3 ALGEBRA

INTRODUCTION

3.1 In the first term, we learnt about Polynomials, classification of polynomials based on degrees and number of terms, zeros of polynomial, basic operations on polynomials, remainder theorem and their applications.

Polynomial : A polynomial is an expression consisting of variables and constants that involves four fundamental arithmetic operations and non-negative integer exponents of variables.

3.2 **Factor Theorem :**

If $p(x)$ is a polynomial of degree $n \geq 1$ and 'a' is any real number then

(i) $p(a) = 0$ implies $(x - a)$ is a factor of $p(x)$.

(ii) $(x - a)$ is a factor of $p(x)$ implies $p(a) = 0$.

Significance of Factor Theorem

It enables us to find whether the given linear polynomial is a factor or not without actually following the process of long division.

Exercise 3.1

1. **Determine whether $(x - 1)$ is a factor of the following polynomials:**

(i) $x^3 + 5x^2 - 10x + 4$ (ii) $x^4 + 5x^2 - 5x + 1$

Sol. (i) Let $P(x) = x^3 + 5x^2 - 10x + 4$

By factor theorem $(x - 1)$ is a factor of $P(x)$, if $P(1) = 0$

$$P(1) = 1^3 + 5(1^2) - 10(1) + 4$$
$$= 1 + 5 - 10 + 4$$
$$P(1) = 0$$

$\therefore (x - 1)$ is a factor of $x^3 + 5x^2 - 10x + 4$

(ii) Let $P(x) = x^4 + 5x^2 - 5x + 1$

By factor theorem, $(x - 1)$ is a factor of $P(x)$, if $P(1) = 0$

$$P(1) = 1^4 + 5(1^2) - 5(1) + 1$$
$$= 1 + 5 - 5 + 1$$
$$= 2 \neq 0$$

$\therefore (x - 1)$ is not a factor of $x^4 + 5x^2 - 5x + 1$

2. Determine whether $(x + 2)$ is a factor of $2x^4 + x^3 + 4x^2 - x - 7$.

Sol.

$$\text{Let } P(x) = 2x^4 + x^3 + 4x^2 - x - 7$$

By factor theorem, $(x + 2)$ is a factor of $P(x)$, if $P(-2) = 0$

$$\begin{aligned} P(-2) &= 2(-2)^4 + (-2)^3 + 4(-2)^2 - (-2) - 7 \\ &= 2(16) + (-8) + 16 + 2 - 7 \\ &= 32 - 8 + 18 - 7 \\ &= 50 - 15 = 35 \neq 0 \end{aligned}$$

$\therefore (x + 2)$ is not a factor of $2x^4 + x^3 + 4x^2 - x - 7$

3. Using factor theorem, show that $(x - 5)$ is a factor of the polynomial $2x^3 - 5x^2 - 28x + 15$

Sol.

$$\text{Let } P(x) = 2x^3 - 5x^2 - 28x + 15$$

By factor theorem, $(x - 5)$ is a factor of $P(x)$, if $P(5) = 0$

$$\begin{aligned} P(5) &= 2(5)^3 - 5(5)^2 - 28(5) + 15 \\ &= 2 \times 125 - 5 \times 25 - 140 + 15 \\ &= 250 - 125 - 140 + 15 \\ &= 265 - 265 = 0 \end{aligned}$$

$\therefore (x - 5)$ is a factor of $2x^3 - 5x^2 - 28x + 15$

4. Determine the value of m, if $(x + 3)$ is a factor of $x^3 - 3x^2 - mx + 24$.

Sol.

$$\text{Let } P(x) = x^3 - 3x^2 - mx + 24$$

By using factor theorem,

$(x + 3)$ is a factor of $P(x)$, then $P(-3) = 0$

$$\begin{aligned} P(-3) &= (-3)^3 - 3(-3)^2 - m(-3) + 24 = 0 \\ \Rightarrow -27 - 3 \times 9 + 3m + 24 &= 0 \\ \Rightarrow \quad 3m &= 54 - 24 \\ \Rightarrow \quad m &= \frac{30}{3} = 10 \end{aligned}$$

5. If both $(x - 2)$ and $\left(x - \frac{1}{2}\right)$ are the factors of $ax^2 + 5x + b$, then show that $a = b$.

Sol.

$$\begin{aligned} \text{Let } P(x) &= ax^2 + 5x + b \\ (x-2) \text{ is a factor of } P(x), \text{ if } P(2) &= 0 \\ P(2) &= a(2)^2 + 5(2) + b = 0 \\ 4a + 10 + b &= 0 \\ 4a + b &= -10 \qquad \ldots (1) \end{aligned}$$

$$\begin{aligned} \left(x - \frac{1}{2}\right) \text{ is a factor of } P(x), \text{ if } P\left(\frac{1}{2}\right) &= 0 \\ P\left(\frac{1}{2}\right) &= a\left(\frac{1}{2}\right)^2 + 5\left(\frac{1}{2}\right) + b = 0 \\ \frac{a}{4} + \frac{5}{2} + b &= 0 \\ \frac{a}{4} + b &= \frac{-5}{2} \end{aligned}$$

$$\frac{a+4b}{4} = \frac{-5}{2}$$

$$2a+8b = -20$$

$$a+4b = -10 \quad \dots (2)$$

From (1) and (2)

$$4a+b = -10 \quad \dots (1)$$

$$a+4b = -10 \quad \dots (2)$$

(1) and (2) $\Rightarrow$

$$4a+b = a+4b$$

$$3a = 3b$$

$$\therefore a = b. \quad \text{Hence it is proved.}$$

Is $(2x-3)$ a factor of $p(x) = 2x^3 - 9x^2 + x + 12$?

Sol. Let $P(x) = 2x^3 - 9x^2 + x + 12$

By factor theorem,

$(2x-3)$ is a factor of $P(x)$, if $P\left(\frac{3}{2}\right) = 0$

> To find the zero of $2x-3$, put
> $2x-3 = 0$
> $2x = 3$
> $x = \frac{3}{2}$

$$P\left(\frac{3}{2}\right) = 2\left(\frac{3}{2}\right)^3 - 9\left(\frac{3}{2}\right)^2 + \frac{3}{2} + 12$$

$$= \cancel{2}\left(\frac{27}{\cancel{8}_4}\right) - 9\left(\frac{9}{4}\right) + \frac{3}{2} + 12 = \frac{27}{4} - \frac{81}{4} + \frac{3}{2} + 12 = \frac{27-81}{4} + \frac{3}{2} + 12$$

$$= \frac{\cancel{54}^{27}}{\cancel{4}_2} + \frac{3}{2} + 12 = \frac{-27}{2} + \frac{3}{2} + 12 = \frac{-27}{2} + \frac{3+24}{2} = \frac{-27}{2} + \frac{27}{2} = 0$$

$\therefore (2x-3)$ is a factor of $P(x) = 2x^3 - 9x^2 + x + 12$

7. **If $(x-1)$ divides the polynomial $kx^3 - 2x^2 + 25x - 26$ without remainder, then find the value of k.**

Sol. Let $P(x) = kx^3 - 2x^2 + 25x - 26$

By factor theorem, $(x-1)$ divides $P(x)$ without remainder, $P(1) = 0$

$$P(1) = k(1)^3 - 2(1)^2 + 25(1) - 26 = 0$$

$$k - 2 + 25 - 26 = 0$$

$$k - 3 = 0$$

$$k = 3$$

Check if $(x+2)$ and $(x-4)$ are the sides of a rectangle whose area is $x^2 - 2x - 8$ by using factor theorem.

Sol. Let $P(x) = x^2 - 2x^2 - 8$

By using factor theorem, $(x+2)$ is a factor of $P(x)$, if $P(-2) = 0$

$$P(-2) = (-2)^2 - 2(-2) - 8 = 4 + 4 - 8 = 0$$

and also $(x-4)$ is a factor of $P(x)$, if $P(4) = 0$

$$P(4) = 4^2 - 2(4) - 8 = 16 - 8 - 8 = 0$$

$\therefore (x+2)$, $(x-4)$ are the sides of a rectangle whose area is $x^2 - 2x - 8$.

3.3 Algebraic Identities :

An identity is an equality that remains true regardless of the values the for its variables.

We have already learnt about the following identities:

(1) $(a+b)^2 \equiv a^2 + 2ab + b^2$

(2) $(a-b)^2 \equiv a^2 - 2ab + b^2$

(3) $(a+b)(a-b) \equiv a^2 - b^2$

(4) $(x+a)(x+b) \equiv x^2 + (a+b)x + ab.$

Exercise 3.2

1. Expand the following :

(i) $(2x+3y+4z)^2$ (ii) $(2a-3b+4c)^2$

(iii) $(-p+2q+3r)^2$ (iv) $\left(\frac{a}{4}+\frac{b}{3}+\frac{c}{2}\right)^2$

Sol. (i) $(2x+3y+4z)^2$

$$\boxed{(a+b+c)^2 \equiv a^2+b^2+c^2+2ab+2bc+2ca}$$

$$\therefore (2x+3y+4z)^2 = (2x)^2+(3y)^2+(4z)^2+2(2x)(3y)+2(3y)(4z)+2(4z)$$

$$= 4x^2+9y^2+16z^2+12xy+24yz+16zx$$

(ii) $(2a-3b+4c)^2$

$$\boxed{(a+b+c)^2 \equiv a^2+b^2+c^2+2ab+2bc+2ca}$$

$$(2a-3b+4c)^2 = (2a+(-3b)+4c)^2 = (2a)^2+(-3b)^2+(4c)^2+2(2a)(-3b)+2(-3b)(4c)+2(4c)$$

$$= 4a^2+9b^2+16c^2+(-12ab)+(-24bc)+16ca$$

$$= 4a^2+9b^2+16c^2+-12ab-24bc+16ca$$

(iii) $(-p+2q+3r)^2$

$$\boxed{(a+b+c)^2 \equiv a^2+b^2+c^2+2ab+2bc+2ca}$$

$$(-p+2q+3r)^2 = (-p)^2+(2q)^2+(3r)^2+2(-p)(2q)+2(2q)(3r)+2(-p$$

$$= p^2+4q^2+9r^2-4pq+12qr-6rp$$

(iv) $\left(\frac{a}{4}+\frac{b}{3}+\frac{c}{2}\right)^2$

$$\boxed{(a+b+c)^2 \equiv a^2+b^2+c^2+2ab+2bc+2ca}$$

$$\left(\frac{a}{4}+\frac{b}{3}+\frac{c}{2}\right)^2 = \left(\frac{a}{4}\right)^2+\left(\frac{b}{3}\right)^2+\left(\frac{c}{2}\right)^2+\cancel{2}\left(\frac{a}{\cancel{4}}\right)\left(\frac{b}{3}\right)+\cancel{2}\left(\frac{b}{3}\right)\left(\frac{c}{\cancel{2}}\right)+\cancel{2}\left(\frac{c}{\cancel{2}}\right)$$

$$= \frac{a^2}{16} + \frac{b^2}{9} + \frac{c^2}{4} + \frac{ab}{6} + \frac{bc}{3} + \frac{ca}{4}$$

$$\therefore \left(\frac{a}{4} + \frac{b}{3} + \frac{c}{2}\right)^2 = \frac{a^2}{16} + \frac{b^2}{9} + \frac{c^2}{4} + \frac{ab}{6} + \frac{bc}{3} + \frac{ca}{4}$$

Find the expansion of the following:

(i) $(x + 4)(x + 5)(x + 6)$ (ii) $(2p + 3)(2p - 4)(2p - 5)$

(iii) $(3a + 1)(3a - 2)(3a + 4)$ (iv) $(5 + 4m)(4m + 4)(-5 + 4m)$

(i) $(x + 4)(x + 5)(x + 6)$

$$\boxed{(x+a)(x+b)(x+c) \equiv x^3 + (a+b+c)x^2 + (ab+bc+ca)x + abc}$$

$$(x+4)(x+5)(x+6) = x^3 + (4+5+6)x^2 + (4\times 5 + 5\times 6 + 6\times 4)x + 4\times 5\times 6$$

$$= x^3 + 15x^2 + (20+30+24)x + 120$$

$$(x+4)(x+5)(x+6) = x^3 + 15x^2 + 74x + 120$$

(ii) $(2p + 3)(2p - 4)(2p - 5)$

$$\boxed{(x+a)(x+b)(x+c) \equiv x^3 + (a+b+c)x^2 + (ab+bc+ca)x + abc}$$

$$(2p+3)(2p-4)(2p-5) = (2p)^3 + (3-4-5)(2p)^2 + [(3\times -4)^2 + (-4\times -5) + (-5\times 3)]\,2p + 3\times -4\times -5$$

$$= 8p^3 + (-6)(4p^2) + [-12 + 20 + (-15)]\,2p + 60$$

$$= 8p^3 - 24p^2 + (-7)2p + 60$$

$$(2p+3)(2p-4)(2p-5) = 8p^3 - 24p^2 - 14p + 60$$

(iii) $(3a + 1)(3a - 2)(3a + 4)$

$$\boxed{(x+a)(x+b)(x+c) \equiv x^3 + (a+b+c)x^2 + (ab+bc+ca)x + abc}$$

$$(3a+1)(3a-2)(3a+4) = (3a)^3 + (1-2+4)(3a)^2 + [1\times(-2) + (-2\times 4) + 4\times 1](3a) + 1\times -2\times 4$$

$$= 27a^3 + 3(9a^2) + (-2-8+4)3a - 8$$

$$= 27a^3 + 27a^2 - 8a - 8$$

(iv) $(5 + 4m)(4m + 4)(-5 + 4m)$

$$\boxed{(x+a)(x+b)(x+c) \equiv x^3 + (a+b+c)x^2 + (ab+bc+ca)x + abc}$$

$$(4m+5)(4m+4)(4m-5) = (4m)^3 + (5+4-5)(4m)^2 + [(5\times 4) + (4\times -5) + (-5\times 5)](4m) + (5\times 4\times -5)$$

$$= 64m^3 + 64m^2 + (20-20-25)4m - 100$$

$$= 64m^3 + 64m^2 - 100m - 100$$

3. **Using algebraic identity, find the coefficients of x^2 , x and constant term with actual expansion.**

