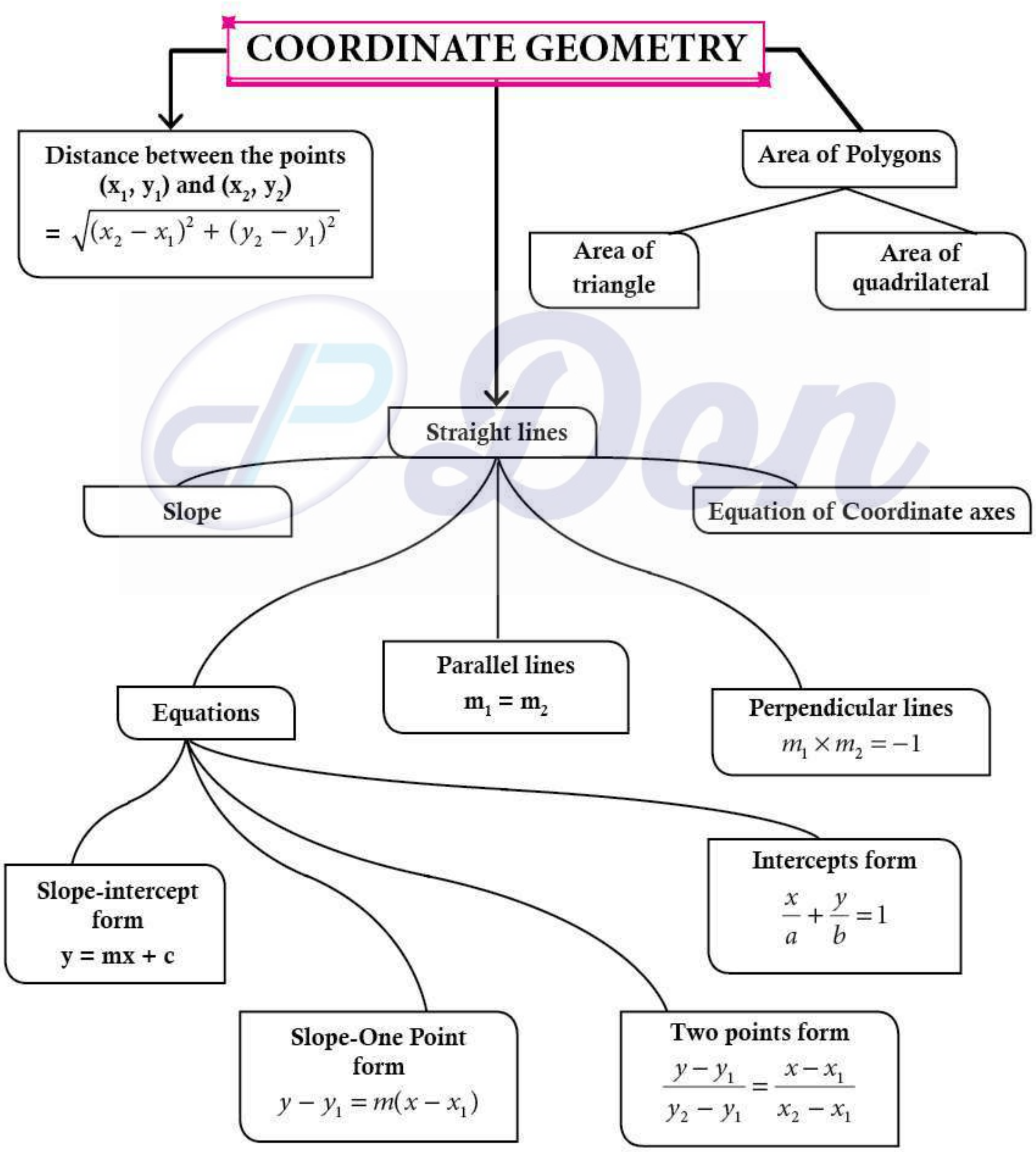
UNIT 5 COORDINATE GEOMETRY

MIND MAP



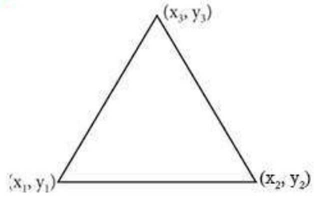
Area of a Triangle

Key Points

Area of triangle with the given vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) is

$$= \frac{1}{2}\,[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] \text{ sq. units}$$

(or) $$\frac{1}{2}\,[(x_1y_2 + x_2y_3 + x_3y_1) - (x_2y_1 + x_3y_2 + x_1y_3)] \text{ sq. units.}$$



Condition for collinearity

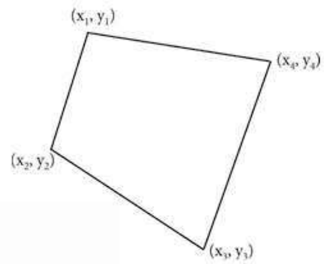
- If the three distinct points (x_1, y_1), (x_2, y_2) and (x_3, y_3) are collinear, then area of the triangle formed by the points is zero.

  i.e., $x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$

Area of Quadrilateral formed by the points

(x_1, y_1), (x_2, y_2), (x_3, y_3) and (x_4, y_4) is

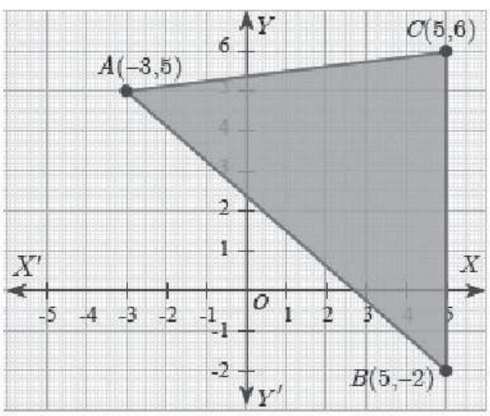
$$= \frac{1}{2}\,[(x_1 - x_3)(y_2 - y_4) - (x_2 - x_4)(y_1 - y_3)] \text{ sq. units.}$$



Worked Examples

5.1 **Find the area of the triangle whose vertices are at (–3, 5), (5, 6) and (5, –2).**

Sol :



Plot the points in a rough diagram and take them in counter-clockwise order.

Let the vertices be A(–3, 5), B(5, – 2), C(5, 6)

$\downarrow$ (x_1, y_1) $\quad\downarrow$ (x_2, y_2) $\quad\downarrow$ (x_3, y_3)

The area of ΔABC is

$$= \frac{1}{2}\left\{(x_1y_2 + x_2y_3 + x_3y_1) - (x_2y_1 + x_3y_2 + x_1y_3)\right\}$$

$$= \frac{1}{2}\ \{(6 + 30 + 25) - (25 - 10 - 18)\}$$

$$= \frac{1}{2}\ \{61 + 3\}$$

$$= \frac{1}{2}(64) = 32 \text{ sq. units}$$

5.2 **Show that the points P(–1.5, 3), Q(6, – 2), R(–3, 4) are collinear.**

Sol :

The points are P(–1.5, 3), Q(6, – 2), R(–3, 4)

Area of ΔPQR

$$= \frac{1}{2}\left\{(x_1y_2 + x_2y_3 + x_3y_1) - (x_2y_1 + x_3y_2 + x_1y_3)\right\}$$

$$= \frac{1}{2}\ \{(3 + 24 - 9) - (18 + 6 - 6)\}$$

$$= \frac{1}{2}\ \{18 - 18\} = 0 \text{ sq. units}$$

Therefore, the given points are collinear.

5.3 **If the area of the triangle formed by the vertices A(–1, 2), B(k, – 2) and C(7, 4) (taken in order) is 22 sq. units, find the value of k.**

Sol :

The vertices are A(–1, 2), B(k, – 2) and C(7, 4)

Area of triangle ABC is 22 sq. units

$$\frac{1}{2}\{(x_1y_2 + x_2y_3 + x_3y_1) - (x_2y_1 + x_3y_2 + x_1y_3)\} = 22$$

$$\frac{1}{2}\{(2 + 4k + 14) - (2k - 14 - 4)\} = 22$$

$$2k + 34 = 44 \Rightarrow 2k = 10 \Rightarrow k = 5$$

5.4 If the points P(–1, –4), Q(b, c) and R(5, – 1) are collinear and 2b + c = 4, then find the values of b and c.

Sol :

Since the three points P(–1, –4), Q(b, c) and R(5, –1) are collinear,

Area of triangle PQR = 0

$$\frac{1}{2}\{(x_1y_2 + x_2y_3 + x_3y_1) - (x_2y_1 + x_3y_2 + x_1y_3)\} = 0$$

$$\frac{1}{2}\{(-c - b - 20) - (-4b + 5c + 1)\} = 0$$

$$-c - b - 20 + 4b - 5c - 1 = 0$$

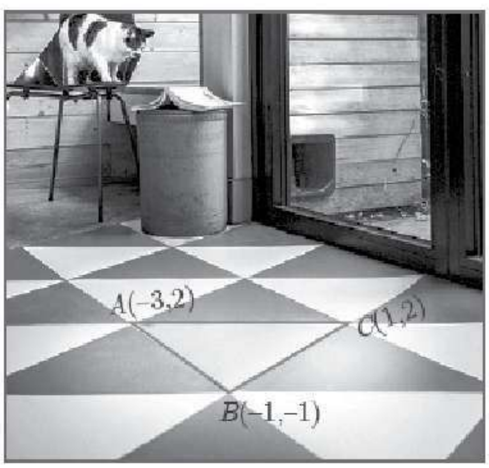
$$b - 2c = 7 \quad \dots (1)$$

Also, 2b + c =4 (from given information) ... (2)

Solving (1) and (2) we get b = 3, c = – 2

5.5 The floor of a hall is covered with identical tiles which are in the shapes of triangles. One such triangle has the vertices at (– 3, 2), (– 1, – 1) and (1, 2). If the floor of the hall is completely covered by 110 tiles, find the area of the floor.

Sol :



Vertices of one triangular tile are at (– 3, 2), (– 1, – 1) and (1, 2)

∴ Area of this tile

$$= \frac{1}{2}\{(3 - 2 + 2) - (-2 - 1 - 6)\} \text{ sq. units}$$

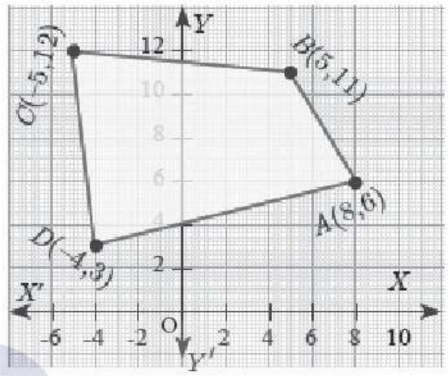
$$= \frac{1}{2}(12) = 6 \text{ sq. units}$$

Since the floor is covered by 110 triangle shaped identical tiles,

Area of floor = $110 \times 6 = 660$ sq. units

5.6 Find the area of the quadrilateral formed by the points (8, 6), (5, 11), (– 5, 12) and (– 4, 3).

Sol :



Before determining the area of quadrilateral, plot the vertices in a graph.

Let the vertices be A(8, 6), B(5, 11), C(– 5, 12) and D(– 4, 3)

Therefore, area of the quadrilateral ABCD

$$= \frac{1}{2}\{(x_1y_2 + x_2y_3 + x_3y_4 + x_4y_1) - (x_2y_1 + x_3y_2 + x_4y_3 + x_1y_4)\}$$

$$= \frac{1}{2}\{(88 + 60 - 15 - 24) - (30 - 55 - 48 + 24)\}$$

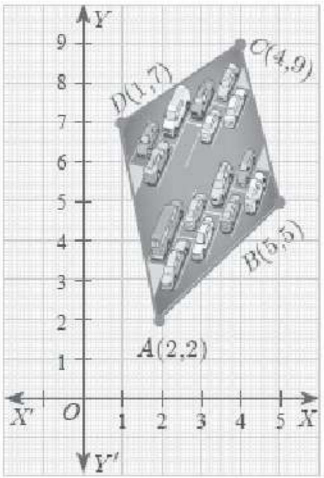
$$= \frac{1}{2}\{109 + 49\}$$

$$= \frac{1}{2}\{158\} = 79 \text{ sq. units}$$

5.7 The given diagram shows a plan for constructing a new parking lot at a campus. It is estimated that such construction would cost ₹ 1300 per square feet. What will be the total cost for making the parking lot?

Sol :

The new parking lot is a quadrilateral whose vertices are at A(2, 2), B(5, 5), C(4, 9) and D(1, 7).



Therefore, Area of parking lot

$= \frac{1}{2}\ \{(10 + 45 + 28 + 2) - (10 + 20 + 9 + 14)\}$

$= \frac{1}{2}\ \{85 - 53\}$

$= \frac{1}{2}(32) = 16$ sq. units.

Therefore, area of parking lot = 16 sq. ft

Construction rate per square feet = ₹ 1300

Therefore, total cost for constructing the parking lot = 16 × 1300 = ₹ 20800.

Progress Check

1. Complete the following table.

Sl. No.	Points	Dis-tance	Mid Point	Internal		External	
				Point	Ratio	Point	Ratio
(i)	(3, 4), (5, 5)	...	...	...	2 : 3	...	2 : 3
(ii)	(– 7, 13), (– 3, 1)	...	...	$\left(-\frac{13}{3}, 5\right)$	...	(– 13, 15)	...

Ans : (i) (3, 4), (5, 5)

Distance $= \sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

$= \sqrt{(3-5)^2+(4-5)^2}$

$= \sqrt{4+1} = \sqrt{5}$ units

Mid point $= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{3+5}{2}, \frac{4+5}{2}\right)$

$= \left(4, \frac{9}{2}\right)$

Internal division $= \left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$

$= \left(\frac{2(5)+3(3)}{2+3}, \frac{2(5)+3(4)}{2+3}\right) = \left(\frac{19}{5}, \frac{22}{5}\right)$

External division $= \left(\frac{mx_2-nx_1}{m-n}, \frac{my_2-ny_1}{m-n}\right)$

$= \left(\frac{2(5)-3(3)}{2-3}, \frac{2(5)-3(4)}{2-3}\right)$

$= \left(\frac{1}{-1}, \frac{-2}{-1}\right) = (-1, 2).$

(ii) (– 7, 13), (– 3, 1)

Distance $= \sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

$= \sqrt{(-7+3)^2+(13-1)^2}$

$= \sqrt{16+144} = \sqrt{160} = 4\sqrt{10}$ units.

Mid point $= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$= \left(\frac{-7-3}{2}, \frac{13+1}{2}\right) = (-5, 7)$

Internal Division:

Let the Ratio be k : 1, given point $\left(-\frac{13}{3}, 5\right)$

Now $\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right) = \left(-\frac{13}{3}, 5\right)$

$\left(\frac{k(-3)+1(-7)}{k+1}, \frac{k(1)+1(13)}{k+1}\right) = \left(-\frac{13}{5}, 5\right)$

$\left(\frac{-3k-7}{k+1}, \frac{k+13}{k+1}\right) = \left(-\frac{13}{3}, 5\right)$

$\Rightarrow \quad \frac{k+13}{k+1} = 5$

k + 13 = 5 k + 5

8 = 4 k ⇒ 2 = k

∴ Ratio is 2 : 1

External Division:

Let the ratio be k : 1 and given point (– 13, 15)

Now $\left(\frac{mx_2-nx_1}{m-n}, \frac{my_2-ny_1}{m-n}\right) = (-13, 15)$

$$\left[\frac{k(-3)-1(-7)}{k-1}, \frac{k(1)-1(13)}{k-1}\right] = (-13, 15)$$

$$\Rightarrow \quad \frac{k-13}{k-1} = 15$$

$$k - 13 = 15k - 15$$

$$15 - 13 = 15k - k$$

$$2 = 14k$$

$$k = \frac{2}{14} = \frac{1}{7}$$

$\therefore$ Ratio is 1 : 7

2. A (0, 5), B (5, 0) and C (– 4, – 7) are vertices of a triangle then its centroid will be at _____.

Ans : A (0, 5), B (5, 0) and C (– 4, – 7)

Centroid of a triangle $= \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

$$= \left(\frac{0+5-4}{3}, \frac{5+0-7}{3}\right)$$

$$= \left(\frac{1}{3}, \frac{-2}{3}\right)$$

3. The vertices of ΔPQR are P (0, – 4), Q (3, 1) and R (– 8, 1)

(i) Draw ΔPQR on a graph paper.

(ii) Check if ΔPQR is equilateral.

(iii) Find the area of ΔPQR.

(iv) Find the co-ordinates of M, the mid-point of QP.

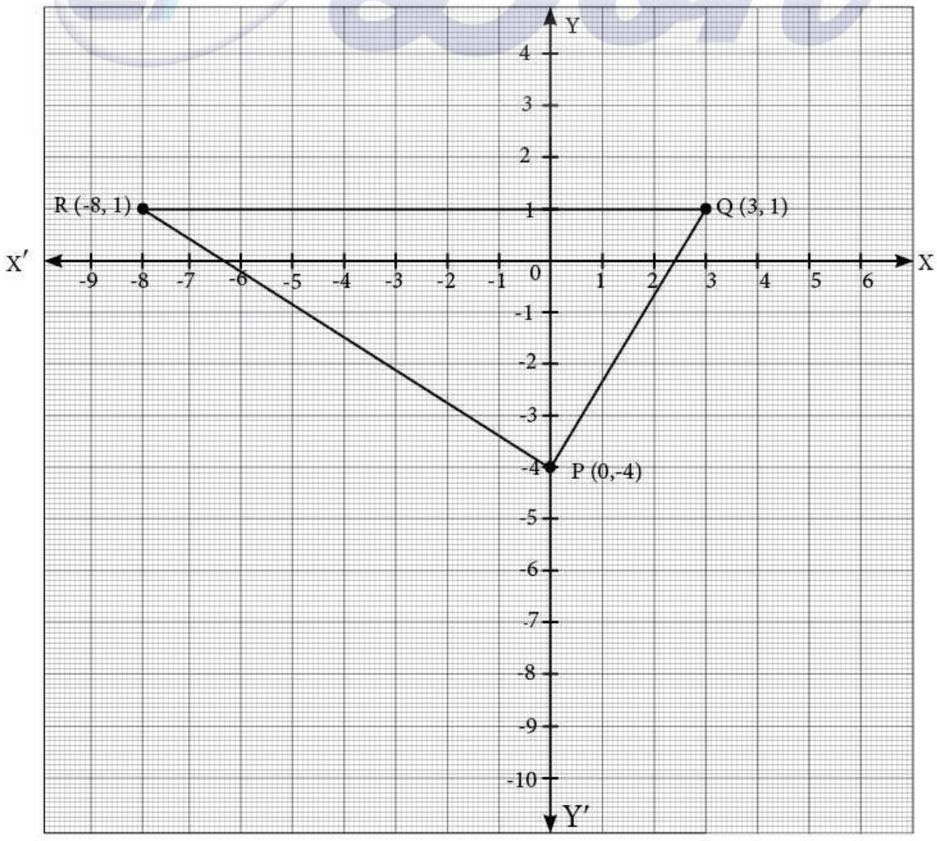
(v) Find the co-ordinates of N, the mid-point of QR.

(vi) Find the area of ΔMPN.

(vii) What is the ratio between the areas of ΔMPN and ΔPQR?

Ans : Given vertices are P (0, – 4), Q (3, 1) and R (– 8, 1)

(i)



(ii) ΔPQR is not equilateral.

(iii) Area of ΔPQR

$= \frac{1}{2}\ [x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)]$

$= \frac{1}{2}\ [0 (1 - 1) + 3 (1 + 4) - 8 (- 4 - 1)]$

$= \frac{1}{2}\ [15 + 40] = \frac{55}{2}$ sq. units.

(iv) Mid point of

$QP = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$= M\left(\frac{0+3}{2}, \frac{-4+1}{2}\right) = M\left(\frac{3}{2}, -\frac{3}{2}\right)$

(v) Mid point of

$QR = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$= N\left(\frac{3-8}{2}, \frac{1+1}{2}\right) = N\left(-\frac{5}{2}, 1\right)$

(vi) Area of ΔMPN

$M\left(\frac{3}{2}, -\frac{3}{2}\right), P(0, -4), N\left(-\frac{5}{2}, 1\right)$

Area $= \frac{1}{2}\left[\frac{3}{2}(-4-1) + 0\left(1+\frac{3}{2}\right) - \frac{5}{2}\left(-\frac{3}{2}+4\right)\right]$

$= \frac{1}{2}\left[-\frac{15}{2} - \frac{25}{4}\right]$

$= -\frac{1}{2}\left[\frac{30+25}{4}\right] = \frac{55}{8}$ sq. units

[$\because$ area cannot be negative.]

(vii) Area of ΔPQR > area of ΔMPN.

Area of PQR : Area of MPN $= \frac{55}{2} : \frac{55}{4} = 4 : 2$

4. In a quadrilateral ABCD with vertices A (– 3, – 8), B (6, – 6), C (4, 2), D (– 8, 2)

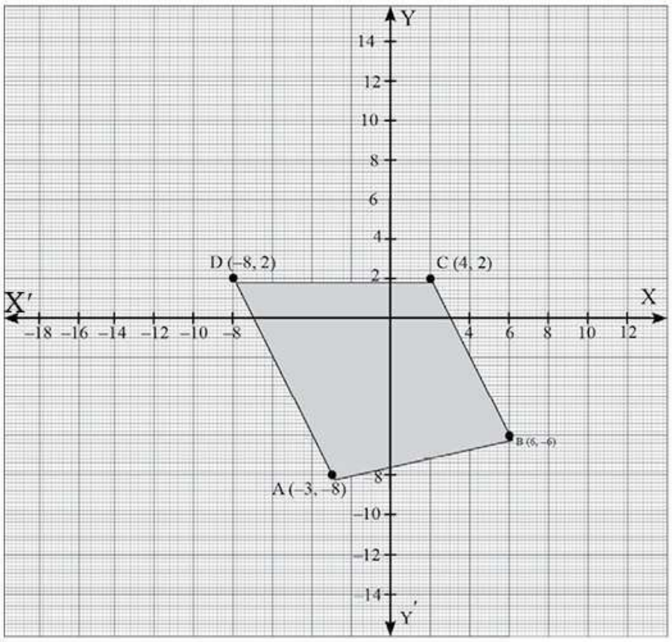
(i) Find the area of ΔABC

(ii) Find the area of ΔACD

(iii) Calculate area of ΔABC + area of ΔACD

(iv) Find the area of quadrilateral ABCD

(v) Compare the answer (iii) and (iv)



Ans : Quadrilateral ABCD with vertices A (– 3, – 8), B (6, – 6), C (4, 2) and D (– 8, 2)

(i) Area of $\Delta ABC = \frac{1}{2}\ [x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)]$ sq. units.

$= \frac{1}{2}\ [- 3 (- 6 - 2) + 6 (2 + 8) + 4 (- 8 + 6)]$

$= \frac{1}{2}\ (24 + 60 - 8) = \frac{1}{2}\ (76) = 38$ sq. units.

(ii) Area of ΔACD

$= \frac{1}{2}\ [- 3 (2 - 2) + 4 (2 + 8) - 8 (- 8 - 2)]$

$= \frac{1}{2}\ [40 + 80] = 60$ sq. units.

(iii) Area of ΔABC + Area of ΔACD

$= 38 + 60 = 98$ sq. units.

(iv) Area of Quadrilateral ABCD.

$= \frac{1}{2}\ [(x_1 - x_3)(y_2 - y_4) - (x_2 - x_4)(y_1 - y_3)]$ sq. units

$= \frac{1}{2}\ [(- 3 - 4)(- 6 - 2) - (6 + 8)(- 8 - 2)]$

$= \frac{1}{2}\ [56 + 140] = \frac{196}{2} = 98$ sq. units.

(v) From **(iii)** and **(iv)**, we know that Area of Quadrilateral ABCD = Area of ΔABC + Area of ΔACD.

Thinking Corner

1. How many triangles exist, whose area is zero?

Ans : No such triangles exist.

2. If the area of a quadrilateral formed by the points (a, a), (– a, a), (a, – a) and (– a, – a), where $a \neq 0$ is 64 square units, then

(i) identify the type of the quadrilateral

(ii) find all possible values of a.

Ans : Given points (a, a), (– a, a), (a, – a) and (– a, –a)

Area of the quadrilateral = 64 sq. units

$\frac{1}{2}[(x_1 - x_3)(y_2 - y_4) - (x_2 - x_4)(y_1 - y_3)] = 64$

$(a + a)(a + a) - (-a - a)(a + a) = 128$

$4a^2 + 4a^2 = 128$

$8a^2 = 128$

$a^2 = 16$

$a = \pm 4$

Since area = 64 [Perfect square and points are equidistant], the quadrilateral is a square.

