1 SET LANGUAGE

1.1 INTRODUCTION

A set, we know, is a "well-defined collection of objects". We are aware of different sets such as empty set, finite set, infinite set, subset, power set, equal sets, equivalent sets and universal set.

1.2 Properties of Set Operations :

We first take up the properties of set operations on union and intersection

1.2.1 Commutative Property

For any two sets A and B

(i) $A \cup B = B \cup A$ (ii) $A \cap B = B \cap A$

1.2.2 Associative Property

For any three sets A, B and C

(i) $A \cup (B \cup C) = (A \cup B) \cup C$ (ii) $A \cap (B \cap C) = (A \cap B) \cap C$

Exercise 1.1

1. If $P = \{1, 2, 5, 7, 9\}$, $Q = \{2, 3, 5, 9, 11\}$, $R = \{3, 4, 5, 7, 9\}$ and $S = \{2, 3, 4, 5, 8\}$, the find

(i) $(P \cup Q) \cup R$ **(ii) $(P \cap Q) \cap S$** **(iii) $(Q \cap S) \cap R$**

Sol. **(i) $(P \cup Q) \cup R$**

$(P \cup Q) = \{1, 2, 5, 7, 9\} \cup \{2, 3, 5, 9, 11\} = \{1, 2, 3, 5, 7, 9, 11\}$

$(P \cup Q) \cup R = \{1, 2, 3, 5, 7, 9, 11\} \cup \{3, 4, 5, 7, 9\} = \{1, 2, 3, 4, 5, 7, 9, 11\}$

(ii) $(P \cap Q) \cap S$

$(P \cap Q) = \{1, \underline{2}, \underline{5}, 7, \underline{9}\} \cap \{\underline{2}, 3, \underline{5}, \underline{9}, 11\} = \{2, 5, 9\}$

$(P \cap Q) \cap S = \{\underline{2}, \underline{5}, 9\} \cap \{\underline{2}, 3, 4, \underline{5}, 8\} = \{2, 5\}$

(iii) $(Q \cap S) \cap R$

$(Q \cap S) = \{\underline{2}, \underline{3}, \underline{5}, 9, 11\} \cap \{\underline{2}, \underline{3}, 4, \underline{5}, 8\} = \{2, 3, 5\}$

$(Q \cap S) \cap R = \{2, \underline{3}, \underline{5}\} \cap \{\underline{3}, 4, \underline{5}, 7, 9\} = \{3, 5\}$

2. **Test for the commutative property of union and intersection of the sets**
P = { x : x is a real number between 2 and 7} and
Q = { x : x is an irrational number between 2 and 7}

Sol. **Commulative Property of union of sets**

$\boxed{(A \cup B) = (B \cup A)}$

Here P $= \{3, 4, 5, 6\}$, Q $= \{\sqrt{3}, \sqrt{5}, \sqrt{6}\}$

P∪Q $= \{3, 4, 5, 6\} \cup \{\sqrt{3}, \sqrt{5}, \sqrt{6}\} = \{3, 4, 5, 6, \sqrt{3}, \sqrt{5}, \sqrt{6}\}$ ...(1

Q∪P $= \{\sqrt{3}, \sqrt{5}, \sqrt{6}\} \cup \{3, 4, 5, 6\} = \{\sqrt{3}, \sqrt{5}, \sqrt{6}, 3, 4, 5, 6\}$ ...(2

(1) = (2)

∴ P∪Q = Q∪P

∴ It is verified that union of sets is commutative.

Commutative Property of intersection of sets

$\boxed{(P \cap Q) = (Q \cap P)}$

P∩Q $= \{3, 4, 5, 6\} \cap \{\sqrt{3}, \sqrt{5}, \sqrt{6}\} = \{\ \}$ ...(1

Q∩P $= \{\sqrt{3}, \sqrt{5}, \sqrt{6}\} \cap \{3, 4, 5, 6\} = \{\ \}$ ...(2

From (1) and (2)

P∩Q = Q∩P

∴ It is verified that intersection of sets is commutative.

3. **If A = $\{p, q, r, s\}$, B = $\{m, n, q, s, t\}$ and C = $\{m, n, p, q, s\}$, then verify the associative property of union of sets.**

Sol. **Associative Property of union of sets**

$\boxed{A \cup (B \cup C) = (A \cup B) \cup C)}$

$B \cup C = \{m, n, q, s, t\} \cup \{m, n, p, q, s\} = \{m, n, p, q, s, t\}$

$A \cup (B \cup C) = \{p, q, r, s\} \cup \{m, n, p, q, s, t\} = \{m, n, p, q, r, s, t\}$ ...(1

$(A \cup B) = \{p, q, r, s\} \cup \{m, n, q, s, t\} = \{p, q, r, s, m, n, t\}$

$(A \cup B) \cup C = \{p, q, r, s, m, n, t\} \cup \{m, n, p, q, s\} = \{p, q, r, s, m, n, t\}$ ...(2

From (1) & (2)

It is verified that $A \cup (B \cup C) = (A \cup B) \cup C$

4. **Verify the associative property of intersection of sets for A=$\{-11, \sqrt{2}, \sqrt{5},$ B = $\{\sqrt{3}, \sqrt{5}, 6, 13\}$ and C = $\{\sqrt{2}, \sqrt{3}, \sqrt{5}, 9\}$.**

Sol. **Associative Property of intersection of sets**

$\boxed{A \cap (B \cap C) = (A \cap B) \cap C)}$

$B \cap C = \{\sqrt{3}, \sqrt{5}, 6, 13\} \cap \{\sqrt{2}, \sqrt{3}, \sqrt{5}, 9\} = \{\sqrt{3}, \sqrt{5}\}$

$A \cap (B \cap C) = \{-11, \sqrt{2}, \sqrt{5}, 7\} \cap \{\sqrt{3}, \sqrt{5}\} = \{\sqrt{5}\}$ ...(1

$A \cap B = \{-11, \sqrt{2}, \sqrt{5}, 7\} \cap \{\sqrt{3}, \sqrt{5}, 6, 13\} = \{\sqrt{5}\}$

$(A \cap B) \cap C = \{\sqrt{5}\} \cap \{\sqrt{2}, \sqrt{3}, \sqrt{5}, 9\} = \{\sqrt{5}\}$ ...(2

From (1) and (2), it is verified that $A \cap (B \cap C) = (A \cap B) \cap C$

5. **If $A = \{x : x = 2^n, n \in \mathbb{W} \text{ and } n < 4\}$, $B = \{x : x = 2n, n \in \mathbb{N} \text{ and } n \leq 4\}$ and $C = \{0, 1, 2, 5, 6\}$, then verify the associative property of intersection of sets.**

Sol.

$$\begin{aligned} A &= \{x : x = 2^n, n \in \mathbb{W}, n < 4\} \\ \Rightarrow \quad x &= 2^0 = 1 \\ x &= 2^1 = 2 \\ x &= 2^2 = 4 \\ x &= 2^3 = 8 \\ \therefore A &= \{1, 2, 4, 8\} \\ B &= \{x : x = 2n, n \in \mathbb{N} \text{ and } n \leq 4\} \\ \Rightarrow \quad x &= 2 \times 1 = 2 \\ x &= 2 \times 2 = 4 \\ x &= 2 \times 3 = 6 \\ x &= 2 \times 4 = 8 \\ \therefore B &= \{2, 4, 6, 8\} \\ C &= \{0, 1, 2, 5, 6\} \end{aligned}$$

Associative property of intersection of sets

$$\begin{aligned} A \cap (B \cap C) &= (A \cap B) \cap C \\ B \cap C &= \{2, 6\} \\ A \cap (B \cap C) &= \{1, 2, 4, 8\} \cap \{2, 6\} = \{2\} && \text{... (1)} \\ A \cap B &= \{1, 2, 4, 8\} \cap \{2, 4, 6, 8\} = \{2, 4, 8\} \\ (A \cap B) \cap C &= \{2, 4, 8\} \cap \{0, 1, 2, 5, 6\} = \{2\} && \text{... (2)} \end{aligned}$$

From (1) and (2), It is verified that $A \cap (B \cap C) = (A \cap B) \cap C$

1.2.3 Distributive Property

In lower classes, we have studied distributive property of multiplication over addition on numbers. That is, $a \times (b + c) = (a \times b) + (a \times c)$.

