Question Bank in Page 45

Classification of numbers in Decimal Number System.

Real Numbers .

- Rational Numbers
  - Integer
    - Positive integer
    - Negative integer
    - Whole Number
    - Natural Number .
  - Non Integer Rational Number
- Irrational Numbers .

1) **Natural Number :** Numbers starting from 1, having no fraction part which we use in counting the objects denoted by N .

$$N = \{1, 2, 3, \ldots\}.$$

2) **Whole number :** The system of Natural numbers along with 0 is called whole Number (N) $W = \{0, 1, 2, 3, \ldots\}$.

3) **Types of Natural Number :** 1. Even Number , 2. Odd Number

3). Prime Number : The number which can be divided only by itself and 1 is called prime Number 2, 3, 5, 7, 11, ...

4) Composite Number : The number which can be divided by a number other than 1 and the number it self is called Composite number

5) Consecutive Number : A series of Numbers in which each number is greater by 1 than the number which proceeds it .

Note : 0 is neither even nor odd number .

Division Algorithm : let a, b be two integers s.t $b \neq 0$ on dividing a by b . Let q be the quotient and r be the remainder then the relationship between a, b, q and r is $a = bq + r$.

(ie) Dividend = Divisor × quotient + remainder .

Integers Any number having sign + or − without having any fractional part is called integer (including zero)

$$Z = \{\ldots -3, -2, -1, 0, 1, 2, 3 \ldots\}.$$

Rational Number : A number which can be written in the form of P/q where $P, q \in Z$ and $q \neq 0$ is called Rational Number .

Rational Number can be expressed as decimal based .

1. Terminating: If the prime factors of denominator contains no factor other than 2 and 5 it is terminating.

2. Non terminating and Recurring: If the prime factors of denominator contains factor other than 2 and 5 is non-terminating recurring rational Number.

Rational Number between two rational Number:

If a and b are two distinct rational Numbers s.t $a<b$ then n rational Numbers between a and b may be

$$a_i = a + \frac{b-a}{n+1} \times i \quad \text{where } i = 1, 2, 3, \ldots n.$$

Irrational Number: An irrational number is a non-terminating, non-recurring decimal which cannot written in the form of p/q is called irrational number.

1. The number $\sqrt{x}$ x is not perfect square is an Irrational number and $\sqrt{x}+y$ is also irrational Number.
2. π is an irrational number. which is the ratio of the circumference of a circle to its diameter.
3. 0 is not an irrational Number
※ 4. Sum, difference, product, quotient two irrational number may be rational (or) irrational
※ 5. Sum, difference, product, quotient of one rational and other irrational numbers is always irrational
6. Every real number is either a rational number or an irrational number but not both.

   $\therefore R = Q \cup Q'$, $Q \cap Q' = \varphi$
7. Every terminating or infinite periodic decimal is a rational Number
8. The decimal representation of an irrational number will neither be terminating nor infinite periodic.

Absolute value: The distance of a number $a \in R$ from 0 on the number line is called the absolute value of that number and is denoted by $|a|$ (ie)

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

and Hence 1.1 defines a function known as absolute value fn. from R onto $[0, \infty)$

1. For any $x \in R$ we have $|x| = |-x|$ and thus $|x| = |y|$ iff $x = y$ or $x = -y$.

2. For $|x-a| = r$ iff $r \geq 0$ and $x - a = r$ (or) $x - a = -r$

Some Results For Absolute value.

1) If x and $y \in R$ $|x+y| = |y-x|$ then $xy = 0$

$$|xy| = |x|\,|y|$$

$$\left|\frac{x}{y}\right| = \frac{|x|}{|y|} \quad x, y \in R \text{ and } y \neq 0$$

$$|x+y| \leq |x| + |y|$$

Inequalities involving Absolute values.

1. If $|x| < r$ iff $-r < x < r$ (ie) $x \in (-r, r)$
2. If $|x| > r$ iff $x < -r$ or $x > r$ $\therefore x \in (-\infty, -r) \cup (r, \infty)$
3. If For any $a \in R$ $|x-a| \leq r$ iff $-r \leq x - a \leq r$
4. If For any $a \in R$ $|x-a| \geq r$ iff $x - a \leq -r$ or $x - a > r$ (ie) $x \in (-\infty, a-r] \cup [a+r, \infty]$.

Note: If $|x| > r$, If $r < 0$ then every $x \in R$ satisfies the inequality.

Quadratic Equation: General form of quadratic Eqn

$ax^2 + bx + c = 0$. where $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

1. $\sqrt{u}$ is defined as a real number only for $u \geq 0$
2. when we write $\sqrt{u}$, mean only the non negative root.

3) If $b^2 - 4ac > 0$ The roots are real and distinct and intersect the x axis at two points.

4) If $b^2 - 4ac = 0$ The roots are real and equal.

5) If $b^2 - 4ac < 0$ The roots are imaginary ~~that means~~ no real roots.

The $b^2 - 4ac \geq 0$ The curve touches the ~~curve~~ x axis at one point and if $b^2 - 4ac < 0$ the curve does not touch at any point on x axis.

6) If α, β are the roots of the equation then SR $\alpha + \beta = \dfrac{-b}{a}$ and $\alpha\beta = \dfrac{c}{a}$.

7) If the curve doesnot intersect the x axis then it has no real root.

8) If $1+\sqrt{5}$ is one root of the equation $1-\sqrt{5}$ is also a root of the equation. Since 5 is prime. This is only possible when it is $1+\sqrt{c}$, c is prime.

9) If $f(a)=0$ $f(b)=0$ then a, b are zero of the polynomial $f(x)$

If $f(a)=0$, $x-a$ is a factor of $f(x)$

If $(x-a)$ is a factor of $f(x)=0$ then $x=a$.

10) If we express $f(x)$ as $f(x)=(x-a)^k g(x)$ $g(x)\neq 0$. Then the value of k is called the multiplicity of the zero a.

11) The conjugate of $a+b\sqrt{p}$ is $a-b\sqrt{p}$ where p is prime.

12) $y=a^x \equiv \log_a y = x$.

13) $\log_a(\,)$ is defined only for positive real number.
Also $a^0=1$ $\log(1)=0$. for any base a.

14) 1) $a^{\log_a x} = x$ $\forall x \in (0,\infty)$ and $\log_a a^y = y\log_a a = y$. $\because \log_a a = 1$

2) $\log_a(xy) = \log_a x + \log_a y$ $\forall x, y > 0$

3) $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$ "

4) $\log_a x^n = n\log_a x$ "

5) $\frac{\log_a x}{\log_a b} = \log_b x \Rightarrow \log_b x \cdot \log_a b = \log_a x$.

Note: $\log_{10} x$ is called common logarithm.
$\log_e x$ is called natural logarithm.

6) $\log_a x = \frac{1}{\log_x a}$

7) $\log_{a^n} x = \frac{1}{n}\log_a x$

8) $\log_{a^n} x^m = \frac{m}{n}\log_a x$.

Properties of exponential function

For $a, b > 0$ and $a \neq 1$ $b \neq 1$

1. $a^{x+y} = a^x \cdot a^y$ $x, y \in R$
2. $\frac{a^x}{a^y} = a^{x-y}$
3. $(a^x)^y = a^{xy}$
4. $(ab)^x = a^x b^x$
5. $a^x = 1$ iff $x = 0$.

1. P.T $\sqrt{2}$ is an irrational Number.

Proof: suppose if $\sqrt{2}$ is rational Number

Let $\sqrt{2} = \frac{m}{n}$ where m and n are positive integers with no common factors greater than 1.

TBP

$\Rightarrow m^2 = 2n^2$

$\Rightarrow m^2$ is even, hence m is also even.

Let $m = 2K$.

$2n^2 = 4K^2$

$n^2 = 2K^2 \Rightarrow n$ is also even.

which implies that m, and n are even numbers having Common factor 2.

which is a contradiction to the fact that m and n have no Common factor greater than 1.

$\therefore \sqrt{2}$ is irrational number.

2. P.T $\sqrt{3}$ is irrational number:

Proof: Suppose $\sqrt{3}$ is rational number.

Let $\sqrt{3} = \frac{m}{n}$ where m, n are positive integers with no common factor greater than 1.

TBP

$m^2 = 3n^2$

$2m^2 = 6n^2$

$= 3(2n^2) \Rightarrow m^2$ is even $\because 2n^2$ is divisible by 2.

$\therefore$ m is also even $\Rightarrow m = 2k$

Next $3n^2 = 4k^2$

$\therefore$ n is also even.

$\therefore$ both m and n are even having Common factors 2.

which is the Contradiction to the fact that m, n have no common factor greater than 1.

$\therefore \sqrt{3}$ is irrational number.

2) Are there two distinct irrational numbers s.t their difference is
TBP rational number Justify

Sol: Let the two distinct rational numbers are $3+\sqrt{5}$, $6+\sqrt{5}$.
Their difference is $(3+\sqrt{5})-(6+\sqrt{5})=-3$ which is rational number

3. Find two irrational numbers s.t their sum is rational number
TBP Can you find the irrational number whose product is a rational number.

Sol: let $5+\sqrt{7}$, $7-\sqrt{7}$ are two irrational number.

Sum $5+\sqrt{7}+7-\sqrt{7}=12$ which is rational.

Consider $5+\sqrt{7}$ and $5-\sqrt{7}$

Product $(5+\sqrt{7})(5-\sqrt{7})=25-7=18$ is rational number.

4. Find the positive number smaller than $\frac{1}{2^{1000}}$. Justify.
TBP

Sol: Given number $\frac{1}{2^{1000}}$

We know $1000<1001$

But $\frac{1}{2^{1001}}<\frac{1}{2^{1000}}$ (ie) $\frac{1}{2^{1001}}$ is less than $\frac{1}{2^{1000}}$.

5) classify each element of $\{\sqrt{7}, -\frac{1}{4}, 0, 3.14, 4, \frac{22}{7}\}$ as a member of
$N, Q, R-Q$ (or) Z.

$\sqrt{7}$ is an irrational Number $\sqrt{7}\in R$.

$-\frac{1}{4}$ negative ration number $-\frac{1}{4}\in Q$

0 is an integer $0\in Z$.

3.14 is an irrational number $R-Q$

4 is positive integ $4\in R-Q$

$\frac{22}{7}=3.14\in R$ which is irrational number.

1\. Solve $|2x-17|=3$ for x.

TBP

$|2x-17|=3$ then $2x-17=\pm 3$. (or) $2x-14=0$

$2x-20=0$ $\quad 2x=14$

$x=\frac{20}{2}=10$ $\quad x=7$.

$\therefore x=10$ (or) $x=7$.

2\) Solve: $3|x-2|+7=19$ for x.

TBP

$3|x-2|+7=19$

$3|x-2|=19-7$

$=12$

$|x-2|=\frac{12}{3}=4$

$\therefore |x-2|=4 \Rightarrow x-2=\pm 4$

$x=6$ or $x=-2$

3\) Solve $|2x-3|=|x-5|$

We know that $|u|=|v|$ iff $u=v$ or $u=-v$

TBP

(ie) $|2x-3|=|x-5|$

$2x-3=x-5$ (or) $2x-3=5-x$.

$x=-2$ $\quad 3x=8$

$x=8/3$.

4\) Solve $|x-9|<2$ for x

TBP

Sol: $|x-9|<2$

$\Rightarrow -2<(x-9)<2$

$-2+9<x<2+9$

$+7<x<11 \Rightarrow +7\leq x<11$

5\) Solve $\left|\frac{2}{x-4}\right|>1$ $x\neq 4$.

Sol: $\left|\frac{2}{x-4}\right|>1 \Rightarrow |2|>|x-4|$

$\Rightarrow -2<x-4<2$. $\quad x\neq 4$

TBP

$-2+4<x<2+4$.

$2<x<6$. But $x\neq 4$

$\therefore x\in (2,4)\cup(4,6)$

6\) Solve: $|3-x|<7$.

TBP

$\Rightarrow -7<3-x<7 \Rightarrow$

$+7>x-3>-7 \Rightarrow$ $\quad \Rightarrow 10>x>-4 \Rightarrow -4<x<10$

7) $|4x-5| \geq -2$.

Sol: $|4x-5| \geq -2$

TBP

If $|x| > r$ and if $r < 0$ then every $x \in R$ satisfies the inequality.

$\therefore$ Result is R.

8) $\left|3-\frac{3}{4}x\right| \leq \frac{1}{4}$

Sol: $\left|3-\frac{3}{4}x\right| \leq 1/4$

TBP

$$-\frac{1}{4} \leq \left(3-\frac{3}{4}x\right) \leq 1/4$$

$$-\frac{1}{4}-3 \leq -\frac{3}{4}x \leq \frac{1}{4}-3$$

$$-\frac{13}{4} \leq -\frac{3}{4}x \leq -\frac{11}{4}$$

$$\frac{13}{4} \geq \frac{3}{4}x - \frac{11}{4}$$

$\div 3/4$ $$\frac{13}{3} \geq x \geq \frac{11}{3}.$$

$$\Rightarrow \frac{11}{3} \leq x \leq \frac{13}{3}$$

$\therefore$ The solution set is $\frac{11}{3} \leq x \leq \frac{13}{3}$

9) Solve $\frac{1}{|2x-1|} < 6$ and express the solution using the interval notation.

Sol: $\frac{1}{|2x-1|} < 6 \Rightarrow |2x-1| > \frac{1}{6}.$

TBP

$$\Rightarrow -\frac{1}{6} > (2x-1) > \frac{1}{6}$$

$$\Rightarrow -\frac{1}{6}+1 > 2x > \frac{1}{6}+1$$

$$\frac{5}{6} > 2x > \frac{7}{6}$$

$$\frac{5}{12} > x > \frac{7}{12}$$

$$\Rightarrow x \in (-\infty, 5/12) \cup (7/12, \infty)$$

10) Solve $-3|x|+5 \le -2$ and Graph the solution set in the number line.

Sol: $-3|x|+5 \le -2$

TBP

$$-3|x| \le -7$$

$$|x| \le 7/3$$

$$\Rightarrow -7/3 \le x \le 7/3$$

$\therefore$ The solution set is $(-\infty, -7/3] \cup [7/3, \infty)$

11) Solve $2|x+1|-6 \le 7$ and Graph the solution set in number line

Sol: $2|x+1|-6 \le 7$

TBP

$$2|x+1| \le 13$$

$$|x+1| \le \frac{13}{2}$$

$$-\frac{13}{2} \le (x+1) \le 13/2$$

$$-\frac{13}{2}-1 \le x \le \frac{13}{2}-1$$

$$-\frac{15}{2} \le x \le \frac{11}{2} \quad \therefore x \in \left[-15/2, \frac{11}{2}\right]$$

12) Solve $\frac{1}{5}|10x-2| < 1$.