Exercise 5.1

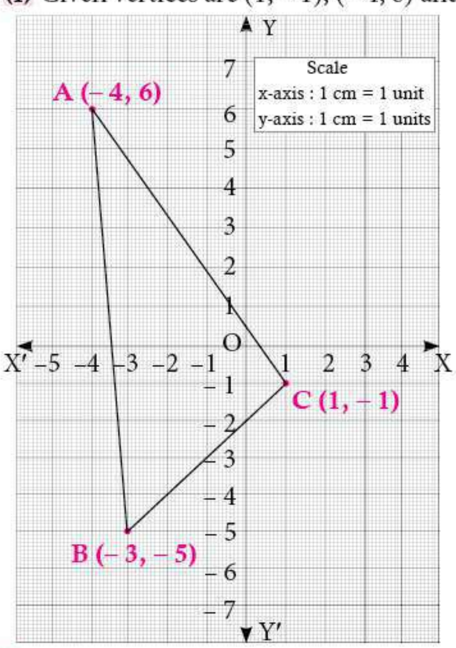
1. Find the area of the triangle formed by the points

(i) (1, – 1), (– 4, 6) and (– 3, – 5)

(ii) (– 10, – 4), (– 8, – 1) and (– 3, – 5)

Sol :

(i) Given vertices are (1, – 1), (– 4, 6) and (– 3, – 5)



A(–4, 6), B(–3, –5), C(1, –1)

Area of triangle ABC $= \frac{1}{2}[(x_1 y_2 + x_2 y_3 + x_3 y_1) - (x_2 y_1 + x_3 y_2 + x_1 y_3)]$

$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$ sq. units

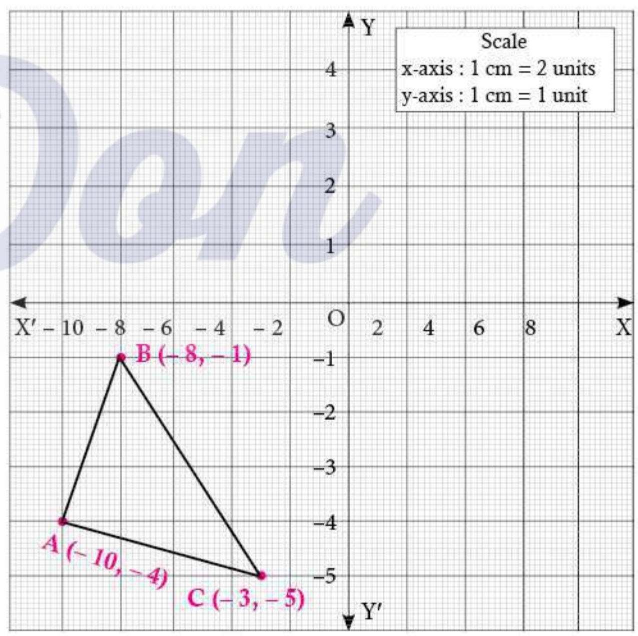
$= \frac{1}{2}[-4(-5+1) - 3(-1-6) + 1(6+5)]$

$= \frac{1}{2}[-4 \times (-4) - 3 \times (-7) + 1 \times (11)]$

$= \frac{1}{2}[16 + 21 + 11]$

$= \frac{1}{2}(48) = 24$ sq. units.

(ii) Given vertices are A(– 10, – 4), B(– 8, – 1) and B(– 3, – 5)



Area of triangle $= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$ sq. units

$= \frac{1}{2}[-10(-1+5) - 8(-5+4) - 3(-4+1)]$

$= \frac{1}{2}[-10(4) - 8(-1) - 3(-3)]$

$= \frac{1}{2}[-40 + 8 + 9]$

$= \frac{1}{2}[-40 + 17] = -\frac{23}{2} = -11.5$

[$\because$ Area cannot be negative]

$\therefore$ Area of triangle = 11.5 sq. units.

2. Determine whether the set of points are collinear.

(i) $\left(-\frac{1}{2}, 3\right)$, **(– 5, 6) and (– 8, 8)**

(ii) (a, b + c), (b, c + a) and (c, a + b)

Sol :

(i) Given points are $\left(-\frac{1}{2}, 3\right)$, (– 5, 6) and (– 8, 8).

Let us use area of triangle formula

Area of triangle= $\frac{1}{2}$ $[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}\left[-\frac{1}{2}(6-8) - 5(8-3) - 8(3-6)\right]$

$= \frac{1}{2}\left[-\frac{1}{2}(-2) - 5(5) - 8(-3)\right]$

$= \frac{1}{2}[1 - 25 + 24] = \frac{1}{2}(0) = 0$

Since, the area of triangle is zero, the given points are collinear.

(ii) Given points are (a, b + c), (b, c + a) and (c, a + b)

Area of triangle= $\frac{1}{2}$ $[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}[a(c + a - a - b) + b(a + b - b - c) + c(b + c - c - a)]$

$= \frac{1}{2}[a(c - b) + b(a - c) + c(b - a)]$

$= \frac{1}{2}[ac - ab + ab - bc + bc - ac]$

$= \frac{1}{2}(0) = 0$

Since, the area of triangle is zero, the given points are collinear.

3. Vertices of given triangles are taken in order and their areas are provided below. In each of the following find the value of 'p'.

Sl. No.	Vertices	Area (sq. units)
(i)	(0, 0), (p, 8), (6, 2)	20
(ii)	(p, p), (5, 6), (5, – 2)	32

Sol :

(i) Given vertices are (0, 0), (p, 8) and (6, 2)

Area of triangle= 20 sq. units.

Area of triangle= $\frac{1}{2}$ $[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$\frac{1}{2}[0(8-2) + p(2-0) + 6(0-8)] = 20$

$2p - 48 = 40$

$2p = 40 + 48$

$2p = 88$

$p = \frac{88}{2} = 44$

(ii) Given vertices are (p, p), (5, 6) and (5, – 2)

Area of triangle= –32 sq. units

Area of triangle= $\frac{1}{2}$ $[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$\frac{1}{2}[p(6+2) + 5(-2-p) + 5(p-6)] = 32$

$8p - 10 - 5p + 5p - 30 = 64$

$8p - 40 = 64$

$8p = 64 + 40 = 104$

$p = \frac{104}{8} = 13$

4. In each of the following, find the value of 'a' for which the given points are collinear.

(i) (2, 3), (4, a) and (6, – 3)

(ii) (a, 2 – 2a), (– a + 1, 2a) and (– 4 – a, 6 – 2 a)

Sol :

(i) Given points are (2, 3), (4, a) and (6, – 3)

Since the points are colinear, Area of triangle is zero.

i.e., $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$

$\Rightarrow 2(a + 3) + 4(-3 - 3) + 6(3 - a) = 0$

$2a + 6 - 24 + 18 - 6a = 0$

$-4a + 0 = 0$

$-4a = 0$

$a = 0$

(ii) Given points are (a, 2 – 2a), (– a + 1, 2a) and (– 4 – a, 6 – 2a)

Since the points are collinear, Area of triangle is zero.

i.e., $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$

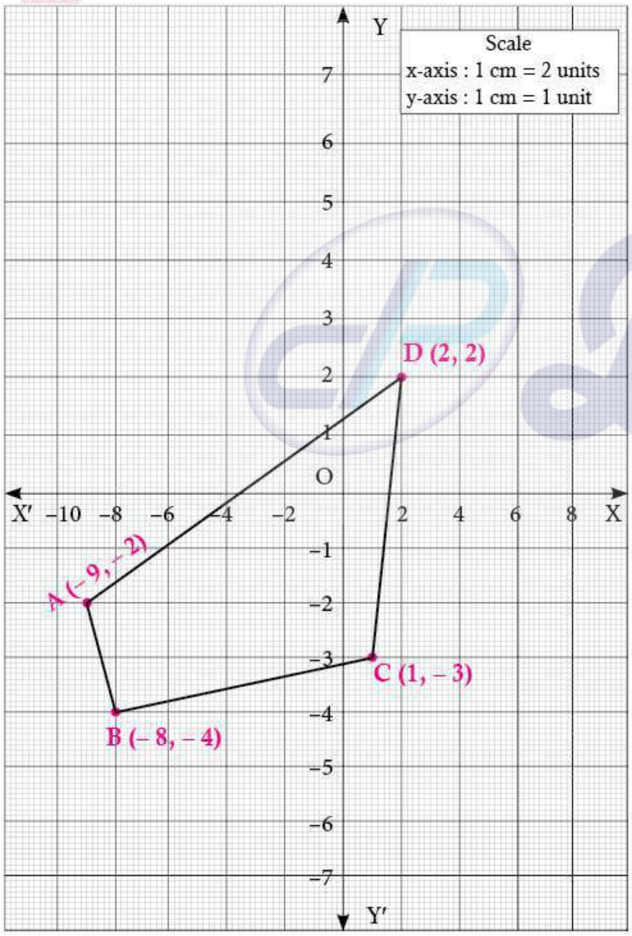
$a(2a - 6 + 2a) + (-a + 1)(6 - 2a - 2 + 2a) + (-4 - a)(2 - 2a - 2a) = 0$

$a(4a - 6) + (-a + 1)(4) + (-4 - a)(2 - 4a) = 0$

$$4a^2 - 6a - 4a + 4 - 8 + 16a - 2a + 4a^2 = 0$$
$$8a^2 + 4a - 4 = 0$$
(Dividing by 4)
$$2a^2 + a - 1 = 0$$
$$(2a - 1)(a + 1) = 0$$
$$2a - 1 = 0,\ a + 1 = 0 \Rightarrow a = \frac{1}{2}, -1.$$

5. Find the area of the quadrilateral whose vertices are at
(i) (– 9, – 2), (– 8, – 4), (2, 2) and (1, – 3)
(ii) (– 9, 0), (– 8, 6), (– 1, – 2) and (– 6, – 3)

Sol :



(i) Given vertices are A(– 9, – 2), B(– 8, – 4), C(1, –3) and D(2, 2)

Area of the Quadrilateral ABCD

$= \frac{1}{2}\ [(x_1 y_2 + x_2 y_3 + x_3 y_4 + x_4 y_1) - (x_2 y_1 + x_3 y_2 + x_4 y_3 + x_1 y_4)]$ sq. units

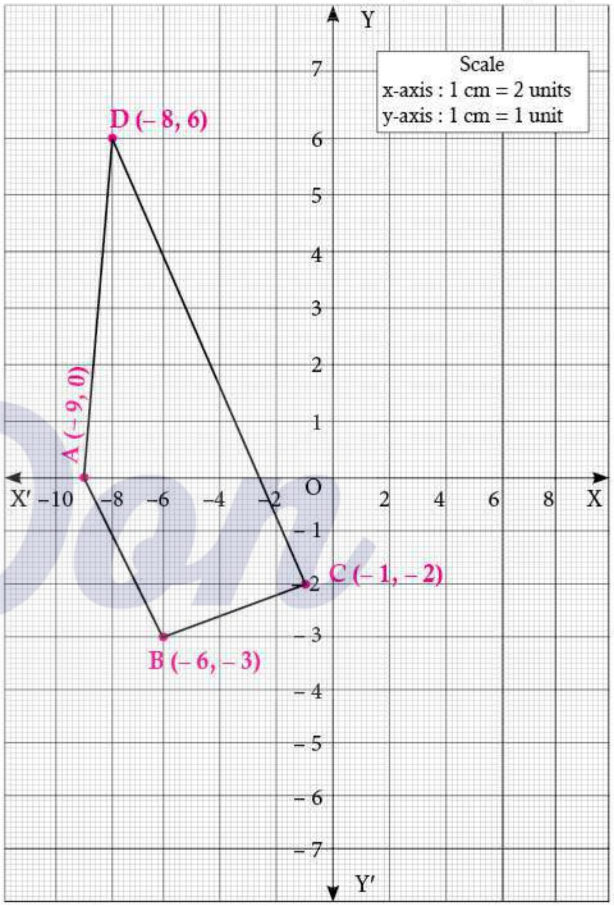
$= \frac{1}{2}\ [(-9)(-4) + (-8)(-3) + (1)(2) + (2)(-2)] - [(-8)(-2) + (1)(-4) + (2)(-3) + (-9)(2)]$

$= \frac{1}{2}\ [(36 + 24 + 2 - 4) - (16 - 4 - 6 - 18)]$

$= \frac{1}{2}\ [58 - (-12)] = \frac{1}{2}\ [70] = 35$ sq. units

$\therefore$ Area of quadrilateral = 35 sq. units.

(ii) Given vertices are (– 9, 0), (– 8, 6), (– 1, – 2) and (– 6, – 3)



Area of quadrilateral ABCD

$= \frac{1}{2}\ [(x_1 y_2 + x_2 y_3 + x_3 y_4 + x_4 y_1) - (x_2 y_1 + x_3 y_2 + x_4 y_3 + x_1 y_4)]$

(or) $\frac{1}{2} = [(x_1 - x_3)(y_2 - y_4) - (x_2 - x_4)(y_1 - y_3)]$ sq. units

$= \frac{1}{2}\ [(-9 + 1)(-3 - 6) - (-6 + 8)(0 + 2)]$

$= \frac{1}{2}\ [(-8)(-9) - (2)(2)]$

$= \frac{1}{2}\ [72 - 4] = \frac{68}{2} = 34$

$\therefore$ Area of quadrilateral = 34 sq. units.

6. Find the value of k, if the area of a quadrilateral is 28 sq. units, whose vertices are (– 4, – 2), (– 3, k), (3, – 2) and (2, 3)

Sol : Given vertices are (– 4, – 2), (– 3, k), (3, – 2) and (2, 3) and area of quadrilateral is 28 sq. units.

$$\text{Area of quadrilateral} = \frac{1}{2}\,[(x_1 - x_3)(y_2 - y_4) - (x_2 - x_4)(y_1 - y_3)]$$

$$\frac{1}{2}\,[(-4-3)(k-3) - (-3-2)(-2+2)] = 28$$

$$(-7)(k-3) - (-5)(0) = 56$$

$$-7k + 21 = 56$$

$$-7k = 56 - 21 = 35$$

$$k = \frac{35}{-7} = -5$$

$$k = -5$$

7. If the points A (– 3, 9), B (a, b) and C (4, – 5) are collinear and if a + b = 1, then find a and b.

Sol :

Given points A (– 3, 9), B(a, b) and C (4, – 5)

Since the points are collinear, Area of triangle ABC = 0

$$\therefore \frac{1}{2}\,[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$$

$$\Rightarrow \quad -3(b+5) + a(-5-9) + 4(9-b) = 0$$

$$-3b - 15 - 14a + 36 - 4b = 0$$

$$(\div -7) \quad -14a - 7b + 21 = 0$$

$$2a + b = 3 \quad \ldots (1)$$

$$\text{Given that } a + b = 1 \quad \ldots (2)$$

Solving (1) and (2)

$$2a + b = 3 \quad \ldots (1)$$

$$a + b = 1 \quad \ldots (2)$$

$$(1) - (2) \Rightarrow \quad a = 2$$

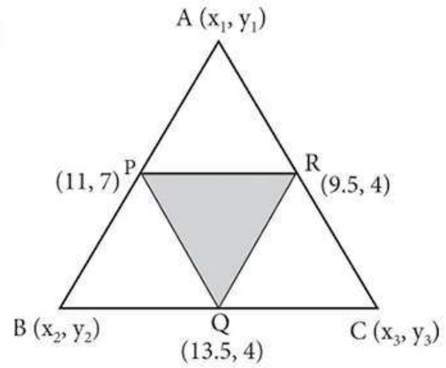
Substituting in (2)

$$2 + b = 1 \Rightarrow b = 1 - 2 = -1$$

$\therefore$ a = 2, b = – 1.

8. Let P (11, 7), Q (13.5, 4) and R (9.5, 4) be the mid-points of the sides AB, BC and AC respectively of ΔABC. Find the coordinates of the vertices A, B and C. Hence find the area of ΔABC and compare this with area of ΔPQR.

Sol :



Given P, Q, R are the mid points of the sides of a triangle.

Let the vertices of triangle ABC be A (x_1, y_1), B (x_2, y_2) and C (x_3, y_3)

P is the mid point of AB

$$\Rightarrow \quad \left[\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right] = (11, 7)$$

Comparing the co-ordinates, we get

$$x_1 + x_2 = 22 \text{ and} \quad \ldots (1)$$

$$y_1 + y_2 = 14 \quad \ldots (2)$$

Q is the mid point of BC

$$\Rightarrow \quad \left[\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2}\right] = (13.5, 4)$$

$$x_2 + x_3 = 27 \quad \ldots (3)$$

$$y_2 + y_3 = 8 \quad \ldots (4)$$

R is the mid point of AC

$$\Rightarrow \quad \left[\frac{x_1 + x_3}{2}, \frac{y_1 + y_3}{2}\right] = (9.5, 4)$$

$$x_1 + x_3 = 19 \quad \ldots (5)$$

$$y_1 + y_3 = 8 \quad \ldots (6)$$

Solving (1), (3) and (5), we get

$$x_1 = 7, x_2 = 15 \text{ and } x_3 = 12.$$

Solving (2), (4) and (6)

we get $y_1 = 7, y_2 = 7, y_3 = 1$

$\therefore$ The vertices are A (7, 7), B (15, 7) and C (12, 1)

$$\text{Area of } \Delta ABC = \frac{1}{2}\,[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2}\,[7(7-1) + 15(1-7) + 12(7-7)]$$

$$= \frac{1}{2}\,[42 - 90] = \frac{1}{2}(-48) = -24$$

$$= 24 \text{ sq. units.} \quad [\because \text{ Area cannot be negative}]$$

Area of $\Delta\ PQR$

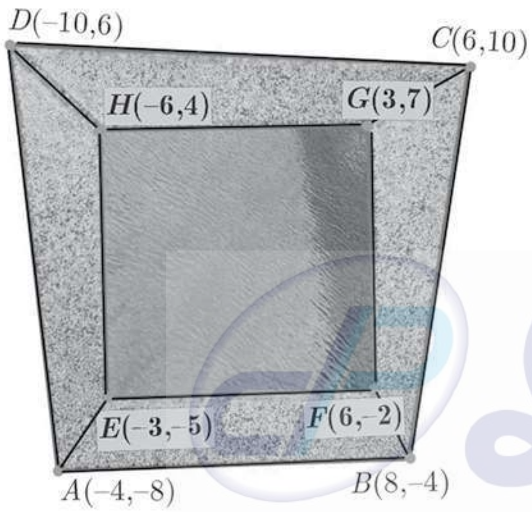
$$= \frac{1}{2}\ [11\ (4-4) + 13.5\ (4-7) + 9.5\ (7-4)]$$

$$= \frac{1}{2}\ [0 - 40.5 + 28.5] = \frac{-12}{2} = -6$$

Area $= 6$ sq. units [$\because$ area cannot be negative]

$\therefore$ Area of $\Delta ABC = 4$ (Area of $\Delta\ PQR$).

9. In the figure, the quadrilateral swimming pool shown is surrounded by concrete patio. Find the area of the patio.



Sol : Area of patio = Area of Quadrilateral ABCD − Area of Quadrilateral EFGH

Area of Quadrilateral ABCD

A (− 4, − 8), B (8, − 4), C (6, 10) and D (− 10, 6)

$$\text{Area} = \frac{1}{2}\ [(x_1 - x_3)\ (y_2 - y_4) - (x_2 - x_4)\ (y_1 - y_3)]$$

$$= \frac{1}{2}\ [(-4-6)\ (-4-6) - (8+10)\ (-8-10)]$$

$$= \frac{1}{2}\ [100 + 324] = \frac{424}{2} = 212 \text{ sq. units}$$

Area of Quadrilateral EFGH

E (− 3, − 5), F (6, − 2), G (3, 7) and H (− 6, 4)

$$\text{Area} = \frac{1}{2}\ [(-3,-3)\ (-2-4) - (6+6)\ (-5-7)]$$

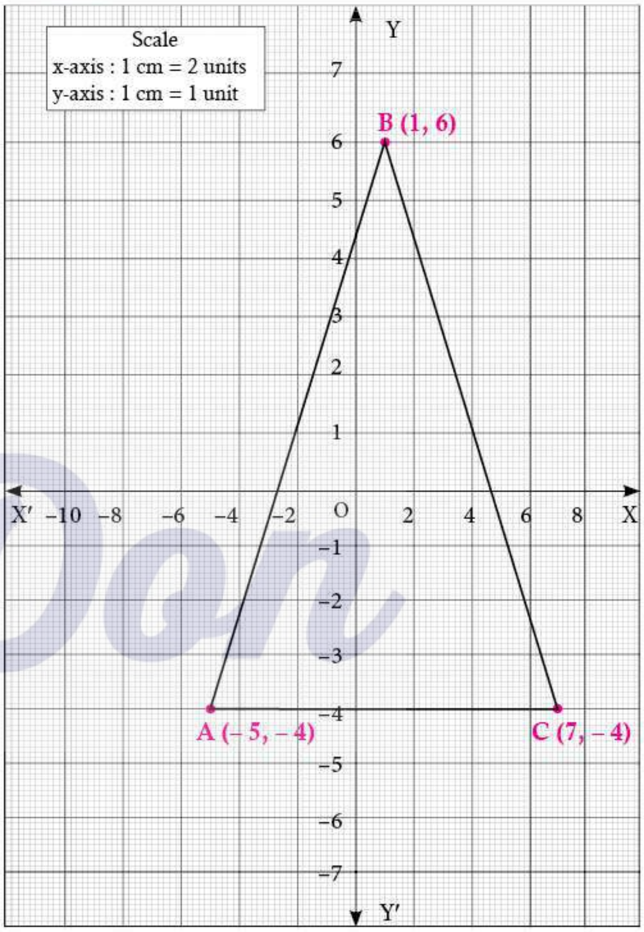
$$= \frac{1}{2}\ [36 + 144] = \frac{180}{2} = 90 \text{ sq. units}$$

$\therefore$ Area of patio = Area of Quadrilateral ABCD − Area of Quadrilateral EFGH

$= 212 - 90 = 122$ sq. units

10. A triangular shaped glass with vertices at A(− 5, − 4), B(1, 6) and C(7, − 4) has to be painted. If one bucket of paint covers 6 square feet, how many buckets of paint will be required to paint the whole glass, if only one coat of paint is applied.

Sol :



Given vertices are A (− 5, − 4), B (1, 6) and C (7, − 4)

$$\text{Area of triangle} = \frac{1}{2}\ [x_1\ (y_2 - y_3) + x_2\ (y_3 - y_1) + x_3\ (y_1 - y_2)] \text{ sq. units}$$

$$\text{Area of triangle ABC} = \frac{1}{2}\ [-5\ (6+4) + 1\ (-4+4) + 7\ (-4-6)]$$

$$= \frac{1}{2}\ [-50 + 0 - 70] = \frac{-120}{2} = -60$$

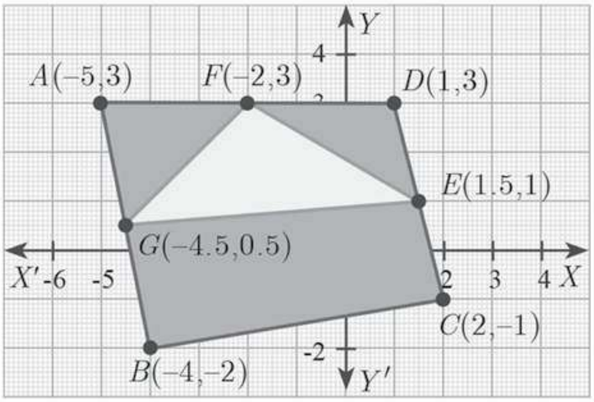
[$\because$ Area cannot be negative].

$\therefore$ Area $= 60$ sq. units.

Given that one bucket of paint can be applied for 6 sq. ft

$\therefore$ No . of buckets $= \frac{60}{6} = 10.$

11. In the figure, find the area of (i) triangle AGF (ii) triangle FED (iii) quadrilateral BCEG.



Sol :

(i) Area of triangle AGF

Vertices A (– 5, 3), G (– 4.5, 0.5) and F (– 2, 3).

Area of triangle = $\frac{1}{2}$ [x_1 (y_2 – y_3) + x_2 (y_3 – y_1) + x_3 (y_1 – y_2)] sq. units

$= \frac{1}{2}$ [– 5 (0.5 – 3) – 4.5 (3 – 3) – 2 (3 – 0.5)]

$= \frac{1}{2}$ [12.5 – 5] = $\frac{7.5}{2}$ = 3.75 sq. units

(ii) Area of triangle FED

Vertices are F (– 2, 3), E (1.5, 1) and D (1, 3)

Area of triangle = $\frac{1}{2}$ [x_1 (y_2 – y_3) + x_2 (y_3 – y_1) + x_3 (y_1 – y_2)] sq. units

$= \frac{1}{2}$ [– 2 (1 – 3) + 1.5 (3 – 3) + 1 (3 – 1)]

$= \frac{1}{2}$ [4 + 2] = $\frac{6}{2}$ = 3 sq. units

(iii) Area of quadrilateral BCEG

Vertices are B (– 4, – 2), C (2, – 1), E (1.5, 1) and G (– 4.5, 0.5)

Area of quadrilateral = $\frac{1}{2}$ [(x_1 – x_3) (y_2 – y_4) – (x_2 – x_4) (y_1 – y_3)] sq. units

$= \frac{1}{2}$ [(– 4 – 1.5) (– 1 – 0.5) – (2 + 4.5) (– 2 – 1)]

$= \frac{1}{2}$ [(– 5.5) (– 1.5) – (6.5) (– 3)]

$= \frac{1}{2}$ [8.25 + 19.5] = $\frac{1}{2}$ [27.75]

= 13.875 ≈ 13.88 sq. units

Inclination of a Line

Key Points

The angle of inclination of a line is the angle which a straight line makes with the positive direction of X-axis and it is usually denoted by θ.

- The angle of inclination of any line parallel to X-axis is 0°.
- The angle of inclination of any line perpendicular to X-axis (or) parallel to Y-axis is 90°.
- Slope: If 'θ' is the angle of inclination, then '$\tan\theta$' is called slope of a straight line and it is usually denoted by 'm'.
- Slope m = $\tan\theta$, $0^o \le \theta \le 180^o$, $\theta \ne 90^o$
- Slope = $\dfrac{\textit{change in ordinates}}{\textit{change in abscissae}} = \dfrac{y_2 - y_1}{x_2 - x_1}$
- The slope of a vertical line is undefined as tan 90° = undefined.
- The slope of the line through the points (x_1, y_1) and (x_2, y_2) is $\dfrac{y_2 - y_1}{x_2 - x_1}$ (or) $\dfrac{y_1 - y_2}{x_1 - x_2}$
- If two lines are parallel, then their slopes are equal. i.e., $m_1 = m_2$
- If two lines are perpendicular, then the product of their slopes is – 1 i.e., $m_1 \times m_2 = -1$.
- Slope is also called Gradient of the line.

