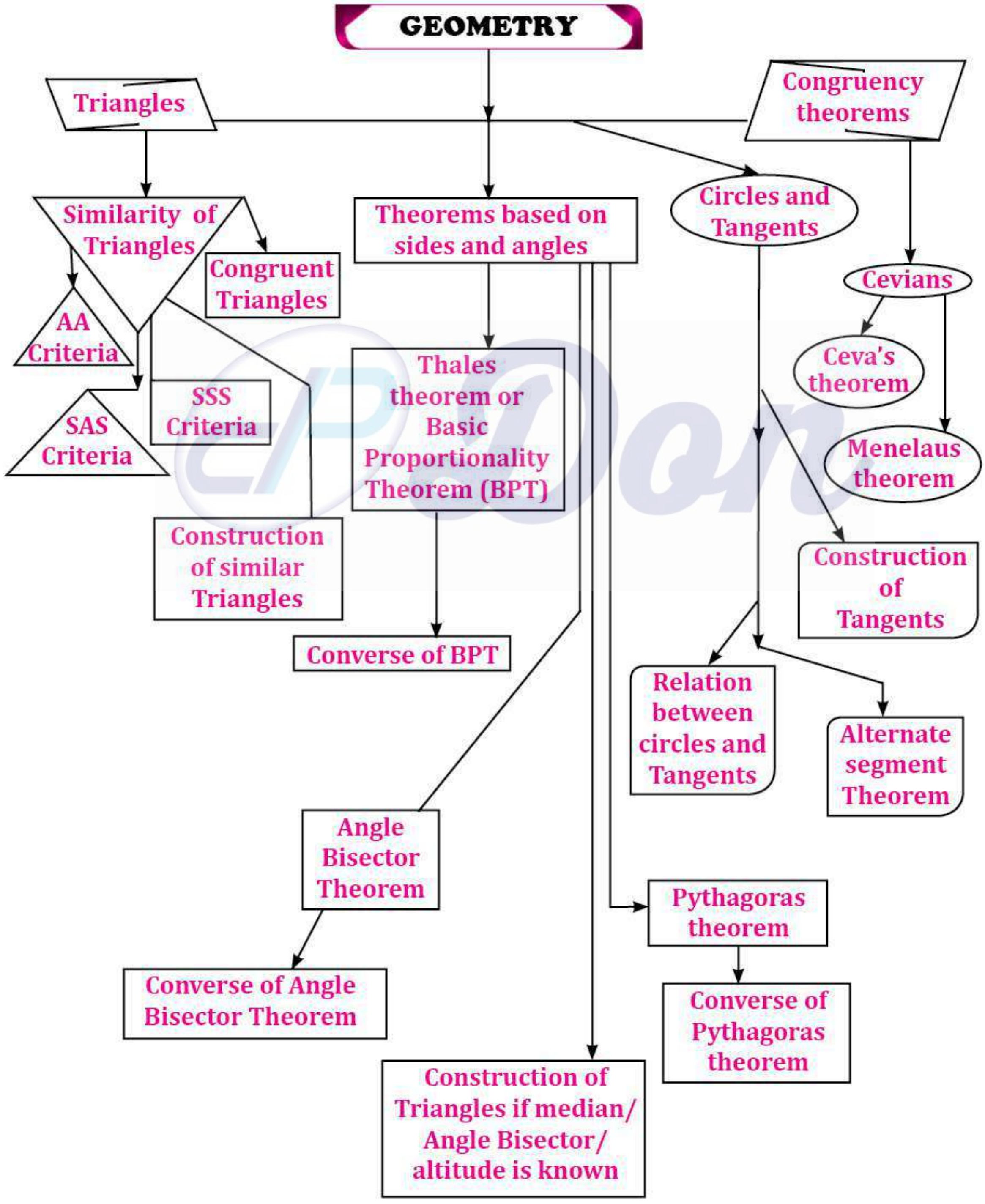
UNIT 4 GEOMETRY

MIND MAP



SIMILARITY

Key Points

SIMILARITY

- The ratio of the corresponding measurements of two **similar** objects must be proportional.
- Two geometrical figures are **congruent,** if they have same size and shape.

SIMILAR TRIANGLES

- If two triangles are similar then their corresponding angles are **equal** and their corresponding sides are proportional.
- The triangles are equiangular if the corresponding angles are equal
- If triangle ABC and ΔPQR are similar, they can be written as $\Delta ABC \sim \Delta PQR$.

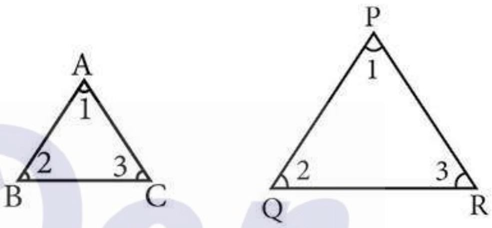
If $\Delta ABC \sim \Delta PQR$ then

$\angle A = \angle P$; $\angle B = \angle Q$; $\angle C = \angle R$

Also AB ≠ PQ; BC ≠ QR; CA ≠ RP

But $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP} > 1$ or < 1.

Shapes are same but not the sizes.



CRITERIA OF SIMILARITY

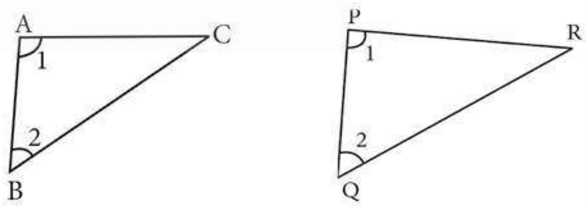
1. AA criterion of similarity

If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.

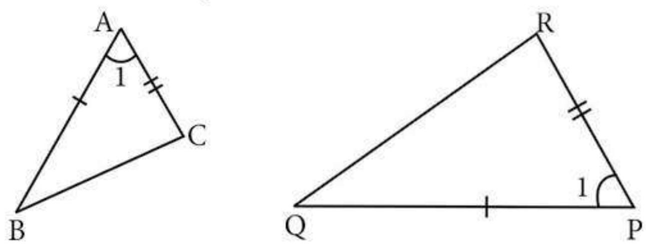
i.e., If $\angle A = \angle P = 1$

$\angle B = \angle Q = 2$ then $\Delta ABC \sim \Delta PQR$ then $\angle C$ must be equal to $\angle R$ [By angle sum property of triangles]

$\therefore$ This is AA similarity or AAA similarity.



2. SAS criterion of similarity



If one angle of a triangle is equal to one angle of another triangle and if the sides including them are proportional then the two triangles are similar.

If $\angle A = \angle P = 1$ and

$$\frac{AB}{PQ} = \frac{AC}{PR} \text{ then}$$

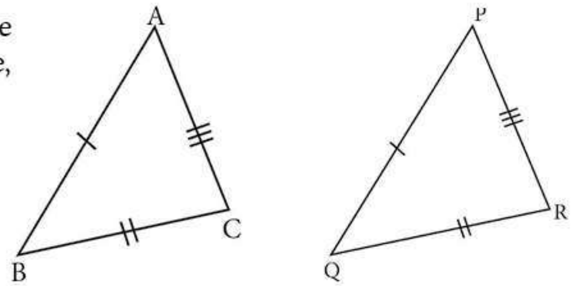
$\Delta ABC \sim \Delta PQR$

3. **SSS criterion of similarity**

   If three sides of a triangle are proportional to the three corresponding sides of another triangle, then two triangles are similar.

   So if $\frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR}$ then

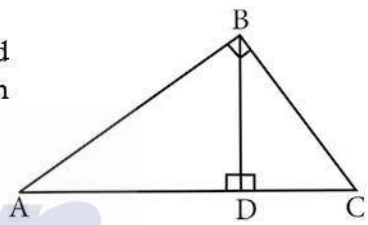
   $\Delta ABC \sim \Delta PQR$



SOME USEFUL RESULTS ON SIMILAR TRIANGLES

1. A perpendicular line drawn from the vertex of a right angled triangle divides the triangle into two triangles similar to each other and also to original triangle.

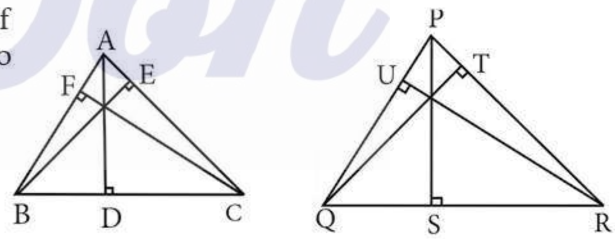
   $\Delta ADB \sim \Delta BDC$, $\Delta ABC \sim \Delta ADB$, $\Delta ABC \sim \Delta BDC$



2. If two triangles are similar, then the ratio of the corresponding sides are equal to the ratio of their corresponding altitudes

   i.e., if $\Delta ABC \sim \Delta PQR$ then

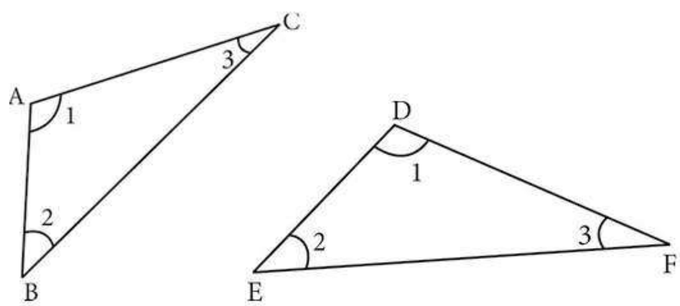
   $$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{PR} = \frac{AD}{PS} = \frac{BE}{QT} = \frac{CF}{RU}$$



3. If two triangles are similar, then the ratio of the corresponding sides are equal to the ratio of the corresponding perimeters.

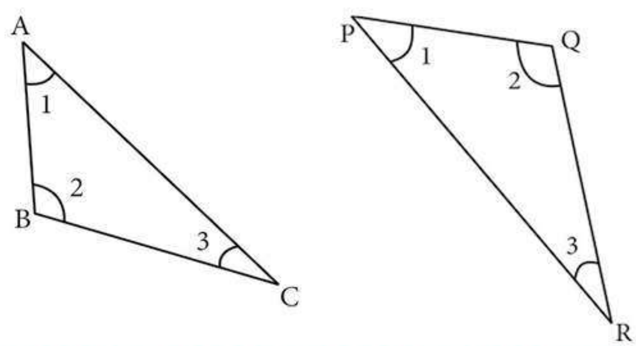
   $\Delta ABC \sim \Delta DEF$ then

   $$\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD} = \frac{AB + BC + CA}{DE + EF + FD}$$



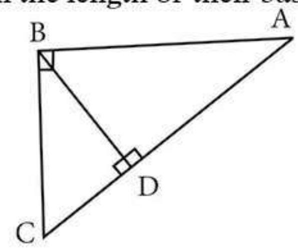
4. The ratio of the area of two similar triangles are equal to the ratio of the squares of their corresponding sides

   $$\frac{area(\Delta ABC)}{area(\Delta \text{PQR})} = \frac{AB^2}{\text{PQ}^2} = \frac{BC^2}{\text{QR}^2} = \frac{AC^2}{\text{PR}^2}$$



5. If two triangles have common vertex and their bases are on the same straight line, the ratio between their areas is equal to the ratio between the length of their bases.

Here, $\dfrac{area(\Delta ABD)}{area(\Delta BDC)} = \dfrac{AD}{DC}$



Note:

i) A pair of equiangular triangles are **similar.**

ii) If two triangles are similar, then they are equiangular.

CONSTRUCTION OF SIMILAR TRIANGLES

"Scale Factor" measures the ratio of the sides of a triangle to be constructed with the corresponding sides of the given triangle.

CONGRUENCY OF TRIANGLES

- Congruency is a particular case of similarity. Two triangles are said to be congruent if
  - (a) their corresponding angles are equal.
  - (b) their corresponding sides are also equal i.e., they have the same shape and size, if they are congruent

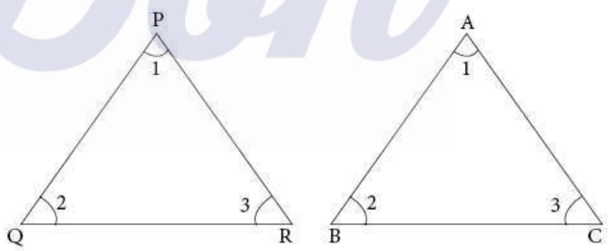
  $\Delta PQR \cong \Delta ABC$

  If $\angle P = \angle A$; $\angle Q = \angle B$; $\angle R = \angle C$

  and AB = PQ; BC = QR; CA = RP

  $\dfrac{AB}{PQ} = \dfrac{BC}{QR} = \dfrac{CA}{RP} = 1$

  Same shape and same size.



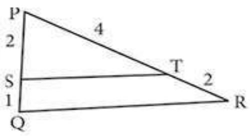
- Two polygons are similar if
  - (i) The lengths of their corresponding sides are proportional.
  - (ii) Their corresponding angles are equal.

- Two congruent figures are always similar, but two similar figures need not be congruent.
  - ☆ All line segments are similar.
  - ☆ All equilateral triangles are similar.
  - ☆ All squares are similar.
  - ☆ All circles are similar.

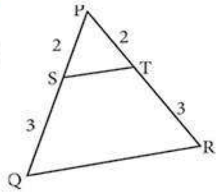
Worked Examples

4.1 **Show that $\Delta PST \sim \Delta PQR$.**

(i)



(ii)



Sol :

(i) In ΔPST and ΔPQR,

$$\frac{PS}{PQ} = \frac{2}{2+1} = \frac{2}{3}, \quad \frac{PT}{PR} = \frac{4}{4+2} = \frac{2}{3}$$

Thus, $\frac{PS}{PQ} = \frac{PT}{PR}$ and $\angle P$ is common.

Therefore, by SAS similarity,

$\Delta PST \sim \Delta PQR$

(ii) In ΔPST and ΔPQR,

$$\frac{PS}{PQ} = \frac{2}{2+3} = \frac{2}{5}, \quad \frac{PT}{PR} = \frac{2}{2+3} = \frac{2}{5}$$

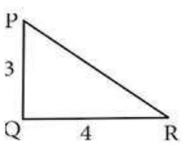
Thus, $\frac{PS}{PQ} = \frac{PT}{PR}$ and $\angle P$ is common

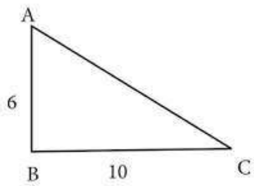
Therefore, by SAS similarity,

$\Delta PST \sim \Delta PQR$

4.2 **Is $\Delta ABC \sim \Delta PQR$?**

Sol :





In ΔABC and ΔPQR,

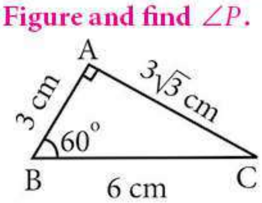
$$\frac{PQ}{AB} = \frac{3}{6} = \frac{1}{2}; \quad \frac{QR}{BC} = \frac{4}{10} = \frac{2}{5}$$

Since $\frac{1}{2} \neq \frac{2}{5}$, $\frac{PQ}{AB} \neq \frac{QR}{BC}$.

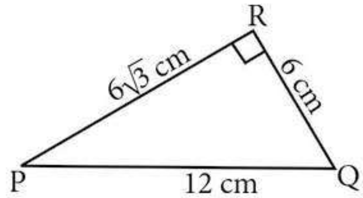
The corresponding sides are not proportional.

Therefore ΔABC is not similar to ΔPQR.

4.3 **Observe Figure and find $\angle P$.**



Sol :



In ΔBAC and ΔPQR, $\frac{AB}{RQ} = \frac{3}{6} = \frac{1}{2}$;

$$\frac{BC}{QP} = \frac{6}{12} = \frac{1}{2}; \quad \frac{CA}{PR} = \frac{3\sqrt{3}}{6\sqrt{3}} = \frac{1}{2}$$

Therefore, $\frac{AB}{RQ} = \frac{BC}{QP} = \frac{CA}{PR}$

By SSS similarity, we have $\Delta BAC \sim \Delta QRP$

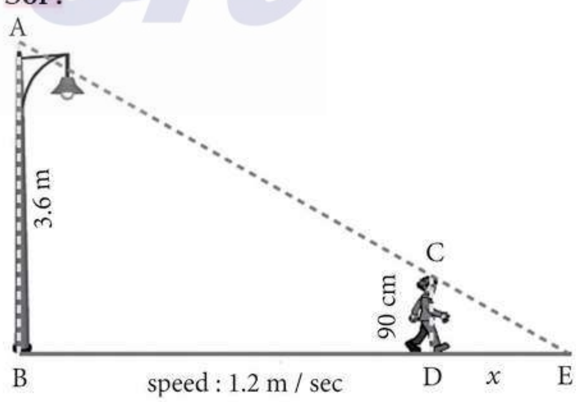
$\angle P = \angle C$ (since the corresponding parts of similar triangle)

$$\angle P = \angle C = 180^\circ - (\angle A + \angle B)$$
$$= 180^\circ - (90^\circ + 60^\circ)$$

$\angle P = 180^\circ - 150^\circ = 30^\circ$

4.4 **A boy of height 90 cm is walking away from the base of a lamppost at a speed of 1.2 m/sec. If the lamppost is 3.6 m above the ground, find the length of his shadow cast after 4 seconds.**

Sol :



Given Speed = 1.2 m/s

time = 4 seconds

Distance = speed × time

= 1.2 × 4 = 4.8 m

Let x be the length of the shadow after 4 seconds

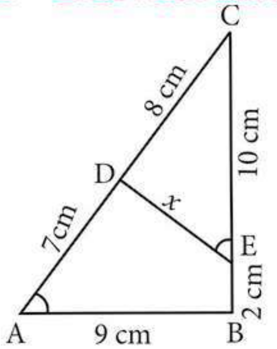
Since, $\Delta ABE \sim \Delta CDE$, $\frac{BE}{DE} = \frac{AB}{CD}$

$\Rightarrow \frac{4.8 + x}{x} = \frac{3.6}{0.9} = 4$. (since 90 cm = 0.9 m)

$= 4.8 + x = 4x \Rightarrow 3x = 4.8 \Rightarrow x = 1.6$ m

The length of his shadow DE = 1.6 m

4.5 In Figure $\angle A = \angle CED$ prove that $\Delta CAB \sim \Delta CED$. Also find the value of x.



Sol :

In ΔCAB and ΔCED, $\angle C$ is common,

$\angle A = \angle CED$

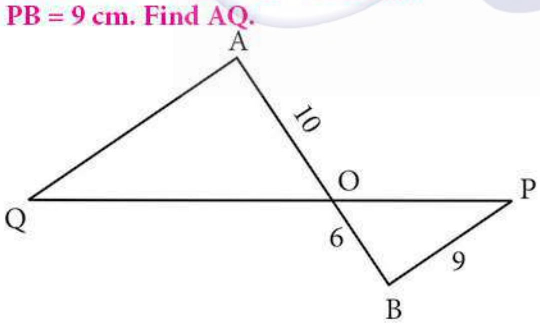
Therefore, $\Delta CAB \sim \Delta CED$ (By AA similarity)

Hence, $\frac{CA}{CE} = \frac{AB}{DE} = \frac{CB}{CD}$

$$\frac{AB}{DE} = \frac{CB}{CD} \Rightarrow \frac{9}{x} = \frac{10+2}{8}$$

$$\Rightarrow x = \frac{8 \times 9}{12} = 6 \text{ cm.}$$

4.6 In Figure QA and PB are perpendiculars to AB. If AO = 10 cm, BO = 6 cm and PB = 9 cm. Find AQ.



Sol :

In ΔAOQ and ΔBOP, $\angle OAQ = \angle OBP = 90^\circ$

$\angle AOQ = \angle BOP$ (Vertically opposite angles)

Therefore, by AA criterion of similarity,

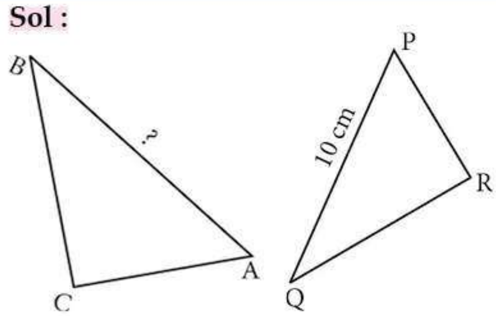
$\Delta AOQ \sim \Delta BOP$

$$\frac{AO}{BO} = \frac{OQ}{OP} = \frac{AQ}{BP}$$

$$\frac{10}{6} = \frac{AQ}{9} \Rightarrow AQ = \frac{10 \times 9}{6} = 15 \text{ cm}$$

4.7 The perimeters of two similar triangles ABC and PQR are respectively 36 cm and 24 cm. If PQ = 10 cm, find AB.

Sol :



The ratio of the corresponding sides of similar triangles is same as the ratio of their perimeters.

Since $\Delta ABC \sim \Delta PQR$,

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{36}{24}$$

$$\frac{AB}{PQ} = \frac{36}{24} \Rightarrow \frac{AB}{10} = \frac{36}{24}$$

$$AB = \frac{36 \times 10}{24} = 15 \text{ cm}$$

4.8 If ΔABC is similar to ΔDEF such that BC = 3 cm, EF = 4 cm and area of ΔABC = 54 cm². Find the area of ΔDEF.

Sol :

Since the ratio of area of two similar triangles is equal to the ratio of the squares on any two corresponding sides, we have

$$\frac{Area(\Delta ABC)}{Area(\Delta DEF)} = \frac{BC^2}{EF^2}$$

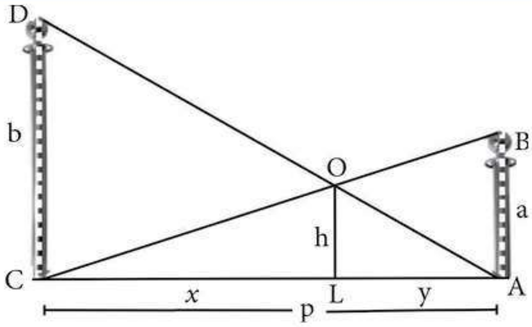
$$\Rightarrow \frac{54}{Area(\Delta DEF)} = \frac{3^2}{4^2}$$

$$Area\ (\Delta DEF) = \frac{16 \times 54}{9} = 96 \text{ cm}^2$$

4.9 Two poles of height 'a' metres and 'b' metres are 'p' metres apart. Prove that the height of the point of intersection of the lines joining the top of each pole to the foot of the opposite pole is given by $\frac{ab}{a+b}$ metres.

Sol :

Let AB and CD be two poles of height 'a' metres and 'b' metres respectively such that the poles are 'P' metres apart. That is AC = p metres. Suppose the lines AD and BC meet at O, such that OL = h metres.



Let CL = x and LA = y.

Then, x + y = p

In ΔABC and ΔLOC,

we have $\angle CAB = \angle CLO$ [each equal to 90°]

$\angle C = \angle C$ [C is common]

$\Delta CAB \sim \Delta CLO$ [By AA similarity]

$$\frac{CA}{CL} = \frac{AB}{LO} \Rightarrow \frac{p}{x} = \frac{a}{h} \Rightarrow x = \frac{ph}{a} \ldots (1)$$

In ΔALO and ΔACD, we have

$\angle ALO = \angle ACD$ [each equal to 90°]

$\angle A = \angle A$ [A is common]

$\Delta ALO \sim \Delta ACD$ [by AA similarity]

$$\frac{AL}{AC} = \frac{OL}{DC} \Rightarrow \frac{y}{p} = \frac{h}{b} \Rightarrow y = \frac{ph}{b} \ldots (2)$$

$$(1) + (2) \Rightarrow x + y = \frac{ph}{a} + \frac{ph}{b}$$

$$p = ph\left(\frac{1}{a}+\frac{1}{b}\right) \qquad \text{(Since } x + y = p\text{)}$$

$$1 = h\left(\frac{a+b}{ab}\right)$$

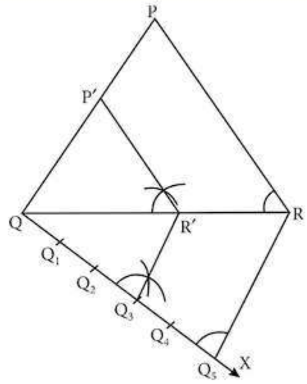
Therefore, $h = \frac{ab}{a+b}$

Hence, the height of the intersection of the lines joining the top of each pole to the foot of the opposite pole is $\frac{ab}{a+b}$ metres.

4.10 Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{3}{5}$ of the corresponding sides of the triangle PQR (scale factor $\frac{3}{5} < 1$).

Sol :

Given a triangle PQR we are required to construct another triangle whose sides are $\frac{3}{5}$ of the corresponding sides of the triangle PQR



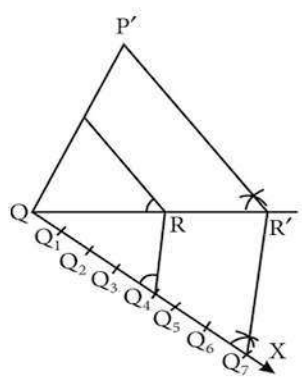
Steps of construction:

1. Construct a ΔPQR with any measurement.
2. Draw a ray QX making an acute angle with QR on the side opposite to the vertex P.
3. Locate 5 (the greater of 3 and 5 in $\frac{3}{5}$) Points. Q_1, Q_2, Q_3, Q_4 and Q_5 on QX so that $QQ_1 = Q_1Q_2 = Q_2Q_3 = Q_3Q_4 = Q_4Q_5$
4. Join Q_5R and draw a line through Q_3 (the third point, 3 being smaller of 3 and 5 in $\frac{3}{5}$) parallel to Q_5R to intersect QR at R'.
5. Draw line through R' parallel to the line RP to intersect QP at P'.

   Then, $\Delta P'QR'$ is the required triangle each of whose sides is three-fifths of the corresponding sides of ΔPQR.

4.11 Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{7}{4}$ of the corresponding sides of the triangle PQR (scale factor $\frac{7}{4} > 1$)

Sol :



Given a triangle PQR, we are required to construct another triangle whose sides are $\frac{7}{4}$ of the corresponding sides of the triangle PQR.

Steps of construction

1. Construct a ΔPQR with any measurement.
2. Draw a ray QX making an acute angle with QR on the side opposite to the vertex P.
3. Locate 7 points (the greater of 7 and 4 in $\frac{7}{4}$) $Q_1, Q_2, Q_3, Q_4, Q_5, Q_6$ and Q_7 on QX so that $QQ_1 = Q_1Q_2 = Q_2Q_3 = Q_3Q_4$ $= Q_4Q_5 = Q_5Q_6 = Q_6Q_7$.
4. Join Q_4 (the fourth point, 4 being smaller of 4 and 7 in $\frac{7}{4}$) to R and draw a line through Q_7 parallel to Q_4R, intersecting the extended line segment QR at R'.
5. Draw a line through R' parallel to RP inter-secting the extended line segment QP at P'.

   Then $\Delta P'QR'$ is the required triangle each of whose sides is seven-fourths of the corresponding sides of ΔPQR.

Progress Check

1. All circles are _____ (congruent/similar).

Ans : similar

2. All squares are _____(similar/congruent).

Ans : similar

3. Two triangles are similar, if their corresponding angles are _____ and their corresponding sides are _____.

Ans : equal, proportional

4. (a) All similar triangles are congruent True/ False.

Ans : False

(b) All congruent triangles are similar True/ False.

Ans : True

5. Give two different examples of pair of non-similar figures.

Ans : (i) A circle and a triangle are non similar figures.

(ii) An isosceles triangle and a scalene triangle are non-similar figures.

Thinking Corner

1. Are square and a rhombus similar or congruent. Discuss.

Ans :

For a rhombus

☆ All sides are equal.

☆ The diagonals of a rhombus are perpendicular bisectors of one another.

☆ Opposite angles are equal.

For a square

☆ All sides are equal.

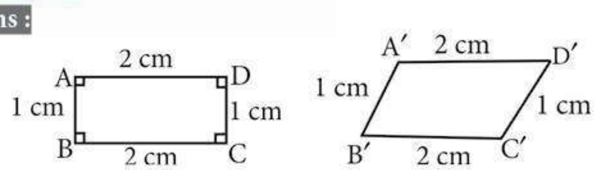
☆ All angles are 90°.

☆ Diagonals are equal and perpendicular bisectors of each other.

For both the shapes the angles are not equal, but corresponding sides are proportional. so they are neither similar nor congruent.

2. Are a rectangle and a parallelogram similar. Discuss.

Ans :



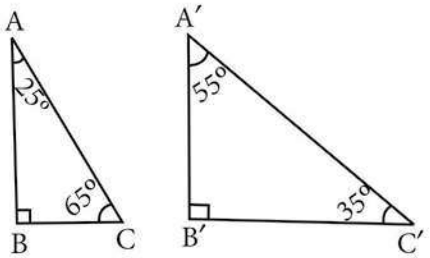
On observing the given figures.

Their corresponding sides are proportional but their corresponding angles are not equal.

∴ The shapes parallelogram and rectangle are not similar.

3. Are any two right angled triangles similar? Why?

Ans :



For any two right angled triangles.

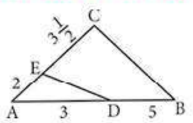
☆ The angles other than right angle need not be equal.

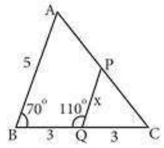
☆ The corresponding sides need not be proportional.

± ∴ Any two right angled triangles need not be similar.

Exercise 4.1

1. Check whether the given triangles are similar and find the value of x.

(i) 

(ii) 

Sol :

(i) In ΔABC and ΔADE $\angle A$ is common

$$\frac{AE}{EC} = \frac{2}{3\frac{1}{2}} = \frac{2}{\frac{7}{2}} = \frac{2\times 2}{7} = \frac{4}{7}$$

$$\frac{AD}{DB} = \frac{3}{5}$$

Here $\frac{4}{7} \pm \frac{3}{5}$

$$\frac{AE}{EC} \neq \frac{AD}{DB}$$

The corresponding sides are not proportional.

∴ ΔABC and ΔADE are not similar.

(ii) In ΔCPQ and ΔCAB $\angle C$ is common.

$\angle PQC = 180^\circ - 110^\circ = 70^\circ$

[∵ $\angle PQC$ and $\angle PQB$ are liner pair of angles]

$\angle ABC = 70^\circ$

∴ $\angle BAC = \angle QPC$

[∵ sum of three angles of a triangle are 180°]

$\angle PQC = 180^\circ - (\angle QPC + 70^\circ)$

∴ $\angle ABC = \angle PQC = 70^\circ$

$\angle C$ common and $\angle BAC = \angle QPC$

By AAA similarity criteria, $\Delta ABC \sim \Delta PQC$

∴ Corresponding sides are proportional

$$\frac{AB}{PQ} = \frac{BC}{QC}$$

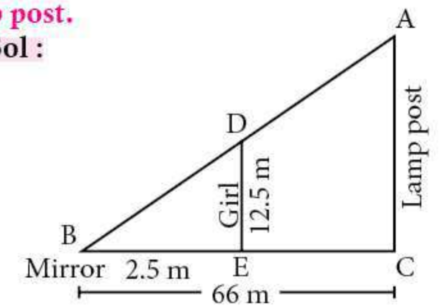
$$\frac{5}{x} = \frac{6}{3}$$

[∵ BC = BQ + QC = 3 + 3 = 6]

$$x = \frac{5}{6} \times 3 = 2.5 \qquad \therefore x = 2.5$$

2. A girl looks the reflection of the top of the lamp post on the mirror which is 66 m away from the foot of the lamp post. The girl whose height is 12.5 m is standing 2.5 m away from the mirror. Assuming the mirror is placed on the ground facing the sky and the girl, mirror and the lamp post are in a same line, find the height of the lamp post.

Sol :



Let AC is the lamp post and ED is the girl.

From the triangles ΔABC and ΔDBE

$\angle B$ is common. $\angle DEB = \angle ACB = 90^\circ$

By AA criteria

$\Delta ABC \sim \Delta DBE$

∴ Their sides are proportional

i.e., $\frac{AC}{DE} = \frac{BC}{BE}$

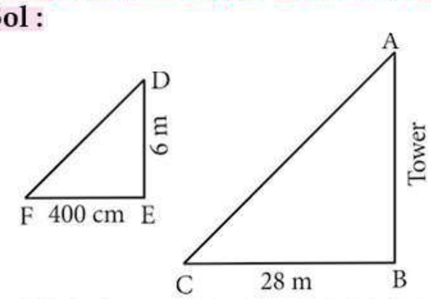
$$\frac{AC}{12.5} = \frac{66}{2.5}$$

$$AC = \frac{66\times 12.5}{2.5} = \frac{66\times \cancel{12.5}^{5}}{\cancel{2.5}_{1}} = 330 \text{ m}$$

∴ Height of the lamp post = 330 m

3. A vertical stick of length 6 m casts a shadow 400 cm long on the ground and at the same time a tower casts a shadow 28 m long. Using similarity, find the height of the tower.

Sol :



Let DE is the vertical stick and AB is the tower.

