UNIT – 10 OSCILLATIONS

TWO MARKS AND THREE MARKS:

01. What is meant by periodic and non-periodic motion?. Give any two examples, for each motion.

1) Periodic motion Any motion which repeats itself in a fixed time interval is known as periodic motion.

Examples : Hands in pendulum clock, swing of a cradle, the revolution of the Earth around the Sun, waxing and waning of Moon, etc.

2) Non-Periodic motion Any motion which does not repeat itself after a regular interval of time is known as non-periodic motion.

Example : Occurrence of Earth quake, eruption of volcano, etc.

02. What is meant by force constant of a spring?

The displacement of the particle is measured in terms of linear displacement $\vec{r}$. The restoring force is $\vec{F} = -k\vec{r}$, where k is a spring constant or force constant.

1) Oscillations of a loaded spring 2) Vibrations of a turning force

03. Define time period of simple harmonic motion.

The time period is defined as the time taken by a particle to complete one oscillation. It is usually denoted by T. For one complete revolution, the time taken is t = T, therefore, $\omega T = 2\pi \Rightarrow T = \frac{2\pi}{\omega}$

04. Define frequency of simple harmonic motion.

The number of oscillations produced by the particle per second is called frequency. It is denoted by f. SI unit for frequency is s^{-1} or hertz
(In symbol, Hz).

Angular frequency is related to time period by $f = \frac{1}{T}$

The number of cycles (or revolutions) per second is called angular frequency.

It is usually denoted by the Greek small letter 'omega', ω.

Angular frequency and frequency are related by $\omega = 2\pi f$

SI unit for angular frequency is rad s^{-1}.

05. What is an epoch?

The displacement time t = 0 s (initial time), the phase $\varphi = \varphi_0$ is called epoch. (initial phase) where φ_0 is called the angle of epoch.

06. Write short notes on two springs connected in series.

Consider only two springs whose spring constant are k_1 and k_2 and which can be attached to a mass m. The results thus obtained can be generalized for any number of springs in series.

07. Write short notes on two springs connected in parallel.

Consider only two springs of spring constants k_1 and k_2 attached to a mass m. The results can be generalized to any number of springs in parallel.

08. Write down the time period of simple pendulum.

The angular frequency of this oscillator (natural frequency of this system) is $\omega^2 = \frac{g}{l} \Rightarrow \omega = \sqrt{\frac{g}{l}}$ in rads^{-1}

The frequency of oscillations is $f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$ in Hz, and time period of oscillations is $T = 2\pi\sqrt{\frac{l}{g}}$

09. State the laws of simple pendulum?

Law of length: For a given value of acceleration due to gravity, the time period of a simple pendulum is directly proportional to the square root of length of the pendulum. $T \,\alpha\, \sqrt{l}$

Law of acceleration: For a fixed length, the time period of a simple pendulum is inversely proportional to square root of acceleration due to gravity. $T \,\alpha\, \frac{1}{\sqrt{g}}$

10. Write down the equation of time period for linear harmonic oscillator.

From Newton's second law, we can write the equation for the particle executing simple harmonic motion $m\frac{d^2x}{dt^2} = k\,x$;

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$

Comparing the equation with simple harmonic motion equation,

we get, $\omega = \sqrt{\frac{k}{m}}$ rad s^{-1}

Natural frequency of the oscillator is $f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ Hertz.

and the time period of the oscillation is $T = \frac{1}{f} = 2\pi\sqrt{\frac{m}{k}}$ second.

11. What is meant by free oscillation?

When the oscillator is allowed to oscillate by displacing its position from equilibrium position, it oscillates with a frequency which is equal to the natural frequency of the oscillator. Such an oscillation or vibration is known as free oscillation or free vibration.

12. Explain damped oscillation. Give an example.

1) Due to the presence of friction and air drag, the amplitude of oscillation decreases as time progresses. It implies that the oscillation is not sustained and the energy of the SHM decreases gradually indicating the loss of energy.
2) The energy lost is absorbed by the surrounding medium. This type of oscillatory motion is known as damped oscillation.

Examples (i) The oscillations of a pendulum (including air friction) or pendulum oscillating inside an oil filled container. (ii) Electromagnetic oscillations in a tank circuit. (iii) Oscillations in a dead beat and ballistic galvanometers.

13. Define forced oscillation. Give an example.

In this type of vibration, the body executing vibration initially vibrates with its natural frequency and due to the presence of external periodic force, the body later vibrates with the frequency of the applied periodic force. Such vibrations are known as forced vibrations. **Example:** Sound boards of stringed instruments.

14. What is meant by maintained oscillation? Give an example.

While playing in swing, the oscillations will stop after a few cycles, this is due to damping. To avoid damping we have to supply a push to sustain oscillations. By supplying energy from an external source, the amplitude of the oscillation can be made constant. Such vibrations are known as maintained vibrations.

Example: The vibration of a tuning fork getting energy from a battery or from external power supply.

15. Explain resonance. Give an example.

The frequency of external periodic force (or driving force) matches with the natural frequency of the vibrating body (driven). As a result the oscillating body begins to vibrate such that its amplitude increases at each step and ultimately it has a large amplitude. Such a phenomenon is known as resonance and the corresponding vibrations are known as resonance vibrations.