Worked Examples

5.8 **(i) What is the slope of a line whose inclination is 30°?**

(ii) What is the inclination of a line whose slope is $\sqrt{3}$?

Sol :

(i) Here $\theta = 30^o$

Slope m = $\tan\theta$

Therefore, slope = m = $\tan 30^o = \dfrac{1}{\sqrt{3}}$

(ii) m = $\sqrt{3}$, let θ be the inclination of the line

$\tan\theta = \sqrt{3} \quad \Rightarrow \theta = 60^o$

5.9 **Find the slope of a line joining the given points**

(i) (– 6, 1) and (– 3, 2)

(ii) $\left(-\frac{1}{3}, \frac{1}{2}\right)$ *and* $\left(\frac{2}{7}, \frac{3}{7}\right)$

(iii) (14, 10) and (14, – 6)

Sol :

(i) (– 6, 1) and (– 3, 2)

The slope $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{2-1}{-3+6} = \dfrac{1}{3}$

(ii) $\left(-\frac{1}{3}, \frac{1}{2}\right)$ *and* $\left(\frac{2}{7}, \frac{3}{7}\right)$

The slope $= \dfrac{\frac{3}{7} - \frac{1}{2}}{\frac{2}{7} + \frac{1}{3}} = \dfrac{\frac{6-7}{14}}{\frac{6+7}{21}}$

$= -\dfrac{1}{14} \times \dfrac{21}{13} = -\dfrac{3}{26}$.

(iii) (14, 10) and (14, – 6)

The slope $= \dfrac{-6-10}{14-14} = \dfrac{-16}{0}$

which is undefined.

The slope is undefined.

5.10 **The line r passes through the points (– 2, 2) and (5, 8) and the line s passes through the points (– 8, 7) and (– 2, 0). Is the line r perpendicular to s?**

Sol :

The slope of line r is $m_1 = \dfrac{8-2}{5+2} = \dfrac{6}{7}$

The slope of line s is $m_2 = \dfrac{0-7}{-2+8} = \dfrac{-7}{6}$

The product of slopes $= \dfrac{6}{7} \times \dfrac{-7}{6} = -1$

That is, $m_1 m_2 = -1$

Therefore, the line r is perpendicular to line s.

5.11 **The line P passes through the points (3, – 2), (12, 4) and the line Q passes through the points (6, – 2) and (12, 2). Is P parallel to Q?**

Sol :

The slope of line P is $m_1 = \dfrac{4+2}{12-3} = \dfrac{6}{9} = \dfrac{2}{3}$

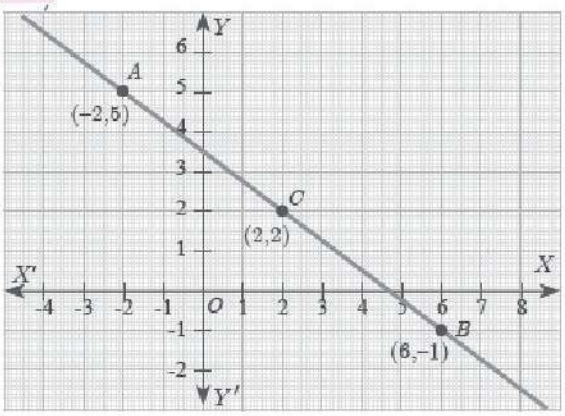
The slope of line Q is $m_2 = \dfrac{2+2}{12-6} = \dfrac{4}{6} = \dfrac{2}{3}$

Thus slope of line P = slope of line Q

Therefore, the line P is parallel to the line Q.

5.12 **Show that the points (– 2, 5), (6, – 1) and (2, 2) are collinear.**

Sol :



The vertices are A(– 2, 5), B(6, – 1) and C(2, 2).

Slope of AB $= \dfrac{-1-5}{6+2}$

$= \dfrac{-6}{8} = \dfrac{-3}{4}$

Slope of BC $= \dfrac{2+1}{2-6}$

$= \dfrac{3}{-4} = \dfrac{-3}{4}$

$\Rightarrow$ Slope of AB = Slope of BC

Therefore, the points A, B, C all lie in a same straight line.

Hence the points A, B, and C are collinear.

5.13 **Let A(1, – 2), B(6, – 2), C(5, 1) and D(2, 1) be four points**

(i) Find the slope of the line segment (a) AB (b) CD

(ii) Find the slope of the line segment (a) BC (b) AD

(iii) What can you deduce from your answer.

Sol :

(i) **(a)** Slope of AB $= \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{-2+2}{6-1} = 0$

(b) Slope of CD $= \dfrac{1-1}{2-5}$

$= \dfrac{0}{-3} = 0$

(ii) **(a)** Slope of BC $= \dfrac{1+2}{5-6}$

$= \dfrac{3}{-1} = -3$

(b) Slope of AD $= \dfrac{1+2}{2-1}$

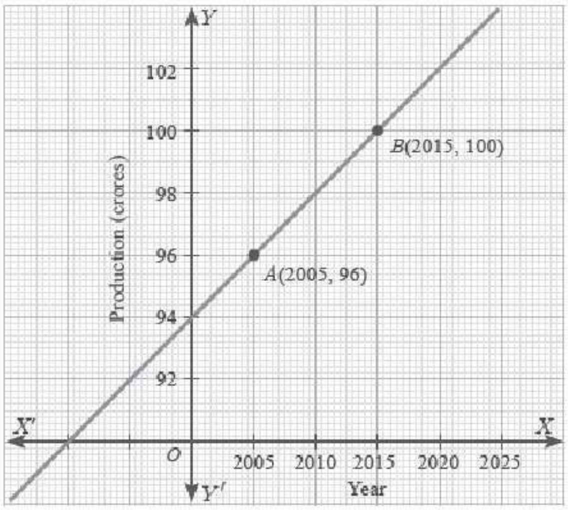
$= \dfrac{3}{1} = 3$

(iii) The slope of AB and CD are equal so they are parallel.

Similarly the lines AD and BC are not parallel, since their slopes are not equal. So we can deduce that the quadrilateral ABCD is a trapezium.

5.14 **Consider the given graph representing (in crores) growth of population graph. Find the slope of the line AB and hence estimate the population in the year 2030.**

Sol :



The points A(2005, 96) and B(2015, 100) are on the line AB.

$$\text{Slope of AB} = \frac{100-96}{2015-2005} = \frac{4}{10} = \frac{2}{5}$$

Let the growth of population in 2030 be k crores. Assuming that the point C(2030, k) is on AB, we have

$$\text{slope of AC} = \text{slope of AB}$$

$$\Rightarrow \quad \frac{k-96}{2030-2005} = \frac{2}{5}$$

$$\Rightarrow \quad \frac{k-96}{25} = \frac{2}{5}$$

$$\Rightarrow \quad k-96 = 10$$

$$\Rightarrow \quad k = 106$$

Hence the estimated population in 2030 = 106 Crores.

5.15 Without using Pythagoras theorem, show that the vertices (1, – 4), (2, – 3) and (4, – 7) form a right angled triangle.

Sol :

The vertices are A(1, – 4), B(2, – 3) and C(4, – 7).

$$\text{The slope of AB} = \frac{-3+4}{2-1} = \frac{1}{1} = 1$$

$$\text{The slope of BC} = \frac{-7+3}{4-2} = \frac{-4}{2} = -2$$

$$\text{The slope of AC} = \frac{-7+4}{4-1} = \frac{-3}{3} = -1$$

Slope of AB × Slope of AC = (1) (–1) = – 1

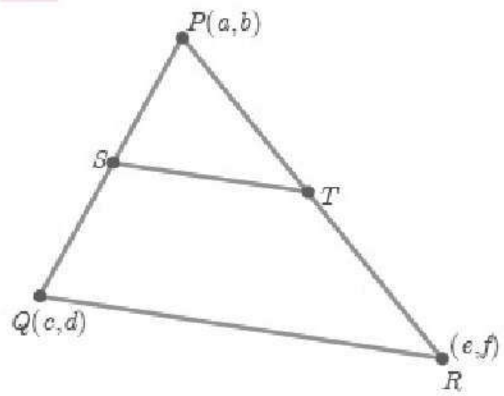
That is, $m_1 \times m_2 = -1$

Therefore, AB is perpendicular to AC. $\angle A = 90^\circ$

Therefore, ΔABC is a right angled triangle.

5.16 Prove analytically that the line segment joining the mid-points of two sides of a triangle is parallel to the third side and is equal to half of its length.

Sol :



Let P(a, b), Q(c, d) and R(e, f) be the vertices of a triangle.

Let S be the mid-point of PQ and T be the mid-point of PR

Therefore,

$$S = \left(\frac{a+c}{2}, \frac{b+d}{2}\right) \text{ and } T = \left(\frac{a+e}{2}, \frac{b+f}{2}\right)$$

$$\text{Now, Slope of ST} = \frac{\frac{b+f}{2} - \frac{b+d}{2}}{\frac{a+e}{2} - \frac{a+c}{2}} = \frac{f-d}{e-c}$$

$$\text{And slope of QR} = \frac{f-d}{e-c}$$

Therefore, ST is parallel to QR. (since, their slopes are equal)

Also,

$$ST = \sqrt{\left(\frac{a+e}{2} - \frac{a+c}{2}\right)^2 + \left(\frac{b+f}{2} - \frac{b+d}{2}\right)^2}$$

$$= \frac{1}{2}\sqrt{(e-c)^2 + (f-d)^2}$$

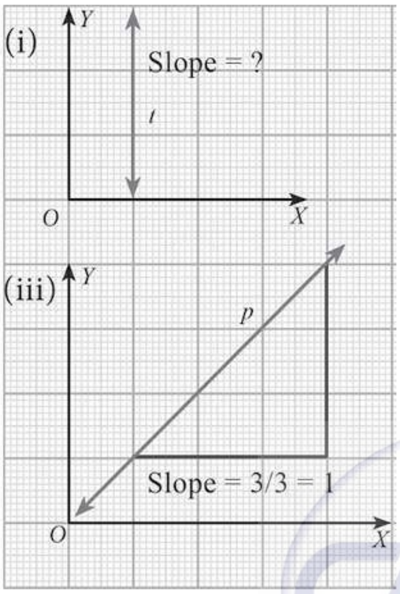
$$ST = \frac{1}{2} QR$$

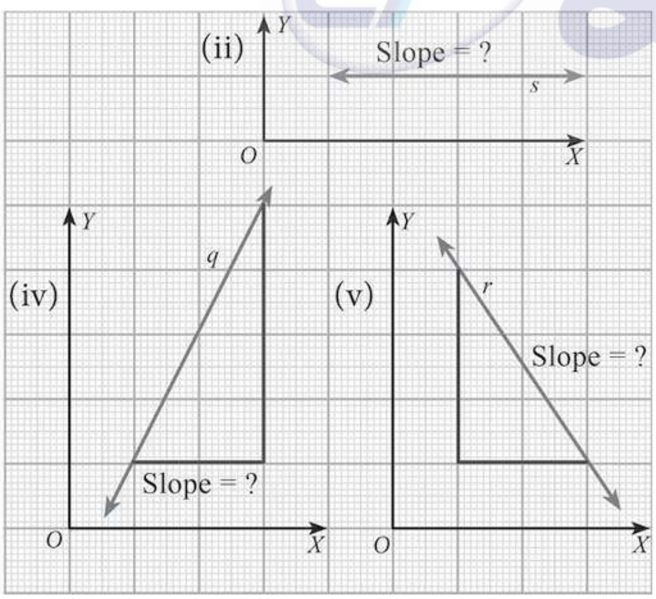
Thus ST is parallel to QR and half of it.

Progress Check

1. Write down the slope of each of the lines shown on the grid below.

Write down the slope of the lines shown on the grid below.





Sol : (iii) Slope of the line

$$P = \frac{\text{change in } y \text{ coordinate}}{\text{change in } x \text{ coordinate}} = \frac{3}{3} = 1.$$

Ans : Slope of T is undefined.

Slope of S is 0.

2. Fill the missing boxes.

Sl. No.	Points	Slope
1	A (– a, b), B (3a, – b)	...
2	A (2, 3), B (–, –)	2
3	...	0
4	...	undefined

Ans :

1. Points A (–a, b), B (3a, – b)

$$\text{Slope} = \frac{y_1 - y_2}{x_1 - x_2} = \frac{b+b}{-a-3a} = \frac{2b}{-4a} = -\frac{b}{2a}$$

2. A (2, 3), B (___, ___), slope = 2

$$\text{Slope} = \frac{y_1 - y_2}{x_1 - x_2} = 2$$

$$\frac{3 - y_2}{2 - x_2} = 2$$

Any values satisfying this, can be the second point. For example, $x_2 = -2$, $y_2 = -5$

3. Any line parallel to X-axis, is having slope '0'.
4. Any line perpendicular to X-axis, is having slope 'undefined'.

Thinking Corner

1. The straight lines x-axis and y-axis are perpendicular to each other. Is the condition $m_1m_2 = -1$ true?

Ans : $m_1\, m_2 = -1$ is true only when well-defined slopes are given.

X-axis with slope '0' and y-axis with slope 'undefined'.

2. Provide three examples of using the concept of slope in real-life situations.

Ans : Real life situation of concept of slope.

(i) While building the roads, need to consider the slope.

(ii) Wheel-chair ramp in Hospitals.

(iii) While constructing Bridges.

Exercise 5. 2

1. What is the slope of a line whose inclination with positive direction of x-axis is

(i) 90° **(ii) 0°**

Sol :

(i) Given angle of inclination $\theta = 90^\circ$

$\therefore$ Slope of a line 'm' $= \tan\theta$

$= \tan 90^\circ = \infty$ (undefined)

(ii) Angle of inclination $\theta = 0^\circ$

Slope of a line 'm' $= \tan\theta = \tan 0^\circ = 0$

2. What is the inclination of a line whose slope is

(i) 0 **(ii) 1**

Sol :

(i) Given slope 'm' = 0

$\Rightarrow \tan\theta = 0 = \tan 0^\circ$

$\therefore \theta = 0^\circ$

(ii) Slope 'm' = 1

$\tan\theta = 1 = \tan 45^\circ$

$\therefore \theta = 45^\circ$

3. Find the slope of a line joining the points

(i) $(5, \sqrt{5})$ **with the origin**

(ii) $(\sin\theta, -\cos\theta)$ **and** $(-\sin\theta, \cos\theta)$

Sol :

(i) Given points $(5, \sqrt{5})$ and (0, 0)

Slope of a line $= \dfrac{y_1 - y_2}{x_1 - x_2}$ (or) $\dfrac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{\sqrt{5} - 0}{5 - 0} = \frac{\sqrt{5}}{5} = \frac{1}{\sqrt{5}}$$

(ii) Given points $(\sin\theta, -\cos\theta)$ and $(-\sin\theta, \cos\theta)$

Slope of a line $= \dfrac{y_1 - y_2}{x_1 - x_2}$

$$= \frac{-\cos\theta - \cos\theta}{\sin\theta + \sin\theta} = -\frac{2\cos\theta}{2\sin\theta} = -\cot\theta$$

4. What is the slope of a line perpendicular to the line joining A (5, 1) and P where P is the mid-point of the segment joining (4, 2) and (– 6, 4).

Sol : Mid point of line segment joining (4, 2) and (– 6, 4)

$$\text{Mid point} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$= P\left(\frac{4-6}{2}, \frac{2+4}{2}\right)$$

$$= P\left(-\frac{2}{2}, \frac{6}{2}\right) = P(-1, 3)$$

Now, slope of a line joining A (5, 1) and P (– 1, 3)

$$m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{1-3}{5+1} = \frac{-2}{6} = -\frac{1}{3}$$

$\therefore$ Slope of a perpendicular to the line joining A and P

$$= -\frac{1}{m} = -\frac{1}{\left(-\frac{1}{3}\right)} = 3$$

5. Show that the given points are collinear: (– 3, – 4), (7, 2) and (12, 5)

Sol :

Given points (– 3, – 4), (7, 2) and (12, 5)

Let the points be A (– 3, – 4), B (7, 2) and C (12, 5)

Slope of a line $= \dfrac{y_1 - y_2}{x_1 - x_2}$

Slope of AB $= \dfrac{-4-2}{-3-7} = \dfrac{-6}{-10} = \dfrac{3}{5}$

Slope of BC $= \dfrac{2-5}{7-12} = \dfrac{-3}{-5} = \dfrac{3}{5}$

Slope of AB = Slope of BC

$\therefore$ The points A, B and C are collinear.

6. If the three points (3, – 1), (a, 3) and (1, – 3) are collinear, find the value of a.

Sol :

Given points (3, – 1), (a, 3) and (1, – 3)

Let the points be A (3, – 1), B (a, 3) and C (1, – 3)

Slope of AB $= \dfrac{y_1 - y_2}{x_1 - x_2} = \dfrac{-1-3}{3-a} = \dfrac{-4}{3-a}$

Slope of BC $= \dfrac{y_1 - y_2}{x_1 - x_2} = \dfrac{3+3}{a-1} = \dfrac{6}{a-1}$

Since, the points A, B and C are collinear.

Slope of AB = Slope of BC

$$\frac{-4}{3-a} = \frac{6}{a-1}$$

$$-2(a-1) = 3(3-a)$$
$$-2a+2 = 9-3a$$
$$3a-2a = 9-2$$
$$a = 7.$$

7. The line through the points (– 2, a) and (9, 3) has slope $-\frac{1}{2}$. Find the value of a.

Sol : Given points (– 2, a) and (9, 3) and

Slope = $-\frac{1}{2}$

$$\text{Slope of the line} = -\frac{1}{2}$$
$$\frac{y_1-y_2}{x_1-x_2} = -\frac{1}{2}$$
$$\frac{a-3}{-2-9} = -\frac{1}{2}$$
$$\frac{a-3}{-11} = -\frac{1}{2}$$
$$2(a-3) = 11$$
$$2a = 11+6$$
$$a = \frac{17}{2}$$

8. The line through the points (– 2, 6) and (4, 8) is perpendicular to the line through the points (8, 12) and (x, 24). Find the value of x.

Sol :

$$\text{Slope of the line} = \frac{y_1-y_2}{x_1-x_2}$$

Slope of the line joining the points (– 2, 6) and (4, 8)

$$m_1 = \frac{6-8}{-2-4} = \frac{-2}{-6} = \frac{1}{3}$$

Slope of the line joining the points (8, 12) and (x, 24)

$$m_2 = \frac{12-24}{8-x} = -\frac{12}{8-x}$$

Given that the lines are perpendicular

$\therefore\ m_1 \times m_2 = -1.$

$$\frac{1}{3} \times \frac{-12}{8-x} = -1$$
$$4 = 8-x$$
$$x = 8-4$$
$$x = 4.$$

9. Show that the given vertices form a right angled triangle and check whether its satisfies Pythagoras theorem

(i) A (1, – 4), B (2, – 3) and C (4, – 7)

(ii) L (0, 5), M (9, 12) and N (3, 14)

Sol :

(i) Given vertices A (1, – 4), B (2, – 3) and C (4, – 7)

$$\text{Slope of the line} = \frac{y_1-y_2}{x_1-x_2}$$
$$\text{Slope of AB} = \frac{-4+3}{1-2} = \frac{-1}{-1} = 1$$
$$\text{Slope of BC} = \frac{-3+7}{2-4} = \frac{4}{-2} = -2$$
$$\text{Slope of AC} = \frac{-4+7}{1-4} = \frac{3}{-3} = -1$$

(Slope of AB) × (Slope of AC) = – 1

$\therefore$ AB is perpendicular to AC.

Hence, the given vertices form a right angled triangle.

Distance between the points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$ units.

$$AB = \sqrt{(1-2)^2+(-4+3)^2} = \sqrt{1+1} = \sqrt{2}$$
$$AB^2 = (\sqrt{2})^2 = 2$$
$$BC = \sqrt{(2-4)^2+(-3+7)^2} = \sqrt{4+16} = \sqrt{20}$$
$$BC^2 = 20$$
$$AC = \sqrt{(1-4)^2+(-4+7)^2} = \sqrt{9+9} = \sqrt{18}$$
$$AC^2 = 18$$

Now, $AB^2 + AC^2 = BC^2$

Hence, the Pythagoras theorem is satisfied.

(ii) L (0, 5), M (9, 12) and N (3, 14)

$$\text{Slope of a line} = \frac{y_1-y_2}{x_1-x_2}$$
$$\text{Slope of LM} = \frac{5-12}{0-9} = \frac{-7}{-9} = \frac{7}{9}$$
$$\text{Slope of MN} = \frac{12-14}{9-3} = \frac{-2}{6} = \frac{-1}{3}$$
$$\text{Slope of LN} = \frac{5-14}{0-3} = \frac{-9}{-3} = 3$$

$$\text{(Slope of MN)} \times \text{(Slope of LN)} = \left(-\frac{1}{3}\right) \times 3 = -1.$$

$\therefore$ MN is perpendicular to LN.

Hence, the given vertices form a right angled triangle.

Distance formula $= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$$LM = \sqrt{(0-9)^2 + (5-12)^2} = \sqrt{81+49} = \sqrt{130}$$

$$LM^2 = 130$$

$$MN = \sqrt{(9-3)^2 + (12-14)^2} = \sqrt{36+4} = \sqrt{40}$$

$$MN^2 = 40$$

$$LN = \sqrt{(0-3)^2 + (5-14)^2} = \sqrt{9+81} = \sqrt{90}$$

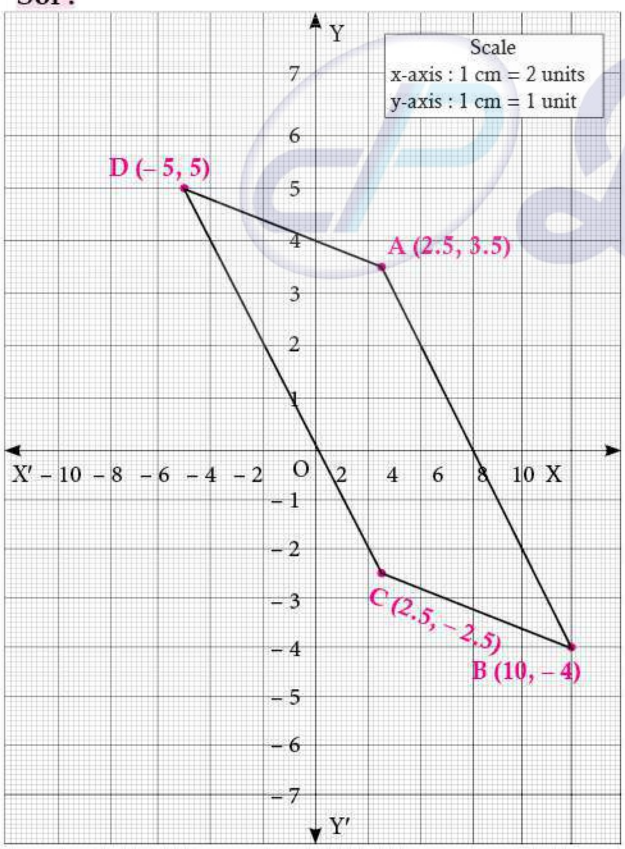
$$LN^2 = 90$$

$\therefore\ LN^2 + MN^2 = LM^2$

Hence, the Pythagoras theorem is satisfied.

10. Show that the given points form a parallelogram: A (2.5, 3.5), B (10, – 4), C (2.5, – 2.5) and D (– 5, 5)

Sol :



Given points A (2.5, 3.5), B (10, – 4), C (2.5, – 2.5) and D (– 5, 5)

$$\text{Slope of a line} = \frac{y_1 - y_2}{x_1 - x_2}$$

$$\text{Slope of AB} = \frac{3.5+4}{2.5-10} = \frac{7.5}{-7.5} = -1$$

$$\text{Slope of BC} = \frac{-4+2.5}{10-2.5} = \frac{-1.5}{7.5} = -\frac{1}{5}$$

$$\text{Slope of CD} = \frac{-2.5-5}{2.5+5} = -\frac{7.5}{7.5} = -1$$

$$\text{Slope of AD} = \frac{3.5-5}{2.5+5} = -\frac{1.5}{7.5} = -\frac{1}{5}$$

Slope of AB = Slope of CD = – 1

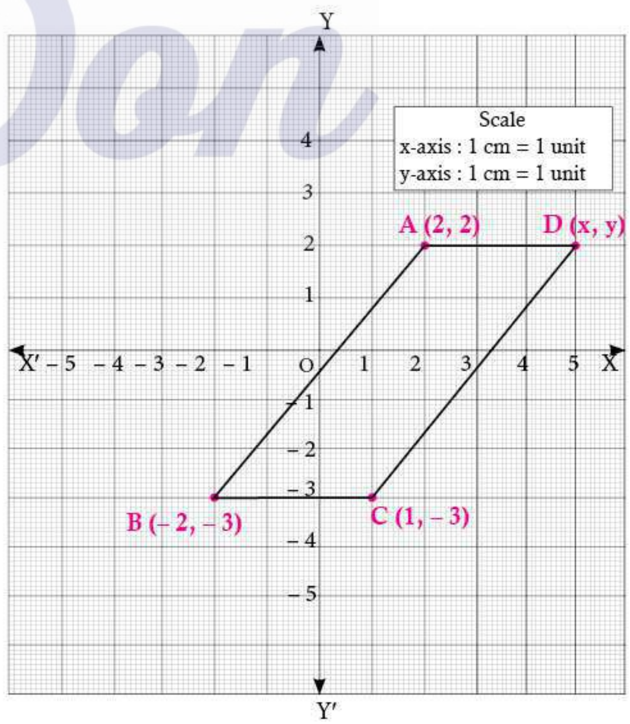
Slope of BC = Slope of AD $= -\frac{1}{5}$

$\therefore$ AB is parallel to CD and BC is parallel to AD.

