Chapter 6: Superposition of Waves

EXERCISES [PAGES 156 - 157]

Exercises | Q 1.1 | Page 156

Choose the correct option:
When an air column in a pipe closed at one end vibrates such that three nodes are formed in it, the frequency of its vibrations is _____ times the fundamental frequency.

1. 2
2. 3
3. 4
4. **5**

SOLUTION

When an air column in a pipe closed at one end vibrates such that three nodes are formed in it, the frequency of its vibrations is **5** times the fundamental frequency.

Exercises | Q 1.2 | Page 156

Choose the correct option.
If two open organ pipes of length 50 cm and 51 cm sounded together produce 7 beats per second, the speed of sound is.

1. 307 m/s
2. 327 m/s
3. 350 m/s
4. **357 m/s**

SOLUTION

357 m/s

Exercises | Q 1.3 | Page 156

Choose the correct option.
The tension in a piano wire is increased by 25%. Its frequency becomes ______ times the original frequency.

1. 0.8
2. **1.12**
3. 1.25
4. 1.56

SOLUTION

The tension in a piano wire is increased by 25%. Its frequency becomes **1.12** times the original frequency.

Exercises | Q 1.4 | Page 156

Choose the correct option:

Which of the following equations represents a wave travelling along Y-axis?

1. **x = A sin (ky - ωt)**
2. y = A sin (kx - ωt)
3. y = A sin (ky) cos (ωt)
4. y = A cos (ky) sin (ωt)

SOLUTION

x = A sin (ky - ωt)

Here x is the particle displacement of the wave and the wave is travelling along the Y-axis because the particle displacement is perpendicular to the direction of wave motion.

Exercises | Q 1.5 | Page 156

Choose the correct option:
A standing wave is produced on a string clamped at one end and free at the other. The length of the string

must be an odd integral multiple of $\frac{\lambda}{4}$

must be an odd integral multiple of $\frac{\lambda}{2}$

must be an odd integral multiple of λ

must be an even integral multiple of λ

SOLUTION

must be an odd integral multiple of $\frac{\lambda}{4}$

A standing wave is produced on a string clamped at one end and free at the other. Its fundamental frequency is given by

$$\nu = \left(n + \frac{1}{2}\right)\frac{v}{2L}$$

$$\Rightarrow v = \nu\lambda$$

$$\Rightarrow \nu = \left(n + \frac{1}{2}\right)\frac{\nu\lambda}{2L}$$

$$\Rightarrow L = \left(\frac{2n+1}{4}\right)\lambda$$

$$\Rightarrow L = \frac{\lambda}{4}, \frac{3\lambda}{4}, \ldots$$

Exercises | Q 2.1 | Page 156

Answer in brief:

A wave is represented by an equation y = A sin (Bx + Ct). Given that the constants A, B and C are positive, can you tell in which direction the wave is moving?

SOLUTION

Equation of the wave is y = A sin (Bx + Ct).

When the variable of the equation is (Bx + Ct)

then the wave must be moving in the negative x direction.

Exercises | Q 2.2 | Page 156

Answer in brief:

A string is fixed at the two ends and is vibrating in its fundamental mode. It is known that the two ends will be at rest. Apart from these, is there any position on the string which can be touched so as not to disturb the motion of the string? What will be the answer to this question if the string is vibrating in its first and second overtones?

SOLUTION

No. We know that in case of the fundamental mode, all points except the ends are vibrating with different amplitudes.

Now, touching any point other than the ends will disturb the motion.

When the string will vibrate in the first overtone there is a node also at the midpoint which is stationary.

This midpoint can be touched without disturbing the motion.

Thus, we can say that there is no point in the string that can satisfy the given condition.

Exercises | Q 2.3 | Page 156

Answer in brief:

What are harmonics and overtones?

SOLUTION

Harmonic frequencies are whole number multiples of the fundamental frequency or the lowest frequency of vibration.

Consider a vibrating string. The modes of vibration are all multiples of the fundamental and are related to the string length and wave velocity. Higher frequencies are found via the relationship $f_n = nf_1$, wavelength = $2\frac{L}{n}$ where L is the string length.

An overtone is a name given to any resonant frequency above the fundamental frequency or fundamental tone.

The list of successive overtones for an object is called the overtone series. The first overtone as well as all subsequent overtones in the series may or may not be an integer multiple of the fundamental. Sometimes the relationship is that simple, and other times it is more complex, depending on the properties and geometry of the vibrating object.

