UNIT – 06 GRAVITATION

TWO MARKS AND THREE MARKS:

01. State Kepler's three laws.

1. **Law of orbits:** Each planet moves around the Sun in an elliptical orbit with the Sun at one of the foci.
2. **Law of area:**
   The radial vector (line joining the Sun to a planet) sweeps equal areas in equal intervals of time
3. **Law of period:**
   The square of the time period of revolution of a planet around the Sun in its Elliptical orbit is directly proportional to the cube of the semi-major axis of The ellipse.

02. State Newton's Universal law of gravitation.

Newton's law of gravitation states that a particle of mass M_1 attracts any other particle of mass M_2 in the universe with an attractive force. The strength of this force of attraction was found to be directly proportional to the product of their masses and is inversely proportional to the square of the distance between them.

03. Will the angular momentum of a planet be conserved? Justify your answer.

Yes, Because $\vec{\tau} = \vec{r} \times \vec{F}$; $\vec{r} \times \left(\frac{GM_S M_E}{r^2}\hat{r}\right) = 0$

Since $\vec{r} = r\,\hat{r}$, $(\hat{r} \times \hat{r}) = 0$ So, $\vec{\tau} = \frac{d\vec{L}}{dt} = 0$

It implies that angular momentum is a constant vector. The angular momentum of the Earth about the Sun is constant throughout the motion.

04. Define the gravitational field. Give its unit.

The gravitational force experienced by unit mass placed at that point.

Unit $\vec{E}_1 = \frac{\vec{F}21}{m_2}$ in equation we get, $\vec{E} = -\frac{Gm_1}{r^2}\hat{r}$. its unit is N kg^{-1} (or) m s^{-2} .

05. What is meant by superposition of gravitational field?

Consider 'n' particles of masses, m_1 , m_2 ...m_n distributed in space at positions $\hat{r}_1, \hat{r}_2, \hat{r}_3$ ....etc, with respect to point P. The total gravitational field at a point P due to all the masses is given by the vector sum of the gravitational field due to the individual masses. This principle is known as superposition of gravitational fields.

$$\vec{E}_{total} = \vec{E}_1 + \vec{E}_2 + ...\vec{E}_n$$

$$= -\frac{Gm_1}{r_1^2}\vec{r}_1 - \frac{Gm_2}{r_2^2}\vec{r}_2 - .... \frac{Gm_n}{r_n^2}\vec{r}_n \; ; = -\sum_{i=1}^{n}\frac{Gm_i}{r_i^2}\vec{r}_i$$

06. Define gravitational potential energy.

Gravitational potential energy associated with this conservative force field. The gravitational potential energy is defined as the work done to bring the mass m_2 from infinity to a distance 'r' in the gravitational field of mass m_1. Its unit is joule.

07. Is potential energy the property of a single object? Justify.

Potential energy is a property of a system rational than of a single object due to its physical position. Because gravitational potential energy depends on relative position. So, a reference level at which to set the potential energy equal to zero.

08. Define gravitational potential.

The gravitational potential at a distance r due to a mass is defined as the amount of work required to bring unit mass from infinity to the distance r.

09. What is the difference between gravitational potential and gravitational potential energy?

Gravitational potential:

The amount of work done in bringing a body of unit mass from infinity to that point without acceleration. $V = -\frac{GM}{R}$

Gravitational potential Energy:

The energy stored in the body at that point. If the position of the body changes due to force acting on it, then change in its potential energy is equal to the amount of work done on the body by the forces acting on it. $U = -\frac{GMm}{R}$

10. What is meant by escape speed in the case of the Earth?

The minimum speed required by an object to escape from Earth's gravitational field.

ie. $V_e = \sqrt{2gR_E}$; $V_e = 11.2\ kms^{-1}$

11. Why is the energy of a satellite (or any other planet) negative?

The negative sign in the total energy implies that the satellite is bound to the Earth and it cannot escape from the Earth.

As h approaches, ∞ the total energy tends to zero. Its physical meaning is that the satellite is completely free from the influence of Earth's gravity and is not bound to Earth at large distances.

12. What are geostationary and polar satellites?

Geostationary satellites:

The satellites revolving the Earth at the height of 36000 km above the equator, are appear to be stationary when seen from Earth is called geo-stationary satellites.

