Matrics

EXERCISE 2.1 [PAGES 39 - 40]

Exercise 2.1 | Q 1 | Page 39

Apply the given elementary transformation of the following matrix.

$A = \begin{bmatrix} 1 & 0 \\ -1 & 3 \end{bmatrix}$, $R_1 \leftrightarrow R_2$

Solution:

$A = \begin{bmatrix} 1 & 0 \\ -1 & 3 \end{bmatrix}$

By $R_1 \leftrightarrow R_2$, we get,

$A \sim \begin{bmatrix} -1 & 3 \\ 1 & 0 \end{bmatrix}$

Exercise 2.1 | Q 2 | Page 39

Apply the given elementary transformation of the following matrix.

$B = \begin{bmatrix} 1 & -1 & 3 \\ 2 & 5 & 4 \end{bmatrix}$, $R_1 \rightarrow R_1 - R_2$

Solution:

$B = \begin{bmatrix} 1 & -1 & 3 \\ 2 & 5 & 4 \end{bmatrix}$

$R_1 \rightarrow R_1 - R_2$ gives,

$B \sim \begin{bmatrix} -1 & -6 & -1 \\ 2 & 5 & 4 \end{bmatrix}$

Exercise 2.1 | Q 3 | Page 39

Apply the given elementary transformation of the following matrix.

$A = \begin{bmatrix} 5 & 4 \\ 1 & 3 \end{bmatrix}$, $C_1 \leftrightarrow C_2$; $B = \begin{bmatrix} 3 & 1 \\ 4 & 5 \end{bmatrix}$ $R_1 \leftrightarrow R_2$.

What do you observe?

Solution:

$A = \begin{bmatrix} 5 & 4 \\ 1 & 3 \end{bmatrix}$

By $C_1 \leftrightarrow C_2$, we get,

$A \sim \begin{bmatrix} 4 & 5 \\ 3 & 1 \end{bmatrix}$ ....(1)

$B = \begin{bmatrix} 3 & 1 \\ 4 & 5 \end{bmatrix}$

By $R_1 \leftrightarrow R_2$, we get,

$B \sim \begin{bmatrix} 4 & 5 \\ 3 & 1 \end{bmatrix}$ ....(2)

From (1) and (2), we observe that the new matrices are equal.

Exercise 2.1 | Q 4 | Page 39

Apply the given elementary transformation of the following matrix.

$A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 3 \end{bmatrix}$, $2C_2$

$B = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 4 & 5 \end{bmatrix}$, $-3R_1$

Find the addition of the two new matrices.

Solution:

A = $\begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 3 \end{bmatrix}$

By $2C_2$, we get,

A ~ $\begin{bmatrix} 1 & 4 & -1 \\ 0 & 2 & 3 \end{bmatrix}$

B = $\begin{bmatrix} 1 & 0 & 2 \\ 2 & 4 & 5 \end{bmatrix}$

By $-3R_1$, we get,

B ~ $\begin{bmatrix} -3 & 0 & -6 \\ 2 & 4 & 5 \end{bmatrix}$

Now, addition of the two new matrices

$= \begin{bmatrix} 1 & 4 & -1 \\ 0 & 2 & 3 \end{bmatrix} + \begin{bmatrix} -3 & 0 & -6 \\ 2 & 4 & 5 \end{bmatrix}$

$= \begin{bmatrix} -2 & 4 & -7 \\ 2 & 6 & 8 \end{bmatrix}$

Exercise 2.1 | Q 5 | Page 39

Apply the given elementary transformation of the following matrix.

A = $\begin{bmatrix} 1 & -1 & 3 \\ 2 & 1 & 0 \\ 3 & 3 & 1 \end{bmatrix}$, $3R_3$ and then $C_3 + 2C_2$

Solution:

A = $\begin{bmatrix} 1 & -1 & 3 \\ 2 & 1 & 0 \\ 3 & 3 & 1 \end{bmatrix}$

By $3R_3$, we get,

A ~ $\begin{bmatrix} 1 & -1 & 3 \\ 2 & 1 & 0 \\ 9 & 9 & 3 \end{bmatrix}$

By $C_3 + 2C_2$, we get,

A ~ $\begin{bmatrix} 1 & -1 & 3+2(-1) \\ 2 & 1 & +2(1) \\ 9 & 9 & +2(9) \end{bmatrix}$

∴ A ~ $\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & 2 \\ 9 & 9 & 21 \end{bmatrix}$

Exercise 2.1 | Q 6 | Page 39

Apply the given elementary transformation of the following matrix.

A = $\begin{bmatrix} 1 & -1 & 3 \\ 2 & 1 & 0 \\ 3 & 3 & 1 \end{bmatrix}$, $3R_3$ and then $C_3 + 2C_2$

and A = $\begin{bmatrix} 1 & -1 & 3 \\ 2 & 1 & 0 \\ 3 & 3 & 1 \end{bmatrix}$, $C_3 + 2C_2$ and then $3R_3$

What do you conclude.

Solution:

$$A = \begin{bmatrix} 1 & -1 & 3 \\ 2 & 1 & 0 \\ 3 & 3 & 1 \end{bmatrix}$$

By $3R_3$, we get,

$$A \sim \begin{bmatrix} 1 & -1 & 3 \\ 2 & 1 & 0 \\ 9 & 9 & 3 \end{bmatrix}$$

By $C_3 + 2C_2$, we get,

$$A \sim \begin{bmatrix} 1 & -1 & 3+2(-1) \\ 2 & 1 & +2(1) \\ 9 & 9 & +2(9) \end{bmatrix}$$

$$\therefore A \sim \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & 2 \\ 9 & 9 & 21 \end{bmatrix} \text{..............(i)}$$

And

$$A = \begin{bmatrix} 1 & -1 & 3 \\ 2 & 1 & 0 \\ 3 & 3 & 1 \end{bmatrix}$$

By $C_3 + 2C_2$, we get,

$$A \sim \begin{bmatrix} 1 & -1 & 3+2(-1) \\ 2 & 1 & 0+2(1) \\ 3 & 3 & +1+2(3) \end{bmatrix}$$

$$\therefore A \sim \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & 2 \\ 3 & 3 & 7 \end{bmatrix}$$

$$\therefore A \sim \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & 2 \\ 3 & 3 & 7 \end{bmatrix}$$

By $3R_3$, we get

$$A \sim \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & 2 \\ 9 & 9 & 21 \end{bmatrix} \quad \text{......(ii)}$$

We conclude from (i) and (ii) the matrix remains the same by interchanging the order of the elementary transformations. Hence, the transformations are commutative.

Exercise 2.1 | Q 7 | Page 39

Apply the given elementary transformation of the following matrix.

Use suitable transformation on $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ to convert it into an upper triangular matrix.

Solution:

Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

By $R_2 - 3R_1$, we get,

$$A \sim \begin{bmatrix} 1 & 2 \\ 0 & -2 \end{bmatrix}$$

This is an upper triangular matrix.

Exercise 2.1 | Q 8 | Page 39

Apply the given elementary transformation of the following matrix.

Convert $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ into an identity matrix by suitable row transformations.

Solution:

convert $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ into identify matrix by suitable row motion

$\therefore AA^{-1} = I$

$$A^{-1}\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Applying Elementary Row operation.

$R_1 \to R_1 + \dfrac{1}{3}R_2$

$$A6-1\begin{bmatrix} \frac{5}{3} & 0 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{3} \\ 0 & 1 \end{bmatrix}$$

$R_1 \to \dfrac{3}{5}R_1$

$$A^{-1}\begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ 0 & 1 \end{bmatrix}$$

$$A_1\begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ \frac{-6}{5} & \frac{3}{5} \end{bmatrix}$$

$R_2 \to \dfrac{1}{3}R_2$

$$A^{-1}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ \frac{-2}{5} & \frac{1}{5} \end{bmatrix}$$

$$\text{So, } A^{-1}\begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ \frac{-2}{5} & \frac{1}{5} \end{bmatrix}$$

Exercise 2.1 | Q 9 | Page 40

Apply the given elementary transformation of the following matrix.

Transform $\begin{bmatrix} 1 & -1 & 2 \\ 2 & 1 & 3 \\ 3 & 2 & 4 \end{bmatrix}$ into an upper triangular matrix by suitable column transformations.

Solution:

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 3R_1$

$$\begin{bmatrix} 1 & -1 & 2 \\ 2 & 1 & 3 \\ 3 & 2 & 4 \end{bmatrix}$$

$R_3 \to R_3 - \dfrac{5}{3} R_2$

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & -1 \\ 0 & 5 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & -1 \\ 0 & 0 & \frac{17}{3} \end{bmatrix}$$

It is an upper triangular matrix.

EXERCISE 2.2 [PAGES 51 - 52]

Exercise 2.2 | Q 1.1 | Page 51

Find the co-factor of the element of the following matrix.

$$\begin{bmatrix} -1 & 2 \\ -3 & 4 \end{bmatrix}$$

Solution:

Let A = $\begin{bmatrix} -1 & 2 \\ -3 & 4 \end{bmatrix}$

Here, $a_{11} = -1$, $M_{11} = 4$

$\therefore A_{11} = (-1)^{1+1}(4) = 4$

$a_{12} = 2$, $M_{12} = -3$

$\therefore A_{12} = (-1)^{1+2(}-3) = 3$

$a_{21} = -3$, $M_{21} = 2$

$\therefore A_{21} = (-1)^{2+1}(2) = -2$

$a_{22} = 4$, $M_{22} = -1$

$\therefore A_{22} = (-1)^{2+2}(-1) = -1$

Exercise 2.2 | Q 1.2 | Page 51

Find the co-factor of the element of the following matrix.

$$\begin{bmatrix} 1 & -1 & 2 \\ -2 & 3 & 5 \\ -2 & 0 & -1 \end{bmatrix}$$

Solution:

Let A = $\begin{bmatrix} 1 & -1 & 2 \\ -2 & 3 & 5 \\ -2 & 0 & -1 \end{bmatrix}$

The co-factor of a_{ij} is given by $A_{ij} = (-1)^{i+j} M_{ij}$

Now, $M_{11} = \begin{vmatrix} 3 & 5 \\ 0 & -1 \end{vmatrix} = -3 - 0 = -3$

$\therefore A_{11} = (-1)^{1+1}(-3) = -3$

$M_{12} = \begin{vmatrix} -2 & 5 \\ -2 & -1 \end{vmatrix} = 2 + 10 = 12$

$\therefore A_{12} = (-1)^{1+2}(12) = -12$

$M_{13} = \begin{vmatrix} -2 & 3 \\ -2 & 0 \end{vmatrix} = 0 + 6 = 6$

$\therefore A_{13} = (-1)^{1+3}(6) = 6$

$M_{21} = \begin{vmatrix} -1 & 2 \\ 2 & -1 \end{vmatrix} = 1 - 0 = 1$

$\therefore A_{21} = (-1)^{2+1}(1) = -1$

$M_{22} = \begin{vmatrix} 1 & 2 \\ -2 & -1 \end{vmatrix} = -1 + 4 = 3$

$\therefore A_{22} = (-1)^{2+2}(3) = 3$

$M_{23} = \begin{vmatrix} 1 & -1 \\ -2 & 0 \end{vmatrix} = 0 - 2 = -2$

$\therefore A_{23} = (-1)^{2+3}(-2) = 2$

$M_{31} = \begin{vmatrix} -1 & 2 \\ 3 & 5 \end{vmatrix} = -5 - 6 = -11$

$\therefore A_{31} = (-1)^{3+1}(-11) = -11$

$M_{32} = \begin{vmatrix} 1 & 2 \\ -2 & 5 \end{vmatrix} = 5 + 4 = 9$

$\therefore A_{32} = (-1)^{3+2}(9) = -9$

$M_{33} = \begin{vmatrix} 1 & -1 \\ -2 & 3 \end{vmatrix} = 3 - 2 = 1$

$\therefore A_{33} = (-1)^{3+3}(1) = 1$

Exercise 2.2 | Q 2.1 | Page 51

Find the matrix of the co-factor for the following matrix.

$$\begin{bmatrix} 1 & 3 \\ 4 & -1 \end{bmatrix}$$

Solution:

Let A = $\begin{bmatrix} 1 & 3 \\ 4 & -1 \end{bmatrix}$

Here, $a_{11} = 1$, $M_{11} = -1$

$\therefore A_{11} = (-1)^{1+1}(-1) = -1$

$a_{12} = 3$, $M_{12} = 4$

$\therefore A_{12} = (-1)^{1+2}(4) = -4$

$a_{21} = 4$, $M_{21} = 3$

$\therefore A_{21} = (-1)^{2+1}(3) = -3$

$a_{22} = -1$, $M_{22} = 1$

$\therefore A_{22} = (-1)^{2+2}(1) = 1$

$\therefore$ the co-factor matrix = $\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$

$= \begin{bmatrix} -1 & -4 \\ -3 & 1 \end{bmatrix}$

Exercise 2.2 | Q 2.2 | Page 51

Find the matrix of the co-factor for the following matrix.

$$\begin{bmatrix} 1 & 0 & 2 \\ -2 & 1 & 3 \\ 0 & 3 & -5 \end{bmatrix}$$

Solution:

Let A = $\begin{bmatrix} 1 & 0 & 2 \\ -2 & 1 & 3 \\ 0 & 3 & -5 \end{bmatrix}$

Here, a_{11} = 1,

$A_{11} = \begin{vmatrix} 1 & 3 \\ 3 & -5 \end{vmatrix} = -14$

a_{12} = 0,

$A_{12} = \begin{vmatrix} -2 & 0 \\ 3 & -5 \end{vmatrix} = -10$

a_{13} = 2,

$A_{13} = \begin{vmatrix} -2 & 1 \\ 0 & 3 \end{vmatrix} = -6$

$a_{21} = -2$

$A_{21} = \begin{vmatrix} 0 & 2 \\ 3 & -5 \end{vmatrix} = 6$

$a_{22} = 1$

$A_{22} = \begin{vmatrix} 1 & 0 \\ 2 & -5 \end{vmatrix} = -5$

$a_{23} = 3$

$A_{23} = \begin{vmatrix} 1 & 0 \\ 0 & 3 \end{vmatrix} = -3$

$a_{31} = 0$

$A_{31} = \begin{vmatrix} 0 & 2 \\ 1 & 3 \end{vmatrix} = -2$

$a_{32} = 3$

$$A_{32} = \begin{vmatrix} 1 & 2 \\ -2 & 3 \end{vmatrix} = -7$$

$a_{33} = -5$

$$A_{33} = \begin{vmatrix} 1 & 0 \\ -2 & 1 \end{vmatrix} =$$

$\therefore$ the co-factor matrix

$$= \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} = \begin{bmatrix} -14 & -10 & -6 \\ 6 & -5 & -3 \\ -2 & -7 & 1 \end{bmatrix}$$

[Note: Answer in the textbook is incorrect.]

Exercise 2.2 | Q 3.1 | Page 51

Find the adjoint of the following matrix.

$$\begin{bmatrix} 2 & -3 \\ 3 & 5 \end{bmatrix}$$

Solution:

Let $A = \begin{bmatrix} 2 & -3 \\ 3 & 5 \end{bmatrix}$

Here, $a_{11} = 2$, $M_{11} = 5$

$\therefore A_{11} = (-1)^{1+1}(5) = 5$

$a_{12} = -3$, $M_{12} = 3$

$\therefore A_{12} = (-1)^{1+2}(3) = -3$

$a_{21} = 3$, $M_{21} = -3$

$\therefore A_{21} = (-1)^{2+1}(-3) = 3$

$a_{22} = 5$, $M_{22} = 2$

$\therefore A_{22} = (-1)^{2+2} = 2$

$\therefore$ the co-factor matrix = $\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$

= $\begin{bmatrix} 5 & -3 \\ 3 & 2 \end{bmatrix}$

$\therefore$ adj A = $\begin{bmatrix} 5 & 3 \\ -3 & 2 \end{bmatrix}$

Exercise 2.2 | Q 3.2 | Page 51

Find the adjoint of the following matrix.

$$\begin{bmatrix} 1 & -1 & 2 \\ -2 & 3 & 5 \\ -2 & 0 & -1 \end{bmatrix}$$

Solution:

Let A = $\begin{bmatrix} 1 & -1 & 2 \\ -2 & 3 & 5 \\ -2 & 0 & -1 \end{bmatrix}$

Now, M_{11} = $\begin{vmatrix} 3 & 5 \\ 0 & -1 \end{vmatrix}$ = – 3 – 0 = – 3

$\therefore A_{11} = (-1)^{1+1}(-3) = -3$

M_{12} = $\begin{vmatrix} -2 & 5 \\ -2 & -1 \end{vmatrix}$ = 2 + 10 = 12

$\therefore A_{12} = (-1)^{1+2}(12) = -12$

M_{13} = $\begin{vmatrix} -2 & 3 \\ -2 & 0 \end{vmatrix}$ = 0 + 6 = 6

$\therefore A_{13} = (-1)^{1+3}(6) = 6$

$M_{21} = \begin{vmatrix} -1 & 2 \\ 2 & -1 \end{vmatrix} = 1 - 0 = 1$

$\therefore A_{21} = (-1)^{2+1}(1) = -1$

$M_{22} = \begin{vmatrix} 1 & 2 \\ -2 & -1 \end{vmatrix} = -1 + 4 = 3$

$\therefore A_{22} = (-1)^{2+2}(3) = 3$

$M_{23} = \begin{vmatrix} 1 & -1 \\ -2 & 0 \end{vmatrix} = 0 - 2 = -2$

$\therefore A_{23} = (-1)^{2+3}(-2) = 2$

$M_{31} = \begin{vmatrix} -1 & 2 \\ 3 & 5 \end{vmatrix} = -5 - 6 = -11$

$\therefore A_{31} = (-1)^{3+1}(-11) = -11$

$M_{32} = \begin{vmatrix} 1 & 2 \\ -2 & 5 \end{vmatrix} = 5 + 4 = 9$

$\therefore A_{32} = (-1)^{3+2}(9) = -9$

$M_{33} = \begin{vmatrix} 1 & -1 \\ -2 & 3 \end{vmatrix} = 3 - 2 = 1$

$\therefore A_{33} = (-1)^{3+3}(1) = 1$

the co-factor matrix = $\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$

$$= \begin{bmatrix} -3 & -12 & 6 \\ -1 & 3 & 2 \\ -11 & -9 & 1 \end{bmatrix}$$

$$\therefore \text{adj } A = \begin{bmatrix} -3 & -1 & -11 \\ -12 & 3 & -9 \\ 6 & 2 & 1 \end{bmatrix}$$

Exercise 2.2 | Q 4 | Page 51

If ∴ verify that A (adj A) = (adj A) A = | A | I

Solution:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$$

$$\therefore |A| = \begin{vmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{vmatrix}$$

$= 1(0 + 0) + 1(9 + 2) + 2(0 - 0)$

$= 0 + 11 + 0$

$= 11$

First we have to find the co-factor matrix $= [A_{ij}]_{3\times3'}$

where $A_{ij} = (-1)^{i+j} M_{ij}$

Now, $A_{11} = (-1)^{1+1} M_{11} = \begin{vmatrix} 0 & -2 \\ 0 & 3 \end{vmatrix} = 0 + 0 = 0$

$$A_{12} = (-1)^{1+2}M_{12} = -\begin{vmatrix} 3 & -2 \\ 1 & 3 \end{vmatrix} = -(9+2) = -11$$

$$A_{13} = (-1)^{1+3}M_{13} = \begin{vmatrix} 3 & 0 \\ 1 & 0 \end{vmatrix} = 0 - 0 = 0$$

$$A_{21} = (-1)^{2+1}M_{21} = -\begin{vmatrix} -1 & 2 \\ 0 & 3 \end{vmatrix} = -(-3-0) = 3$$

$$A_{22} = (-1)^{2+2}M_{22} = \begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix} = 3 - 2 = 1$$

$$A_{23} = (-1)^{2+2}M_{23} = -\begin{vmatrix} 1 & -1 \\ -1 & 0 \end{vmatrix} = -(0+1) = -1$$

$$A_{31} = (-1)^{3+1}M_{31} = \begin{vmatrix} -1 & 2 \\ 0 & -2 \end{vmatrix} = 2 - 0 = 2$$

$$A_{32} = (-1)^{3+2}M_{32} = -\begin{vmatrix} 1 & 2 \\ 3 & -2 \end{vmatrix} = -(-2-6) = 8$$

$$A_{33} = (-1)^{3+3}M_{33} = \begin{vmatrix} 1 & -1 \\ 3 & 0 \end{vmatrix} = 0 + 3 = 3$$

Hence the co-factor matrix

$$= \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} = \begin{bmatrix} 0 & -11 & 0 \\ 3 & 1 & -1 \\ 2 & 8 & 3 \end{bmatrix}$$

$$\therefore \text{adj } A = \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix}$$

$\therefore$ A(adj A)

$$= \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix} \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0+11+0 & 3-1-2 & 2-8+6 \\ 0+0-0 & 9+0+2 & 6+0-6 \\ 0+0+0 & 3+0-3 & 2+0+9 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix} \text{..................(1)}$$

(adj A) A

$$= \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0+9+2 & 0+0+0 & 0-6+6 \\ -11+3+8 & 11+0+0 & -22-2+24 \\ 0-3+3 & 0-0+0 & 0+2+9 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix} \text{................(2)}$$

$$|A|\ I = 11 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix} \text{............(3)}$$

From (1), (2) and (3), we get,

A (adj A) = (adj A)A = |A| I.

