Combinatorics and Mathematical Induction. ஸ்ரீ
Chapter – 4

Fundamental Rules of counting

1) **Sum Rule:** Let us consider two tasks which need to be completed in m different way and the second in n different ways if these cannot be performed simultaneously (they are independent) then there are m+n ways of doing either task. This is the sum of rule of counting. (clue word for Sum Rule (either) or (or))

This Rule is allowed in one task after another task.

2) **The product Rule:** Let us suppose that a task comprises of two procedures. If the first procedure can be completed in m different ways and the second procedure can be done in n different ways after the first procedure is done then the total number of ways of completing the task is taken together. m×n ways. (clue word for product Rule is (and))

3) **The inclusion-Exclusion principle:** suppose two tasks A and B can be performed simultaneously. Let n(A), n(B) represents the number of ways of performing the tasks A and B independent of each other. Also let n(A∩B) be the number of ways of performing the two tasks simultaneously. the sum of ways completing the tasks. (Both and, or are included)

$$n(A \cup B) = n(A) + n(B) - n(A \cap B).$$

4) **The Pigeonhole principle** - suppose a flock of pigeon fly into set of pigeonholes. If there are more pigeons than pigeonholes then there must be at least one pigeonhole with at least two pigeons in it. If K+1 (or) more objects are placed in K boxes, then there is at least one box containing two or more of the objects.

5) $n! = n \cdot (n-1)(n-2) \cdots 3 \cdot 2 \cdot 1.$

$n! = n(n-1)!$

$n! = n(n-1)(n-2)!$

Permutations: The number of permutations in n things taken r at a time is $nP_r = \frac{n!}{(n-r)!}$ $(r \leq n)$

1) $nP_r = \frac{n!}{(n-r)!}$ $r \le n$; 0 $r > n$ 2) $nP_r = nP_n = n!$ $r = n$; $nP_0 = 1$ $r = 0$

Note: The n different objects arranged in a row is $nP_n = n!$ ways.

I 2) The number of permutations of n different objects taken r at a time where repetitions is allowed is n^r.

3) $nP_n = nP_{n-1}$

4) $nP_r = n \cdot (n-1)P_{r-1}$

5) $nP_r = (n-1)P_r + r \cdot (n-1)P_{r-1}$

Objects always together:

II The number of permutations of n different objects, taken all at a time when m specified objects are always together is $= m! \times (n-m+1)!$

No two things are together: To obtain the number of permutation of n different objects when no two of k given objects occur

III together and there are no restrictions on the remaining $m = n-k$ objects is $m! \times (m+1)P_k$

Permutations of not all distinct objects

Number of permutations of n objects where p objects are of the same kind and rest are all distinct is $\frac{n!}{p!}$

Note: The number of permutations of n objects where

IV p_1 objects are one kind, p_2 objects are of second kind ---- p_k are of k^{th} kind and rest of the other are distinct kind is

$$= \frac{n!}{p_1!\, p_2! \cdots p_k!}$$

Combinations: The number of combinations of n distinct objects taken r at a time is given by $nC_r = \frac{n!}{(n-r)!\, r!}$ $0 \le r \le n$

Note $r!\, nC_r = nP_r$

Properties of combinations:

1) $nC_0 = 1$, 2) $nC_n = 1$ 3) $nC_r = \frac{n(n-1)(n-2)\ldots(n-r+1)}{r!}$

4) $nC_r = nC_{n-r}$

5) $nC_r + nC_{r-1} = (n+1)C_r$.

6) $nC_r = \frac{n}{r} \times (n-1)C_{(r-1)}$

Important results on permutations:

1. The number of permutations of n different things taken r at a time when each thing may be repeated any number of times is n^r

2. The number of permutations of n different things taken all at a time is $nP_n = n!$

3. The number of permutations of n things taken all at a time in which p are alike of one kind, q are alike of second kind and r alike of third kind and the rest are different is $= \frac{n!}{p!q!r!}$.

4) The number of permutations of n things of which P_1 are alike of one kind, P_2 are alike of second kind P_3 are alike of third kind ... P_r are alike of r^{th} kind s.t $P_1 + P_2 + P_3 + \ldots + P_r = n$. Then $\frac{n!}{P_1!P_2!P_3!\ldots P_r!}$.

5. Number of permutations of n different things taken r at a time in each arrangements is

a) when a particular thing to be included
$r \cdot (n-1)P_{r-1}$

b) when the particular thing is always excluded then the number of arrangements $= n-1P_r$.

c) Number of permutations of n different things, taken r at a time when P particular things are to be always included in each arrangements $P!(r-(P-1))^{n-p}P_{r-p}$.

6. Number of permutations of n different things taken all at a time when m specified things always come together is $= m!(n-m+1)!$.

7) Number of permutations of n different things taken all at a time, when m specified things never come together is $n! - m! \times (n-m+1)!$.

8) The number of ways in which $(m+n)$ different things can be divided into two groups which contain m and n things respectively $= \frac{(m+n)!}{m!\,n!}$.

9) circular permutations: In a circular permutations, firstly we fix the position of one of the objects and then arrange the other objects in all possible ways

1) Number of circular permutations of n different things taken all at a time is $(n-1)!$ If clockwise and anticlockwise orders are taken as different.

2) Number of circular permutations of n different things taken all at a time when clockwise or anticlockwise is not different $= \frac{1}{2}(n-1)!$

3) Number of circular permutation of n different things taken r at a time, when clockwise or anticlockwise orders are not different is $\frac{{}^nP_r}{r}$

4) Number of circular permutation of n different things taken r at a time when clockwise (or) anticlockwise orders are not different is $\frac{{}^nP_r}{2r}$.

5) If we make 1 to n on chairs in a round table, then n persons sitting around table is $n!$.

1. The sum of all r digit numbers that can be formed using the given n non zero digits is ${}^{(n-1)}P_{r-1} \times$ Sum of the digits $\times 111 \cdots r$ times.

2) If 0 is one digit among the given n digits then we get the sum of the r digits number.

$({}^{n-1}P_{r-1} \times (S.D) \times 11 \cdots r \text{ times}) - ({}^{n-2}P_{r-2} \times S.D \times 111 \cdots r-1 \text{ times})$

1. Number of diagonals in a polygon with n sides $= \frac{n(n-3)}{2}$

Penta — 5 (sides)

Septa = hepta = 7 (sides)

Fundamental principles of counting.

Eg: 4.1. Suppose one girl or one boy has to be selected for a competition from a class comprising 17 boys and 29 girls. In how many different ways can this selection be made.

Sum Rule: The first task of selecting girls in 29 ways

The second task of selecting boys in 17 ways.

Total number of ways $: 29+17$
$= 46$ ways.

Eg 4.2 Consider the three cities chennai, Tiruchy and Tirunelveli. In order to reach Tirunelveli from chennai one has to pass through Tiruchy. There 2 roads connecting chennai to Tiruchy and there are 3 roads connecting Tiruchy with Tirunelveli. What are the total number of ways of travelling from chennai to Tirunelveli. (one after another)

Product Rule: Number of ways to chennai to Tiruchy $= 2$

Number of ways to Tiruchy to Tirunelveli $= 3$.

∴ Total number of ways $: 2\times 3 = 6$.

Eg: 4.3. A school library has 75 books of Mathematics, 35 books of physics. A student can choose only one book. In how many ways a student can choose a book on Maths or physics.

Sum Rule: Number of ways of choosing Maths Books $= 75$

Number of ways of choosing physics Bo. $= 35$

∴ Total ways $= 75+35$.
$= 110$.

Eg: 4.4: In an electricity consumer has the consumer number say 238:110:29, then describe the linking and count the number of house connections up to 29th consumer connecting linked to the larger capacity transformer number 238 subject to the condition that each smaller capacity transformer can have a maximal consumer link of say 100.

Eg: 4.5 A person wants to buy a car. There are two brands of car available in the market and each brand has 3 variant models and each model comes to five different colours. In how many ways she can choose a car to buy?

one after the other { Product Rule }. A brand can be chosen in 2 ways.
Model can be chosen in 3 ways.
colour can be chosen in 5 ways.

$\therefore$ Total number of ways : $2 \times 3 \times 5 = 30$.

Eg: 4.6. A woman wants to select one silk saree and one Sungudi saree from a textile shop located at Kanchipuram. In that shop there are 20 different varieties of silk sarrees and 8 different varities of sungudi sarees. In how many ways she can select her saries.

one after the other { Product Rule }. The woman can select a silk saries in 20 ways.
and Sungudi saree can select in 8 ways

$\therefore$ Total number of ways $= 20 \times 8$
$= 160$ ways.

Eg: 4.7: In a village out of the total number of people, 80% of the people own coconut groves and 65% of the people own Paddy fields. What is minimum percentage of people own both.

Sol: $n(C) = 80$, $n(P) = 65$

By Inclusion - Exclusion Rule

$$n(C \cap P) = n(C) + n(P) - n(C \cup P)$$

Here $n(C \cup P) = 100$

$$n(C \cap P) = 80 + 65 - 100 = \underline{\underline{45}}$$

Eg: 4.8: 1) Find the number of strings of length 4, which can be formed using the letters of the word BIRD, without repetition of the letters.

2) How many strings of length 5 can be formed out of the letters of the word PRIME taking all the letters at a time without repetition.

Sol:

1) $\boxed{4}\ \boxed{3}\ \boxed{2}\ \boxed{1}$

By the Rule of Multiplication rule the number of ways in which the 4 places can be filled $= 4\times3\times2\times1 = 24$

$\therefore$ The number of Strings $= 24$

2) $\boxed{5}\ \boxed{4}\ \boxed{3}\ \boxed{2}\ \boxed{1}$

The number of ways in which the 5 places can be filled

$= 5\times4\times3\times2\times1$

$= 120$.

$\therefore$ The number of strings: 120.

Eg: 4.9. How many strings of length 6 can be formed using letters of the word FLOWER.

1) either starts with F or ends with R

2) neither starts with F nor ends with R.

In any such string each of the letters FLOWER is used exactly once.

1. If F is in the first place the remaining places are filled in 5, 4, 3, 2, 1 ways resp.

using product Rule Total ways $5\times4\times3\times2 = 120$

2. If R is in the end place the remaining places are filled in 5, 4, 3, 2, 1, ways resp

$\therefore$ using product Rule Total ways $= 5\times4\times3\times2 = 120$

3. Either F in the first place and R in the the last place the remaining places are filled in 4, 3, 2, 1

Total ways $= 4\times3\times2\times1 = 24$.

By Inclusion Exclusion Rule

number of ways with either F in the first place or R in the last place $= 120+120-24 = 216$.

Total number of strings formed by six letters.

$= 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$= 720$

$\therefore$ Number of string with neither F in the first place nor R in the last place

$= 720 - 216.$

$= 504.$

Eg: 4.10. How many licence plates may be made either two district letters followed by four digits or two digits followed by 4 district letters where all digits and strings letters are district.

Case 1: The number of plates having two letters followed by four digits is

$= 26 \times 25 \times 10 \times 9 \times 8 \times 7 = 32,76,000$

Case 2: The number of plates having 2 digits followed by ~~two~~ 4 letters.

$= 10 \times 9 \times 26 \times 25 \times 24 \times 23 = 3,22,99,000.$

Tototal number of licence plates $= 32,76,000 + 3,22,99,000$

(addition Rule since or) $= 3,55,68,000.$

Eg. 11. Count the number of positive integers greater than 7000 and less than 8000 which are divisible by 5 provided that no digits are repeated.

Sol: $\because$ The number should be divisible by 5 or 0

$\because$ The number is greater than 7000 and less than 8000 in can be filled by 7.

The remaining places are filled by 8, 7 ways

1	8	7	2

$\therefore$ Total number of Numbers greater than 7000 and less than 8000 which are divisible by 5 $= 1 \times 8 \times 7 \times 2$

$= 112.$

Eg: 12 How many 4 digits even number can be formed using the digits 0, 1, 2, 3 and 4 if the repetition of digits are not permitted.

Sol: 1) It is 4 digit number and hence its 1000th place cannot be 0

2) It is even number and hence its unit place can be either 0, 2, 4.

4	3	2	1

Case i) when the unit place is 0

Number of ways $4 \times 3 \times 2 \times 1 = 24$

Case ii) If the unit place is filled by non zero even number.

∴ Number of even numbers $= 3\times3\times2\times2$ | 3 | 3 | 2 | 2 |

$= 36.$

By addition Rule (either 0 or 2 or 4 at the unit place)

Total number of even numbers $= 24 + 36 = 60.$

Eg 4·13. Find the Total number of out comes when 5 coins are tossed once:

If n coins are tossed then the number of out comes $= 2^n$

When 5 coins are tossed no of out comes : 2^5
$= 32$

Note: If n different objects are to be placed in m places then the number of ways of placing is m^n $(\text{place}^{\text{object}})$.

Eg 4·14 In how many ways 1) 5 different balls be distributed among 3 boxes

2) 3 different balls be distributed among 5 boxes.

5 different balls be distributed among 3 boxes is $= 3^5 = 243.$

3 different balls be distributed among 5 boxes is $= 5^3 = 125$

Eg 4·15: There are 10 bulbs in a room. Each one of them can be operated independently. Find the number of ways in which the room can be illuminated.

The each bulbs can be operated in two ways. That is in off mode on mode.

∴ The total number of doing this is : 2^{10} ways.

When all 10 bulbs are in off mode the room cannot be illuminated

∴ Total Number of ways is $= 2^{10} - 1 = 1024 - 1$
$= 1023.$

Exercise – 4·1

1) 1) A person went to a restaurant for dinner. In the menu card the person saw 10 Indian and 7 chinese food items. In how many ways the person can select either an Indian or a chinese food.

Indian food selected in 10 ways.
Chinese food selected in 7 ways.

∴ Total ways $= 10 + 7 = 17$ ways

2) There are 3 types of toy car and 2 types of toy train available in a shop. Find the number of ways a baby can buy a toy car and toy Train

The number of ways of buying toy Train 2 ways

The number of ways of buying toy car 3 ways.

Total Number of ways = $2 \times 3 = 6$ ways.

3) How many two digits numbers can be formed using 1, 2, 3, 4, 5 without repetition of digits (NCERT)

$\boxed{5}\boxed{4}$

10^{th} place can be placed in 5 ways

unit place can be placed in 4 ways

Total number of Two digits number = $5 \times 4 = 20$

4) Three persons having enter into a confrence hall in which there are 10 seats. In how many ways They can take their seats.

The first person take seats in 10 ways.

Second person can take seats in 9 ways.

Third person can take seats in 8 ways.

$\therefore$ Total number of ways $10 \times 9 \times 8 = 720$.

5) In how many ways 5 persons can be seated in a row.

5 person can be seated in $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$ ways.

2. 1. A mobile phone has passcode of 6 distinct digits. What is the maximum number of attempts one makes to retrieve the pass code

2. Given four flags of different colours, how many different signals can be generated if each signal requires the use of the three flags, one below the other?

1) Maximum number of attempts = $10 \times 9 \times 8 \times 7 \times 6 \times 5$

$= 1,51,200$.

2) Number of ways for Top signal = 4

" middle signals = 3

" lower signals = 2

Total number of ways = $4 \times 3 \times 2 = 24$.

3. Four children are running a race
1) In how many ways can the first two places be filled?
2) In how many different ways could they finish the race?

1) The first place can be filled in 4 ways.
Second place can be filled in 3 ways.
Total number of ways $= 4 \times 3 = 12$

2. The winner must be 4 ways
Runner (second place) " 3 ways.
Third place " 2 ways.
last place " 1 way
Total number of ways $= 4 \times 3 \times 2 \times 1$
$= 24.$

4) Count the number of three digits numbers which can be formed from the digits 2, 4, 6, 8 if 1) Repetitions of digits allowed
2) repetitions of digits not allowed.

1) without repetition:
100^{th} place can be filled in 4 ways
10^{th} place can be filled in 3 ways.
unit place can be filled in 2 ways.
∴ The number of three digits number $= 4 \times 3 \times 2 = 24$.

2) with repetition
100^{th} place can be filled in 4 ways.
10^{th} " 4 ways.
unit " 4 ways.
∴ The number of 3 digits number: $4 \times 4 \times 4 = 64$.

5) How many three digit number are there with 3 in the unit place.
1) with repetition 2) without repetition.