For any three sets A, B and C

(i) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ [Intersection over union]

(ii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ [Union over intersection]

Exercise 1.2

1. **If $K = \{a, b, d, e, f\}$, $L = \{b, c, d, g\}$ and $M \{a, b, c, d, h\}$ then find the following:**
   (i) $K \cup (L \cap M)$ (ii) $K \cap (L \cup M)$ (iii) $(K \cup L) \cap (K \cup M)$
   (iv) $(K \cap L) \cup (K \cap M)$

Sol. $K = \{a, b, d, e, f\}$, $L = \{b, c, d, g\}$ and $M \{a, b, c, d, h\}$

(i) $K \cup (L \cap M)$

$$\begin{aligned} L \cap M &= \{b, c, d, g\} \cap \{a, b, c, d, h\} = \{b, c, d\} \\ K \cup (L \cap M) &= \{a, b, d, e, f\} \cup \{b, c, d\} = \{a, b, c, d, e, f\} \end{aligned}$$

(ii) $K\cap(L\cup M)$

$L\cup M = \{a, b, c, d, g, h\}$

$K\cap(L\cup M) = \{a, b, d, e, f\} \cap \{a, b, c, d, g, h\} = \{a, b, d\}$

(iii) $(K\cup L)\cap(K\cup M)$

$K\cup L = \{a, b, c, d, e, f, g\}$

$K\cup M = \{a, b, c, d, e, f, h\}$

$(K\cup L)\cap(K\cup M) = \{a, b, c, d, e, f\}$

(iv) $(K\cap L)\cup(K\cap M)$

$(K\cap L) = \{b, d\}$

$(K\cap M) = \{a, b, d\}$

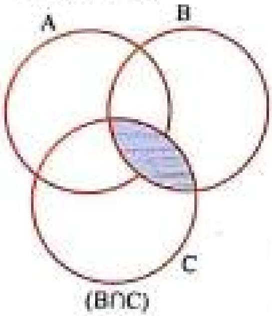
$(K\cap L)\cup(K\cap M) = \{b, d\}\cup \{a, b, d\} = \{a, b, d\}$

2. Draw Venn diagram for each of the following:

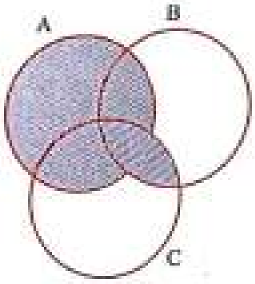
(i) $A\cup(B\cap C)$ **(ii)** $A\cap(B\cup C)$ **(iii)** $(A\cup B)\cap C$

(iv) $(A\cap B)\cup C$

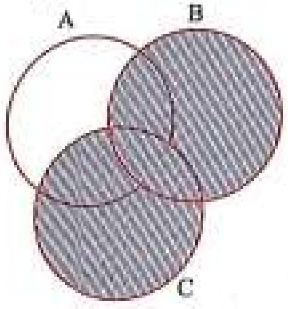
Sol. **(i)** $A\cup(B\cap C)$ **(ii)** $A\cap(B\cup C)$



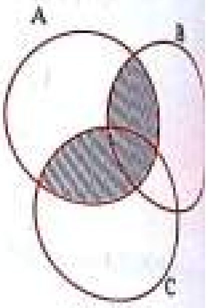
(B∩C)



A∪(B∩C)

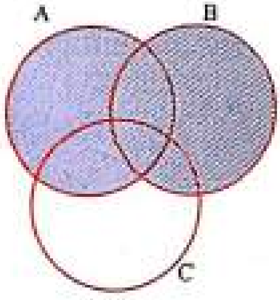


(B∪C)

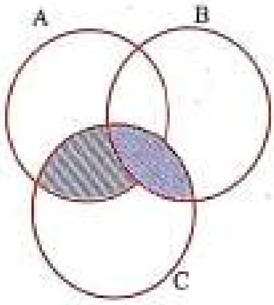


A∩(B∪C)

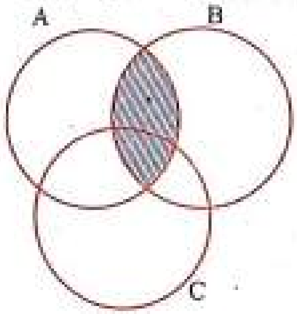
(iii) $(A\cup B)\cap C$ **(iv)** $(A\cap B)\cup C$



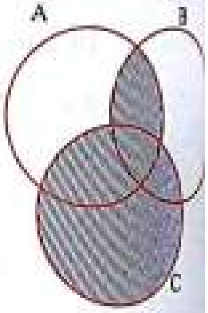
A∪B



(A∪B)∩C



(A∩B)



(A∩B)∪C

3. If $A = \{11, 13, 14, 15, 16, 18\}$, $B = \{11, 12, 15, 16, 17, 19\}$ and $C = \{13, 15, 16, 17, 18, 2$ then verify $A\cap(B\cup C) = (A\cap B)\cup(A\cap C)$.

Sol.

$A = \{11, 13, 14, 15, 16, 18\}$

$B = \{11, 12, 15, 16, 17, 19\}$, $C = \{13, 15, 16, 17, 18, 20\}$

$(B\cup C) = \{11, 12, 13, 15, 16, 17, 18, 19, 20\}$

$A\cap(B\cup C) = \{11, 13, 14, 15, 16, 18\}\cap \{11, 12, 13, 15, 16, 17, 18, 19, 2$

$= \{11, 13, 15, 16, 18\}$ ... (1)

$A\cap B = \{11, 15, 16\}$; $A\cap C = \{13, 15, 16, 18\}$

$(A\cap B)\cup(A\cap C) = \{11, 13, 15, 16, 18\}$ ... (2)

From (1) and (2) it is verified that $A\cap(B\cup C) = (A\cap B)\cup(A\cap C)$.

4. **If $A=\{x: x\in \mathbb{Z}, -2<x\leq 4\}$, $B=\{x: x\in W, x\leq 5\}$, $C=\{-4,-1,0,2,3,4\}$, then verify $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$.**

Sol.
$A = \{x: x\in \mathbb{Z}, -2<x\leq 4\} = \{-1, 0, 1, 2, 3, 4\}$
$B = \{x: x\in W,\ x\leq 5\} = \{0, 1, 2, 3, 4, 5\}$
$C = \{-4, -1, 0, 2, 3, 4\}$

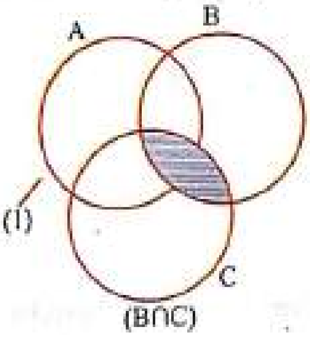
$A\cup(B\cap C)$
$B\cap C = \{0, 1, 2, 3, 4, 5\}\cap\{-4, -1, 0, 2, 3, 4\} = \{0, 2, 3, 4\}$
$A\cup(B\cap C) = \{-1, 0, 1, 2, 3, 4\}\cup(0, 2, 3, 4\} = \{-1, 0, 1, 2, 3, 4\}$ ... (1)

$(A\cap B)\cup(A\cap C)$
$A\cap B = \{0, 1, 2, 3, 4\}$
$A\cap C = \{-1, 0, 2, 3, 4\}$
$(A\cap B)\cup(A\cap C) = \{0, 1, 2, 3, 4\}\cup\{-1, 0, 2, 3, 4\} = \{-1, 0, 1, 2, 3, 4\}$ ... (2)

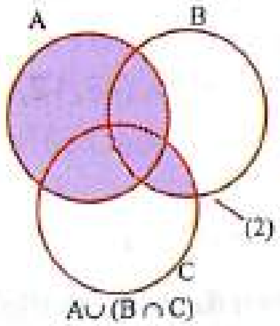
From (1) and (2), it is verified that
$A\cup(B\cap C) = (A\cup B)\cap(A\cup C)$

5. **Verify $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$ using Venn diagrams.**

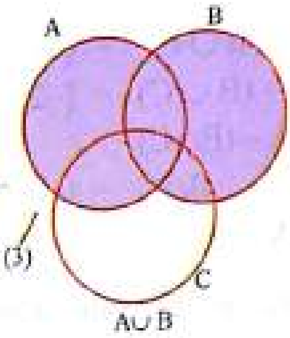
Sol. L.H.S. $A\cup(B\cap C)$



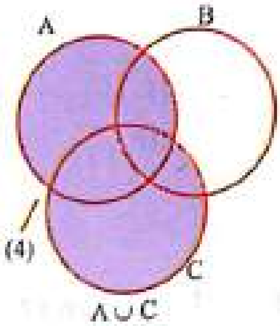
$(B\cap C)$



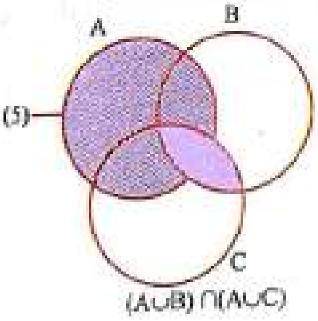
$A\cup(B\cap C)$



$A\cup B$



$A\cup C$



$(A\cup B)\cap(A\cup C)$

From (2) and (5), it is verified that $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$

1.3 De Morgan's Law

1.3.1 De Morgan's Law for Set Difference

These laws relate the set operations union, intersection and set difference

1.3.2 De Morgan's Law for Set Difference

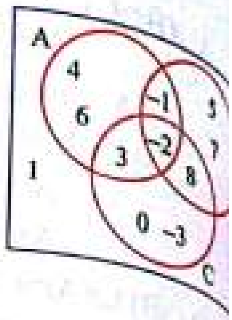
These laws relate the set operations union, intersection and complementation.

