Chapter 8: Binomial Distribution

EXERCISE 8.1 [PAGES 251 - 252]

Exercise 8.1 | Q 1.1 | Page 251

A die is thrown 6 times. If 'getting an odd number' is a success, find the probability of 5 successes.

SOLUTION

Let X = number of successes, i.e. number of odd numbers.

p = probability of getting an odd number in a single throw of a die

$\therefore$ p = $\frac{3}{6} = \frac{1}{2}$ and q = $1 - p = 1 - \frac{1}{2} = \frac{1}{2}$

Given: n = 6

$\therefore$ X ~ B$\left(6, \frac{1}{2}\right)$

The p.m.f. of X is given by

$p(X = x) = {}^nC_x\ p^x\ q^{n-x}$

i.e. p(x) = ${}^6C_x\left(\frac{1}{2}\right)^x\left(\frac{1}{2}\right)^{6-x}$

$= {}^6C_x\left(\frac{1}{2}\right)^6$, x = 0, 1, 2, ...,6

P(5 successes) = P[X = 5]

$= p(5) = {}^6C_5\left(\frac{1}{2}\right)^6$

$= {}^6C_1 \times \frac{1}{64}$ .....$[\because {}^nC_x = {}^nC_{n-x}]$

$= \frac{6}{64} = \frac{3}{32}$

Hence, the probability of 5 successes is $\frac{3}{32}$

Exercise 8.1 | Q 1.2 | Page 251

A die is thrown 6 times. If ‘getting an odd number’ is a success, find the probability of at least 5 successes.

SOLUTION

Let X = number of successes, i.e. number of odd numbers.

p = probability of getting an odd number in a single throw of a die

$\therefore$ p = $\frac{3}{6} = \frac{1}{2}$ and q = $1 - \mathrm{p} = 1 - \frac{1}{2} = \frac{1}{2}$

Given: n = 6

$\therefore$ X ~ B$\left(6, \frac{1}{2}\right)$

The p.m.f. of X is given by

$p(\mathrm{X} = \mathrm{x}) = {}^{n}C_{x}\, p^{x}\, q^{n-x}$

i.e. p(x) = ${}^{6}C_{x}\left(\frac{1}{2}\right)^{x}\left(\frac{1}{2}\right)^{6-x}$

$= {}^{6}C_{x}\left(\frac{1}{2}\right)^{6}$, x = 0, 1, 2, ...,6

P(at least 5 successes) = P[X ≥ 5]

= p(5) + p(6)

$= {}^{6}C_{5}\left(\frac{1}{2}\right)^{6} + {}^{6}C_{6}\left(\frac{1}{2}\right)^{6}$

$= \left({}^{6}C_{5} + {}^{6}C_{6}\right)\left(\frac{1}{2}\right)^{6}$

$$= (6+1)\frac{1}{64} = \frac{7}{64}$$

Hence, the probability of at least 5 successes is $\frac{7}{64}$.

Exercise 8.1 | Q 1.3 | Page 251

A die is thrown 6 times. If 'getting an odd number' is a success, find the probability of at most 5 successes.

SOLUTION

Let X = number of successes, i.e. number of odd numbers.

p = probability of getting an odd number in a single throw of a die

$\therefore$ p = $\frac{3}{6} = \frac{1}{2}$ and q = $1 - \mathrm{p} = 1 - \frac{1}{2} = \frac{1}{2}$

Given: n = 6

$\therefore$ X ~ B$\left(6, \frac{1}{2}\right)$

The p.m.f. of X is given by

$$p(\mathbf{X} = \mathbf{x}) = {}^nC_x\, p^x\, q^{n-x}$$

i.e. p(x) = ${}^6C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{6-x}$

$= {}^6C_x \left(\frac{1}{2}\right)^6$, x = 0, 1, 2, ...,6

P(at most 5 successes) = P[X ≤ 5]

= 1 - P [X > 5]

= 1 - p(6) = $1 - {}^6C_6 \left(\frac{1}{2}\right)^6$

$$= 1 - 1 \times \frac{1}{64} = \frac{63}{64}$$

Hence, the probability of at most 5 successes is $\frac{63}{64}$.