Example The breaking of glass due to sound.

16. Define oscillatory or vibratory motion.

When an object or a particle moves back and forth repeatedly for some duration of time its motion is said to be oscillatory (or vibratory).

Examples: our heart beat, swinging motion of the wings of an insect, grandfather's clock (pendulum clock), etc.

17. State five characteristics of SHM.

Displacement: The distance travelled by the vibrating particle at any instant of time t from its mean position is known as displacement.
Velocity: The rate of change of displacement of the particle is velocity.
Acceleration: The rate of change of velocity of the particle is acceleration.
Amplitude: The maximum displacement on either side of mean position.
Time Period: The time taken by the particle executing SHM to complete one vibration.

18. Will a pendulum clock lose or gain time when taken to the top of a mountain?

On the top of the mountain, the value of g is less than that on the surface of the earth the decreases in the value of g increases the time period of the pendulum on the top of the mountain. So the pendulum clock loses time.

19. Why are army troops not allowed to march in steps while crossing the bridge?

Army troops are not allowed to march in steps because it is quite likely that the frequency of the footsteps may match with the natural frequency of the bridge and due to resonance the bridge may pick up large amplitude and break.

20. How can earthquakes cause disaster sometimes?

The resonance may cause disaster during the earthquake, if the frequency of oscillation present within the earth per chance coincides with natural frequency of some building, which may start vibrating with large amplitude due to resonance and may get damaged.

21. Every simple harmonic motion is periodic motion but every periodic motion need not be simple harmonic motion. Do you agree? Give example.

Yes, every periodic motion need not be simple harmonic motion. The motion of the earth round the sun is a period motion, but not simple harmonic motion as the back and forth motion is not taking place.

22. Glass windows may be broken by a far away explosion. Explain why?

A large amplitude in all directions. As these sound waves strike the glass windows, they set them into forced oscillations.

Since glass is brittle, so the glass windows break as soon as they start oscillating due to forced oscillations.

FIVE MARKS

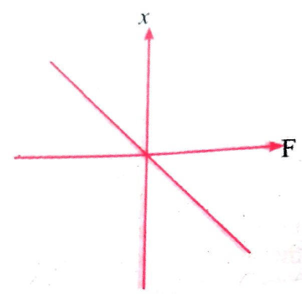
01. What is meant by simple harmonic oscillation? Give examples and explain why every simple harmonic motion is a periodic motion whereas the converse need not be true.

1) Simple harmonic motion is a special type of oscillatory motion in which the acceleration or force on the particle is directly proportional to its displacement from a fixed point and is always directed towards that fixed point.

2) In one dimensional case, let x be the displacement of the particle and a_x be the acceleration of the particle, then $a_x \propto$; $a_x = -\,b\,x$
where b is a constant which measures acceleration per unit displacement and dimensionally it is equal to T^{-2}. By multiplying by mass of the particle on both sides of equation and from Newton's second law, the force is $F_x = -\,kx$
where k is a force constant which is defined as force per unit length.

3) The negative sign indicates that displacement and force (or acceleration) are in opposite directions.

4) This means that when the displacement of the particle is taken towards right of equilibrium position (x takes positive value), the force (or acceleration) will point towards equilibrium (towards left) and similarly, when the displacement of the particle is taken towards left of equilibrium position (x takes negative value), the force (or acceleration) will point towards equilibrium (towards right).



5) This type of force is known as **restoring force** because it always directs the particle executing simple harmonic motion to restore to its original (equilibrium or mean) position. This force (restoring force) is central and attractive whose center of attraction is the equilibrium position.

6) In order to represent in two or three dimensions, we can write using vector notation $\vec{F} = -\,k\vec{r}$, where $\vec{r}$ is the displacement of the particle from the chosen origin. Note that the force and displacement have a linear relationship.

7) This means that the exponent of force $\vec{F}$ and the exponent of displacement $\vec{r}$ are unity. The sketch between cause (magnitude of force $|\vec{F}|$) and effect (magnitude of displacement $|\vec{r}|$) is a straight line passing through second and fourth quadrant

By measuring slope $\frac{1}{k}$, one can find the numerical value of force constant k.

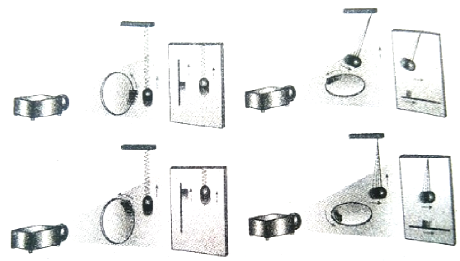
02. Describe Simple Harmonic Motion as a projection of uniform circular motion.

1) Consider a particle of mass m moving with uniform speed v along the circumference of a circle whose radius is r in anti-clockwise direction . Let us assume that the origin of the coordinate system coincides with the center O of the circle.

2) If ω is the angular velocity of the particle and θ the angular displacement of the particle at any instant of time t, then $\theta = \omega t$. By projecting the uniform circular motion on its diameter gives a simple harmonic motion.

