CHAPTER 5: Two Dimensional Analytical Geometry II (1)

1. Eqn of the circle $(x-h)^2+(y-k)^2 = r^2$
   Centre (h,k) and radius $= r$.
2. Eqn of a circle in general form $x^2+y^2+2gx+2fy+c = 0$
   centre $(-g,-f)$ & radius $= \sqrt{g^2+f^2-c}$.
3. Equation of a circle with $(x_1,y_1)\,(x_2,y_2)$ as extremities of one of the diameter is $(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$
4) Equation of tangent and normal at (x_1,y_1) on circle $x^2+y^2+2gx+2fy+c=0$
   is $xx_1+yy_1+g(x+x_1)+f(y+y_1)+c = 0$
   and $yx_1-xy_1+g(y-y_1)-f(x-x_1)=0$.
5) Condition for the line $y=mx+c$ to be a tangent to the conics:

	Conic	Equation	Condition to be tangent	Point of contact	Equation of tangent
1.	circle	$x^2+y^2=a^2$	$c^2=a^2(1+m^2)$	$\left(\frac{\mp am}{\sqrt{1+m^2}}, \frac{\pm a}{\sqrt{1+m^2}}\right)$	$y=mx \pm a\sqrt{1+m^2}$
2.	Parabola	$y^2=4ax$	$c=\frac{a}{m}$	$\left(\frac{a}{m^2}, \frac{2a}{m}\right)$	$y=mx+\frac{a}{m}$
3)	Ellipse	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$	$c^2=a^2m^2+b^2$	$\left(-\frac{a^2m}{c}, \frac{b^2}{c}\right)$	$y=mx\pm\sqrt{a^2m^2+b^2}$
4)	Hyperbola	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$	$c^2=a^2m^2-b^2$	$\left(-\frac{a^2m}{c}, -\frac{b^2}{c}\right)$	$y=mx\pm\sqrt{a^2m^2-b^2}$

6) Tangent and Normal.

	Curve	Equation	Equation of tangent	Equation of normal
i)	Circle	$x^2+y^2=a^2$	cartesian form (i) $xx_1+yy_1=a^2$ (ii) Parametric form $x\cos\theta+y\sin\theta=a$	(i) $xy_1-yx_1=0$ (ii) $x\sin\theta-y\cos\theta=0$
(ii)	Parabola	$y^2=4ax$	(i) $yy_1=2a(x+x_1)$ (ii) $yt=x+at^2$	(i) $xy_1+2ay=2ay_1+x_1y_1$ (ii) $y+xt=at^3+2at$
(iii)	Ellipse	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$	(i) $\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$ (ii) $\frac{x\cos\theta}{a}+\frac{y\sin\theta}{b}=1$	(i) $\frac{a^2x}{x_1}-\frac{b^2y}{y_1}=a^2-b^2$ (ii) $\frac{ax}{\cos\theta}-\frac{by}{\sin\theta}=a^2-b^2$
(iv)	Hyperbola	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$	(i) $\frac{xx_1}{a^2}-\frac{yy_1}{b^2}=1$ (ii) $\frac{x\sec\theta}{a}-\frac{y\tan\theta}{b}=1$	(i) $\frac{a^2x}{x_1}+\frac{b^2y}{y_1}=a^2+b^2$ (ii) $\frac{ax}{\sec\theta}+\frac{by}{\tan\theta}=a^2+b^2$

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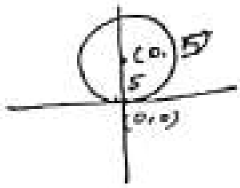
Chapter 5: Two Dimensional Analytical Geometry - II (2)

Exercise 5.1

1. Obtain the equation of the circle with radius 5 cm and touching x axis at the origin in general form.



Soln radius = 5 cm

Passing through point (0,0)

Centre at y axis be $(0,k) = (0,\pm 5)$

Equation of circle be —①

$$(x-h)^2+(y-k)^2 = r^2$$

$$(x-0)^2+(y-k)^2 = r^2 = 5^2$$

At (0,0)

$$x^2+(y-k)^2 = 25$$

$$0+(0-k)^2 = 25$$

$$k^2 = 25$$

$$k = \pm 5$$

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$(0,k) = (0,\pm 5)$

$\therefore$ ① $\Rightarrow$ $(x-0)^2+(y\pm 5)^2 = r^2$

$$x^2+y^2\pm 10y+\cancel{25} = \cancel{25}$$

$$x^2+y^2\pm 10y = 0$$

2. Find the equation of the circle with centre (2,−1) and passing through the point (3,6) in standard form.

Soln Centre = (2,−1) = (h,k)

Passing through the point (3,6)

Eqn of the circle $(x-h)^2+(y-k)^2 = r^2$ —①

$$(3-2)^2+(6+1)^2 = r^2$$

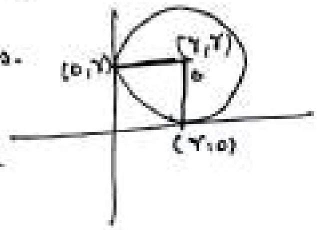
$$1^2+7^2 = r^2$$

$$1+49 = r^2$$

$$r^2 = 50$$

① $\Rightarrow$ $(x-2)^2+(y+1)^2 = 50$

3. Find the equation of circles that touch both the axes and pass through (−4,−2) in general form.



Soln Let the circle touch both the axis.

Centre be (r, r). = (h,k)

Eqn of the circle $(x-h)^2+(y-k)^2 = r^2$

$$(x-r)^2+(y-r)^2 = r^2 \quad —①$$

Passing through (−4,−2)

$$(-4-r)^2+(-2-r)^2 = r^2$$

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$4+r^2+8r+4+\cancel{r^2}+4r=\cancel{r^2}$

$r^2+12r+20=0$

$(r+10)(r+2)=0$

$r=-10,\ r=-2.$

(i) $r=-10$ in ①

$(x+10)^2+(y+10)^2=10^2$

$x^2+100+20x+y^2+100+20y=100$

$x^2+y^2+20x+20y+100=0$

(ii) $r=-2$

$(x+2)^2+(y+2)^2=(-2)^2$

$x^2+4+4x+y^2+4y+4=4$

$x^2+y^2+4x+4y+4=0.$

4) Find the equation of the circle with centre (2, 3) and passing through the intersection of the lines $3x-2y-1=0$ and $4x+y-27=0$

Soln centre $(2,3)=(h,k)$

Point of intersection :- — ③

Solve $3x-2y-1=0$ — ①

$4x+y-27=0$ — ②

①⇒ $3x-2y=1$

②×2⇒ $8x+2y=54$

$11x=55$

$x=5.$ put in ①

$15-2y-1=0$

$14=2y$

$y=7.$

Passing through point is $(5,7)$

Eqn of circle be $(x-h)^2+(y-k)^2=r^2$ — ③

$(5-2)^2+(7-3)^2=r^2$

$3^2+4^2=r^2$

$r^2=25$

∴ ③ ⇒ $(x-2)^2+(y-3)^2=25.$

$x^2-4x+4+y^2-6y+9-25=0$

$x^2+y^2-4x-6y-12=0$

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5) Obtain the equation of the circle for which (3, 4) and (2, −7) are the ends of a diameter.

Soln $(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$ $(x_1,y_1)=(3,4)$ $(x_2,y_2)=(2,-7)$

$(x-3)(x-2)+(y-4)(y+7)=0$

$x^2-5x+6+y^2+3y-28=0$

$x^2+y^2-5x+3y-28=0.$

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6) Find the equation of the circle through the points $(1,0)$ $(-1,0)$ and $(0,1)$ (4)

Soln Let the points $(1,0)$ $(-1,0)$ & $(0,1)$
x_1, y_1 $x_2\ y_2$ $x_3\ y_3$.

If two points lie any where on the circumference but diametrically opposite each other. Then the radius of the circle won't be minimum

$\therefore$ $\overset{x_1\ y_1}{(1,0)}$ & $\overset{x_2\ y_2}{(0,1)}$ form the end points of diameter

$\cancel{(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0}$

$\cancel{(x-1)(x-0)+(y-0)(y-1)=0}$

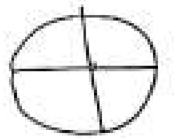
Radius be unit circle $(r=1)$ centre $(0,0)$

$$(x-h)^2+(y-k)^2=r^2$$
$$(x-0)^2+(y-0)^2=1^2$$
$$x^2+y^2=1$$

$(-1,0)$ also pass through this circle. $(-1)^2+0^2=1^2$.

$\therefore$ $x^2+y^2=1$ be the equation of the circle through the points $(1,0)$ & $(-1,0)$ & $(0,1)$

7) A circle of area 9π square units has two of its diameters along the lines $x+y=5$ & $x-y=1$ Find the equation of the circle



Soln area of a circle $\cancel{\pi}r^2=9\cancel{\pi}$

$r^2=9$.

Solve $x+y=5$ — ①
$x-y=1$ — ② we get centre of the circle.

Add ① & ② $2x=6$

$\boxed{x=3}$

① $\Rightarrow$ $3+y=5$

$\boxed{y=2}$

centre $(h,k)=(3,2)$ & $r^2=9$

$$(x-h)^2+(y-k)^2=r^2$$
$$(x-3)^2+(y-2)^2=9.$$
$$x^2+\cancel{9}-6x+y^2-4y+4=\cancel{9}$$
$$x^2-6x+y^2-4y+4=0.$$

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8) If $y = 2\sqrt{2}x + c$ is a tangent to the circle $x^2+y^2=16$ find the value of c.

Soln The condition of the line $y = mx + c$ to be a tangent to the circle $x^2+y^2=a^2$ is

$a^2 = 16$ $\quad m = 2\sqrt{2}$. $\quad m^2 = 4 \times 2 = 8$.

$$c^2 = a^2(1+m^2)$$
$$c^2 = 16(1+8)$$
$$c^2 = 16(9)$$
$$c = \pm 4 \times 3 = \pm 12$$

$$\boxed{\therefore c = \pm 12}$$

9. Find the equation of the tangent and normal to the circle $x^2+y^2-6x+6y-8=0$ at $(2,2)$.

Soln Eqn of the tangent be

$$xx_1 + yy_1 - \cancel{6}^{3}\left(\frac{x+x_1}{2}\right) + \cancel{6}^{3}\left(\frac{y+y_1}{2}\right) - 8 = 0$$

At $(2,2)$
$$2x + 2y - 3(x+2) + 3(y+2) - 8 = 0.$$
$$2x + 2y - 3x - \cancel{6} + 3y + \cancel{6} - 8 = 0$$
$$-x + 5y - 8 = 0$$
$$x - 5y + 8 = 0.$$

Eqn of the normal be

$$yx_1 - xy_1 + g(y-y_1) - f(x-x_1) = 0$$

$2g = -6$, $g = -3$, $2f = 6$, $f = 3$

At $(2,2)$
$$y(2) - x(2) - 3(y-2) - 3(x-2) = 0$$
$$2y - 2x - 3y + 6 - 3x + 6 = 0$$
$$-5x - y + 12 = 0$$
$$5x + y - 12 = 0.$$

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Another method for normal

Eqn of tangent is $\perp r$ to normal.

$$x - 5y + 8 = 0$$

$\perp r$ eqn be $5x + y + k = 0$

At $(2,2)$
$$10 + 2 + k = 0$$
$$k = -12$$

$\therefore 5x + y - 12 = 0$ be eqn of normal

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10) Determine whether the points $(-2,1)$, $(0,0)$ and $(-4,-3)$ lie out-side, on or inside the circle $x^2+y^2-5x+2y-5=0$

Soln $x^2+y^2-5x+2y-5=0$

At $(-2,1)$ $(-2)^2+1^2-5(-2)+2(1)-5$

$= 4+1+10+2-5 = 12 > 0.$

$\therefore (-2,1)$ lies out-side the circle

(ii) At $(0,0)$ $0+0-0+0-5 = -5 < 0$

$(0,0)$ lies in-side the circle

(iii) At $(-4,-3)$ $(-4)^2+(-3)^2-5(-4)+2(-3)-5$

$= 16+9+20+6-5 = 34 > 0$

$(-4,-3)$ lies out side the circle.

