 1. <u>Reflexive</u> $a \in A$, $(a, a) \in R$ $\forall a \in A$.

$|a-a| = 0$ which is even $\therefore$ R is reflexive

2) <u>symmetric</u>: $\forall a, b \in A$, $|a-b|$ is even then $|b-a|$ is also even.

$\therefore (a, b) \in R \Rightarrow (b, a) \in R$.

3) <u>transitive</u>: Let $a, b, c \in A$ $(a, b) \in R$ and $(b,c) \in R$

$|a-b|$ is even and $|b-c|$ is even.

then $|a-b| + |b-c|$ is also even

$|a-c|$ is even $\therefore$ R is Transitive.

Hence R is an equivalence relation.

7) Show that the relation R 'is perpendicular to' on the set of all straight line in a plane is symmetric, but it is neither reflexive nor transitive

Sol: R is the relation 'is perpendicular to' on the set of lines L in a plane is symmetric because $l_1 \perp l_2$ and $l_2 \perp l_1$ $\forall (l_1, l_2) \in R$.

For $(l_1, l_1) \in R$ l_1 is not $\perp r$ to l_1. $\therefore$ it is not reflexive

For $l_1, l_2, l_3 \in R$ $l_1 \perp l_2$, and $l_2 \perp l_3$ But l_1 not $\perp r$ to l_3

(ie) $(l_1, l_2) \in R$ and $(l_2, l_3) \in R$ but $(l_1, l_3) \notin R$.

$\therefore$ R is not transitive.

Hence R is symmetric but neither reflexive nor transitive.

8) S.T the relation 'is greater than' in a set R of all real numbers is transitive but it is neither reflexive nor transitive.

Sol: 1) ... : As no real number can be greater than itself $\therefore$ it is not reflexive.

2) Symmetric: $(a, b) \in R$ $a > b$. but $b \not> a$

$\therefore (a, b) \in R$ $(b, a) \notin R$.

$\therefore$ it is not symmetric.

3) transitive : As $a, b, c \in R$.

$a > b$, $b > c \Rightarrow a > c$

$\therefore$ The relation greater than is transitive.

Hence the relation 'greater than' is transitive but neither reflexive nor transitive.

9) Discuss the relation reflexive, symmetric, transitive for the relation R defined on the set of all positive integers mRn if m divides n.

Sol: Reflexive: Since every positive integer divides itself (ie) $\forall a \in Z^+$ $(a, a) \in R$. So R is reflexive.

Symmetric : $\because$ 2 divides 6 but 6 does not divide 2

$\therefore (a, b) \in R^+$ a divides b but b does not divide a Hence R is not symmetric.

transitive : 2 divides 4 and 4 divides 8 $\Rightarrow$ 2 divides 8

(ie) $\forall a, b, c \in Z^+$ a divides b, and b divides c $\Rightarrow$ a divides c

$\therefore$ R is transitive.

$\therefore$ R is reflexive and transitive.

10) The non empty set consisting of children in a family and a relation R defined as aRb if a is a brother of b then R is_ _ _ _ _

Sol: R : {(a, b) a is a brother of b}.

1) Reflexive: a ∈ R, (a, a) ∉ R ∵ a being not at all brother to it self -

∴ not reflexive.

2) Symmetric: (a, b) ∈ R, a is a brother b but b may or may not be brother of a (ie) b may be sister of a.

∴ it is not symmetric.

3) transitive: a is a brother of b and b is a brother of c. ⟹ a is a brother of c

(a, b) ∈ R, (b, c) ∈ R ⟹ (a, c) ∈ R.

Hence transitive.

∴ It is not symmetric but transitive.

11) Let A be the set consisting of all the female members of a family. The relation R is defined by aRb if a is not a sister of b.

Sol: R : {(a, b) a is not a sister of b.

1) Reflexive a ∈ R (a, a) ∉ R. a being a not at all not sister of a.

∴ not reflexive

2) Symmetric : a, b ∈ R, a is not a sister b hence b is also not a sister of a

∴ (a, b) ∈ R and (b, a) ∈ R.

Hence Symmetric.

3) Transitive : If a is not a sister of b and b is not a sister of c

a may be sister of c

(ie) (a, b) ∈ R, (b, c) R and (a, c) ∉ R -

∴ R is not reflexive.

$(P_1 P_3) \in R$ Hence R is reflexive. ($\because$ Triangle is also polygon with 3 sides)
The elements in A related to the right angled triangle T with sides 3, 4, 5 are those polygons which have 3 sides. Hence, the set of all elements in A related to T is the set of triangle.

Theorem: 1) The number of relations from a set containing m elements to a set containing n elements is 2^{mn}. In particular the number of relations on a set containing n elements is 2^{n^2}

2) The number of reflexive relations on a set containing n elements is 2^{n^2-n}

3) The number of symmetric relations on a set containing n elements is $2^{\frac{n^2+n}{2}}$

Theorem: If R is the relation from A to B then the relation R^{-1} defined from B to A by $R^{-1} = \{(b,a) : (a,b) \in R\}$ is called the inverse relation of R.

The domain of R becomes range of R^{-1} and the range of R^{-1} becomes domain of R^{-1}

29) Let A be the set of first 10 natural numbers and let R be relation on A defined by $\{(x,y) \in R \Leftrightarrow x+2y = 10\}$ Express R and R^{-1} as sets of ordered pairs. Also determine 1) domain of R and R^{-1} 2) Range of R and R^{-1}.

Sol: $R = \{(x,y), x \in A, y \in A \quad x+2y=10$

$\therefore y = \frac{10-x}{2}$ when $x=2 \quad y=4$
$x=4 \quad y=3$
$x=6 \quad y=2$
$x=8 \quad y=1$

$\therefore R = \{(2,4)\ (4,3)\ (6,2)\ (8,1)\}$

$R^{-1} = \{(4,2)\ (3,4)\ (2,6)\ (1,8)\}$.

Domain of $R = \{2, 4, 6, 8\}$ = range of R^{-1}.

range of $R = \{4, 3, 2, 1\}$ = Domain of R^{-1}.

30) If $A = \{a, b\}$ $B = \{2, 3\}$ find the number of relations from A to B.

Sol: Number of relations $= 2^{2\times 2} = 2^4 = 16$.

31) If $n(A) = 3$ and $B = \{2, 3, 4, 6, 7, 8\}$ then find the number of relations from A to B.

Sol: $n(A) = 3 \quad n(B) = 6$

$\therefore$ Number of relations $= 2^{3\times 6} = 2^{18}$

Functions.

32) Find the domain of each of the following:

1) $\frac{1}{\sqrt{x-2}}$ 2) $\sqrt{4-x^2}$

1) Sol: For $x-2>0$ $f(x)$ is real value $f(x)=\frac{1}{\sqrt{x-2}}$

(or) $x>2$. (ie) $x \in (2,\infty)$ $f(x)=\frac{1}{\sqrt{x-2}}$ is real

$\therefore$ the domain of $\frac{1}{\sqrt{x-2}}$ is $(2,\infty)$

2) $f(x)=\sqrt{4-x^2}$ For $f(x)\geq 0$ $4-x^2\geq 0$

(or) $x^2-4\leq 0 \Rightarrow (x+2)(x-2)\leq 0$

$x=[-2,2]$

$\therefore$ The domain is $[-2,2]$

33) Find the domain for which the functions $f(x)=2x^2-1$ and $g(x)=1-3x$ are equal.

Sol: $f(x)=2x^2-1$ $g(x)=1-3x$

Given $f(x)=g(x)$

$2x^2-1=1-3x \Rightarrow 2x^2+3x-2=0$ -4 $+4$ -1

$2x^2+4x-x-2=0$

$2x(x+2)-1(x+2)=0$

$(x+2)(2x-1)=0$

$x=1/2, -2$

$\therefore$ domain for $f(x)=g(x)$ is $\{\frac{1}{2},-2\}$

34) Find the domain of $f(x)=\frac{1}{\sqrt{x+|x|}}$

Sol: ~~Find~~ $f(x)=\frac{1}{\sqrt{x+|x|}}$ $|x|=\begin{cases} x & x\geq 0 \\ -x & x<0 \end{cases}$

$x+|x|=\begin{cases} x+x & x\geq 0 \\ 0 & x<0 \end{cases} \Rightarrow x+|x|=\begin{cases} 2x & \geq 0 \\ 0 & <0 \end{cases}$

$\therefore$ when $\sqrt{x+|x|}>0$ $f(x)$ is real.

$x+|x|>0$

$2x>0 \Rightarrow x>0$ $\therefore x\in(0,\infty)$

Hence the domain $=(0,\infty)$

Note Domain of modulus function is always R (ie) $(-\infty,\infty)$

35) Find the domain of $f(x)=\frac{1}{x+2}$

Sol: For $f(x)$ ~~≥ 0~~ is real, $x+2\neq 0 \Rightarrow$ if $x+2=0$ $x=-2$

$\therefore$ Domain $=\{R-(-2)\}$

36) Find the domain of $f(x) = \frac{x-1}{x-3}$.

Sol: For $f(x)$ is real $f(x) \neq$ ~~0~~ $x-3 \neq 0 \Rightarrow x = 3$.

$\therefore$ Domain $(f) = R-(3)$

37) Find the domain of $\frac{2x-3}{x^2-3x+2}$

Sol: $f(x) = \frac{2x-3}{x^2-3x+2}$

Put $x^2-3x+2=0$ $\quad$ 2 $\quad -2 \; -1$

$(x-2)(x-1)=0$

$x=1, 2$

$\therefore$ Except $x=1, 2$ other all values of x gives $f(x)$ is real

$\therefore$ Domain $R-\{1,2\}$.