Exercises | Q 2.4 | Page 156

Answer in brief.

For a stationary wave set up in a string having both ends fixed, what is the ratio of the fundamental frequency to the second harmonic?

SOLUTION

The fundamental is the first harmonic. Therefore, the ratio of the fundamental frequency (n) to the second harmonic (n_1) is 1 : 2.

Exercises | Q 2.5 | Page 156

Answer in brief.

The amplitude of a wave is represented by y = 0.2 sin 4π $\left[\frac{t}{0.08} - \frac{x}{0.8}\right]$ in SI units. Find (a) wavelength, (b) frequency, and (c) amplitude of the wave.

SOLUTION

Data: $y = 0.2 \sin 4\pi \left[\frac{t}{0.08} - \frac{x}{0.8}\right]$

$$y = 0.2 \sin 2\pi \left[\frac{2t}{0.08} - \frac{2x}{0.8}\right]$$

$$y = 0.2 \sin 2\pi \left[\frac{t}{0.04} - \frac{x}{0.4}\right]$$

Let us compare above equation with the equation of a simple harmonic progressive wave:

$$y = A \sin 2\pi \left[\frac{t}{T} - \frac{x}{\lambda}\right] = 0.2 \sin 2\pi \left[\frac{t}{0.04} - \frac{x}{0.4}\right]$$

Comparing the quantities on both sides, we get,

$A = 0.2$ m, $T = 0.04$ s, $\lambda = 0.4$ m

$\therefore$ (a) Wavelength (λ) = 0.4 m

(b) Frequency (n) = $\frac{1}{T} = \frac{1}{0.04}$ = 25 Hz

(c) Amplitude (A)= 0.2 m

Exercises | Q 3 | Page 156

Answer in brief:
State the characteristics of progressive waves.

SOLUTION

Progressive wave

A progressive wave is defined as the onward transmission of the vibratory motion of a body in an elastic medium from one particle to the successive particle.

Characteristics of a progressive wave

- Energy is transmitted from particle to particle without the physical transfer of matter.
- The particles of the medium vibrate periodically about their equilibrium positions.
- In the absence of dissipative forces, every particle vibrates with the same amplitude and frequency but differs in phase from its adjacent particles. Every particle lags behind in its state of motion compared to the one before it.
- Wave motion is doubly periodic, i.e., it is periodic in time and periodic in space.
- The velocity of propagation through a medium depends upon the properties of the medium.
- A transverse wave can propagate only through solids, but not through liquids and gases while a longitudinal wave can propagate through any material medium.
- **Progressive waves are of two types:** transverse and longitudinal. In a transverse mechanical wave, the individual particles of the medium vibrate perpendicular to the direction of propagation of the wave. The progressively changing phase of the successive particles results in the formation of alternate crests and troughs that are periodic in space and time.

  In an em wave, the electric and magnetic fields oscillate in mutually perpendicular directions, perpendicular to the direction of propagation. In a longitudinal mechanical wave, the individual particles of the medium vibrate along the line of propagation of the wave. The progressively changing phase of the successive particles results in the formation of typical alternate regions of compressions and rarefactions that are periodic in space and time. Periodic compressions and rarefactions result in periodic pressure and density variations in the medium. There is no longitudinal em wave.

Exercises | Q 4 | Page 156

Answer in brief:
State the characteristics of stationary waves.

SOLUTION

Stationary wave:

When two, identical, progressive waves of equal amplitudes and equal wavelengths and traveling in a similar medium, along the similar straight line, but in opposite directions, interfere, and then the wave formed is called a standing wave or a stationary wave.

Characteristics of stationary waves:

1. Stationary waves are produced by the interference of two identical progressive waves travelling in opposite directions, under certain conditions.

2. The overall appearance of a standing wave is of alternate intensity maximum (displacement antinode) and minimum (displacement node).

3. The distance between adjacent nodes (or antinodes) is $\lambda/2$.

4. The distance between successive node and antinode is $\lambda/4$.

5. There is no progressive change of phase from particle to particle. All the particles in one loop, between two adjacent nodes, vibrate in the same phase, while the particles in adjacent loops are in opposite phase.

6. A stationary wave does not propagate in any direction and hence does not transport energy through the medium.

Exercises | Q 5 | Page 156

Derive an expression for the equation of stationary wave on a stretched string.