Exercise 8.1 | Q 2 | Page 251

A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability of two successes.

SOLUTION

Let X = number of doublets.

p = probability of getting a doublet when a pair of dice is thrown

$\therefore$ p = $\frac{6}{36} = \frac{1}{6}$ and

q = 1 - p = $1 - \frac{1}{6} = \frac{5}{6}$

Given: n = 4

$\therefore$ X ~ B$\left(4, \frac{1}{6}\right)$

The p.m.f. of X is given by

P(X = x) = ${}^nC_x\ p^x\ q^{n-x}$

i.e. p(x) = ${}^nC_x \left(\frac{1}{6}\right)^x \left(\frac{5}{6}\right)^{4-x}$, x = 0, 1, 2, 3, 4

$\therefore$ P(2 successes) = P(X = 2)

= p(2) = ${}^4C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^{4-2}$

$= \frac{4!}{2! \cdot 2!} \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^2$

$= \frac{4 \cdot 3 \cdot 2!}{2!.2.1} \times \frac{1}{36} \times \frac{25}{36}$

$= \frac{25}{216}$

Hence, the probability of two successes is $\frac{25}{216}$.

Exercise 8.1 | Q 3 | Page 251

There are 5% defective items in a large bulk of items. What is the probability that a sample of 10 items will include not more than one defective item?

SOLUTION

Let X = number of defective items

p = probability of defective item

$\therefore$ p = 5% = $\frac{5}{100} = \frac{1}{20}$

and q = 1 - p = $1 - \frac{1}{20} = \frac{19}{20}$

$\therefore$ X ~ B $\left(10, \frac{1}{20}\right)$

The p.m.f. of X is given by

P(X = x) = ${}^{n}C_{x}\, p^{x}\, q^{n-x}$

i.e. p(x) = ${}^{10}C_{x}\left(\frac{1}{20}\right)^{x}\left(\frac{19}{20}\right)^{10-x}$, x = 0, 1, 2, ...,10

P(sample of 10 items will include not more than one defective item) = P[X ≤ 1]

= P(x = 0) + P(x = 1)

$$= {}^{10}C_{0}\left(\frac{1}{20}\right)^{0}\left(\frac{19}{20}\right)^{10-0} + {}^{10}C_{1}\left(\frac{1}{20}\right)^{1}\left(\frac{19}{20}\right)^{10-1}$$

$$= 1 \cdot 1 \cdot \left(\frac{19}{20}\right)^{10} + 10 \times \left(\frac{1}{20}\right) \times \left(\frac{19}{20}\right)^{9}$$

$$= \left(\frac{19}{20}\right)^{9}\left[\frac{19}{20} + \frac{10}{20}\right]$$

$$= \left(\frac{19}{20}\right)^{9}\left(\frac{29}{20}\right) = 29\left(\frac{19^{9}}{20^{10}}\right)$$

Hence, the probability that a sample of 10 items will include not more than one defective item = $29\left(\frac{19^9}{20^{10}}\right)$.

Exercise 8.1 | Q 4.1 | Page 251

Five cards are drawn successively with replacement from a well-shuffled deck of 52 cards, find the probability that all the five cards are spades.

SOLUTION 1

Let X = number of spade cards.

p = probability of drawing a spade card from pack of 52 cards.

Since, there are 13 spade cards in the pack of 52 cards,

$$\therefore \text{p} = \frac{13}{52} = \frac{1}{4} \quad \text{and} \quad \mathrm{q} = 1 - \mathrm{p} = 1 - \frac{1}{4} = \frac{3}{4}$$

Given: n = 5

$$\therefore X \sim B\left(5, \frac{1}{4}\right)$$

The p.m.f. of X is given by

$$P(X = x) = {}^{n}C_x\, p^x\, q^{n-x}$$

$$\text{i.e. } p(x) = {}^{5}C_x \left(\frac{1}{4}\right)^x \left(\frac{3}{5}\right)^{5-x}, x = 0, 1, 2, \ldots, 5$$

P(all five cards are spade)

$$= P(X = 5) = p(5) = {}^{5}C_5\left(\frac{1}{4}\right)^5\left(\frac{3}{4}\right)^{5-5}$$

$$= 1\left(\frac{1}{4}\right)^5\left(\frac{3}{4}\right)^0$$

$$= 1 \times \frac{1}{1024} \times 1 = \frac{1}{1024}$$

Hence, the probability of all the five cards are spades = $\frac{1}{1024}$

SOLUTION 2

Let X denote the number of spades.

