eg 8.19

$\boxed{n=50}$ $\boxed{\bar{x}=9.3}$ $\boxed{\mu=8.9}$ $\boxed{\sigma=1.6}$

Null hypothesis;

$H_0 : \mu = 8.9$ Accepted.

Alternative hypothesis:

$H_1 : \mu \neq 8.9$ rejected.

level of significance;

5% (1.96)

Z-test

$$Z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} = \frac{9.3-8.9}{\frac{1.6}{\sqrt{50}}}$$

$$|Z| = 1.7678$$

$|Z| < Z_{\alpha/2}$ H_0 accepted.

Exercise 4.1

Arbitrary Constant:

2 (i) $y = cx + c - c^3$

$\boxed{y' = c} + 0 - 0$

$$\boxed{y = y'x + y' - (y')^3}$$

(iii) $\frac{xy}{uv} = c^2$ //

$$\boxed{xy' + y = 0}$$

(iv) $x^2 + y^2 = a^2$

$2x + 2yy' = 0$

$(\div 2)$

$$\boxed{x + yy' = 0}$$

(ii) $y = c(x-c)^2 \rightarrow (1)$

$d(x^n) = nx^{n-1}$

$y' = 2c(x-c)'.(1-0)$

$y' = 2c(x-c) \rightarrow (2)$

$\frac{(1)}{(2)} = \frac{y}{y'} = \frac{\cancel{c}(x-c)^{\cancel{2}}}{2\cancel{c}\cancel{(x-c)}}$

$\frac{2y}{y'} = x - c$

$c = x - \frac{2y}{y'}$

$$\boxed{c = \frac{xy' - 2y}{y'}}$$

$$y = \left(\frac{xy'-2y}{y'}\right)\left(x - \frac{(xy'-2y)}{y'}\right)^2$$

$$= \left(\frac{xy'-2y}{y'}\right)\left(\frac{xy'-xy'+2y}{y_1}\right)^2$$

$$y = \left(\frac{xy'-2y}{y'}\right)\frac{4y^2}{(y')^2}$$

$$(y')^3 = 4xy'y - 8y^2$$

$$\boxed{(y')^3 - 4xyy' + 8y^2 = 0}$$

eg 4.5

$$y = e^x(a\cos x + b\sin x) \quad \text{(2 time diff)}$$

$$y' = e^x(-a\sin x + b\cos x) + (a\cos x + b\sin x)e^x$$

$$y' = e^x(-a\sin x + b\cos x) + y.$$

$$y' - y = e^x(-a\sin x + b\cos x)$$

$$-y'' - y' = e^x(-a\cos x - b\sin x) + (-a\sin x + b\cos x)e^x$$

$$y'' - y' = -y + y' - y$$

$$\boxed{y'' - 2y' + 2y = 0}$$

//.

(3) $(x-\alpha)^2 + (y-\beta)^2 = r^2 \longrightarrow (1)$ (2 time diff)

$$\cancel{2(x-\alpha)(1-0) + 2(y-\beta)(y'-0) = 0}$$

$(\div 2)$

$$\cancel{(x-\alpha) + (y-\beta)(y') = 0}$$

$$\cancel{(1-0) + (y-\beta)y'' + y'(y'-0) = 0}$$

$$\cancel{(y-\beta)y'' = -(y')^2 - 1}$$

$$\cancel{(y-\beta) = -\frac{(1+(y')^2)}{y''}}$$

$$\cancel{(x-\alpha) - \frac{(1+(y_1)^2}{2y''}y' = 0}$$

$$\cancel{(x-\alpha) = \frac{(1+y')^2}{y''}y'}$$

(4) $y = mx + c$ at origin $(0,0)$

$$0 = 0 + c$$

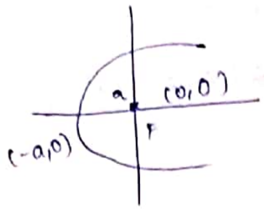
$$\boxed{c = 0}$$

$$\boxed{y = mx} \to (1)$$

$$y' = m$$

$$(1) \Rightarrow \boxed{y = y'x}$$

(4)



$(y-k)^2 = 4a(x-h)$ (1 time)