Sol: $\frac{1}{5}|10x-2| < 1$

TBP

$$\Rightarrow |10x-2| < 5$$

$$\Rightarrow -5 < (10x-2) < 5$$

$$-5+2 < 10x < 5+2$$

$$-3 < 10x < 7$$

$$-\frac{3}{10} < x < \frac{7}{10}$$

13) Solve $|5x-12| < -2$

Solu:

TBP

$|5x-12| < -2$ is not possible

$\therefore$ no Solution.

Linear inequalities:

14) Represent the following inequalities in the interval notation.

TBP

1) $x \geq -1$ and $x < 4$

$x \in [-1, 4)$

2. $x \leq 5$ and $x \geq -3$

$x \in [-3, 5]$

3. $x < -1$ or $x < 3$

$x \in (-\infty, 3)$

4) $-2x > 0$ (or) $3x - 4 < 11$

$x < 0$ $\quad$ $3x < 15$

$x < 5$

$\therefore x \in (-\infty, 5)$

---

15) Solve $23x < 100$ $\quad$ 1) x is natural number

2) x is an integer.

TBP When x is natural number x satisfies for 1, 2, 3, 4

When x is an integer x satisfies for ....$-3, -2, -1, 0, 1, 2, 3, 4$.

---

16) Solve $-2x \geq 9$ when i) x is real number

TBP

2) x is an integer

3) x is natural number.

$-2x \geq 9$

$x \leq -\frac{9}{2}$ $\quad$ 1. $x \in \left(-\infty, -\frac{9}{2}\right]$

2) .... $-7, -6, -5$

3. No solution.

---

17) Solve: $\dfrac{3(x-2)}{5} \leq \dfrac{5(2-x)}{3}$

TBP

$9(x-2) \leq 25(2-x)$

$9x - 18 \leq 50 - 25x$

$34x \leq 68$

$x \leq 2.$ $\quad$ $\therefore x \in (-\infty, 2]$.

18) $\frac{5-x}{3} < \frac{x}{2} - 4$.

TBP: $\frac{5-x}{3} < \frac{x-8}{2}$

$10 - 2x < 3x - 24$

$34 < 5x$

$5x > 34$

$x > \frac{34}{5}$ $\quad \therefore x \in \left(\frac{34}{5}, \infty\right)$

19) To secure A grade one must obtain an average of 90 marks or more in 5 subjects each of max 100 marks. If one score 84, 87, 95, 91 in first four subjects what is the minimum marks one scored in the fifth subject to get A Grade in the course.

TBP

Sol: given $\frac{84+87+95+91+x}{5} \geq 90$

$\frac{357+x}{5} \geq 90$

$357 + x \geq 450$

$x \geq 450 - 357$

$x \geq 93$.

20) A manufacturer has 600 litres of a 12 percent solution of acid. How many litres of a 30% acid solution must be added to it so that the acid content in the resulting mixture will be more than 15% but less than 18%.

TBP

Sol: Let x be the number of litters of 30% acid solution.

Given: 30% of $x + 12\%$ of $600 > 15\%(600 + x)$

$\frac{30x}{\cancel{100}} + \frac{12}{\cancel{100}} \times 600 > \frac{15}{\cancel{100}}(600 + x)$

$30x + 7200 > 9000 + 15x$.

$15x > 1800$

$x > 120$

2) $30\% x + 12\% \cdot 600 < 18\% (600 + x)$

$$\frac{30x}{100} + \frac{12}{100} \times 600 < \frac{18}{100}(600 + x)$$

$$30x + 7200 < 10800 + 18x.$$

$$12x < 3600$$

$$x < 300$$

21) Find all pairs of consecutive odd natural numbers
TBP. both of which are larger than 10 and their sum is less than 40.

Sol: Let x be the small odd number and another one is $x+2$.

Given $x > 10$ and $x+2 > 10$.

$$\Rightarrow x > 10$$

Bont $x + (x+2) < 40$

$$2x < 38$$

$$x < 19$$

$\Rightarrow$ $10 < x < 19$. Since x is odd 11, 13, 15, 17, 19

Hence the required possible pairs is (11,13) (13,15) (15,17) (17,19)

22) A model Rocket is launched from the ground. the height h reached by the rocket after t seconds from lift off is given by $h(t) = -5t^2 + 100t$ $0 \le t \le 20$. At what time the rocket is 495 ft above the ground..

$$h(t) = -5t^2 + 100t$$

Given $0 < h(t) < 495$

$$0 < -5t^2 + 100t < 495$$

$$0 < -5t^2 + 100t - 495 < 0$$

$$\Rightarrow \cancel{5}t^2 - \overset{20}{\cancel{100}}t + \overset{99}{\cancel{495}} = 0$$

$$t^2 - 20t + 99 = 0$$

$$(t - 11)(t - 9) = 0$$

$$t = 9, \text{ or } 11.$$

$t = 9$ sec or 11 sec..

23) A plumber can be paid according to the following schemes. In the first scheme he will be paid rupees 500 plus rupees 70/- per hour and in the second scheme he will be paid rupees 120 per hr. If he works x hrs, then for what value of x does the first give better wages.

TBP Sol: wages from the first scheme $500 + 70x$.

" Second scheme $120x$.

$$500 + 70x > 120x.$$

$$500 > 50x$$

$$50x < 500$$

$$x < 10$$

The value of x so that the first scheme gives better wages is $x = 1, 2, 3, 4, 5, 6, 7, 8, 9$.

24) A and B are working on a similar jobs but their monthly salaries differ by more than Rs 6000/=. If B earns rupees 27,000 per month what are the possibilities of A's salary per month.

TBP

Let the salary of A be x.

Then B's salary will be more than $6000 + x$.

But B's salary is 27,000

$$\therefore \ 6000 + x < 27,000$$

$$x < 21,000$$

(or)

$$x - 6000 > 27,000$$

$$x > 33,000.$$

25) If a and b are the roots of the equation $x^2 - px + q = 0$ find

TBP $\frac{1}{a} + \frac{1}{b}$.

Sol: $x^2 - px + q = 0$

$SR.: a + b = -\frac{b}{a} = p$.

$PR \quad ab = \frac{c}{a} = q$.

$\frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab} = \frac{p}{q}$.

26) Find the complete set of values of a for which the quadratic

TBP $x^2 - ax + a + 2 = 0$ has equal roots.

Sol: $x^2 - ax + (a+2) = 0 \quad a = 1$

$b = -a$

$c = a + 2$

For Equal roots

$b^2 - 4ac = 0$

$a^2 - 4(a+2) = 0$

$a^2 - 4a - 8 = 0 \quad a = 1, \ b = -4, c = -8$

$a = \frac{4 \pm \sqrt{16+32}}{2} = \frac{4 \pm \sqrt{48}}{2} = \frac{4 \pm 4\sqrt{3}}{2}$

$= 2 \pm 2\sqrt{3}$.

27) Find the number of solutions $x^2 + |x-1| = 1$.

TBP

Case i: For $x \geq 1 \quad |x-1| = (x-1)$

$x^2 + x - 1 = 1$

$x^2 + x - 2 = 0 \Rightarrow (x+2)(x-1) = 0$

$x = -2, 1$

$\therefore x \geq 1 \quad x = 1$ only.

For $x < 1 \qquad |x-1| = -(x-1)$

$\therefore \quad x^2 - x + 1 = 1$

$x^2 - x = 0$

$x(x-1) = 0 \qquad \therefore x < 1$ choose $x = 0$

$x = 0 \quad x = 1$

$\therefore$ The solution is $= \{0, 1\}$. $\therefore$ The equation has two solutions.

28) Construct a quadratic with roots 7 and −3.

TBP Sol: $SR = 7 - 3 = 4$

$PR = 7(-3) = -21$.

Equation $x^2 - (SR)x + PR = 0$

$x^2 - 4x - 21 = 0$.

29) A quadratic polynomial has one of its roots $1+\sqrt{5}$ and it satisfies $P(1) = 2$ find the quadratic polynomial.

Sol: Since $1+\sqrt{5}$ is one root $1-\sqrt{5}$ is also a root.

∴ Let the polynomial be $P(x) = a\left(x - (1+\sqrt{5})\right)\left(x - (1-\sqrt{5})\right)$

$= a\left((x-1) - \sqrt{5}\right)\left((x-1) + \sqrt{5}\right)$

$= a\left((x-1)^2 - 5\right)$

$= a\left(x^2 - 2x + 1 - 5\right)$

$= a\left(x^2 - 2x - 4\right)$

∵ $P(1) = 2$ $\quad P(1) = a(1 - 2 - 4) = 2$

$-5a = 2$

∴ The polynomial $p(x) = -\frac{2}{5}\left(x^2 - 2x - 4\right)$ $\quad a = \frac{2}{-5}$

29) a) Discuss the nature of roots.

1) $-x^2 + 3x + 1 = 0$ $\quad a = -1$, $b = 3$, $c = 1$

$\Delta = b^2 - 4ac$

$= 9 + 4 > 0$

The roots are real and unequal.

2) $4x^2 - x - 2 = 0$ $\quad a = 4$, $b = -1$, $c = -2$

$\Delta = b^2 - 4ac$

$= 1 + 32 > 0$

∴ The roots are real and unequal.

3) $9x^2 + 5x = 0$ $\quad a = 9$, $b = 5$, $c = 0$

$\Delta = b^2 - 4ac$

$= 25 - 0 > 0$

The roots are real and unequal.

30) If α, β are the roots of the quadratic eqn $x^2+\sqrt{2}x+3=0$

TBP Form the quadratic polynomial with zero $1/\alpha, 1/\beta$.

Eqn $x^2 + \sqrt{2}x + 3 = 0$ $\quad a = 1$, $b = \sqrt{2}$, $c = 3$.

$SR = \alpha+\beta = -\frac{b}{a} = -\sqrt{2}$

$PR = \alpha\beta = \frac{c}{a} = 3$.

Now the given roots are $\frac{1}{\alpha}, \frac{1}{\beta}$.

$SR = \frac{1}{\alpha}+\frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{-\sqrt{2}}{3}$.

$P.R. \quad \frac{1}{\alpha\beta} = \frac{1}{3}$.

$\therefore$ polynomial eqn $p(x) = x^2 - \left(\frac{-\sqrt{2}}{3}\right)x + \frac{1}{3} = 0$

$= 3x^2 + \sqrt{2}x + 1 = 0$

31) If one root of $k(x-1)^2 = 5x-7$ is double the other root

TBP. S.T $k = 2$, or -25

Sol: $k(x-1)^2 = 5x-7$

$k(x^2+1-2x) = 5x-7$

$kx^2+k-2kx-5x+7=0$.

$kx^2-2kx-5x+k+7=0$

$kx^2-x(2k+5)+(k+7)=0$.

$a = k$, $b = -(2k+5)$, $c = k+7$.

Given one root is double the other

$\alpha, 2\alpha$.

$\alpha+2\alpha = \frac{2k+5}{k}$

$3\alpha = \frac{2k+5}{k}$

$\alpha = \frac{2k+5}{3k}$

$2\alpha^2 = \frac{k+7}{k}$

$\alpha^2 = \frac{k+7}{2k}$.

$\therefore \frac{k+7}{2\cancel{k}} = \frac{(2k+5)^2}{9k\cancel{2}}$

$\frac{k+7}{2} = \frac{4k^2+20k+25}{9k} \Rightarrow 9k^2+63k = 8k^2+40k+50$

$k^2+23k-50=0$

$(k+25)(k-2)=0$

$\Rightarrow k = 2, -25$

32) If the difference of the roots of the equation $2x^2-(a+1)x+(a-1)=0$ is equal to their product then prove that $a=2$.

TBP

Sol: $2x^2-(a+1)x+(a-1)=0$

$a=2$

$b=-(a+1)$

$c=a-1$.

Let α, β are the roots

$$\alpha+\beta=\frac{a+1}{2}$$

$$\alpha\beta=\frac{a-1}{2}.$$

Given: $\alpha-\beta=\alpha\beta. \Longrightarrow$

$$(\alpha-\beta)^2=\alpha^2\beta^2$$

$$\alpha^2+\beta^2-2\alpha\beta=(\alpha\beta)^2$$

$$(\alpha+\beta)^2-4\alpha\beta=(\alpha\beta)^2$$

$$\left(\frac{a+1}{2}\right)^2-\overset{2}{\cancel{4}}\left(\frac{a-1}{\cancel{2}}\right)=\frac{(a-1)^2}{4}$$

$$\frac{a^2+1+2a}{4}-(2a-2)=\frac{a^2+1-2a}{4}$$

$$\frac{a}{2}+\frac{a}{2}-2a+2=0$$

$$-a=-2$$

$$a=2$$

33) Find the condition that one of the root of $ax^2+bx+c=0$ may be i) negative of the other, ii) thrice the other iii) reciprocal of the other.

TBP

Given Eqn: $ax^2+bx+c=0$.

1) If one root is negative of other

$\alpha, -\alpha$. $\quad \alpha-\alpha=\frac{-b}{a}$

$0=-b \Longrightarrow b=0$

2) $\alpha=3\alpha$. $\quad \alpha+3\alpha=-b/a$

$4\alpha=-\frac{b}{a}$

$\alpha=-\frac{b}{4a}$.

Product $\alpha\cdot 3\alpha=\frac{c}{a}$.

$$3\alpha^2=\frac{c^2}{a^2}$$

$$\alpha^2=\frac{c^2}{3a^2}$$

But $\alpha=-\frac{b}{4a}$

$$\alpha^2=\frac{b^2}{16a^2}$$

$$\therefore \frac{b^2}{16\cancel{a^2}}=\frac{c^2}{3\cancel{a^2}}$$

$$3b^2=16c^2$$

3) $\alpha, \frac{1}{\alpha}$

$$\cancel{\alpha}\cdot\frac{1}{\cancel{\alpha}}=\frac{c}{a}$$

$$\underline{\underline{a=c}}$$

34) If the equations $x^2-ax+b=0$ and $x^2-ex+f=0$ have one root is common and if the second eqn has equal roots then P.T $ae=2(b+f)$.

TBP

Let α, β are the roots of the eqn $x^2-ax+b=0$ and α be the common root

$$\therefore \alpha+\beta = a \quad \text{---} (1)$$
$$\alpha\beta = b. \quad \text{---} (2)$$

Let α, α are the roots of the second eqn $x^2-ex+f=0$

$$\alpha+\alpha = 2\alpha = e$$
$$\alpha = \frac{e}{2} \quad \text{---} (3)$$
$$\alpha^2 = f$$
$$\frac{e^2}{4} = f$$
$$\Rightarrow e^2 = 4f$$

$\therefore$ sub in (2)

$$\frac{e}{2}\beta = b$$
$$\beta = \frac{2b}{e}.$$

Now $\alpha+\beta = a$

$$\frac{e}{2}+\frac{2b}{e} = a.$$
$$\frac{e^2+4b}{2e} = a$$
$$e^2+4b = 2ae$$
$$4f+4b = 2ae$$
$$\cancel{4}^{2}(f+b) = \cancel{2}ae$$
$$ae = 2(f+b).$$

35) Without sketching the graphs find whether the graphs of the following functions will intersect the x axis and if so in how many points.