Exercise 1.3

1. **Using the adjacent venn diagram, find the following sets :**

(i) $A - B$ **(ii)** $B - C$ **(iii)** $A' \cup B'$ **(vi)** $A' \cap B'$

(v) $(B \cup C)'$ **(vi)** $A - (B \cup C)$ **(vii)** $A - (B \cap C)$



Sol. (i) $A - B = \{3, 4, 6\}$

(ii) $B - C = \{-1, 5, 7\}$

(iii) $A' \cup B'$

$A' = \{1, 2, 0, -3, 5, 7, 8\}$

$B' = \{-3, 0, 1, 2, 3, 4, 6\}$

$A' \cup B' = \{-3, 0, 1, 2, 3, 4, 5, 6, 7, 8\}$

(iv) $A' \cap B'$

$A' \cap B' = \{-3, 0, 1, 2\}$

(v) $(B \cup C)'$

$B \cup C = \{-3, -2, -1, 0, 3, 5, 7, 8\}$

$(B \cup C)' = U - (B \cup C)$

$= \{-3, -2, -1, 0, 1, 2, 3, 4, 5, .6, 7, 8\} - \{-3, -2, -1, 0, 3, 5, 7,$

$(B \cup C)' = \{1, 2, 4, 6\}$

(vi) $A - (B \cup C) = \{-2, -1, 3, 4, 6\} - \{-3, -2, -1, 0, 3, 5, 7, 8\} = \{4, 6$

(vii) $A - (B \cap C)$

$B \cap C = \{-2, 8\}$

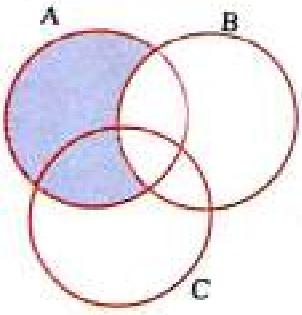
$A - (B \cap C) = \{-2, -1, 3, 4, 6\} - \{-2, 8\} = \{-1, 3, 4, 6\}$

2. **If A, B and C are overlapping sets, then draw Venn diagram for the following**

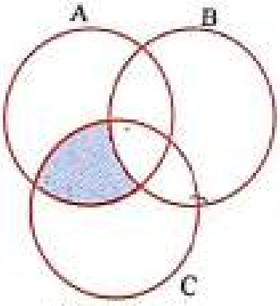
(i) $(A - B) \cap C$ **(ii)** $(A \cup C) - B$ **(iii)** $A - (A \cap C)$ **(iv)** $A (B \cup C)$-

(v) $A \cap B \cap C$

Sol. (i)

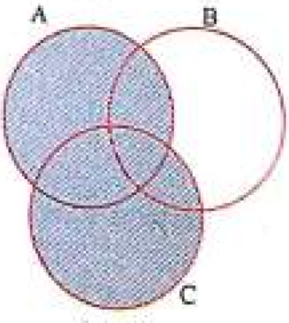


A – B

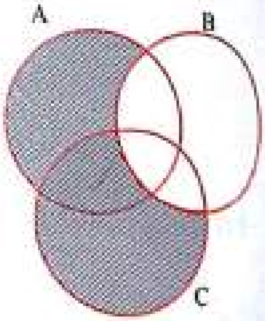


(A – B) ∩ C

(ii)

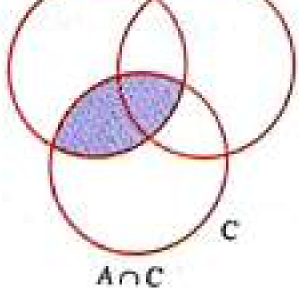


(A ∪ C)



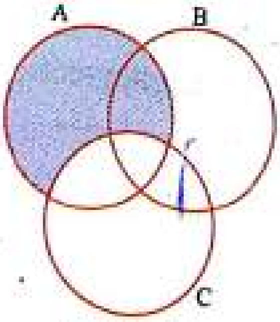
(A ∪ C) – B

(iii)

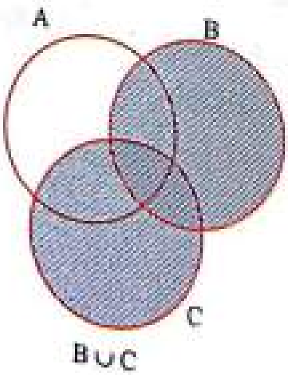


A ∩ C

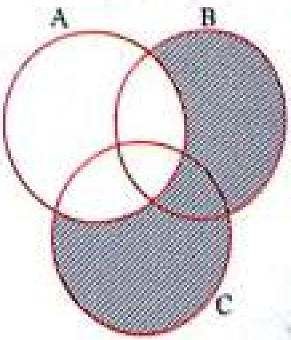
(iv)



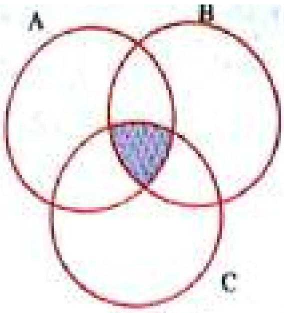
A – (A ∩ C)



B ∪ C



(v)



$A \cap B \cap C$

If $A = \{b, c, e, g, h\}$, $B = \{a, c, d, g, i\}$ and $C = \{a, d, e, g, h\}$, then show that $A - (B \cap C) = (A - B) \cup (A - C)$.

Sol.

$$A = \{b, c, e, g, h\}$$
$$B = \{a, c, d, g, i\}$$
$$C = \{a, d, e, g, h\}$$
$$B \cap C = \{a, d, g\}$$
$$A - (B \cap C) = \{b, c, e, g, h\} - \{a, d, g\} = \{b, c, e, h\} \quad \ldots (1)$$
$$A - B = \{b, c, e, g, h\} - \{a, c, d, g, i\} = \{b, e, h\}$$
$$A - C = \{b, c, e, g, h\} - \{a, d, e, g, h\} = \{b, c\}$$
$$(A - B) \cup (A - C) = \{b, c, e, h\} \quad \ldots (2)$$

From (1) and (2) it is verified that

$$A - (B \cap C) = (A - B) \cup (A - C)$$

If $A = \{x : x = 6n, n \in \mathbb{W} \text{ and } n < 6\}$, $B = \{x : x = 2n, n \in \mathbb{N} \text{ and } 2 < n \leq 9\}$ and $C = \{x : x = 3n, n \in \mathbb{N} \text{ and } 4 \leq n < 10\}$, then show that $A-(B \cap C) = (A-B) \cup (A-C)$

Sol.

$$A = \{x : x = 6n, n \in \mathbb{W}, n < 6\}$$
$$x = 6n$$
$$n = \{0, 1, 2, 3, 4, 5\}$$
$$\Rightarrow \quad x = 6 \times 0 = 0$$
$$x = 6 \times 1 = 6$$
$$x = 6 \times 2 = 12$$
$$x = 6 \times 3 = 18$$
$$x = 6 \times 4 = 24$$
$$x = 6 \times 5 = 30$$
$$\therefore A = \{0, 6, 12, 18, 24, 30\}$$
$$B = \{x : x = 2n, n \in \mathbb{N}, 2 < n \leq 9\}$$
$$n = \{3, 4, 5, 6, 7, 8, 9\}$$
$$x = 2n$$
$$\Rightarrow \quad x = 2 \times 3 = 6 ; 2 \times 4 = 8 ; 2 \times 5 = 10; 2 \times 6 = 12; \quad 2 \times 7 = 14$$
$$2 \times 8 = 16 ; 2 \times 9 = 18$$
$$\therefore B = \{6, 8, 10, 12, 14, 16, 18\}$$

$C = \{x : x = 3n, n \in \mathbb{N}, 4 \le n < 10\}$

$N = \{4, 5, 6, 7, 8, 9\}$

$\Rightarrow \quad x = 3 \times 4 = 12 ;\ x = 3 \times 5 = 15 ;\ x = 3 \times 6 = 18$

$x = 3 \times 7 = 21 ;\ x = 3 \times 8 = 24 ;\ x = 3 \times 9 = 27$

$x = 2 \times 9 = 18$

$\therefore C = \{12, 15, 18, 21, 24, 27\}$

$A-(B\cap C) = (A-B)\cup(A-C)$

L.H.S R.H.S

$B\cap C = \{12, 18\}$

$A-(B\cap C) = \{0, 6, 12, 18, 24, 30\} - \{12, 18\}$ ... (1)

$= \{0, 6, 24, 30\}$

$(A-B) = \{0, 24, 30\}$

$(A-C) = \{0, 6, 30\}$

$(A-B)\cup(A-C) = \{0, 6, 24, 30\}$ ... (2)

From (1) and (2), it is verified that

$A-(B\cap C) = (A-B)\cup(A-C).$

5. **If $A = \{-2, 0, 1, 3, 5\}$, $B = \{-1, 0, 2, 5, 6\}$ and $C = \{-1, 2, 5, 6, 7\}$, then show that $A-(B\cup C) = (A-B)\cap(A-C)$.**

Sol.

