UNIT – 07 PROPERTIES OF MATTER

TWO MARKS AND THREE MARKS:

01. Define stress and strain.

The force per unit area is called as stress. Stress, $\sigma = \frac{Force}{Area} = \frac{F}{A}$

The SI unit of stress is N m^{-2} or Pascal (Pa) and its dimension is $[ML^{-1}T^{-2}]$.

The fractional change in the size of the object, in other words, strain measures the degree of deformation. Strain, $e = \frac{Change\ in\ Size}{Original\ size} = \frac{\Delta l}{l}$

02. State Hooke's law of elasticity.

Hooke's law is for a small deformation, when the stress and strain are proportional to each other.

03. Define Poisson's ratio.

The ratio of relative contraction (lateral strain) to relative expansion (longitudinal strain). It is denoted by the symbol μ.

Poisson s ratio, μ = Lateral strain / Longitudinal strain

04. Explain elasticity using intermolecular forces.

In a solid, inter-atomic forces bind two or more atoms together and the atoms occupy the positions of stable equilibrium. When a deforming force is applied on a body, its atoms are pulled apart or pushed closer. When the deforming force is removed, inter-atomic forces of attraction or repulsion restore the atoms to their equilibrium positions. If a body regains its original shape and size after the removal of deforming force, it is said to be elastic and the property is called elasticity.

05. Which one of these is more elastic, steel or rubber? Why?

Steel is more elastic than rubber because the steel has higher young's modulus than rubber. That's why, if equal stress is applied on both steel and rubber, the steel produces less strain.

06. A spring balance shows wrong readings after using for a long time. Why?

When the spring balances have been used for a long time they develop elastic fatigue in them and therefore the reading shown by such balances will be wrong.

07. What is the effect of temperature on elasticity?

If the temperature of the substance increases, its elasticity decreases.

08. Write down the expression for the elastic potential energy of a stretched wire.

Consider a wire whose un-stretch length is L and area of cross section is A. Let a force produce an extension l and further assume that the elastic limit of the wire has not been exceeded and there is no loss in energy. Then, the work done by the force F is equal to the energy gained by the wire.

The work done in stretching the wire by dl, dW = F dl

The total work done in stretching the wire from 0 to l is

$$W = \int_0^l F\,dl \text{ ------------------1}$$

From Young's modulus of elasticity, $Y = \frac{F}{A} \times \frac{L}{l} \Rightarrow F = \frac{Y\,Al}{L}$ --------- 2

Substituting equation (2) in equation (1), we get

$$W = \int_0^l \frac{Y\,Al}{L} dl = \frac{Y\,Al^2}{L.2} = ½ .Fl$$

$$W = \int \frac{Y\,Al'}{L} dl' = \frac{YA}{L}\frac{l'^2}{2}\bigg|_0^l = \frac{YA}{L}\frac{l^2}{2} = \frac{1}{2}\left(\frac{YAl}{L}\right)l = \frac{1}{2}\,Fl$$

$W = \frac{1}{2}$ Fl = Elastic potential energy.

09. State Pascal's law in fluids.

If the pressure in a liquid is changed at a particular point, the change is transmitted to the entire liquid without being diminished in magnitude.

10. State Archimedes principle.

It states that when a body is partially or wholly immersed in a fluid, it experiences an upward thrust equal to the weight of the fluid displaced by it and its up-thrust acts through the centre of gravity of the liquid displaced.

11. What do you mean by up-thrust or buoyancy?

The upward force exerted by a fluid that opposes the weight of an immersed object in a fluid is called up-thrust or buoyant force and the phenomenon is called buoyancy.

12. State the law of floatation.

The law of floatation states that a body will float in a liquid if the weight of the liquid displaced by the immersed part of the body equals the weight of the body.

13. Define coefficient of viscosity of a liquid.

The coefficient of viscosity is defined as the force of viscosity acting between two layers per unit area and unit velocity gradient of the liquid. Its unit is Nsm^{-2} and dimension is $[ML^{-1}T^{-1}]$.

14. Distinguish between streamlined flow and turbulent flow.

Streamlined flow: When a liquid flows such that each particle of the liquid passing through a point moves along the same path with the same velocity as its predecessor then the flow of liquid is said to be a streamlined flow.

The velocity of the particle at any point is constant. It is also referred to as steady or laminar flow.

The actual path taken by the particle of the moving fluid is called a streamline, which is a curve, the tangent to which at any point gives the direction of the flow of the fluid at that point.

Turbulent flow: When the speed of the moving fluid exceeds the critical speed, v_c the motion becomes turbulent.

The velocity changes both in magnitude and direction from particle to particle.

The path taken by the particles in turbulent flow becomes erratic and whirlpool-like circles called eddy current or eddies.

15. What is Reynold's number? Give its significance.

Reynold's number(R_C) is a dimensionless number, which is used to find out the nature of flow of the liquid. $R_C = \frac{\rho v D}{\eta}$

Where, ρ- density of the liquid, v –The velocity of flow of liquid.

D- Diameter of the pipe, η - The coefficient of viscosity of the fluid.

16. Define terminal velocity.

The maximum constant velocity acquired by a body while falling freely through a viscous medium is called the terminal velocity.