(i) $(x+5)(x+6)(x+7)$ (ii) $(2x+3)(2x-5)(2x-6)$

Sol. (i) $(x+5)(x+6)(x+7)$

$$\boxed{(x+a)(x+b)(x+c) \equiv x^3+(a+b+c)x^2+(ab+bc+ca)x+abc}$$

Co-efficient of x^2 $= a+b+c = 5+6+7 = 18$

Co-efficient of x^2 $= ab+bc+ca = (5\times 6)+(6\times 7)+(7\times 5)$

$= 30+42+35 = 107$

Constant term $= abc = 5\times 6\times 7$

Co-efficient of constant term $= 210$

(ii) $(2x+3)(2x-5)(2x-6)$

$\therefore$ Co-efficient of x^2 $= 4(a+b+c) = 4(3+(-5)+(-6))$

$= 4\times(-8) = -32$

Co-efficient of x $= 2(ab+bc+ca)$

$= 2[3\times(-5)+(-5)(-6)+(-6)(3)]$

$= 2[-15+30-18] = 2\times(-3) = -6$

Constant term $= abc = 3\times(-5)\times(-6) = 90$

4. **If $(x+a)(x+b)(x+c) = x^3+14x^2+59x+70$, find the value of**

(i) $a+b+c$

(ii) $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$

(iii) $a^2+b^2+c^2$

(iv) $\frac{a}{bc}+\frac{b}{ac}+\frac{c}{ab}$

Sol. $(x+a)(x+b)(x+c) = x^3+14x^2+59x+70$ ...(1)

$$\boxed{(x+a)(x+b)(x+c) \equiv x^3+(a+b+c)x^2+(ab+bc+ca)x+abc}$$ ...(2)

(i) Comparing (1) & (2)

We get, $a+b+c = 14$

(ii) $\frac{1}{a}+\frac{1}{b}+\frac{1}{c} = \frac{bc+ac+ab}{abc} = \frac{59}{70}$

(iii) $\boxed{(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca}$

$a^2+b^2+c^2 = (a+b+c)^2-2(ab+bc+ca)$

$= 14^2-2(59) = 196-118 = 78$

(iv) $\frac{a}{bc}+\frac{b}{ac}+\frac{c}{ab} = \frac{a^2+b^2+c^2}{abc} = \frac{78}{70} = \frac{39}{35}$

5. Expand

(i) $(2a+3b)^3$ (ii) $(3a-4b)^3$

(iii) $\left(x+\frac{1}{y}\right)^3$ (iv) $\left(a+\frac{1}{a}\right)^3$

Sol. (i) $(2a+3b)^3$

We know that

$$\boxed{(a+b)^3 \equiv a^3+3a^2b+3ab^2+b^3}$$

$$(2a+3b)^3 = (2a)^3+3(2a)^2(3b)+3(2a)(3b)^2+(3b)^3$$
$$= 8a^3+36a^2b+54ab^2+27b^3$$

(ii) $(3a-4b)^3$

We know that

$$\boxed{(x-y)^3 \equiv x^3-3x^2y+3xy^2-y^3}$$

$$(3a-4b)^3 = (3a)^3-3(3a)^2(4b)+3(3a)(4b)^2-(4b)^3$$
$$= 27a^3-108a^2b+144ab^2-64b^3$$

(iii) $\left(x+\frac{1}{y}\right)^3$

$$\boxed{(x+y)^3 \equiv x^3+3x^2y+3xy^2+y^3}$$

$$\left(x+\frac{1}{y}\right)^3 = x^3+\frac{3x^2}{y}+\frac{3x}{y^2}+\frac{1}{y^3}$$

(iv) $\left(a+\frac{1}{a}\right)^3$

$$\boxed{(x+y)^3 \equiv x^3+3x^2y+3xy^2+y^3}$$

$$\left(a+\frac{1}{a}\right)^3 = a^3+\frac{3a^{\cancel{2}}}{\cancel{a}}+3\cancel{a}\times\frac{1}{a^{\cancel{2}}}+\frac{1}{a^3}=a^3+3a+\frac{3}{a}+\frac{1}{a^3}$$

6. **Evaluate the following by using identities:**

(i) 98^3 (ii) 103^3 (iii) 99^3 (iv) 1001^3

Sol. (i)

$$98^2 = (100-2)^3$$

$$\boxed{(a-b)^3 \equiv a^3-3a^2b+3ab^2-b^3}$$

$$98^3 = (100-2)^3 = 100^3-3\times100^2\times2+3\times100\times2^2-2^3$$
$$= 1000000-3\times10000\times2+300\times4-8$$
$$= 1000000-60000+1200-8 = 1001200-60008 = 941192$$

(ii)

$$(103)^3 = (100+3)^3$$

$$\boxed{(a+b)^3 \equiv a^3+3a^2b+3ab^2+b^3}$$

$$(100+3)^3 = 100^3+3(100)^2\times3+3\times100\times3^2+3^3$$
$$= 1000000+9(10000)+2700+27 = 1092727$$

(iii) $99^3 = (100-1)^3$

$$\boxed{(a-b)^3 \equiv a^3+3a^2b+3ab^2-b^3}$$

$$(100-1)^3 = 100^3 - 3(100^2)(1) + 3 \times 100 \times 1^2 - 1^3$$

$$= 1000000 - 3 \times 10000 + 300 - 8 = 970299$$

(iv) $1001^3 = (1000+1)^3$

$$\boxed{(a+b)^3 \equiv a^3+3a^2b+3ab^2+b^3}$$

$$(1000+1)^3 = 1000^3 + 3(1000^2) \times 1 + 3 \times 1000 \times 1^2 + 1^3$$

$$= 1000,000,000 + 3,000,000 + 3000 + 1 = 1,003,003,001$$

7. If $(x+y+z) = 9$ and $(xy+yz+zx) = 26$ then find the value of $x^2+y^2+z^2$.

Sol.

$$(x+y+z) = 9 \text{ and } (xy+yz+zx) = 26$$

$$x^2+y^2+z^2 = (x+y+z)^2 - 2(xy+yz+zx)$$

$$= 9^2 - 2 \times 26 = 81 - 52 = 29$$

8. Find $27a^3+64b^3$, if $3a+4b = 10$ and $ab = 2$.

Sol.

$$3a+4b = 10, ab = 2$$

$$(3a+4b)^3 = (3a)^3 + 3(3a)^2(4b) + 3(3a)(4b)^2 + (4b)^3$$

$$(27a^3+64b^3) = (3a+4b)^3 - 3(3a)(4b)(3a+4b)$$

$$\boxed{\because x^3+y^3 \equiv (x+y)^3 - 3xy-(x+y)}$$

$$= 10^3 - 36\,ab\,(10) = 1000 - 36 \times 2 \times 10$$

$$= 1000 - 720 = 280$$

9. Find x^3-y^3, if $x-y = 5$ and $xy = 14$.

Sol.

$$x-y = 5, xy = 14$$

$$x^3-y^3 = (x-y)^3 + 3xy(x-y) = 5^3 + 3 \times 14 \times 5$$

$$= 125 + 210 = 335$$

10. If $a+\frac{1}{a}=6$, then find the value of $a^3+\frac{1}{a^3}$.

Sol.

$$\boxed{a^3+b^3 = (a+b)^3 - 3ab(a+b)}$$

$$a^3+\left(\frac{1}{a}\right)^3 = \left(a+\frac{1}{a}\right)^3 - 3\cancel{a} \times \frac{1}{\cancel{a}}\left(a+\frac{1}{a}\right)$$

$$a^3+\frac{1}{a^3} = 6^3 - 3 \times 6 = 216 - 18 = 198$$

11. If $x^2+\frac{1}{x^2} = 23$, then find the value of $x+\frac{1}{x}$ and $x^3+\frac{1}{x^3}$.

Sol.

$$\left(x+\frac{1}{x}\right)^2 = x^2 + 2\cancel{x} \times \frac{1}{\cancel{x}} + \frac{1}{x^2}$$

$$\left(x+\frac{1}{x}\right)^2 = x^2 + 2 + \frac{1}{x^2} = \left(x^2+\frac{1}{x^2}\right) + 2$$

$$\left(x+\frac{1}{x}\right)^2 = 23 + 2 = 25$$

$$\left(x+\frac{1}{x}\right) = \sqrt{25} = 5$$

$$x^3+\frac{1}{x^3} = \left(x+\frac{1}{x}\right)^3 - 3\cancel{x}\times\frac{1}{\cancel{x}}\left[x+\frac{1}{x}\right]$$

$$= 5^3 - 3\,(5) = 125 - 15 = 110$$

13. If $\left(y-\frac{1}{y}\right)^3 = 27$, then find the value of $y^3 - \frac{1}{y^3}$.

Sol.

$$\left(y-\frac{1}{y}\right)^3 = 27 \text{ (Given)}$$

$$y^3-\frac{1}{y^3} = \left(y-\frac{1}{y}\right)^3 + 3y + \frac{1}{y}\left(y-\frac{1}{y}\right)$$

$$\boxed{\because x^3 - y^3 \equiv (x-y)^3 + 3xy\,(x-y)}$$

$$= 27 + 3\left(y-\frac{1}{y}\right)$$

$$\left[\because \left(y-\frac{1}{y}\right)^3 = 27\,;\; y-\frac{1}{y} = \sqrt[3]{27} = 3\right]$$

$$= 27 + 3\times 3 = 27 + 9 = 36$$

13. **Simplify : (i) $(2a+3b+4c)(4a^2+9b^2+16c^2-6ab-12bc-12bc-8ca)$**

(ii) $(x-2y+3z)\,(x^2+4y^2+9z^2+2xy+6yz-3xz)$

Sol. (i) $(2a+3b+4c)(4a^2+9b^2+16c^2-6ab-12bc-12bc-8ca)$

We know that

$$\boxed{(a+b+c)\,(a^2+b^2+c^2-ab-bc-ca) = a^3+b^3+c^3\times 3abc}$$

$\therefore (2a+3b+4c)\,(4a^2+9b^2+16\,c^2-6\,ab-12\,bc-8\,ca)$

$$= (2a)^3+(3b)^3+(4c)^3-3\times 2a\times 3b\times 4c$$

$$= 8a^3+27b^3+64c^3-72\,abc$$

(ii) $(x-2y+3z)\,(x^2+4y^2+9z^2+2xy+6yz-3xz)$

$$\boxed{(a+b+c)\,(a^2+b^2+c^2-ab-bc-ca) = a^3+b^3+c^3-3abc}$$

$\therefore (x-2y+3z)\,(x^2+4y^2+9z^2+2\,xy+6yz-3\,xz)$

$$= x^3+(-2y)^3+(3z)^3-3\times x\times(-2y)\,(3z)$$

$$= x^3-8y^3+27z^3+18\,xyz$$

14. **By using identity evaluate the following:**

(i) $7^3-10^3+3^3$ **(ii)** $729-216-27$

(iii) $\left(\frac{1}{3}\right)^3+\left(\frac{1}{2}\right)^3-\left(\frac{5}{6}\right)^3$ **(iv)** $1+\frac{1}{8}-\frac{27}{8}$