$A = \{-2, 0, 1, 3, 5\}, B = \{-1, 0, 2, 5, 6\}$

$C = \{-1, 2, 5, 6, 7\}$

$B \cup C = \{-1, 0, 2, 5, 6, 7\}$

$A - (B \cup C) = \{-2, 1, 3\}$ ... (1)

$(A - B) = \{-2, 1, 3\}$

$(A - C) = \{-2, 0, 1, 3\}$

$(A - B) \cap (A - C) = \{-2, 1, 3\}$ ... (2)

From (1) and (2), it is verified that

$A-(B\cup C) = (A-B)\cap(A-C)$

6. **If $A = \left\{y : y = \frac{a+1}{2}, a \in W \text{ and } a \le 5\right\}$, $B = \left\{y : y = \frac{2n-1}{2}, n \in W \text{ and } n < 5\right\}$ and $C = \left\{-1, -\frac{1}{2}, 1, \frac{3}{2}, 2\right\}$, then show that $A-(B \cup C) = (A-B) \cap (A-C)$.**

Sol.

$A = \left\{y : y = \frac{a+1}{2}, a \in W, a \le 5\right\}$

$a = \{0, 1, 2, 3, 4, 5\}$

$\Rightarrow \quad y = \frac{a+1}{2} = \frac{a+1}{2} = \frac{1}{2}$

$y = \frac{1+1}{2} = \frac{2}{2} = 1$

$y = \frac{2+1}{2} = \frac{3}{2} ;\ y = \frac{3+1}{2} = \frac{4}{2} = 2$

$y = \frac{4+1}{2} = \frac{5}{2} ;\ y = \frac{5+1}{\ }\ 6$

$$\therefore A = \left\{\frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3\right\}$$

$$B = \left\{y : y = \frac{2n-1}{2}, n \in W \text{ and } n < 5\right\}$$

$$n = \{0, 1, 2, 3, 4\}$$

$$\Rightarrow \quad y = \frac{2\times0-1}{2} = \frac{-1}{2}$$

$$y = \frac{2\times1-1}{2} = \frac{1}{2}\ ;\ y = \frac{2\times2-1}{2} = \frac{3}{2}$$

$$y = \frac{2\times3-1}{2} = \frac{5}{2}\ ;\ y = \frac{2\times4-1}{2} = \frac{7}{2}$$

$$\therefore B = \left\{-\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}\right\}$$

$$C = \left\{-1, -\frac{1}{2}, 1, \frac{3}{2}, 2\right\}$$

$$B \cup C = \left\{-1, -\frac{1}{2}, \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, \frac{7}{2}\right\}$$

$$A-(B\quad C) = \{3\} \qquad \text{... (1)}$$

$$A - B = \{1, 2, 3\}$$

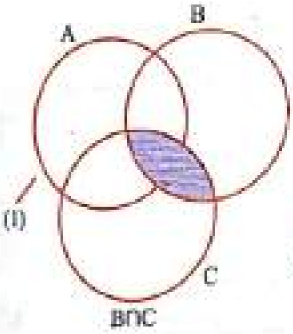
$$A - C = \left\{\frac{1}{2}, \frac{5}{2}, 3\right\}$$

$$(A - B) \cap (A - C) = \{3\} \qquad \text{... (2)}$$

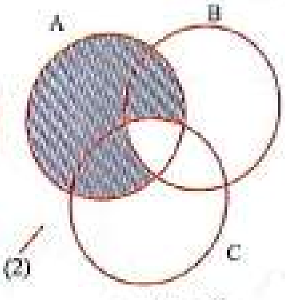
From (1) and (2), it is verified that $A-(B\quad C) = (A-B)\quad (A-C)$

7. **Verify $A-(B \cap C) = (A - B) \cup (A - C)$ using Venn diagrams.**

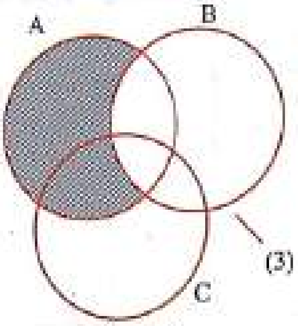
Sol.



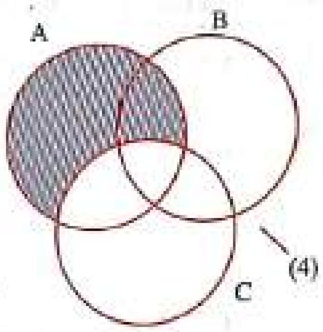
B∩C



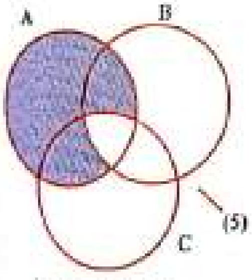
A − (B ∩ C)



(A − B)



(A − C)



(A − B) ∪ (A − C)

$\therefore A-(B \cap C) = (A - B) \cup (A - C)$

Hence it is proved.

8. If U = {4, 7, 8, 10, 11, 12, 15, 16}, A = {7, 8, 11, 12} and B = {4, 8, 12, 15}, then ve De Morgan's Laws for complementation.

Sol. $U = \{4, 7, 8, 10, 11, 12, 15, 16\}$

$A = \{7, 8, 11, 12\}, B = \{4, 8, 12, 15\}$

De Morgan's Laws for complementation.

$(A \cup B)' = A' \cap B'$

$A \cup B = \{4, 7, 8, 11, 12, 15\}$

$(A \cup B)' = \{4, 7, 8, 10, 11, 12, 15, 16\} - \{4, 7, 8, 11, 12, 15\} = \{10, 16\}$ ...(

$A' = \{4, 10, 15, 16\}; \; B' = \{7, 10, 11, 16\}$

$A' \cap B' = \{10, 16\}$ ...(

From (1) and (2) it is verified that

$(A \cup B)' = A' \cap B'$.

9. If U = {x : −4 ≤ x ≤ 4, x ∈ ℤ}, A = {x : −4 < x ≤ 2, x ∈ ℤ} a B = {x : −2 ≤ x ≤ 3, x ∈ ℤ}, then verify De Morgan's laws for complementation.

Sol. $U = \{x : -4 \le x \le 4, x \in \mathbb{Z}\}, U = \{-4, -3, -2, -1, 0, 1, 2, 3, 4$

$A = \{x : -4 < x \le 2, x \in \mathbb{Z}\}, A = \{-3, -2, -1, 0, 1, 2\}$

$B = \{x : -2 \le x \le 3, x \in \mathbb{Z}\}, B = \{-2, -1, 0, 1, 2, 3\}$

$(A \cup B)' = A' \cap B'$

$A \cup B = \{-3, -2, -1, 0, 1, 2, 3\}$

$(A \cup B)' = \{-4, 4\}$ ...(

$A' = \{-4, 3, 4\}; \; B' = \{-4, -3, 4\}$

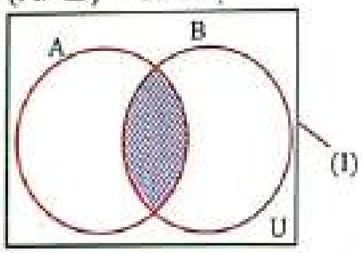
$A' \cap B' = \{-4, 4\}$ ...(

From (1) and (2) it is verified that

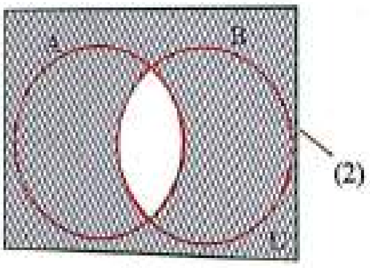
$(A \cup B)' = A' \cap B'$.