SOLUTION

When two progressive waves having the same amplitude, wavelength and speed propagate in opposite directions through the same region of a medium, their superposition under certain conditions creates a stationary interference pattern called a stationary wave.

Consider two simple harmonic progressive waves, of the same amplitude A, wavelength λ, and frequency $n = \omega/2\pi$, traveling on a string stretched along the x-axis in opposite directions. They may be represented by

$y_1 = A \sin(\omega t - kx)$ (along the + x-axis) and ....(1)

$y_2 = A \sin(\omega t + kx)$ (along the − x-axis) ....(2)

where $k = 2\pi/\lambda$ is the propagation constant.

By the superposition principle, the resultant displacement of the particle of the medium at the point at which the two waves arrive simultaneously is the algebraic sum

$Y = Y_1 + Y_2 = A[\sin(\omega t - kx) + \sin(\omega t + kx)]$

Using the trigonometrical identity,

$$\sin C + \sin D = 2 \sin\left(\frac{C + D}{2}\right) \cos\left(\frac{C - D}{2}\right),$$

$y = 2A \sin \omega t \cos(-kx)$

$= 2A \sin \omega t \cos kx$ ....[$\therefore \cos(-kx) = \cos(kx)$]

$= 2A \cos kx \sin \omega t$, ...(3)

$\therefore y = R \sin \omega t$, ...(4)

where $R = 2A \cos kx$ ....(5)

Equation (4) is the equation of a stationary wave.

Exercises | Q 6 | Page 156

Find the amplitude of the resultant wave produced due to interference of two waves given as $y_1 = A_1 \sin \omega t$, $y_2 = A_2 \sin(\omega t + \varphi)$

SOLUTION

The amplitude of the resultant wave produced due to the interference of the two waves is

$$A = \sqrt{A_1^2 + 2A_1A_2 \cos \varphi + A_2^2}$$

Exercises | Q 7 | Page 156

State the laws of vibrating strings

SOLUTION

The fundamental frequency of vibration of a stretched string or wire of uniform cross-section is

$$n = \frac{1}{2L}\sqrt{\frac{T}{m}}$$

where L is the vibrating length, m the mass per unit length (linear density) of the string, and T the tension in the string. From the above expression, we can state the following three laws of vibrating strings.

1. **Law of length:** The fundamental frequency of vibrations of a stretched string is inversely proportional to its vibrating length if the tension and mass per unit length are kept constant. If T and m are constant,
   $n \propto \frac{1}{L}$ or nL = constant.
2. **Law of tension:** The fundamental frequency of vibrations of a stretched string is directly proportional to the square root of the applied tension if the length and mass per unit length are kept constant. If L and m are constant,
   $n \propto \sqrt{T}$ or n^2/T = constant.
3. **Law of mass or (the law of linear density):** The fundamental frequency of vibrations of a stretched string is inversely proportional to the square root of its mass per unit length if the length and tension are kept constant. If L and T are constant,
   $\propto \frac{1}{\sqrt{m}}$ or n^2m = constant

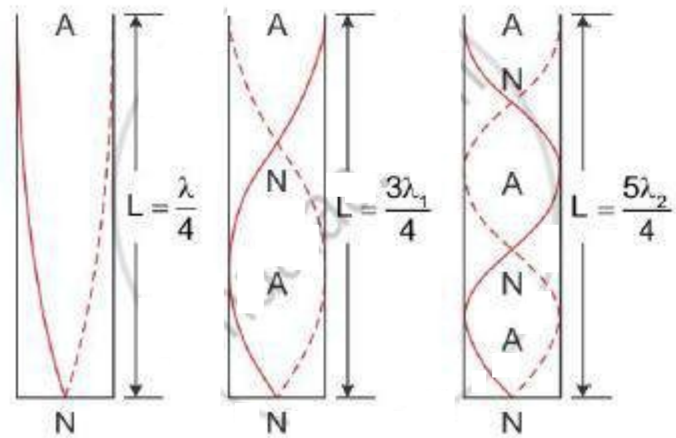
Exercises | Q 8 | Page 156

Show that only odd harmonics are present in an air column vibrating in a pipe closed at one end.

SOLUTION

Consider the different modes of vibration of an air column within a pipe closed at one end. Let L be the length of the pipe.