38) Find the domain of $\frac{x^2+3x+5}{x^2-5x+4}$

Sol: Let $f(x) = \frac{x^2+3x+5}{x^2-5x+4}$

Put $x^2-5x+4=0$ $\quad$ 4 $\quad -4 \; -1$

$(x-4)(x-1)=0 \Rightarrow x = 1, 4$

$\therefore R-\{1,4\}$ is the domain

39) Find the domain of $\sqrt{4-x} + \frac{1}{\sqrt{x^2-1}}$

Sol: clearly $f(x)$ is satisfying $\forall x$ when

$4-x \geq 0$ and $x^2-1 \geq 0$

$x \leq 0$ and $(x+1)(x-1) > 0$

when $x < -1$ or $x > 1$

$\therefore x \in (-\infty, 0)$ and $x \in (1,4)$

$\therefore$ The domain of f is $(-\infty, 0) \cup (1,4)$

40) Find the domain of $f(x) = \frac{x^2+3x+5}{x^2-5x+4}$.

Sol: For $f(x)$ is real the value of x without $x^2-5x+4=0$

$(x-1)(x-4)=0 \quad x = 1, 4$

$\therefore f(x)$ is real $\forall x \in R-\{1,4\}$.

Hence the domain $= R-\{1,4\}$.

41) Find the domain of the function $f(x) = \frac{x^2+2x+1}{x^2-8x+12}$

Sol: Put $x^2-8x+12=0$

$(x-6)(x-2)=0 \Rightarrow x=2,6$

$\therefore$ The domain of $f(x) = R-\{2,6\}$.

42) Find the domain and Range of the function $f(x) = -|x|$

Sol: $f(x) = -|x|$

x can have all real values

$\therefore f(x)$ is defined $\in R$.

$\therefore$ Domain of $f(x) = R$.

Range: $\because |x|$ cannot be negative

$-|x| \leq 0$ $\therefore$ Range $= (-\infty, 0]$

43) Find the domain and range of the real function $f(x) = \sqrt{x-1}$

Sol: f is defined for $x-1 \geq 0 \Rightarrow x \geq 1$

Hence the domain is $[1, \infty)$

Range: $\forall x \in [1, \infty)$ the range is $[0, \infty)$

44) Find the domain and Range of the real function $f(x) = |x-1|$

Sol: $f(x) = |x-1| \geq 0$ $\forall$ real value of x

Hence the domain is R.

Range: $\forall x \in R$ $|x-1|$ cannot be negative

Hence the range is $[0, \infty)$ (ie) non negative real number.

45) Find the domain and range of 1) $f(x) = \{x, \sqrt{9-x^2}\ x \in R\}$.

2) $f(x) = \{x, \frac{x^2-1}{x-1}\ x \in R, x \neq 1$

Sol: 1) $f(x) = \sqrt{9-x^2}$

$9-x^2 \geq 0$ $x^2 \leq 9$ (or) $-3 \leq x \leq 3$.

$\therefore$ Domain of $f = [-3, 3]$

Range Take $f(x) = y$

$y = \sqrt{9-x^2} \Rightarrow y^2 = 9-x^2 \Rightarrow x^2 = 9-y^2$

Now $x^2 \geq 0$ $9-y^2 \geq 0$ $y^2 \leq 9 \Rightarrow -3 \leq y \leq 3$

$\therefore$ Range for $y \geq 0$ $\therefore$ Range $[0, 3]$.

2) $f(x) = \dfrac{x^2-1}{x-1}$ $x \in R$ $x \neq 1$

when $x-1 \neq 0$ $f(x)$ is defined

$x \neq 1$ $f(x)$ is defined.

$\therefore$ Domain of $f = R - \{1\}$.

Range: $y = \dfrac{x^2-1}{x-1} = \dfrac{(x+1)\cancel{(x-1)}}{\cancel{(x-1)}}$

$y = x+1$

to get $y=2$ we have to put $x=1$ which is absurd.

$\therefore$ Range $= R - \{2\}$.

46) Let $f = \left\{\left[x, \dfrac{x^2}{1+x^2}\right]\right\}$ $x \in R$ be a function $R \to R$. Determine the range of R.

Sol: $f(x) = \dfrac{x^2}{1+x^2}$ $\forall x \in R$ $f(x)$ is defined.

Let $y = \dfrac{x^2}{1+x^2} \Rightarrow y(1+x^2) = x^2$

$y + yx^2 = x^2$

$y = x^2(1-y)$

$x^2 = \dfrac{y}{1-y}$ $\therefore x = \pm\sqrt{\dfrac{y}{1-y}}$

For Real value of x $y \geq 0$, $1-y > 0 \Rightarrow 1 > y \Rightarrow y < 1$

$\therefore 0 \leq y < 1$

$\therefore$ Range is $\{y = f(x) : 0 \leq y < 1\}$.

47) Find the domain and Range of the function $f(x) = \dfrac{x-2}{3-x}$.

Sol: Let $f(x) = \dfrac{x-2}{3-x}$

$f(x)$ is defined for all $x \in R$ except $3 - x \neq 0$

$x \neq 3$.

$\therefore$ domain of $f = R - \{3\}$.

Range: Let $y = f(x) = \dfrac{x-2}{3-x}$

$yx - 3y = x - 2$

$yx - x = 3y - 2$

$x(y-1) = 3y - 2$

$x = \dfrac{3y-2}{y-1}$

$\Rightarrow 3y - yx = x - 2$

$x = \dfrac{3y+2}{y+1}$

clearly x has real value when

$y+1 \neq 0 \Rightarrow y \neq -1$

$\therefore$ The range is $R - \{-1\}$.

48) Find the domain and range of the function $f(x) = \frac{4-x}{x-4}$

Sol: clearly $f(x)$ is defined $\forall x \in R$ except $x-4 \neq 0 \Rightarrow x \neq 4$.

$\therefore$ Domain of $f = R - \{4\}$.

Range : $f(x) = -\frac{(x-4)}{x-4} = -1$

$\therefore$ The range of $f = \{-1\}$

49) Find the Domain and range of $f(x) = \frac{1}{2-\sin 3x}$.

Sol: $f(x) = \frac{1}{2-\sin 3x}$.

$\because \quad -1 \leq \sin 3x \leq 1 \quad \forall x \in R$

$-1 \leq -\sin 3x \leq 1 \quad \forall x \in R$

$1 \leq 2 - \sin 3x \leq 3$ add 2 on both sides.

$\Rightarrow \quad 2 - \sin 3x \neq 0 \quad \therefore \frac{1}{2-\sin 3x}$ is defined $\forall x \in R$.

$\therefore$ domain of $f = R$.

Range: $\because \quad 1 \leq 2 - \sin 3x \leq 3$

$\frac{1}{3} \leq \frac{1}{2-\sin 3x} \leq 1$ (on reciprocal) $\forall x \in R$.

$\therefore$ The range is $[1/3, 1]$.

Find the Range of $\frac{1}{1-2\cos x}$

Sol: $-1 \leq \cos x \leq 1$

$-2 \leq -2\cos x \leq 2$

$-1 \leq 1 - 2\cos x \leq 3$

$-1 \geq \frac{1}{1-2\cos x} \geq \frac{1}{3}$

Range

$(-\infty, -1] \cup [1/3, \infty)$

50) Find the domain of the function $f(x) = \frac{1}{1-2\cos x}$

Sol: $f(x) = \frac{1}{1-2\cos x}$

$f(x)$ is defined only $\forall x \in R$ except $1 - 2\cos x = 0$

3P (ie) $2\cos x = 1$

$\cos x = 1/2$

(ie) except $x = (2n\pi \pm \pi/3)\ n \in z$.

Hence the domain is $R - \{(2n\pi \pm \pi/3)\}\ n \in z$.

51) Find the range of the function $f(x) = \frac{1}{1-3\cos x}$.

Sol: $f(x) = \frac{1}{1-3\cos x}$

3P we know $-1 \leq \cos x \leq 1$

$-3 \leq -3\cos x \leq 3$

$-2 \leq 1 - 3\cos x \leq 4$

Taking the reciprocals

$-\frac{1}{2} \geq \frac{1}{1-3x}$ and $\frac{1}{1-3\cos x} \geq \frac{1}{4}$

$\therefore$ Range is $(-\infty, -1/2] \cup [1/4, \infty)$

52) Find the domain of $\cancel{\frac{1}{1+2\cos x}}$. $\frac{1}{1-2\sin x}$.

Sol: Let $f(x) = \dfrac{1}{1-2\sin x}$

We know that $-1 \le \sin x \le 1$

$$-2 \le -2\sin x \le 2 \quad \forall x \in R$$

$$-1 \le 1-2\sin x \le 3 \quad \forall x \in R$$

$$\therefore \cancel{1-2\sin x \ne 0} \quad -1 \ge \frac{1}{1-2\sin x} \ge \frac{1}{3}$$

Suppose $1-2\sin x = 0$

$$2\sin x = 1$$

$$\sin x = 1/2$$

$$x = n\pi + (-1)^n \cdot \frac{\pi}{3}.$$

$\therefore$ Domain of f = $R - \left\{ n\pi + (-1)^n \frac{\pi}{3} \right\}$ $n \in Z$.

53) Find the largest possible domain for the real valued fn

$$f(x) = \frac{\sqrt{9-x^2}}{\sqrt{x^2-1}}$$

sol: $f(x) = \dfrac{\sqrt{9-x^2}}{\sqrt{x^2-1}}$.

$9-x^2$ will not be negative for $9-x^2 \ge 0$
$x^2 = \pm 3 \quad -3 \le x \le 3$.

$\therefore x \in [-3, 3]$ ——— ①

x^2-1 will not be negative and $\ne 0$

Suppose $x^2-1=0 \quad x = \pm 1$

$\therefore x$ lies out side $[-1, 1]$

$\therefore x \in (-\infty, -1) \cup (1, \infty)$ ——— ②

Combining ① and ②

The domain of f is $[-3, 3] \cap [(-\infty, -1) \cup (1, \infty)]$

$$\Rightarrow [-3, -1) \cup (1, 3]$$

54) Find the largest possible domain of the real valued fn

$$f(x) = \frac{\sqrt{4-x^2}}{\sqrt{x^2-9}}.$$

Sol: $f(x) = \frac{\sqrt{4-x^2}}{\sqrt{x^2-9}}$.