DE = 6 m, EF = 400 cm = 4 m, BC = 28 m

From ΔDFE and ΔACB

Using similarity criteria

$$\frac{AB}{DE} = \frac{BC}{EF}$$

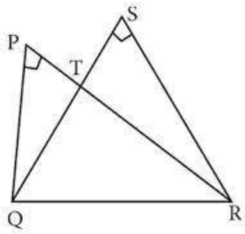
$$\frac{AB}{6} = \frac{28}{4}$$

$$AB = \frac{28\times6}{4} = 42 \text{ m}$$

$\therefore$ Height of the tower = 42 m

4. Two triangles QPR and QSR, right angled at P and S respectively are drawn on the same base QR and on the same side of QR. If PR and SQ intersect at T, prove that PT × TR = ST × TQ.

Sol :



Given $\angle QSR = \angle QPR = 90^\circ$

PR and SQ intersect at T.

In ΔQPT and ΔRST

$\angle QSR = \angle QPR = 90^\circ$

Given

$\angle PTQ = \angle STR$

[$\because$ Vertically opposite angles]

$\therefore$ By AA similarity criteria

$\Delta PQT \sim \Delta SRT$

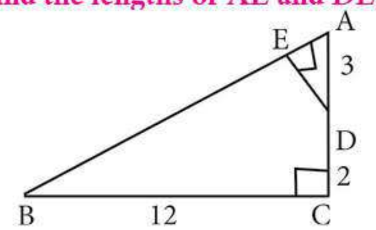
$\therefore$ Their corresponding sides are proportional

$$\therefore \frac{PT}{ST} = \frac{TQ}{TR}$$

$$PT \times TR = ST \times TQ$$

Hence proved.

5. In the adjacent figure ΔABC is right angled at C and DE $\perp$ AB. Prove that $\Delta ABC \sim \Delta ADE$ and hence find the lengths of AE and DE?



Sol :

Given $\angle C = 90^\circ = \angle DEA$

$\angle A$ is common to both the triangles ΔABC and ΔADE

$\therefore$ By AA criteria for similarity

$\Delta ABC \sim \Delta ADE$

$\therefore$ Their corresponding sides are proportional.

$$\therefore \frac{BC}{DE} = \frac{AC}{AE} = \frac{AB}{AD}$$

$$\frac{12}{DE} = \frac{3+2}{AE} = \frac{AB}{3} \quad \text{... (1)}$$

Also in right ΔABC, $\angle C = 90^\circ$

$\therefore$ using Pythagoras theorem, we have

$$AB^2 = BC^2 + AC^2 = 12^2 + 5^2$$

$$= 144 + 25 = 169$$

$$\therefore AB = \sqrt{169}$$

$$AB = 13 \text{ cm}$$

Now from (1), we get

$$\frac{12}{DE} = \frac{13}{3}$$

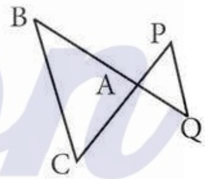
$$DE = \frac{12\times3}{13} = \frac{36}{13}$$

$$DE = 2.77 \text{ cm}$$

Also from (1), $\dfrac{5}{AE} = \dfrac{13}{3}$

$$AE = \frac{5\times3}{13} = \frac{15}{13} = 1.15 \text{ cm}$$

6. If figure $\Delta ACB \sim \Delta APQ$. If BC = 8 cm, PQ = 4 cm, BA = 6.5 cm and AP = 2.8 cm, find CA and AQ.



Sol : Given $\Delta ACB \sim \Delta APQ$

$\therefore$ Their corresponding sides are proportional

$$\frac{AC}{AP} = \frac{CB}{PQ} = \frac{AB}{AQ}$$

$$\frac{AC}{2.8} = \frac{8}{4} = \frac{6.5}{AQ}$$

Taking $\dfrac{AC}{2.8} = \dfrac{8}{4}$

$$AC = \frac{8 \times 2.8}{4} = 5.6 \text{ cm}$$

$$AC = 5.6 \text{ cm}$$

Also taking $\dfrac{8}{4} = \dfrac{6.5}{AQ}$

$$AQ = \frac{6.5}{8} \times 4 = 3.25 \text{ cm}$$

$$AQ = 3.25 \text{ cm}$$

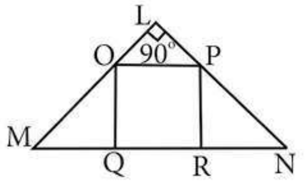
7. In figure OPQR is a square and $\angle MLN = 90^\circ$. Prove that

(i) $\Delta LOP \sim \Delta QMO$ **(ii)** $\Delta LOP \sim \Delta RPN$

(iii) $\Delta QMO \sim \Delta RPN$

(iv) $QR^2 = MQ \times RN$

Sol :



(i) In ΔLOP and ΔQMO

$\angle OLP = \angle MQO = 90^\circ$

$\angle LOP = \angle QMO$ (corresponding angles)

$\therefore$ By AA criterion of similarity.

$\Delta LOP \sim \Delta QMO$

(ii) In ΔLOP and ΔRPN,

we have $\angle PLO = \angle NRP = 90^\circ$

and $\angle LPO = \angle RNP$

(corresponding angles)

$\therefore$ By AA criterion of similarity

$\Delta LOP \sim \Delta RPN$

(iii) Also in ΔQMO and ΔRPN

$\angle OQM = \angle NRP = 90^\circ$

we have $\Delta LOP \sim \Delta QMO$ and $\Delta LOP \sim \Delta RPN$

$\Delta QMO \sim \Delta RPN$

(iv) We have $\Delta QMO \sim \Delta RPN$

$$\therefore \frac{MQ}{PR} = \frac{QO}{RN}$$

$$\frac{MQ}{QR} = \frac{QR}{RN}$$

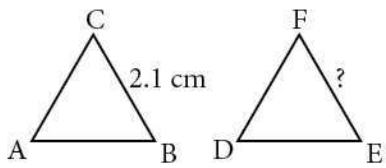
[$\because$ OQRP is a square PR = QR and QO = QR]

$\therefore QR \times QR = MQ \times RN$

$QR^2 = MQ \times RN$

8. If $\Delta ABC \sim \Delta DEF$ such that area of ΔABC is 9 cm^2 and the area of ΔDEF is 16 cm^2 and BC = 2.1 cm. Find the length of EF.

Sol :



Given $\Delta ABC \sim \Delta DEF$

then we have

$$\frac{Area(\Delta ABC)}{Area(\Delta DEF)} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$$

$$\frac{9}{16} = \frac{BC^2}{EF^2}$$

$$\frac{9}{16} = \frac{2.1 \times 2.1}{EF^2}$$

$$EF^2 = \frac{2.1 \times 2.1 \times 16}{9} = \frac{2.1 \times 2.1 \times 4 \times 4}{3 \times 3}$$

$$EF^2 = \left(\frac{2.1 \times 4}{3}\right)^2$$

$$EF = \frac{2.1 \times 4}{3} = 2.8$$

EF = 2.8 cm

Another method:

$$\frac{9}{16} = \frac{BC^2}{EF^2}$$

Taking square root

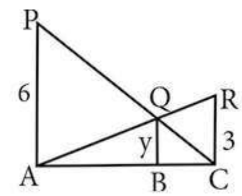
$$\frac{3}{4} = \frac{BC}{EF}$$

$$\frac{3}{4} = \frac{2.1}{EF}$$

$$3 \times EF = 2.1 \times 4$$

$$EF = \frac{2.1 \times 4}{3} = 2.8 \text{ cm}$$

9. Two vertical poles of heights 6 m and 3 m are erected above a horizontal ground AC. Find the value of y.



Sol :

Let AP and CR be the vertical poles of height 6 m and 3 m respectively

Let BQ = y m

In ΔCRA and ΔBQA

$\angle A = \angle A$ common

$\angle ACR = \angle ABQ = 90^\circ$

$\therefore$ By AA criterion of similarity

$\Delta CRA \sim \Delta BQA$

$\therefore$ Their corresponding sides are proportional.

$$\frac{AC}{AB} = \frac{CR}{BQ}$$

$$\frac{AC}{AB} = \frac{3}{y}$$

$$AB = \frac{AC \times y}{3} \quad \ldots (1)$$

In ΔCBQ and ΔCAP

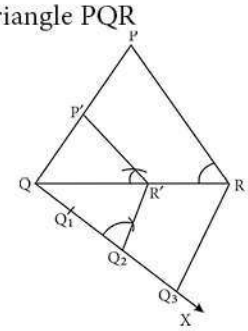
$$\angle CBQ = \angle CAP = 90^{\circ}$$
$$\angle C = \angle C \quad \text{[common]}$$
$$\therefore \Delta CBQ \sim \Delta CAP$$
$$\frac{CB}{CA} = \frac{BQ}{AP}$$
$$\frac{CB}{CA} = \frac{y}{6}$$
$$BC = \frac{y \times CA}{6} \quad \ldots (2)$$

$$(1) + (2) \Rightarrow AB + BC = \frac{AC \times y}{3} + \frac{y \times CA}{6}$$
$$AC = y \times AC\left(\frac{1}{3} + \frac{1}{6}\right)$$
$$\frac{AC}{AC} = y\left(\frac{2+1}{6}\right)$$
$$1 = y\left(\frac{3}{6}\right)$$
$$1 = \frac{1}{2}y$$
$$y = 2 \text{ m.}$$

10. Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{2}{3}$ of the corresponding sides of the triangle PQR (scale factor $\frac{2}{3}$)

Sol :

Given a triangle PQR we are required to construct another triangle whose sides are $\frac{2}{3}$ of the corresponding sides of the triangle PQR



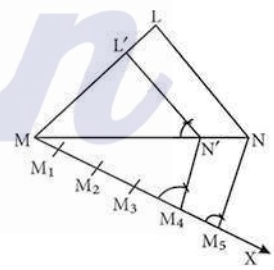
Steps of construction:

1. Constructed a ΔPQR with any measurement.
2. Drawn a ray QX making an acute angle with QR on the side opposite to the vertex P.
3. Located 3 points Q_1, Q_2 and Q_3 on QX so that $QQ_1 = Q_1Q_2 = Q_2Q_3$.
4. Joined Q_3R and drawn a line through Q_2 parallel to Q_3R to intersect QR at R'
5. Drawn a line through R' parallel to the line RP to intersect QP at P'. Then $\Delta P'QR'$ is the required triangle each of whoses sides is two third of the corresponding sides of ΔPQR

11. Construct a triangle similar to a given triangle LMN with its sides equal to $\frac{4}{5}$ of the corresponding sides of the triangle LMN (scale factor $\frac{4}{5}$)

Sol :

Given a triangle LMN. We are required to construct another triangle whose sides are $\frac{4}{5}$ of the corresponding sides of the ΔLMN .



Steps of construction:

1. Constructed a ΔLMN with any measurement.
2. Drawn a ray MX making an acute angle with MN on the side opposite to the vertex L.
3. Located 5 points M_1, M_2, M_3, M_4, M_5 on MX so that

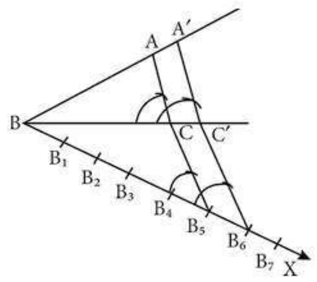
   $MM_1 = M_1M_2 = M_2M_3 = M_3M_4 = M_4M_5$
4. Joined M_5N and drawn a line through M_4 parallel to M_5N to intersect MN at N'..
5. Drawn a line through N' parallel to the line NL to intersect ML at L'.

   Then $\Delta L'MN'$ is the required triangle each of whose sides is four fifth of the corresponding sides of ΔLMN ..

12. Construct a triangle similar to a given triangle ABC with its sides equal to 6/5 of the corresponding sides of the triangle ABC (scale factor 6/5)

Sol :

Given a triangle ABC, we are required to construct another triangle whose sides are $\frac{6}{5}$ of the corresponding sides of the triangle ABC



Steps of construction:

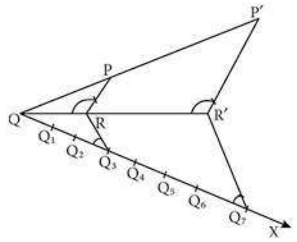
1. Constructed a $\Delta\, ABC$ with any measurement.
2. Drawn a ray BX making an acute angle with BC on the side opposite to the vertex A.
3. Located 6 points B_1, B_2, B_3, B_4, B_5 and B_6 so that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5 = B_5B_6$.
4. Joined B_5 to C and drawn a line through B_6 parallel to B_5C intersecting the extended line segment BC at C'
5. Drawn a line through C' parallel to CA intersecting the extended BA at A'.

   Then $\Delta A'BC'$ is the required triangle each of whose sides is six fifth of the corresponding sides of ΔABC.

13. Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{7}{3}$ of the corresponding sides of the triangle PQR (scale factor $\frac{7}{3}$)

Sol :

Given a triangle PQR, we are required to construct another triangle whose sides are $\frac{7}{3}$ of the corresponding sides of the triangle PQR



Steps of construction:

1. Constructed a ΔPQR with any measurement.
2. Drawn a ray QX making an acute angle with QR on the side opposite to the vertex P.
3. Located 7 points $Q_1, Q_2, Q_3, Q_4, Q_5, Q_6$ and Q_7 on QX so that $QQ_1 = Q_1Q_2 = Q_2Q_3 = Q_3Q_4 = Q_4Q_5 = Q_5Q_6 = Q_6Q_7$.
4. Joined Q_3 to R and drawn a line through Q_7 parallel to Q_3R, intersecting the extended line segment QR at R'
5. Drawn a line through R' parallel to RP intersecting the extended line segment QP at P'.

Then $\Delta P'QR'$ is the required triangle each of whose sides is seven-thirds of the corresponding sides of ΔPQR.

THALES THEOREM AND ANGLE BISECTOR THEOREM

Key Points

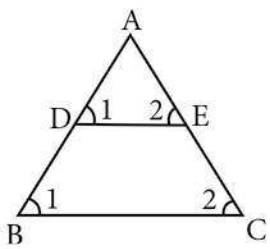
Thales theorem and angle bisector theorem

- Thales was the first man to announce that any idea that emerged should be tested scientifically and only then it can be accepted.
- Thales was credited for providing first proof in mathematics called "Basic proportionality theorem" or otherwise "Thales theorem".

Basic proportionality theorem or thales theorem

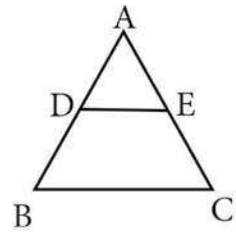
"A straight line drawn parallel to a side of triangle, divides the other two sides proportionally".

$$\frac{AD}{DB} = \frac{AE}{EC}$$



Corollary

If in a ΔABC, a straight line DE parallel to BC, intersects AB at D and AC at E, then



(i) $\frac{AB}{AD} = \frac{AC}{AE}$

(ii) $\frac{AB}{DB} = \frac{AC}{EC}$

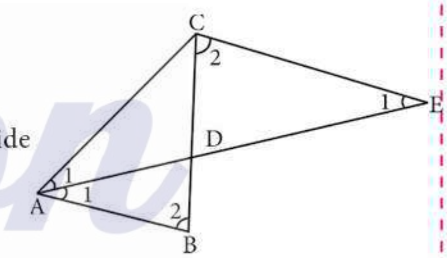
Converse of Thales theorem

If a straight line divides two sides of a triangle proportionally then the straight line is parallel to the third side.

Angle bisector theorem

The internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the corresponding sides containing the angle.

i.e., $\frac{AB}{AC} = \frac{BD}{CD}$



Converse of Angle bisector theorem

"If a straight line through one vertex of a triangle divides the opposite side internally in the ratio of the other two sides, then the line bisects the angle internally at the vertex".

Theorem 1:

Basic proportionality theorem or Thales theorem.

Statement

"A straight line drawn parallel to a side of triangle intersecting the other two sides, divides the sides in the same ratio.

Proof

Given : In ΔABC, D is a point on AB and E is a point on AC.

To prove : $\frac{AD}{DB} = \frac{AE}{EC}$

Construction : Draw a line DE || BC

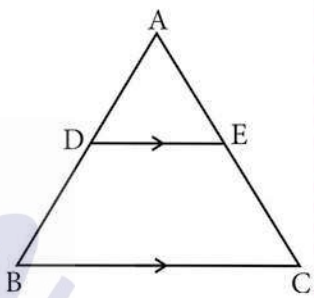
Sl. No.	Statement	Reason
1	$\angle ABC = \angle ADE = \angle 1$	Corresponding angles are equal because $DE \parallel BC$
2	$\angle ACB = \angle AED = \angle 2$	Corresponding angles are equal because $DE \parallel BC$
3	$\angle DAE = \angle BAC = \angle 3$	Both triangles have a common angle

4	$\Delta ABC \sim \Delta ADE$ $\frac{AB}{AD} = \frac{AC}{AE}$ $\frac{AD + DB}{AD} = \frac{AE + EC}{AE}$ $1 + \frac{DB}{AD} = 1 + \frac{EC}{AE}$ $\frac{DB}{AD} = \frac{EC}{AE}$ $\frac{AD}{DB} = \frac{AE}{EC}$	By AAA similarity Corresponding sides are proportional Split AB and AC Simplification Cancelling 1 on both sides Taking reciprocals

Hence proved.

Corollary

If in a ΔABC, a straight line DE parallel to BC, intersects AB at D and AC at E, then



(i) $\frac{AB}{AD} = \frac{AC}{AE}$ **(ii)** $\frac{AB}{DB} = \frac{AC}{EC}$.

Proof

In ΔABC, DE || BC,

therefore, $\frac{AD}{DB} = \frac{AE}{EC}$ (by B.P.T. theorem)

(i) Taking reciprocals, we get $\frac{DB}{AD} = \frac{EC}{AE}$

Add 1 to both in the sides $\frac{DB}{AD} + 1 = \frac{EC}{AE} + 1$

$$\Rightarrow \frac{DB + AD}{AD} = \frac{EC + AE}{AE} \Rightarrow \frac{AB}{AD} = \frac{AC}{AE}$$

(ii) Add 1 to both the sides

$$\frac{AD}{DB} + 1 = \frac{AE}{EC} + 1$$

Therefore, $\frac{AB}{DB} = \frac{AC}{EC}$

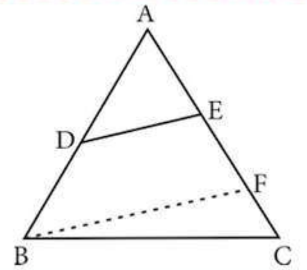
Theorem 2:

Converse of Basic Proportionality Theorem (or) Thales Theorem.

Statement:

"If a straight line divides any two sides of a triangle in the same ratio, then the line must be parallel to the third side."

Proof:



Given : In ΔABC, $\frac{AD}{DB} = \frac{AE}{EC}$

To prove : DE || BC

Construction : Draw BF || DE

Sl. No.	Statement	Reason
1	In ΔABC, $BF \parallel DE$	Construction
2	$\frac{AD}{DB} = \frac{AE}{EC}$ ... (1)	Thales theorem (In ΔABC taking D in AB and E in AC)
3	$\frac{AD}{DB} = \frac{AF}{FC}$ ... (2)	Thales theorem (In ΔABC taking F in AC)
4	$\frac{AE}{EC} = \frac{AF}{FC}$ $\frac{AE}{EC} + 1 = \frac{AF}{FC} + 1$ $\frac{AE + EC}{EC} = \frac{AF + FC}{FC}$ $\frac{AC}{EC} = \frac{AC}{FC}$ EC = FC Therefore, F = C Thus $DE \parallel BC$	From (1) and (2) Adding 1 to both sides Cancelling AC on both sides F lies between E and C. Hence proved

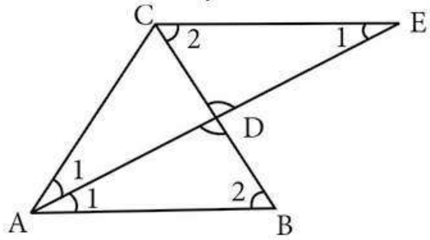
Theorem 3:

Angle Bisector Theorem

Statement:

"The internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the corresponding sides containing the angle"

Proof:



Given : In ΔABC, AD is the internal bisector

To prove : $\frac{AB}{AC} = \frac{BD}{CD}$

Construction : Draw a line through C parallel to AB.

Extend AD to meet line through C at E.

Sl. No.	Statement	Reason
1	$\angle AEC = \angle BAE = \angle 1$	Two parallel lines cut by a transversal make alternate equal angles

2	ΔACE is isosceles AC = CE ... (1)	In ΔACE, $\angle CAE = \angle CEA$
3	$\Delta ABD \sim \Delta ECD$ $\frac{AB}{CE} = \frac{BD}{CD}$	By AA Similarity
4	$\frac{AB}{AC} = \frac{BD}{CD}$	From (1) AC = CE Hence proved.

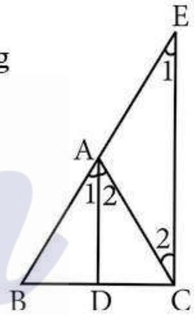
Theorem 4:

Converse of Angle Bisector Theorem

Statement:

"If a straight line through one vertex of a triangle divides the opposite side internally in the ratio of the other two sides, then the line bisects the angle internally at the vertex."

Proof:



Given, ABC is a triangle. AD divides BC in the ratio of the sides containing the angles $\angle A$ to meet BC at D.

That is $\frac{AB}{AC} = \frac{BD}{DC}$ ... (1)

To prove: AD bisects $\angle A$ i.e., $\angle 1 = \angle 2$

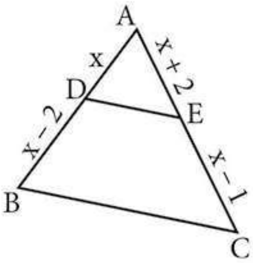
Construction: Draw CE || DA. Extend BA to meet at E.

Sl. No.	Statement	Reason
1	Let $\angle BAD = \angle 1$ and $\angle DAC = \angle 2$	Assumption
2	$\angle BAD = \angle AEC = \angle 1$ ... (1)	Since $DA \parallel CE$, corresponding angles are equal
3	$\angle DAC = \angle ACE = \angle 2$	Since $DA \parallel CE$, Alternate angles are equal
4	$\frac{BA}{AE} = \frac{BD}{DC}$ ... (2)	In ΔBCE by Thales theorem
5	$\frac{AB}{AC} = \frac{BD}{DC}$	From (1)
6	$\frac{AB}{AC} = \frac{BA}{AE}$	From (1) and (2)
7	AC = AE ... (3)	Cancelling AB
8	$\angle 1 = \angle 2$	ΔACE is isosceles by (3)
9	AD bisects $\angle A$	Since, $\angle 1 = \angle BAD = \angle 2 = \angle DAC$ Hence proved.

Worked Examples

4.12 **In ΔABC DE || BC, if AD = x, DB = x – 2, and EC = x – 1 then find the lengths of the sides AB and AC.**

Sol :



In ΔABC

we have DE || BC.

By Thales theorem, we have $\dfrac{AD}{DB} = \dfrac{AE}{EC}$

$$\frac{x}{x-2} = \frac{x+2}{x-1}$$

$\Rightarrow$ x (x – 1) = (x – 2) (x + 2)

$\Rightarrow$ $x^2 - x = x^2 - 4 \Rightarrow x = 4$

When x = 4, AD = 4, DB = x – 2 = 2,

AE = x + 2 = 6; EC = x – 1 = 3.

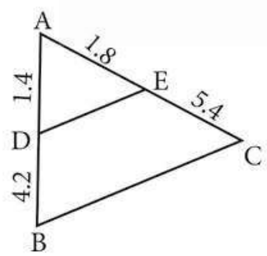
Hence, AB = AD + DB = 4 + 2 = 6,

AC = AE + EC = 6 + 3 = 9.

Therefore, AB = 6, AC = 9.

4.13 **D and E are respectively the points on the sides AB and AC of a ΔABC such that AB = 5.6 cm, AD = 1.4 cm, AC = 7.2 cm and AE = 1.8 cm, show that DE || BC.**

Sol :



We have AB = 5.6 cm, AD = 1.4 cm, AC = 7.2 cm and AE = 1.8 cm.

BD = AB – AD = 5.6 – 1.4 = 4.2 cm

and EC = AC – AE = 7.2 – 1.8 = 5.4 cm.

$$\frac{AD}{DB} = \frac{1.4}{4.2} = \frac{1}{3} \text{ and } \frac{AE}{EC} = \frac{1.8}{5.4} = \frac{1}{3}$$

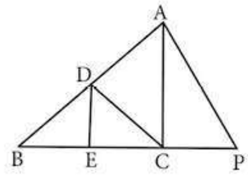
$$\frac{AD}{DB} = \frac{AE}{EC}$$

Therefore, by converse of Basic Proportionality theorem, we have DE is parallel to BC.

Hence proved.

4.14 **In the Figure, DE || AC and DC || AP. Prove that $\dfrac{BE}{EC} = \dfrac{BC}{CP}$.**

Sol :



In ΔBPA we have DC || AP. By Basic proportionality Theorem,

we have $\dfrac{BC}{CP} = \dfrac{BD}{DA}$ ... (1)

In ΔBCA, we have DE || AC. By Basic Proportionality Theorem, we have

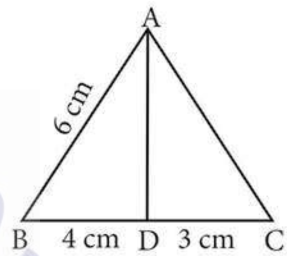
$$\frac{BE}{EC} = \frac{BD}{DA} \quad \text{... (2)}$$

From (1) and (2) we get, $\dfrac{BE}{EC} = \dfrac{BC}{CP}$

Hence proved.

4.15 **In the Figure AD is the bisector of $\angle A$. If BD = 4 cm, DC = 3 cm and AB = 6 cm, find AC.**

Sol :



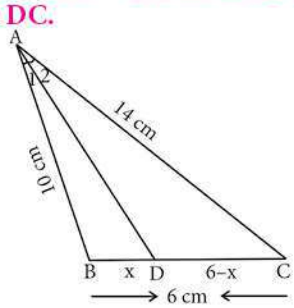
In ΔABC, AD is the bisector of $\angle A$

Therefore by Angle Bisector Theorem

$$\frac{BD}{DC} = \frac{AB}{AC}$$

$$\frac{4}{3} = \frac{6}{AC} \Rightarrow 4\,AC = 18 \Rightarrow \text{Hence } AC = \frac{9}{2} = 4.5 \text{ cm}$$

4.16 **In the Figure AD is the bisector of $\angle BAC$ if AB = 10 cm, AC = 14 cm and BC = 6 cm. Find BD and DC.**



Sol :

Let BD = x cm, then DC = (6 – x) cm

AD is the bisector of $\angle A$

Therefore by Angle Bisector Theorem

$$\frac{AB}{AC} = \frac{BD}{DC}$$

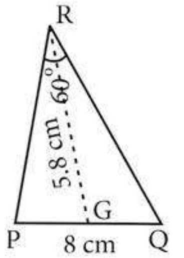
$$\frac{10}{14} = \frac{x}{6-x} \Rightarrow \frac{5}{7} = \frac{x}{6-x}$$

$$\Rightarrow 12x = 30 \Rightarrow x = \frac{30}{12} = 2.5 \text{ cm}$$

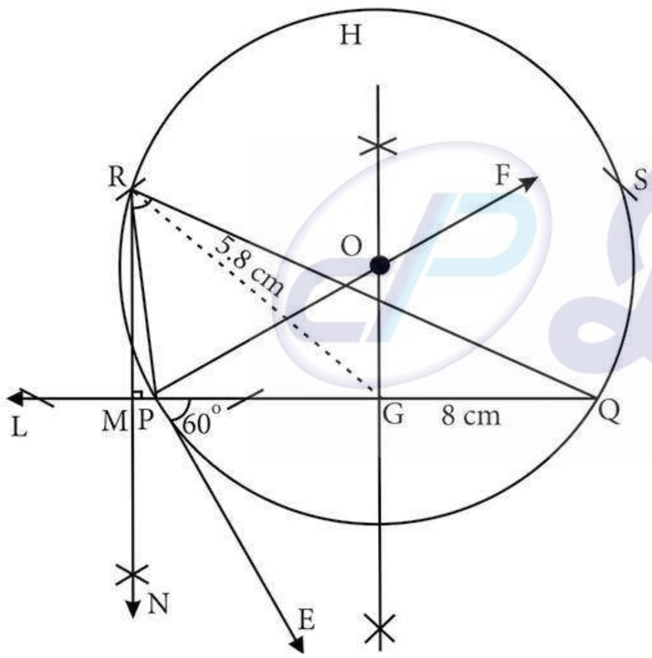
Therefore, BD = 2.5 cm,
DC = 6 – x = 6 – 2.5 = 3.5 cm

4.17 **Construct a ΔPQR in which PQ = 8 cm, $\angle R = 60^{o}$ and the median RG from R to PQ is 5.8 cm. Find the length of the altitude from R to PQ.**

Sol :



Rough diagram



Construction :

Step 1: Draw a line segment PQ = 8 cm.

Step 2: At P, draw PE such that $\angle QPE = 60^{o}$.

Step 3: At P, draw PF such that $\angle EPF = 90^{o}$.

Step 4: Draw the perpendicular bisector to PQ, which intersects PF at O and PQ at G.

Step 5: With O as centre and OP as radius draw a circle.

Step 6: From G mark arcs of radius 5.8 cm on the circle. Mark them as R and S.

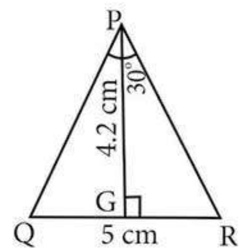
Step 7: Join PR and RQ. Then ΔPQR is the required triangle.

Step 8: From R drawn a line RN which perpendicular to LQ. LQ meets RN at M.

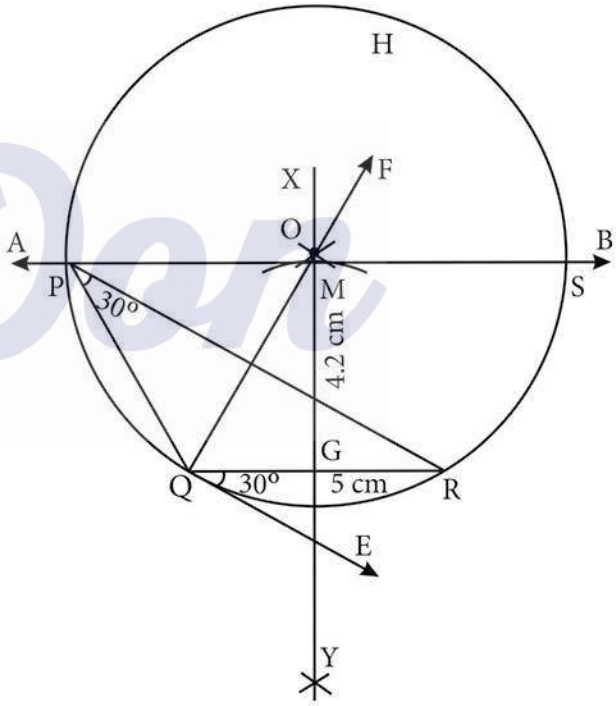
Step 9: The length of the altitude is RM = 3.5 cm.

4.18 **Construct a triangle ΔPQR such that QR = 5 cm, $\angle P = 30^{o}$ and the altitude from P to QR is of length 4.2 cm.**

Sol :



Rough diagram



Construction:

Step 1: Draw a line segment QR = 5 cm.

Step 2: At Q draw QE such that $\angle RQE = 30^{o}$.

Step 3: At Q draw QF such that $\angle EQF = 90^{o}$.

Step 4: Draw the perpendicular bisector XY to QR which intersects QF at O and QR at G.

Step 5: With O as centre and OQ as radius draw a circle.

Step 6: XY intersects QR at G. On XY, from G mark an arc at M, such that GM = 4.2 cm.

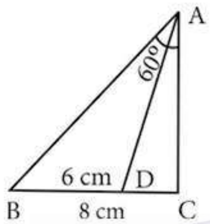
Step 7: Draw AB through M which is parallel to QR.

Step 8: AB meets the circle at P and S.

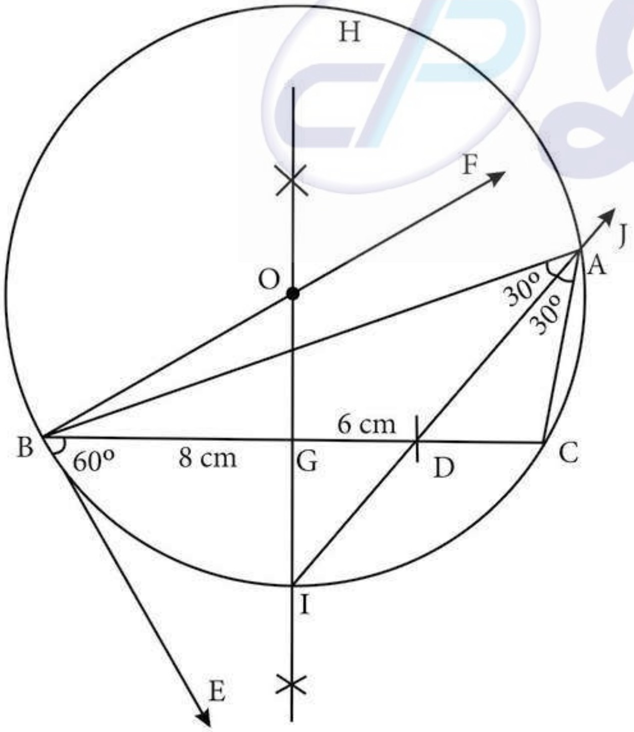
Step 9: Join QP and RP. Then ΔPQR is the required triangle.