3) This means that we can associate map (or a relationship) between uniform circular (or revolution) motion to vibratory motion. Conversely, any vibratory motion or revolution can be mapped to uniform circular motion. In other words, these two motions are similar in nature.

4) Let us first project the position of a particle moving on a circle, on to its vertical diameter or on to a line parallel to vertical diameter .Similarly, we can do it for horizontal axis or a line parallel to horizontal axis.



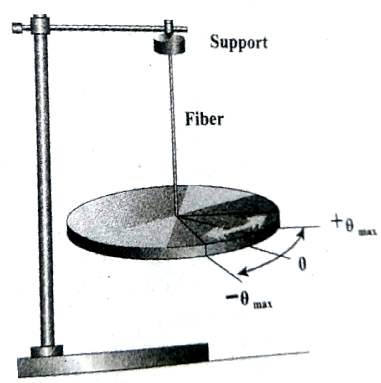
5) As a specific example, consider a spring mass system (or oscillation of pendulum) .When the spring moves up and down (or pendulum moves to and fro), the motion of the mass or bob is mapped to points on the circular motion.

6) Thus, if a particle undergoes uniform circular motion then the projection of the particle on the diameter of the circle (or on a line parallel to the diameter) traces straightline motion which is simple harmonic in nature.

7) The circle is known as reference circle of the simple harmonic motion. The simple harmonic motion can also be defined as the motion of the projection of a particle on any diameter of a circle of reference.

03. What is meant by angular harmonic oscillation? Compute the time period of angular harmonic oscillation.

1) When a body is allowed to rotate freely about a given axis then the oscillation is known as the angular oscillation. The point at which the resultant torque acting on the body is taken to be zero is called mean position.



2) If the body is displaced from the mean position, then the resultant torque acts such that it is proportional to the angular displacement and this torque has a tendency to bring the body towards the mean position. Let $\vec{\theta}$ be the angular displacement of the body and the resultant torque $\vec{\tau}$ acting on the body is $\vec{\tau} \ \alpha \ \vec{\theta}$ ------------- 1

$$\vec{\tau} = -k\vec{\theta} \quad \text{------------- 2}$$

k is the restoring torsion constant, which is torque per unit angular displacement. If *I* is the moment of inertia of the body and $\vec{\alpha}$ is the angular acceleration then $\vec{\tau} = I\vec{\alpha} = -k\vec{\theta}$.

But $\vec{\alpha} = \frac{d^2\vec{\theta}}{dt^2}$ and therefore,

$$\vec{\alpha} = \frac{d^2\vec{\theta}}{dt^2} = \frac{k}{I}\vec{\theta} \text{ ------------- } 3$$

3) This differential equation resembles simple harmonic differential equation. So, comparing equation with simple harmonic motion given in equation,

we have $\omega = \sqrt{\frac{k}{I}}$ rad s^{-1} ----------- 4

The frequency of the angular harmonic motion is $f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{I}}$ Hz……5

and the time period of the oscillation is $T = \frac{1}{f} = 2\pi\sqrt{\frac{I}{k}}$ second.

04. Write down the difference between simple harmonic motion and angular simple harmonic motion.

S. No.	Simple Harmonic Motion	Angular Harmonic Motion
1	The displacement of the particle is measured in terms of linear displacement $\vec{r}$	The displacement of the particle is measured in terms of angular displacement $\vec{\theta}$
2	Acceleration of the particle is $\vec{a} = -\omega^2\vec{r}$	Angular Acceleration of the particle is $\vec{\alpha} = -\omega^2\vec{\theta}$
3	Force , $\vec{F} = m\vec{a}$ where m is called mass of the particle.	Torque, $\vec{\tau} = I\vec{\alpha}$ where I is called moment of inertia of a body.
4	The restoring force $\vec{F} = -k\,\vec{r}$ where k is restoring force constant	The restoring torque $\vec{\tau} = -k\,\vec{\theta}$ where k is restoring torsion constant. Note: k pronounced “kappa”
5	Angular frequency $\omega = \sqrt{\frac{k}{m}}$ rad^{-1}	Angular frequency $\omega = \sqrt{\frac{k}{I}}$ rad^{-1}

05. Discuss the simple pendulum in detail.

1) A pendulum is a mechanical system which exhibits periodic motion. It has a bob with mass m suspended by a long string (assumed to be mass less and inextensible string) and the other end is fixed on a stand.

2) When a pendulum is displaced through a small displacement from its equilibrium position and released, the bob of the pendulum executes to and fro motion. Let l be the length of the pendulum which is taken as the distance between the point of suspension and the centre of gravity of the bob.

3) Two forces act on the bob of the pendulum at any displaced position.

(i) The gravitational force acting on the body ($\vec{F}=-m\vec{g}$) which acts vertically downwards. (ii) The tension in the string $\vec{F}$ which acts along the string to the point of suspension.