11) Find the centre and radius of the following circles

(i) $x^2+(y+2)^2=0.$

Compare with $(x-h)^2+(y-k)^2=r^2.$

$h=0$ $k=-2$ $r^2=0$

$r=0.$ = radius

Centre $(h,k) = (0,-2)$

(ii) $x^2+y^2+6x-4y+4=0$

$2g=6$ $2f=-4$ $c=4.$

$g=3$ $f=-2.$

Centre $(-g,-f) = (-3,2)$

Radius $r = \sqrt{g^2+f^2-c} = \sqrt{9+4-4} = 3.$

(iii) $x^2+y^2-x+2y-3=0.$

$2g=-1$ $2f=2$ $c=-3.$

$g=-1/2$ $f=1$

Centre $(-g,-f) = (1/2,-1)$

Radius $\sqrt{g^2+f^2-c} = \sqrt{\frac{1}{4}+1+3} = \sqrt{\frac{1+4+12}{4}} = \sqrt{\frac{17}{4}} = \frac{\sqrt{17}}{2}$

(iv) $2x^2+2y^2-6x+4y+2=0$

$(\div 2)$ $x^2+y^2-3x+2y+1=0$ $c=1$

$2g=-3$ $2f=2$

$g=-3/2$ $f=1$ $c=1$

7

Centre $(-g,-f) = (3/2, -1)$

$r = \sqrt{g^2+f^2-c} = \sqrt{\frac{9}{4}+1-x} = \frac{3}{2}$.

12) If the equation $3x^2+(3-p)xy+qy^2-2px = 8pq$ represents a circle find p & q. Also determine the centre and radius of the circle.

Soln $3x^2+(3-p)xy+qy^2-2px = 8pq$ represent a circle means

coefft of x^2 = coefft of y^2

$3 = q$

$\boxed{\therefore q = 3}$

coefft of $xy = 0$.

$3-p = 0$

$\boxed{p = 3}$

$3x^2+3y^2-6x = 8(3)(3)$

$3x^2+3y^2-6x-72 = 0$

$(\div 3)$ $x^2+y^2-2x-24 = 0$.

$2g = -2$ $\quad 2f = 0$ $\quad c = -24$

$g = -1$ $\quad f = 0$

centre $(-g, -f) = (1, 0)$

radius $= \sqrt{g^2+f^2-c} = \sqrt{1+0+24} = \sqrt{25} = 5$.

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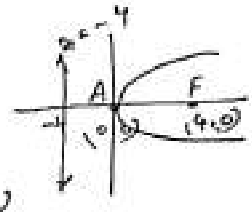
Exercise 5.2

1\. Find the equation of the parabola in each of the cases given below.

(i) focus (4, 0) and directrix $x = -4$.



Equation of a parabola be

$y^2 = 4ax$

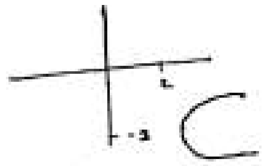
$a = 4$. (Distance $AF = 4$ unit)

$\therefore y^2 = 4(4)x$

$y^2 = 16x$.

(ii) passes through $(2, -3)$ and symmetric about y axis

Soln Eqn of a parabola be



$x^2 = -4ay$

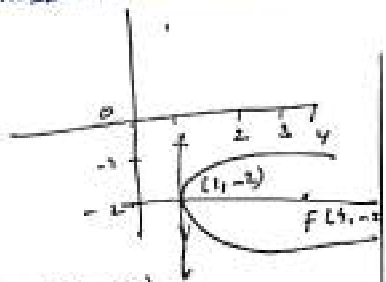
At $(2,-3)$ $\quad 4 = -4a(-3)$

$1 = 3a \Rightarrow \boxed{a = \frac{1}{3}}$

$x^2 = -4\left(\frac{1}{3}\right)y$

$\boxed{3x^2 = -4y}$

(iii) vertex $(1, -2)$ and focus $(4, -2)$



Soln $a =$ Distance $VF = 3$ units.

Eqn of a parabola be

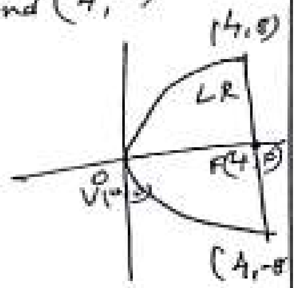
$(y-k)^2 = 4a(x-h)$.

$(h,k) = (1,-2)$

$(y+2)^2 = 4(3)(x-1)$

$(y+2)^2 = 12(x-1)$

(iv) End points of latus rectum $(4, -8)$ and $(4, 8)$



Soln Distance between VF is 4 unit.

$a = 4$.

$y^2 = 4ax$

$y^2 = 4(4)x$

$\boxed{y^2 = 16x}$

2\) Find the equation of the ellipse in each of the cases given below

(i) foci $(\pm 3, 0)$, $e = 1/2$

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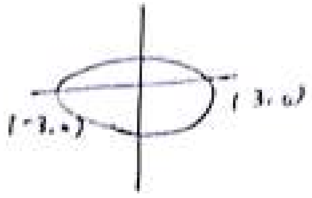
Soln $ae = 3$

$e = 1/2$.

$a\left(\frac{1}{2}\right) = 3$

$a = 6 \quad \therefore a^2 = 36$



$b^2 = a^2(1-e^2)$

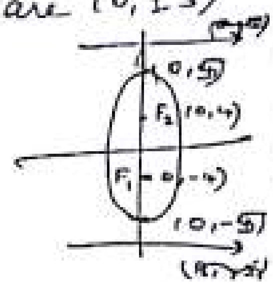
$b^2 = 36\left(1-\frac{1}{4}\right) = \overset{9}{\cancel{36}}\left(\frac{3}{\cancel{4}}\right) = 27.$

$b^2 = 27$

Eqn of the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$\boxed{\frac{x^2}{36} + \frac{y^2}{27} = 1}$$

(ii) foci $(0, \pm 4)$ and end points of major axis are $(0, \pm 5)$



Soln $ae = 4$ | $2a = 10$

$5e = 4$ | $a = 5$

$e = \frac{4}{5}$. | $a^2 = 25$

$b^2 = a^2(1-e^2) = 25\left(1-\frac{16}{25}\right) = \cancel{25}\left(\frac{9}{\cancel{25}}\right)$

$b^2 = 9.$

Eqn of the ellipse be $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

$$\frac{x^2}{9} + \frac{y^2}{25} = 1$$

(iii) Length of Latus Rectum 8, eccentricity $= \frac{3}{5}$ and major axis on x axis

Soln $e = \frac{3}{5}$

Latus Rectum $\frac{2b^2}{a} = 8$.

$\frac{2b^2}{a} = 8$.

$b^2 = \frac{\overset{4}{\cancel{8}}a}{\cancel{2}} = 4a = \cancel{4}\left(\frac{25}{\cancel{4}}\right) = 25$

$b^2 = a^2(1-e^2)$

$\cancel{4}a = a^2\left(1-\frac{9}{25}\right)$

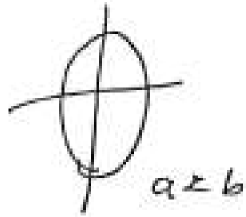
$\cancel{4} = a\left(\frac{16^4}{25}\right) \Rightarrow \frac{25}{4} = a. \quad \therefore a^2 = \frac{625}{16}$

$\therefore \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \Rightarrow \frac{x^2}{\frac{625}{16}} + \frac{y^2}{25} = 1$

$$\frac{16x^2}{625} + \frac{y^2}{25} = 1$$

(v) length of latus rectum 4, distance between foci $4\sqrt{2}$ and major axis as y axis

Soln $2ae = 4\sqrt{2}$ | $\frac{2b^2}{a} = 4$
$ae = 2\sqrt{2}$ | $b^2 = 2a$.



$b^2 = a^2(1-e^2)$

$2a = a^2 - a^2e^2$

$2a = a^2 - (2\sqrt{2})^2 \Rightarrow a^2 - 8 - 2a = 0$

$(a-4)(a+2) = 0$

$a = 4 \quad a = -2$ (is not possible)

$\therefore a = 4 \Rightarrow a^2 = 16$

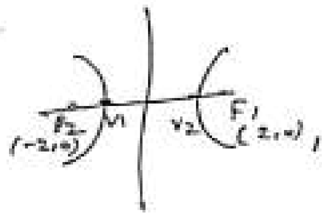
$b^2 = 2a = 2(4) = 8$.

$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

$\frac{x^2}{8} + \frac{y^2}{16} = 1$.

3) Find the equation of the hyperbola in each of the cases given below.

(i) foci $(\pm 2, 0)$ eccentricity $= 3/2$.



Soln $ae = 2 \quad e = 3/2$

$a\left(\frac{3}{2}\right) = 2$

$a = \frac{4}{3}$. $\quad a^2 = \frac{16}{9}$

$b^2 = a^2(e^2 - 1) = \frac{16}{9}\left(\frac{9}{4} - 1\right) = \frac{16}{9}\left(\frac{5}{4}\right)$

$= \frac{20}{9}$.

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

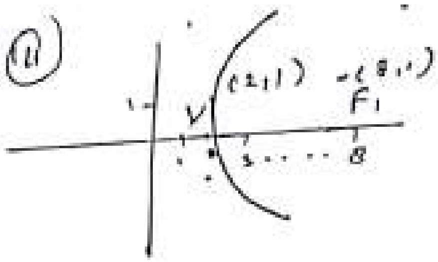
$\frac{x^2}{\frac{16}{9}} - \frac{y^2}{\frac{20}{9}} = 1$

$\frac{9x^2}{16} - \frac{9y^2}{20} = 1$.

(ii) centre (2,1) one of the foci (8,1) and corresponding directrix $x = 4$.