17. Write down the expression for the Stoke's force and explain the symbols involved in it.

Viscous force F acting on a spherical body of radius r depends directly on

i) radius (r) of the sphere
ii) velocity (v) of the sphere and
iii) coefficient of viscosity η of the liquid F = $6\pi\eta r v$

18. State Bernoulli's theorem.

According to Bernoulli's theorem, the sum of pressure energy, kinetic energy, and potential energy per unit mass of an incompressible, non-viscous fluid in a streamlined flow remains a constant.

19. What are the energies possessed by a liquid? Write down their equations.

A liquid in a steady flow can possess three kinds of energy. They are (1) Kinetic energy, (2) Potential energy, and (3) Pressure energy, respectively.

KE = ½ mv^2 ------------ 1

PE = mgh --------------- 2 F x d = w = PV = pressure energy --------------3

20. Two streamlines cannot cross each other. Why?

No two streamlines can cross each other. If they do so, the particles of the liquid at the point of intersection will have two different directions for their flow, which will destroy the steady nature of the liquid flow.

21. Define surface tension of a liquid. Mention its S.I unit and dimension.

The surface tension of a liquid is defined as the energy per unit area of the surface of a liquid. (or) The surface tension of a liquid is defined as the force of tension acting perpendicularly on both sides of an imaginary line of unit length drawn on the free surface of the liquid.

Its unit is N m^{-1} and dimension is $[MT^{-2}]$.

22. How is surface tension related to surface energy?

Consider a rectangular frame of wire ABCD in a soap solution. Let AB be the movable wire. Suppose the frame is dipped in soap solution, soap film is formed which pulls the wire AB inwards due to surface tension. Let F be the force due to surface tension, then F = (2T)l

Here, 2 is introduced because it has two free surfaces. Suppose AB is moved by a small distance Δx to new a position A'B'. Since the area increases, some work has to be done against the inward force due to surface tension.

Work Done = Force x distance = (2T)l (Δx)

Increases in area of the film $\Delta A = (2l)\,(\Delta x) = 2l\Delta x$

Therefore, Surface energy = $\frac{\text{Work Done}}{\text{Increase in Surface area}}$

$= \frac{2Tl\Delta x}{2l\Delta x} = T$

Hence, the surface energy per unit area of a surface is numerically equal to the surface tension.

23. Define angle of contact for a given pair of solid and liquid.

The angle between the tangent to the liquid surface at the point of contact and the solid surface is known as the angle of contact.

24. Distinguish between cohesive and adhesive forces.

The force between the like molecules which holds the liquid together is called '*cohesive force*'. When the liquid is in contact with a solid, the molecules of the these solid and liquid will experience an attractive force which is called '*adhesive force*'.

25. What are the factors affecting the surface tension of a liquid?

(1) *The presence of any contamination or impurities* considerably affects the force of surface tension depending upon the degree of contamination.

(2) *The presence of dissolved substances* can also affect the value of surface tension. For example, a highly soluble substance like sodium chloride (NaCl) when dissolved in water (H_20) increases the surface tension of water. But the sparingly soluble substance like phenol or soap solution when mixed in water decreases the surface tension of water.

(3) *Electrification* affects the surface tension. When a liquid is electrified, surface tension decreases. Since external force acts on the liquid surface due to electrification, area of the liquid surface increases which acts against the contraction phenomenon of the surface tension. Hence, it decreases.

(4) *Temperature* plays a very crucial role in altering the surface tension of a liquid. Obviously, the surface tension decreases linearly with the rise of temperature.

26. What happens to the pressure inside a soap bubble when air is blown into it?

Pressure is greater inside the small build.

27. What do you mean by capillarity or capillary action?

The rise or fall of a liquid in a narrow tube is called capillarity or capillary action.

28. A drop of oil placed on the surface of water spreads out. But a drop of water place on oil contracts to a spherical shape. Why?

A drop of oil placed on the surface of water spreads because the force of adhesion between water and oil molecules dominates the cohesive force of oil molecules.

On the other hand, cohesive force of water molecules dominates the adhesive force between water and oil molecules. So drop of water on oil contracts to a spherical shape.

29. State the principle and usage of Venturimeter.

Bernoulli's theorem is the principle of Venturimeter.

Venturimeter is used to measure the rate of flow or flow speed of the incompressible fluid flowing through a pipe.

30. What are the applications of surface tension?

1) Oil pouring on the water reduces surface tension. So that the floating mosquitoes eggs drown and killed.
2) Finely adjusted surface tension of the liquid makes droplets of desired size, which helps in desktop printing, automobile painting and decorative items.
3) Specks of dirt are removed from the cloth when it is washed in detergents added hot water, which has low surface tension.
4) A fabric can be made waterproof, by adding suitable waterproof material (wax) to the fabric. This increases the angle of contact due to surface tension.

31. What physical quantity actually do we check by pressing the tyre after pumping?

After pumping the tyre, we actually check the compressibility of air by pressing the tyre. For smooth riding, rear tyre should have less compressibility than the front.

32. Give some examples for surface tension.

Clinging of painting brush hairs, when taken out of water.
Needle float on the water, Camphor boat.

33. How do water bugs and water striders walk on the surface of water?

When the water bugs or water striders are on the surface of the water, its weight is balanced by the surface tension of the water. Hence, they can easily walk on it.