Polar satellites:

The satellites which revolve from north to south of the Earth at the height of 500 to 800 km from the Earth surface are called Polar satellites.

13. Define weight

The weight of an object is defined as the downward force whose magnitude W is equal to that of upward force that must be applied to the object to hold it at rest or at constant velocity relative to the earth. The magnitude of weight of an object is denoted as, W=N=mg.

14. Why is there no lunar eclipse and solar eclipse every month?

Moon's orbit is tilted 5^0 with respect to Earth's orbit, only during certain periods of the year; the Sun, Earth and Moon align in straight line leading to either lunar eclipse or solar eclipse depending on the alignment.

15. How will you prove that Earth itself is spinning?

The Earth's spinning motion can be proved by observing star's position over a night. Due to Earth's spinning motion, the stars in sky appear to move in circular motion about the pole star.

16. What is meant by state of weightlessness?

When downward acceleration of the object is equal to the acceleration due to the gravity of the Earth, the object appears to be weightless

17. Why do we have seasons on Earth?

The seasons in the Earth arise due to the rotation of Earth around the Sun with 23.5° tilt. Due to this 23.5° tilt, when the northern part of Earth is farther to the Sun, the southern part is nearer to the Sun. So when it is summer in the northern hemisphere, the southern hemisphere experience winter.

18. Water falls from the top of a hill to the ground. Why?

This is because the top of the hill is a point of higher gravitational potential than the surface of the Earth. i.e. $V_{hill} > V_{ground}$.

19. What is the effect of rotation of the earth on the acceleration due to gravity?

The acceleration due to gravity decreases due to rotation of the earth. This effect is zero at poles and maximum at the equator.

20. A satellite does not need any fuel to aide around the earth. Why?

The gravitational force between satellite and earth provides the centripetal force required by the satellite to move in a circular orbit.

21. Why does a tide arise in the ocean?

Tides arise in the ocean due to the force of attraction between the moon and sea water.

22. Water falls from the top of a hill to the ground. Why?

This is because the top of the hill is a point of higher gravitational potential than the surface of the Earth. i.e. $V_{hill} > V_{ground}$.

23. When a man is standing in the elevator, what are forces acting on him.

1. Gravitational force which acts downward. If we take the vertical direction as positive y direction, the gravitational force acting on the man is $\vec{F}_G = -mg\hat{j}$
2. The normal force exerted by floor on the man which acts vertically upward, $\vec{N} = N\hat{j}$

24. Find the distance between Venus and Sun.

1) The distance between Venus and Sun. The distance between Earth and Sun is taken as one Astronomical unit (1 AU).
2) The trigonometric relation satisfied by this right angled triangle is $\sin\theta = \frac{r}{R}$
3) Where R = 1 AU. r = R $\sin\theta$ = (1 AU) ($\sin 46^0$). Here $\sin.46^0 = 0.77$. Hence Venus is at a distance of 0.77 AU from Sun.

25. Find the expression of the orbital speed of satellite revolving around the earth.

Satellite of mass M to move in a circular orbit, centripetal force must be acting on the satellite. This centripetal force is provided by the Earth's gravitational force.

$$\frac{MV^2}{(R_E+h)} = \frac{GMM_E}{(R_E+h)^2}$$

$$V^2 = \frac{GM_E}{(R_E+h)} \ ;$$

$$V = \sqrt{\frac{GM_E}{(R_E+h)}}$$

As h increases, the speed of the satellite decreases.

26. What are the points to be noted to study about gravitational field?

Case 1: If $r < r'$

Since gravitational force is attractive, m_2 is attracted by m_1.Then m_2 can move from r' to r without any external work. Here work is done by the system spending its internal energy and hence the work done is said to be negative.

Case 2: If $r > r'$

Work has to be done against gravity to move the object from r' to r. Therefore work is done on the body by external force and hence work done is positive.

27. What is meant by retrograde motion of planet?

1) The planets move eastwards and reverse their motion for a while and return to eastward motion again. This is called **"retrograde motion"** of planets.