1) with repetition: 3 is fixed in unit place.
100^{th} place can be filled in 9 ways (excluding 0 and 3)
10^{th} " 10 ways $10 \times 9 \times 1 = 90$.
Total Number of three digit numbers $= \cancel{8 \times 8 \times 1} = 64$

2) without Repetition: 3 is fixed in unit place.
100^{th} place can be filled in 8 ways. (excluding 0 and 3)
10^{th} place " 8 ways (remaining 8 number.
Total ways $= 8 \times 8 \times 1 = 64$

6) How many numbers are there between 100 and 500 with the digits 0, 1, 2, 3, 4, 5? 1) if repetition allowed 2) if repetition not allowed!

1) With repetition:

100th place can be filled in 4 ways (1, 2, 3, 4)

10th place can be filled in 6 ways

unit place can be filled in 6 ways.

$\therefore$ Total number of 3 digit numbers $= 4 \times 6 \times 6 = 144$

2) without repetitions:

100th place can be filled in 4 ways. (1, 2, 3, 4)

10th place " 5 ways

unit place " 4 ways.

Total Number of 3 digits Number $= 4 \times 5 \times 4 = 80$

7) How many 3 digits odd number can be formed by using the digits 0, 1, 2, 3, 4, 5? 1) If the repetition of digits is not allowed 2) the repetition of digits is allowed.

Repetition is not allowed:

unit place place can be filled in 3 ways (only by 1, 3, 5)

100th place can be filled in 4 ways. (excluding 0)

10th place can be filled in 4 ways.

$\therefore$ Total number of ways $= 3 \times 4 \times 4 = 48$.

Repetition is allowed:

unit place can be filled in 3 ways.

100th place can be filled in 5 ways. (without 0)

10th ~~unit~~ place " 6 ways.

Total ways : $3 \times 5 \times 6 = 90$.

8) Count the numbers between 999 and 10,000 subject to the condition that there are 1) no restriction 2) no digit is repeated 3) the last one of the digit is repeated.

$\square\square\square\square$

1) 1000th place can be filled in 9 ways.

100th " " 10 ways.

10th " " 10 ways.

unit " " 10

Total Number of ways $9 \times 10 \times 10 \times 10 = 9000$.

2) No digit is repeated.

1000^{th} place can be filled 9 ways.

100^{th} " " 9 ways.

10^{th} " " 8 ways

unit " " 7 ways.

Total ways $9 \times 9 \times 8 \times 7 = 4536$.

3) Atleast one of the digit is repeated $= 9000 - 4536$
$= 4464$.

9) How many three digit numbers which are divisible by 5 can be formed using the digits 0, 1, 2, 3, 4 5 if 1) repetition of digit are not allowed 2) repetition of digits are allowed.

Sol: 1) when repetition is not allowed:

□□□

when the unit place be filled by 0 ~ 1 way.

100^{th} place can be filled in 5 ways.

10^{th} place " 4 ways. Total ways $= 5 \times 4 = 20$

(or) when the unit place be filled by 5 ~~ways~~ 1 way

100^{th} place be filled by 4 ways

10^{th} " 4 ways. Total ways $4 \times 4 = 16$

∴ Total number of digits which are divisible by 5 is $= 20 + 16 = 36$.

2) when the repetition is allowed.

when the unit place be filled by 0 (ie) one way.

100^{th} place can be filled in 5 ways.

10^{th} " " 6 ways. $6 \times 5 \times 1 = 30$

Illy when unit place be filled by 5 (one way)

100^{th} place be filled by 5 ways.

10^{th} place " 6 ways $5 \times 6 \times 1 = 30$

Total Number of 3 digit numbers which are divisible by 5 is $30 + 30 = 60$.

10) To travel from place A to place B there are two different bus routes B_1, B_2 and two T_1, T_2 different trainroutes and one air route A_1. From place B to place C there is one busroute B_1, two different trainroutes T_1 and T_2 and another airroute A_1. Find the number of routes of commuting from place A to place C via B without using similar mode of transportation.

Number of routes to $A \to B$ $\quad 2+2+1 = 5$

" $\quad B \to C \quad 1+2+1 = 4$.

Total number of Rutes from $A \to C$ is $5 \times 4 = 20$.

By given condition

$$B \to B = 2 \times 1 = 2$$
$$T \to T = 2 \times 2 = 4$$
$$A \to A = 1 \times 1 = \underline{1}$$
$$7$$

∴ Required number of ways $= 20 - 7 = 13$ ways.

11) How many numbers are there between 1 and 1000 (both inclusive) which are divisible neither by 2 nor by 5.

$n(A)$ = number of numbers which are divisible by 2 $= 500$

$n(B)$ " 5 $= 200$

$n(A \cap B)$ " 10 $= 100$.

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$
$$= 500 + 200 - 100$$
$$= 600.$$

Number of numbers divisible by neither 2 nor 5 is $= 1000 - 600 = 400$.

12) How many strings can be formed using the letters of words Lotus of the word i) either starts with L or ends with S ii) neither starts with L nor ends with S.

$n(A)$: words starting with L $= 1 \times 4 \times 3 \times 2 \times 1 = 24$

$n(B)$: " S $= 1 \times 4 \times 3 \times 2 \times 1 = 24$

number of words starting with L and ending with S $\quad 1 \times 3 \times 2 \times 1 = 6$

$$n(A\cup B) = n(A)+n(B)-n(A\cap B)$$
$$= 24+24-6 = 42.$$

Total number of words : $5\times4\times3\times2\times1 = 120$

Required number of words $= 120-42 = 78$.

13) 1 Count the total number of ways of answering 6 objective type questions, each having 4 choices.

2) In how many ways 10 Pigeons can be placed in 3 different Pigeon holes.

3) Find the number of distributing 12 distinct prizes to 10 students

Sol: 1 Number of ways of answering 6 questions each of having 4 choice $= 4^6$ (object 6, event 4)

2. Number of ways 10 pigeons placed in 3 different holes $= 3^{10}$ (object 10, event 3)

3) Number of ways of distributing 12 distinct prizes to 10 students $= 10^{12}$ (object 12, event 10)

Factorials: $n! = 1\cdot2\cdot3\cdot\ldots\cdot(n-3)(n-2)(n-1)\cdot n$.

$$= n(n-1)!$$
$$= n(n-1)(n-2)!$$
$$= n(n-1)(n-2)(n-3)!$$

Eg 4.16: Find the value of 1) $5!$ 2) $6!-5!$ 3) $\dfrac{8!}{5!\,2!}$

1) $5! = 5\cdot4\cdot3\cdot2\cdot1 = 120$.

2) $6!-5! = 6\cdot5!-5!$
$$= (6-1)5!$$
$$= 5\cdot5! = 5\times120 = 600$$

3) $\dfrac{8!}{5!\,2!} = \dfrac{8\cdot7\cdot6\cdot\cancel{5!}}{\cancel{5!}\,2!} = 168$.

Eg: 4.17. Simplify $\frac{7!}{2!}$

Sol: $\frac{7!}{2!} = \frac{7\cdot6\cdot5\cdot4\cdot3\cdot\cancel{2!}}{\cancel{2!}} = 2520$

Eg 4.18: Evaluate $\frac{n!}{r!(n-r)!}$ when i) $n=7, r=5$ 2) $n=50, r=47$ 3) For any n with $r=3$.

1) when $n=7, r=5$

$$\frac{n!}{r!(n-r)!} = \frac{7!}{5!(7-5)!} = \frac{7\cdot6\cdot\cancel{5!}}{\cancel{5!}\,2!} = \frac{42}{2} = 21.$$

2) when $n=50$ $r=47$.

$$\frac{n!}{r!(n-r)!} = \frac{50!}{47!(50-47)!} = \frac{50\cdot49\cdot\overset{8}{\cancel{48}}\cdot\cancel{47!}}{\cancel{47!}\,\cancel{3!}} = 19600$$

3) For any n and $r=3$.

$$\frac{n!}{3!(n-3)!} = \frac{n(n-1)(n-2)\cancel{(n-3)!}}{3!\,\cancel{(n-3)!}}$$

$$= \frac{n(n-1)(n-2)}{3!}$$

4.19) Let N denote the number of days. If the value of $N!$ is equal to the total number of hrs in N days then find the value of N.

$N! = 24\times N.$ $\qquad 1\cdot2\cdot3\cdot4$

$\cancel{N}(N-1)! = 24\times\cancel{N}$

$= 4!$

$N-1 = 4$

$N = 5.$

4.20) If $\frac{6!}{n!} = 6$ find the value of n.

$$\frac{6!}{6} = n! \Rightarrow \frac{\cancel{6}\cdot5!}{\cancel{6}} = n! \Rightarrow n=5.$$

4.21) If $n! + (n-1)! = 30$ then find the value of n.

$n! + (n-1)! = 30$

$n(n-1)! + (n-1)! = 6\times5$

$(n-1)!\,(n+1) = 3!\times5 \Rightarrow n-1 = 3$

$n = 4$

$n+1 = 5$

$\therefore n = 4.$

4.22) What is the unit digit of the sum $2! + 3! + 4! + 5! + \cdots + 22!$

Sol: From $5!$ onwards for all $n!$ the unit digit is zero.

$\therefore 2! + 3! + 4! = 2 + 6 + 24 = 32.$

$\therefore$ The required unit digit is 2.

4.23) If $\frac{1}{7!} + \frac{1}{8!} = \frac{A}{9!}$ Find the value of A.

$$\frac{A}{9!} = \frac{1}{7!} + \frac{1}{8!}$$

$$\frac{A}{9\cdot8\cdot7!} = \frac{1}{7!} + \frac{1}{8\cdot7!}$$

$$\frac{A}{9\cdot8\cdot\cancel{7!}} = \frac{1}{\cancel{7!}}\left[1+\frac{1}{8}\right] \Rightarrow \frac{A}{\cancel{72}_9} = \frac{9}{\cancel{8}}$$

$A = 81$

4.24) P.T $\frac{(2n)!}{n!} = 2^n(1\cdot3\cdot5\cdots(2n-1))$

Sol: $$(2n)! = \frac{1\cdot2\cdot3\cdots(2n-4)(2n-3)(2n-2)(2n-1)\cdot2n}{n!}$$

$$= \frac{[1\cdot3\cdot5\cdots(2n-3)(2n-1)][2\cdot4\cdot6\cdots(2n-2)2n]}{n!}$$

$$= \frac{[1\cdot3\cdot5\cdots(2n-3)(2n-1)]2^n\cancel{[1\cdot2\cdot3\cdots(n-1)n]}}{\cancel{n!}}$$

$$= 2^n(1\cdot3\cdot5\cdots(2n-1))$$

EX 4.1

14) Find the value of i) $6!$ 2) $4!+5!$ 3) $3!-2!$ 4) $3! \times 4!$

5) $\frac{12!}{9! \times 3!}$ 6) $\frac{(n+3)!}{(n+1)!}$

1. $6! = 6\cdot5\cdot4\cdot3\cdot2\cdot1 = 720$

2) $4!+5! = 4!+5\cdot4!$
$= 4!(1+5)$
$= 1\cdot2\cdot3\cdot4\cdot6$
$= 144$

3) $3!-2! = 3\cdot2!-2!$
$= 2!(3-1)$
$= 2!\cdot2$
$= 4.$

4) $3! \times 4! = 1\cdot2\cdot3 \times 1\cdot2\cdot3\cdot4$
$= 144$

$\cancel{36} \times 4$

5) $\frac{12!}{9!3!} = \frac{\cancel{12}\cdot11\cdot10\cdot\cancel{9!}}{\cancel{9!}\ \cancel{3}\cancel{\times4}} = 110.$

6) $\frac{(n+3)!}{(n+1)!} = \frac{(n+3)(n+2)\cancel{(n+1)!}}{\cancel{(n+1)!}} = (n+3)(n+2)$

15) Evaluate $\frac{n!}{r!(n-r)!}$

1) when $n=6, r=2$ $= \frac{6!}{2!4!} = \frac{\overset{3}{\cancel{6}}\times5\times\cancel{4!}}{\cancel{2!}\cancel{4!}} = 15$

2) when $n=10, r=3$ $= \frac{10!}{3!7!} = \frac{10\cdot\overset{3}{\cancel{9}}\cdot\overset{4}{\cancel{8}}\cdot\cancel{7!}}{1\cdot\cancel{2}\cdot\cancel{3}\cdot\cancel{7!}}$
$= 120.$

3) For any n with $r=2$

$= \frac{n!}{2!(n-2)!} = \frac{n(n-1)\cancel{(n-2)!}}{2!\cancel{(n-2)!}}$

$= \frac{n(n-1)}{2}$

16) Find the value of n if

1) $(n+1)! = 20(n-1)!$

$(n+1)(n)\cancel{(n-1)!} = 20\cancel{(n-1)!}$

$n^2+n-20=0$

$(n-4)(n+5)=0 \quad \therefore n=-5,4$

2) $\frac{1}{8!}+\frac{1}{9!}=\frac{n}{10!}$

$\frac{1}{\cancel{8!}}+\frac{1}{9\cdot\cancel{8!}}=\frac{n}{10\cdot9\cdot\cancel{8!}}$

$\frac{10}{\cancel{9}}=\frac{n}{10\cdot\cancel{9}} \quad \therefore n=\underline{\underline{100}}$

Theorem 4.1: If n, r are positive integers and $r \leq n$ then the number of permutations of n distinct objects taken r at a time is $n(n-1)(n-2)\ldots(n-r+1)$.

Proof: Permutations of n distinct objects taken r at a time is formed by filling of r positions in a row with objects chosen from the given n distinct objects...

1	2	3		r^{th} place
n	$n-1$	$n-2$		$n-r+1$

The first place can be filled in n different ways.

Second " " $n-1$ "

Third " " $n-2$ "

- - - - - - - - - -

r^{th} place can be filled in $n-(r-1) = n-r+1$ ways.

$$\therefore nP_r = n(n-1)(n-2)\ldots(n-r+1)$$

Theorem 4.2 If $n \geq 1$ and $0 \leq r \leq n$ then $nP_r = \frac{n!}{(n-r)!}$

Proof: $nP_r = n(n-1)(n-2)\ldots(n-r+1)$

$$= \frac{n(n-1)(n-2)\ldots(n-r+1)(n-r)(n-r-1)\ldots 2\cdot 1}{(n-r)(n-r-1)\ldots 2\cdot 1}$$

$$= \frac{n!}{(n-r)!}$$

Properties: 1) $nP_n = nP_{n-1}$

Proof: $nP_{n-1} = \frac{n!}{[n-(n-1)]!} = \frac{n!}{1!} = n! = nP_n$.

2) $nP_r = n \times {}^{n-1}P_{r-1}$

Proof: $n \times (n-1)P_{r-1} = n \times \frac{(n-1)!}{[(n-1)-(r-1)]!} = \frac{(n-1)! \times n}{(n-r)!} = \frac{n!}{(n-r)!}$

$= nP_r$.

3) $nP_r = (n-1)P_r + r \times (n-1)P_{(r-1)}$

Proof:

LHS $(n-1)P_r + r\,(n-1)P_{r-1}$

$$= \frac{(n-1)!}{(n-1-r)!} + r \cdot \frac{(n-1)!}{((n-1)-(r-1))!}$$

$$= \frac{(n-1)!}{((n-1)-r))!} + r \cdot \frac{(n-1)!}{(n-r)!}$$

$$= \frac{(n-1)!\,(n-r)}{(n-1-r)!\,(n-r)} + \frac{r\,(n-1)!}{(n-r)!}$$

$$= \frac{(n-1)!\,(n-r)}{(n-r)!} + \frac{r\,(n-1)!}{(n-r)!}$$

$$= \frac{(n-1)!\,(n-\cancel{r}+\cancel{r})}{(n-r)!}$$

$$= \frac{n!}{(n-r)!} = nPr.$$

4.25) Evaluate: i) $4P_4$ 2) $5P_3$ 3) $8P_4$ 4) $6P_5$

1) $4P_4 = 4! = 4\times3\times2\times1 = 24$

2) $5P_3 = 5\times4\times3 = 60$

3) $8P_4 = 8\times7\times6\times5 = 1680$

4) $6P_5 = 6\times5\times4\times3\times2 = 720.$

4.26) If $(n+2)P_4 = 42\,nP_2$ find n.

Sol: $(n+2)P_4 = 42\,nP_2$

$(n+2)(n+1)\cancel{(n)(n-1)} = 42 \cdot \cancel{n(n-1)}$

$(n+2)(n+1) = 42$

$n^2+3n+2 = 42$

$n^2+3n-40=0$

$(n+8)(n-5)=0$

$n=-8,\ n=5$

n should not be $-ve$

$\therefore n = 5.$

4.27) If $^{10}P_r = {^7P_{r+2}}$ find r.