Hence, the given points form a parallelogram.

11. If the points A (2, 2), B (– 2, – 3), C (1, – 3) and D (x, y) form a parallelogram then find the value of x and y.

Sol : Given points A (2, 2), B (– 2, – 3), C (1, – 3) and D (x, y)



$$\text{Slope of a line} = \frac{y_1 - y_2}{x_1 - x_2}$$

$$\text{Slope of AB} = \frac{2+3}{2+2} = \frac{5}{4}$$

$$\text{Slope of BC} = \frac{-3+3}{-2-1} = 0$$

$$\text{Slope of CD} = -\frac{-3-y}{1-x}$$

$$\text{Slope of AD} = \frac{2-y}{2-x}$$

Since, the points form a parallelogram

AB is parallel to CD and BC is parallel to AD

$\therefore$ Slope of AB = Slope of CD

$$\frac{5}{4} = \frac{-3-y}{1-x}$$

$$5(1-x) = 4(-3-y)$$

$$5-5x = -12-4y$$

$$\Rightarrow 5x-4y = 17 \qquad \ldots (1)$$

Slope of BC = Slope of AD

$$0 = \frac{2-y}{2-x}$$

$$\Rightarrow 2-y = 0$$

$$y = 2$$

Substituting in (1)

$$5x - 4(2) = 17$$

$$5x = 17+8 = 25$$

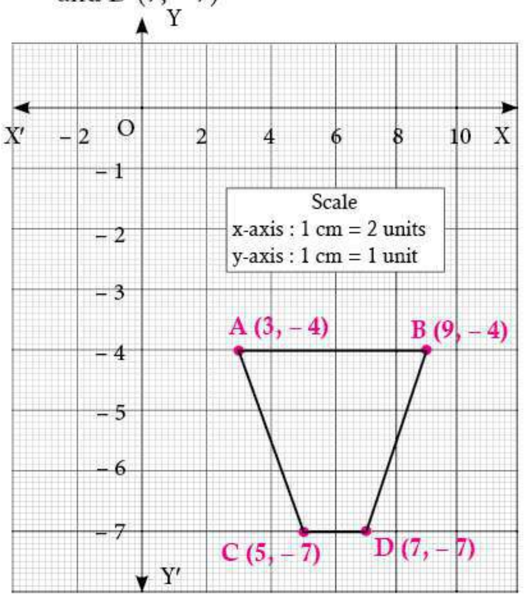
$$x = \frac{25}{5} = 5$$

$\therefore$ x = 5, y = 2.

12. Let A (3, – 4), B (9, – 4), C (5, – 7) and D (7, – 7). Show that ABCD is a trapezium.

Sol :

(i) Given points A (3, – 4), B (9, – 4), C (5, – 7) and D (7, – 7)



$$\text{Slope of a line} = \frac{y_1-y_2}{x_1-x_2}$$

$$\text{Slope of AB} = \frac{-4+4}{3-9} = 0$$

$$\text{Slope of BD} = \frac{-4+7}{9-7} = \frac{3}{2}$$

$$\text{Slope of CD} = \frac{-7+7}{5-7} = 0$$

$$\text{Slope of AC} = \frac{-4+7}{3-5} = \frac{3}{-2} = \frac{-3}{2}$$

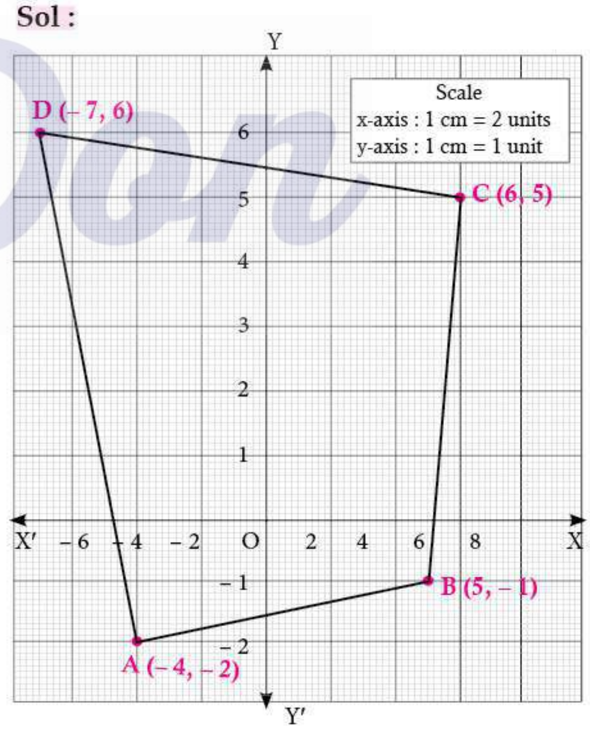
Slope of AB = Slope of CD

$\therefore$ AB is parallel to CD

Hence, the given points form a Trapezium.

13. A quadrilateral has vertices A(– 4, – 2), B(5, – 1), C(6, 5) and D(– 7, 6). Show that the mid-points of its sides form a parallelogram.

Sol :



Given, vertices of a quadrilateral are

A(– 4, – 2), B(5, – 1), C(6, 5) and D(– 7, 6).

Let P, Q, R and S be the mid points of the sides AB, BC, CD and AD respectively

Mid point of

$$\text{AB} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \text{P}\left(\frac{-4+5}{2}, \frac{-2-1}{2}\right)$$

$$= \text{P}\left(\frac{1}{2}, -\frac{3}{2}\right)$$

Mid point of BC

$$= Q\left(\frac{5+6}{2}, \frac{-1+5}{2}\right) = Q\left(\frac{11}{2}, 2\right)$$

Mid point of CD

$$= R\left(\frac{6-7}{2}, \frac{5+6}{2}\right) = R\left(-\frac{1}{2}, \frac{11}{2}\right)$$

Mid point of AD

$$= S\left(\frac{-4-7}{2}, \frac{-2+6}{2}\right) = S\left(-\frac{11}{2}, 2\right)$$

$$\text{Slope of PQ} = \frac{y_1-y_2}{x_1-x_2} = \frac{-\frac{3}{2}-2}{\frac{1}{2}-\frac{11}{2}} = \frac{-\frac{7}{2}}{-\frac{10}{2}} = \frac{7}{10}$$

$$\text{Slope of QR} = \frac{2-\frac{11}{2}}{\frac{11}{2}+\frac{1}{2}} = \frac{-\frac{7}{2}}{\frac{12}{2}} = -\frac{7}{12}$$

$$\text{Slope of RS} = \frac{\frac{11}{2}-2}{-\frac{1}{2}+\frac{11}{2}} = \frac{\frac{7}{2}}{\frac{10}{2}} = \frac{7}{10}$$

$$\text{Slope of PS} = \frac{-\frac{3}{2}-2}{\frac{1}{2}+\frac{11}{2}} = \frac{-\frac{7}{2}}{\frac{12}{2}} = -\frac{7}{12}$$

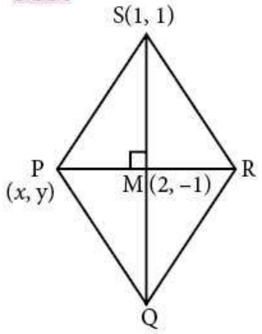
Slope of PQ = Slope of RS $\Rightarrow$ PQ || RS

Slope of QR = Slope of PS $\Rightarrow$ QR || PS

Hence, the mid points form a parallelogram.

14. PQRS is a rhombus. Its diagonals PR and QS intersect at the point M and satisfy QS = 2PR. If the co-ordinates of S and M are (1, 1) and (2, – 1) respectively, find the co-ordinates of P.

Sol :



PQRS is a rhombus

Slope of QS=Slope of SM

$$\text{Slope SM} = \frac{-1-1}{2-1} = -\frac{2}{1} = -2$$

Diagonals PR and QS intersect at right angle.

$\therefore$ Slope of PR = Slope of PM = $\frac{1}{2}$

Let P be (x, y)

$$\text{Then the slope of PM} = \frac{y_1-y_2}{x_1-x_2} = \frac{y+1}{x-2} = \frac{1}{2}$$

$$y+1 = \frac{x-2}{2}$$

$$y = \frac{x-2}{2} - 1 = \frac{x-4}{2}$$

$$\therefore \text{ P is } \left(x, \frac{x-4}{2}\right)$$

Given QS = 2 PR

$$\Rightarrow \frac{QS}{2} = PR$$

$$\Rightarrow SM = PR$$

$$\Rightarrow \frac{SM}{2} = PM \text{ (M is the midpoint of PR)}$$

$$\therefore \frac{SM}{2} = PM = \frac{\sqrt{(2-1)^2+(-1-1)^2}}{2}$$

$$\frac{SM}{2} = PM = \frac{\sqrt{5}}{2}$$

$$PM^2 = \frac{5}{4}$$

Now using distance formula,

$$PM^2 = (x-2)^2 + \left(\frac{x-4}{2}+1\right)^2 = \frac{5}{4}$$

$$x^2 - 4x + 4 + \frac{x^2-4x+4}{4} = \frac{5}{4}$$

$$x^2 - 4x + 4 = 1$$

$$x^2 - 4x + 3 = 0$$

$$(x-1)(x-3) = 0$$

$$x = 1, 3$$

When x = 1, $y = -\frac{3}{2}$

When x = 3, $y = -\frac{1}{2}$

$\therefore$ The co-ordinates of P can be $\left(1, -\frac{3}{2}\right), \left(3, -\frac{1}{2}\right)$

Straight Line

Key Points

- The linear equation is of the form ax + by + c = 0 is the equation of straight line where a, b and c are real numbers.
- Equation of X-axis is y = 0
- Equation of Y-axis is x = 0
- Equation of any line parallel to X-axis is y = b
- Equation of any line parallel to Y-axis is x = a
- Equation of straight line in slope-intercept form is y = mx + c where 'm' is slope and 'c' is y intercept.
- Equation of straight line in slope-point form is $y - y_1 = m(x - x_1)$ where 'm' is the slope and (x_1, y_1) is the point that the line passes through.
- Equation of straight line in two points form is $\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$ where (x_1, y_1) and (x_2, y_2) are the points.
- Equation of straight line in intercepts form is $\frac{x}{a} + \frac{y}{b} = 1$ where 'a' is x-intercept and 'b' is y-intercept.
- y = mx is the equation of straight line passing through origin.
- In the point (x, y), 'x' is called 'Abscissa' and 'y' is called 'Ordinate'.

Worked Examples

5.17 Find the equation of the straight line passing through (5,7) and is (i) parallel to X-axis (ii) parallel to Y-axis.

Sol :

(i) The equation of any straight line parallel to X-axis is y = b.
Since it passes through (5, 7), b = 7 .
Therefore, the required equation of the line is $y = 7$.

(ii) The equation of any straight line parallel toY-axis is $x = a$
Since it passes through (5, 7), a = 5
Therefore, the required equation of the line is $x = 5$.

5.18 Find the equation of a straight line whose
(i) Slope is 5 and Y-intercept is – 9
(ii) Inclination is 45° and Y-intercept is 11

Sol :

(i) Given, Slope = 5, Y- intercept c = – 9
Therefore, equation of a straight line is

$$y = mx + c$$

$$y = 5x - 9 \Rightarrow 5x - y - 9 = 0$$

(ii) Given, $\theta = 45^\circ$, Y-intercept c = 11
Slope m = tan θ = tan 45° = 1

Therefore, equation of a straight line is y = mx + c

$$\Rightarrow \quad y = x + 11 \Rightarrow x - y + 11 = 0$$

5.19 Calculate the slope and Y-intercept of the straight line 8x – 7y + 6 = 0

Sol :

Equation of the given straight line is

$$8x - 7y + 6 = 0$$

$$\Rightarrow \quad 7y = 8x + 6$$

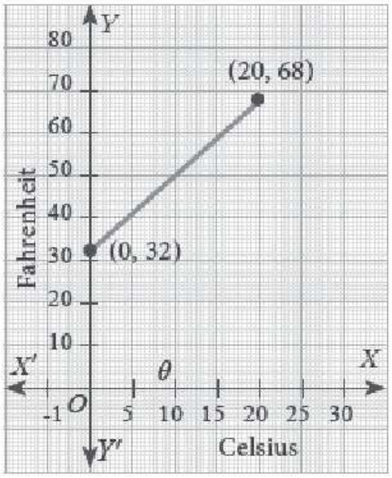
$$y = \frac{8}{7}x + \frac{6}{7} \quad \text{... (1)}$$

Comparing (1) with y = mx + c

Slope m = $\frac{8}{7}$ and Y-intercept c = $\frac{6}{7}$

5.20 The graph relates temperatures y (in Fahrenheit degree) to temperatures x (in Celsius degree) (a) Find the slope and Y intercept (b) Write an equation of the line (c) What is the mean temperature of the earth in Fahrenheit degree if its mean temperature is 25° Celsius?

Sol :



(a) Slope = $\frac{\text{Change in } y \text{ co-ordinate}}{\text{Change in } x \text{ co-ordinate}}$

$= \frac{68-32}{20-0} = \frac{36}{20} = \frac{9}{5} = 1.8$

The line crosses the Y-axis at (0, 32)

$\Rightarrow$ So the slope is $\frac{9}{5}$ and Y-intercept is 32.

(b) Use the slope and Y-intercept to write an equation

$\Rightarrow$ The equation is $y = \frac{9}{5}x + 32$

(c) In Celsius, the mean temperature of the earth is 25°. To find the mean temperature in Fahrenheit, we find the value of y when x = 25

$$y = \frac{9}{5}x + 32$$

$$y = \frac{9}{5}(25) + 32$$

$$y = 77$$

Therefore, the mean temperature of the earth is 77°F

5.21 Find the equation of a line passing through the point (3, – 4) and having slope $\frac{-5}{7}$

Sol : $(x_1, y_1) = (3, -4)$ and $m = \frac{-5}{7}$

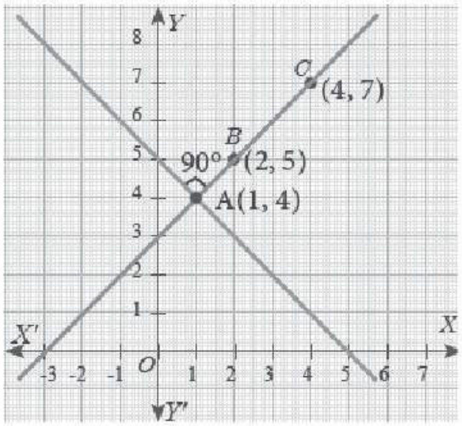
The equation of the point-slope form of the straight line is $y - y_1 = m(x - x_1)$

$$y + 4 = -\frac{5}{7}(x-3)$$

$$5x + 7y + 13 = 0$$

5.22 Find the equation of a line passing through the point A(1, 4) and perpendicular to the line joining points (2, 5) and (4, 7).

Sol :



Let the given points be A(1, 4), B(2, 5) and C(4, 7).

Slope of line BC $= \frac{7-5}{4-2} = \frac{2}{2} = 1$

Let m be the slope of the required line

Since the required line is perpendicular to BC,

$\Rightarrow \quad m \times 1 = -1 \quad \Rightarrow m = -1$

The required line also passes through the point A(1, 4).

The equation of the required straight line is

$$y - y_1 = m(x - x_1)$$

$$y - 4 = -1(x - 1)$$

$$y - 4 = -x + 1$$

$$x + y - 5 = 0$$

5.23 Find the equation of a straight line passing through (5, – 3) and (7, – 4)

Sol : The equation of a straight line passing through the two points (x_1, y_1) and (x_2, y_2) is

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

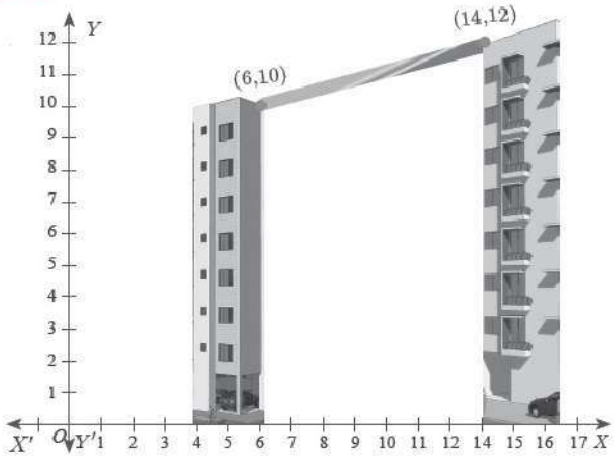
$$\Rightarrow \frac{y+3}{-4+3} = \frac{x-5}{7-5}$$

$$\Rightarrow 2y + 6 = -x + 5$$

Therefore, $x + 2y + 1 = 0$

5.24 Two buildings of different heights are located at opposite sides of each other. If a heavy rod is attached joining the terrace of the buildings from (6, 10) to (14, 12), find the equation of the rod joining the buildings.

Sol :



Let A(6, 10), B(14, 12) be the points denoting the terrace of the buildings.

The equation of the rod is the equation of the straight line passing through A(6, 10) and B(14, 12)

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1} \Rightarrow \frac{y-10}{12-10} = \frac{x-6}{14-6}$$

$$\frac{y-10}{2} = \frac{x-6}{8}$$

Therefore, $x - 4y + 34 = 0$

Hence, equation of the rod is $x - 4y + 34 = 0$

5.25 Find the equation of a line which passes through (5, 7) and makes intercepts on the axes equal in magnitude but opposite in sign.

Sol :

Let the X-intercept be 'a' and Y-intercept be '– a'.

The equation of the line in intercept form is

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\Rightarrow \quad \frac{x}{a} + \frac{y}{-a} = 1 \quad \text{(Here } b = -a\text{)}$$

Therefore, $x - y = a$ ... (1)

Since (1) passes through (5, 7)

Therefore, $5 - 7 = a \Rightarrow a = -2$

Thus the required equation of the straight line is $x - y = -2$; or $x - y + 2 = 0$

5.26 Find the intercepts made by the line $4x - 9y + 36 = 0$ on the co-ordinate axes.

Sol :

Equation of the given line is

$4x - 9y + 36 = 0 \Rightarrow 4x - 9y = -36$

Dividing by – 36 we get, $\frac{x}{-9} + \frac{y}{4} = 1$ ... (1)

Comparing (1) with intercept form, we get

X-intercept $a = -9$; Y-intercept $b = 4$.

5.27 A mobile phone is put to use when the battery power is 100%. The percent of battery power 'y' (in decimal) remaining after using the mobile phone for x hours is assumed as $y = -0.25x + 1$

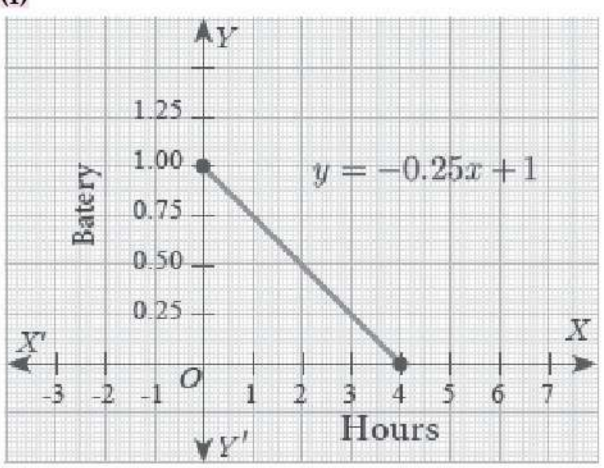
(i) Draw a graph of the equation.

(ii) Find the number of hours elapsed if the battery power is 40%.

(iii) How much time does it take so that the battery has no power?

Sol :

(i)



(ii) To find the time when the battery power is 40%, we have to take $y = 0.40$

$0.40 = -0.25x + 1 \Rightarrow 0.25x = 0.60$

$$\Rightarrow x = \frac{0.60}{0.25} = 2.4 \text{ hours.}$$

(iii) If the battery power is 0 then y = 0

Therefore, $0 = -0.25x + 1 \Rightarrow -0.25x = 1$

$\Rightarrow$ x = 4 hours.

Thus, after 4 hours, the battery of the mobile phone will have no power.

5.28 A line makes the positive intercepts on co-ordinate axes whose sum is 7 and it passes through (– 3, 8). Find its equation.

Sol :

If a and b are the intercepts then

a + b = 7 or b = 7 – a

By intercept form $\frac{x}{a} + \frac{y}{b} = 1$ ... (1)

We have $\frac{x}{a} + \frac{y}{7-a} = 1$

As this line passes through the point (– 3, 8), we have

$$\frac{-3}{a} + \frac{8}{7-a} = 1 \Rightarrow -3(7-a) + 8a = a(7-a)$$

$$\Rightarrow -21 + 3a + 8a = 7a - a^2$$

$$\Rightarrow a^2 + 4a - 21 = 0$$

Solving this equation (a – 3) (a + 7) = 0

a = 3 or a = – 7

Since a is positive,

we have a = 3 and b = 7 – a = 7 – 3 = 4.

Hence $\frac{x}{3} + \frac{y}{4} = 1$

Therefore, 4x + 3y – 12 = 0 is the required equation.

5.29 A circular garden is bounded by East Avenue and Cross Road. Cross Road intersects North Street at D and East Avenue at E. AD is tangential to the circular garden at A(3, 10) using the figure.

(a) Find the equation of

(i) East Avenue

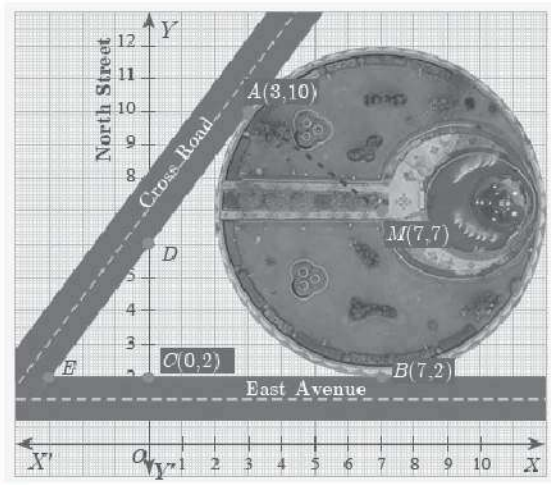
(ii) North Street

(iii) Cross Road

(b) Where does the Cross Road intersect the

(i) East Avenue?

(ii) North Street



Sol :

(a) (i) East Avenue is the straight line joining C(0, 2) and B(7, 2). Thus the equation of East Avenue is obtained by using two-point form which is

$$\frac{y-2}{2-2} = \frac{x-0}{7-0}$$

$$\frac{y-2}{0} = \frac{x}{7} \Rightarrow y = 2$$

(ii) Since the point D lie vertically above C(0, 2). The x-coordinate of D is 0. Since any point on North Street has x-coordinate value 0.

Therefore, the equation of North Street is X = 0

(iii) To find equation of Cross Road.

Center of circular garden M is (7, 7), A is (3, 10)

We first find slope of MA, which we call m_1

Thus $m_1 = \frac{10-7}{3-7} = \frac{-3}{4}$.

Since the Cross Road is perpendicular to MA, if m_2 is the slope of the Cross Road then, $m_1m_2 = 1$

$$\Rightarrow \frac{-3}{4} m_2 = -1 \Rightarrow m_2 = \frac{4}{3}.$$

Now, the corss road has slope $\frac{4}{3}$ and it passes through the point A(3, 10).

The equation of the Cross Road is

$$y - 10 = \frac{4}{3}(x-3)$$

$$\Rightarrow 3y - 30 = 4x - 12$$

$$\Rightarrow 4x - 3y + 18 = 0$$

(b) **(i)** If D is (0, k) then D is a point on the Cross Road.

Therefore, substituting x = 0, y = k in the equation of Cross Road, we get

$0 - 3k + 18 = 0$

$\Rightarrow \quad k = 6$

D is (0, 6)

(ii) To find E, Let E be (q, 2)

put y = 2 in the equation of the Cross Road, we get

$4q - 6 + 18 = 0$

$4q = -12 \Rightarrow q = -3$

Therefore, The point E is (– 3, 2)

Thus the Cross Road meets the North Street at D(0, 6) and East Avenue at E (– 3, 2).

Progress Check

1. Fill the details in Respective boxes.

Form	When to use?	Name
y = mx + c	Slope = m, Intercept = c are given	...
$\dfrac{y-y_1}{y_2-y_1} = \dfrac{x-x_1}{x_2-x_1}$	...	...
...	The intercepts are given	Intercept form

Ans :

y = mx + c	Slope m, y Intercept c	Slope-intercept form
$\dfrac{y-y_1}{y_2-y_1} = \dfrac{x-x_1}{x_2-x_1}$	When two points (x_1, y_1) and (x_2, y_2) are given.	Two points form
$\dfrac{x}{a} + \dfrac{y}{b} = 1$	Intercepts are given	Intercepts form

2.