$4-x^2$ will not be negative suppose $4-x^2=0$ $x=\pm 2$

$\therefore x \in [-2, 2]$ —— ①

and x^2-9 will not be $-$ve and $\neq 0$.

suppose $x^2-9=0$

$x^2=9 \Rightarrow x=-3, 3$

$\therefore$ x lies out side $[-3, 3]$

$\therefore x \in (-\infty, -3) \cup (3, \infty)$ —— ②

Combining ① and ② $x \in \{[-2, 2] \cap [(-\infty, -3) \cup (3, \infty)]\}$

$\therefore x \in (-3, -2] \cup [2, 3)$

$\therefore$ The domain is $(-3, -2] \cup [2, 3)$

55) Find the range of the function $\frac{1}{2\cos x - 1}$

Sol: $f(x) = \frac{1}{2\cos x - 1}$

We know $-1 \leq \cos x \leq 1$

$-2 \leq 2\cos x \leq 2$. Add (-1) on both sides

$-3 \leq 2\cos x - 1 \leq 1$

$-\frac{1}{3} \geq \frac{1}{2\cos x - 1} \geq 1$

(ie) $\frac{1}{2\cos x - 1} \leq -1/3 \Rightarrow (-\infty, -1/3]$

$\frac{1}{2\cos x - 1} \geq 1 \Rightarrow [1, \infty)$

$\therefore$ The range is $(-\infty, -1/3] \cup [1, \infty)$

56) Find the largest possible domain for the real valued function f defined by $f(x) = \sqrt{x^2-5x+6}$

Sol: x^2-5x+6 should not be negative (ie) $x^2-5x+6 \geq 0$.

$x^2-5x+6=0 \Rightarrow (x-2)(x-3)=0$

$\Rightarrow x = 2, 3$

$\therefore$ The domain is out side $(2, 3)$ (ie) $(-\infty, 2] \cup [3, \infty)$

57) Determine whether the function $f: A \to B$ defined by $f(x) = 4x+7$ $x \in A$ is one - one.

Sol: Let $f: A \to B$ defined by $f(x) = 4x+7$

Model-1

Let $x_1, x_2 \in A$ s.t $f(x_1) = f(x_2)$

$$4x_1 + 7 = 4x_2 + 7$$

$$4x_1 = 4x_2$$

$$x_1 = x_2$$ $\therefore$ f is one-one function.

58) S.T the function $f: R \to R$ given by $f(x) = \begin{cases} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases}$

Sol: $\because f(x) = 1$ for $x > 0$ (ie) $\forall$ positive real values $f(x) = 1$

(ie) more than one element have the same image

$f(x) = -1$ for $x < 0$ (ie) $\forall$ negative real values $f(x) = -1$.

(ie) more than one element have same image

$\therefore$ f is not one-one.

59) check which of the following function is onto and into

i) $f: A \to B$ given by $f(x) = 3x$ where $A = \{0, 1, 2\}$ and $B = \{0, 3, 6\}$.

ii) $f: Z \to Z$ given by $f(x) = 3x+2$ {Z is the set of all integers

Sol: i) $f: A \to B$, $f(x) = 3x$. $A = \{0, 1, 2\}$

$f(0) = 0$, $f(1) = 3$ $f(2) = 6$. $B = \{0, 3, 6\}$.

$\therefore$ Range and Codomain are equal (or) every element B has pre image in A. $\therefore$ it is onto.

ii) $f: Z \to Z$, $f(x) = 3x+2$ Z = set of all integers

let $y = f(x) \Rightarrow y = 3x+2$

$$3x = y - 2$$

$$x = \frac{y-2}{3}$$

When $y = 0$ $x = -\frac{2}{3} \notin Z$. $\therefore 0 \in$ codomain does not have any pre image

Model-I

$\therefore$ f is not onto function $\therefore$ it is into fn.

60) Let R be the set of all non-zero real number. Then S.T $f: R \to R$ given by $f(x) = \frac{1}{x}$ is one-one and onto

Sol: $f: R \to R$ Let $x_1, x_2 \in R$ $x_1 \neq 0$, $x_2 \neq 0$

Model I

$f(x_1) = f(x_2)$

$\frac{1}{x_1} = \frac{1}{x_2} \Rightarrow x_1 = x_2$ $\therefore$ f is one-to-one.

Model-II

Let $y = f(x) = \frac{1}{x} \Rightarrow x = \frac{1}{y}$

$f(x) = f\left(\frac{1}{y}\right) = \frac{1}{1/y} = y \Rightarrow$ every element in the co-domain has pre image $\therefore$ it is onto

61) S.T the function $f: R \to R$ defined as $f(x) = x^2$ is neither one-one nor onto

Sol: $f: R \to R$ $f(x) = x^2$

For $f(1) = 1$ and $f(2) = 4$

$f(-1) = 1$ $f(-2) = 4$.

$\because$ more than one element in the domain have same image

$\therefore$ it is not one-one.

$\because$ in the Co-domain the negative real numbers have no Pre images $\therefore$ it is not onto.

62) S.T the function $f: N \to N$ given by $f(x) = 2x$ is one to one but not onto.

Sol: $f: N \to N$ $f(x) = 2x$.

$\forall x \in N$ there is a unique image in co-domain

(or) for $f(x_1) = f(x_2)$

$2x_1 = 2x_2$

$x_1 = x_2$ $\therefore$ f is one-one.

But $\forall$ odd natural numers in codomain there will be no Pre image in domain.

(ie) Let $y = f(x) = 2x \Rightarrow x = \frac{y}{2}$

For $y = 1$ $x = \frac{1}{2} \notin N$.

$\therefore$ f is not onto.

63) S.T the function $f: N \to N$ given by $f(1) = f(2) = 1$ and $f(x) = x-1$ for every $x > 2$ is onto but not one-one.

Sol: $f: N \to N$. $f(1) = 1$

$f(2) = 1$ $\because$ for More than one element in the domain has same image

$\therefore$ it is not one-one.

for $x > 2$ Let $y = f(x) = x-1$

$x = y+1$

$f(y+1) = y+1-1$

$= y$ $\therefore$ f is onto.

(64) Let $f: R \to R$ be defined by $f(x) = x^2+1$. Then find the pre images of 17 and -3

Sol: Consider $x^2+1 = 17$ and $x^2+1 = -3$

$\Rightarrow x^2 = 16$ $x^2 = -4$

$x = \pm 4$ Pre image of (-3) does not exist.

65) $A = \{1, 2, 3\}$ $B = \{4, 5, 6, 7\}$ and $f = \{(1,4)(2,5)(3,6)\}$ is a function from A to B. State whether f is one-one or not

Sol: $f = \{(1,4)(2,5)(3,6)\}$

$\because$ Since every element of A has unique image in B f is one-one

66) Check the injectivity of the following function $f: R \to R$ is given by $f(x) = x^3$

Sol: Let $x_1 x_2 \in R$ $f(x_1) = f(x_2)$

$x_1^3 = x_2^3$

$x_1 = x_2$ taking cube roots on both sides.

$\therefore$ f is injective

67) S.T The function $f: R \to R$ given by $f(x) = |x|$ is neither one-one nor onto

Sol: $f(x) = |x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$ $\therefore$ Range is $[0, \infty)$

For more than one element in R there is unique image.

(ie) $x = 1$ $f(1) = 1$
$x = -1$ $f(-1) = 1$. $\therefore$ it is not one-one.

Also Range $[0, \infty) \neq R$ $\therefore$ range $\neq$ codomain Hence it is not onto.

68) S.T The relation $f: R \to R$ given by $f(x) = \cos x$ $\forall x \in R$ is neither one to one nor onto

Sol: Let $f: R \to R$ and $f(x) = \cos x$.

$f(0) = \cos 0 = 1$

$f(2\pi) = \cos 2\pi = 1$

$\therefore$ For 2 values of x there is unique image and $\therefore$ it is not one-one

Also $\{-1, 1\} \neq R$ (ie) Range is not equal to co-domain

Hence it is not onto.

69) State whether the function $f: R \to R$ defined by $f(x) = 3 - 4x$ is onto or not

Sol: Let $y \in R$ be the codomain.

$y = f(x) = 3 - 4x$.

$x = \frac{3-y}{4}$

$= f\left(\frac{3-y}{4}\right) = 3 - 4\left(\frac{3-y}{4}\right)$

$= y$.

$\therefore$ f is onto.

70) Let us consider some illustration.

1. $X = \{1, 2, 3, 4\}$, $Y = \{a, b, c, d, e\}$

BP

$f = \{(1,a)\ (2,c)\ (3,e)\ (4,b)\}$

clearly it is one-one $\because$ every element of X has unique image in Y.

But not onto $\because$ in Y, d does not have any preimage

2. $X = \{1, 2, 3, 4\}$ $Y = \{a, b\}$

$f : \{(1,a)\ (2,a)\ (3,a)\ (4,a)\}$

It is not one-one $\because$ more than one element in X have same image

Also it is not onto $\because$ in Y, b does not have any pre image

3. $X = \{1, 2, 3, 4\}$ $Y = \{a\}$.

$f : \{(1,a)\ (2,a)\ (3,a)\ (4,a)\}$

f is not one-one since more than one element in X have same image

But onto since co-domain = range.

4. $X = \{1, 2, 3, 4\}$ $Y = \{a, b, c, d, e\}$.

$f = (1,a)\ (2,c)\ (3,b)\ (4,b)$

f is neither one-one nor onto.