Sol: If $^{10}P_r = {^7P_{r+2}}$

$$\frac{10!}{(10-r)!} = \frac{7!}{(7-(r+2))!}$$

$$\frac{10\cdot 9\cdot 8\cdot \cancel{7!}}{(10-r)!} = \frac{\cancel{7!}}{(5-r)!}$$

$$\frac{10\cdot 9\cdot 8\cdot}{(10-r)(9-r)(8-r)(7-r)(6-r)\cancel{(5-r)!}} = \frac{1}{\cancel{(5-r)!}}$$

$$\therefore (10-r)(9-r)(8-r)(7-r)(6-r) = 10\times 9\times 8$$

$$= 5\times 2\times 3\times 3\times 4\times 2$$

$$= 6\times 5\times 4\times 3\times 2$$

$$\Rightarrow 10-r = 6 \quad \therefore r = 4$$

4.28) How many letter strings together can be formed with the letters of the word 'VOWELS' so that

1) the string begin with E
2) the strings begin with E and end with W.

Sol:

E					
1	5	4	3	2	1.

The string begin with E

The first plance can be filled in 1 way
5 ways.

Second place "

- - - - - .

- - - - -

6th place can be filled in 1 way.

$\therefore$ Number of ways. $= 1\times 5\times 4\times 3\times 2\times 1$

$= 120$

The string begin with E and end with W.

E					W
1	4	3	2	1	1

Total no of ways: $1\times 4\times 3\times 2\times 1\times 1 = 24$

4.29) A number of four different digits is formed with the use of the digits 1, 2, 3, 4, 5 in all possible ways. Find the following

1. How many such numbers can be formed?
2. How many of these are even?
3. How many of these are exactly divisible by 4.

1) No of four digit numbers.

5	4	3	2	

1000th place can be filled in 5 ways
100th " 4 ways
10th " 3 ways.
unit " 2 ways.

∴ Total number of 4 digit numbers $= 5 \times 4 \times 3 \times 2$
$= 120.$

2) No of even numbers.

			2/4

unit place can be filled in 2 ways.
(2 or 4)

1000th place " " 4 ways.
100th place " " 3 ways.
10th place " " 2 ways.

Total number of even numbers : $4 \times 3 \times 2 \times 2$
$= 48.$

3) Numbers divisible by 4.

The last two digits will be 12, 24, 32, 52.

∴ The last two digits can be filled in 4 ways.
1000th place can be filled in 3 ways.
100th place can be filled in 2 ways.

Total number of Numbers which are divisible by 4
is $3 \times 2 \times 4 = 24$

4.30) How many different strings can be formed together using the letters of the word 'EQUATION' So that

1) The vowels always come together?

2) The vowels never come together?

---

1) The number of permutations of n things taken all at a time when m specified objects are always together is

$$= m! \times (n-m+1)! \qquad m = EUAIO = 5$$
$$n = 8$$
$$= 5!\,(8-5+1)!$$
$$= 5!\,4!$$
$$= 120 \times 24 = 2880.$$

2) The number of permutations of n different things taken all at a time when m specified things never come together is $n! - m!\,(n-m+1)!$

$$= 8! - 2880$$
$$= 40320 - 2880$$
$$= 37440.$$

4.31) There are 15 candidates for an examination. 7 candidates are appearing for mathematics examination 8 are appearing for different subjects. In how many ways can they be seated in a row so that no two mathematic candidates are together.

To obtain the number of permutation of n things when no two of K ~~of~~ given thing are together $m = n - k$.

$$= m! \times {}^{(m+1)}P_K.$$

$$n = 15 \qquad m = 15 - \cancel{8}\,7 = 8.$$
$$K = \cancel{8}\,7$$

$$= 8! \times {}^{9}P_{\cancel{8}}.$$

$$= 8! \times \frac{9!}{2!}$$

4.32) In how many ways 5 boys and 4 girls can be seated in a row so that no two girls are together.

$$m = 5,\ n = 4$$
$$= m! \times {}^{(m+1)}P_K \quad = 5!\ {}^{6}P_4 = 120 \times 360$$
$$= 43200$$

4.33) 4 boys and 4 girls form a line with boys and girls alternating. Find the number of ways of making this line.

If we form the line in this way we get

$B_1 - B_2 - B_3 - B_4 - = 4! \times 4!$

we arrange this way

(or) $G_1 - G_2 - G_3 - G_4 - = 4! \times 4!$

∴ Total number of ways $= (4! \times 4!) + (4! \times 4!)$

$(24 \times 24) \times 2$

$= 1152$

$$\begin{array}{r} 48 \times 24 \\ \hline 192 \\ 96 \\ \hline 1152 \end{array}$$

4.34) A van has 8 seats. It has two seats ~~in the front~~ with two rows of three seats behind. The van belongs to a family, consisting of seven members F, M, S_1, S_2, S_3, D_1, D_2. How many ways can the family sit in the van if

1) There are no restriction.
2. Either M or F drives the van
3. D_1, D_2 sits next to window and F is driving.

∴ 1) There is no restriction: driver seat can be occupied in 7 ways.

Then the 7 seats can be seated in $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$ ways.

∴ Total number of ways: $7 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$= 35280.$

2) Either F or M drives the van

Driver seat can be seated in 2 ways.

Family members can be seated in 7! ways.

∴ Total ways $= 2 \times 7!$

$= 2 \times 5040 = 1080$

3) D_1, D_2 sits next to window and F is driving

∴ 5 window seats available D_1, D_2 can be sit in $5C_2$ ways

F is a driver — 1 way.

$= \frac{5!}{3!} = 5 \cdot 4$

$= 20.$

The remaining 4 peoples can be seated in 5 seats in $5P_4$ way $= \frac{5!}{1!} = 120$

∴ Total No of ways: $20 \times 120 \times 1 = 2400$

4.36) Find the number of ways arranging the letters of the word BANANA.

B – 1
A – 3 $\quad n = 6.$
N – 2

The number of ways of arrangements $= \frac{6!}{3!\,2!}$

$= \frac{1\cdot\cancel{2}\cdot\cancel{3}\cdot 4\cdot 5\cdot\cancel{6}}{\cancel{1}\cdot\cancel{2}\cdot\cancel{3}\cdot\cancel{2}} = 60$ ways.

4.37) Find the number of ways of arranging the letters of the word RAMANUJAM so that the relative positions of vowels and consonants are not changed.

Vowels : A A A U.

Vowels can be arranged in $\frac{4!}{3!} = 4$ ways.

Consonants R M N J M

consonants can be arranged in $\frac{5!}{2!} = 5\times4\times3 = 60$

Total number of ways $= 60\times4 = 240$

4.38) 3 twins pose for a photograph standing in a line. How many arrangements are there 1) when there are no restrictions. 2) when each persons is standing next to his or her twin.

1) <u>when there is no restriction:</u> 6 person can be arranged in $6!$ ways
$= 6\times5\times4\times3\times2\times1$
$= 720.$

2) Since there are 3 sets of twins. These three sets can be arranged in $3!$ ways.
Each twins can be arranged in $2!$ ways.
$\therefore$ Total number of ways $= 3!\times2!\times2!\times2!$
$= 48.$

4.39) How many numbers can be formed using the digits 1, 2, 3, 4, 2, 1 such that even digits can occupy even places.

Even numbers 2, 4, 2.

3 even places can be arranged in $\frac{3!}{2!} = 3$ ways.

The remaining three numbers 1, 3, 1 can be arranged in $= \frac{3!}{2!}$
$= 3$ ways

$\therefore$ Total ways $= 3\times3 = 9$

4.40) How many paths are there from start to end of 6×4 grid as shown in the picture.

There 6 horizontal and 4 vertical unit lengths.

Total number of unit lengths = 10

$\therefore$ Total number of paths = $\frac{10!}{6!\,4!} = \frac{10\times \overset{3}{\cancel{9}}\times \cancel{8}\times 7\times \cancel{6}\times \cancel{5}\times \cancel{4}\times \cancel{2}\times 1}{\cancel{1\times 2\times 3\times 4\times 5\times 6}\times 1\times \cancel{2}\times \cancel{3}\times \cancel{4}}$

$= 210.$

4-41) If the different permutations of all letters of the word BHASKARA are listed as in a dictionary how many strings are there in this list before the first word starting with B.

BHASKARA

= AAABHKRS

Number of strings starting with A and using the letters A, A, B H K R S

$= \frac{7!}{2!} = \frac{7\cdot 6\cdot 5\cdot 4\cdot 3\cdot \cancel{2!}}{\cancel{2!}} = 2520.$

4.42: If the letters of the word IIIJEE are permuted in all possible ways and the strings thus formed are arranged in the lexicographic order, find the rank of the word IITJEE

EEIIJT

E(EIIIJ) $= \frac{5!}{2!} = 60$

IIE(EIJ) $= 3! = 6.$

IIJ(EEI) $= \frac{3!}{2!} = 3$

IITE(EJ) $= 2! = 2$

IITJEE $= 1$ $\quad = 1$

$\overline{72}$

$\therefore$ The rank is $= \underline{\underline{72}}.$

4.43: Find the sum of all 4 digits numbers that can be formed using the digits 1, 2, 4, 6, 8:

Total number of numbers $5P_4$ (or) $\boxed{5|4|3|2} = 120$.

From this 120 numbers when the unit place filled by 1. Remaining 3 places can be fill by 4 numbers with $4P_3$ ways. $= 4\times3\times2 = 24$.

This 24 numbers contains unit place $= 1$.

III^ly 2, 4, 6, 8 contains in the unit place each 24 times.

$\therefore$ Sum of Total of unit places $= (24\times1)+(24\times2)+(24\times4)+(24\times6)+(24\times8)$

$= 24(1+2+4+6+8)$

$= 24\times21$

III^ly Sum of 10^{th} contains $= 24\times21\times10$

Sum of 100^{th} place contains $= 24\times21\times100$

Sum of 1000^{th} place contains : $24\times21\times1000$

Sum of Total numbers $= 24\times21\times(1+10+100+1000)$

$= 24\times21\times1111$

$= 559944$.

(or)

$n=5, r=4$

sum of the digits $= 1+2+4+6+8 = 21$.

Sum $n-1P_{r-1}(S.D)\times1111$

$= 4P_3\times21\times1111$

$= 4\cdot3\cdot2\times21\times1111$

$= 559944$

Exercise – 4.2

1) If $(n-1)P_3 : nP_4 = 1:10$ find n.

$(n-1)P_3\times10 = nP_4\times1$

$(n-1)(n-2)(n-3)\times10 = n(n-1)(n-2)(n-3)$

$n=10$

2) If $10P_{r-1} = 2\times6P_r$ find r.

$$\frac{10!}{(10-(r-1))!} = 2\times\frac{6!}{(6-r)!}$$

$$\frac{10!}{(11-r)!} = 2\times\frac{6!}{(6-r)!}$$

$$\frac{10.9.8.7.\cancel{6!}}{(11-r)(10-r)(9-r)(8-r)(7-r)\cancel{(6-r)!}} = 2\times\frac{\cancel{6!}}{\cancel{(6-r)!}}$$

$(11-r)(10-r)(9-r)(8-r)(7-r) = 2520$

$= 3\times4\times5\times6\times7$

$\Rightarrow 11-r=7 \Rightarrow r=4$

3) i) Suppose 8 people enter an event in a swimming meet. In how many ways could the Gold, silver, and bronze prizes be awarded:

1. 8 people and 3 prizes.

No of ways : $8P_3 = 8\cdot7\cdot6 = 336$.

ii) Three men have 4 coats, 5 waist coats, 6 caps. In How many ways can they wear them.

Number of ways $4P_3 \times 5P_3 \times 6P_3$.

$= 4\times3\times2 \times 5\times4\times3\times6\times5\times4$

$= 24\times60\times120$

$= 172800$.

4) Determine the number of permutations of the letters of the word SIMPLE if all are taken at a time.

$n = 6 \quad r = n$

$\therefore \quad nP_n = 6P_6 = 6! = 6\times5\times4\times3\times2\times1$

$= 720$

5) A test consists of 10 multiple choice question. In how many ways can the test be answered if

1) Each questions have 4 choices.

2) The first ~~4~~ questions have 3 choices and the remaining have 5 choices.

3) question number n has n+1 choices.

1) Number of ways 4^{10} (coin type)

2) Number of ways $3^4\times5^6$

3) 1st question has two choices

2 " 3 "

- - - - - - - -

$\therefore 2\times3\times4\times\cdots\times11$

$= 11!$

6) A student appears in an objective type question test which contain 5 multiple choice question. Each questions have four choices out of which one correct answer.

1) What is the max-number of different answers can the student give

2) How will the answer change if each question may have more than one correct answer.

Sol: 1. Max. Number of answers $= 4^5$

2. In one question may correct 1 answer, or 2 or 3 or 4 or 5

$\therefore$ Total Correct answers $1+2+3+4+5 = 15$

$\therefore$ Number of choices $= 15^5$

7. How many strings can be formed from the letters of the word ARTICLE. So that the vowels occupy even places.

Vowels : A E I

Consonants : R T C L

$\square \; \boxtimes \; \square \; \boxtimes \; \square \; \boxtimes \; \square$

Vowels can occupy $3!$ ways.

other letters occupy $4!$ ways.

Total ways $= 3! \times 4!$

$= 144$

8) 8 women and 6 men are standing in a line

1) How many arrangements are possible if any individual can stand in any position.

2. In how many arrangements will all 6 men be standing next to one another

3. In how many arrangements will no two men be standing next to one another.

1) 14 Persons can stand in $14!$ ways.

2. $n = 14$, $m = 6$.

Number of ways $= m! \times (n-m+1)!$

$6! \times (14-6+1)! = 6!\,9!$

3) $n = 14$, $K = 6$, $m = n - K = 8$

Number of ways $m!\,{}^{(m+1)}P_K$

$= 8!\,{}^{9}P_6$

9) Find the distinct permutations of the letters of the word MISSISSIPPI

$M - 1$
$S - 4$
$I - 4$
$P = 2$

No of distinct permutations $= \dfrac{11!}{1!\,4!\,4!\,2!}$

$= 34650.$

10) How many ways can the product $a^2b^3c^4$ be expressed without exponents.

Total letters $- 9.$
$a = 2$
$b = 3$
$c = 4:$

Total number of ways $= \dfrac{9!}{2!\,3!\,4!}$

$= 1260.$

11) In how many ways 4 Mathematics, 3 physics, 2 chemistry, 1 Bio books can be arranged on a shelf so that all books of the same subject are together -

There are 4 units (M, P, C, B)

units may be arranged in $= 4!$ ways.

Maths " $= 4!$ ways.

Physics " $= 3!$ ways.

Chemistry " $= 2!$ ways.

Biology " $= 1!$ way.

$\therefore$ Total number of ways $= 4! \times 4! \times 3! \times 2! \times 1!$

$= 6912$

12) In How many ways can the letters of the word SUCCESS so that all S s are together

SUCCESS.

Consider 3 S's as one unit

$\therefore$ Total 5 units But 2 c's

$\therefore$ Total ways $= \dfrac{5!}{2!} = \dfrac{\cancel{1}\cdot\cancel{2}\cdot 3\cdot 4\cdot 5}{\cancel{1\cdot 2}} = 60.$

13) A coin is tossed 8 times

1) How many different sequences of heads and tails are possible

2) How many different sequences containing 6 heads and two tails are possible.

1. Total possible ways : 2^8

2. Number of ways : $\frac{8!}{6!\,2!} = 28$

14) How many strings are there using the letters of the word INTERMEDIATE

1) The vowels and consonants are alternative

2) All the vowels are together

3) Vowels are never together

4) No two vowels are together.

1) First place vowel and next place consonants.

$$= \frac{6!}{2!\,3!} \times \frac{6!}{2!}$$

(or) First place consonants and second place vowel.

$$= \frac{6!}{2!} \times \frac{6!}{2!\,3!}$$

∴ Total ways : $2 \times \frac{6!}{2!} \times \frac{6!}{2!\times 3!}$

$$= 2(360 * 60) = 43200.$$

2) All vowels are together:

$n = 12, \quad m = 6$

$$= m!\,(n-m+1)!$$

$$= \frac{6! \times 7!}{3! \times 2! \times 2!} = 151200$$

3) Vowels are never together: $\frac{12!}{2!\,2!\,3!} - 1,51,200$

$$= 19958400 - 1,51,200$$

$$= 19807200$$

4 No two vowels are together $= m! \times (m+1)P_k$ $\quad m = n-k = 12-6 = 6.$

$$= \frac{6! \times 7P_6}{2!\,2!\,3!} = 1,51,200$$

15) Each digits 1, 1, 2, 3, 3, 4 is written on separate card. The six cards are then laid out in a row to form a 6 digit number

1. How many 6 digit number are there
2. How many of them are even
3. How many of them are divisible by 4.

1, 1, 2, 3, 3, 4

1) Number of 6 digits number = $\frac{6!}{2!.2!} = 180$

2) Number of even digits number = $\frac{5!}{2!2!} = \frac{120}{4}$

If 2 is in the unit place

(or) If 4 is in the unit place = $\frac{5!}{2!2!} = \frac{120}{4}$.