2) To explain this retrograde motion, Ptolemy introduced the concept of "epicycle" in his geocentric model. According to this theory, while the planet orbited the Earth, it also underwent another circular motion termed as "epicycle". A combination of epicycle and circular motion around the Earth gave rise to retrograde motion of the planets with respect to Earth.

CONCEPTUAL QUESTIONS

01. In the following, what are the quantities which that are conserved?

a) Linear momentum of planet b) Angular momentum of planet
c) Total energy of planet d) Potential energy of a planet

Ans. (b & d) Angular momentum of planet, Potential energy of a Planet.

02. The work done by Sun on Earth in one year will be

a) Zero b) Non zero c) positive d) negative Ans . : Zero

03. The work done by Sun on Earth at any finite interval of time is

a) positive, negative or zero b) Strictly positive
c) Strictly negative d) It is always zero

Ans. d) it is always zero

04. If a comet suddenly hits the Moon and imparts energy which is more than the total energy of the Moon, what will happen?

If a comet hits the moon with large mass and with large velocity may destroy the moon completely or its impact makes the moon, go out of the orbit.

05. If the Earth's pull on the Moon suddenly disappears, what will happen to the Moon?

If the gravitational force suddenly disappears, moon will stop revolving around the earth and it will move in a direction tangential to its original orbit with a speed with which it was revolving around the earth.

06. If the Earth has no tilt, what happens to the seasons of the Earth?

If the Earth has no tilt, there will be no season as like now and the duration of day and night will be equal throughout the year.

07. A student was asked a question 'why are there summer and winter for us? He replied as 'since Earth is orbiting in an elliptical orbit, when the Earth is very far away from the Sun(aphelion) there will be winter, when the Earth is nearer to the Sun(perihelion) there will be winter'. Is this answer correct? If not, what is the correct explanation for the occurrence of summer and winter?

No, The seasons in the Earth arise due to the rotation of Earth around the Sun with 23.5° tilt. Due to this 23.5° tilt, when the northern part of Earth is farther to the Sun, the southern part is nearer to the Sun. So when it is summer in the northern hemisphere, the southern hemisphere experience winter.

08. The following photographs are taken from the recent lunar eclipse which occurred on January 31, 2018. Is it possible to prove that Earth is a sphere from these photographs?

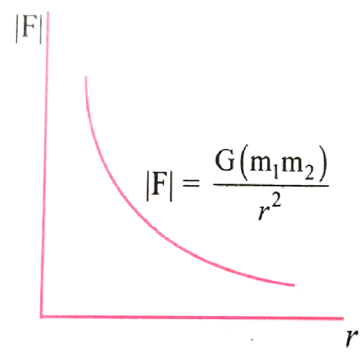
No. the moon goes around the earth in an elliptical orbit. This means its distance from us varies periodically as it goes around us.

FIVE MARKS:

01. Discuss the important features of the law of gravitation.

Important features of gravitational force:

1) As the distance between two masses increases, the strength of the force tends to decrease because of inverse dependence on r^2. Physically it implies that Ex. The planet Uranus experiences less gravitational force from the Sun than the Earth since Uranus is at larger distance from the Sun compared to the Earth.



2) The gravitational forces between two particles always constitute an action-reaction pair. It implies that the gravitational force exerted by the Sun on the Earth is always towards the Sun. The reaction-force is exerted by the Earth on the Sun. The direction of this reaction force is towards Earth.

3) The torque experienced by the Earth due to the gravitational force of the Sun is given by

$$\vec{\tau} = \vec{r} \times \vec{F} \,;\, \vec{r} \times \left(\frac{GM_SM_E}{r^2}\hat{r}\right) = 0$$

Since $\vec{r} = r\,\hat{r}$, $(\hat{r} \times \hat{r}) = 0$ So, $\vec{\tau} = \frac{d\vec{L}}{dt} = 0$

It implies that angular momentum is a constant vector. The angular momentum of the Earth about the Sun is constant throughout the motion

4) Earth orbits around the Sun due to Sun's gravitational force, we assumed Earth and Sun to be point masses. This assumption is a good approximation because the distance between the two bodies is very much larger than their diameters.

