12/2/2020

Chapter - 8

Sampling Techniques & statistical Inference

Standard Errors (SE)

n = sample size. σ = Standard error

σ^2 = variance ($\sigma + 6$)

	Standard error.
① sample mean ($\bar{x}$)	$\frac{\sigma}{\sqrt{n}}$
②. Observed sample proportion (p)	$\sqrt{\frac{PQ}{n}}$
③ Sample standard deviation (s)	$\sqrt{\frac{\sigma^2}{2n}}$ (or) $\frac{\sigma}{\sqrt{2n}}$
④ sample variance (s^2)	$\sigma^2\sqrt{\frac{2}{n}}$
⑤ sample quartiles	$1.36263 \frac{\sigma}{\sqrt{n}}$
⑥ sample median	$1.25331 \frac{\sigma}{\sqrt{n}}$
⑦ sample correlation coefficient (r)	$\frac{(1-r^2)}{\sqrt{n}}$

eg 8.6

$\bar{x} = 20/\text{min}$

$Var = \sigma^2 = 4 \Rightarrow \boxed{\sigma = 2}$

$\boxed{n = 1}$ $\boxed{hr = 60\,min}$

Standard error $= \frac{\sigma}{\sqrt{n}} = \frac{2}{\sqrt{60}}$

$= \frac{2}{7.745} = \boxed{0.258}$

eg 8.7

$n = ?$ $\boxed{\sigma = 10}$ $\boxed{S.E \text{ of } \bar{x} = 3}$

$S.E = \frac{\sigma}{\sqrt{n}}$

$3 = \frac{10}{\sqrt{n}}$

$\sqrt{n} = \frac{10}{3}$

Take Square :

$n = \frac{100}{9}$

$n = 11.11$

$\boxed{n = 11}$

eg 8.8

$\boxed{n = 9000}$

$p = \frac{3240}{9000} = 0.36$

Probability :

$P = 3 \text{ (or) } 4$

$= \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \boxed{0.33}$

$\boxed{Q = 0.67}$

$S.E = \sqrt{\frac{PQ}{n}}$

$= \sqrt{\frac{0.33 \times 0.67}{9000}}$

$= \boxed{0.00496}$ //

(E). $\boxed{n = 600}$

$P = 4\% = \boxed{0.04}$

$\boxed{Q = 0.96}$

$S.E_{(P)} = \sqrt{\frac{PQ}{n}}$

$= \sqrt{\frac{0.04 \times 0.96}{600}}$

$= \boxed{0.008}$ //

(18) $\boxed{n = 1000}$ $\boxed{\sigma = 30}$

$$S.E_{(\bar{x})} = \frac{\sigma}{\sqrt{n}} = \frac{30}{\sqrt{1000}} = \boxed{0.948}$$

(19) $\boxed{n = 60}$ $\boxed{\sigma = 3}$

$$S.E_{(s)} = \sqrt{\frac{\sigma^2}{2n}} = \sqrt{\frac{9}{2 \times 60}} = \boxed{0.2738}$$

(20) $\boxed{n = 400}$ $p = \frac{230}{400} = 0.575$

$\boxed{P = 0.58}$

$\boxed{Q = 0.42}$

$$S.E_{(P)} = \sqrt{\frac{PQ}{n}} = \sqrt{\frac{0.58 \times 0.42}{400}} = \boxed{0.0248}$$

eg 8.9

$\boxed{n = 50}$ $\boxed{\sigma = 6}$

$$S.E_{(s)} = \sqrt{\frac{\sigma^2}{2n}} = \sqrt{\frac{36}{2(50)}} = \sqrt{\frac{36}{100}} = \sqrt{0.36} = \boxed{0.6}$$

13/02/2020

Statistical Inference :

(i) Estimation

(ii) Testing Hypothesis.