Soln Distance VF = ae = 6 —(1)

Directrix $\frac{a}{e} = 4.$ —(2)

(ii) 

(1)×(2) $ae \times \frac{a}{e} = 24$

$a^2 = 24.$

$\frac{(1)}{(2)} \quad \frac{ae}{a/e} = \frac{6}{4}$

$e^2 = \frac{3}{2} \qquad e = \frac{\sqrt{3}}{\sqrt{2}}$

$b^2 = a^2(e^2-1)$

$b^2 = 24\left(\frac{3}{2}-1\right) = 24\left(\frac{1}{2}\right) = 12.$

$(h,k) = (2,1)$

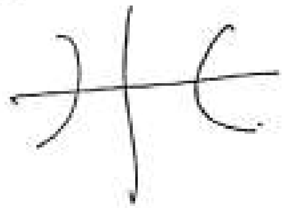
$\therefore \frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1.$

$\frac{(x-2)^2}{24} - \frac{(y-1)^2}{12} = 1.$

(iii) passing through (5,−2) and length of the transverse axis along x axis and length 8 units

Soln Transverse length $2a = 8$

$a = 4$



$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ —(1)

at (5,−2) $\frac{25}{16} - \frac{4}{b^2} = 1$

$\frac{25}{16} - 1 = \frac{4}{b^2}.$

$\frac{25-16}{16} = \frac{4}{b^2} \Rightarrow \frac{9}{16} = \frac{4}{b^2}$

$b^2 = \frac{16\times4}{9} = \frac{64}{9}$

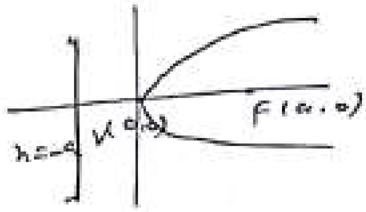
(1) $\Rightarrow \therefore \frac{x^2}{16} - \frac{y^2}{\frac{64}{9}} = 1$

$\boxed{\frac{x^2}{16} - \frac{9y^2}{64} = 1}$

2) Find the vertex, focus, equation of directrix and length of the latus rectum of the following.

Soln (i) $y^2 = 16x = 4(4)x$

$a = 4.$



(i) Vertex $V(0,0)$

(ii) Focus $F(a,0) = F(4,0)$

(iii) Eqn of the directrix $x = -a$

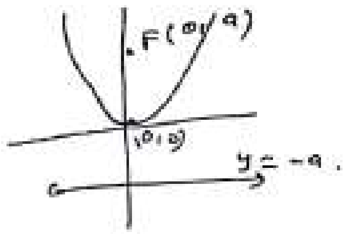
$x = -4 \Rightarrow x+4=0$

(iv) Length of the latus rectum $= 4a = 4(4) = 16.$

(ii) $x^2 = 24y.$

$4a = 24$

$a = 6.$



(i) Vertex $V(0,0)$, (ii) Focus $F(0,a)$

$= F(0,6)$

(iii) Eqn of the directrix $y = -a = -6$

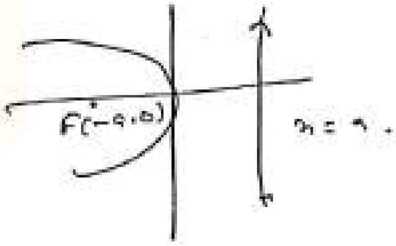
$\Rightarrow y+6=0$

(iv) Length of the latus rectum $= 4a = 4(6) = 24$

(iii) $y^2 = -8x$ (i)

$4a = 8$

$a = 2$



(i) Vertex $(0,0) = (0,0)$

(ii) Focus $(-a,0) = (-2,0).$

(iii) Eqn of the directrix $x = a = 2$

$x-2=0$

(iv) length of the latus rectum $4a = 8.$

(iv) $x^2 - 2x + 8y + 17 = 0$

$x^2 - 2x = -8y - 17$

$(x-2)^2 = -8y - 17 + 1$

$(x-2)^2 = -8y - 16 = -8(y+2)$

$X^2 = -8Y.$ $\quad X = x-2 \quad Y = y+2$

$x = X+2 \quad y = Y-2.$

$4a = 8$

$\boxed{a = 2}$

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(i) Vertex be $(1, -2) = (h, k)$

(ii) Focus $(0+h, +a+k)$

$= (0+1, +2-2) = (1, -2)$

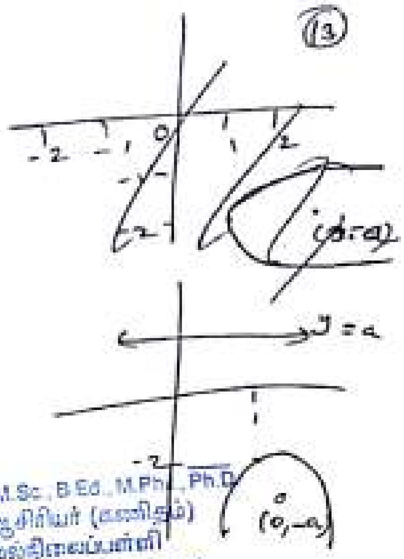
(iii) Eqn of the directrix $= y+k+a = 0$

$y-2+2=0$

$y=0$

(iv) Length of latus rectum is $4a$

$= 4 \times 2 = 8$ units.



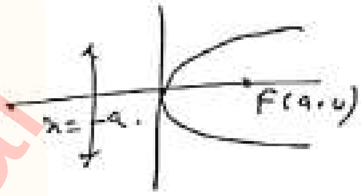
(ii) $y^2-4y-8x+12=0$.

$y^2-4y = 8x-12$.

$(y-2)^2 = 8x-12+4$. $= 8x-8 = 8(x-1)$

$(y-2)^2 = 8(x-1)$

$Y^2 = 8X$



$Y = y-2$ $\quad X = x-1$ $\quad 4a = 8$

$y = Y+2$ $\quad x = X+1$ $\quad a = 2$

vertex $(1, 2) = (h, k)$

(ii) Focus $(a+h, 0+k) = (2+1, 0+2) = (3, 2)$

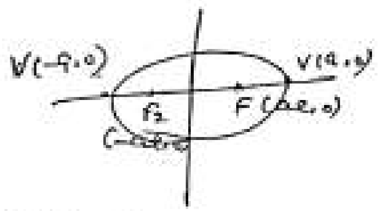
(iii) Eqn of the directrix $x = -a+h = -2+1 = -1$

$\boxed{x+1=0}$

(iv) Length of latus rectum is $4a = 4 \times 2 = 8$ units.

5) Identify the type of conic and find centre, foci, vertices and directrices of each of the following.

(i) $\frac{x^2}{25} + \frac{y^2}{9} = 1$.



it is ellipse.

$a^2 = 25$ $\quad b^2 = 9$.

$a = 5$

$e^2 = \left(1-\frac{b^2}{a^2}\right) = \left(1-\frac{9}{25}\right) = \frac{25-9}{25} = \frac{16}{25}$

$e = \frac{4}{5}$ $\quad ae = 5\left(\frac{4}{5}\right) = 4$

~~Vert~~ centre $= (0,0)$, vertex $(\pm a, 0) = (\pm 5, 0)$

Foci $(\pm ae, 0) = (\pm 4, 0)$

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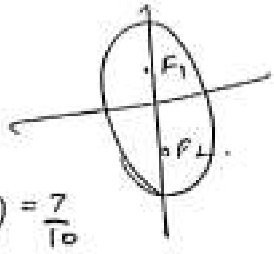
Eqn of directrix $x = \pm a/e = \pm \frac{5}{4/5} = \pm \frac{25}{4}$

$$\boxed{x = \pm \frac{25}{4}}$$

(ii) $\frac{x^2}{3} + \frac{y^2}{10} = 1$ $\quad a^2 < b^2$ it is ellipse.



$a^2 = 10$ $\quad b^2 = 3.$

$a = \pm\sqrt{10}$ $\quad e^2 = \left(1 - \frac{b^2}{a^2}\right) = \left(1 - \frac{3}{10}\right) = \frac{7}{10}$

$e^2 = \frac{7}{10}$ $\quad \therefore e = \sqrt{\frac{7}{10}}.$

$ae = \sqrt{10} \cdot \frac{\sqrt{7}}{\sqrt{10}} = \sqrt{7}.$

(i) centre $(0,0)$

(ii) vertex $(0, \pm a) = (0 \pm \sqrt{10})$

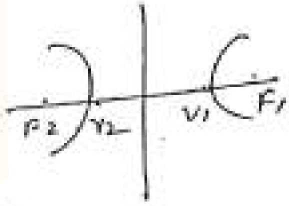
(iii) Foci $(0, \pm ae) = (0, \pm\sqrt{7})$

(iv) Eqn of the directrix $y = \pm a/e = \pm \frac{\sqrt{10}}{\sqrt{7}} \cdot \sqrt{10} = \pm \frac{10}{\sqrt{7}}$

$y = \pm \frac{10}{\sqrt{7}}.$

(iii) $\frac{x^2}{25} - \frac{y^2}{144} = 1.$

it is Hyperbola.



$a^2 = 25$ $\quad b^2 = 144$

$e^2 = \left(1 + \frac{b^2}{a^2}\right) = \left(1 + \frac{144}{25}\right) = \left(\frac{169}{25}\right)$

$e^2 = \frac{169}{25}$ $\quad \therefore e = \frac{13}{5}$

$a = 5$ $\quad ae = 5\left(\frac{13}{5}\right) = 13$

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(i) centre $(0,0)$

(ii) Vertex $(\pm a, 0) = (\pm 5, 0)$

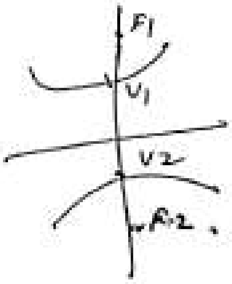
(iii) Foci $(\pm ae, 0) = (\pm 13, 0)$

(iv) Eqn of the directrix $x = \pm a/e = \pm \frac{5}{13/5} = \pm \frac{25}{13}$

$x = \pm \frac{25}{13}.$

(iv) $\frac{y^2}{16} - \frac{x^2}{9} = 1.$

it is Hyperbola about y axis



$a^2 = 16$ $\quad b^2 = 9.$

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i) Vertex be $(1, -2) = (h, k)$

ii) Focus $(a+h, a+k)$

$= (\cancel{a}+1, \cancel{a}-2) = (1, -2)$

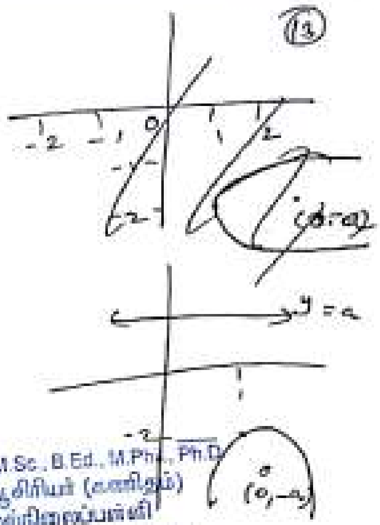
iii) Eqn of the directrix $= y+k+a = 0$

$y-2+2=0$

$y=0$

iv) Length of latus rectum is $4a$

$= 4 \times 2 = 8$ units.



(ii) $y^2-4y-8x+12=0.$

$y^2-4y = 8x-12.$

$(y-2)^2 = 8x-12+4. = 8x-8 = 8(x-1)$

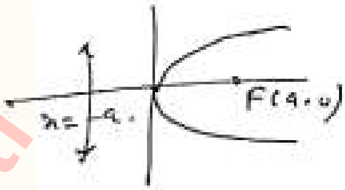
$(y-2)^2 = 8(x-1)$

$Y^2 = 8X$

$Y = y-2 \qquad X = x-1 \qquad 4a = 8$

$y = Y+2 \qquad x = X+1 \qquad a = 2$

vertex $(1, 2) = (h, k)$



(ii) Focus $(a+h, 0+k) = (2+1, 0+2) = (3, 2)$

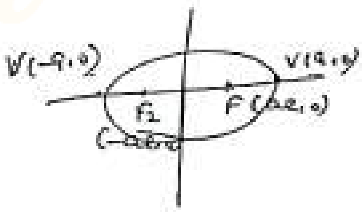
(iii) Eqn of the directrix $x = -a+h = -2+1 = -1$

$\boxed{x+1=0}$

iv) Length of latus rectum is $4a = 4 \times 2 = 8$ units.