4.19 Draw a triangle ABC of base BC = 8 cm, $\angle A = 60^o$ and the bisector of $\angle A$ meets BC at D such that BD = 6 cm.

Sol :



Rough diagram



Construction:

Step 1: Draw a line segment BC = 8 cm.

Step 2: At B, draw BE such that $\angle CBE = 60^o$.

Step 3: At B, draw BF such that ∠EBF = 90°.

Step 4: Draw the perpendicular bisector to BC, which intersects BF at O and BC at G.

Step 5: With O as centre and OB as radius draw a circle.

Step 6: From B, mark an arc of 6 cm on BC at D.

Step 7: The perpendicular bisector intersects the circle at I. Joint ID.

Step 8: ID produced meets the circle at A. Now join AB and AC.

Then ΔABC is the required triangle.

Progress Check

1. A straight line drawn _____ to a side of a triangle divides the other two sides proportionally.

Ans : Parallel

2. Basic Proportionality Theorem is also known as _____.

Ans : Thales Theorem

3. Let ΔABC be equilateral. Using Angle Bisector Theorem, $\frac{BD}{DE}$ is _____ where D is a point on BC and AD is the internal bisector of $\angle A$.

Ans : 1

4. The _____ of an angle of a triangle divides the opposite side internally in the ratio of the corresponding sides containing the angle.

Ans : Internal bisector

5. If the median AD to the side BC of a ΔABC is also an angle bisector of $\angle A$ then $\frac{AB}{AC}$ is _______.

Ans : 1

Exercise 4. 2

1. In ΔABC, D and E are points on the sides AB and AC respectively such that DE || BC

(i) If $\frac{AD}{DB} = \frac{3}{4}$ and AC = 15 cm find AE.

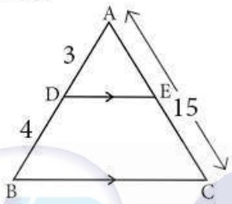
(ii) If AD = 8x – 7, DB = 5x – 3, AE = 4x – 3 and EC = 3x – 1, find the value of x.

Sol :

(i) Given $\frac{AD}{DB} = \frac{3}{4}$

$$AC = 15 \text{ cm}$$
$$EC = AC - AE$$
$$= 15 - AE$$

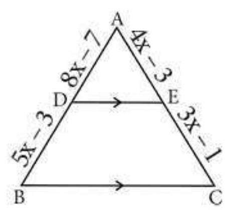
By Basic proportionality theorem.



We have $\frac{AD}{DB} = \frac{AE}{EC}$

$$\frac{3}{4} = \frac{AE}{AC - AE}$$
$$\frac{3}{4} = \frac{AE}{15 - AE}$$
$$3(15 - AE) = 4AE$$
$$45 - 3AE = 4AE$$
$$45 = 4AE + 3AE$$
$$7AE = 45$$
$$AE = \frac{45}{7} = 6.43 \text{ cm}$$

(ii) $\frac{AD}{DB} = \frac{AE}{EC}$



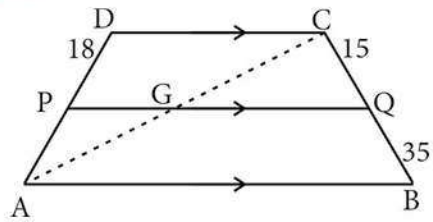
Given
$$AD = 8x - 7$$
$$DB = 5x - 3$$
$$AE = 4x - 3$$
$$EC = 3x - 1$$
$$\frac{8x - 7}{5x - 3} = \frac{4x - 3}{3x - 1}$$
$$(8x - 7)(3x - 1) = (4x - 3)(5x - 3)$$
$$24x^2 - 21x - 8x + 7 = 20x^2 - 15x - 12x + 9$$
$$24x^2 - 20x^2 - 29x + 27x + 7 - 9 = 0$$
$$4x^2 - 2x - 2 = 0$$
$$\div \text{ by } 2 \Rightarrow 2x^2 - x - 1 = 0$$
$$2x^2 - 2x + x - 1 = 0$$
$$2x(x - 1) + 1(x - 1) = 0$$
$$(x - 1)(2x + 1) = 0$$
$$x - 1 = 0 \text{ or } 2x + 1 = 0$$
$$x = 1 \quad \text{or} \quad x = -\frac{1}{2}$$

x cannot be negative

$$\therefore \ x = 1$$

2. ABCD is a trapezium in which AB || DC and P, Q are points on AD and BC respectively, such that PQ || DC if PD = 18 cm, BQ = 35 cm and QC = 15 cm. Find AD.

Sol :



Join AC such that it meets PQ at G.

$\therefore$ AB || DC and PQ || DC

PQ || AB

In ΔADC

PG || DC

$$\frac{AP}{PD} = \frac{AG}{GC} \quad \text{... (1)}$$

In ΔCAB

$$\frac{AG}{GC} = \frac{BQ}{QC} \quad \text{... (2)}$$

From (1) and (2) we get

$$\frac{AP}{PD} = \frac{BQ}{QC}$$
$$\frac{AP}{18} = \frac{35}{15}$$
$$AP = \frac{35 \times 18}{15}$$
$$AP = 42 \text{ cm}$$

Now AD = AP + PD

$$= 42 + 18 = 60 \text{ cm}$$
$$AD = 60 \text{ cm}$$

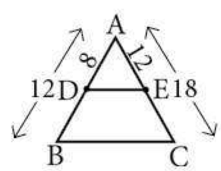
3. In a ΔABC D and E are points on the sides AB and AC respectively. For each of the following cases show that DE || BC.

(i) AB = 12 cm, AD = 8 cm, AE = 12 cm and AC = 18 cm.

(ii) AB = 5.6 cm, AD = 1.4 cm, AC = 7.2 cm and AE = 1.8 cm.

Sol :

(i)



Given AB = 12 cm, AD = 8 cm, AE = 12 cm, AC = 18 cm

By basic proportionality theorem $\frac{AD}{DB} = \frac{AE}{EC}$

$$\frac{AD}{DB} = \frac{8}{AB-AD} = \frac{8}{12-8}$$

$$\frac{AD}{DB} = \frac{8}{4} = 2 \quad \ldots (1)$$

$$\frac{AE}{EC} = \frac{12}{AC-AE}$$

$$\frac{AE}{EC} = \frac{12}{18-12} = \frac{12}{6} = 2 \quad \ldots (2)$$

From (1) and (2)

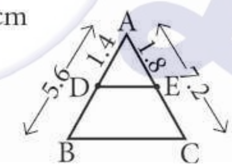
$$\frac{AD}{DB} = \frac{AE}{EC}$$

$\therefore$ DE || BC

(ii) Given AB = 5.6 cm; AD = 1.4 cm; AC = 7.2 cm; AE = 1.8 cm

DB = AB – AD
= 5.6 – 1.4 = 4.2 cm

EC = AC – AE
= 7.2 – 1.8 = 5.4 cm



Now $\frac{AD}{DB} = \frac{1.4}{4.2}$

$$\frac{AD}{DB} = \frac{1}{3} \quad \ldots (1)$$

$$\frac{AE}{EC} = \frac{1.8}{5.4}$$

$$\frac{AE}{EC} = \frac{1}{3} \quad \ldots (2)$$

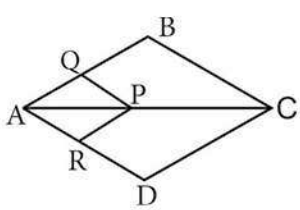
From (1) and (2)

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$\therefore$ DE || BC

4. In figure if PQ || BC and PR || CD prove that

(i) $\frac{AR}{AD} = \frac{AQ}{AB}$ **(ii)** $\frac{QB}{AQ} = \frac{DR}{AR}$



Sol :

(i) In ΔABC

PQ || CB

Using Basic Proportionality theorem, we have

$$\frac{AQ}{AB} = \frac{AP}{AC} \quad \ldots (1)$$

Again in ΔACD PR || CD

Using Basic Proportionality theorem

$$\frac{AP}{AC} = \frac{AR}{AD} \quad \ldots (2)$$

From (1) and (2)

$$\frac{AQ}{AB} = \frac{AP}{AC} = \frac{AR}{AD}$$

Thus we have $\frac{AR}{AD} = \frac{AQ}{AB}$

(ii) From (1) and (2) we have

$$\frac{AQ}{AB} = \frac{AR}{AD}$$

$$\frac{AB}{AQ} = \frac{AD}{AR}$$

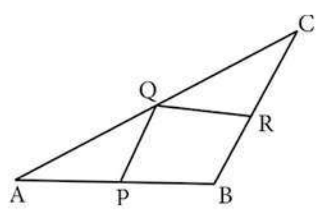
$$\frac{AQ+QB}{AQ} = \frac{AR+RD}{AR}$$

$$1+\frac{QB}{AQ} = 1+\frac{RD}{AR}$$

$$\Rightarrow \frac{QB}{AQ} = \frac{DR}{AR}$$

5. Rhombus PQRB is inscribed in ΔABC such that $\angle B$ is one of its angle. P, Q and R lie on AB, AC and BC respectively. If AB = 12 cm and BC = 6 cm, find the sides PQ, RB of the rhombus.

Sol :



Given AB = 12 cm, BC = 6 cm

In ΔAPQ and ΔQRC

$\angle QAP = \angle CQR$
($\because$ PB || QR, corresponding angles)

$\angle PQA = \angle RCQ$
($\because$ PQ || BR, corresponding angles)

$\therefore$ By AA criterion of similarity, we have

$\Delta APQ \sim \Delta QRC$

$$\Rightarrow \frac{AP}{QR} = \frac{PQ}{RC} = \frac{AQ}{QC}$$

$$\frac{AP}{QR} = \frac{PQ}{RC}$$

$$\frac{PQ}{AP} = \frac{RC}{QR} \quad \text{... (1)}$$

Now in ΔAPQ and ΔABC , we have

$$\angle CAB = \angle QAP \quad \text{(common)}$$

$$\angle AQP = \angle ACB$$

$\therefore$ By AA criterion of similarity,

we have $\Delta APQ \sim \Delta ABC$

$$\frac{AP}{AB} = \frac{PQ}{BC} = \frac{AQ}{AC}$$

$$\frac{AP}{AB} = \frac{PQ}{BC}$$

$$\frac{PQ}{AP} = \frac{BC}{AB} \quad \text{... (2)}$$

$$\frac{PQ}{AP} = \frac{6}{12}$$

Since PQRB is a rhombus, PQ = QR = RB = PB

$$\frac{PQ}{AB-PB} = \frac{6}{12}$$

$$\frac{PQ}{AB-PQ} = \frac{6}{12}$$

$$\frac{PQ}{12-PQ} = \frac{6}{12}$$

$$12\ PQ = 6(12 - PQ)$$

$$12\ PQ = 72 - 6\ PQ$$

$$12\ PQ + 6\ PQ = 72$$

$$18\ PQ = 72$$

$$PQ = \frac{72}{18} = 4$$

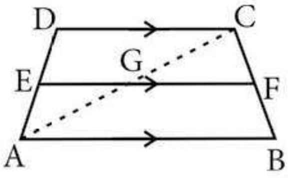
PQ = 4 cm

Since PQ = RB we have

PQ = RB = 4 cm

6. In trapezium ABCD, AB || DC, E and F are points on non-parallel sides AD and BC respectively, such that EF || AB. Show that $\frac{AE}{ED} = \frac{BF}{FC}$.

Sol :



Given: ABCD is a trapezium in which DC || AB and EF || AB

To prove : $\frac{AE}{ED} = \frac{BF}{FC}$

Construction: Join AC meeting EF at G

Proof:

In ΔADC , we have

EG || DC

$$\Rightarrow \frac{AE}{ED} = \frac{AG}{GC} \quad \text{[By thales theorem] ..... (1)}$$

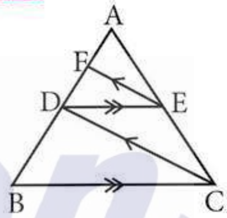
In ΔABC , we have

$$\frac{AG}{GC} = \frac{BF}{FC} \quad \text{[By thales theorem] ... (2)}$$

From (1) and (2), we get

$$\frac{AE}{ED} = \frac{BF}{FC}$$

7. In figure DE || BC and CD || EF. Prove that $AD^2 = AB \times AF$.



Sol :

In ΔABC , we have DE || BC

$$\frac{AB}{AD} = \frac{AC}{AE} \quad \text{[By Thales Theorem] ... (1)}$$

In ΔADC , we have

$$\frac{AD}{AF} = \frac{AC}{AE} \quad \text{[By Thales Theorem] ... (2)}$$

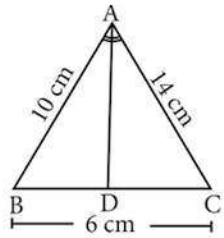
From (1) and (2) we get

$$\frac{AB}{AD} = \frac{AD}{AF}$$

$$AD^2 = AB \times AF$$

8. In a ΔABC, AD is the bisector of $\angle A$ meeting side BC at D, if AB = 10 cm, AC = 14 cm and BC = 6 cm, find BD and DC.

Sol :



By angle bisector theorem, we have

$$\frac{AB}{AC} = \frac{BD}{CD} \quad \text{... (1)}$$

$$\frac{10}{14} = \frac{BD}{CD}$$

[$\because$ BC = BD + CD = 6; DC = 6 – BD]

$$\frac{10}{14} = \frac{BD}{6 - BD}$$

$$\frac{5}{7} = \frac{BD}{6 - BD}$$

$$5(6 - BD) = 7\,BD$$

$$30 - 5\,BD = 7\,BD$$

$$30 = 7\,BD + 5\,BD$$

$$30 = 12\,BD$$

$$BD = \frac{30}{12} = \frac{5}{2}$$

$$BD = 2.5 \text{ cm}$$

Also from (1)

$$\frac{10}{14} = \frac{6 - CD}{CD}$$

$$\frac{5}{7} = \frac{6 - CD}{CD}$$

$$5\,CD = 42 - 7\,CD$$

$$5\,CD + 7\,CD = 42$$

$$12\,CD = 42$$

$$CD = \frac{42}{12} = \frac{7}{2} = 3.5 \text{ cm}$$

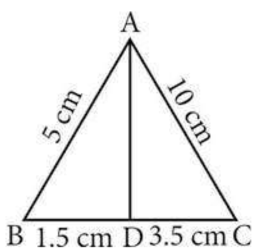
$$CD = 3.5 \text{ cm}$$

9. Check whether AD is bisector of $\angle A$ of ΔABC in each of the following

(i) AB = 5 cm, AC = 10 cm, BD = 1.5 cm and CD = 3.5 cm.

(ii) AB = 4 cm, AC = 6 cm, BD = 1.6 cm and CD = 2.4 cm.

Sol :



(i) We have AB = 5 cm, AC = 10 cm

BD = 1.5 cm, CD = 3.5 cm

$$\frac{AB}{AC} = \frac{5}{10} = \frac{1}{2}$$

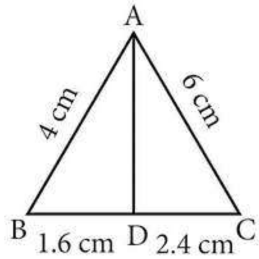
$$\frac{BD}{DC} = \frac{1.5}{3.5} = \frac{3}{7}$$

$$\frac{AB}{AC} \neq \frac{BD}{DC}$$

$\therefore$ By the converse of the angle bisector theorem, AD is not a bisector of $\angle A$

(ii) We have AB = 4 cm, AC = 6 cm,

BD = 1.6 cm, CD = 2.4 cm



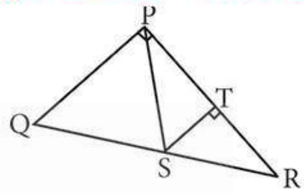
$$\frac{AB}{AC} = \frac{4}{6} = \frac{2}{3}$$

$$\frac{BD}{DC} = \frac{1.6}{2.4} = \frac{2}{3}$$

$$\frac{AB}{AC} = \frac{BD}{DC}$$

$\therefore$ By the converse of the Angle Bisector theorem AD is the bisector of $\angle A$

10. In figure $\angle QPR$ = 90°, PS is its bisector. If ST $\perp$ PR, prove that ST × (PQ + PR) = PQ × PR.



Sol :

Given that PS is the bisector of $\angle P$ of ΔPQR

$$\frac{PQ}{PR} = \frac{QS}{SR} \quad \text{[By Angle Bisector Theorem]}$$

Adding 1 on both the sides

$$\frac{PQ}{PR} + 1 = \frac{QS}{SR} + 1$$

$$\frac{PQ + PR}{PR} = \frac{QS + SR}{SR}$$

$$\frac{PQ + PR}{PR} = \frac{QR}{SR} \qquad \text{... (1)}$$

In ΔRST and ΔRQP we have

$$\angle SRT = \angle QRP = \angle R \qquad \text{[common]}$$

$$\angle QPR = \angle STR = 90^\circ$$

By AA criterion for similarity, we have

$\Delta RST \sim \Delta RQP$

$$\frac{RS}{RQ} = \frac{ST}{QP}$$

$$\frac{QP}{ST} = \frac{QR}{RS} \qquad \text{... (2)}$$

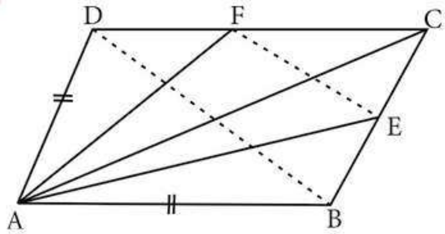
From (1) and (2)

$$\frac{QP}{ST} = \frac{PQ + PR}{PR}$$

$$PQ \times PR = ST\,(PQ + PR)$$

11. ABCD is a quadrilateral in which AB = AD, the bisector of $\angle$BAC and $\angle CAD$ intersect the sides BC and CD at the point E and F respectively. Prove that EF || BD.

Sol :



Given: A quadrilateral ABCD in which AB = AD and the bisectors of $\angle BAC$ and $\angle CAD$ meet the sides BC and CD at E and F respectively.

To prove: EF || BD

Construction: Join AC, BD and EF.

Proof: In ΔCAB, AE is the bisector of $\angle BAC$

$$\therefore \frac{AC}{AB} = \frac{CE}{BE} \quad \ldots (1)$$

In ΔACD, AF is the bisector of $\angle CAD$

$$\frac{AC}{AD} = \frac{CF}{DF}$$

$$\Rightarrow \frac{AC}{AB} = \frac{CF}{DF} \quad \ldots (2)$$

[$\because$ AD = AB]

From (1) and (2) we get

$$\frac{CE}{BE} = \frac{CF}{DF}$$

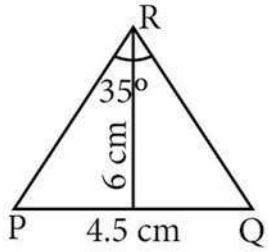
$$\Rightarrow \frac{CE}{EB} = \frac{CF}{FD}$$

Thus in ΔCBD, E and F divide the sides CB and CD respectively in the same ratio.

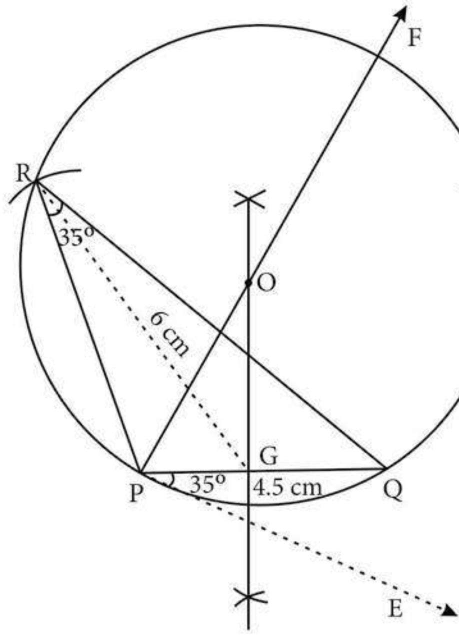
$\therefore$ By the converse of Thales theorem, we have EF || BD.

12. Construct a ΔPQR which the base PQ = 4.5 cm, $\angle R$ = 35° and the median from R to PQ is 6 cm.

Sol :



Rough Diagram



Construction:

Step 1: Drawn a line segment PQ = 4.5 cm

Step 2: At P, drawn PE such that $\angle QPE$ = 35°

Step 3: At P, drawn PF such that $\angle EPF$ = 90°

Step 4: Drawn the perpendicular bisector to PQ, which intersects PF at O and PQ at G

Step 5: With O as center and OP as radius drawn a circle

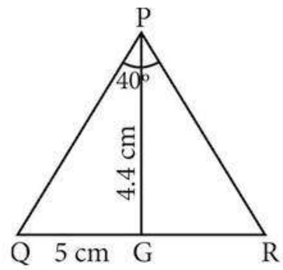
Step 6: From G, marked arcs of radius 6 cm on the circle marked them as R and S.

Step 7: Joined PR and RQ. Then ΔPQR is the required triangle.

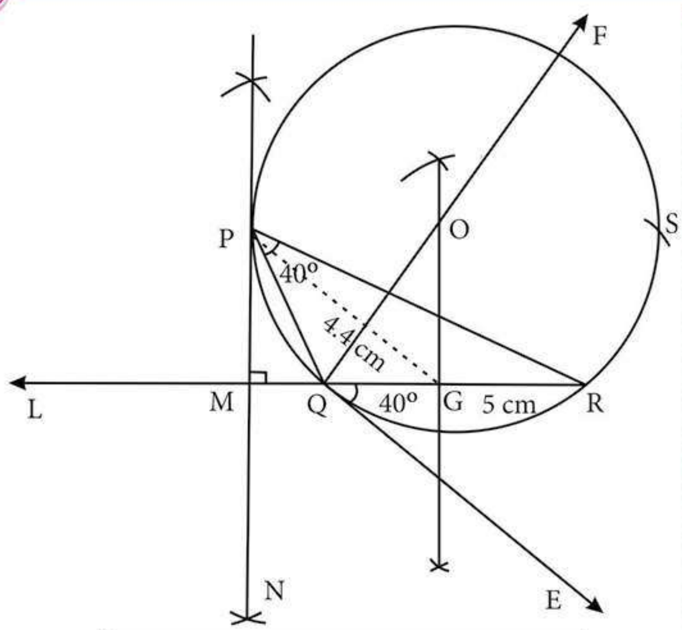
Step 8: ΔPQS is also another required triangle.

13. Construct a ΔPQR in which QR = 5 cm, $\angle P$ = 40° and the median PG from P to QR is 4.4 cm. Find the length of the altitude from P to QR.

Sol :



Rough Diagram



Length of the altitude = PM = 2.1 cm.

Construction:

Step 1: Drawn a line segment QR = 5 cm

Step 2: At Q, drawn QE such that $\angle RQE = 40^\circ$

Step 3: At Q, drawn AF such that $\angle EQF = 90^\circ$

Step 4: Drawn a perpendicular bisector to QR, which intersects QF at 'O' and QR at G.

Step 5: With 'O' as center and OQ as radius drawn a circle.

Step 6: From G marked arcs of radius 4.4 cm on the circle. Marked them as P and S.

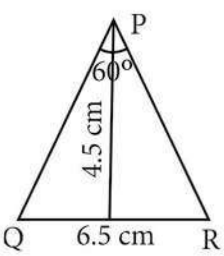
Step 7: Joined QP and PR. Now ΔPQR is the required triangle.

Step 8: From P drawn a line PN which is perpendicular to LR. LR meets PN at M.

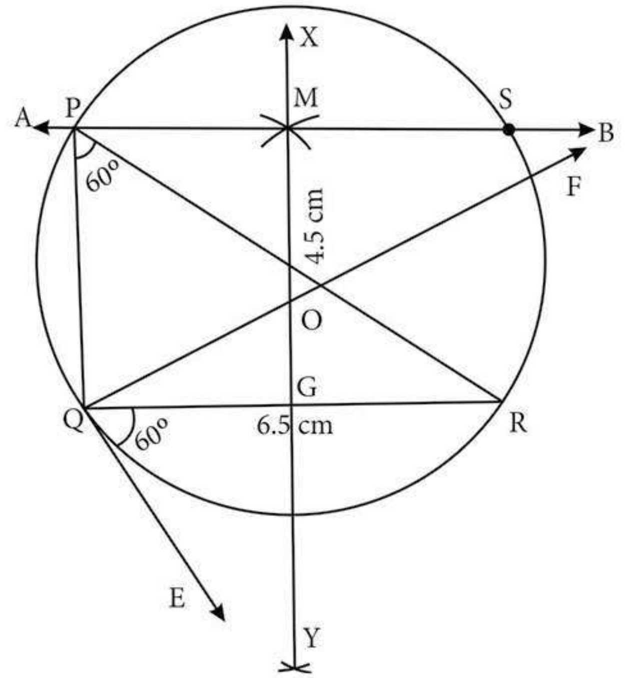
Step 9: The length of the altitude is PM = 2.1 cm.

14. Construct a ΔPQR such that QR = 6.5 cm, $\angle P = 60^\circ$ and the altitude from P to QR is of length 4.5 cm.

Sol :



Rough diagram



Construction:

Step 1: Drawn a line segment QR = 6.5 cm

Step 2: At Q drawn QE such that $\angle RQE = 60^\circ$

Step 3: At Q drawn QF such that $\angle EQF = 90^\circ$

Step 4: Drawn the perpendicular bisector XY to QR which intersects QF at O and QR at G.

Step 5: With O as center and OQ as radius drawn a circle

Step 6: XY intersects QR at G. On XY, from G marked an arc at M, such that GM = 4 .5 cm

Step 7: Drawn AB through M which is parallel to QR

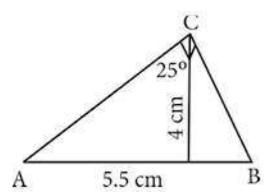
Step 8: AB meets the circle at P and S

Step 9: Joined QP and RP . Then ΔPQR is the required triangle.

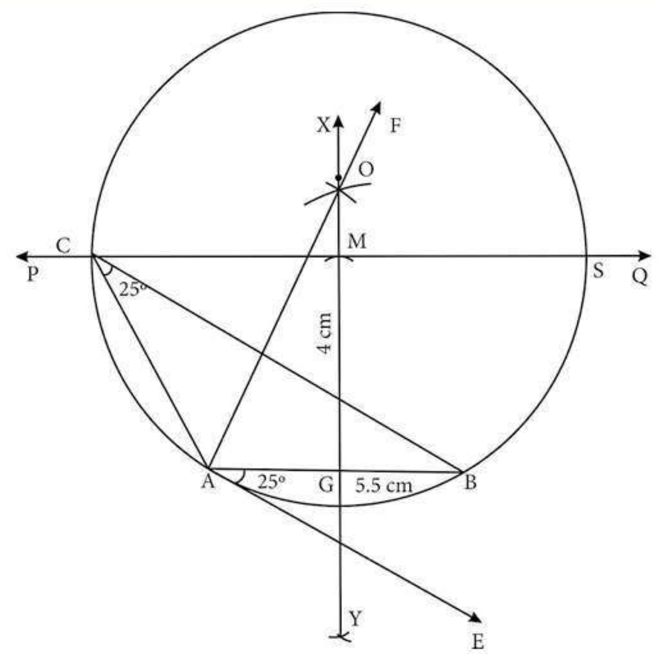
Step 10: Here ΔSQR is also another required triangle.

15. Construct a ΔABC such that AB = 5.5 cm, $\angle C = 25^\circ$ and the altitude from C to AB is 4 cm.

Sol :



Rough diagram



Construction:

Step 1: Drawn a line segment AB = 5.5 cm

Step 2: At A drawn AE such that $\angle BAE = 25^\circ$

Step 3: At A drawn AF such that $\angle FAE = 90^\circ$

Step 4: Drawn the perpendicular bisector XY to AB which intersects AF at O and AB at G.

Step 5: With O as center and OA as radius drawn a circle

Step 6: XY intersects AB at G. On XY from G marked an arc at M such that GM = 4 cm

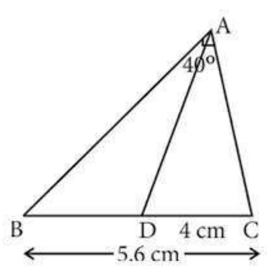
Step 7: Drawn PQ through M which is parallel to AB.

Step 8: PQ meets the circle at C and S.

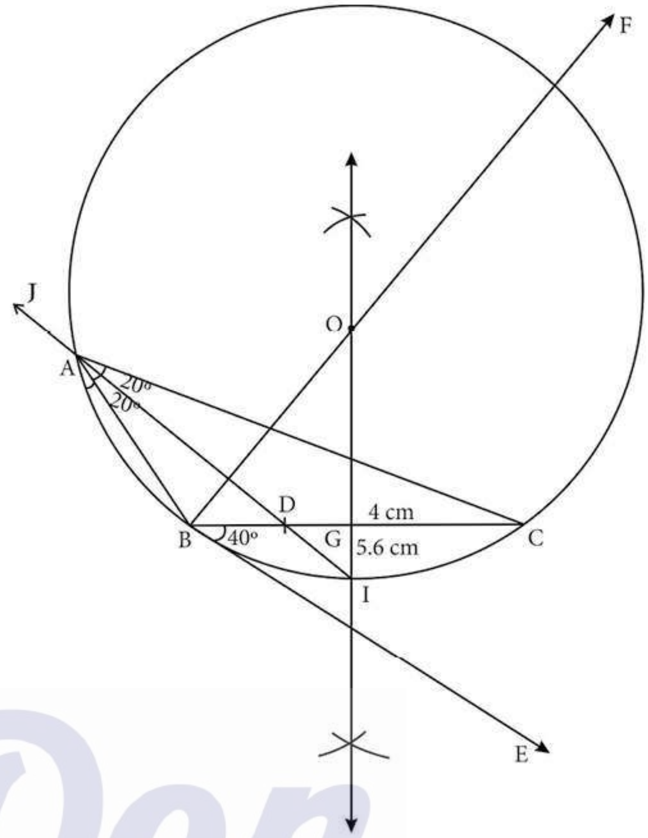
Step 9: Joined AC and BC. Now ΔABC is the required triangle.

16. Draw a triangle ABC of base BC = 5.6 cm, $\angle A = 40^\circ$ and the bisector of $\angle A$ meets BC at D such that CD = 4 cm.

Sol :



Rough Diagram



Construction:

Step 1: Drawn a line segment BC = 5.6 cm

Step 2: At B, drawn BE such that $\angle CBE = 40^\circ$

Step 3: At B, drawn BF such that $\angle EBF = 90^\circ$

Step 4: Drawn the perpendicular bisector to BC, which intersects BF at O and BC at G.

Step 5: With O as center and OB as radius drawn a circle.

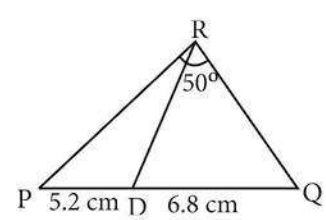
Step 6: From B, marked an arc of 4 cm on BC at D.

Step 7: The perpendicular bisector intersects the circle at I. Joined ID.

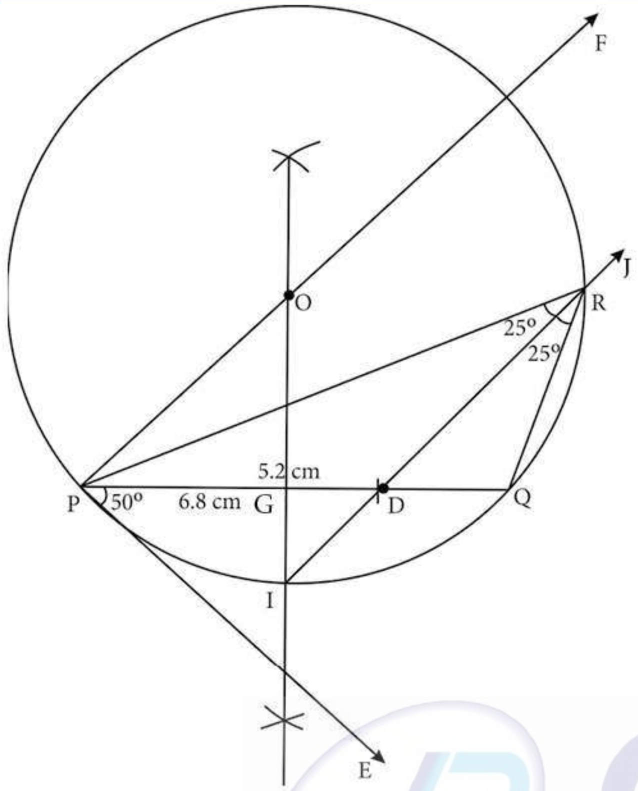
Step 8: ID produced meets the circle at A. Now joined AB and AC. Then ΔABC is the required triangle.

17. Draw ΔPQR such that PQ = 6.8 cm, vertical angle is 50° and the bisector of the vertical angle meets the base at D where PD = 5.2 cm.

Sol :



Rough Diagram



Construction:

Step 1: Drawn a line segment PQ = 6.8 cm

Step 2: At P, drawn PE such that $\angle QPE = 50^{\circ}$

Step 3: At P, drawn PF such that $\angle EPF = 90^{\circ}$

Step 4 : Drawn the perpendicular bisector to PQ, which intersects PF at 'O' and PQ at G.

Step 5: With O as center and OP as radius drawn a circle.