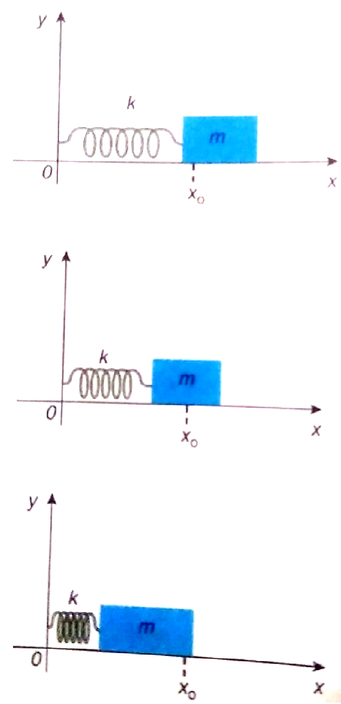
4) Resolving the gravitational force into its components:

a. **Normal component:** The component along the string but in opposition to the direction of tension, F_{as} = mg cosθ.

b. **Tangential component:** The component perpendicular to the string i.e., along tangential direction of arc of swing, F_{ps} = mg sinθ.

06. Explain the horizontal oscillations of a spring.

1) Consider a system containing a block of mass m attached to a mass less spring with stiffness constant or force constant or spring constant k placed on a smooth horizontal surface (frictionless surface) as shown in Figure.



2) Let x_0 be the equilibrium position or mean position of mass m when it is left undisturbed. Suppose the mass is displaced through a small displacement x towards right from its equilibrium position and then released, it will oscillate back and forth about its mean position x_0.

3) Let F be the restoring force (due to stretching of the spring) which is proportional to the amount of displacement of block. For one dimensional motion, mathematically, we have $F \alpha\, x$; $F = -k\,x$

4) Where negative sign implies that the restoring force will always act opposite to the direction of the displacement. Notice that, the restoring force is linear with the displacement.

5) This is not always true; in case if we apply a very large stretching force, then the amplitude of oscillations becomes very large (which means, force is proportional to displacement containing higher powers of x) and therefore, the oscillation of the system is not linear and hence, it is called non-linear oscillation.

6) We restrict ourselves only to linear oscillations throughout our discussions, which means Hooke's law is valid (force and displacement have a linear relationship).

From Newton's second law, we can write the equation for the particle executing simple harmonic motion m $\frac{d^2x}{dt^2} = -k\,x$;

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x \quad \text{----------------}1$$

Comparing the equation (1) with simple harmonic motion equation, we get

$$\omega^2 = \frac{k}{m}$$

Which means the angular frequency or natural frequency of the oscillator is

$\omega = \sqrt{\frac{k}{m}}$ rsd s^{-1} -------------- 2

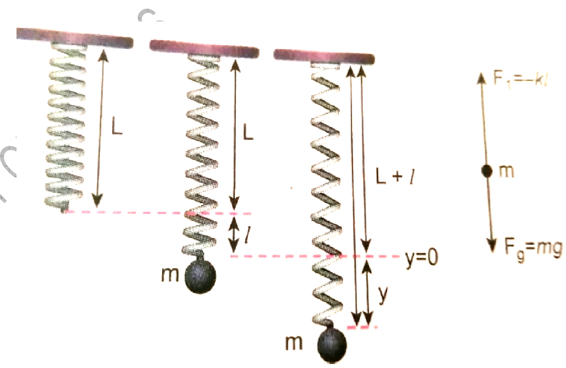
Natural frequency of the oscillator is $f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ Hertz --------- 3

and the time period of the oscillation is $T = \frac{1}{f} = 2\pi\sqrt{\frac{m}{k}}$ second --------- 4

Notice that **in simple harmonic motion, the time period of oscillation is independent of amplitude.** This is valid only if the amplitude of oscillation is small.

07. Describe the vertical oscillations of a spring.

1) Consider a mass less spring with stiffness constant or force constant k attached to a ceiling as shown in Figure. Let the length of the spring before loading mass m be L. If the block of mass m is attached to the other end of spring, then the spring elongates by a length l.



2) Let F1 be the restoring force due to stretching of spring. Due to mass m, the gravitational force acts vertically downward. We can draw free-body diagram for this system as shown in Figure. When the system is under equilibrium, $F_1 + mg = 0$---------------- 1

3) But the spring elongates by small displacement l,

therefore, $F_1 \propto l \Rightarrow F_1 = -kl$ ------------ 2

Substituting equation (2) in equation (1), we get $-kl + mg = 0$

$mg = kl$ or $\frac{m}{k} = \frac{l}{g}$ ----------- 3

4) Suppose we apply a very small external force on the mass such that the mass further displaces downward by a displacement y, then it will oscillate up and down. Now, the restoring force due to this stretching of spring (total extension of spring is $y + l$) is $F_2 \propto (y + l)$ $F_2 = -k(y + l) = -ky - kl$ ------4

Since, the mass moves up and down with acceleration $\frac{d^2y}{dt^2}$, by drawing the free body diagram for this case, we get $-ky - kl + mg = m\frac{d^2y}{dt^2}$--------- 5

The net force acting on the mass due to this stretching is $F = F_2 + mg$

$F = -ky - kl + mg$ ------------- 6

5) The gravitational force opposes the restoring force. Substituting equation (3) in equation (6), we get F = $-ky - kl + kl = -ky$

Applying Newton's law, we get m$\frac{d^2y}{dt^2} = -ky$

m$\frac{d^2y}{dt^2} = -\frac{k}{m}$ y ------------- 7

6) The above equation is in the form of simple harmonic differential equation. Therefore, we get the time period as T $= \frac{1}{f} = 2\pi\sqrt{\frac{m}{k}}$ second