5) Identify the type of conic and find centre, foci, vertices and directrices of each of the following.

(i) $\frac{x^2}{25} + \frac{y^2}{9} = 1.$



it is ellipse.

$a^2 = 25 \qquad b^2 = 9.$

$a = 5$

$e^2 = \left(1 - \frac{b^2}{a^2}\right) = \left(1 - \frac{9}{25}\right) = \frac{25-9}{25} = \frac{16}{25}$

$e = \frac{4}{5} \qquad ae = 5\left(\frac{4}{5}\right) = 4$

centre $= (0,0)$, vertex $(\pm a, 0) = (\pm 5, 0)$

Foci $(\pm ae, 0) = (\pm 4, 0)$

$a = 4.$ $e^2 = \left(1 + \frac{b^2}{a^2}\right) = \left(1 + \frac{9}{16}\right) = \frac{25}{16}$ (5)

$e = 5/4$

$ae = 4(5/4) = 5$

(i) Centre $(0,0)$

(ii) Vertex $(0, \pm a) = (0, \pm 4)$

(iii) Foci $(0, \pm ae) = (0, \pm 5)$

(iv) Eqn of the directrix $y = \pm a/e = \pm \frac{4}{5/4} = \pm \frac{16}{5}$

$y = \pm \frac{16}{5}.$

6) Prove that the length of the latus rectum of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $\frac{2b^2}{a}.$

Soln The latus rectum LL' of an hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ passes through (ae, y_1).

$$\frac{a^2e^2}{a^2} - \frac{y_1^2}{b^2} = 1$$

$$e^2 - 1 = \frac{y_1^2}{b^2} \qquad \left(\because e^2 = 1 + \frac{b^2}{a^2}\right)$$

$$y_1^2 = b^2(e^2 - 1)$$

$$= b^2\left(\cancel{1} + \frac{b^2}{a^2} - \cancel{1}\right)$$

$$y_1^2 = \frac{b^4}{a^2}.$$

$$y_1 = \pm \frac{b^2}{a}.$$

end points of latus rectums are

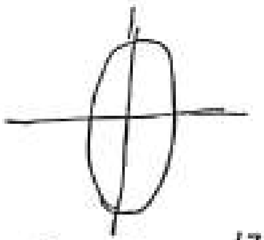
$(ae, b^2/a)$ & $(ae, -b^2/a)$ $\therefore LL' = \frac{b^2}{a} + \frac{b^2}{a}.$

$LL' = 2b^2/a.$ Hence proved.

7) Show that the absolute value of difference of the focal distances of any point P on the hyperbola is the length of its transverse axis

8. Identify the type of conic and find centre, foci, vertices and directrices of each of the following.

(i) $\frac{(x-3)^2}{225} + \frac{(y-4)^2}{289} = 1$ $\quad a^2 < b^2$



It is ellipse.

$a^2 = 289 \qquad b^2 = 225$

$a = \pm 17 \qquad b = \pm 15$, $ae = 17\left(\frac{8}{17}\right) = 8$, $a/e = \frac{17}{8/17} = \frac{289}{8}$

$e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{225}{289}} = \sqrt{\frac{289-225}{289}} = \sqrt{\frac{64}{289}} = \frac{8}{17}$.

(i) Centres $(3, 4) = (h, k)$

(ii) Vertices $(h+0, \pm a + k) = (3, \pm 17 + 4)$
$= (3, 21)$ & $(3, -13)$

(iii) Foci $(h+0, \pm ae + k) = (3, \pm 8 + 4)$
$= (3, 12)$ & $(3, -4)$

(iv) directrices $y = \pm a/e + k = \pm \frac{289}{8} + 4$
$= \frac{289}{8} + 4$ or $-\frac{289}{8} + 4$

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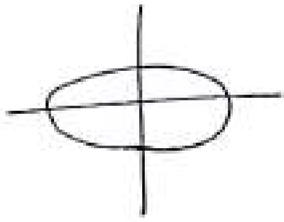
$= \frac{289+32}{8}$ or $\frac{-289+32}{8}$ $= \frac{321}{8}$ or $-\frac{257}{8}$.

(ii) $\frac{(x+1)^2}{100} + \frac{(y-2)^2}{64} = 1$

it is ellipse $a^2 > b^2$



$a^2 = 100$ $b^2 = 64$

$a = 10$

$e^2 = \left(1 - \frac{b^2}{a^2}\right) = 1 - \frac{64}{100} = \frac{36}{100}$

$e = 3/5$.

$ae = 10(3/5) = 6$; $a/e = \frac{10}{3/5} = \frac{50}{3}$.

$X = x+1$ $Y = y-2$

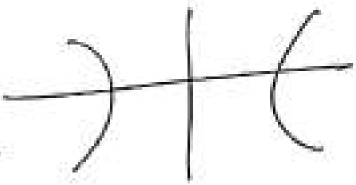
$x = X-1$ $y = Y+2$

	Referred to X & Y	Referred to x & y $x = X-1$ & $y = Y+2$
(i) centre	$(0,0)$	$(-1, 2)$
(ii) Vertices	V : $(\pm a, 0)$ V_1 : $(+10, 0)$ V_2 : $(-10, 0)$	V_1 $(+9, 2)$ V_2 $(-11, 2)$
(iii) Focus	$F(\pm ae, 0)$ $F_1(+6, 0)$ $F_2(-6, 0)$	$F_1(5, 2)$ $F_2(-7, 2)$.
(iv) Directrices	$X = \pm a/e = \pm \frac{50}{3}$	$x = \frac{50}{3} - 1 = \frac{47}{3}$ $x = \frac{50}{-3} - 1 = -\frac{53}{3}$

(iii) $\frac{(x+3)^2}{225} - \frac{(y-4)^2}{64} = 1$ it is hyperbola on x axis

$a^2 = 225$ $b^2 = 64$

$a = \pm 15$



$e^2 = 1 + \frac{b^2}{a^2} = 1 + \frac{64}{225} = \frac{225+64}{225} = \frac{289}{225}$

$e^2 = \frac{289}{225}$ $\therefore e = \frac{17}{15}$

$ae = 15\left(\frac{17}{15}\right) = 17$

$a/e = \frac{15}{\frac{17}{15}} = \frac{225}{17}$.

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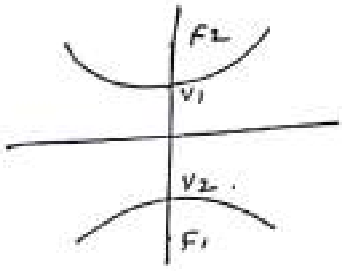
$X = x+3 \qquad Y = y-4$

$x = X-3 \qquad y = Y+4.$

	Referred to X & Y	Referred to x & y $x = X-3$ $y = Y+4$
(i) centre	$C(0,0)$	$C(-3,4)$
(ii) vertices	$V(\pm a, 0)$ $V_1(+15,0)$ $V_2(-15,0)$	$V_1(12,4)$ $V_2(-18,4)$
(iii) Foci	$F(\pm ae, 0)$ $F_1(17,0)$ $F_2(-17,0)$	$F_1(14,4)$ $F_2(-20,4)$
(iv) Directrix	$X = \pm a/e = \pm \frac{225}{17}$	$x = \frac{225}{17} - 3 = \frac{174}{17}$ $x = -\frac{225}{17} - 3 = -\frac{276}{17}$

(iv) $\frac{(y-2)^2}{25} - \frac{(x+1)^2}{16} = 1$



it is hyperbola on Y axis

$a^2 = 25 \qquad b^2 = 16$

$a = \pm 5$

$e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{16}{25}} = \sqrt{\frac{41}{25}} = \frac{\sqrt{41}}{5}$

$ae = 5\left(\frac{\sqrt{41}}{5}\right) = \sqrt{41}$

$a/e = 5 / \frac{\sqrt{41}}{5} = \frac{25}{\sqrt{41}}$

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$X = x+1 \qquad Y = y-2$

$x = X-1 \qquad y = Y+2$

	Referred to X & Y	Referred to x & y $x = X-1$ & $y = Y+2$
(i) centre	$C(0,0)$	$(-1,2)$
(ii) vertices $(0, \pm a)$	$V_1(0,5)$ $V_2(0,-5)$	$V_1(-1,7)$ $V_2(-1,-3)$
(iii) Foci $(0, \pm ae)$	$F_1(0, +\sqrt{41})$ $F_2(0, -\sqrt{41})$	$F_1(-1, \sqrt{41}+2)$ $F_2(-1, -\sqrt{41}+2)$
(iv) Directrix	$Y = \pm a/e = \pm \frac{25}{\sqrt{41}}$	$y = \frac{25}{\sqrt{41}} + 2$ $y = \frac{-25}{\sqrt{41}} - 2$

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(v) $18x^2+12y^2-144x+48y+120=0$

$18x^2-144x+12y^2+48y=-120$

$18(x^2-8x)+12(y^2+4y)=-120$

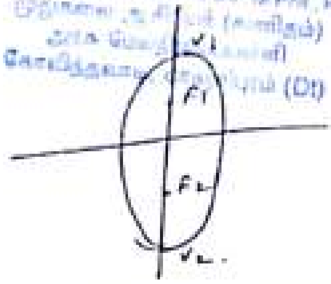
$18(x-4)^2+12(y+2)^2=-120+288+48$

$18(x-4)^2+12(y+2)^2=216$

$18(x-4)^2+12(y+2)^2=216$

$\frac{18(x-4)^2}{216}+\frac{12(y+2)^2}{216}=1$

$\frac{(x-4)^2}{12}+\frac{(y+2)^2}{18}=1$



it is ellipise on y axis

$a^2=18 \quad b^2=12$

$a=\sqrt{18}=3\sqrt{2}$

$e=\sqrt{1-\frac{b^2}{a^2}}=\sqrt{1-\frac{12}{18}}=\sqrt{1-\frac{2}{3}}$

$=\sqrt{\frac{1}{3}}=\frac{1}{\sqrt{3}}$

$ae=3\sqrt{2}\cdot\frac{1}{\sqrt{3}}=\sqrt{6}$

$a/e=\frac{3\sqrt{2}}{1/\sqrt{3}}=3\sqrt{6}$

$X=x-4 \quad Y=y+2$

$x=X+4 \quad y=Y-2$

	Referred to X & Y	Referred to x & y
(i) Centre	$C(0,0)$	$(4,-2)$
(ii) Vertices $(0,\pm a)$	$V_1(0,+3\sqrt{2})$ $V_2(0,-3\sqrt{2})$	$V_1(4,3\sqrt{2}-2)$ $V_2(4,-2-3\sqrt{2})$
(iii) Foci $(0,\pm ae)$	$F_1(0,\sqrt{6})$ $F_2(0,-\sqrt{6})$	$F_1(4,-2+\sqrt{6})$ $F_2(4,-2-\sqrt{6})$
(iv) Directric $Y=\pm a/e$	$Y=\pm 3\sqrt{6}$	$y=3\sqrt{6}-2$ (or) $=-3\sqrt{6}-2$