Sol. **(i)** $\mathbf{7^3 - 10^3 + 3^3}$

$$\boxed{(a+b+c)(a^2+b^2+c^2-ab-bc-ca) = a^3+b^3+c^3-3abc}$$

$$\text{If } a+b+c=0, \text{ then } a^3+b^3+c^3 = 3\,abc$$

$$\therefore 7-10+3 = 0$$

$$\Rightarrow 7^3-10^3+3^3 = 3\times 7\times -10\times 3$$

$$= 9\times -70 = -630$$

(ii) $\mathbf{729-216-27}$

$$= 9^3-6^3-3^3 \qquad \text{(here } 9-6-3 = 9-9 = 0\text{)}$$

$$\text{If } a+b+c = 0 \text{ then } a^3+b^3+c^3 = 3\,abc$$

$$729-216-27 = 9^3+(-6)^3+(-3)^3 = 3\times 9\times -6\times -3$$

$$= 27\times 18 = 486$$

(iii) $\left(\frac{1}{3}\right)^3+\left(\frac{1}{2}\right)^3-\left(\frac{5}{6}\right)^3$

$$\boxed{\text{Here } \frac{1}{3}+\frac{1}{2}-\frac{5}{6} = \frac{2+3-5}{6} = \frac{5-5}{6} = \frac{0}{6} = 0}$$

$$\left(\frac{1}{3}\right)^3+\left(\frac{1}{2}\right)^3-\left(\frac{5}{6}\right)^3 = \cancel{3}\times\frac{1}{\cancel{3}}\times\frac{1}{2}\times\frac{-5}{6} = \frac{-5}{12}$$

(iv) $1+\frac{1}{8}-\frac{27}{8}$

$$1+\frac{1}{8}-\frac{27}{8} = 1^3+\left(\frac{1}{2}\right)^3+\left(\frac{-3}{2}\right)^3$$

$$\boxed{\text{Here } 1+\frac{1}{2}-\frac{3}{2} = \frac{2+1-3}{2} = \frac{0}{2} = 0}$$

$$\therefore 1+\frac{1}{8}-\frac{27}{8} = 1^3+\left(\frac{1}{2}\right)^3+\left(\frac{-3}{2}\right)^3 = 3\times 1\times\frac{1}{2}\times\frac{-3}{2} = \frac{-9}{4}$$

15. **Simplify :** $\left[\frac{(x^2-y^2)^3+(y^2-z^2)^3+(z^2-x^2)^3}{(x-y)^3+(y-z)^3+(z-x)^3}\right]$ **by using identity.**

Sol. Let $\frac{(x^2-y^2)^3+(y^2-z^2)^3+(z^2-x^2)^3}{(x-y)^3+(y-z)^3+(z-x^3)}$ ... (1)

$$\text{here } \cancel{x^2}-\cancel{y^2}+\cancel{y^2}-\cancel{z^2}+\cancel{z^2}-\cancel{x^2} = 0 \text{ and}$$

$$\cancel{x}+\cancel{y}+\cancel{y}-\cancel{z}+\cancel{z}-\cancel{x} = 0$$

$$(x^2-y^2)^3+(y^2-z^2)^3+(z^2-x^2)^3 = 3\,(x^2-y^2)(y^2-z^2)(z^2-x^2)$$

$$(x-y)^3+(y-z)^3+(z-x)^3 = 3\,(x-y)(y-z)(z-x)$$

$$\therefore (1) = \frac{\cancel{3}(x+y)\cancel{(x-y)}(y+z)\cancel{(y-z)}\cancel{(z-x)}(z+x)}{\cancel{3}\cancel{(x-y)}\cancel{(y-z)}\cancel{(z-x)}}$$

$$= (x+y)(y+z)(z+x)$$

If $a=4$, $b=5$ and $c=6$, then find the value of $\dfrac{(ab+bc+ca-a^2-b^2-c^2)}{(3abc-a^3-b^3-c^3)}$.

Sol.

$$\boxed{\begin{aligned} x^3+y^3+z^3-3xyz &= (x+y+z)(x^2+y^2+z^2-xy-yz-zx) \\ \frac{1}{(x+y+z)} &= \frac{x^2+y^2+z^2-xy-yz-zx}{x^3+y^3+z^3-3xyz} \end{aligned}}$$

$$ab+bc+ca-a^2-b^2-c^2 = -[a^2+b^2+c^2-ab-bc-ca]$$

$$3abc-a^3-b^3-c^3 = -[a^3+b^3+z^3-3abc]$$

$$\therefore \frac{ab+bc+ca-a^2-b^2-c^2}{3abc-a^2-b^3-c^3} = \frac{\cancel{-}[a^2+b^2+c^2-ab-bc-ca]}{\cancel{-}[a^3+b^3+z^3-3abc]}$$

$$= \frac{1}{a+b+c} = \frac{1}{4+5+6} = \frac{1}{15}$$

17. **Verify $x^3+y^3+z^3-3xyz = \dfrac{1}{2}[x+y+z][(x-y)^2+(y-z)^2+(z-x)^2]$.**

Sol.

$$\text{R.H.S} = \frac{1}{2}(x+y+z)[(x-y)^2+(y-z)^2+(z-x)^2]$$

$$= \frac{1}{2}(x+y+z)[x^2-2xy+y^2+y^2-2yz+z^2+z^2-2zx+x^2]$$

$$= \frac{1}{2}[(x+y+z)[2x^2+2y^2+2z^2-2xy-2yz-2zx^2]]$$

$$= \frac{1}{\cancel{2}}[(x+y+z)\cancel{2}(x^2+y^2+z^2-xy-yz-zx)]$$

$$= (x+y+z)(x^2+y^2+z^2-xy-yz-zx]$$

$$= [x^3+y^3+z^3+\cancel{x^2y}+\cancel{y^2z}+\cancel{y^2x}+\cancel{y^2z}+\cancel{z^2x}+\cancel{z^2y}-\cancel{x^2y}-\cancel{xy^2}$$
$$-xyz-xyz-\cancel{y^2z}-\cancel{yz^2}-zx^2-xyz-\cancel{z^2x}]$$

$$= x^3+y^3+z^3-3xyz = \text{L. H. S}$$

Hence it is verified.

18. **If $2x-3y-4z=0$, then find $8x^3-27y^3-64z^3$.**

Sol.

$$\text{If } 2x-3y-4z = 0 \text{ then } 8x^3-27y^3-64z^3 = ?$$

$$\boxed{\text{If } x+y+z = 0 \text{ then } x^3+y^3+z^3 = 3xyz}$$

$$8x^3-27y^3-64z^3 = (2x)^3+(-3y)^3+(-4z)^3$$

$$= 3\times 2x\times -3y\times -4z = 72\,xyz$$

Exercise 3.3

1. Find the GCD for the following:

(i) p^5, p^{11}, p^9 (ii) $4x^3, y^3, z^3$

(iii) $9a^2b^2c^3, 15a^3b^2c^4$ (iv) $64x^8, 240x^6$

(v) $ab^2c^3, a^2b^3c, a^3bc^2$ (vi) $35x^5y^3z^4, 49x^2yz^3, 14xy^2z^2$

(vii) $25ab^2c, 100a^2bc, 125ab$ (viii) $3abc, 5xyz, 7pqr$

Sol. (i) p^5, p^{11}, p^9

G.C.D. of $p^5, p^{11}, p^9 = p^5$

$p^5 = \underline{p^5}$
$p^9 = \underline{p^5} \times p^4$
$p^{11} = \underline{p^5} \times p^6$

(ii) $4x^3, y^3, z^3$

$$\text{G.C.D of } 4x^3 = \underline{1} \times 4x^3$$
$$y^3 = \underline{1} \times y^3$$
$$z^3 = \underline{1} \times z^3$$
$$\therefore \text{G.C.D} = 1$$

(iii) $9a^2b^2c^3, 15a^3b^2c^4$

$$9a^2b^2c^3 = \underline{3} \times 3a^2b^2c^3$$
$$15a^3b^2c^4 = \underline{3} \times 5a^3b^2c^4$$
$$\text{G.C.D. of } 9a^2b^2c^3, 15a^3b^2c^4 = 3 \times a^2b^2c^3 = 3a^2b^2c^3$$

(iv) $64x^8, 240x^6$

$$64x^8 = \underline{2} \times \underline{2} \times \underline{2} \times \underline{2} \times 2 \times 2 \times \underline{x^6} \times x^2$$
$$240x^6 = \underline{2} \times \underline{2} \times \underline{2} \times \underline{2} \times 3 \times 5\,\underline{x^6}$$
$$\text{G.C.D. of } 64x^8, 240x^6 \text{ is} = 2 \times 2 \times 2 \times 2\,x^6 = 16x^6$$

2	240
2	120
2	60
2	30
3	15
5	5
	1

(v) $ab^2c^3, a^2b^3c, a^3bc^2$

$$ab^2c^3 = \underline{a} \times \underline{b} \times b \times \underline{c} \times c \times c$$
$$a^2b^3c = \underline{a} \times a \times \underline{b} \times b \times b \times \underline{c}$$
$$a^3bc^2 = \underline{a} \times a \times a \times \underline{b} \times \underline{c} \times c$$
$$\text{G.C.D. of } ab^2c^3, a^2b^3c, a^3bc^2 \text{ is} = a \times b \times c = abc$$

(vi) $35x^5y^3z^4, 49x^2yz^3, 14xy^2z^2$

$$35x^5y^3z^4 = 5 \times \underline{7}x^5y^3z^4$$
$$49x^2yz^3 = \underline{7} \times 7x^2y\,z^3$$
$$14xy^2z^2 = 2 \times \underline{7}x\,y^2\,z^2$$
$$\text{G.C.D is} = 7x\,y\,z^2$$

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(vii) $25\,ab^3c$, $100\,a^2bc$, $125\,ab$

$$\begin{aligned} 25\,ab^3c &= \underline{5} \times \underline{5}\,ab^3c \\ 100a^2bc &= 2 \times \underline{5} \times 2 \times \underline{5}\,a^2bc \\ 125\,ab &= \underline{5} \times \underline{5} \times 5\,ab \\ \text{G.C.D is} &= 5 \times 5\,ab = 25\,ab \end{aligned}$$

(viii) $3abc$, $5xyz$, $7pqr$

$$\begin{aligned} 3\,abc &= \underline{1} \times \underline{3}\,abc \\ 5\,xyz &= \underline{1} \times 5\,xyz \\ 7\,pqr &= \underline{1} \times 7\,pqr \\ \text{G.C.D is} &= 1 \end{aligned}$$

Find the GCD of the following :

(i) $(2x+5)$, $(5x+2)$ (ii) $a^{m+1}, a^{m+2}, a^{m+3}$

(iii) $2a^2+a$, $4a^2-1$ (iv) $3a^2, 5b^3, 7c^4$

(v) x^4-1, x^2-1 (vi) a^3-9ax^2, $(a-3x)^2$

Sol. (i) $(2x+5)$, $(5x+2)$

$$\begin{aligned} (2x+5) &= \underline{1} \times (2x+5) \\ (5x+2) &= \underline{1} \times (5x+2) \\ \therefore \text{G.C.D} &= 1 \end{aligned}$$

(ii) $a^{m+1}, a^{m+2}, a^{m+3}$

$$\begin{aligned} a^{m+1} &= \underline{a}^m \times \underline{a}^1 \\ a^{m+2} &= \underline{a}^m \times \underline{a}^1 \times a^1 \\ a^{m+3} &= \underline{a}^m \times \underline{a}^1 \times a^1 \times a^1 \\ \text{G.C.D} &= a^m \times a^1 = a^{m+1} \end{aligned}$$

(iii) $2a^2+a$, $4a^2-1$

$$\begin{aligned} 2a^2+a &= a\,(2a+1) \\ 4a^2-1 &= (2a)^2-1^2 = (2a+1)\,(2a-1) \\ \text{G.C.D} &= (2a+1) \end{aligned}$$

(iv) $3a^2, 5b^3, 7c^4$

$$\begin{aligned} 3a^2 &= \underline{1} \times 3a \times a \\ 5b^3 &= \underline{1} \times 5\,b \times b \times b \\ 7c^4 &= \underline{1} \times 7\,c \times c \times c \times c \\ \therefore \text{G.C.D} &= 1 \end{aligned}$$

(v) x^4-1, x^2-1

$$\begin{aligned} x^4-1 &= (x^2)^2-1^2 = (x^2+1)\,(x^2-1) \\ &= (x^2+1)\,(x^2-1^2) = (x^2+1)\,\underline{(x+1)}\,\underline{(x-1)} \\ x^2-1 &= x^2-1^2 = (x+1)\,(x-1) \\ \text{G.C.D} &= (x+1)\,(x-1) = x^2-1 \end{aligned}$$

(vi) a^3-9ax^2, $(a-3x)^2$

$$\begin{aligned} a^3-9ax^2 &= a\,(a^2-(3x)^2) = a\,(a+3x)\,\underline{(a-3x)} \\ (a-3x)^2 &= \underline{(a-3x)}\,(a-3x) \\ \text{G.C.D} &= (a-3x) \end{aligned}$$