The time period can be rewritten using equation (3)

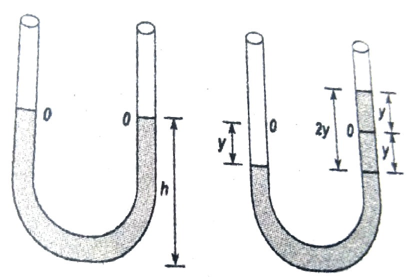
T$= 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{l}{g}}$ second

The acceleration due to gravity g can be computed from the formula

$g = 4\pi^2\left(\frac{l}{T^2}\right)$ms^{-1}

08. Write short notes on the oscillations of liquid column in U-tube.

1) Consider a U-shaped glass tube which consists of two open arms with uniform cross-sectional area A. Let us pour a non-viscous uniform incompressible liquid of density ρ in the U-shaped tube to a height h as shown in the Figure.



2) If the liquid and tube are not disturbed then the liquid surface will be in equilibrium position O. It means the pressure as measured at any point on the liquid is the same and also at the surface on the arm (edge of the tube on either side), which balances with the atmospheric pressure.

3) Due to this the level of liquid in each arm will be the same. By blowing air one can provide sufficient force in one arm, and the liquid gets disturbed from equilibrium position O, which means, the pressure at blown arm is higher than the other arm.

4) This creates difference in pressure which will cause the liquid to oscillate for a very short duration of time about the mean or equilibrium position and finally comes to rest.

Time period of the oscillation is T $= 2\pi\sqrt{\frac{l}{g}}$ second

09. Discuss in detail the energy in simple harmonic motion.

a. Expression for Potential Energy

1) For the simple harmonic motion, the force and the displacement are related by Hooke's law $\vec{F} = -k\vec{r}$

2) Since force is a vector quantity, in three dimensions it has three components. Further, the force in the above equation is a conservative force field; such a force can be derived from a scalar function which has only one component. In one dimensional case $F = -kx$ ----------------- (1)

The work done by the conservative force field is independent of path. The potential energy U can be calculated from the following expression.

$F = \frac{dU}{dx}$ -------------- 2

Comparing (1) and (2), we get $-\frac{dU}{dx} = -kx$; $dU = kx dx$

3) This work done by the force F during a small displacement dx stores as potential energy $U(x) = \int_0^x kx' dx = \frac{1}{2}(x')^2 \Big|_0^x = \frac{1}{2} kx^2$ ------- 3

From equation $\omega = \sqrt{\frac{k}{m}}$,

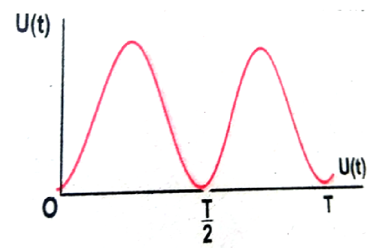
we can substitute the value of force constant $k = m\omega^2$ in equation (3),

$U(x) = m\,\omega^2 x^2$

4) where ω is the natural frequency of the oscillating system. For the particle executing simple harmonic motion from equation $x = A \sin \omega t$

$U(t) = \frac{1}{2} m\omega^2 A^2 \sin^2 \omega t$ -------------4

This variation of U is shown below.



b. Expression for Kinetic Energy

Kinetic energy $KE = \frac{1}{2} m v_x^2 = \frac{1}{2} m \left(\frac{dx}{dt}\right)^2$

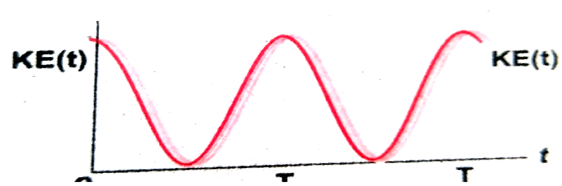
Since the particle is executing simple harmonic motion, from equation $y = A \sin \omega t$; $x = A \sin \omega t$ Therefore, velocity is $v_x = \frac{dx}{dt}$ A ωcosωt

$= a\omega\sqrt{1 - \left(\frac{x}{A}\right)^2}$; $v_x = \omega\sqrt{A^2 - x^2}$ --------------- 5

Hence, $KE = \frac{1}{2} m v_x^2 = \frac{1}{2} m\omega^2 (A^2 - x^2)$ ------------- 6

$KE = \frac{1}{2} m\omega^2 A^2 \cos^2 \omega t$ -------------- 7

This variation with time is shown below.



c. Expression for Total Energy

Total energy is the sum of kinetic energy and potential energy

E = KE + U -------------8 ; $E = \frac{1}{2}m\,\omega^2 (A^2 - x^2) + \frac{1}{2}m\,\omega^2 x^2$

Hence, cancelling x^2 term, $E = \frac{1}{2}m\,\omega^2 A^2$ = Constant ----------9

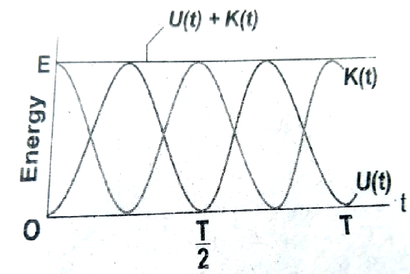
Alternatively, from equation (4) and equation (7),

we get the total energy as $E = \frac{1}{2} m\omega^2 A^2 \sin^2\omega t + \frac{1}{2} m\omega^2 A^2 \cos^2\omega t$

$$E = \frac{1}{2} m\omega^2 A^2(\sin^2\omega t + \cos^2\omega t)$$

From trigonometry identity, $(\sin^2\omega t + \cos^2\omega t) = 1$

$E = \frac{1}{2} m\omega^2 A^2$ = Constant.



which gives the law of conservation of total energy.