Total number of even numbers : $\frac{120}{4} + \frac{120}{4} = \frac{240}{4} = 60$

3. Number of the number divisible by 4.

If the last two digits be 12, = $\frac{4!}{2!} = 12$ (1, 3, 3, 4)

If the last two digits be 24 = $\frac{4!}{2!2!} = 6$ (1, 1, 3, 3)

Total = 12+6 = 18.

19) Find the sum of all 4 digits numbers that can be formed using the digits 1, 2, 3, 4, 5 repetition not allowed.

Sum of the digits 1 + 2 + 3 + 4 + 5 = 15

$n = 5 \quad r = 4$.

Sum $= (n-1)P_{r-1} \times S.D \times 111 \ldots r$ times

$= 4P_3 \times 15 \times 1111$

$= 4 \times 3 \times 2 \times 15 \times 1111 = 399960$.

20) Find the sum of all 4 digits that can be formed using the digits 0, 2, 5, 7, 8 without repetition,

Sum of the digits = 0 + 2 + 5 + 7 + 8 = 22

$n = 5 \quad r = 4$.

If 0 is included

Sum $= (n-1P_{r-1} \times SD \times 11 \ldots r$ times$) - (n-2)P_{r-2} \times SD \times 1.1 \ldots r-1$ times$)$

$= 4P_3 \times 22 \times 1111 - 3P_2 \times 22 \times 111$

$= 586608 - 14652 = 571956$.

16) If the letters of the word GARDEN are permuted in all possible ways and the strings thus formed are arranged in the dictionary order then find the rank of the word

1) GARDEN 2) DANGER.

1) $\overset{4}{G}\overset{1}{A}\overset{6}{R}\overset{2}{D}\overset{3}{E}\overset{5}{N}$

Rank 3 0 3 0 0 0

5! 4! 3! 2! 1! 0! $= 3\times5! + 3\times3!$

$= 3\times1\times2\times3\times4\times5 + 3\times3\times2$

$360 \quad +18$

$= 378+1 = 379$

Rank 2) $\overset{2}{D}\overset{1}{A}\overset{5}{N}\overset{4}{G}\overset{3}{E}\overset{6}{R}$.

1 0 2 1 0 0

5! 4! 3! 2! 1! 0! $= 5! + 2\times3! + 1\cdot2!$

$= 120+12+2 = 134+1 = 135.$

18) If the letters of the word FUNNY are permitted in all possible ways and the strings thus formed are arranged in the dictionary order Find the rank of the word FUNNY

$\overset{1}{F}\overset{3}{U}\overset{2}{N}\overset{2}{N}\overset{4}{Y}$

0 $\frac{2}{2!}$ 0 0 0

4! 3! 2! 1! 0 $= \frac{\cancel{2}}{\cancel{2!}}\times3! = 6+1 = 7.$

4.35. If the letters of the word table are permuted in all possible ways and the words thus formed are arranged in the dictionary order find the rank of the words. 1) TABLE 2) BLEAT

1) $\overset{5}{T}\overset{1}{A}\overset{2}{B}\overset{4}{L}\overset{3}{E}$

4 0 0 1 0

4! 3! 2! 1! 0!

$= 4\times4! + 1\times!$

$= 96+1+1 = 98$

2) $\overset{2}{B}\overset{4}{L}\overset{3}{E}\overset{1}{A}\overset{5}{T}$

1 2 1 0 0

4! 3! 2! 1! 0!

$= 1\times4! + 2\times3! + 1\times2!$

$= 24+12+2+1$

$= 39.$

Combinations:

The number of combinations of n distinct objects taken r at a time is given by nC_r.

$$nC_r = \frac{n!}{(n-r)!\,r!} \qquad (0 \le r \le n)$$

Relationship between nP_r and nC_r

$$nP_r = nC_r \times r!$$

Properties of combinations:

1. $nC_0 = 1$ 2) $nC_n = 1$ 3) $nC_r = \frac{n(n-1)(n-2)\cdots(n-r+1)}{r!}$

1) $nC_0 = \frac{n!}{(n-0)!\,0!} = \frac{n!}{n!} = 1$

2) $nC_n = \frac{n!}{(n-n)!\,n!} = \frac{n!}{0!\,n!} = 1$

3) $nC_r = \frac{n(n-1)(n-2)\cdots(n-r+1)}{r!}$

2) $nC_r = nC_{n-r}$

Proof: $nC_{n-r} = \frac{n!}{(n-(n-r))!\,(n-r)!}$

$$= \frac{n!}{r!\,(n-r)!} = nC_r$$

$\therefore nC_r = nC_{n-r}$.

3) If $nC_x = nC_y$ then either $x = y$ or $x + y = n$.

$$nC_x = nC_y = nC_{n-y} \Rightarrow x = y \text{ (or) } x = n - y$$
$$x + y = n.$$

5) $nC_r + nC_{r-1} = (n+1)C_r$

Proof $nC_r + nC_{r-1} = \frac{n!}{r!\,(n-r)!} + \frac{n!}{(r-1)!\,(n-r+1)!}$

$$\frac{n!}{r(r-1)!\,(n-r)!} + \frac{n!}{(r-1)!\,(n-r+1)(n-r)!}$$

$$= \frac{n!}{(n-r)!\,(r-1)!}\left[\frac{1}{r} + \frac{n}{n-r+1}\right]$$

$$\frac{n!}{(n-r)!\,(r-1)!}\left[\frac{(n-\cancel{r}+1)+r}{r\,(n-r+1)}\right]$$

$$= \frac{n!}{(n-r)!\,(r-1)!}\cdot\frac{(n+1)}{r\,(n-r+1)}$$

$$= \frac{(n+1)!}{(n-r+1)!\;r!}$$

$$= (n+1)C$$

5) $nC_r = \frac{n}{r} \times {}^{n-1}C_{r-1}$

$$= \frac{n}{r}\left[\frac{(n-1)!}{((n-1)-(r-1))!\,(r-1)!}\right]$$

$$= \frac{n!}{r\,(r-1)!\,(n-r)!} = \frac{n!}{(n-r)!\,r!} = nC_r.$$

4.44! Evaluate: 1) $10C_3$ 2) $15C_{13}$ 3) $100C_{99}$ 4) $50C_{50}$

1. $10C_3 = \frac{10\times\cancel{9}^{3}\times\cancel{8}^{4}}{1\cdot\cancel{2}\cdot\cancel{3}} = 120$

2) $15C_{13} = 15C_2 = \frac{15\cdot\cancel{14}}{1\cdot\cancel{2}} = 105$

3) $100C_{99} = 100C_1 = 100$

4) $50C_{50} = 1.$

4.46) If $nC_4 = 495$. Find n.

$$nC_4 = \frac{n!}{(n-4)!\,4!} = 495$$

$$\frac{n(n-1)(n-2)(n-3)\cancel{(n-4)!}}{\cancel{(n-4)!}\cdot 4!} = 495$$

$$\Rightarrow n(n-1)(n-2)(n-3) = 495\times4\times3\times2\times1$$

$$= 3\times3\times5\times11\times4\times3\times2\times1$$

$$= 12\times11\times10\times9$$

$\therefore n = 12$

4.47. If $nP_r = 11880$ and $nC_r = 495$ find r.

Sol: $\frac{nPr}{nCr} = r!$

(ie) $r! = \frac{11880}{495} = 24$

$= 4!$

$\begin{array}{r|l} 2 & 24 \\ 3 & 12 \\ & 4 \end{array}$

$\therefore r = 4.$

4.48) P.T $24C_4 + \sum_{r=0}^{4} (28-r)C_3 = 29C_4$.

$24C_4 + \sum_{r=0}^{4} (28-r)C_3 = 24C_4 + 28C_3 + 27C_3 + 26C_3 + 25C_3 + 24C_3$

$= 24C_4 + 24C_3 + 25C_3 + 26C_3 + 27C_3 + 28C_3.$

$\boxed{nC_r + nC_{r-1} = (n+1)C_r}$

$= 25C_4 + 25C_3 + 26C_3 + 27C_3 + 28C_3$

$= 26C_4 + 26C_3 + 27C_3 + 28C_3$

$= 27C_4 + 27C_3 + 28C_3$

$= 28C_4 + 28C_3$

$= 29C_4.$

4.49. P.T $10C_2 + 2 \times 10C_3 + 10C_4 = 12C_4$.

LHS: $10C_2 + 2 \times 10C_3 + 10C_4$

$= 10C_2 + 10C_3 + 10C_3 + 10C_4$

$= (10C_3 + 10C_2) + (10C_4 + 10C_3)$

$nC_r + nC_{r-1} = n+1C_r$

$= 11C_3 + 11C_4$

$= 12C_4.$

4.50: If $(n+2)C_7 : (n-1)P_4 = 13 : 24$ find n.

$\frac{(n+2)C_7}{(n-1)P_4} = \frac{13}{24}$

$\frac{(n+2)!}{(n-5)! \times 7!} \Big/ \frac{(n-1)!}{(n-5)!} = \frac{13}{24}$

$$\frac{(n+2)!}{(\cancel{n-5})!\,7!} \times \frac{\cancel{(n-5)!}}{(n-1)!} = \frac{13}{24}$$

$$\frac{(n+2)(n+1)(n)\cancel{(n-1)!}}{7!\,\cancel{(n-1)!}} = \frac{13}{24}$$

$$n(n+1)(n+2) = \frac{13\times 7\times \cancel{6}\times 5\times \cancel{4}\times 3\times 2\times 1}{\cancel{24}}$$

$$= 13\times 14\times 15$$

$$\Rightarrow n = 13$$

EXERCISE - 4.3.

1. If $nC_{12} = nC_9$ Find $21C_n$.

Sol: $nC_{12} = nC_9$ $\qquad nC_r = nC_{n-r}$.

$nC_{n-12} = nC_9$

$n-12 = 9 \Rightarrow n = 21$. $\therefore 21C_{21} = 1$

2) If $15C_{2r-1} = 15C_{2r+4}$ find r.

Sol. $15C_{2r-1} = 15C_{2r+4}$

$15C_{15-(2r-1)} = 15C_{2r+4}$

$15 - 2r + 1 = 2r + 4$

$12 = 4r \Rightarrow r = 3$.

3) If $nP_r = 720$, $nC_r = 120$ find n, r.

Sol: $\frac{nP_r}{nC_r} = r! = \frac{72\cancel{0}}{12\cancel{0}} = 6$.

$= 1\cdot 2\cdot 3$

$r! = 3!$

$r = 3$.

$nC_3 = 120$

$\frac{n(n-1)(n-2)}{1\cdot 2\cdot 3} = 120$

$n(n-1)(n-2) = 720$

$n(n-1)(n-2) = 10\times 9\times 8$

$n = 10$.

4) P.T $15C_3 + 2\times 15C_4 + 15C_5 = 17C_5$

LHS: $15C_3 + 15C_4 + 15C_4 + 15C_5$

$= \underline{15C_4 + 15C_3} + \underline{15C_4 + 15C_5}$ $\quad nC_r + nC_{r-1} = n+1C_r.$

$= 16C_4 + 16C_5 = 17C_5$

5) P.T $35C_5 + \sum_{r=0}^{4} {}^{(39-r)}C_4 = 40C_5.$

LHS: $35C_5 + \sum_{r=0}^{4} {}^{(39-r)}C_4$

$= 35C_5 + 39C_4 + 38C_4 + 37C_4 + 36C_4 + 35C_4$

$= 35C_5 + 35C_4 + 36C_4 + 37C_4 + 38C_4 + 39C_4$

$= 36C_5 + 36C_4 + 37C_4 + 38C_4 + 39C_4$

$= 37C_5 + 37C_4 + 38C_4 + 39C_4$

$= 38C_5 + 38C_4 + 39C_4$

$= 39C_5 + 39C_4$

$= 40C_5$

6) If $(n+1)C_8 : (n-3)P_4 = 57:16$ find the value of n.

$$\frac{(n+1)C_8}{(n-3)P_4} = \frac{57}{16}.$$

$$\frac{(n+1)!}{\cancel{(n-7)!}\,8!} \times \frac{\cancel{(n-7)!}}{(n-3)!} = \frac{57}{16}$$

$$\frac{(n+1)(n)(n-1)(n-2)\cancel{(n-3)!}}{8!\,\cancel{(n-3)!}} = \frac{57}{16}.$$

$$= \frac{57 \times 8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}{16}$$

$= 19 \times 3 \times 7 \times 6 \times 5 \times 4 \times 3$

$= 21 \times 20 \times 19 \times 18$

$\underline{\underline{n = 20}}$

7) PT $2nC_n = \dfrac{2^n \, 1 \cdot 3 \cdots (2n-1)}{n!}$

$$2nC_n = \frac{2n!}{(2n-n)!\,n!}$$

$$= \frac{2n!}{n!\,n!}$$

$$\frac{2n(2n-1)(2n-2)(2n-3)\cdots 2\cdot 1}{n!\,n!}$$

$$= \frac{2^n \cdot n(2n-1)(n-1)(2n-3)\cdots 3\cdot 1}{n!\,n!}$$

$$= \frac{2^n \cdot (n(\cancel{n-1})(\cancel{n-2})\cdots 1)(2n-1)(2n-3)\cdots 3\cdot 1}{\cancel{n!}\,n!}$$

$$= \frac{2^n \cdot 1\cdot 3 \cdots (2n-3)(2n-1)}{n!}$$

8) PT if $1 \le r \le n$ then $n \times {}^{n-1}C_{r-1} = (n-r+1)\,{}^{n}C_{r-1}$.

LHS: $n \times {}^{n-1}C_{r-1} = n \cdot \dfrac{(n-1)!}{(n-\cancel{1}-r+\cancel{1})!\,(r-1)!}$

$$= \frac{n(n-1)!}{(n-r)!\,(r-1)!} = \frac{n!}{(n-r)!\,(r-1)!}$$

$$= \frac{n!\,(n-r+1)}{(n-r)!\,(n-r+1)\,(r-1)!}$$

$$= \frac{n!\,(n-r+1)}{(n-r+1)!\,(r-1)!}$$

$$= (n-r+1)\cdot {}^{n}C_{r-1}$$

9) 1. A Kabadi Coach has 14 players ready to play. How many different teams of 7 players could the coach put on the court.

2. There are 15 persons in a party and if each two of them shakes hands with each other - How many handshakes happen in the party

3. How many chords can be drawn through 20 points on the circle

4. In a Parking lot one hundred, one year old cars, are parked out of them five are to be chosen at random for to check its pollution devices. How many different set of five

5) How many ways can a team of 3 boys 2 girls and 1 transgender be selected from 5 boys, 4 girls and 2 transgender.

Sol: 1. Number of ways of selection from 14 players 7 players are selected : $14C_7$

$$= \frac{14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7}$$

$$= 13 \times 11 \times 2 \times 3 \times 4$$

$$= 3432.$$

2. Number of hand shakes : $15C_2 = \frac{15 \times 14}{2} = 105$

3. For drawing chords we need two points

$\therefore$ Number of chords $= 20C_2 = \frac{20 \times 19}{2} = 190.$

4) Number of selections : $100C_5 =$

5) Number of selections $= 5C_3 \times 4C_2 \times 2C_1$

$$= \frac{5 \cdot 4 \cdot 3}{1 \cdot 2 \cdot 3} \times \frac{4 \cdot 3}{1 \cdot 2} \times 2$$

$$= 120$$

10) Find the total number of subsets with 1) 4 elements 2) 5 elements 3) n elements.

1. with 4 elements $= 2^4$ subsets.

2. with 5 elements : 2^5 subsets

3. with n elements : 2^n subsets.

11) The trust has 25 members

1) How many ways 3 officers can be selected

2. In how many ways can a president, Vice president, and a secretary to be selected.

1) Number of ways $25C_3 = \frac{25 \times 24 \times 23}{1 \cdot 2 \cdot 3} = 2300$

2. Number of ways $25C_3 = 2300.$

12) How many ways a commitee of six person from 10 persons can be chosen along with chairperson and a secoretary.