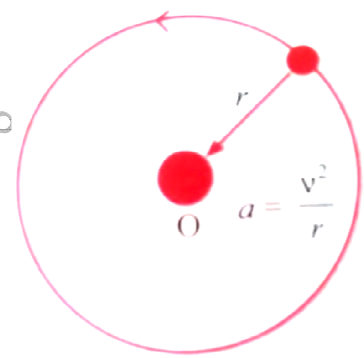
5) To calculate force of attraction between a hollow sphere of mass M with uniform density and point mass m kept outside the hollow sphere, we can replace the hollow sphere of mass M as equivalent to a point mass M located at the center of the hollow sphere.

6) If we place another object of mass 'm' inside this hollow sphere, the force experienced by this mass 'm' will be zero.

02. Explain how Newton arrived at his law of gravitation from Kepler's third law.

Newton's inverse square Law:

Newton considered the orbits of the planets as circular. For circular orbit of radius r, the centripetal acceleration towards the center is



$$a = \frac{v^2}{r} \quad \text{----------------------- 1}$$

Here v is the velocity and r, the distance of the planet from the center of the orbit. The velocity in terms of known quantities r and T, is

$$V = \frac{2\pi r}{T} \quad \text{-------------------- 2}$$

Here T is the time period of revolution of the planet. Substituting this value of v in equation (1) we get,

$$a = \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = -\frac{4\pi^2 r}{T^2} \quad \text{------- 3}$$

Substituting the value of 'a' from (3) in Newton's second law, F = ma, where 'm' is the mass of the planet.

$$F = \frac{4\pi m r}{T^2} \quad \text{--------------------- 4}$$

From Kepler's third law,

$$\frac{r^3}{T^2} = k \text{ (Constant)} \quad \text{----------- 5}$$

$$\frac{r}{T^2} = \frac{k}{r^2} \quad \text{------------------ 6}$$

By substituting equation 6 in the force expression, we can arrive at the law of gravitation.

$$F = \frac{4\pi^2 m k}{r^2} \quad \text{----------7}$$

Here negative sign implies that the force is attractive and it acts towards the center. In equation (7), mass of the planet 'm' comes explicitly. But Newton strongly felt that according to his third law, if Earth is attracted by the Sun, then the Sun must also be attracted by the Earth with the same magnitude of force. So he felt that the Sun's mass (M) should also occur explicitly in the expression for force. From this insight, he equated the constant $4\pi^2 k$ to GM which turned out to be the law of gravitation.

$$F = \frac{GMm}{r^2}$$

Again the negative sign in the above equation implies that the gravitational force is attractive.

03. Explain how Newton verified his law of gravitation.

1) Newton verified his law of universal gravitation by comparing the acceleration of a terrestrial object to the acceleration of the moon.

2) He knew that the distance from the center of earth to the center of two spheres of known mass at either end of a light rod suspended by a then fiber from the center of the rod.

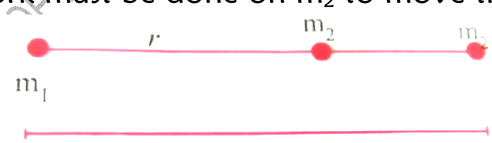
3) He had earlier found the small force that was needed to twist the fiber.

4) By bringing a third sphere close to one of the suspended spheres.

5) He was able to measure the force of gravity between the spheres and hence gravitation.

04. Derive the expression for gravitational potential energy.

1) Two masses m_1 and m_2 are initially separated by a distance r'. Assuming m_1 to be fixed in its position, work must be done on m_2 to move the distance from r' to r as shown in Figure (a)



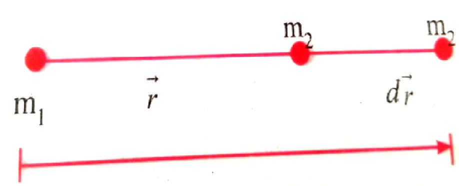
(a)

2) To move the mass m_2 through an infinitesimal displacement $d\vec{r}$ from r to $\vec{r} + d\vec{r}$ (shown in the Figure (b), work has to be done externally.