Step 6: From P, marked an arc of 5.2 cm on PQ at D

Step 7: The perpendicular bisector intersects the circle at I. Joined ID

Step 8: ID produced meets the circle at R now joining PR and QR, we get the required ΔPQR

Pythagoras Theorem

Key Points

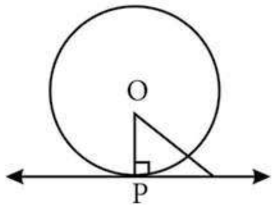
Pythagoras theorem

- Pythagoras theorem has the maximum number of proofs.
- Three numbers (a, b, c) are said to form Pythagorean Triple, if they form sides of a right triangle.
- (a, b, c) is a Pythagorean Triplet if and only if $c^2 = a^2 + b^2$.
- In a right angled triangle, the side opposite to 90° (the right angle) is called the **hypotenuse**.
- The hypotenuse will be the **longest** side of the triangle.
- Pythagoras theorem states that "In a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides".
- In India pythagoras theorem is also referred as **"Baudhyana Theorem".**
- If the square of the longest side of a triangle is equal to the sums of squares of other two sides, then the triangle is a right angle triangle.

Circles and tangents

- If a line and a circle are in the same plane
  - (i) the line may touch the circle at a point on it.
  - (ii) the line intersects the circle at two points.
  - (iii) the line may not touch the circle.

- If the line touches the circle at a point then we say the line is the **tangent** to the circle. In this case the number of point of intersection is **one.**
- If the line intersects the circle at two points then we say that the line is the secant of the circle.
- The chord of a circle is a subsection of a **secant**.
- A tangent at any point on a circle and the radius through the point are **perpendicular** to each other.



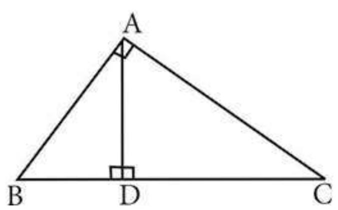
- **No tangents** can be drawn from an interior point of the circle.
- **Only one** tangent can be drawn at any point on the circle.
- **Two tangents** can be drawn from any exterior point of a circle.
- The lengths of the two tangents drawn from an exterior point to a circle are **equal.**
- If two circles touch externally, the distance between their centres is equal to the **sum of their radii.**
- It two circles touch internally, the distance between their centres is equal to the **difference of their radii.**
- The two direct common tangents drawn to the circles are **equal.**

Theorem 5: Pythagoras Theorem

Statement:

"In a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides."

Proof:



Given: In ΔABC, $\angle A = 90^\circ$

To prove: $AB^2 + AC^2 = BC^2$

Construction: Draw $AD \perp BC$

Sl. No.	Statement	Reason
1	Compare ΔABC and ΔABD $\angle B$ is common $\angle BAC = \angle BDA = 90^o$ Therefore $\Delta ABC \sim \Delta ABD$ $\Rightarrow \frac{AB}{BD} = \frac{BC}{AB}$ $\Rightarrow AB^2 = BC \times BD$ ... (1)	Given $\angle BAC = 90^o$ and by construction $\angle BDA = 90^o$ BA AA similarity

2	Compare ΔABC and ΔADC $\angle C$ is common $\angle BAC = \angle ADC = 90^o$ Therefore, $\Delta ABC \sim \Delta ADC$ $\frac{BC}{AC} = \frac{AC}{DC}$ $AC^2 = BC \times DC$ ...(2)	Given $\angle BAC = 90^o$ and by construction $\angle CDA = 90^o$ By AA similarity

Adding (1) and (2) we get

$$AB^2 + AC^2 = BC \times BD + BC \times DC$$
$$= BC [BD + DC] = BC \times BC$$
$$AB^2 + AC^2 = BC^2$$

Hence the theorem is proved.

Converse of pythagoras Theorem

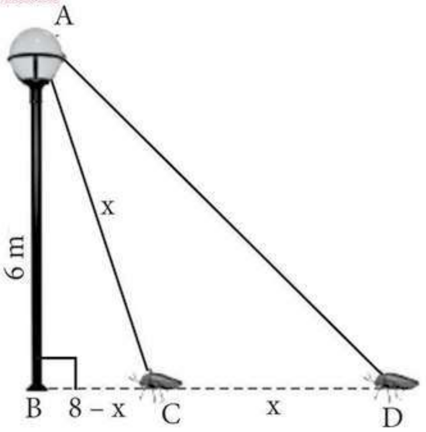
Statement

"If the square of the longest side of a triangle is equal to sums of squares of other two sides, then the triangle is a right angled triangle."

Worked Examples

4.20 **An insect 8 m away initially from the foot of a lamp post which is 6 m tall, crawls towards it moving through a distance. If its distance from the top of the lamp post is equal to the distance it has moved, how far is the insect away from the foot of the lamp post?**

Sol :



Distance between the insect and the foot of the lamp post = BD = 8 m

The height of the lamp post AB = 6 m

After moving a distance of x m, let the insect be at C

Let, AC = CD = x.

Then BC = BD – CD = 8 – x

In ΔABC, $\angle B = 90^o$

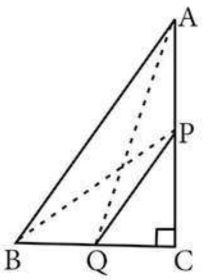
$$AC^2 = AB^2 + BC^2 \Rightarrow x^2 = 6^2 + (8 - x)^2$$
$$x^2 = 36 + 64 - 16x + x^2$$
$$16x = 100 \Rightarrow x = 6.25$$

Then, BC = 8 – x = 8 – 6.25 = 1.75 m

Therefore the insect is 1.75 m away from the foot of the lamp post.

4.21 **P and Q are the mid-points of the sides CA and CB respectively of a ΔABC, right angled at C. Prove that $4(AQ^2 + BP^2) = 5AB^2$.**

Sol :



Since, ΔAQC is a right triangle at C,

$$AQ^2 = AC^2 + QC^2 \quad \text{... (1)}$$

Since, ΔBPC is a right triangle at C,

$$BP^2 = BC^2 + CP^2 \quad \text{... (2)}$$

From (1) and (2),

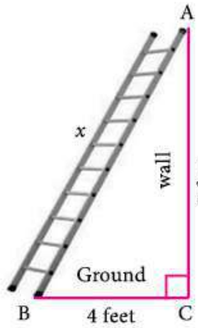
$$AQ^2 + BP^2 = AC^2 + QC^2 + BC^2 + CP^2$$
$$4(AQ^2 + BP^2) = 4AC^2 + 4QC^2 + 4BC^2 + 4CP^2$$
$$= 4AC^2 + (2QC)^2 + 4BC^2 + (2CP)^2$$
$$= 4AC^2 + BC^2 + 4BC^2 + AC^2$$

(Since P and Q are mid points)

$$= 5(AC^2 + BC^2)$$
$$4(AQ^2 + BP^2) = 5AB^2$$
(By Pythagoras Theorem)

4.22 What length of ladder is needed to reach a height of 7 ft along the wall when the base of the ladder is 4 ft from the wall? Round off your answer to the next tenth place.

Sol :



Let x be the length of the ladder, BC = 4 ft,
AC = 7 ft.

By Pythagoras theorem we have,

$$AB^2 = AC^2 + BC^2$$
$$x^2 = 7^2 + 4^2 \Rightarrow x^2 = 49 + 16$$
$$x^2 = 65 \Rightarrow x = \sqrt{65}$$

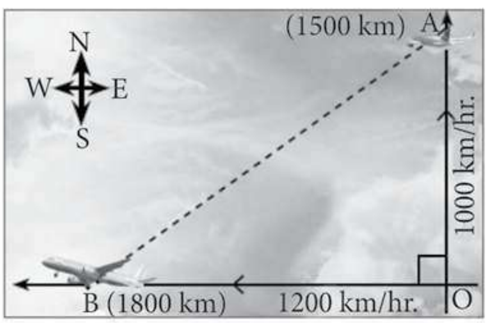
The number $\sqrt{65}$ is between 8 and 8.1.

$$8^2 = 64 < 65 < 65.61 = 8.1^2$$

Therefore, the length of the ladder is approximately 8.1 ft.

4.23 An Aeroplane leaves an airport and flies due north at a speed of 1000 km/hr. At the same time, another aeroplane leaves the same airport and flies due west at a speed of 1200 km/hr. How far apart will be the two planes after $1\frac{1}{2}$ hours?

Sol :



Let the first aeroplane starts from O and goes upto A towards north,

(Distance = Speed × time)

where OA $= \left(1000 \times \frac{3}{2}\right)$ km = 1500 km

Let the second aeroplane starts from O at the same time and goes upto B towards west,

where OB $= \left(1200 \times \frac{3}{2}\right)$ km = 1800 km

The required distance = BA. In a right angled triangle AOB, $AB^2 = OA^2 + OB^2$

$$AB^2 = (1500)^2 + (1800)^2 = 100^2 (15^2 + 18^2)$$
$$= 100^2 \times 549 = 100^2 \times 9 \times 61$$
$$AB = 100 \times 3 \times \sqrt{61} = 300\sqrt{61} \text{ kms.}$$

Progress Check

1. **_____ is the longest side of the right angled triangle.**

   Ans : Hypotenuse

2. **In India, Pythagoras theorem is called as _____**

   Ans : Baudhayana Theorem

3. **The first theorem in mathematics is _____**

   Ans : Thales Theorem

4. **If the square of the longest side of a triangle is equal to sums of squares of other two sides, then the triangle is _____**

   Ans : Right angled Triangle

5. **State True or False**

   (i) Pythagoras Theorem is applicable to all triangles.

   Ans : False

   (ii) One side of a right angled triangle must be a multiple of 4.

   Ans : True

Thinking Corner

1. **Write down any five Pythagorean triplets?**

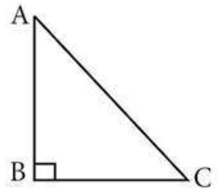
   Ans : (3, 4, 5), (9, 12, 15), (12, 16, 20), (8, 15, 17) (15, 20, 25)

2. **In a right angle triangle the sum of other two angles is _____**

   Ans : 90^o.

3. Can all the three sides of a right angled triangle be odd numbers? Why?

Ans :



No, All the three sides of a right angled triangle cannot be odd numbers.

If ΔABC is right angled triangle with $\angle B = 90^\circ$, then $AC^2 = AB^2 + BC^2$

If AB and BC are odd, then their squares AB^2 and BC^2 are odd numbers.

Their sum $AB^2 + BC^2$ is an even number.

Square root of $AB^2 + BC^2$ is also an even number.

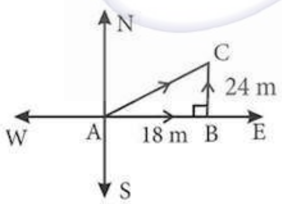
$\therefore$ AC is even.

So three measures cannot be odd.

Exercise 4. 3

1. A man goes 18 m due east and then 24 m due north. Find the distance of his current position from the starting point?

Sol :



Let the initial position of the man be A and his final position be C

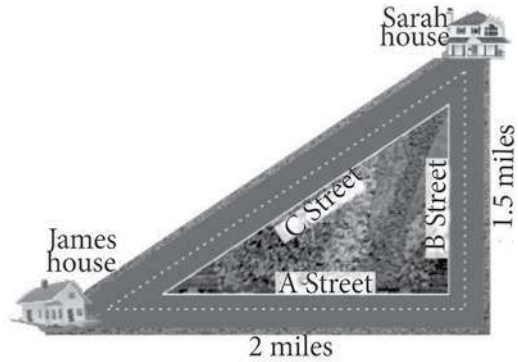
Since the man goes 18 m east and then 24 m north, ΔABC is a right angled triangle with $\angle B = 90^\circ$; AB = 18 m and BC = 24 m

By pythagoras theorem, we have

$$AC^2 = AB^2 + BC^2$$
$$AC^2 = 18^2 + 24^2$$
$$AC^2 = 324 + 576$$
$$AC^2 = 900 = 30 \times 30$$
$$AC = 30 \text{ m}$$

$\therefore$ His current distance from starting point = 30 m

2. There are two paths that one can choose to go from Sarah's house to James house. One way is to take C street and the other way requires to take A street and then B street. How much shorter is the direct path along C street? (Using figure).



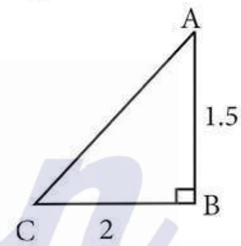
Sol :

Let Sarah's house is at 'A' and James's house is at 'B' from the picture.

Distance between Sarah's house to James house through Street B and C

= 1.5 miles + 2 miles = 3.5 miles

Distance through street C is $AC^2 = AB^2 + BC^2$



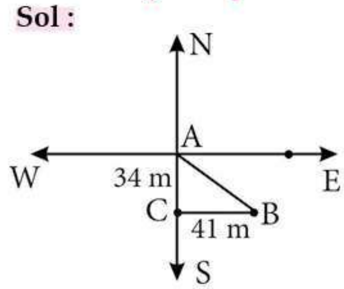
$$AC^2 = (1.5)^2 + (2)^2$$
$$= 2.25 + 4$$
$$AC^2 = 6.25$$
$$AC = \sqrt{6.25} = 2.5$$
$$AC = 2.5 \text{ miles}$$

Difference between two paths = 3.5 – 2.5 = 1 mile

$\therefore$ Direct path along C street is 1 mile shorter.

3. To get from point A to point B you must avoid walking through a pond. You must walk 34 m south and 41 m east. To the nearest meter, how many meters would be saved if it were possible to walk through the pond?

Sol :



Let A be the starting position and 'B' be the final position. C be the position south of A at 34 m distance.

Clearly $\angle C = 90^\circ$ in ΔACB

$$AC^2 + CB^2 = AB^2$$
[By pythagoras theorem]
$$34^2 + 41^2 = AB^2$$
$$1156 + 1681 = AB^2$$
$$2837 = AB^2$$
$$AB = 53.26 \text{ m}$$

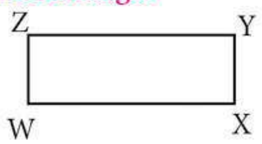
Distance from A to B through the pond = 53.26 m

Distance through C = 34 m + 41 m = 75 m

Difference = 75 – 53.26 = 21.74 m

21.74 m would be saved if it is possible to walk through the pond.

4. In the rectangle WXYZ, XY + YZ = 17 cm, and XZ + YW = 26 cm. Calculate the length and breadth of the rectangle?



Sol :

Given XY + YZ = 17 cm

XZ + YW = 26 cm

We know that diagonals if a rectangle bisect each other and the diagonals have equal length.

$$\therefore \text{Each diagonal} = \frac{26}{2} = 13 \text{ cm}$$

i.e., X Z = 13 cm and YW = 13 cm

Also given XY + YZ = 17 cm

Squaring on both sides $(XY + YZ)^2 = 17^2$

$$(XY)^2 + (YZ)^2 + 2 \times (XY) \times (YZ) = 289$$

By Pythagoras theorem $(XY)^2 + (YZ)^2 = XZ^2$

$$\therefore [XZ]^2 + 2\,(XY) \times (YZ) = 289$$
$$13^2 + 2 \times \text{length} \times \text{breadth} = 289$$
$$2 \times \text{Area} = 289 - 169$$
$$\text{Area} = \frac{289 - 169}{2} = \frac{120}{2}$$
$$\text{Area} = 60 \text{ cm}^2$$

$\therefore$ The possible length and breadth are (1, 60) (2, 30) (3, 20) (4, 15), (5, 12) (6, 10).

In this pair the length and breadth should satisfy pythagoras theorem for diagonal.

$\therefore$ 5, 12 is the possible length and breadth.

5. The hypotenuse of a right triangle is 6 m more than twice of the shortest side. If the third side is 2 m less than the hypotenuse, find the sides of the triangle?

Sol :

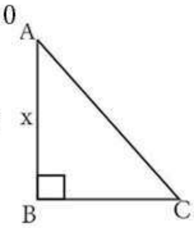
Let ΔABC be the right triangle with $\angle B = 90^\circ$.

Let the shortest side of the right triangle be x m.

Then Hypotenuse AC = (2x + 6) m

Third side BC $= [(2x + 6) - 2]$ m
$= (2x + 6 - 2)$ m
$= (2x + 4)$ m

By pythagoras theorem
$$AC^2 = AB^2 + BC^2$$
$$(2x + 6)^2 = x^2 + [2x + 4]^2$$
$$4x^2 + 36 + 24x = x^2 + 4x^2 + 16 + 16x$$
$$4x^2 + 24x + 36 = 5x^2 + 16x + 16$$
$$5x^2 + 16x + 16 - 4x^2 - 24x - 36 = 0$$
$$x^2 - 8x - 20 = 0$$
$$(x + 2)(x - 10) = 0$$
$$x = -2 \text{ or } x = 10$$



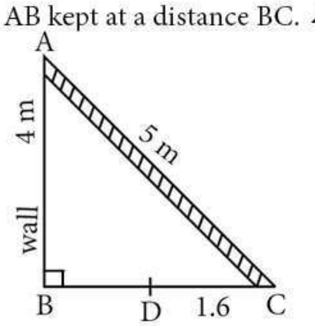
Length cannot be negative.
$$\therefore x = 10 \text{ m}$$
$$\therefore AB = 10 \text{ m}$$
$$BC = 2x + 4 = 2(10) + 4$$
$$= 24 \text{ m}$$
$$AC = 2x + 6 = 2(10) + 6 = 26 \text{ m}$$

$\therefore$ The sides of the triangle are 10 m, 24 m, 26 m.

6. 5 m long ladder is placed learning towards a vertical wall such that it reaches the wall at a point 4 m high. If the foot of the ladder is moved 1.6 m towards the wall, then find the distance by which the top of the ladder would slide upwards on the wall.

Sol :

Clearly the ladder AC make a right triangle with the wall AB kept at a distance BC. $\angle B = 90^\circ$



$\therefore$ By Pythagoras theorem
$$AC^2 = AB^2 + BC^2$$
$$5^2 = 4^2 + BC^2$$
$$BC^2 = 25 - 16$$
$$BC^2 = 9$$
$$BC = 3 \text{ m}$$

If C moves 1.6 m towards the wall BC becomes
$$3\text{m} - 1.6 \text{ m} = 1.4 \text{ m}$$

Now in ΔABC
$$AC^2 = AB^2 + BC^2$$
$$5^2 = AB^2 + (1.4)^2$$
$$25 - 1.96 = AB^2$$

$$AB^2 = 23.04$$
$$AB = 4.8 \text{ m}$$

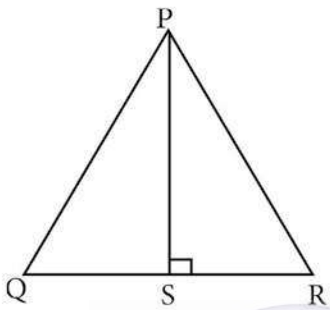
The new height of the wall = 4.8 m

Difference = 4.8 – 4 = 0.8 m

∴ The ladder would be placed 0.8 m upward the wall.

7. The perpendicular PS on the base QR of a ΔPQR intersects QR at S, such that QS = 3 SR. Prove that $2PQ^2 = 2PR^2 + QR^2$.

Sol :



We have QS = 3 SR

$$QR = QS + SR = 3SR + SR$$
$$QR = 4SR$$
$$SR = \frac{1}{4} QR \quad \text{... (a)}$$
$$SR = \frac{1}{4} QR \text{ and}$$
$$QS = 3SR = 3 \times \frac{1}{4} QR$$
$$QS = \frac{3}{4} QR \quad \text{... (1)}$$

Since ΔPSQ is a right triangle with $\angle S = 90^\circ$

$$PQ^2 = PS^2 + SQ^2 \quad \text{... (2)}$$

Similarly in ΔPSR, $\angle S = 90^\circ$

$$\therefore \quad PR^2 = PS^2 + SR^2 \quad \text{... (3)}$$

(2) – (3) $\Rightarrow$ $PQ^2 - PR^2 = SQ^2 - SR^2$

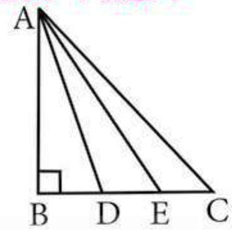
$$PQ^2 - PR^2 = \left(\frac{3}{4}QR\right)^2 - \left(\frac{1}{4}QR\right)^2$$

(∵ from (a) and (1))

$$= \frac{9}{16}QR^2 - \frac{QR^2}{16}$$
$$= \frac{1}{16}(9QR^2 - QR^2) = \frac{8\,QR^2}{16}$$
$$PQ^2 - PR^2 = \frac{1}{2} QR^2$$
$$2(PQ^2 - PR^2) = QR^2$$
$$2PQ^2 - 2PR^2 = QR^2$$
$$2PQ^2 = 2PR^2 + QR^2$$

Hence proved.

8. In figure, ABC is a right angled triangle with right angle at B and points D, E trisect BC. Prove that $8AE^2 = 3AC^2 + 5AD^2$.



Sol :

Since D and E are the points of trisection of BC.

∴ BD = DE = CE

Let BD = DE = CE = x,

then BE = 2x and BC = 3x

In right triangles ABD, ABE and ABC, we have

$$\Rightarrow \quad AD^2 = AB^2 + BD^2$$
$$\Rightarrow \quad AD^2 = AB^2 + x^2 \quad \text{... (1)}$$

In rtΔ ABE, $AE^2 = AB^2 + BE^2$

$$\Rightarrow \quad AE^2 = AB^2 + (2x)^2$$
$$\Rightarrow \quad AE^2 = AB^2 + 4x^2 \quad \text{... (2)}$$

In rtΔ ABC, $AC^2 = AB^2 + BC^2$

$$\Rightarrow \quad AC^2 = AB^2 + (3x)^2$$
$$\Rightarrow \quad AC^2 = AB^2 + 9x^2 \quad \text{... (3)}$$

Now $8AE^2 - 3AC^2 - 5AD^2 = 8(AB^2 + 4x^2) - 3(AB^2 + 9x^2) - 5(AB^2 + x^2)$

$$= 8AB^2 + 32x^2 - 3AB^2 - 27x^2 - 5AB^2 - 5x^2$$
$$\Rightarrow 8AE^2 - 3AC^2 - 5AD^2 = 0$$
$$8AE^2 = 3AC^2 + 5AD^2$$

Hence proved.

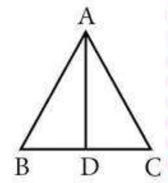
ALTERNATE SEGMENT

Key Points

Alternate segment theorem

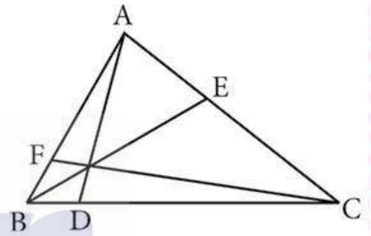
- If a line touches a circle and from the point of contact a chord is drawn, the angles between the tangent and the chord are respectively equal to the angles in the corresponding alternate segments.

Concurrency



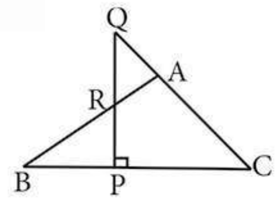
- A **cevian** is a line segment that extends from one vertex of a triangle to the opposite side. Here AD is a cevian.
- A **median** is a cevian that divides the opposite side into two congruent (equal) lengths.
- An **altitude** is a cevian that is perpendicular to the opposite side.
- An **angle bisector** is a cevian that bisects the corresponding angle.

Ceva's theorem



- Let ABC be a triangle and D, E, F be points on lines BC, CA and AB respectively.
- Then the cevians AD, BE, CF are concurrent if and only if $\frac{BD}{DC} \times \frac{CE}{EA} \times \frac{AF}{FB} = 1$ where the lengths are directed.
- The reciprocal also true in this case.
- The cevians do not necessarily lie within the triangle.

Manelaus theorem



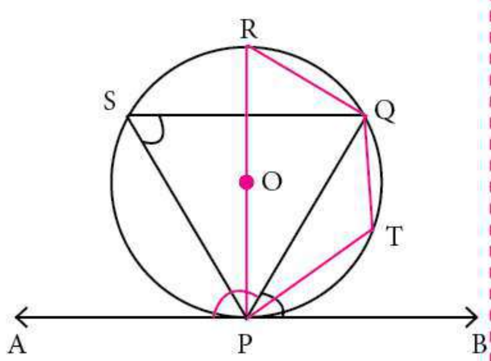
- A necessary and sufficient condition for points P, Q, R on the respective sides BC, CA, AB (or their extension) of a triangle ABC to be collinear is that $\frac{BP}{PC} \times \frac{CQ}{QA} \times \frac{AR}{RB} = -1$, where all the segments are directed.
- It can also be given as BP × CQ × AR = – PC × QA × RB.
- **Centroid** is the point of concurrence of the **medians.**

Theorem 6: Alternate segment theorem

Statement

"If a line touches a circle and from the point of contact a chord is drawn, the angles between the tangent and the chord are respectively equal to the angles in the corresponding alternate segments."

Proof



Given : A circle with centre at O, tangent AB touches the circle at P and PQ is a chord. S and T are two points on the circle in the opposite sides of chord PQ.

To prove : (i) $\angle QPB = \angle PSQ$ and (ii) $\angle QPA = \angle PTQ$

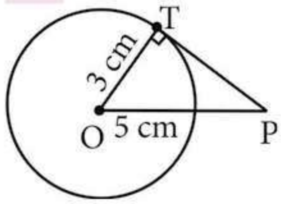
Construction : Draw the diameter POR. Draw QR, QS and PS.

Sl. No.	Statement	Reason
1	$\angle RPB = 90^o$ Now, $\angle RPQ + \angle QPB = 90^o$ ...(1)	Diameter RP is perpendicular to tangent AB.
2	In ΔRPQ, $\angle PQR = 90^o$ ...(2)	Angle in a semicircle is 90°.
3	$\angle QRP + \angle RPQ = 90^o$ ...(3)	In a right angled triangle, sum of the two acute angles is 90°.
4	$\angle RPQ + \angle QPB = \angle QRP + \angle RPQ$ $\angle QPB = \angle QRP$ ...(4)	From (1) and (3).
5	$\angle QRP = \angle PSQ$ ... (5)	Angles in the same segment are equal.
6	$\angle QPB = \angle PSQ$ ...(6)	From (4) and (5); Hence (i) is proved.
7	$\angle QPB + \angle QPA = 180^o$ ...(7)	Linear pair.
8	$\angle PSQ + \angle PTQ = 180^o$ ... (8)	Sum of opposite angles of a cyclic quadrilateral is 180°.
9	$\angle QPB + \angle QPA = \angle PSQ + \angle PTQ$	From (7) and (8).
10	$\angle QPB + \angle QPA = \angle QPB + \angle PTQ$	$\angle QPB = \angle PSQ$ from (6)
11	$\angle QPA = \angle PTQ$	Hence (ii) is proved.

Worked Examples

4.24 **Find the length of the tangent drawn from a point whose distance from the centre of a circle is 5 cm and radius of the circle is 3 cm.**

Sol :



Given OP = 5 cm, radius $r = 3$ cm

To find tangent PT = ?

In right angled ΔOTP, $\angle T = 90^o$

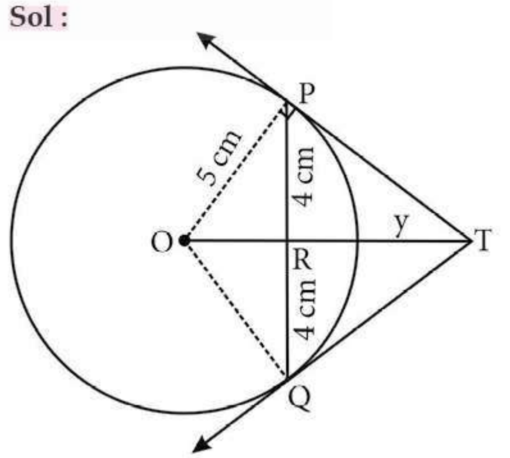
$$OP^2 = OT^2 + PT^2 \text{ (by Pythagoras theorem)}$$

$$5^2 = 3^2 + PT^2 \Rightarrow PT^2 = 25 - 9 = 16$$

Length of the tangent PT = 4 cm

4.25 **PQ is a chord of length 8 cm to a circle of radius 5 cm. The tangents at P and Q intersect at a point T. Find the length of the tangent TP.**

Sol :



Let TR = y. Since, OT is perpendicular bisector of PQ.

$$PR = QR = 4 \text{ cm}$$

In ΔORP, $OP^2 = OR^2 + PR^2$

$$OR^2 = OP^2 - PR^2$$

$$OR^2 = 5^2 - 4^2 = 25 - 16 = 9$$

$\Rightarrow$ OR = 3 cm

$$OT = OR + RT = 3 + y \quad \text{... (1)}$$

In ΔPRT, $TP^2 = TR^2 + PR^2$ ... (2)

and ΔOPT we have,

$$OT^2 = TP^2 + OP^2$$

$$OT^2 = (TR^2 + PR^2) + OP^2$$

(substitute for TP^2 from (2))

$$(3 + y)^2 = y^2 + 4^2 + 5^2$$

(substitute for OT from (1))

$$9 + 6y + y^2 = y^2 + 16 + 25$$

Therefore $y = TR = \frac{16}{3}$

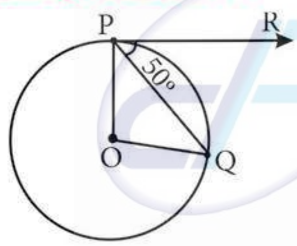
$$6y = 41 - 9 \Rightarrow y = \frac{16}{3}$$

From (2), $TP^2 = TR^2 + PR^2$

$$TP^2 = \left(\frac{16}{3}\right)^2 + 4^2 = \frac{256}{9} + 16 = \frac{400}{9}$$

$$\Rightarrow TP = \frac{20}{3} \text{ cm}$$

4..26 **In Figure, O is the centre of a circle. PQ is a chord and the tangent PR at P makes an angle of 50° with PQ. Find $\angle POQ$.**



Sol :

$\angle OPQ = 90° - 50° = 40°$

(angle between the radius and tangent is 90°)

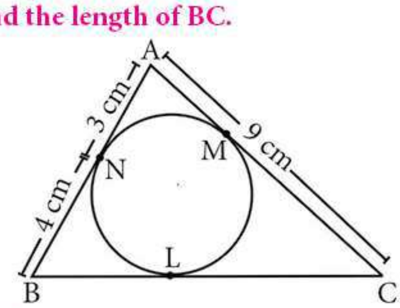
OP = OQ (Radii of a circle are equal)

$\angle OPQ = \angle OQP = 40°$ (ΔOPQ is isosceles)

$$\angle POQ = 180° - \angle OPQ - \angle OQP$$

$$\angle POQ = 180° - 40° - 40° = 100°$$

4.27 **In Figure, ΔABC is circumscribing a circle. Find the length of BC.**



Sol :

AN = AM = 3 cm (Tangents drawn from same external point are equal)

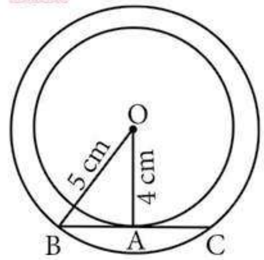
BN = BL = 4 cm;

CL = CM = AC − AM = 9 − 3 = 6 cm

$\Rightarrow$ BC = BL + CL = 4 + 6 = 10 cm

4.28 **If radii of two concentric circles are 4 cm and 5 cm then find the length of the chord of one circle which tangent to the other circle.**

Sol :



OA = 4 cm, OB = 5 cm; also $OA \perp BC$.

$$OB^2 = OA^2 + AB^2$$

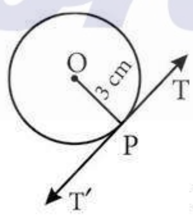
$$5^2 = 4^2 + AB^2 \Rightarrow AB^2 = 9$$

Therefore AB = 3 cm

$$BC = 2AB \Rightarrow BC = 2 \times 3 = 6 \text{ cm}$$

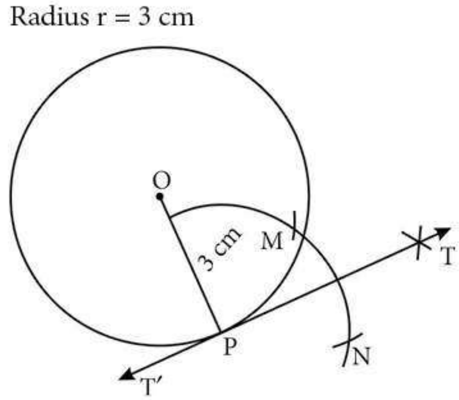
4.29 **Draw a circle of radius 3 cm. Take a point P on this circle and draw a tangent at P.**

Sol :



Rough diagram

Radius r = 3 cm



Construction:

Step 1: Draw a circle with centre at O of radius 3 cm.

Step 2: Take a point P on the circle. Join OP.

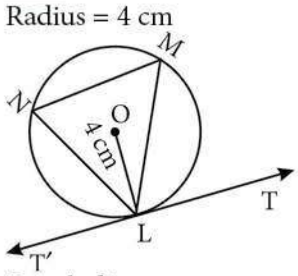
Step 3: Draw perpendicular line TT' to OP which passes through P.

Step 4: TT' is the required tangent.