10. Verify (A∩B)′ = A′∪B′ using Venn diagrams.

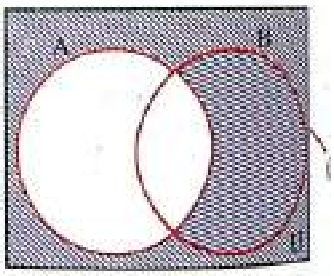
Sol. $(A \cap B)' = A' \cup B'$



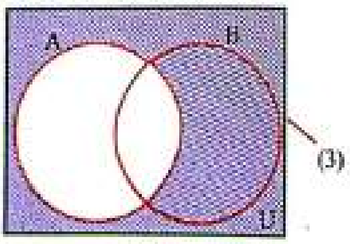
(A∩B)



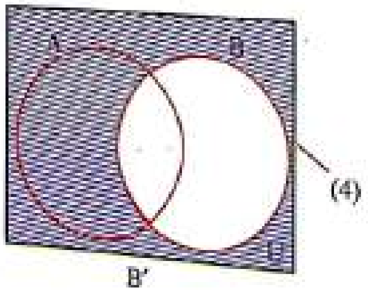
(A∩B)′



A′



A′



B′

$(2) = (5)$

$\therefore (A \cap B)' = A' \cup B'$

1.4 Cardinality and Practical Problems on Set Operations :

In the first term, we have learn to solve problems involving two sets using the formula $n(A\cup B) = n(A) + n(B) - n(A\cap B)$. Suppose we have three sets, we can apply this formula to get a similar formula for three sets.

For any three finite sets A, B and C

$$n(A\cup B\cup C) = n(A) + n(B) + n(C) - n(A\cap B) - n(B\cap C) - n(A\cap C) + n(A\cap B\cap C)$$

Exercise 1.4

1. **Verify $n(A\cup B\cup C) = n(A) + n(B) + n(C) - n(A\cap B) - n(B\cap C) - n(A\cap C) + n(A\cap B\cap C)$ for the following sets.**

(i) $A\{a, c, e, f, h\}$, $B = \{c, d, e, f\}$ and $C = \{a, b, c, f\}$

(ii) $A = \{1, 3, 5\}$ $B = \{2, 3, 5, 6\}$ and $C = \{1, 5, 6, 7\}$.

Sol

$$n(A\cup B\cup C) = n(A) + n(B) + n(C) - n(A\cap B) - n(B\cap C) - n(A\cap C) + n(A\cap B\cap C)$$

(i)

$$\begin{aligned}
A &= \{a, c, e, f, h\}, B = \{c, d, e, f\}, C = \{a, b, c, f\} \\
n(A) &= 5, n(B) = 4, n(C) = 4 \\
n(A\cap B) &= 3 \\
n(B\cap C) &= 2 \\
n(A\cap C) &= 3 \\
n(A\cap B\cap C) &= 2 \\
A\cap B &= \{c, e, f\} \\
B\cap C &= \{c, f\} \\
A\cap C &= \{a, c, f\} \\
A\cap B\cap C &= \{c, f\} \\
A\cup B\cup C &= \{a, c, d, e, f, b, h\} \\
\therefore n(A\cup B\cup C) &= 7 \qquad \ldots (1) \\
n(A) + n(B) + n(C) - n(A\cap B) - n(B\cap C) - n(A\cap C) + n(A\cap B\cap C) & \\
&= 5 + 4 + 4 - 3 - 2 - 3 + 2 = 15 - 8 = 7 \qquad \ldots (2) \\
(1) &= (2) \\
\Rightarrow n(A\cup B\cup C) &= n(A) + n(B) + n(C) - n(A\cap B) - n(B\cap C) - n(A\cap C) + n(A\cap B\cap C)
\end{aligned}$$

Hence it is verified.

(ii)

$$\begin{aligned}
A &= \{1, 3, 5\}, B = \{2, 3, 5, 6\}, C = \{1, 5, 6, 7\} \\
n(A) &= 3, n(B) = 4, n(C) = 4 \\
n(A\cap B) &= 2 \\
n(B\cap C) &= 2 \\
n(C\cap A) &= 2 \\
n(A\cap B\cap C) &= 1 \\
n(A\cup B\cup C) &= 6 \\
n(A\cup B\cup C) &= n(A) + n(B) + n(C) - n(A\cap B) - n(B\cap C) - n(A\cap C) + n(A\cap B\cap C) \\
6 &= 3 + 4 + 4 - 2 - 2 - 2 + 1 = 12 - 6 = 6
\end{aligned}$$

Hence it is verified.

2. **In a colony, 275 families buy Tamil newspaper, 150 fa newspa 45 families buy Hindi newspaper, 125 families buy Tamil and English newspap 17 families buy English and Hindi newspapers, 5 families buy Tamil and Hi newspapers and 3 families buy all the three newspapers. If each family buy atle one of these newspapers then find**

(i) Number of families buy only one newspaper

(ii) Number of families buy atleast two newspapers

(iii) Total number of families in the colony.

Sol. (i)

Tamil Newspaper buyers $n(A) = 275$

English Newspaper buyers $n(B) = 150$

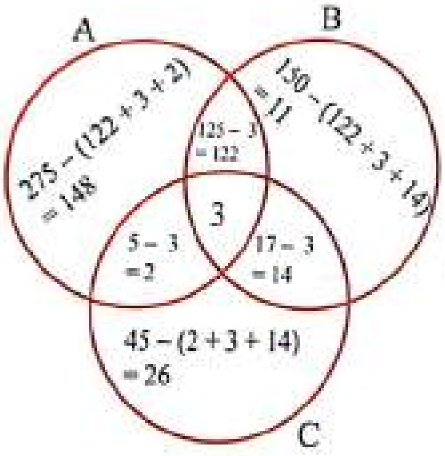
Hindi Newspaper buyers $n(C) = 45$

Tamil and English Newspaper buyers $n(A \cap B) = 125$

English and Hindi Newspaper buyers $n(B \cap C) = 17$

Hindi and Tamil Newspaper buyers $n(C \cap A) = 5$

All the three Newspaper buyers $n(A \cap B \cap C) = 3$



(i) Number of families buy only one newspaper = 148 + 11 + 26 = 185

(ii) Number of families buy atleast two news papers = 122 + 14 + 2 + 3 = 141

(iii) Total number of families in the colony = 148 + 11 + 26 + 122 + 14 + 2 + 3 =

3. **A soap company interviewed 800 people in a city. It was found out that $\frac{3}{8}$ brand A soap, $\frac{1}{5}$ use brand B soap, $\frac{1}{2}$ use brand C soap, 70 use brand A and soap, 55 use brand B and C soap, 60 use brand A and C soap and $\frac{1}{40}$ use all three brands. Find,**

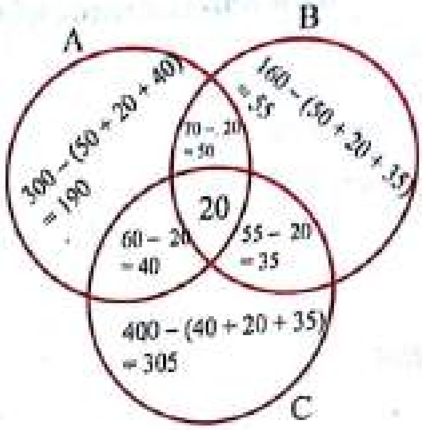
(i) Number of people who use exactly two branded soaps,

(ii) Number of people who use atleast one branded soap,

(iii) Number of people who do not use any one of these brands.

Sol.

$$\begin{aligned}
n(A) &= \overset{100}{\cancel{800}} \times \frac{3}{\cancel{8}} = 300 \\
n(B) &= \overset{160}{\cancel{800}} \times \frac{1}{\cancel{5}} = 160 \\
n(C) &= 800 \times \frac{1}{2} = 400 \\
n(A \cap B) &= 70 \\
n(B \cap C) &= 55 \\
n(C \cap A) &= 60 \\
n(A \cap B \cap C) &= \overset{20}{\cancel{800}} \times \frac{1}{\cancel{40}} = 20
\end{aligned}$$



(i) Number of people who use exactly two branded soaps

$$= 50 + 35 + 40 = 125$$

(ii) Number of people who use atleast one branded soaps

$$= 190 + 55 + 305 + 50 + 35 + 40 + 20 = 695$$

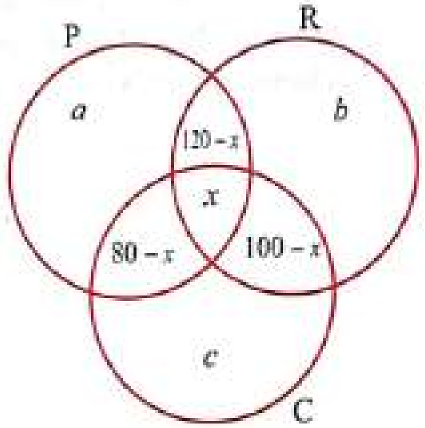
(iii) Number of people who do not use any of these brands

$$= 800 - [695] = 105$$

4. **A survey of 1000 farmers found that 600 grew paddy, 350 grew ragi, 280 grew corn, 120 grew paddy and ragi, 100 grew ragi and corn, 80 grew paddy and corn. If each farmer grew atleast any one of the above three, then find the number of farmers who grew all the three.**