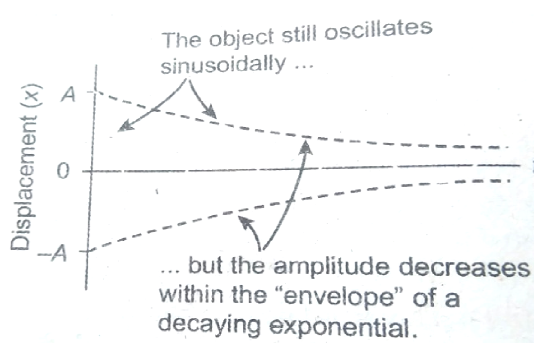
This is depicted in Figure. Thus the amplitude of simple harmonic oscillator, can be expressed in terms of total energy.

$$A = \sqrt{\frac{2E}{m\omega^2}} = \sqrt{\frac{2E}{k}}$$

10. Explain in detail the four different types of oscillations.

Damped oscillations:

1) During the oscillation of a simple pendulum, we have assumed that the amplitude of the oscillation is constant and also the total energy of the oscillator is constant. But in reality, in a medium, due to the presence of friction and air drag, the amplitude of oscillation decreases as time progresses.



2) It implies that the oscillation is not sustained and the energy of the SHM decreases gradually indicating the loss of energy. The energy lost is absorbed by the surrounding medium. This type of oscillatory motion is known as damped oscillation.
3) In other words, if an oscillator moves in a resistive medium, its amplitude goes on decreasing and the energy of the oscillator is used to do work against the resistive medium.
4) The motion of the oscillator is said to be damped and in this case, the resistive force (or damping force) is proportional to the velocity of the oscillator.

Examples (i) The oscillations of a pendulum (including air friction) or pendulum oscillating inside an oil filled container. (ii) Electromagnetic oscillations in a tank circuit. (iii) Oscillations in a dead beat and ballistic galvanometers.

Maintained oscillations:

1) While playing in swing, the oscillations will stop after a few cycles, this is due to damping. To avoid damping we have to supply a push to sustain oscillations.
2) By supplying energy from an external source, the amplitude of the oscillation can be made constant. Such vibrations are known as maintained vibrations.

Example: The vibration of a tuning fork getting energy from a battery or from external power supply.

Forced oscillations:

1) Any oscillator driven by an external periodic agency to overcome the damping is known as forced oscillator or driven oscillator.
2) In this type of vibration, the body executing vibration initially vibrates with its natural frequency and due to the presence of external periodic force, the body later vibrates with the frequency of the applied periodic force. Such vibrations are known as forced vibrations.

Example: Sound boards of stringed instruments.

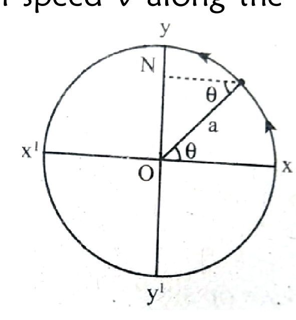
Resonance:

1) It is a special case of forced vibrations where the frequency of external periodic force (or driving force) matches with the natural frequency of the vibrating body (driven).
2) As a result the oscillating body begins to vibrate such that its amplitude increases at each step and ultimately it has a large amplitude. Such a phenomenon is known as resonance and the corresponding vibrations are known as resonance vibrations.

Example The breaking of glass due to sound.

11. Show that the projection of uniform circular motion on a diameter is SHM.

1) Consider a particle of mass m moving with uniform speed v along the circumference of a circle whose radius is r in anti-clockwise direction as shown in Figure . Let us assume that the origin of the coordinate system coincides with the center O of the circle.



2) If ω is the angular velocity of the particle and θ the angular displacement of the particle at any instant of time t, then $\theta = \omega t$.

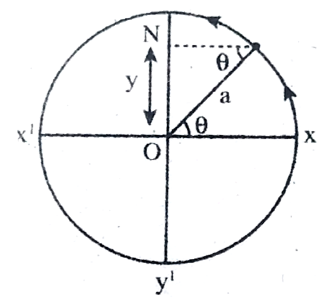
3) By projecting the uniform circular motion on its diameter gives a simple harmonic motion. This means that we can associate a map (or a relationship) between uniform circular (or revolution) motion to vibratory motion.

4) Conversely, any vibratory motion or revolution can be mapped to uniform circular motion. In other words, these two motions are similar in nature.