TBP

1) $y = x^2+x+2$

$x^2+x+2=0$ $\quad a=1, b=1, c=2$

$\Delta = b^2-4ac$
$= 1-8 < 0$

$\therefore$ The curve does not even touch the curve

2) $y = x^2-3x-7$ $\quad a=1, b=-3, c=-7$

$x^2-3x-7=0$

$\Delta = 9+28 > 0$

$\therefore$ The curve intersect the x axis at two points.

3) $y = x^2+6x+9$ $\quad a=1, b=6, c=9$

$\Delta = 36-36 = 0.$

$\therefore$ The curve touch the x axis at one point

36) Write $f(x) = x^2+5x+4$ in completed square form

$$x^2+5x+4 = \left(x+\frac{5}{2}\right)^2+4-\frac{25}{4}$$
$$= \left(x+\frac{5}{2}\right)^2-\left(\frac{3}{2}\right)^2$$

Quadratic inequalities

36) Solve $3x^2+5x-2 \le 0$

Sol: let $3x^2+5x-2=0$

TBP

$3x^2+6x-x-2=0$

$3x(x+2)-1(x+2)=0$

$(x+2)(3x-1)=0$

$x=-2, 1/3$

$-2 \quad 1/3$

Given $(x+2)(3x-1) \le 0$

	$x+2$	$3x-1$	$3x^2+5x-2$
$(-\infty,-2)$	$-$	$-$	$+$
$[-2,3]$	$+$	$-$	$-$
$(3,\infty)$	$+$	$+$	$+$

$\therefore$ The solution set is $[-2, 3]$

37. Solve $\sqrt{x+14} < x+2$.

Sol: $\sqrt{x+14}$ is defined only for $x+14 \ge 0$

TBP

$\therefore \quad x \ge -14$

and $x+2>0 \Rightarrow x \ge -2$

But $(x+14) < (x+2)^2$

$(x+14) < x^2+4x+4$

$0 < x^2+3x-10$

(ie) $x^2+3x-10>0$

$(x+5)(x-2)>0$

$(-5, 2)$

when $x \in (-5,2)$ $\quad (+)(-)<0$

when $x \in (-\infty,-5)$ $\quad (-)(-)>0$

$x \in (2,\infty)$ $\quad (+)(+)>0$

$\therefore$ $x \ge -2$ the solution to be $x>2$.

38) Solve the equation $\sqrt{6-4x-x^2} = x+4$

TBP

$x+4 \geq 0$ and $6-4x-x^2 = (x+4)^2$

$6-4x-x^2 = x^2+8x+16$

$\Rightarrow x \geq -4$

$2x^2+12x+10=0$

$x^2+6x+5=0$

$(x+5)(x+1)=0$

$x=-1,-5$

But $x \geq -4$ Hence $x=-1$ is the solution.

39) Solve $2x^2+x-15 \leq 0$

TBP

Sol: let $2x^2+x-15=0$ $\quad -30$

$2x^2+6x-5x-15=0$

$2x(x+3)-5(x+3)=0$

Given. $(2x-5)(x+3) \leq 0$ $\quad [-3, 5/2]$

When $x \in (-\infty, -3)$ $\quad (-)(-1) = +$

$x \in [-3, 5/2]$ $\quad (-1)(+) = -$

$x \in (5/2, \infty)$ $\quad (+)(+1) = +$

$\therefore x \in [3, 5/2]$ is the solution.

40) Solve $-x^2+3x-2 \geq 0$

TBP

Sol: $-x^2+3x-2 \geq 0$

$x^2-3x+2 \leq 0$

Let $x^2-3x+2=0$

Given. $(x-2)(x-1) \leq 0$ $\quad [1, 2]$

when $x \in (-\infty, 1)$ $\quad (-)(-) = +$

$x \in [1, 2]$ $\quad (-)(+) \leq 0 = -$

$x \in (2, \infty)$ $\quad (+)(+) > 0 = +$

$\therefore x \in [1, 2]$ is the solution.

Polynomial function.

Remainder Theorem: If a polynomial $f(x)$ is divided by $x-a$ then the remainder is $f(a)$. Thus the remainder $c = f(a) = 0$ iff $x-a$ factor of $f(x)$.

Definition: A real number a is said to be a zero of the polynomial $f(x)$ if $f(a) = 0$. If $x = a$ is a zero of $f(x)$ then $x-a$ is a factor of $f(x)$

41) Find the zeros of the polynomial function $f(x) = 4x^2 - 25$

TBP

$$f(x) = 4x^2 - 25$$
$$= (2x+5)(2x-5)$$

$\therefore$ The zeros of the polynomial are $x = -5/2$, $x = 5/2$

42) If $x = -2$ is one root of $x^3 - x^2 - 17x = 22$ then find the other roots of the equation.

TBP

$$x^3 - x^2 - 17x - 22 = 0$$

$$\begin{array}{r|rrrr} -2 & 1 & -1 & -17 & -22 \\ & & -2 & 6 & +22 \\ \hline & 1 & -3 & -11 & \boxed{0} \end{array}$$

$\therefore$ $x^3 - x^2 - 17x - 22 = (x+2)(x^2 - 3x - 11) = 0$

$\Rightarrow$ $x^2 - 3x - 11 = 0$. $a = 1$, $b = -3$, $c = -11$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{3 \pm \sqrt{9+44}}{2} = \frac{3 \pm \sqrt{53}}{2}$$

43) Find the real roots of $x^4 - 16 = 0$

TBP

$$x^4 - 16 = 0$$
$$(x^2)^2 - 4^2 = 0$$
$$(x^2+4)(x^2-4) = 0$$
$$x^2 = -4 \quad x^2 = 4$$
$$x = \pm 2.$$

when $x^2 = -4$

x is imaginary

44) Solve $(2x+1)^2 - (3x+2)^2 = 0$

TBP

$$(2x+1)^2 - (3x+2)^2 = 0$$
$$(4x^2+4x+1) - (9x^2+12x+4) = 0$$
$$-5x^2 - 8x - 3 = 0$$

$$5x^2 + 8x + 3 = 0$$
$$5x^2 + 5x + 3x + 3 = 0$$
$$5x(x+1) + 3(x+1) = 0$$
$$(x+1)(5x+3) = 0 \Rightarrow x = -1, -3/5$$

Method of undetermined co-efficients.

45) Find the quadratic polynomial $f(x)$ s.t $f(0)=1$, $f(-2)=0$ and $f(1)=0$

Sol: Let the polynomial be $f(x) = ax^2+bx+c$.

TBP

$$f(0)=1 \Rightarrow 0+0+c=1$$
$$c=1$$

$$f(1) = a+b+c = 0$$
$$f(-2) = 4a-2b+c=0$$

$\because c=1$ $\quad a+b=-1$

$$4a-2b=-1$$

$$2a+2b=-2$$
$$4a-2b=-1$$
$$\overline{a=-3.} \Rightarrow a=-\frac{3}{6}=-1/2$$

$$-\frac{1}{2}+b=-1$$
$$b=-1+1/2=-1/2$$

$\therefore P(x) = -\frac{1}{2}x^2-\frac{1}{2}x+1$

(or) Since $f(-2)=0$
$f(1)=0$
$x+2$, $x-1$ are the factors.

$\therefore f(x) = a(x+2)(x-1)$

and $f(0) = a(2)(-1) = 1$

$a=-\frac{1}{2}$

$\therefore f(x) = -\frac{1}{2}(x+2)(x-1)$

$= -\frac{1}{2}(x^2+x-2)$ ✓

46) Construct a cubic polynomial functions having zeros $2/5$, $1+\sqrt{3}$ s.t $f(0)=-8$

TBP $\because$ $1+\sqrt{3}$ is one root $1-\sqrt{3}$ is also another root.

$\therefore$ The given roots are $x = 2/5, 1+\sqrt{3}, 1-\sqrt{3}$.

$$P(x) = a(x-2/5)(x-(1+\sqrt{3}))(x-(1-\sqrt{3}))$$
$$= a(x-2/5)((x-1)-\sqrt{3})((x-1)+\sqrt{3})$$
$$= a(x-2/5)\left[(x-1)^2-3\right]$$

$\therefore P(0)=-8$ $\quad a(-2/5)(-2) = -8$

$$a = -\frac{\cancel{8}^{2}\times 5}{\cancel{4}} = -10$$

$$\therefore P(x) = -10(x-2/5)((x-1)^2-3)$$
$$= -10(x-2/5)(x^2-2x+\underbrace{1-3}_{-2})$$
$$= -10\left(x^3-2x^2-2x-\frac{2}{5}x^2+\frac{4}{5}x+\frac{4}{5}\right)$$
$$= -10x^3+24x^2+12x-8)$$

47) Find the quadratic polynomial $f(x)$ s.t $f(0)=1$, $f(-2)=0$, $f(1)=0$.

47) P.T $ap+q=0$ if $f(x)=x^3-3px+2q$ is divisible by $x^2+2ax+a^2$

TBP

Sol: The degree of the $f(x)$ is 3 and the leading Co.eff. is 1.

If $g(x)$ divides $f(x)$ ⇒ $f(x)=g(x)(x+b)$ where b is any const.

$$\therefore\ x^3-3px+2q=(x^2+2ax+a^2)(x+b)$$

Co.eff. x^2 $\quad 2a+b=0 \Rightarrow b=-2a$.

Co.eff. x $\quad 2ab+a^2=-3p \Rightarrow -4a^2+a^2=-3p$

$\cancel{-}3a^2=\cancel{-}3p$

$a^2=p$.

$a^2b=2q$

$\cancel{-2}a^3=\cancel{-}2q \Rightarrow a^3=q$

$$\therefore\ ap+q=a^3-a^3=0$$

48) Find the roots of the polynomial equation $(x-1)^3(x+1)^2(x+5)=0$

TBP and state their multiplicites

Sol: Let $f(x)=(x-1)^3(x+1)^2(x+5)$

clearly $x=1,-1,-5$

Hence the roots are 1 with multiplicites 3 and -1 with 2 and -5 with one.

49) Solve $x=\sqrt{x+20}$ $x\in R$.

TBP.

Sol: $\sqrt{x+20}$ defined when $x+20\geq 0$ (Positive)

$\therefore\ x=\sqrt{x+20}$

$x^2=x+20 \Rightarrow x^2-x-20=0$

$(x-5)(x+4)=0$

$x=-4,5$

$\therefore$ x is positive $x=5$.

50. The equation $x^2-bx+a=0$ and $x^2-bx+b=0$ have one root in common. The other root of first and second equations are integers in the ratio 4:3 Find the common root.

Sol: Let α be the common root of the equs.

Let $\alpha, 4\beta$ are the roots of $x^2 - 6x + a = 0$ $\quad a=1$, $b=-6$, $c=a$.

$$\alpha + 4\beta = 6 \quad \text{--- (1)}$$

$$\alpha \cdot 4\beta = a \quad \text{--- (2)}$$

Let $\alpha, 3\beta$ are the roots of $x^2 - bx + 6 = 0$. $\quad a=1$, $b=-b$, $c=6$.

$$\alpha + 3\beta = b \quad \text{--- (3)}$$

$$\alpha \cdot 3\beta = 6 \quad \text{--- (4)}$$

$$\frac{(2)}{(4)} \quad \frac{4\cancel{\alpha\beta}}{3\cancel{\alpha\beta}} = \frac{a}{\cancel{6}\,2}$$

$$\Rightarrow a = 8$$

$-4, -2$

$$\therefore x^2 - 6x + 8 = 0$$

$$(x-4)(x-2) = 0$$

$$x = 2, 4.$$

If $\alpha = 2, \quad \beta = 1$

$\alpha = 4 \quad \beta = \frac{1}{2}$ $\quad \because$ The roots are integer

The common root is 2.

51) Find the values of p for which the difference between the roots of the equation $x^2 + px + 8 = 0$ is 2.

Sol: $x^2 + px + 8 = 0$ $\quad a = 1$, $b = p$, $c = 8$

Let α, β are the roots of the eqn.

$$\alpha + \beta = -p, \quad \alpha\beta = 8$$

But $\alpha - \beta = 2$

$$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$$

$$4 = p^2 - 32$$

$$p^2 = 36 \quad p = \pm 6.$$

52. Factorize: $x^4 + 1$

$$x^4 + 1 = 0$$

$$(x^2)^2 + 2x^2 + 1 - 2x^2 = 0$$

$$(x^2+1)^2 - (\sqrt{2}x)^2 = 0$$

$$(x^2 + \sqrt{2}x - 1)(x^2 - \sqrt{2}x - 1) = 0.$$

53) If x^2+x+1 is a factor of the polynomial $3x^3+8x^2+8x+a$
TBP then find the value of a.

$$\begin{array}{r|l} & 3x+5 \\ x^2+x+1 & 3x^3+8x^2+8x+a \\ & 3x^3+3x^2+3x \\ \hline & 5x^2+5x+a \\ & 5x^2+5x+5 \\ \hline & 0 \end{array}$$

$\Rightarrow a-5=0$

$a=5$.

$\because x^2+x+1$ is a factor the remainder is zero.

Rational inequatities.

54) solve: $\frac{x+1}{x+3}<3$
TBP

Sol: $\frac{x+1}{x+3}<3 \Rightarrow \frac{x+1}{x+3}-3<0$

$$\frac{x+1-3x-9}{x+3}<0$$

$$\frac{-2x-8}{x+3}<0$$

$$\frac{x+4}{x+3}>0.$$

$\because x+4, x+3$ both are Positive (or) both are negative.

	$x+3$	$x+4$	$\frac{x+4}{x+3}$
when $x\in(-\infty,-4)$	$-$	$-$	$+$
$x\in(-4,-3)$	$-$	$+$	$-$
$x\in(-3,\infty)$	$+$	$+$	$+$

$-4 \quad -3$

$\therefore$ The solution set is given by $(-\infty,-4)\cup(-3,\infty)$

55) Find all values of x for which $\frac{x^3(x-1)}{x-2}>0$

$\frac{x^3(x-1)}{x-2}>0 \Rightarrow x^3(x-1)\geq 0$ when $x\neq 2$

$\Rightarrow x=0, x=1$

$x\neq 2$.