This infinitesimal work is given by

$dW = \vec{F}_{ext} . d\vec{r}$ --------------- 1

3) The work is done against the gravitational force, therefore,



(b)

$|\vec{F}_{ext}| = |\vec{F}_G| = \frac{Gm_1m_2}{r^2}$ ----------- 2

Substituting equation (2) in (1), we get

$dW = \frac{Gm_1m_2}{r^2}\hat{r} . d\vec{r}$; $d\vec{r} = dr\,\hat{r}$

$\frac{Gm_1m_2}{r^2}\hat{r} . d\vec{r}$; $\hat{r}.\hat{r} = 1$ (Since both are unit vectors)

$dW = \frac{Gm_1m_2}{r^2}\,dr$

4) Thus the total work done for displacing the particle from r' to r is $W = \int_{r'}^{r} dW = \int_{r'}^{r} \frac{Gm_1m_2}{r^2}\,dr$

$$W = -\left(\frac{Gm_1m_2}{r^2}\right)_{r'}^{r}$$

$$W = -\frac{Gm_1m_2}{r} + \frac{Gm_1m_2}{r'}$$

W = U (r) – U (r′)

Where U (r) $= \frac{Gm_1m_2}{r}$

5) This work done W gives the gravitational potential energy difference of the system of masses m_1 and m_2 when the separation between them are r and r′ respectively.

05. Prove that at points near the surface of the Earth, the gravitational potential energy of the object is U = mgh.

1) Consider the Earth and mass system, with *r*, the distance between the mass m and the Earth's centre. Then the gravitational potential energy,

$$U = -\frac{GM_em}{r} \quad \text{-----------------} \quad 1$$

2) Here r = Re+h, where Re is the radius of the Earth. h is the height above the Earth's surface, $U = -G\frac{M_em}{(R_e+h)}$ ------------- 2

If h << Re, equation (2) can be modified as

$$U = -G\frac{M_em}{R_e\left(1+\frac{h}{R_e}\right)} \; ; \quad U = -G\frac{M_em}{R_e}\left(1+\frac{h}{R_e}\right)^{-1} \quad \text{----------} \quad 3$$

3) By using Binomial expansion and neglecting the higher order terms, we get $U = -G\frac{M_em}{R_e}\left(1-\frac{h}{R_e}\right)$ ---------- 4

We know that, for a mass m on the Earth's surface,

$$G\frac{M_em}{R_e} = mgR_e \quad \text{----------------} \quad 5$$

Substituting equation (5) in (4) we get, $U = -mgR_e + mgh$

It is clear that the first term in the above expression is independent of the height h. For example, if the object is taken from h and it can be omitted.

U = mgh

06. Explain in detail the idea of weightlessness using lift as an example.

i) When the lift falls (when the lift wire cuts) with downward acceleration a = g, the person inside the elevator is in the state of weightlessness or free fall.

ii) As they fall freely, they are not in contact with any surface (by neglecting air friction). The normal force acting on the object is zero. The downward acceleration is equal to the acceleration due to the gravity of the Earth. i.e (a = g). From equation N = m (g − a) we get, a = g ∴ N = m (g – g) = 0. This is called the state of weightlessness.

07. Derive an expression for escape speed.

1) Consider an object of mass M on the surface of the Earth. When it is thrown up with an initial speed v_i , the initial total energy of the object is

$$E_i = \frac{1}{2} MV_i^2 - \frac{GMM_E}{R_E} \text{ ---------------- } 1$$

Where M_E, is the mass of the Earth and R_E - the radius of the Earth.

The term $-\frac{GMM_E}{R_E}$ is the potential energy of the mass M.

2) When the object reaches a height far away from Earth and hence treated as approaching infinity, the gravitational potential energy becomes zero [U (∞) = 0] and the kinetic energy becomes zero as well. Therefore the final total energy of the object becomes zero. This is for minimum energy and for minimum speed to escape. Otherwise Kinetic energy can be non-zero.