Stationary waves are formed within the air column when the time taken by the sound waves to produce a compression and rarefaction becomes equal to the time taken by the wave to travel twice the length of the tube. The standing waves are formed only for certain discrete frequencies.



In the first mode of vibration of the air column, there is one node and one antinode as shown in the figure above.

If λ is the length of the wave in the fundamental mode of vibration,

Then, Length of the air column, $L=\lambda/4$

$\Rightarrow \lambda = 4L$ .... (1)

As velocity of the wave, $v = n\lambda$

$n = v/\lambda$

substituting (1) we get:

$n = v/4L$; Frequency of the fundamental mode

In the **second mode of vibration** of the air column, two nodes and two antinodes are formed.

In this case:

The length of the air column, $L=3\lambda_1/4$

where λ_1 is the wavelength of the wave in the second mode of vibration,

$\Rightarrow\lambda = 4L/3$ ....(1)

$v = n_1\lambda_1$

$n_1 = v/\lambda_1$

substituting (1), we get

$$n_1 = \frac{v}{\frac{4L}{3}} = \frac{3v}{4L} = 3n$$

This frequency is called the third harmonics or first overtone

Similarly, during the third mode of vibration of the air column, three nodes and three antinodes are formed.

Here,

The length of the air column, L = $5\lambda_2/4$

where λ_2 is the wavelength of the wave in third mode of vibration,

⇒ $\lambda_2 = 4L/5$ ....(1)

$v = n_2\lambda_2$

$n_2 = v/\lambda_2$

substituting (1), we get

$$n_2 = \frac{\mathrm{v}}{\frac{4L}{5}} = \frac{5\mathrm{v}}{4L} = 5\mathrm{n}$$

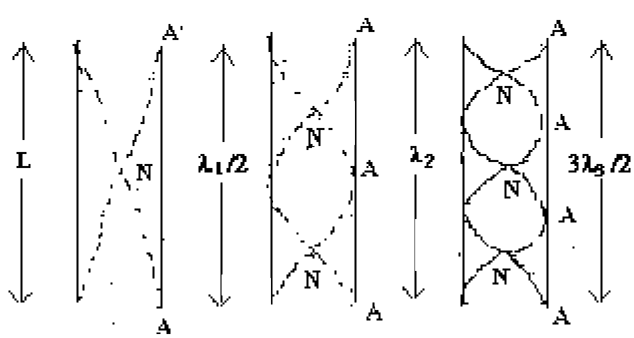
This frequency is called the fifth harmonic or second overtone.

Thus, we see that the frequencies of the modes of vibrations are in the ratio $n:n_1:n_2$ = 1:3:5. This shows that only odd harmonics are present in the modes of vibrations of the air column closed at one end.

Exercises | Q 9 | Page 156
With a neat labelled diagram, show that all harmonics are present in an air column contained in a pipe open at both the ends. Define end correction.

SOLUTION

Stationary waves are produced due to the superposition of two identical simple harmonic progressive waves traveling through the same part of the medium in opposite direction. In the case of pipe open at both ends, the air molecules near the open end are free to vibrate, hence they vibrate with maximum amplitude. Therefore the open end becomes an antinode.



The simplest mode of vibration is the fundamental mode of vibration. (in above fig a) In this case one node & two antinodes are formed. If λ_1 be the corresponding wavelength &

$$\text{L be length of the pipe L} = \frac{\lambda_1}{2}$$

$$\lambda_1 = 2L$$

If n_1 be corresponding frequency and v be velocity of sound in air $v = n_1\lambda_1$

$$n_1 = \frac{v}{\lambda_1}$$

$$n_1 = \frac{v}{2L}......(1)$$ This is called 1st harmonic.

The next possible mode of vibration is called the 1st overtone. (in above fig) In this case, two nodes and three antinodes are formed. If λ_2 and n_2 be the corresponding wavelength and frequency,

$L = \lambda_2$

$\lambda_2 = L$

But

v $n_2\lambda_2$ i.e = $\lambda_2 = \frac{v}{\lambda_2}$ $n_2 = \frac{v}{L}$

This is called 2nd harmonic.

The next possible mode of vibration is called 2nd overtone. (in above fig c) In this case three nodes & four antinodes are formed. If λ_3 and n_3 be the corresponding wavelength & frequency,

$$L = 3\frac{\lambda_3}{2}$$

$$\lambda_3 = \frac{2L}{3}$$

$$v = n_3\lambda_3$$

i.e

$$n_3 = \frac{v}{\lambda_3}$$

$n_3 = 3\frac{v}{2}L$.