$0 \quad 1 \quad 2$

	x^3	$x-1$	$x^3(x-1)$	$\frac{x^3(x-1)}{x-2}$
$x\in(-\infty,0)$	$-$	$-$	$+$	$-$
$x\in(0,1)$	$+$	$-$	$-$	$+$
$x\in(1,2)$	$+$	$+$	$+$	$-$

$\therefore$ The solution set is $x \in (0,1) \cup (2,\infty)$

56) Find all values of x that satisfies the inequality

TBP $\dfrac{2x-3}{(x-2)(x-4)} < 0$

Sol: $\dfrac{2x-3}{(x-2)(x-4)} < 0$ $\quad 2x-3=0 \quad x \neq 2,\ x \neq 4$

$2x = 3$

$x = 3/2$

$3/2 \quad 2 \quad 4$

x	$2x-3$	$x-2$	$x-4$	$\dfrac{2x-3}{(x-2)(x-4)}$
when $x \in (-\infty, 3/2)$	$-$	$-$	$-$	$-$
$x \in (3/2, 2)$	$+$	$-$	$-$	$+$
$x \in (2,4)$	$+$	$+$	$-$	$-$
$x \in (4,\infty)$	$+$	$+$	$+$	$+$

$\therefore \dfrac{2x-3}{(x-2)(x-4)} < 0 \quad x \in (-\infty, 3/2) \cup (2,4)$

57) Solve $\dfrac{x^2-4}{x^2-2x-15} \leq 0.$

$$\frac{x^2-4}{x^2-2x-15} = \frac{(x+2)(x-2)}{(x-5)(x+3)} \leq 0$$

$(x+2)(x-2) = 0 \quad x = -2 \text{ or } 2$

$x \neq -3,\ x \neq 5$

$-3 \quad -2 \quad 2 \quad 5$

x	$x+2$	$x-2$	$x-5$	$x+3$	$\dfrac{(x+2)(x-2)}{(x-5)(x+3)}$
$(-\infty,-3)$	$-$	$-$	$-$	$-$	$+$
$(-3,-2]$	$-$	$-$	$-$	$+$	$-$
$[-2,2)$	$+$	$-$	$-$	$+$	$+$
$(2,5)$	$+$	$+$	$-$	$+$	$-$
$(5,\infty)$	$+$	$+$	$+$	$+$	$+$

$\therefore x \quad \dfrac{(x+2)(x-2)}{(x-5)(x+3)} \leq 0 \in (-3,-2] \cup [2,5)$

Partial Fraction.

58) Resolve into partial fraction. $\frac{x}{(x+3)(x-4)}$

let $\frac{x}{(x+3)(x-4)} = \frac{A}{x+3} + \frac{B}{x-4}$

TBP

$$x = A(x-4) + B(x+3)$$

Put $x=4$ $\quad 4 = 7B \Rightarrow B = 4/7$

$x=-3$ $\quad -3 = -7A \quad A = 3/7$

$$\therefore \frac{x}{(x+3)(x-4)} = \frac{3/7}{x+3} + \frac{4/7}{x-4}$$

$$= \frac{3}{7(x+3)} + \frac{4}{7(x-4)}$$

59) $\frac{1}{x^2-a^2} = \frac{1}{(x+a)(x-a)}$

TBP

$$\frac{1}{x^2-a^2} = \frac{A}{x+a} + \frac{B}{x-a}$$

$$1 = A(x-a) + B(x+a)$$

Put $x=a$ $\quad 1 = 2aB \Rightarrow B = 1/2a.$

$x=-a$ $\quad 1 = -2aA \Rightarrow A = \frac{-1}{2a}$

$$\therefore \frac{1}{x^2-a^2} = -\frac{1}{2a(x+a)} + \frac{1}{2a(x-a)}$$

60) $\frac{3x+1}{(x-2)(x+1)} = \frac{A}{x-2} + \frac{B}{x+1}$

TBP $\quad 3x+1 = A(x+1) + B(x-2)$

Put $x=2$ $\quad 7 = 3A \Rightarrow A = 7/3.$

$x=-1$ $\quad \cancel{+}2 = \cancel{+}3B \quad B = 2/3.$

$$\therefore \frac{3x+1}{(x-2)(x+1)} = \frac{7}{3(x-2)} + \frac{2}{3(x+1)}$$

61) $\frac{2x}{(x^2+1)(x-1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$

$$2x = A(x^2+1) + (Bx+C)(x-1)$$

Put $x=1$ $\quad 2 = A\cdot 2 \Rightarrow A = 1.$

Co-eff x^2 $\quad 0 = A+B$

$0 = 1+B$

$\Rightarrow B = -1$

Const: $\quad 0 = A-C$

$0 = 1-C \Rightarrow C = 1$

$$\therefore \frac{2x}{(x^2+1)(x-1)} = \frac{1}{x-1} + \frac{1-x}{x^2+1}$$

62) Resolve into partial fraction - $\frac{x+1}{x^2(x-1)}$

TBP.

Sol: Let $\frac{x+1}{x^2(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{c}{x-1}$

$$x+1 = Ax(x-1) + B(x-1) + c \cdot x^2$$

Put $x=0$ $\quad 1 = -B \Rightarrow B = -1$

Put $x=1$ $\quad 2 = c = c = 2.$

Coeff. x^2: $\quad 0 = A + C$

$\quad 0 = A + 2 \Rightarrow A = -2$

$$\therefore \frac{x+1}{x^2(x-1)} = \frac{-2}{x} - \frac{1}{x^2} + \frac{2}{x-1}$$

63) $\frac{x}{(x^2+1)(x-1)(x-2)}$

Let $\frac{x}{(x^2+1)(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{Cx+D}{x^2+1}.$

$$x = A(x-2)(x^2+1) + B(x-1)(x^2+1) + (Cx+D)(x-1)(x-2)$$

Put $x=1$ $\quad 1 = A(-1)(2)$

$= -2A \Rightarrow A = -1/2$

Put $x=2$ $\quad 2 = B(1)(5) \Rightarrow B = 2/5$

Const: $\quad 0 = -2A - B + 2D$

$= -2(-1/2) - \frac{2}{5} + 2D$

$1 + \frac{2}{5} = 2D \Rightarrow 2D = \frac{7}{5}$

$D = 7/10$

Co-eff x^3 $\quad 0 = A + B + C$

$0 = -\frac{1}{2} + \frac{2}{5} + C \Rightarrow C = \frac{1}{2} - \frac{2}{5} = \frac{\cancel{3}}{10} \, 3/5$

$$\therefore \frac{x}{(x^2+1)(x-1)(x-2)} = \frac{-1/2}{x-1} + \frac{2/5}{x-2} + \frac{3/5 x + \frac{7}{10}}{x^2+1}.$$

64) $\dfrac{x}{(x^2+1)(x-1)(x+2)}$

TBP

Let $\dfrac{x}{(x^2+1)(x-1)(x+2)} = \dfrac{A}{x-1} + \dfrac{B}{x+2} + \dfrac{Cx+D}{x^2+1}$

$x = A(x+2)(x^2+1) + B(x-1)(x^2+1) + (Cx+D)(x-1)(x+2)$

Put $x=1$ $\quad 1 = A(3)(2) \Rightarrow A = 1/6$

$x=-2 \quad \cancel{-}2 = B(\cancel{-}3)(5)$

$B = 2/15$

Cons:

$0 = 2A - B - 2D.$

$2D = 2 \cdot \frac{1}{\cancel{6}3} - \frac{2}{15}$

$= \frac{5-2}{15} = \frac{3}{15}$

$D = \cancel{\frac{3}{30}}\ 1/10.$

Co-eff. x^3

$0 = A + B + C.$

$0 = \frac{1}{6} + \frac{2}{15} + C.$

$C = -\frac{5+4}{30} = -\cancel{9}3/\cancel{30}10.$

$\therefore \dfrac{x}{(x^2+1)(x-1)(x+2)} = \dfrac{1}{6(x-1)} + \dfrac{2}{15(x+2)} + \dfrac{-\frac{3}{10}x + \frac{1}{10}}{x^2+1}$

$= \dfrac{1}{6(x-1)} + \dfrac{2}{15(x+2)} + \dfrac{-3x+1}{10(x^2+1)}$

65) $\dfrac{x}{(x-1)^3}$

TBP

Sol: Let $\dfrac{x}{(x-1)^3} = \dfrac{A}{x-1} + \dfrac{B}{(x-1)^2} + \dfrac{C}{(x-1)^3}.$

$x = A(x-1)^2 + B(x-1) + C.$

Put $x=1 \quad 1 = C$

Coff. $x^2 = 0 = A$

Consf: $0 = A - B + C$

$= 0 - B + 1 \Rightarrow B = 1$

$\therefore \dfrac{x}{(x-1)^3} = \dfrac{1}{(x-1)^2} + \dfrac{1}{(x-1)^3}.$

66) Resolve into partial fraction. $\dfrac{x^2+x+1}{x^2-5x+6}$.
TBP

Sol: $\dfrac{x^2+x+1}{x^2-5x+6} = 1 + \dfrac{6x-5}{x^2-5x+6}$

$$\begin{array}{r|l} & 1 \\ \hline x^2-5x+6 & x^2+x+1 \\ & x^2-5x+6 \\ \hline & 6x-5 \end{array}$$

$\dfrac{6x-5}{x^2-5x+6} = \dfrac{6x-5}{(x-3)(x+2)} = \dfrac{A}{x-3} + \dfrac{B}{x+2}$

$x^2-5x+6 = (x-3)(x+2)$

$6x-5 = A(x+2) + B(x-3)$

Put $x=3$ $\quad 13 = A \Rightarrow A = 13$

Put $x=2$ $\quad 7 = -B \Rightarrow B = -7$.

$\therefore \dfrac{6x-5}{(x-3)(x-2)} = \dfrac{13}{x-3} - \dfrac{7}{x+2}$

$\therefore \dfrac{x^2+x+1}{x^2-5x+6} = 1 + \dfrac{13}{x-3} - \dfrac{7}{x+2}$

67) $\dfrac{x^3+2x+1}{x^2+5x+6}$
TBP

$$\begin{array}{r|l} & x-5 \\ \hline x^2+5x+6 & x^3+0+2x+1 \\ & x^3+5x^2+6x \\ \hline & -5x^2-4x+1 \\ & -5x^2-25x-30 \\ \hline & 21x+31 \end{array}$$

$\dfrac{x^3+2x+1}{x^2+5x+6} = (x-5) + \dfrac{21x+31}{(x+2)(x+3)}$

$\dfrac{21x+31}{(x+2)(x+3)} = \dfrac{A}{x+2} + \dfrac{B}{x+3}$

$21x+31 = A(x+3) + B(x+2)$

Put $x=-2$ $\quad -42+31 = +A$

$A = -11$.

$x=-3$ $\quad -63+31 = -B$

$-32 = -B \Rightarrow B = 32$

$\therefore \dfrac{21x+31}{(x+2)(x+3)} = \dfrac{-11}{x+2} + \dfrac{32}{x+3}$.

$\therefore \dfrac{x^3+2x+1}{x^2+5x+6} = (x-5) - \dfrac{11}{x+2} + \dfrac{32}{x+3}$.

68) TBP $\dfrac{x+12}{(x+1)^2(x-2)}$

$$\frac{x+12}{(x+1)^2(x-2)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x-2}$$

$$x+12 = A(x+1)(x-2) + B(x-2) + C(x+1)^2$$

Put $x=2$ $\quad 14 = 9C \Rightarrow C = \frac{14}{9}$.

$x=-1$ $\quad 11 = -3B \Rightarrow B = -\frac{11}{3}$

Const: $\quad 12 = -2A - 2B + C$

$$= -2A + \frac{22}{3} + \frac{14}{9}.$$

$$\cancel{2}A = \frac{\cancel{22}^{11}}{3} + \frac{\cancel{14}^{7}}{9} - \cancel{12}^{6}$$

$$A = \frac{33+7-54}{9} = \frac{-14}{9}.$$

$\therefore \dfrac{x+12}{(x+1)^2(x-2)} = \dfrac{-14}{9(x+1)} - \dfrac{11}{3(x+1)^2} + \dfrac{14}{9(x-2)}$

69) TBP $\dfrac{6x^2-x+1}{x^3+x^2+x+1}$.

x^3+x^2+x+1
$= x^2(x+1)+1(x+1)$
$= (x+1)(x^2+1)$

Sol:

$$\frac{6x^2-x+1}{x^3+x^2+x+1} = \frac{Ax+B}{x^2+1} + \frac{C}{x+1}$$

$$6x^2-x+1 = (Ax+B)(x+1) + C(x^2+1)$$

$x=-1$ $\quad 6+1+1 = 2C \Rightarrow C = 4$

Const. $\quad 1 = B+C \Rightarrow B = 1-C$
$= 1-4 = -3$

coeff x^2 $\quad 6 = A+C$
$= A+4 \Rightarrow A = 2$

$\therefore \dfrac{6x^2-x+1}{x^3+x^2+x+1} = \dfrac{2x-3}{x^2+1} + \dfrac{4}{x+1}$.

70) TBP $\dfrac{2x^2+5x-11}{x^2+2x-3}$

$$\frac{2x^2+5x-11}{x^2+2x-3} = 2+\frac{x-5}{x^2+2x-3}$$

$$\begin{array}{r|l} & \quad 2 \\ x^2+2x-3 & 2x^2+5x-11 \\ & 2x^2+4x-6 \\ \hline & \quad x-5 \end{array}$$

$$= 2+\frac{x-5}{(x+3)(x-1)}$$

$x^2+2x-3=(x+3)(x-1)$

$$\frac{x-5}{(x+3)(x-1)} = \frac{A}{x+3}+\frac{B}{x-1}$$

$$x-5 = A(x-1)+B(x+3)$$

Put $x=1$

$-4 = 4B \Rightarrow B=-1$

$x=-3$ $\cancel{-}8 = \cancel{-}4A$ $A=2$

$$\therefore \frac{2x^2+5x-11}{x^2+2x-3} = 2+\frac{2}{x+3}-\frac{1}{x-1}.$$

71) TBP $\dfrac{7+x}{(1+x)(1+x^2)}$

Sol: $$\frac{7+x}{(1+x)(1+x^2)} = \frac{Ax+B}{x^2+1}+\frac{C}{x+1}$$

$$7+x = (Ax+B)(x+1)+C(x^2+1)$$

Put $x=-1$ $7-1 = 2C$

$2C=6$

$C=3.$

Const: $7 = B+C$

$B=7-3=7-3=4.$

Co-eff. x^2 $0 = A+C.$

$A=-C=-3.$

$$\therefore \frac{7+x}{(1+x)(1+x^2)} = \frac{-3x+4}{x^2+1}+\frac{3}{x+1}.$$

72) TBP $\dfrac{(x-1)^2}{x^3+x}$.

$$\frac{(x-1)^2}{x^3+x} = \frac{x^2 \cancel{+} 2x+1}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1}$$

$$x^2-2x+1. = A(x+1) + Bx$$

Put $x=-1$ $\quad 1+2+1 = -B$

$$B = -4.$$

Cons: $\quad 1 = A$

$$\therefore \quad \frac{x^2-2x+1}{x(x^2+1)} = \frac{1}{x} - \frac{4}{x+1}.$$

73) TBP $\dfrac{1}{x^4-1}$

$$\frac{1}{x^4-1} = \frac{1}{(x^2)^2-1^2} = \frac{1}{(x^2+1)(x^2-1)} = \frac{1}{(x^2+1)(x+1)(x-1)}$$

$$\frac{1}{(x^2+1)(x^2-1)} = \frac{Ax+B}{x^2+1} + \frac{C}{x+1} + \frac{D}{x-1}$$

$$1 = (Ax+B)(x^2-1) + C(x^2+1)(x-1) + D(x^2+1)(x+1)$$

Put $\underline{x=1}$

$$1 = 4D \Rightarrow D = \frac{1}{4}.$$

Co-eff. x^3 $\quad 0 = A + C + D \Rightarrow A + C = -\frac{1}{4}.$ —— ①

Coef. x^2 $\quad 0 = B - C + D \quad B - C = -1/4.$ —— ③

Coeff x $\quad 0 = -A + C + D \quad -A + C = -1/4.$ —— ②

① + ② $\quad \cancel{2}C = -\frac{2}{4} \Rightarrow C = -1/4$

$$A = -\frac{1}{4} + \frac{1}{4} = 0.$$

$$B = -\frac{2}{4} = -1/4$$

$$\therefore \quad \frac{1}{x^4-1} = \frac{-1}{(x^2+1)} - \frac{1}{4(x+1)} + \frac{1}{4(x-1)}.$$

Logarithm:

74) Find the logarithm of 1728 to the base $2\sqrt{3}$.