[Note: This relation is valid for any non-singular matrix A]

Exercise 2.2 | Q 5.1 | Page 52

Find the inverse of the following matrix by the adjoint method.

$$\begin{bmatrix} -1 & 5 \\ -3 & 2 \end{bmatrix}$$

Solution:

Let A = $\begin{bmatrix} -1 & 5 \\ -3 & 2 \end{bmatrix}$

$\therefore$ |A| = $\begin{vmatrix} -1 & 5 \\ -3 & 2 \end{vmatrix}$ = – 2 + 15 = 13 $\neq$ 0

$\therefore$ A^{-1} exists.

First we have to find the co-factor matrix

= $[A_{ij}]_{2\times2}$, where $A_{ij} = (-1)^{i+j}M_{ij}$

Now, $A_{11} = (-1)^{1+1}M_{11} = 2$

$A_{12} = (-1)^{1+2}M_{12} = -(-3) = 3$

$A_{21} = (-1)^{2+1}M_{21} = -5$

$A_{22} = (-1)^{2+2}M_{22} = -1$

Hence, the co-factor matrix

= $\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$ = $\begin{bmatrix} 2 & 3 \\ -5 & -1 \end{bmatrix}$

$\therefore$ adj A = $\begin{bmatrix} 2 & -5 \\ 3 & -1 \end{bmatrix}$

$\therefore$ $A^{-1} = \dfrac{1}{|A|}$ (adj A) = $\dfrac{1}{13}\begin{bmatrix} 2 & -5 \\ 3 & -1 \end{bmatrix}$

Exercise 2.2 | Q 5.2 | Page 52

Find the inverse of the following matrix by the adjoint method.

$$\begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}$$

Solution:

Let $A = \begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}$

$|A| = \begin{vmatrix} 2 & -2 \\ 4 & 3 \end{vmatrix} = 6 + 8 = 14 \neq 0$

$\therefore$ A^{-1} exist

First we have to find the co-factor matrix

$= [A_{ij}]_{2\times2'}$, where $A_{ij} = (-1)^{i+j}M_{ij}$

Now, $A_{11} = (-1)^{1+1}M_{11} = 3$

$A_{12} = (-1)^{1+2}M^{12} = -4$

$A_{21} = (-1)^{2+1}M_{21} = (-2) = 2$

$A_{22} = (-1)^{2+2}M_{22} = 2$

Hence, the co-factor matrix

$$= \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} 3 & -4 \\ 2 & 2 \end{bmatrix}$$

$$\therefore \text{adj } A = \begin{bmatrix} 3 & 2 \\ -4 & 2 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}(\text{adj } A) = \frac{1}{14}\begin{bmatrix} 3 & 2 \\ -4 & 2 \end{bmatrix}$$

Exercise 2.2 | Q 5.3 | Page 52

Find the inverse of the following matrix by the adjoint method.

$$\begin{bmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{bmatrix}$$

Solution:

Let A = $\begin{bmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{bmatrix}$

$\therefore$ |A| = $\begin{vmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{vmatrix}$

= 1 (– 3 – 0) – 0 + 0

= – 3 $\neq$ 0

$\therefore$ A^{-1} exist

First we have to find the co-factor matrix

= $[A_{ij}]_{3\times3}$, where $A_{ij} = (-1)^{i+j}M_{ij}$

Now, $A_{11} = (-1)^{1+1}M_{11} = \begin{vmatrix} 3 & 0 \\ 2 & -1 \end{vmatrix} = -3 - 0 = -3$

$A_{12} = (-1)^{1+2}M_{12} = -\begin{vmatrix} 3 & 0 \\ 5 & -1 \end{vmatrix} = -(-3 - 0) = 3$

$A_{13} = (-1)^{1+3}M_{13} = \begin{vmatrix} 3 & 3 \\ 5 & 2 \end{vmatrix} = 6 - 15 = -9$

$A_{21} = (-1)^{2+1}M_{21} = -\begin{vmatrix} 0 & 0 \\ 2 & -1 \end{vmatrix} = -(0 - 0) = 0$

$A_{22} = (-1)^{2+2}M_{22} = \begin{vmatrix} 1 & 5 \\ 0 & -1 \end{vmatrix} = -1 - 0 = -1$

$A_{23} = (-1)^{2+3}M_{23} = -\begin{vmatrix} 1 & 0 \\ 5 & 2 \end{vmatrix} = -(2 - 0) = -2$

$A_{31} = (-1)^{3+1}M_{31} = \begin{vmatrix} 0 & 0 \\ 3 & 0 \end{vmatrix} = 0 - 0 = 0$

$A_{32} = (-1)^{3+2}M_{32} = -\begin{vmatrix} 1 & 0 \\ 3 & 0 \end{vmatrix} = -(0 - 0) = 0$

$A_{33} = (-1)^{3+3}M_{33} = \begin{vmatrix} 1 & 0 \\ 3 & 3 \end{vmatrix} = 3 - 0 = 3$

∴ the co-factor matrix

$$= \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} = \begin{bmatrix} -3 & 3 & -9 \\ 0 & -1 & -2 \\ 0 & 0 & 3 \end{bmatrix}$$

$$\therefore \text{adj } A = \begin{bmatrix} -3 & 0 & 0 \\ 3 & -1 & 0 \\ -9 & -2 & 3 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} (\text{adj } A)$$

$$= \frac{1}{-3} \begin{bmatrix} -3 & 0 & 0 \\ 3 & -1 & 0 \\ -9 & -2 & 3 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{3} \begin{bmatrix} 3 & 0 & 0 \\ -3 & 1 & 0 \\ 9 & 2 & -3 \end{bmatrix}$$

Exercise 2.2 | Q 5.4 | Page 52

Find the inverses of the following matrices by the adjoint method:

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix}$$

Solution:

Let A = $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix}$

∴ |A| = $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix}$

= 1(10 - 0) - 0 + 0

= 1(10) - 0 + 0

= 10 ≠ 0

∴ A^{-1} exists.

First we have to find the co-factor matrix

$= [A_{ij}]_{3\times3}$, where $A_{ij} = (-1)^{i+j}M_{ij}$

Now, $A_{11} = (-1)^{1+1}M_{11} = \begin{vmatrix} 2 & 4 \\ 0 & 5 \end{vmatrix} = 10 - 0 = 10$

$A_{12} = (-1)^{1+2}M_{12} = -\begin{vmatrix} 0 & 4 \\ 0 & 5 \end{vmatrix} = -0 - 0 = 0$

$A_{13} = (-1)^{1+3}M_{13} = \begin{vmatrix} 0 & 2 \\ 0 & 0 \end{vmatrix} = 0 - 0 = 0$

$A_{21} = (-1)^{2+1}M_{21} = -\begin{vmatrix} 2 & 3 \\ 0 & 5 \end{vmatrix} = -10 - 0 = -10$

$A_{22} = (-1)^{2+2}M_{22} = \begin{vmatrix} 1 & 3 \\ 0 & 5 \end{vmatrix} = 5 - 0 = 5$

$A_{23} = (-1)^{2+3}M_{23} = -\begin{vmatrix} 1 & 2 \\ 0 & 0 \end{vmatrix} = -0 - 0 = 0$

$A_{31} = (-1)^{3+1}M_{31} = \begin{vmatrix} 2 & 3 \\ 2 & 4 \end{vmatrix} = 8 - 6 = 2$

$A_{32} = (-1)^{3+2}M_{32} = -\begin{vmatrix} 1 & 3 \\ 0 & 4 \end{vmatrix} = -4 - 0 = -4$

$A_{33} = (-1)^{3+3}M_{33} = \begin{vmatrix} 1 & 2 \\ 0 & 2 \end{vmatrix} = 2 - 0 = 2$

$\therefore$ the co-factor matrix

$$= \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} = \begin{bmatrix} 10 & 0 & 0 \\ -10 & 5 & 0 \\ 2 & -4 & 2 \end{bmatrix}$$

$$\therefore \text{adj } A = \begin{bmatrix} 10 & -10 & 2 \\ 0 & 5 & -4 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}(\text{adj } A)$$

$$= \frac{1}{10}\begin{bmatrix} 10 & -10 & 2 \\ 0 & 5 & -4 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{10}\begin{bmatrix} 10 & -10 & 2 \\ 0 & 5 & -4 \\ 0 & 0 & 2 \end{bmatrix}$$

Exercise 2.2 | Q 6.1 | Page 52

Find the inverse of the following matrix.

$$\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$$

Solution:

Let A = $\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

$\therefore$ |A| = $\begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix}$ = – 1 – 4 = – 5 $\neq$ 0

$\therefore$ A^{-1} exists.

consider AA^{-1} = I

$\therefore$ $\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

By R_2 – $2R_1$, we get

$\begin{bmatrix} 1 & 2 \\ 0 & -5 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix}$

By $\left(-\frac{1}{5}\right)R_2$, we get,

$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ 2/5 & -1/5 \end{bmatrix}$

By R_1 – $2R_2$, we get,

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1/5 & 2/5 \\ 2/5 & -1/5 \end{bmatrix}$

$\therefore$ $A^{-1} = \frac{1}{5}\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

The answer can be checked by finding the product AA^{-1}

$AA^{-1} = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 1/5 & 2/5 \\ 2/5 & -1/5 \end{bmatrix}$

= $\begin{bmatrix} 1\left(\frac{1}{5}\right) + 2\left(\frac{2}{5}\right) & 1\left(\frac{2}{5}\right) + 2\left(-\frac{1}{5}\right) \\ 2\left(\frac{1}{5}\right) - 1\left(\frac{2}{5}\right) & 2\left(\frac{2}{5}\right) - 1\left(-\frac{1}{5}\right) \end{bmatrix}$

$$= \begin{bmatrix} \frac{1}{5} + \frac{4}{5} & \frac{2}{5} - \frac{2}{5} \\ \frac{2}{5} - \frac{2}{5} & \frac{4}{5} + \frac{1}{5} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

Hence, A^{-1} is the required answer.

Exercise 2.2 | Q 6.2 | Page 52

Find the inverse of the following matrix.

$$\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$$

Solution:

Let $A = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

$\therefore |A| = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix} = 4 - 3 = 1 \neq 0$

$\therefore A^{-1}$ exists.

consider $AA^{-1} = I$

$\therefore \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

By $R_1 + R_2$, we get,

$$\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

By $R_2 + R_1$, we get,

$$\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$$

By $R_1 + R_2$, we get,

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

Exercise 2.2 | Q 6.3 | Page 52

Find the inverse of the following matrix.

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

Solution:

Let $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

$\therefore |A| = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

$= 0(2 - 3) - 1(1 - 9) + 2(1 - 6)$

$= 0 + 8 - 10$

$= -2 \neq 0$

$\therefore A^{-1}$ exists.

consider $AA^{-1} = I$

$$\therefore \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_1 \leftrightarrow R_2$, we get,

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 3 & 1 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_3 - 3R_1$, we get,

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & -5 & -8 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & -3 & 1 \end{bmatrix}$$

By $R_1 - 2R_2$ and $R_3 + 5R_2$, we get,

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} A^{-1} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 5 & -3 & 1 \end{bmatrix}$$

By $\left(\frac{1}{2}\right)R_3$, we get,

$$\begin{bmatrix}1 & 0 & -1\\0 & 1 & 2\\0 & 0 & 1\end{bmatrix}A^{-1} = \begin{bmatrix}-2 & 1 & 0\\1 & 0 & 0\\5/2 & -3/2 & 1/2\end{bmatrix}$$

By $R_1 + R_3$ and $R_2 - 2R_3$, we get,

$$\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}A^{-1} = \begin{bmatrix}1/2 & -1/2 & 1/2\\-4 & 3 & -1\\5/2 & -3/2 & 1/2\end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{2}\begin{bmatrix}1 & -1 & 1\\-8 & 6 & -2\\5 & -3 & 1\end{bmatrix}$$

[Note: Answer in the textbook is incorrect.]

Exercise 2.2 | Q 6.4 | Page 52

Find the inverse of the following matrix.

$$\begin{bmatrix}2 & 0 & -1\\5 & 1 & 0\\0 & 1 & 3\end{bmatrix}$$

Solution:

Let A = $\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

$\therefore$ |A| = $\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

= 0(3 – 0) – 0(15 – 0) – 1(5 – 0)

= 6 – 0 – 5

= 1 $\neq$ 0

$\therefore$ A^{-1} exists.

consider $AA^{-1} = I$

$\therefore \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

By $3R_1$, we get,

$\begin{bmatrix} 6 & 0 & -3 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

By $R_1 - 3_2$, we get,

$\begin{bmatrix} 1 & -1 & -3 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 3 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

By $R_2 - 5R_1$, we get,

$$\begin{bmatrix}1 & -1 & -3\\0 & 6 & 0\\0 & 1 & 3\end{bmatrix}A^{-1} = \begin{bmatrix}3 & -1 & 0\\-15 & 6 & -5\\0 & 0 & 1\end{bmatrix}$$

By R2 – 5R3, we get,

$$\begin{bmatrix}1 & -1 & -3\\0 & 1 & 0\\0 & 1 & 3\end{bmatrix}A^{-1} = \begin{bmatrix}3 & -1 & 0\\-15 & 6 & -5\\0 & 0 & 1\end{bmatrix}$$

By $R_1 + R_2$ and $R_3 - R_2$, we get,

$$\begin{bmatrix}1 & 0 & -3\\0 & 1 & 0\\0 & 0 & 3\end{bmatrix}A^{-1} = \begin{bmatrix}-12 & 5 & -5\\-15 & 6 & -5\\15 & -6 & 6\end{bmatrix}$$

By $\left(\frac{1}{3}\right)R_3$, we get,

$$\begin{bmatrix}1 & 0 & -3\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}A^{-1} = \begin{bmatrix}-12 & 5 & -5\\-15 & 6 & -5\\5 & -2 & 2\end{bmatrix}$$

By $R_1 + 3R_3$, we get,

$$\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}A^{-1} = \begin{bmatrix}3 & -1 & 1\\-15 & 6 & -5\\5 & -2 & 2\end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix}3 & -1 & 1\\-15 & 6 & -5\\5 & -2 & 2\end{bmatrix}$$

MISCELLANEOUS EXERCISE 2 (A) [PAGES 52 - 54]

Miscellaneous exercise 2 (A) | Q 1 | Page 52

If A = $\begin{pmatrix}1 & 0 & 0\\2 & 1 & 0\\3 & 3 & 1\end{pmatrix}$, then reduce it to I_3 by using column transformations.

Solution:

$|A| = \begin{vmatrix}1 & 0 & 0\\2 & 1 & 0\\3 & 3 & 1\end{vmatrix}$

$= 1(1 - 0) - 0 + 0 = 1 \neq 0$

$\therefore$ A is a non-singular matrix.

Hence, the required transformation is possible.

Now, A = $\begin{bmatrix}1 & 0 & 0\\2 & 1 & 0\\3 & 3 & 1\end{bmatrix}$

By $C_1 - 2C_2$, we get, A ~ $\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\-3 & 3 & 1\end{bmatrix}$

By $C_1 + 3C_3$ and $C_2 - 3C_3$, we get,

$A \sim \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} = I_3.$

Miscellaneous exercise 2 (A) | Q 2 | Page 52

If A = $\begin{pmatrix} 2 & 1 & 3 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}$, then reduce it to I_3 by using row transformations.

Solution:

$|A| = \begin{vmatrix} 2 & 1 & 3 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{vmatrix}$

= 2(0 - 1)- 1(1 - 1) + 3(1 - 0)

= - 2 - 0 + 3

= 1 ≠ 0

∴ A is a non-singular matrix.

Hence, the required transformation is possible.

Now, A = $\begin{bmatrix} 2 & 1 & 3 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

By R_1 - R_2, we get,

$A \sim \begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

By R_2 - R_1 and R_3 - R_1, we get,

$A \sim \begin{bmatrix} 1 & 1 & 2 \\ 0 & -1 & -1 \\ 0 & 0 & -1 \end{bmatrix}$

By $(-1)R_2$ and $(-1)R_3$, we get,

$$A \sim \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_1 - R_2$, we get,

$$A \sim \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_1 - R_3$, and $R_2 - R_3$, we get,

$$A \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Miscellaneous exercise 2 (A) | Q 3.1 | Page 52

Check whether the following matrix is invertible or not:

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Solution:

Let A = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Then, $|A| = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1 \neq 0.$

$\therefore$ A is a non-singular matrix.