P(getting spade) = p = $\frac{13}{52} = \frac{1}{4}$

$\therefore$ q= 1 – p = $1 - \frac{1}{4} = \frac{3}{4}$

Given, n = 5

$\therefore$ X ~ B$\left(5, \frac{1}{4}\right)$

The p.m.f. of X is given by

P(X = x) = ${}^5C_x \left(\frac{1}{4}\right)^x \left(\frac{3}{4}\right)^{5-x}, x$ = 0, 1, ...,5

P(all five cards are spades)

= P(X = 5)

= ${}^5C_5 \left(\frac{1}{4}\right)^5 \left(\frac{3}{4}\right)^0$

= $\frac{1}{4^5}$.

Exercise 8.1 | Q 4.2 | Page 251

Five cards are drawn successively with replacement from a well-shuffled deck of 52 cards, find the probability that only 3 cards are spades

SOLUTION 1

Let X = number of spade cards.

p = probability of drawing a spade card from pack of 52 cards.

Since, there are 13 spade cards in the pack of 52 cards,

$\therefore$ p = $\frac{13}{52} = \frac{1}{4}$ and $q = 1 - p = 1 - \frac{1}{4} = \frac{3}{4}$

Given: n = 5

$\therefore$ X ~ B$\left(5, \frac{1}{4}\right)$

The p.m.f. of X is given by

P(X = x) = ${}^{n}C_{x}\, p^{x}\, q^{n-x}$

i.e. p(x) = ${}^{5}C_{x}\, \left(\frac{1}{4}\right)^{x}\, \left(\frac{3}{5}\right)^{5-x}$, x = 0, 1, 2,...,5

P(only 3 cards are spades) = P(X = 3)

= p(3) = ${}^{5}C_{3}\left(\frac{1}{4}\right)^{3}\left(\frac{3}{4}\right)^{5-3}$

$= \frac{5!}{3!\cdot 2!}\left(\frac{1}{4}\right)^{3}\left(\frac{3}{4}\right)^{2}$

$= \frac{5\cdot 4\cdot 3!}{3!\cdot 2\cdot 1}\times\frac{1}{64}\times\frac{9}{16}=\frac{45}{512}$

Hence, the probability of only 3 cards are spades = $\frac{45}{512}$

SOLUTION 2

Let X denote the number of spades.

P(getting spade) = p = $\frac{13}{52}=\frac{1}{4}$

$\therefore$ q= 1 – p = $1-\frac{1}{4}=\frac{3}{4}$

Given, n = 5

$\therefore$ X ~ B$\left(5, \frac{1}{4}\right)$

The p.m.f. of X is given by

$$P(X = x) = {}^5C_x \left(\frac{1}{4}\right)^x \left(\frac{3}{4}\right)^{5-x}, x = 0, 1, ..., 5$$

P(only 3 cards are spades)

$= P(X = 3)$

$$= {}^5C_3 \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^2$$

$$= \frac{5!}{3! \times 2!} \times \frac{3^2}{4^3 \times 4^2}$$

$$= \frac{5 \times 4 \times 3!}{3! \times 2 \times 1} \times \frac{9}{4^5}$$

$$= \frac{90}{4^5}.$$

Exercise 8.1 | Q 4.3 | Page 251

Five cards are drawn successively with replacement from a well-shuffled deck of 52 cards, find the probability that none is a spade.

SOLUTION 1

Let X = number of spade cards.

p = probability of drawing a spade card from pack of 52 cards.