$\because$ in X, 3 and 4 have same image and d and e in Y does not have any pre image

5) $X = \{1, 2, 3, 4\}$ $Y = \{a, b, c, d\}$

$f = \{(1,a)(2,b)\ (3,c)\ (4,d)\}$

f is both one-one and onto

Because every element in X has unique image in Y and Range = codomain.

6) $X = \{1, 2, 3, 4\}$ $Y = \{a, b, c, d, e\}$

$f = (1,a)\ (2,c)\ (3,e)$

This is not at all a function $\because 4 \in X$ has no image

But it is a relation.

(2) Find linear maps $T : \mathbb{R}^2 \to \mathbb{R}^3$ so that the following hold, if possible. If it is not possible, explain why.

(a) T is both 1-1 and onto.

(b) T is 1-1 but not onto.

(c) T is not 1-1, but is onto.

(d) T is neither 1-1 nor onto.

(3) Do the same as Exercise 2, but this time for $T : \mathbb{R}^3 \to \mathbb{R}^2$.

(4) Do the same as Exercise 2, but this time for $T : \mathbb{R}^3 \to \mathbb{R}^3$.

Problems :

1. (2) Show that if $\{\vec{a_1}, \ldots, \vec{a_n}\}$ spans $\mathbb{R}^m$ and is linearly independent, $n = m$.
2. (2) Show that if $\{\vec{a_1}, \ldots, \vec{a_n}\}$ spans $\mathbb{R}^m$ and is linearly dependent, $n > m$.
3. (2) Show that if $\{\vec{a_1}, \ldots, \vec{a_n}\}$ does not span $\mathbb{R}^m$ and is linearly independent, $n < m$.
4. (2) Given any $n, m \geq 1$, find a linearly dependent set of vectors $\{\vec{a_1}, \ldots, \vec{a_n}\} \subseteq \mathbb{R}^m$ which does not span $\mathbb{R}^m$.

71) Let $A = \{1, 2, 3, 4\}$ and $B = \{a, b, c, d\}$ Give a function for each of the following 1) neither one-one nor onto 2) not one-one but onto 3) One-one but not onto 4) one-one and onto.

TBP

1) neither one-one nor onto.

$f = \{(1,a)\ (4,b)\ (2,c)\ (3,c)\}$

more than one element in A have the same image $\therefore$ it is not one-one

$\therefore$ d has no pre image in A it is not onto.

2) not one-one but onto

$f = \{(1,a)\ (2,a)\ (3,b)\ (2,c)\ (3,d)\}$

not one-one because more than one element in the set A has same image the set B. But every element in the set B has pre-image in the set A. $\therefore$ it is onto.

3) one-one but not onto

$f = \{(1,a)\ (2,b)\ (3,d), (4,c)\}$

it is one-one and onto

$\because$ Every element in the set A has unique image in the set B. also every element in the set B has pre image in the set A (ex) range = Co domain Hence it is onto.

72) Check whether the following functions are one-one and onto

TBP:

1) $f: N \to N$ define by $f(n) = n+2$

2) $f = N \cup \{-1, 0\} \to N$ defined by $f(n) = n+2$.

Sol: 1. $f: N \to N \quad f(n) = n+2$

Let $f(n) = f(m)$

$n+2 = m+2 \Rightarrow n = m \quad \therefore f$ is One-one

But $f(n) = n+2 \Rightarrow f(1) = 3$

$f(2) = 4$ and so on.

$\therefore \{1, 2\}$ in the Codomain have no pre image in the domain

Hence it is not onto.

2). If m is the codomain $m-2$ is in the domain

$f(m-2) = m-2+2 = m$ thus m has pre image in domain

Hence this function is onto.

$\{\vec{a_1},\dots,\vec{a_n}\}$ spans $\mathbb{R}^m$	$\{\vec{a_1},\dots,\vec{a_n}\}$ is LI	Example	Necessary conditons
True	True	$\begin{bmatrix}1\\0\end{bmatrix},\begin{bmatrix}0\\1\end{bmatrix}$	$n = m$
True	False	$\begin{bmatrix}1\\0\end{bmatrix},\begin{bmatrix}0\\1\end{bmatrix},\begin{bmatrix}1\\1\end{bmatrix}$	$n > m$
False	True	$\begin{bmatrix}1\\0\end{bmatrix}$	$n < m$
False	False	$\begin{bmatrix}1\\0\end{bmatrix},\begin{bmatrix}2\\0\end{bmatrix}$	$n, m \geq 1$

We leave it to the reader to verify that these necessary conditions are necessary for the given properties to hold (See Problems). Note that these necessary conditions are not sufficient. For example, for the (True, True) case, $\begin{bmatrix}1\\0\end{bmatrix},\begin{bmatrix}2\\0\end{bmatrix}$ has the same number of vectors as number of components, but $\left\{\begin{bmatrix}1\\0\end{bmatrix},\begin{bmatrix}2\\0\end{bmatrix}\right\}$ is not linearly independent.

<u>Exercises</u> :

(1) Determine if the following sets of vectors span $\mathbb{R}^m$, where m is the number of componenets they each have. Also, determine if the set if linearly independent.

(a) $\left\{\begin{bmatrix}1\\2\end{bmatrix}\right\}$

(b) $\left\{\begin{bmatrix}1\\5\end{bmatrix},\begin{bmatrix}-4\\7\end{bmatrix}\right\}$

(c) $\left\{\begin{bmatrix}1\\2\end{bmatrix},\begin{bmatrix}2\\4\end{bmatrix},\begin{bmatrix}3\\7\end{bmatrix}\right\}$

(d) $\left\{\begin{bmatrix}1\\2\\5\end{bmatrix},\begin{bmatrix}-4\\-8\\-20\end{bmatrix}\right\}$

(e) $\left\{\begin{bmatrix}1\\4\\0\end{bmatrix},\begin{bmatrix}2\\3\\5\end{bmatrix},\begin{bmatrix}-1\\-7\\3\end{bmatrix}\right\}$

(f) $\left\{\begin{bmatrix}1\\4\\3\end{bmatrix},\begin{bmatrix}-2\\1\\5\end{bmatrix},\begin{bmatrix}1\\7\\8\end{bmatrix}\right\}$

(g) $\left\{\begin{bmatrix}1\\1\\1\end{bmatrix},\begin{bmatrix}2\\2\\2\end{bmatrix},\begin{bmatrix}1\\1\\0\end{bmatrix},\begin{bmatrix}0\\1\\1\end{bmatrix}\right\}$

(h) $\left\{\begin{bmatrix}1\\1\\1\\1\end{bmatrix},\begin{bmatrix}2\\2\\2\\2\end{bmatrix},\begin{bmatrix}-6\\1\\2\\7\end{bmatrix},\begin{bmatrix}0\\3\\2\\5\end{bmatrix}\right\}$

(i) $\left\{\begin{bmatrix}1\\2\\1\\0\end{bmatrix},\begin{bmatrix}-3\\4\\1\\0\end{bmatrix},\begin{bmatrix}2\\5\\-1\\0\end{bmatrix},\begin{bmatrix}9\\8\\7\\0\end{bmatrix}\right\}$

$\therefore f(x) = |x| + x = \begin{cases} x + x = 2x & x \geq 0 \\ -x + x = 0 & x < 0 \end{cases}$

$g(x) = |x| - x = x - x = 0 \quad x \geq 0$

$= -x - x = -2x \quad x < 0.$

$f \circ g(x) = f(g(x)) = \begin{cases} f(0) = 0 & x \geq 0 \\ f(-2x) & x < 0 \\ = 2(-2x) & \text{when } x<0,\ -2x>0 \\ = -4x & \end{cases}$

$g \circ f(x) = g(f(x)) = g(2x) = 0 \quad x \geq 0$

$g(0) = \quad x < 0 \quad$ when $x < 0 \ -2x > 0$

$= g(0) = 0$

$\therefore g \cdot f(x) = 0 \quad \forall x \in R.$

93) If $f: R \to R$ defined by $f(x) = 3x - 5$ P.T f is bijection and find its inverse.

TBP

Sol: Let $f: R \to R$ defined by $f(x) = 3x - 5$

Let $f(x) = f(y)$

$3x - 5 = 3y - 5$

$3x = 3y$

$x = y \quad \therefore f$ is one-one.

Let $y = f(x) = 3x - 5 \Rightarrow y + 5 = 3x$

$\frac{y+5}{3} = x.$

$\therefore f(x) = f\left(\frac{y+5}{3}\right) = \cancel{3} \cdot \frac{(y+5)}{\cancel{3}} - 5 = y$

$\therefore f$ is onto

$\therefore f$ is bijection. and

$f^{-1}(y) = \frac{y+5}{3}$

$f^{-1}(x) = \frac{x+5}{3}.$

94) Let f, g and h are the functions from $R \to R$ S.T

TBP. 1) $(f+g) \circ h = (f \circ h) + (g \circ h)$

2) $(f \cdot g) \circ h = (f \circ h) \cdot (g \circ h)$

1) LHS: $(f+g) \circ h(x) = (f+g) h(x)$

$= f(h(x)) + g(h(x))$

$= (f \circ h)(x) + (g \circ h)(x)$

$= (f \circ h) + (g \circ h)$

2) LHS: $(f \cdot g) h(x) = f(h(x)) \cdot g(h(x))$

$= (f \circ h) x \cdot (g \circ h)(x)$

$= (f \circ h)(g \circ h)$

95) Let f and g be two functions given by

$f = \{(2,4)(5,6)(8,-1)(10,-3)\}$ and $g = \{(2,5)(7,1)(8,4)(10,13)(11,-5)\}$

Find the domain of $f+g$

Sol: Domain of $f+g = D_1 \cap D_2$

$D_1 = \{2,5,8,10\}$ $D_2 = \{2,7,8,10,11\}$.

$D_1 \cap D_2 = \{2,8,10\}$.