This is called 3rd harmonic.

$n_1 : n_2 : n_3 :: 1 : 2 : 3$

Thus in case of pipe open at both end, all harmonics are present.
In this case, the node is forming at the closed-end i.e. at the water surface & antinode is forming at the open end. But antinode is not formed exactly at the open end but slightly outside it since air molecules are free to vibrate. The correction has to be applied is called end correction.

Exercises | Q 10 | Page 156

A wave of frequency 500 Hz is traveling with a speed of 350 m/s. (a) What is the phase difference between two displacements at a certain point at times 1.0 ms apart? (b) what will be the smallest distance between two points which are 45° out of phase at an instant of time?

SOLUTION

Data: n = 500 Hz, v = 350 m/s

$v = n \times \lambda$

$\therefore \lambda = \frac{350}{500} = 0.7$ m

(a) t = 1.0 ms = 0.001 s, the path difference is the distance covered v x t = 350 x 0.001 = 0.35 m

$\therefore$ Phase difference = $\frac{2\pi}{\lambda} \times \text{Path difference}$

$= \frac{2\pi}{0.7} \times 0.35 = \pi$ rad

(b) Phase difference = 45° = $\frac{\pi}{4}$ rad

$\therefore$ Path difference = $\frac{\lambda}{2\pi} \times \text{Phase difference}$

$= \frac{0.7}{2\pi} \times \frac{\pi}{4} = 0.0875$ m

Exercises | Q 11 | Page 157

A sound wave in a certain fluid medium is reflected at an obstacle to form a standing wave. The distance between two successive nodes is 3.75 cm. If the velocity of sound is 1500 m/s, find the frequency.

SOLUTION

Data: Distance between two successive nodes

$$= \frac{\lambda}{2} = 3.75 \times 10^{-2} \text{ m}, v = 1500 \text{ m/s}$$

$$\therefore \lambda = 7.5 \times 10^{-2} \text{ m}$$

$$v = n \times \lambda$$

$$\therefore n = \frac{1500}{7.5 \times 10^{-2}} = 20 \text{ kHz}$$

Exercises | Q 12 | Page 157

Two sources of sound are separated by a distance of 4 m. They both emit sound with the same amplitude and frequency (330 Hz), but they are 180° out of phase. At what points between the two sources, will the sound intensity be maximum?

SOLUTION

$$\therefore \lambda = \frac{v}{n} = \frac{330}{330} = 1 \text{ m}$$

Directly at the center of two sources of sound, path difference is zero. But since the waves are 180° out of phase, two maxima on either side should be at a distance of $\lambda/4$ from the point at the center. Other maxima will be located each $\lambda/2$ further along.

Thus, the sound intensity will be maximum at ± 0.25, ± 0.75, ± 1.25, ± 1.75 m from the point at the center.

Exercises | Q 13 | Page 157

Two sound waves travel at a speed of 330 m/s. If their frequencies are also identical and are equal to 540 Hz, what will be the phase difference between the waves at points 3.5 m from one source and 3 m from the other if the sources are in phase?

SOLUTION

Data: $v = 330$ m/s, $n_1 = n_2 = 540$ Hz

$v = n \times \lambda$

$\therefore \lambda = \frac{330}{540} = 0.61$ m

Here, the path difference = 3.5 − 3 m = 0.5 m

Phase difference = $\frac{2\pi}{\lambda} \times \text{Path difference}$

$= \frac{2\pi}{0.61} \times 0.5 = 1.64\pi$ rad

Exercises | Q 14 | Page 157

Two wires of the same material and the same cross-section are stretched on a sonometer. One wire is loaded with 1.5 kg and another is loaded with 6 kg. The vibrating length of the first wire is 60 cm and its fundamental frequency of vibration is the same as that of the second wire. Calculate the vibrating length of the other wire.