12. Explain briefly about the graphical representation of Displacement, Velocity and Acceleration in SHM.

Displacement:

1) The distance travelled by the vibrating particle at any instant of time t from its mean position is known as displacement. Let P be the position of the particle on a circle of radius A at some instant of time t as shown in Figure. Then its displacement y at that instant of time t can be derived as follows



In ΔOPN $\text{Sin}\,\theta = \frac{ON}{OP} \Rightarrow ON = OP\ \text{Sin}\,\theta$

But θ = ωt, ON = y and OP = A

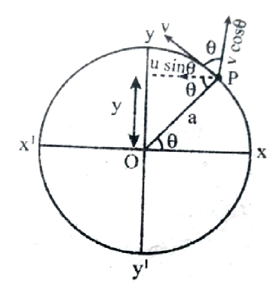
y = A sin ωt --------------1

2) The displacement y takes maximum value (which is equal to A) when sin ωt = 1. This maximum displacement from the mean position is known as amplitude (A) of the vibrating particle. For simple harmonic motion, the amplitude is constant. But, in general, for any motion other than simple harmonic, the amplitude need not be constant, it may vary with time.

Velocity :

1) The rate of change of displacement is velocity. Taking derivative of equation (1) with respect to time, we get $v = \frac{dy}{dt} = \frac{d}{dt}(A \sin \omega t)$

For circular motion (of constant radius), amplitude A is a constant and further, for uniform circular motion, angular velocity ω is a constant. Therefore, $v = \frac{dy}{dt} = (A\,\omega \cos \omega t)$ ------------2



Using trigonometry identity, $\sin^2 \omega t + \text{con}^2\, \omega t = 1$

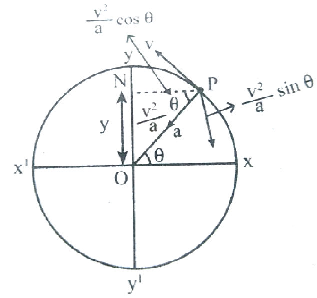
$\Rightarrow \cos \omega t = \sqrt{1 - Sin^2 \omega t}$, we get, $v = A\omega\sqrt{1 - Sin^2\omega t}$

From equation(1) $\sin \omega t = \frac{y}{A}$; $v = A\omega\sqrt{1 - \left(\frac{y}{A}\right)^2}$

$v = \omega\sqrt{A^2 - y^2}$ ------------3

2) From equation (3), when the displacement y = 0, the velocity v = ωA (maximum) and for the maximum displacement y = A, the velocity v = 0 (minimum).

3) As displacement increases from zero to maximum, the velocity decreases from maximum to zero. This is repeated.
Since velocity is a vector quantity, equation (2) can also be deduced by resolving in to components.



Acceleration:

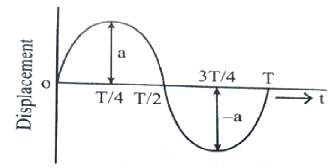
The rate of change of velocity is acceleration.

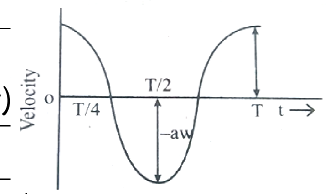
$$a = \frac{dv}{dt} = \frac{d}{dt}(A\,\omega\cos\omega t)\ ;$$

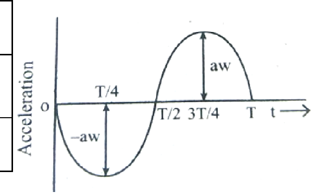
$$a = -\omega^2 A\sin\omega t\ = -\omega^2 y \text{ ----------4}$$

$$a = \frac{d^2y}{dt^2} = -\omega^2 y \text{ -------------5}$$

The mean position($y = 0$), velocity of the particle is maximum but the acceleration of the particle is zero. At the extreme position ($y = \pm A$), the velocity of the particle is zero but the acceleration is maximum $\pm A\omega^2$ acting in the opposite direction.





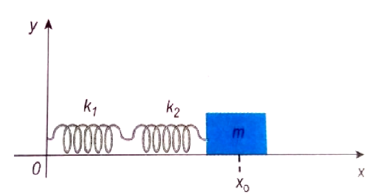


Time	ωt	Displacement ($y = A\sin\omega t$)	Velocity ($v = A\,\omega\cos\omega t$)	Acceleration ($a = -\omega^2 A\sin\omega t$)
$t=0$	0	0	$A\omega$	0
$t=\frac{T}{4}$	$\frac{\pi}{2}$	$+A$	0	$-A\omega^2$
$t=\frac{T}{2}$	π	0	$-A\omega$	0
$t=\frac{3T}{4}$	$\frac{3\pi}{2}$	$-A$	0	$A\omega^2$
$t = T$	2π	0	$A\omega$	0

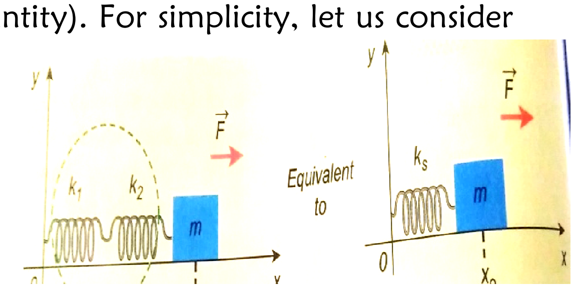
13. Explain the effective spring constant in series connection and parallel connection

a) Springs connected in series

1) When two or more springs are connected in series, all the springs in series with an equivalent spring (effective spring) whose net effect is the same as if all the springs are in series connection.