Exercise 3.4

1. Factorise the following expressions:

(i) $2a^2 + 4a^2b + 8a^2c$

(ii) $ab - ac - mb + mc$

(iii) $pr + qr + pq + p^2$

(iv) $2y^3 + y^2 - 2y - 1$

Sol. (i) $2a^2 + 4a^2b + 8a^2c = 2a^2[1 + 2b + 4c]$

(ii) $ab - ac - mb + mc = a(b - c) - m(b - c) = (b - c)(a - m)$

(iii) $pr + qr + pq + p^2 = p(p + r) + q(p + r) = (p + r)(p + q)$

(iv) $2y^3 + y^2 - 2y - 1 = 2y^3 - 2y + y^2 - 1 = 2y(y^2 - 1) + (y^2 - 1)$

$= (y^2 - 1)(2y + 1) = (y + 1)(y - 1)(2y + 1)$

2. Factorise the following:

(i) $x^2 + 4x + 4$

(ii) $3a^2 - 24ab + 48b^2$

(iii) $x^5 - 16x$

(iv) $m^2 + \frac{1}{m^2} - 23$

(v) $6 - 216x^2$

(vi) $a^2 + \frac{1}{a^2} - 18$

(vii) $m^4 - 7m^2 + 1$

(viii) $x^{2n} + 2x^n + 1$

(ix) $\frac{1}{3}a^2 - 2a + 3$

(x) $a^4 + a^2b^2 + b^4$

(xi) $x^4 + 4y^4$

Sol. (i) $x^2 + 4x + 4 = (x + 2)(x + 2) = (x + 2)^2$

$$\boxed{\because (a + b)^2 \equiv a^2 + 2ab + b^2}$$

(ii) $3a^2 - 24ab + 48b^2 = 3[a^2 - 8ab + 16b^2]$

$= 3[a - 4b]^2 \qquad (\because (a - b)^2 = a^2 - 2ab + b^2)$

(iii) $x^5 - 16x = x[x^4 - 16] = x[(x^2)^2 - 4^2]$

$= x(x^2 + 4)(x^2 - 4) = x(x^2 + 4)(x + 2)(x - 2)$

(iv) $m^2 + \frac{1}{m^2} - 23 = \left(m + \frac{1}{m}\right)^2 - 2 - 23 = \left(m + \frac{1}{m}\right)^2 - 25$

$= \left(m + \frac{1}{m}\right)^2 - 5^2 = \left(m + \frac{1}{m} - 5\right)\left(m + \frac{1}{m} + 5\right)$

(v) $6 - 216x^2 = 6(1 - (6x)^2) = 6(1 + 6x)(1 - 6x)$

(vi) $a^2 + \frac{1}{a^2} - 18 = \left(a - \frac{1}{a}\right)^2 + 2 - 18$

$= \left(a - \frac{1}{a}\right)^2 - 16 = \left(a - \frac{1}{a} + 4\right)\left(a - \frac{1}{a} - 4\right)$

(vii) $m^4 - 7m^2 + 1 = (m^2 + 3m + 1)(m^2 - 3m + 1)$

(viii) $x^{2n} + 2x^n + 1 = (x^n)^2 + 2\,x^n + 1^2 = (x^n + 1)^2$

(ix) $\frac{1}{3}a^2 - 2a + 3 = \left(\frac{1}{\sqrt{3}}a\right)^2 - 2\times\frac{1}{\sqrt{3}}a\times\sqrt{3} + (\sqrt{3})^2$

$= \left(\frac{1}{\sqrt{3}}a - \sqrt{3}\right)^2$

(x) $a^4 + a^2b^2 + b^4 = (a^2 + b^2)^2 - a^2b^2 = (a^2 + b^2)^2 - (ab)^2$

$= (a^2 + b^2 + ab)(a^2 + b^2 - ab)$

(xi) $x^4 + 4y^4 = (x^2 + 2y^2)^2 - 4x^2y^2 = (x^2 + 2y^2)^2 - (2xy)^2$

$= (x^2 + 2y^2 + 2xy)(x^2 + 2y^2 - 2xy)$

3. **Factorise the following:**

(i) $\mathbf{4x^2 + 9y^2 + 25z^2 + 12xy + 30yz + 20\,xz}$

(ii) $\mathbf{1 + x^2 + 9y^2 + 2x - 6xy - 6y}$

(iii) $\mathbf{25x^2 + 4y^2 + 9z^2 - 20xy + 12yz + 30xz}$

(iv) $\mathbf{\frac{1}{x^2} + \frac{4}{y^2} + \frac{9}{z^2} + \frac{4}{xy} + \frac{12}{yz} + \frac{6}{xz}}$

Sol. (i) $4x^2 + 9y^2 + 25z^2 + 12\,xy + 30\,yz + 20\,xz$

$= (2x)^2 + (3y)^2 + (5z)^2 + 2\,(2x)\,(3y) + 2\,(3y) + (5z) + 2\times 3y\times 5z$

$= (2x + 3y + 5z)^2$

$\because (a + b + 1)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

(ii) $1 + x^2 + 9y^2 + 2\,x - 6xy - 6y$

$= 1^2 + x^2 + (3y)^2 + 2\times 1\times x + 2\times x\times -3y + 2\times(-3y)\times 1$

$= (1 + x - 3y)^2$

(iii) $25x^2 + 4y^2 + 9z^2 - 20xy + 12\,yz - 30xz$

$= (5x)^2 + (-2y)^2 + (-3z)^2 + 2(5x)(-2y) + 2\,(-2y)(-3z) + 2\,(-3z)(5x)$

$= (5x - 2y - 3z)^2$

(iv) $\frac{1}{x^2} + \frac{4}{y^2} + \frac{9}{z^2} + \frac{4}{xy} + \frac{12}{yz} + \frac{6}{xz}$

$= \left(\frac{1}{x}\right)^2 + \left(\frac{2}{y}\right)^2 + \left(\frac{3}{2}\right)^2 + 2\left(\frac{1}{x}\right)\left(\frac{2}{y}\right) + 2\left(\frac{2}{y}\right)\left(\frac{3}{z}\right) + 2\left(\frac{3}{2}\right)\left(\frac{1}{x}\right) = \left(\frac{1}{x} + \frac{2}{y} + \frac{3}{z}\right)$

4. **Factorise the following:**

(i) $\mathbf{8x^2 + 125y^3}$ (ii) $\mathbf{a^3 - 729}$

(iii) $\mathbf{27x^3 - 8y^3}$ (iv) $\mathbf{m^3 + 512}$

(v) $\mathbf{a^3 + 3a^2b + 3ab^2 + 2b^3}$ (vi) $\mathbf{a^6 - 64}$

$\boxed{\therefore a^3 - b^3 = (a+b)(a^2 - ab + b^2)}$

Sol. (i) $8x^3 + 125y^3 = (2x)^3 + (5y)^3$

$= (2x + 5y)[(2x)^2 - (2x)(5y) + (5y)^2]$

$= (2x + 5y)^2 (4x^2 - 10xy + 25y^2]$

(ii) $a^3 - 729 = a^3 - 9^3$

$= (a-9)(a^2 + a \times 9 + 9^2)$

$= (a-9)(a^2 + 9a + 81)$ $\boxed{\therefore a^3 - b^3 = (a-b)(a^2 + ab + b^2)}$

(iii) $27x^3 - 8y^3 = (3x)^3 - (2y)^2$

$= (3x - 2y)((3x)^2 + 3x \times 2y + (2y)^2)$

$= (3x - 2y)(9x^2 + 6xy + 4xy + 4y^2)$

(iv) $m^3 + 512 = m^3 + 8^3$

$\boxed{a^3 + b^3 = (a+b)(a^2 - ab + b^2)}$

$= (m+8)(m^2 - 8m + 64)$

(v) $a^3 + 3a^2 b + 3ab^2 + 2b^3 = (a + 2b)(a^2 + b^2 + ab)$

(vi) $a^6 - 64 = (a^2)^3 - 4^3$ $(a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$= (a^2 - 4)(a^4 + 4a^2 + 4^2)$

$= (a+2)(a-2)(a^2 + 4 - 2a)(a^2 - 4 + 2a)$

5. Factorise the following:

(i) $x^3 + 8y^3 + 27z^3 - 18xyz$ **(ii)** $a^3 + b^3 - 3ab + 1$

(iii) $x^3 + 8y^3 + 6xy - 1$ **(iv)** $l^3 - 8m^3 - 27n^3 - 18lmn$

Sol. (i) $x^3 + 8y^3 + 27z^3 - 18xyz$

$= x^3 + (2y)^3 + (3z)^3 - 3(x)(2y)(3z)$

$= [(x + 2y + 3z)(x^2 + (2y)^2 + (3z)^2 - x \times 2y - 2y \times 3z - 3z \times x]$

$= (x + 2y + 3z)(x^2 + 4y^2 + 9z^2 - 2xy - 6yz - 3xz)$

(ii) $a^3 + b^3 - 3ab + 1 = a^3 + b^3 + 1^3 - 3ab \times 1^3$

$= (a + b + 1)(a^2 + b^2 + 1^2 - ab - b \times 1 - 1 \times a)$

$= (a + b + 1)(a^2 + b^2 + 1 - ab - b - a)$

$= (1 + x - 3y)^2$

(iii) $x^3 + 8y^3 + 6xy - 1 = x^3 + (2y)^3 + (-1)^3 - 3(x)(2y)(-1)$

$= (x + 2y - 1)(x^2 + 4y^2 + 1 - 2xy + 2y + x)$

(iv) $l^3 - 8m^3 - 27n^3 - 18lmn = l^3 + (-2m)^3 + (-3n)^3 - 3(l)(-2m)(-3n)$

$= (l - 2m - 3n)(l^2 + (-2m)^2 + (-3n)^2 - l \times -2m -$

$(-2m \times -3n) - (-3n \times l))$

$= (l - 2m - 3n)(l^2 + 4m^2 + 9n^2 + 2lm - 6mn + 3nl)$

Exercise 3.5

1. Factorise the following:

(i) $x^2 + 10x + 24$ (ii) $x^2 - 2x - 99$

(iii) $z^2 + 4z - 12$ (iv) $x^2 + 14x - 15$

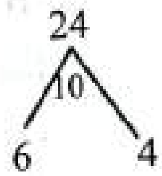
(v) $p^2 - 6p - 16$ (vi) $t^2 + 72 - 17t$

(vii) $x^2 - 8x + 15$ (viii) $y^2 - 16y - 80$

(ix) $a^2 + 10a - 600$

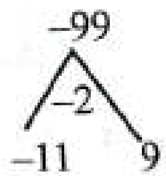
Sol. (i) $x^2 + 10x + 24$

$$\begin{aligned} x^2 + 10x + 24 &= x^2 + 6x + 4x + 24 \\ &= x(x+6) + 4(x+6) \\ &= (x+6)(x+4) \end{aligned}$$



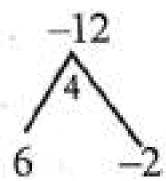
(ii) $x^2 - 2x - 99$

$$\begin{aligned} x^2 - 2x - 99 &= x^2 - 11x + 9x - 99 \\ &= x(x-11) + 9(x-11) \\ &= (x-11)(x+9) \end{aligned}$$



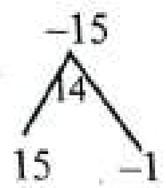
(iii) $z^2 + 4z - 12$

$$\begin{aligned} z^2 + 4z - 12 &= z^2 + 6z - 2z - 12 \\ &= z(z+6) - 2(z+6) \\ &= (z+6)(z-2) \end{aligned}$$



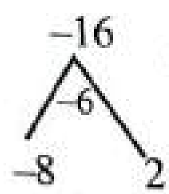
(iv) $x^2 + 14x - 15$

$$\begin{aligned} x^2 + 14x - 15 &= x^2 + 15x - x - 15 \\ &= x(x+15) - 1(x+15) \\ &= (x+15)(x-1) \end{aligned}$$



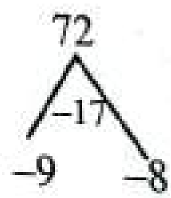
(v) $p^2 - 6p - 16$

$$\begin{aligned} p^2 - 6p - 16 &= p^2 - 8p + 2p - 16 \\ &= p(p-8) + 2(p-8) \\ &= (p-8)(p+2) \end{aligned}$$



(vi) $t^2 + 72 - 17t$

$$\begin{aligned} t^2 + 72 - 17t &= t^2 - 17t + 72 \\ &= t^2 - 9t - 8t + 72 \\ &= t(t-9) - 8(t-9) \\ &= (t-9)(t-8) \end{aligned}$$



(vii) $x^2 - 8x + 15$

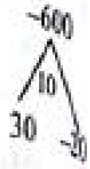
$$\begin{aligned} x^2 - 8x + 15 &= x^2 - 5x - 3x + 15 \\ &= x(x-5) - 3(x-5) \\ &= (x-5)(x-3) \end{aligned}$$

(viii) $y^2 - 16y - 80$

$$\begin{aligned} y^2 - 16y - 80 &= y^2 - 20y + 4y - 80 \\ &= y(y-20) + 4(y-20) \\ &= (y-20)(y+4) \end{aligned}$$