Hence, A^{-1} exists.

Miscellaneous exercise 2 (A) | Q 3.2 | Page 52

Check whether the following matrix is invertible or not:

$$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

Solution:

Let A = $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

Then, |A| = $\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 1 - 1 = 0$

$\therefore$ A is a singular matrix.

Hence, A^{-1} does not exist.

Miscellaneous exercise 2 (A) | Q 3.3 | Page 52

Check whether the following matrix is invertible or not:

$$\begin{pmatrix} 1 & 2 \\ 3 & 3 \end{pmatrix}$$

Solution:

Let A = $\begin{pmatrix} 1 & 2 \\ 3 & 3 \end{pmatrix}$

Then, |A| = $\begin{vmatrix} 1 & 2 \\ 3 & 3 \end{vmatrix} = 3 - 6 = -3 \neq 0$

$\therefore$ A is a non - singular matrix.

Hence, A^{-1} exists.

Miscellaneous exercise 2 (A) | Q 3.4 | Page 52

Check whether the following matrix is invertible or not:

$$\begin{pmatrix} 2 & 3 \\ 10 & 15 \end{pmatrix}$$

Solution:

Let A = $\begin{pmatrix} 2 & 3 \\ 10 & 15 \end{pmatrix}$

Then, |A| = $\begin{vmatrix} 2 & 3 \\ 10 & 15 \end{vmatrix} = 30 - 30 = 0$

∴ A is a singular matrix.

Hence, A^{-1} does not exist.

Miscellaneous exercise 2 (A) | Q 3.5 | Page 52

Check whether the following matrix is invertible or not:

$\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

Solution:

Let A = $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

Then, |A| = $\begin{vmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{vmatrix} = \cos^2\theta + \sin^2\theta = 1 \neq 0$

∴ A is a non - singular matrix.

Hence, A^{-1} exists.

Miscellaneous exercise 2 (A) | Q 3.6 | Page 52

Check whether the following matrix is invertible or not:

$\begin{pmatrix} \sec\theta & \tan\theta \\ \tan\theta & \sec\theta \end{pmatrix}$

Solution:

Let A = $\begin{pmatrix} \sec\theta & \tan\theta \\ \tan\theta & \sec\theta \end{pmatrix}$

Then, |A| = $\begin{vmatrix} \sec\theta & \tan\theta \\ \tan\theta & \sec\theta \end{vmatrix} = \sec^2\theta - \tan^2\theta = 1 \neq 0$

∴ A is a non - singular matrix.

Hence, A^{-1} exists.

Miscellaneous exercise 2 (A) | Q 3.7 | Page 52

Check whether the following matrix is invertible or not:

$$\begin{pmatrix} 3 & 4 & 3 \\ 1 & 1 & 0 \\ 1 & 4 & 5 \end{pmatrix}$$

Solution:

Let A = $\begin{pmatrix} 3 & 4 & 3 \\ 1 & 1 & 0 \\ 1 & 4 & 5 \end{pmatrix}$

Then, |A| = $\begin{vmatrix} 3 & 4 & 3 \\ 1 & 1 & 0 \\ 1 & 4 & 5 \end{vmatrix}$

= 3(5 - 0) - 4(5 - 0) + 3(4 - 1)

= 15 - 20 + 9

= 4 ≠ 0`

∴ A is a non - singular matrix.

Hence, A^{-1} exists.

Miscellaneous exercise 2 (A) | Q 3.8 | Page 52

Check whether the following matrix is invertible or not:

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & -1 & 3 \\ 1 & 2 & 3 \end{pmatrix}$$

Solution:

Let A = $\begin{pmatrix} 1 & 2 & 3 \\ 2 & -1 & 3 \\ 1 & 2 & 3 \end{pmatrix}$

Then, |A| = $\begin{vmatrix} 1 & 2 & 3 \\ 2 & -1 & 3 \\ 1 & 2 & 3 \end{vmatrix}$

= 1(- 3 - 6) - 2(6 - 3) + 3(4 + 1)

= - 9 - 6 + 15

= 0

∴ A is a singular matrix.

Hence, A^{-1} does not exist.

Miscellaneous exercise 2 (A) | Q 3.9 | Page 52

Check whether the following matrix is invertible or not:

$$\begin{pmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 4 & 6 & 8 \end{pmatrix}$$

Solution:

Let A = $\begin{pmatrix}1 & 2 & 3\\ 3 & 4 & 5\\ 4 & 6 & 8\end{pmatrix}$

Then, |A| = $\begin{vmatrix}1 & 2 & 3\\ 3 & 4 & 5\\ 4 & 6 & 8\end{vmatrix}$

= 1 (32 - 30) - 2 (24 - 20) + 3(18 - 16)

= 2 - 8 + 6

= 0

∴ A is a singular matrix.

Hence, A^{-1} does not exist.

Miscellaneous exercise 2 (A) | Q 4 | Page 52

Find AB, if A = $\begin{pmatrix}1 & 2 & 3\\ 1 & -2 & -3\end{pmatrix}$ and B = $\begin{pmatrix}1 & -1\\ 1 & 2\\ 1 & -2\end{pmatrix}$. Examine whether AB has inverse or not.

Solution:

AB = $\begin{pmatrix}1 & 2 & 3\\ 1 & -2 & -3\end{pmatrix} \times \begin{pmatrix}1 & -1\\ 1 & 2\\ 1 & -2\end{pmatrix}$

$$= \begin{bmatrix}1(1)+2(1)+3(1) & 1(-1)+2(2)+3(-2)\\ 1(1)+(-2)(1)+(-3)(1) & 1(-1)+(-2)(2)+(-3)(-2)\end{bmatrix}$$

$$= \begin{bmatrix}1+2+3 & -1+4-6\\ 1-2-3 & -1-4+6\end{bmatrix}$$

$$= \begin{bmatrix} 6 & -3 \\ -4 & 1 \end{bmatrix}$$

∴ |AB| = $\begin{vmatrix} 6 & -3 \\ -4 & 1 \end{vmatrix} = 6 - 12 = -6 \neq 0$

∴ AB is a non-singular matrix.

Hence, $(AB)^{-1}$ exists.

Miscellaneous exercise 2 (A) | Q 5 | Page 52

If A = $\begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$ is a non-singular matrix, then find A^{-1} by using elementary row transformations. Hence, find the inverse of $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$.

Solution:

Since A is a non-singular matrix, A^{-1} exists. We write $AA^{-1} = I$

$$\therefore \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $\left(\frac{1}{x}\right)R_1, \left(\frac{1}{y}\right)R_2$ and $\left(\frac{1}{z}\right)R_3$, we get,

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} \frac{1}{x} & 0 & 0 \\ 0 & \frac{1}{y} & 0 \\ 0 & 0 & \frac{1}{z} \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} \frac{1}{x} & 0 & 0 \\ 0 & \frac{1}{y} & 0 \\ 0 & 0 & \frac{1}{z} \end{bmatrix}$$

Comparing $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ with $\begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$

we get, x = 2, y = 1, z = - 1

$$\therefore \frac{1}{x} = \frac{1}{2}, \frac{1}{y} = \frac{1}{1} = 1, \frac{1}{z} = \frac{1}{-1} = -1$$

Hence, the inverse of

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \text{ is } \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

Miscellaneous exercise 2 (A) | Q 6 | Page 53

If A = $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and X is a 2 × 2 matrix such that AX = I, find X.

Solution: We will reduce the matrix A to the identity matrix by using row transformations. During this process, I will be converted to matrix X.

We have AX = I

$$\therefore \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} X = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

By $R_2 - 3R_1$, we get,

$$\begin{bmatrix} 1 & 2 \\ 0 & -2 \end{bmatrix} X = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}$$

By $\left(-\frac{1}{2}\right)R_2$, we get,

$$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} X = \begin{bmatrix} 1 & 0 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}$$

By $R_1 - 2R_2$, we get,

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} X = \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}$$

$\therefore$ X = $\begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}$

Miscellaneous exercise 2 (A) | Q 7.01 | Page 53

Find the inverse of the following matrix (if they exist):

$$\begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$$

Solution:

Let A = $\begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$

$\therefore$ |A| = $\begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix} = 3 + 2 = 5 \neq 0$

$\therefore$ A^{-1} exists.

Consider $AA^{-1} = I$

$$\therefore \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix} A^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

By $R_2 - 2R_1$, we get,

$$\begin{pmatrix} 1 & -1 \\ 0 & 5 \end{pmatrix} A^{-1} = \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix}$$

By $\left(\frac{1}{5}\right)\mathrm{R}_2$, we get,

$$\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}\mathbf{A}^{-1} = \begin{pmatrix} 1 & 0 \\ -\frac{2}{5} & \frac{1}{5} \end{pmatrix}$$

By $R_1 + R_2$, we get,

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\mathbf{A}^{-1} = \begin{pmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{pmatrix}$$

$\therefore A^{-1} = \frac{1}{5}\begin{pmatrix} 3 & 1 \\ -2 & 1 \end{pmatrix}$

Miscellaneous exercise 2 (A) | Q 7.02 | Page 53

Find the inverse of the following matrix (if they exist):

$$\begin{pmatrix} 2 & 1 \\ 1 & -1 \end{pmatrix}$$

Solution:

Let A = $\begin{pmatrix} 2 & 1 \\ 1 & -1 \end{pmatrix}$

$\therefore |A| = \begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} = -2 - 1 = -3 \neq 0$

$\therefore A^{-1}$ exists.

Consider $AA^{-1} = I$

$\therefore \begin{pmatrix} 2 & 1 \\ 1 & -1 \end{pmatrix}\mathbf{A}^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

By $R_1 \leftrightarrow R_2$, we get,

$$\begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix}\mathbf{A}^{-1} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

By R_2 - $2R_1$, we get,

$$\begin{pmatrix}1 & -1\\0 & 3\end{pmatrix}\mathbf{A}^{-1} = \begin{pmatrix}0 & 1\\1 & -2\end{pmatrix}$$

By $\left(\frac{1}{3}\right)\mathbf{R}_2$, we get,

$$\begin{pmatrix}1 & -1\\0 & 1\end{pmatrix}\mathbf{A}^{-1} = \begin{pmatrix}0 & 1\\\frac{1}{3} & -\frac{2}{3}\end{pmatrix}$$

By $R_1 + R_2$, we get,

$$\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\mathbf{A}^{-1} = \begin{pmatrix}\frac{1}{3} & \frac{1}{3}\\\frac{1}{3} & -\frac{2}{3}\end{pmatrix}$$

$\therefore A^{-1} = \frac{1}{3}\begin{pmatrix}1 & 1\\1 & -2\end{pmatrix}$

Miscellaneous exercise 2 (A) | Q 7.03 | Page 53

Find the inverse of the following matrix (if they exist):

$$\begin{pmatrix}1 & 3\\2 & 7\end{pmatrix}$$

Solution:

Let A = $\begin{bmatrix}1 & 3\\2 & 7\end{bmatrix}$

$\therefore |A| = \begin{vmatrix}1 & 3\\2 & 7\end{vmatrix} = 7 - 6 = 1 \neq 0$

$\therefore A^{-1}$ exists.

Consider $AA^{-1} = I$

$$\therefore \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

By $R_2 - 2R_1$, we get,

$$\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$$

By $R_1 - 3R_2$ we get,

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix}$$

Miscellaneous exercise 2 (A) | Q 7.04 | Page 53

Find the inverse of the following matrix (if they exist):

$$\begin{bmatrix} 2 & -3 \\ 5 & 7 \end{bmatrix}$$

Solution:

Let A = $\begin{bmatrix} 2 & -3 \\ 5 & 7 \end{bmatrix}$

$\therefore$ |A| = $\begin{vmatrix} 2 & -3 \\ 5 & 7 \end{vmatrix} = 14 + 15 = 29 \neq 0$

$\therefore$ A^{-1} exists.

Consider $AA^{-1} = I$

$$\therefore \begin{bmatrix} 2 & -3 \\ 5 & 7 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

By $3R_1$, we get,

$$\begin{bmatrix} 6 & -9 \\ 5 & 7 \end{bmatrix} A^{-1} = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$$

By R_1 - R_2, we get,

$$\begin{bmatrix}1 & -16\\5 & 7\end{bmatrix}A^{-1} = \begin{bmatrix}3 & -1\\0 & 1\end{bmatrix}$$

By R_2 - $5R_1$, we get,

$$\begin{bmatrix}1 & -16\\0 & 87\end{bmatrix}A^{-1} = \begin{bmatrix}3 & -1\\-15 & 6\end{bmatrix}$$

By $\left(\frac{1}{87}\right)R_2$,we get

$$\begin{bmatrix}1 & -16\\0 & 1\end{bmatrix}A^{-1} = \begin{bmatrix}3 & -1\\-\frac{5}{29} & \frac{2}{29}\end{bmatrix}$$

By R_1 + $16R_2$, we get,

$$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}A^{-1} = \begin{bmatrix}\frac{7}{29} & \frac{3}{29}\\-\frac{5}{29} & \frac{2}{29}\end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{29}\begin{bmatrix}7 & 3\\-5 & 2\end{bmatrix}$$

Miscellaneous exercise 2 (A) | Q 7.05 | Page 53

Find the inverse of the following matrix (if they exist):

$$\begin{bmatrix}2 & 1\\7 & 4\end{bmatrix}$$

Solution:

Let A = $\begin{bmatrix}2 & 1\\ 7 & 4\end{bmatrix}$

∴ |A| = $\begin{vmatrix}2 & 1\\ 7 & 4\end{vmatrix} = 8 - 7 = 1 \neq 0$

∴ A^{-1} exists.

Consider AA^{-1} = I

∴ $\begin{bmatrix}2 & 1\\ 7 & 4\end{bmatrix}A^{-1} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

By $R_1 \to R_1 - \dfrac{1}{7}R_2$ we get,

$$\begin{bmatrix}1 & \frac{3}{7}\\ 7 & 4\end{bmatrix}A^{-1} = \begin{bmatrix}1 & \frac{-1}{7}\\ 0 & 1\end{bmatrix}$$

By $R_2 \to R_2 - 7R_1$ we get,

$$\begin{bmatrix}1 & \frac{3}{7}\\ 0 & 1\end{bmatrix}A^{-1} = \begin{bmatrix}1 & \frac{-1}{7}\\ -7 & 2\end{bmatrix}$$

By $R_1 \to R_1 - \dfrac{3}{7}R_2$

$$\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}A^{-1} = \begin{bmatrix}4 & -1\\ -7 & 2\end{bmatrix}$$

∴ $A^{-1} = \begin{bmatrix}4 & -1\\ -7 & 2\end{bmatrix}$

Miscellaneous exercise 2 (A) | Q 7.06 | Page 53

Find the inverse of the following matrix (if they exist):

$\begin{bmatrix}3 & -10\\ 2 & -7\end{bmatrix}$

Solution:

Let A = $\begin{bmatrix}3 & -10\\2 & -7\end{bmatrix}$

$\therefore$ |A| = $\begin{vmatrix}3 & -10\\2 & -7\end{vmatrix} = -21 + 20 = -1 \neq 0$

$\therefore$ A^{-1} exists.

Consider $AA^{-1} = I$

$\therefore \begin{bmatrix}3 & -10\\2 & -7\end{bmatrix}A^{-1} = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

By $R_1 - R_2$, we get,

$\therefore \begin{bmatrix}1 & -3\\2 & -7\end{bmatrix}A^{-1} = \begin{bmatrix}1 & -1\\0 & 1\end{bmatrix}$

By $R_2 - 2R_1$, we get,

$\begin{bmatrix}1 & -3\\0 & -1\end{bmatrix}A^{-1} = \begin{bmatrix}1 & -1\\-2 & 3\end{bmatrix}$

By $(-1)R_2$, we get,

$\begin{bmatrix}1 & -3\\0 & 1\end{bmatrix}A^{-1} = \begin{bmatrix}1 & -1\\2 & -3\end{bmatrix}$

By $R_1 + 3R_2$, we get,

$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}A^{-1} = \begin{bmatrix}7 & -10\\2 & -3\end{bmatrix}$

$\therefore A^{-1} = \begin{bmatrix}7 & -10\\2 & -3\end{bmatrix}$

Miscellaneous exercise 2 (A) | Q 7.07 | Page 53

Find the inverse of the following matrix (if they exist):

$$\begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$$

Solution:

Let A = $\begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$

$\therefore$ |A| = $\begin{vmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{vmatrix}$

= 2(4 + 6) +3(4 - 9)+3(- 4 - 6)

= 20 - 15 - 30

= - 25 $\neq$ 0

$\therefore$ A^{-1} exists.

Consider $AA^{-1} = I$

$$\therefore \begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_1 \leftrightarrow R_3$, we get,

$$\begin{bmatrix} 3 & -2 & 2 \\ 2 & 2 & 3 \\ 2 & -3 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

By $R_1 \to R_1 - R_2$, we get

$$\begin{bmatrix} 1 & -4 & -1 \\ 2 & 2 & 3 \\ 2 & -3 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & -1 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

By $R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 - 2R_1$, we get,

$$\begin{bmatrix} 1 & -4 & -1 \\ 0 & 10 & 5 \\ 0 & 5 & 0 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & -1 & 1 \\ 0 & 3 & -2 \\ 1 & 2 & -2 \end{bmatrix}$$

By $R_2 \to \left(\frac{1}{10}\right)R_2$, we get,

$$\begin{bmatrix} 1 & -4 & -1 \\ 0 & 1 & \frac{1}{2} \\ 0 & 5 & 0 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & -1 & 1 \\ 0 & \frac{3}{10} & -\frac{1}{5} \\ 1 & 2 & -2 \end{bmatrix}$$

By $R_1 \to R_1 + 4R_2$ and $R_3 \to R_3 - 5R_2$, we get,

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & \frac{1}{2} \\ 0 & 0 & \frac{5}{2} \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & \frac{1}{5} & \frac{1}{5} \\ 0 & \frac{3}{10} & -\frac{1}{5} \\ 1 & \frac{1}{2} & -1 \end{bmatrix}$$

By $R_3 \to \left(\frac{2}{5}\right)R_3$, we get,

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & \frac{1}{2} \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & \frac{1}{5} & \frac{1}{5} \\ 0 & \frac{3}{10} & -\frac{1}{5} \\ \frac{2}{5} & \frac{1}{5} & -\frac{2}{5} \end{bmatrix}$$

By $R_1 \to R_3$ and $R_2 \to R_2 - \frac{1}{2}R_3$, we get,

$$\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}A^{-1} = \begin{bmatrix}-\frac{2}{5} & 0 & \frac{3}{5}\\-\frac{1}{5} & \frac{1}{5} & 0\\\frac{2}{5} & \frac{1}{5} & -\frac{2}{5}\end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix}-\frac{2}{5} & 0 & \frac{3}{5}\\-\frac{1}{5} & \frac{1}{5} & 0\\\frac{2}{5} & \frac{1}{5} & -\frac{2}{5}\end{bmatrix}$$

Miscellaneous exercise 2 (A) | Q 7.08 | Page 53

Find the inverse of the following matrix (if they exist).

$$\begin{bmatrix}1 & 3 & -2\\-3 & 0 & -5\\2 & 5 & 0\end{bmatrix}$$

Solution:

Let A = $\begin{bmatrix}1 & 3 & -2\\-3 & 0 & -5\\2 & 5 & 0\end{bmatrix}$

$\therefore$ |A| = $\begin{vmatrix}1 & 3 & -2\\-3 & 0 & -5\\2 & 5 & 0\end{vmatrix}$

= 1(0 + 25) + 3(0 + 10) + 2(- 15 - 0)

= 25 + 30 - 30

= 25 ≠ 0

$\therefore$ A^{-1} exists.

