(Chapter 6)(Electromagnetic Induction)

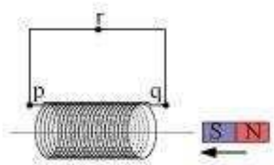
XII

Exercises

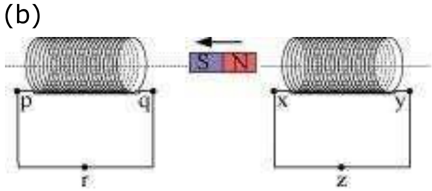
Question 6.1:

Predict the direction of induced current in the situations described by the following Figs. 6.18(a) to (f).

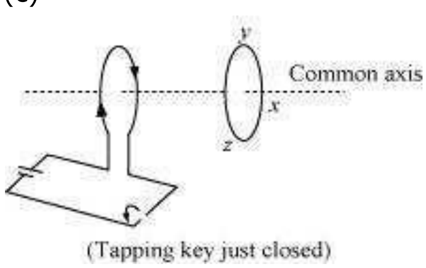
(a)



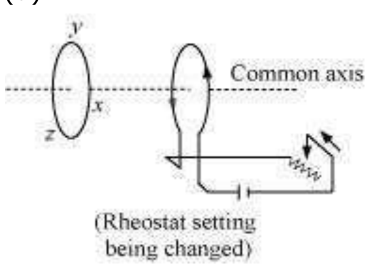
(b)



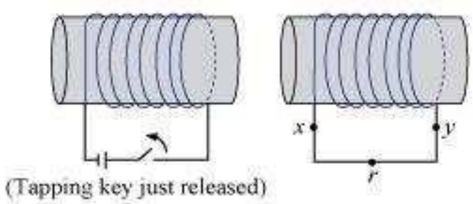
(c)



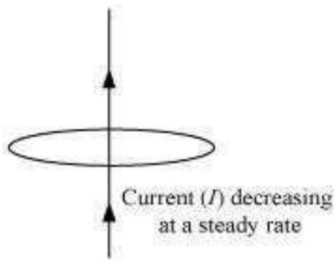
(d)



(e)

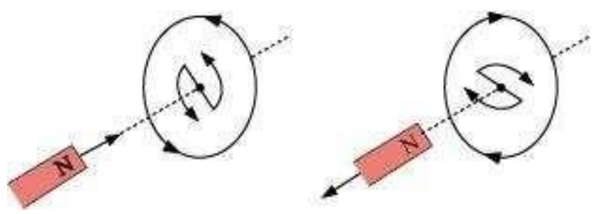


(f)



Answer

The direction of the induced current in a closed loop is given by Lenz's law. The given pairs of figures show the direction of the induced current when the North pole of a bar magnet is moved towards and away from a closed loop respectively.



Using Lenz's rule, the direction of the induced current in the given situations can be predicted as follows:

(a) The direction of the induced current is along qrpq.

(b) The direction of the induced current is along prqp.

(c) The direction of the induced current is along yzxy.

(d) The direction of the induced current is along zyxz.

(e) The direction of the induced current is along xryx.

(f) No current is induced since the field lines are lying in the plane of the closed loop.

Question 6.2:

A 1.0 m long metallic rod is rotated with an angular frequency of 400 rad s^{-1} about an axis normal to the rod passing through its one end. The other end of the rod is in contact with a circular metallic ring. A constant and uniform magnetic field of 0.5 T parallel to the axis exists everywhere. Calculate the emf developed between the centre and the ring.

Answer

Length of the rod, l = 1 m

Angular frequency, ω = 400 rad/s

Magnetic field strength, B = 0.5 T

One end of the rod has zero linear velocity, while the other end has a linear velocity of $l\omega$.

Average linear velocity of the rod, $v = \frac{l\omega + 0}{2} = \frac{l\omega}{2}$

Emf developed between the centre and the ring,

$$e = Blv = Bl\left(\frac{l\omega}{2}\right) = \frac{Bl^2\omega}{2}$$

$$= \frac{0.5 \times (1)^2 \times 400}{2} = 100 \text{ V}$$

Hence, the emf developed between the centre and the ring is 100 V.