(ix) $a^2 + 10a - 600$

$$\begin{aligned} a^2 + 10a - 600 &= a^2 + 30a - 20a - 600 \\ &= a(a+30) - 20(a+30) \\ &= (a+30)(a-20) \end{aligned}$$



2. Factorise the following:

(i) $2a^2 + 9a + 10$

(ii) $11 + 5m - 6m^2$

(iii) $4x^2 - 20x + 25$

(iv) $32 + 8x - 60x^2$

(v) $5x^2 - 29xy - 42y^2$

(vi) $9 - 18x + 18x^2$

(vii) $6x^2 + 16xy + 8y^2$

(viii) $9 + 3x - 12x^2$

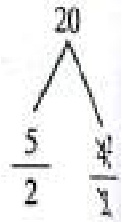
(ix) $10 - 7a - 3a^2$

(x) $12x^2 + 36x^2y + 27y^2x^2$

(xi) $(a+b)^2 + 9(a+b) + 18$

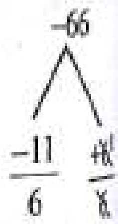
Sol. (i) $2a^2 + 9a + 10$

$$\begin{aligned} 2a^2 + 9a + 10 &= 2a^2 + 4a + 5a + 10 \\ &= 2a(a+2) + 5(a+2) \\ &= (a+2)(2a+5) \end{aligned}$$



(ii) $11 + 5m - 6m^2$

$$\begin{aligned} 11 + 5m - 6m^2 &= -(6m^2 - 5m - 11) \\ &= -(6m^2 - 6m - 11m - 11) \\ &= -(6m(m+1) - 11(m+1)) \\ &= -((m+1)(6m-11)) = (m+1)(11-6m) \end{aligned}$$



Aliter : $= 6m^2 - 5m - 11 = -\left(m - \frac{11}{6}\right)(m+1) = -(6m-11)(m+1)$

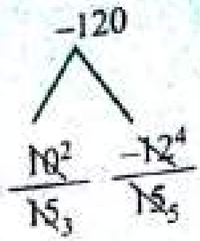
(iii) $4x^2 - 20x + 25$

$$\begin{aligned} 4x^2 - 20x + 25 &= 4x^2 - 10x - 10x + 25 \\ &= 2x(2x-5) - 5(2x-5) \\ &= (2x-5)(2x-5) \end{aligned}$$

Aliter : $= \left(x - \frac{5}{2}\right)\left(x - \frac{5}{2}\right) = (2x-5)(2x-5)$

(iv) $32 + 8x - 60x^2$

$$32 + 8x - 60x^2 = -60x^2 + 8x + 32$$
$$= -4\,(15x^2 - 2x - 8)$$
$$= -4\,(15x^2 + 10\,x - 12\,x - 8)$$
$$= -4\,(5x\,(3x + 2) - 4\,(3x + 2))$$
$$= -4\,(3x + 2)\,(5x - 4)$$



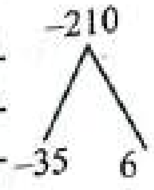
Aliter : $= 32 + 8x - 60x^2 = -60x^2 + 8x + 32$

$$= -4\,(15x^2 - 2x - 8) = -4\left(x + \frac{2}{3}\right)\left(x - \frac{4}{5}\right)$$
$$= -4\,(3x + 2)\,(5x - 4)$$

(v) $5x^2 - 29xy - 42y^2$

$$= 5x^2 - 35xy + 6xy - 42y^2$$
$$= 5x\,(x - y)\,y\,(x - y)$$
$$= (x - y)\,(5x - y)$$

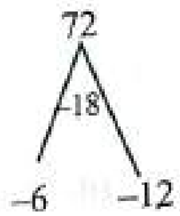
5	210
2	42
3	21
7	7
	1



(vi) $9 - 18x + 8x^2$

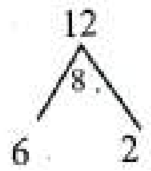
$$= 8x^2 - 18x + 9$$
$$= 8x^2 - 6x - 12x + 9$$
$$= 2x\,(4\,x - 3) - 3\,(4x - 3)$$
$$= (4x - 3)\,(2x - 3)$$

2	72
2	36
3	18
2	6
3	3
	1



(vii) $6x^2 + 16xy + 8y^2$

$$= 2\,(3x^2 + 8xy + 4y^2)$$
$$= 2\,(3x^2 + 8xy + 4y^2)$$
$$= 2\,(3x^2 + 6xy + 2xy + 4y^2)$$
$$= 2\,(3x\,(x + 2y) - 2y\,(x + 2y))$$
$$= 2\,(x + 2y)\,(3x + 2y)$$

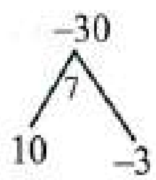


(viii) $9 + 3x - 12x^2$

$$= -3\,(4x^2 - x - 3)$$
$$= -3\,(4x^2 - 4x + 3x - 3) = -3\,(4x\,\,(x - 1) + 3\,(x - 1)$$
$$= -3\,(x - 1)\,(4x + 3)$$

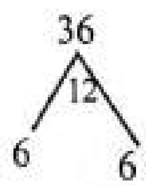
(ix) $10 - 7a - 3a^2$

$$= -(3a^2 + 7a - 10)$$
$$= -(3a^2 + 10a - 3a - 10) = [-a\,(3a + 10) - 1\,(3a + 10)]$$
$$= -(3a + 10)\,(a - 1)$$



(x) $12x^2 + 36x^2\,y + 27y^2\,x^2$

$$= 27y^2\,x^2 + 36x^2\,y + 12x^2 = 3x^2\,(9y^2 + 12y + 4)$$
$$= 3x^2\,(9y^2 + 6y + 6y + 4) = 3x^2\,(3y\,(3y + 2) + 2\,(3y + 2))$$
$$= 3x^2\,(3y + 2)\,(3y + 2) = 3x^2\,(3y + 2)\,(3y + 2)$$



(xi) $(a+b)^2+9(a+b)+18$

$= (a+b)^2+6(a+b)+3(a+b)+18$

$= (a+b)^2+6(a+b)+3((a+b)+6)$

$= (a+b)((a+b)+6)+3((a+b)+3) = (a+b+6)\ (a+b+3)$

$= ((a+b)+6)((a+b)+3)$

3. **Factorise the following:**

(i) $(p-q)^2 - 6(p-q) - 16$

(ii) $9(2x-y)^2 - 4(2x-y) - 13$

(iii) $m^2 + 2mn - 24n^2$

(iv) $\sqrt{5}a^2 + 2a - 3\sqrt{5}$

(v) $a^4 - 3a^2 + 2$

(vi) $8m^3 - 2m^2n - 15mn^2$

(vii) $4\sqrt{3}x^2 + 5x - 2\sqrt{3}$

(viii) $a^4 - 7a^2 + 1$

(ix) $a^2 + \dfrac{1}{a^2} - 18$

(x) $\dfrac{1}{x^2} + \dfrac{1}{y^2} + \dfrac{2}{xy}$

(xi) $\dfrac{3}{x^2} + \dfrac{8}{xy} + \dfrac{4}{y^2}$

Sol. (i) $(p-q)^2 - 6(p-q) - 16$

$= (p-q)^2 - 8(p-q) + 2(p-q) - 16$

$= (p-q)((p-q)-8) + 2((p-q)-8)$

$= (p-q-8)(p-q+2)$

(ii) $9(2x-y)^2 - 4(2x-y) - 13$

Here put $(2x-y) = a$

Then $9(2x-y)^2 - 4(2x-y) - 13 = 9a^2 - 4a - 13$

$= 9a^2 + 9a - 13a - 13$

$= 9a(a+1) - 13(a+1)$

$= (a+1)(9a-13)$

$= (2x-y+1)(9(2x-y)-13)$

$= (2x-y+1)(18x-9y-13)$

(iii) $m^2 + 2mn - 24n^2$

$= m^2 + 6mn - 4mn - 24n^2$

$= m(m+6n) - 4n(m+6n)$

$= (m+6n)(m-4n)$

(iv) $\sqrt{5}a^2 + 2a - 3\sqrt{5}$

$= \sqrt{5}\,a^2 + 2a - 3\sqrt{5}$ $\qquad \sqrt{5} \times -3\sqrt{5}$

$= \sqrt{5}\,a^2 + 5a - 3a - 3\sqrt{5}$ $\qquad = -3 \times 5$

$= \sqrt{5}\,a(a+\sqrt{5}) - 3(a+\sqrt{5})$ $\qquad = -15$

$= (a+\sqrt{5})(\sqrt{5}\,a - 3)$

(v) $a^4 - 3a^2 + 2$

$= a^4 - 2a^2 - 1\,a^2 + 2$

$= a^2\,(a^2 - 2) - 1\,(a^2 - 2)$

$= (a^2 - 2)\,(a^2 - 1) = (a^2 - 2)\,(a + 1)\,(a - 1)$

(vi) $8m^3 - 2m^2n - 15mn^2$

$= m\,(8m^2 - 2\,mn - 15n^2\,)$

$= m\,(8m^2 - 12mn + 10\,mn - 15n^2)$

$= m\;(4m\,(2m - 3n) + 5n\,(2m - 3n)$

$= m(4m + 5n)\,(2m - 3n)$

(vii) $4\sqrt{3}x^2 + 5x - 2\sqrt{3}$

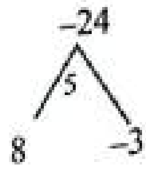
$= +\,4\sqrt{3}\,x^2 + 8x - 3x - 2\sqrt{3}$

$= +\,4\sqrt{3}\,x^2 + 8x - 3x - 2\sqrt{3}$

$= 4x\,(\sqrt{3}\,x + 2) - \sqrt{3}\;(\sqrt{3}\,x + 2)$

$= (\sqrt{3}\,x + 2)\,(4x - \sqrt{3}\,)$

$4\sqrt{3} \times 2\sqrt{3}$
$= -\,8 \times 3$
$= -\,24$



(viii) $a^4 - 7a^2 + 1$

$= (a^2 + 1)^2 - 2a^2 - 7a^2 = (a^2 + 1)^2 - 9a^2 = (a^2 + 1)^2 - (3a)^2$

$= (a^2 + 3a + 1)\,(a^2 - 3a + 1)$

(ix) $a^2 + \dfrac{1}{a^2} - 18$

$= \left(a - \dfrac{1}{a} + 4\right)\left(a - \dfrac{1}{a} - 4\right)$

(x) $\dfrac{1}{x^2} + \dfrac{1}{y^2} + \dfrac{2}{xy}$

$= \left(\dfrac{1}{x}\right)^2 + 2\left(\dfrac{1}{x}\right)\left(\dfrac{1}{y}\right) + \left(\dfrac{1}{y}\right)^2 = \left(\dfrac{1}{x} + \dfrac{1}{y}\right)^2 \quad (\because (a+b)^2 = a^2 + 2ab + b^2)$

(xi) $\dfrac{3}{x^2} + \dfrac{8}{xy} + \dfrac{4}{y^2}$

$= \dfrac{3y^2 + 8xy + 4y^2}{x^2y^2} = \dfrac{1}{x^2y^2}\,[3y^2 + 6xy + 2xy + 4x^2]$

$= \dfrac{1}{x^2y^2}\,[3y\,(y + 2x) + 2x\,(y + 2x)]$

$= \dfrac{1}{x^2y^2}\,(y + 2x)\,(3y + 2x) = \left(\dfrac{y + 2x}{xy}\right)\left(\dfrac{3y + 2x}{xy}\right)$

$= \left(\dfrac{1}{x} + \dfrac{2}{y}\right)\left(\dfrac{3}{x} + \dfrac{2}{y}\right)$

Exercise 3.6

1. Find the quotient and remainder for the following using synthetic division:

(i) $(x^2 + x^2 - 7x - 3) \div (x - 3)$ **(ii)** $(x^2 + 2x^2 - x - 4) \div (x + 2)$

(iii) $(x^3 + 4x^2 + 16x + 61) \div (x - 4)$ **(iv)** $(3x^3 - 2x^2 + 7x - 5) \div (x + 3)$

(v) $(3x^3 - 4x^2 - 10x + 8) \div (3x - 2)$ **(vi)** $(8x^3 - 2x^2 + 6x + 5) \div (4x + 1)$

Sol. **(i)** $(x^2 + x^2 - 7x - 3) \div (x - 3)$

Let $p(x) = x^3 + x^2 - 7x - 3$

$q(x) = x - 3$ To find the zero of $x - 3$:

$p(x)$ in standard form ((i.e.) descending order)

$x^3 + x^2 - 7x - 3$

Co-efficients are 1 1 –7 – 3

$$
\begin{array}{r|rrrr}
3 & 1 & 1 & -7 & -3 \\
 & 0 & 3 & 12 & 15 \\
\hline
 & 1 & 4 & 5 & \boxed{12}
\end{array}
\quad \text{(remainder)}
$$

Quotient is $x^2 + 4x + 5$

Remainder is 12

(ii) $(x^2 + 2x^2 - x - 4) \div (x + 2)$

$p(x) = x^3 + 2x^2 - x - 4$

Co-efficients are 1 2 –1 – 4

To find zero of $x + 2$, put $x + 2 = 0$; $x = -2$

$$
\begin{array}{r|rrrr}
-2 & 1 & 2 & -1 & -4 \\
 & 0 & -2 & 0 & 2 \\
\hline
 & 1 & 0 & -1 & \boxed{-2}
\end{array}
$$

$\therefore$ Quotient is $x^2 - 1$

Remainder is –2

(iii) $(x^3 + 4x^2 + 16x + 61) \div (x - 4)$

To find zero of the divisor $(x - 4)$, put $x - 4 = 0$; $x = 4$.