Number of ways of chairperson and secoretary can be selected = $10P_2$

From the remaining 8 persons Number of selection : $8C_4$

$\therefore$ Total number of selection = $10P_2 \times 8C_4$

$= \frac{10 \times 9 \times \cancel{8}^{2} \times 7 \times \cancel{6} \times 5}{2 \times 1 \cdot \cancel{2} \cdot \cancel{3} \cdot \cancel{4}}$

= $= 3150$

13) How many different selections of 5 Books can be made from 12 different books if 1) Two particular Books are always selected
2) Two particular books are never selected

1) For two particular books are always selected $\Big\} = 10C_3 = \frac{10 \times \cancel{9}^{3} \times \cancel{8}^{4}}{1 \cdot \cancel{2} \cdot \cancel{3}}$

$= 120$

2) Two particular books are never selected

$= 10C_5 = \frac{\cancel{10}^{2} \times 9 \times \cancel{8} \times 7 \times \cancel{6}}{1 \cdot \cancel{2} \cdot \cancel{3} \cdot \cancel{4} \cdot \cancel{5}}$

$= 252.$

14) There are 5 teachers and 20 students. out of them a commitee of 2 teachers and 3 students is to be formed. Find the number of ways in which this can be done. Further find in howmany of these Commitees 1) a particular teacher is included
2) a particular student is excluded.

The number of selection of 2 T and 3 S from 5 T and 20 S is

$= 5C_2 \times 20C_3$

$= \frac{5 \times \cancel{4}^{2}}{1 \cdot \cancel{2}} \times \frac{20 \times 19 \times \cancel{18}^{3}}{1 \cdot \cancel{2} \cdot \cancel{3}} = 11400$

1) Particular teacher is included. (ie) out of 5 teachers 1 T is must $\Big\} \therefore 4C_1 \times 20C_3 = 4 \times \frac{20 \times 19 \times \cancel{18}^{3}}{1 \cdot \cancel{2} \cdot \cancel{3}} = 4560$

2) one student tobe exclucded $= 5C_2 \times 19C_3 = \frac{5 \times \cancel{4}^{2}}{1 \cdot \cancel{2}} \times \frac{19 \times \cancel{18}^{3} \times 17}{1 \cdot \cancel{2} \cdot \cancel{3}}$

$= 9690$

15) In an examination a student has to answer 5 questions out of 9 questions ~~with~~ in which 2 are compulsary. In howmany ways a student can answer the questions.

Total number of questions = 9.

5 questions to be selected.

in which 2 is compulsary.

$\therefore$ The number of ways $= {}^7C_3$

$= \frac{7\times \cancel{6}\times 5}{1.\cancel{2}.\cancel{3}} = 35$

16) Determine the number of 5 card combinations out of deck of 52 cards if there is exactly three aces in each combination

Total cards = 52

Aces = 4

Non aces = 48.

From 4 aces 3 has to be select and from 48 2 has to select $\Big\} = 4C_3 \times 48C_2$

$= \overset{2}{\cancel{4}} \times \frac{48\times 47}{1.\cancel{2}}$

$= 4512$.

17) Find the number of ways of forming of 5 members out of 7 Indians and 5 Americans so that always Indian will be the majority in the committee.

Indian (7)	American (5)	Combinations.
5	—	$7C_5 = 21$
4	1	$7C_4 \times 5C_1 = 175$
3	2	$7C_3 \times 5C_2 = 350$

Total number of Commities $\underline{546}$

18) A Committee of 7 peoples has to be formed from 8 men and 4 women In howmany ways can this be done when the committee consist of

1) exactly 3 women

2) at least 3 women

3) atmost 3 women.

1. Exactly 3 women.

Men (8)	Women(4)	Combinations
4	3	$8C_4 \times 4C_3 = 280$.

2) Atleast 3 women

4	3	$8C_4 \times 4C_3 = 280$
3	4	$8C_3 \times 4C_4 = 56$

Total = 336.

3) At most 3 womens

4	3	$8C_4 \times 4C_3 = 280$
5	2	$8C_5 \times 4C_2 = 336$
6	1	$8C_6 \times 4C_1 = 112$
7	0	$8C_7 \times 4C_0 = 8$

Total = 736

19) 7 relatives of of a man comprises 4 ladies and 3 gentlemen his wife also has 7 relatives, 3 of then are ladies and 4 gentlemen. In how many ways can they invite a dinner party of 3 ladies and 3 gentlemen so that there are 3 of Men's relative and 3 of wifes relative.

Man		Woman		
Gents 3	Ladies - 4	Gents 4	Ladies - 3.	
3	–	–	3	$3C_3 \times 3C_3 = 1$
2	1	1	2	$3C_2 \times 4C_1 \times 4C_1 \times 3C_2 = 144$
1	2	2	1	$3C_1 \times 4C_2 \times 4C_2 \times 3C_1 = 324$
	3	3		$4C_3 \times 4C_3 = 16$

Total: 485.

20) A box contains two white balls, three black balls and 4 red balls. In how many ways can three balls be drawn from the box, if atleast one black ball is to be included in the draw?

2W	3B	4R	Combinations.
2	1		$2C_2 \times 3C_1 = 3$
1	1	1	$2C_1 \times 3C_1 \times 4C_1 = 24$
	1	2	$3C_1 \times 4C_2 = 18$
1	2		$2C_1 \times 3C_2 = 6$
	2	1	$3C_2 \times 4C_1 = 12$
	3		$3C_3 = 1$ Total 64

21) Find the number of strings of 4 letters that can be formed with the letters of the word EXAMINATION. N N A, A

See Sheet-24 Back Side.

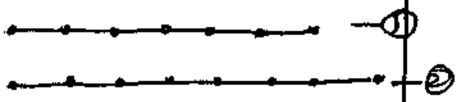
22) How many Δ's can be formed by joining 15 points on the plane in which no line joining any three points.

Sol: To get the Δle we need 3 points

$\therefore$ Number of Δles are $^{15}C_3 = 455$

23) How many Δles can be formed by 15 points in which 7 of them lie on the one line and the remaining 8 on another parallel line

Sol: Select 2 points from the line ① and select 1 point in line two.



Number of Δles $= {}^7C_2 \times {}^8C_1 = 168$.

and select 1 point in line ① and 2 points in line ②.

Number of Δles ${}^7C_1 \times {}^8C_2 = 196$

$\therefore$ Total number of Δles $= 168 + 196 = 364$

24) There are 11 points, No three of these lies in the same straight line except 4 points which are collinear.

1. The number of st. lines that can be obtained from the pairs of these points

2. The number of Δ's that can be formed for which the points are their vertices.

Sol: From the 11 points when joining two we get $^{11}C_2$ st. lines.

But 4 points are collinear $^4C_2$ lines diminished

But the 4 points make one line

$\therefore$ Total number of lines $= {}^{11}C_2 - {}^4C_2 + 1 = 50$

To get the Δle we need 3 points $\therefore$ $^{11}C_3$.

But 4 points are collinear $\therefore$ $^4C_3$

Total Δles $^{11}C_3 - {}^4C_3 = 165 - 4 = 161$.

25) A polygon has 90 diagonals find the number of sides.

Let there are n noncollinear points.

$\therefore$ Number of lines $= nC_2$

$\therefore$ Number of lines $= n$.

$\therefore$ Number of diagonals $= nC_2 - n$

$$= \frac{n(n-1)}{2} - n.$$

Given $\frac{n(n-1)}{2} - n = 90$

$$n(n-1) - 2n = 180$$

$$n^2 - 3n - 180 = 0$$

$$(n-15)(n+12) = 0$$

$$n = \cancel{-12}, 15$$

180 → −15, 12

$\therefore$ number of sides $= 15$.

Example: 4.51. A salad at a certain Restaurant consists of 4 of the following fruits, apple, banana, guava, pomegranate grapes, papaya and pineapple. Find the number of fruit salads.

Number of fruits $= 7$.

Number of combinations in 7 things taken 4 at a time

$$= 7C_4 = 7C_3$$

$nC_r = nC_{n-r}$

$$= \frac{7 \times 6 \times 5}{1 \cdot 2 \cdot 3} = 35.$$

4.52. A Mathematics club has 15 members in that 8 girls and 6 of the members are to be selected for a compition and half of them should be girls. How many ways of these selections are possible.

From 8 girls 3 has to be selected $= 8C_3$ ways.

From 7 boys 3 has to be selected $=$ $7C_3$ ways

Total number of ways $= 8C_3 \times 7C_3 = 56 \times 35 = 1960$

4.53. In rating 20 brands of cars, a car magazine picks a first, second, third, fourth and fifth best brand and then 7 more as acceptable. In how many ways can it be done.

From the 20 brands 5 has to be selected in $20C_5$ ways.

From the remaining 7 has to be selected in $15C_7$ ways

Total ways $= 20C_5 \times 15C_7$ ways

4.54) From a class of 25 students 10 students are to be chosen for an excursion party. There are 4 students who decide that either all of them will join or none of them will join. In how many ways can the excursion party be chosen.

1. If all the 4 students included $21C_6$ ways.

2 If all the 4 students be excluded $21C_{10}$ ways.

$\therefore$ Total number of ways : $21C_6 + 21C_{10}$

$$= \frac{21!}{15!\,6!} + \frac{21!}{11!\,10!}.$$

4.55 A box of on dozen apple contain a rotten apple. If we are choosing 3 apples simultaneously, in how many ways one can get only good apple.

Total number of ways of selecting 3 apples from 12 apples.
$= 12C_3$ ways.

Number of selecting rotten apple is $11C_2 = 55$.

$\therefore$ Number of ways of getting good apples $= 12C_3 - 11C_2$
$= 220 - 55$
$= 165.$

4.56) An exam paper contains 8 questions. 4 in part A and 4 in part B. Examiners are required to answer 5 questions. In how many can this be done if
1) There are no restrictions of choosing a number of questions in either part
2. At least two questions from part A must be answered.

1. If there is no restrictions No ways : $8C_5 = 8C_3 = 56$.

2)

A	B	No of combinations.
2	3	$4C_2 \times 4C_3 = 24$
3	2	$4C_3 \times 4C_2 = 24$
4	1	$4C_4 \times 4C_1 = 4$

Total number of ways. $\underline{52}$

4.57) out of 7 consonants and 4 vowels, how many strings of 3 consonants and 2 vowels can be formed.

Number of ways of selecting 3 consonents from 7 and 2 vowels from 4

$$ is = 7C_3 \times 4C_2$$

The number of ways of arranging 5 letters among themselves $= 5!$

Total number of ways of of selection : $7C_3 \times 4C_2 \times 5!$

$= 35 \times 6 \times 120$

$= 25200$.

4.59) If the set of m parallel lines intersect another set of n parallel lines (not parallel to the lines in the first set) then find the number of parallelograms formed in this lattice structure.

If we select 2 lines from the set of m lines and 2 lines from the second set of n lines one parallelogram is formed.

$\therefore$ Number of parallelogram is $mC_2 \times nC_2$.

4.60) How many diagonals are there in a polygon with n sides.

A polygon of n sides has n vertices. By joining any two vertices of a polygon. We obtain either a side or a diagonal of the polygon. Number of line segments obtained by joining the vertices of an n sided polygon taken two at a time is nC_2. out of these lines there are n sides of the polygon.

$$\therefore \text{No of diagonals} = nC_2 - n$$

$$= \frac{n(n-1)}{2} - n = \frac{n(n-1)-2n}{2}$$

$$= \frac{n(n-3)}{2}$$

4.58) Find the number of strings of 5 letters that can be formed with the letters of the word PROPOSITION.

Sol: PP, II, OOO, R, S, T, N.

Sol: Letter options	Selections	Arrangements.
5 distinct (RSTNPIO)	$^7C_5$	$^7C_5 \times 5! = 2520$
1 set of 3 alike (OOO) 1 set of 2 alike (PP, II)	$^1C_1 \times {}^2C_1$	$^1C_1 \times {}^2C_1 \times \frac{5!}{3!2!} = 20$
1 set of 3 alike (OOO) 2 distinct (RSTNPI)	$^1C_1 \times {}^6C_2$	$^1C_1 \times {}^6C_2 \times \frac{5!}{3!} = 300$
2 sets of 2 alike (PP II OO) 1 distinct (RSTN) and remaining one in 2 alike.	$^3C_2 \times {}^5C_1$	$^3C_2 \times {}^5C_1 \times \frac{5!}{2!2!} = 450$
1 set of two alike (PP II OO) 3 distinct RSTN and remaining 2 in 2 alike	$^3C_1 \times {}^6C_3$	$^3C_1 \times {}^6C_3 \times \frac{5!}{2!} = 3600$
∴ Total number of strings = 6890		6890

2) Find the number of strings of 4 letters that can be formed with the letters of the word EXAMINATION?

Sol: Letter option.	Selection	Arrangements.
4 distinct EXMTO	$^8C_4$	$^8C_4 \times 4! = 1680$
2 sets of two alike (AA, II, NN)	$^3C_2$	$^3C_2 \times \frac{4!}{2!2!} = 18$
1 set of 2 alike (AA, II, NN) 2 distinct EXMTONI	$^3C_1 \times {}^7C_2$	$^3C_1 \times {}^7C_2 \times \frac{4!}{2!} = 756$
		2454

Mathematical Induction.

Example: 4.61) By principle of mathematical induction P.T for all integers $n \geq 1$. $1+2+3+\dots+n = \frac{n(n+1)}{2}$

Sol Let $P(n) = 1+2+3+\dots+n = \frac{n(n+1)}{2}$

For $n=1$ $P(1) = 1 = \frac{1(1+1)}{2} = 1$ $\therefore P(1)$ is True.

For $n=K$ $P(K) = 1+2+3+\dots+K = \frac{K(K+1)}{2}$ is True

T.P.T For $n=K+1$ $P(K+1) = 1+2+3+\dots+K+(K+1) = \frac{(K+1)(K+2)}{2}$

LHS $1+2+3+\dots+K+(K+1) = \frac{K(K+1)}{2}+(K+1)$

$= \frac{K(K+1)+2(K+1)}{2}$

$= \frac{(K+1)(K+2)}{2}$ is True

$\therefore P(n) = 1+2+\dots+n = \frac{n(n+1)}{2}$

4.62) P.T the sum of first n positive odd numbers is n^2

Sol: Let $P(n) = 1+3+5+\dots+(2n-1) = n^2$

For $n=1$ $P(1) = 1 = 1^2 = 1$ $\therefore P(1)$ is True

For $n=K$ $P(K) = 1+3+\dots+(2K-1) = K^2$ is True.

P.T For $n=K+1$ $P(K+1) = 1+3+\dots+(2K-1)+(2(K+1)-1) = (K+1)^2$

LHS $1+3+\dots(2K-1)+2(K+1)-1$

$= K^2+2(K+1)-1$

$= K^2+2K+\cancel{2}^{1}-\cancel{1}$

$= (K+1)^2$ $\therefore P(K+1)$ is True.

$\therefore P(n) = 1+3+\dots+(2n-1) = n^2$.

4.63) P.T $1^2+2^2+\dots+n^2 = \frac{n(n+1)(2n+1)}{6}$ $n \geq 1$.

(or) P.T Sum of the squares of the first n natural number $= \frac{n(n+1)(2n+1)}{6}$.

Sol: Let $P(n) = 1^2+2^2+3^2+\dots+n^2 = \frac{n(n+1)(2n+1)}{6}$

For $n=1$ $P(1) = 1^2 = \frac{1(1+1)(2+1)}{6} = \frac{1\times2\times3}{6} = 1$ $\therefore P(1)$ is True.

For $n=K$ $P(K) = 1^2+2^2+3^2+\dots+K^2 = \frac{K(K+1)(2K+1)}{6}$ is True

For $n = k+1$

TPT $P(k+1) = 1^2+2^2+3^2+\dots+k^2+(k+1)^2 = \frac{(k+1)(k+2)(2(k+1)+1)}{6}$

$= \frac{(k+1)(k+2)(2k+3)}{6}$.

LHS: $1^2+2^2+3^2+\dots+k^2+(k+1)^2$

$= \frac{k(k+1)(2k+1)}{6} + (k+1)^2$

$2k^2+4k+3k+6$
$= 2k(k+2)+3(k+2)$
$(k+2)(2k+3)$

$= \frac{k(k+1)(2k+1)+6(k+1)^2}{6}$

$= \frac{(k+1)[k(2k+1)+6(k+1)]}{6} = \frac{(k+1)(2k^2+k+6k+6)}{6}$

$= \frac{(k+1)(2k^2+7k+6)}{6}$

$\therefore P(k+1)$ is True. $= \frac{(k+1)(k+2)(2k+3)}{6}$

Hence $P(n) = 1^2+2^2+3^2+\dots+n^2 = \frac{n(n+1)(2n+1)}{6}$

4.64) Using the mathematical induction S.T for any natural number $\frac{1}{1\cdot2}+\frac{1}{2\cdot3}+\frac{1}{3\cdot4}+\dots+\frac{1}{n(n+1)} = \frac{n}{n+1}$.