Sol.

$$\begin{aligned}
a &= 600 - (120 - x + x + 80 - x) \\
&= 600 - (200 - x) \\
&= 600 - 200 + x = 400 + x \\
b &= 350 - (120 - x + x + 100 - x) \\
&= 350 - (220 - x) \\
&= 350 - 230 + x = 130 + x \\
c &= 280 - (80 - x + x + 100 - x) \\
&= 2800 - (180 - x) \\
&= 280 - 180 + x = 100 + x
\end{aligned}$$



Each farmer grew atleast one of the above three, the number of farmers who grew all the three is x.

$$\begin{aligned}
&= a + b + c + 120 - x + 100 - x + 80 - x + x = 1000 \\
&\quad 400 + x + 130 + x + 100 + x + 120 - x + 100 - x + 80 - x + x = 1000 \\
\therefore 930 + x &= 1000 \\
x &= 1000 - 930 = 70
\end{aligned}$$

∴ 70 farmers grew all the three crops

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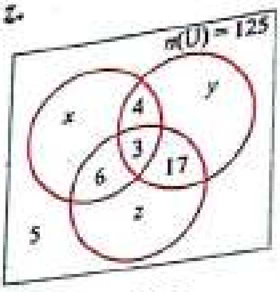
5. **In the adjacent diagram, if n(U) = 125, y is two times of x and z is 10 more th then find the value of x, y and z.**



Sol.

$$\begin{aligned}
n(U) &= 125\\
y &= 2x\\
z &= x + 10\\
\therefore x+y+z+4+17+6+3+5 &= 125\\
x+2x+x+10+35 &= 125\\
4x+45 &= 125\\
4x &= 125-45\\
4x &= 80\\
x &= 20\\
\therefore y &= 2x = 2\times 20 = 40\\
z &= x+10 = 20+10 = 30\\
\text{Hence } x &= 20; y = 40; z = 30
\end{aligned}$$

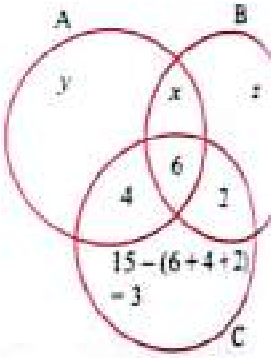
6. **Each student in a class of 35 plays atleast one game among chess, carrom a table tennis. 22 play chess, 21 play carrom, 15 play table tennis, 10 play chess a table tennis, 8 play carrom and table tennis and 6 play all the three games. Fi the number of students who play (i) chess and carrom but not table tennis (ii) o chess (iii) only carrom (Hint: Use Venn diagram)**

Sol.

A – Chess

B – Carrom

C – Table Tennis



$$\begin{aligned}
n(A) &= 22\\
n(B) &= 21\\
n(C) &= 15\\
n(A\cap C) &= 10\\
n(B\cap C) &= 8\\
n(A\cap B\cap C) &= 6
\end{aligned}$$

(i)

$$\begin{aligned}
y &= 22-(x+6+4) = 22-(x+10)\\
&= 22-x-10 = 12-x\\
z &= 21-(x+6+2) = 21-(8+x)\\
&= 21-8-x = 13-x\\
y+z+3+x+2+4+6 &= 35\\
12-x+13-\cancel{x}+15+\cancel{x} &= 35;\ 40-x = 35\\
x &= 40-35 = 5
\end{aligned}$$

(i) Number of students who pay only chess and Carrom but not table tennis = 5

(ii) Number of students who play only chess = $12 - x = 12 - 5 = 7$

(iii) Number of students who play only carrom = $13 - x = 13 - 5 = 8$

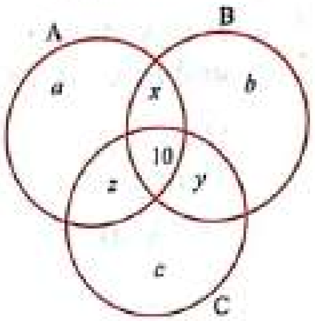
7. **In a class of 50 students, each one come to school by bus or by bicycle or on foot. 25 by bus, 20 by bicycle, 30 on foot and 10 students by all the three. Now how many students come to school exactly by two modes of transport?**

Sol.

A – by bus

B – by bicycle

C – on foot



$n(A) = 25$

$n(B) = 20$

$n(C) = 30$

$n(A \cap B \cap C) = 10$

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$$

$$50 = 25 + 20 + 30 - (10 + x) - (10 + y) - (10 + z) + 10$$

$$50 = 75 - \cancel{10} - x - 10 - y - 10 - z \cancel{+10} = 75 - 20 - (x + y + z)$$

$$= 55 - (x + y + z)$$

$$x + y + z = 55 - 50 = 5$$

∴The number of students who come to school exactly by two modes of transport = 5.

Exercise 1.5

MULTIPLE CHOICE QUESTIONS :

1. **If U = $\{x : x \in \mathbb{N}$ and $x < 10\}$, A = $\{1, 2, 3, 5, 8\}$ and B = $\{2, 5, 6, 7, 9\}$, then $n[(A \cup B)']$ is**

(1) 1 (2) 2 (3) 4 (4) 8

Hint : $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$A = \{1, 2, 3, 5, 8\}$

$B = \{2, 5, 6, 7, 9\}$

$A \cup B = \{1, 2, 3, 5, 6, 7, 8, 9\}$

$(A \cup B)' = \{4\}$,

$n(A \cup B)' = 1$

[Ans. (1) 1]

2. **For any three sets P, Q and R, P−(Q∩R) is**

(1) $P - (Q \cup R)$ (2) $(P \cap Q) - R$

(3) $(P - Q) \cup (P - R)$ (4) $(P - Q) \cap (P - R)$

Hint : $P - (Q \cap R) = (P - Q) \cup (P - R)$

[Ans. (3) (P − Q)∪(P − R)]

3. **Which of the following is true?**

(1) $A - B = A \cap B$ (2) $A - B = B - A$

(3) $(A \cup B)' = A' \cup B'$ (4) $(A \cap B)' = A' \cup B'$

Hint : (1) $(A - B) = A \cap B$ ×

(2) $A - B = B - A$ ×

(3) $(A \cup B)' = A' \cup B'$ ×

(4) $(A \cap B)' = A' \cup B'$ ✓

[Ans. (4) $(A \cap B)' = A'\cup$

4. **If $n(A \cup B \cup C) = 40$, $n(A) = 30$, $n(B) = 25$, $n(C) = 20$, $n(A \cap B) = 12$, $n(B \cap C)$, and $n(A \cap C) = 15$, then $n(A \cap B \cap C)$ is**

(1) 5 (2) 10 (3) 15 (4) 20

Hint :

$$\begin{aligned} n(A\cap B\cap C) &= n(A\cup B\cup C) - n(A) - n(B) - n(C) + n(A\cap B) + n(B\cap C) + n(C\cap A) \\ &= 40 - 30 - 25 - 20 + 12 + 18 + 15 \\ &= 87 - 75 = 10 \end{aligned}$$

[Ans. (2)

5. **If $n(A \cup B \cup C) = 100$, $n(A) = 4x$, $n(B) = 6x$, $n(C) = 5x$, $n(A \cap B) = 20$, $n(B \cap C)=$ $n(A \cap C) = 25$ and $n(A \cap B \cap C) = 10$, then the value of x is**

(1) 10 (2) 15 (3) 25 (4) 30

Hint :

$$\begin{aligned} n(A\cup B\cup C) &= n(A) + n(B) + n(C) - n(A\cap B) - n(B\cap C) - n(C\cap A) + n(A\cap B\cap C) \\ 100 &= 4x + 6x + 5x - 20 - 15 - 25 + 10 \\ 100 &= 15x - 60 + 10 \\ 100 &= 15x - 50 \\ \therefore 15x &= 100 + 50 = 150 \\ x &= 10 \end{aligned}$$

[Ans. (1)

6. **For any three sets A, B and C, $(A - B) \cap (B - C)$ is equal to**

(1) A only (2) B only (3) C only (4) ϕ

Hint : $(A - B) \cap (B - C)$ is equal to ϕ

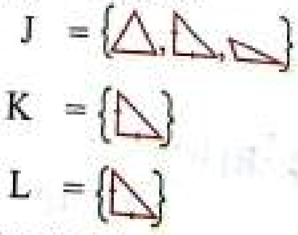
[Ans. (4

7. **If J = Set of three sided shapes, K = Set of shapes with two equal sides and L = of shapes with right angle, then $J \cap K \cap L$ is**

(1) Set of isoceles triangles (2) Set of equilateral triangles

(3) Set of isoceles right triangles (4) Set of right angled triangles

Hint :