Sl. No.	Equation	Slope	x intercept	y intercept
1	3x – 4y + 2 = 0	...	...	...
2	y = 14 x	...	...	0
3	...	...	2	– 3

Ans :

1. $3x - 4y + 2 = 0$

$\Rightarrow \quad \text{Slope} = \dfrac{-3}{-4} = \dfrac{3}{4}$

$3x - 4y = -2$

$\dfrac{x}{\left(-\dfrac{2}{3}\right)} + \dfrac{y}{\left(\dfrac{1}{2}\right)} = 1 \Rightarrow$ x intercept $-\dfrac{2}{3}$

y intercept $\dfrac{1}{2}$

2. $y = 14x \Rightarrow$ Slope m = 14

[$\because$ y = mx + c form]

x intercept is '0'

y intercept is '0'

3. Given 'x' intercept is 2, 'y' intercept is – 3

Equation of the line is $\dfrac{x}{a} + \dfrac{y}{b} = 1$

$\Rightarrow \quad \dfrac{x}{2} + \dfrac{y}{-3} = 1$

$3x - 2y - 6 = 0$

Thinking Corner

1. Is it possible to express, the equation of a straight line in slope-Intercept form, when it is parallel to Y-axis?

Ans :

Any line parallel to Y-axis, is of the form x = a.

$\therefore$ It cannot be expressed in slope-intercept form.

Exercise 5. 3

1. Find the equation of a straight line passing through the mid-point of a line segment joining the points (1, – 5), (4, 2) and parallel to: (i) X-axis (ii) Y-axis.

Sol : Given points (1, – 5) and (4, 2)

Mid-point of the line joining the points (1, – 5), (4, 2)

$= \left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right) = \left(\dfrac{1+4}{2}, \dfrac{-5+2}{2}\right)$

$= \left(\dfrac{5}{2}, -\dfrac{3}{2}\right)$

(i) Equation of a straight line passing through $\left(\frac{5}{2}, -\frac{3}{2}\right)$ and parallel to X-axis is $y = b$.

i.e., $y = -\frac{3}{2} \Rightarrow 2y + 3 = 0$.

(ii) Equation of a straight line passing through $\left(\frac{5}{2}, -\frac{3}{2}\right)$ and parallel to Y-axis is $x = a$.

i.e., $x = \frac{5}{2} \Rightarrow 2x - 5 = 0$

2. The equation of a straight line is 2 (x – y) + 5 = 0. Find its slope, inclination and intercept on the Y-axis.

Sol :

Given line is $2(x - y) + 5 = 0$

$\Rightarrow 2x - 2y + 5 = 0$ is in the form $ax + by + c = 0$

$$\text{Slope} = -\frac{a}{b} = \frac{-2}{-2} = 1$$

$$\text{Slope} = 1$$

Inclination:

$$\text{i.e.,} \quad \tan\theta = 1 = \tan 45^\circ$$

$$\theta = 45^\circ$$

$$2x - 2y + 5 = 0$$

$$2x - 2y = -5 \text{ (Dividing by } -5)$$

$$\frac{x}{\left(-\frac{5}{2}\right)} + \frac{y}{\left(\frac{5}{2}\right)} = 1$$

y intercept is $\frac{5}{2}$

3. Find the equation of a whose inclination is 30° and making an intercept – 3 on the Y-axis.

Sol :

Given inclination $\theta = 30^\circ$

$\therefore$ Slope $m = \tan 30^\circ = \frac{1}{\sqrt{3}}$

y intercept $c = -3$

Equation of straight line in slope and 'y' intercept form is

$$y = mx + c$$

$$\Rightarrow \quad y = \frac{1}{\sqrt{3}}x - 3$$

$$\sqrt{3}y = x - 3\sqrt{3}$$

$$x - \sqrt{3}y - 3\sqrt{3} = 0$$

4. Find the slope and Y intercept of $\sqrt{3}x + (1-\sqrt{3})$ y = 3.

Sol :

Given line is $\sqrt{3}x + (1-\sqrt{3})y = 3$.

$\sqrt{3}x + (1-\sqrt{3})y - 3 = 0$ is in the form

$$ax + by + c = 0$$

$$\text{Slope} = -\frac{a}{b}$$

$$= -\frac{\sqrt{3}}{1-\sqrt{3}}$$

$$= -\frac{\sqrt{3}}{1-\sqrt{3}} \times \frac{1+\sqrt{3}}{1+\sqrt{3}}$$

$$= \frac{\sqrt{3}+3}{2}$$

$\sqrt{3}x + (1-\sqrt{3})y = 3$ [Dividing throughout by '3'

$$\frac{\sqrt{3}x}{3} + \frac{(1-\sqrt{3})}{3}y = 1$$

$$\frac{x}{(\sqrt{3})} + \frac{y}{\left(\frac{3}{1-\sqrt{3}}\right)} = 1$$

$$\text{y intercept} = \frac{3}{1-\sqrt{3}} \times \frac{1+\sqrt{3}}{1+\sqrt{3}}$$

$$= \frac{3+3\sqrt{3}}{1-3} = \frac{3+3\sqrt{3}}{-2}$$

5. Find the value of 'a', the line through (– 2, 3) and (8, 5) is perpendicular to y = ax + 2.

Sol :

Slope of a line passing through (– 2, 3) and (8, 5) is

$$\frac{y_1 - y_2}{x_1 - x_2} = \frac{3-5}{-2-8} = \frac{-2}{-10} = \frac{1}{5} = m_1$$

Slope of the line $y = ax + 2$ is 'a' $= m_2$

Given that the lines are perpendicular

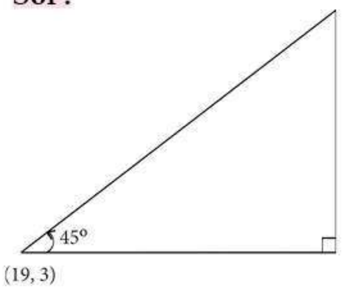
$$\therefore \quad m_1 \times m_2 = -1$$

$$\frac{1}{5} \times a = -1$$

$$a = -5.$$

6. The hill in the form of a triangle has its foot at (19, 3). The inclination of the hill to the ground is 45°. Find the equation of the hill joining the foot and top.

Sol :



Foot of the hill is at (19, 3)

Angle of inclination $\theta = 45°$

Slope $m = \tan 45° = 1$

Equation of line passing through (x_1, y_1) and having

Slope 'm' is $y - y_1 = m(x - x_1)$

$\Rightarrow y - 3 = 1(x - 19)$

$y - 3 = x - 19 \Rightarrow x - y - 16 = 0.$

7. Find the equation of a line through the given pair of points

(i) $\left(2, \frac{2}{3}\right)$ **and** $\left(\frac{-1}{2}, -2\right)$

(ii) **(2, 3) and (– 7, – 1)**

Sol :

(i) Given points $\left(2, \frac{2}{3}\right)$ and $\left(-\frac{1}{2}, -2\right)$

Equation of the line passing through (x_1, y_1) and (x_2, y_2) is

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\Rightarrow \frac{y - \frac{2}{3}}{-2 - \frac{2}{3}} = \frac{x - 2}{-\frac{1}{2} - 2}$$

$$\Rightarrow \frac{3y - 2}{-6 - 2} = \frac{2x - 4}{-1 - 4}$$

$\Rightarrow -5(3y - 2) = -8(2x - 4)$

$\Rightarrow -15y + 10 = -16x + 32$

$\Rightarrow 16x - 15y - 22 = 0$

(ii) Given points (2, 3) and (– 7, – 1)

Equation of the line passing through (x_1, y_1) and (x_2, y_2) is

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 3}{-1 - 3} = \frac{x - 2}{-7 - 2}$$

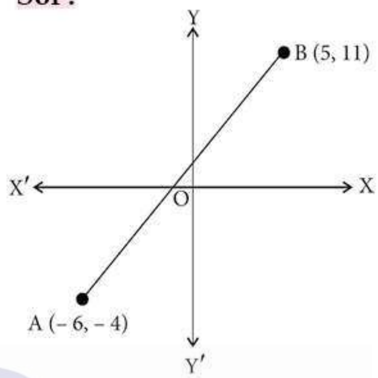
$9(y - 3) = 4(x - 2)$

$9y - 27 = 4x - 8$

$4x - 9y + 19 = 0$

8. A cat is located at the point (– 6, – 4) in XY-plane. A bottle of milk is kept at (5, 11). The cat wishes to consume the milk travelling through shortest possible distance. Find the equation of the path it needs to take its milk.

Sol :



Let A be the location of the cat and B be the point where bottle of milk is kept.

Given A (–6, – 4), B (5, 11)

Equation of line passing through (x_1, y_1) and (x_2, y_2) is

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y + 4}{11 + 4} = \frac{x + 6}{5 + 6}$$

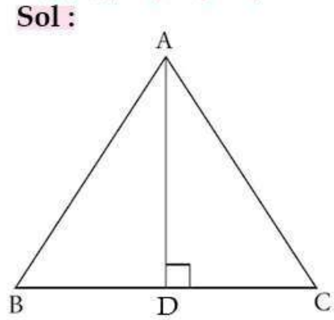
$11(y + 4) = 15(x + 6)$

$11y + 44 = 15x + 90$

$15x - 11y + 46 = 0$

9. Find the equation of the median and altitude of ΔABC through A where the vertices are A (6, 2), B (– 5, – 1) and C (1, 9).

Sol :



Given vertices are A (6, 2), B (– 5, – 1) and C (1, 9)

Median through A:

Let D be the mid point of BC

Mid point of BC $= D\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$= D\left(\frac{-5+1}{2}, \frac{-1+9}{2}\right)$

$= D(-2, 4)$

Now AD is the median.

Equation of AD $\Rightarrow \frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$

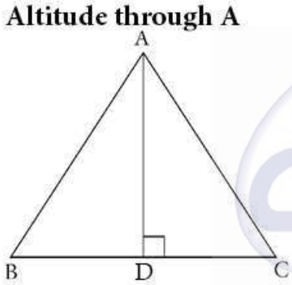
$$\frac{y-2}{4-2} = \frac{x-6}{-2-6}$$

$$\frac{y-2}{2} = \frac{x-6}{-8}$$

$$-4y+8 = x-6$$

$$x+4y-14 = 0$$

Altitude through A



Altitude is passing through 'A' and perpendicular to BC.

Now,

Slope of BC $= \frac{y_1-y_2}{x_1-x_2} = \frac{-1-9}{-5-1} = \frac{-10}{-6} = \frac{5}{3}$

$\therefore$ Slope of Altitude $= -\frac{3}{5}$

Equation of the altitude which is passing through A (6, 2) and having slope $-\frac{3}{5}$ is

$$y-y_1 = m(x-x_1)$$

$$y-2 = -\frac{3}{5}(x-6)$$

$$5y-10 = -3x+18$$

$$3x+5y-28 = 0$$

10. Find the equation of a straight line which has slope $\frac{5}{-4}$ and passing through the point (– 1, 2).

Sol : Given point (– 1, 2), Slope m $= -\frac{5}{4}$

Equation of the line passing through (x_1, y_1) and having slope 'm' is

$$y-y_1 = m(x-x_1)$$

$$y-2 = -\frac{5}{4}(x+1)$$

$$4y-8 = -5x-5$$

$$5x+4y-3 = 0$$

11. You are downloading a song. The percent y (in decimal form) of mega bytes remaining to get downloaded in x seconds is given by $y = -0.1x + 1$. Find

(i) graph the equation.

(ii) the total MB of the song.

(iii) after how many seconds will 75% of the song gets downloaded.

(iv) after how many seconds the song will be downloaded completely.

Sol :

Given equation is y = – 0.1x + 1 where 'x' is time (in seconds) and 'y' is percentage of megabytes remaining.

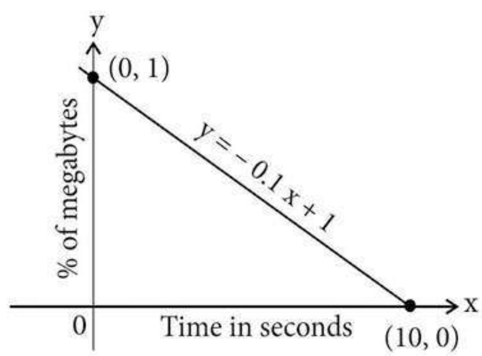
(i) Graph of y = – 0.1x + 1

$\Rightarrow \quad y = -\frac{x}{10}+1$

$\Rightarrow \quad 10y = -x+10$

Points to be plotted

x	0	10
y	1	0



(ii) y = – 0.1x + 1

Initially, time x = 0 $\Rightarrow$ y = 1

$\therefore$ Total MB of the song is 1.

(iii) After how many seconds will 75% of the song gets downloaded.

$\therefore$ 25% is remaining $\Rightarrow$ y = 0.25

$$y = -0.1x+1$$

$$0.25 = -0.1x+1$$

$$0.25-1 = -0.1x$$

$$x = \frac{-0.75}{-0.1} = \frac{75}{10} = 7.5 \text{ seconds.}$$

(iv) After how many seconds, the song will be downloaded completely.

$\therefore$ Remaining megabytes is 0

$$y = 0.1\,x + 1$$
$$0 = -0.1\,x + 1$$
$$0.1\,x = 1$$
$$x = \frac{1}{0.1} = 10 \text{ seconds.}$$

12. Find the equation of a line whose intercepts on the axes are given below

(i) 4, – 6 **(ii) $-5, \frac{3}{4}$**

Sol :

(i) Given intercepts are 4, – 6

$$a = 4, b = -6$$

Equation of the line in the intercepts form is

$$\frac{x}{a} + \frac{y}{b} = 1$$
$$\frac{x}{4} + \frac{y}{-6} = 1$$
$$3x - 2y - 12 = 0$$

(ii) Given intercepts are $-5, \frac{3}{4}$

$$\Rightarrow \quad a = -5, b = \frac{3}{4}$$

Equation of the line in the intercepts form is

$$\frac{x}{a} + \frac{y}{b} = 1$$
$$\Rightarrow \quad \frac{x}{-5} + \frac{y}{\left(\frac{3}{4}\right)} = 1$$
$$\Rightarrow \quad \frac{x}{-5} + \frac{4y}{3} = 1$$
$$\Rightarrow \quad 3x - 20y = -15$$
$$\Rightarrow \quad 3x - 20y + 15 = 0$$

13. Find the intercepts made by the following lines on the co-ordinate axes

(i) $3x - 2y - 6 = 0$ **(ii) $4x + 3y + 12 = 0$**

Sol :

(i) Given line is $3x - 2y - 6 = 0$

$$\Rightarrow \quad 3x - 2y = 6$$

Dividing by 6

$$\Rightarrow \quad \frac{3x}{6} - \frac{2y}{6} = 1$$
$$\Rightarrow \quad \frac{x}{2} + \frac{y}{(-3)} = 1 \text{ is in the form}$$
$$\frac{x}{a} + \frac{y}{b} = 1$$

$\therefore$ 'x' intercept $= a = 2$

y intercept $= b = -3$

(ii) $4x + 3y + 12 = 0$

$$\Rightarrow \quad 4x + 3y = -12$$

Dividing by – 12

$$\Rightarrow \quad \frac{4x}{-12} + \frac{3y}{-12} = 1$$
$$\Rightarrow \quad \frac{x}{(-3)} + \frac{y}{(-4)} = 1 \text{ is in the form}$$
$$\frac{x}{a} + \frac{y}{b} = 1$$

$\therefore$ x intercept $= a = -3$

y intercept $= b = -4.$

14. Find the equation of a straight line

(i) Passing through (1, – 4) and has intercepts which are in the ratio 2 : 5

(ii) Passing through (– 8, 4) and making equal intercepts on the co-ordinate axes.

Sol :

(i) Given that intercepts are in the ratio 2 : 5

i.e., $\frac{a}{b} = \frac{2}{5}$

$$a = \frac{2b}{5}$$

Equation of the line in Intercepts form is $\frac{x}{a} + \frac{y}{b} = 1$

$$\Rightarrow \quad \frac{x}{\left(\frac{2b}{5}\right)} + \frac{y}{b} = 1$$
$$\Rightarrow \quad \frac{5x}{2b} + \frac{y}{b} = 1$$
$$\Rightarrow \quad 5x + 2y = 2b$$

This passes through (1, – 4)

$\therefore$ $5(1) + 2(-4) = 2b$

$$5 - 8 = 2b \quad \Rightarrow \quad b = -\frac{3}{2}$$

$$a = \frac{2b}{5} = \frac{2\left(-\frac{3}{2}\right)}{5} = -\frac{3}{5}$$

∴ Equation of a straight line is

$$\frac{x}{a}+\frac{y}{b} = 1 \Rightarrow \frac{x}{\left(-\frac{3}{5}\right)}+\frac{y}{\left(-\frac{3}{2}\right)} = 1$$

$$\Rightarrow \quad \frac{5x}{-3}+\frac{2y}{-3} = 1$$

$$\Rightarrow \quad 5x + 2y + 3 = 0.$$

(ii) Given that intercepts are equal.

$$\therefore \quad a = b$$

Equation of the line in intercepts form is

$$\frac{x}{a}+\frac{y}{b} = 1$$

$$\Rightarrow \quad \frac{x}{a}+\frac{y}{a} = 1$$

$$\Rightarrow \quad x + y = a$$

This passes through $(-8, 4)$

$$\therefore \quad -8 + 4 = a$$

$$\Rightarrow \quad a = -4 \quad [\because \; a = b]$$

then, $\quad b = -4$

∴ Equation of the straight line is

$$\frac{x}{a}+\frac{y}{b} = 1$$

$$\Rightarrow \quad \frac{x}{-4}+\frac{y}{-4} = 1$$

$$\Rightarrow \quad x + y + 4 = 0.$$

General form of a straight line

Key Points

The Linear equation $ax + by + c = 0$ is known as general form of the straight line where a, b and c are real numbers.

- Condition for parallelism:

  If two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are parallel, then $\frac{a_1}{a_2} = \frac{b_1}{b_2}$.

- Condition for perpendicularity:

  If two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are perpendicular, then $a_1a_2 + b_1b_2 = 0$.

- Slope of the line $ax + by + c = 0$

  $$\text{Slope} = -\frac{\text{Co-efficient of } x}{\text{Co-efficient of } y} = -\frac{a}{b}$$

Worked Examples

5.30 **Find the slope of the straight line $6x + 8y + 7 = 0$.**

Sol :

Given $6x + 8y + 7 = 0$

$$\text{slope } m = \frac{-\text{Co-efficient of } x}{\text{Co-efficient of } y}$$

$$= -\frac{6}{8} = -\frac{3}{4}$$

Therefore, the slope of the straight line is $-\frac{3}{4}$.

5.31 **Find the slope of the line which is**
(i) parallel to $3x - 7y = 11$
(ii) perpendicular to $2x - 3y + 8 = 0$

Sol :

(i) Given straight line is $3x - 7y = 11$

$$\Rightarrow \quad 3x - 7y - 11 = 0$$

$$\text{Slope } m = \frac{-3}{-7} = \frac{3}{7}$$

Since parallel lines have same slopes, slope of any line parallel to

$$3x - 7y = 11 \text{ is } \frac{3}{7}$$

(ii) Given straight line is $2x - 3y + 8 = 0$

$$\text{Slope} \quad m = \frac{-2}{-3} = \frac{2}{3}$$

Since product of slopes is -1 for perpendicular lines, slope of any line perpendicular to

$$2x - 3y + 8 = 0 \text{ is } \frac{-1}{\frac{2}{3}} = \frac{-3}{2}$$

5.32 Show that the straight lines $2x + 3y - 8 = 0$ and $4x + 6y + 18 = 0$ are parallel.

Sol : Slope of the straight line $2x + 3y - 8 = 0$ is

$$m_1 = \frac{-co\text{-}efficient\ of\ x}{co\text{-}efficient\ of\ y}$$

$$m_1 = \frac{-2}{3}$$

Slope of the straight line $4x + 6y + 18 = 0$ is

$$m_2 = \frac{-4}{6} = \frac{-2}{3}$$

Here, $m_1 = m_2$

That is, slopes are equal. Hence, the two straight lines are parallel.

5.33 Show that the straight lines $x - 2y + 3 = 0$ and $6x + 3y + 8 = 0$ are perpendicular.

Sol :

Slope of the straight line $x - 2y + 3 = 0$ is

$$m_1 = \frac{-1}{-2} = \frac{1}{2}$$

Slope of the straight line $6x + 3y + 8 = 0$ is

$$m_2 = \frac{-6}{3} = -2$$

$$\text{Now,} \quad m_1 \times m_2 = \frac{1}{2} \times (-2) = -1$$

Hence, the two straight lines are perpendicular.

5.34 Find the equation of a straight line which is parallel to the line $3x - 7y = 12$ and passing through the point (6, 4).

Sol : Equation of the straight line, parallel to

$3x - 7y - 12 = 0$ is $3x - 7y + k = 0$.

Since it passes through the point (6, 4).

$$3(6) - 7(4) + k = 0$$

$$k = 28 - 18 = 0$$

Therefore, equation of the required straight line is $3x - 7y + 10 = 0$.

5.35 Find the equation of a straight line perpendicular to the line $y = \frac{4}{3}x - 7$ and passing through the point (7, – 1).

Sol :

The equation $y = \frac{4x}{3} - 7$ can be written as

$$4x - 3y - 21 = 0.$$

Equation of a straight line perpendicular to

$4x - 3y - 21 = 0$ is $3x + 4y + k = 0$

Since it is passes through the point (7, – 1),

$$21 - 4 + k = 0 \Rightarrow k = -17$$

Therefore, equation of the required straight line is $3x + 4y - 17 = 0$.

5.36 Find the equation of a straight line parallel to Y-axis and passing through the point of intersection of the lines $4x + 5y = 13$ and $x - 8y + 9 = 0$.

Sol :

Given lines $4x + 5y - 13 = 0$ ... (1)

$x - 8y + 9 = 0$ ... (2)

To find point of intersection, solve equation (1) and (2)

$$\begin{array}{cccccc} & x & & y & & 1 \\ 5 & & -13 & & 4 & & 5 \\ -8 & & 9 & & 1 & & -8 \end{array}$$

$$\frac{x}{45-104} = \frac{y}{-13-36} = \frac{1}{-32-5}$$

$$\frac{x}{-59} = \frac{y}{-49} = \frac{1}{-37}$$

$$x = \frac{59}{37}, \ y = \frac{49}{37}$$

Therefore, the point of intersection

$$(x, y) = \left(\frac{59}{37}, \frac{49}{37}\right)$$

The equation of line parallel to Y-axis is $x = c$.

It passes through $(x, y) = \left(\frac{59}{37}, \frac{49}{37}\right)$.

$$\text{Therefore,} \quad c = \frac{59}{37}.$$

The equation of the line is $x = \frac{59}{37}$

$$\Rightarrow \quad 37x - 59 = 0.$$

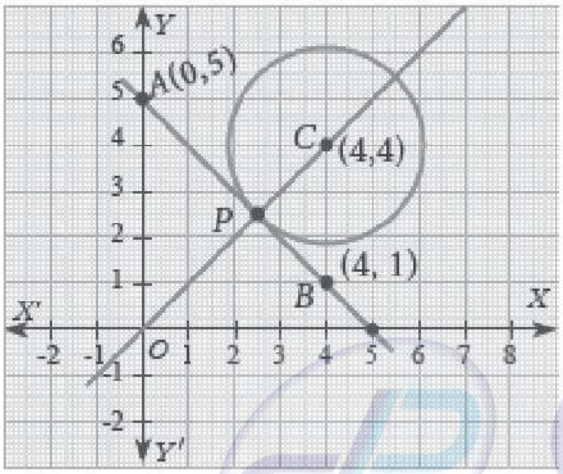
5.37 The line joining the points A (0, 5) and B (4, 1) is a tangent to a circle whose centre C is at the point (4, 4).

(i) Find the equation of the line AB.

(ii) Find the equation of the line through C which is perpendicular to the line AB.

(iii) Find the co-ordinates of the point of contact of tangent line AB with the circle.

Sol :



(i) Equation of line AB, A (0, 5) and B (4, 1)

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

$$\frac{y-5}{1-5} = \frac{x-0}{4-0} \Rightarrow 4(y-5) = -4x$$

$$y - 5 = -x$$

$$x + y - 5 = 0.$$

(ii) The equation of a line which is perpendicular to the line AB : $x + y - 5 = 0$ is $x - y + k = 0$

Since it is passing through the point (4, 4), we have

$$4 - 4 + k = 0 \Rightarrow k = 0$$

The equation of a line which is perpendicular to AB and through C is

$$x - y = 0 \quad \text{... (2)}$$

(iii) The co-ordinate of the point of contact P of the tangent line AB with the circle point of intersection of lines.

$$x + y - 5 = 0 \text{ and } x - y = 0$$

Solving, we get $x = \frac{5}{2}$ and $y = \frac{5}{2}$

Therefore, the co-ordinate of the point of contact is $P\left(\frac{5}{2}, \frac{5}{2}\right)$.

Progress Check

1.

Sl. No.	Equations	Parallel or perpendicular
1	$5x + 2y + 5 = 0$ $5x + 2y - 3 = 0$	
2	$3x - 7y - 6 = 0$ $7x + 3y + 8 = 0$	
3	$8x - 10y + 11 = 0$ $4x - 5y + 16 = 0$	
4	$2y - 9x - 7 = 0$ $27y + 6x - 21 = 0$	

Ans :

1. $5x + 2y + 5 = 0 \quad a_1 = 5, \; b_1 = 2$

   $5x + 2y - 3 = 0 \quad a_2 = 5, \; b_2 = 2$

   $\frac{a_1}{a_2} = \frac{5}{5} = 1, \; \frac{b_1}{b_2} = \frac{2}{2} = 1$

   $\therefore \quad \frac{a_1}{a_2} = \frac{b_1}{b_2}$

   $\therefore$ Lines are parallel.