Question 6.3:

A long solenoid with 15 turns per cm has a small loop of area 2.0 cm^2 placed inside the solenoid normal to its axis. If the current carried by the solenoid changes steadily from 2.0 A to 4.0 A in 0.1 s, what is the induced emf in the loop while the current is changing?

Answer

Number of turns on the solenoid = 15 turns/cm = 1500 turns/m

Number of turns per unit length, n = 1500 turns

The solenoid has a small loop of area, A = 2.0 cm^2 = 2×10^{-4} m^2

Current carried by the solenoid changes from 2 A to 4 A.

$\therefore$ Change in current in the solenoid, di = 4 − 2 = 2 A

Change in time, dt = 0.1 s

Induced emf in the solenoid is given by Faraday's law as:

$$e = \frac{d\phi}{dt} \qquad \ldots (i)$$

Where,

ϕ = Induced flux through the small loop

= BA ... (ii)

B = Magnetic field

$= \mu_0 n i \qquad \ldots (iii)$

μ_0 = Permeability of free space

= $4\pi \times 10^{-7}$ H/m

Hence, equation (i) reduces to:

$$e = \frac{d}{dt}(BA)$$

$$= A\mu_0 n \times \left(\frac{di}{dt}\right)$$

$$= 2\times10^{-4} \times 4\pi \times 10^{-7} \times 1500 \times \frac{2}{0.1}$$

$$= 7.54\times10^{-6} \text{ V}$$

Hence, the induced voltage in the loop is 7.54×10^{-6} V.

Question 6.4:

A rectangular wire loop of sides 8 cm and 2 cm with a small cut is moving out of a region of uniform magnetic field of magnitude 0.3 T directed normal to the loop. What is the emf developed across the cut if the velocity of the loop is 1 cm s^{-1} in a direction normal to the (a) longer side, (b) shorter side of the loop? For how long does the induced voltage last in each case?

Answer

Length of the rectangular wire, l = 8 cm = 0.08 m

Width of the rectangular wire, b = 2 cm = 0.02 m

Hence, area of the rectangular loop,

A = lb

= 0.08 × 0.02

= 16×10^{-4} m^2

Magnetic field strength, B = 0.3 T Velocity of the loop, v = 1 cm/s = 0.01 m/s (a)

Emf developed in the loop is given as:

e = Blv

= 0.3 × 0.08 × 0.01 = 2.4×10^{-4} V

$$\text{Time taken to travel along the width, } t = \frac{\text{Distance travelled}}{\text{Velocity}} = \frac{b}{v}$$

$$= \frac{0.02}{0.01} = 2 \text{ s}$$

Hence, the induced voltage is 2.4×10^{-4} V which lasts for 2 s.

(b) Emf developed, e = Bbv

= 0.3 × 0.02 × 0.01 = 0.6×10^{-4} V

$$\text{Time taken to travel along the length, } t = \frac{\text{Distance traveled}}{\text{Velocity}} = \frac{l}{v}$$

$$= \frac{0.08}{0.01} = 8 \text{ s}$$

Hence, the induced voltage is 0.6×10^{-4} V which lasts for 8 s.

Question 6.6:

A circular coil of radius 8.0 cm and 20 turns is rotated about its vertical diameter with an angular speed of 50 rad s^{-1} in a uniform horizontal magnetic field of magnitude 3.0×10^{-2} T. Obtain the maximum and average emf induced in the coil. If the coil forms a closed loop of resistance 10Ω, calculate the maximum value of current in the coil. Calculate the average power loss due to Joule heating. Where does this power come from?

Answer

Max induced emf = 0.603 V

Average induced emf = 0 V

Max current in the coil = 0.0603 A

Average power loss = 0.018 W

(Power comes from the external rotor)

Radius of the circular coil, r = 8 cm = 0.08 m

Area of the coil, $A = \pi r^2 = \pi \times (0.08)^2\ m^2$

Number of turns on the coil, N = 20

Angular speed, ω = 50 rad/s

Magnetic field strength, $B = 3 \times 10^{-2}$ T

Resistance of the loop, R = 10 Ω

Maximum induced emf is given as:

e = Nω AB

$= 20 \times 50 \times \pi \times (0.08)^2 \times 3 \times 10^{-2}$

= 0.603 V

The maximum emf induced in the coil is 0.603 V.