Consider $AA^{-1} = I$

$$\therefore \begin{bmatrix}1 & 3 & -2\\-3 & 0 & -5\\2 & 5 & 0\end{bmatrix}A^{-1} = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

By $R_2 \rightarrow R_2 + 3R_1$

$$\begin{bmatrix}1 & 3 & -2\\0 & 9 & -11\\2 & 5 & 0\end{bmatrix}A^{-1} = \begin{bmatrix}1 & 0 & 0\\3 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

By $R_3 \rightarrow R_3 - 2R_1$

$$\begin{bmatrix}1 & 3 & -2\\0 & 9 & -11\\0 & -1 & 4\end{bmatrix}A^{-1} = \begin{bmatrix}1 & 0 & 0\\3 & 1 & 0\\-2 & 0 & 1\end{bmatrix}$$

By $\mathrm{R} \rightarrow \frac{1}{9}\mathrm{R}_2$

$$\begin{bmatrix}1 & 3 & -2\\0 & 1 & -\frac{11}{9}\\0 & -1 & 4\end{bmatrix}A^{-1} = \begin{bmatrix}1 & 0 & 0\\\frac{1}{3} & \frac{1}{9} & 0\\-2 & 0 & 1\end{bmatrix}$$

By $\mathrm{R}_3 \rightarrow \mathrm{R}_3 + \mathrm{R}_2$

$$\begin{bmatrix}1 & 3 & -2\\0 & 1 & -\frac{11}{9}\\0 & 0 & \frac{25}{9}\end{bmatrix}A^{-1} = \begin{bmatrix}1 & 0 & 0\\\frac{1}{3} & \frac{1}{9} & 0\\-\frac{5}{3} & \frac{1}{9} & 1\end{bmatrix}$$

By $\mathrm{R}_1 \rightarrow \mathrm{R}_1 + 3\mathrm{R}_2$

$$\begin{bmatrix}1 & 0 & \frac{5}{3}\\0 & 1 & -\frac{11}{9}\\0 & 0 & \frac{25}{9}\end{bmatrix}A^{-1} = \begin{bmatrix}0 & -\frac{1}{3} & 0\\\frac{1}{3} & \frac{1}{9} & 0\\-\frac{5}{3} & \frac{1}{9} & 1\end{bmatrix}$$

By $\mathrm{R}_3 \rightarrow \frac{9}{25}\mathrm{R}_3$

$$\begin{bmatrix}1 & 0 & \frac{5}{3}\\0 & 1 & -\frac{11}{9}\\0 & 0 & 1\end{bmatrix}A^{-1} = \begin{bmatrix}0 & -\frac{1}{3} & 0\\\frac{1}{3} & \frac{1}{9} & 0\\-\frac{3}{5} & \frac{1}{25} & \frac{9}{25}\end{bmatrix}$$

By $\mathrm{R}_2 \rightarrow \mathrm{R}_2 + \frac{11}{9}\mathrm{R}_3$

$$\begin{bmatrix} 1 & 0 & \frac{5}{3} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & -\frac{1}{3} & 0 \\ -\frac{2}{5} & \frac{4}{25} & \frac{11}{25} \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix}$$

By $R_1 \to R_1 - \frac{5}{3} R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} \frac{5}{5} & -\frac{6}{15} & -\frac{3}{5} \\ -\frac{2}{5} & \frac{4}{25} & \frac{11}{25} \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1 & -\frac{2}{5} & -\frac{3}{5} \\ -\frac{2}{5} & \frac{4}{25} & \frac{11}{25} \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix}$$

$$A^{-1} = \frac{1}{25} \begin{bmatrix} 25 & -10 & -15 \\ -10 & 4 & 11 \\ -15 & 1 & 9 \end{bmatrix}$$

Miscellaneous exercise 2 (A) | Q 7.09 | Page 53

Find the inverse of the following matrix (if they exist):

$$\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

Solution:

Let A = $\begin{bmatrix}2 & 0 & -1\\5 & 1 & 0\\0 & 1 & 3\end{bmatrix}$

$\therefore$ |A| = $\begin{vmatrix}2 & 0 & -1\\5 & 1 & 0\\0 & 1 & 3\end{vmatrix}$

= 2(3 - 0) - 0 - 1(5 - 0)

= 6 - 0 - 5

= 1 $\neq$ 0

$\therefore$ A^{-1} exists.

Consider $AA^{-1} = I$

$\therefore \begin{bmatrix}2 & 0 & -1\\5 & 1 & 0\\0 & 1 & 3\end{bmatrix} A^{-1} = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$

By $R_1 \leftrightarrow R_2$, we get,

$$\begin{bmatrix} 5 & 1 & 0 \\ 2 & 0 & -1 \\ 0 & 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By R_1 - $2R_2$, we get,

$$\begin{bmatrix} 1 & 1 & 2 \\ 2 & 0 & -1 \\ 0 & 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By R_2 - $2R_1$, we get,

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & -2 & -5 \\ 0 & 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} -2 & 1 & 0 \\ 5 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By R_2 + $3R_3$, we get,

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 4 \\ 0 & 1 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} -2 & 1 & 0 \\ 5 & -2 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

By R_1 - R_2 and R_3 - R_2, we get,

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 4 \\ 0 & 0 & -1 \end{bmatrix} A^{-1} = \begin{bmatrix} -7 & 3 & -3 \\ 5 & -2 & 3 \\ -5 & 2 & -2 \end{bmatrix}$$

By (- 1)R_3, we get,

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} -7 & 3 & -3 \\ 5 & -2 & 3 \\ 5 & -2 & 2 \end{bmatrix}$$

By $R_1 + 2R_3$ and $R_2 - 4R_3$, we get,

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

Miscellaneous exercise 2 (A) | Q 7.1 | Page 54

Find the inverse of the following matrix by elementary row transformations if it exists.

$$A = \begin{bmatrix} 1 & 2 & -2 \\ 0 & -2 & 1 \\ -1 & 3 & 0 \end{bmatrix}$$

Solution:

$$A = \begin{bmatrix} 1 & 2 & -2 \\ 0 & -2 & 1 \\ -1 & 3 & 0 \end{bmatrix}$$

$$\therefore A = \begin{vmatrix} 1 & 2 & -2 \\ 0 & -2 & 1 \\ -1 & 3 & 0 \end{vmatrix}$$

$$= 1\begin{vmatrix} -2 & 1 \\ 3 & 0 \end{vmatrix} - 2\begin{vmatrix} 0 & 1 \\ -1 & 1 \end{vmatrix} - 2\begin{vmatrix} 0 & -2 \\ -1 & 3 \end{vmatrix}$$

$$|A| = 1(0-3) - 2(0+1) - 2(0-2)$$

$$= -3 - 2 + 4$$

$$= -1 \neq 0$$

$$\therefore A^{-1} \text{ exist}$$

We have

$AA^{-1} = I$

$$\begin{bmatrix} 1 & 2 & -2 \\ 0 & -2 & 1 \\ -1 & 3 & 0 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_3 \to R_3 + R_1$

$$\begin{bmatrix} 1 & 2 & -2 \\ 0 & -2 & 1 \\ 0 & 5 & -2 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$R_3 \to R_3 + 2R_2$

$$\begin{bmatrix} 1 & 2 & -2 \\ 0 & -2 & 1 \\ 0 & 1 & -0 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

$R_2 \leftrightarrow R_3$

$$\begin{bmatrix} 1 & 2 & -2 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$R_1 \to R_1 - 2R_2 \qquad R3 \to R_3 + 2R_2$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} -1 & -4 & -2 \\ 1 & 2 & 1 \\ 2 & 5 & 2 \end{bmatrix}$$

$R_1 \to R_1 + 2R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 3 & 6 & 2 \\ 1 & 2 & 1 \\ 2 & 5 & 2 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 3 & 6 & 2 \\ 1 & 2 & 1 \\ 2 & 5 & 2 \end{bmatrix}$$

Miscellaneous exercise 2 (A) | Q 8.1 | Page 53

Find the inverse of A = $\begin{bmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}$ by elementary row transformations.

Solution:

$|A| = \begin{vmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{vmatrix}$

$= \cos\theta(\cos\theta - 0) + \sin\theta(\sin\theta - 0) + 0$

$= \cos^2\theta + \sin^2\theta = 1 \neq 0$

$\therefore A^{-1}$ exists.

Consider $AA^{-1} = I$

$$\therefore \begin{bmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}A^{-1} = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

By $\cos\theta \times R_1$, we get,

$$\begin{bmatrix}\cos^2\theta & -\sin\theta.\cos\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}A^{-1} = \begin{bmatrix}\cos\theta & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

By $R_1 + \sin\theta \times R_2$, we get,

$$\begin{bmatrix} 1 & 0 & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By R_2 - sin θ × R_1, we get,

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta\cos\theta & \cos^2\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $\left(\dfrac{1}{\cos\theta}\right) \times R_2$, we get,

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Miscellaneous exercise 2 (A) | Q 8.2 | Page 53

Find the inverse of A = $\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ by elementary column transformations.

Solution:

$$|A| = \begin{vmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

= cos θ(cos θ - 0) + sin θ(sin θ - 0) + 0

= $\cos^2\theta + \sin^2\theta = 1 \neq 0$

∴ A^{-1} exists.

Consider $A^{-1}A = I$

$$\therefore \mathbf{A}^{-1} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By (cos θ) × C_1, we get,

$$\mathbf{A}^{-1} = \begin{bmatrix} \cos^2\theta & -\sin\theta & 0 \\ \sin\theta\cos\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By C_1 - sin θ × C_2, we get,

$$\mathbf{A}^{-1} \begin{bmatrix} 1 & -\sin\theta & 0 \\ 0 & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & 0 \\ -\sin\theta & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By C_2 + sin θ × C_1, we get,

$$\mathbf{A}^{-1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta\cos\theta & 0 \\ -\sin\theta & \cos^2\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $\left(\frac{1}{\cos\theta}\right)C_2$, we get,

$$A^{-1}\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} = \begin{bmatrix}\cos\theta & \sin\theta & 0\\ -\sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}$$

Miscellaneous exercise 2 (A) | Q 9 | Page 53

If A = $\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}$, B = $\begin{bmatrix}1 & 0\\ 3 & 1\end{bmatrix}$, find AB and $(AB)^{-1}$. Verify that $(AB)^{-1} = B^{-1}.A^{-1}$.

Solution:

AB = $\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}\begin{bmatrix}1 & 0\\ 3 & 1\end{bmatrix}$

$$= \begin{bmatrix}2+9 & 0+3\\ 1+6 & 0+2\end{bmatrix} = \begin{bmatrix}11 & 3\\ 7 & 2\end{bmatrix}$$

∴ |AB| = $\begin{vmatrix}11 & 3\\ 7 & 2\end{vmatrix} = 22 - 21 = 1 \neq 0$

∴ $(AB)^{-1}$ exists.

Now, $(AB)(AB)^{-1} = I$

$$\therefore \begin{bmatrix}11 & 3\\ 7 & 2\end{bmatrix}(AB)^{-1} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

By $2R_1$, we get,

$$\begin{bmatrix}22 & 6\\ 7 & 2\end{bmatrix}(AB)^{-1} = \begin{bmatrix}2 & 0\\ 0 & 1\end{bmatrix}$$

By $R_1 - 3R_2$, we get,

$$\begin{bmatrix}1 & 0\\7 & 2\end{bmatrix}(AB)^{-1} = \begin{bmatrix}2 & -3\\0 & 1\end{bmatrix}$$

By $R_2 - 7R_1$, we get,

$$\begin{bmatrix}1 & 0\\0 & 2\end{bmatrix}(AB)^{-1} = \begin{bmatrix}2 & -3\\-14 & 22\end{bmatrix}$$

By $\left(\frac{1}{2}\right)R_2$, we get,

$$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}(AB)^{-1} = \begin{bmatrix}2 & -3\\-7 & 11\end{bmatrix}$$

$\therefore (AB)^{-1} = \begin{bmatrix}2 & -3\\-7 & 11\end{bmatrix}$ ....(1)

$|A| = \begin{vmatrix}2 & 3\\1 & 2\end{vmatrix} = 4 - 3 = 1 \neq 0$

$\therefore A^{-1}$ exists.

Consider, $AA^{-1} = I$

$$\therefore \begin{bmatrix}2 & 3\\1 & 2\end{bmatrix}A^{-1} = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

By $R_1 \leftrightarrow R_2$, we get,

$$\begin{bmatrix}1 & 2\\2 & 3\end{bmatrix}A^{-1} = \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$$

By $R_2 - 2R_1$,we get,

$$\begin{bmatrix}1 & 2\\0 & -1\end{bmatrix}A^{-1} = \begin{bmatrix}0 & 1\\1 & -2\end{bmatrix}$$

By $(-1)R_2$, we get,

$$\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}A^{-1} = \begin{bmatrix}0 & 1\\-1 & 2\end{bmatrix}$$

By $R_1 - 2R_2$, we get,

$$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}A^{-1} = \begin{bmatrix}2 & -3\\-1 & 2\end{bmatrix}$$

$\therefore A^{-1} = \begin{bmatrix}2 & -3\\-1 & 2\end{bmatrix}$

$|B| = \begin{vmatrix}1 & 0\\3 & 1\end{vmatrix} = 1 - 0 = 1 \neq 0$

$\therefore B^{-1}$ exists.

Consider, $BB^{-1} = I$

$\therefore \begin{bmatrix}1 & 0\\3 & 1\end{bmatrix}B^{-1} = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

By $R_2 - 3R_1$, we get,

$$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}A^{-1} = \begin{bmatrix}1 & 0\\-3 & 1\end{bmatrix}$$

$\therefore B^{-1} = \begin{bmatrix}1 & 0\\-3 & 1\end{bmatrix}$

$\therefore B^{-1}.A^{-1} = \begin{bmatrix}1 & 0\\-3 & 1\end{bmatrix}, \begin{bmatrix}2 & -3\\-1 & 2\end{bmatrix}$

$= \begin{bmatrix}2-0 & -3+0\\-6-1 & 9+2\end{bmatrix}$

$= \begin{bmatrix}2 & -3\\-7 & 11\end{bmatrix}$ ....(2)

From (1) and (2), $(AB)^{-1} = B^{-1}.A^{-1}$.

Miscellaneous exercise 2 (A) | Q 10 | Page 53

If A = $\begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix}$, show that $\mathbf{A}^{-1} = \frac{1}{6}(\mathbf{A} - 5\mathbf{I})$.

Solution:

|A| = $\begin{vmatrix} 4 & 5 \\ 2 & 1 \end{vmatrix} = 4 - 10 = -6 \neq 0$

∴ A^{-1} exists.

consider $AA^{-1} = I$

∴ $\begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix} \mathbf{A}^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

By $\left(\frac{1}{4}\right)\mathbf{R}_1$, we get,

$$\begin{bmatrix} 1 & \frac{5}{4} \\ 2 & 1 \end{bmatrix} \mathbf{A}^{-1} = \begin{bmatrix} \frac{1}{4} & 0 \\ 0 & 1 \end{bmatrix}$$

By $R_2 - 2R_1$ we get,

$$\begin{bmatrix} 1 & \frac{5}{4} \\ 0 & -\frac{3}{2} \end{bmatrix} \mathbf{A}^{-1} = \begin{bmatrix} \frac{1}{4} & 0 \\ -\frac{1}{2} & 1 \end{bmatrix}$$

By $\left(-\frac{2}{3}\right)\mathrm{R}_2$, we get,

$$\begin{bmatrix}1 & \frac{5}{4}\\0 & 1\end{bmatrix}\mathrm{A}^{-1} = \begin{bmatrix}\frac{1}{4} & 0\\\frac{1}{3} & -\frac{2}{3}\end{bmatrix}$$

By $\mathrm{R}_1 - \frac{5}{4}\mathrm{R}_2$, we get,

$$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\mathrm{A}^{-1} = \begin{bmatrix}-\frac{1}{6} & \frac{5}{6}\\\frac{1}{3} & -\frac{2}{3}\end{bmatrix}$$

$\therefore$ $A^{-1} = \frac{1}{6}\begin{bmatrix}-1 & 5\\2 & -4\end{bmatrix}$ ....(1)

$$\frac{1}{6}(\mathrm{A}-5\mathrm{I}) = \frac{1}{6}\left\{\begin{bmatrix}4 & 5\\2 & 1\end{bmatrix} - 5\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\right\}$$

$$= \frac{1}{6}\left\{\begin{bmatrix}4 & 5\\2 & 1\end{bmatrix} - \begin{bmatrix}5 & 0\\0 & 5\end{bmatrix}\right\}$$

$$= \frac{1}{6}\begin{bmatrix}-1 & 5\\2 & -4\end{bmatrix}$$ ....(2)

From (1) and (2), $\mathrm{A}^{-1} = \frac{1}{6}(\mathrm{A}-5\mathrm{I})$

Miscellaneous exercise 2 (A) | Q 11 | Page 53

Find the matrix X such that

AX = B, where A = $\begin{bmatrix}1 & 2\\-1 & 3\end{bmatrix}$ and B = $\begin{bmatrix}0 & 1\\2 & 4\end{bmatrix}$

Solution:

AX = B

$$\therefore \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} X = \begin{bmatrix} 0 & 1 \\ 2 & 4 \end{bmatrix}$$

By $R_2 + R_1$, we get,

$$\begin{bmatrix} 1 & 2 \\ 0 & 5 \end{bmatrix} X = \begin{bmatrix} 0 & 1 \\ 2 & 5 \end{bmatrix}$$

By $\left(\frac{1}{5}\right) R_2$, we get,

$$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} X = \begin{bmatrix} 0 & 1 \\ \frac{2}{5} & 1 \end{bmatrix}$$

By $R_1 - 2R_2$, we get,

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} X = \begin{bmatrix} -\frac{4}{5} & -1 \\ \frac{2}{5} & 1 \end{bmatrix}$$

$$\therefore X = \begin{bmatrix} -\frac{4}{5} & -1 \\ \frac{2}{5} & 1 \end{bmatrix}$$

Miscellaneous exercise 2 (A) | Q 12 | Page 53

Find X, if AX = B, where A = $\begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}$ and B = $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

Solution:

AX = B

$$\therefore \begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix} \mathrm{X} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

By $R_2 + R_1$ and $R_3 - R_1$, we get,

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & 0 & 1 \end{bmatrix} \mathrm{X} = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$$

By $\left(\frac{1}{3}\right)\mathrm{R}_2$, we get,

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & \frac{5}{3} \\ 0 & 0 & 1 \end{bmatrix} \mathrm{X} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

By $R_1 - 2R_2$, we get,

$$\begin{bmatrix} 1 & 0 & -\frac{1}{3} \\ 0 & 1 & \frac{5}{3} \\ 0 & 0 & 1 \end{bmatrix} \mathrm{A}^{-1} = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$$

By $\mathrm{R}_1 + \dfrac{1}{3}\mathrm{R}_3$ and $\mathrm{R}_3 - \dfrac{5}{3}\mathrm{R}_3$ we get,

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathrm{A}^{-1} = \begin{bmatrix} -\frac{1}{3} \\ -\frac{7}{3} \\ 2 \end{bmatrix}$$

$$\therefore \mathrm{X} = \begin{bmatrix} -\frac{1}{3} \\ -\frac{7}{3} \\ 2 \end{bmatrix}$$

Miscellaneous exercise 2 (A) | Q 13 | Page 54

If A = $\begin{bmatrix}1 & 1\\ 1 & 2\end{bmatrix}, \mathrm{B} = \begin{bmatrix}4 & 1\\ 3 & 1\end{bmatrix}$ and C = $\begin{bmatrix}24 & 7\\ 31 & 9\end{bmatrix}$, then find the matrix X such that AXB = C.