Since, there are 13 spade cards in the pack of 52 cards,

$$\therefore p = \frac{13}{52} = \frac{1}{4} \text{ and } q = 1 - p = 1 - \frac{1}{4} = \frac{3}{4}$$

Given: n = 5

$$\therefore X \sim B\left(5, \frac{1}{4}\right)$$

The p.m.f. of X is given by

$$P(X = x) = {}^nC_x\, p^x\, q^{n-x}$$

i.e. $p(x) = {}^5C_x \left(\frac{1}{4}\right)^x \left(\frac{3}{5}\right)^{5-x}, x = 0, 1, 2, ..., 5$

P(none of cards is spade) = P(X = 0)

$$= p(0) = {}^5C_0\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^{5-0}$$

$$= 1 \times 1 \times \left(\frac{3}{4}\right)^5 = \frac{243}{1024}$$

Hence, the probability of only 3 cards are spades = $\frac{243}{1024}$

SOLUTION 2

Let X = number of spade cards.

p = probability of drawing a spade card from pack of 52 cards.

Since, there are 13 spade cards in the pack of 52 cards,

$$\therefore p = \frac{13}{52} = \frac{1}{4} \quad \text{and} \quad q = 1 - p = 1 - \frac{1}{4} = \frac{3}{4}$$

Given: n = 5

$$\therefore X \sim B\left(5, \frac{1}{4}\right)$$

The p.m.f. of X is given by

$$P(X = x) = {}^nC_x\, p^x\, q^{n-x}$$

i.e. $p(x) = {}^5C_x \left(\frac{1}{4}\right)^x \left(\frac{3}{5}\right)^{5-x}$, x = 0, 1, 2,...,5

P(none is a spade)

= P(X = 0)

$$= {}^5C_0\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^5$$

$$= \left(\frac{3}{4}\right)^5.$$

Exercise 8.1 | Q 5 | Page 251

The probability that a bulb produced by a factory will fuse after 150 days of use is 0.05. What is the probability that out of 5 such bulbs

(i) none

(ii) not more than one

(iii) more than one

(iv) at least one

will fuse after 150 days of use.

SOLUTION

Let X represent the number of bulbs that will fuse after 150 days of use in an experiment of 5 trials. The trials are Bernoulli trials.

It is given that, p = 0.05

∴ q = 1 - p = 1- 0.05 = 0.95`

X has a binomial distribution with n = 5 and p = 0.05

∴ P (X = x) = ${}^nC_x\ p^x\ q^{n-x}$ where x = 1, 2, ...n

$= {}^5C_x(0.05)^x(0.95)^{n-x}$

(i) **P (none)** = P(X = 0)

= p(0) = ${}^5C_0(0.05)^0(0.95)^{5-0}$

$= 1 \times 1 \times (0.95)^5$

= $(0.95)^2$

(ii) **P (not more than one)** = P(X ≤ 1)

= p(0) + p(1)

$= {}^5C_0 \cdot (0.05)^0(0.95)^{5-0} + {}^5C_1(0.05)^1(0.95)^4$

$= 1 \times 1 \times (0.95)^5 + 5 \times (0.05) \times (0.95)^4$

$= (0.95)^4[0.95 + 5(0.05)]$

$= (0.95)^4(0.95 + 0.25)$

$= (0.95)^4(1.20) = (1.2)(0.95)^4$

(iii) **P (more than 1)** = P(X > 1)

$= 1 - P[X \leq 1]$

$= 1 - (1.2)(0.95)^4$

(iv) **P (at least one)** = $P(X \geq 1)$

$= 1 - P[X = 0]$

$= 1 - p(0)$

$= 1 - {}^5C_0(0.05)^0(0.95)^{5-0}$

$$= 1 - 1 \times 1 \times (0.95)^5$$

$$= 1 - (0.95)^5$$

Exercise 8.1 | Q 6 | Page 252

A bag consists of 10 balls each marked with one of the digits 0 to 9. If four balls are drawn successively with replacement from the bag, what is the probability that none is marked with the digit 0?

SOLUTION

Let X denote the number of balls marked with the digit 0 among the 4 balls drawn.