Dividend in Standard form

$x^3 + 4x^2 + 16x + 61$

Co-efficients are 1 4 16 61

Synthetic Division

4	1	4	16	61	
	0	4	32	192	
	1	8	48	253	Remainder

Quotient = $x^2 + 8x + 48$

Remainder = 253

(iv) $(3x^3 - 2x^2 + 7x - 5) \div (x + 3)$

To find zero of the divisor $(x + 3)$, put $x + 3 = 0$; $\therefore x = -3$

Dividend in Standard form $3x^3 - 2x^2 + 7x - 5$

Co-efficients are 3 −2 7 −5

Synthetic Division

−3	3	−2	7	−5
	0	−9	33	−120
	3	−11	40	−125

Quotient is $3x^2 - 11x + 40$

Remainder is -125

(v) $(3x^3 - 4x^2 - 10x + 8) \div (3x - 2)$

To find zero of the divisor $3x - 2$, put $3x - 2 = 0$; $3x = 2$; $x = \frac{2}{3}$.

Dividend in Standard form $3x^3 - 4x^2 - 10x + 8$

Co-efficients are 3 −4 −10 8

Synthetic Division

$\frac{2}{3}$	3	−4	−10	8
	0	2	$-\frac{4}{3}$	$-\frac{68}{9}$
	3	−2	$-\frac{34}{3}$	$\frac{4}{9}$

$$3x^3-4x^2-10x+8 = \left(x-\frac{2}{3}\right)\left(3x^2-2x+\frac{34}{3}\right)+\frac{4}{9}$$

$$= \left(\frac{3x-2}{\cancel{3}}\right)\cancel{3}\left(x^2-\frac{2}{3}x+\frac{34}{9}\right)+\frac{4}{9}$$

Hence

Quotient is $=x^2-\frac{2}{3}x+\frac{34}{9}$

Remainder is $\frac{4}{9}$

(vi) $(8x^4-2x^2+6x+5)\div(4x+1)$

To find zero of the divisor $4x+1$, put $4x+1=0$; $4x=-1$; $x=-\frac{1}{4}$.

Dividend in Standard form $8x^4+0x^3-2x^2+6x+5$

Co-efficients are 8 0 −2 6 5

Synthetic Division

$$\begin{array}{c|ccccc}
-\frac{1}{4} & 8 & 0 & -2 & 6 & 5 \\
 & 0 & -2 & \frac{1}{2} & +\frac{3}{8} & -\frac{51}{32} \\
\hline
 & 8 & -2 & -\frac{3}{2} & \frac{51}{8} & \boxed{\frac{109}{32}}
\end{array}$$

$$8x^4+0x^3-2x^2+6x+5 = \left(x+\frac{1}{4}\right)\left(8x^3-2x^2-\frac{3x}{2}+\frac{51}{8}\right)+\frac{109}{32}$$

$$= \frac{(4x+1)}{\cancel{4}}\times\cancel{4}\left(2x^3-\frac{x^2}{2}-\frac{3x}{8}+\frac{51}{32}\right)+\frac{109}{32}$$

$$= (4x+1)\left(2x^3-\frac{x^2}{2}-\frac{3x}{8}+\frac{51}{32}\right)+\frac{109}{32}$$

$\therefore$ Quotient is $=2x^3-\frac{x^2}{2}-\frac{3x}{8}+\frac{51}{32}$

Remainder is $\frac{109}{32}$

2. **If the quotient obtained on dividing $(8x^4-2x^2+6x-7)$ by $(2x+1)$ is $(4x^3+px^2-qx+3)$, then find p, q and also the remainder.**

Sol. Let $p(x) = 8x^4-2x^2+6x-7$

Standard form $= 8x^4+0x^3-2x^2+6x-7$

Co-efficients are 8 0 −2 6 −7

$Q(x) = 2x+1,$

To find zero of $2x+1$, put $2x+1=0$; $2x=-1$; $x=-\frac{1}{2}$

Synthetic division

$$
\begin{array}{c|ccccc}
 & 8 & 0 & -2 & 6 & -7 \\
-\frac{1}{2} & 0 & -4 & 2 & 0 & -3 \\
\hline
 & 8 & -4 & 0 & 6 & \boxed{-10}
\end{array}
$$

Quotient $= \frac{1}{2}[8x^3-4x^2+6] = 4x^3-2x^2+3$

Quotient $4x^3-2x^2+3$ is compared with the given quotient $4x^3+px^2-qx+3$

Co-efficients of x^2 is $p = -2$

Co-efficients of x is $q = 0$

Remainder is -10

$$\boxed{\begin{aligned} p &= -2 \\ q &= 0 \\ r &= -10 \end{aligned}}$$

If the quotient obtained on dividing $3x^3+11x^2+34x+106$ by $x-3$ is $3x^2+ax+b$, then find a, b and also the remainder.

Let $p(x) = 3x^3+11x^2+34x+106$

$p(x)$ in standard form

Co-efficients are 3 11 34 106

$q(x) = x-3$, its zero $x=3$

Synthetic division

$$
\begin{array}{c|cccc}
3 & 3 & 11 & 34 & 106 \\
 & 0 & 9 & 60 & 282 \\
\hline
 & 3 & 20 & 94 & \boxed{388}
\end{array}
$$

Quotient is $3x^2+20x+94$, it is compared with the given quotient $3x^2+ax+b$

Co-efficient of x is $a = 20$

Constant term is $b = 94$

Remainder $r = 388$

Exercise 3.7

1. **Factorise each of the following polynomials using synthetic division:**

(i) $x^3-3x^2-10x+24$ (ii) $2x^3-3x^2-3x+2$

(iii) $4x^3-5x^2+7x-6$ (iv) $-7x+3+4x^3$

(v) $x^3+x^2-14x-24$ (vi) x^3-7x+6

(vii) x^3-10x^2-x+10 (viii) x^3-5x+4

Sol. **(i)** $x^3 - 3x^2 - 10x + 24$

$$\text{Let } p(x) = x^3 - 3x^2 - 10x + 24$$

Sum of all the co-efficients $= 1 - 3 - 10 + 24 = 25 - 13 = 12 \neq 0$

Hence $(x - 1)$ is not a factor.

Sum of co-efficient of even powers with constant $= -3 + 24 = 21$

Sum of co-efficients of odd powers $= 1 - 10 = -9$

$$21 \neq -9$$

Hence $(x + 1)$ is not a factor.

$$p(2) = 2^3 + -3(2^2) - 10 \times 2 + 24 = 8 - 12 - 20 + 24$$
$$= 32 - 32 = 0 \quad \therefore (x-2) \text{ is a factor.}$$

Now we use synthetic division to find other factor

$$\begin{array}{r|rrrr}
2 & 1 & -3 & -10 & 24 \\
 & 0 & 2 & -2 & -24 \\
\hline
-3 & 1 & -1 & -12 & \boxed{0} \\
 & 0 & -3 & 12 & \\
\hline
 & 1 & -4 & \boxed{0} &
\end{array}$$

Thus $(x - 2)(x + 3)(x - 4)$ are the factors.

$$\therefore x^3 - 3x^2 - 10x + 24 = (x-2)(x+3)(x-4)$$

(ii) $2x^2 - 3x^2 - 3x + 2$

$$\text{Let } p(x) = 2x^3 - 3x^2 - 3x + 2$$

Sum of all the co-efficients are

$$2 - 3 - 3 + 2 = 4 - 6 = -2 \neq 0$$

$\therefore (x - 1)$ is not a factor

Sum of co-efficients of even powers of x with constant $= -3 + 2 = -1$

Sum of co-efficients of odd powers of $x = 2 - 3 = -1$

$$(-1) = (-1)$$

$\therefore (x + 1)$ is a factor

Let us find the other factors using synthetic division

$$\begin{array}{r|rrrr}
-1 & 2 & -3 & -3 & 2 \\
 & 0 & -2 & 5 & -2 \\
\hline
 & 2 & -5 & 2 & \boxed{0}
\end{array}$$

Quotient is $2x^2 - 5x + 2 = 2x - 4x - x + 2 = 2x(x-2) - 1(x-2)$

$$= (x-2)(2x-1)$$

$$\therefore 2x^3 - 3x^2 - 3x + 2 = (x+1)(x-2)(2x-1)$$

(iii) $4x^3 - 5x^2 + 7x - 6$

Let $p(x) = 4x^3 - 5x^2 + 7x - 6$

Sum of all the co-efficients are $= 4 - 5 + 7 - 6 = 11 - 11 = 0$

$\therefore (x-1)$ is a factor

Sum of co-efficients of even powers of x with constant

$-5 - 6 = -11$

Sum of co-efficients of odd powers of x

$4 + 7 = 11$, $\boxed{-11 \neq 11}$

$\therefore (x+1)$ is not a factor

To find the other factors, using synthetic division

$$\begin{array}{r|rrrr} 1 & 4 & -5 & 7 & -6 \\ & 0 & 4 & -1 & 6 \\ \hline & 4 & -1 & 6 & \boxed{0} \end{array}$$

Quotient $4x^2 - x + 6$ cannot be split into factors.

Hence the factors are $(x-1)$ and $(4x^2 - x + 6)$

$\therefore 4x^3 - 5x^2 + 7x - 6 = (x-1)(4x^2 - x + 6)$

(iv) $-7x + 3 + 4x^3$

Let $p(x) = 4x^3 + 0x^2 - 7x + 3$

Sum of the co-efficients are $= 4 + 0 - 7 + 3$

$= 7 - 7 = 0$

$\therefore (x-1)$ is a factor

Sum of co-efficients of even powers of x with constant $= 0 + 3 = 3$

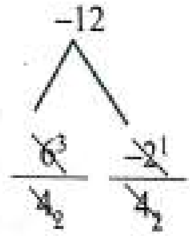
Sum of co-efficients of odd powers of x with constant $= 4 - 7 = -3$

$3 \neq -3$

$\therefore (x+1)$ is not a factor

Using synthetic division, let us find the other factors.

$$\begin{array}{r|rrrr} 1 & 4 & 0 & -7 & 3 \\ & 0 & 4 & 4 & -3 \\ \hline & 4 & 4 & -3 & \boxed{0} \end{array}$$



Quotient is $4x^2 + 4x - 3$

$= 4x^2 + 6x - 2x - 3$

$= 2x(2x+3) - 1(2x+3)$

$= (2x+3)(2x-1)$

$\therefore$ The factors are $(x-1)$, $(2x+3)$ and $(2x-1)$

(v) $x^3 + x^2 - 14x - 24$

$$\text{Let } p(x) = x^3 + x^2 - 14x - 24$$

$$\text{Sum of the co-efficients are} = 1 + 1 - 14 - 24 = -36 \neq 0$$

$\therefore (x-1)$ is not a factor

Sum of co-efficients of even powers of x with constant $= 1 - 24 = -23$

Sum of co-efficients of odd powers of $x = 1 - 14 = -3$

$$-23 \neq -13$$

$\therefore (x+1)$ is also not a factor

$$\begin{aligned} p(2) &= 2^3 + 2^2 - 14(2) - 24 = 8 + 4 - 28 - 24 \\ &= 12 - 52 \neq 0, (x-2) \text{ is a not a factor} \\ p(-2) &= (-2)^3 + (-2)^2 - 14(-2) - 24 \\ &= -8 + 4 + 28 - 24 = 32 - 32 = 0 \end{aligned}$$

$\therefore (x+2)$ is a factor

To find the other factors let us use synthetic division.

$x^3 + x^2 - 14x - 24$

-2	1	1	-14	-24
	0	-2	2	24
-3	1	-1	-12	0
	0	-3	12	
	1	-4	0	

-12

$-4 \quad 3$

$\therefore$ The factors are $(x+2), (x+3), (x-4)$

$$\therefore x^3 + x^2 - 14x - 24 = (x+2)(x+3)(x-4)$$

(vi) $x^3 - 7x + 6$

$$\text{Let } p(x) = x^3 + 0x^2 - 7x + 6$$

$$\text{Sum of the co-efficients are} = 1 + 0 - 7 + 6 = 7 - 7 = 0$$

$\therefore (x-1)$ is a factor

Sum of co-efficients of even powers of x with constant $= 0 + 6 = 6$

Sum of coefficient of odd powers of $x = 1 - 7 = -7$

$$6 \neq -7$$

$\therefore (x+1)$ is not a factor

To find the other factors, let us use synthetic division.