Sol: let $P(n) = \frac{1}{1\cdot2}+\frac{1}{2\cdot3}+\frac{1}{3\cdot4}+\dots+\frac{1}{n(n+1)} = \frac{n}{n+1}$.

For $n=1$ $\frac{1}{1\cdot2} = \frac{1}{1+1} \Rightarrow \frac{1}{2}=\frac{1}{2}$ $\therefore P(1)$ is True.

for $n=k$ $\frac{1}{1\cdot2}+\frac{1}{2\cdot3}+\frac{1}{3\cdot4}+\dots+\frac{1}{k(k+1)} = \frac{k}{k+1}$ is True.

for $n=k+1$

T.P.T $P(k+1) = \frac{1}{1\cdot2}+\frac{1}{2\cdot3}+\dots+\frac{1}{k(k+1)}+\frac{1}{(k+1)(k+2)} = \frac{k+1}{k+2}$

LHS: $\frac{k}{(k+1)}+\frac{1}{(k+1)(k+2)}$

$= \frac{k(k+2)+1}{(k+1)(k+2)}$

$= \frac{k^2+2k+1}{(k+1)(k+2)}$

$= \frac{(k+1)^{\cancel{2}}}{\cancel{(k+1)}(k+2)} = \frac{k+1}{k+2}$.

4.65) Prove that for any natural number n, $a^n - b^n$ is divisible by $a-b$ where $a > b$.

Sol: Let $P(n) = a^n - b^n$ is divisible by $a-b$.

For $n=1$ $P(1) = a-b$ which is divisible by $a-b$ $\therefore P(1)$ is True.

For $n=k$ $P(k) = a^k - b^k$ is divisible by $a-b$ is True.

(ie) $P(k) = \lambda(a-b)$.

For $n = k+1$ $P(k+1) = a^{k+1} - b^{k+1}$ is divisible by $a-b$.

T.P.T: (ie) $a^{k+1} - b^{k+1} = \lambda_1(a-b)$

$$\begin{aligned}
LHS &= a^{k+1} - b^{k+1} \\
&= a^{k+1} - ab^k + ab^k - b^{k+1} \\
&= a(a^k - b^k) + b^k(a-b) \\
&= a(\lambda(a-b)) + b^k(a-b) \\
&= (a-b)(\lambda a + b^k) \\
&= (a-b)\lambda_1 \quad \text{where } \lambda_1 = \lambda a + b^k
\end{aligned}$$

$\therefore a^{k+1} - b^{k+1}$ is divisible by $a-b$.

$\therefore P(k+1)$ is True.

Hence $P(n) = a^n - b^n$ is divisible by $a-b$.

4.66) P.T $3^{2n+2} - 8n - 9$ is divisible by 8 for all $n \geq 1$.

Let $P(n) = 3^{2n+2} - 8n - 9$ which is divisible by 8.

For $n=1$ $P(1) = 3^{2+2} - 8 - 9 = 81 - 8 - 9$

$= 64$ which is divisible by 8

$\therefore P(1)$ is True.

For $n=k$ $P(k) = 3^{2k+2} - 8k - 9$ which is divisible by 8 is True

(ie) $3^{2k+2} - 8k - 9 = 8\lambda$

T.P.T

For $n = k+1$ $P(k+1) = 3^{2(k+1)+2} - 8(k+1) - 9$ is divisible by 8.

$$\begin{aligned}
&= 3^2\, 3^{2(k+1)} - 8k - 8 - 9 \\
&= 3^2\, 3^{2(k+1)} - 8k - 17 \\
&= 3^2(8\lambda + 8k + 9) - 8k - 17 \\
&= (72\lambda + 72k + 81 - 8k - 17
\end{aligned}$$

$= 72\lambda + 64k + 56$

$= 8(9\lambda + 8k + 7)$

$= 8 * \lambda_1 \Rightarrow$ which is divisible by 8.

$\therefore P(k+1)$ is True.

Hence $P(n) = 3^{2n+2} - 8n - 9$ is divisible by 8.

4.67) using Mathematical induction. S.T for any integer $n \geq 2$ $3n^2 > (n+1)^2$.

Sol: Let $P(n)$ $3n^2 > (n+1)^2$

$\because n \geq 2$.

For $n = 2$ $P(2) =$ $3 \times 4 > (2+1)^2$ is True $\therefore P(2)$ is True.

$12 > 9$

For $n = k$ $P(k)$ $3k^2 > (k+1)^2$ is True.

TPT

For $n = k+1$, $P(k+1)$ $3(k+1)^2 > (k+2)^2$

$= 3(k^2 + 2k + 1)$

$= 3k^2 + 6k + 3$

$> (k+1)^2 + 6x + 3$

$= k^2 + 2k + 1 + 6k +$

$= k^2 + 8k + 4$

$= k^2 + 4k + 4 + 4k$

$= (k+2)^2 + 4k$ $\because k > 0$.

$> (k+2)^2$

$\therefore P(k+1)$ is True.

Hence $P(n)$ $3n^2 > (n+1)^2$

4.68) using Mathematical induction S.T for any integer $n \geq 2$ $3^n > n^2$.

Sol: Let $P(n)$ $3^n > n^2$ $\because n \geq 2$

For $n = 2$, $P(2)$: $3^2 > 2^2$

$9 > 4$ is True $\therefore P(2)$ is True.

For $n = k$ $P(k) = 3^k > k^2$ is True

For $n = k+1$, T.P.T $P(k+1)$ $3^{k+1} > (k+1)^2$

$= 3^{k+1} = 3^k \cdot 3$

$> k^2 \cdot 3$

$> (k+1)^2$ (by 4.67)

$\therefore P(k+1)$ is True

Hence $3^n > n^2$.

4.69) By the principle of Mathematical Induction P.T for $n \in N$

$$\cos\alpha + \cos(\alpha+\beta) + \cos(\alpha+2\beta) + \cdots \cos(\alpha+(n-1)\beta) = \cos\left(\alpha + \frac{(n-1)\beta}{2}\right) \times \frac{\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}}$$

Sol: Let $P(n) = \cos\alpha + \cos(\alpha+\beta) + \cos(\alpha+2\beta) + \cdots + \cos(\alpha+(n-1)\beta) = \cos\left(\alpha + \frac{(n-1)\beta}{2}\right) \times \frac{\sin\frac{n\beta}{2}}{\sin\beta/2}$

For $n=1$ $P(1) = \cos\alpha = \cos\alpha \cdot \frac{\sin\beta/2}{\sin\beta/2} = \cos\alpha$ $\therefore P(1)$ is True.

For $n=k$ $P(k) = \cos\alpha + (\cos(\alpha+\beta)) + \cos(\alpha+2\beta) + \cdots + \cos(\alpha+(k-1)\beta) = \cos\left(\alpha + \frac{(k-1)\beta}{2}\right)\frac{\sin\frac{k\beta}{2}}{\sin\beta/2}$ is True

For $n=k+1$ T.P.T $P(k+1) = \cos\alpha + \cos(\alpha+\beta) + \cos(\alpha+2\beta) + \cdots + \cos(\alpha+(k-1)\beta) + \cos(\alpha+k\beta)$

$$= \cos\left(\alpha + \frac{k\beta}{2}\right)\frac{\sin\frac{(k+1)\beta}{2}}{\sin\beta/2}$$

LHS $\cos\alpha + \cos(\alpha+\beta) + \cos(\alpha+2\beta) + \cdots + \cos(\alpha+(k-1)\beta) + \cos(\alpha+k\beta)$

$$= \cos\left(\alpha + \frac{(k-1)\beta}{2}\right)\frac{\sin\frac{k\beta}{2}}{\sin\beta/2} + \cos(\alpha+k\beta)$$

$$= \frac{1}{\sin\beta/2}\left[\cos\left(\left(\alpha+\frac{k\beta}{2}\right) - \beta/2\right)\cdot\sin\frac{k\beta}{2} + \cos(\alpha+k\beta)\sin\beta/2\right]$$

$$= \frac{1}{\sin\beta/2}\left\{\left[\cos\left(\alpha+\frac{k\beta}{2}\right)\cos\beta/2 + \sin\left(\alpha+\frac{k\beta}{2}\right)\sin\beta/2\right]\sin\frac{k\beta}{2} + \cos(\alpha+k\beta)\sin\beta/2\right\}$$

$$= \frac{1}{\sin\beta/2}\left[\cos\left(\alpha+\frac{k\beta}{2}\right)\cos\beta/2\sin\frac{k\beta}{2} + \sin\left(\alpha+\frac{k\beta}{2}\right)\sin\beta/2\sin\frac{k\beta}{2} + \cos(\alpha+k\beta)\sin\beta/2\right]$$

$$= \frac{1}{\sin\beta/2}\left[\cos\left(\alpha+\frac{k\beta}{2}\right)\cos\beta/2\sin\frac{k\beta}{2} + \sin\beta/2\left(\sin\alpha+\frac{k\beta}{2}\sin\frac{k\beta}{2} + \cos(\alpha+k\beta)\right)\right]$$

$$= \frac{1}{\sin\beta/2}\left[\cos\left(\alpha+\frac{k\beta}{2}\right)\cos\beta/2\sin\frac{k\beta}{2} + \frac{\sin\beta/2}{2}\left(2\sin\frac{\alpha+k\beta}{2}\sin\frac{k\beta}{2} + 2\cos(\alpha+k\beta)\right)\right]$$

$$= \frac{1}{\sin\beta/2}\left[\cos\left(\alpha+\frac{k\beta}{2}\right)\cos\beta/2\sin\frac{k\beta}{2} + \frac{\sin\beta/2}{2}\left(\cos\alpha - \cos(\alpha+k\beta) + 2\cos(\alpha+k\beta)\right)\right] \rightarrow \left(\because 2\sin C\sin D = \cos(D-C) - \cos(C+D)\right)$$

$$= \frac{1}{\sin\beta/2}\left\{\cos\left(\alpha+\frac{k\beta}{2}\right)\cos\beta/2\sin\frac{k\beta}{2} + \frac{\sin\beta/2}{2}\left[\cos\alpha + \cos(\alpha+k\beta)\right]\right\} \longrightarrow \left(\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}\right)$$

$$= \frac{1}{\sin\beta/2}\left\{\cos(\alpha+k\beta/2)\cos\beta/2\sin\frac{k\beta}{2} + \frac{\sin\beta/2}{2}\left[\cos\alpha + \cos(\alpha+k\beta)\right]\right\}$$

$$= \frac{1}{\sin\beta/2}\left[\cos(\alpha+k\beta/2)\cos\beta/2\sin k\beta/2 + \frac{\sin\beta/2}{2}\left(2\cos\left(\alpha+\frac{k\beta}{2}\right)\cos\frac{k\beta}{2}\right)\right]$$

$$= \frac{1}{\sin\beta/2}\cos(\alpha+k\beta/2)\left[\sin\frac{k\beta}{2}\cos\beta/2 + \cos\frac{k\beta}{2}\sin\beta/2\right] = \frac{\cos\left(\alpha+\frac{k\beta}{2}\right)\sin\frac{(k+1)\beta}{2}}{\sin\beta/2}$$

$\therefore P(k+1)$ is True.

4.70) Using Mathematical Induction. S.T for any natural number n with the assumption $i^2=-1$ $(r(\cos\theta+i\sin\theta))^n = r^n(\cos n\theta+i\sin n\theta)$ (Demoivres Theorem)

Sol: Let $P(n) = (r(\cos\theta+i\sin\theta))^n = r^n(\cos n\theta+i\sin n\theta)$

For $n=1$ $P(1) = r(\cos\theta+i\sin\theta) = r(\cos\theta+i\sin\theta)$ $\therefore$ $P(1)$ is True.

For $n=k$ $P(k) = (r(\cos\theta+i\sin\theta))^k = r^k(\cos k\theta+i\sin k\theta)$ is True.

For $n=k+1$ T.P.T $P(k+1) = (r(\cos\theta+i\sin\theta))^{k+1} = r^{k+1}(\cos(k+1)\theta+i\sin(k+1)\theta)$

$\underline{\underline{\text{LHS}}}$ $(r(\cos\theta+i\sin\theta))^{k+1} = r^{k+1}(\cos\theta+i\sin\theta)^{k+1}$

$$= r^{k+1}(\cos\theta+i\sin\theta)^k(\cos\theta+i\sin\theta)$$

$$= r^{k+1}(\cos k\theta+i\sin k\theta)(\cos\theta+i\sin\theta)$$

$$= r^{k+1}(\cos k\theta\cos\theta+i\cos k\theta\sin\theta+i\sin k\theta\cos\theta+i^2\sin k\theta\sin\theta)$$

$$= r^{k+1}[(\cos k\theta\cos\theta-\sin k\theta\sin\theta)+i(\sin k\theta\cos\theta+\cos k\theta\sin\theta)]$$

$$= r^{k+1}[\cos(k+1)\theta+i\sin(k+1)\theta]$$ $\therefore$ $P(k+1)$ is True

$\therefore$ $P(n) = (r(\cos\theta+i\sin\theta))^n = r^n(\cos n\theta+i\sin n\theta)$

EXERCISE - 4.4

1. By the principle of mathematical induction P.T for $n\geq 1$ $1^3+2^3+3^3+\dots+n^3 = \left[\frac{n(n+1)}{2}\right]^2$

Sol: Let $P(n) = 1^3+2^3+3^3+\dots+n^3 = \left[\frac{n(n+1)}{2}\right]^2$

For $n=1$ $P(1) = 1 = \left[\frac{1(1+1)}{2}\right]^2 = 1^2 = 1$ $\therefore$ $P(1)$ is True

For $n=k$ $P(k) = 1^3+2^3+\dots+k^3 = \left[\frac{k(k+1)}{2}\right]^2$ is True

For $n=k+1$ TPT $P(k+1) = 1^3+2^3+\dots+k^3+(k+1)^3 = \left[\frac{(k+1)(k+2)}{2}\right]^2$

LHS: $\left(\frac{k(k+1)}{2}\right)^2+(k+1)^3 = \frac{k^2(k+1)^2+4(k+1)^3}{4}$

$$= \frac{(k+1)^2(k^2+4k+4)}{4}$$

$$= \frac{(k+1)^2(k+2)^2}{4} = \text{RHS}.$$

$\therefore$ $P(k+1)$ is True.

Hence $1^3+2^3+3^3+\dots+n^3 = \left[\frac{n(n+1)}{2}\right]^2$

2) By the principle of mathematical induction P.T for $n \geq 1$

$$1^2+3^2+5^2+\cdots+(2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$$

Sol: Let $P(n) = 1^2+3^2+5^2+\cdots+(2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$.

For $n=1$ $P(1) = 1 = \frac{1(2-1)(2+1)}{3} = 1$ $\therefore P(1)$ is True.

For $n=k$ $P(k) = 1^2+3^2+5^2+\cdots+(2k-1)^2 = \frac{k(2k-1)(2k+1)}{3}$ is True

For $n=k+1$ T.P.T $P(k+1) = 1^2+3^2+5^2+\cdots+(2k-1)^2+(2(k+1)-1)^2$

$$= \frac{(k+1)(2(k+1)-1)(2k+3)}{3}$$

$$= \frac{(k+1)(2k+1)(2k+3)}{3}$$

LHS: $1^2+3^2+5^2+\cdots+(2k-1)^2+(2(k+1)-1)^2$

$$= \frac{k(2k-1)(2k+1)}{3} + (2k+1)^2$$

$$= \frac{k(2k-1)(2k+1)+3(2k+1)^2}{3}$$

$$= \frac{(2k+1)(k(2k-1)+3(2k+1))}{3}$$

$$= \frac{(2k+1)(2k^2+5k+3)}{3}$$

$$= \frac{(2k+1)(k+1)(2k+3)}{3} = RHS$$

$\cancel{2k^2+4k+6}$

$= \cancel{2k^2+6k-2k+6}$

$= \cancel{2k(k+3)-2(k}$

$2k^2+5k+3 = 2k^2+2k+3k+3$

$= 2k(k+1)+3(k+1)$

$= (k+1)(2k+3)$

$\therefore P(k+1)$ is True.

$\therefore 1^2+3^2+5^2+\cdots+(2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$

3) P.T the ~~first~~ sum of n non zero even numbers is n^2+n.

Sol: $P(n)$: $2+4+6+\cdots+2n = n^2+n$.

For $n=1$ $P(1) = 2 = 1^2+1 = 2$ $\therefore P(1)$ is True.