$E_f = 0$, According to the law of energy conservation, $E_i = E_f$ -------- 2

Substituting (1) in (2) we get,

$$\frac{1}{2} MV_i^2 - \frac{GMM_E}{R_E} = 0$$

$$\frac{1}{2} MV_i^2 = \frac{GMM_E}{R_E} \text{ ----------------- } 3$$

3) The escape speed, the minimum speed required by an object to escape Earth's gravitational field, hence replace, V_i with V_e. i.e,

$$\frac{1}{2} MV_e^2 = \frac{GMM_E}{R_E}$$

$$V_e^2 = \frac{GMM_E}{R_E} \cdot \frac{2}{M} \ ; \ V_e^2 = \frac{2GM_E}{R_E} \text{ ------------- } 4$$

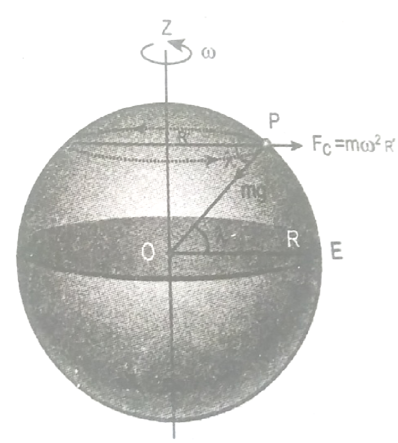
$$\text{Using } g = \frac{GM_E}{R_e} \text{ ---------------- } 5$$

$$V_e^2 = 2gR_E \ ; \ V_e = \sqrt{2gRE} \text{ -------------- } 6$$

From equation (6) the escape speed depends on two factors: acceleration due to gravity and radius of the Earth. It is completely independent of the mass of the object.

08. Explain the variation of g with latitude.

Variation of g with latitude:



Whenever we analyze the motion of objects in rotating frames, we must take into account the centrifugal force. Even though we treat the Earth as an inertial frame, it is not exactly correct because the Earth spins about its own axis. So when an object is on the surface of the Earth, it experiences a centrifugal force that depends on the latitude of the object on Earth. If the Earth were not spinning, the force on the object would have been *mg*. However, the object experiences an additional centrifugal force due to spinning of the Earth.

This centrifugal force is given by $m\omega^2 R'$

$R' = R\cos\lambda$ ------------ 1

Where λ is the latitude. The component of centrifugal acceleration experienced by the object in the direction opposite to g is $a_c = \omega^2 R' \cos\lambda$

$= \omega^2 R\cos^2\lambda$ since $R' = R\cos\lambda$ Therefore,

$g' = g - \omega^2 R\cos^2\lambda$ ----------- 2

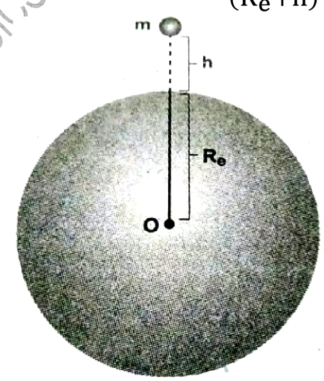
From the expression (2), we can infer that at equator, $\lambda = 0$; $g' = g - \omega^2 R$. The acceleration due to gravity is minimum. At poles λ = 90; $g' = g$, it is maximum. At the equator, g' is minimum.

09. Explain the variation of g with altitude.

Variation of g with altitude:

Consider an object of mass m at a height h from the surface of the Earth. Acceleration experienced by the object due to Earth is $g' = \frac{GM}{(R_e+h)^2}$

$$g' = \frac{GM}{R_e^2\left(1+\frac{h}{R_e}\right)^2} \quad ; \quad g' = \frac{GM}{R_e^2}\left(1+\frac{h}{R_e}\right)^{-2}$$



If h << R_e. We can use Binomial expansion.
Taking the terms upto first order

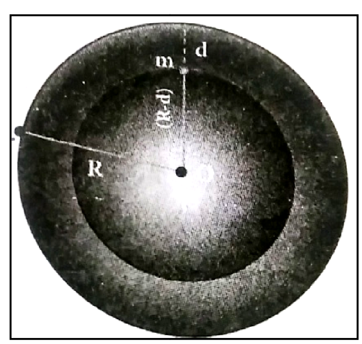
$$g' = \frac{GM}{R_e^2}\left(1 - 2\frac{h}{R_e}\right);$$

$$g' = g\left(1 - 2\frac{h}{R_e}\right)$$

We find that $g' < g$. This means that as altitude h increases the acceleration due to gravity g decreases.