$x^3 + x^2 - 14x - 24$

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$$
\begin{array}{r|rrrr}
-1 & 1 & 0 & -7 & 6 \\
 & 0 & 1 & 1 & -6 \\
\hline
2 & 1 & 1 & -6 & \boxed{0} \\
 & 0 & 2 & 6 & \\
\hline
 & 1 & 3 & 0 &
\end{array}
$$

$\therefore$ The factors are $(x-1), (x-2), (x+3)$

$$\therefore x^3 + 0x^2 - 7x + 6 = (x-1)(x-2)(x+3)$$

(vii) $x^3 - 10x^2 - x + 10$

$$
\begin{aligned}
\text{Let } p(x) &= x^3 - 10x^2 - x + 10 \\
\text{Sum of the co-efficients} &= 1 - 0 - 1 + 10 \\
&= 11 - 11 = 0
\end{aligned}
$$

$\therefore (x-1)$ is a factor

Sum of co-efficients of even powers of x with constant $= -10 + 10 = 0$

Sum of co-efficients of odd powers of $= 1 - 1 = 0$

$\therefore (x+1)$ is a factor

Synthetic division

$$
\begin{array}{r|rrrr}
-1 & 1 & -10 & -1 & 10 \\
 & 0 & 1 & -9 & -10 \\
\hline
-1 & 1 & -9 & -10 & \boxed{0} \\
 & 0 & -1 & 10 & \\
\hline
 & 1 & -10 & \boxed{0} &
\end{array}
$$

$$\therefore x^3 + 10x^2 - x + 10 = (x-1)(x+1)(x-10)$$

(viii) $x^3 - 7x + 6$

$$
\begin{aligned}
\text{Let } p(x) &= x^3 - 5x + 4 \\
&= x^3 - 0x^2 - 5x + 4 \\
\text{Sum of the co-efficients} &= 1 + 0 - 5 + 4 = 5 - 5 = 0
\end{aligned}
$$

$\therefore (x-1)$ is a factor

Sum of co-efficients of even powers of x with constant $= 0 + 4 = 4$

Sum of co-efficient of odd powers of x $= 1 - 5 = -4$

$$4 \neq -4$$

$\therefore (x+1)$ is not a factor

Using Synthetic division

$$\begin{array}{r|rrrr} 1 & 1 & 0 & -5 & 4 \\ & 0 & 1 & 1 & -4 \\ \hline & 1 & 1 & -4 & \boxed{0} \end{array}$$

Quotient is $x^2 + x - 4$

$\therefore x^3 - 5x + 4 = (x - 1)(x^2 + x - 4)$

Exercise 3.8

MULTIPLE CHOICE QUESTIONS :

1. **If $p(a) = 0$ then $(x - a)$ is a ________ of $p(x)$**

(1) divisor (2) quotient (3) remainder (4) factor

[Ans. (4) factor]

2. **Zeros of $(2 - 3x)$ is ________**

(1) 3 (2) 2 (3) $\frac{2}{3}$ (4) $\frac{3}{2}$

Hint :

$$\begin{aligned} 2 - 3x &= 0 \\ -3x &= -2 \\ x &= \frac{2}{3} \end{aligned}$$

[Ans. (3) $\frac{2}{3}$]

3. **Which of the following has $x - 1$ as a factor?**

(1) $2x - 1$ (2) $3x - 3$ (3) $4x - 3$ (4) $3x - 4$

Hint :

$$\begin{aligned} p(x) &= 3x - 3 \\ p(1) &= 3(1) - 3 = 0 \end{aligned}$$

$\therefore (x - 1)$ is a factor $p(x)$

[Ans. (2) $3x - 3$]

4. **If $x - 3$ is a factor of $p(x)$, then the remainder is**

(1) 3 (2) -3 (3) $p(3)$ (4) $p(-3)$

[Ans. (3) $p(3)$]

5. **$(x + y)(x^2 - xy + y^2)$ is equal to**

(1) $(x + y)^3$ (2) $(x - y)^3$ (3) $x^3 + y^3$ (4) $x^3 - y^3$

[Ans. (3) $x^3 + y^3$]

6. **If one of the factors of $x^2 - 6x - 16$ is $x - 8$ then the other factor is**

(1) $(x + 6)$ (2) $(x - 2)$ (3) $(x + 2)$ (4) $(x - 16)$

Hint : $p(x) = x^2 - 6x - 16$

8	1	–6	–16
	0	8	+16
	1	2	0

[Ans. (3) $x+2$]

$(a+b-c)^2$ is equal to ________

(1) $(a-b+c)^2$ (2) $(-a-b+c)^2$ (3) $(a+b+c)^2$ (4) $(a-b-c)^2$

Hint : $(a+b-c)^2 = [-(-a-b+c)]^2 = (-a-b+c)^2$

[Ans. (2) $(-a-b+c)^2$]

In an expression ax^2+bx+c the sum and product of the factors respectively,

(1) a, bc (2) b, ac (3) ac, b (4) bc, a

[Ans. (2) b, ac]

If $(x+5)$ and $(x-3)$ are the factors of ax^2+bx+c, then values of a, b and c are

(1) 1,2,3 (2) 1,2,15 (3) 1,2, –15 (4) 1, –2,15

Hint :

$$\begin{aligned}
p(-5) &= a(-5^2)+b(-5)+c \\
&= 25a-5b+c=0 \quad \ldots (1) \\
p(+3) &= a(+3^2)+bc+3+c \\
&= 9+3b+c=0 \quad \ldots (2) \\
25a-5b &= 9a+3b \\
25a-9a &= 3b+5b \\
16a &= 8b \\
\frac{a}{b} &= \frac{8}{16}=\frac{1}{2}
\end{aligned}$$

Substitute

$$\begin{aligned}
a &= 1, b=2 \text{ in (1)} \\
25(1)-5(2) &= -c \\
25-10 &= 15=-c \\
c &= -15
\end{aligned}$$

[Ans. (3) 1, 2, –15]

Cubic polynomial may have maximum of ________ linear factors

(1) 1 (2) 2 (3) 3 (4) 4

[Ans. (3) 3]

Degree of the constant polynomial is ________

(1) 3 (2) 2 (3) 1 (4) 0

[Ans. (4) 0]

GCD of any two prime numbers is ________

(1) –1 (2) 0 (3) 1 (4) 2

[Ans. (3) 1]

13. The remainder when $(x^2 - 2x + 7)$ is divided by $(x + 4)$ is

(1) 28 (2) 31 (3) 30 (4) 29

Hint :

$$p(x) = x^2 - 2x + 7$$
$$p(-4) = (-4)^2 - 2(-4) + 7 = 16 + 8 + 7$$
$$= 31$$

[Ans. (2)

14. The GCD of a^k, a^{k+1}, a^{k+5} where, $k \in \mathbb{N}$.

(1) a^k (2) a^{k+1} (3) a^{k+5} (4) 1

[Ans. (1)

15. The GCD of $x^4 - y^4$ and $x^2 - y^2$ is

(1) $x^4 - y^4$ (2) $x^2 - y^2$ (3) $(x + y)^2$ (4) $(x + y)$

Hint :

$$x^4 - y^4 = (x^2)^2 - (y^2)^2 = (x^2 + y^2)(x^2 - y^2)$$
$$x^2 - y^2 = x^2 - y^2$$
$$\text{G.C.D. is} = x^2 - y^2$$

[Ans. (2) $x^2 - y^2$

16. If there are 36 students of class 9 and 48 students of class 10, what is the minimum number of rows to arrange them in which each row consists of same class with same number of students.

(1) 12 (2) 144 (3) 7 (4) 72

Hint :

$$48 = 2 \times 2 \times 2 \times 2 \times 3$$
$$36 = 2 \times 2 \times 3 \times 3$$
$$\text{G.C.D. is} = 2 \times 2 \times 3$$
$$= 12$$
$$\frac{48 + 36}{12} = \frac{84}{12} = 7$$

[Ans. (3)

Additional Questions and Answers

EXERCISE 3.1

1. **Show that $x + 4$ is a factor of $x^3 + 6x^2 - 7x - 60$.**

Sol. Let $p(x) = x^3 + 6x^2 - 7x - 60$

By factor theorem $(x + 4)$ is a factor of $p(x)$, if $p(-4) = 0$

$$p(-4) = (-4)^3 + 6(-4)^2 - 7(-4) - 60 = -64 + 96 + 28 - 60 = 0$$

Therefore, $(x + 4)$ is a factor of $x^3 + 6x^2 + - 7x - 60$

2. **In $(5x + 4)$ a factor of $5x^3 + 14x^2 - 32x - 32$.**

Sol. Let $p(x) = 5x^3 + 14x^2 - 32x - 32$

By factor theorem, $5x + 4$ is a factor, if $p\left(\frac{-4}{5}\right) = 0$

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$$p\left(\frac{-4}{5}\right) = 5\left(\frac{-4}{5}\right)^3 + 14\left(\frac{-4}{5}\right)^2 - \left(32\left(\frac{-4}{5}\right) - 32\right.$$

$$= 5\left(\frac{-64}{125}\right) + 14\left(\frac{16}{25}\right) + 32\left(\frac{4}{5}\right) - 32$$

$$= \frac{-64}{25} + \frac{224}{25} + \frac{128}{5} - 32 = \frac{-65}{25} + \frac{224}{25} + \frac{640}{25} - \frac{800}{25}$$

$$= \frac{-65 + 224 + 640 - 800}{25} = 0$$

$$p\left(\frac{-4}{5}\right) = 0$$

Therefore, $5x + 4$ is a factor of $5x^3 + 14x^2 - 32x - 32$

Find the value of k, if $(x - 3)$ is a factor of polynomial $x^3 - 9x^2 + 26x + k$.

Sol. Let $p(x) = x^3 - 9x^2 + 26x + k$

By factor theorem, $(x - 3)$ is a factor of $p(x)$, if $p(3) = 0$

$$p(3) = 0$$

$$3^3 - 9(3)^2 + 26(3) + k = 0$$

$$27 - 81 + 78 + k = 0$$

$$k = -24$$

To find the zero of $x - 3$:
Put $x - 3 = 0$
we get $x = 3$

Show that $(x - 3)$ is a factor of $x^3 + 9x^2 - x - 105$.

Sol. Let $p(x) = x^3 + 9x^2 - x - 105$

By factor theorem, $x - 3$ is a factor of $p(x)$, if $p(3) = 0$

$$p(3) = 3^3 + 9(3)^3 - 3 - 105$$

$$= 27 + 81 - 3 - 105$$

$$= 108 - 108$$

$$p(3) = 0$$

To find the zero of $x - 3$:
Put $x - 3 = 0$
we get $x = 3$

Therefore, $x - 3$ is a factor of $x^3 + 9x^2 - x - 105$

Is $(4x + 3)$ a factor of $4x^3 + 15x^2 - 31x - 30$.

Sol. Let $p(x) = 4x^3 + 15x^2 - 31x - 30$

By factor theorem, $(4x + 3)$ is a factor, if $p\left(\frac{-3}{4}\right) = 0$

To find the zero of $4x + 3$:
Put $4x - 3 = 0$; $4x = -3$
we get $x = \frac{-3}{4}$

$$p\left(\frac{-3}{4}\right) = 4\left(\frac{-3}{4}\right) + 15\left(\frac{-3}{4}\right)^2 - 31\left(\frac{-3}{4}\right) - 30$$

$$= 4\left(\frac{-27}{64}\right) + 15\left(\frac{9}{16}\right) + 31\left(\frac{3}{4}\right) - 30$$

$$= \frac{-27}{16} + \frac{135}{16} + \frac{372}{16} - \frac{480}{16} = \frac{-507 + 507}{16} - P\frac{-3}{4} = 0$$

Therefore, $4x + 3$ is a factor of $4x^3 + 15x^2 - 31x - 30$

EXERCISE 3.2

1. **Expand the following using identities :**

(i) $(7x + 2y)^2$ (ii) $(4m - 3m)^2$ (iii) $(4a + 3b)(4a - 3b)$

(iv) $(k + 2)(k - 3)$

Sol. (i) $(7x + 2y)^2 = (7x)^2 + 2(7x)(2y) + (2y)^2 = 49x^2 + 28xy + 4y^2$

(ii) $(4m - 3m)^2 = (4m)^2 - 2(4m)(3m) + (3m)^2 = 16m^2 - 24mn + 9$

(iii) $(4a + 3b)(4a - 3b)$ [We have $(a + b)(a - b) = a^2$

Put [$a = 4a, b$

$(4a + 3b)(4a - 3b) = (4a)^2 - (3b)^2 = 16a^2 - 9b^2$

[We have $(x + a)(x - b) = x^2 + (a - b)x$

(iv) $(k + 2)(k - 3)$ Put [$x = k, a = 2, b$

$(k + 2)(k - 3) = k^2 + (2 - 3)x - 2 \times 3 = k^2 - x - 6$

2. **Expand : $(a + b - c)^2$**

Sol. Replacing 'c' by '$-c$' in the expansion of

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

$$(a + b + (-c))^2 = a^2 + b^2 + (-c)^2 + 2ab + 2b(-c) + 2(-c)a$$

$$= a^2 + b^2 + c^2 + 2ab - 2bc - 2ca$$

3. **Expand : $(x + 2y + 3z)^2$**

Sol. We know that,

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

Substituting, $a = x, b = 2y$ and $c = 3z$

$$(x + 2y + 3z)^2 = x^2 + (2y)^2 + (3z)^2 + 2(x)(2y) + 2(2y)(3z) + 2(3z)(x)$$

$$= x^2 + 4y^2 + 9z^2 + 4xy + 12y^2 + 6zx$$

4. **Find the area of square whose side length is $m + n - q$.**

Sol. Area of square = side × side

$$= (m + n - q)^2$$

We know that,

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

$$[m + n + (-q)]^2 = m^2 + n^2 + (-q)^2 + 2mn + 2n(-q) + 2(-q)m$$

$$= m^2 + n^2 + q^2 + 2mn - 2nq - 2qm$$

Therefore, Area of square $= [m^2 + n^2 + q^2 + 2mn + 2nq - 2qm]$ sq. units.