Solution:

AXB = C

$$\therefore \begin{bmatrix}1 & 1\\ 1 & 2\end{bmatrix}(\mathrm{XB}) = \begin{bmatrix}24 & 7\\ 31 & 9\end{bmatrix}$$

First we perform the row transformations.

By $R_2 - R_1$, we get,

$$\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}(\mathrm{XB}) = \begin{bmatrix}24 & 7\\ 7 & 2\end{bmatrix}$$

By $R_1 - R_2$, we get,

$$\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}(\mathrm{XB}) = \begin{bmatrix}17 & 5\\ 7 & 2\end{bmatrix}$$

$\therefore$ XB = $\begin{bmatrix}17 & 5\\ 7 & 2\end{bmatrix}$

Now, we perform the column transformations.

By $C_1 \leftrightarrow C_3$, we get,

$$\mathrm{X}\begin{bmatrix}1 & 4\\ 1 & 3\end{bmatrix} = \begin{bmatrix}5 & 17\\ 2 & 7\end{bmatrix}$$

By $C_2 - 4C_1$, we get,

$$X\begin{bmatrix}1 & 0\\ 1 & -1\end{bmatrix} = \begin{bmatrix}5 & -3\\ 2 & -1\end{bmatrix}$$

By $(-1)C_2$ we get,

$$X\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix} = \begin{bmatrix}5 & 3\\ 2 & 1\end{bmatrix}$$

By $C_1 - C_2$ we get,

$$X\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} = \begin{bmatrix}2 & 3\\ 1 & 1\end{bmatrix}$$

$\therefore X = \begin{bmatrix}2 & 3\\ 1 & 1\end{bmatrix}$

Miscellaneous exercise 2 (A) | Q 14 | Page 54

Find the inverse of $\begin{bmatrix}1 & 2 & 3\\ 1 & 1 & 5\\ 2 & 4 & 7\end{bmatrix}$ by the adjoint method.

Solution:

Let $A = \begin{bmatrix}1 & 2 & 3\\ 1 & 1 & 5\\ 2 & 4 & 7\end{bmatrix}$

$\therefore |A| = \begin{vmatrix}1 & 2 & 3\\ 1 & 1 & 5\\ 2 & 4 & 7\end{vmatrix}$

$= 1(7 - 20) - 2(7 - 10) + 3(4 - 2)$

$= -13 + 6 + 6 = -1 \neq 0$

$\therefore A^{-1}$ exists.

Let A = $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{bmatrix}$

∴ |A| = $\begin{vmatrix} 1 & 2 & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{vmatrix}$

= 1(7 - 20) - 2(7 - 10) + 3(4 - 2)

= - 13 + 6 + 6 = - 1 ≠ 0

∴ A^{-1} exists.

First we have to find the cofactor matrix

$= [\mathrm{A_{ij}}]_{3\times 3}$, where $\mathrm{A_{ij}} = (-1)^{\mathrm{i+j}}\mathrm{M_{ij}}$

Now $\mathrm{A_{11}} = (-1)^{1+1}\mathrm{M_{11}} = \begin{vmatrix} 1 & 5 \\ 4 & 7 \end{vmatrix} = 7 - 20 = -13$

$$\mathrm{A_{12}} = (-1)^{1+2}\mathrm{M_{12}} = -\begin{vmatrix} 1 & 5 \\ 2 & 7 \end{vmatrix} = -(7 - 10) = 3$$

$$\mathrm{A_{13}} = (-1)^{1+3}\mathrm{M_{13}} = \begin{vmatrix} 1 & 1 \\ 2 & 4 \end{vmatrix} = 4 - 2 = 2$$

$$\mathrm{A_{21}} = (-1)^{2+1}\mathrm{M_{21}} = -\begin{vmatrix} 2 & 3 \\ 4 & 7 \end{vmatrix} = -(14 - 12) = -2$$

$$\mathrm{A_{22}} = (-1)^{2+2}\mathrm{M_{22}} = \begin{vmatrix} 1 & 3 \\ 2 & 7 \end{vmatrix} = 7 - 6 = 1$$

$$\mathrm{A_{23}} = (-1)^{2+3}\mathrm{M_{23}} = -\begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix} = -(4 - 4) = 0$$

$$\mathrm{A_{31}} = (-1)^{3+1}\mathrm{M_{31}} = \begin{vmatrix} 2 & 3 \\ 1 & 5 \end{vmatrix} = 10 - 3 = 7$$

$$A_{32} = (-1)^{3+2}M_{32} = -\begin{vmatrix} 1 & 3 \\ 1 & 5 \end{vmatrix} = -(5-3) = -2$$

$$A_{33} = (-1)^{3+3}M_{33} = \begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} = 1 - 2 = -1$$

∴ the co-factor matrix =

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} = \begin{bmatrix} -13 & 3 & 2 \\ -2 & 1 & 0 \\ 7 & -2 & -1 \end{bmatrix}$$

∴ adj A = $\begin{bmatrix} -13 & -2 & 7 \\ 3 & 1 & -2 \\ 2 & 0 & -1 \end{bmatrix}$

∴ A^{-1} = $\dfrac{1}{|A|}$(adj A)

$$= \frac{1}{-1}\begin{bmatrix} -13 & -2 & 7 \\ 3 & 1 & -2 \\ 2 & 0 & -1 \end{bmatrix}$$

∴ A^{-1} = $\begin{bmatrix} -13 & -2 & 7 \\ 3 & 1 & -2 \\ 2 & 0 & -1 \end{bmatrix}$

Miscellaneous exercise 2 (A) | Q 15 | Page 54

Find the inverse of matrix A by using adjoint method; where

A = $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$

Solution 1:

where A = $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$

|A| = l(2 -6) - 0(0 -3) + 1(0 - 2)

|A| = -4 -2

|A| = -6 ≠ 0

$\therefore A^{-1}$ exists.

Minors	Co-factors
M_{11} = -4	A_{11} = -4
M_{12} = -3	A_{12} = 3
M_{13} = -2	A_{13} = -2
M_{21} = -2	A_{21} = 2
M_{22} = 0	A_{22} = 0
M_{23} = 2	A_{23} = -2
M_{31} = -2	A_{31} = -2
M_{32} = 3	A_{32} = -3
M_{33} = 2	A_{33} = 2

adj (A) = $\begin{bmatrix} -4 & 2 & -2 \\ 3 & 0 & -3 \\ -2 & -2 & 2 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|}$. adj(A)

$A^{-0} = \frac{-1}{6}\begin{bmatrix} -4 & 2 & -2 \\ 3 & 0 & -3 \\ -2 & -2 & 2 \end{bmatrix}$

Solution 2:

where A = $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$

|A| = l(2 -6) - 0(0 -3) + 1(0 - 2)

|A| = -4 -2

|A| = -6 ≠ 0

$\therefore A^{-1}$ exists.

First we have to find the cofactor matrix

$= [\mathrm{A_{ij}}]_{3\times3}$, where $\mathrm{A_{ij}} = (-1)^{\mathrm{i+j}}\mathrm{M_{ij}}$

Now $\mathrm{A}_{11} = (-1)^{1+1}\mathrm{M}_{11} = \begin{vmatrix} 2 & 3 \\ 2 & 1 \end{vmatrix} = 2 - 6 = -4$

$\mathrm{A}_{12} = (-1)^{1+2}\mathrm{M}_{12} = -\begin{vmatrix} 0 & 3 \\ 1 & 1 \end{vmatrix} = 0 - 3 = -3$

$\mathrm{A}_{13} = (-1)^{1+3}\mathrm{M}_{13} = \begin{vmatrix} 0 & 2 \\ 1 & 2 \end{vmatrix} = 0 - 2 = -2$

$\mathrm{A}_{21} = (-1)^{2+1}\mathrm{M}_{21} = -\begin{vmatrix} 0 & 1 \\ 2 & 1 \end{vmatrix} = -0 - 2 = -2$

$\mathrm{A}_{22} = (-1)^{2+2}\mathrm{M}_{22} = \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 1 - 1 = 0$

$\mathrm{A}_{23} = (-1)^{2+3}\mathrm{M}_{23} = -\begin{vmatrix} 1 & 0 \\ 1 & 2 \end{vmatrix} = -2 - 0 = -2$

$\mathrm{A}_{31} = (-1)^{3+1}\mathrm{M}_{31} = \begin{vmatrix} 0 & 1 \\ 2 & 3 \end{vmatrix} = 0 - 2 = -2$

$$A_{32} = (-1)^{3+2}M_{32} = -\begin{vmatrix} 1 & 1 \\ 0 & 3 \end{vmatrix} = -3 - 0 = -3$$

$$A_{33} = (-1)^{3+3}M_{33} = \begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix} = 2 - 0 = 2$$

$$\text{adj (A)} = \begin{bmatrix} -4 & -3 & -2 \\ -2 & 0 & -2 \\ -2 & -3 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} . \text{adj(A)}$$

$$A^{-0} = \frac{1}{6}\begin{bmatrix} -4 & -3 & -2 \\ -2 & 0 & -2 \\ -2 & -3 & 2 \end{bmatrix}$$

Miscellaneous exercise 2 (A) | Q 16 | Page 54

Find A^{-1} by the adjoint method and by elementary transformations, if

$$A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}$$

Solution:

$$|A| = \begin{vmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \\ 1 & 2 & 4 \end{vmatrix}$$

$= 1(4 - 4) - 2(- 4 - 2) + 3(- 2 - 1)$

$= 0 + 12 - 9$

$= 3 \neq 0$

$\therefore A^{-1}$ exists.

∴ A^{-1} exists.

A^{-1} by adjoint method:

We have to find the cofactor matrix

$= [A_{ij}]_{3\times 3}$, where $A_{ij} = (-1)^{i+j} M_{ij}$

Now $A_{11} = (-1)^{1+1} M_{11} = \begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix} = 4 - 4 = 0$

$$A_{12} = (-1)^{1+2} M_{12} = -\begin{vmatrix} -1 & 2 \\ 1 & 4 \end{vmatrix} = -(-4 - 2) = 6$$

$$A_{13} = (-1)^{1+3} M_{13} = \begin{vmatrix} -1 & 1 \\ 1 & 2 \end{vmatrix} = -2 - 1 = -3$$

$$A_{21} = (-1)^{2+1} M_{21} = -\begin{vmatrix} 2 & 3 \\ 2 & 4 \end{vmatrix} = -(8 - 6) = -2$$

$$A_{22} = (-1)^{2+2} M_{22} = \begin{vmatrix} 1 & 3 \\ 1 & 4 \end{vmatrix} = 4 - 3 = 1$$

$$A_{23} = (-1)^{2+3} M_{23} = -\begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix} = -(2 - 2) = 0$$

$$A_{31} = (-1)^{3+1} M_{31} = \begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} = 4 - 3 = 1$$

$$A_{32} = (-1)^{3+2} M_{32} = -\begin{vmatrix} 1 & 3 \\ -1 & 2 \end{vmatrix} = -(2 + 3) = -5$$

$$A_{33} = (-1)^{3+3} M_{33} = \begin{vmatrix} 1 & 2 \\ -1 & 1 \end{vmatrix} = 1 + 2 = 3$$

∴ the cofactor matrix =

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} = \begin{bmatrix} 0 & 6 & -3 \\ -2 & 1 & 0 \\ 1 & -5 & 3 \end{bmatrix}$$

$$\therefore \text{adj A} = \begin{bmatrix} 0 & -2 & 1 \\ 6 & 1 & -5 \\ -3 & 0 & 3 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}(\text{adj A})$$

$$= \frac{1}{3}\begin{bmatrix} 0 & -2 & 1 \\ 6 & 1 & -5 \\ -3 & 0 & 3 \end{bmatrix}$$

A^{-1} by elementary transformations:

Consider $AA^{-1} = I$

$$\therefore \begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_2 + R_1$ and $R_3 - R_1$we get,

$$\therefore \begin{bmatrix} 1 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

By $\left(\frac{1}{3}\right)R_2$ we get,

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & \frac{5}{3} \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

By $R_1 - 2R_2$ we get,

$$\begin{bmatrix} 1 & 0 & -\frac{1}{3} \\ 0 & 1 & \frac{5}{3} \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} \frac{1}{3} & -\frac{2}{3} & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

By $R_1 + \dfrac{1}{3}R_3$ and $R_2 - \dfrac{5}{3}R_3$, we get,

$$\therefore \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & -\frac{2}{3} & \frac{1}{3} \\ 2 & \frac{1}{3} & -\frac{5}{3} \\ -1 & 0 & 1 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{3}\begin{bmatrix} 0 & -2 & 1 \\ 6 & 1 & -5 \\ -3 & 0 & 3 \end{bmatrix}$$

Miscellaneous exercise 2 (A) | Q 17 | Page 54

Find the inverse of A = $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$ by using elementary column transformations.

Solution:

|A| = $\begin{vmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ 1 & 2 & 1 \end{vmatrix}$

= 1(2 - 6) - 0 + 1(0 - 2)

= - 4 - 2

= - 6 ≠ 0

∴ A^{-1} exists.

Consider $A^{-1}A = I$

$$\therefore A^{-1}\begin{bmatrix}1 & 0 & 1\\ 0 & 2 & 3\\ 1 & 2 & 1\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

By $C_3 - C_1$, we get,

$$A^{-1}\begin{bmatrix}1 & 0 & 0\\ 0 & 2 & 3\\ 1 & 2 & 0\end{bmatrix} = \begin{bmatrix}1 & 0 & -1\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

By $C_2 \leftrightarrow C_3$we get

$$A^{-1}\begin{bmatrix}1 & 0 & 0\\ 0 & 3 & 2\\ 1 & 0 & 2\end{bmatrix} = \begin{bmatrix}1 & -1 & 0\\ 0 & 0 & 1\\ 0 & 1 & 0\end{bmatrix}$$

By $C_2 - C_3$we get

$$A^{-1}\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 2\\ 1 & -2 & 2\end{bmatrix} = \begin{bmatrix}1 & -1 & 0\\ 0 & -1 & 1\\ 0 & 1 & 0\end{bmatrix}$$

By $C_3 - 2C_2$we get

$$A^{-1}\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 1 & -2 & 6\end{bmatrix} = \begin{bmatrix}1 & -1 & 2\\ 0 & -1 & 3\\ 0 & 1 & -2\end{bmatrix}$$

By $\left(\frac{1}{6}\right) C_3$ we get

$$A^{-1}\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\1 & -2 & 1\end{bmatrix} = \begin{bmatrix}1 & -1 & \frac{1}{3}\\0 & -1 & \frac{1}{2}\\0 & 1 & -\frac{1}{3}\end{bmatrix}$$

By $C_1 - C_3$ and $C_2 + 2C_3$ we get

$$A^{-1}\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\1 & -2 & 1\end{bmatrix} = \begin{bmatrix}\frac{2}{3} & -\frac{1}{3} & \frac{1}{3}\\-\frac{1}{2} & 0 & \frac{1}{2}\\\frac{1}{3} & \frac{1}{3} & -\frac{1}{3}\end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{6}\begin{bmatrix}4 & -2 & 2\\-3 & 0 & 3\\2 & 2 & -2\end{bmatrix}$$

Miscellaneous exercise 2 (A) | Q 18 | Page 54

Find the inverse of $\begin{bmatrix}1 & 2 & 3\\1 & 1 & 5\\2 & 4 & 7\end{bmatrix}$ by using elementary row transformations.

Solution:

Let $A = \begin{bmatrix}1 & 2 & 3\\1 & 1 & 5\\2 & 4 & 7\end{bmatrix}$

$|A| = \begin{vmatrix}1 & 2 & 3\\1 & 1 & 5\\2 & 4 & 7\end{vmatrix}$

$= 1(7 - 20) - 2(7 - 10) + 3(4 - 2)$

= - 13 + 6 + 6

= - 1 ≠ 0

∴ A^{-1} exists.