Estimation :

Population mean $= \mu$

S.D $= \sigma$

mean $= \bar{x}$.

confindence Interval :

$$\bar{x} - Z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \le \mu \le \bar{x} + Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}.$$

Z_α table :

Critical values Z_α	level of significance (α) 1%	2%	5%	10%
Two tailed Test $\|Z_\alpha\|$	2.58	2.33	1.96	1.645
Right tailed Test Z_α	2.33	2.055	1.645	1.28
left tailed Test Z_α	-2.33	-2.055	-1.645	-1.28

eg 8.11

$\boxed{\sigma = 1.6}$ $\boxed{n = 64}$ $\boxed{\bar{x} = 90}$

$\left[Z_{\alpha/2} = 1.96 \ (5\%) \right]$

$\bar{x} - Z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$

$= 90 - 1.96 \times \frac{1.6}{\sqrt{64}} \leq \mu \leq 90 + 1.96 \times \frac{1.6}{\sqrt{64}}$

$= 90 - 1.96 \times 0.2 \leq \mu \leq 90 + 1.96 \times 0.2$

$= 90 - 0.392 \leq \mu \leq 90 + 0.392$

$= 89.608 \leq \mu \leq 90.392$

$\therefore$ fall in the interval $(89.6, 90.39)$ at 95%.

eg 8.12

$\boxed{n = 100}$ $\boxed{\bar{x} = 7.4}$ $\boxed{\sigma = 1.2}$

$\left[Z_{\alpha/2} = 1.96 \ (5\%) \right]$

$\bar{x} - Z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$

$7.4 - 1.96 \times \frac{1.2}{10} \leq \mu \leq 7.4 + 1.96 \times \frac{1.2}{10}$

$7.4 - 0.2352 \leq \mu \leq 7.4 + 0.2352$

$7.1648 \leq \mu \leq 7.6352$

fall in the interval $(7.16, 7.63)$ at 95%.

eg 8.13

$\boxed{n = 169}$ $\boxed{\bar{x} = 1350}$ $\boxed{\sigma = 100}$

$\left[Z_{\alpha/2} = 1.645 \ (10\%) \right]$

$\bar{x} - Z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$

$1350 - 1.645 \times \frac{100}{13} \leq \mu \leq 1350 + 1.645 \times \frac{100}{13}$

$1350 - 12.6538 \leq \mu \leq 1350 + 12.6538$

$1337.3462 \leq \mu \leq 1362.6538$

fall in the interval.

$1337.35 \leq \mu \leq 1362.65$

at 90%.

19/02/2020.

Test of Hypothesis

Z = test.

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} \sim N(0,1).$$

Null hypothesis:

$H_0 : \mu = \mu_0$ accepted.

Alternate hypothesis:

$H_1 : \mu \neq \mu_0$ rejected. (Two tail).

$H_1 : \mu > \mu_0$ Right tail.

$H_1 : \mu < \mu_0$ left tail.

Test of Hypothesis:

Type I : The error of rejecting H_0 when it is true.

Type II : The error of accepting H_0 when is false.

Critical region (or) significant values:

$$z = \frac{t - E(t)}{\sqrt{Var(t)}} = \frac{t - E(t)}{S.E} \sim N(0,1)$$

Conclusion:

(i) $|z| < z_{\alpha/2}$, H_0 accepted

(ii) $|z| > z_{\alpha/2}$, H_0 rejected.

(1A). $\boxed{n = 100}$ $\boxed{\mu = 4}$ $\boxed{\sigma = 3}$ $\boxed{\bar{x} = 3.5}$

Null hypothesis: $H_0 : \mu = 4$, accepted.

Alternative hypothesis: $H_1 : \mu \neq 4$, rejected.

level of significance 5%

Z-test

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{3.5 - 4}{\frac{3}{\sqrt{100}}} = \frac{-0.5}{\frac{3}{10}}$$

$$= \frac{-0.5}{0.3} = \boxed{-1.667}$$

$\boxed{|z| = 1.667}$

Conclusion:

$|z| = 1.66 < 1.96 = 5\%$.

$|z| < z_{\alpha/2}$ it is accepted.