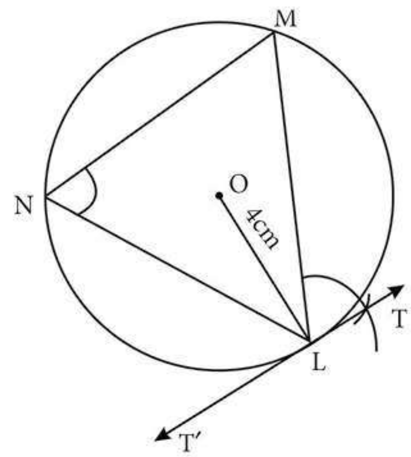
4.30 **Draw a circle of radius 4 cm. At a point L on it draw a tangent to the circle using the alternate segment.**

Sol :

Radius = 4 cm



Rough diagram



Construction:

Step 1: With O as the centre, draw a circle of radius 4 cm.

Step 2: Take a point L on the circle. Through L draw any chord LM.

Step 3: Take a point M distinct from L and N on the circle, so that L, M and N are in anti-clockwise direction. Join LN and NM.

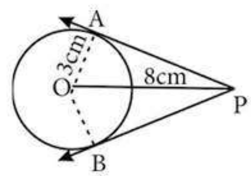
Step 4: Through L draw a tangent TT' such that $\angle TLM = \angle MNL$.

Step 5: TT' is the required tangent.

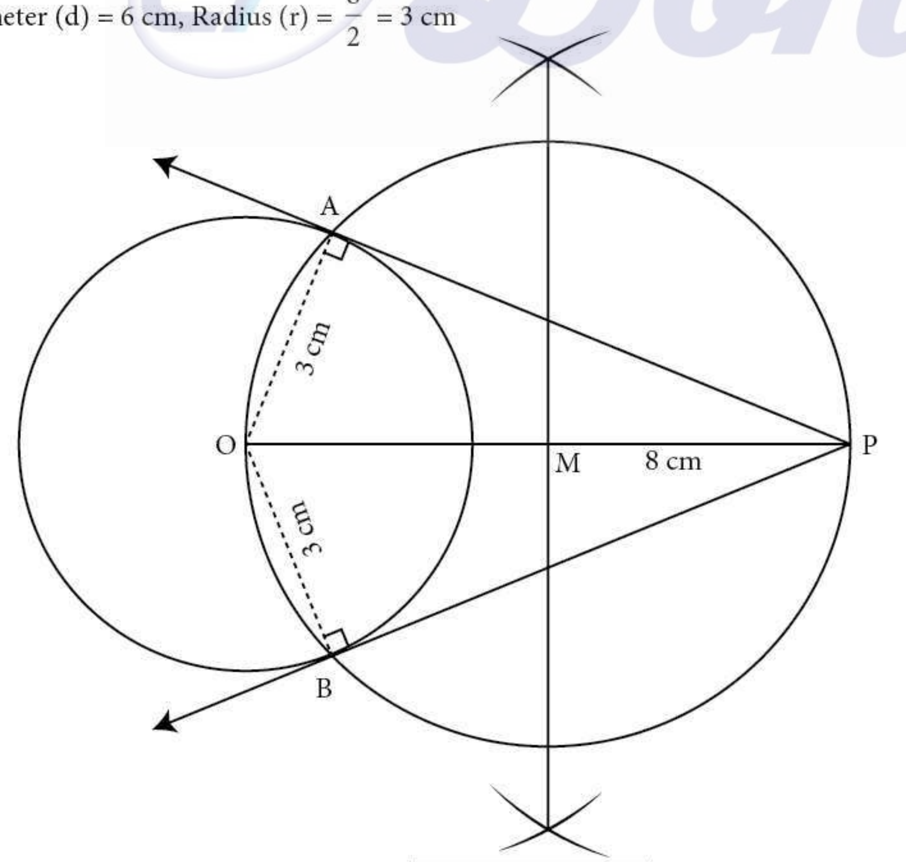
4.31 **Draw a circle of diameter 6 cm from a point P, which is 8 cm away from its centre. Draw the two tangents PA and PB to the circle and measure their lengths.**

Sol :

Diameter (d) = 6 cm, Radius (r) = $\frac{6}{2}$ = 3 cm



Rough diagram



PA = PB = 7.4 cm

Construction:

Step 1: With centre at O, draw a circle of radius 3 cm .

Step 2: Draw a line OP = 8 cm.

Step 3: Draw a perpendicular bisector of OP, which cuts OP at M.

Step 4: With M as centre and MO as radius, draw a circle which cuts the previous circle at A and B.

Step 5: Join AP and BP. AP and BP are the required tangents. Thus length of the tangents are PA = PB = 7.4 cm.

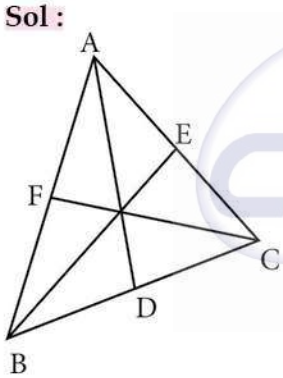
Verification: In the right angle triangle OAP,

$PA^2 = OP^2 - OA^2 = 64 - 9 = 55$

$PA = \sqrt{55} = 7.4$ cm (approximately).

4.32 Show that in a triangle, the medians are concurrent.

Sol :



Medians are line segments joining each vertex to the midpoint of the corresponding opposite sides.

Thus medians are the cevians where D, E, F are midpoints of BC, CA and AB respectively.

Since D is a midpoint of BC,

$$BD = DC \Rightarrow \frac{BD}{DC} = 1 \quad \text{... (1)}$$

Since, E is a midpoint of CA, CE = EA

$$\Rightarrow \frac{CE}{EA} = 1 \quad \text{... (2)}$$

Since, F is a midpoint of AB,

$$AF = FB \Rightarrow \frac{AF}{FB} = 1 \quad \text{... (3)}$$

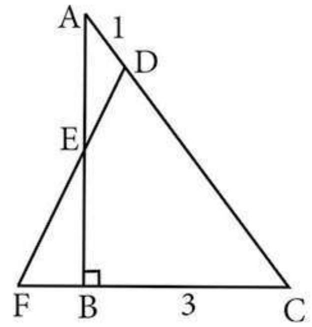
Thus, multiplying (1), (2) and (3) we get,

$$\frac{BD}{DC} \times \frac{CE}{EA} \times \frac{AF}{FB} = 1 \times 1 \times 1 = 1$$

And so, Ceva's theorem is satisfied.

Hence the Medians are concurrent.

4.33 In Figure, ABC is a triangle with $\angle B = 90^o$, BC = 3 cm and AB = 4 cm. D is point on AC such that AD = 1 cm and E is the midpoint of AB. Join D and E and extend DE to meet CB at F. Find BF.



Sol :

Consider ΔABC . Then D, E and F are respective points on the sides CA, AB and BC. By construction D, E, F are collinear.

By Menelaus' theorem

$$\frac{AE}{EB} \times \frac{BF}{FC} \times \frac{CD}{DA} = 1 \quad \text{... (1)}$$

By assumption, AE = EB = 2, DA = 1 and

FC = FB + BC

= BF + 3

By Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$= 16 + 9 = 25.$

Therefore AC = 5

and so, CD = AC – AD = 5 – 1 = 4.

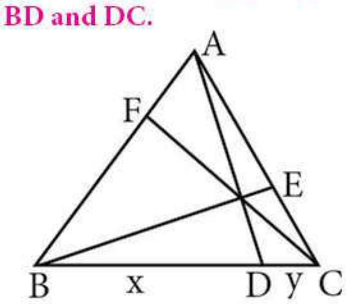
Substituting the values of FC, AE, EB, DA, CD in (1),

we get, $\frac{2}{5} \times \frac{BF}{BF+3} \times \frac{4}{1} = 1$

4BF = BF + 3

4BF – BF = 3 $\Rightarrow$ BF = 1

4.34 Suppose AB, AC and BC have lengths 13, 14 and 15 respectively. If $\frac{AF}{FB} = \frac{2}{5}$ and $\frac{CE}{EA} = \frac{5}{8}$. Find BD and DC.



Sol :

Given that AB = 13, AC = 14 and BC = 15.

Let BD = x and DC = y

Using Ceva's theorem,

we have, $\frac{BD}{DC} \times \frac{CE}{EA} \times \frac{AF}{FB} = 1$ ... (1)

Substitute the value of $\frac{AF}{FB}$ and $\frac{CE}{EA}$ in (1),

we have $\frac{BD}{DC} \times \frac{5}{8} \times \frac{2}{5} = 1$

$\frac{x}{y} \times \frac{10}{40} = 1 \Rightarrow \frac{x}{y} \times \frac{1}{4} = 1$

$\Rightarrow \quad x = 4y$ ... (2)

$BC = 15 \Rightarrow BD + DC = 15$

$\Rightarrow \quad x + y = 15$ ... (3)

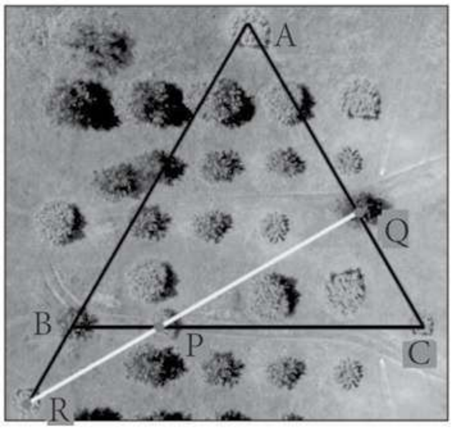
From (2), using x = 4y in (3) we get,

$4y + y = 15 \Rightarrow 5y = 15 \Rightarrow y = 3$

Substitute y = 3 in (3) we get, x = 12.

Hence BD = 12, DC = 3.

4.35 **In a garden containing several trees, three particular trees P, Q, R at located in the following way, BP = 2 m, CQ = 3 m, RA = 10 m, PC = 6 m, QA = 5m, RB = 2 m, where A, B, C are points such that P lies on BC, Q lies on AC and R lies on AB. Check whether the trees P, Q, R lie on a same straight line.**



Sol :

By Menelaus' theorem, the trees P, Q, R will be collinear (lie on same straight line)

if $\frac{BP}{PC} \times \frac{CQ}{QA} \times \frac{RA}{RB} = 1$ ... (1)

Given BP = 2 m, CQ = 3 m, RA = 10 m, PC = 6m, QA = 5 m and RB = 2 m

Substituting these values in (1) we get,

$$\frac{BP}{PC} \times \frac{CQ}{QA} \times \frac{RA}{RB} = \frac{2}{6} \times \frac{3}{5} \times \frac{10}{2} = \frac{60}{60} = 1$$

Hence the trees P, Q, R lie on a same straight line.

Progress Check

1. A straight line that touches a circle at a common point is called a _____

Ans : tangent

2. A chord is a subsection of _____

Ans : a secant

3. The lengths of the two tangents drawn from _____ point to a circle are equal.

Ans : exterior

4. No tangent can be drawn from _____ of the circle.

Ans : an interior point

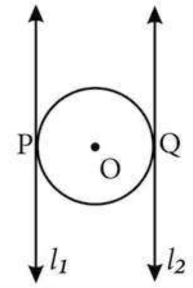
5. An _____ is a cevian that divides the angle, into two equal halves.

Ans : Angle Bisector

Thinking Corner

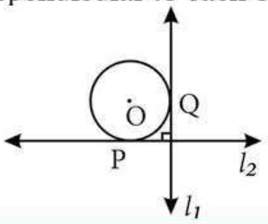
1. Can we draw two tangents parallel to each other on a circle?

Ans : Yes. We can draw two tangents parallel to each other on a circle in two different points on the circle.



2. Can we draw two tangents perpendicular to each other on a circle?

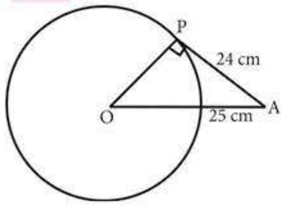
Ans : Yes. We can draw two tangents perpendicular to each other.



Exercise 4. 4

1. The length of the tangent to a circle from a point P, which is 25 cm away from the centre is 24 cm. What is the radius of the circle?

Sol :



Let 'O' be the center of the circle AP be the tangent.

Given OA = 25 cm; AP = 24 cm. OP is the radius.

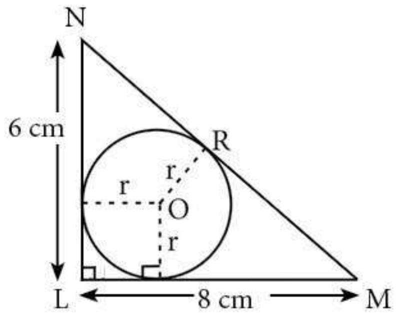
Tangent and radius through the point are perpendicular to each other.

$\therefore$ In the right triangle ΔOPA,

$$OA^2 = OP^2 + PA^2$$
$$25^2 = OP^2 + 24^2$$
$$625 = OP^2 + 576$$
$$OP^2 = 625 - 576 = 49$$
$$OP = 7 \text{ cm}$$
$$\therefore \text{Radius} = 7 \text{ cm}$$

2. ΔLMN is a right angled triangle with $\angle L = 90^\circ$. A circle is inscribed in it. The lengths of the sides containing the right angle are 6 cm and 8 cm. Find the radius of the circle.

Sol :



Given ΔLMN is a right angled triangle with $\angle L = 90^\circ$

Since PL $\parallel$ OQ, PL = r = OP

we have NR = NP = NL – PL [$\because$ NR and NP are tangents]

= (6 – r) cm

MR = MQ = ML – LQ = (8 – r) cm

[$\because$ MR and MQ are tangents]

NM = NR + RM

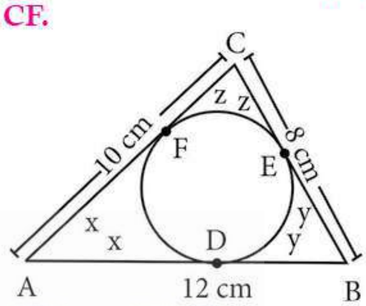
= (6 – r + 8 – r) cm

= (14 – 2r) cm

Now $NM^2 = NL^2 + LM^2$ [By Pythagoras Theorem]

$$\Rightarrow (14 - 2r)^2 = 8^2 + 6^2$$
$$(14 - 2r)^2 = 64 + 36$$
$$(14 - 2r)^2 = 100$$
$$14 - 2r = 10$$
$$-2r = 10 - 14$$
$$-2r = -4$$
$$r = 2$$
$$\therefore \text{Radius} = 2 \text{ cm}$$

3. A circle is inscribed in ΔABC having sides 8 cm, 10 cm and 12 cm as shown in figure, Find AD, BE and CF.



Sol :

We know that the tangents drawn from an external point to a circle are equal.

Let AD = AF = x

BD = BE = y

and CE = CF = z

Now AB = 12 cm, BC = 8 cm, and CA = 10 cm

x + y = 12; y + z = 8, and z + x = 10

(x + y) + (y + z) + (z + x) = 12 + 8 + 10

2(x + y + z) = 30

$\Rightarrow$ x + y + z = 15

Now x + y = 12 and x + y + z = 15

$\Rightarrow$ 12 + z = 15

z = 3

y + z = 8 and x + y + z = 15

$\Rightarrow$ x + 8 = 15 $\Rightarrow$ x = 7

and z + x = 10 and x + y + z = 15

$\Rightarrow$ y + 10 = 15

y = 5

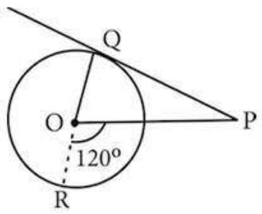
Hence AD = x = 7 cm

BE = y = 5 cm

CF = z = 3 cm

4. PQ is a tangent drawn from a point P to a circle with centre O and QOR is a diameter of the circle such that $\angle POR = 120^\circ$. Find $\angle OPQ$.

Sol :



Given PQ is the tangent from the point P outside the circle and OQ is the radius.

We know that tangent meet the radius perpendicularly

$$\therefore \angle PQO = 90^\circ$$
$$\angle POQ = 180 - 120 = 60^\circ$$

[$\because \angle POQ$ and $\angle POR$ are linear pair of angles]

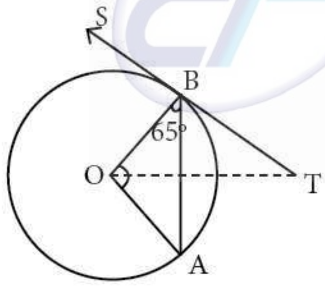
In ΔPQO $\angle POQ + \angle PQO + \angle OPQ = 180^\circ$

[$\because$ sum of angles of a triangle]

$$60^\circ + 90^\circ + \angle OPQ = 180^\circ$$
$$\angle OPQ = 180^\circ - 150^\circ$$
$$\angle OPQ = 30^\circ$$

5. A tangent ST to a circle touches it at B. AB is a chord such that $\angle ABT = 65^\circ$. Find $\angle AOB$, where "O" is the centre of the circle.

Sol :



Given TB is the tangent from the external point T and OB-radius

$$\therefore \angle OBT = 90^\circ$$

Also $\angle ABT = 65^\circ$ (given)

$$\therefore \angle OBA = \angle OBT - \angle ABT$$
$$= 90^\circ - 65^\circ = 25^\circ$$

Since OA = OB [radius]

ΔABO is an isosceles triangle.

$\therefore$ Angles opposite to equal sides are equal.

$$\therefore \angle OBA = \angle BAO = 25^\circ$$

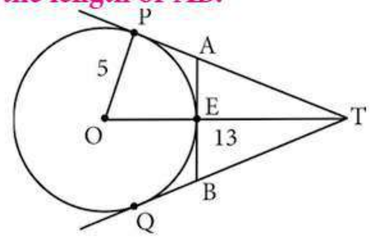
Now in ΔAOB,

$$\angle AOB + \angle OBA + \angle OAB = 180^\circ$$
$$\angle AOB + 25^\circ + 25^\circ = 180^\circ$$

[sum of angles of a triangle]

$$\angle AOB + 50^\circ = 180^\circ$$
$$\angle AOB = 180^\circ - 50^\circ = 130^\circ$$
$$\therefore \angle AOB = 130^\circ$$

6. In figure, O is the centre of radius 5 cm. T is a point such that OT = 13 cm and OT intersects the circle E, if AB is the tangent to the circle at E, find the length of AB.



Sol :

Since OP is the radius and PT tangent.

$$\angle OPT = 90^\circ$$

Applying Pythagoras theorem in ΔOPT, we have

$$OT^2 = OP^2 + PT^2$$
$$13^2 = 5^2 + PT^2$$
$$PT^2 = 169 - 25$$
$$PT^2 = 144$$
$$PT = 12 \text{ cm}$$

Since the lengths of tangents drawn from an exterior point to a circle are equal.

$$\therefore \quad AP = AE = x \text{ (say)}$$
$$\Rightarrow \quad AT = PT - AP = (12 - x) \text{ cm}$$

Since AB is the tangent to the circle at E

$\therefore$ OE $\perp$ AB

$$\Rightarrow \angle OEA = 90^\circ$$
$$\Rightarrow \angle AET = 90^\circ$$
$$AT^2 = AE^2 + ET^2$$

[Applying pythagoras theorem in ΔAET]

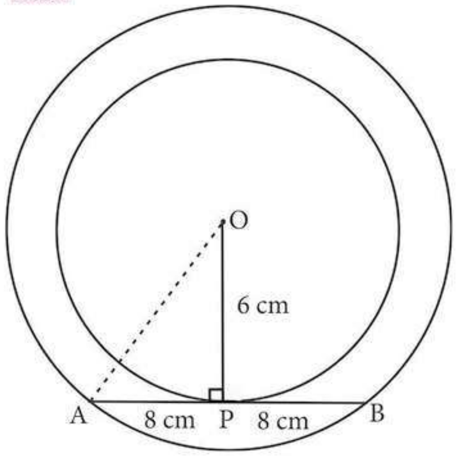
$$(12 - x)^2 = x^2 + (13 - 5)^2$$
$$144 - 24x + x^2 = x^2 + 64$$
$$24x = 144 - 64$$
$$24x = 80$$
$$\Rightarrow 3x = 10$$
$$x = \frac{10}{3} \text{ cm}$$

Similarly $BE = \frac{10}{3}$ cm

$$\therefore \quad AB = AE + BE$$
$$= \left(\frac{10}{3} + \frac{10}{3}\right) \text{ cm}$$
$$AB = \frac{20}{3} \text{ cm}$$

7. In two concentric circles, a chord of length 16 cm of larger circle becomes a tangent to the smaller circle whose radius is 6 cm. Find the radius of the larger circle.

Sol :



Let O be the center of concentric circles and APB be the chord of length 16 cm of the larger circle touching the smaller circle at P.

Then OP ⊥ AB and P is the midpoint of AB.

$\therefore$ AP = PB = 8 cm

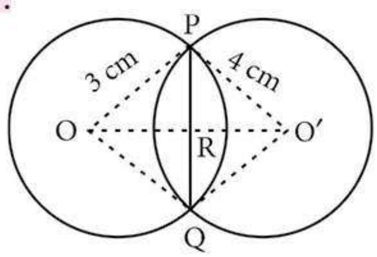
In ΔOPA, we have

$$OA^2 = OP^2 + AP^2 \text{ [By Pythagoras Theorem]}$$
$$OA^2 = 6^2 + 8^2$$
$$OA^2 = 36 + 64$$
$$OA^2 = 100$$
$$OA = 10 \text{ cm}$$

$\therefore$ Radius of the larger circle in 10 cm

8. Two circles with centres O and O' of radii 3 cm and 4 cm, respectively intersect at two points P and Q, such that OP and $O'P$ are tangents to the two circles. Find the length of the common chord PQ.

Sol :



Since the tangents at a point to a circle is perpendicular to the radius through the point of contact

$\therefore \angle OPO' = 90°$

In $\Delta OPO'$, we have

$$OP^2 + O'P^2 = (OO')^2$$
[By Pythagoras theorem]
$$3^2 + 4^2 = (OO')^2$$
$$9 + 16 = (OO')^2$$
$$25 = (OO')^2$$
$$OO' = 5 \text{ cm}$$

Since the line joining the centres of two intersecting circles is perpendicular bisector of their common chord.

OR ⊥ PQ and $O'R$ ⊥ PQ

Also PR = QR

Let OR = x, then $O'R = 5 - x$

Also at PR = QR = y cm

In ΔORP and $\Delta O'RP$,

Applying pythagoras theorem

$OP^2 = OR^2 + RP^2$ and $O'P^2 = O'R^2 + RP^2$

$$3^2 = x^2 + y^2 \text{ and } 4^2 = (5 - x)^2 + y^2$$

Subtracting $\Rightarrow 4^2 - 3^2 = \{(5 - x)^2 + y^2\} - (x^2 + y^2)$

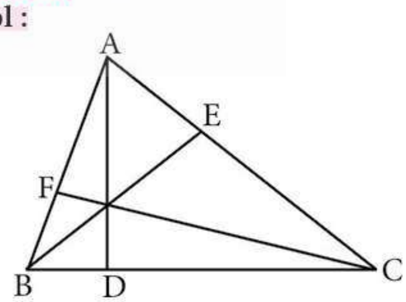
$$16 - 9 = 25 - 10x + x^2 + y^2 - x^2 - y^2$$
$$7 = 25 - 10x$$
$$10x = 25 - 7$$
$$10x = 18$$
$$\Rightarrow x = 1.8 \text{ cm}$$
$$3^2 = x^2 + y^2$$
$$\Rightarrow y = \sqrt{9 - (1.8)^2} = \sqrt{5.76}$$
$$y = 2.4 \text{ cm}$$

Hence PR = QR = 2.4 cm

PQ = 2y = 4.8 cm.

9. Show that the angle bisectors of a triangle are concurrent.

Sol :



Let ΔABC be B triangle points D, E, F are angular bisectors of $\angle A$, $\angle B$ and $\angle C$ respectively. By angular bisector theorem we have

$$\frac{BD}{DC} = \frac{AB}{AC} \Rightarrow AB = \frac{BD \times AC}{DC} \quad \text{... (1)}$$

$$\frac{AC}{BC} = \frac{AF}{FB} \Rightarrow AC = \frac{AF \times BC}{FB} \quad \text{... (2)}$$

$$\frac{AE}{EC} = \frac{AB}{BC} \Rightarrow AB = \frac{AE \times BC}{EC} \quad \text{... (3)}$$

From (1) and (3), we have

$$\frac{BD \times AC}{DC} = \frac{AE \times BC}{EC} \quad \text{... (4)}$$

Now substituting (2) in (4) we have

$$\frac{BD \times \left(\frac{AF \times BC}{FB}\right)}{DC} = \frac{AE \times BC}{EC}$$

$$\frac{BD \times AF \times BC}{DC \times FB} = \frac{AE \times BC}{EC}$$

$$BD \times AF \times EC = \frac{AE \times BC \times DC \times FB}{BC}$$

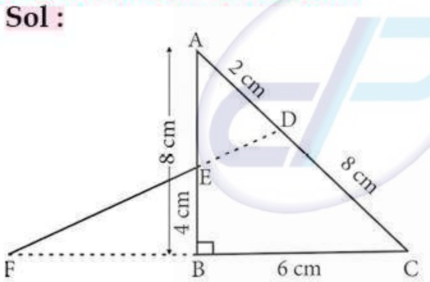
$$BD \times AF \times CE = EA \times FB \times DC$$

$$\therefore \frac{BD \times AF \times CE}{EA \times FB \times DC} = 1$$

Hence by Ceva's theorem we conclude that the angle bisectors of a triangle are concurrent.

10. In ΔABC, with $\angle B = 90°$, BC = 6 cm and AB = 8 cm, D is a point on AC such that AD = 2 cm and E is the midpoint of AB. Join D to E and extend it to meet at F. Find BF.

Sol :



Given AB = 8 cm, AD = 2 cm;

BC = 6 cm; AE = $\frac{AB}{2} = \frac{8}{2} = 4$ cm

We have ΔABC is a right triangle

$$AC^2 = AB^2 + BC^2 = 8^2 + 6^2$$
$$= 64 + 36 = 100$$
$$AC^2 = 100$$
$$AC = 10 \text{ cm}$$
$$DC = AC - AD = 10 - 2 = 8 \text{ cm}$$

In ΔABC, D, E, F are respective points on the sides CA, AB and BC.

By construction D, E, F are colinear

$\therefore$ By Menelaus' theorem $\frac{AE}{EB} \times \frac{BF}{FC} \times \frac{CD}{DA} = 1$

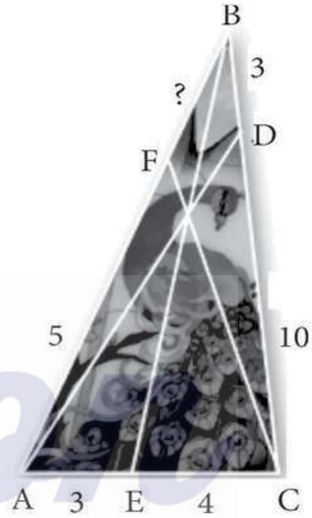
$$\frac{4}{4} \times \frac{BF}{(FB + BC)} \times \frac{8}{2} = 1$$

$$\Rightarrow 1 \times \frac{BF}{FB + 6} \times 4 = 1$$

$$4BF = FB + 6$$

$$4FB - FB = 6$$
$$3BF = 6$$
$$BF = \frac{6}{3}$$
$$BF = 2 \text{ cm}$$

11. An artist has created a triangular stained glass window and has one strip of small length left before completing the window. She needs to figure out the length of left out portion based on the lengths of the other sides as shown in the figure.



Sol :

Clearly In ΔABC, D, E, F are points on lines BC, CA, AB respectively using Ceva's theorem, we have

$$\frac{AE}{EC} \times \frac{CD}{DB} \times \frac{BF}{FA} = 1 \qquad \ldots (1)$$

From the diagram it is clear that

AE = 3, EC = 4, CD = 10, DB = 3, FA = 5

Substituting these values in (1)

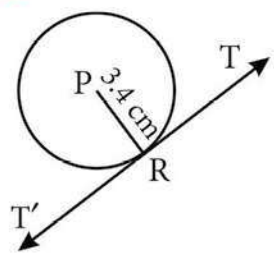
$$\frac{3}{4} \times \frac{10}{3} \times \frac{BF}{5} = 1$$

$$BF = \frac{1 \times 4 \times 3 \times 5}{3 \times 10} = 2 \text{ cm}$$

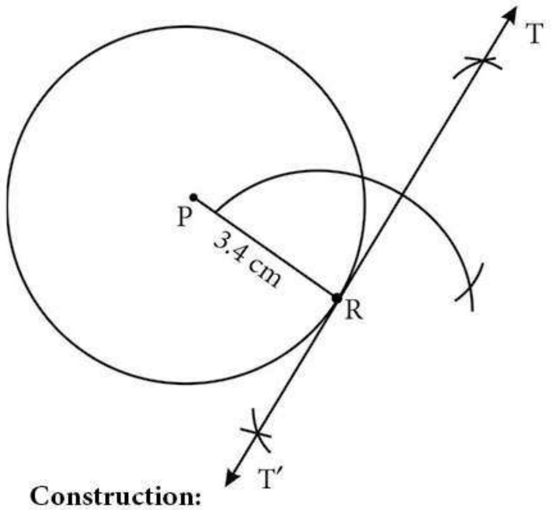
12. Draw a tangent at any point R on the circle of radius 3.4 cm and centre at P.

Sol :

Radius = 3.4 cm



Rough diagram



Construction:

Step 1: Drawn a circle with center at P of radius 3.4 cm

Step 2: Taken a point R on the circle joined PR

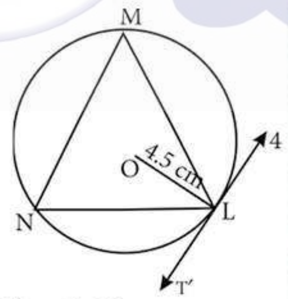
Step 3: Drawn perpendicular line TT' to PR which passes through R.

Step 4: TT' is the required tangent

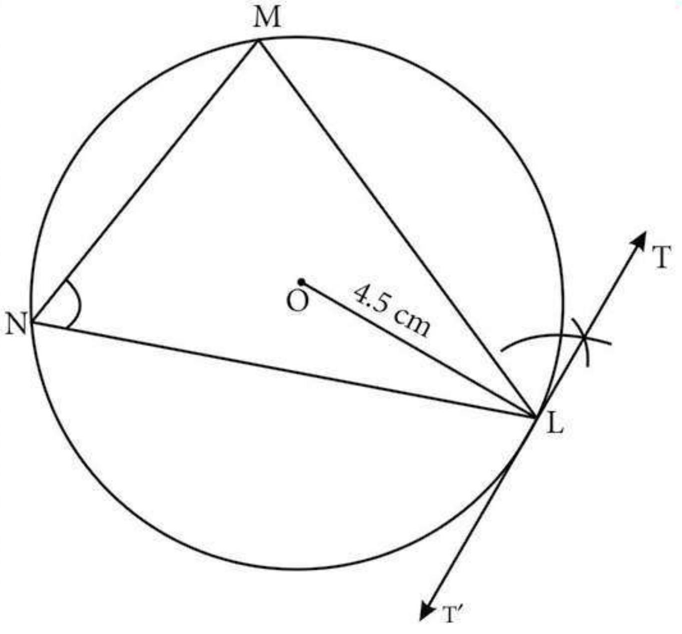
13. Draw a circle of radius 4.5 cm. Take a point on the circle. Draw the tangent at that point using the alternate-segment theorem.

Sol :

Radius = 4.5 cm



Rough diagram



Construction:

Step 1: With O as centre, drawn a circle of radius 4.5 cm

Step 2: Taken a point L on the circle through L drawn as chord LM

Step 3: Taken a point M distinct from L and N on the circle so that L, M, N are anti-clock wise direction. Joined LN and NM

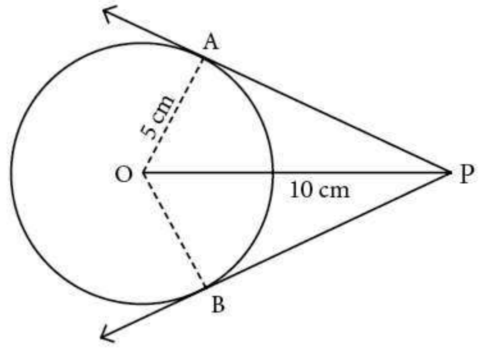
Step 4: Through 'L' drawn a tangent TT' such that $\angle TLM = \angle MNL$

Step 5: TT' is the required tangent.