Consider $AA^{-1} = I$

$$\therefore \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_2 - R_1$ and $R_3 - 2R_1$, we get,

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 2 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

By $(-1)R_2$we get

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

By $R_1 - 2R_2$we get

$$\begin{bmatrix} 1 & 0 & 7 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} -1 & 2 & 0 \\ 1 & -1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

By $R_1 - 7R_3$ and $R_2 + 2R_3$ we get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 13 & 2 & -7 \\ -3 & -1 & 2 \\ -2 & 0 & 1 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} 13 & 2 & -7 \\ -3 & -1 & 2 \\ -2 & 0 & 1 \end{bmatrix}$$

Miscellaneous exercise 2 (A) | Q 19.1 | Page 54

Show with the usual notation that for any matrix

$A = [a_{ij}]_{3\times3}$ is $a_{11}A_{21} + a_{12}A_{22} + a_{13}A_{23} = 0$

Solution:

$$A = [a_{ij}]_{3\times3} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$A_{21} = (-1)^{2+1} M_{21} = -\begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix}$$

$$= -(a_{12}a_{33} - a_{13}a_{32})$$

$$= -a_{12}a_{33} + a_{13}a_{32}$$

$$A_{22} = (-1)^{2+2} M_{22} = -\begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix}$$

$$= a_{11}a_{33} - a_{13}a_{31}$$

$$A_{23} = (-1)^{2+3} M_{23} = -\begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix}$$

$= - (a_{11}a_{32} - a_{12}a_{31})$

$= - a_{11}a_{32} + a_{12}a_{31}$

$\therefore a_{11}A_{21} + a_{12}A_{22} + a_{13}A_{23}$

$= a_{11}(-a_{12}a_{33} + a_{13}a_{32}) + a_{12}(a_{11}a_{33} - a_{13}a_{31}) + a_{13}(-a_{11}a_{32} + a_{12}a_{31})$

$= -a_{11}a_{12}a_{33} + a_{11}a_{13}a_{32} + a_{11}a_{12}a_{33} - a_{12}a_{13}a_{31} - a_{11}a_{13}a_{32} + a_{12}a_{13}a_{31}$

$= 0$

Miscellaneous exercise 2 (A) | Q 19.2 | Page 54

Show with the usual notation that for any matrix

$A = [a_{ij}]_{3\times 3}$ is $a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13} = |A|$

Solution:

$$A = [a_{ij}]_{3\times 3} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$A_{11} = (-1)^{1+1} M_{11} = -\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$$

$$A_{12} = (-1)^{1+2} M_{12} = -\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$$

$$A_{13} = (-1)^{1+3} M_{13} = -\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$\therefore a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$

$$= a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$= \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = |A|$$

Miscellaneous exercise 2 (A) | Q 20 | Page 54

If A = $\begin{bmatrix}1 & 0 & 1\\0 & 2 & 3\\1 & 2 & 1\end{bmatrix}$ and B = $\begin{bmatrix}1 & 2 & 3\\1 & 1 & 5\\2 & 4 & 7\end{bmatrix}$, then find a matrix X such that XA = B.

Solution:

Consider XA = B

$$\therefore X\begin{bmatrix}1 & 0 & 1\\0 & 2 & 3\\1 & 2 & 1\end{bmatrix} = \begin{bmatrix}1 & 2 & 3\\1 & 1 & 5\\2 & 4 & 7\end{bmatrix}$$

By $C_3 - C_1$ we get,

$$X\begin{bmatrix}1 & 0 & 1\\0 & 2 & 3\\1 & 2 & 0\end{bmatrix} = \begin{bmatrix}1 & 2 & 2\\1 & 1 & 4\\2 & 4 & 7\end{bmatrix}$$

By $\left(\frac{1}{2}\right)C_2$ we get,

$$X\begin{bmatrix}1 & 0 & 0\\0 & 1 & 3\\1 & 1 & 0\end{bmatrix} = \begin{bmatrix}1 & 1 & 2\\1 & \frac{1}{2} & 4\\2 & 2 & 5\end{bmatrix}$$

By $C_3 - 3C_2$ we get,

$$X\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\1 & 1 & -3\end{bmatrix} = \begin{bmatrix}1 & 1 & -1\\1 & \frac{1}{2} & \frac{5}{2}\\2 & 2 & -1\end{bmatrix}$$

By $\left(-\frac{1}{3}\right)C_3$, we get

$$X\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\1 & 1 & 1\end{bmatrix} = \begin{bmatrix}1 & 1 & \frac{1}{3}\\1 & \frac{1}{2} & -\frac{5}{6}\\2 & 2 & \frac{1}{3}\end{bmatrix}$$

By $C_1 - C_3$ and $C_2 - C_3$ we get,

$$X\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} = \begin{bmatrix}\frac{2}{3} & \frac{2}{3} & \frac{1}{3}\\\frac{11}{6} & \frac{4}{3} & -\frac{5}{6}\\\frac{5}{3} & \frac{5}{3} & \frac{1}{3}\end{bmatrix}$$

$$\therefore X = \frac{1}{6}\begin{bmatrix}4 & 4 & 2\\11 & 8 & -5\\10 & 10 & 2\end{bmatrix}$$

EXERCISE 2.3 [PAGES 59 - 60]

Exercise 2.3 | Q 1.1 | Page 59

Solve the following equations by inversion method.

x + 2y = 2, 2x + 3y = 3

Solution:

The given equations can be written in the matrix form as:

$$\begin{bmatrix}1 & 2\\2 & 3\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}2\\3\end{bmatrix}$$

This is of the form AXB, where

$A = \begin{bmatrix}1 & 2\\2 & 3\end{bmatrix}$, $x = \begin{bmatrix}x\\y\end{bmatrix}$ and $B = \begin{bmatrix}2\\3\end{bmatrix}$

Let us find A^{-1}

$|A| = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} = 3 - 4 = -1 \neq 0$

$\therefore A^{-1}$ exists.

Consider $AA^{-1} = I$

$\therefore \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Bu $R_2 - 2R_1$, we get,

$\begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$

By $(-1)R_2$, we get,

$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} A-1 = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}$

By $R_1 - 2R_2$, we get,

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}$

$\therefore A^{-1} = \begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}$

Now, premultiply AX = B by A^{-1}, we get,

$A^{-1}(AX) = A^{-1}B$

$\therefore (A^{-1}A)X = A^{-1}B$

$\therefore IX = A^{-1}B$

$\therefore X = \begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \end{bmatrix}$

$\therefore \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -6+6 \\ 4-3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

By equality of matrices,

x = 0, y = 1 is the required solution.

Exercise 2.3 | Q 1.2 | Page 59

Solve the following equations by inversion method:

x + y = 4, 2x - y = 5

Solution:

x + y = 4, 2x - y = 5

The given equations can be written in the matrix form as:

$$\begin{bmatrix}1 & 1\\2 & -1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}4\\5\end{bmatrix}$$

This is of the form AX = B ⇒ X ⇒ $A^{-1}B$

A = $\begin{bmatrix}1 & 1\\2 & -1\end{bmatrix}$

|A| = - 1 - 2 = - 3 ≠ 0

Adj A = $\begin{bmatrix}-1 & -1\\-2 & 1\end{bmatrix}$

$$= \frac{1}{-3}\begin{bmatrix}-1 & -1\\-2 & 1\end{bmatrix} = \begin{bmatrix}1 & 1\\2 & -1\end{bmatrix}$$

X = A^{-1}B = $\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}1 & 1\\2 & -1\end{bmatrix}\begin{bmatrix}4\\5\end{bmatrix}$

$$= \begin{bmatrix}4 & 5\\8 & -5\end{bmatrix} = \begin{bmatrix}9\\3\end{bmatrix}$$

$$= \begin{bmatrix}3\\1\end{bmatrix}$$

By equality of matrices,

x = 3, y = 1.

Exercise 2.3 | Q 1.3 | Page 59

Solve the following equations by inversion method.

2x + 6y = 8, x + 3y = 5

Solution:

The given equations can be written in the matrix form as:

$$\begin{bmatrix} 2 & 6 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 5 \end{bmatrix}$$

This is of the form AXB, where

A = $\begin{bmatrix} 2 & 6 \\ 1 & 3 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \end{bmatrix}$ and B = $\begin{bmatrix} 8 \\ 5 \end{bmatrix}$

Let us find A^{-1}

$|A| = \begin{bmatrix} 2 & 6 \\ 1 & 3 \end{bmatrix} = 6 - 6 = 0$

$\therefore$ A^{-1} does not exist.

Hence, x and y do not exist.

Exercise 2.3 | Q 2.1 | Page 60

Solve the following equations by the reduction method.

2x + y = 5, 3x + 5y = – 3

Solution: The given equations can be written in the matrix form as:

$$\begin{bmatrix} 2 & 1 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -3 \end{bmatrix}$$

By $2R_2$, we get,

$$\begin{bmatrix} 2 & 1 \\ 6 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -6 \end{bmatrix}$$

By $R_2 - 3R_1$, we get,

$$\begin{bmatrix} 2 & 1 \\ 0 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -21 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 2x + y \\ 0 + 7y \end{bmatrix} = \begin{bmatrix} 5 \\ -21 \end{bmatrix}$$

By equality of matrices,

$2x + y = 5$ ....(1)

$7y = -21$.............(2)

From (2), $y = -3$

Substituting $y = -3$ in (1), we get,

$2x - 3 = 5$

$\therefore 2x = 8$

$\therefore x = 4$

Hence, $x = 4$, $y = -3$ is the required solution.

Exercise 2.3 | Q 2.2 | Page 60

Solve the following equations by the reduction method.

$x + 3y = 2$, $3x + 5y = 4$

Solution: The given equations can be written in the matrix form as:

$$\begin{bmatrix}1 & 3\\3 & 5\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}2\\4\end{bmatrix}$$

By $R_2 - 3R_1$, we get,

$$\begin{bmatrix}1 & 3\\0 & -4\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}2\\-2\end{bmatrix}$$

$$\therefore \begin{bmatrix}x + 3y\\0 - 4y\end{bmatrix} = \begin{bmatrix}2\\-2\end{bmatrix}$$

By equality of matrices,

x + 3y = 2 ....(1)

– 4y = – 2 ....(2)

From (2), y = $\frac{1}{2}$

Substituting y = $\frac{1}{2}$ in (1), we get,

x + $\frac{3}{2}$ = 2

$\therefore$ x = $2 - \frac{3}{2} = \frac{1}{2}$

Hence, x = $\frac{1}{2}$, y = $\frac{1}{2}$ is the required solution.

Exercise 2.3 | Q 2.3 | Page 60

Solve the following equations by the reduction method.

3x – y = 1, 4x + y = 6

Solution: The given equations can be written in the matrix form as:

$$\begin{bmatrix} 3 & -1 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 6 \end{bmatrix}$$

By $4R_1$ and $3R_2$, we get,

$$\begin{bmatrix} 12 & -4 \\ 12 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 18 \end{bmatrix}$$

By $R_2 - R_1$, we get,

$$\begin{bmatrix} 12 & -4 \\ 0 & 7 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 14 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 12x - 4y \\ 0 + 7y \end{bmatrix} = \begin{bmatrix} 4 \\ 14 \end{bmatrix}$$

By equality of matrices,

$12x - 4y = 4$ ....(1)

$7y = 14$ ....(2)

From (2), $y = 2$

Substituting $y = 2$ in (1), we get,

$12x - 8 = 4$

$\therefore 12x = 12$

$\therefore x = 1$

Hence, $x = 1$, $y = 2$ is the required solution.

Exercise 2.3 | Q 2.4 | Page 60

Solve the following equations by the reduction method.

$5x + 2y = 4$, $7x + 3y = 5$

Solution: $5x + 2y = 4$ ....(1)

$7x + 3y = 5$ ....(2)

Multiplying Eq. (1) with 7 and Eq. (2) with 5

35x + 14 y = 28
35x + 15 y = 25
− −

− 1y = 3
y = − 3

Put y = − 3 into Eq. (1)
5x + 2y = 4
5x + 2(− 3) = 4
5x − 6 = 4
5x = 4 + 6
5x = 10
x = 10/5
x = 2

Hence, x = 2, y = − 3 is the required solution.

Exercise 2.3 | Q 3 | Page 60

The cost of 4 dozen pencils, 3 dozen pens and 2 dozen erasers is Rs. 60. The cost of 2 dozen pencils, 4 dozen pens and 6 dozen erasers is Rs. 90 whereas the cost of 6 dozen pencils, 2 dozen pens and 3 dozen erasers is Rs. 70. Find the cost of each item per dozen by using matrices.

Solution: Let Rs.'x', Rs.'y' and Rs.'z' be the cost of one dozen pencils, one dozen pens and one dozen erasers.

Thus, the system of equations are:

$$4x + 3y + 2z = 60$$
$$2x + 4y + 6z = 90$$
$$6x + 2y + 3z = 70$$

Let us write the above equations in the matrix form as:

$$\begin{bmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix} \quad \text{i.e} \quad AX = B$$

$$\text{Using} \quad R_z \to R_2 - \frac{1}{2}R_1 \text{ and } R_3 \to R_3 - \frac{3}{2}R_1$$

$$\begin{bmatrix} 4 & 3 & 2 \\ 0 & \frac{5}{2} & 5 \\ 0 & -\frac{5}{2} & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 60 \\ 60 \\ -20 \end{bmatrix}$$

Using $R_3 \to R_3 + R_2$

$$\begin{bmatrix} 4 & 3 & 2 \\ 0 & \frac{5}{2} & 5 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 60 \\ 60 \\ 40 \end{bmatrix}$$

As matrix A is reduced to its upper triangular form we can write

4x + 3y + 2z = 60..........(i)

$\frac{5}{2}$y + 5z = 60..........(ii)

0x + 0y + 5z = 40

z = 8.....(iii)

Substituting (iii) in (ii) we get,

$\frac{5}{2}$y + 5(8) = 60

y = $\frac{60-40}{5} \times 2 = 8$

y = 8 .....(iv)

Substituting (iii) and (iv) in (i) we get,

4x + 3 (8) + 2 (8) = 60

$4x = 60 - 24 - 16$

$$x = \frac{20}{4} = 5$$

$\therefore x = 5$

$\therefore$ Cost of one dozen pencils, one dozen pens and one dozen erasers is Rs. 5, Rs. 8 and Rs. 8 respectively

Exercise 2.3 | Q 4 | Page 60

If three numbers are added, their sum is 2. If two times the second number is subtracted from the sum of first and third numbers we get 8 and if three times the first number is added to the sum of second and third numbers we get 4. Find the numbers using matrices.

Solution: Let the three numbers x , y , z.

From given condition, we have

$x + y + z = 2$ ....(1)

$x + z - 2y = 8$

$x - 2y + z = 8$ ......(2)

And

$3x + y + z = 4$ .....(3)

Given all equation can be written in matrix form as ,

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & 1 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 8 \\ 4 \end{bmatrix}$$

Consider , AX = B

On multiplying A^{-1} both sides , we get

$A^{-1} . A.X = A^{-1}.B$

$X = A^{-1} . B$ ......(4)

Now

$A^{-1} = \frac{1}{|A|}\,\mathbf{Adj\ (A)}$

$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -2 & 1 \\ 3 & 1 & 1 \end{vmatrix}$

$|A| = 6$

Now , For Adj (A) , we need minors and co-factors.

$M_{11} = -3$, $M_{12} = -2$, $M_{13} = 7$

$M_{21} = 0$, $M_{22} = -2$, $M_{23} = -2$

$M_{31} = 3$, $M_{32} = 0$,$M_{33} = -3$

Therefore ,

$A^{-1} = \frac{1}{6}\begin{bmatrix} -3 & 0 & 3 \\ 2 & -2 & 0 \\ 7 & 2 & -3 \end{bmatrix}$

From equation (4),

$$X = \frac{1}{6}\begin{bmatrix} -3 & 0 & 3 \\ 2 & -2 & 0 \\ 7 & 2 & -3 \end{bmatrix}\begin{bmatrix} 2 \\ 8 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$$

Hence , x = 1 , y = -2 , z = 3

Exercise 2.3 | Q 5 | Page 60

The total cost of 3 T.V. sets and 2 V.C.R.'s is ₹ 35,000. The shopkeeper wants a profit of ₹ 1000 per T.V. set and ₹ 500 per V.C.R. He sells 2 T.V. sets and 1 V.C.R. and gets the total revenue as ₹ 21,500. Find the cost price and the selling price of a T.V. set and a V.C.R.

Solution: Let the cost of each T.V. set be ₹ x and each V.C.R. be ₹ y. Then the total cost of 3 T.V. sets and 2 V.C.R.'s is ₹ (3x + 2y) which is given to be ₹35,000.

∴ 3x + 2y = 35000

The shopkeeper wants profit of ₹ 1000 per T.V. set and of ₹ 500 per V.C.R.

∴ the selling price of each T.V. set is ₹ (x + 1000) and each V.C.R. is ₹ (y + 500).

∴ selling price of 2 T.V. set and 1 V.C.R. is

₹ [2(x + 1000) + (y + 500)] which is given to be ₹ 21500

∴ 2(x + 1000) + (y + 500) = 21500

∴ 2x + 2000 + y + 500 = 21500

∴ 2x + y = 19000

Hence, the system of linear equations is

3x + 2y = 35000

2x + y = 19000

These equations can be written in the matrix form as:

$$\begin{bmatrix}3 & 2\\2 & 1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}35000\\19000\end{bmatrix}$$

By $R_1 \leftrightarrow R_2$, we get,

$$\begin{bmatrix}2 & 1\\3 & 2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}19000\\35000\end{bmatrix}$$

By $R_2 - 2R_1$, we get,

$$\begin{bmatrix}2 & 1\\-1 & 0\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}19000\\-3000\end{bmatrix}$$

$$\therefore \begin{bmatrix}2x + y\\-x + 0\end{bmatrix} = \begin{bmatrix}19000\\-3000\end{bmatrix}$$

By equality of matrices,

2x + y = 19000 ....(1)

- x = - 3000 ....(2)

From (2), x = 3000

Substituting x = 3000 in (1), we get,

2(3000) + y = 19000

∴ y = 13000

∴ the cost price of one T.V. set is ₹ 3000 and of one V.C.R. is ₹ 13000 and the selling price of one T.V. set is ₹ 4000 and of one V.C.R. is ₹ 13500.