TBP

Sol: Let $\log_{2\sqrt{3}} 1728 = x$

$1728 = (2\sqrt{3})^x$.

But $1728 = 2^6 3^3$

$= 2^6 \sqrt{3}^6$

$(2\sqrt{3})^x = (2\sqrt{3})^6$

$\Rightarrow x = 6$.

$\therefore \log_{2\sqrt{3}} 1728 = 6$

$$\begin{array}{r|l} 2 & 1728 \\ 2 & 864 \\ 2 & 432 \\ 2 & 216 \\ 2 & 108 \\ 2 & 54 \\ & 27 \end{array}$$

75) If the logarithm of 324 to the base a is 4 find a.

TBP

Sol: $\log_a 324 = 4 \Rightarrow 324 = a^4$

$= 2^2 3^4$

$= (\sqrt{2})^4 3^4$

$= (\sqrt{2} \cdot 3)^4$

$\Rightarrow a = 3\sqrt{2}$

$$\begin{array}{r|l} 2 & 324 \\ 2 & 162 \\ 9 & 81 \\ & 9 \end{array}$$

76) P.T $\log \frac{75}{16} - 2\log\frac{5}{9} + \log\frac{32}{243} = \log 2$.

TBP

Sol: LHS: $\log\frac{75}{16} - 2\log\frac{5}{9} + \log\frac{32}{243}$

$= \log\frac{75}{16} - \log\frac{25}{81} + \log\frac{32}{243}$

$= \log\left[\frac{\cancel{75}^{3}}{\cancel{16}} \times \frac{\cancel{32}^{2}}{\cancel{243}_{3}} \times \frac{\cancel{81}}{\cancel{25}}\right]$

$= \log 2$

77) $\log_2 x + \log_4 x + \log_{16} x = \frac{7}{2}$ find the value of x.

TBP

$\underline{\underline{LHS}}$ $\log_2 x + \log_4 x + \log_{16} x = \log_2 x + \log_{2^2} x + \log_{2^4} x$

$\because \boxed{\log_{a^n} x = \frac{1}{n}\log_a x}$

$= \log_2 x + \frac{1}{2}\log_2 x + \frac{1}{4}\log_2 x$

$= \log_2 x\left[1 + \frac{1}{2} + \frac{1}{4}\right]$

$= \log_2 x\left[\frac{4+2+1}{4}\right]$

$= \frac{\cancel{7}}{\cancel{4}_2}\log_2 x = \frac{\cancel{7}}{\cancel{2}}1$

$\log_2 x = 1$

$\Rightarrow x = (2^2)^1$

$\underline{\underline{= 4}}$

78) Solve $x^{\log_3 x} = 9$.

TBP

Sol: Let $\log_3 x = y \Rightarrow x = 3^y$

$\therefore x^{\log_3 x} = 3^{y \cdot y} = 3^{y^2} = 9$ $\qquad 9 = 3^2$

$= 3^{y^2} = 3^2$

$\Longrightarrow y^2 = 2 \quad \therefore y = \pm\sqrt{2}$

Hence $x = 3^{\sqrt{2}}, 3^{-\sqrt{2}}$

79) Compute $\log_3 5 \cdot \log_{25} 27$

TBP

Sol: $\log_3 5 \; \log_{25} 27 = \log_3 5 \cdot \log_{5^2} 27$

$= \log_3 5 \cdot \frac{1}{2} \log_5 27$

$= \frac{1}{2} \log_3 27 = \frac{1}{2} \log_3 3^3 = \frac{3}{2} \log_3 3$

$= 3/2$

80) Given that $\log_{10} 2 = 0.30103$ $\log_{10} 3 = 0.47712$ find the number of digits in $2^8 \cdot 3^{12}$

TBP

Sol: Let $N = 2^8 3^{12}$

$\log N = \log(2^8 \cdot 3^{12}) = \log 2^8 + \log 3^{12}$

$= 8 \log 2 + 12 \log 3$

$= 8 \times 0.30103 + 12 \times 0.47712$

$\log N = 8.13368$ on seeing Antilog

$\therefore$ N has 9 digits.

82) Let $b > 0$ and $b \neq 1$ Express $y = b^x$ in logarithmic form Also state the domain and range the logarithmic fn

TBP

Sol: $y = b^x$

$\log_b y = x$.

Domain $(0, \infty)$ range $(-\infty, \infty)$

83) Compute $\log_9 27 - \log_{27} 9$

TBP

$\log_9 27 = \log_{3^2} 3^3$ $\qquad$ But $\log_{27} 9 = \frac{1}{\log_9 27}$

$= \frac{1}{2} \log_3 3^3 = \frac{3}{2} \log_3 3 = 3/2$ $\qquad = \frac{1}{3/2} = 2/3$

$\therefore \log_9 27 - \log_{27} 9 = \frac{3}{2} - \frac{2}{3} = \frac{9-4}{6} = 5/6$

84) solve $\log_2 x + \log_4 x + \log_8 x = 11$

TBP

Sol: $\log_2 x + \log_4 x + \log_8 x = 11$

$\log_2 x + \log_{2^2} x + \log_{2^3} x = 11$

$\log_2 x + \frac{1}{2}\log_2 x + \frac{1}{3}\log_2 x = 11$

$\log_2 x\left[1 + \frac{1}{2} + \frac{1}{3}\right] = 11$

$\log_2 x\left[\frac{6+3+2}{6}\right] = 11$

$\log_2 x = \cancel{11} \times \frac{6}{\cancel{11}}$

$x = 2^6 = 64.$

85 solve $\log_4 2^{8x} = \log_2 2^8$ $\qquad a^{\log_a x} = x.$

TBP

Sol: $\log_4 2^{8x} = 8$

$\log_{2^2} 2^{8x} = 8$

$\frac{1}{2}\log_2 2^{8x} = 8 \Rightarrow \log_2 2^{8x} = 16$

$8x \log_2 2 = 16 \Rightarrow 8x = 16$

$x = 2$

(or) $\log_4 2^{8x} = 8$

$2^{8x} = 4^8 = 2^{16}$

$8x = 16$

$x = 2$

86) If $a^2+b^2 = 7ab$ S.T $\log\frac{a+b}{3} = \frac{1}{2}(\log a + \log b)$

TBP

Sol: $a^2+b^2 = 7ab.$

$a^2+b^2+2ab = 7ab+2ab$

$(a+b)^2 = 9ab$

$\log(a+b)^2 = \log(9ab)$

$2\log(a+b) = \log 9 + \log a + \log b.$

$\log(a+b) = \frac{1}{2}\log 9 + \frac{1}{2}(\log a + \log b)$

$= \log 9^{1/2} + \frac{1}{2}(\log a + \log b)$

$= \log 3 + \frac{1}{2}(\log a + \log b)$

$\log(a+b) - \log 3 = \frac{1}{2}(\log a + \log b)$

$\log\left(\frac{a+b}{3}\right) = \frac{1}{2}(\log a + \log b).$

87) P.T $\log \frac{a^2}{bc} + \log \frac{b^2}{ca} + \log \frac{c^2}{ab} = 0$

TBP:

LHS $\log \frac{a^2}{bc} + \log \frac{b^2}{ca} + \log \frac{c^2}{ab}$

$= \log \left(\frac{a^2}{bc} \times \frac{b^2}{ca} \times \frac{c^2}{ab} \right)$

$= \log 1 = 0.$

88) P.T $\log 2 + 16 \log \frac{16}{15} + 12 \log \frac{25}{24} + 7 \log \frac{81}{80} = 1$

TBP

LHS: $\log 2 + 16 \log \frac{16}{15} + 12 \log \frac{25}{24} + 7 \log \frac{81}{80}$

$= \log 2 + \log \left(\frac{16}{15}\right)^{16} + \log \left(\frac{25}{24}\right)^{12} + \log \left(\frac{81}{80}\right)^{7}$

$= \log \left[2 \times \frac{16^{16}}{15^{16}} \times \frac{(25)^{12}}{24^{12}} \times \frac{81^7}{80^7} \right]$

$= \log \left[2 \times \frac{4^{32}}{5^{16} 3^{16}} \times \frac{5^{24}}{4^{12} 3^{12} 2^{12}} \times \frac{3^{28}}{4^7 \times 5^7 \times 4^7} \right]$

$= \log \left[2 \times \frac{\cancel{4^{32}} \times \cancel{5^{24}} \times \cancel{3^{28}}}{\cancel{5^{23}}\, \cancel{4^{26}}\, \cancel{3^{28}} . 2^{12}} \right]$

$= \log 10 = 1$

89) P.T $\log_{a^2} a . \log_{b^2} b . \log_{c^2} c = \frac{1}{8}$

TBP

$= \frac{1}{2} \log_a a . \frac{1}{2} \log_b b . \frac{1}{2} \log_c c$

$= \frac{1}{8} . 1 . 1 . 1 . = 1/8$

90) P.T $\log a + \log a^2 + \log a^3 + \dots + \log a^n = \frac{n(n+1)}{2} \log a$.

TBP

Sol: LHS: $\log a + \log a^2 + \log a^3 + \dots + \log a^n$

$= \log a + 2 \log a + 3 \log a + \dots + n \log a.$

$= \log a [1 + 2 + 3 + \dots + n]$

$= \frac{n(n+1)}{2} \log a.$

91) Solve: $\log_2 x - 3 \log_{\frac{1}{2}} x = 6.$

TBP

Sol: LHS: $\log_2 x - 3 \log_{1/2} x$

$= \frac{1}{\log_x 2} - 3 . \frac{1}{\log_x 1/2} = 6$

$= \frac{1}{\log_x 2} - 3 . \frac{1}{\cancel{\log 1} - \log 2} = 6$

$\frac{1}{\log_x 2} + \frac{3}{\log_x 2} = 6$

$\frac{4}{\log_x 2} = 6$

$\cancel{4}_2 \log_2 x = \cancel{6}^3$

$x = 2^{3/2}$

$\therefore x = \sqrt{8}.$

92) solve $\log_{5-x}(x^2-6x+65)=2$
TBP

$$x^2-6x+65=(5-x)^2$$
$$=x^2+25-10x$$
$$4x=-40$$
$$x=-10.$$

Exponents and Radicals:

93) Simplify by rationalising the denominator: $\frac{7+\sqrt{6}}{3-\sqrt{2}}$
TBP

Sol: $\frac{7+\sqrt{6}}{3-\sqrt{2}}\times\frac{3+\sqrt{2}}{3+\sqrt{2}}$

$$=\frac{21+7\sqrt{2}+3\sqrt{6}+\sqrt{12}}{9-2}$$
$$=\frac{21+7\sqrt{2}+3\sqrt{6}+2\sqrt{3}}{7}$$
$$=$$

94 If $x=\sqrt{2}+\sqrt{3}$ find $\frac{x^2+1}{x^2-2}$.
TBP.

Sol: $\frac{x^2+1}{x^2-2}=\frac{(\sqrt{2}+\sqrt{3})^2+1}{(\sqrt{2}+\sqrt{3})^2-2}=\frac{2+3+2\sqrt{6}+1}{2+3+2\sqrt{6}-2}$

$$=\frac{6+2\sqrt{6}}{3+2\sqrt{6}}\times\frac{3-2\sqrt{6}}{3-2\sqrt{6}}$$
$$=\frac{18-12\sqrt{6}+6\sqrt{6}-24}{9-24}$$
$$=\frac{-6-6\sqrt{6}}{-15}=\frac{\cancel{+6}^{2}(1+\sqrt{6})}{\cancel{+15}_{5}}$$
$$=\frac{2+2\sqrt{6}}{5}$$

95 Rationalize the denominator of $\frac{\sqrt{5}}{\sqrt{5}+\sqrt{2}}$, $\frac{\sqrt{5}}{\sqrt{6}+\sqrt{2}}$
TBP

Sol: $\frac{\sqrt{5}}{\sqrt{5}+\sqrt{2}}\times\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}$

$$=\frac{5-\sqrt{10}}{5-2}=\frac{5-\sqrt{10}}{3}.$$

$$\frac{\sqrt{5}}{\sqrt{6}+\sqrt{2}}\times\frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}$$
$$\frac{\sqrt{30}-\sqrt{10}}{4}$$

96) Find the square root of $7-4\sqrt{3}$.

TBP

Let $\sqrt{7-4\sqrt{3}} = a+b\sqrt{3}$

Squaring on both sides.

$$7-4\sqrt{3} = (a+b\sqrt{3})^2$$

$$7-4\sqrt{3} = a^2+3b^2+2ab\sqrt{3}$$

$\Rightarrow a^2+3b^2=7$ | $2ab\sqrt{3} = 4\sqrt{3}$

$2ab=4$

$a = \frac{4}{2b} = \frac{2}{b}$.

$\therefore \frac{4}{b^2}+3b^2=7$

$$4+3b^4=7b^2$$

$\Rightarrow 3b^4-7b^2+4=0$ which is quadratic in b^2

Let $b^2=t$ $\quad 3t^2-7t+4=0$ $\quad a=1$, $b=-7$

$$3t^2-3t-4t+4=0$$

$$3t(t-1)-4(t-1)=0$$

$$(3t-4)(t-1)=0$$

$$t=1, 4/3.$$

(ie) $b^2=1$, $b=\pm 1$ | $b=\frac{4}{3}$, $b=\pm\frac{2}{\sqrt{3}}$

$\because$ b is rational $b=\pm 1$ then $a=\mp 2$. and $b=\pm\frac{2}{\sqrt{3}}$ $a=\pm\frac{\cancel{2\sqrt{3}}}{\cancel{2}}$

$\therefore \sqrt{7-4\sqrt{3}}>0$ where $a+b\sqrt{3}$

$= 2-\sqrt{3}$

97) Simplify $\frac{1}{3-\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-2}$

TBQ.