Since the balls are drawn with replacement, the trials are Bernoulli trials.

X has a binomial distribution with n = 4 and p = 1/10

and $q = 1 - p = 1 - \frac{1}{10} = \frac{9}{10}$

The p.m.f. of X is given by

$P(X = x) = {}^nC_x\, p^x\, q^{n-x}$

i.e. $p(x) = {}^4C_x\left(\frac{1}{10}\right)^x\left(\frac{9}{10}\right)^{4-x}$, x = 0, 1, ...,4

P(none of the ball marked with digit 0) = P(X = 0)

$= p(0) = {}^4C_x\left(\frac{1}{10}\right)^0\left(\frac{9}{10}\right)^{4-0}$

$$= 1 \times 1 \times \left(\frac{9}{10}\right)^4 = \left(\frac{9}{10}\right)^4$$

Hence, the probability that none of the bulb marked with digit o is $\left(\frac{9}{10}\right)^4$.

Exercise 8.1 | Q 7 | Page 252

On a multiple choice examination with three possible answers for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing?

SOLUTION

The repeated guessing of correct answers from multiple-choice questions is Bernoulli trials. Let X represent the number of correct answers by guessing in the set of 5 multiple-choice questions.

Probability of getting a correct answer is, p = 1/3

$$\therefore q = 1 - p = 1 - \frac{1}{3} = \frac{2}{3}$$

Clearly, X has a binomial distribution with n = 5 and p = $\frac{1}{3}$.

The p.m.f. of X is given by

P(X = x) = ${}^nC_x\, p^x\, q^{n-x}$, x = 0, 1, 2, 4, 5

i.e. p(x) = ${}^nC_x \left(\frac{1}{3}\right)^x \left(\frac{2}{3}\right)^{5-x}$ x = 0, 1, 2, 3, 4, 5

P(four or more correct answers) = P[X ≥ 4] = p(4) + p(5)

$$= {}^5C_4\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^{5-4} + {}^5C_5\left(\frac{1}{3}\right)^5\left(\frac{2}{3}\right)^{5-5}$$

$$= 5 \times \left(\frac{1}{3}\right)^4 \times \left(\frac{2}{3}\right)^1 + 1 \times \left(\frac{1}{3}\right)^5\left(\frac{2}{3}\right)^0$$

$$= \left(\frac{1}{3}\right)^4 \left[5 \times \frac{2}{3} + \frac{1}{3}\right]$$

$$= \left(\frac{1}{3}\right)^4 \left[\frac{10}{3} + \frac{1}{3}\right] = \frac{1}{81} \times \frac{11}{3} = \frac{11}{243}$$

Hence, the probability of getting four or more correct answers $\frac{11}{243}$.

Exercise 8.1 | Q 8.1 | Page 252

A person buys a lottery ticket in 50 lotteries, in each of which his chance of winning a prize is 1/100. What is the probability that he will win a prize at least once.

SOLUTION

Let X denote the number of times the person wins the lottery.
Then, X follows a binomial distribution with n = 50.

Let p be the probability of winning a prize .

$$\therefore p = \frac{1}{100}, q = 1 - \frac{1}{100} = \frac{99}{100}$$

Hence, the distribution is given by

$$P(X = r) = {}^{50}C_r \left(\frac{1}{100}\right)^r \left(\frac{99}{100}\right)^{50-r}, r = 0, 1, 2 \ldots 50$$

$$P(\text{winning at least once}) = P(X \geq 0)$$

$$= 1 - P(X - 0)$$

$$= 1 - \left(\frac{99}{100}\right)^{50}$$

Hence, probability of winning a prize at least once $= 1 - \left(\frac{99}{100}\right)^{50}$

Exercise 8.1 | Q 9.1 | Page 252

In a box of floppy discs, it is known that 95% will work. A sample of three of the discs is selected at random. Find the probability that none of the floppy disc work.