MISCELLANEOUS EXERCISE 2 (B) [PAGES 61 - 63]

Miscellaneous exercise 2 (B) | Q 1.01 | Page 61

Choose the correct answer from the given alternatives in the following question:

If A = $\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}$, adj A = $\begin{bmatrix}4 & a\\-3 & b\end{bmatrix}$, then the values of a and b are

1. **a = - 2, b = 1**
2. a = 2, b = 4
3. a = 2, b = - 1

4. a = 1, b = - 2

Solution: a = - 2, b = 1

Miscellaneous exercise 2 (B) | Q 1.02 | Page 61

Choose the correct answer from the given alternatives in the following question:

The inverse of $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ is

Options

$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

none of these

Solution:

$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

Miscellaneous exercise 2 (B) | Q 1.03 | Page 61

Choose the correct answer from the given alternatives in the following question:

If A = $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ and A(adj A) = k I, then the value of k is

1. 1
2. -1
3. 0
4. **-3**

Solution: - 3

Miscellaneous exercise 2 (B) | Q 1.04 | Page 61

Choose the correct answer from the given alternatives in the following question:

If A = $\begin{bmatrix} 2 & -4 \\ 3 & 1 \end{bmatrix}$, then the adjoint of matrix A is

Options

$\begin{bmatrix} -1 & 3 \\ -4 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 4 \\ -3 & 2 \end{bmatrix}$

$\begin{bmatrix} 1 & 3 \\ 4 & -2 \end{bmatrix}$

$\begin{bmatrix} -1 & -3 \\ -4 & 2 \end{bmatrix}$

Solution:

$\begin{bmatrix} 1 & 4 \\ -3 & 2 \end{bmatrix}$

Miscellaneous exercise 2 (B) | Q 1.05 | Page 62

Choose the correct answer from the given alternatives in the following question:

If A = $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, and A (adj A) = kI, then the value of k is

1. 2
2. **- 2**
3. 10
4. - 10

Solution: - 2

Miscellaneous exercise 2 (B) | Q 1.06 | Page 62

Choose the correct answer from the given alternatives in the following question:

If A = $\begin{bmatrix} \lambda & 1 \\ -1 & -\lambda \end{bmatrix}$, and A^{-1} does not exist if λ = ______

1. 0
2. **± 1**
3. 2
4. 3

Solution:

If A = $\begin{bmatrix} \lambda & 1 \\ -1 & -\lambda \end{bmatrix}$, and A^{-1} does not exist if λ = **<u>± 1</u>**

Miscellaneous exercise 2 (B) | Q 1.07 | Page 62

Choose the correct answer from the given alternatives in the following question:

If A = $\begin{bmatrix} \cos\alpha & \sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, then A^{-1} = ____

Options

$\begin{bmatrix} \frac{1}{\cos\alpha} & -\frac{1}{\sin\alpha} \\ \frac{1}{\sin\alpha} & \frac{1}{\cos\alpha} \end{bmatrix}$

$\begin{bmatrix} \cos\alpha & -\sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

$\begin{bmatrix} -\cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

$\begin{bmatrix} -\cos\alpha & \sin\alpha \\ \sin\alpha & -\cos\alpha \end{bmatrix}$

Solution:

$$\begin{bmatrix} \cos\alpha & -\sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

Miscellaneous exercise 2 (B) | Q 1.08 | Page 62

Choose the correct answer from the given alternatives in the following question:

If A = $\begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$ where α ∈ R, then $[F(\alpha)]^{-1}$ is

1. **F(- α)**
2. $F(\alpha^{-1})$
3. F(2α)
4. none of these

Solution: F(- α)

Miscellaneous exercise 2 (B) | Q 1.09 | Page 62

Choose the correct answer from the given alternatives in the following question:

The inverse of A = $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is

1. 1
2. **A**
3. A'
4. - I

Solution: A

Miscellaneous exercise 2 (B) | Q 1.1 | Page 63

Choose the correct answer from the given alternatives in the following question:

The inverse of a symmetric matrix is

1. **symmetric**
2. non-symmetric
3. null matrix
4. diagonal matrix

Solution: symmetric

Miscellaneous exercise 2 (B) | Q 1.11 | Page 63

Choose the correct answer from the given alternatives in the following question:

For a 2 × 2 matrix A, if A(adj A) = $\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$, then determinant A equals

1. 20
2. **10**
3. 30
4. 40

Solution: 10

Miscellaneous exercise 2 (B) | Q 1.12 | Page 63

Choose the correct answer from the given alternatives in the following question:

If A' = $-\frac{1}{2}\begin{bmatrix} 1 & -4 \\ -1 & 2 \end{bmatrix}$, then A = _____.

Options

$\begin{bmatrix} 2 & 4 \\ -1 & 1 \end{bmatrix}$

$\begin{bmatrix} 2 & 4 \\ 1 & -1 \end{bmatrix}$

$\begin{bmatrix} 2 & -4 \\ 1 & 1 \end{bmatrix}$

$\begin{bmatrix} 2 & 4 \\ 1 & 1 \end{bmatrix}$

Solution:

$\begin{bmatrix} 2 & 4 \\ 1 & 1 \end{bmatrix}$

MISCELLANEOUS EXERCISE 2 (B) [PAGE 63]

Miscellaneous exercise 2 (B) | Q 1.1 | Page 63

Solve the following equation by the method of inversion:

2x - y = - 2, 3x + 4y = 3

Solution: The given equations can be written in the matrix form as:

The given equations can be written in the matrix form as:

$$\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

This is of the form AX = B, where

$$A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

Let us find A^{-1}.

$$|A| = \begin{vmatrix} 2 & -1 \\ 3 & 4 \end{vmatrix} = 8 + 3 = 11 \neq 0$$

$\therefore$ A^{-1} exists.

Consider $AA^{-1} = I$

$$\therefore \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

By $R_1 \leftrightarrow R_2$ we get,

$$\begin{bmatrix} 3 & 4 \\ 2 & -1 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

By $R_1 - R_2$, we get,

\

$$\begin{bmatrix} 1 & 5 \\ 2 & -1 \end{bmatrix} \mathbf{A}^{-1} = \begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix}$$

By R_2 - $2R_1$, we get,

$$\begin{bmatrix} 1 & 5 \\ 0 & -11 \end{bmatrix} \mathbf{A}^{-1} = \begin{bmatrix} -1 & 1 \\ 3 & -2 \end{bmatrix}$$

By $\left(-\frac{1}{11}\right)\mathbf{R}_2$, we get,

$$\begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix} \mathbf{A}^{-1} = \begin{bmatrix} -1 & 1 \\ -\frac{3}{11} & \frac{2}{11} \end{bmatrix}$$

By R_1 - $5R_2$ we get

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{A}^{-1} = \begin{bmatrix} \frac{4}{11} & \frac{1}{11} \\ -\frac{3}{11} & \frac{2}{11} \end{bmatrix}$$

$\therefore$ A^{-1} = $\begin{bmatrix} \frac{4}{11} & \frac{1}{11} \\ -\frac{3}{11} & \frac{2}{11} \end{bmatrix}$

Now, premultiply AX = B by A^{-1} , we get,

A^{-1} (AX) = A^{-1} B

$\therefore$ (A^{-1} A)X = A^{-1} B

$\therefore$ IX = A^{-1} B

$$\therefore X = \begin{bmatrix} \frac{4}{11} & \frac{1}{11} \\ -\frac{3}{11} & \frac{2}{11} \end{bmatrix} \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -\frac{8}{11} + \frac{3}{11} \\ \frac{6}{11} + \frac{6}{11} \end{bmatrix} = \begin{bmatrix} -\frac{5}{11} \\ \frac{12}{11} \end{bmatrix}$$

By equality of matrices,

$x = -\dfrac{5}{11}$, $y = \dfrac{12}{11}$ is the required solution.

Miscellaneous exercise 2 (B) | Q 1.2 | Page 63

Solve the following equations by the method of inversion:

x + y+ z = 1, 2x + 3y + 2x = 2, ax + ay + 2az = 4, a ≠ 0.

Solution:

The given equations can be written in the matrix form as:

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ a & a & 2a \end{bmatrix}, \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$$

This is of the form AX = B, where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ a & a & 2a \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$$

Let us find A^{-1}

$$|A| = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ a & a & 2a \end{bmatrix}$$

= 1(6a - 2a) - 1(4a - 2a) + 1(2a - 3a)

= 4a - 2a - a

= a ≠ 0 ∴ A^{-1} exists.

Consider $AA^{-1} = I$

$$\therefore \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ a & a & 2a \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By R_2 - $2R_1$ and R_3 - aR_1, we get

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & a \end{bmatrix} = A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -a & 0 & 1 \end{bmatrix}$$

By R_1 - R_2, we get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & a \end{bmatrix} = A^{-1} = \begin{bmatrix} 3 & -1 & 0 \\ -2 & 1 & 0 \\ -a & 0 & 1 \end{bmatrix}$$

By $\left(\frac{1}{a}\right)R_3$, we get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 4 & -1 & -\frac{1}{a} \\ -2 & 1 & 0 \\ -1 & 0 & \frac{1}{a} \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} 4 & -1 & -\frac{1}{a} \\ -2 & 1 & 0 \\ -1 & 0 & \frac{1}{a} \end{bmatrix}$$

ow, premultiply AX = B by A^{-1}, we get,

$A^{-1}(AX) = A^{-1}B$

$\therefore (A^{-1}A)X = A^{-1}B$

$\therefore IX = A^{-1}B$

$$\therefore X = \begin{bmatrix} 4 & -1 & -\frac{1}{a} \\ -2 & 1 & 0 \\ -1 & 0 & \frac{1}{a} \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 - 2 - \frac{4}{a} \\ -2 + 2 + 0 \\ -1 + 0 + \frac{4}{a} \end{bmatrix} = \begin{bmatrix} 2 - \frac{4}{a} \\ 0 \\ \frac{4}{a} - 1 \end{bmatrix}$$

By equality of matrices,

$x = 2 - \dfrac{4}{a}$ $y = 0$, $z = \dfrac{4}{a} - 1$ is the required solution.

Miscellaneous exercise 2 (B) | Q 1.3 | Page 63

Solve the following equations by the method of inversion:

5x - y + 4z = 5, 2x + 3y + 5z = 2, 5x - 2y + 6z = - 1

Solution:

The given equations can be written in the matrix form as:

$$\begin{bmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix}$$

This is of the form AX = B, where

A = $\begin{bmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix}$

Let us find A^{-1}.

|A| = $\begin{vmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{vmatrix}$

= 5(18 + 10)+1(12 - 25)+4(- 4 - 15)

= 140 - 13 - 76
= 51 ≠ 0

∴ A^{-1} exists.

Now, we have to find the cofactor matrix

$$= [A_{ij}]_{3\times 3}, \text{where} Aij = (-1)^{i+j} M_{ij}$$

$$A_{11} = (-1)^{1+1} M_{11} = \begin{vmatrix} 3 & 5 \\ -2 & 6 \end{vmatrix} = 18 + 10 = 28$$

$$A_{12} = (-1)^{1+2} M_{12} = -\begin{vmatrix} 2 & 5 \\ 5 & 6 \end{vmatrix} = -(12 - 25) = 13$$

$$A_{13} = (-1)^{1+3} M_{13} = \begin{vmatrix} 2 & 3 \\ 5 & -2 \end{vmatrix} = -4 - 15 = -19$$

$$A_{21} = (-1)^{2+1} M_{21} = -\begin{vmatrix} -1 & 4 \\ -2 & 6 \end{vmatrix} = -(-6 + 8) = -2$$

$$A_{22} = (-1)^{2+2} M_{22} = \begin{vmatrix} 5 & 4 \\ 5 & 6 \end{vmatrix} = (30 - 20) = 10$$

$$A_{23} = (-1)^{2+3}M_{23} = -\begin{vmatrix} 5 & -1 \\ 5 & -2 \end{vmatrix} = -(-10+5) = 5$$

$$A_{31} = (-1)^{3+1}M_{31} = \begin{vmatrix} -1 & 4 \\ 3 & 5 \end{vmatrix} = -5 - 12 = -17$$

$$A_{32} = (-1)^{3+2}M_{32} = -\begin{vmatrix} 5 & 4 \\ 2 & 5 \end{vmatrix} = -(25-8) = -17$$

$$A_{33} = (-1)^{3+3}M_{33} = \begin{vmatrix} 5 & -1 \\ 2 & 3 \end{vmatrix} = 15 + 2 = 17$$

∴ the cofactor matrix =

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} = \begin{bmatrix} 28 & 13 & -19 \\ -2 & 10 & 5 \\ -17 & -17 & 17 \end{bmatrix}$$

∴ adj A = $\begin{bmatrix} 28 & 13 & -19 \\ -2 & 10 & 5 \\ -17 & -17 & 17 \end{bmatrix}$

∴ $A^{-1} = \dfrac{1}{|A|}$(adj A)

$$= \frac{1}{51}\begin{bmatrix} 28 & -2 & -17 \\ 13 & 10 & -17 \\ -19 & 5 & 17 \end{bmatrix}$$

Now, premultiply AX = B by A^{-1}, we get,

$$A^{-1}(AX) = A^{-1}B$$

∴ $(A^{-1}A)X = A^{-1}B$

$\therefore$ IX = $\mathrm{A}^{-1}\mathrm{B}$

$$\therefore X = \frac{1}{51}\begin{bmatrix} 28 & -2 & -17 \\ 13 & 10 & -17 \\ -19 & 5 & 17 \end{bmatrix}\begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix}$$

$$= \frac{1}{51}\begin{bmatrix} 140 - 4 + 17 \\ 65 + 20 + 17 \\ -95 + 10 - 17 \end{bmatrix}$$

$$= \frac{1}{51}\begin{bmatrix} 153 \\ 102 \\ -102 \end{bmatrix}$$

$$\therefore \begin{bmatrix} \mathrm{x} \\ \mathrm{y} \\ \mathrm{z} \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ -2 \end{bmatrix}$$

By equality of matrices,

x = 3, y = 2, z = - 2 is the required solution.

Miscellaneous exercise 2 (B) | Q 1.4 | Page 63

Solve the following equations by the method of inversion:

2x - y = - 2, 3x + 4y = 3

Solution:

The given equations can be written in the matrix form as:

$$\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

This is of the form AX = B, where

$$A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

Let us find A^{-1}.

$|A| = \begin{vmatrix} 2 & 3 \\ 3 & 1 \end{vmatrix} = 2 - 9 = -7 \neq 0$

A^{-1} exists.

Consider $AA^{-1} = I$

$$\therefore \begin{bmatrix} 2 & 3 \\ 3 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

By $R_1 \leftrightarrow R_2$ we get,

$$\begin{bmatrix} 3 & 1 \\ 2 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

By $R_1 - R_2$, we get,

$$\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix}$$

By $R_2 - 2R_1$, we get,

$$\begin{bmatrix} 1 & -2 \\ 0 & 7 \end{bmatrix} A^{-1} = \begin{bmatrix} -1 & 1 \\ 3 & -2 \end{bmatrix}$$

By $\left(\frac{1}{7}\right)\mathbf{R_2}$we get,

$$\begin{bmatrix}1 & -2\\0 & 1\end{bmatrix}\mathbf{A}^{-1} = \begin{bmatrix}-1 & 1\\\frac{3}{7} & -\frac{2}{7}\end{bmatrix}$$

By $R_1 + 2R_2$ we get,

$$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\mathbf{A}^{-1} = \begin{bmatrix}-\frac{1}{7} & \frac{3}{7}\\\frac{3}{7} & -\frac{2}{7}\end{bmatrix}$$

$$\therefore \mathbf{A}^{-1} = \begin{bmatrix}-\frac{1}{7} & \frac{3}{7}\\\frac{3}{7} & -\frac{2}{7}\end{bmatrix}$$

Now, premultiply AX = B by $\mathbf{A}^{-1}$, we get,

$A^{-1}(AX) = A^{-1}B$

$\therefore (A^{-1}A)X = A^{-1}B$

$\therefore IX = A^{-1}B$

$$\therefore X = \begin{bmatrix}-\frac{1}{7} & \frac{3}{7}\\\frac{3}{7} & -\frac{2}{7}\end{bmatrix}\begin{bmatrix}-5\\3\end{bmatrix}$$

$$\therefore \begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}\frac{5}{7}+\frac{9}{7}\\-\frac{15}{7}-\frac{6}{7}\end{bmatrix} = \begin{bmatrix}\frac{14}{7}\\-\frac{21}{7}\end{bmatrix}$$

$$\therefore \mathbf{A}^{-1} = \frac{1}{|\mathbf{A}|}\text{(adj A)}$$

$$= \frac{1}{51}\begin{bmatrix} 28 & -2 & -17 \\ 13 & 10 & -17 \\ -19 & 5 & 17 \end{bmatrix}$$

Now, premultiply AX = B by A^{-1}, we get,

$$\mathbf{A}^{-1}(\mathbf{AX}) = \mathbf{A}^{-1}\mathbf{B}$$

$$\therefore \left(\mathbf{A}^{-1}\mathbf{A}\right)\mathbf{X} = \mathbf{A}^{-1}\mathbf{B}$$

$$\therefore \text{IX} = \mathbf{A}^{-1}\mathbf{B}$$

$$\therefore \text{X} = \frac{1}{51}\begin{bmatrix} 28 & -2 & -17 \\ 13 & 10 & -17 \\ -19 & 5 & 17 \end{bmatrix}\begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix}$$

$$= \frac{1}{51}\begin{bmatrix} 140 - 4 + 17 \\ 65 + 20 + 17 \\ -95 + 10 - 17 \end{bmatrix}$$

x = 2, y = - 3 is the required solution.

Miscellaneous exercise 2 (B) | Q 1.5 | Page 63

Solve the following equations by the method of inversion:

x + y + z = - 1, y + z = 2, x + y - z = 3

Solution:

The given equations can be written in the matrix form as:

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$$

This is of the form AX = B, where

A = $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & -1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$

Let us find A^{-1}.

|A| = $\begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$

= 1(- 1 - 1) - 1(0 - 1) + 1(0 - 1)

= - 2 + 1 - 1

= - 2 ≠ 0

∴ A^{-1} exists.

Consider $AA^{-1} = I$

∴ $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & -1 \end{bmatrix} A - 1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

By $R_3 - R_1$, we get

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & -2 \end{bmatrix} A - 1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

By $\mathrm{R_1 - R_2}$, we get,

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & -2 \end{bmatrix} A - 1 = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

By $\left(-\frac{1}{2}\right)\mathrm{R_3}$, we get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \mathrm{A}^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ \frac{1}{2} & 0 & -\frac{1}{2} \end{bmatrix}$$

By $\mathrm{R_2 - R_3}$, we get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathrm{A}^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ -\frac{1}{2} & 1 & \frac{1}{2} \\ \frac{1}{2} & 0 & -\frac{1}{2} \end{bmatrix}$$

$$\mathrm{A}^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ -\frac{1}{2} & 1 & \frac{1}{2} \\ \frac{1}{2} & 0 & -\frac{1}{2} \end{bmatrix}$$

Now, premultiply AX = B by A-1 , we get

A-1(AX) = A-1B

∴ (A-1A)X = A-1B

∴ IX = A-1B

$$\therefore X = \begin{bmatrix} -1-2+0 \\ \frac{1}{2}+2+\frac{3}{2} \\ -\frac{1}{2}+0-\frac{3}{2} \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 \\ 4 \\ -2 \end{bmatrix}$$

∴ by equality of the matrices, x = -3, y = 4, z = - 2 is the required solution.

Miscellaneous exercise 2 (B) | Q 2.1 | Page 63

Express the following equations in matrix form and solve them by the method of reduction:

x - y + z = 1, 2x - y = 1, 3x + 3y - 4z = 2

Solution:

The given equations can be written in the matrix form as:

$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 3 & 3 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

By R_2 - $2R_1$ and R_3 - $3R_1$, we get,

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & -2 \\ 0 & 6 & -7 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$$

By R_3 - $6R_2$, we get,

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 5 \end{bmatrix}$$

∴ [("x" - "y" + "z"),(0+"y" - 2"z"),(0 + 0 + "5z")] = [(1),(-1),(5)]

By equality of matrices,

x - y + z = 1 ...(1)

y - 2z = - 1 ....(2)

5z = 5 ...(3)

From (3), z = 1

Substituting z = 1 in (2), we get,

y - 2 = - 1

∴ y = 1

Substituting y = 1, z = 1 in (1), we get,

x - 1 + 1 = 1

∴ x = 1

Hence, x = 1, y = 1, z = 1 is the required solution.