For $n=k$ $P(k) = 2+4+6+\cdots+2k = k^2+k$.

For $n=k+1$

TPT $P(k+1) = 2+4+6+\cdots+2k+2(k+1) = (k+1)^2+(k+1)$

$= (k+1)(k+2)$

LHS $= k^2+k+2(k+1) = k(k+1)+2(k+1)$

$= (k+1)(k+2)$

$\therefore P(k+1)$ is True.

Hence $2+4+6+\cdots+2n = n^2+n$.

4) By the priniple of Mathematical induction P.T for $n \geq 1$

$$1\cdot 2 + 2\cdot 3 + 3\cdot 4 + \cdots + n(n+1) = \frac{n(n+1)(n+2)}{3}$$

Sol: Let $P(n) = 1\cdot 2 + 2\cdot 3 + 3\cdot 4 + \cdots + n(n+1) = \frac{n(n+1)(n+2)}{3}$.

For $n = \cancel{k}1$ $P(1) = 1\cdot 2 = \frac{1(1+1)(1+2)}{3} = 2$ $\therefore P(1)$ is True.

For $n = K$ $P(K) = 1\cdot 2 + 2\cdot 3 + \cdots + K(K+1) = \frac{K(K+1)(K+2)}{3}$.

For $n = K+1$
TPT $P(K+1) = 1\cdot 2 + 2\cdot 3 + \cdots + K(K+1) + (K+1)(K+2)$
$= \frac{(K+1)(K+2)(K+3)}{3}$.

LHS: $\frac{K(K+1)(K+2)}{3} + (K+1)(K+2)$

$= \frac{K(K+1)(K+2) + 3(K+1)(K+2)}{3}$

$= \frac{(K+1)(K+2)(K+3)}{3}$ $\therefore P(K+1)$ is True.

$\therefore 1\cdot 2 + 2\cdot 3 + 3\cdot 4 + \cdots + n(n+1) = \frac{n(n+1)(n+2)}{3}$

5) Using the mathematical induction P.T for any natural number $n \geq 2$

$$\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)\cdots\left(1-\frac{1}{n^2}\right) = \frac{n+1}{2n}.$$

Sol: Let $P(n) = \left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\cdots\left(1-\frac{1}{n^2}\right) = \frac{n+1}{2n}$.

For $n=2$ $P(2) = 1-\frac{1}{2^2} = \frac{2+1}{4} = \frac{3}{4}$

$\frac{3}{4} = \frac{3}{4}$ $\therefore P(1)$ is True.

For $n = K$ $P(K) = \left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\cdots\left(1-\frac{1}{K^2}\right) = \frac{K+1}{2K}$ is True.

For $n = K+1$
TPT $P(K+1) = \left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\cdots\left(1-\frac{1}{K^2}\right)\left(1-\frac{1}{(K+1)^2}\right) = \frac{K+2}{2(K+1)}$

LHS: $\left(\frac{K+1}{2k}\right)\left(1-\frac{1}{(K+1)^2}\right) = \left(\frac{\cancel{K+1}}{2k}\right)\left(\frac{k^2+2k+\cancel{1}-\cancel{1}}{(K+1)^{\cancel{2}}}\right)$

$= \frac{\cancel{K}(K+2)}{2\cancel{K}(K+1)} = \frac{K+2}{2(K+1)} = \text{RHS}$

$\therefore P(K+1)$ is True

Hence $\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\cdots\left(1-\frac{1}{n^2}\right) = \frac{n+1}{2n}$.

6) using the mathematical induction, S.T for any natural number $n \geq 2$

$$\frac{1}{1+2} + \frac{1}{1+2+3} + \frac{1}{1+2+3+4} + \cdots + \frac{1}{1+2+3+\cdots+n} = \frac{n-1}{n+1}.$$

Sol: Let $P(n) = \frac{1}{1+2} + \frac{1}{1+2+3} + \cdots \frac{1}{1+2+3+\cdot+n} = \frac{n-1}{n+1}$

For $n=2$ $P(2) = \frac{1}{1+2} = \frac{1}{3}$ $RHS = \frac{2-1}{2+1} = \frac{1}{3}$ $\therefore P(1)$ is True

For $n=k$ $P(k) = \frac{1}{1+2} + \frac{1}{1+2+3} + \cdots + \frac{1}{1+2+3+\cdots+k} = \frac{k-1}{k+1}$ is True.

$\frac{\text{For } n=k+1}{\text{T.P.T}}$ $P(k+1) = \frac{1}{1+2} + \frac{1}{1+2+3} + \cdots + \frac{1}{1+2+3+k} + \frac{1}{1+2+\cdot+k+1} = \frac{k+1-1}{k+1+1}$

$= \frac{k}{k+2}$

$1+2+\cdots+n = \frac{n(n+1)}{2}$

$$LHS = \frac{k-1}{k+1} + \frac{1}{1+2+\cdots+k+1}$$

$$= \frac{k-1}{k+1} + \frac{1}{\frac{(k+1)(k+2)}{2}}$$

$$= \frac{k-1}{k+1} + \frac{2}{(k+1)(k+2)}$$

$$= \frac{(k-1)(k+2)+2}{(k+1)(k+2)} = \frac{k^2+k-\cancel{2}+\cancel{2}}{(k+1)(k+2)}$$

$$= \frac{k\cancel{(k+1)}}{\cancel{(k+1)}(k+2)} = \frac{k}{k+2}$$

$\therefore P(k+1)$ is True.

$$\therefore \frac{1}{1+2} + \frac{1}{1+2+3} + \cdots + \frac{1}{1+2+3+\cdots+n} = \frac{n-1}{n+1}$$

7) using Mathematical induction S.T for any natural number n

$$\frac{1}{1\cdot2\cdot3} + \frac{1}{2\cdot3\cdot4} + \cdots + \frac{1}{n(n+1)(n+2)} = \frac{n(n+3)}{4(n+1)(n+2)}$$

Sol: Let $P(n) = \frac{1}{1\cdot2\cdot3} + \frac{1}{2\cdot3\cdot4} + \cdots + \frac{1}{n(n+1)(n+2)} = \frac{n(n+3)}{4(n+1)(n+2)}$

For $n=1$ $P(1) = \frac{1}{1\cdot2\cdot3} = \frac{1(1+3)}{4(1+1)(1+2)} = \frac{1\times\cancel{4}}{\cancel{4}\times2\times3}$

$\frac{1}{6} = \frac{1}{6}$ $\therefore P(1)$ is True.

For $n=k$

$P(k) = \frac{1}{1\cdot2\cdot3} + \frac{1}{2\cdot3\cdot4} + \cdots + \frac{1}{k(k+1)(k+2)} = \frac{k(k+3)}{4(k+1)(k+2)}$ is True.

For $n = k+1$

T.P.T $P(k+1) = \frac{1}{1\cdot2\cdot3} + \frac{1}{2\cdot3\cdot4} + \cdots + \frac{1}{k(k+1)(k+2)} + \frac{1}{(k+1)(k+2)(k+3)} = \frac{(k+1)(k+4)}{4(k+2)(k+3)}$

LHS: $\frac{k(k+3)}{4(k+1)(k+2)} + \frac{1}{(k+1)(k+2)(k+3)}$

$= \frac{k(k+3)^2 + 4}{4(k+1)(k+2)(k+3)} = \frac{k(k^2+6k+9)+4}{4(k+1)(k+2)(k+3)}$

$= \frac{k^3+6k^2+9k+4}{4(k+1)(k+2)(k+3)}$

$$\begin{array}{r|rrrr} -1 & 1 & 6 & 9 & 4 \\ & & -1 & -5 & -4 \\ \hline & 1 & 5 & 4 & 10 \end{array}$$

$(k+1)(k^2+5k+4)$

$= (k+1)(k+1)(k+4)$

$= \frac{(k+1)^2(k+4)}{4\cancel{(k+1)}(k+2)(k+3)}$

$= \frac{(k+1)(k+4)}{4(k+2)(k+3)}$ $\therefore$ $P(k+1)$ is True.

$\therefore$ $\frac{1}{1\cdot2\cdot3} + \frac{1}{2\cdot3\cdot4} + \cdots + \frac{1}{n(n+1)(n+2)} = \frac{n(n+3)}{4(n+1)(n+2)}$

8) Using Mathematical induction, show that for any natural number n

$\frac{1}{2\cdot5} + \frac{1}{5\cdot8} + \frac{1}{8\cdot11} + \cdots + \frac{1}{(3n-1)(3n+2)} = \frac{n}{6n+4}$.

Sol: Let $P(n) = \frac{1}{2\cdot5} + \frac{1}{5\cdot8} + \cdots + \frac{1}{(3n-1)(3n+2)} = \frac{n}{6n+4}$.

For $n=1$ $P(1) = \frac{1}{2\cdot5} = \frac{1}{6+4}$

$\frac{1}{10} = \frac{1}{10}$ $\therefore$ $P(1)$ is True.

For $n=k$ $P(k) = \frac{1}{2\cdot5} + \frac{1}{5\cdot8} + \cdots + \frac{1}{(3k-1)(3k+2)} = \frac{k}{6k+4}$. is True.

For $n=k+1$

T.P.T $P(k+1) = \frac{1}{2\cdot5} + \frac{1}{5\cdot8} + \cdots + \frac{1}{(3k-1)(3k+2)} + \frac{1}{(3(k+1)-1)(3(k+1)+2)}$

$= \frac{k+1}{6(k+1)+4}$

$= \frac{k+1}{6k+10}$

LHS: $\frac{k}{6k+4} + \frac{1}{(3k+2)(3k+5)}$

$\frac{k}{2(3k+2)} + \frac{1}{(3k+2)(3k+5)}$

$$\frac{k(3k+5)+2}{2(3k+2)(3k+5)} = \frac{3k^2+5k+2}{2(3k+2)(3k+5)}$$

$$= \frac{(k+1)(\cancel{3k+2})}{2(\cancel{3k+2})(3k+5)}$$

$$= \frac{k+1}{6k+10} = RHS.$$

6
3 2

$3k^2+3k+2k+2$
$=3k(k+1)+2(k+1)$
$(k+1)(3k+2)$

$\therefore P(k+1)$ is True

Hence $\frac{1}{2\cdot5}+\frac{1}{5\cdot8}+\cdots+\frac{1}{(3n-1)(3n+2)} = \frac{n}{6n+4}$.

9) Prove by mathematical induction that
$1!+(2+2!)+(3+3!)+\cdots+(n+n!) = (n+1)!-1$.

Sol: Let $P(n) = 1!+(2+2!)+(3+3!)+\cdots+(n+n!) = (n+1)!-1$.

For $n=1$ $P(1) = 1 = (1+1)!-1$
$= 2-1$
$1 = 1$ $\therefore P(1)$ is True.

For $n=k$ $P(k) = 1!+(2+2!)+(3+3!)+\cdots+(k+k!) = (k+1)!-1$ is True.

For $n=k+1$
TPT: $P(k+1) = 1!+(2+2!)+\cdots+(k+k!)+((k+1)+(k+1)!)$
$= (k+2)!-1$.

LHS: $(k+1)!\cancel{-1}+(k+1)+(k+1)!$

12) Use induction to prove that n^3-7n+3 is divisible by 3 $\forall n \in N$.

Sol: Let $P(n) = n^3-7n+3$.

For $n=1$ $P(1) = 1-7+3 = -3$ which is divisible by 3.
$\therefore P(1)$ is True.

For $n=k$ $P(k) = k^3-7k+3$ which is divisible by 3
(ie) $k^3-7k+3 = 3t$.

For $n=k+1$
T.P.T $P(k+1) = (k+1)^3-7(k+1)+3$ which must be divisible by 3.
(ie) $(k+1)^3-7(k+1)+3 = 3\cdot$const.
$k^3+3k^2+3k+1-7k-4$
$(k^3-7k+3)+3(k^2+k)-3$

$= 3t + 3(k^2+k)+3$

$= 3(t+k^2+k+1)$ which is divisible by 3.

$\therefore n^3 - 7n + 3$ is divisible by 3.

14) Use Induction method $10^n + 3\times 4^{n+2} + 5$ is divisible by 9 $\forall n \in N$.

Let $P(n) = 10^n + 3\times 4^{n+2} + 5$

$\frac{64\times 3}{192}$

For $n=1$

$P(1) = 10 + 3\times 4^3 + 5$

$= 207$ which is divisible by 9 $\therefore P(1)$ is True

For $n=k$

$P(k) = 10^k + 3\times 4^{k+2} + 5$ is divisible by 9.

(ie) $10^k + 3\times 4^{k+2} + 5 = 9t$.

For $n = k+1$

TPT $P(k+1) = 10^{k+1} + 3\times 4^{k+3} + 5 = 9 \times (\text{const})$

$= 10\cdot 10^k + 3\cdot 4\cdot 4^{k+2} + 5$

$= 10\cdot 10^k + \cancel{3\cdot 4}\, 4^{k+2} + \cancel{9\cdot 4^{k+2}} + 5$

$= 10(10^k + 3\cdot 4^{k+2} + 5) - 18\cdot 4^{k+2} - 45$

$= 10\cdot 9t - 18\, 4^{k+2} - 45$ which is divisible by 9.

$= 9(10t - 2\, 4^{k+2} - 5)$

$= 9\lambda$ $\quad\therefore P(k+1)$ is True.

Hence $10^n + 3\times 4^{n+2} + 5$ is divisible by 9 $\forall n \in N$.

13. Using induction P.T $5^{n+1} + 4\times 6^n$ when divided by 20 leaves a remainder 9 for all natural numbers.

Sol: Let $P(n) = 5^{n+1} + 4\times 6^n$ ~~which~~ when divided by 20 leaves the remainder 9.

For $n=1$ $\quad 5^2 + 4\times 6 = 25 + 24$

$= 49$ when divided by leaves 9 reminder $\therefore P(1)$ is True

For $n=k$ $\quad P(k) = 5^k + 4\times 6^k$ when divided by 20 leaves 9

(ie) $5^k + 4\times 6^k = 20q + 9$.

for $n = k+1$

T.P.T when $P(k+1)$ is divided by 20 leaves reminder 9.

(ie) $5^{k+1} + 4\times 6^{k+1} = 20\lambda + 9$.

$5 \cdot 5^k + 4 \times 6 \cdot 6^k = 5 \cdot (5^k + 4 \cdot 6^k) + 4 \cdot 6^k$.

$= 5(20q + 9) + 4\times 6^k$.

$= 100q + 45 + 4\cdot 6^k$.

$= 100q + 49 + 4\cdot 6^k - 4$

$= 100q + 49 + 4(6^k - 1)$ — ⊛

$= (100q + 40) + 9 + 4(6^k - 1)$

No we have to prove that $6^n - 1$ is divisible by 5.

$n = 1$ $P(1) = 6 - 1 = 5$ which is divisible by 5

$n = k$ $P(k)$ $6^k - 1$ is divisible by 5 is True.

(ie) $6^k - 1 = 5t \Rightarrow 6^k = 5t + 1$

$n = k+1$ T.P.T $6^{k+1} - 1$ is divisible by 5

$6 \cdot 6^k - 1 = 6(5t + 1) - 1$

$= 6 \cdot 5t - 5$

$= 5(6t - 1)$ which is divisible by 5

$\therefore$ by ⊛ $P(k+1)$ is True Hence $5^{n+1} + 4\cdot 6^n$ when divided by 20 leaves reminder 9.

11) By the principle of Mathematical induction P.T for $n \geq 1$

$1^2 + 2^2 + 3^2 + \dots + n^2 > \frac{n^3}{3}$.

Let $P(n) = 1^2 + 2^2 + 3^2 + \dots + n^2 > n^3/3$.

For $n = 1$ $P(1) = 1 > \frac{1}{3}$ $\therefore P(1)$ is True.

For $n = k$ $P(k) = 1^2 + 2^2 + \dots + k^2 > k^3/3$ is True.

For $n = k+1$ TPT $P(k+1) = 1^2 + 2^2 + \dots + k^2 + (k+1)^2 > \frac{(k+1)^3}{3}$.

LHS: $1^2 + 2^2 + \dots + k^2 + (k+1)^2 > \frac{k^3}{3} + (k+1)^2$

$> \frac{k^3 + 3(k+1)^2}{3}$

$> \frac{k^3 + 3k^2 + 3k + 3}{3}$

$> \frac{(k+1)^3}{3}$ $\therefore P(k+1)$ True

$\therefore 1^2 + 2^2 + 3^2 + \dots + n^2 > \frac{n^3}{n}$.

to 10) using mathematical Induction. S.T for any natural number n $x^{2n}-y^{2n}$ is divisible by $x+y$.