SOLUTION

Let X = number of working discs.

p = probability that a floppy disc works

$\therefore$ p = 95% = $\frac{95}{100} = \frac{19}{20}$

and q = 1 - p = $1 - \frac{19}{20} = \frac{1}{20}$

Given: n = 3

$\therefore$ X ~ B$\left(3, \frac{19}{20}\right)$

The p.m.f. of X is given by

P(X = x) = ${}^nC_x\, p^x q^{n-x}$

i.e. p(x) = ${}^3C_x\left(\frac{19}{20}\right)^x\left(\frac{1}{20}\right)^{3-x}$, x = 0, 1, 2, 3

P(none of the floppy discs work) = P(X = 0)

= p(0) = ${}^3C_0\left(\frac{19}{20}\right)^0\left(\frac{1}{20}\right)^{3-0}$

$= 1 \times 1 \times \frac{1}{20^3} = \frac{1}{20^3}$

Hence, the probability that none of the floppy disc will work = $\frac{1}{20^3}$

Exercise 8.1 | Q 9.2 | Page 252

In a box of floppy discs, it is known that 95% will work. A sample of three of the discs is selected at random. Find the probability that exactly one floppy disc work.

SOLUTION

Let X = number of working discs.

p = probability that a floppy disc works

∴ p = 95% = $\frac{95}{100}=\frac{19}{20}$

and q = 1 - p = $1-\frac{19}{20}=\frac{1}{20}$

Given: n = 3

∴ X ~ B$\left(3, \frac{19}{20}\right)$

The p.m.f. of X is given by

P(X = x) = ${}^nC_x\ p^x q^{n-x}$

i.e. p(x) = ${}^3C_x\left(\frac{19}{20}\right)^x\left(\frac{1}{20}\right)^{3-x}$, x = 0, 1, 2, 3

P(exactly one floppy discs work) = P(X = 1)

= p(1) = ${}^3C_1\left(\frac{19}{20}\right)^1\left(\frac{1}{20}\right)^{3-1}$

$=3\times\frac{19}{20}\times\left(\frac{1}{20}\right)^2$

$=3\left(\frac{19}{20^3}\right)$

Hence, the probability that none of the floppy disc will work = $3\left(\frac{19}{20^3}\right)$

Exercise 8.1 | Q 9.3 | Page 252

In a box of floppy discs, it is known that 95% will work. A sample of three of the discs is selected at random. Find the probability that exactly two floppy disc work.

SOLUTION

Let X = number of working discs.

p = probability that a floppy disc works

$\therefore$ p = 95% = $\frac{95}{100} = \frac{19}{20}$

and q = 1 - p = $1 - \frac{19}{20} = \frac{1}{20}$

Given: n = 3

$\therefore$ X ~ B$\left(3, \frac{19}{20}\right)$

The p.m.f. of X is given by

P(X = x) = ${}^nC_x\ p^x q^{n-x}$

i.e. p(x) = ${}^3C_x \left(\frac{19}{20}\right)^x \left(\frac{1}{20}\right)^{3-x}$, x = 0, 1, 2, 3

P(exactly two floppy discs work) = P(X = 2)

= p(2) = ${}^3C_2 \left(\frac{19}{20}\right)^2 \left(\frac{1}{20}\right)^{3-2}$

$= \frac{3 \cdot 2!}{2! \cdot 1!} \times \frac{(19)^2}{(20)^2} \times \left(\frac{1}{20}\right)$

$= 3\left(\frac{19^2}{20^3}\right)$

Hence, the probability that none of the floppy disc will work = 3$\left(\frac{19^2}{20^3}\right)$

Exercise 8.1 | Q 9.4 | Page 252

In a box of floppy discs, it is known that 95% will work. A sample of three of the discs is selected at random. Find the probability that all 3 of the sample will work.