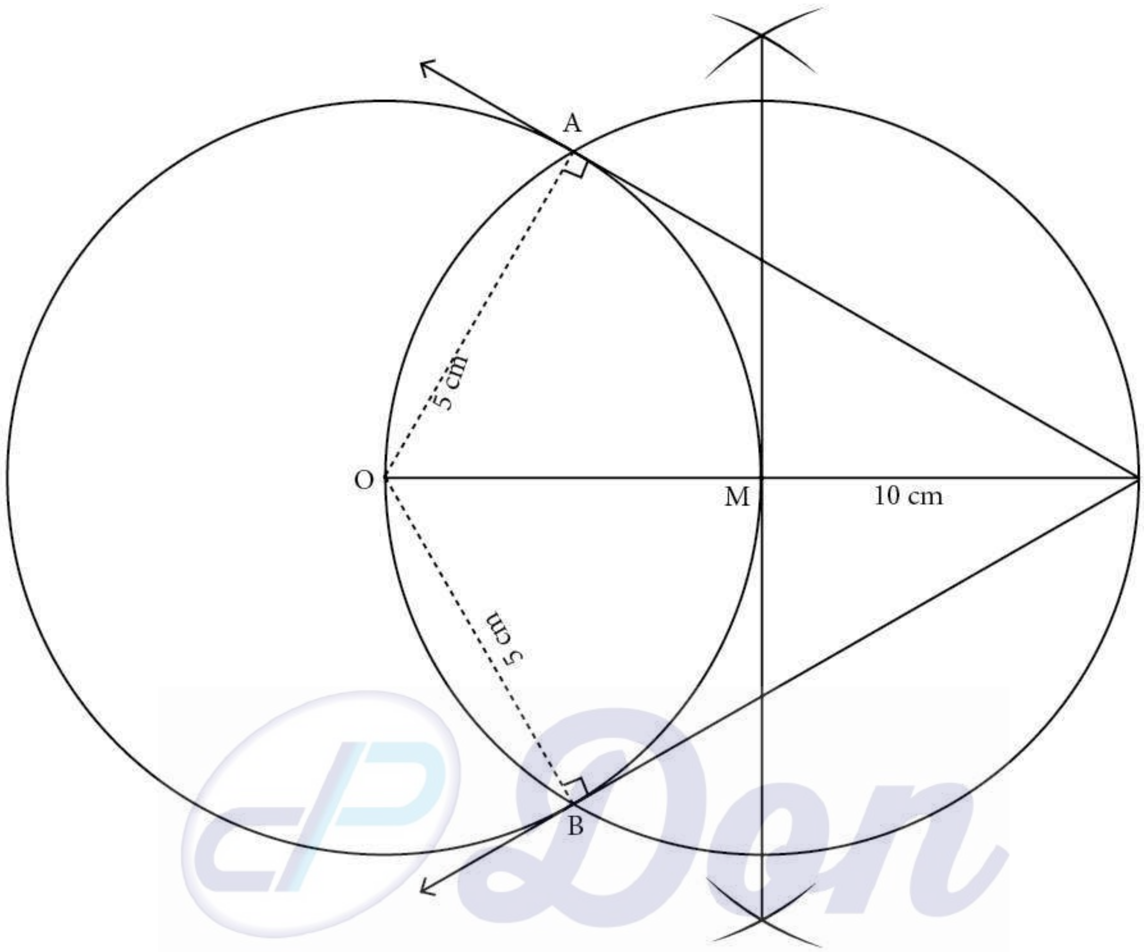
14. Draw the two tangents from a point which is 10 cm away from the centre of a circle of radius 5 cm. Also, measure the lengths of the tangents.

Sol :

Given radius r = 5 cm



Rough diagram



Length of the tangents PA = PB = 8.7 cm

Construction:

Step 1: With center at 'O', drawn a circle of radius 5 cm

Step 2: Drawn a line OP = 10 cm

Step 3: Drawn a perpendicular bisector to OP, which cuts OP at M.

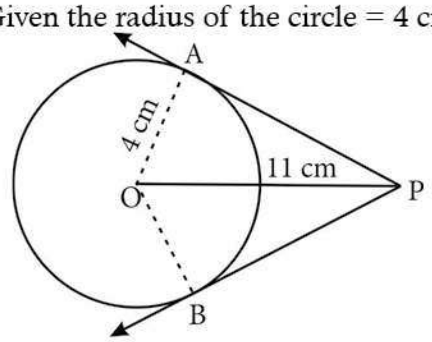
Step 4: With M as center and MO as radius, drawn a circle which cuts previous circle at A and B

Step 5: Joined AP and BP. AP and BP are the required tangents. The length of the tangents PA = PB = 8.7 cm.

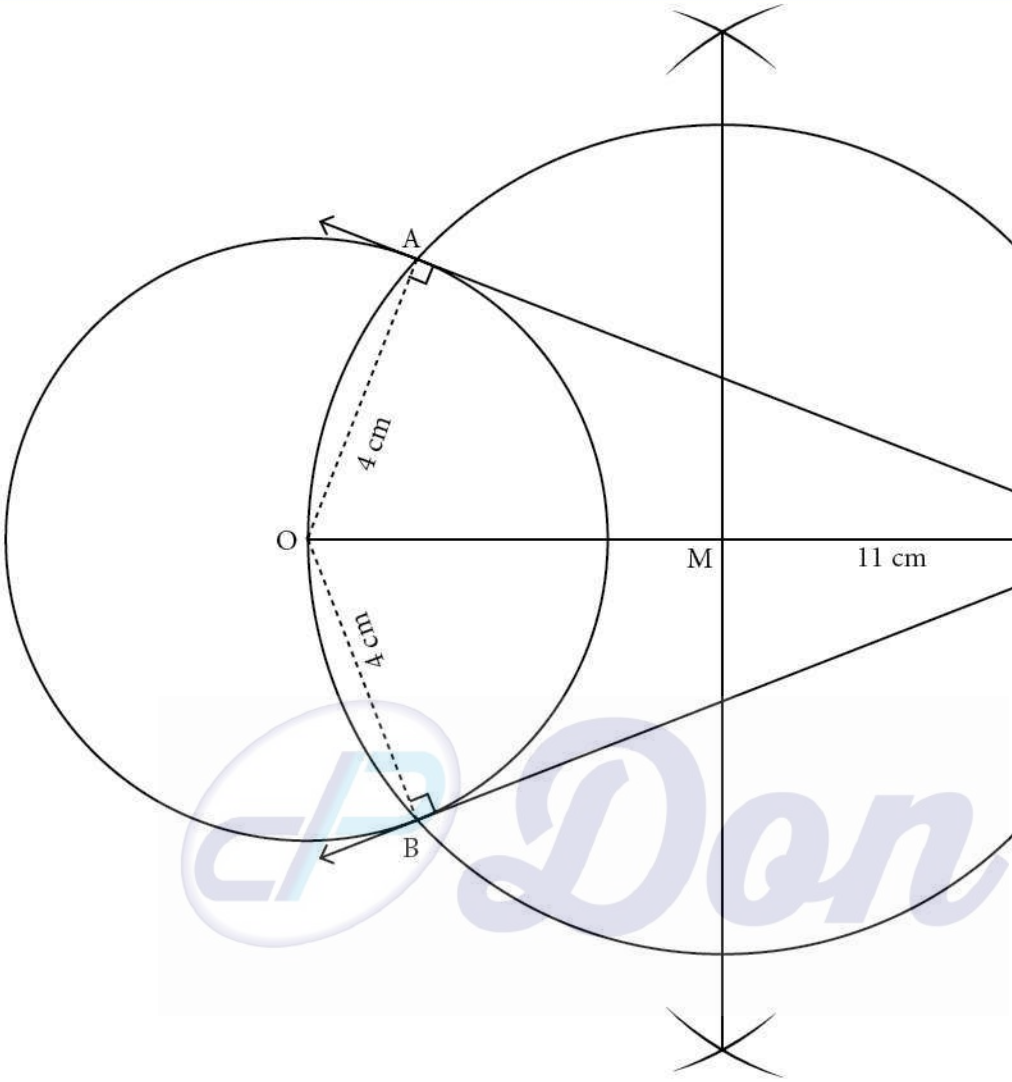
15. Take a point which is 11 cm away from the centre of a circle of radius 4 cm and draw the two tangents to the circle from that point.

Sol :

Given the radius of the circle = 4 cm



Rough diagram



Length of the tangents PA = PB = 10.2 cm

Construction:

Step 1: With center at 'O' drawn a circle of radius 4 cm

Step 2: Drawn a line OP = 11 cm

Step 3: Drawn a perpendicular bisector to OP, which cuts OP at M

Step 4: With M as center and MO as radius drawn a circle which cuts previous circle at A and B.

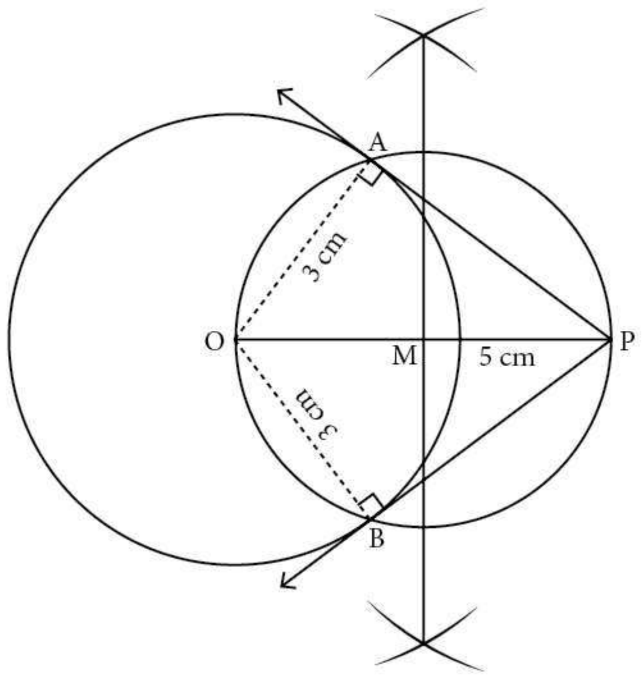
Step 5: Joined AP and BP. AP and BP are the required tangents. Thus the length of the tangents are PA = PB = 10.2 cm.

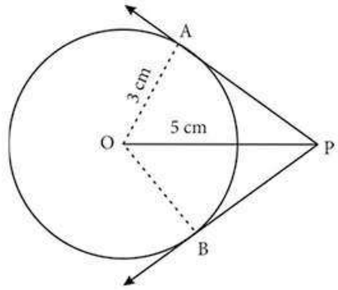
16. Draw the two tangents from a point which is 5 cm away from the centre of a circle of diameter 6 cm. Also, measure the lengths of the tangents.

Sol :

Given diameter = 6 cm

$\therefore$ radius = $\dfrac{6}{2}$ = 3 cm





Rough diagram

Length of the tangents PA = PB = 4 cm

Construction:

Step 1: With centre at O drawn a circle of radius 3 cm

Step 2: Drawn a line OP = 5 cm

Step 3: Drawn a perpendicular bisector to OP, which cuts OP at M.

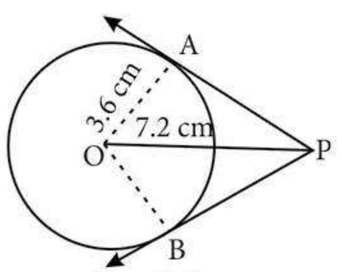
Step 4: With M as center and MO as radius drawn a circle which cuts previous circle at A and B.

Step 5: Joined AP and BP. AP and BP are the required tangents. Thus the length of the tangents are PA = PB = 4 cm

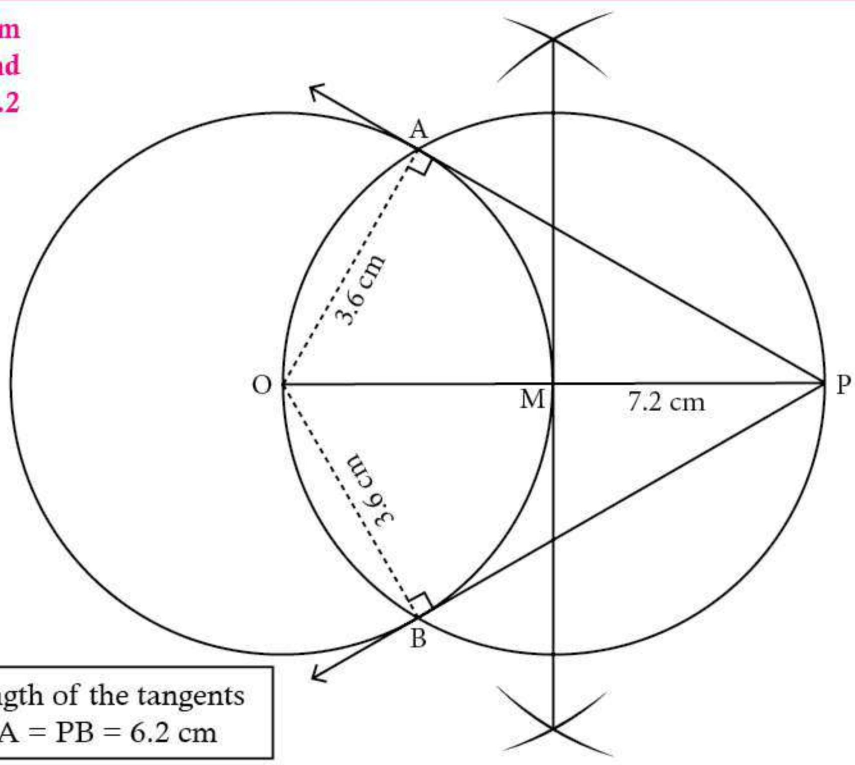
17. Draw a tangent to the circle from the point P having radius 3.6 cm and centre at O point P is at a distance 7.2 cm from the centre.

Sol :

Given radius r = 3.6 cm



Rough diagram



Length of the tangents
PA = PB = 6.2 cm

Construction:

Step 1: With centre at O, drawn a circle of radius 3.6 cm.

Step 2: Drawn a line OP = 7.2 cm.

Step 3: Drawn a perpendicular bisector of OP which cuts OP at M.

Step 4: With M as center and MO as radius, drawn a circle which cuts previous circle at A and B.

Step 5: Joined AP and BP. AP and BP are the required tangents.

Length of the tengents PA = PB = 6.2 cm

Exercise 4. 5

1. If in triangles ABC and EDF, $\frac{AB}{DE} = \frac{BC}{FD}$ then they will be similar, when

(1) $\angle B = \angle E$ (2) $\angle A = \angle D$

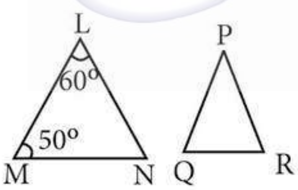
(3) $\angle B = \angle D$ (4) $\angle A = \angle F$ **[Ans: 3]**

2. In ΔLMN, $\angle L = 60^{\circ}$, $\angle M = 50^{\circ}$. If $\Delta LMN \sim \Delta PQR$ then the value of $\angle R$ is

(1) 40° (2) 70°

(3) 30° (4) 110° **[Ans: 2]**

Sol :



If $\Delta LMN \sim \Delta PQR$

$\angle R = \angle N = 180 - (60 + 50)$

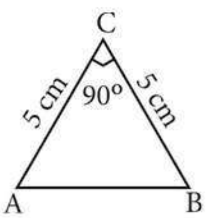
$= 180 - 110 = 70^{\circ}$

3. If ΔABC is an isosceles triangle with $\angle C = 90^{\circ}$ and AC = 5 cm, then AB is

(1) 2.5 cm (2) 5 cm

(3) 10 cm (4) $5\sqrt{2}$ cm **[Ans: 4]**

Sol :

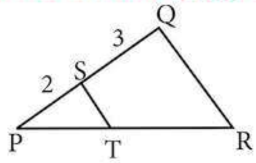


$AB^2 = AC^2 + BC^2$

$= 5^2 + 5^2 = 50$

$AB = \sqrt{50} = 5\sqrt{2}$ cm

4. In a given figure ST || QR, PS = 2 cm and SQ = 3 cm. Then the ratio of the area of ΔPQR to the area of ΔPST is



(1) 25 : 4 (2) 25 : 7

(3) 25 : 11 (4) 25 : 13 **[Ans: 1]**

Sol :

$$\left(\frac{PQ}{SP}\right)^2 = \frac{Area\ \Delta PQR}{Area\ \Delta PST}$$

$$PQ = PS + SQ = 2 + 3 = 5$$

$$\therefore \left(\frac{5}{2}\right)^2 = \frac{Area\ (\Delta PQR)}{Area\ (\Delta PST)} = \frac{25}{4} \Rightarrow 25 : 4$$

5. The perimeters of two similar triangles ΔABC and ΔPQR are 36 cm and 24 cm respectively. If PQ = 10 cm, then the length of AB is

(1) $6\frac{2}{3}$ cm (2) $\frac{10\sqrt{6}}{3}$ cm

(3) $66\frac{2}{3}$ cm (4) 15 cm **[Ans: 4]**

Sol :

If $\Delta ABC \sim \Delta PQR$

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PQ} = \frac{Perimeter\ of\ \Delta ABC}{Perimeter\ of\ \Delta PQR}$$

Given AB + BC + CA = 36 cm

PQ + QR + PR = 24 cm

$$\frac{AB}{PQ} = \frac{36}{24} \Rightarrow \frac{AB}{10} = \frac{36}{24}$$

$$AB = \frac{36 \times 10}{24}$$

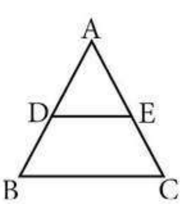
$$AB = 15 \text{ cm}$$

6. If in ΔABC, DE || BC. AB = 3.6 cm, AC = 2.4 cm and AD = 2.1 cm then the length of AE is

(1) 1.4 cm (2) 1.8 cm

(3) 1.2 cm (4) 1.05 cm **[Ans: 1]**

Sol :



Let AE = x, then EC = 2.4 – x

AB = 3.6, AD = 2.1, ∴ DB = 3.6 – 2.1 = 1.5

$$\text{Now} \quad \frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{2.1}{1.5} = \frac{x}{2.4-x}$$

$$\frac{7}{5} = \frac{x}{2.4-x}$$

$$7[2.4 - x] = 5x$$

$$16.8 = 5x + 7x = 12x$$

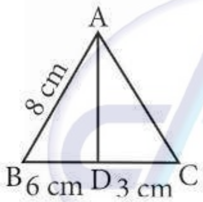
$$x = \frac{16.8}{12} = 1.4 \text{ cm}$$

7. In a ΔABC, AD is the bisector of $\angle BAC$. If AB = 8 cm, BD = 6 cm and DC = 3 cm. The length of the side AC is

(1) 6 cm (2) 4 cm

(3) 3 cm (4) 8 cm **[Ans: 2]**

Sol :

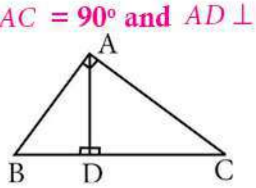


$$\frac{AB}{AC} = \frac{BD}{DC}$$

$$\frac{8}{AC} = \frac{6}{3}$$

$$AC = \frac{8}{6} \times 3 = 4 \text{ cm}$$

8. In figure $\angle BAC = 90^\circ$ and $AD \perp BC$ then



(1) $BD \cdot CD = BC^2$

(2) $AB \cdot AC = BC^2$

(3) $BD \cdot CD = AD^2$

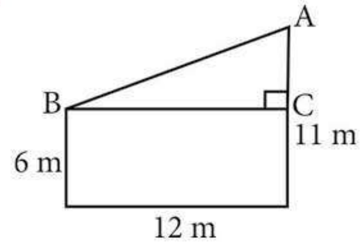
(4) $AB \cdot AC = AD^2$ **[Ans: 3]**

9. Two poles of heights 6 m and 11 m stand vertically on a plane ground. If the distance between their feet is 12 m, what is the distance between their tops?

(1) 13 cm (2) 14 m

(3) 15 m (4) 12.8 m **[Ans: 1]**

Sol :



AC = 11 – 6 = 5 cm

In right ΔABC

$$AB^2 = BC^2 + AC^2 = 12^2 + 5^2$$

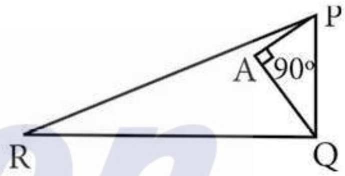
$$= 144 + 25 = 169$$

$$AB = 13 \text{ cm}$$

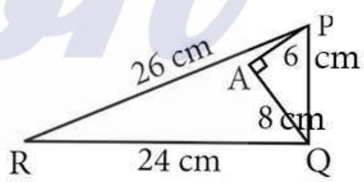
10. In the given figure PR = 26 cm, QR = 24 cm, $\angle PAQ = 90^\circ$, PA = 6 cm and QA = 8 cm. Find $\angle PQR$

(1) 80° (2) 85°

(3) 75° (4) 90° **[Ans: 4]**



Sol :



$$PQ^2 = 6^2 + 8^2 = 36 + 64 = 100$$

$$PQ = 10 \text{ cm}$$

$$\text{Now } PQ^2 + QR^2 = 10^2 + 24^2$$

$$= 100 + 576 = 676$$

$$= 26^2 = PR^2$$

$$\therefore \quad \angle PQR = 90^\circ$$

11. A tangent is perpendicular to the radius at the

(1) centre (2) point of contact

(3) infinity (4) chord **[Ans: 2]**

12. How many tangents can be drawn to the circle from an exterior point?

(1) one (2) two

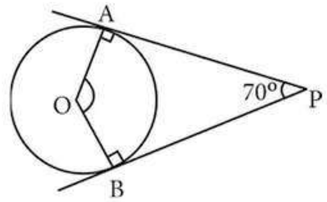
(3) infinite (4) zero **[Ans: 2]**

13. The two tangents from an external points P to a circle with centre at O are PA and PB. If $\angle APB = 70^\circ$ then the value of $\angle AOB$ is

(1) 100° (2) 110°

(3) 120° (4) 130° **[Ans: 2]**

Sol :

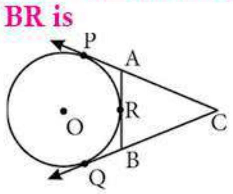


In quadrilateral APBO

$70° + 90° + 90° + \angle AOB = 360°$

$\angle AOB = 360° - 250° = 110°$

14. In figure CP and CQ are tangents to a circle with centre at O. ARB is another tangent touching the circle at R. If CP = 11 cm and BC = 7 cm, then the length of BR is



(1) 6 cm (2) 5 cm

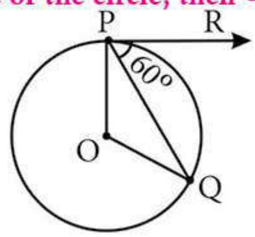
(3) 8 cm (4) 4 cm **[Ans: 4]**

Sol :

CP = 11 cm; BC = 7 cm

$\therefore$ CQ = 11 cm BQ = 11 − 7 = 4 cm = BR

15. In figure if PR is tangent to the circle at P and O is the centre of the circle, then $\angle POQ$ is



(1) 120° (2) 100°

(3) 110° (4) 90° **[Ans: 1]**

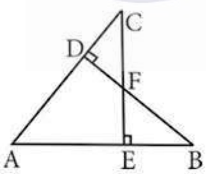
Sol :

$60 \times 2 = 120°$

UNIT EXERCISE - 4

1. In the figure, if BD $\perp$ AC and CE $\perp$ AB, prove that

(i) $\Delta AEC \sim \Delta ADB$ **(ii)** $\dfrac{CA}{AB} = \dfrac{CE}{DB}$



Sol :

(i) In ΔAEC and ΔADB

$\angle AEC = \angle ADB = 90°$

$\angle CAE = \angle BAD$

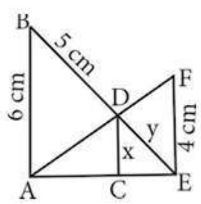
[common By AA similarity criteria]

$\Delta AEC \sim \Delta ADB$

(ii) Their corresponding sides are proportional

$\therefore \dfrac{CA}{AB} = \dfrac{CE}{DB}$ Hence proved.

2. In the given figure AB || CD || EF. If AB = 6 cm, CD = x cm, EF = 4 cm, BD = 5 cm and DE = y cm. find x and y.



Sol :

Given AB || CD || EF

AB = 6 cm, BD = 5 cm, EF = 4 cm, CD = x cm, DE = y cm

In ΔECD and ΔEAB

$\angle CED = \angle AEB$ [common]

$\angle ECD = \angle EAB$ [corresponding angles]

$\Delta ECD \sim \Delta EAB$

[By AA similarity criteria] ... (1)

$\therefore \dfrac{EC}{EA} = \dfrac{CD}{AB}$

[$\because$ Corresponding parts of similar triangles are proportional]

$\dfrac{EC}{EA} = \dfrac{x}{6}$ ... (2)

In ΔACD and ΔAEF

$\angle CAD = \angle EAF$ [Common]

$\angle ACD = \angle AEF$ [Corresponding angles]

$\Delta ACD \sim \Delta AEF$ [By AA similarity]

$\dfrac{AC}{AE} = \dfrac{CD}{EF}$

[$\therefore$ Corresponding parts of similar triangle are proportional]

$\therefore \dfrac{AC}{AE} = \dfrac{x}{4}$ ... (3)

Adding (2) and (3)

$$\frac{EC}{EA}+\frac{AC}{AE} = \frac{x}{6}+\frac{x}{4}$$

$$\frac{EC+AC}{AE} = \frac{4x+6x}{24}$$

$$\frac{AE}{AE} = \frac{10x}{24}$$

$$1 = \frac{10x}{24}$$

$$x = \frac{24}{10} = \frac{12}{5} \text{ cm}$$

From (1) $\Delta ECD \sim \Delta EAB$

$$\frac{DC}{AB} = \frac{ED}{EB}$$

$$\frac{x}{6} = \frac{y}{5+y}$$

$$\because x=\frac{12}{5} \Rightarrow \frac{12/5}{6} = \frac{y}{5+y}$$

$$\frac{12}{5\times 6} = \frac{y}{5+y}$$

$$12(5+y) = 30y$$

$$60+12y = 30y$$

$$60 = 30y-12y$$

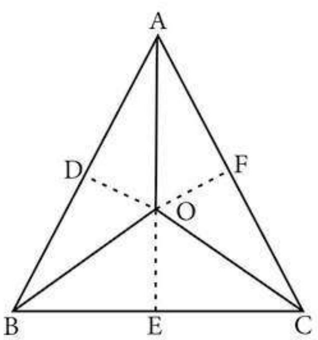
$$18y = 60$$

$$y = \frac{60}{18}$$

$$y = \frac{10}{3} \text{ cm}$$

3. O is any point inside a triangle ABC. The bisectors of $\angle AOB, \angle BOC$ and $\angle COA$ meet the sides AB, BC and CA in points D, E and F respectively. Show that AD × BE × CF = DB × EC × FA

Sol :



In ΔAOB, OD is the bisector of $\angle AOB$.

$$\frac{OA}{OB} = \frac{AD}{DB} \quad \text{... (1)}$$

In ΔBOC, OE is the bisector of $\angle BOC$

$$\therefore \quad \frac{OB}{OC} = \frac{BE}{EC} \quad \text{... (2)}$$

In ΔCOA, OF is the bisector of $\angle COA$

$$\therefore \quad \frac{OC}{OA} = \frac{CF}{FA} \quad \text{... (3)}$$

Multiplying the corresponding sides of (1), (2) and (3) we get

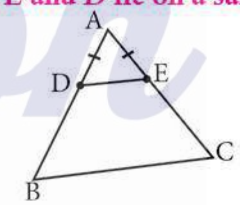
$$\frac{OA}{OB}\times\frac{OB}{OC}\times\frac{OC}{OA} = \frac{AD}{DB}\times\frac{BE}{EC}\times\frac{CF}{FA}$$

$$\Rightarrow \quad 1 = \frac{AD}{DB}\times\frac{BE}{EC}\times\frac{CF}{FA}$$

$\Rightarrow$ DB × EC × FA = AD × BE × CF

$\Rightarrow$ AD × BE × CF = DB × EC × FA; Hence proved

4. In the figure, ABC is a triangle in which AB = AC. Points D and E are points on the sides AB and AC respectively such that AD = AE. Show that the points B, C, E and D lie on a same circle.



Sol :

To prove the points B, C, E and D lie on a same circle, it is sufficient to show that $\angle ABC + \angle CED = 180^\circ$ and $\angle ACB + \angle BDE = 180^\circ$.

i.e., to prove opposite angles of the quadrilateral BCED are supplementary.

In ΔABC we have

AB = AC and AD = AE

$\Rightarrow$ AB – AD = AC – AE

$\Rightarrow$ DB = EC

Thus we have

AD = AE and DB = EC

$$\Rightarrow \quad \frac{AD}{DB} = \frac{AE}{EC}$$

$\Rightarrow$ DE || BC [$\because$ By the converse of Thale's theorem]

$\Rightarrow$ $\angle ABC = \angle ADE$ [$\because$ Corresponding angles]

$\Rightarrow$ $\angle ABC + \angle BDE = \angle ADE + \angle BDE$

[Adding $\angle BDE$ on both sides]

$\Rightarrow \quad \angle ABC + \angle BDE = 180^\circ$

$\Rightarrow \quad \angle ACB + \angle BDE = 180^\circ \quad ...(1)$

[$\because$ AB = AC, $\angle ABC = \angle ACB$]

Again DE $\parallel$ BC

$\Rightarrow \quad \angle ACB = \angle AED$

$\Rightarrow \quad \angle ACB + \angle CED = \angle AED + \angle CED$

[Adding $\angle CED$ on both sides]

$\Rightarrow \quad \angle ACB + \angle CED = 180^\circ$

[$\because$ $\angle ABC = \angle ACB$]

$\Rightarrow \quad \angle ABC + \angle CED = 180^\circ \quad ...(2)$

From (1) and (2),

Thus BDEC is a quadrilateral such that

$\angle ACB + \angle BDE = 180^\circ$

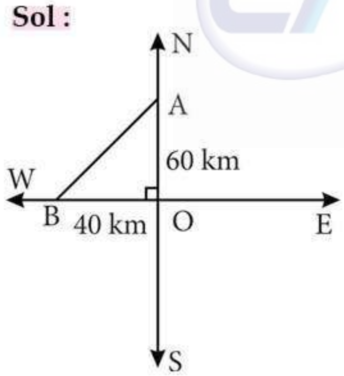
and $\angle ABC + \angle CED = 180^\circ$

$\therefore$ BDCE is a cyclic quadrilateral.

$\therefore$ Points B, C, E and D lie on a same circle.

5. Two trains leave a railway station at the same time. The first train travels due west and the second train due north. The first train travels at a speed of 20 km/hr and the second train travels at 30 km/hr. After 2 hours, what is the distance between them?

Sol :



Distance travelled by the first train in 2 hours

$= 2 \times 20 = 40$ km

Distance travelled by the second train in 2 hours

$= 2 \times 30 = 60$ km

Let the distances are represents by OB and OA respectively

Now applying pythagoras theorem,

Distance between the trains after 2 hours is AB.

we have $AB^2 = OA^2 + OB^2 = 60^2 + 40^2$

$= 3600 + 1600$

$= 5200$

$AB = \sqrt{5200}$

$= \sqrt{2^2 \times 2^2 \times 5 \times 5 \times 13}$

$= 2^2 \times 5\sqrt{13}$

$= 20\sqrt{13}$ km

$\therefore$ Distance between the trains after 2 hrs

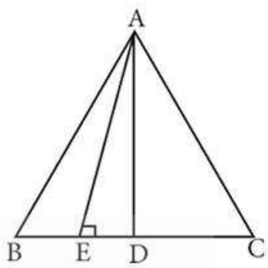
$= 20\sqrt{13}$ kms

6. D is the mid point of sides BC and AE $\perp$ BC. If BC = a, AC = b, AB = c, ED = x, AD = p and AE = h, prove that

(i) $b^2 = p^2 + ax + \frac{a^2}{4}$ (ii) $c^2 = p^2 - ax + \frac{a^2}{4}$

(iii) $b^2 + c^2 = 2p^2 + \frac{a^2}{2}$

Sol :



We have $\angle AED = 90^\circ$ [$\because$ AE $\perp$ BC]

Given BC = a, AC = b, AB = c, ED = x, AD = p and AE = h

(i) In ΔAEC, by Pythagoras theorem

$AC^2 = AE^2 + EC^2$

$AC^2 = AE^2 + (ED + DC)^2$

$AC^2 = AE^2 + ED^2 + DC^2 + 2 . ED . DC$

$AC^2 = (AE^2 + ED^2) + DC^2 + 2\, ED . DC$

$AC^2 = AD^2 + DC^2 + 2\, ED . DC$

$$AC^2 = AD^2 + \left(\frac{1}{2}BC\right)^2 + 2\left(\frac{1}{2}BC\right)DE$$

[$\because$ D is the mid point of BC, BD = DC]

$$AC^2 = AD^2 + BC . DE + \frac{1}{4}BC^2 \quad .... (1)$$

i.e., $b^2 = p^2 + ax + \frac{1}{4}a^2$

$b^2 = p^2 + ax + \frac{a^2}{4}$

(ii) Again in ΔABC, $\angle AED = \angle AEB = 90^\circ$.

By Pythagoras theorem.

$AB^2 = AE^2 + EB^2$

[$\because$ $AE^2 = AD^2 - DE^2$]

$= AD^2 - DE^2 + (BD - DE)^2$

$= AD^2 - DE^2 + BD^2 + DE^2 - 2\, BD . DE$

$= AD^2 + BD^2 - 2BD . DE$

$$AB^2 = AD^2 + \left(\frac{1}{2}BC\right)^2 - 2 . \frac{1}{2} BC . DE$$

$$AB^2 = AD^2 + \frac{1}{4}BC^2 - BC \cdot DE$$

$$AB^2 = AD^2 - BC \cdot DE + \frac{1}{4}BC^2 \quad ...(2)$$

$$c^2 = p^2 - ax + \frac{a^2}{4}$$

(iii) From (1) and (2)

$$AC^2 + AB^2 = AD^2 + BC \cdot DE + \frac{1}{4}BC^2 + AD^2 - BC \cdot DE + \frac{BC^2}{4}$$

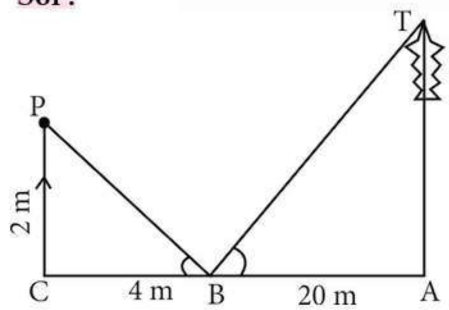
$$= 2\,AD^2 + 2\left(\frac{BC^2}{4}\right)$$

$$AC^2 + AB^2 = 2\,AD^2 + \frac{BC^2}{2}$$

$$b^2 + c^2 = 2p^2 + \frac{a^2}{2}$$

7. A clever outdoorsman whose eye-level is 2 m above the ground wishes to find the height of a tree. He places a mirror horizontally on the ground 20 m from the tree and finds that if he stands at a point C which is 4 m from the mirror B, he can see the reflection of the top of the tree. How height is the tree?