SOLUTION

Data: $m_1 = m_2 = m$, $L_1 = 60$ cm $= 0.6$ m,

$T_1 = 1.5$ kg $= 14.7$ N, $T_2 = 6$ kg $= 58.8$ N

$n_1 = \frac{1}{2L_1}\sqrt{\frac{T_1}{m}}$ and $n_2 = \frac{1}{2L_2}\sqrt{\frac{T_2}{m}}$

But, $n_1 = n_2$

$$\therefore \frac{1}{2L_1}\sqrt{\frac{T_1}{m}} = \frac{1}{2L_2}\sqrt{\frac{T_2}{m}}$$

$$L_2 = \sqrt{\frac{T_2}{T_1}} \times L_1$$

$$= \sqrt{\frac{58.8}{14.7}} \times 0.6 = \sqrt{4} \times 0.6 = 1.2 \text{ m}$$

The vibrating length of the second wire is 1.2 m.

Exercises | Q 15 | Page 157

A pipe closed at one end can produce overtones at frequencies 640 Hz, 896 Hz, and 1152 Hz. Calculate the fundamental frequency.

SOLUTION

The difference between the given frequencies of the overtones is 256 Hz. This implies that they are consecutive overtones. Let n_c be the fundamental frequency of the closed pipe and n_q, n_{q-1} = the frequencies of the q^{th}, $(q + 1)^{th}$ and $(q + 2)^{th}$ consecutive overtones, where q is an integer.

Data: n_q = 640 Hz, n_{q-1} = 896 Hz, n_{q+2} = 1152 Hz Since only odd harmonics are present as overtones,

$nq = (2q + 1)\ n_c$

and $n_{q+t} = [2(q + 1) + 1]\ n_c = (2q + 3)\ n_c$

$$\therefore \left(\frac{n_{q+1}}{n_q}\right) = \frac{2q+3}{2q+1} = \frac{896}{640} = \frac{7}{5}$$

$$\therefore 14q + 7 = 10q + 15$$

$$\therefore 4q = 8$$

$$\therefore q = 2$$

Therefore, the three given frequencies correspond to the second, third and fourth overtones, i.e., the fifth, seventh and ninth harmonics, respectively.

$\therefore 5n_c = 640 \therefore b_c = 128$ Hz

Exercises | Q 16 | Page 157

A standing wave is produced in a tube open at both ends. The fundamental frequency is 300 Hz. What is the length of the tube? (speed of the sound = 340 m s^{-1}).

SOLUTION

Data: For the tube open at both the ends,

n = 300 Hz and v = 340 m s^{-1} Ignoring end correction, the fundamental frequency of the tube is

$$n = \frac{v}{2L}$$

$$\therefore L = \frac{v}{2n} = \frac{340}{2 \times 300} = 0.566 \text{ m}$$

The length of the tube open at both the ends is 0.5667 m.

Exercises | Q 17 | Page 157

Find the fundamental, first overtone, and second overtone frequencies of a pipe, open at both the ends, of length 25 cm if the speed of sound in air is 330 m/s.

SOLUTION

Data: Open pipe, L = 25 cm = 0.25 m, v = 330 m/s The fundamental frequency of an open pipe ignoring end correction,

$$n_O = \frac{v}{\lambda} = \frac{v}{2L}$$

$$\therefore n_O = \frac{330}{2 \times 0.25} = 660 \text{ Hz}$$

Since all harmonics are present as overtones, the first overtone is,

$$n_1 = 2n_O = 2 \times 660 = 1320 \text{ Hz}$$

The second overtone is

$$n_2 = 3n = 3 \times 660 = 1980 \text{ Hz}$$

Exercises | Q 18 | Page 157

A pipe open at both the ends has a fundamental frequency of 600 Hz. The first overtone of a pipe closed at one end has the same frequency as the first overtone of the open pipe. How long are the two pipes?

SOLUTION

Data: Open pipe, n_O = 600 Hz, $n_{C,1} = n_{O,1}$ (first overtones)

For an open pipe, the fundamental frequency,

$nO = \frac{v}{2L_O}$

∴ The length of the open pipe is

$LO = \frac{v}{2n_O} = \frac{330}{2 \times 600} = 0.275$ m

For the open pipe, the frequency of the first overtone is

$2n_O = 2 \times 600 = 1200$ Hz

For the pipe closed at one end, the frequency of the first overtone is $\frac{3v}{L_O}$

By the data, $\frac{3v}{4L} = 1200$

$\therefore L_C = \frac{3 \times 330}{4 \times 1200} = 0.206$ m

The length of the pipe open at both ends is 27.5 cm and the length of the pipe closed at one end is 20.6 cm.