34. What are the applications of viscosity?

1) Viscosity of liquids helps in choosing the lubricants for various machinery parts. Low viscous lubricants are used in light machinery parts and high viscous lubricants are used in heavy machinery parts.
2) As high viscous liquids damp the motion, they are used in hydraulic brakes as brake oil.
3) Blood circulation through arteries and veins depends upon the viscosity of fluids.
4) Viscosity is used in Millikan's oil-drop method to find the charge of an electron.

35. Explain the Stoke's law application in raindrop falling.

According to Stoke's law, terminal velocity is directly proportional to square of radius of the spherical body. So that smaller raindrops having less terminal velocity float as cloud in air. When they gather as bigger drops get higher terminal velocity and start falling.

36. Define Young's modulus. Give its unit.

Young's modulus is defined as the ratio of tensile or compressive stress to the tensile or compressive strain. Its unit is N m^{-2} or pascal.

37. What are the applications of elasticity?

Elasticity is used in structural engineering in which bridges and buildings are designed such a way that it can withstand load of flowing traffic, the force of winds and even its own weight.

The material of high Young's modulus is used in constructing beams.

38. Define Pressure. Give its unit and dimension.

The pressure is defined as the force acting per unit area. Its unit is N m^{-2} or pascal and dimension is $[ML^{-1}T^{-2}]$.

39. What is elasticity? Give examples.

Elasticity is the property of a body in which it regains its original shape and size after the removal of deforming force. Ex: Rubber, metals, steel ropes.

40. What is plasticity? Give an example.

Plasticity is the property of a body in which it does not regains its original shape and size after the removal of deforming force.
Ex: Glass.

CONCEPTUAL QUESTIONS

01. Why coffee runs up into a sugar lump (a small cube of sugar) when one corner of the sugar lump is held in the liquid?

The coffee runs up into the pores of sugar lump due to capillary action of the liquid.

02. Why two holes are made to empty an oil tin?

When oil comes out from a hole of an oil tin, pressure inside it decreased than the atmosphere. Therefore, the surrounding air rush up into the same hole prevents the oil to come out. Hence two holes are made to empty the oil tin.

03. We can cut vegetables easily with a sharp knife as compared to a blunt knife. Why?

Since the stress produced on the vegetables by the sharp knife is higher than the blunt knife, vegetables can be cut easily with the sharp knife.

04. Why the passengers are advised to remove the ink from their pens while going up in an aero-plane?

When an aero-plane ascends, the atmospheric pressure is decreased. Hence, the ink from the pen will leak out. So that, the passengers are advised to remove the ink from their pens while going up in the aero-plane.

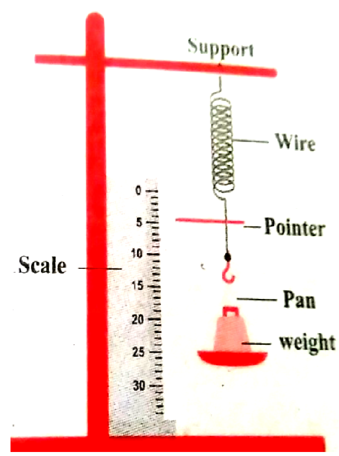
05. We use straw to suck soft drinks, why?

When we suck the soft drinks through the straw, the pressure inside the straw becomes less than the atmospheric pressure. Due to the difference in pressure, the soft drink rises in the straw and we are able to enjoy it conveniently.

FIVE MARKS

01. State Hooke's law and verify it with the help of an experiment.

1) Hooke's law is for a small deformation, when the stress and strain are proportional to each other.

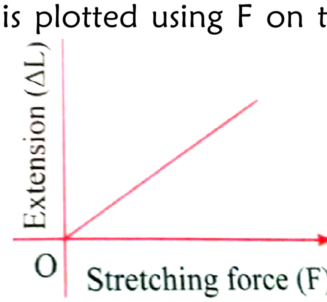


2) It can be verified in a simple way by stretching a thin straight wire (stretches like spring) of length L and uniform cross-sectional area A suspended from a fixed point O.

3) A pan and a pointer are attached at the free end of the wire as shown in Figure (a).

4) The extension produced on the wire is measured using a vernier scale arrangement. The experiment shows that for a given load, the corresponding stretching force is F and the elongation produced on the wire is ΔL.

5) It is directly proportional to the original length L and inversely proportional to the area of cross section A. A graph is plotted using F on the X- axis and ΔL on the Y- axis.



6) This graph is a straight line passing through the origin as shown in Figure (b).

Therefore, ΔL = (slope)F

Multiplying and dividing by volume,

V = A L,

F (slope) $= \frac{AL}{AL}\Delta L$

Rearranging, we get, $\frac{F}{A} = \left[\frac{L}{A(slope)}\right]\frac{\Delta L}{L}$ Therefore, , $\frac{F}{A}\ \alpha \left[\frac{\Delta L}{L}\right]$

Comparing with stress equation and strain equation, we get $\sigma\ \alpha\ \varepsilon$

i.e., the stress is proportional to the strain in the elastic limit.

02. Explain the different types of modulus of elasticity.

There are three types of elastic modulus.