2) Given the value of individual spring constants k_1, k_2, k_3,... (known quantity), we can establish a mathematical relationship to find out an effective (or equivalent) spring constant k_s (unknown quantity). For simplicity, let us consider only two springs whose spring constant are k_1 and k_2 and which can be attached to a mass m as shown in Figure.



3) The results thus obtained can be generalized for any number of springs in

series. Let F be the applied force towards right as shown in Figure.

4) Since the spring constants for different spring are different and the connection points between them is not rigidly fixed, the strings can stretch in different lengths.

5) Let x_1 and x_2 be the elongation of springs from their equilibrium position (un-stretched position) due to the applied force F. Then, the net displacement of the mass point is $x = x_1 + x_2$ ------------1

From Hooke's law, the net force

$F = -k_s(x_1+x_2) \Rightarrow x_1 + x_2 = -\frac{F}{k_s}$ ------------2

For springs in series connection

$-k_1x_1 = -k_2x_2 = F$

$x_1 = -\frac{F}{k_1}$ and $x_2 = -\frac{F}{k_2}$ -----------3

Therefore, substituting equation (3) in equation (2), the effective spring constant can be calculated as $-\frac{F}{k_1} - \frac{F}{k_2} = \frac{F}{k_s}$

$\frac{1}{k_s} = \frac{1}{k_1} + \frac{1}{k_2}$ or $k_s = \frac{k_1k_2}{k_1+k_2}$ Nm^{-1} -------------4

6) Suppose we have n springs connected in series, the effective spring constant in series is $\frac{1}{k_s} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + ... + \frac{1}{k_n} = \sum_{i=1}^{n}\frac{1}{k_i}$ ----------5

If all spring constants are identical i.e., $k_1 = k_2 = ... = k_n = k$

then $\frac{1}{k_s} = \frac{n}{k} \Rightarrow k_s = \frac{k}{n}$ ----------- 6

7) This means that the effective spring constant. reduces by the factor n. Hence, for springs in series connection, the effective spring constant is lesser than the individual spring constants.

8) From equation (3), we have, $k_1x_1 = k_2x_2$ Then the ratio of compressed distance or elongated distance x_1 and x_2 is $\frac{x_2}{x_1} = \frac{k_1}{k_2}$ ------------7

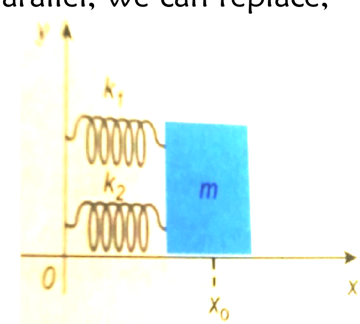
The elastic potential energy stored in first and second springs are

$v_1 = \frac{1}{2}k_1x_1^2$ and $v_2 = \frac{1}{2}k_2x_2^2$ respectively.

Then, their ratio is $\frac{v_1}{v_2} = \frac{\frac{1}{2}k_1x_1^2}{\frac{1}{2}k_2x_2^2}$ ------------ 8

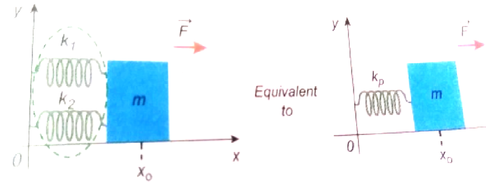
b) Springs connected in parallel

1) When two or more springs are connected in parallel, we can replace, all these springs with an equivalent spring (effective spring) whose net effect is same as if all the springs are in parallel connection.



2) Given the values of individual spring constants to be k_1, k_2, k_3, ... (known quantities), we can establish a mathematical relationship to find out an effective (or equivalent) spring constant k_p (unknown quantity).

3) For simplicity, let us consider only two springs of spring constants k_1 and k_2 attached to a mass m as shown in Figure. The results can be generalized to any number of springs in parallel.



4) Let the force F be applied towards right as shown in Figure. In this case, both the springs elongate or compress by the same amount of displacement. Therefore, net force for the displacement of mass m is

$F = -k_p x$ -------------- 1

where k_p is called **effective spring constant.**

5) Let the first spring be elongated by a displacement x due to force F_1 and second spring be elongated by the same displacement x due to force F_2, then the net force $F = -k_1x - k_2x$ ------------- 2

Equating equations (2) and (1), we get

$$k_p = k_1 + k_2 \text{ --------------------- } 3$$

Generalizing, for n springs connected in parallel, $k_p = \sum_{i=1}^{n} k_i$ ---------- 4

If all spring constants are identical i.e., $k_1 = k_2 = \ldots = k_n = k$

then $k_p = nk$ -------------- 5.

6) This implies that the effective spring constant increases by a factor n. Hence, for the springs in parallel connection, the effective spring constant is greater than individual spring constant.