Exercises | Q 19 | Page 157

A string 1m long is fixed at one end. The other end is moved up and down with frequency of 15 Hz. Due to this, a stationary wave with four complete loops gets produced on the string. Find the speed of the progressive wave which produces the stationary wave.
[Hint: Remember that the moving end is an antinode.]

SOLUTION

Data: L = 1 m, n = 15 Hz.

The string is fixed only at one end. Hence, an antinode will be formed at the free end. Thus, with four and a half loops on the string, the length of the string is

$L = \frac{\lambda}{4} + 4\left(\frac{\lambda}{2}\right) = \frac{9}{4}\lambda$

$\therefore \lambda = \frac{4L}{9} = \frac{4}{9} \times 1 = \frac{4}{9}$ m

$v = n\lambda$

$\therefore$ Speed of the progressive wave

$$v = 15 \times \frac{4}{9} = \frac{60}{9} = 6.667 \text{ m/s}$$

Exercises | Q 20 | Page 157

A violin string vibrates with fundamental frequency of 440Hz. What are the frequencies of the first and second overtones?

SOLUTION

Data: n = 440 Hz

The first overtone, $n_1 = 2n = 2 \times 400 = 880$ Hz

The second overtone, $n_2 = 3n = 3 \times 400 = 1320$ Hz

Exercises | Q 21 | Page 157

A set of 8 tuning forks is arranged in a series of increasing order of frequencies. Each fork gives 4 beats per second with the next one and the frequency of last fork is twice that of the first. Calculate the frequencies of the first and the last fork.

SOLUTION

Data: $n_8 = 2n_1$, beat frequency = 4Hz

The set of a tuning fork is arranged in the increasing order of their frequencies.

$\therefore n_2 = n_1 + 4$

$n_3 = n_2 + 4 = n_1 + 2 \times 4$

$n_4 = n_3 + 4 = n_1 + 3 \times 4$

$\therefore n_8 = n_7 + 4 = n_1 + 7 \times 4 = n_1 + 28$

Since $n_8 = 2n_1$,

$2n_1 = n_1 + 28$

$\therefore$ The frequency of the first fork, $n_1 = 28$ Hz

$\therefore$ The frequency of the last fork,

$n_8 = n_1 + 28 = 28 + 28 = 56$ Hz

Exercises | Q 22 | Page 157

A sonometer wire is stretched by the tension of 40 N. It vibrates in unison with a tuning fork of frequency 384 Hz. How many numbers of beats get produced in two seconds if the tension in the wire is decreased by 1.24 N?

SOLUTION

Data: T_1 =40 N, n_1 = 384 Hz,

T_2 = 40 − 1.24 = 38.76 N

n_1 = $\frac{1}{\sqrt{2l}} = \sqrt{\frac{T_1}{m}}$ and n_2 = $\frac{1}{2l}$ = $\sqrt{\frac{T_2}{m}}$

$\therefore \frac{n_2}{n_1} = \sqrt{\frac{T_2}{T_1}}$

$\therefore \frac{n_2}{384} = \sqrt{\frac{38.76}{40}} = \sqrt{0.969}$ = 0.9844

∴ n_2 = 384 × 0.9844 = 378.0 Hz

∴ $n_1 - n_2$ = 384 − 378 = 6 Hz

∴ The number of beats produced in two seconds

= 2 × 6 = 12

Exercises | Q 23 | Page 157

A sonometer wire of length 0.5 m is stretched by a weight of 5 kg. The fundamental frequency of vibration is 100 Hz. Calculate the linear density of wire.

SOLUTION

Data: L = 0.5 m, T = 5 kg = 5 x 9.8 = 49 N,

n = 100 Hz

$n = \frac{1}{2L}\sqrt{\frac{T}{m}}$

∴ Linear density, $m = \frac{T}{4L^2n^2}$

$= \frac{49}{4(0.5)^2(100)^2}$

$= 4.9 \times 10^{-3}$ kg/m

Exercises | Q 24 | Page 157

The string of a guitar is 80 cm long and has a fundamental frequency of 112 Hz. If a guitarist wishes to produce a frequency of 160 Hz, where should the person press the string?

SOLUTION

Data: L_1 = 80 cm, n_1 = 112 Hz, n_2 = 160 Hz

According to the law of length, $n_1L_1 = n_2L_2$.

∴ The vibrating length to produce the fundamental frequency of 160 Hz,

$$L_2 = \frac{n_1L_1}{n_2} = \frac{112(80)}{160} = 56 \text{ cm}$$