(vi) $9x^2-y^2-36x-6y+18=0$

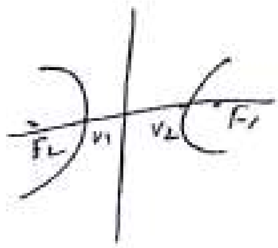
$9x^2-36x-y^2-6y=-18$

$9(x^2-4x)-(y^2+6y)=-18$

$9(x-2)^2-(y+3)^2=-18+36-9$

$9(x-2)^2-(y+3)^2=9$

$\frac{(x-2)^2}{1}-\frac{(y+3)^2}{9}=1$



it is hyperbola on x axis

$a^2 = 9 \quad b^2 = 1 \qquad c = \sqrt{1+\frac{b^2}{a^2}} = \sqrt{1+\frac{1}{9}} = \sqrt{\frac{10}{9}} = \frac{\sqrt{10}}{3}$ (20)

$a = 3$

$ae = 3 \times \frac{\sqrt{10}}{3} = \sqrt{10}$

$X = x-2 \qquad Y = y+3$

$a/e = \frac{3}{\sqrt{10}/3} = \frac{9}{\sqrt{10}}$

$x = X+2 \qquad y = Y-3$

	Referred to X & Y	Referred to x & y $x = X+2 \quad y = Y-3$
(i) Centre	$C(0,0)$	$(2,-3)$
(ii) vertices $(\pm a, 0)$	$V_1(3,0)$ $V_2(-3,0)$	$V_1(5,-3)$ $V_2(-1,-3)$
(iii) Foci $(\pm ae, 0)$	$F_1(\sqrt{10},0)$ $F_2(-\sqrt{10},0)$	$F_1(2+\sqrt{10},-3)$ $F_2(2-\sqrt{10},-3)$
(iv) directrix	$X = \pm a/e = \pm\frac{9}{\sqrt{10}}$	$x = \frac{9}{\sqrt{10}}+2$ (or) $= -\frac{9}{\sqrt{10}}+2$

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Exercise 5.3

Identifying the conics from the general equation of the Conic.

$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

(i) $A = C = 1$, $B = 0$ $D = -2h$, $E = -2k$, $F = (h^2 + k^2 - r^2)$ which is a circle $(x-h)^2 + (y-k)^2 = r^2$.

(ii) $B = 0$ and either A or $C = 0$ which is parabola.

(iii) $A \neq C$ & A and C are the same signs it is ellipse

(iv) $A \neq C$ & A and C are of opposite signs. it is hyperbola

(v) $A = C$ and $B = D = E = F = 0$ it is $x^2 + y^2 = 0$.

(vi) $A = C = F$ & $B = D = E = 0$ it is $x^2 + y^2 + 1 = 0$. as There is no real solution.

(vii) $A = 0$ or $C \neq 0$ and others are zeros the general equation yield coordinate axes.

(viii) $A = -C$ and rests are zero. $\Rightarrow x^2 - y^2 = 0$

I. identify the type of conic section of each of the equations.

	Equation	Condition	Types of the Conic.
1.	$2x^2 - y^2 = 7$.	4	Hyperbola
2.	$3x^2 + 3y^2 - 4x + 3y + 10 = 0$	1	Circle.
3.	$3x^2 + 2y^2 = 14$	3	ellipse
4.	$x^2 + y^2 + x - y = 0$	1	circle.
5.	$11x^2 - 25y^2 - 44x + 50y - 256 = 0$	4	hyperbola.
6.	$y^2 + 4x + 3y + 4 = 0$	2	Parabola

Exercise 5.4

1. Find the equation of the two tangents that can be drawn from (5, 2) to the ellipse $2x^2+7y^2=14$.

Soln $2x^2+7y^2=14$

$\frac{x^2}{7}+\frac{y^2}{2}=1$

$y=mx+\sqrt{a^2m^2+b^2}$

$a^2=7 \quad b^2=2$

it passes through (5, 2)

$2=5m+\sqrt{7m^2+2}$

$2-5m=\sqrt{7m^2+2}$

$(2-5m)^2=(7m^2+2)$

$4+25m^2-20m=7m^2+2$

$18m^2-20m+2=0$

$9m^2-10m+1=0$

$(9m-1)(m-1)=0$

$m=1$ (or) $m=\frac{1}{9}$.

Eqn of tangent

(i) $m=1$ $\quad (y-2)=1(x-5)$

$\boxed{x-y-3=0}$

(ii) $m=\frac{1}{9}$ $\quad y-2=\frac{1}{9}(x-5)$

$9y-18=x-5$

$\boxed{x-9y+13=0}$

∴ Eqn of tangent be $x-y-3=0$ & $x-9y+13=0$.

2. Find the equation of tangents to the hyperbola $\frac{x^2}{16}-\frac{y^2}{64}=1$ which is parallel to $10x-3y+9=0$.

Soln $\frac{x^2}{16}-\frac{y^2}{64}=1$

$10x-3y+9=0$

$m=\frac{-10}{-3}=\frac{10}{3}$ $\quad c=\frac{9}{10}$.

$a^2=16 \quad b^2=64$.

$\sqrt{a^2m^2-b^2}=\sqrt{16\left(\frac{10}{3}\right)^2-64}=\sqrt{\frac{1600}{9}-64}=\sqrt{\frac{1600-576}{9}}$

$=\sqrt{\frac{1024}{9}}=\frac{32}{3}$

Eqn of tangent to the hyperbola be

$y=mx\pm\sqrt{a^2m^2-b^2}$

$y=\frac{10}{3}x\pm\frac{32}{3}$

$y=\frac{10}{3}x+\frac{32}{3}$ or $y=\frac{10}{3}x-\frac{32}{3}$

$3y=10x+32$ $\qquad 3y=10x-32$

$\boxed{10x-3y+32=0}$ $\qquad \boxed{10x-3y-32=0.}$

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4) Find the equation of the tangent to the parabola $y^2 = 16x$ perpendicular to $2x + 2y + 3 = 0$.

Soln $y^2 = 16x$ $\qquad$ $2x + 2y + 3 = 0$

$4a = 16$ $\qquad$ $x + y + \frac{3}{2} = 0$

$a = 4$

The equation of the line which is perpendicular to $x + y + 3/2 = 0$ is

$x - y + c = 0$. $\quad$ — (1)

$m = \frac{-1}{-1} = 1$ $\qquad$ $c = \frac{a}{m} = \frac{4}{1} = 4$

$\therefore$ (1) $\Rightarrow$ $\boxed{x - y + 4 = 0}$

3) Show that the line $x - y + 4 = 0$ is a tangent to the ellipse $x^2 + 3y^2 = 12$ Also find the co-ordinates of the point of contact.

Soln $y = mx + c$ be tangent to $x^2 + 3y^2 = 12$

The condition is $c^2 = a^2m^2 + b^2$.

$x - y + 4 = 0$

$x + 4 = y$

$m = 1$ $\quad$ $c = 4$

$x^2 + 3y^2 = 12$

$\frac{x^2}{12} + \frac{y^2}{4} = 1$

$a^2 = 12$ $\qquad$ $b^2 = 4$.

$c^2 = a^2m^2 + b^2$

$4^2 = 12(1)^2 + 4$

$16 = 16$.

$\therefore$ Given line touches tangent $x^2 + 3y^2 = 12$.

Point of contact $\left(-\frac{a^2m}{c}, -\frac{b^2}{c}\right)$

$= \left(\frac{-12(1)}{4}, \frac{4}{4}\right) = (-3, 1)$

5) Find the equation of the tangent and normal to hyperbola $12x^2 - 9y^2 = 108$ at $\theta = \pi/3$

(Hint use parametric form)

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Soln Eqn of the tangent to hyperbola be

$$\frac{x\sec\theta}{a} - \frac{y\tan\theta}{b} = 1$$

Given. $12x^2 - 9y^2 = 108$

$$\frac{12x^2}{108} - \frac{9y^2}{108} = 1$$

$$\frac{x^2}{9} - \frac{y^2}{12} = 1$$

$a^2 = 9$ $\quad b^2 = 12$ $\quad \theta = \pi/3$

$a = 3$ $\quad b = 2\sqrt{3}$

$$\therefore \frac{x\sec\pi/3}{3} - \frac{y\tan\pi/3}{2\sqrt{3}} = 1$$

$$\frac{x(2)}{3} - \frac{y\sqrt{3}}{2\sqrt{3}} = 1$$

$$\frac{2x}{3} - \frac{y}{2} = 1$$

$$\frac{4x-3y}{6} = 1 \Rightarrow \boxed{4x-3y=6}$$

(ii) Eqn of the normal to hyperbola be

$$\frac{ax}{\sec\theta} + \frac{by}{\tan\theta} = a^2+b^2.$$

$$\frac{3x}{\sec\pi/3} + \frac{2\sqrt{3}\,y}{\tan\pi/3} = 9+12.$$

$$\frac{3x}{2} + \frac{2\sqrt{3}\,y}{\sqrt{3}} = 21$$

$$\frac{3x+4y}{2} = 21$$

$$\boxed{3x+4y=42}$$

5) Find the equation of the tangent at $t=2$ to the parabola $y^2 = 8x$ (Hint use parametric form).

Soln Eqn of a tangent be

$$yt = x + at^2 \qquad \text{Here } t = 2$$

$y^2 = 8x \quad \therefore 4a = 8$

$a = 2$

$$y(2) = x + 2(4)$$
$$2y = x + 8$$
$$\boxed{x - 2y + 8 = 0}$$

7) Prove that the point of intersection of the tangents at t_1 and t_2 on the parabola $y^2 = 4ax$ is $[at_1t_2,\ a(t_1+t_2)]$

Soln Eqn of the tangent of parabola $y^2 = 4ax$ be

$$yt = x + at^2$$

at t_1 $\quad yt_1 = x + at_1^2 \quad$ — ①

at t_2 $\quad yt_2 = x + at_2^2 \quad$ — ②

① − ② $\quad y(t_1 - t_2) = a(t_1^2 - t_2^2)$

$$y\,\cancel{(t_1 - t_2)} = a(t_1 + t_2)\,\cancel{(t_1 - t_2)}$$

$$y = a(t_1 + t_2)$$

① ⇒ $t_1 a(t_1 + t_2) = x + at_1^2$

$$x = a\cancel{t_1^2} + at_1t_2 - \cancel{at_1^2}$$

$$x = at_1t_2$$

∴ Point of Intersection be $(at_1t_2,\ a(t_1+t_2))$

8) If the normal at the point t_1 on the parabola $y^2 = 4ax$ meets the parabola again at the point t_2 then prove that $t_2 = -\left(t_1 + \frac{2}{t_1}\right)$

Soln $\quad y^2 = 4ax$

$$yy_1 = \cancel{4}a\left(\frac{x + x_1}{\cancel{2}}\right)$$

$$yy_1 = 2a(x + x_1)$$

At $(at_1^2, 2at_1)$ ⇒ $y(2at_1) = 2a(x + at_1^2)$

$$y = \frac{1}{t_1}x + at_1$$

∴ slope of tangent be $\frac{1}{t_1}$

Slope of normal be $\frac{-1}{1/t_1} = -t_1.$

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Eqn of normal at $(at_1^2, 2at_1)$ is

$$\frac{y-2at_1}{x-at_1^2} = -t_1$$

$$y-2at_1 = -t_1(x-at_1^2)$$

$(at_2^2, 2at_2)$ lies on parabola.

$$2at_2-2at_1 = -t_1(at_2^2-at_1^2)$$

$$2a(t_2-t_1) = -t_1 a(t_2^2-t_1^2)$$

$$2a(t_2-t_1) = -at_1(t_2-t_1)(t_1+t_2)$$

$$2(t_2-t_1) + (t_2-t_1)t_1(t_1+t_2) = 0$$

$$(t_2-t_1)(2+t_1(t_1+t_2)) = 0$$

$t_1 \neq t_2$ $\therefore$ $2+t_1(t_1+t_2) = 0$

$$t_1(t_1+t_2) = -2$$

$$t_1+t_2 = -\frac{2}{t_1}$$

$$t_2 = -\frac{2}{t_1} - t_1$$

$$\boxed{t_2 = -\left(\frac{2}{t_1}+t_1\right)}$$

Hence proved.