(a) Young's modulus, (b) Rigidity modulus (or Shear modulus)

(c) Bulk modulus

Young's modulus:

When a wire is stretched or compressed, then the ratio between tensile stress (or compressive stress) and tensile strain (or compressive strain) is defined as Young's modulus.

$$= \frac{\text{Tensile stress or compressive stress}}{\text{Tensile starin or compressive strain}} \quad Y = \frac{\sigma_t}{\varepsilon_t} \text{ or } Y = \frac{\sigma_c}{\varepsilon_c}$$

The unit for Young modulus has the same unit of stress because, strain has no unit. So, S.I. unit of Young modulus is N m^{-2} or pascal.

Bulk modulus:

Bulk modulus is defined as the ratio of volume stress to the volume strain.

Bulk modulus, $K = \frac{\text{Normal (Perpendicular)stress or pressure}}{\text{Volume strain}}$

The normal stress or pressure is $\sigma_n = \frac{F_n}{\Delta A} = \Delta p$

The volume strain is $\varepsilon_v = \frac{\Delta V}{V}$

Therefore, Bulk modulus is $K = -\frac{\sigma_n}{\varepsilon_v} = -\frac{\Delta p}{\frac{\Delta V}{V}}$

The negative sign in the equation means that when pressure is applied on the body, its volume decreases. Further, the equation implies that a material can be easily compressed if it has a small value of bulk modulus.

The rigidity modulus or shear modulus:

The rigidity modulus is defined as Rigidity modulus or Shear modulus,

$$\eta_R = \frac{\text{Shearing stress}}{\text{Angle of shear or shearing strain}}$$

The shearing stress is $\sigma_s = \frac{\text{Trangential force}}{\text{Area over which it is applied}} = \frac{F_t}{\Delta A}$

The angle of shear or shearing strain $\varepsilon_s = \frac{x}{h} = \theta$

Therefore, Rigidity modulus is $\eta = \frac{\sigma_s}{\varepsilon_s} = \frac{\frac{F_t}{\Delta A}}{\frac{x}{h}} = \frac{\frac{F_t}{\Delta A}}{\theta}$

Further, the equation (7.9) implies, that a material can be easily twisted if it has small value of rigidity modulus. For example, consider a wire, when it is twisted through an angle θ, a restoring torque is developed, that is

$$\tau \propto \theta$$

This means that for a larger torque, wire will twist by a larger amount (angle of shear θ is large). Since, rigidity modulus is inversely proportional to angle of shear, the modulus of rigidity is small.

03. Derive an expression for the elastic energy stored per unit volume of a wire.

When a body is stretched, work is done against the restoring force (internal force). This work done is stored in the body in the form of elastic energy. Consider a wire whose un-stretch length is L and area of cross section is A. Let a force produce an extension *l* and further assume that the elastic limit of the wire has not been exceeded and there is no loss in energy. Then, the work done by the force F is equal to the energy gained by the wire.

The work done in stretching the wire by dl, dW = F dl

The total work done in stretching the wire from 0 to l is

$W = \int_0^l F\,dl$ ------------------1

From Young's modulus of elasticity, $Y = \frac{F}{A} \times \frac{L}{l} \Rightarrow F = \frac{YAl}{L}$ --------- 2

Substituting equation (2) in equation (1), we get

$W = \int_0^l \frac{YAl}{L} dl = \frac{YAl^2}{L.2} = ½ .Fl$

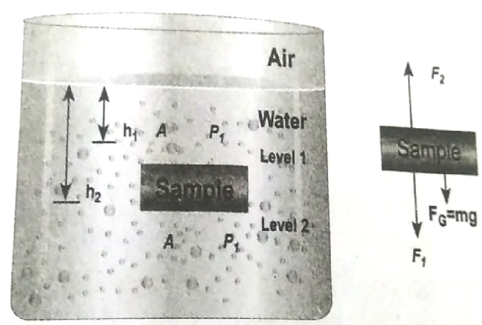
$W = \int \frac{YAl'}{L} dl' = \frac{YA}{L}\frac{l'^2}{2}\Big|_0^l = \frac{YA}{L}\frac{l^2}{2} = \frac{1}{2}\left(\frac{YAl}{L}\right) l = \frac{1}{2} Fl$

$W = \frac{1}{2} Fl$ = Elastic potential energy.

Energy per unit volume is called energy density,

$u = \frac{\text{Elastic potential energy}}{\text{Volume}} = \frac{\frac{1}{2}Fl}{AL} \quad \frac{1}{2}\frac{F}{A}\frac{l}{L} = \frac{1}{2}$ (Stress x Strain)

04. Derive an equation for the total pressure at a depth 'h' below the liquid surface.



Consider a water sample of cross sectional area in the form of a cylinder. Let h_1 and h_2 be the depths from the air-water interface to level 1 and level 2 of the cylinder, respectively as shown in Figure (a). Let F1 be the force acting downwards on level 1 and F_2 be the force acting upwards on level 2, such that, $F_1 = P_1 A$ and $F_2 = P_2 A$ Let us assume the mass of the sample to be *m* and under equilibrium condition, the total upward force (F_2) is balanced by the total downward force (F_1 + mg), in other words, the gravitational force will act downward which is being exactly balanced by the difference between the force. $F_2 - F_1$

$F_2 - F_1 = mg = F_G$

Where *m* is the mass of the water available in the sample element. Let ρ be the density of the water then, the mass of water available in the sample element is $m = \rho V = \rho A (h_2 - h_1)$ $V = A (h_2 - h_1)$