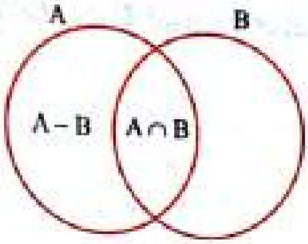


[Ans. (3) Set of isoceles right triang

8. **If A and B are two non-empty sets, then $(A - B) \cup (A \cap B)$ is**

(1) A (2) B (3) ϕ (4) U

Hint : $(A - B) \cup (A \cap B)$ is

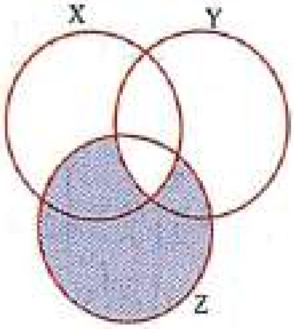


[Ans. (1) A]

9. **The shaded region in the Venn diagram is**

(1) $Z-(X \cup Y)$ (2) $(X \cup Y) \cap Z$ (3) $Z-(X \cap Y)$ (4) $Z \cup (X \cap Y)$

Hint : $Z-(X \cap Y)$



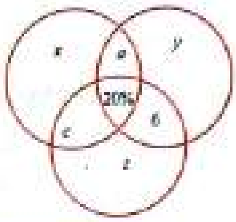
[Ans. (3) $Z-(X \cap Y)$]

10. **In a city, 40% people like only one fruit, 35% people like only two fruits, 20% people like all the three fruits. How many percentage of people do not like any one of the above three fruits?**

(1) 5 (2) 8 (3) 10 (4) 15

Hint :

$$40+35+20+x = 100\%$$
$$95\%+x = 100\%$$
$$x = 5\%$$



[Ans. (1) 5]

Additional Questions and Answers

EXERCISE 1.1

1. **If A = {2, 5, 6, 7} and B = {3, 5, 7, 8}, then verify the commutative property of**

(i) union of sets (ii) intersection of sets

Sol. Given, A = {2, 5, 6, 7} and B = {3, 5, 7, 8}

(i) $A \cup B = \{2, 3, 5, 6, 7, 8\}$ ... (1)

$B \cup A = \{2, 3, 5, 6, 7, 8\}$ ... (2)

From (1) and (2) we have $A \cup B = B \cup A$

It is verified that union of sets is commutative.

(ii) $A \cap B = \{5, 7\}$ ... (3)

$B \cap A = \{5,7\}$ ... (4)

From (3) and (4) we get, $A \cap B = B \cap A$

It is verified that intersection of sets is commutative.

2. **If A = {b, c, d, e} and B = {b, c, e, g} and C = {a, c, e}, then verify A∪(B∪C) = (A∪B)∪C.**

Sol.

$$
\begin{aligned}
\text{Given, } A &= \{b, c, d, e\} \text{ and } B = \{b, c, e, g\} \text{ and } C = \{a, c, e\}\\
\text{Now } B \cup C &= \{a, b, c, e, g\}\\
A \cup (B \cup C) &= \{a, b, c, d, e, g\} & \dots (1)\\
\text{Then, } A \cup B &= \{b, c, d, e, g\}\\
(A \cup B) \cup C &= \{a, b, c, d, e, g\} & \dots (2)
\end{aligned}
$$

From (1) and (2) it is verified that

$$A \cup (B \cup C) = (A \cup B) \cup C$$

3. **If** $A = \left\{\frac{1}{2}, 1, \frac{5}{4}, \frac{7}{4}, 3\right\}$, $B = \left\{1, \frac{5}{4}, \frac{7}{4}, 3, \frac{7}{2}\right\}$ **and** $C = \left\{\frac{1}{2}, \frac{5}{4}, 2, 3, \frac{7}{2}\right\}$, **then verify A∩(B∩C) = (A∩B)∩C.**

Sol.

$$
\begin{aligned}
\text{Now } B \cap C &= \left\{1, \underline{\tfrac{5}{4}}, \tfrac{7}{4}, \underline{3}, \underline{\tfrac{7}{2}}\right\} \cap = \left\{\tfrac{1}{2}, \underline{\tfrac{5}{4}}, 2, \underline{3}, \underline{\tfrac{7}{2}}\right\} = \left\{3, \tfrac{7}{2}, \tfrac{5}{4}\right\}\\
A \cap (B \cap C) &= \left\{\tfrac{1}{2}, 1, \underline{\tfrac{5}{4}}, \tfrac{7}{4}, \underline{3}\right\} \cap \left\{\underline{3}, \tfrac{7}{2}, \tfrac{5}{4}\right\} = \left\{\tfrac{5}{4}, 3\right\} & \dots (1)\\
\text{Then } A \cap B &= \left\{\tfrac{1}{2}, \underline{1}, \underline{\tfrac{5}{4}}, \tfrac{7}{4}, \underline{3}\right\} \cap \left[\underline{1}, \underline{\tfrac{5}{4}}, \underline{\tfrac{7}{4}}, \underline{3}, \tfrac{7}{2}\right] = \left\{1, \tfrac{5}{4}, \tfrac{7}{4}, 3\right\}\\
A \cap B \cap C &= \left\{1, \underline{\tfrac{5}{4}}, \tfrac{7}{4}, \underline{3}\right\} \cap \left\{\tfrac{1}{2}, \tfrac{5}{4}, 2, \underline{3}, \tfrac{7}{2}\right\} = \left\{\tfrac{5}{4}, 3\right\} & \dots (2)
\end{aligned}
$$

From (1) and (2) it is verified that

$$(A \cap B) \cap C = A \cap (B \cap C)$$

4. **State Associative property of sets.**

Sol. For any three sets A, B and C

(i) $A \cap (B \cap C) = (A \cap B) \cap C$

(ii) $A \cup (B \cup C) = (A \cup B) \cup C$

EXERCISE 1.2

1. **If A = {1, 3, 5, 7, 9}, B = {x ; x is a composite number and x < 12}** **C = {x : x ∈ N and 6 < x < 10} then verify A ∪ (B ∩ C) = (A ∪B) ∩ (A ∪C).**

Sol.

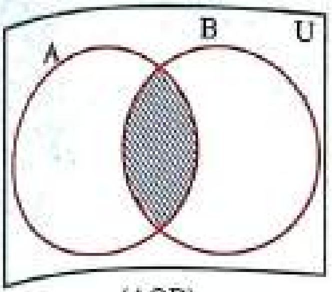
$$
\begin{aligned}
\text{Given, } A &= \{1, 3, 5, 7, 9\} \text{ and } B = \{4, 6, 8, 9, 10\} \text{and } C = \{6, 7, 8, 9\}\\
B \cap C &= \{4, \underline{6}, \underline{8}, \underline{9}, 10\} \cap \{\underline{6}, 7, \underline{8}, \underline{9}\} = \{6, 8, 9\}\\
A \cup (B \cap C) &= \{1, 3, 5, 6, 7, 8, 9\} & \dots (1)\\
\text{Then } (A \cup B) &= \{1, 3, 5, 7, 9\} \cup \{4, 6, 8, 9, 10\} = \{1, 3, 4, 5, 6, 7, 8, 9, 10\}\\
(A \cup C) &= \{1, 3, 5, 7, 9\} \cup \{6, 7, 8, 9\} = \{1, 3, 5, 6, 7, 8, 9\}\\
(A \cup B) \cap (A \cup C) &= \{\underline{1}, \underline{3}, 4, \underline{5}, \underline{6}, \underline{7}, \underline{8}, \underline{9}, 10\} \cap \{\underline{1}, \underline{3}, \underline{5}, \underline{6}, \underline{7}, \underline{8}, \underline{9}\}\\
&= \{1, 3, 5, 6, 7, 8, 9\} & \dots (2)
\end{aligned}
$$

From (1) and (2), it is verified that

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

2. **If A, B overlapping sets, draw venn diagram for : $A \cap B$**

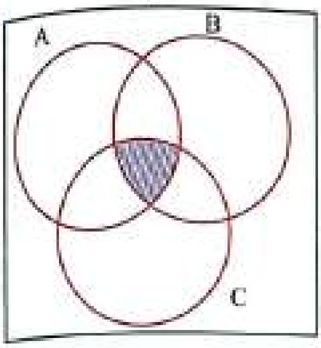
Sol.



(A∩B)

3. **Draw Venn diagram for $A \cap B \cap C$**

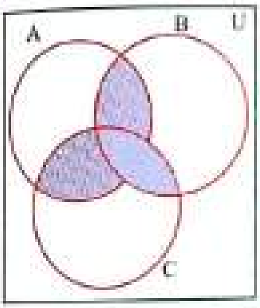
Sol.



$A \cap B \cap C$

4. **Draw Venn diagram for $(A \cap B) \cup (B \cap C) \cup (C \cap A)$**

Sol.