Sol: Let $P(n) = x^{2n}-y^{2n}$ is divisible by $x+y$.

For $n=1$ $\quad P(1) = x^2-y^2 = (x+y)(x-y)$ which is divisible by $x+y$

$\therefore P(1)$ is True.

for $n=k$ $\quad P(k) = x^{2k}-y^{2k}$ is divisible by $x+y$.

(ie) $x^{2k}-y^{2k} = (x+y)t \Rightarrow x^{2k} = (x+y)t+y^{2k}$.

For $n=k+1$ $\quad P(k+1) = x^{2(k+1)} - y^{2(k+1)}$ is divisible by $x+y$.

T-P-T

$$= x^{2k}\cdot x^2 - y^{2k}\cdot y^2$$
$$= x^2\left[t(x+y)+y^{2k}\right]-y^{2k}\cdot y^2$$
$$= tx^2(x+y)+x^2y^{2k}-y^2y^{2k}$$
$$= tx^2(x+y)+y^{2k}(x^2-y^2))$$
$$= tx^2(x+y)+y^{2k}(x+y)(x-y))$$
$$= (x+y)\left(tx^2+y^{2k}(x-y)\right) \Rightarrow \text{divisible by } x+y.$$

$\therefore P(k+1)$ is True $\quad \therefore x^{2n}-y^{2n}$ is divisible by $x+y$.

EXERCISE 4.5 (One Mark)

1. The sum of the digits at the 10^{th} place of all numbers formed with the help of 2, 4, 5, 7 taken all at a time is

a) 432 2) 1080 3) 36 4) 18

Hint

Total number of numbers $= 4\times3\times2\times1 = 24$.

In the 10^{th} place 2 contains 6 times

4 " 6 times

5 " 6 times

7 " 6 times.

Sum of the digits $= 10(6\times2 + 4\times4 + 4\times5 + 4\times7)$

$= 60(2+4+5+7)$

$= 60\times18 = 1080$

2) In an examination there are three multiple choice question and each question has 5 choices. Number of ways in which a student can fail to get all answer is correct.

1) 124 2) 125 3) 64 4) 63.

Hint

Total number of ways $= 5^3 = 125$

Correct answer $\quad 1$

Number of incorrect answer $\quad 124$

3) The num of ways in which the following prize be given to a class of 30 students (boys) first and second in mathematics, first and second in physics, first in chemistry first in English.

1) $30^4\times29^2$ 2) $30^3\times29^3$ 3) $30^2\times29^4$ 4) 30×29^5

Hint

First in Maths = 30 ways

Second in Maths - 29 "

First in physics = 30 "

Second " 29 "

First in Chem. 30

First in Engh 30

$\therefore$ Total ways $30^4\times29^2$

4) The number of 5 digits number all digits of which are odd.

1) 25 2) 5^5 3) 5^6 4) 625

Hint The number of odd numbers 1, 3, 5, 7, 9

$5\times5\times5\times5\times5 = 5^5$

5) In three fingers, the number of ways four rings can be worn is ___ ways.

1) 4^3-1 2) ✓ 3^4 3) 6^8 4) 6^4.

Hint 4 rings can be worn in 3 fingers is 3^4 ways.

6) If $^{n+5}P_{n+1} = \frac{11(n-1)}{2}\, ^{n+3}P_3$ Then the value of n are

1) 7 and 11 2) ✓ 6 and 7 3) 2 and 11 4) 2 and 6.

Hint $\frac{(n+5)!}{4!} = \frac{11(n-1)}{2} \cdot \frac{(n+3)!}{3!}$

$\frac{(n+5)(n+4)\cancel{(n+3)!}}{2 \cdot \cancel{4} \times 3!} = \frac{11(n-1)}{\cancel{2}} \frac{\cancel{(n+3)!}}{\cancel{3!}}$

$n^2+9n+20 = 22n-22$

$n^2-13n+42=0 \Rightarrow (n-6)(n-7)=0 = n=6,7$.

7) The product of r consecutive positive integers is divisible by

1) $r!$ 2) $(r-1)!$ 3) $(r+1)!$ 4) r^r.

Hint Theorem! Product of r consecutive positive integers is divisible by $r!$

8) The number of telephone numbers (5 digits) having atleast one of their digits repeated is

1) 90,000 2) 10,000 3) 30240 4) ✓ 69760

When 0 is allowed in the first place.

Number of 5 digits number in the digits 0, 1, 2 . . . 9 is 10^5.

Number of 5 digits number of five digits which have none of their digits repeated $= {}^{10}P_5 = 30240$

Required Number: $10^5 - 30240 = 69760$.

9) If $^{a^2-a}C_2 = {}^{a^2-a}C_4$ then the value of a is. 1) ✓ 2 2) 3 3) 4 4) 5

$^{a^2-a}C_2 = {}^{a^2-a}C_{a^2-a-4} \Rightarrow a^2-a-4=2$ $-3, 2$.

$a^2-a-6=0$

$(a-3)(a+2)=0$

$a=-2, 3$

10) There are 10 points in a plane and 4 of them are collinear. The number of straight lines joining any two points is

1) 45 2) ✓ 40 3) 39 4) 38

$\underbrace{^{10}C_2}_{\text{Total}} - \underbrace{^4C_2}_{\text{Collinear}} + \underbrace{1}_{\text{Collinear points line}} = 45-6+1 = 40$

11) The number of ways in which a host lady invite for a party of 8 out of 12 people of whom two do not want to attend the party together.

1) $a \times {}^{11}C_7 \times {}^{10}C_8$ 2) ${}^{11}C_7 \times {}^{10}C_8$ 3) ${}^{12}C_8 - {}^{10}C_6$ d) ${}^{10}C_6 + 2!$.

Number of selecting 8 out of 12 is ${}^{12}C_8$

Two of them do not attend

$\therefore$ out of 10 selecting 6 is ${}^{10}C_6$

$\therefore$ Required value $= {}^{12}C_8 - {}^{10}C_6$

12) The number of parallelograms that can be formed from a set of four ||el lines intersecting another set of 3 ||el lines

1) 6 2) 9 3) 12 4) 18.

$${}^4C_2 \times {}^3C_2 = \frac{\cancel{4}^2 \cdot 3}{1 \cdot \cancel{2}} \times 3 = 18$$

13) Everybody in a room shakes hands with everybody else. The total number of shakes hands is 66. The number of person in the room is

1) 11 2) 12 3) 10 4) 6

Let n be the person.

number of shakes $= (n-1)(n-2) \cdots 2 \cdot 1$

$$= \frac{(n-1)n}{2} = 66$$

$$n^2 - n = 132 \Rightarrow n^2 - n - 132 = 0$$

$$(n-12)(n+11) = 0$$

$$n = \cancel{-11}, 12$$

$\therefore$ The number of persons is 12.

14) The num of sides of the polygon having 44 diagonals is

1) 4 2) 4! 3) 11 4) 22

Number of sides of the polygon whose sides is n $= \frac{n(n-3)}{2} = 44$

$$n(n-3) = 88$$

$$n^2 - 3n - 88 = 0$$

$$(n-11)(n+8) = 0 \Rightarrow n = 11, \cancel{-8}$$

15) 10 lines are drawn in a plane s.t no two of them are ||el and no three of them are concurrent then the number of point of intersection

1) 45 2) 40 3) 10! 4) 2^{10}

$${}^{10}C_2 = \frac{\cancel{10}^5 \times 9}{1 \cdot \cancel{2}} = 45$$

16) In a plane there are $\underline{10}$ points are there out of which $\underline{4}$ points are collinear. Then the number of $\triangle$le formed is

1) 110 2) $10C_3$ 3) 120 4) 116.

$$10C_3 - 4C_3 = \frac{10\cdot \cancel{9}^3\cdot \cancel{8}^4}{1\cdot 2\cdot \cancel{3}} - 4 = 116$$

17) If $2nC_3 : nC_3 = 11:1$ find n

1) 5 2) 6 3) 11 4) 7.

$$\frac{2nC_3}{nC_3} = \frac{11}{1} \Rightarrow \frac{2\cancel{n}(2n-1)(2n-2)}{\cancel{1\cdot 2\cdot 3}} = 11\cdot \frac{\cancel{n}(n-1)(n-2)}{\cancel{1\cdot 2\cdot 3}}$$

$$2nC_3 = 11\cdot nC_3 \qquad 4(2n-1)\cancel{(n-1)} = 11\cancel{(n-1)}(n-2)$$

$$8n-4 = 11n-22$$

$$3n = 18 \quad n = 6$$

18) $(n-1)C_r + (n-1)C_{r-1}$ is

1) $(n+1)Cr$ 2) $(n-1)Cr$ 3) nCr 4) $nCr-1$.

Property $(n-1)C_r + (n-1)C_{r-1} = nCr$.

19) The number of ways of choosing 5 cards out of a deck of 52 cards which include at least one King is.

1) $52C_5$ 2) $48C_5$ 3) $52C_5 + 48C_5$ 4) $52C_5 - 48C_5$.

Total selection $52C_5$.

in which $48C_5$ has no king. $\therefore$ Required value : $52C_5 - 48C_5$

20) The number of rectangles in a chess board is

1) 81 2) 9^9 3) 1296 4) 6561.

$$9C_2 \times 9C_2 = 1296$$

21) The number of 10 digits number that can be written by using the digits 2 and 3.

1) $10C_2 + 9C_2$ 2) 2^{10} 3) $2^{10} - 2$ 4) 10!

$$2^{10} \qquad = (\text{event})^{\text{object}}.$$

23) The product of the n natural numbers is equal to $(1/2)^n \times 2nC_n \times nP_n$.

25) $1+3+5+7 \cdots +17$ is equal to $9^2 = 81$

Students are advised to study the formulas and tips first and try to understand properly and then try to do the problems. Mostly problems are formula oriented.

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TIPS on PERMUTATION – XI Std -

Permutations: A permutations is an arrangement of objects in definite order. Arrangements can be made by taking some or all objects at a time.

1. The number of permutations of n different objects taken r at a time where $0 \le r \le n$ and the objects do not repeat is $n(n-1)(n-2)\ldots(n-r+1)$ which is denoted by nP_r.

   (ie) $nP_r = \dfrac{n!}{(n-r)!}$

2. The number of permutation of n different objects taken r at a time, when each may be repeated any number of times in each arrangements is n^r. (Permutation with repetitions)

3. If m particular things out of n different things are to be together then we count these r particular things as one thing and the remaining $n-m$ things are separate things.

   ∴ The total number of things is $(n-m)+1$.

   ∴ These are all different things the number of permutations is $(n-m+1)!$ But r particular things can also be arrange in $m!$ ways. ∴ The total number of permutations taken all at a time are :- $(n-m+1)!\,m!$

   Note if m particular things are identical the required number of permutations are $(n-m+1)!$

4) The number of permutations of n objects taken r at a time when the particular object is taken in each arrangements is $r \cdot n-1P_{r-1}$.

5) The number of permutations of n things taken r at a time when the particular object is never taken in each arrangements is $n-1P_r$.

6) The number of permutations of n different objects taken r at a time in which two specific objects always occur together is $2!\,(n-2)P_{(r-2)}\,(r-1)$

7) The number of permutations in n things taken (all) n at a time is $nP_n = n!$

3(A) The number of permutations of n different things taken all at a time when m specified things never come together is

8) The number of diagonals in a polygon with n sides is

$$= \frac{n(n-3)}{2}$$

Note: Penta = 5 (sides)
Septa = hepta = 7 (sides)

9) The sum of all r digit numbers that can be formed using the given n nonzero digits is $(n-1)P_{r-1} \times$ Sum of the digits $\times 111\ldots r$ times

(10) If 0 is one digit among the given n digits then we get the sum of their digits numbers

$$\left((n-1)P_{r-1} \times S.D \times 111\ldots r \text{ times}\right) - \left(n-2\,P_{r-2} \times S.D \times 111\ldots (r-1) \text{ times}\right)$$

11) $\frac{(2n)!}{n!} = 1.3.5\ldots(2n-1)\,2^n$.

3 B) The number of permutations of n things taken all at a time in which p are alike of one kind, q are alike of second kind and r are alike of third kind and the rest are different is

$$= \frac{n!}{p!\,q!\,r!}$$

3 C) The number of permutations of n things of which P_1 are alike of one kind, P_2 are alike of second kind --- P_K are alike of K^{th} kind s.t $P_1 + P_2 + P_3 + \cdots + P_K = n$ then

$$= \frac{n!}{P_1!\,P_2!\ldots P_K!}$$

Properties of permutations.

1) $nP_n = nP_{n-1}$ 2) $nP_r = n(n-1)P_{r-1}$ 3) $nP_r = (n-1)P_r + r.(n-1)P_{r-1}$.

4) $nP_n = n!$ 5) $nP_0 = 1$

Combinations: The number of combinations of n distinct objects taken r at a time is given by $nC_r = \frac{n!}{(n-r)!\,r!}$ $0 \le r \le n$

Note $nC_r = \frac{nP_r}{r!}$

Properties of combination: 1) $nC_0 = 1$, 2) $nC_n = 1$ 3) $nC_r = \frac{n(n-1)\cdots(n-r+1)}{r!}$

4) $nC_r = nC_{n-r}$ 5) $nC_r + nC_{r-1} = (n+1)C_r$ 6) $nC_r = \frac{n}{r} \times n-1\,C_{r-1}$

The number of $\triangle$s formed by joining the n non collinear points is nC_3

The number of $\triangle$s formed by joining the n points in which 4 points are collinear is $nC_3 - 4C_3$.

XI Std Maths - Work Sheet.

Important problems other than Text Book. (Permutations)

AP 1. How many different words can be formed with the letters of the word MISSISSIPPI? In how many of these permutations four I's do not come together. (NCERT)

2. How many different words can be formed with the letters of the word HARYANA
ii) How many of these begin with H and end with N.
iii) In how many of these H and N are together.

3) How many different words can be formed by using all the letters of the word ALLAHABAD? (NCERT)

4) Find the number of arrangements of the letters of the word INDEPENDENCE. In how many of the arrangements 1) do the word starts with P 2) do all the vowels always occur together 3) do all the vowels never occur together 4) do the word begin with I and end with P. (NCERT)

5) If all the letters of the word AGAIN be arranged as in the dictionary what is the fiftieth word. (NCERT)

6) The letters of the word RANDOM are written in all possible orders and these words are written out as in a dictionary. Find the rank of the word RANDOM.

7) If the different permutations of the word EXAMINATION are listed as in the dictionary find the rank of the word.
1) EXAMNATION. 2) MISSISSIPPI

8) In how many ways can the letters of the word ASSASSINATION be arranged so that all the S are together. (NCERT)

9) Find the total number of permutations of the letters of the word INSTITUTE. (NCERT)

10) What is the rank of the word ZENITH?

AP 11) How many numbers greater than 10,00,000 can be formed by using the digits 2, 3, 0, 3, 4, 2, 3? (HOTS)

12) Three Married couples are to be seated in a row having six seats in the cinema halls. If spouses are are to be seated next to each other, in how many was can they be seated. Also find the number of ways of their seating if all the ladies sit together. (NCERT)

13) How many numbers lying between 100 and 1000 can be formed with the digits 0, 1, 2, 3, 4, 5 if the repetitions of the digits are not allowed? (NCERT)

(14) How many 4 digit Numbers are there with no digit repeated? (NCERT)

15) In how many ways can 5 Girls and 3 boys be seated in a row so that no two boys are together. (NCERT)

16) In how many ways of 3 Maths, 4 History Books, 3 chemistry books and 2 Biology books can be arranged on a shelf so that all books of the same subjects are together NCERT

17) In How many ways can the letters of the word PERMUTATIONS be arranged if

1) word start with P and end with S
2) Vowels are all together. (NCERT)

18) Find the number of different words that can be formed from the letters of the word INTERMEDIATE such that two vowels are never come together. (NCERT)

19) Find the number of ways in which 5 boys and 5 Girls be seated in a row so that 1) no two girls sit together
2) boys and Girls sit alternatively.

20) It is required to seat 5 Men and 4 women in a row so that the women occupy the even places. How many such arrangements are possible? (NCERT)

21) How many words with (or) without meaning, can be formed ~~by~~ using all the letters of the word EDUCATION at a time so that the vowels and consonants occurs together? NCERT

22) All the letters of the word EAMCOT are arranged in different possible ways. Find the number of such arrangements in which no two vowels are adjacent to each other. NCERT

23) In how many ways can 5 children be arranged in a line such that 1) Two particular children of them always together
2) Two particular children of them are never together? NCERT

Students are advised ~~to~~ try to work out the work sheet problems your own.

(*) If you want solution to these problems I will send you soon call me.

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