96) Let f and g be two real functions defined by

$f = \{(0,1)(2,0)(3,-4)(4,2)(5,1)\}$ $g = \{(1,0)(2,2)(3,-1)(4,4)(5,3)\}$

Find the domain of fg.

Domain of $fg = D_1 \cap D_2$

$D_1 = \{0,2,3,4,5\}$ $D_2 = \{1,2,3,4,5\}$

$D_1 \cap D_2 = \{2,3,4,5\}$

97) Find the set of values of x for which the function $f(x) = 3x^2 - 1$ $g(x) = 3 + x$ are equal.

$3x^2 - 1 = 3 + x$ $\quad -12$

$3x^2 - x - 4 = 0$ $\quad -4 \; 3$

$3x^2 + 3x - 4x - 4 = 0$

$3x(x+1) - 4(x+1) = 0$

$(x+1)(3x-4) = 0$

$x = -1, 4/3$ $\quad \therefore \{-1, 4/3\}$.

In a survey, it is found that 21 like product A, 26 like product B and 29 like product C. If 14 people like products A and B, 12 people like products C and A, 14 people like products B and C and 8 people like all three. Find How many like product C only.

Sol: $n(C) = 29$, $n(C \cap A) = 12$ $n(B \cap C) = 14$ $n(A \cap B \cap C) = 8$

$$n(C \text{ only}) = n(C) - n(C \cap A) - n(C \cap B) + n(A \cap B \cap C)$$

$$= 29 - 12 - 14 + 8$$

$$= 37 - 26 = 11.$$

From 50 students taking examinations in Maths, physics, chemistry each of the student pass atleast one of the subject. 37 passed in Maths 24 in physics, 43 in chemistry. Almost 19 passes in M and P. and 29 in M and C and 20 P and C. Find the largest possible number that could have passed all three subjects.

Sol: $n(M) = 37$, $n(P) = 24$ $n(C) = 43$ $n(M \cap P) \leq 19$, $n(M \cap C) \leq 29$ and $n(P \cap C) \leq 20$ and $n(M \cup P \cup C) = 50$

$$n(M \cup P \cup C) = n(M) + n(P) + n(C) - n(M \cap P) - n(M \cap C) - n(P \cap C) + n(M \cap P \cap C) \leq 50$$

$$37 + 24 + 43 - 19 - 29 - 20 + n(M \cap P \cap C) \leq 50$$

$$32 + n(M \cap P \cap C) \leq 50$$

$$\Rightarrow n(M \cap P \cap C) = 14.$$

25) If $A = \{a, b, c\}$ then the relation $R = \{(b,c)\}$ on A is

1) reflexive only 2) symmetric only 3) ~~t~~transitive only 4) reflexive and transitive only.

26) Let $A = \{2, 3, 4, 5 \cdots 17, 18\}$ Let $\simeq$ be the equivalance relation on $A \times A$ Cartesian product of A with itself defined by $(a, b) \simeq (c, d)$ iff $ad = bc$ Then the number of ordered pairs of the equivalance classes of $(3, 2)$ is

1) 4 2) 5 3) 6 4) 7.

27) The relation R in $N \times N$ s.t $(a, b) R (c, d) \Longleftrightarrow a + d = b + c$ is

1) reflexive but not symmetric 2) reflexive and transitive but not symmetric

3) equivalance relation 3) non of these

$(a, b) R (a, b) = a + b = b + a$ reflexive $\cdot \in N$

$(a, b) R (c, d) = a + d = b + a$ symmetric $\cdot \in N$.

$(a, b) R (c, d), (c, d) R (e, f) \Rightarrow (a, b) R (e, f) = a + f = b + e$ transitive $\in N$.

$\therefore$ equivalance relation.

28) If $A = \{1, 2, 3\}$ $B = \{1, 4, 6, 9\}$ and R is a relation from A to B defined by $x > y$ The range of R is

1) $\{1, 4, 6, 9\}$ 2) $\{4, 6, 9\}$ 3) $\{1\}$ 4) non of these

29) A relation R is defined from $\{2, 3, 4, 5\}$ to $\{3, 6, 7, 10\}$ by $x R y \Leftrightarrow x$ is relatively prime to y then the domain of R is

1) $\{2, 3, 5\}$ 2) $\{3, 5\}$ 3) $\{2, 3, 4\}$ 4) $\{2, 3, 4, 5\}$.

$R = \{(2, 3) (2, 7) (3, 7) (3, 10) (4, 3) (4, 6) (4, 7) (4, 10) (5, 6)\}$

$\therefore$ Domain is $\{2, 3, 4, 5\}$.

30) Let R be the relation on N defined by $x + 2y = 8$ the domain of R is

1) $\{2, 4, 8\}$ 2) $\{2, 4, 6, 8\}$ 3) $\{2, 4, 6\}$ 4) $\{1, 2, 3, 4\}$.

$2y = 8 - x \overset{x=}{=} \{2, 4, 6\}$

31) R is a relation from $\{11, 12, 13\}$ to $\{8, 10, 12\}$ defined by $y = x - 3$. then R^{-1} is

1) $\{(8, 11) (10, 13)\}$ 2) $\{(11, 8) (13, 10)\}$ 3) $\{(10, 13) (8, 11) (8, 10)\}$ 4) non of these

$R = \{(11, 8) (13, 10)\} \Rightarrow R^{-1} = \{(8, 11) (10, 13)\}$

32) Let $R = \{(a, a) (b, b) (c, c) (a, b)\}$ be a relation on set $A = \{a, b, c\}$ then R is 1) identity relation 2) reflexive 3) symmetric

4) equivalance.

2) reflexive $\because A = \{a, b, c\}$ and $\{(a,a)\ (b\,b)\ (c,c)\} \in R$.

33) Let $A = \{1,2,3\}$ and $R = \{(1,2)\ (2,3)\ (1,3)\}$ be a relation on A. then R is

1) neither reflexive nor transitive 2) neither symmetric nor transitive

3) transitive 4) non of these

3) transitive $(1,2) \in R,\ (1,3) \in R \Rightarrow (1,3) \in R$.

34) If R is a relation on the set $A = \{1,2,3,4,5,6,7,8,9\}$ given by $xRy \Leftrightarrow y = 3x$

then R = 1) $\{(3,1)\ (6,2)\ (8,2)\ (9,3)$ 2) $\{(3,1)\ (6,2)\ (9,3)\}$

3) $\{(3,1)\ (2,6)\ (3,9)$ 4) non of these

(x, one time, y 3 times)

35) If R is a relation on the set $A = \{1,2,3\}$ given by $R = \{(1,1)\ (2,2)\ (3,3)\}$ then R is

1) reflexive 2) symmetric 3) transitive 4) All of these

$R = \{1,2,3\}$ $f = \{(1,1)\ (2,2)\ (3,3)\}$ $\therefore$ R is reflexive, symmetric and transitive (ie) All.

If there is $(2,3) \in R$ then only we have check symmetric or transitive.

36) If $A = \{a, b, c, d\}$ then the relation $\{(a,b)\ (b,a)\ (a,a)\}$ on A is

1) symmetric and transitive only 2) reflexive and transitive only

3) symmetric only 4) transitive only.

$(a,b) \in R \Rightarrow (b,a) \in R$ $\therefore$ symmetric.

37) If $A = \{1,2,3\}$ then the relation $R = \{(2,3)\}$ on A is

1) symmetric and transitive only 2) symmetric only

3) transitive only 4) non of these

38) Let R be the relation on the set $A = \{1,2,3,4\}$ given by

$R = \{(1,2)\ (2,2)\ (1,1)\ (4,4)\ (1,3)\ (3,3)\ (3,2)\}$ then

1) R is reflexive and symmetric but not transitive

2) R is reflexive and transitive but not symmetric

3) R is symmetric and transitive but not reflexive

4) R is equivalence relation.

$\{(1\,1)\ (2\,2)\ (3\,3)\ (4\,4)\} \in R$ $\therefore$ reflexive

$(3,2) \in R$ but $(2,3) \notin R$ $\therefore$ not symmetric.

$(1,3) \in R\ (3,2) \in R \Rightarrow (1,2) \in R$ $\therefore$ transitive

39) The relation $R = \{(1,1)\ (2,2)\ (3,3)\}$ on the set $\{1,2,3\}$ is

1) Symmetric only 2) reflexive only 3) equivalance relation

4) transitive only

40) Let $A = \{1,2,3\}$ and Consider the relation $R = \{(1,1)\ (2,2)\ (3,3)\ (1,2)\ (2,3)$ and $(1,3)\}$

1) reflexive but not symmetric 2) reflexive but not transitive

3) symmetric and transitive 4) neither symmetric nor transitive

$\because \{(1,1)\ (2,2)\ (33)\} \in R$. reflexive

$(1,2) \in R$ but $(2,1) \in R$ not symmetric

$(1,2) \in R\ (2,3) \in R \Rightarrow (1,3) \in R$ ∴ transitive.

41. The relation S defined on the set R of all real numbers by the rule aRb iff $a \geq b$ is

1. an equivalance relation 2) reflexive, transitive but not symmetric

3) Symmetric transitive but not reflexive 4) neither transitive nor reflexive but symmetric.

$a \geq a$ ∴ reflexive (this possible only for equality)

$a \geq b$ but $b \ngeq a$ not symmetric

$a \geq b,\ b \geq c \Rightarrow a \geq c \in R$ Transitive

42) Let R be a relation on the set N of natural numbers defined by nRm iff n divides m then R is

1) Reflexive and symmetric 2) Transitive and symmetric

3) Equivalance 4) Reflexive, transitive but not symmetric

n divides n ∴ reflexive

n divides m $\not\Rightarrow$ m divides n Hence not symmetric.

$n|m$ and $m|l \Rightarrow n|l$ ∴ transitive.