Eg 8.1 A

$\boxed{n = 50}$ $\boxed{\sigma = 3.5}$ $\boxed{\bar{x} = 10}$ $\boxed{\mu = 9.5}$

Null hypothesis: $H_0 : \mu = 9.5$, accepted.

Alternative hypothesis: $H_1 : \mu \neq 9.5$ rejected.

level of sign - 5%.

Z-test $z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{10 - 9.5}{\frac{3.5}{\sqrt{50}}} = \frac{0.5}{\frac{3.5}{7.07}}$

$$= \frac{0.5}{0.495} = 1.01$$

Conclusion:

$|z| = 1.01 < 1.96 = 5\%$. H_0 accepted.

15. $\boxed{n = 400}$ $\boxed{\bar{x} = 67.47}$ $\boxed{\mu = 67.39}$ $\boxed{\sigma = 1.30}$

Null hypothesis:

$H_0 : \mu = 67.39$ accepted.

Alternative hypothesis:

$H_1 : \mu \neq 67.39$ rejected.

level of sign $= 5\%$

Z - test:

$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{67.47 - 67.39}{\frac{1.30}{\sqrt{400}}}$$

$$= \frac{0.08}{\frac{1.30}{20}} = \frac{0.08}{0.065} = \boxed{1.231}$$

Conclusion:

$|Z| = 1.231 < 1.96 = 5\%$. Ho accepted.

$|Z| > Z_{\alpha/2}$, Ho accepted.

16. $\boxed{n = 100}$ $\boxed{\bar{x} = 72}$ $\boxed{\mu = 76}$ $\boxed{\sigma = 8}$

Null hypothesis:

$H_0 : \mu = 76$ accepted.

Alternative hypothesis:

$H_1 : \mu \neq 76$ rejected (Two tail)

level of signi $= 5\%$.

Z - test $Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{72 - 76}{\frac{8}{\sqrt{100}}} = \frac{-4}{\frac{8}{10}} = \frac{-4}{0.8}$

$Z = -5$, $\boxed{|Z| = 5}$

Conclusion:

$|Z| = 5 > 1.96 = 5\%$.

$|Z| = Z_{\alpha/2}$. Ho rejected.

17. $\boxed{n = 50}$ $\boxed{\bar{x} = 1850}$ $\boxed{\mu = 1800}$ $\boxed{\sigma = 100}$

Null hypothesis:

$H_0 : \mu = 1800$ accepted.

Alternative hypothesis:

$H_1 : \mu \neq 1800$ rejected (Two tail)

level of sign 1%.

Z-test

$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{1850 - 1800}{\frac{100}{\sqrt{50}}}$$

$$= \frac{50}{\frac{100}{7.07}} = \frac{50}{14.14} = 3.536$$

Conclusion,

$|Z| = 3.536 > 2.58 = 1\%$

$|Z| > Z_{\alpha/2}$. Ho rejected.

eg 8.15

$\sigma = 20 \quad \bar{x} = 390 \quad \mu = 400 \quad n = 100$

Null hypothesis :

$H_0 : \mu = 400$ accepted.

Alternative hypothesis :

$H_1 : \mu \neq 400$ rejected.

level of significance : 1% (2.58)

Z-test :

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{390 - 400}{\frac{20}{\sqrt{100}}}$$

$$= \frac{-10}{\frac{20}{\sqrt{100}}}$$

$\boxed{z = -5}$

$\boxed{|z| = 5}$

Conclusion :

$|z| = 5$

$|z| > z_{\alpha/2}$ Ho rejected.

eg : 8.16.

$\sigma = 2.61, \quad \mu = 3.25 \quad \bar{x} = 3.4 \quad n = 900$

(i) Null hypothesis :

$H_0 : \mu = 3.25$ accepted.

Alternative hypothesis :

$H_1 : \mu \neq 3.25$ rejected.

level of significance : 5% (1.96)

z-test :

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{3.4 - 3.25}{\frac{2.61}{\sqrt{900}}}$$

$$= \frac{0.15}{0.087}$$

$\boxed{z = 1.724}$

Conclusion :

$|z| = 1.724$

$|z| < z_{\alpha/2}$ Ho accepted.