Consider $\frac{1}{3-\sqrt{8}} \times \frac{3+\sqrt{8}}{3+\sqrt{8}} = \frac{\sqrt{3}+\sqrt{8}}{9-8} = 3+\sqrt{8}$

$$\frac{1}{\sqrt{8}-\sqrt{7}} \times \frac{\sqrt{8}+\sqrt{5}}{\sqrt{8}+\sqrt{7}} = \frac{\sqrt{8}+\sqrt{7}}{8-7} = \sqrt{8}+\sqrt{7}$$

$$\frac{1}{\sqrt{7}-\sqrt{6}} \times \frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}-\sqrt{6}} = \frac{\sqrt{7}+\sqrt{6}}{7-6} = \sqrt{7}+\sqrt{6}.$$

$$\frac{1}{\sqrt{6}-\sqrt{5}} \times \frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}+\sqrt{5}} = \frac{\sqrt{6}+\sqrt{5}}{6-5} = \sqrt{6}+\sqrt{5}$$

$$\frac{1}{\sqrt{5}-2} \cdot \frac{\sqrt{5}+2}{\sqrt{5}+2} = \frac{\sqrt{5}+2}{5-4} = \sqrt{5}+2$$

$\therefore \frac{1}{3-\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-2}$

$= (3+\cancel{\sqrt{8}}) - (\cancel{\sqrt{8}}+\cancel{\sqrt{7}}) + (\cancel{\sqrt{7}}+\cancel{\sqrt{6}}) - (\cancel{\sqrt{6}}+\cancel{\sqrt{5}}) + (\cancel{\sqrt{5}}+2)$

$= 3+2 = 5$

98) Find the radius of the spherical tank whose volume is $\frac{32\pi}{3}$ units.
TBP

Volume of the sphere $\frac{\cancel{4}}{\cancel{3}}\cancel{\pi} r^3 = \frac{\overset{8}{\cancel{32}}\cancel{\pi}}{\cancel{3}}$

$r^3 = 8$

$r = 2$

99) Simplify and hence find the value of n : $3^{2n}9^{2}3^{-n}/3^{3n} = 27$
TBP

Sol: $\frac{3^{2n}\,9^{2}\,3^{-n}}{3^{3n}} = 27$

$\frac{3^{2n}\cdot 3^{4}\,3^{-n}}{3^{3n}} = 27$

$3^{2n+4-n-3n} = 27$

$3^{4-2n} = 3^{3} \Rightarrow 4-2n = 3.$

$-2n = -1$

$n = 1/2$

100 If $\left(x^{1/2}+\frac{1}{x^{1/2}}\right)^2 = 9/2$ find the value of $\left(x^{1/2}-\frac{1}{x^{1/2}}\right)^2$ for $x > 1$.
TBP

$\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2 = \left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2 - 4$

$= \frac{9}{2} - 4 = 1/2$

$\sqrt{x} - \frac{1}{\sqrt{x}} = \frac{1}{\sqrt{2}}$

101) Evaluate $\left(\left((256)^{-1/2}\right)^{-1/4}\right)^3 = (256)^{1/8\cdot 3} = (256)^{3/8}$ 256

$= (2^8)^{3/8} = 2^3$

$= 8$

BASIC ALGEBRA – 1-Mark.

1. If $|x+2| \le 9$ then x belongs to

TBP 1) $(-\infty, -7)$ 2) $[-11, 7]$ 3) $(-\infty, -7) \cup (11, 8)$ 4) $(-11, 7)$

Hint $-9 \le (x+2) \le 9$

$-11 \le x \le 7 \Rightarrow x \in [-11, 7]$

2. Given that x, y and b are real numbers $x < y$, $b > 0$ then

TBP 1) $xb < yb$ 2) $xb > yb$ 3) $xb \le yb$ 4) $\frac{x}{b} \ge \frac{y}{b}$.

Hint: It is clear that when $x < y$ and $b > 0$

$xb < yb$.

3) If $\frac{|x-2|}{x-2} \ge 0$ then x belongs to

TBP 1) $[2, \infty)$ 2) $(2, \infty)$ 3) $(-\infty, 2)$ 4) $(-2, \infty)$

Hint $\frac{|x-2|}{x-2} \ge 0 \Rightarrow x-2 \ge 0$, $x \ge 2$ and $x-2 \ne 0 \Rightarrow x \ne 2$

(ie) $x \in (2, \infty)$

4) The solution of $5x-1 < 24$ and $5x+1 > -24$ is

1) $(4, 5)$ 2) $(-5, -4)$ 3) $(-5, 5)$ 4) $(-5, 4)$

TBP

$5x - 1 < 24$	$5x + 1 > -24$
$5x < 25$	$5x > -25$
$x < 5$	$x > -5$

$\therefore x \in (-5, 5)$

5) The value of $\log_{\sqrt{2}} 512$ is a) 16 b) 18 c) 9 d) 12.

Hint: Let $\log_{\sqrt{2}} 512 = x \Rightarrow 512 = (\sqrt{2})^x$.

TBP

$2^9 = (2)^{x/2}$

$\Rightarrow \frac{x}{2} = 9 \Rightarrow x = 18$

2 | 512
2 | 256
2 | 128
2 | 64
2 | 32
2 | 16
2 | 8
2 | 4
2

6) The solution set of $|x-1| \ge |x-3|$ is

a) $[0, 2]$ b) $[2, \infty)$ c) $(0, 2)$ d) $(-\infty, 2)$

Hint when $x = 0$ $|-1| \ge |-3|$ is not true.

TBP

$\therefore$ a), c), d) are wrong.

$\therefore [2, \infty)$ is correct answer.

7) The value of $\log_3 \frac{1}{81}$ is a) -2 b) -8 c) -4 d) -9.

Hint Let $\log_3 \frac{1}{81} = x \Rightarrow \frac{1}{81} = 3^x \Rightarrow \frac{1}{3^4} = 3^x \Rightarrow 3^{-4} = 3^x \Rightarrow x = -4$

TBP

8) If $\log_{\sqrt{x}} 0.25 = 4$ then the value of x is Hint $0.25 = (\sqrt{x})^4$

TBP a) 0.5 b) 2.5 c) 1.5 d) 1.25. $0.25 = x^2$

$x = .5$

9) The value of $\log_a b \cdot \log_b c \cdot \log_c a$ is

TBP a) 2 b) 1 c) 3 d) 4.

Hint $\log_a b \cdot \log_b c \cdot \log_c a = 1$

10) If 3 is the logarithm of 343 then the base

TBP a) 5 b) 7 c) 6 d) 9

Hint $\log_x 343 = 3 \Rightarrow x^3 = 343$

$7^3 = 343 \Rightarrow x = 7.$

$$\begin{array}{r|l} 7 & 343 \\ \hline 7 & 49 \\ \hline & 7 \end{array}$$

Otherwise we cannot write 343 as $5^{(\,)}$ $6^{(\,)}$, $9^{(\,)}$

∴ Correct answer is $x = 7$

11) Find a so that the sum and product of the roots of the equation

TBP $2x^2 + (a-3)x + (3a-5) = 0$ are equal is

a) 1 b) 2 c) 0 d) 4

$SR = \frac{-(a-3)}{2}$ $PR = \frac{3a-5}{2}$

$\frac{3a-5}{\cancel{2}} = \frac{-(a-3)}{\cancel{2}}$ $4a = 3+5$

$a = 2$

12) If a and b are the roots of the Eqn $x^2 - kx + 16 = 0$ satisfy $a^2 + b^2 = 32$ then the value of k is

a) $k = 10$ b) $k = -8$ c) $k = -8, 8$ d) $k = 6$.

Hint: $a + b = k$, $ab = 16$.

$a^2 + b^2 = 32$

$(a+b)^2 - 2ab = 32 \Rightarrow k^2 - 32 = 32$

$k^2 = 64$ $k = \pm 8$

13) The number of solutions of $x^2 + |x-1| = 1$ is

a) 1 b) 0 c) 2 d) 3.

$|x-1| = 1 - x^2$

$1 - x^2 = x - 1$

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$x = 1, -2$

$1 - x^2 = -(x-1)$

$= -x + 1$

$x^2 - x = 0$

$x(x-1) = 0$

$x = 0, x = 1$

From this -2 not satisfied the solutions are 0, 1

∴ number of solutions 2.

14) The equations whose roots are numerically equal but opposite in sign to the roots of $3x^2-5x-7=0$ is :

TBP a) $3x^2+5x-7=0$ b) $3x^2+5x-7=0$ c) $3x^2-5x+7=0$ d) $3x^2+x-7$

Let α, β are the roots of the eqn. $3x^2-5x-7=0$ $a=3$, $b=-5$, $c=-7$.

$\alpha+\beta = 5/3$ $\alpha\beta = -7/3$.

If $-\alpha, -\beta$ are roots SR: $(-\alpha+\beta) = -(\alpha+\beta)$
$= -5/3$.

PR $(-\alpha)(-\beta) = \alpha\beta = -7/3$.

$\therefore$ Eqn. $x^2-(SR)x+PR=0$ $x^2-\frac{5}{3}x-\frac{7}{3}=0$

$3x^2-5x-7=0$.

15) If 8 and 2 are the roots of $x^2+ax+c=0$ and 3,3 are the roots of $x^2+dx+b=0$ then the roots of the equation $x^2+ax+b=0$.

TBP a) (1,2 b) $-1,1$ c) 9,1 d) $-1,2$.

Hint: 8, 2 are the roots of $x^2+ax+c=0$

SR = $10 = -a \Rightarrow a=-10$

PR : $16 = c$

3, 3 are the roots of the equation $x^2+dx+b=0$

SR $6=-d \Rightarrow d=-6$.

$9 = b$

$\therefore x^2-10x+9=0$.

$(x-1)(x-9)=0 \Rightarrow x=1,9$.

16) If a and b are the real roots of the Eqn. $x^2-kx+c=0$ then the distance between the points $(a,0)$ and $(b,0)$ is

TBP a) $\sqrt{k^2-4c}$ b) $\sqrt{4k^2-c}$ c) $\sqrt{4c-k^2}$ d) $\sqrt{k-8c}$.

Hint $a+b=k$
$ab=c$

$d=\sqrt{(a-b)^2+(0-0)^2}$

$=\sqrt{(a+b)^2-4ab} = \sqrt{k^2-4c}$

17) $\frac{kx}{(x+2)(x-1)} = \frac{2}{x+2}+\frac{1}{x-1}$ then k is a) 1 b) 2 c) 3 d) 4.

TBP $kx = 2(x-1)+(x+2)$

$k=3$

18) If $\frac{1-2x}{3+2x-x^2} = \frac{A}{3-x} + \frac{B}{x+1}$ then the value of $A+B$ is

TBP

a) $-\frac{1}{2}$ b) $-\frac{2}{3}$ c) $\frac{1}{2}$ d) $\frac{2}{3}$.

$1-2x = A(x+1) + B(3-x)$

$1 = A + 3B$

$-2 = A - B$

$3 = 4B \quad B = \frac{3}{4}$

$\therefore A = 1 - 3 \cdot \frac{3}{4}$

$= -\frac{5}{4}$

$A+B = -\frac{5}{4} + \frac{3}{4} = \frac{-2}{4} = -\frac{1}{2}$

19) The number of real roots of $(x+3)^4 + (x+5)^4 = 16$ is

TBP

a) 4 b) 2 c) 3 d) 0.

$\because$ The degree of the eqn is 4. it has 4 roots.

20) The value of $\log_3 11 . \log_{11} 13 . \log_{13} 15 . \log_{15} 27 . \log_{27} 81$ is

TBP

a) 1 b) 2 c) 3 d) 4.

Hint $\log_3 11 . \log_{11} 13 . \log_{13} 15 . \log_{15} 27 . \log_{27} 81 = \log_3 81$

$= \log_3 3^4 = 4 \log_3 3$

$= 4.$

T.G. Venkatesan
9444209677

www.nammakalvi.org

2018-2019
XI-STD MATHEMATICS
BASIC ALGEBRA (Q B)

63 x 2 = 126

1) Classify each element of $\left\{\sqrt{7}, \frac{-1}{4}, 0, 3.14, 4, \frac{22}{7}\right\}$ as a member of N, Q, R, -Q or Z.

2) Prove that $\sqrt{3}$ is an irrational number. (Hint: Follow the method that we have used to prove $\sqrt{2} \notin Q$)

3) Are there two distinct irrational numbers such that their difference is a rational number? Justify.

4) Find two irrational numbers such that their sum is a rational number. Can you find two irrational numbers whose product is a rational number?

5) Find a positive number small than $\frac{1}{2^{1000}}$. Justify

6) Construct a quadratic equation with roots 7 and -3

7) A quadratic polynomial has one of its zeros as $1+\sqrt{5}$ and it satisfies p(1) = 2. Find the quadratic polynomial.

8) If one root of $k(x-1)^2 = 5x-7$ is double the other root, show that k=2 or -25

9) If the difference of the roots of the equation $2x^2-(a+1)x+a-1=0$ is equal to their product, then prove that a=2.

10) Find the condition that one of the roots of ax^2+bx+c may be negative of the other

11) Determine the region in the Plane determined by the inequalities. $x \le 3y, x \ge y$

12) Determine the region in the Plane determined by the inequalities $y \ge 2x, \ -2x+3y \le 6$

13) Determine the region in the Plane determined by the inequalities.

$3x+5y \ge 45, \ x \ge 0, \ y \ge 0$

14) Factorize: x^4+1

$3x^3+8x^2+8x+a$

15) if x^2+x+1 is a factor of the polynomial 3x2+8x2+8x+a, then find the value of a.

16) If the equations $x^2 - ax + b = 0$ and $x^2 - ex + f = 0$ have one root in common and if the second equation has equal roots, then prove that ae = 2 (b + f).

17) Discuss the nature of roots of $-x^2+3x+1=0$

18) Discuss the nature of roots of $4x^2-x-2=0$

19) Discuss the nature of roots of $9x^2+5x=0$.

20) Without sketching the graphs, find whether the graphs of the following functions will intersect the x-axis and if so in how many points. $y = x^2+x+2$

21) Without sketching the graphs, find whether the graphs of the following functions will intersect the x-axis and if so in how many points. $y = x^2-3x-7$

22) Without sketching the graphs, find whether the graphs of the following functions will intersect the x-axis and if so in how many points. $y = x^2+6x+9$

23) Write $f(x) = x^2 + 5x + 4$ in completed square form.

24) If α and β are the roots of the quadratic equation $x^2 + \sqrt{2x} + 3 = 0$, form a quadratic polynomial with zeros $\frac{1}{\alpha}, \frac{1}{\beta}$

25) Determine the region in the plane determined by the inequalities.

$2x + 3y \leq 35, \quad y \geq 2, \quad x \geq 5.$

26) Determine the region in the plane determined by the inequalities.

$2x + 3y \leq 6, \quad x + 4y \leq 4, \quad x \geq 0, \quad y \geq 0.$

27) Determine the region in the plane determined by the inequalities.

$x - 2y \geq 0,\ 2x - y \leq -2,\ x \geq 0,\ y \geq 0.$

28) Determine the region in the plane determined by the inequalities.

$2x + y \geq 8, \quad x + 2y \geq 8, \quad x + y \leq 6$

29) Find the condition that one of the roots of ax^2+bx+c may be thrice the other

30) Find the condition that one of the roots of ax^2+bx+c may be reciprocal of the other.