EXERCISE 3.3

1. **Find GCD of $25x^3y^2z$, $45x^2y^4z^3b$**

Sol.

$$25x^3y^2z = 5\times 5x^3y^2z = \underline{5}\times 5\times \underline{x^2}\times x\times \underline{y^2}\times z$$

$$45x^3y^4z^3 = 5\times 3\times 3\times x^2y^4z^3 = \underline{5}\times 9\times \underline{x^2}\times \underline{y^2}\times y^2\times \underline{z}\times z^2$$

$$\text{Therefore GCD} = 5x^2y^2z$$

2. **Find the GCD of (y^3-1) and $(y-1)$.**

Sol.

$$y^3-1 = (y-1)(y^2+y+1)$$

$$y-1 = y-1$$

$$\text{Therefore, GCD} = y-1$$

3. **Find the GCD of $3x^2-48$ and $x^2-7x+12$.**

Sol.

$$3x^2-48 = 3(x^2-16) = 3(x^2-4^2) = 3(x+4)(x-4)$$

$$x^2-7x+12 = x^2-3x-4x+12$$

$$= x(x-3)-4(x-3) = (x-3)(x-4)$$

$$\text{Therefore, GCD} = x-4$$

4. **Find the GCD of $(x-7)^2$, $(x+7)^2$, $(x-4)^3$.**

Sol.

$$(x-7)^2 = (x-7)(x-7)$$

$$(x+7)^2 = (x+7)(x+7)$$

$$(x-4)^3 = (x-4)(x-4) = x(x-3)-4(x-3) = (x-3)(x-4)$$

There is no common factor other than one.

$$\text{Therefore, GCD} = 1$$

5. **Find the GCD of a^x, a^{x+y}, a^{x+y+z}.**

Sol.

$$a^x = \underline{a^x}$$

$$a^{x+y} = \underline{a^x}.a^y$$

$$a^{x+y+z} = \underline{a^x}.a^y.a^z \qquad \therefore \text{GCD} = a^x$$

EXERCISE 3.4

1. **Factorise the following**

(i) $25m^{-2}-16n^2$ (ii) x^4-9x^2

$[\because a^2-b^2=(a-b)(a+b)]$

Sol. (i)

$$25m^2-16n^2 = (5m)^2-(4n)^2$$

$$= (5m-4n)(5m+4n)$$

(ii)

$$x^4-9x^2 = x^2(x^2-9) = x^2(x^2-3^2) = x^2(x-3)(x+3)$$

2. **Factorise the following.**

(i) $64m^3+27n^3$

Sol. (i)

$$64m^3+27n^3 = (4m)^3+(3n)^3$$

$$= (4m+3n)((4m)^2-(4m)(3n)+(3n)^2)$$

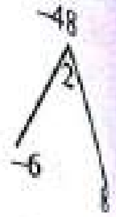
$[\because a^3+b^3=(a+b)(a^2-ab+b^2)]$

$$= (4m+3n)(16m^2-12mn+9n^2)$$

EXERCISE 3.5

1. Factorise $4x^2 + 2x - 12$

2	48
2	24
2	12
2	6
	3

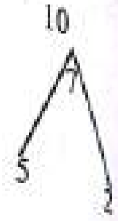


Sol.

$$\begin{aligned} 4x^2 + 2x - 12 &= 4x^2 + 8x - 6x - 12 \\ &= 4x(x+2) - 6(x+2) \\ &= (x+2)(4x-6) \end{aligned}$$

Therefore $x - 2$ and $4x - 6$ are factors if $4x^2 + 2x - 12$

2. Factorise $(a-b)^2 + 7(a-b) + 10$

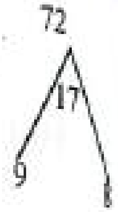


Sol.

$$\begin{aligned} \text{Let } a - b &= p, \text{ we get } p^2 + 7p + 10, \\ p^2 + 7p + 10 &= p^2 + 5p + 2p + 10 \\ &= p(p+5) + 2(p+5) = (p+5)(p+2) \\ \text{Put } p &= a - b \text{ we get,} \\ (a-b)^2 + 7(a-b) + 10 &= (a-b+5)(a-b+2) \end{aligned}$$

3. Factorise $6x^2 + 17x + 12$

2	72
2	36
2	18
3	9
	3



Sol.

$$\begin{aligned} 6x^2 + 17x + 12 &= 6x^2 + 9x + 8x + 12 \\ &= 3x(2x+3) + 4(2x+3) \\ &= (2x+3)(3x+4) \end{aligned}$$

EXERCISE 3.6

1. Find the quotient and remainder when $5x^3 - 9x^2 + 10x + 2$ is divided by $x + 2$ using synthetic division.

Sol.

$$\begin{aligned} p(x) &= 5x^3 - 9x^2 + 10x + 2 \\ d(x) &= x + 2 \\ \text{Standard form of } p(x) &= 5x^2 - 9x^2 + 10x + 2 \text{ and} \\ d(x) &= x + 2 \end{aligned}$$

To find the zero of $x + 2$:
Put $x + 2 = 0$
we get $x = -2$

–2	5	–9	10	2	
	0	–10	38	–96	
	5	–19	48	–94	(remainder)

$$5x^3 - 9x^2 + 10x + 2 = (x+2)(5x^2 - 19x + 48) - 94$$

Hence the quotient is $5x^2 - 19x + 48$ and remainder is -94.

2. Find the quotient and remainder when $4x^3 + 6x^2 + 7x + 2$ is divided by $x - 2$

Sol.

$$\begin{aligned} p(x) &= 4x^3 + 6x^2 + 7x + 2 \\ d(x) &= x - 2 \\ \text{Standard form of } p(x) &= 4x^3 + 6x^2 + 7x + 2 \text{ and} \\ d(x) &= x - 2 \end{aligned}$$

2	4	6	7	2	
	0	8	28	70	
	4	14	35	72	(remainder)

$$4x^2 + 6x^2 + 7x + 2 = (x-2)(4x^3 + 14x + 35) + 72$$

Hence the quotient is $4x^3 + 14x + 35$ and remainder is 72.

Find the quotient and remainder when $5x^3 + 7x^2 + 3x + 2$ is divided by $3x + 2$

Sol.

$$p(x) = 5x^3 + 7x^2 + 3x + 2$$
$$d(x) = 3x + 2$$
$$\text{Standard form of } p(x) = 5x^3 + 7x^2 + 3x + 2$$
$$\text{and } d(x) = 3x + 2$$

To find the zero of $3x + 2$:
Put $3x + 2 = 0$
we get $3x = -2$; $x = \frac{-2}{3}$

$\frac{-2}{3}$	5	7	3	2
	0	$\frac{-10}{3}$	$\frac{-22}{9}$	$\frac{-10}{27}$
	5	$\frac{11}{3}$	$\frac{5}{9}$	$\frac{44}{27}$

$$5x^3 + 7x^2 + 3x + 2 = \left(x+\frac{2}{3}\right)\left(5x^2 - \frac{11}{3}x + \frac{5}{9}\right) + \frac{44}{27} = \left(\frac{3x+2}{3}\right)\left(5x^2 - \frac{11}{3}x + \frac{5}{9}\right) + \frac{44}{27}$$
$$= \left(\frac{3x+2}{3}\right)3\left(\frac{5}{3}x^2 - \frac{11}{9}x + \frac{5}{27}\right) + \frac{44}{27}$$
$$= (3x+2)\left(\frac{5}{3}x^2 - \frac{11}{9}x + \frac{5}{27}\right) + \frac{44}{27}$$

Hence the quotient $\frac{5}{3}x^2 - \frac{11}{9}x + \frac{5}{21}$ and remainder is $\frac{44}{27}$

EXERCISE 3.7

1. **Factorise $2x^3 - x^2 - 12x - 9$ into linear factors.**

Sol.

$$\text{Let } p(x) = 2x^3 - x^2 - 12x - 9$$
$$\text{Sum of the co-efficients} = 2 - 1 - 12 - 9 = -20 \neq 0$$

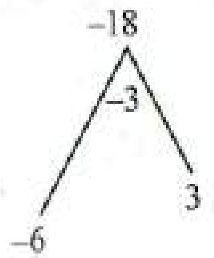
Hence $x - 1$ is not a factor

Sum of co-efficients of even powers with constant $= -1 - 9 = -10$

Sum of co-efficients of odd powers $= 2 - 12 = -10$

Hence $x + 1$ is a factor of x.

Now we use synthetic division to find the other factors.



-1	2	-1	-12	-9
	0	-2	3	9
	2	-3	-9	0

$$\text{Then } p(x) = (x+1)(2x^2 - 3x - 9)$$
$$\text{Now } 2x^2 - 3x - 9 = 2x^2 - 6x + 3x - 9 = 2x(x-3) + 3(x-3)$$
$$= (x-3)(2x+3)$$
$$\text{Hence } 2x^3 - x^2 - 12x - 9 = (x+1)(x-3)(2x+3)$$

2. **Prove that $x - 1$ is a factor $x^5 - 45x^4 + 36x^3 + 45x^2 - 36x - 1$.**

Sol.

$$\text{Let } p(x) = x^5 - 45x^4 + 36x^3 + 45x^2 - 36x - 1$$
$$\text{Sum of co-efficients} = 1 - 45 + 36 + 45 - 36 - 1 = 0$$

Thus $x - 1$ is a factor of $p(x)$

EXERCISE 3.8

MULTIPLE CHOICE QUESTIONS :

1. Zero of $(7 + 4x)$ is ______
(1) $\frac{4}{7}$ (2) $\frac{-7}{4}$ (3) 7 (4) 4
[Ans. (2) $\frac{-7}{4}$]

2. Which of the following has as a factor?
(1) $x^2 + 2x$ (2) $(x-1)^2$ (3) $(x+1)^2$ (4) $(x^2 - 2^2)$
[Ans. (1) $x^2 + 2x$]

3. If $x - 2$ is a factor of $q(x)$, then the remainder is ______
(1) $q(-2)$ (2) $x - 2$ (3) 0 (4) -2
[Ans. (3) 0]

4. $(a-b)(a^2 + ab + b^2) =$ ______
(1) $a^3 + b^3 + c^3 - 3abc$ (2) $a^2 - b^2$
(3) $a^3 + b^3$ (4) $a^3 - b^3$
[Ans. (4) $a^3 - b^3$]

5. The polynomial whose factors are $(x+2)(x+3)$ is ______
(1) $x^2 + 5x + 6$ (2) $x^2 - 4$ (3) $x^2 - 9$ (4) $x^2 + 6x + 5$
[Ans. (1) $x^2 + 5x + 6$]

6. $(-a-b-c)^2$ is equal to ______
(1) $(a-b+c)^2$ (2) $(a+b-c)^2$ (3) $(-a+b+c)^2$ (4) $(a+b+c)^2$
[Ans. (4) $(a+b+c)^2$]

7. Degree of the linear polynomial is ______
(1) 1 (2) 2 (3) 3 (4) 4
[Ans. (1) 1]

8. Quadratic polynomial may have maximum of ______ linear factors.
(1) 1 (2) 2 (3) 3 (4) 4
[Ans. (2) 2]

9. If $p(a) = 0$, then one of a factor of $p(x)$ is ______
(1) $x + a$ (2) $x^2 - a^2$ (3) $x - a$ (4) $x^2 + a^2$
[Ans. (3) $x - a$]

10. If one of the factor of $x^2 - 9x + 18$ is $(x-3)$ then the other factor is ______
(1) $x - 9$ (2) $x + 6$ (3) $(x - 6)$ (4) $x - 18$
[Ans. (3) $(x - 6)$]