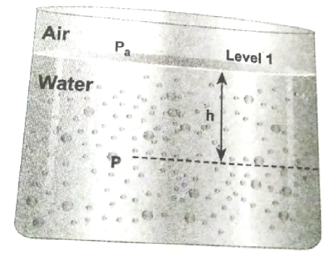
Hence, gravitational force,

$F_G = \rho A (h_2 - h_1) g$

On substituting the value of W in equation

$$F_2=F_1+m\,g \Rightarrow P_2A = P_1A+ \rho A(h_2 - h_1)g$$

Cancelling out A on both sides, $P_2 = P_1 + \rho(h_2 - h_1)g$



If we choose the level 1 at the surface of the liquid (i.e., air-water interface) and the level 2 at a depth 'h' below the surface (as shown in Figure (b), then the value of h1 becomes zero ($h_1 = 0$) and in turn P1 assumes the value of atmospheric pressure (say Pa). In addition, the pressure (P_2) at a depth becomes P. Substituting these values in equation, we get

$$P = Pa + \rho gh$$

Which means, the pressure at a depth h is greater than the pressure on the surface of the liquid, where Pa is the atmospheric pressure which is equal to 1.013×10^5 Pa. If the atmospheric pressure is neglected or ignored then

$$P = \rho gh$$

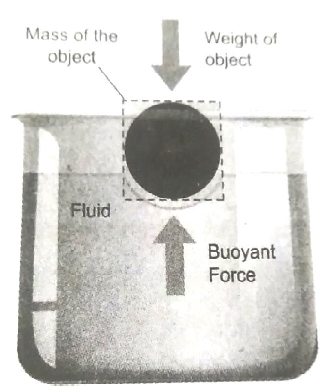
05. State and prove Pascal's law in fluids.

Hydraulic lift which is used to lift a heavy load with a small force. It is a force multiplier. It consists of two cylinders A and B connected to each other by a horizontal pipe, filled with a liquid (Figure). They are fitted with frictionless pistons of cross sectional areas A_1 and A_2 ($A_2 > A_1$). Suppose a downward force F is applied on the smaller piston, the pressure of the liquid under this piston increases to $P\left(\text{where}, P = \frac{F_1}{A_1}\right)$. But according to Pascal's law, this increased pressure P is transmitted undiminished in all directions. So a pressure is exerted on piston B. Upward force on piston B is

$$F_2 = P \times A_2 = \frac{F_1}{A_1} \times A_2 \Rightarrow F_2 = \frac{A_2}{A_1} \times F_1$$

Therefore by changing the force on the smaller piston A, the force on the piston B has been increased by the factor $\frac{A_2}{A_1}$ and this factor is called the mechanical advantage of the lift.

06. State and prove Archimedes principle.



It states that when a body is partially or wholly immersed in a fluid, it experiences an upward thrust equal to the weight of the fluid displaced by it and its up-thrust acts through the centre of gravity of the liquid displaced.

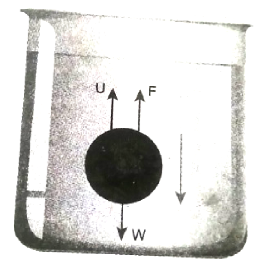
Up-thrust or buoyant force = weight of liquid displaced.

07. Derive the expression for the terminal velocity of a sphere moving in a high viscous fluid using stokes force.

Expression for terminal velocity:

Consider a sphere of radius r which falls freely through a highly viscous liquid of coefficient of viscosity η. Let the density of the material of the sphere be ρ and the density of the fluid be σ.

Gravitational force acting on the sphere, F_G = mg = $\frac{4}{3}\pi r^3 \rho g$ (downward force)



Up thrust, U = $\frac{4}{3}\pi r^3 \sigma g$ (upward force)

Viscous force F = $6\pi\eta r v_t$

At terminal velocity v_t, downward force = upward force

$F_G - U = F \Rightarrow \frac{4}{3}\pi r^3 \rho g - = \frac{4}{3}\pi r^3 \sigma g \;\; = 6\pi\eta r v_t$

$V_t = \frac{2}{9} \times \frac{r^2(\rho-\sigma)}{\eta} g \Rightarrow V_t \propto r^2$

Here, it should be noted that the terminal speed of the sphere is directly proportional to the square of its radius. If σ is greater than ρ, then the term (ρ - σ) becomes negative leading to a negative terminal velocity.

08. Derive Poiseuille's formula for the volume of a liquid flowing per second through a pipe under streamlined flow.

Consider a liquid flowing steadily through a horizontal capillary tube. Let $v = \left(\frac{V}{t}\right)$ be the volume of the liquid flowing out per second through a capillary tube. It depends on (1) coefficient of viscosity (η) of the liquid, (2) radius of the tube (r), and (3) the pressure gradient$\left(\frac{P}{l}\right)$. Then, $v \; \alpha \; \eta^a r^b \left(\frac{P}{l}\right)^c$;

$v = k \; \eta^a r^b \left(\frac{P}{l}\right)^c$ where, k is a dimensionless constant. Therefore,

$[v] = \frac{\text{Volume}}{\text{time}} = [L^3T^{-1}]$, $\left[\frac{dP}{dx}\right] = \frac{\text{Pressure}}{\text{distance}} = [ML^{-2}T^{-2}]$,