43) Let L denotes the set of all straight lines in a plane. Let a relation R be defined by lRm iff l is $\perp r$ to m $\forall l, m \in R$ then R is

1) Reflexive 2) Symmetric 3) transitive, 4) non of these

l is not $\perp r$ to l. not reflexive

$l \perp m \Rightarrow m \perp l$ symmetric

$l \perp m$ and $m \perp n$ but l is not $\perp r$ n not transitive.

44) Let T be the set of all triangles in the Euclidean plane and let a relation R on T defined as aRb if a is congruent b for all $a, b \in T$ then R is

1. Symmetric but not transitive 2) transitive but not symmetric

3) neither symmetric nor transitive 4) both symmetric and transitive

45) Consider a non empty set consisting of children in a family and a relation R defined as aRb if a is a brother of b then R is
1) Symmetric but not transitive 2) transitive but not symmetric
3) neither symmetric nor transitive 3) both symmetric and transitive

2) transitive but not symmetric.

$\because$ a is a brother of b, b is a brother of c $\Rightarrow$ a is a brother of c.
(b may sister also)
$\therefore$ not symmetric.

46) For real number x and y define xRy iff $x-y+\sqrt{2}$ is an irrational number. Then the relation R is
1) reflexive 2) symmetric 3) transitive 4) non of these.

Clearly R is reflexive $\because xRx = x-x+\sqrt{2} = \sqrt{2}$ is irrational

Functions.

47) $f: R \to R$ given by $f(x) = x + \sqrt{x^2}$ is
1) injective 2) surjective 3) bijective 4) None of these

$\because f(-1) = 0$ and $f(-2) = 0$ it is not injective
There is no pre images for all negative numbers which are in co-domain
$\therefore$ it is not onto. $\therefore$ It is not bijective. Hence none of these is answer.

48) The function $f(x) = 2^x + 2^{|x|}$ if $f: R \to R-R$.
1) one to one 2) many-one and onto 3) one-one and into
4) many one and into.

$f(x) = 2^x + 2^{-x}$ $x<0$ $f(-1) = 2^{-1}+2 = 5/2$ one-one
$2^x + 2^x$ $x \geq 0$ $f(1) = 2+2 = 4$

$\forall x \in R$ $f(x) = +ve$. $\therefore$ -ve values in co-domain has no pre image
hence it is not onto $\therefore$ one-one and into

49) Let $f: R-(-b) \to R-(1)$ be defined by $f(x) = \frac{x+a}{x+b}$ $a \neq b$ then
1. f is one one but not onto 2) f is onto but not one-one
3. f is both one-one and onto 4) none of these.

$\forall a, b \in R$ $f(a) = \frac{2a}{a+b}$ and $f(b) = \frac{a+b}{2b}$. Suppose $\frac{a+b}{2b} = +1$
$a+b = +2b$
$\therefore$ f is one-one and clearly f is onto
$\because a \neq b$ $a-b \neq 0$
$\therefore$ f is both one one and onto.
$\therefore \frac{a+b}{2b} \neq +1$

50) The function $f : A \to B$ defined by $f(x) = -x^2+6x-8$ is bijection if

1) $A = (-\infty, 3)$ $B = (-\infty, 1]$ 2) $A = [-3, \infty)$ and $B = (-\infty, 1)$ 3) $A = (-\infty\ 3]$ $B = [1, \infty)$ 4) $A = [3, \infty)$ and $B = [1, \infty)$

Put the values of A in $f(x)$ and find B. see which one is satisfied

$x^2 - 6x + 8 = 0$
$(x-4)(x-2) = 0$
$x = (2, 4)$

51) Let $A : \{x \in R : -1 \le x \le 1\} = B$ then the mapping $f : A \to B$ given by $f(x) = x|x|$ 1) injective but not surjective 2) surjective but not injective 3) bijective 4) non of these.

$f(x) = \begin{cases} x^2 & x \geq 0 \\ -x^2 & x < 0 \end{cases}$ clearly it is one-one $f(1) = 1$, $f(-1) = -1$

and every element in $-1 \le x \le 1$ have image $\therefore$ onto.

$\therefore$ it is bijective

52) The function $f : [0, \infty) \to R$ given by $\frac{x}{x+1} = f(x)$ is

1) one-one and onto 2) one-one but not onto 3) onto but not one-one 4) neither one-one nor onto.

For every element $[0, \infty)$ there is a unique image in R. $\therefore$ one-one but the negative value in the Co-domain (R) does not have any pre image $\therefore$ it is not onto.

$\therefore$ it is one-one but not onto

53) The range of the function $f(x) = {}^{7-x}P_{x-3}$ is

1) $\{1, 2, 3, 4, 5\}$ 2) $\{1, 2, 3, 4, 5, 6\}$ 3) $\{1, 2, 3, 4\}$ 4) $\{1, 2, 3\}$.

We can find nP_r for $n \geq r$ only. $nP_r = \frac{n!}{(n-r)!}$

Put $x = 3$ $4P_0 = 1$ $x = 4$, $3P_1 = \frac{3!}{2!} = 3$ $x = 5$, $2P_2 = 2$

range $= \{1, 2, 3\}$.

54) A function f from the set of natural numbers to set of integers defined by $f(n) = \frac{n-1}{2}$ when n is odd

$= -\frac{n}{2}$ when n is even.

is 1) neither one-one nor onto 2) one-one but not onto 3) onto but not one-one 4) one and onto

Every different odd numbers in N has unique image and every element in integers have pre image in N. $\therefore$ this one-one and onto.

55) Which of the following functions from Z to itself are bijections

1) $f(x) = x^3$ 2) $f(x) = x+2$ 3) $f(x) = 2x+1$ 4) $f(x) = x^2+x$.

$f(x) = x^2+x$ is neither one-one and onto.

$f(x) = 2x+1$ there is no pre image for 0 in co-domain

$f(x) = x+2$ it is one-one and onto (bijective)

$f(x) = x^3$ = All elements in the Co domain does not have pre image

56) Which of the following function from $A: \{x : -1 \le x \le 1\}$ to itself are bejection.

1) $f(x) = x/2$ 2) $g(x) = \sin\left(\frac{\pi x}{2}\right)$ 3) $h(x) = |x|$ 4) $K(x) = x^2$

For $K(x) = x^2$ there is no different image for $f(-1)$ and $f(1)$ ∴ not one-one

$h(x) = |x| = \begin{cases} x & x \ge 0 \\ -x & x < 0 \end{cases}$ For -1 there is no pre image ∴ not onto

$f(x) = \frac{x}{2}$: for -1 is no pre image ∴ not onto

$g(x) = \sin\left(\frac{\pi x}{2}\right)$ it is clearly one-one and onto.

57) Let $A: \{x : -1 \le x \le 1\}$ and $f: A \to A$ s.t $f(x) = x|x|$ Then f is

1) bijection 2) injective but not surjective

3) Surjective but not injective 4) neither injective nor surjective

$f(x) = \begin{cases} x^2 & x \ge 0 \\ -x^2 & x < 0 \end{cases}$ clearly bijection.

58) If a function $f: R \to A$ given by $\frac{x^2}{x^2+1}$ is a surjection then $A =$

1) R 2) $[0,1]$ 3) $(0,1]$ 4) $[0,1)$

59) If $f: R \to R$ is given by $f(x) = 3x-5$ then $f^{-1}(x)$.

1) $\frac{1}{3x-5}$ 2) $\frac{x+5}{3}$ 3) does not exist 4) does not exist because of not onto.

Let $y = f(x) = 3x-5 \Rightarrow \frac{y+5}{3} = x \Rightarrow f^{-1}(y) = \frac{y+5}{3}$

$f^{-1}(x) = \frac{x+5}{3}$

60) A function $f: R \to R$ defined by $f(x) = 6^x + 6^{|x|}$ is

1) one-one and onto 2) many one and onto

3) one-one and into 4) many one and into

61) $f: Z \to Z$ be given by $f(x) = \begin{cases} x/2 & \text{if } x \text{ is even} \\ 0 & \text{if } x \text{ is odd.} \end{cases}$

1) onto but not one-one 2) one-one but not onto

3) one-one and onto 4) neither one-one nor onto.

For different odd numbers no different image ∴ not one-one and different odd number in the codomain has no pre image Hence not onto

62) which of the following function from $A = \{x \in R : -1 \le x \le 1\}$ to itself are bijections 1) $f(x) = |x|$ 2) ✓ $f(x) = \sin\frac{\pi x}{2}$

3) $f(x) = \sin\frac{\pi \pi}{4}$ 4) none of These

63) Let $f : R \to R$ defined by $f(x) = \frac{x^2-8}{x^2+2}$ then f is

1) one-one but not onto 2) ✓ one-one and onto

3) onto but not one-one 4) neither one-one nor onto.

$f(x_1) = f(x_2)$

$$\frac{x_1^2-8}{x_1^2+2} = \frac{x_2^2-8}{x_2^2+2} \Rightarrow \cancel{x_1^2x_2^2}+2x_1^2-8x_2^2-\cancel{16} = \cancel{x_1^2x_2^2}-8x_1^2+2x_2^2-\cancel{16}$$

$$10x_1^2 = 10x_2^2 \Rightarrow x_1 = x_2 \quad \therefore \text{ one-one}.$$

$$y = f(x) = \frac{x^2-8}{x^2+2} \Rightarrow x^2y+2y = x^2-8$$

$$x^2(y-1) = -2y-8$$

$$x^2 = \frac{2y+8}{1-y}$$

$$\therefore f\left(\frac{2y+8}{1-y}\right) = \left(\frac{2y+8}{1-y}-8\right) \Big/ \left(\frac{2y+8}{1-y}+2\right)$$

$$= \frac{2y+\cancel{8}-\cancel{8}+8y}{\cancel{2y}+8+2-\cancel{2y}} = \frac{10y}{10} = y \quad \therefore \text{ onto}.$$

$\therefore$ one-one and onto.