Miscellaneous exercise 2 (B) | Q 2.2 | Page 63

Express the following equations in matrix form and solve them by the method of reduction:

x + y = 1, y + z = 3, z + x = 3.

Solution:

The given equations can be written in the matrix form as:

$$\begin{bmatrix}1 & 1 & 0\\0 & 1 & 1\\1 & 0 & 1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\\frac{5}{3}\\\frac{4}{3}\end{bmatrix}$$

By $R_3 - R_1$, we get,

$$\begin{bmatrix}1 & 1 & 0\\0 & 1 & 1\\0 & -1 & 1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\\frac{5}{3}\\\frac{1}{3}\end{bmatrix}$$

By $R_3 + R_2$, we get,

$$\begin{bmatrix}1 & 1 & 0\\0 & 1 & 1\\0 & 0 & 2\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\\frac{5}{3}\\2\end{bmatrix}$$

$$\therefore \begin{bmatrix}x+y+0\\0+y+z\\0+0+2z\end{bmatrix}=\begin{bmatrix}1\\\frac{5}{3}\\2\end{bmatrix}$$

By equality of matrices,

$x + y = 1$ ...(1)

$y + z = \frac{5}{3}$ .....(2)

$2z = 2$ ....(3)

From (3), $z = 1$

Substituting $z = 1$ in (2), we get,

$y + 1 = \frac{5}{3}$

$\therefore y = \frac{2}{3}$

Substituting y = 2/3 in (1), we get,

$x + \frac{2}{3} = 1$

$\therefore x = \frac{1}{3}$

Hence, $x = \frac{1}{3}$, $y = \frac{2}{3}$, z = 1 is the required solution.

Miscellaneous exercise 2 (B) | Q 2.3 | Page 63

Express the following equations in matrix form and solve them by the method of reduction:

2x - y + z = 1, x + 2y + 3x = 8, 3x + y - 4z = 1.

Solution:

The given equations can be written in the matrix form as:

$$\begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 8 \\ 1 \end{bmatrix}$$

By $R_1 \leftrightarrow R_2$, we get,

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 1 \\ 3 & 1 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z/ \end{bmatrix} = \begin{bmatrix} 8 \\ 1 \\ 1 \end{bmatrix}$$

By $R_2 - 2R_1$ and $R_3 - 3R_1$, we get,

$$\begin{bmatrix}1 & 2 & 3\\0 & -5 & -5\\0 & -5 & -13\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}8\\-15\\-23\end{bmatrix}$$

By R_3 - R_2, we get,

$$\begin{bmatrix}1 & 2 & 3\\0 & -5 & -5\\0 & 0 & -8\end{bmatrix}[x, y, z] = [8, -15, -8]$$

$$\therefore \begin{bmatrix}x + 2y + 3z\\0 - 5y - 5z\\0 + 0 - 8z\end{bmatrix} = \begin{bmatrix}8\\-15\\-8\end{bmatrix}$$

By equality of matrices,

x + 2y + 3z = 8 ...(1)

- 5y - 5z = - 15 ....(2)

- 8z = - 8 ....(3)

From (3), z = 1

Substituting z = 1 in (2), we get,

- 5y - 5 = - 15

∴ - 5y = - 10

∴ y = 2

Substituting y = 2, z = 1 in (1), we get,

x + 4 + 3 = 8

∴ x = 1

Hence, x = 1, y = 2, z = 1 is the required solution.

Miscellaneous exercise 2 (B) | Q 2.5 | Page 63

Express the following equations in matrix form and solve them by the method of reduction:

x + 2y + z = 8, 2x + 3y - z = 11, 3x - y - 2z = 5.

Solution:

The given equations can be written in the matrix form as:

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & -1 \\ 3 & -1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 11 \\ 5 \end{bmatrix}$$

By R_1 - $2R_1$ and R_3 - $3R_1$, we get,

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & -3 \\ 0 & -7 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ -5 \\ -19 \end{bmatrix}$$

By R_3 - $7R_2$, we get,

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & -3 \\ 0 & 0 & 16 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ -5 \\ 16 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x + 2y + z \\ 0 - y - 3z \\ 0 + 0 + 16z \end{bmatrix} = \begin{bmatrix} 8 \\ -5 \\ 16 \end{bmatrix}$$

By equality of matrices,

x + 2y + z = 8 ...(1)

- y - 3z = - 5 ....(2)

16z = 16 ...(3)

From (3), z = 1

Substituting z = 1 in (2), we get,

- y - 3 = - 5,

∴ y = 2

Substituting y = 2, z = 1 in (1), we get,

x + 4 + 1 = 8

$\therefore$ x = 3

Hence, x = 3, y = 2, z = 1 is the required solution.

Miscellaneous exercise 2 (B) | Q 2.6 | Page 63

Express the following equations in matrix form and solve them by the method of reduction:

x + 3y + 2z = 6, 3x - 2y + 5z = 5, 2x - 3y + 6z = 7

Solution:

The given equations can be written in the matrix form as:

$$\begin{bmatrix} 1 & 3 & 2 \\ 3 & -2 & 5 \\ 2 & -3 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \\ 7 \end{bmatrix}$$

By $R_3 - 2R_1$, $R_2 - 3R_1$ we get

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & 11 & 1 \\ 0 & -9 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 13 \\ -5 \end{bmatrix}$$

By $R_2 + R_3$, we get,

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & 2 & 3 \\ 0 & -9 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \\ -5 \end{bmatrix}$$

By $\dfrac{R_2}{2}$ we get

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & 1 & \frac{3}{2} \\ 0 & -9 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ -5 \end{bmatrix}$$

By $R_3 + 9R_2$ we get,

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & \frac{31}{2} \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ 31 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x + 3y + 2z \\ 0 + y + \frac{3}{2}z \\ 0 + 0 + \frac{31}{2}z \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ 31 \end{bmatrix}$$

By equality of matrices,

$x + 3y + 2z = 6$ ...(1)

$y + \dfrac{3}{2}z = 4$ ....(2)

$$\frac{31}{2}z = 31 \quad .....(3)$$

From (3), **z = 2**

Substituting z = 2 in (2), we get,

$y + \frac{3}{2}z = 4$

$y + \frac{3}{2}(2) = 4$

y + 3 = 4

y = 1

Substituting y = 1, z = 2 in (1), we get,

x + 3y + 2z = 6

x + 3(1) + 2(2) = 6

x + 3 + 4 = 6

x = - 1

Hence, x = -1, y = 1, z = 2 is the required solution.

Miscellaneous exercise 2 (B) | Q 3 | Page 63

The sum of three numbers is 6. If we multiply the third number by 3 and add it to the second number we get 11. By adding first and third numbers we get a number, which is double than the second number. Use this information and find a system of linear equations. Find these three numbers using matrices.

Solution: Let the first , second & third number be x, y, z respectively

Given,

∴ x + y + z = 6

y + 3z = 11

x + z = 2y or x - 2y + z = 0

Step 1

write equation as AX = B

A = $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$

Hence A = $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \& B = \begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$

Step 2

calculate |A|

|A| = $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix}$

= 1(1 + 6) - 0 (1 + 2) + 1(3 + 1)

= 7 + 2

= 9

So, |A| ≠0

∴ The system of equation is consistent & has a unique solutions

Now , AX = B

X = A^{-1} B

Hence A = $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \& B = \begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$

= 1 (1+6)-0(1+2)+1(3-1)

=7+2

$=9\neq 0$

Since determinant is not equal to O , A^{-1} exists

Now find adj (A)

now AX = B

$X = A^{-1} B$

Step 3

Calculating $X= A^{-1} B$

Calculating A^{-1}

$$\text{Now } A^{-1} = \frac{1}{|A|}\text{adj (A)}$$

$$\text{adj } A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}' = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix}$$

$A_{11} = 1 \times 1-3\times(-2)=1+6=7$

$A_{12} = - [0\times1-3\times1]=-(-3)=3$

$A_{13} =- 0\times(-2) -1\times1=-1$

$A_{21} = [1\times1-(-2)\times1]=-[1+2]=-3$

$A_{22}=1\times1-1\times1=1-1=0$

$A_{23} = [1\times(-2)-1\times1]=-[-2-1]=-(-3)=3$

$A_{31} = 1\times3-1\times1=3-1=2$

$A_{32} =-[1\times3-0\times1]=-[3-0]=-3$

$A_{33}=1\times1-1\times0=1-0=1$

Hence , adj (A) = $\begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$

Now ,

$A^{-1} = \frac{1}{|A|}$ adj (A)

$A^{-1} = \frac{1}{9}\begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$

Solution of given system of equations is

X = A^{-1} B

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}\begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 42 & -33 & +0 \\ 18 & +0 & +0 \\ -6 & +33 & +0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 9 \\ 18 \\ 27 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

∴ x = 1, y = 2, z = 3

Miscellaneous exercise 2 (B) | Q 4 | Page 63

The cost of 4 pencils, 3 pens, and 2 books is ₹ 150. The cost of 1 pencil, 2 pens, and 3 books is ₹ 125. The cost of 6 pencils, 2 pens, and 3 books is ₹ 175. Find the cost of each item by using matrices.

Solution: Let the cost of 1 pencil, 1 pen and 1 book be ₹ x, ₹ y, ₹ z respectively.

According to the given conditions,

$4x + 3y + 2z = 150$

$x + 2y + 3z = 125$

$6x + 2y + 3z = 175$

The equations can be written in matrix form as:

$$\begin{bmatrix} 4 & 3 & 2 \\ 1 & 2 & 3 \\ 6 & 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 150 \\ 125 \\ 175 \end{bmatrix}$$

By $R_1 \leftrightarrow R_2$, we get,

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 3 & 2 \\ 6 & 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 125 \\ 150 \\ 175 \end{bmatrix}$$

By $R_2 - 4R_1$ and $R_3 - 6R_1$, we get,

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -5 & -10 \\ 0 & -10 & -15 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 125 \\ -350 \\ -575 \end{bmatrix}$$

By $R_3 - 2R_2$, we get,

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -5 & -10 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 125 \\ -350 \\ 125 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x + 2y + 3z \\ 0 - 5y - 10z \\ 0 + 0 + 5z \end{bmatrix} = \begin{bmatrix} 125 \\ -350 \\ 125 \end{bmatrix}$$

By equality of matrices,

x + 2y + 3z = 125 ...(1)

- 5y - 10z = - 350 ...(2)

5z = 125 ...(3)

From (3), z = 25

Substituting z = 25 in (2), we get

- 5y - 10(25) = - 350

∴ - 5y = - 350 + 250 = - 100

∴ y = 20

Substituting y = 20, z = 25 in (1), we get

x + 2(20) + 3(25) = 125

∴ x = 125 - 40 - 75 = 10

∴ x = 10, y = 20, y = 25

Hence, the cost of 1 pencil is ₹ 10, 1 pen is ₹ 20 and 1 book is ₹ 25.

[**Note:** Answer to cost of a pen in the textbook is incorrect.]

Miscellaneous exercise 2 (B) | Q 5 | Page 63

The sum of three numbers is 6. Thrice the third number when added to the first number, gives 7. On adding three times the first number to the sum of second and third numbers, we get 12. Find the three number by using matrices.

Solution: Let the numbers be x, y and z. According to the given conditions, x + y + z = 6

3z + x = 7, i.e. x + 3z = 7

and 3x + y + z = 12

Hence, the system of linear equations is

x + y + z = 6

x + 3z = 7

3x + y + z = 12

These equations can be written in matrix form as:

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 3 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix}$$

By R_2 - R_1 and R_3 - $3R_1$, we get,

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & 2 \\ 0 & -2 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \\ -6 \end{bmatrix}$$

By R_3 + R_2, we get,

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & 2 \\ 0 & -3 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \\ -5 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x + y + z \\ 0 - y + 2z \\ 0 - 3y + 0 \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \\ -5 \end{bmatrix}$$

By equality of matrices,

x + y + z = 6   ...(1)

- y + 2z = 1   ...(2)

- 3y = - 5   ...(3)

From (3), y = $\frac{5}{3}$

Substituting y = $\frac{5}{3}$ in (2), we get,

$$-\frac{5}{3} + 2z = 1$$

∴ 2z = $1 + \frac{5}{3} = \frac{8}{3}$

∴ z = $\frac{4}{3}$

Substituting y = $\frac{5}{3}$, y = $\frac{4}{3}$ in (1), we get,

$$x + \frac{5}{3} + \frac{4}{3} = 6$$

∴ x = 3

∴ x = 3, y = $\frac{5}{3}$, z = $\frac{4}{3}$

Hence, the required numbers are $3, \frac{5}{3}$ and $\frac{4}{3}$.

Miscellaneous exercise 2 (B) | Q 6 | Page 63

The sum of three numbers is 2. If twice the second number is added to the sum of first and third, the sum is 1. By adding second and third number to five times the first number, we get 6. Find the three numbers by using matrices.

Solution:

Let the three numbers be x, y and z.

According to the question,

$x+y+2$

$x+2y+z=1$

$5x+y+z=6$

The given system of equations can be written in matrix form as follows

$$\begin{bmatrix}1 & 1 & 1\\1 & 2 & 1\\5 & 1 & 1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}2\\1\\6\end{bmatrix}$$

$AX=B$

Here,

$$A=\begin{bmatrix}1 & 1 & 1\\1 & 2 & 1\\5 & 1 & 1\end{bmatrix}, X=\begin{bmatrix}x\\y\\z\end{bmatrix} \text{ and } B=\begin{bmatrix}2\\1\\6\end{bmatrix}$$

$|A|=1(2-1)-1(1-5)+1(1-10)$

$=1+4-9$

$=-4$

$\text{Let } C_{ij} \text{ be the cofactors of t he elements a }_{ij} \text{ in } A=[a_{ij}].\ \text{Then,}$

$$C_{11}=(-1)^{1+1}\begin{vmatrix}2 & 1\\ 1 & 1\end{vmatrix}=1, C_{12}=(-1)^{1+2}\begin{vmatrix}1 & 1\\ 5 & 1\end{vmatrix}=4, C_{13}=(-1)^{1+3}\begin{vmatrix}1 & 2\\ 5 & 1\end{vmatrix}=-9$$

$$C_{21}=(-1)^{2+1}\begin{vmatrix}1 & 1\\ 1 & 1\end{vmatrix}=0, C_{22}=(-1)^{2+2}\begin{vmatrix}1 & 1\\ 5 & 1\end{vmatrix}=-4, C_{23}=(-1)^{2+3}\begin{vmatrix}1 & 1\\ 5 & 1\end{vmatrix}=4$$

$$C_{31}=(-1)^{3+1}\begin{vmatrix}1 & 1\\ 2 & 1\end{vmatrix}=-1, C_{32}=(-1)^{3+2}\begin{vmatrix}1 & 1\\ 1 & 1\end{vmatrix}=0, C_{33}=(-1)^{3+3}\begin{vmatrix}1 & 1\\ 1 & 2\end{vmatrix}=1$$

$$adjA=\begin{bmatrix}1 & 4 & -9\\ 0 & -4 & 4\\ -1 & 0 & 1\end{bmatrix}^T$$

$$=\begin{bmatrix}1 & 0 & -1\\ 4 & -4 & 0\\ -9 & 4 & 1\end{bmatrix}$$

$$\Rightarrow A^{-1}=\frac{1}{|A|}adjA$$

$$=\frac{1}{-4}\begin{bmatrix}1 & 0 & -1\\ 4 & -4 & 0\\ -9 & 4 & 1\end{bmatrix}$$

$$X=A^{-1}B$$

$$\Rightarrow X=\frac{1}{-4}\begin{bmatrix}1 & 0 & -1\\ 4 & -4 & 0\\ -9 & 4 & 1\end{bmatrix}\begin{bmatrix}2\\ 1\\ 6\end{bmatrix}$$

$$\Rightarrow X=\frac{1}{-4}\begin{bmatrix}2+0-6\\ 8-4+0\\ -18+4+6\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}x\\ y\\ z\end{bmatrix}=\frac{1}{-4}\begin{bmatrix}-4\\ 4\\ -8\end{bmatrix}$$

$$\therefore x=1, y=-1 \text{ and } z=2$$

Miscellaneous exercise 2 (B) | Q 7 | Page 63

An amount of ₹ 5000 is invested in three types of investments, at interest rates 6%, 7%, 8% per annum respectively. The total annual income from these investments is ₹ 350. If the total annual income from the first two investments is ₹ 70 more than the income from the third, find the amount of each investment using matrix method.

Solution: Let the amounts in three investments by ₹ x, ₹ y and ₹ z respectively.

Then x + y + z = 5000

Since the rate of interest in these investments are 6%, 7%and 8% respectively, the annual income of the three investments are 6x/100,7y/100, and 8z/100 respectively.

According to the given conditions,

$$\frac{6x}{100}+\frac{7y}{100}+\frac{8z}{100}=350$$

i.e. 6x + 7y + 8z = 35000

Also, $\frac{6x}{100}+\frac{7y}{100}+\frac{8z}{100}+70$

i.e. 6x + 7y - 8z = 7000

Hence, the system of linear equation is

x + y + z = 5000

6x + 7y + 8z = 35000

6x + 7y - 8z = 7000

These equations can be written in matrix form as:

$$\begin{bmatrix}1 & 1 & 1\\ 6 & 7 & 8\\ 6 & 7 & -8\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}5000\\ 35000\\ 7000\end{bmatrix}$$

By R_3 - R_2, we get,

$$\begin{bmatrix}1 & 1 & 1\\ 6 & 7 & 8\\ 0 & 0 & -16\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}5000\\ 35000\\ -28000\end{bmatrix}$$

By $R_2 - 6R_1$, we get,

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & -16 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5000 \\ 5000 \\ -28000 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x + y + z \\ 0 + y + 2z \\ 0 + 0 - 16z \end{bmatrix} = \begin{bmatrix} 5000 \\ 5000 \\ -28000 \end{bmatrix}$$

By equality of matrices,

$x + y + z = 5000$ ...(1)

$y + 2z = 5000$ ....(2)

$- 16z = - 28000$ ...(3)

From (3), $z = 1750$

Substituting $z = 1750$ in (2), we get,

$y + 2(1750) = 5000$

$\therefore y = 5000 - 3500 = 1500$

Substituting $y = 1500$, $z = 1750$ in (1), we get,

$x + 1500 + 1750 = 5000$

$\therefore x = 5000 - 3250 = 1750$

$\therefore x = 1750, y = 1500, z = 1750$

Hence, the amounts of the three investments are ₹ 1750, ₹ 1500 and ₹ 1750 respectively.