$[\eta] = [ML^{-1}T^{-1}]$ and $[r] = [L]$

Substituting in equation, So, equating the powers of M, L, and T on both sides, we get a + c = 0, −a + b −2c =3, and −a −2c = −1

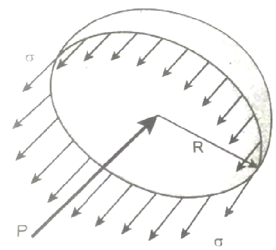
We have three unknowns a, b, and c. We have three equations, on solving, we get a = −1, b = 4, and c = 1

Therefore, equation becomes, $v = k\eta^{-1} r^4 \left(\frac{P}{l}\right)^1$

Experimentally, the value of k is shown to be $\frac{\pi}{8}$, we have $v = \frac{\pi r^4 P}{8\eta l}$

09. Obtain an expression for the excess of pressure inside a i) liquid drop ii) liquid bubble iii) air bubble.

i) Excess of pressure inside air bubble in a liquid.

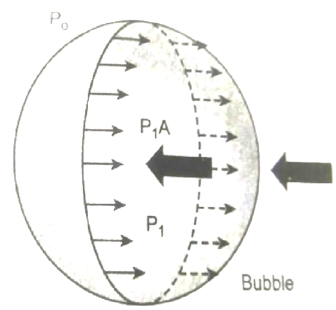


Consider an air bubble of radius R inside a liquid having surface tension T as shown in Figure (a). Let P_1 and P_2 be the pressures outside and inside the air bubble, respectively. Now, the excess pressure inside the air bubble is

$\Delta P = P_1 - P_2$. To find the excess pressure inside the air bubble, let us consider the forces acting on the air bubble.

ii) Excess pressure inside a soap bubble.

Consider a soap bubble of radius R and the surface tension of the soap bubble be T as shown in Figure (b). A soap bubble has two liquid surfaces in contact with air, one inside the bubble and other outside the bubble. Therefore, the force on the soap bubble due to surface tension is $2\times2\pi RT$. The various forces acting on the soap bubble are,



i) Force due to surface tension $F_T = 4\pi RT$ towards right
ii) Force due to outside pressure $F_{P1} = P_1\pi R^2$ towards right
iii) Force due to inside pressure $F_{P2} = P_2\pi R^2$ towards left

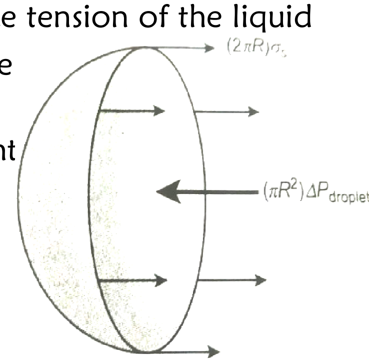
As the bubble is in equilibrium, $F_{P2} = F_T + F_{P1}$

$P_2\pi R^2 = 4\pi RT + P_1\pi R^2 \Rightarrow (P_2 - P_1)\pi R^2 = 4\pi RT$

Excess pressure is $\Delta P = P_2 - P_1 = \frac{4T}{R}$

iii) Excess pressure inside the liquid drop

Consider a liquid drop of radius R and the surface tension of the liquid is T as shown in Figure. The various forces acting on the



i) Force due to surface tension $F_T = 2\pi RT$ towards right
ii) Force due to outside pressure $F_{P1} = P_1\pi R^2$ towards right
iii) Force due to inside pressure $F_{P2} = P_2\pi R^2$ towards left

As the liquid drop is in equilibrium, $F_{P2} = F_T + F_{P1}$

$P_2\pi R^2 = 2\pi RT + P_1\pi R^2 \Rightarrow (P_2 - P_1)\pi R^2 = 2\pi RT$

Excess pressure is $\Delta P = P_2 - P_1 = \frac{2T}{R}$

10. What is capillarity? Obtain an expression for the surface tension of a liquid by capillary rise method.

Consider a capillary tube which is held vertically in a beaker containing water; the water rises in the capillary tube to a height h due to surface tension.

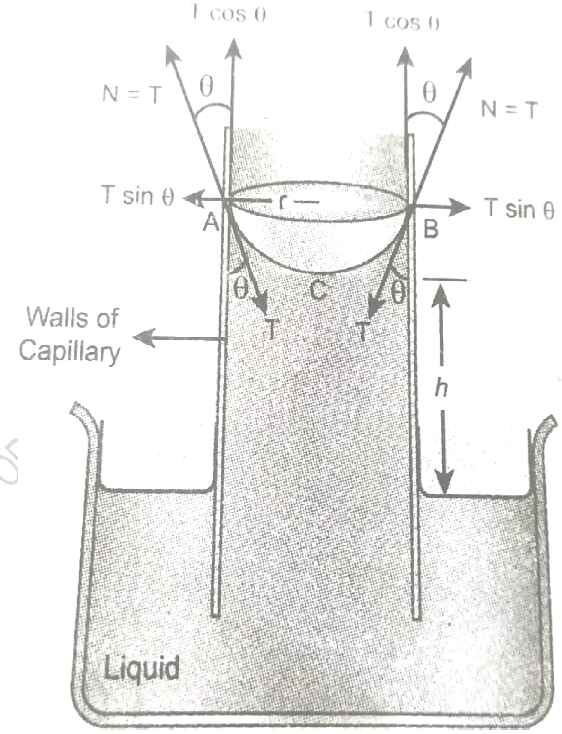
The surface tension force F_T, acts along the tangent at the point of contact downwards and its reaction force upwards. Surface tension T, is resolved into two components i) Horizontal component T sinθ and ii) Vertical component T cosθ acting upwards, all along the whole circumference of the meniscus.