2. $3x - 7y - 6 = 0 \quad a_1 = 3, \; b_1 = -7$

   $7x + 3y + 8 = 0 \quad a_2 = 7, \; b_2 = 3$

   $a_1 a_2 + b_1 b_2 = (3)(7) + (-7)(3) = 0$

   $\therefore$ The lines are perpendicular.

3. $8x - 10y + 11 = 0 \quad a_1 = 8, \; b_1 = -10$

   $4x - 5y + 16 = 0 \quad a_2 = 4, \; b_2 = -5$

   $\frac{a_1}{a_2} = \frac{8}{4} = 2, \quad \frac{b_1}{b_2} = \frac{-10}{-5} = 2$

   $\frac{a_1}{a_2} = \frac{b_1}{b_2}$

   $\therefore$ The lines are parallel.

4. $2y - 9x - 7 = 0 \quad a_1 = 2, \; b_1 = -9$

   $27y + 6x - 21 = 0 \quad a_2 = 27, \; b_2 = 6$

   $a_1 a_2 + b_1 b_2 = (2)(27) + (-9)(6) = 54 - 54 = 0$

   $\therefore$ The lines are perpendicular.

Thinking Corner

1. How many straight lines do you have with slope 1?

Ans : Many

2. Find the number of point of intersection of two straight lines.

Ans : One

3. Find the number of straight lines perpendicular to the line 2x – 3y + 6 = 0.

Ans : Many

Exercise 5. 4

1. Find the slope of the following straight lines

(i) 5y – 3 = 0 (ii) $7x - \frac{3}{17} = 0$

Sol :

(i) 5y – 3 = 0

$$y = \frac{3}{5}$$

$$y = (0)x + \frac{3}{5}$$

Slope m = 0

(or) Comparing with ax + by + c = 0

$$\text{Slope} = -\frac{a}{b} = -\frac{0}{5} = 0$$

(ii) $7x - \frac{3}{17} = 0$

Comparing with ax + by + c = 0

$$\text{Slope} = -\frac{a}{b} = -\frac{7}{0} = \text{undefined}$$

2. Find the slope of the line which is

(i) parallel to y = 0.7x – 11

(ii) perpendicular to the line x = – 11

Sol :

(i) Given line is y = 0.7x – 11 is in the form

y = mx + c

Slope m = 0.7

∴ Slope of the line parallel to y = 0.7 x – 11 is m = 0.7.

(∵ Slopes are equal when the lines are parallel)

(ii) Given line is x = – 11

(Comparing with ax + by + c = 0)

$$\text{Slope } m = -\frac{a}{b} = -\frac{1}{0} = \text{undefined.}$$

∴ Slope of the line perpendicular to x = – 11 is 0.

3. Check whether the given lines are parallel or perpendicular

(i) $\frac{x}{3}+\frac{y}{4}+\frac{1}{7}=0$ and $\frac{2x}{3}+\frac{y}{2}+\frac{1}{10}=0$

(ii) 5x + 23y + 14 = 0 and 23x – 5y + 9 = 0.

Sol :

(i) Given lines are $\frac{x}{3}+\frac{y}{4}+\frac{1}{7}=0$ and

$$\frac{2x}{3}+\frac{y}{2}+\frac{1}{10}=0$$

$$a_1=\frac{1}{3}, b_1=\frac{1}{4} \text{ and } a_2=\frac{2}{3}, b_2=\frac{1}{2}$$

$$\text{Now, } \frac{a_1}{a_2}=\frac{\frac{1}{3}}{\frac{2}{3}}=\frac{1}{2}, \frac{b_1}{b_2}=\frac{\frac{1}{4}}{\frac{1}{2}}=\frac{1}{2}$$

Since $\frac{a_1}{a_2}=\frac{b_1}{b_2}$, the given lines are parallel.

(ii) Given lines 5x + 23y + 14 = 0 and 23x – 5y + 9 = 0

∴ $a_1 = 5, b_1 = 23$ and $a_2 = 23, b_2 = -5$

Now $a_1a_2 + b_1b_2 = (5)(23) + (23)(-5) = 0$

∴ The given lines are perpendicular.

4. If the straight lines 12y = – (p + 3)x + 12, 12x – 7y = 16 are perpendicular then find 'p'.

Sol :

Given lines are 12y = – (p + 3)x + 12 and

12x – 7y = 16

12y = – (p + 3)x + 12

⇒ (p + 3)x + 12y – 12 = 0 ... (1)

12x – 7y – 16 = 0 ... (2)

∴ $a_1 = (p+3), b_1 = 12$ and $a_2 = 12, b_2 = -7$

Given that the lines are perpendicular,

∴ $a_1a_2 + b_1b_2 = 0$

⇒ (p + 3)(12) + (12)(– 7) = 0

⇒ 12[p + 3 – 7] = 0

⇒ p – 4 = 0

⇒ p = 4

5. Find the equation of a straight line passing through the point P(– 5, 2) and parallel to the line joining the points Q(3, – 2) and R(– 5, 4).

Sol :

The vertices Q(3, – 2) and R(– 5, 4)

$$\text{Slope of the line QR} = \frac{y_1 - y_2}{x_1 - x_2} = \frac{-2-4}{3+5} = \frac{-6}{8} = \frac{-3}{4}$$

Slope of the line parallel to QR is $-\frac{3}{4}$

$\therefore$ Equation of the line passing through P(– 5, 2) and having slope $-\frac{3}{4}$ is

$$y - y_1 = m(x - x_1)$$
$$y - 2 = -\frac{3}{4}(x+5)$$
$$4y - 8 = -3x - 15$$
$$3x + 4y + 7 = 0$$

6. Find the equation of a line passing through (6, – 2) and perpendicular to the line joining the points (6, 7) and (2, – 3).

Sol :

Slope of the line joining the points (6, 7) and (2, – 3) is

$$\frac{y_1 - y_2}{x_1 - x_2} = \frac{7+3}{6-2} = \frac{10}{4} = \frac{5}{2} = m$$

Slope of the perpendicular line is

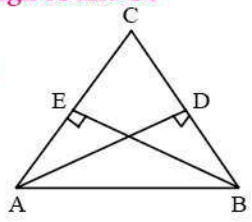
$$-\frac{1}{m} = -\frac{1}{(5/2)} = -\frac{2}{5}$$

Now, equation of the line passing through (6, – 2) and having slope $-\frac{2}{5}$ is

$$y - y_1 = m(x - x_1)$$
$$y + 2 = -\frac{2}{5}(x-6)$$
$$5y + 10 = -2x + 12$$
$$2x + 5y - 2 = 0$$

7. A(– 3, 0) B(10, – 2) and C(12, 3) are the vertices of ΔABC. Find the equation of the altitude through A and B.

Sol :



Given vertices are A(– 3, 0), B(10, – 2) and C(12, 3).

$$\text{Slope of BC} = \frac{y_1 - y_2}{x_1 - x_2} = \frac{-2-3}{10-12} = \frac{-5}{-2} = \frac{5}{2}$$

Altitude AD is perpendicular to BC and passing through A(– 3, 0)

$$\therefore \text{ Slope of AD} = -\frac{2}{5}$$

Equation of AD $\Rightarrow$ $y - y_1 = m(x - x_1)$

$$y - 0 = -\frac{2}{5}(x+3)$$
$$5y = -2x - 6$$
$$2x + 5y + 6 = 0$$

$$\text{Slope of AC} = \frac{y_1 - y_2}{x_1 - x_2} = \frac{0-3}{-3-12} = \frac{-3}{-15} = \frac{1}{5}$$

Altitude BE is perpendicular to AC and passing through B(10, – 2). Slope of BE = 5

$\therefore$ Equation of BE $\Rightarrow$ $y - y_1 = m(x - x_1)$

$$y + 2 = -5(x - 10)$$
$$y + 2 = -5x + 50$$
$$5x + y - 48 = 0$$

8. Find the equation of the perpendicular bisector of the line joining the points A(– 4, 2), B(6, – 4).

Sol :

Given points A(– 4, 2) and B(6, – 4)

$$\text{Mid point of AB} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-4+6}{2}, \frac{2-4}{2}\right) = (1, -1)$$

$$\text{Slope of AB} = \frac{y_1 - y_2}{x_1 - x_2} = \frac{2+4}{-4-6} = \frac{6}{-10} = \frac{-3}{5}$$

Slope of perpendicular line = $\frac{5}{3}$

Perpendicular bisector is passing through (1, – 1) and having slope $\frac{5}{3}$.

$\therefore$ The equation of perpendicular bisector is

$$y - y_1 = m(x - x_1)$$
$$y + 1 = \frac{5}{3}(x-1)$$
$$3y + 3 = 5x - 5$$
$$5x - 3y - 8 = 0$$

9. Find the equation of a straight line through the intersection of lines 7x + 3y = 10, 5x – 4y = 1 and parallel to the line 13x + 5y + 12 = 0.

Sol :

Given lines are $7x + 3y = 10$ and $5x - 4y = 1$.
Let us solve the equations to get the point of intersection.

$$7x + 3y = 10 \quad \text{... (1)}$$
$$5x - 4y = 1 \quad \text{... (2)}$$
$$(1) \times 4 \Rightarrow 28x + 12y = 40 \quad \text{... (3)}$$
$$(2) \times 3 \Rightarrow 15x - 12y = 3 \quad \text{... (4)}$$
$$(3) + (4) \Rightarrow 43x = 43$$
$$x = \frac{43}{43} = 1$$

Substituting $x = 1$ in (1)

$$7(1) + 3y = 10 \Rightarrow y = 1$$

The point of intersection is (1, 1)

Slope of the line $13x + 5y + 12 = 0$ is $-\frac{a}{b} = \frac{-13}{5}$

$\therefore$ Slope of the parallel line is $\frac{-13}{5}$

Now, equation of the line passing through (1, 1) and having slope $\frac{-13}{5}$ is

$$y - y_1 = m(x - x_1)$$
$$y - 1 = \frac{-13}{5}(x - 1)$$
$$5y - 5 = -13x + 13$$
$$13x + 5y - 18 = 0$$

10. Find the equation of a straight line through the intersection of lines 5x – 6y = 2, 3x + 2y = 10 and perpendicular to the line 4x – 7y + 13 = 0.

Sol :

Given lines are $5x - 6y = 2$ and $3x + 2y = 10$.
Let us solve the equations to get the point of intersection.

$$5x - 6y = 2 \quad \text{... (1)}$$
$$3x + 2y = 10 \quad \text{... (2)}$$
$$(2) \times 3 \Rightarrow 9x + 6y = 30 \quad \text{... (3)}$$
$$5x - 6y = 2 \quad \text{... (1)}$$
$$(3) + (1) \Rightarrow 14x = 32$$
$$x = \frac{32}{14} = \frac{16}{7}$$

Substituting $x = \frac{16}{7}$ in (2)

$$3\left(\frac{16}{7}\right) + 2y = 10$$
$$2y = 10 - \frac{48}{7} = \frac{22}{7}$$
$$y = \frac{11}{7}$$

$\therefore$ The point of intersection is $\left(\frac{16}{7}, \frac{11}{7}\right)$

Slope of the line $4x - 7y + 13 = 0$ is

$$-\frac{a}{b} = \frac{-4}{-7} = \frac{4}{7}$$

Slope of the perpendicular line is $-\frac{7}{4}$.

Now, equation of the line passing through $\left(\frac{16}{7}, \frac{11}{7}\right)$ and having slope $m = -\frac{7}{4}$ is

$$y - y_1 = m(x - x_1)$$
$$y - \frac{11}{7} = \frac{-7}{4}\left(x - \frac{16}{7}\right)$$
$$\frac{7y - 11}{7} = \frac{-7x}{4} + 4$$
$$28y - 44 = -49x + 112$$
$$49x + 28y - 156 = 0$$

11. Find the equation of a straight line joining the point of intersection of 3x + y + 2 = 0 and x – 2y – 4 = 0 to the point of intersection of 7x – 3y = – 12 and 2y = x + 3.

Sol :

Let us solve $3x + y + 2 = 0$ and $x - 2y - 4 = 0$

$$3x + y = -2 \quad \text{... (1)}$$
$$x - 2y = 4 \quad \text{... (2)}$$
$$(1) \times 2 \Rightarrow 6x + 2y = -4 \quad \text{... (3)}$$
$$x - 2y = 4 \quad \text{... (2)}$$
$$(3) + (2) \Rightarrow 7x = 0$$
$$x = 0$$

Substituting $x = 0$ in (1)

$$3(0) + y = -2$$
$$y = -2$$

The point of intersection of (1) and (2) is (0, – 2)

Now, Solving $7x - 3y = -12$ and $2y = x + 3$

$$7x - 3y = -12 \quad \text{... (5)}$$
$$x - 2y = -3 \quad \text{... (2)}$$

$(2) \times 7 \Rightarrow \quad 7x - 14y = -21 \quad \ldots (6)$

$(5) \Rightarrow \quad 7x - 3y = -12$

$(6) - (5) \Rightarrow \quad -11y = -9$

$$y = \frac{9}{11}$$

Substituting $y = \frac{9}{11}$ in (2)

$$x - 2\left(\frac{9}{11}\right) = -3$$

$$x = -3 + \frac{18}{11} = \frac{-15}{11}$$

The point of intersection of (5) and (2) is $\left(\frac{-15}{11}, \frac{9}{11}\right)$

Equation of the line joining (0, – 2) and $\left(\frac{-15}{11}, \frac{9}{11}\right)$ is

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\Rightarrow \frac{y+2}{\frac{9}{11}+2} = \frac{x-0}{-\frac{15}{11}-0}$$

$$\frac{11(y+2)}{9+22} = \frac{-11x}{15}$$

$$15(y+2) = -31x$$

$$15y + 30 = -31x$$

$$\Rightarrow 31x + 15y + 30 = 0$$

12. Find the equation of a straight line through the point of intersection of the lines 8x + 3y = 18, 4x + 5y = 9 and bisecting the line segment joining the points (5, – 4) and (– 7, 6).

Sol :

Let us solve 8x + 3y = 18 and 4x + 5y = 9

$8x + 3y = 18 \quad \ldots (1)$

$4x + 5y = 9 \quad \ldots (2)$

$(2) \times 2 \Rightarrow \quad 8x + 10y = 18 \quad \ldots (3)$

$(1) \Rightarrow \quad 8x + 3y = 18 \quad \ldots (1)$

$(3) - (1) \Rightarrow \quad 7y = 0$

$$y = 0$$

Substituting in (1)

$$8x + 3(0) = 18$$

$$x = \frac{18}{8} = \frac{9}{4}$$

The point of intersection of (1) and (2) is $\left(\frac{9}{4}, 0\right)$

Mid point of the line segment joining (5, – 4) and (– 7, 6) is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{5-7}{2}, \frac{-4+6}{2}\right) = (-1, 1)$$

Equation of the straight line joining $\left(\frac{9}{4}, 0\right)$ and (– 1, 1) is

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\Rightarrow \frac{y-0}{1-0} = \frac{x - \frac{9}{4}}{-1 - \frac{9}{4}}$$

$$\Rightarrow \frac{y}{1} = \frac{4x-9}{-13}$$

$$\Rightarrow -13y = 4x - 9$$

$$\Rightarrow 4x + 13y - 9 = 0$$

Exercise 5.5

Multiple Choice Questions:

1. The area of triangle formed by the points (–5, 0), (0, – 5) and (5, 0) is

(1) 0 sq. units (2) 25 sq. units

(3) 5 s.q. units (4) none of these

[Ans : (2)]

Sol :

Points (– 5, 0), (0, – 5) and (5, 0)

Area of triangle

$$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] \text{ sq.units}$$

$$= \frac{1}{2}[-5(-5-0) + 0(0-0) + 5(0+5)]$$

$$= \frac{1}{2}[25+25] = \frac{50}{2} = 25 \text{ Sq.units.}$$

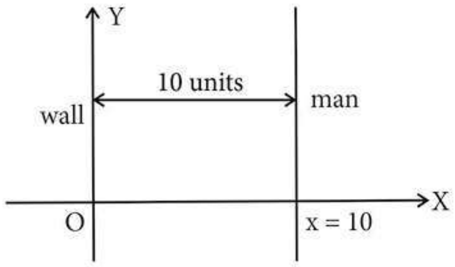
2. A man walks near a wall, such that the distance between him and the wall is 10 units. Consider the wall be the Y axis. The path travelled by the man is

(1) x = 10 (2) y = 10

(3) x = 0 (4) y = 0 **[Ans : (1)]**

Sol : x = 10

A line parallel to Y-axis.



3. The straight line given by the equation x = 11 is

(1) Parallel to X-axis
(2) Parallel to Y-axis
(3) Passing through the origin
(4) Passing through the point (0, 11)

[Ans : (2)]

Sol :

Any line is of the form

x = a is parallel to y-axis

4. If (5, 7), (3, p) and (6,6) are collinear, then the value of p is

(1) 3 (2) 6
(3) 9 (4) 12 **[Ans : (3)]**

Sol :

Given (5, 7), (3, p) and (6, 6) are Collinear.

∴ Area of the triangle formed by the points is zero.

i.e., $\frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\} = 0$

$5(p - 6) + 3(6 - 7) + 6(7 - p) = 0$

$5p - 30 - 3 + 42 - 6p = 0$

$\Rightarrow -p + 9 = 0 \Rightarrow \therefore p = 9$

5. The point of intersection of 3x – y = 4 and x + y = 8 is

(1) (5, 3) (2) (2, 4)
(3) (3, 5) (4) (4, 4) **[Ans : (3)]**

Sol :

Solving 3x – y = 4 and x + y = 8

$3x - y = 4$ ...(1)

$x + y = 8$ ...(2)

(1) + (2) ⇒ $4x = 12$

$x = 3$

Substituting in (2)

$3 + y = 8$

$y = 5$

∴ the point of intersection is (3, 5)

6. The slope of the line joining (12, 3), (4, a) is $\frac{1}{8}$. The value of 'a' is

(1) 1 (2) 4
(3) – 5 (4) 2 **[Ans : (4)]**

Sol :

Slope of the line joining (12, 3) and (4, a) is 1/8

$$\frac{y_1 - y_2}{x_1 - x_2} = \frac{3-a}{12-4} = \frac{1}{8}$$

$$\frac{3-a}{8} = \frac{1}{8}$$

$$3 - a = 1 \Rightarrow a = 2$$

7. The slope of the line which is perpendicular to line joining the points (0, 0) and (– 8, 8) is

(1) – 1 (2) 1
(3) $\frac{1}{3}$ (4) – 8 **[Ans : (2)]**

Sol :

Slope of the joining (0, 0) and (– 8, 8) is

$$\frac{y_1 - y_2}{x_1 - x_2} = \frac{0-8}{0+8} = \frac{-8}{8} = -1$$

∴ Slope of the perpendicular line is 1.

8. If slope of the line PQ is $\frac{1}{\sqrt{3}}$ then the slope of perpendicular bisector of PQ is

(1) $\sqrt{3}$ (2) $-\sqrt{3}$
(3) $\frac{1}{\sqrt{3}}$ (4) 0

[Ans : (2)]

Sol :

Slope of PQ = $\frac{1}{\sqrt{3}} = m$

Slope of perpendicular bisector = $\frac{-1}{m} = -\sqrt{3}$

9. If A is a point on the Y-axis whose ordinate is 8 and B is a point on the X-axis whose abscissa is 5 then the equation of the line AB is

(1) 8x + 5y = 40 (2) 8x – 5y = 40
(3) x = 8 (4) y = 5 **[Ans : (1)]**

Sol :

Given A is a point on y-axis and its ordinate is 8

B is a point on X-axis and its abscissa is 5

∴ The point A and B are (0, 8) and (5, 0) respectively.

Equation of AB is $\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$

$$\frac{y-8}{0-8} = \frac{x-0}{5-0}$$
$$5(y-8) = -8x$$
$$8x + 5y - 40 = 0$$
$$8x + 5y = 40$$

10. The equation of a line passing through the origin and perpendicular to the line $7x - 3y + 4 = 0$ is

(1) $7x - 3y + 4 = 0$ (2) $3x - 7y + 4 = 0$
(3) $3x + 7y = 0$ (4) $7x - 3y = 0$

[Ans : (3)]

Sol :

Slope of the line $7x - 3y + 4 = 0$ is

$$\frac{-a}{b} = \frac{-7}{-3} = \frac{7}{3}$$

Slope of the perpendicular line = $\frac{-3}{7}$

equation of the line passing through origin (0, 0) and having slope $\frac{-3}{7}$ is

$$y - y_1 = m(x - x_1)$$
$$y - 0 = \frac{-3}{7}(x - 0)$$
$$7y = -3x$$
$$3x + 7y = 0$$

11. Consider four straight line

(i) $l_1 : 3y = 4x + 5$ (ii) $l_2 : 4y = 3x - 1$
(iii) $l_3 : 4y + 3x = 7$ (iv) $l_4 : 4x + 3y = 2$

Which of the following statement is not true?

(1) l_1 and l_2 are perpendicular
(2) l_1 and l_4 are parallel
(3) l_2 and l_4 are perpendicular
(4) l_2 and l_3 are parallel **[Ans : (3)]**

Sol :

Given lines $l_1 : 3y = 4x + 5 \Rightarrow 4x - 3y + 5 = 0$

$l_2 : 4y = 3x - 1 \Rightarrow 3x - 4y - 1 = 0$

$l_3 : 4y + 3x = 7 \Rightarrow 3x + 4y - 7 = 0$

$l_4 : 4x + 3y = 2 \Rightarrow 4x + 3y - 2 = 0$

From l_2, $a_2 = 3$, $b_2 = -4$ and for l_4, $a_4 = 4$, $b_4 = 3$

$a_1a_2 + b_1b_2 = 3(4) + (-4)(3) = 0$

$\therefore$ l_2 and l_4 are perpendicular.

12. A straight line has equation $8y = 4x + 21$. Which of the following is true

(1) The slope is 0.5 and the Y-intercept is 2.6
(2) The Slope is 5 and the Y-intercept is 1.6
(3) The Slope is 0.5 and the Y-intercept is 1.6
(4) The Slope is 5 and the Y-intercept is 2.6

[Ans : (1)]

Sol : Given line is $8y = 4x + 21$

$$y = \frac{x}{2} + \frac{21}{8}$$
$$y = \frac{x}{2} + 2.625 \text{ is in the form}$$
$$y = mx + c$$

where $m = \frac{1}{2} = 0.5$, $c = 2.625$

$\therefore$ Slope $m = \frac{1}{2} = 0.5$

y intercept = 2.6

13. When proving that a quadrilateral is a trapezoid, it is necessary to show

(1) Two lines are parallel.
(2) Two parallel and two non-parallel sides.
(3) Opposite sides are parallel.
(4) All sides are of equal length. **[Ans : (2)]**

Sol :

Two parallel and two non-parallel sides.

14. When proving that a quadrilateral is a parallelogram by using slopes you must find

(1) The slopes of all four sides
(2) The slopes of any one pair of opposite sides
(3) The lengths of all four sides
(4) Both the lengths and slopes of all four sides

[Ans : (1)]

Sol : The slopes of all four sides.

15. (2, 1) is the point of intersection of two lines.

(1) $x - y - 3 = 0$; $3x - y - 7 = 0$
(2) $x + y = 3$; $3x + y = 7$
(3) $3x + y = 3$; $x + y = 7$
(4) $x + 3y - 3 = 0$; $x - y - 7 = 0$ **[Ans : (2)]**

Sol :

Substituting (2, 1) in equation (2)

$x + y = 3$ and $3x + y = 7$

both the equations are satisfied by (2, 1)

UNIT EXERCISE - 5

1. PQRS is a rectangle formed by joining the points P(– 1, –1), Q(–1, 4), R (5, 4) and S(5, – 1). A, B, C and D are the mid-points of PQ , QR , RS and SP respectively. Is the quadrilateral ABCD a square, a rectangle or a rhombus? Justify your answer.

Sol :

Given vertices of a rectangle are P (–1, –1), Q (– 1, 4), R (5, 4) and S (5, –1)

Mid point of PQ $= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$= A\left(\frac{-1-1}{2}, \frac{-1+4}{2}\right) = A\left(-1, \frac{3}{2}\right)$

Mid point of QR $= B\left(\frac{-1+5}{2}, \frac{4+4}{2}\right) = B\ (2, 4)$

Midpoint of RS $= C\left(\frac{5+5}{2}, \frac{4-1}{2}\right) = C\left(5, \frac{3}{2}\right)$

Midpoint of SP $= D\left(\frac{-1+5}{2}, \frac{-1-1}{2}\right) = D\ (2, -1)$

$$AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$= \sqrt{(2+1)^2+(4-3/2)^2} = \sqrt{9+\frac{25}{4}} = \frac{\sqrt{61}}{2}$$

$$BC = \sqrt{(5-2)^2+(3/2-4)^2} = \frac{\sqrt{61}}{2}$$

$$CD = \sqrt{(2-5)^2+(-1-3/2)^2} = \frac{\sqrt{61}}{2}$$

$$DA = \sqrt{(2+1)^2+(-1-3/2)^2} = \frac{\sqrt{61}}{2}$$

$$AC = \sqrt{(5+1)^2+(3/2-3/2)^2} = \sqrt{36} = 6$$

From triangle ABC,

$$AB^2+BC^2 = \frac{61}{4}+\frac{61}{4} = \frac{122}{4} = \frac{61}{2} \neq 36$$

i.e., $AB^2 + BC^2 \neq AC^2$

$\therefore$ The points A, B, C and D Cannot be the vertices of a square or Rectangle.