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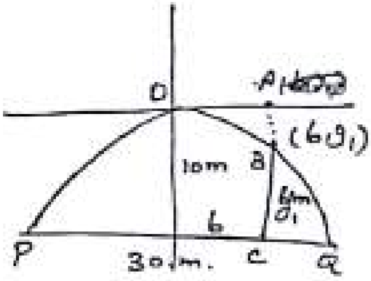
Exercise 5.5

1. A bridge has a parabolic arch that is 10m high in the centre and 30m wide at the bottom. Find the height of the arch 6m. from the centre on either sides.

Soln



$PQ = 2a = 30m$

$a = 15m.$

Point Q be $(15, -10)$.

Eqn of the parabola

$x^2 = -4ay$. — (1)

Q lies on parabola.

$15^2 = -4a(-10)$

$\frac{225}{40} = a \Rightarrow a = \frac{225}{40}$

(1) $\Rightarrow x^2 = -4\left(\frac{225}{40}\right) y = \cancel{-800y}$.

$x^2 = -\frac{225}{10} y$

Let $B(6, y_1)$ lies on parabola

$6^2 = -\frac{225}{10} y_1$

$y_1 = \frac{36 \times 10}{-225} = \frac{-8}{5} = \frac{8}{5} m.$

$AB = AC - BC = 10 - \frac{8}{5} = \frac{50-8}{5} = \frac{42}{5}$

$AB = 8.4m$

∴ The heigh of the arch 6m from the centre is 8.4m.

2. A tunel through a mountain for a four lane highway is to have a elliptical opening. The total width of the highway (not the opening) is to be 16m. and the height at the edge of the road must be sufficient for a truck 4m high to clear if the highest point of the opening is to be 5m approximately. How wide must the opening be?

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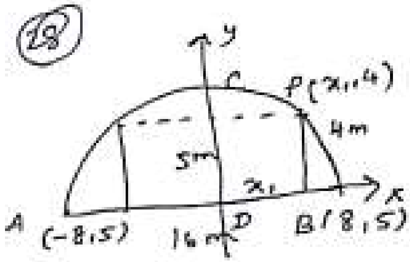


Soln $AB = 2a = 16$

$a = 8$

$b = 5$.

Eqn of the ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{64} + \frac{y^2}{25} = 1.$$

let $P(x_1, 4)$ lies on ellipse.

$$\frac{x_1^2}{64} + \frac{4^2}{25} = 1$$

$$\frac{x_1^2}{64} = 1 - \frac{16}{25} = \frac{25-16}{25} = \frac{9}{25}$$

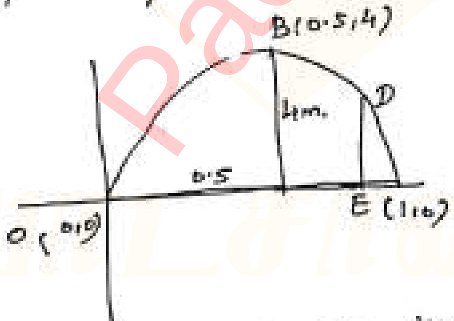
$$x_1^2 = 64 \times \frac{9}{25}$$

$$x_1 = \frac{8 \times 3}{5} = \frac{24}{5} = 4.8.$$

wide $= 2x_1 = 2 \times 4.8 = 9.6m.$

3) At a water fountain, water attains a maximum height of 4m at horizontal distance of 0.5m from its origin. If the path of water is a parabola. find the height of water at a horizontal distance of 0.75m from the point of origin

Soln



$y = ax^2 + bx + c$. it passes through $O(0,0)$, $B(0.5,4)$ And $E(1,0)$.

(i) At $(0,0)$ $c = 0$

(ii) At $E(1,0)$ $a+b+c = 0$

$a+b = 0$ — ①

(iii) At $B(0.5,4)$ $4 = 0.25a + 0.5b + c = 0$

$0.25a + 0.5b = 4$ — ②

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$① \times 0.5 \Rightarrow \quad 0.5a + 0.5b = 6$

$\quad 0.25a + 0.5b = 4$

$\quad (-) \quad (-) \quad (-)$

$-0.25a = -4$

$a = \frac{4}{0.25} = \frac{400}{25} = 16$

$a + b = 0$

$b = -a = -16.$

$\therefore c = 0 \quad a = +16, \quad b = -16.$

$\therefore y = -16x^2 + 16x.$

when $x = 0.75$ $\quad y = -16(0.75)^2 + 16(0.75)$

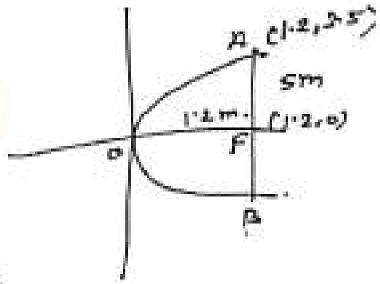
$= -16(0.5625) + 12$

$y = -9 + 12$

$y = 3m.$

Height of water at a horizontal distance of 0.75m from the point of origin is 3m.

4) An engineer designs a satellite dish with a parabolic cross section. The dish is 5m wide at the opening and the focus is placed 1.2m from the vertex.
a) Position a co-ordinate system with the origin at the vertex and the x axis on the parabola's axis of symmetry and find an equation of the parabola
b) find the depth of the satellite dish at the vertex.



soln Eqn of the parabola be

$y^2 = 4ax$ — ①

$a = 1.2.$ $\quad \therefore y^2 = 4 \times 1.2 x$

$\Rightarrow y^2 = 4.8x$

$AB = 5$

$AF = 5/2 = 2.5$

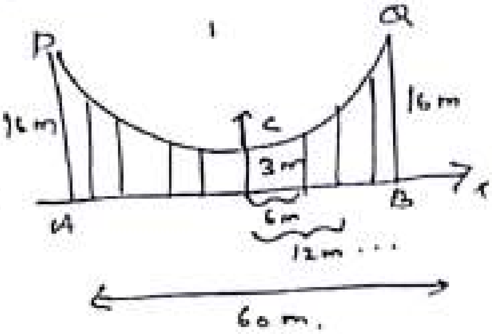
Point A be $(1.2, 2.5)$ lies on parabola.

$(2.5)^2 = 4(1.2)x$

$\frac{2.5 \times 2.5}{4 \times 1.2} = x \quad \Rightarrow \quad x = \frac{25 \times 25}{40 \times 12} = 1.3m.$

$\boxed{\begin{array}{l} \therefore x = 1.3m \\ y^2 = 4.8x \end{array}}$

5) Parabolic cable of a 60m portion of the roadbed of a suspension bridge are positioned as shown below. Vertical cables are to be spaced every 6m along this portion of the roadbed. Calculate the lengths of first two of these vertical cables from the vertex.



soln

vertex be $V(0,3) = (h,k)$

$2a = 60m$

$a = 30.$

$\therefore$ Let A & B be $(30,-16)$ and $(30,16)$.

Eqn of the parabola be

$(x-h)^2 = 4a(y-k)$. —①

B(30,16) lies on ①

$(30-0)^2 = 4a(16-3)$

$30^2 = 4a(13)$

$a = \frac{30\times 30}{4\times 13}$.

(i) when $x = 6$

$(x-h)^2 = 4a(y-k)$

$(6-0)^2 = \cancel{4}\left(\frac{30\times 30}{\cancel{4}\times 13}\right)(y-3)$

$\frac{\cancel{36}^{\cancel{12}\,4} \times 13}{30\times 30} = y-3$

$\frac{52}{100} + 3 = y$

$\therefore y = 0.52 + 3 = 3.52m.$

$\boxed{\therefore y = 3.52m}$

(ii) when $x = 12$

$(12-0)^2 = 4a(y-3)$

$144 = \cancel{4}\left(\frac{30\times 30}{\cancel{4}\times 13}\right)(y-3)$

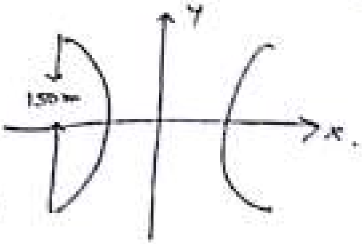
$\frac{\cancel{144}^{\cancel{48}\,16}\times 13}{30\times 30} = y-3$

$\frac{208}{100} + 3 = y$ $\Rightarrow$ $2.08 + 3 = y$

$\boxed{\therefore y = 5.08m}$

6) Cross section of a Nuclear cooling tower is in the shape of a hyperbola with equation $\frac{x^2}{30^2} - \frac{y^2}{44^2} = 1$. The tower is 150m tall. Find the distance from the top of the tower to the centre of the hyperbola is

half the distance from the base of the tower to the centre of the hyperbola. Find the diameter of the top and base of the tower. (31)



Soln Given. $p + 2p = 150$

$3p = 150$

$p = 50.$

Distance from the top of the tower $= 50m$.

Distance from the base of the tower be 100.

Eqn of hyperbola is

$$\frac{x^2}{30^2} - \frac{y^2}{44^2} = 1$$

i) $y = 50m$ Then $\frac{x^2}{30^2} - \frac{50^2}{44^2} = 1$

$$\frac{x^2}{30^2} = 1 + \frac{50^2}{44^2} = 1 + 1.29$$

$$\frac{x^2}{30^2} = 2.29$$

$$x^2 = 30^2(2.29)$$

$$x = 30\sqrt{2.29} = 45.35$$

$$\boxed{x = 45.35\,m}$$

ii) when $y = -100$.

$$\frac{x^2}{30^2} - \frac{(-100)^2}{44^2} = 1$$

$$\frac{x^2}{30^2} = 1 + \frac{(100)^2}{44^2} = 1 + 5.17 = 6.17$$

$$x^2 = 30^2(6.17)$$

$$x = 30\sqrt{6.17}$$

$$\boxed{x = 74.51\,m}$$

7) A rod of length 1.2m moves with its ends always touching the coordinate axes. The locus of a point P on the rod, which is 0.3 m from the end in contact with x axis is an ellipse. Find the eccentricity.