10. Explain the variation of g with depth from the Earth's surface.

Variation of g with depth:



Consider a particle of mass m which is in a deep mine on the earth. Ex. Coal mines – in Neyveli). Assume the depth of the mine as d.
To Calculate g at a depth d, consider the following points. The part of the Earth which is above the radius (R_e – d) do not contribute to the acceleration. The result is proved earlier and is given as $g' = \frac{GM'}{(R_e-d)^2}$ Here M is the mass of the Earth of radius (R_e – d). Assuming the density of earth ρ to be constant,

$$\rho = \frac{M'}{V'} \; ; \; \frac{M'}{V'} = \frac{M}{V} \text{ and } M' = \frac{M}{V}V'$$

$M' = \left(\frac{M}{\frac{4}{3}\pi R_e^3}\right)\left(\frac{4}{3}\pi(R_e - d)^3\right)$;

$M' = \frac{M}{R_e^3}(R_e - d)^3$

$g' = G\frac{M}{R_e^3}(R_e - d)^3 \cdot \frac{1}{(R_e - d)^2}$;

$g' = GM\frac{R_e\left(1-\frac{d}{R_e}\right)}{R_e^3}$

$g' = GM\frac{\left(1-\frac{d}{R_e}\right)}{R_e^2}$ thus $g' = g\left(1-\frac{d}{R_e}\right)$. Here also $g' < g$.

As depth increases, g′ decreases.

11. Derive the time period of satellite orbiting the Earth.

Time period of the satellite:

The distance covered by the satellite during one rotation in its orbit is equal to 2π (R_E +h) and time taken for it is the time period, T. Then

$$\frac{\text{Distance travelled}}{\text{Time taken}} = \frac{2\pi\,(R_E + h)}{T}$$

From equation, $\sqrt{\frac{GM_E}{(R_E+h)}} = \frac{2\pi\,(R_E + h)}{T}$ --------------- 1

$T = \frac{2\pi}{\sqrt{GM_E}}(R_E + h)^{\frac{3}{2}}$ ------------------- 2

Squaring both sides of the equation (2), we get $T^2 = \frac{4\pi^2}{GM_E}(R_E + h)^3$

$\frac{4\pi^2}{GM_E}$ = Constant say c, $T^2 = c\,(R_E + h)^3$ ---------------- 3

Equation (3) implies that a satellite orbiting the Earth has the same relation between time and distance as that of Kepler's law of planetary motion. For a satellite orbiting near the surface of the Earth, h is negligible compared to the radius of the Earth R_E . Then, $T^2 = \frac{4\pi^2}{GM_E}R_E^3$; $T^2 = \frac{4\pi^2}{\frac{GM_E}{R_E^2}}$

$T^2 = \frac{4\pi^2}{g}R_E$ Since $\frac{GM_E}{R_E^2} = g$; $T = 2\pi\sqrt{\frac{R_E}{g}}$

12. Derive an expression for energy of satellite.

Energy of an Orbiting Satellite

The total energy of a satellite orbiting the Earth at a distance h from the surface of Earth is calculated as follows; The total energy of the satellite is the sum of its kinetic energy and the gravitational potential energy. The potential energy of the satellite is, $U = \frac{GM_SM_E}{(R_E+h)}$

Here M_S - mass of the satellite, M_E- mass of the Earth, R_E - radius of the Earth.

The Kinetic energy of the satellite is KE = ½ M_SV^2 ------------1

Here ν is the orbital speed of the satellite and is equal to $\nu = \frac{GM_E}{(R_E+h)}$

Substituting the value of ν in (1), the kinetic energy of the satellite becomes,

$$KE = \frac{1}{2}\,\frac{GM_SM_E}{2(R_E+h)}$$

Therefore the total energy of the satellite is $E = \frac{1}{2}\,\frac{GM_SM_E}{(R_E+h)} - \frac{GM_SM_E}{(R_E+h)}$

$$E = -\frac{GM_SM_E}{2(R_E+h)}$$

The negative sign in the total energy implies that the satellite is bound to the Earth and it cannot escape from the Earth.