Hence, ABCD is a Rhombus.

2. The area of a triangle is 5. Two of its vertices are (2, 1) and (3, – 2). The third vertex is (x, y) where y = x + 3. Find the coordinates of the third vertex.

Sol :

Given area of triangle is 5

Vertices of triangle are (2, 1), (3, – 2) and (x, y)

Area of triangle =

$$\frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)] = 5$$

$$2(-2-y)+3(y-1)+x(1+2) = 10$$
$$-4-2y+3y-3+3x = 10$$
$$3x+y = 17 \quad \text{...(1)}$$

Given that y = x + 3, Substituting in (1)

$$\therefore\ 3x+x+3 = 17$$
$$4x = 14$$
$$x = 7/2$$
$$\text{and } y = 7/2+3$$
$$= 13/2$$

Third vertex is (7/2, 13/2)

3. Find the area of a triangle formed by the lines $3x + y - 2 = 0$, $5x + 2y - 3 = 0$, and $2x - y - 3 = 0$.

Sol : Given sides of a triangle are

$$3x+y-2 = 0 \Rightarrow 3x+y = 2 \quad \text{...(1)}$$
$$5x+2y-3 = 0 \Rightarrow 5x+2y = 3 \quad \text{...(2)}$$
$$2x-y-3 = 0 \Rightarrow 2x-y = 3 \quad \text{...(3)}$$

Solving (1) and (2)

$$(1)\times(2) \Rightarrow 6x+2y = 4 \quad \text{...(4)}$$
$$5x+2y = 3 \quad \text{...(2)}$$

$$(4)-(2) \Rightarrow x = 1$$

Substituting in (1)

$$\Rightarrow y = 2-3 = -1$$

Point of intersection of (1) and (2) is (1, – 1)

Now, solving (2) and (3)

$$(3)\times(2) \Rightarrow 4x-2y = 6 \quad \text{...(5)}$$
$$5x+2y = 3 \quad \text{...(2)}$$

$$(5)+(2) \Rightarrow 9x = 9$$
$$x = 1$$

Substituting in (3) $\Rightarrow$ y = –1

Point of intersection of (2) and (3) is (1, – 1). Since the point of intersection of (1), (2) and (2), (3) is same, No such triangle is possible.

Hence, area of triangle is zero.

4. If vertices of quadrilateral are at A (– 5, 7), B(– 4, k), C(– 1, – 6) and D(4, 5) and its area is 72 sq.units. Find the value of k.

Sol :

Given vertices of a quadrilateral are A (–5, 7), B (– 4, k), C (– 1, – 6) and D (4, 5)

Area of quadrilateral is 72 sq. units

$$\frac{1}{2}[(x_1 - x_3)(y_2 - y_4) - (x_2 - x_4)(y_1 - y_3)] = 72$$

$$(-5+1)(k-5) - (-4-4)(7+6) = 144$$

$$-4k + 20 + 104 = 144$$

$$-4k = 144 - 124 = 20$$

$$k = \frac{-20}{4} = -5$$

5. Without using distance formula, show that points (–2, – 1), (4, 0), (3, 3) and (–3, 2) are the vertices of a parallelogram.

Sol :

Given points (–2, –1), (4, 0), (3, 3) and (–3, 2) let the points be A (–2, –1), B (4, 0), C (3, 3) and D (–3, 2)

$$\text{Slope} = \frac{y_1 - y_2}{x_1 - x_2}$$

$$\text{Slope of AB} = \frac{-1-0}{-2-4} = \frac{1}{6}$$

$$\text{Slope of BC} = \frac{0-3}{4-3} = \frac{-3}{1} = -3$$

$$\text{Slope of CD} = \frac{3-2}{3+3} = \frac{1}{6}$$

$$\text{Slope of DA} = \frac{2+1}{-3+2} = \frac{3}{-1} = -3$$

Slope of AB = Slope of CD $\Rightarrow$ AB is parallel to CD

Slope of BC = Slope of DA $\Rightarrow$ BC is parallel to DA

Hence, the given points form a parallelogram

6. Find the equations of the lines, whose sum and product of intercepts are 1 and – 6 respectively.

Sol : Let a, b be the intercepts

Given, sum of the intercepts a + b = 1 ...(1)

Product of the intercepts ab = – 6 ...(2)

$$b = \frac{-6}{a}$$

Substituting in (1)

$$a - \frac{6}{a} = 1$$

$$a^2 - a - 6 = 0$$

$$(a-3)(a+2) = 0$$

$$a = -2, 3$$

When $a = -2$, $b = \frac{-6}{-2} = 3$

When $a = 3$, $b = \frac{-6}{-2} = -2$

Equation of the line in intercepts form is

$$\frac{x}{a} + \frac{y}{b} = 1$$

(i) $a = -2$, $b = 3$ $\Rightarrow \frac{x}{-2} + \frac{y}{3} = 1$

$3x - 2y + 6 = 0$

(ii) $a = 3, b = -2$ $\Rightarrow \frac{x}{3} + \frac{y}{-2} = 1$

$-2x + 3y + 6 = 0$

$\Rightarrow 2x - 3y - 6 = 0$

7. The owner of a milk store finds that, he can sell 980 litres of milk each week at ₹ 14/litre and 1220 litres of milk each week at ₹16/litre. Assuming a linear relationship between selling price and demand, how many litres could sell weekly at ₹ 17/litre?

Sol :

The relationship between selling price and demand is linear. Taking selling price along x-axis and demand along y-axis. We have two points from the data. (14, 980) and (16, 1220)

Equation of a straight line joining the points (x_1, y_1) and (x_2, y_2) is

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 980}{1220 - 980} = \frac{x - 14}{16 - 14}$$

$$\frac{y - 980}{240} = \frac{x - 14}{2}$$

$$y - 980 = 120x - 1680$$

$$120x - y - 700 = 0$$

(or) $y = 120x - 700$

when x = 17, $y = 120(17) - 700$

$= 2040 - 700 = 1340$

Hence the owner could sell 1340 litres of milk weekly at Rs.17/litre.

8. Find the image of the point (3, 8) with respect to the line x + 3y = 7 assuming the line to be a plane mirror.

Sol :

Let P be (3, 8) and Q (a, b) be the image of P.

$$\text{Slope of PQ} = \frac{y_1 - y_2}{x_1 - x_2} = \frac{8-b}{3-a} = m_1$$

Slope of the line x + 3y = 7 is $\frac{-1}{3} = m_2$

Since PQ is perpendicular to the given line,

then $m_1 \times m_2 = -1$

$$\frac{8-b}{3-a} \times \left(\frac{-1}{3}\right) = -1$$

$$8 - b = 9 - 3a$$

$$3a - b = 1 \quad \text{...(1)}$$

$$\text{Mid point of PQ} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{3+a}{2}, \frac{8+b}{2}\right)$$

Since the line x + 3y = 7 is the perpendicular bisector of PQ, the midpoint of PQ lies on the line.

$$\therefore \quad \left(\frac{3+a}{2}\right) + 3\left(\frac{8+b}{2}\right) = 7$$

$$3 + a + 24 + 3b = 14$$

$$a + 3b = -13 \quad \text{...(2)}$$

Solving (1) and (2)

$(1) \times (3) \Rightarrow \quad 9a - 3b = 3 \quad \text{...(3)}$

$a + 3b = -13 \quad \text{...(2)}$

$(3) + (2) \Rightarrow \quad 10a = -10$

$a = -1$

Substituting in (1)

$b = -4$

$\therefore$ Image of (3, 8) is (– 1, – 4).

9. Find the equation of a line passing through the point of intersection of the lines 4x + 7y – 3 = 0 and 2x – 3y + 1 = 0 that has equal intercepts on the axes.

Sol :

Given lines $4x + 7y - 3 = 0 \Rightarrow 4x + 7y = 3 \quad \text{... (1)}$

$2x - 3y + 1 = 0 \Rightarrow 2x - 3y = -1 \quad \text{... (2)}$

Solving (1) and (2)

$4x + 7y = 3 \quad \text{... (1)}$

$(2) \times 2 \Rightarrow \quad 4x - 6y = -2 \quad \text{... (3)}$

$(1) - (3) \Rightarrow \quad 13y = 5$

$$y = \frac{5}{13}$$

Substituting in (1) $\Rightarrow \quad 4x + 7\left(\frac{5}{13}\right) = 3$

$$\Rightarrow \quad x = \frac{1}{13}$$

The point intersection of (1) and (2) is $\left(\frac{1}{13}, \frac{5}{13}\right)$

Given that the line has equal intercepts on the axes.

i.e., a = b

Now,intercept form of the line is $\frac{x}{a} + \frac{y}{b} = 1$

$$\Rightarrow \quad \frac{x}{a} + \frac{y}{a} = 1 \Rightarrow x + y = a$$

This passes through $\left(\frac{1}{13}, \frac{5}{13}\right)$

$$\Rightarrow \quad \frac{1}{13} + \frac{5}{13} = a \Rightarrow a = \frac{6}{13}$$

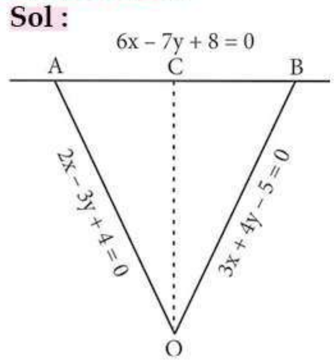
$$\therefore \quad b = \frac{6}{13}$$

Hence the equation of a line is $\frac{x}{\frac{6}{13}} + \frac{y}{\frac{6}{13}} = 1$

$\Rightarrow \quad 13x + 13y - 6 = 0.$

10. A person standing at a junction (crossing) of two straight paths represented by the equations 2x – 3y + 4 = 0 and 3x + 4y – 5 = 0 seek to reach the path whose equation is 6x – 7y + 8 = 0 in the least time. Find the equation of the path that he should follow.

Sol :



Let the person standing at 'O' and the two straight paths are OA and OB

Where OA: $2x - 3y + 4 = 0 \quad \text{... (1)}$

and OB: $3x + 4y - 5 = 0$ ... (2)

Given that the person seeks to reach the path

AB: $6x - 7y = 8 = 0$

To reach AB in least time, he needs to choose the least distance (perpendicular distance) OC.

Let as find out equation of OC

Solving OA and OB

$(1) \times (3) \Rightarrow \quad 6x - 9y = -12$ ... (3)

$(2) \times (2) \Rightarrow \quad 6x + 8y = 10$ ... (4)

$(3) - (4) \Rightarrow \quad -17y = -22$

$$y = \frac{22}{17}$$

Substituting in (2)

$$3x + 4\left(\frac{22}{17}\right) = 5$$

$$3x = 5 - \frac{88}{17} = -\frac{3}{17}$$

$$x = -\frac{1}{17}$$

The point of intersection of OA and OB is

$$O\left(-\frac{1}{17}, \frac{22}{17}\right)$$

$$\text{Slope of AB} = \frac{-6}{-7} = \frac{6}{7}$$

Now, OC is perpendicular to AB, $\therefore$ Slope of OC $= -\frac{7}{6}$

OC is passing through $\left(-\frac{1}{17}, \frac{22}{17}\right)$ and having slope $-\frac{7}{6}$

Equation of OC $\Rightarrow \quad y - y_1 = m(x - x_1)$

$$\Rightarrow \quad y - \frac{22}{17} = -\frac{7}{6}\left(x + \frac{1}{17}\right)$$

$$\Rightarrow \quad \frac{17y - 22}{17} = -\frac{7}{6}\left(\frac{17x + 1}{17}\right)$$

$$\Rightarrow \quad 102y - 132 = -119x - 7$$

$$\Rightarrow \quad 119x + 102y - 125 = 0$$

CREATIVE QUESTIONS

I. Multiple Choice Questions

Area of Triangle and Area of parallelogram

1. The area of the triangle whose vertices are (2, – 3), (3, 2) and (– 2, 5) is

(1) 11 (2) 12
(3) 14 (4) 13 **[Ans: (3)]**

Sol :

Area of triangle

$= \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$ sq. units

$= \frac{1}{2}\left[2(2 - 5) + 3(5 + 3) - 2(-3 - 2)\right]$

$= \frac{1}{2}(28) = 14$ sq. units

2. AD is the median of triangle ABC with vertices A (– 3, 2), B (5, – 2) and C (1, 3) The area of triangle ABD is

(1) 5 (2) 6
(3) 7 (4) 8 **[Ans: (2)]**

Sol :

$$\text{Area of } \Delta ABC = \frac{1}{2}\left[-3(-2-3) + 5(3-2) + 1(2+2)\right]$$

$$= \frac{1}{2}(24) = 12$$

$$\text{Area of } \Delta ABD = \frac{1}{2}(Area\ of\ \Delta ABC) = \frac{12}{2}$$

$= 6$ sq.units

3. If the points (2, 1), (3, – 2) and (a, b) are collinear, then

(1) a + b = 7 (2) 3a + b = 7
(3) a – b = 7 (4) 3a – b = 7 **[Ans: (2)]**

Sol : Since the points are collinear, Area of triangle is zero.

$$\therefore \frac{1}{2}\left[2(-2 - b) + 3(b - 1) + a(1 + 2\right] = 0$$

$$3a + b = 7$$

4. If (a, b), (c, d) and (a – c, b – d) are collinear, then

(1) $\frac{a}{b} = \frac{c}{d}$ (2) $\frac{a}{d} = \frac{b}{c}$

(3) $\frac{a}{c} = \frac{d}{b}$ (4) $\frac{a}{b} = \frac{b}{c}$ **[Ans: (1)]**

Sol : Area of triangle is zero, when the points are collinear.

$\therefore a(d-b+d)+c(b-d-b)+(a-c)(b-d) = 0$

$2ad - ab - cd + ab - ad - bc + cd = 0$

$ad - bc = 0$

$\frac{a}{b} = \frac{c}{d}$

5. If the area of the triangle formed by the points (– 2, 3), (4, – 5) and (– 3, y) is 10 square units, then y =

(1) 1 (2) – 1

(3) $\frac{23}{3}$ (4) $\frac{-22}{3}$ **[Ans: (3)]**

Sol : Area of triangle = 10

$\frac{1}{2}\left[-2(-5-y)+4(y-3)-3(3+5)\right] = 10$

$\frac{1}{2}(6y-26) = 10$

$3y - 13 = 10$

$y = \frac{23}{3}$

6. The area of quadrilateral formed by the points (0, 0), (1, 0), (1, 4) and (0, 2) is

(1) 4 (2) 8

(3) 12 (4) 16 **[Ans: (3)]**

Sol : Area of quadrilateral

$= \frac{1}{2}\left[(x_1-x_3)(y_2-y_4)-(x_2-x_4)(y_1-y_3)\right]$ sq. units

$= 8$ sq. units

(or) Plotting the points on the graph, we get the rectangle with length 4 and breadth 2.

$\therefore$ Area = 8 sq.units

7. The area of the rhombus formed by the points (3, 0), (0, 4), (– 3, 0) and (0, – 4) is

(1) 24 (2) 30

(3) 32 (4) 36 **[Ans: (1)]**

Sol : Area of Quadrilateral

$= \frac{1}{2}[(x_1-x_3)(y_2-y_4)-(x_2-x_4)(y_1-y_3)]$

$= \frac{1}{2}\left[(3+3)(4+4)-(0-0)(0-0)\right]$

$= \frac{1}{2}[48] = 24$ sq. units

8. The point (x, y) lies on the line joining (3, 4) and (– 5, – 6) if

(1) $4x - 5y = 1$ (2) $5x - 4y = 1$

(3) $5x - 4y + 1 = 0$ (4) $4x + 5y = 1$

[Ans: (3)]

Sol :

Points are collinear, $\therefore$ Area of triangle is zero.

$\frac{1}{2}\left[x(4+6)+3(-6-y)-5(y-4)\right] = 0$

$10x - 8y + 2 = 0$

$5x - 4y + 1 = 0$

Angle of Inclination and Slope of a Straight line

9. What can be said regarding a line if its slope is negative?

(1) acute (2) obtuse

(3) zero (4) None of these

[Ans: (2)]

Sol :

If slope is negative i.e., $\tan\theta$ is negative then 'θ' is obtuse.

10. What is the slope of a line whose inclination is 45°?

(1) 1 (2) 2

(3) 0 (4) $\frac{1}{2}$ **[Ans: (1)]**

Sol :

Slope m = tan 45° = 1

11. Find the inclination whose slope is $\frac{1}{\sqrt{3}}$

(1) 30° (2) 60°

(3) 90° (4) 45° **[Ans: (1)]**

Sol : Slope $m = \frac{1}{\sqrt{3}}$

i.e., $\tan\theta = \frac{1}{\sqrt{3}} = \tan 30^\circ$

$\theta = 30^\circ$

12. Slope of the line joining the points (4, – 6) and (– 2, – 5) is

(1) $\frac{1}{6}$ (2) $\frac{-1}{6}$

(3) 6 (4) – 6 **[Ans: (2)]**

Sol : Slope $= \frac{y_1-y_2}{x_1-x_2} = \frac{-6+5}{4+2} = \frac{-1}{6}$

13. The points A (1, – 2), B (3, 4) and C (4, 7)

(1) form a right triangle
(2) form an isosceles triangle
(3) form an equilateral triangle
(4) collinear **[Ans: (4)]**

Sol :

Slope of AB $= \frac{-2-4}{1-3} = \frac{-6}{-2} = 3$

Slope of BC $= \frac{4-7}{3-4} = \frac{-3}{-1} = 3$

Slope of AB = Slope of BC

$\therefore$ Points are collinear.

14. The points A (4, 4), B (3, 5) and C (– 1, – 1) form

(1) right triangle
(2) isosceles triangle
(3) equilateral triangle
(4) None of these **[Ans: (1)]**

Sol :

Slope of AB $= -1$

Slope of BC $= \frac{3}{2}$

Slope of AC $= 1$

Here, Slope of AB × Slope of AC = – 1 × 1 = – 1

$\therefore$ Points form a right triangle.

15. The value of 'x' if the slope of the line joining (2, 5) and (x, 3) is 2

(1) 4 (2) 3
(3) 2 (4) 1 **[Ans: (4)]**

Sol :

$$\text{Slope} = 2$$
$$\frac{5-3}{2-x} = 2$$
$$2 = 4 - 2x$$
$$2x = 2$$
$$x = 1$$

16. Slope of the median through B if the vertices of ΔABC are A (2, 4), B (– 3, 1) and C (4, – 7) is

(1) $\frac{12}{5}$ (2) $\frac{-12}{5}$
(3) $\frac{5}{12}$ (4) $\frac{-5}{12}$ [Ans: (4)]

Sol :

Mid-point of AC $= \left(\frac{2+4}{2}, \frac{4-7}{2}\right) = \left(3, \frac{-3}{2}\right)$

$\therefore$ Slope of median through B $= \dfrac{1+\frac{3}{2}}{-3-3} = \frac{-5}{12}$

17. The slopes of two line segments are equal. Which of the following is correct?

(1) The line segments are parallel
(2) The end points of the line segments are collinear
(3) The line segments are perpendicular
(4) The end points of line segments are non-collinear [Ans: (1) and (2)]

18. What does a linear equation with a slope of zero look like on a plane?

(1) Vertical line
(2) Horizontal line
(3) line passing through the origin
(4) y-axis [Ans: (2)]

Equations of Straight lines

19. The equation of straight line which passes through the point (2, – 3) and parallel to x-axis is

(1) x = – 2 (2) x = 2
(3) y = – 3 (4) y = 3 **[Ans: (3)]**

Sol :

Equation of any line parallel to X-axis y = b

It passes through (2, – 3)

$$y = -3$$

Hence the equation of straight line is y = – 3

20. The equation of straight line parallel to y-axis and at a distance 3 units to the right is

(1) x = 1 (2) x = 2
(3) x = – 3 (4) x = 3 **[Ans: (4)]**

Sol :

Equation of any line parallel to y-axis at a distance of 'a' units to the right is x = a

$\therefore$ Required equation is x = 3

21. The equation of straight line having slope 3 and making intercept 4 on the y-axis is

(1) 3x – y – 4 = 0 (2) 3x – y + 4 = 0
(3) 3x + y – 4 =0 (4) 3x + y + 4 = 0
[Ans: (2)]

Sol :

Given slope 'm' = 3, y intercept 'c' = 4

$\therefore$ equation of straight line is

$$y = mx + c$$
$$y = 3x + 4$$
$$3x - y + 4 = 0$$

22. The equation of straight line which passes through (– 4, 3) and having slope $\frac{1}{2}$ is

(1) $x - 2y + 10 = 0$ (2) $x - 2y - 10 = 0$
(3) $x + 2y + 10 = 0$ (4) $x + 2y - 10 = 0$

[Ans: (1)]

Sol :
Equation of straight line in slope-point form is

$$y - y_1 = m(x - x_1)$$
$$y - 3 = \frac{1}{2}(x+4)$$
$$x - 2y + 10 = 0$$

23. Equation of straight line passes through the points (0, – a) and (b, 0) is

(1) $bx - ay = ab$ (2) $ax - by = ab$
(3) $x - y = ab$ (4) $ax + by = 1$

[Ans: (2)]

Sol : Equation of straight line in two points form is

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$
$$\frac{y+a}{0+a} = \frac{x-0}{b-0}$$
$$ax - by = ab$$

24. Slope of the line $\frac{x}{a}+\frac{y}{b}=1$ is

(1) $\frac{b}{a}$ (2) $\frac{a}{b}$
(3) $\frac{-b}{a}$ (4) $\frac{-a}{b}$ **[Ans: (3)]**

Sol : Equation is $\frac{x}{a}+\frac{y}{b} = 1$

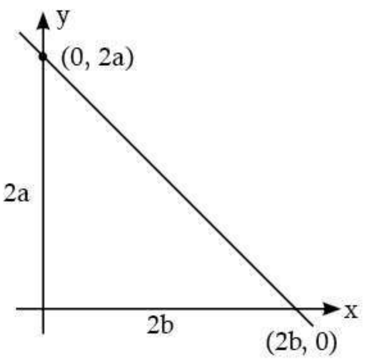
$$\text{Slope} = \frac{-\textit{Coefficient of 'x'}}{\textit{Coefficient of 'y'}} = \frac{\frac{-1}{a}}{\frac{1}{b}} = \frac{-b}{a}$$

25. Area of the triangle formed by the co-ordinate axes and the line ax + by = 2ab is

(1) ab (2) 2ab
(3) $\frac{ab}{2}$ (4) 4ab **[Ans: (2)]**

Sol : Equation of straight line is $ax + by = 2ab$
$(\div 2ab)$

$$\frac{x}{2b} + \frac{y}{2a} = 1 \text{ which is in intercept form}$$



x intercept is '2b'
y intercept is '2a'

$$\text{Area of triangle} = \frac{1}{2}\times 2b \times 2a$$
$$= 2\,ab$$

26. If the line y = mx meets the lines x + 2y – 1 = 0 and 2x – y + 3 = 0 at the same point, then m is

(1) 1 (2) – 1
(3) 2 (4) – 2 **[Ans: (2)]**

Sol :
Solving $x + 2y - 1 = 0$ and $2x - y + 3 = 0$, the point of intersection is (– 1, 1)

$y = mx$ passes through (– 1, 1)

$\therefore$ $1 = m(-1) \Rightarrow m = -1$

27. Equation of the line perpendicular to x = 2 and passing through the point (2, – 8) is

(1) y = 8 (2) y = – 8
(3) x = 8 (4) x = – 2 **[Ans: (2)]**

Sol :
Slope of any line perpendicular to the line of the form x = a is '0'

$\therefore$ Equation is

$$y - y_1 = m(x - x_1)$$
$$y + 8 = 0\,(x - 2)$$
$$y + 8 = 0 \Rightarrow y = -8$$

28. Equation of straight line which cuts off intercepts 2 and 3 from the co-ordinate axes is

(1) $2x - 3y - 6 = 0$ (2) $2x + 3y - 6 = 0$
(3) $3x - 2y - 6 = 0$ (4) $3x + 2y - 6 = 0$

[Ans: (4)]

Sol : Equation of line in intercept form is

$$\frac{x}{a}+\frac{y}{b} = 1$$
$$\frac{x}{2}+\frac{y}{3} = 1$$
$$3x + 2y = 6$$
$$3x + 2y - 6 = 0$$

General form of Straight line

29. General equation of a straight line is

(1) $\frac{-a}{b}+by+\frac{c}{b}=0$

(2) $ax^2 + by^2 + c = 0$

(3) $y = mx + c$

(4) $ax + by + c = 0$ [Ans: (4)]

30. Equation of line parallel to $ax + by + c = 0$ is

(1) $x + y + k = 0$

(2) $ax + by + k = 0$

(3) $x + y = - c$

(4) $bx + ay = c$ [Ans: (2)]

31. $ax + by + c = 0$ represents a line parallel to x-axis if

(1) $a = 0, b = 0$ (2) $a=0, b\neq 0$

(3) $a\neq 0, b=0$ (4) $c = 0$ **[Ans: (2)]**

32. The condition for the lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ to be perpendicular is

(1) $a_1a_2 + b_1b_2 = 0$ (2) $a_1b_1 + a_2b_2 = 0$

(3) $a_1a_2 - b_1b_2 = 0$ (4) $a_1b_1 - a_2b_2 = 0$

[Ans: (1)]

33. The lines $3x + 4y + 7 = 0$ and $4x - 3y + 5 = 0$ are

(1) Parallel

(2) Perpendicular

(3) Neither parallel nor perpendicular

(4) Parallel and Perpendicular **[Ans: (2)]**

Sol :

$3x + 4y + 7 = 0$ and $4x - 3y + 5 = 0$

$a_1 = 3, b_1 = 4, a_2 = 4, b_2 = -3$

Now $a_1a_2 + b_1b_2 = (3)(4) + (4)(-3)$

$= 12 - 12 = 0$

$\therefore$ The lines are perpendicular.