Total upward force = $(T\cos\theta)\,(2\pi r) = 2\pi r T\cos\theta$

Where θ is the angle of contact, r is the radius of the tube. Let ρ be the density of water and h be the height to which the liquid rises inside the tube.

Then, $\begin{pmatrix}\text{the volume of}\\ \text{liquid column in}\\ \text{the tube, V}\end{pmatrix} = \begin{pmatrix}\text{Volume of the liquid}\\ \text{column of radius r}\\ \text{height h}\end{pmatrix} + \begin{pmatrix}\text{Volume of liquid of}\\ \text{radius r and height}\\ \text{r} - \text{Volume of the}\\ \text{hemisphere of radius r}\end{pmatrix}$

V = $\pi r^2 h + \left(\pi r^2 \text{x}\, r - \frac{2}{3}\pi r^3\right) \Rightarrow \pi r^2 h + \frac{1}{3}\pi r^3$



The upward force supports the weight of the liquid column above the free surface, therefore, $2\pi r T\cos\theta = \pi r^2\left(h + \frac{1}{3}r\right)\rho g \Rightarrow$

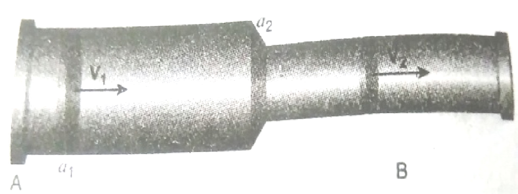
T = $\dfrac{r\left(h+\frac{1}{3}r\right)\rho g}{2\cos\theta}$

If the capillary is a very fine tube of radius (i.e., radius is very small) then $\frac{r}{3}$ can be neglected when it is compared to the height h. Therefore,

$$T = \frac{r\rho g h}{2\cos\theta}$$

11. Obtain an equation of continuity for a flow of fluid on the basis of conservation of mass.

Consider a pipe AB of varying cross sectional area a_1 and a_2 such that $a_1 > a_2$. A non-viscous and incompressible liquid flows steadily through the pipe, with velocities v_1 and v_2 in area a_1 and a_2, respectively as shown in Figure.



Let m_1 be the mass of fluid flowing through section A in time Δt, $m_1 = (a_1 v_1 \Delta t)\,\rho$

Let m_2 be the mass of fluid flowing through section B in time Δt, $m_2 = (a_2 v_2 \Delta t)\,\rho$

For an incompressible liquid, mass is conserved $m_1 = m_2$

$a_1 v_1 \Delta t\,\rho = a_2 v_2 \Delta t\,\rho$

$a_1 v_1 = a_2 v_2 \Rightarrow a\,v = \text{constant}$

which is called the equation of continuity and it is a statement of conservation of mass in the flow of fluids.

In general, a v = constant, which means that the volume flux or flow rate remains constant throughout the pipe. In other words, the smaller the cross section, greater will be the velocity of the fluid.

12. State and prove Bernoulli's theorem for a flow of incompressible, non-viscous, and streamlined flow of fluid.

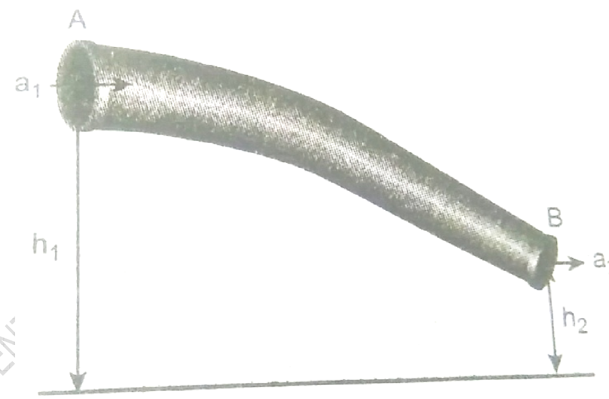
Bernoulli's theorem :

According to Bernoulli's theorem, the sum of pressure energy, kinetic energy, and potential energy per unit mass of an incompressible, non-viscous fluid in a streamlined flow remains a constant.

$\frac{P}{\rho} + \frac{1}{2}v^2 + gh =$ Constant, this is known as Bernoulli's equation.

Proof:

Let us consider a flow of liquid through a pipe AB as shown in Figure. Let V be the volume of the liquid when it enters A in a time t which is equal to the volume of the liquid leaving B in the same time. Let a_A, v_A and P_A be the area of cross section of the tube, velocity of the liquid and pressure exerted by the liquid at A respectively.



Let the force exerted by the liquid at A is $F_A = P_A a_A$

Distance travelled by the liquid in time t is $d = v_A t$

Therefore, the work done is $W = F_A d = P_A a_A v_A t$

But $a_A v_A t = a_A d = V$, volume of the liquid entering at A.