13. Explain in detail the geostationary and polar satellites.

Geo-stationary and polar satellite

1) The satellites orbiting the Earth have different time periods corresponding to different orbital radii. Can we calculate the orbital radius of a satellite if its time period is 24 hours is calculated below. Kepler's third law is used to find the radius of the orbit.

$$T^2 = \frac{4\pi^2}{GM_E}(R_E + h)^3 \ ; \ (R_E + h)^3 = \frac{GM_ET^2}{4\pi^2}$$

$$(R_E + h) = \left(\frac{GM_ET^2}{4\pi^2}\right)^{\frac{1}{3}}$$

2) Substituting for the time period (24 hrs = 86400 seconds), mass, and radius of the Earth, h turns out to be 36,000 km. Such satellites are called **"geo-stationary satellites"**, since they appear to be stationary when seen from Earth.

3) geo-stationary satellites for the purpose of telecommunication. Another type of satellite which is placed at a distance of 500 to 800 km from the surface of the Earth orbits the Earth from north to south direction.

4) This type of satellite that orbits Earth from North Pole to South Pole is called a polar satellite. The time period of a polar satellite is nearly 100 minutes and the satellite completes many revolutions in a day.

5) A Polar satellite covers a small strip of area from pole to pole during one revolution. In the next revolution it covers a different strip of area since the Earth would have moved by a small angle. In this way polar satellites cover the entire surface area of the Earth.

14. Explain how geocentric theory is replaced by heliocentric theory using the idea of retrograde motion of planets.

1) To explain this retrograde motion, Ptolemy introduced the concept of "epicycle" in his geocentric model. According to this theory, while the planet orbited the Earth, it also underwent another circular motion termed as "epicycle".

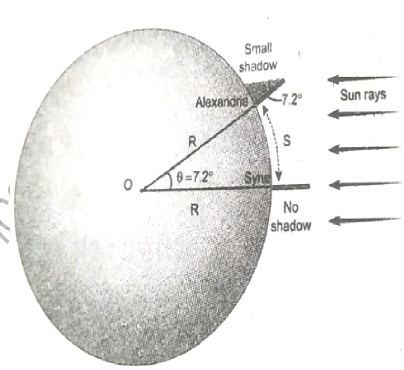
2) A combination of epicycle and circular motion around the Earth gave rise to retrograde motion of the planets with respect to Earth.

3) But Ptolemy's model became more and more complex as every planet was found to undergo retrograde motion. In the 15^{th} century, the Polish astronomer Copernicus proposed.

4) The heliocentric model to explain this problem in a simpler manner. According to this model, the Sun is at the center of the solar system and all planets orbited the Sun.

5) The retrograde motion of planets with respect to Earth is because of the relative motion of the planet with respect to Earth.

15. Explain in detail the Eratosthenes method of finding the radius of Earth.



During noon time of summer solstice the Sun's rays cast no shadow in the city Syne which was located 500 miles away from Alexandria. At the same day and same time he found that in Alexandria the Sun's rays made 7.2 degree with local vertical as shown in the Figure. This difference of 7.2 degree was due to the curvature of the Earth.

The angle 7.2 degree is equivalent to $\frac{1}{8}$ radian. So, $\theta = \frac{1}{8}$ rad.

If S is the length of the arc between the cities of Syne and Alexandria, and if R is radius of Earth, then $S = R\,\theta = 500$ miles, so radius of the Earth

$R = \frac{500}{\theta}$ miles , $R = 500 \frac{miles}{\frac{1}{8}}$ $R = 4000$ miles.

1 mile is equal to 1.609 km. So, he measured the radius of the Earth to be equal to R = 6436 km, which is amazingly close to the correct value of 6378 km.

16. Describe the measurement of Earth's shadow (umbra) radius during total lunar eclipse

1) It is possible to measure the radius of shadow of the Earth at the point where the Moon crosses.

2) When the Moon is inside the umbra shadow, it appears red in color. As soon as the Moon exits from the umbra shadow, it appears in crescent shape.

3) By finding the apparent radii of the Earth's umbra shadow and the Moon, the ratio of the these radii can be calculated.

4) The apparent radius of Earth's umbra shadow = R_s = 13.2 cm

The apparent radius of the Moon = R_m = 5.15 cm

The ratio $\frac{R_S}{R_m} \approx 2.56$.

The radius of the Earth's umbra shadow is $R_s = 2.56 \times R_m$.