34. Equation of line perpendicular to $2x + 5y = 7$ and passing through the point $(-1, 4)$ is

(1) $x - y + 13 = 0$ (2) $x + y + 13 = 0$

(3) $2x + 5y + 13 = 0$ (4) $5x - 2y + 13 = 0$

[Ans: (4)]

Sol :

Slope of the line $2x + 5y = 7$ is $-\frac{2}{5}$

$\therefore$ Slope of perpendicular line is $\frac{5}{2}$

Required equation is

$$y - 4 = \frac{5}{2}(x+1)$$

$$2y - 8 = 5x + 5$$

$$\Rightarrow 5x - 2y + 13 = 0.$$

35. Find the value of k if the staight lines $(2 + 6k)x + (3 - k)y + (4 + 12k) = 0$ and $7x + 5y - 4 = 0$ are perpendicular.

(1) $\frac{29}{37}$ (2) $-\frac{29}{37}$

(3) $\frac{37}{29}$ (4) $-\frac{37}{29}$ [Ans: (2)]

Sol :

Since the lines are perpendicular

$$a_1a_2 + b_1b_2 = 0$$

$$(2 + 6k)(7) + (3 - k)(5) = 0$$

$$14 + 42k + 15 - 5k = 0$$

$$29 + 37k = 0 \Rightarrow k = -\frac{29}{37}$$

36. The value of k if the lines $4x + ky = 8$ and $4x + 3y = 5$ are parallel is

(1) 3 (2) 5

(3) 4 (4) 2 [Ans: (1)]

Sol :

When two lines are parallel, the Co-efficients of 'x' are equal and Co-efficients of 'y' are equal.

II. Very Short Answer Questions

1. Find the area of triangle whose vertices are (1, 1), (2, 3) and (4, 5).

Sol :

$$\text{Area of triangle} = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] \text{ sq. units}$$

$$= \frac{1}{2}[1(3-5) + 2(5-1) + 4(1-3)]$$

$$= \frac{1}{2}[-2 + 8 - 8]$$

$$= \frac{-2}{2} = -1$$

$= 1$ sq. unit [$\because$ area cannot be negative].

2. Find the area of triangle ABC, where A (0, 0), B(3, 4) and C (0, 3).

Sol :

$$\text{Area of triangle} = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] \text{ sq. units}$$

$$= \frac{1}{2}[0(4-3)+3(3-0)+0(0-4)]$$

$$= \frac{1}{2}(9) = \frac{9}{2} \text{ sq. units}$$

3. Find the value of k, for which the points (7, – 2), (5, 1) and (3, – k) are collinear?

Sol : For collinear points, Area of triangle is zero.

$$\text{i.e., } \frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)] = 0$$

$$7(1+k)+5(-k+2)+3(-2-1) = 0$$

$$7+7k-5k+10-9 = 0$$

$$2k = -8$$

$$k = -4$$

4. Find the area of quadrilateral whose vertices are (– 1, – 1), (– 1, 4), (5, 4) and (5, – 1)

Sol :

$$\text{Area of Quadrilateral} = \frac{1}{2}[(x_1-x_3)(y_2-y_4) - (x_2-x_4)(y_1-y_3)] \text{ sq. units}$$

$$= \frac{1}{2}[(-1-5)(4+1)-(-1-5)(-1-4)]$$

$$= \frac{1}{2}[-30-30]$$

$$= -\frac{60}{2} = -30$$

Area = 30 sq. units

[$\because$ area cannot be negative.]

5. Find the Slope or Gradient of a line whose angle of inclination is (i) 45° (ii) 60°.

Sol :

(i) Angle of inclination $\theta = 45^\circ$

Slope $m = \tan\theta$

$m = \tan 45^\circ = 1$

(ii) When $\theta = 60^\circ$,

slope $m = \tan 60^\circ = \sqrt{3}$

6. Find the slope of the line that passes through the points (2, 0) and (3, 4).

Sol : Slope of the line joining two points (x_1, y_1) and (x_2, y_2) is

$$\frac{y_1-y_2}{x_1-x_2} = \frac{0-4}{2-3} = \frac{-4}{-1} = 4 = m.$$

7. What is the slope of the line parallel to the line whose slope is 2 ?

Sol :

When the lines are parallel, their slopes are equal.

Slope of the required line = 2.

8. Are the three points A (2, 3), B (5, 6) and C (0, – 2) collinear?

Sol :

$$\text{Slope of AB} = \frac{y_1-y_2}{x_1-x_2} = \frac{3-6}{2-5} = \frac{-3}{-3} = 1$$

$$\text{Slope of BC} = \frac{y_1-y_2}{x_1-x_2} = \frac{6+2}{5-0} = \frac{8}{5}$$

Slope of AB $\neq$ Slope of BC

$\therefore$ The points are not collinear.

9. Find the equation of the straight line passing through the points (a, b) and (a + b, a – b).

Sol :

Equation of the straight line passing through the points (x_1, y_1) and (x_2, y_2) is

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

$$\Rightarrow \frac{y-b}{a-b-b} = \frac{x-a}{a+b-a}$$

$$\Rightarrow \frac{y-b}{a-2b} = \frac{x-a}{b}$$

$$\Rightarrow b(y-b) = (x-a)(a-2b)$$

$$\Rightarrow by-b^2 = ax-2bx-a^2+2ab$$

$$\Rightarrow (a-2b)x-by-a^2+2ab+b^2 = 0$$

10. Find the equation of the line passing through (1, 2) and making an angle of 30° with Y-axis.

Sol :

Given that the line makes an angle 30° with Y-axis. i.e., the line makes 60° with the positive direction of X-axis.

$\therefore$ Slope of the line $= m = \tan 60^\circ = \sqrt{3}$

Equation of the line passing through the point (x_1, y_1) and having slope 'm' is $y - y_1 = m(x - x_1)$

$$\Rightarrow y - 2 = \sqrt{3}(x-1)$$

$$\Rightarrow y - 2 = \sqrt{3}x - \sqrt{3}$$

$$\Rightarrow \sqrt{3}x - y + 2 - \sqrt{3} = 0.$$

11. Find the equation of the line whose x intercept is 4 and y intercept is $-\frac{3}{2}$.

Sol :

Equation of straight line is $\frac{x}{a} + \frac{y}{b} = 1$

$$\Rightarrow \frac{x}{4} + \frac{y}{-\frac{3}{2}} = 1$$

$$\Rightarrow \frac{x}{4} - \frac{2y}{3} = 1$$

$$\Rightarrow 3x - 8y - 12 = 0.$$

12. Find the values of k if the straight line $2x + 3y + 4 + k(6x - y + 12) = 0$ is perpendicular to the line $7x + 5y - 4 = 0$.

Sol :

The two lines are $x(2 + 6k) + y(3 - k) + 4 + 12k = 0$ and $7x + 5y - 4 = 0$

Slope of the first line $= m_1 = -\frac{(2+6k)}{3-k}$

Slope of the second line $= m_2 = -\frac{7}{5}$

The lines are perpendicular, $\therefore$ $m_1 \times m_2 = -1$

$$-\frac{(2+6k)}{3-k} \times -\frac{7}{5} = -1$$

$$14 + 42k = -15 + 5k$$

$$k = -\frac{29}{37}$$

13. A line passing through the points (a, 2a) and (–2, 3) is perpendicular to the line $4x + 3y + 5 = 0$, find the values of 'a'.

Sol :

Slope of the line $4x + 3y + 5 = 0$ is $-\frac{4}{3} = m_1$

Slope of the line joining (a, 2a) and (– 2, 3) is $\frac{2a-3}{a+2} = m_2$

Since, the lines are perpendicular, $m_1 \times m_2 = -1$

$$\Rightarrow -\frac{4}{3} \times \frac{2a-3}{a+2} = -1$$

$$\Rightarrow 8a - 12 = 3a + 6$$

$$\Rightarrow a = \frac{18}{5}$$

14. Find the angle between the lines $x = a$ and $by + c = 0$.

Sol :

$x = a$ is parallel to Y-axis and

$by + c = 0 \Rightarrow y = -\frac{c}{b}$ is parallel to X-axis

$\therefore$ The angle between the two lines is 90°.

III. Short Answer Questions:

1. Find the area of the triangle formed by the points (a, c + a), (a, c) and (– a, c – a).

Sol :

Area of triangle $= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$ sq. units

$= \frac{1}{2}[a(c - c + a) + a(c - a - c - a) - a(c + a - c)]$

$= \frac{1}{2}[a(a) + a(-2a) - a(a)]$

$= \frac{1}{2}(-2a^2) = -a^2$

Area $= a^2$ sq. units. [$\because$ Area cannot be negative]

2. Find the value of p if the points (p + 1, 1), (2p + 1, 3) and (2p + 2, 2p) are collinear.

Sol :

Area of triangle is zero, Since the points are collinear.

i.e., $x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$

$(p + 1)(3 - 2p) + (2p + 1)(2p - 1) + (2p + 2)(1 - 3) = 0$

$3p - 2p^2 + 3 - 2p + 4p^2 - 1 - 4p - 4 = 0$

$$2p^2 - 3p - 2 = 0$$
$$(2p + 1)(p - 2) = 0$$
$$2p + 1 = 0, \quad p - 2 = 0$$
$$p = -\frac{1}{2}, \quad p = 2$$

3. Prove that the points A (0, – 1), B (2, 1) and C(– 4, 3) form a right angled triangle.

Sol :

$$\text{Slope of AB} = \frac{y_1 - y_2}{x_1 - x_2} = \frac{-1-1}{0-2} = \frac{-2}{-2} = 1$$

$$\text{Slope of BC} = \frac{1-3}{2+4} = -\frac{2}{6} = -\frac{1}{3}$$

$$\text{Slope of AC} = \frac{-1-3}{0+4} = -\frac{4}{4} = -1$$

(Slope of AB) × (Slope of AC) = 1 × – 1 = – 1

$[\therefore \ m_1 \times m_2 = -1]$

∴ The given points form a right triangle.

4. Show that the points A (– 2, 0), B (2, 4), C (4, 1) and D (0, – 3) form a parallelogram.

Sol :

$$\text{Slope of AB} = \frac{y_1 - y_2}{x_1 - x_2} = \frac{0-4}{-2-2} = \frac{-4}{-4} = 1$$

$$\text{Slope of BC} = \frac{4-1}{2-4} = -\frac{3}{2}$$

$$\text{Slope of CD} = \frac{1+3}{4-0} = \frac{4}{4} = 1$$

$$\text{Slope of AD} = \frac{0+3}{-2-0} = -\frac{3}{2}$$

Slope of AB = Slope of CD,
Slope of BC = Slope of AD

∴ The given points form a parallelogram.

5. The Fahrenheit temperature F and absolute temperature K satisfy a linear equation. Given that K = 273 when F = 32 and that K = 373 when F = 212. Express K in terms of F and find the value of F when K = 0.

Sol : Assuming F along X-axis and K along Y-axis.

The two points are (32, 273) and (212, 373).

The equation of straight line passing through the points (32, 273) and (212, 373) is

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\Rightarrow \frac{K - 273}{373 - 273} = \frac{F - 32}{212 - 32}$$

$$K = \frac{5}{9}(F - 32) + 273 \qquad \text{.......(1)}$$

Putting K = 0 in (1), we get

$$0 = \frac{5}{9}(F - 32) + 273$$

$$\Rightarrow F = 32 - 491.4 = -459.4$$

6. If the intercept of a line between the co-ordinate axes is divided by point (– 5, 4) in the ratio 1 : 2, then find the equation of a line.

Sol :

Let the equation of the line be $\frac{x}{a} + \frac{y}{b} = 1$

Which meets X-axis at A (a, 0) and Y-axis at B (0, b).

Given that P (– 5, 4) dividing AB in the ratio 1 : 2

Using section formula, $\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$

$$\Rightarrow \left(\frac{1(0) + 2(a)}{1+2}, \frac{1(b) + 2(0)}{1+2}\right) = (-5, 4)$$

$$\Rightarrow \left(\frac{2a}{3}, \frac{b}{3}\right) = (-5, 4)$$

$$\frac{2a}{3} = -5, \quad \frac{b}{3} = 4$$

$$a = -\frac{15}{2}, \quad b = 12$$

∴ The equation of line is $\frac{x}{a} + \frac{y}{b} = 1.$

$$\text{i.e., } -\frac{2x}{15} + \frac{y}{12} = 1$$

$$\Rightarrow 8x - 5y + 60 = 0.$$

7. Find the equations of the straight lines which pass through (4, 3) and are respectively parallel and perpendicular to the x-axis.

Sol :

(i) Slope of a line parallel to X-axis is '0'

i.e., $m = 0$

Equation of a line passing through (x_1, y_1) and having slope 'm' is

$$y - y_1 = m(x - x_1)$$
$$\Rightarrow y - 3 = 0(x - 4)$$
$$\Rightarrow y - 3 = 0$$
$$\Rightarrow y = 3$$

(ii) Slope of a line perpendicular to X-axis is undefined i. e., $\frac{1}{0} = m$

Equation of a straight line is $y - 3 = \frac{1}{0}(x - 4)$

$$\Rightarrow x - 4 = 0$$
$$\Rightarrow x = 4$$

8. If the straight line y = mx + c passes through the points (2, 4) and (– 3, 6). Find the values of m and c.

Sol :

$y = mx + c$ passes through (2, 4)

$\therefore\ 4 = 2m + c$ ... (1)

Again $y = mx + c$ passes through (– 3, 6)

Then, $6 = -3m + c$ ... (2)

Solving (1) and (2)

$$\Rightarrow m = -\frac{2}{5}$$

Substituting in (1)

$$\Rightarrow c = \frac{24}{5}$$

9. Find the equation of the line passing through the point (2, 1) and parallel to join of the points (1, 3) and (– 3, 1).

Sol : Slope of the line joining the points (1, 3) and (– 3, 1) is

$$\frac{y_1 - y_2}{x_1 - x_2} = \frac{3-1}{1+3} = \frac{2}{4} = \frac{1}{2}$$

Required line is parallel to the line joining the points (1, 3) and (– 3, 1).

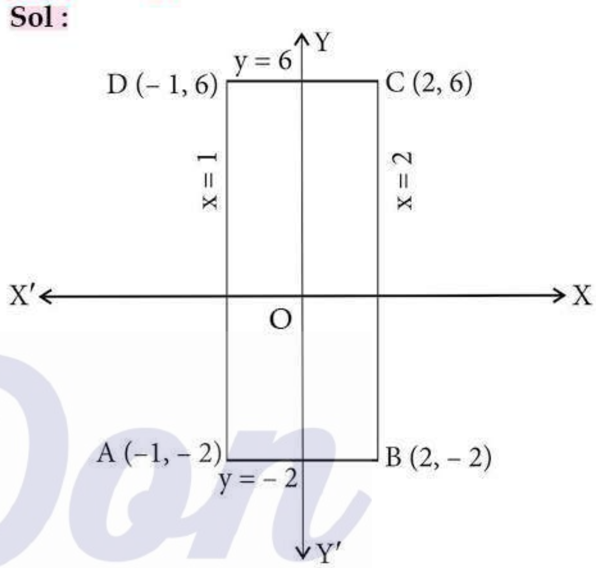
$\therefore$ Slope of the required line $= \frac{1}{2}$ as slopes are equal Since the lines are parallel.

Equation of line passing through the point (x_1, y_1) and having slope 'm' is

$$y - y_1 = m(x - x_1)$$
$$\Rightarrow y - 1 = \frac{1}{2}(x - 2)$$
$$\Rightarrow 2(y - 1) = x - 2$$
$$\Rightarrow x - 2y = 0.$$

10. Find the equation of the diagonals of the rectangle whose sides are x = 2, x = – 1, y = 6 and y = – 2.

Sol :



$$\text{Equation of AC} = \frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$
$$\Rightarrow \frac{y+2}{6+2} = \frac{x+1}{2+1}$$
$$\Rightarrow 3(y + 2) = 8(x + 1)$$
$$\Rightarrow 8x - 3y + 2 = 0$$

$$\text{Equation of BD} \Rightarrow \frac{y+2}{6+2} = \frac{x-2}{-1-2}$$
$$\Rightarrow -3(y + 2) = 8(x - 2)$$
$$\Rightarrow 8x + 3y - 10 = 0.$$

11. Show that the line $a^2x + ay + 1 = 0$ is perpendicular to the line x – ay = 0 for all non-zero real values of 'a'.

Sol :

$$\text{Slope of } a^2x + ay + 1 = 0 \text{ is } -\frac{a^2}{a} = -a = m_1$$

$$\text{Slope of } x - ay = 1 \text{ is } \frac{-1}{-a} = \frac{1}{a} = m_2$$

$$\text{Now, } m_1 \times m_2 = -a \times \left(\frac{1}{a}\right) = -1$$

$\therefore$ The lines are perpendicular.

12. Find the equation of the perpendicular bisector of the line segment joining the points (1, 1) and (2, 3).

Sol :

Let P be the mid point of the line segment joining A(1, 1) and B (2, 3)

$$\Rightarrow \quad P\left(\frac{3}{2}, 2\right)$$

Let 'm' be the slope of perpendicular bisector of AB .

$$m \times \text{Slope of AB} = -1$$

$$m \times \left(\frac{3-1}{2-1}\right) = -1$$

$$m = -\frac{1}{2}$$

Perpendicular bisector passes through $\left(\frac{3}{2}, 2\right)$ and having slope $m = -\frac{1}{2}$

Equation of perpendicular bisector is

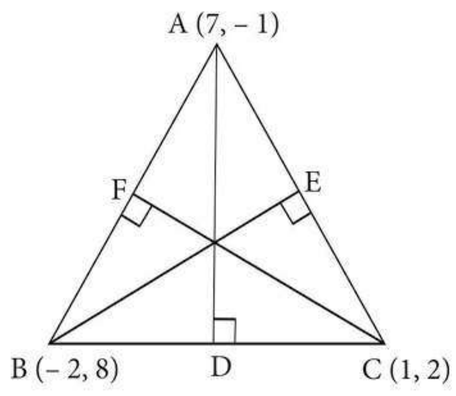
$$y - y_1 = m(x - x_1)$$

$$\Rightarrow \quad y - 2 = -\frac{1}{2}\left(x - \frac{3}{2}\right)$$

$$\Rightarrow \quad 2x + 4y - 11 = 0.$$

13. Find the equations of the altitudes of the triangle whose vertices are A (7, – 1), B (– 2, 8) and C (1, 2).

Sol :



Let AD, BE and CF be three altitudes of triangle ABC. Let m_1, m_2 and m_3 be the slopes of AD, BE and CF respectively.

AD perpendicular to BC

$$\Rightarrow \quad \text{Slope of AD} \times \text{Slope of BC} = -1$$

$$m_1 \times \left(\frac{2-8}{1+2}\right) = -1$$

$$\therefore \quad m_1 = \frac{1}{2}$$

BE perpendicular to AC $\Rightarrow$

$$\text{Slope of BE} \times \text{Slope of AC} = -1$$

$$m_2 \times \left(\frac{-1-2}{7-1}\right) = -1$$

$$\therefore \quad m_2 = 2$$

and CF perpendicular to AB $\Rightarrow$

$$\text{Slope of CF} \times \text{Slope of AB} = -1$$

$$m_3 \times \left(\frac{-1-8}{7+2}\right) = -1$$

$$m_3 = 1$$

Since AD passes through A (7, –1) and has slope $m_1 = \frac{1}{2}$, then its equation is $y - y_1 = m(x - x_1)$

$$y + 1 = \frac{1}{2}(x - 7)$$

$$\Rightarrow \quad x - 2y - 9 = 0$$

BE passes through B (– 2, 8) and having slope $m_2 = 2$,

$$\text{Its equation is } y - 8 = 2(x + 2)$$

$$\Rightarrow \quad 2x - y + 12 = 0$$

CE passes through C (1, 2) and has slope $m_3 = 1$

$$\text{Its equation is } y - 2 = 1(x - 1)$$

$$\Rightarrow \quad x - y + 1 = 0.$$

IV. Long Answer Questions

1. If the points A (0, 1), B (x, y), C (5, – 2) and D (2, – 1) form a parallelogram, then find the values of x, y.

Sol :

$$\text{Slope of AB} = \frac{y_1 - y_2}{x_1 - x_2}$$

$$= \frac{1-y}{0-x}$$

$$= \frac{1-y}{-x}$$

$$\text{Slope of BC} = \frac{y+2}{x-5}$$

$$\text{Slope of CD} = \frac{-2+1}{5-2}$$

$$= -\frac{1}{3}$$

$$\text{Slope of DA} = \frac{-1-1}{2-0}$$

$$= -\frac{2}{2} = -1$$

In parallelogram ABCD, AB is parallel to CD

$$\therefore \text{ Slope of AB} = \text{Slope of CD}$$

$$\frac{1-y}{-x} = -\frac{1}{3}$$

$$\Rightarrow 3 - 3y = x$$

$$\Rightarrow x + 3y = 3 \quad \text{... (1)}$$

BC is parallel to AD

$$\text{Slope of BC} = \text{Slope of AD}$$

$$\frac{y+2}{x-5} = -1$$

$$\Rightarrow y + 2 = -x + 5$$

$$\Rightarrow x + y = 3 \quad \text{... (2)}$$

Solving (1) and (2)

$$x + 3y = 3 \quad \text{... (1)}$$

$$x + y = 3 \quad \text{... (2)}$$

$$(1) - (2) \Rightarrow 2y = 0$$

$$y = 0$$

Substituting in (2)

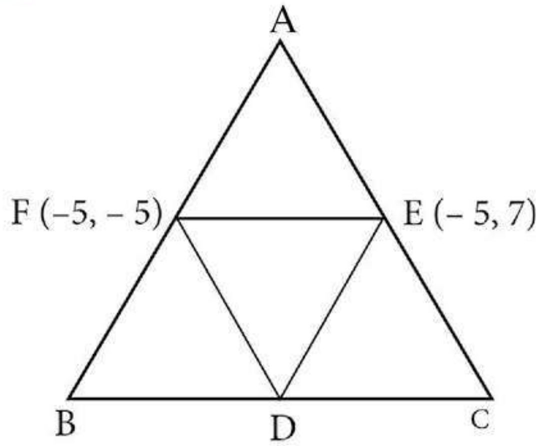
$$x + 0 = 3$$

$$x = 3$$

$\therefore$ x = 3 and y = 0 and the point B is (3, 0).

2. The mid points of the sides of a triangle are (2, 1), (– 5, 7) and (– 5, – 5). Find the equation of the sides.

Sol :



Let D (2, 1), E (– 5, 7) and F (– 5, – 5) be the mid points of the sides BC, CA and AB of a ΔABC.

$$\text{Slope of EF} = \frac{-5-7}{-5+5}$$

$$= -\frac{12}{0} \quad \text{(undefined.)}$$

$\therefore$ EF is parallel to Y-axis

But EF parallel to BC [$\because$ line joining mid points of two sides is parallel to third side and half of it]

Equation of any line parallel to Y-axis at a distance of 'a' units is x = a.

If it passes through (2, 1), then 2 = a

$\therefore$ Equation of side BC is x = 2

$$\text{Now, Slope of FD} = \frac{1+5}{2+5} = \frac{6}{7}$$

By Geometry, CA is parallel to FD

$$\therefore \text{ Slope of CA} = \frac{6}{7}$$

CA passes through the point E (–5, 7)

$$\therefore \text{ Equation of side CA is } y - 7 = \frac{6}{7}(x+5)$$

$$\Rightarrow 6x - 7y + 79 = 0$$

$$\text{Now, slope of DE} = \frac{7-1}{-5-2} = \frac{-6}{7}$$

AB is parallel to DE

$$\text{Slope of AB} = -\frac{6}{7}$$

AB passes through (– 5, – 5)

$$\therefore \text{ Equation of AB is } y + 5 = -\frac{6}{7}(x+5)$$

$$\Rightarrow 6x + 7y + 65 = 0.$$

3. Find the equation to the straight line which passes through the points (3, 4) and have intercepts on the axes.

(i) equal in magnitude but opposite in sign

(ii) such that their sum is 14.

Sol :

(i) Let the intercepts on the axes be 'a' and '– a' respectively.

Equation of line in intercept from is $\frac{x}{a}+\frac{y}{-a}=1$

$$\Rightarrow x - y = a$$

Since, this passes through (3, 4) then

$$a = 3 - 4 = -1$$

$\therefore$ The equation is $x - y + 1 = 0$

(ii) Given sum of the intercepts is 14.

$$\text{i.e., } a + b = 14 \Rightarrow b = 14 - a$$

Equation of straight line is $\frac{x}{a}+\frac{y}{b} = 1$

$$\Rightarrow \frac{x}{a}+\frac{y}{14-a} = 1$$

Since, this passes through (3, 4) then

$$\frac{3}{a}+\frac{4}{14-a} = 1$$

$$\Rightarrow a^2 - 13a + 42 = 0$$

$$\Rightarrow (a-6)(a-7) = 0$$

$$\Rightarrow a = 6, 7$$

When $a = 6, b = 8$

Equation of a straight line is $\frac{x}{6}+\frac{y}{8} = 1$

$$\Rightarrow 4x + 3y = 24$$

When $a = 7, b = 7$

Equation of a straight line is $\frac{x}{7}+\frac{y}{7} = 1$

$$\Rightarrow x + y = 7$$