64) Let $f(x) = x^3$ be the function with its domain $\{0,1,2,3\}$ then the domain of f^{-1} is 1) $\{3,2,1,0\}$ 2) $\{0,-1,-2,-3\}$ 3) ✓ $\{0,1,8,27\}$ 4) $\{0,-1,-8,-27\}$

$f(x) = x^3$

$f(0) = 0$, $f(1) = 1$ $f(2) = 8$, $f(3) = 27$

$f : (0,0)\ (1,1)\ (2,8)\ (3,27)$ domain of $f^{-1} = (0,1,8,27)$

65) If $f : R \to R$ is given by $f(x) = x^3+3$ then $f^{-1}(x)$ is equal to

1) $x^{1/3}-3$ 2) $x^{1/3}+3$ 3) ✓ $(x-3)^{1/3}$ 4) $x+x^{1/3}$.

$$y = f(x) = x^3+3 \Rightarrow y-3 = x^3 \Rightarrow x = (y-3)^{1/3}$$

$$y = f(x) = f(y-3)^{1/3} = \left((y-3)^{1/3}\right)^3+3 = y \text{ (onto)}$$

$$\Rightarrow f^{-1}(y) = (y-3)^{1/3} \Rightarrow f^{-1}(x) = (x-3)^{1/3}$$

66) Let $f : R \to R$ given by $f(x) = x^2-3$ Then f^{-1} is given by

1) ✓ $\sqrt{x+3}$ 2) $\sqrt{x}+3$ 3) $x+\sqrt{3}$ 4) none of these

$$y = f(x) = x^2-3 \Rightarrow x^2 = y+3.$$

$$x = \sqrt{y+3}$$

$$f^{-1}(y) = \sqrt{y+3} \quad \therefore f^{-1}(x) = \sqrt{x+3}.$$

67) If $A = \{1,2,4\}$ $B = \{2,4,5\}$ $C = \{2,5\}$ then $(A-B)\times(B-C)$

1) $\{(1,2)\ (1,5)\ (2,5)\}$ 2) $\{(1,4)\}$ 3) $(1,4)$ 4) non of these

$A-B = \{1\}$ $(A-B)\times(B-C) = \{(1,4)\}$

$B-C = \{4\}$

68) If $A = \{1,2,3\}$ and $B = \{1,4,6,9\}$ and R is a relation from A to B defined by x is greater than y. The range of R is

1) $\{1,4,6,9\}$ 2) $\{4,6,9\}$ 3) $\{1\}$ 4) none of these

$R: (2,1)\ (3,1)$ $\therefore$ The range $= \{1\}$.

69) Let R be the relation on N defined by $x+2y=8$ The domain of R is

1) $\{2,4,8\}$ 2) $\{2,4,6,8\}$ 3) $\{2,4,6\}$ 4) $\{1,2,3,4\}$.

$y = \frac{8-x}{2}$ when we sub. $\{2,4,6\}$ for x we will get $y \in N$. $\therefore$ the domain is $\{2,4,6\}$.

70) If the set A has p elements and B has q elements then the number of elements in $A\times B$ is

a 1) $p+q$ 2) $p+q+1$ 3) pq 4) p^2

71) Let R be a relation from a finite set A to the set B then

1) $R = A\cup B$ 2) $R = A\cap B$ 3) $R \subseteq A\times B$ 4) $R \subseteq B\times A$

72) A set contains n elements then the number of elements in the power set is 2^n

73) The number of elements in the power set of a null set is $2^0 = 1$

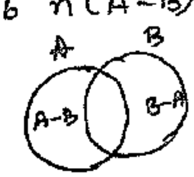
74) If A and B are two sets s.t $n(A)=20$, $n(B) = 25$, $n(A\cup B) = 40$ Then write $n(A\cap B)$

$n(A\cup B) = n(A) + n(B) - n(A\cap B)$

$n(A\cap B) = n(A) + n(B) - n(A\cup B)$

$= 20 + 25 - 40 = 5$

75) If A and B are two sets s.t $n(A) = 115$ $n(B) = 326$ $n(A-B) = 47$ then $n(A\cup B)$ is



$(A\cup B) = (A-B)\cup B$

$n(A\cup B) = n(A-B) + n(B)$

$= 47 + 326 = 373$

76) For any set A, $(A')'$ is equal to

1) A' 2) A 3) φ 4) none of these

77) Let A and B be two sets in the same universal ~~set~~ set then $A-B$

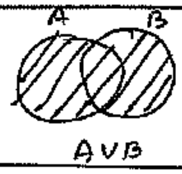
1) $A\cap B$ 2) $A'\cap B$ 3) $A\cap B'$ 4) none of these

78) The number of sub sets of a set containing n elements is

1) n 2) 2^n-1 3) n^2 4) 2^n.

79) For any two sets A and B $A\cap(A\cup B)$

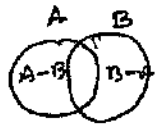
1) A 2) B 3) ϕ 4) none of these



80) The symmetric difference of A and B is not equal to

1) $(A-B)\cap(B-A)$ 2) $(A-B)\cup(B-A)$ 3) $(A\cup B)-(A\cap B)$

4) $\{(A\cup B)-A\}\cup\{(A\cup B)-B\}$.



81) The symmetric difference of $A=\{1,2,3\}$ $B=\{3,4,5\}$ is

1) $\{1,2\}$ 2) $\{1,2,4,5\}$ 3) $\{4,3\}$ 4) $\{2,5,1,4,3\}$.

$A-B=\{1,2\}$ $B-A=\{4,5\}$ $(A-B)\cup(B-A)=\{1,2,4,5\}$

82) For any two sets A and B $(A-B)\cup(B-A)$ is

1) $(A-B)\cup A$ 2) $(B-A)\cup B$ 3) $(A\cup B)-(A\cap B)$ 4) $(A\cup B)\cap(A\cap B)$

Draw the diagram and verify.

83) Which of the following statement is false.

1) $A-B=A\cap B'$ 2) $A-B=A-(A\cap B)$ 3) $A-B=A-B'$

4) $A-B=(A\cup B)-B$ Draw the diagoram and verify.

84) For any three sets A, B and C.

1) $A\cap(B-C)=(A\cap B)-(A\cap C)$ 2) $A\cap(B-C)=(A\cap B)-C$

3) $A\cup(B-C)=(A\cup B)\cap(A\cup C')$ 4) $A\cup(B-C)=(A\cup B)\cap(A\cap C)$

85) Let U be the universal set containing 700 elements. If A, B are the sub sets of U s.t $n(A)=200$, $n(B)=300$ $n(A\cap B)=100$ $n(A'\cap B')=\ldots$.

1) 400 2) 600 3) 300 4) none of these

$n(A'\cap B')=n(A\cup B)'=n(U)-n(A\cup B)$ $n(A\cup B)=200+300-100$
$=400$

$=700-400$

$=300$

86) $A=\{1,2,3,4,5\}$ Then the number of proper subsets of A is

a) 120 b) 30 c) 31 d) 32

Number of proper sub sets $=2^n-1=2^5-1=31$

87)

Book one Mark.

1. If $A=\{(x,y): y=e^{x}\ x\in R\}$ and $B=\{(x,y), y=e^{-x}\ x\in R\}$ then $n(A\cap B)$ is

TBP

1) infinity 2) 0 3) 1 4) 2

$\because f(-x)$ is symmetric about y axis e^{x} and e^{-x} cuts at one point $\therefore n(A\cap B)=1$

2) If $A=\{(x,y): y=\sin x\ x\in R\}$ $B=\{(x,y): y=\cos x, x\in R\}$ then $A\cap B$ contains

TBP

1) no elements 2) infinitely many ~~points~~ elements

3) only one point 4) Cannot be determined.

3) The relation R defined on a set $A=\{0,-1,1,2\}$ by xRy if $|x^2+y^2|\le 2$ then which one of the following is true?

1) $R=\{(0,0)\ (0,-1)\ (0,1)\ (-1,0)\ (1,2)\ (1,0)\}$

TBP 2) $R^{-1}=\{(0,0)\ (0,-1)\ (0,1)\ (-1,0)\ (1,0)\}$.

3) Domain of $R=\{0,-1,1,2\}$

4) Range of R is $=\{0,-1,1\}$.

In one the range is 2 it is not possible $|4|\nleq 2$

In Domain 2 should not be there

4) $f(x)=|x-2|+|x+2|$ if $x\in R$ then

1) $f(x)=\begin{cases}-2x & \text{if } x\in(-\infty,-2] \\ 4 & \text{if } x\in(-2,2] \\ 2x & \text{if } x\in(2,\infty)\end{cases}$

TBP

2) $f(x)=\begin{cases}\cancel{2}x & x\in(-\infty,-2] \\ +4x & x\in \cancel{(-2,\infty)}\ (-2,2] \\ -2x & x\in(2,\infty)\end{cases}$

3) $f(x)=\begin{cases}-2x & x\in(-\infty,-2) \\ -4x & x\in(-2,2] \\ 2x & x\in(2,\infty)\end{cases}$

4) $f(x)=\begin{cases}-2x & x\in(-\infty,-2) \\ 2x & \text{if } x\in(-2,2] \\ 2x & x\in(2,\infty)\end{cases}$

in $(-\infty,-2]$

$f(x)=-(x-\cancel{2})-(x+\cancel{2})$
$=-2x$

in $(-2,2]$

$f(x)=(\cancel{x}+2)-(\cancel{x}-2)=4$

in $(2,\infty)$

$f(x)=(x-\cancel{2})+(x+\cancel{2})$
$=2x$.