Thus, the work done is the pressure energy (at A), $W = F_A d = P_A V$

Pressure energy per unit volume at A $= \frac{\text{Pressure energy}}{\text{Volume}} = \frac{P_A V}{V} = P_A$

Pressure energy per unit mass at A $= \frac{\text{Pressure energy}}{\text{Mass}} = \frac{P_A V}{m} = \frac{P_A}{\frac{m}{V}} = \frac{P_A}{\rho}$

Since m is the mass of the liquid entering at A in a given time, therefore, pressure energy of the liquid at A is $E_{PA} = P_A V = P_A V \times \left(\frac{m}{m}\right) = m\frac{P_A}{\rho}$

Potential energy of the liquid at A, $P_{EA} = mg\,h_A$,

Due to the flow of liquid, the kinetic energy of the liquid at A,

$KE_A = ½\, mV_A^2$

Therefore, the total energy due to the flow of liquid at A,

$E_A = EP_A + KE_A + PE_A$

$E_A = m\frac{P_A}{\rho} + ½\, mV_A^2 + mgh_A$

Similarly, let a_B, v_B, and P_B be the area of cross section of the tube, velocity of the liquid, and pressure exerted by the liquid at B. Calculating the total energy at E_B, we get $E_B = m\frac{P_B}{\rho} + \frac{1}{2} mV_B^2 + mgh_B$

From the law of conservation of energy, $E_A = E_B$

$E_A = m\frac{P_A}{\rho} + \frac{1}{2} mV_A^2 + mgh_A = E_B = m\frac{P_B}{\rho} + \frac{1}{2} mV_B^2 + mgh_B$

$\frac{P_A}{\rho} + \frac{1}{2} V_A^2 + gh_A = \frac{P_B}{\rho} + \frac{1}{2} V_B^2 + gh_B$ = constant

Thus, the above equation can be written as $\frac{P}{\rho g} + \frac{1}{2}\frac{v^2}{g} + h$ = constant

13. Describe the construction and working of venturimeter and obtain an equation for the volume of liquid flowing per second through a wider entry of the tube.

Venturimeter

This device is used to measure the rate of flow (or say flow speed) of the incompressible fluid flowing through a pipe. It works on the principle of Bernoulli's theorem.

Let P_1 be the pressure of the fluid at the wider region of the tube A. Let us assume that the fluid of density 'ρ' flows from the pipe with speed 'v_1' and into the narrow region, its speed increases to 'v_2'. According to the Bernoulli's equation, this increase in speed is accompanied by a decrease in the fluid pressure P_2 at the narrow region of the tube B. Therefore, the pressure difference between the tubes A and B is noted by measuring the height difference ($\Delta P = P_1 - P_2$) between the surfaces of the manometer liquid.

From the equation of continuity, we can say that

$Av_1 = a\, v_2$ which means that $V_2 = \frac{A}{a} v_1$

Using Bernoulli's equation, $P_1 + \rho\frac{V1^2}{2} = P_2 + \rho\frac{V2^2}{2} = P_2 + \rho\frac{1}{2}\left(\frac{A}{a}v_1\right)^2$

From the above equation, the pressure difference,

$\Delta P = P_1 - P_2 = \rho\frac{V1^2}{2}\left(\frac{A^2-a^2}{a^2}\right)$

Thus, the speed of flow of fluid at the wide end of the tube A

$v_1^2 = \frac{2(\Delta P)a^2}{\rho(A^2-a^2)} \Rightarrow V_1 = \sqrt{\frac{2(\Delta P)a^2}{\rho\left(A^2-a^2\right)}}$

The volume of the liquid fl owing out per second is

$AV_1 = \sqrt{\frac{2(\Delta P)a^2}{\rho\left(A^2-a^2\right)}}$

$= aA\sqrt{\frac{2(\Delta P)}{\rho\left(A^2-a^2\right)}}$

14. Write any two applications of Bernoulli's theorem.

(a) Blowing off roofs during wind storm

1) In olden days, the roofs of the huts or houses were designed with a slope. One important scientific reason is that as per the Bernoulli's principle, it will be safeguarded except roof during storm or cyclone.

2) During cyclonic condition, the roof is blown off without damaging the other parts of the house.

3) In accordance with the Bernoulli's principle, the high wind blowing over the roof creates a low-pressure P_1.

4) The pressure under the roof P_2 is greater. Therefore, this pressure difference $(P_2 - P_1)$ creates an up thrust and the roof is blown off.

(b) Aerofoil lift

1) The wings of an airplane (aerofoil) are so designed that its upper surface is more curved than the lower surface and the front edge is broader than the real edge.

2) As the aircraft moves, the air moves faster above the aerofoil than at the bottom.

3) According to Bernoulli's Principle, the pressure of air below is greater than above, which creates an up-thrust called the dynamic lift to the aircraft.

15. Write the applications of elasticity.

1) The elastic behavior is one such property which especially decides the structural design of the columns and beams of a building.

2) As far as the structural engineering is concerned, the amount of stress that the design could withstand is a primary safety factor.

3) A bridge has to be designed in such a way that it should have the capacity to withstand the load of the flowing traffic, the force of winds, and even its own weight.

4) The elastic behavior or in other words the bending of beams is a major concern over the stability of the buildings or bridges.

5) To reduce the bending of a beam for a given load, one should use the material with a higher Young's modulus of elasticity (Y).

6) The Young's modulus of steel is greater than aluminium or copper. Iron comes next to steel.

7)This is the reason why steel is mostly preferred in the design of heavy duty machines and iron rods in the construction of buildings.