Over a full cycle, the average emf induced in the coil is zero.

Maximum current is given as:

$$I = \frac{e}{R}$$

$$= \frac{0.603}{10} = 0.0603 \text{ A}$$

Average power loss due to joule heating:

$$P = \frac{eI}{2}$$

$$= \frac{0.603 \times 0.0603}{2} = 0.018 \text{ W}$$

The current induced in the coil produces a torque opposing the rotation of the coil. The rotor is an external agent. It must supply a torque to counter this torque in order to keep the coil rotating uniformly. Hence, dissipated power comes from the external rotor.

Question 6.7:

A horizontal straight wire 10 m long extending from east to west is falling with a speed of 5.0 m s^{-1}, at right angles to the horizontal component of the earth's magnetic field, 0.30×10^{-4} Wb m^{-2}.

(a) What is the instantaneous value of the emf induced in the wire?

(b) What is the direction of the emf?

(c) Which end of the wire is at the higher electrical potential?

Answer

Length of the wire, l = 10 m

Falling speed of the wire, v = 5.0 m/s

Magnetic field strength, B = 0.3×10^{-4} Wb m^{-2}

(a) Emf induced in the wire, e = Blv

$$= 0.3 \times 10^{-4} \times 5 \times 10$$
$$= 1.5 \times 10^{-3} \text{ V}$$

(b) Using Fleming's right hand rule, it can be inferred that the direction of the induced emf is from West to East.

(c) The eastern end of the wire is at a higher potential.

Question 6.8:

Current in a circuit falls from 5.0 A to 0.0 A in 0.1 s. If an average emf of 200 V induced, give an estimate of the self-inductance of the circuit.

Answer

Initial current, I_1 = 5.0 A

Final current, I_2 = 0.0 A

Change in current, $dI = I_1 - I_2 = 5 \text{ A}$

Time taken for the change, t = 0.1 s

Average emf, e = 200 V

For self-inductance (L) of the coil, we have the relation for average emf as:

$e = L\frac{di}{dt}$

$$L = \frac{e}{\left(\frac{di}{dt}\right)}$$

$$= \frac{200}{\frac{5}{0.1}} = 4 \text{ H}$$

Hence, the self induction of the coil is 4 H.

Question 6.9:

A pair of adjacent coils has a mutual inductance of 1.5 H. If the current in one coil changes from 0 to 20 A in 0.5 s, what is the change of flux linkage with the other coil?

Answer

Mutual inductance of a pair of coils, μ = 1.5 H

Initial current, I_1 = 0 A

Final current I_2 = 20 A

Change in current, $dI = I_2 - I_1 = 20 - 0 = 20 \text{ A}$

Time taken for the change, t = 0.5 s

Induced emf, $e = \frac{d\phi}{dt}$ ... (1)

Where $d\phi$ is the change in the flux linkage with the coil.

Emf is related with mutual inductance as:

$$e = \mu\frac{dI}{dt} \quad \text{... (2)}$$

Equating equations (1) and (2), we get

$$\frac{d\phi}{dt} = \mu\frac{dI}{dt}$$

$$d\phi = 1.5 \times (20)$$

$$= 30 \text{ Wb}$$

Hence, the change in the flux linkage is 30 Wb.

Question 6.10:

A jet plane is travelling towards west at a speed of 1800 km/h. What is the voltage difference developed between the ends of the wing having a span of 25 m, if the Earth's magnetic field at the location has a magnitude of 5×10^{-4} T and the dip angle is 30°.

Answer

Speed of the jet plane, v = 1800 km/h = 500 m/s

Wing spanof jet plane, l = 25 m

Earth's magnetic field strength, B = 5.0×10^{-4} T

Angle of dip, $\delta = 30°$

Vertical component of Earth's magnetic field,

$B_V = B \sin \delta$

$= 5 \times 10^{-4} \sin 30°$

$= 2.5 \times 10^{-4}$ T

Voltage difference between the ends of the wing can be calculated as:

$e = (B_V) \times l \times v$

$= 2.5 \times 10^{-4} \times 25 \times 500$

= 3.125 V

Hence, the voltage difference developed between the ends of the wings is 3.125 V.