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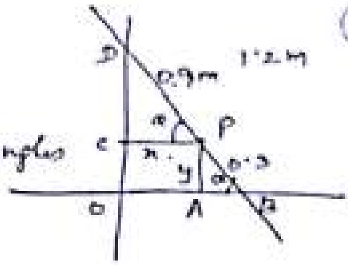
Soln. Length of rod $BD = 1.2\,m$

$PB = 0.3\,m$

$PD = 1.2 - 0.3 = 0.9\,m.$



ΔPAB & ΔPCD are similar triangles

In ΔPAB $\sin\theta = \frac{y}{0.3}$

In ΔPCD $\cos\theta = \frac{x}{0.9}$

We know that $\sin^2\theta + \cos^2\theta = 1$

$$\frac{y^2}{0.3^2} + \frac{x^2}{0.9^2} = 1$$

$$\frac{x^2}{0.9^2} + \frac{y^2}{0.3^2} = 1$$

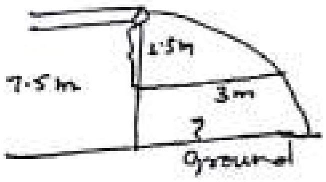
$a^2 = 0.9^2$ $b^2 = 0.3^2$

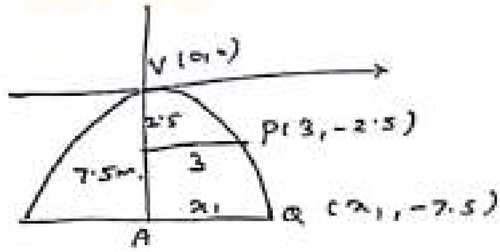
Eccentricity $e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{0.3^2}{0.9^2}} = \sqrt{1 - \left(\frac{1}{3}\right)^2}$

$$= \sqrt{1 - \frac{1}{9}} = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3}.$$

8) Assume that water issuing from the end of a horizontal pipe, 7.5 m above the ground, describes a parabolic path. The vertex of the parabola path is at the end of the pipe. At a position 2.5 m below the line of the pipe. The flow of water has curved outward 3m beyond the vertical line through the end of the pipe. How far beyond this vertical line will the water strike the ground?

Soln





Eqn of the parabola be $x^2 = -4ay$

Let $P(3, -2.5)$ lies on parabola.

$$3^2 = -4(a)(-2.5)$$

$$9 = 10a.$$

$a = \frac{9}{10}$

$\therefore x^2 = -4\left(\frac{9}{10}\right)y$ — ①

Let $Q(x_1, -7.5)$ also lies on parabola

$x_1^2 = -4\left(\frac{9}{10}\right)(-7.5)$

$x_1^2 = 4 \times \frac{9}{10} \times 7.5$

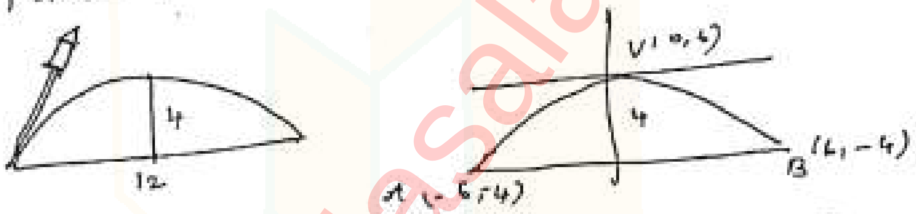
$x_1^2 = 27$

$x = 3\sqrt{3}$ m.

The water strike the ground at $3\sqrt{3}$m.

9) On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of 4m. when it is 6m away from the point of projection. Finally it reaches the ground 12m away from the starting point. Find the angle of projection.

Soln.



Eqn of the parabola be $x^2 = -4ay$. —①

$B(6,-4)$ lies on Parabola.

$6^2 = -4a(-4)$

$\frac{36}{16} = a \qquad \therefore a = \frac{9}{4}$

① ⇒ $x^2 = -4\left(\frac{9}{4}\right)y$

$x^2 = -9y$ — ②

Now need to find slope at $(-6,-4)$

Diff ② w.r.to x

$2x = -9\frac{dy}{dx}$.

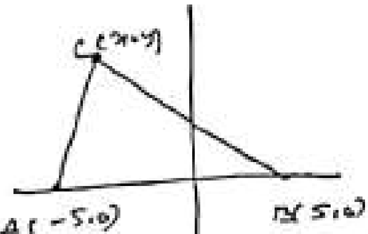
$\frac{dy}{dx} = \frac{2x}{-9}$

At $(-6,-4)$ $\frac{dy}{dx} = \frac{2(-6)}{-9} = \frac{12}{9} = \frac{4}{3}$.

$\tan\alpha = \frac{4}{3}$

$\alpha = \tan^{-1}\left(\frac{4}{3}\right)$

10) Points A and B are 10km apart and it is determined from the sound of an explosion heard at those points at different times that the location of the explosion is 6km closer to A than B. Show that the location of the explosion is restricted to a particular curve and find an equation of it.



Soln As shown in fig. A & B are on both sides of x axis at co-ordinates $(-5,0)$ and $(5,0)$

The distance between A & B is 10.

A point C is on the graph at co-ordinates (x,y)

C is 6km closer to A than B.

Distance between C & A $= \sqrt{(x+5)^2+y^2}$

Distance between C & B $= \sqrt{(x-5)^2+y^2}$.

Given distance between C & B is longer by 6

Hence $\sqrt{(x-5)^2+y^2} - \sqrt{(x+5)^2+y^2} = 6$

$\sqrt{(x-5)^2+y^2} = 6 + \sqrt{(x+5)^2+y^2}$.

Squaring on both sides we get

$(x-5)^2+y^2 = 36 + (x+5)^2+y^2 + 12\left(\sqrt{(x+5)^2+y^2}\right)$

$\cancel{x^2} + \cancel{25} - 10x + \cancel{y^2} = \cancel{x^2} + 10x + \cancel{y^2} + 36 + \cancel{25} + 12\sqrt{(x+5)^2+y^2}$.

$-20x - 36 = 12\sqrt{(x+5)^2+y^2}$.

$-5x - 9 = 3\sqrt{(x+5)^2+y^2}$

Squaring both sides we get.

$25x^2 + 81 + 90x = 9\,(x^2 + 25 + 10x + y^2)$

$25x^2 + 81 + 90x - 9x^2 - 90x - 9y^2 - 225 = 0$

$16x^2 - 9y^2 - 144 = 0$

$16x^2 - 9y^2 = 144$

$(\div 144)$ $\quad \frac{x^2}{9} - \frac{y^2}{16} = 1$ is the required eqn. of hyperbola.

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Exercise 5.6

I. Choose the most appropriate answer.

1. The equation of the circle passing through (1,5) and (4,1) and touching y axis is $x^2+y^2-5x-6y+9+\lambda(4x+3y-19)=0$ where λ is equal to.

2. The eccentricity of the hyperbola whose latus rectum is 8 and conjugate axis is equal to half the distance between the foci is

Soln

$\frac{2b^2}{a} = 8 \qquad b^2 = 4a$

$2b = \frac{1}{2}(2ae) \Rightarrow b = \frac{ae}{2}$

$\therefore \frac{2}{a}\left(\frac{ae}{2}\right)^2 = 8$

$\frac{2}{a}\left(\frac{a^2e^2}{4}\right) = 8 \Rightarrow ae^2 = 16.$

$b^2 = 4a$

$a(e^2-1) = 4$

$ae^2 - a = 4$

$16 - 4 = a$

$\boxed{a = 12}$

$\therefore ae^2 = 16$

$e^2 = \frac{16}{12} = \frac{4}{3} \qquad \therefore e = \frac{2}{\sqrt{3}}$

Ans (2).

3) The circle $x^2+y^2 = 4x+8y+5$ intersects the line $3x-4y=m$ at two distinct points if.

Soln $L: 3x-4y=m$

$C: x^2+y^2-4x-8y-5=0$

$(x-2)^2+(y-4)^2 = 25$

$C(2,4) \quad r=5.$

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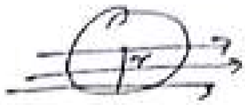
(36)

Distance from centre $< r$



$$\left|\frac{ah+bk+c}{\sqrt{a^2+b^2}}\right| < r$$

$$\left|\frac{3\times 2-4\times 4-10}{\sqrt{9+16}}\right| < r \Rightarrow \left|\frac{-10-m}{5}\right| < 5.$$

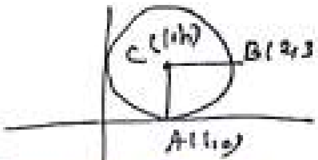
$$-5 < \frac{10+m}{5} < 5$$

$$-25 < 10+m < 25 \Rightarrow -25-10 < m < 25-10$$

$$-35 < m < 15$$

Ans (4)

4) The length of the diameter of the circle which touches the x axis at the point $(1,0)$ and passes through the point $(2,3)$



Soln

$CA = CB$

$CA^2 = CB^2$

$(1-1)^2+(h-0)^2 = (1-2)^2+(h-3)^2$

$h^2 = 1+h^2+9-6h$

$6h = 10$

$h = \frac{10}{6} = \frac{5}{3}.$

Diameter is $2h = 2\left(\frac{5}{3}\right) = \frac{10}{3}$ Ans (3).

5) The radius of the circle $3x^2+by^2+4bx-6by+b^2=0$ is

Soln

$b=3.$

$4b=12$ $6b=18$ $b^2=9.$

$\therefore$ circle be $3x^2+3y^2+12x-18y+9=0$

$(\div 3)$ $x^2+y^2+4x-6y+3=0$

$2g=4$ $2f=-6$ $c=3$

$g=2$ $f=-3$

$r = \sqrt{g^2+f^2-c} = \sqrt{4+9-3} = \sqrt{10}$ Ans (3)

6) The centre of the circle inscribed in a square formed by the lines $x^2-8x-12=0$ and $y^2-14y+45=0$ is

7) The equation of the normal to the circle $x^2+y^2-2x-2y+1=0$ which is parallel to the line $2x+4y=36$

Soln parallel line be $2x+4y+\lambda=0$

Centre be $(-g,-f)=(1,1)$

which lies on line

$2+4+\lambda=0$

$\lambda=-6$

$\therefore 2x+4y-6=0 \Rightarrow x+2y=3$ Ans (1).

$2g=-2$, $g=-1$

$2f=-2$, $f=-1$

8) If $p(x,y)$ be any point on $16x^2+25y^2=400$ with foci $F_1(3,0)$ and $F_2(-3,0)$ then PF_1+PF_2.

Soln $16x^2+25y^2=400$

$\frac{x^2}{25}+\frac{y^2}{16}=1$

$a^2=25 \quad \therefore a=\pm 5$

$PF_1+PF_2=$ major axis $=2a=2\times 5=10.$ Ans (3)

9) The radius of the circle passing through the point $(6,2)$ two of whose diameter are $x+y=6$ and $x+2y=4$ is

Soln solve $x+y=6$ —(1)

$x+2y=4$ —(2)

(1)−(2) $-y=2$, $y=-2$ $\therefore x-2=6$, $x=8$.

point be $(8,-2)$

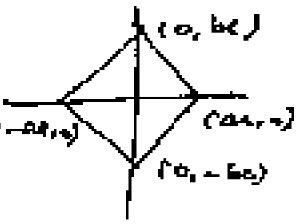
another point $(6,2)$

radius $=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=\sqrt{(8-6)^2+(2+2)^2}=\sqrt{2^2+4^2}$

$=\sqrt{4+16}=\sqrt{20}=2\sqrt{5}.$

10) The area of quadrilateral formed with foci of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and $\frac{x^2}{a^2}-\frac{y^2}{b^2}=-1$

Soln



$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Focus $(\pm ae, 0)$

$e = \sqrt{1 + b^2/a^2}$

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$

Focus $(0, \pm be)$

$e = \sqrt{1 + a^2/b^2}$.

quadrilateral $\square ABCD = \frac{1}{2} AC \times BD = \frac{1}{2} 2ae \times 2be$

$= a\sqrt{1+\frac{b^2}{a^2}} \times 2b\sqrt{1+\frac{a^2}{b^2}} = a\frac{\sqrt{a^2+b^2}}{a} \times 2b\frac{\sqrt{b^2+a^2}}{b}$

$= 2(a^2+b^2)$

Ans (2)

11) If the normals of the parabola $y^2 = 4x$ drawn at the end points of its latus rectum are tangents to the circle $(x-3)^2 + (y+2)^2 = r^2$ Then the value of r^2 is

Soln Eqn of normal at points $(1, \pm 2)$

$y = -x+3$

$x+y-3=0$

$y = x+3$

$x-y-3=0$

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$\left|\frac{3-2-3}{\sqrt{1+1}}\right| = r$

$\left|\frac{-2}{\sqrt{2}}\right| = r \Rightarrow \frac{2}{\sqrt{2}} = r$

$r = \sqrt{2}$

$r^2 = 2$

Ans (1).