Sol :



Let CP be the man whose eye level is 2 m above the ground

AT be the height of the tree

In triangles ΔCBP and ΔABT

$\angle C = \angle A = 90^\circ$

Since CA ⊥ PC and CA ⊥ TA

$\angle CBP = \angle ABT$ as $\angle CBP$ is the angle of reflection and $\angle ABT$ is the angle of incident.

∴ By AA similarity criteria

$\Delta CBP \sim \Delta ABT$

$$\therefore \quad \frac{CB}{AB} = \frac{BP}{BT} = \frac{CP}{AT}$$

$$\frac{CB}{AB} = \frac{CP}{AT}$$

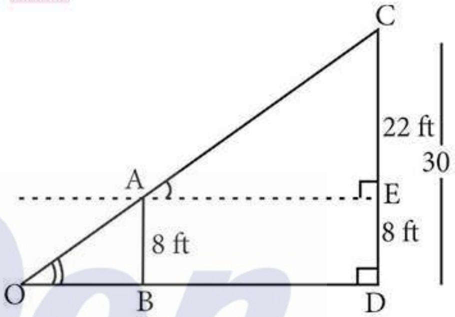
$$\frac{4}{20} = \frac{2}{AT}$$

$$AT = \frac{2 \times 20}{4} = 10 \text{ m}$$

∴ Height of the tree is 10 m.

8. Suppose an emu is 8 ft tall and is walking away from a pillar that is 30 ft tall. At height, the emu will cast a shadow on the ground. What is the relationship between the length of the shadow and the distance from the emu to the pillar?

Sol :



Let AB be the emu = 8 ft

CD be the pillar = 30 ft

OB be the shadow, BD be the distance between the pillar and the emu.

Draw AE ∥ OD at a distance of 8 ft from OD.

Now, $\angle CAE = \angle AOB$ [corresponding angles]

$\angle OBA = \angle AEC = 90^\circ$

∴ By AA similarity criteria

$\Delta CEA \sim \Delta ABO$

$$\frac{CE}{AB} = \frac{EA}{BO}$$

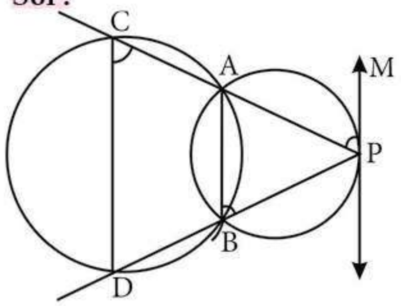
$$\frac{22}{8} = \frac{BD}{OB} \quad [\because EA = BD]$$

$$\frac{11}{4} = \frac{distance}{shadow}$$

$$\therefore \quad \text{Shadow} = \frac{4}{11} \times \text{distance}$$

9. Two circles intersect at A and B. From a point P on one of the circles lines PAC and PBD are drawn intersecting the second circle at C and D. Prove that CD is parallel to the tangent at P.

Sol :



Let MP is the tangent at P Join AB.

$\angle MPA = \angle PBA$ ... (1)

[Angles in the alternate segment]

$\angle PBA = \angle ACD$ ... (2)

[Exterior angle of a cyclic quadrilateral ABCD is equal to its interior opposite angle]

From (1) and (2), we have

$\angle MPA = \angle ACD$

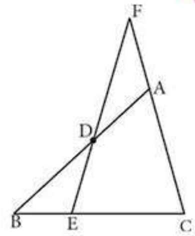
$\Rightarrow \angle MPC = \angle PCD$

But these are a pair of alternate interior angles.

$\therefore$ MP || CD

10. Let ABC be a triangle and D, E, F are points on the respective sides AB, BC, AC (or their extensions). Let AD : DB = 5 : 3, BE : EC = 3 : 2 and AC = 21. Find the length of the line segment CF.

Sol :



Let ΔABC be the given triangle and D, E, F are points on the respectives sides AB, BC AC (or their extension).

Given AD : DB = 5 : 3;

BE : EC = 3 : 2.

Let AD = $5k_1$; DB = $3k_1$; BE = $3k_2$; EC = $2k_2$

Now by Menelaus theorem, we have

$$\frac{BE}{EC} \times \frac{CF}{FA} \times \frac{AD}{DB} = -1$$

$$\frac{3k_2}{2k_2} \times \frac{CF}{FC - AC} \times \frac{5k_1}{3k_1} = -1$$

$$\frac{3}{2} \times \frac{CF}{FC - 21} \times \frac{5}{3} = -1$$

$$\frac{CF}{FC - 21} = \frac{-1 \times 2}{5}$$

$$\frac{CF}{(-CF) - 21} = \frac{-2}{5}$$

$$5\,CF = -2\,[(-CF) - 21]$$

$$5CF = 2\,CF + 42 \quad [\because FC = -CF]$$

$$5CF - 2CF = 42$$

$$3CF = 42$$

$$CF = \frac{42}{3} = 14 \text{ units.}$$

CREATIVE QUESTIONS

I. Multiple Choice Questions

Similar Triangles:

1. Sides of two similar triangle are in the ratio 4 : 9. Areas of these triangles are in the ratio

(1) 2 : 3 (2) 4 : 9

(3) 81 : 16 (4) 16 : 81 **[Ans : (4)]**

Sol :

In similar triangles Ratio of corresponding Area

= Square of ratio of corresponding sides

$$= \left(\frac{4}{9}\right)^2 = \left(\frac{16}{81}\right)$$

Ratio = 16 : 81

2. The areas of two similar triangles are respectively 9 cm² and 16 cm². The ratio the of their corresponding sides is

(1) 3 : 4 (2) 4 : 3

(3) 2 : 3 (4) 4 : 5 **[Ans : (1)]**

Sol :

$$\text{(Ratio of corresponding sides)}^2 = \frac{9}{16} = \frac{3^2}{4^2}$$

$$\therefore \text{ Ratio of corresponding sides} = \frac{3}{4}$$

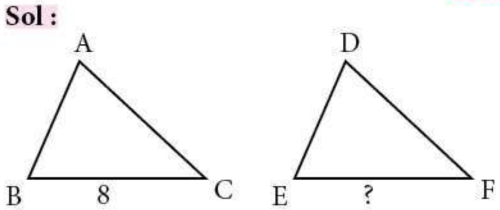
$$\text{Ratio} = 3 : 4$$

3. If ΔABC and ΔDEF are similar such that 2AB = DE and BC = 8 cm and EF =

(1) 16 cm (2) 12 cm

(3) 8 cm (4) 4 cm **[Ans : (1)]**

Sol :



Given $\Delta ABC \sim \Delta DEF$

$$\therefore \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

Given 2AB = DE; BC = 8cm

$$\therefore \frac{AB}{2AB} = \frac{8}{EF}$$

$$EF = 8 \times 2$$

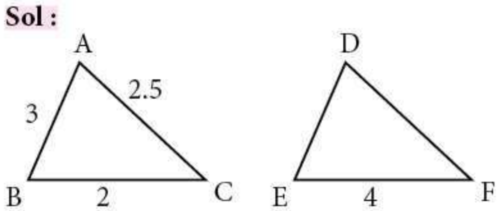
$$EF = 16 \text{ cm}$$

4. ΔABC is such that AB = 3 cm, BC = 2 cm and CA = 2.5 cm. If $\Delta DEF \sim \Delta ABC$ and EF = 4 cm, then perimeter of ΔDEF is

(1) 7.5 cm (2) 15 cm

(3) 22.5 cm (4) 30 cm **[Ans : (2)]**

Sol :



$$\text{Perimeter of } \Delta ABC = AB + BC + CA = 3 + 2 + 2.5 = 7.5 \text{ cm}$$

$$\frac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta DEF} = \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

$[\because \Delta ABC \sim \Delta DEF]$

$$\frac{7.5}{\text{Perimeter of } \Delta DEF} = \frac{BC}{EF} = \frac{2}{4} = \frac{1}{2}$$

$$\therefore \text{ Perimeter of } \Delta DEF = 2 \times 7.5 = 15 \text{ cm}$$

5. If ΔABC and ΔDEF are similar triangles such that $\angle A$ = 47° and $\angle B$ = 83°, then $\angle F$ =

(1) 50° (2) 60°

(3) 70° (4) 80° **[Ans : (1)]**

Sol :

$\Delta ABC \sim \Delta DEF$

$\angle A = \angle D$; $\angle B = \angle E$; $\angle C = \angle F$

$\angle A = \angle D = 47°$ $\angle B = \angle E = 83°$

$$\angle C = 180 - (\angle A + \angle B) = 180° - (47° + 83°) = 180° - 130° = 50°$$

$\therefore \angle C = \angle F = 50°$

$\therefore \angle F = 50°$

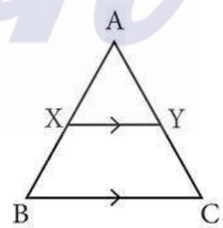
Thales Theorem and Angle Bisector Theorem.

6. XY is drawn parallel to the base BC of a ΔABC cutting AB at x and AC at y. If AB = 4 BX and YC = 2 cm and then AY = _____

(1) 2 cm (2) 4 cm

(3) 6 cm (4) 8 cm **[Ans : (3)]**

Sol :



$$\frac{AX}{XB} = \frac{AY}{YC} \text{ [By Thales Theorem]}$$

$$\frac{AX}{XB} = \frac{AY}{2}$$

$$\frac{AB - XB}{XB} = \frac{AY}{2}$$

$$\frac{4BX - BX}{BX} = \frac{AY}{2}$$

$$\frac{3BX}{BX} = \frac{AY}{2}$$

$$AY = 2 \times 3$$

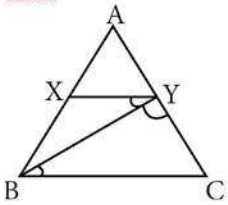
$$AY = 6 \text{ cm}$$

7. In ΔABC, a line XY parallel to BC cuts AB at X and AC at Y. If BY bisects $\angle XYC$, then

(1) BC = CY (2) BC = BY

(3) BC ≠ CY (4) BC ≠ BY **[Ans : (1)]**

Sol :



Given $XY \parallel BC$

$\angle XYB = \angle YBC$ ... (1)

[$\because$ alternate interior angles]

YB bisects $\angle$XYC.

So $\angle XYB = \angle BYC$ ... (2)

$\angle YBC = \angle BYC$ [$\because$ from (1) and (2)]

$\therefore$ BC = YC

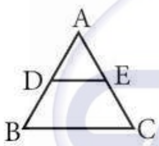
[$\because$ sides opposite to equal angles are equal]

8. In ΔABC, D and E are points on side AB and AC respectively such that $DE \parallel BC$ and AD : DB = 3 : 1. If EA = 3.3 cm then AC = _____

(1) 1.1 cm (2) 4 cm

(3) 4.4 cm (4) 5.5 cm **[Ans : (3)]**

Sol :



$DE \parallel BC$

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{3}{1} = \frac{3.3}{EC}$$

$$EC = \frac{3.3}{3}$$

$$EC = 1.1$$

$$AC = AE + EC$$

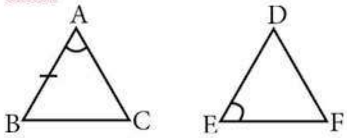
$$= 3.3 + 1.1 = 4.4 \text{ cm}$$

9. In ΔABC and $\angle A = \angle E = 40^\circ$, AB : ED = AC : EF and $\angle F = 65^\circ$, then $\angle B$ = _____

(1) 35° (2) 65°

(3) 75° (4) 85° **[Ans : (3)]**

Sol :



Given $\angle A = \angle E = 40^\circ$

$$\frac{AB}{ED} = \frac{AC}{EF}$$

$\therefore \Delta ABC \sim \Delta EDF$

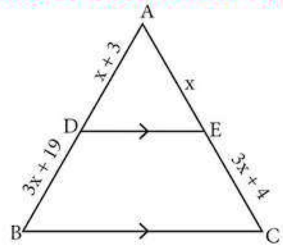
$\therefore \angle F = \angle C = 65^\circ$

$\angle B = 180^\circ - (\angle A + \angle C)$

$= 180^\circ - (\angle 40^\circ + \angle 65^\circ)$

$\angle B = \angle D = 180^\circ - 105^\circ = 75^\circ$

10. Find the value of x for which $DE \parallel AB$ is



(1) 4 (2) 1

(3) 3 (4) 2 **[Ans : (4)]**

Sol :

Since $DE \parallel AB$

$$\frac{AD}{DB} = \frac{AE}{EC} \text{ [By Thales Theorem]}$$

$$\frac{x+3}{3x+19} = \frac{x}{3x+4}$$

$$(x + 3)(3x + 4) = x(3x + 19)$$

$$3x^2 + 13x + 12 = 3x^2 + 19x$$

$$6x = 12$$

$$x = \frac{12}{6}$$

$$x = 2$$

Pythogoras Theorem:

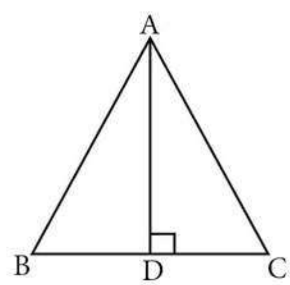
11. In an equilateral triangle ΔABC, if $AD \perp BC$ them

(1) $2AB^2 = 3AD^2$ (2) $4AB^2 = 3AD^2$

(3) $3AB^2 = 4AD^2$ (4) $3AB^2 = 2AD^2$

[Ans : (3)]

Sol :



AB = BC = AC and BD = DC = $\frac{1}{2}$ BC

By Pythogoras theorem

$$AB^2 = BD^2 + AD^2$$

$$AB^2 = \left(\frac{1}{2} BC\right)^2 + AD^2$$

$$AB^2 = \frac{1}{4} BC^2 + AD^2$$

$$4AB^2 = BC^2 + 4AD^2$$

$$4AB^2 - BC^2 = 4AD^2$$

$$4AB^2 - AB^2 = 4AD^2 \qquad [\because \; AB = BC]$$

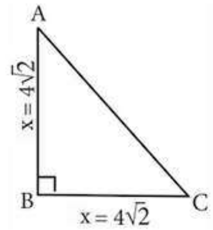
$$3AB^2 = 4AD^2$$

12. The length of the hypotenuse of an isosceles right triangle whose one side is $4\sqrt{2}$ cm is

(1) 12 cm (2) 8 cm

(3) $8\sqrt{2}$ cm (4) $12\sqrt{2}$ cm **[Ans : (2)]**

Sol :



$$AC^2 = AB^2 + BC^2$$
$$= x^2 + x^2$$
$$= 2x^2$$
$$= 2(4\sqrt{2})^2$$
$$= 2 \times 16 \times 2$$
$$AC^2 = 64$$
$$AC = 8 \text{ cm}$$

Another Method:

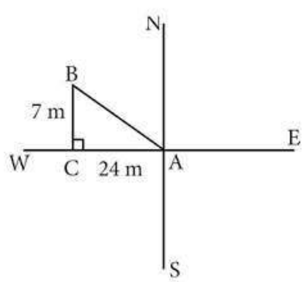
$$AC^2 = AB^2 + BC^2$$
$$= (4\sqrt{2})^2 + (4\sqrt{2})^2$$
$$= (16 \times 2) + (16 \times 2)$$
$$= 32 + 32$$
$$AC^2 = 64$$
$$AC = 8 \text{ cm}$$

13. A man goes 24 m due west and then 7 m due north. How are is he from the starting point?

(1) 31 m (2) 17 m

(3) 25 m (4) 26 m **[Ans : (3)]**

Sol :



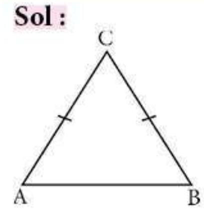
$$AB^2 = AC^2 + BC^2$$
$$= 24^2 + 7^2$$
$$= 576 + 49$$
$$AB^2 = 625$$
$$AB = 25 \text{ m}$$

14. In an isosceles triangle ΔABC if AC = BC and $AB^2 = 2AC^2$, then $\angle C$ = _____

(1) 30° (2) 45°

(3) 90° (4) 60° **[Ans : (3)]**

Sol :



$$AB^2 = 2AC^2$$

$$\Rightarrow \quad AB^2 = AC^2 + BC^2 \quad [\therefore \; AC = BC]$$

By the converse of Pythagoras theorem

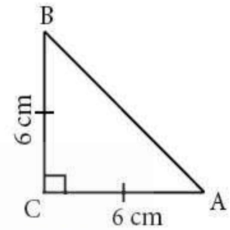
$$\angle C = 90^\circ$$

15. ΔABC is an isosceles triangle in with $\angle C = 90^\circ$, If AC = 6cm, then AB = _____

(1) $6\sqrt{2}$ cm (2) 6 cm

(3) $2\sqrt{6}$ cm (4) $4\sqrt{2}$ **[Ans : (1)]**

Sol :



$$AB^2 = AC^2 + BC^2$$
$$= 6^2 + 6^2$$
$$= 36 + 36$$
$$= 72$$
$$AB = \sqrt{72} = 6\sqrt{2} \text{ cm}$$

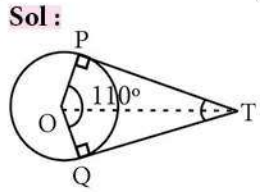
Circles and tangents and Alternate segment theorem:

16. If TP and TQ are two tangents to a circle with centre 'O' so that $\angle POQ = 110^\circ$, then $\angle PTQ$ is

(1) 60° (2) 70°

(3) 80° (4) 90° **[Ans : (2)]**

Sol :



In the quadrilateral POQT

$$\angle O = 110^\circ$$
$$\angle P = 90^\circ$$
$$\angle Q = 90^\circ$$
$$\therefore \angle T = 360^\circ - (110^\circ + 90^\circ + 90^\circ)$$

[Sum of 4 angles of a quadrilateral is 360°]

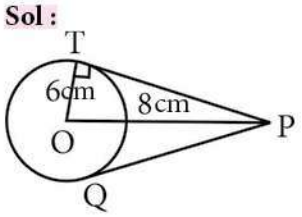
$$= 360^\circ - 290^\circ$$
$$\angle T = 70^\circ$$
$$\angle PTQ = 70^\circ$$

17. The length of the tangent drawn from a point 8 cm away from the centre of a circle of radius 6 cm is _____

(1) $\sqrt{7}$ cm (2) $2\sqrt{7}$ cm

(3) 10 cm (4) 5 cm **[Ans : (2)]**

Sol :



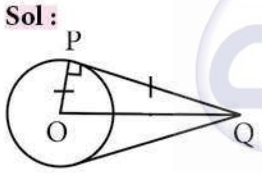
$$OP^2 = PT^2 + TO^2$$
$$8^2 = PT^2 + 6^2$$
$$64 - 36 = PT^2$$
$$PT^2 = 28$$
$$PT = 2\sqrt{7} \text{ cm}$$

18. PQ is a tangent to a circle with center 'O' at the point P, if ΔOPQ is an isosceles triangle, then $\angle OQP$ is =

(1) 30° (2) 45°

(3) 60° (4) 90° **[Ans : (2)]**

Sol :



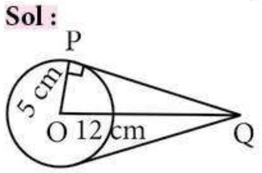
$$\angle P = 90° \ [\because \text{ radius} \perp \text{tangent}]$$
$$OP = PQ \ [\Delta OPQ \text{ is an isosceles}]$$
$$\Rightarrow \angle POQ = \angle PQO$$
$$\angle POQ + \angle PQO = 180° - \angle P$$
$$= 180° - 90° = 90°$$
$$\therefore \angle POQ = \angle PQO = \frac{1}{2} \times 90°$$
$$\therefore \angle OQP = 45°$$

19. A tangent PQ at a point P of circle of radius 5 cm meats a line through the center 'O' at a point Q such that OQ = 12 cm, Length PQ is.

(1) 12 cm (2) 13 cm

(3) 8.5 cm (4) $\sqrt{119}$ cm **[Ans : (4)]**

Sol :



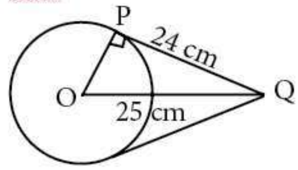
$$OQ^2 = OP^2 + PQ^2 \quad \text{[By Pythagoras Theorem]}$$
$$12^2 = 5^2 + PQ^2$$
$$144 - 25 = PQ^2$$
$$PQ^2 = 119$$
$$PQ = \sqrt{119} \text{ cm}$$

20. From a point Q, the length of the tangent to a circle is 24 cm an d the distance of a Q from the center is 25 cm. The radius of the circle is _____

(1) 7 cm (2) 12 cm

(3) 15 cm (4) 24.5 cm **[Ans : (1)]**

Sol :



$$(\text{radius})^2 + 24^2 = 25^2 \quad \text{[By Pythagoras Theorem]}$$
$$(\text{radius})^2 + 576 = 625$$
$$(\text{radius})^2 = 625 - 576$$
$$= 49$$
$$(\text{radius})^2 = \sqrt{49}$$
$$\text{radius} = 7 \text{ cm}$$

II. Very Short Answer Questions

1. The areas of two similar triangles ΔABC and ΔDEF are 81 cm² and 100 cm² respectively.If EF = 5 cm, then find BC.

Sol :

Given $\Delta ABC \sim \Delta DEF$

$$\frac{area(\Delta ABC)}{area(\Delta DEF)} = \frac{BC^2}{EF^2}$$
$$\frac{81}{100} = \frac{BC^2}{(5)^2}$$
$$\frac{BC}{5} = \frac{9}{10}$$
$$BC = \frac{9}{10} \times 5 = \frac{9}{2} \text{ cm}$$
$$= 4.5 \text{ cm}$$
$$\therefore BC = 4.5 \text{ cm}$$

2. The area of ΔPQR = 64 m². Find the area of ΔLMN if $\frac{PQ}{LM} = \frac{4}{5}$ and $\Delta PQR \sim \Delta LMN$.

Sol :

Given $\Delta PQR \sim \Delta LMN$

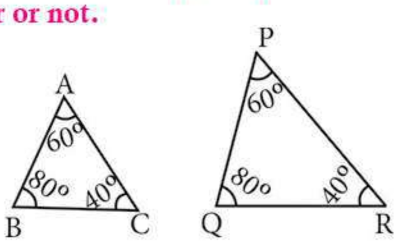
$$\frac{area(\Delta PQR)}{area\ of\ (\Delta LMN)} = \frac{PQ^2}{LM^2}$$
$$\frac{64}{area(\Delta LMN)} = \left(\frac{4}{5}\right)^2 = \frac{16}{25}$$

$$\therefore \quad \text{Area of } \Delta LMN = \frac{64 \times 25}{16}$$

$$= 100 \text{ m}^2$$

$$\text{Area of } \Delta LMN = 100 \text{ m}^2$$

3. Check whether the given pair of triangles are similar or not.



Sol :

In ΔABC and ΔPQR

we have $\angle A = \angle P = 60^\circ$

$\angle B = \angle Q = 80^\circ$

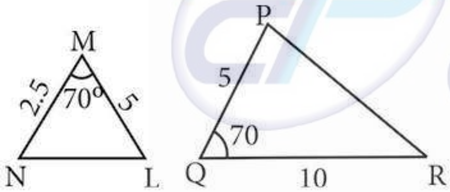
$\angle C = \angle R = 40^\circ$

The corresponding angles are equal

Using AAA similarity rule

$\Delta ABC \sim \Delta PQR$

4. Check the similarity of the given triangles.



Sol :

In ΔMNL and ΔQPR

$$\frac{MN}{QP} = \frac{2.5}{5} = \frac{1}{2}$$

$$\frac{ML}{QR} = \frac{5}{10} = \frac{1}{2}$$

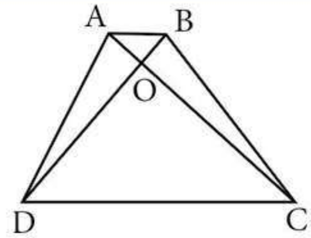
$$\frac{ML}{QR} = \frac{MN}{QP}$$

$$\angle NML = \angle PQR$$

Using SAS criteria of similarity, we have,

$\Delta MNL \sim \Delta QPR$

5. Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at the point O. Using a similarity criterion for two triangles show that $\frac{OA}{OC} = \frac{OB}{OD}$.



Sol :

We have a trapezium ABCD in which AB || DC.

The diagonals AC and BD intersect at O

In ΔOAB and ΔOCD

AB || DC

and BD intersects them

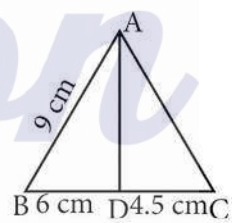
$\therefore \quad \angle OBA = \angle ODC$ (Alternate interior angles)

Also $\angle OAB = \angle OCD$ (Alternate interior angles)

Using AA similarity criteria

$\Delta OAB \sim \Delta OCD$

6. In the figure AD is the bisector of $\angle A$. If BD = 6 cm, DC = 4.5 cm and AB = 9 cm, Find AC.



Sol :

In ΔABC, AD is the bisector of $\angle A$

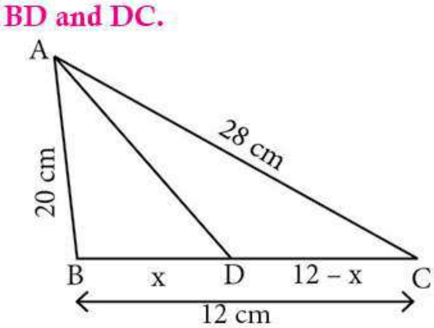
$$\therefore \quad \frac{BD}{DC} = \frac{AB}{AC}$$

$$\frac{6}{4.5} = \frac{9}{AC}$$

$$AC = \frac{9}{6} \times 4.5$$

$$AC = 6.75 \text{ cm}$$

7. In the figure AD is the bisector of $\angle BAC$. If AB = 20 cm, AC = 28 cm and BC = 12 cm, find BD and DC.



Sol :

$$\text{Let } BD = x \text{ cm},$$
$$\text{Then } DC = (12 - x) \text{ cm}$$

Since AD is the bisector of $\angle A$.

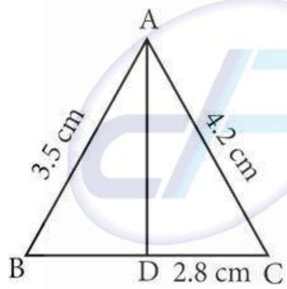
$$\frac{AB}{AC} = \frac{BD}{DC} \Rightarrow \frac{20}{28} = \frac{x}{12-x}$$

$$\frac{5}{7} = \frac{x}{12-x} \Rightarrow 5(12 - x) = 7x$$

$$60 - 5x = 7x$$
$$12x = 60$$
$$x = \frac{60}{12} = 5$$
$$BD = 5 \text{ cm}$$
$$DC = 12 - 5 = 7 \text{ cm}$$

8. In ΔABC, AD is the bisector of $\angle A$, meeting BC at D If AB = 3.5 cm, AC = 4.2 cm and DC = 2.8 cm, find BD.

Sol :



Since AD is the bisector of $\angle A$

$$\frac{AB}{AC} = \frac{BD}{DC} \Rightarrow \frac{3.5}{4.2} = \frac{BD}{2.8}$$

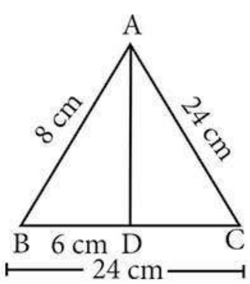
$$BD = \frac{3.5 \times 2.8}{4.2}$$

$$= \frac{0.70}{0.3} = \frac{7}{3} = 2.33$$

$$BD = 2.33 \text{ cm}$$

9. In a ΔABC, if AB = 8 cm, AC = 24 cm, BD = 6 cm and BC = 24 cm. Check whether AD is the bisector of $\angle A$ of ΔABC.

Sol :



$$\text{Given } BC = 24 \text{ cm and } BD = 6 \text{ cm}$$
$$\therefore DC = BC - BD$$
$$= 24 - 6 = 18 \text{ cm}$$

$$\text{Now, } \frac{AB}{AC} = \frac{8}{24} = \frac{1}{3}$$

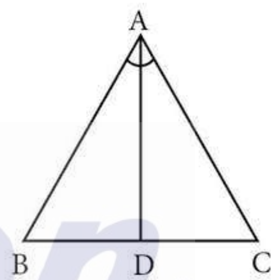
$$\frac{BD}{DC} = \frac{6}{18} = \frac{1}{3}$$

$$\frac{AB}{AC} = \frac{BD}{DC}$$

$\therefore$ By the converse of angular bisector theorem. AD is the bisector of $\angle A$ in ΔABC.

10. If the bisector of an angle of a triangle bisects the opposite side, prove that the triangle is isosceles.

Sol :



Given: In ΔABC, the bisector AD of $\angle A$ bisects the side BC.

To prove: AB = AC

Proof: In ΔABC, AD is the bisector of $\angle A$

$$\frac{AB}{AC} = \frac{BD}{DC}$$

$$\frac{AB}{AC} = 1$$

[$\because$ D is the mid point of BC, BD = DC]

AB = AC.

$\therefore$ The triangle ABC is isosceles.

11. Check the given sides are the sides of a right angled triangle.

(i) a = 6 cm, b = 8 cm and c = 10 cm

(ii) a = 5 cm b = 8 cm and c = 11 cm

Sol :

(i) We have $a = 6$ cm,

$b = 8$ cm and $c = 10$ cm

Here the larger side is $c = 10$ cm

$$\therefore a^2 + b^2 = 6^2 + 8^2$$
$$= 36 + 64 = 100$$
$$= (10)^2 = c^2$$

$\therefore$ The triangle with the given sides is a right triangle.

(ii) We have a = 5 cm,

b = 8 cm and c = 11 cm

Here the larger side is c = 11 cm

$$a^2 + b^2 = 5^2 + 8^2 = 25 + 64$$

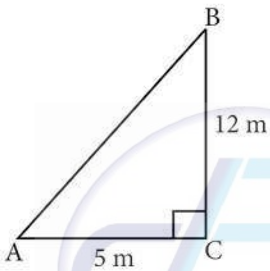
$$a^2 + b^2 = 89$$

$$\text{But } c^2 = 11^2 = 121$$

$\therefore$ $a^2 + b^2 \neq c^2$. The triangle with the given sides are not a right angled triangle.

12. A ladder is placed in such a way that its foot is at a distance of 5 m from a wall and its tip reaches a window 12 m above the ground. Determine the length of the ladder.

Sol :



Let AB be the ladder and B be the window, then

BC = 12 m and AC = 5 m

Since ΔABC is right triangle; right angled at C

$$AB^2 = AC^2 + BC^2$$

$$AB^2 = 5^2 + 12^2 = 25 + 144$$

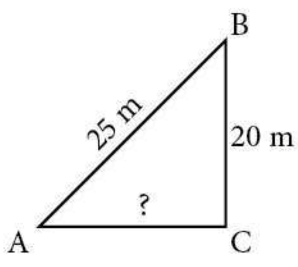
$$AB^2 = 169$$

$$AB = 13 \text{ m}$$

Hence the length of the ladder is 13 m

13. A ladder 25 m long reaches a window of a building 20 m above the ground. Determine the distance of the foot of the ladder from the building.

Sol :



Suppose that AB is the ladder, B is the window and CB is the building.

ΔABC is a right triangle and $\angle C = 90^\circ$

$$AB^2 = AC^2 + BC^2$$

$$(25)^2 = AC^2 + 20^2$$

$$625 = AC^2 + 400$$

$$AC^2 = 625 - 400$$

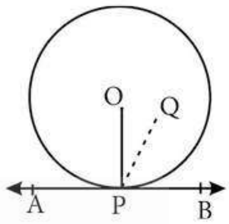
$$AC^2 = 225$$

$$AC = 15$$

Hence the foot of the ladder is at a distance of 15 m from the building.

14. Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre.

Sol :



In the figure, the center of the circle O and tangent AB touches the circle at P. If possible, let PQ be perpendicular to AB such that it is not passing through 'O'. Join OP.

Since tangent at a point to a circle is perpendicular to the radius thought that point

$$AB \perp OP$$

i.e., $\angle OPB = 90^\circ$ ... (1)

But by construction

$AB \perp PQ \Rightarrow \angle QPB = 90^\circ$ ... (2)

From (1) and (2)

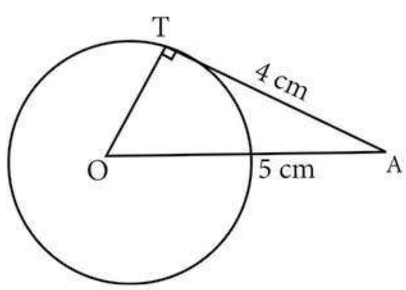
$$\angle QPB = \angle OPB$$

Which is possible only when O and Q coincide

$\therefore$ The perpendicular at the point of contact to the tangent passes through the center.