5) Let R be the set of all real numbers. Consider the following sub sets of the plane $R \times R$ $S = \{(x,y) = x+1$ and $0<x<2)$ and $T = \{(x,y) : x-y$ is an integer$\}$ Then which of the following is True

TBP

1) T is an equivalance relation but S is not an equivalance relation
2) Neither S nor T is an equivalance relation
3) Both S and T are equivalance relation
4) S is equivalence but T is not an equivalence

S is not reflexive $\therefore$ it is not equivalence.

T is equivalence $\because$ $a-a=0 \in Z$. Reflexive

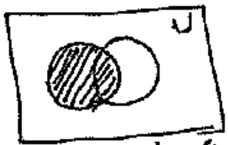
$a-b=+b-a \in Z$ symmetry

$a-b \in Z$, $b-c \in Z$, $a-c \in Z$ transitive

6) Let A and B be subsets of the universal set N, the set of all natural numbers Then $A' \cup ((A \cap B) \cup B')$ is

TBP

1) A 2) A' 3) B 4) N.



$A' \cup ((A \cap B) \cap B') = U$.

7) The number of students who take both the subjects Maths and chemistry is 70. This represents 10% the enrollment in Maths and 14% of the enrollment in chemistry The number of students take at least one of these two subjects is

TBP

1) 1120 2) 1130 3) 1000 4) insufficiant data

8) If $n((A \times B) \cap (A \cap C)) = 8$ and $n(B \cap C) = 2$ Then $n(A)$ is

TBP

1) 6 2) 4 3) 6 4) 5

we know $A \times (B \cap C) = (A \times B) \cap (A \times C)$

$n(A) \cdot n(B \cap C) = n[(A \times B) \cap (A \times C)]$

$n(A) \cdot 2 = 8$

$n(A) = 4$.

9) If $n(A) = 2$ $n(B \cup C) = 3$ then $n[(A \times B) \cup (A \times C)]$ is

TBP

1) 2^3 2) 3^2 3) 6 4) 5

$A \times (B \cup C) = (A \times B) \cup (A \times C)$

$n(A) \cdot n(B \cup C) = n[(A \times B) \cup (A \times C)]$

$2 \times 3 = n[(A \times B) \cup (A \times C)]$

$6 = n[(A \times B) \cup (A \times C)]$.

10) If two sets A and B have 17 elements in common, Then the number of elements common to $A \times B$ and $B \times A$ is

TBP

1) 2^{17} 2) 17^2 ✓ 3) 34 4) insufficient data.

If A and B any two non empty sets having n elements in common then $A \times B$ and $B \times A$ have n^2 elements in common

11) For non empty sets A and B, $A \subseteq B$ then $(A \times B) \cap (B \times A)$ is equal to

TBP

1) $A \cap B$ 2) $A \times A$ ✓ 3) $B \times B$ 4) non of these

Do it yourself by taking suitable sets.

12) The number of relations on a set Containing 3 elements is

TBP

1) 9 2) 81 3) 512 ✓ 3) 1024.

The number of elements in ~~set A~~ is n Then the number of relation on the set itself is 2^{n^2}. Here $2^{3^2} = 2^9 = 512$.

13) Let R be the universal relation on a set X with more than one element. Then R is

TBP

1) not reflexive 2) not symmetric 3) transitive ✓ 4) non of these

Let $A = \{1,2,3\}$ $R = \{(1,1)\ (1,2)\ (1,3)\ (2,1)\ (2,2)\ (2,3)\ (3,1)\ (3,2)\ (3,3)\}$ is universal relation.

It is symmetric, reflexive and transitive.

14) Let $X = \{1,2,3,4\}$ and $R = \{(1,1)\ (1,2)\ (1,3)\ (2,2)\ (3,3)\ (2,1)\ (3,1)\ (1,4)\ (4,1)\}$ Then

TBP

1) reflexive 2) symmetric ✓ 3) transitive 4) equivalence.

There is (4,4) it is not reflexive. $(2,1) \in R$ $(1,3) \in R$ but $(2,3) \notin R$

$\therefore$ not transitive $\therefore$ not equivalence Hence symmetric.

15) The range of the function $\frac{1}{1-2\sin x}$ is

TBP 1) $(-\infty,-1) \cup (1/3,\infty)$ 2) $(-1,1/3)$ 3) $[-1,1/3]$ 4) $(-\infty,-1] \cup [1/3,\infty)$ ✓

clearly $-1 \le \sin x \le 1$

$-2 \le 2\sin x \le 2$

$2 \ge -2\sin x \ge -2$

$3 \ge 1-2\sin x \ge -1$

$\Rightarrow \frac{1}{3} \le \frac{1}{1-2\sin x} \le -1$

$\therefore (-\infty,-1] \cup [1/3,\infty)$

16) The range of the function $f(x) = |\lfloor x \rfloor - x|$ $x \in R$ is

TBP 1) $[0,1]$ 2) $[0,\infty)$ 3) ✓ $[0,1)$ 4) $(0,1)$

$\lfloor 2 \rfloor = 2$ $\lfloor 2.3 \rfloor = 2$. $f(x) = |2-2| = 0$

$|2-2.3| = .3 < 1$

At the most we can put the value of $x = 2.9\ldots$

$\lfloor 2.9 \rfloor = 2$ $\therefore |2-2.9| = .9 < 1$

$\therefore$ The range is $[0,1)$

17) The rule $f(x) = x^2$ is a bijection if the domain and co-domain are given by

TBP 1) R R 2) R, $(0,\infty)$ 3) $(0,\infty)$, R 4) ✓ $[0,\infty)$, $[0,\infty)$

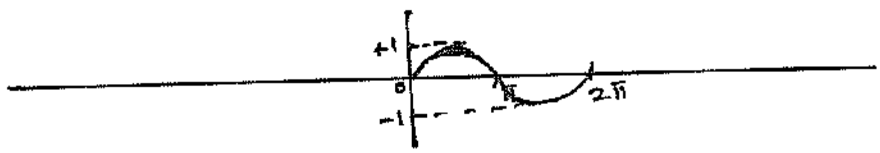
18) The number of constant functions from a set containing m elements to a set containing n elements.

TBP 1) mn 2) m 3) ✓ n 4) m+n.

Similarly we can find n constant functions

19) The function $f : [0, 2\pi] \to [-1, 1]$ defined by $f(x) = \sin x$.

TBP 1) one to one 2) ✓ onto 3) bijection 4) cannot be defined.



$\because$ at $x = 0$ $\sin 0 = 0$ For two values of x same value of y

$x = \pi$ $\sin \pi = 0$ $\therefore$ it not ~~onto~~ one-one.

But every element of y it has unique pre image of x

$\therefore$ onto

20) If the function $f : [-3,3] \to S$ defined by $f(x) = x^2$ is onto then S is ...

TBP 1) $[-9,9]$ 2) R 3) $[-3,3]$ 4) ✓ $[0,9]$.

$\because$ Domain is $[-3,3]$ $\therefore$ the range is $[0,9]$. this fn. is many to one.

21) Let $X = \{1,2,3,4\}$ $Y = \{a,b,c,d\}$ $f = \{(1,a)\ (4,b)\ (2,c)\ (3,d)\ (2,d)\}$ Then f is

TBP 1) one to one function 2) an onto function

3) not one-one 4) ✓ not a function

$\because$ $2 \in$ Domain has different images in the Range

$\therefore$ it is not at all a function.

22) The inverse of $f(x) = \begin{cases} x & x<1 \\ x^2 & 1\le x\le 4 \\ 8\sqrt{x} & x>4 \end{cases}$ is

TBP

1) $f^{-1}(x) = \begin{cases} x & x<1 \\ \sqrt{x} & 1\le x\le 16 \\ \frac{x^2}{64} & x>16 \end{cases}$

2) $f^{-1}(x) = \begin{cases} -x & x<1 \\ \sqrt{x} & 1\le x\le 16 \\ \frac{x^2}{64} & x>16 \end{cases}$

3) $f^{-1}(x) = \begin{cases} x^2 & x<1 \\ \sqrt{x^2} & 1\le x\le 16 \\ \frac{x^2}{64} & x>16 \end{cases}$

4) $f(x) = \begin{cases} 2x & x<1 \\ \sqrt{x} & 1\le x\le 16 \\ \frac{x^2}{8} & x>16 \end{cases}$

Let $y = f(x) = x = y$

$f^{-1}(y) = y \Rightarrow f^{-1}(x) = x.$

$y = f(x) = x^2 \Rightarrow x = \sqrt{y}$

$f^{-1}(y) = \sqrt{y} \Rightarrow f^{-1}(x) = \sqrt{x}.$

$y = f(x) = 8\sqrt{x} \Rightarrow \sqrt{x} = \frac{y}{8}$

$x = \frac{y^2}{64}$

$f^{-1}(y) = \frac{y^2}{64} \Rightarrow f^{-1}(x) = \frac{x^2}{64}.$

24) TBP) Let $f: R\to R$ is defined by $f(x) = \sin x + \cos x$. $f(-x) = -\sin x + \cos x$ $\ne f(x)$ $\ne -f(x)$

1) an odd function 2) neither an odd fn nor an even fn.

3) an even function 4) both odd and even fn.

23) TBP) Let $f: R\to R$ defined by $f(x) = 1-|x|$ then the range of f is

1) R 2) $(1,\infty)$ 3) $(-1,\infty)$ 4) $(-\infty,1]$

$f(x) = \begin{cases} 1-x & x\ge 0 & \forall x\in(0,\infty) \\ 1+x & x<0 & x\in(-\infty,0) \end{cases}$ $f(x) = (-\infty,1]$ $(-\infty,0]$

25) The function $f: R\to R$ is defined by $f(x) = \frac{(x^2+\cos x)(1+x^4)}{(x-\sin x)(2x-x^3)} + e^{-|x|}$

TBP 1) an odd function 2) neither odd nor even fn 3) even fn

4) both odd and even fn

$\frac{(E+E)(E+E)}{(O-O)(O-O)} + E = \frac{E}{E} + E = E+E = E.$