31) Solve $|2x-17| = 3$ for x.

32) Solve $3|x-2| + 7 = 19$ for x.

33) Solve $|2x-3| = |x-5|$.

34) Solve $|x-9| < 2$ for x.

35) Solve $\left|\frac{2}{x-4}\right| > 1, x \neq 4$

36) Our monthly electricity bill contains a basic charge, which does not change with number of units used, and a charge that depends only on how many units we use. Let us say Electricity board charges Rs.110 as basic charge and charges Rs.4 for each unit we use. If a person wants to keep his electricity bill below Rs.250, then what should be his electricity usage?

37) Solve $3x - 5 \leq x + 1$ for x.

38) Solve the following system of linear inequalities $3x-9\geq0$, $4x-10\leq6$;

39) A girl is reading a book having 446 pages and she has already finished reading 271 pages. She wants to finish reading this book within a week. What is the minimum number of pages she should read per day to complete reading the book within a week?

40) If a and b are the roots of the equation $x^2 - px + q = 0$, find the value of $\frac{1}{a} + \frac{1}{b}$

41) Find the complete set of values of a for which the quadratic $x^2 - ax + a + 2 = 0$ has equal roots.

42) Find the number of solutions of $x^2+|x-1|=1$

43) Simplify $\left(x^{\frac{1}{2}}y^{-3}\right)^{\frac{1}{2}}$, where $x,y \geq 0$.

44) Simplify $\sqrt{x^2 - 10x + 25}$

45) Rationalize the denominator of $\frac{\sqrt{5}}{(\sqrt{6}+\sqrt{2})}$

46) Solve $|3x-4|=|x-2|$

47) Solve $2|x+3|-5\leq7$ and graph the solution set in a number line

48) Solve $\frac{1}{|3x+3|} < 2$ and express using interval notching.

49) Solve the linear inequalities $2x - 4\geq0$; $3x + 9 \leq 3$.

50) A and B spend monthly such that the difference between their spending is Rs.600. monthly. If A spends 24,000 per annum what are two possibilities for B's spending?

51) Solve $\frac{6-x}{3} < \frac{x}{4} - 1$

52) If one of the roots of a quadratic equation is $(1-\sqrt{6})$ find the quadratic equation.

53) If α and β are the roots of the equation $x^2 - 2x + 3 = 0$ from the equation where roots are

(a) $\frac{1}{\alpha}$ and $\frac{1}{\beta}$

(b) α^2 and β^2

(c) $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$

54) Draw the graph of $y = x^2 - 3x - 4$, find its line of symmetry, vertex, x intercepts.

55) Solve $\sqrt{x+5} < x - 2$

56) If x = 1 is one root of two equation. $x^3 - 6x + 11x - 6 = 0$ find the other roots.

57) Rationalize the denominator $\frac{1}{\sqrt{5}+\sqrt{4}}$

58) Find the square root of $9-4\sqrt{5}$

59) If $x=\sqrt{3}+\sqrt{5}$ find $x^2 + \frac{1}{x^2} - 2$

60) If $\log_2 x + \log_4 x + \log_8 x = \frac{11}{6}$, find x

61) If $\log_{10}2 = .3010$, $\log_{10}3 = .4771$ find $\log_{10}7^2$

62) Resolve with partial fractions $\frac{1}{(x^2-1)(x+2)}$

63) if α and β are the roots of $ax^2+bx+c=0$ from the equation where roots are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$

43 x 3 = 129

64) Simplify $(125)^{\frac{2}{3}}$

65) Simplify $16^{-\frac{3}{4}}$

66) Simplify $(-1000)^{\frac{-2}{3}}$

67) Simplify $\left(3^6\right)^{\frac{1}{3}}$

68) Simplify $\frac{(27)^{\frac{-2}{3}}}{(27)^{\frac{-1}{3}}}$

69) Evaluate $\left(\left[(256)^{\frac{-1}{4}}\right]^{\frac{-1}{4}}\right)^3$

70) If $\left(x^{+\frac{1}{2}} + x^{-\frac{1}{2}}\right)^2 = \frac{9}{2}$, then find the value of $\left(x^{\frac{1}{2}} - x^{-\frac{1}{2}}\right)$ for x>1

71) Simplify and hence the value of n: $\frac{3^{2n}9^2 3^{-n}}{3^{3n}} = 27$

72) Find the radius of the spherical tank whose volume is $\frac{32\pi}{3}$ units

73) Simplify by rationalising the denominator $\frac{7+\sqrt{6}}{3-\sqrt{2}}$

74) Simplify $\frac{1}{3-\sqrt{8}}-\frac{1}{\sqrt{8}-\sqrt{7}}+\frac{1}{\sqrt{7}-\sqrt{6}}-\frac{1}{\sqrt{6}-\sqrt{5}}+\frac{1}{\sqrt{5}-2}$

75) Solve for x $|3-x|<7$

76) Solve for x $|4x-5|\geq -2$

77) Solve for x $\left|3-\frac{3}{4}x\right|\leq\frac{1}{4}$

78) Solve for x $|x|-10<-3$

79) Solve $\frac{1}{|2x-1|}<6$ and express the solution using the interval notation.

80) Solve $-3|x|+5\leq -2$ and graph the solution set in a number line.

81) Solve $2|x+1|-6\leq 7$ and graph the solution set in a number line.

82) Solve: $\frac{1}{5}|10x-2|<1$

83) Solve: $|5x-12|<-2$

84) If $x=\sqrt{2}+\sqrt{3}$ find $\frac{x^2+1}{x^2-1}$

85) Simplify: $\frac{3\sqrt{2}}{\sqrt{6}-\sqrt{3}}=\frac{4\sqrt{3}}{\sqrt{6}-\sqrt{2}}+\frac{2\sqrt{3}}{\sqrt{6}+\sqrt{2}}$

86) Solve $2x^2+x-15\leq 0$.

87) Solve $-x^2+3x-2\geq 0$

88) Find all values of x for which $\frac{x^3(x-1)}{x-2}>0$.

89) Find all values of x that satisfies the inequality $\frac{2x-3}{(x-2)(x-4)}<0$.

90) Solve: $\frac{x^2-4}{x^2-2x-15}\leq 0$

91) Find a quadratic polynomial f(x) such that, f(0)=1; f(-2)=0 and f(1)=0.

92) Construct a cubic polynomial function having zeros at $x=\frac{2}{5}$, $1+\sqrt{3}$ such that f(0)=-8

93) Prove that ap + q = 0 if $f(x)=x^3-3px+2q$ is divisible by $g(x)=x^2+2ax+a^2$.

94) Use the method of undetermined coefficients to find the sum of $1+2+3+\ldots(n-1)+n$, $n\in N$

95) Find the roots of the polynomial equation $(x-1)^3(x+1)^2(x+5)=0$ and state their multiplicity.

96) Solve $x=\sqrt{x+20}$ for $x\in R$

97) The equations $x^2-6x+a=0$ and $x^2-bx+6=0$ have one root in common. The other root of the first and the second equations are integers in the ratio 4 : 3. Find the common root.

98) Find the value of p for which the difference between the roots of the equation $x^2+px+8=0$ is 2.

99) Solve $\frac{x+1}{x+3}<3$

100) Find the logarithm of 1728 to the base $2\sqrt{3}$

101) If the logarithm of 324 to base a is 4, find a.

102) Prove $log\frac{75}{16}-2log\frac{5}{9}+log\frac{32}{243}=log2$

103) If $\log_2 x+\log_4 x+\log_{16} x=\frac{7}{2}$, find the value of x.

104) Solve $x^{\log_3 x}=9$

105) Compute $\log_3 5\ \log_{25} 27$

106) Given that $\log_{10}2=0.30103$, $\log_{10}3=0.47712$ (approximately), find the number of digits in 28.3^{12}

54 x 5 = 270

107) Represent the following inequalities in the interval notation:

$x \geq -1$ and $x < 4$

108) Represent the following inequalities in the interval notation:

$x \leq 5$ and $x \geq -3$

109) Represent the following inequalities in the interval notation:

$x < -1$ or $x < 3$

110) Represent the following inequalities in the interval notation:

$-2x > 0$ or $3x - 4 < 11$

111) Let b>0 and b≠1. Express $y = b^x$ in logarithmic form.Also state the domain and range of the logarithmic function.

112) Compute $log_9^{27} - log_{27}^{9}$

113) Solve $\log_8 x + \log_4 x + \log_2 x = 11$

114) Solve $\log_4 2^{8x} = 2 \log_2{}^8$

115) Solve 23x<100 when (i) x is a natural number (ii) x is an integer.

116) Solve $3\frac{(x-2)}{5} \leq 5\frac{(2-x)}{3}$

117) Solve: $\frac{5-x}{3} < \frac{x}{2} - 4$

118)

If $a^2+b^2 = 7ab$.Show that $\log\left(\frac{a+b}{3}\right) = \frac{1}{2}$ (log a +log b)

119) Prove log $log\frac{a^2}{bc} + log\frac{b^2}{ca} + log\frac{c^2}{ab} = 0$

120) Prove that $log2 + 16log\frac{16}{15} + 12log\frac{25}{24} + 7log\frac{81}{80} = 1$

121) Prove $log_a 2a \times log_b 2b \times log_c 2c = \frac{1}{8}$

122) Prove log a+log a^2+$\log^3$+... $\log a^n = \frac{n(n+1)}{2} loga$

123) If $\frac{\log x}{y-z} = \frac{\log y}{z-x} = \frac{\log z}{x-y}$, then prove that xyz= 1

124) Solve : $log_2 x - 3log_{\frac{1}{2}} x = 6$

125) Solve $\log_{5-x}(x^2-6x+65)=2$

126) Solve: $\sqrt{x+5} + \sqrt{x+21} = \sqrt{6x+40}$

127) Show that $\frac{1}{3-\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-2} = 5$

128) To secure an A grade one must obtain an average of 90 marks or more in 5 subjects each of maximum 100 marks. If one scored 84, 87, 95, 91 in first four subjects, what is the minimum mark one scored in the fifth subject to get A grade in the course?

129) A manufacturer has 600 litres of a 12 percent solution of acid. How many litres of a 30 percent acid solution must be added to it so that the acid content in the resulting mixture will be more than 15 percent but less than 18 percent?

130) Find all pairs of consecutive odd natural numbers both of which are larger than 10 and their sum is less than 40.

131) A model rocket is launched from the ground. The height 'h' reached by the rocket after t seconds from lift off is given by $h(t) = -5t^2 + 100t$, $0 \le t \le 20$. At what time the rocket is 495 feet above the ground?

132) A plumber can be paid according to the following schemes: In the first scheme he will be paid rupees 500 plus rupees 70 per hour, and in the second scheme he will pay rupees 120 per hour. If he works x hours then for what value of x does the first scheme give better wages?

133) A and B are working on similar jobs but their annual salaries differ by more than Rs 6000. If B earns rupees 27000 per month, then what are the possibilities of A's salary per month?

134) Solve: $2\left(x+\frac{1}{x}\right)^2 - 7\left(x+\frac{1}{x}\right) + 5 = 0.$

135) Find the zeros of the polynomial function $f(x)=4x^2-25$.

136) If $x=-2$ is one root of $x^2-x^2-17x=22$, then find the other roots of the equation.

137) Find the real roots of $x^4=16$

138) Solve $(2x+1)^2-(3x+2)^2=0$

139) Resolve the following rational expressions into partial fractions.

$$\frac{1}{x^2-a^2}$$

140) Resolve the following rational expressions into partial fractions.

$$\frac{3x+1}{(x-2)(x+1)}$$

141) Resolve the following rational expressions into partial fractions.

$$\frac{x}{(x^2+1)(x-1)(x+2)}$$

142) Resolve the following rational expressions into partial fractions.

$$\frac{x}{(x-1)^3}$$

143) Resolve the following rational expressions into partial fractions.

$$\frac{1}{x^4-1}$$

144) Resolve the following rational expressions into partial fractions.

$$\frac{(x-1)^2}{x^3+x}$$

145) Resolve the following rational expressions into partial fractions.

$$\frac{x^2+x+1}{x^2-5x+6}$$

146) Resolve the following rational expressions into partial fractions.

$$\frac{x^2+2x+1}{x^2+5x+6}$$

147) Resolve the following rational expressions into partial fractions.

$$\frac{x+12}{(x+1)^2(x-2)}$$

148) Resolve the following rational expressions into partial fractions.

$$\frac{6x^2-x+1}{x^3+x^2+x+1}$$

149) Resolve the following rational expressions into partial fractions.

$$2x^2+5x-11$$

150) Resolve the following rational expressions into partial fractions.

$$\frac{7+x}{(1+x)(1+x^2)}$$

151) Solve $3x^2 + 5x - 2 \leq 0$.

152) Solve $\sqrt{x+14} < x+2$.

153) Solve the equation $\sqrt{6-4x-x^2} = x+4$

154) Resolve into partial fractions: $\frac{x}{(x+3)(x-4)}$

155) Resolve into partial fractions: $\frac{2x}{(x^2+1)(x-1)}$

156) Resolve into partial fractions: $\frac{x+1}{x^2(x-1)}$

157) Shade the region given by the inequality $x \geq 2$.

158) Shade reaion given by the linear inequality $x + 2y > 3$.

159) Solve the linear inequalities and exhibit the solution set graphically: $x+y \geq 3$, $2x-y \leq 5$, $-x+2y \leq 3$.