15. The length of a tangent from a point at a distance of 5 cm from the center of the circle is 4 cm. Find the radius of the circle.

Sol :



The tangent to a circle is perpendicular to the radius through the point of contact

$\angle OTA = 90^\circ$

Now in right ΔOTA

$OA^2 = OT^2 + TA^2$

$5^2 = OT^2 + 4^2$

$OT^2 = 5^2 - 4^2$

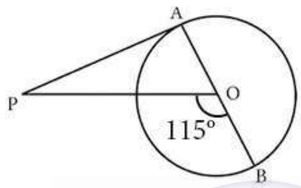
$= 25 - 16 = 9$

$OT = 3$

Thus the radius of the circle is 3 cm.

16. In the figure PA is a tangent from an external point P to a circle with centre O. If $\angle POB = 115^\circ$ then find $\angle APO$.

Sol :



Here PA is a tangent and OA is radius. Also a radius through the point of contact is perpendicular to the tangent.

$OA \perp PA$

$\angle PAO = 90^\circ$

In ΔOAP, $\angle POB$ in an external angle.

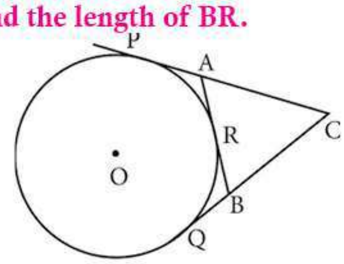
$\therefore \angle APO + \angle PAO = \angle POB$

$\angle APO + 90^\circ = 115^\circ$

$\angle APO = 115^\circ - 90^\circ = 25^\circ$

$\angle APO = 25^\circ$

17. In the figure CP and CQ are tangents to a circle with center O. ARB is another tangent touching the circle at R. If QC = 11 cm, BC = 7 cm. Then find the length of BR.



Sol :

Tangents drawn from an external point are equal.

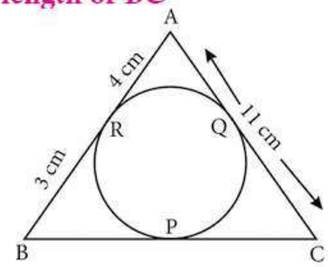
BQ = BR and CQ = CP

Since BC + BQ = QC

7 + BR = 11 [$\because$ BQ = BR]

BR = 11 – 7 = 4 cm

18. In the figure, ΔABC is circumscribing a circle. Find the length of BC



Sol :

Since tangents drawn from an external point to the circle are equal.

AR = AQ = 4 cm

BR = BP = 3 cm

PC = QC

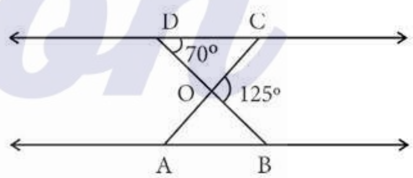
QC = AC – AQ = 11 – 4 = 7 cm

BC = BP + PC = 3 + QC

= (3 + 7) cm = 10 cm

III. Short Answer Questions:

1. In the figure $\Delta ODC \sim \Delta OBA$, $\angle BOC = 125^\circ$ and $\angle CDO = 70^\circ$, find $\angle DOC$, $\angle DCO$ and $\angle OAB$.



Sol :

We have $\angle BOC = 125^\circ$ and

$\angle CDO = 70^\circ$

Since $\angle DOC + \angle BOC = 180^\circ$ [linear pair]

$\angle DOC = 180^\circ - 125^\circ = 55^\circ$ ... (1)

In ΔDOC Using angle sum property, we get

$\angle DOC + \angle ODC + \angle DCO = 180^\circ$

$55^\circ + 70^\circ + \angle DCO = 180^\circ$

$\angle DCO = 180^\circ - 55^\circ - 70^\circ$

$\angle DCO = 55^\circ$ ... (2)

Again $\Delta ODC \sim \Delta OBA$ (given)

$\therefore$ Their corresponding angles are equal.

$\angle OCD = \angle OAB$

$= 55^\circ$ ... (3)

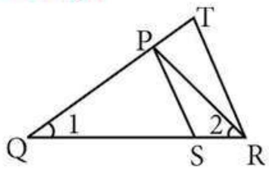
$\therefore$ From (1), (2) and (3)

$\angle DOC = 55^\circ$,

$\angle DCO = 55^\circ$ and

$\angle OAB = 55^\circ$

2. In the figure $\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle 1 = \angle 2$ show that $\Delta PQS \sim \Delta TQR$



Sol :

In ΔPQR $\angle 1 = \angle 2$ (given)

PR = QP ... (1)

[∵ In a triangle, sides opposite to equal angles are equal]

Given $\frac{QR}{QS} = \frac{QT}{PR}$ ... (2)

From (1) and (2)

$$\frac{QR}{QS} = \frac{QT}{QP}$$

$$\frac{QS}{QR} = \frac{QP}{QT} \quad \text{... (3)}$$

Now in ΔPQS and ΔTQR

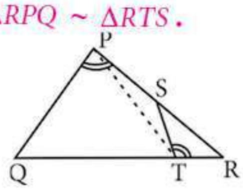
$$\frac{QS}{QR} = \frac{QP}{QT}$$

$\angle SQP = \angle RQT = \angle 1$ [From (3)]

∴ Using SAS similarity criteria

$\Delta PQS \sim \Delta TQR$.

3. S and T are points on sides PR and QR of ΔPQR such that $\angle P = \angle RTS$.
Show that $\Delta RPQ \sim \Delta RTS$.



Sol :

T is a point on QR and S is a point or PR such that

$\angle RTS = \angle P$

Now in ΔRPQ and ΔRTS

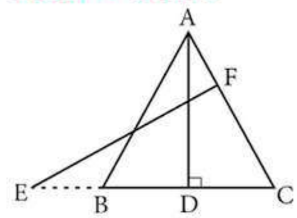
$\angle RPQ = \angle RTS$ (given)

$\angle PRQ = \angle TRS$ [common]

∴ Using AA similarity, we have

$\Delta RPQ \sim \Delta RTS$

4. In the figure E is a point on side CB produced of an isosceles triangle ABC with AB = AC. If AD ⊥ BC and EF ⊥ AC.
Prove that $\Delta ABD \sim \Delta ECF$.



Sol :

We have an isosceles triangle ΔABC in which

AB = AC.

In ΔABD and ΔECF

AB = AC (given)

⇒ Angles opposite to them are equal.

∴ $\angle ACB = \angle ABC$

$\angle ECF = \angle ABD$ ... (1)

AD ⊥ BC and EF ⊥ AC

$\angle ADB = \angle EFC = 90^\circ$ ... (2)

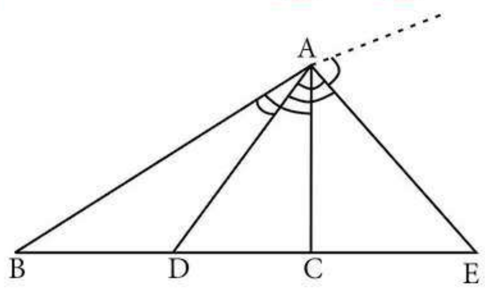
From (1) and (2) we have

By AA criteria of similarity

$\Delta ABD \sim \Delta ECF$

5. The bisector of interior $\angle A$ of ΔABC meets BC in D and the bisector of exterior $\angle A$ meets BC produced in E. Prove that $\frac{BD}{BE} = \frac{CD}{CE}$.

Sol :



Given: In ΔABC AD and AE respectively the bisectors of the interior and exterior angles at A.

To prove: $\frac{BD}{BE} = \frac{CD}{CE}$

Proof: Since AD is the internal bisector of $\angle A$ meeting BC at D.

$$\frac{AB}{AC} = \frac{BD}{DC} \quad \text{... (1)}$$

Since AE is the external bisector of $\angle A$ meeting BC produced in E.

$$\frac{AB}{AC} = \frac{BE}{CE} \quad \ldots (2)$$

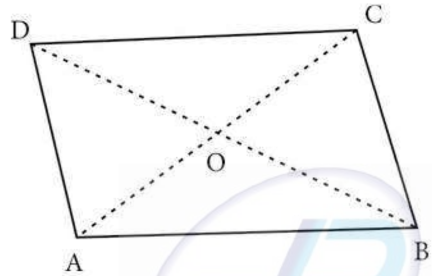
From (1) and (2) we get

$$\frac{BD}{DC} = \frac{BE}{CE}$$

$$\frac{BD}{BE} = \frac{CD}{CE}$$

6. If the diagonal BD of a quadrilateral ABCD bisects both $\angle B$ and $\angle D$, Show that $\frac{AB}{BC} = \frac{AD}{CD}$.

Sol :



Given: A quadrilateral ABCD in which the diagonal BD bisects $\angle B$ and $\angle D$.

To prove: $\frac{AB}{BC} = \frac{AD}{CD}$.

Construction: Join AC intersecting BD in O.

Proof

In ΔABC, BO is the bisector of $\angle B$.

$$\frac{AO}{OC} = \frac{BA}{BC}$$

$$\frac{OA}{OC} = \frac{AB}{BC} \quad \ldots\ldots(1)$$

In ΔADC, DO is the bisector of $\angle D$

$$\frac{AO}{OC} = \frac{DA}{DC}$$

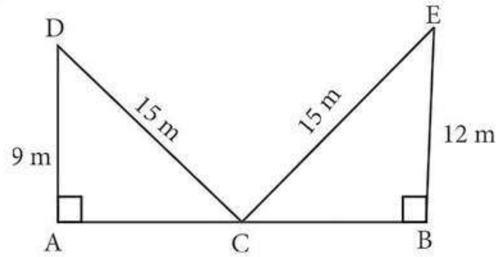
$$\frac{OA}{OC} = \frac{AD}{CD} \quad \ldots\ldots(2)$$

From (1) and (2) we get

$$\frac{AB}{BC} = \frac{AD}{CD}$$

7. A ladder 15 m long reaches a window which is 9 m above the ground on one side of a street. Keeping its foot at the same point, the ladder is turned to other side of the street to reach a window 12 m high. Find the width of the street.

Sol :



Let AB be the width of the street; C be the foot of the ladder. Let D and E be the windows at heights of 9 m and 12 m respectively from the ground.

Then CD and CE are the two positions of the ladder.

Clearly AD = 9 m, BE = 12 m, CD = CE = 15 m

From the right triangle ΔACD we have

$$CD^2 = AC^2 + AD^2$$

$$15^2 = AC^2 + 9^2$$

$$AC^2 = 225 - 81 = 144$$

$$AC = 12 \text{ m}$$

In ΔBCE, we have

$$CE^2 = BC^2 + BE^2$$

$$15^2 = BC^2 + 12^2$$

$$BC^2 = 225 - 144 = 81$$

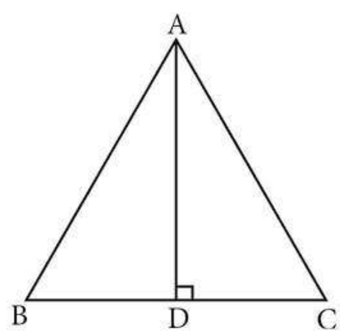
$$BC = 9 \text{ m}$$

Hence width of the street AB = AC + CB

= 12 + 9 = 21 m

8. Prove that three times the square of any side of an equilateral triangle in equal to four times the square of the altitude.

Sol :



Let ABC be an equilateral triangle and AD $\perp$ BC.

In ΔADB and ΔADC, we have

$$AB = AC$$
$$\angle B = \angle C$$
$$\text{and } \angle ADB = \angle ADC$$

$\therefore$ By RHS Criteria

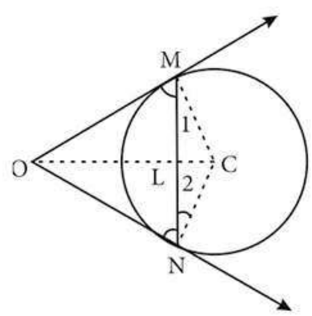
$$\Delta ADB \cong \Delta ADC$$
$$BD = DC \qquad [\because \text{ By CPCTC}]$$
$$\Rightarrow BD = DC = \frac{1}{2} BC$$

Since ΔADB is right triangle right angled at D

$$AB^2 = AD^2 + BD^2$$
$$\Rightarrow AB^2 = AD^2 + \left(\frac{1}{2} BC\right)^2$$
$$\Rightarrow AB^2 = AD^2 + \frac{BC^2}{4}$$
$$AB^2 = AD^2 + \frac{AB^2}{4} \qquad [\because BC = AB]$$
$$AD^2 = AB^2 - \frac{AB^2}{4}$$
$$= \frac{4AB^2 - AB^2}{4}$$
$$AD^2 = \frac{3AB^2}{4}$$
$$4AD^2 = 3AB^2$$

9. Prove that the tangents drawn at the ends of a chord of a circle make equal angles with the chord.

Sol :



Let NM be the chord of a circle with centre C. Let the tangents at M and N meet at O

$\therefore$ OM is a tangent at M

$$\therefore \angle OMC = 90^\circ$$
$$\text{Similarly } \angle ONC = 90^\circ$$
$$\text{Since } CM = CN$$
$$\text{In } \Delta CMN \ \angle 1 = \angle 2$$

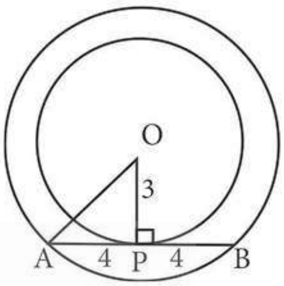
[$\because$ Angles opposite to equal sides are equal in a triangle]

$$\angle OMC - \angle 1 = \angle ONC - \angle 2$$
$$\angle OML = \angle ONL$$

Thus tangents make equal angles with the chord.

10. Two concentric circles have a common center 'O' the chord AB to the bigger circles touches the smaller circle at P. If OP = 3 cm and AB = 8 cm then find the radiu of the bigger circle.

Sol :



$\therefore$ AB touches the smaller circle at P

$\therefore$ OP $\perp$ AB $\Rightarrow$ $\angle OPA = 90^\circ$

Now AB is the chord of the bigger circle. Since the perpendicular from the centre to a chord, bisects the chord.

$\therefore$ P is the mid point of AB

$$AP = \frac{8}{2} = 4 \text{ cm}$$

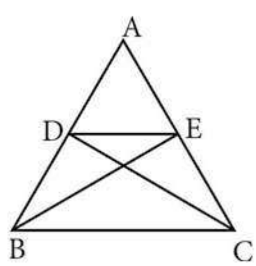
In right ΔAPO, we have

$$AO^2 = OP^2 + AP^2$$
$$AO^2 = 3^2 + 4^2 = 9 + 16$$
$$AO^2 = 25$$
$$AO = 5 \text{ cm}$$

$\therefore$ The radius of the bigger circle = 5 cm.

IV. Long Answer Questions

1. In the figure, if $\Delta ABE \cong \Delta$ ACD, show that $\Delta ADE \sim \Delta ABC$.



Sol :

We have $\Delta ABE \cong \Delta ACD$

Their corresponding parts are equal.

$$AB = AC$$
$$AE = AD$$

$$\therefore \frac{AB}{AC} = \frac{AE}{AD} \Rightarrow \frac{AB}{AE} = \frac{AC}{AD}$$

$$\Rightarrow \frac{AB}{AD} = \frac{AC}{AE} \quad [\because AE = AD] \quad \ldots (1)$$

Now in ΔADE and ΔABC

$$\frac{AB}{AD} = \frac{AC}{AE} \quad \text{[From (1)]}$$

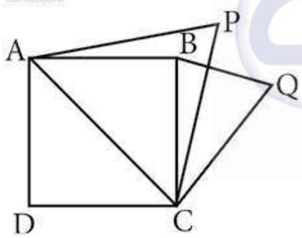
$$\angle DAE = \angle BAC \quad \text{[common]}$$

Using SAS similarity criteria we have

$$\Delta ADE \sim \Delta ABC$$

2. Prove that the area of an equilateral triangle described on one side of a square in equal to half the area of the equilateral triangle described on one of its diagonals

Sol :



We have a square ABCD, whose diagonal AC.

Equilateral triangle ΔBQC is described on the side BC and another equilateral ΔAPC is described on the diagonal AC.

$\therefore$ All equilateral triangles are similar.

$$\Delta APC \sim \Delta BQC$$

$\therefore$ The ratio of their areas is equal to the square of the ratio of their corresponding sides.

$$\frac{area\,(\Delta\,APC)}{area\,(\Delta\,BQC)} = \left(\frac{AC}{BC}\right)^2 \quad \ldots (1)$$

Since the length of a diagonal of a square $= \sqrt{2} \times side$

$$AC = \sqrt{2} \times BC \quad \ldots (2)$$

From (1) and (2) we have

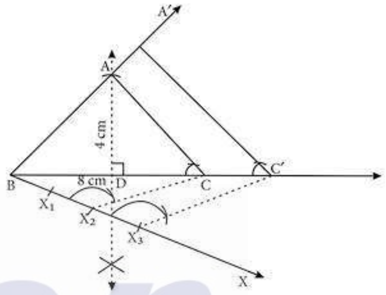
$$\frac{area\,(\Delta\,APC)}{area\,(\Delta\,BQC)} = \left(\frac{\sqrt{2}\,BC}{BC}\right)^2$$

$$= \left(\sqrt{2}\right)^2 = 2$$

$$\therefore \text{area } (\Delta BQC) = \frac{1}{2} \text{ area } (\Delta APC)$$

3. Construct an isosceles triangle whose base is 8 cm and altitude 4 cm and then another triangle whose sides are $1\frac{1}{2}$ times the corresponding sides of the isosceles triangle.

Sol :

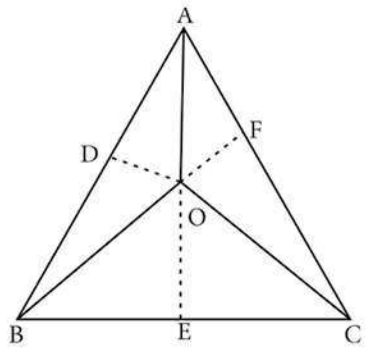


Steps of construction:

1. Drawn BC = 8 cm.
2. Drawn the perpendicular bisector of BC which intersects BC at D.
3. Marked a point A on the above perpendicular such that DA = 4 cm.
4. Joined AB and AC. Thus ΔABC is the required isosceles triangle.
5. Then drawn a ray BX such that $\angle CBX$ is an acute angle.
6. On BX, marked three points (since $1\frac{1}{2} = \frac{3}{2}$) X_1, X_2 and X_3 such that $BX_1 = X_1X_2 = X_2X_3$.
7. Joined X_2 to C.
8. Drawn a line through X_3 parallel to X_2C and intersecting BC extended to C'.
9. Drawn a line through C' parallel to CA intersecting BA extended at A'. Thus $\Delta A'BC'$ is the required triangle.

4. O is any point inside a triangle ΔABC. The bisector of $\angle AOB, \angle BOC, \angle COA$ meet the sides AB, BC and CA in point D, E and F respectively. Show that AD × BE × CF = DB × EC × FA.

Sol :



In ΔAOB, OD is the bisector of $\angle AOB$

$$\frac{OA}{OB} = \frac{AD}{DB} \quad ...(1)$$

In ΔBOC, OE in the bisector of $\angle BOC$

$$\frac{OB}{OC} = \frac{BE}{EC} \quad ...(2)$$

In ΔCOA, OF is the bisector of $\angle COA$

$$\frac{OC}{OA} = \frac{CF}{FA} \quad ...(3)$$

Multiplying the corresponding sides of (1), (2) and (3) we get

$$\frac{OA}{OB} \times \frac{OB}{OC} \times \frac{OC}{OA} = \frac{AD}{DB} \times \frac{BE}{EC} \times \frac{CF}{FA}$$

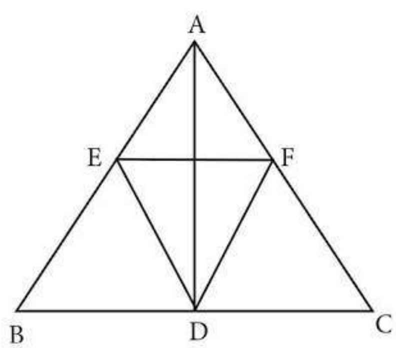
$$1 = \frac{AD}{DB} \times \frac{BE}{EC} \times \frac{CF}{FA}$$

$$DB \times EC \times FA = AD \times BE \times CF$$

$$AD \times BE \times CF = DB \times EC \times FA$$

5. In AD is the median of ΔABC. The bisector of $\angle ADB$ and $\angle ADC$ meet AB and AC in E and F respectively. Prove that EF || BC.

Sol :



Given: In ΔABC, , AD is the median and DE and DF are the bisectors of $\angle ADB$ and $\angle ADC$ respectively meeting AB and AC in E and F respectively.

To prove: $EF \parallel BC$

Proof

In ΔADB, DE is the bisector of $\angle ADB$

$$\therefore \frac{AD}{DB} = \frac{AE}{EB} \quad ...(1)$$

In ΔADC, DF is the bisector of $\angle ADC$

$$\frac{AD}{DC} = \frac{AF}{FC}$$

$$\Rightarrow \frac{AD}{DB} = \frac{AF}{FC}$$

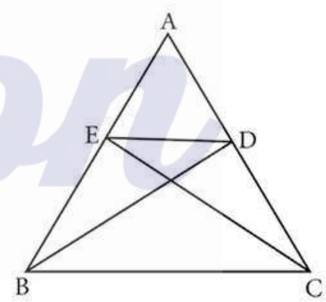
[$\because$ AD is the median BD = DC] ... (2)

From (1) and (2), we get

$$\frac{AE}{EB} = \frac{AF}{FC}$$

6. The bisectors of the angle B and C of a triangle ABC, meet the opposite sides in D and E respectively. If DE $DE \parallel BC$ BC, prove that the triangle is isosceles.

Sol :



Given: ΔABC is a triangle in which the bisectors of $\angle B$ and $\angle C$ meet the sides AC and AB at D and E respectively.

To prove: AB = AC

Construction: Join DE

Proof: In ΔABC, BD is the bisector of $\angle B$

$$\therefore \frac{AB}{BC} = \frac{AD}{DC} \quad ...(1)$$

In ΔABC, CE is the bisector of $\angle C$

$$\therefore \frac{AC}{BC} = \frac{AE}{BE} \quad ...(2)$$

Now DE || BC

$$\Rightarrow \frac{AE}{BE} = \frac{AD}{DC} \quad ...(3)$$

[By thales theorem]

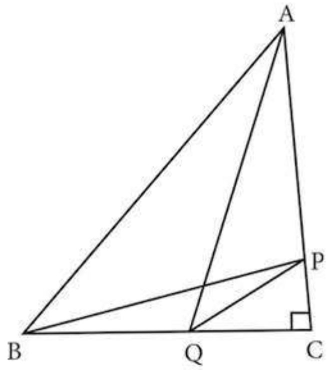
From (1), (2) and (3), we have

$$\frac{AB}{BC} = \frac{AC}{BC} \Rightarrow AB = AC$$

Hence ΔABC is isosceles.

7. P and Q are points on the sides CA and CB respectively of ΔABC right angled at C. Prove that $AC^2 + BP^2 = AB^2 + PQ^2$

Sol :



In right angled triangles ACQ and PCB, we have

$$AQ^2 = AC^2 + CQ^2 \text{ and}$$
$$PB^2 = PC^2 + CB^2$$
$$\Rightarrow AQ^2 + BP^2 = (AC^2 + CQ^2) + (PC^2 + CB^2)$$
$$\Rightarrow AQ^2 + BP^2 = (AC^2 + BC^2) + (PC^2 + QC^2)$$

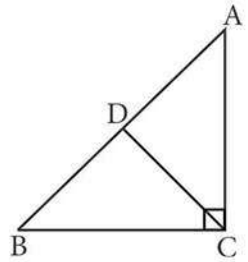
[By pythagoras theorem we have

$AC^2 + BC^2 = AB^2$ and $PC^2 + QC^2 = PQ^2$]

$$\therefore AQ^2 + BP^2 = AB^2 + PQ^2$$

8. ABC is a right triangle right angled at C and $AC = \sqrt{3}$ BC. Prove that $\angle ABC = 60^\circ$.

Sol :



Let D be the midpoint of AB. Join CD.

Since ABC is a right angled triangle, $\angle ACB = 90^\circ$

$$AB^2 = AC^2 + BC^2$$
$$AB^2 = \left(\sqrt{3}\,BC\right)^2 + BC^2$$
$$[\because AC = \sqrt{3}\,BC \text{ given}]$$
$$AB^2 = 3BC^2 + BC^2$$
$$\Rightarrow AB^2 = 4\,BC^2$$
$$AB = 2\,BC \qquad ...(1)$$
$$\text{But } BD = \frac{1}{2}AB$$
$$AB = 2\,BD \qquad ...(2)$$
$$(1) \text{ and } (2) \Rightarrow BD = BC$$

We know that the midpoint of the hypotenuse of a right triangle is equidistance from the vertices

$$\therefore CD = AD = BD$$
$$\Rightarrow CD = BC$$

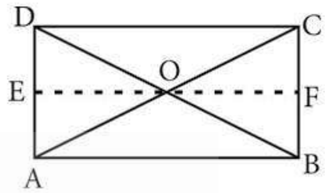
Thus in ΔABC we have BD = CD = BC

$\therefore \Delta BCD$ is equilateral

$$\Rightarrow \therefore \angle ABC = 60^\circ$$

9. A point O in the interior of a rectangle ABCD is joined with each of the vertices A, B, C and D. Prove that $OB^2 + OD^2 = OC^2 + OA^2$

Sol :



Let ABCD be the given rectangle. Let 'O' be the point within it. Join OA, OB, OC and OD.

Through O draw EOF || AB. Then ABFE is a rectangle.

In right triangles ΔOEA and ΔOFC, we have

$$OA^2 = OE^2 + AE^2 \text{ and } OC^2 = OF^2 + CF^2$$
$$OA^2 + OC^2 = (OE^2 + AE^2) + (OF^2 + CF^2)$$
$$OA^2 + OC^2 = OE^2 + OF^2 + AE^2 + CF^2 \qquad ... (1)$$

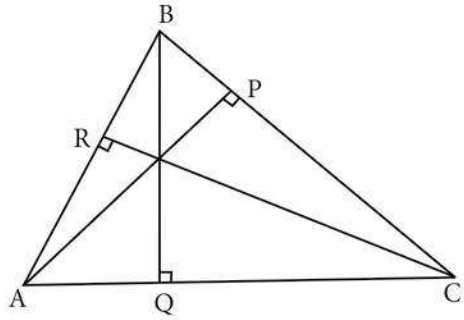
Now in right triangles ΔOFB and ΔDOE we have

$$OB^2 = OF^2 + FB^2 \text{ and } OD^2 = OE^2 + DE^2$$
$$OB^2 + OD^2 = (OF^2 + FB^2) + (OE^2 + DE^2)$$
$$OB^2 + OD^2 = OE^2 + OF^2 + DE^2 + BF^2$$
$$OB^2 + OD^2 = OE^2 + OF^2 + CF^2 + AE^2 \qquad ... (2)$$
$$[\because DE = CF \text{ and } AE = BF]$$

From (1) and (2) we get $OA^2 + OC^2 = OB^2 + OD^2$

10. Show that the altitudes of a triangle are concurrent.

Sol :



Let in ΔABC , P, Q and R are the foot of the perpendiculars drawn from the vertices A, B and C respectively.

$\Delta BRC \sim \Delta BPA$

$\because \angle BRC = \angle BPA = 90^{\circ}$,

$\angle$B is common [$\therefore$ By AA criteria]

$$\frac{BR}{BP} = \frac{BC}{BA} \quad \ldots (1)$$

Similarly $\Delta AQB \sim \Delta ARC$

$$\frac{AQ}{AR} = \frac{AP}{AC} \quad \ldots (2)$$

and $\Delta CPA \sim \Delta CQB$

$$\frac{CP}{CQ} = \frac{AC}{BC} \quad \ldots (3)$$

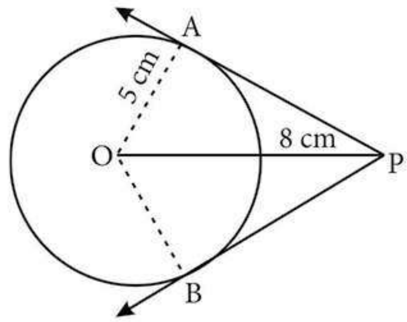
Multiplying (1), (2) and (3), we have

$$\frac{BR}{BP} \times \frac{AQ}{AR} \times \frac{CP}{CQ} = \frac{BC}{AB} \times \frac{AB}{AC} \times \frac{AC}{BC} = 1$$

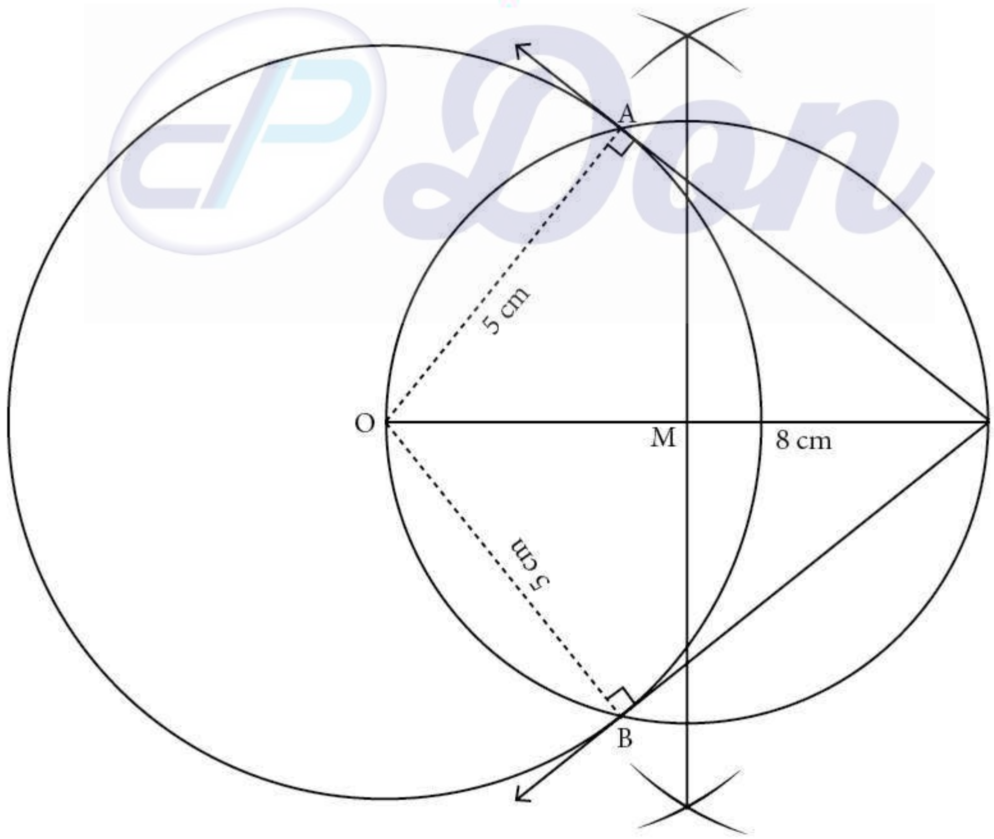
$\therefore$ By Ceva's theorem altitudes are concurrent.

11. Draw a circle of radius 5 cm. Consider a point P at a distance of 8 cm from the circle's centre and draw two tangents from P.

Sol :



Rough Diagram



AP = BP = 6.2 cm

Construction:

Step 1: With center 'O' drawn a circle of radius 5 cm

Step 2: Drawn a line OP = 8 cm

Step 3: Draw a perpendicular bisector of OP which cuts OP at M

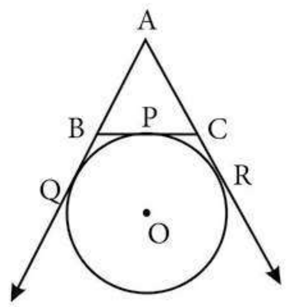
Step 4: With M as center and MO as radius drawn a circle which cuts previous circle at A and B.

Step 5: Joined AP and BP. AP and BP are the required tangents.

AP = BP = 6.2 cm.

12. A circle is touching the side BC of a ΔABC at P and touching AB and AC produced at Q and R. Prove that AQ = $\frac{1}{2}$ (Perimeter of ΔABC)

Sol :



Since the two tangents drawn to a circle from an external point are equal.

$$AQ = AR \qquad \text{... (1)}$$

$$\text{Similarly, } BQ = BP \qquad \text{... (2)}$$

$$\text{and } CR = CP \qquad \text{... (3)}$$

Now perimeter of ΔABC

$= AB + BC + AC$

$= AB + (BP + PC) + AC$

$= AB + (BQ + CR) + AC$ [From (2) and (3)]

$= (AB + BQ) + (CR + AC)$

$= AQ + AR$

$= AQ + AQ$ [From (1)]

$= 2\ AQ$

$$AQ = \frac{1}{2} \text{ (Perimeter of } \Delta ABC\text{)}$$