(ii) Confidental limits :

(a) 95% (5% (1.96))

$\sigma = 2.61, \quad \bar{x} = 3.4, \quad n = 900$

$$\bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

$$3.4 - 1.96(0.087) \leq \mu \leq 3.4 + 1.96(0.087)$$

$$3.2294 \leq \mu \leq 3.5705$$

at 95% (3.23, 3.57).

(b) 98% (2% (2.33))

$$\bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

$$3.4 - 2.33(0.087) \leq \mu \leq 3.4 + 2.3(0.087)$$

$$3.197 \leq \mu \leq 3.6027$$

at 98% (3.197, 3.60).

Eg : 8.17

$\boxed{n = 400}$ $\boxed{\sigma = 17.2}$ $\boxed{\bar{x} = 153.7}$ $\boxed{\mu = 146.3}$

Null hypothesis:

$H_0 : \mu = 146.3$ accepted

Alternative hypothesis

$H_1 : \mu \neq 146.3$ rejected

Level of significance: ((5%) (1.96))

Z - test:

$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$$= \frac{153.7 - 146.3}{\frac{17.2}{\sqrt{400}}}$$

$$= \frac{7.4}{0.86}$$

$\boxed{|Z| = 8.60}$

Conclusion:

$|Z| = 8.60$

$|Z| > Z_{\alpha/2}$ H_0 rejected

Eg 8.18

$\boxed{n = 50}$ var, = 25 $\boxed{\mu = 52}$

S.D = $\sqrt{var}$

$= \sqrt{25}$

$\boxed{\sigma = 5}$

$\bar{x}$ = $\frac{\text{Total Wages}}{n}$

$= \frac{2550}{50}$

$\boxed{\bar{x} = 51}$

Null hypothesis:

$H_0 : \mu = 52$ accepted

Alternative hypothesis

$H_1 : \mu \neq 52$ rejected

level of significance :((1%) 2.58)

Z - test:

$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{51 - 52}{\frac{5}{\sqrt{50}}}$$

$$= \frac{-1}{\frac{5}{7.07}} = \frac{-1}{0.707}$$

$Z = -1.414$

$\boxed{|Z| = 1.414}$

Conclusion:

$|Z| = 1.414$

$|Z| < Z_{\alpha/2}$, H_0 Accepted

eg 8.19

$\boxed{n=50}$ $\boxed{\bar{x}=9.3}$ $\boxed{\mu=8.9}$ $\boxed{\sigma=1.6}$

Null hypothesis :

$H_0 : \mu = 8.9$ Accepted.

Alternative hypothesis :

$H_1 : \mu \neq 8.9$ rejected.

Level of significance :

5% (1.96)

Z-test

$$z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} = \frac{9.3-8.9}{\frac{1.6}{\sqrt{50}}}$$

$|Z| = 1.7676$

$|Z| < Z_{\alpha/2}$ H_0 accepted.

Exercise 4.1

Arbitrary Constant :

(2) (i) $y = cx + c - c^3$

$\boxed{y' = c} + 0 - 0$

$\boxed{y = y'x + y' - (y')^3}$ //

(iii) $\frac{xy}{}= c^2$

$\boxed{xy' + y = 0}$

(iv) $x^2 + y^2 = a^2$

$2x + 2yy' = 0$

$(\div 2)$

$\boxed{x + yy' = 0}$

(ii) $y = c(x-c)^2 \rightarrow (1)$

$d(x^n) = nx^{n-1}$

$y' = 2c(x-c)' . (1-0)$

$y' = 2c(x-c) \rightarrow (2)$

$\frac{(1)}{(2)} = \frac{y}{y'} = \frac{\cancel{c}(x-c)^{\cancel{2}}}{2\cancel{c}\cancel{(x-c)}}$

$\frac{2y}{y'} = x - c$

$c = x - \frac{2y}{y'}$

$$\boxed{c = \frac{xy' - 2y}{y'}}$$