$(h,k) = (-a,0)$

$y^2 = 4a(x+a) \rightarrow ①$

$\cancel{2}yy' = \cancel{4}^{2}a(1+0)$

$$\boxed{\frac{yy'}{2} = a}$$

$① \Rightarrow y^2 = \cancel{4}^{2}\frac{(yy')}{\cancel{2}}\left(x+\frac{yy'}{2}\right)$

$y^{\cancel{2}} = \cancel{2}y'y'\frac{(2x+yy')}{\cancel{2}}$

$$\boxed{y = 2xy' + y(y')^2}$$ //

eg: 4.4

$y = ae^{4x} + be^{-x} \longrightarrow ①$ (2 time)

$y' = 4ae^{4x} - be^{-x} \rightarrow ②$

$y'' = 16ae^{4x} + be^{-x} \longrightarrow ③$

$① + ② \Rightarrow y + y' = 5ae^{4x} + 0$

$② + ③ \rightarrow y' + y'' = 20ae^{4x} + 0$

$y' + y'' = 4 \times 5ae^{4x}$

$y' + y'' = 4(y+y')$

$y' + y'' = 4y + 4y'$

$y'' - 3y' - 4y = 0$

---

(8) $(x-\alpha)^2 + (y-\beta)^2 = r^2 \rightarrow ①$

$2(x-\alpha)(1-0) + 2(y-\beta)(y'-0) = 0$

$(\div 2)$

$(x-\alpha) + (y-\beta)y' = 0$

$(1-0) + (y-\beta)y'' + y'(y'-0) = 0$

$(y-\beta)y'' = -(y')^2 - 1$

$$\boxed{(y-\beta) = -\frac{(1+(y')^2)}{y''}}$$

$(x-\alpha) - \frac{(1+(y')^2)}{y''}y' = 0$

$$\boxed{(x-\alpha) = \frac{(1+y'^2)}{y''}y'}$$

$① \Rightarrow \frac{(1+(y')^2)^2(y')^2}{(y'')^2} + \frac{(1+(y')^2)^2}{(y'')^2} = r^2$

$\frac{(1+(y')^2)^2}{(y'')^2}(1+(y')^2) = r^2$

$$\boxed{\frac{(1+(y')^2)^3}{(y'')^2} = r^2}$$ //

eq of parabola

5. $(y-k)^2 = 4a(x-h) \longrightarrow (1)$ diff

$2(y-k)y' = 4a(1-0)$

$(y-k)y' = 2a. \longrightarrow (2)$

$(y-k)y'' + y'(y'-0) = 0$

$(y-k)y'' = -(y')^2$

$\boxed{(y-k) = \frac{-(y')^2}{y''}}$

model

$y = mx+c$

$y' = m$

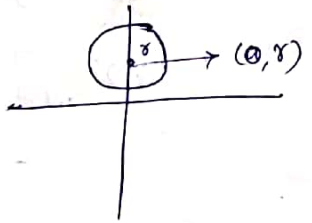
$y'' = 0$

$(2) \rightarrow \frac{-(y')^2}{y''} y' = 2a$

$-(y')^3 = 2ay''$

$\boxed{2ay'' + (y')^3 = 0}$

6.



$(x-h)^2 + (y-k)^2 = r^2$

$(h, k) = (0, r)$ (1 time) diff

$x^2 + (y-r)^2 = r^2 \longrightarrow (1)$

$2x + 2(y-r)(y'-0) = 0$

$(\div 2)$ $x + (y-r)y' = 0$

$(y-r)y' = -x$

$y - r = \frac{-x}{y'}$

$\boxed{r = y + \frac{x}{y'}}$

$(1) \Rightarrow x^2 + \frac{x^2}{(y')^2} = \left(y + \frac{x}{y'}\right)^2$

$x^2 + \cancel{\frac{x^2}{(y')^2}} = y^2 + \cancel{\frac{x^2}{(y')^2}} + \frac{2xy}{y'}.$

$x^2 = y^2 + \frac{2xy}{y'}$

$(\times y')$

$\boxed{x^2y' = y^2y' + 2xy}$

eg 4.3 $y = \frac{a}{x} + b$

$y' = \frac{-a}{x^2} + 0$ | $d\left(\frac{1}{x}\right) = \frac{-1}{x^2}$

$x^2y' = -a$

diff

$\boxed{x^2y'' + 2xy' = 0}$

$\div x$ $\boxed{xy'' + 2y' = 0}$