$(A \cap B) \cup (B \cap C) \cup (C \cap A)$

EXERCISE 1.3

1. **If $P = \{x : x \in \mathbb{N}$ and $1 < x < 11\}$, $Q = \{x : x = 2n, n \in \mathbb{N}$ and $n < 6\}$ and $R = \{4, 6, 8, 9, 10, 12\}$, then verify $P - (Q \cap R) = (P - Q) \cup (P - R)$**

Sol. The roster form of sets P, Q and R are $P = \{2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $Q = \{2, 4, 6, 8, 10\}$ and $R = \{4, 6, 8, 9, 10, 12\}$

First, we find $Q \cap R = \{4, 6, 8, 10\}$

Then, $P - (Q \cap R) = \{2, 3, 5, 7, 9\}$ ... (1)

Next, $P - Q = \{3, 5, 7, 9\}$

and $P - R = \{2, 3, 5, 7\}$

and so, $(P - Q) \cup (P - Q)$

$= \{2, 3, 5, 7, 9\}$ ... (2)

Hence from (1) and (2), it verified that $P - (Q \cap R) = (P - Q) \cup (P - R)$

Finding the elements of set Q

$$\begin{aligned} \text{Given, } x &= 2n \\ n = 1 \rightarrow \quad x &= 2(1) = 2 \\ n = 2 \rightarrow \quad x &= 2(2) = 4 \\ n = 3 \rightarrow \quad x &= 2(3) = 6 \\ n = 4 \rightarrow \quad x &= 2(4) = 8 \\ n = 5 \rightarrow \quad x &= 2(5) = 10 \end{aligned}$$

Therefore, x takes values such as 2, 4, 6, 8, 10

2. **If $U = \{x : x \in \mathbb{Z}, -3 \leq x \leq 9\}$, $A = \{x : x = 2P + 1, P \in \mathbb{Z}, -2 \leq P \leq$ $B = \{x : x = q + 1, q \in \mathbb{Z}, 0 \leq q \leq 3\}$, verify De Morgan's laws for complementati**

Sol.

$$\begin{aligned} \text{Given, } U &= \{-3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9\} \\ A &= \{-3, -1, 1, 3, 5, 7\} \\ \text{and } B &= \{1, 2, 3, 4\} \\ \text{Law (i) } (A \cup B)' &= A' \cap B' \\ \text{Now, } A \cup B &= \{-3, -1, 1, 2, 3, 4, 5, 7\} \\ (A \cup B)' &= \{-2, 0, 6, 8, 9\} \qquad \ldots (1) \\ \text{Then, } A' &= \{-2, 0, 2, 4, 6, 8, 9\} \text{ and} \\ B' &= \{-3, -2, -1, 0, 5, 6, 7, 8, 9\} \\ A' \cap B' &= \{-2, 0, 6, 8, 9\} \qquad \ldots (2) \end{aligned}$$

From (1) and (2) it is verified that

$$\begin{aligned} (A \cup B)' &= A' \cap B' \\ \text{Law (ii) } (A \cap B)' &= A' \cup B' \\ \text{Now, } A \cap B &= \{1, 3\} \\ (A \cap B)' &= \{-3, -2, -1, 0, 2, 4, 5, 6, 7, 8, 9\} \qquad \ldots (3) \\ \text{Then, } A' \cup B' &= \{-3, -2, -1, 0, 2, 4, 5, 6, 7, 8, 9\} \qquad \ldots (4) \end{aligned}$$

From (3) and (4) it is verified that

$$(A \cap B)' = A' \cup B'$$

3. **State De Morgan's laws for set difference.**

Sol. For any three sets, A, B and C

(i) $A - (B \cup C) = (A - B) \cap (A - C)$

(ii) $A - (B \cap C) = (A - B) \cup (A - C)$

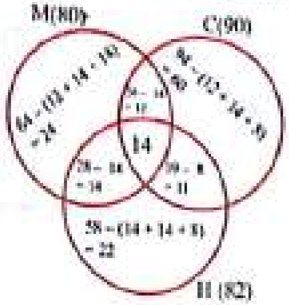
EXERCISE 1.4

1. **In a school, 80 students like Maths, 90 students like Science, 82 students History, 21 like both Maths and Science, 19 like both Science and History like both Maths and History and 8 liked all the three subjects. If each stude like atleast one subject, then find (i) the number of students in the school (ii) number of students who like only one subject.**

Sol. Let M, S and H represent sets of students who like Maths, Science and Histo respectively.

Then, $n(M) = 80$, $n(S) = 90$, $n(H) = 82$, $n(M \cap S) = 21$, $n(S \cap H) =$ $n(M \cap H) = 20$, $n(M \cap S \cap H) = 8$

Let us represents the given data in a venn diagram.



(i) The number of student in the school = 52 + 59 + 55 + 12 + 11 + 8 + 8 = 205

(ii) The number of students who like only one subject = 52 + 59 + 55 = 166

2. State the formula to find $n(A \cup B \cup C)$.

Sol. $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - (B \cap C) - n(A \cap C) + (A \cap B \cap B)$

3. Verify $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - (B \cap C) - n(A \cap C) + (A \cap B \cap C)$ for the following sets A ={1, 3, 5, 6, 8}, B = {3, 4, 5, 6} and C = {1, 2, 3, 6}

Sol. $(A \cup B \cup C) = \{1, 2, 3, 4, 5, 6, 8\}$

$\therefore n(A \cup B \cup C) = 7$

Also, $n(A) = 5, n(B) = 4, n(C) = 4,$

Further, $A \cap B = \{3, 5, 6\} \Rightarrow n(A \cap B) = 3$

$B \cap C = \{3, 6\} \Rightarrow n(B \cap C) = 2$

$A \cap C = \{3, 5, 6\} \Rightarrow n(A \cap C) = 3$

Also, $A \cap B \cap C = \{3, 6\} \Rightarrow n(A \cap B \cap C) = 2$

Now, $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - (B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

$$7 = 5 + 4 + 4 - 3 - 2 - 3 + 2$$
$$7 = 13 - 8 + 2$$
$$7 = 5 + 2$$
$$7 = 7$$

Thus verified

EXERCISE 1.5

MULTIPLE CHOICE QUESTIONS :

1. For any three A, B and C, A – (B ∪ C) is

(1) $(A - B) \cup (A - C)$

(2) $(A - B) \cap (A \cup C)$

(3) $(A - B) \cup C$

(4) $A \cup (B - C)$

[Ans. (2) $(A - B) \cap (A - C)$]

2. Which of the following is true?

(1) $(A \cup B) = B \cup A$

(2) $(A \cup B)' = A' - B'$

(3) $(A \cap B)' = A' \cap B'$

(4) $A - (B \cap C) = (A - B) \cap (A - C)$

[Ans. (1) $(A \cup B) = B \cup A$]

3. **The shaded region in the venn diag**

(1) $A \cup B$

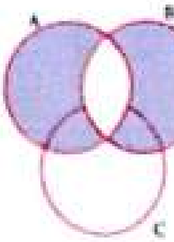
(2) $A \cap B$

(3) $(A \cap B)'$

(4) $(A - B) \cup (B - A)$

Hint :

[Ans. (4) $A - B \cup B$



4. **If A, B, C are non-overlapping sets, then $n(A \cap B \cap C)$ is ________**

(1) $n(A) + n(B) + n(C)$

(2) $n(A \cup B \cup C)$

(3) 0

(4) $n(A \cap B)$

[Ans. (3)

5. **If $U = \{x : x \in W$ and $x < 20\}$, $A = \{2, 4, 6, 8\}$, $B = \{6, 8, 12, 14\}$ then $[n(A \cup B)$**

(1) 14 (2) 6 (3) 8 (4) 2

[Ans. (1)

6. **In a town 30% like only coffee, 20% like only tea, 10% like only milk, 15% only any two of them, 5% only like all the three. What is the percentage of who like none them.**

(1) 15% (2) 20% (3) 10% (4) 5%

[Ans. (2) 20%

7. **$[n(A \cup B \cup C)'] =$ ________**

(1) $n(A \cap B \cap C)$

(2) $n(U) - n(A \cup B \cup C)$

(3) $n(U)$

(4) ϕ

[Ans. (2) $n(U) - n(A \cup B \cup$

8. **$(A \cup B \cup C) \cup (A \cup B \cup C)' =$ ________**

(1) ϕ (2) A (3) U (4) $A \cap B \cap$

[Ans. (3)

9. **$n(A \cup B \cup C) =$ _______**

(1) $n(A) + n(B) + n(C)$

(2) $n(A) + n(B) + n(C) - n(A \cap B \cap$

(3) $n(A \cap B \cap C)$

(4) $n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap$

[Ans. (4) $n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap$

10. **For any three sets, P, Q, R $(P \cap Q)'$**

(1) $P' \cup Q'$ (2) $P \cup Q$ (3) P' (4) Q'

[Ans. (1) $P' \cup$

* * *