12) If $x+y=k$ is normal to the parabola $y^2 = 12x$ then the value of k is

soln $y^2 = 12x \Rightarrow 4a = 12 \Rightarrow a = 3$

$y = mx+c$. $x+y = k \Rightarrow y = -x+k$

$\therefore m = -1 \quad c = k$.

$c = -2am - am^3 \Rightarrow k = -2a(-1) - a(-1)^3$

$k = -6(-1) - 3(-1) = 6+3 = 9$

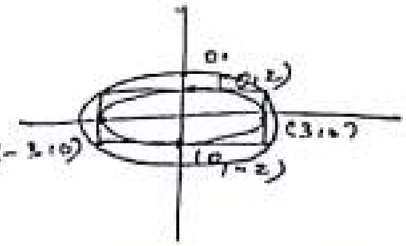
$k = 9$

Ans (4)

13) The ellipse $E_1: \frac{x^2}{9}+\frac{y^2}{4}=1$ is inscribed in a rectangle R whose sides are parallel to the coordinate axes. Another ellipse E_2 passing through the point $(0,4)$ circumscribes the rectangle R. The eccentricity of the ellipse is

soln. Eqn be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

$\frac{9}{a^2} + \frac{4}{16} = 1$



$$\frac{a^2}{b^2} = 1-\frac{1}{4} = \frac{3}{4}$$

$$a^2 = 12 \Rightarrow b^2 = 16.$$

$$e^2 = 1-a^2/b^2 = 1-\frac{12}{16} = 1-\frac{3}{4} = \frac{1}{4}$$

Ans (2)

$$e = \frac{1}{2}$$

14) Tangents are drawn to the hyperbola $\frac{x^2}{9}-\frac{y^2}{4}=1$ parallel to the straight line $2x-y=1$ one of the points of contact of tangents on the hyperbola is

Soln $a^2=9$ $b^2=4$. $2x-y=1$

$y = 2x-1$

$m=2$

$$c = \sqrt{a^2m^2-b^2}$$
$$= \sqrt{9(4)-4} = \sqrt{36-4} = \sqrt{32} = 4\sqrt{2}.$$

Point of contact $\left(\frac{a^2m}{c}, \frac{b^2}{c}\right) = \left(\frac{9\times 2}{4\sqrt{2}}, \frac{4}{4\sqrt{2}}\right)$

$$= \left(\frac{9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$

Ans (3)

15) The equation of the circle passing through the foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ having centre at $(0,3)$ is

Soln $a^2=16$ $b^2=9$ $(h,k)=(0,3)$

$$e = \sqrt{1-\frac{b^2}{a^2}} = \sqrt{1-\frac{9}{16}} = \sqrt{\frac{7}{16}} = \frac{\sqrt{7}}{4}.$$

$ae = 4\sqrt{7}/4 = \sqrt{7}$. $F(\sqrt{7},0)$ lies on circle.

$$(x-h)^2+(y-k)^2 = r^2 \Rightarrow \sqrt{7}^2+3^2 = r^2$$
$$(\sqrt{7}-0)^2+(0-3)^2 = r^2$$
$$7+9 = r^2 \Rightarrow r^2 = 16.$$

$$\therefore (x-0)^2+(y-3)^2 = 16$$
$$x^2+y^2-6y+9 = 16$$
$$x^2+y^2-6y-7 = 0$$

Ans. (1)

16) Let C be the circle with centre at $(1,1)$ and radius $=1$. If T is a circle centred at $(0,y)$ passing through the origin and touching the circle C externally. Then the radius T is equal to.

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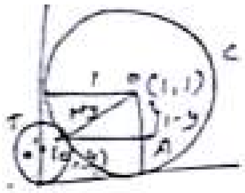
Soln $\Delta OO'A$

$(1+y)^2 = (1-y)^2 + 1$

$1+y^2+2y = 1+y^2-2y+1$

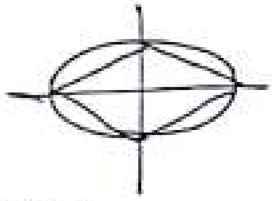
$4y = 1$

$y = 1/4$



Ans (4)

17) Consider an ellipse whose centre is of the origin and its major axis is along x axis. If its eccentricity is $\frac{3}{5}$ and the distance between its foci is 6. Then the area of the quadrilateral inscribed in the ellipse with diagonals as major & minor axis of the ellipse is



Soln $e = 3/5$ $\quad 2ae = 6$

$2a\left(\frac{3}{5}\right) = 6$

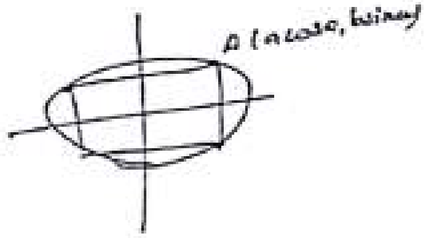
$a = 5$

$a=5 \quad b=4$

Area ABCD $= \cancel{4}^{2} \times \frac{1}{2} \times a\,b = 2ab = 2\times5\times4$

$= 40$ Ans (4)

18) Area of the greatest rectangle inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.



Soln. $x = a\cos\theta \quad y = b\sin\theta$

length $= 2a\cos\theta \quad$ breadth $= 2b\sin\theta$

$A = l\times b = 4ab\sin\theta\cos\theta$

$A = 2ab\sin 2\theta$

$\frac{dA}{dt} = 2ab\cos 2\theta(2) = 4ab\cos 2\theta$

$\frac{dA}{dt} = 0 \quad \cos 2\theta = 0$

$2\theta = \pi/2$

$\theta = \pi/4$

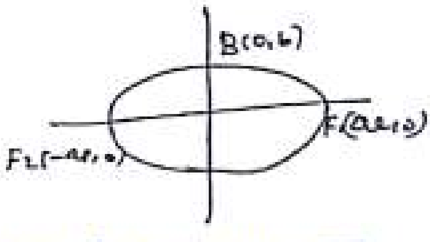
$\therefore A = 2ab\sin 2(\pi/4)$

$= 2ab\sin \pi/2$

$A = 2ab$

Ans (1)

19) An ellipse has OB as semi minor axes F and F' its foci and the angle FBF' is a right angle. Then the eccentricity of the ellipse is



Soln

$\angle FBF' = 90°$

$\left(\sqrt{a^2e^2+b^2}\right)^2 + \left(\sqrt{a^2e^2+b^2}\right)^2 = (2ae)^2$

$\cancel{2}(a^2e^2+b^2) = {}^{2}\cancel{4}a^2e^2$

$2a^2e^2+2b^2 = 4a^2e^2$

$2a^2e^2 = 2b^2$

$e^2 = b^2/a^2$

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WKT $e^2 = 1 - b^2/a^2 = 1 - e^2 \Rightarrow 2e^2 = 1 \Rightarrow e^2 = \frac{1}{2}$ (41)

$e = \frac{1}{\sqrt{2}}$ Ans ①

20) The eccentricity of the ellipse $(x-3)^2 + (y-4)^2 = \frac{y^2}{9}$ is

soln $PF = e^2 PS$

$(x-h)^2 + (y-k)^2 = e^2 \left(\frac{ax+by+c}{\sqrt{a^2+b^2}}\right)^2$

$(h,k) = (3,4)$ $a = 0$ $c = 0$ Ans ②

$e^2 = \frac{1}{9}$

$e = \frac{1}{3}$

21) If the two tangents drawn from a point p to the parabola $y^2 = 4x$ at right angles then the locus of p is

soln locus of p = Directrix of $y^2 = 4x$ $4a = 4$, $a = 1$

Eqn of directrix $x = -a = -1$ Ans ②

$\therefore x = -1$

22) The circle passing through $(1,-2)$ and touching the axis of x at $(3,0)$ passing through the point

soln $(x-3)^2 + (y-0)^2 + \lambda y = 0$

At $(1,-2)$ $(1-3)^2 + (-2-0)^2 + \lambda y = 0$

$4 + 4 - 2\lambda = 0$

$8 = 2\lambda$

$\lambda = 4$

$x^2 - 6x + 9 + y^2 + 4y = 0$

Apply all the point which satisfie that passes through the circle.

At $(5,-2)$

$25 - 30 + 9 + 4 - 8 = 0$

Ans ③

23) The locus of a point whose distance from $(-2,0)$ is $\frac{2}{3}$ times its distance from the line $x = -\frac{9}{2}$ is

soln $P(h,k)$ $Q(-2,0)$

$x = -\frac{9}{2}$ $PQ = \frac{2}{3} PM$

$2x + 9 = 0$ $\sqrt{(h+2)^2 + k^2} = \frac{2}{3} \left|\frac{2h+9}{2}\right|$

$(h+2)^2 + k^2 = \frac{1}{9}(2h+9)^2$

$h^2 + 4 + 4h + k^2 = \frac{1}{9}(4h^2 + 36h + 81)$

$9h^2 + 36 + 36h + 9k^2 = 4h^2 + 36h + 81$

$5h^2 + 9k^2 = 45$

$\frac{h^2}{9} + \frac{k^2}{5} = 1$

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$\frac{x^2}{9} + \frac{y^2}{5} = 1$ which is ellipse Ans (a)

24) The values of m for which the line $y = mx + 2\sqrt{5}$ touches the hyperbola $16x^2 - 9y^2 = 144$ are the roots of $x^2 - (a+b)x - 4 = 0$ Then the value of $(a+b)$ is

Soln $a^2 = 9$ $b^2 = 16$

$a = 3$ $b = 4$

$c^2 = a^2m^2 - b^2$

$(2\sqrt{5})^2 = 9m^2 - 16$

$20 + 16 = 9m^2$

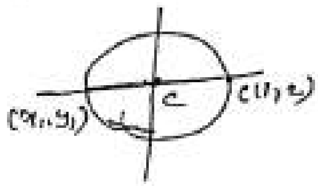
$m^2 = \frac{36}{9} = 4$ $\therefore m = 2$. Which is roots of

$x^2 - (a+b)x - 4 = 0$

$x^2 - (a+b)x - 4 = 0$ Ans (3)

$a + b = 0$

25) If the coordinates at one end of a diameter of the circle $x^2 + y^2 - 8x - 4y + c = 0$ are $(11, 2)$ the coordinates of the other end are



Soln $2g = -8$ $2f = -4$

$g = -4$ $f = -2$

$C\ (-g, -f) = (4, 2)$

$\frac{x_1 + x_2}{2} = 4$ $\frac{y_1 + y_2}{2} = 2$

$\frac{x_1 + 11}{2} = 4$ $\frac{y_1 + 2}{2} = 2$

$x_1 = 8 - 11$ $y_1 = 4 - 2$

$x_1 = -3$ $y_1 = 2$

$\therefore$ other end be $(-3, 2)$