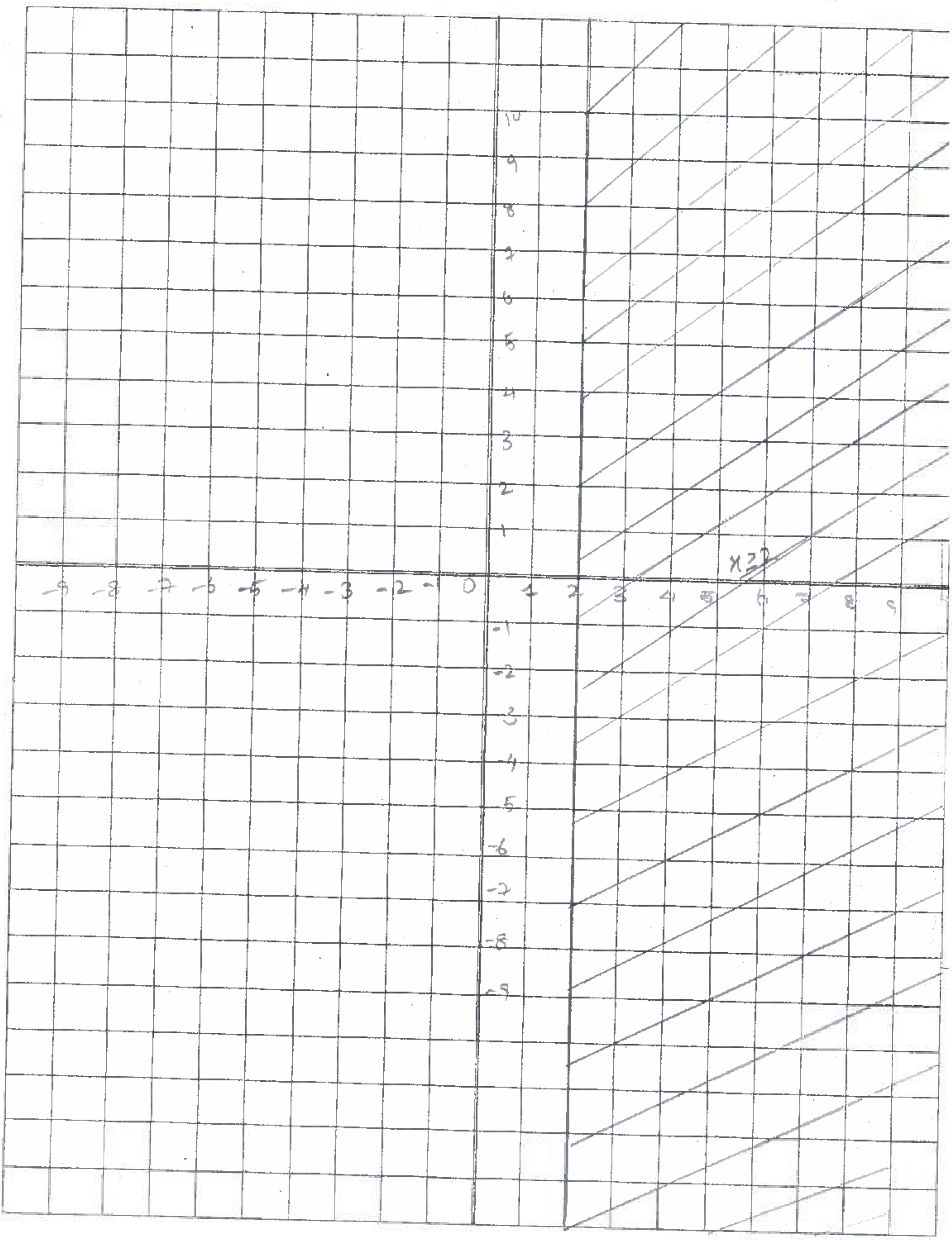
160) Find the square root of $7-4\sqrt{3}$

****

Eg: 2.28

$x \geq 2$

$x = 2$



Eg: 2.29

$x + 2y > 3$

$x + 2y = 3$

x-intercept $\boxed{y=0}$

$x + 2y = 3$

$\boxed{x=3}$

y-intercept $\boxed{x=0}$

$x + 2y = 3$

$2y = 3$

$\boxed{y = 3/2}$

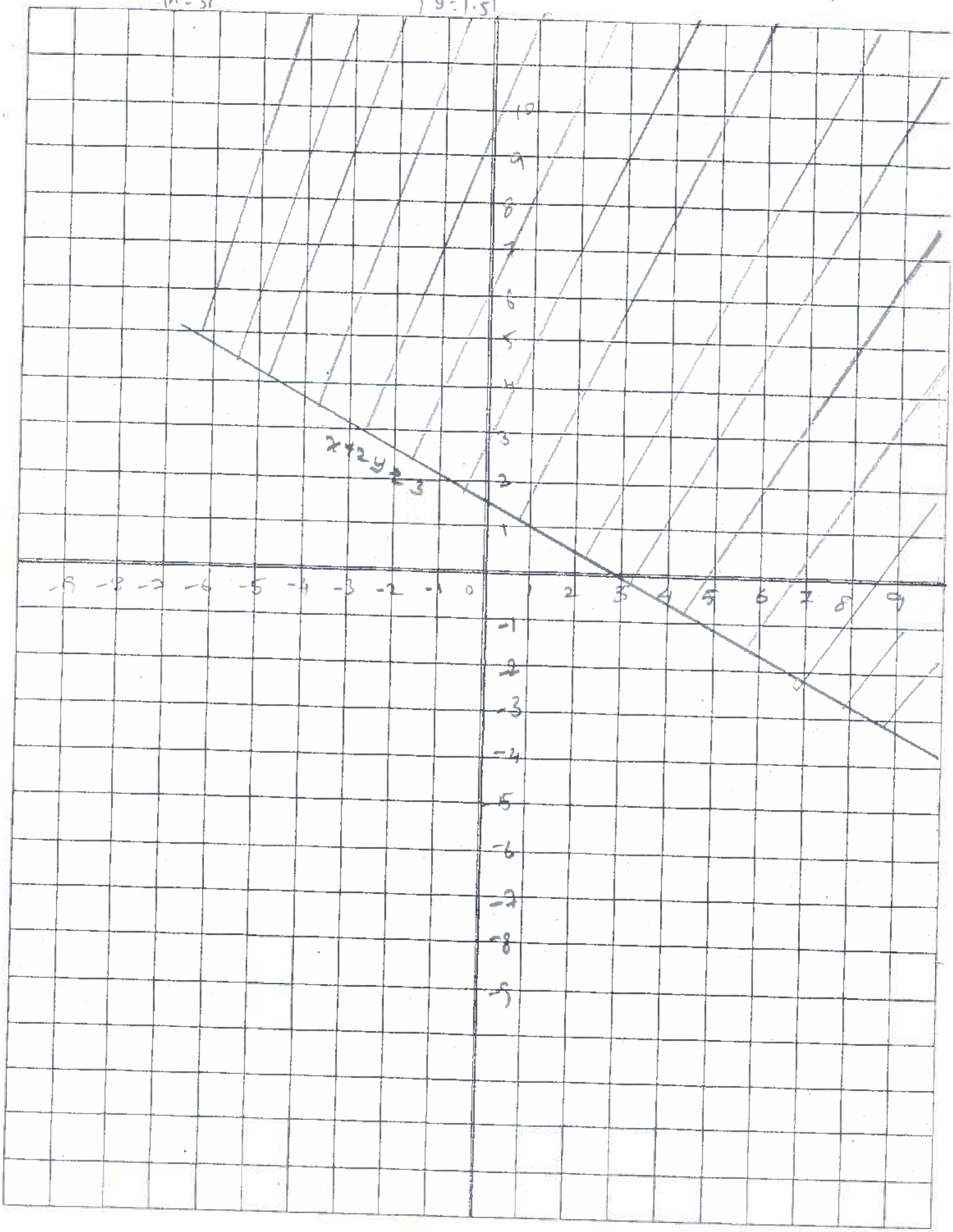
$\boxed{y = 1.5}$

Eg: (3,5)

$3 + 2(5) > 3$

$3 + 10 > 3$

$13 > 3$

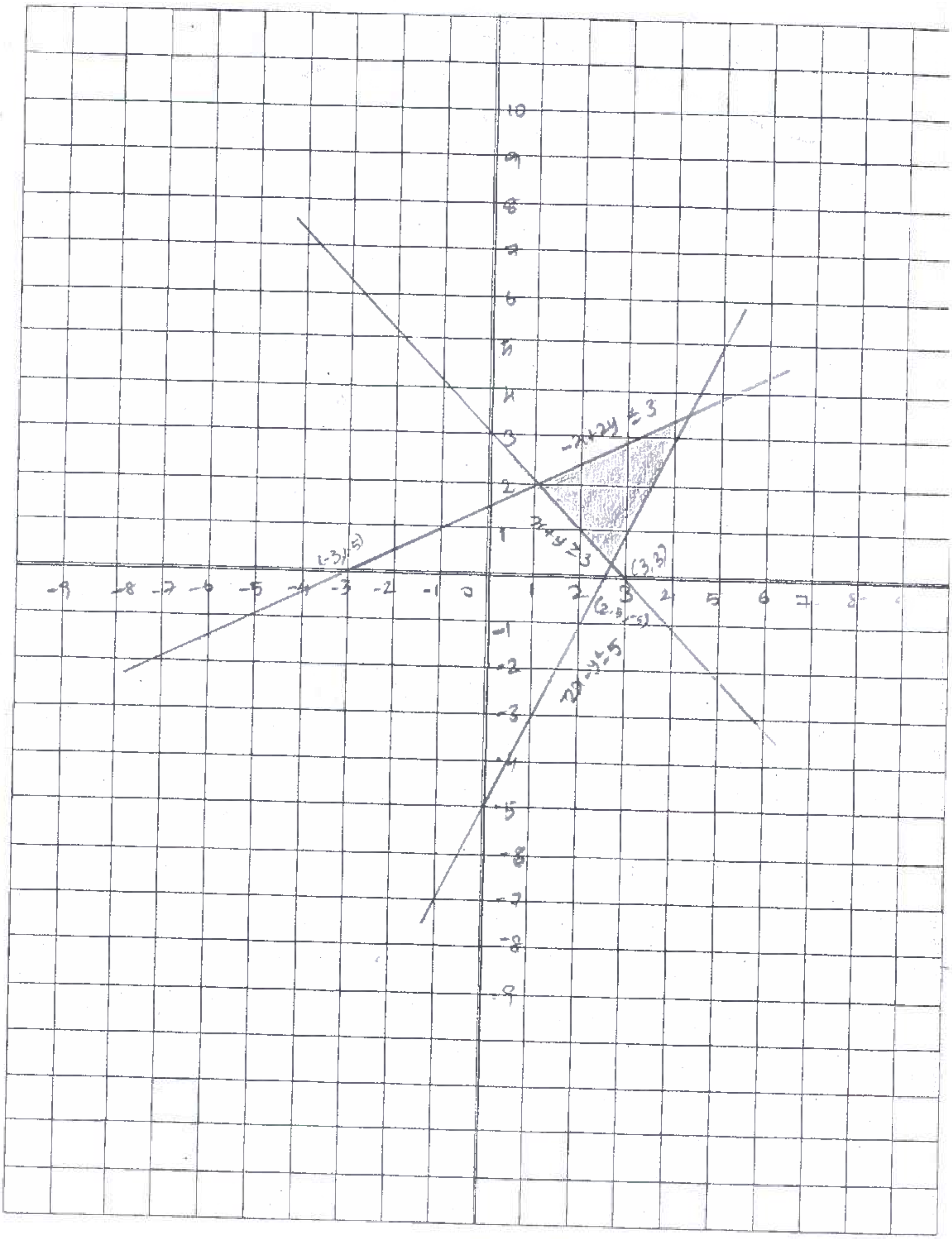


Eg: 2.30

$x+y \geq 3$, $2x-y \leq 5$, $-x+2y \leq 3$

x-intercept $y=0$, $x=3$; x intercept $x = 5/2 = 2.5$; x intercept $x=-3$

y intercept $x=0$, $y=3$; y intercept $y = -5$; y intercept $y = 3 = 1.$



Ex: 2.10

3) $3x + 5y \geq 45$

$x \geq 0$, $y \geq 0$

x intercept Put $y = 0$

$3x = 45$

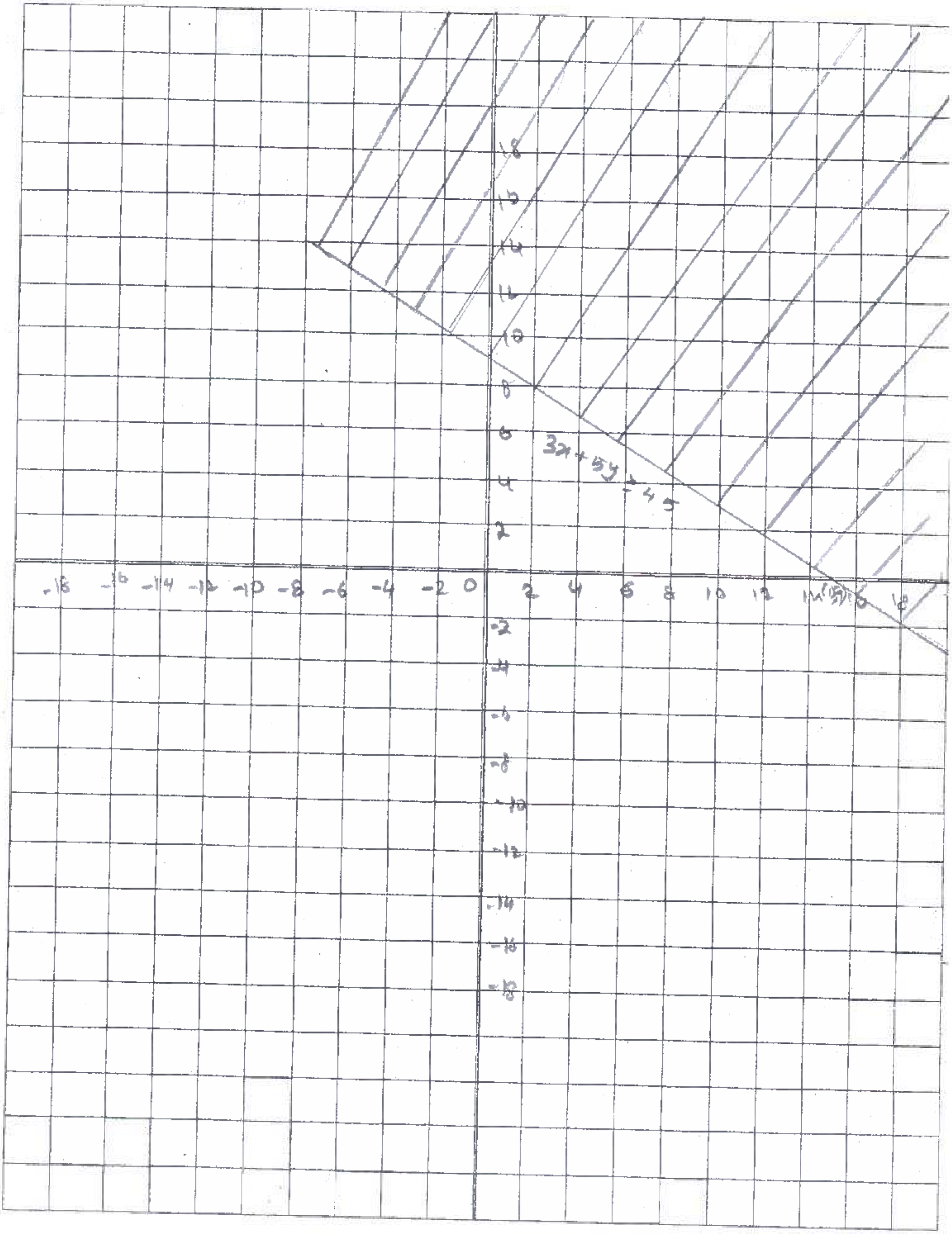
$\boxed{x = 15}$

y-intercept

$x = 0$

$5y = 45$

$\boxed{y = 9}$



5. $2x+3y \le 6$; $x+4y \le 4$, $x \ge 0, y \ge 0$

x intercept
$2x = 6$
$\boxed{x=3}$

y intercept
$3y = 6$
$\boxed{y=2}$

$x+4y \le 4$

x intercept
$\boxed{x=4}$

y intercept
$\boxed{y=1}$