SOLUTION

Let X = number of working discs.

p = probability that a floppy disc works

$\therefore$ p = 95% = $\frac{95}{100} = \frac{19}{20}$

and q = 1 - p = $1 - \frac{19}{20} = \frac{1}{20}$

Given: n = 3

$\therefore X \sim B\left(3, \frac{19}{20}\right)$

The p.m.f. of X is given by

P(X = x) = ${}^{n}C_{x}\, p^{x} q^{n-x}$

i.e. p(x) = ${}^{3}C_{x}\left(\frac{19}{20}\right)^{x}\left(\frac{1}{20}\right)^{3-x}$, x = 0, 1, 2, 3

P(all 3 floppy discs work) = P(X = 3)

= p(3) = ${}^{3}C_{3}\left(\frac{19}{20}\right)^{3}\left(\frac{1}{20}\right)^{3-3}$

$= 1 \times \left(\frac{19}{20}\right)^{3} \times \left(\frac{1}{20}\right)^{0}$

$= \left(\frac{19}{20}\right)^{3}$

Hence, the probability that none of the floppy disc will work = $\left(\frac{19}{20}\right)^{3}$.

Exercise 8.1 | Q 10 | Page 252

Find the probability of throwing at most 2 sixes in 6 throws of a single die.

SOLUTION

The repeated tossing of the die are Bernoulli trials. Let X represent the number of times of getting sixes in 6 throws of the die.

Probability of getting six in a single throw of die, $p = \frac{1}{6}$

$\therefore q = 1 - p = 1 - \frac{1}{6} = \frac{5}{6}$

Clearly, X has a binomial distribution with n = 6

The p.m.f. of X is given by

P(X = x) = ${}^nC_x\, p^x q^{n-x}$

i.e. p(x) = ${}^6C_x \left(\frac{1}{6}\right)^x \left(\frac{5}{6}\right)^{6-x}$, x = 0, 1, 2, ...,6

P(at most 2 sixes) = P[X ≤ 2]

= p(0) + p(1) + p(2)

$$= {}^6C_0 \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^{6-0} + {}^6C_1 \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^{6-1} + {}^6C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^{6-2}$$

$$= 1 \times 1 \times \left(\frac{5}{6}\right)^6 + 6 \times \left(\frac{1}{6}\right) \times \left(\frac{5}{6}\right)^5 + \frac{6!}{2!\,4!} \times \left(\frac{1}{6}\right)^2 \times \left(\frac{5}{6}\right)^4$$

$$= \left(\frac{5}{6}\right)^6 + \left(\frac{5}{6}\right)^5 + \frac{6 \times 5}{2 \times 1} \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^4$$

$$= \left(\frac{5}{6}\right)^6 + \left(\frac{5}{6}\right)^5 + 15 \times \frac{1}{36} \times \left(\frac{5}{6}\right)^4$$

$$= \left[\left(\frac{5}{6}\right)^2 + \left(\frac{5}{6}\right) + \frac{15}{36}\right] \left(\frac{5}{6}\right)^4$$

$$= \left(\frac{25}{36} + \frac{5}{6} + \frac{15}{36}\right) . \left(\frac{5}{6}\right)^4$$

$$= \left(\frac{25+30+15}{36}\right)\left(\frac{5}{6}\right)^4$$

$$= \frac{70}{36}\left(\frac{5}{6}\right)^4$$

$$= \frac{7}{3} \times \frac{10}{12} \times \left(\frac{5}{6}\right)^4$$

$$= \frac{7}{3} \times \frac{5}{6} \times \left(\frac{5}{6}\right)^4 = \frac{7}{3}\left(\frac{5}{6}\right)^5$$

Hence, the probability of throwing at most 2 sixes

$$\frac{7}{3}\left(\frac{5}{6}\right)^5$$

Exercise 8.1 | Q 11 | Page 252

It is known that 10% of certain articles manufactured are defective. What is the probability that in a random sample of 12 such articles, 9 are defective?

SOLUTION

The repeated selections of articles in a random sample space are Bernoulli trails. Let X denote the number of times of selecting defective articles in a random sample space of 12 articles.

Clearly, X has a binomial distribution with n = 12 and p = 10% = $\frac{10}{100} = \frac{1}{10}$

q = 1 - p = $1 - \frac{1}{10} = \frac{9}{10}$

Given: n = 12

∴ X ~ B $\left(12, \frac{1}{10}\right)$

The p.m.f. of X is given by

P[X = x] = ${}^nC_x\ p^x\ q^{n-x}$

i.e. p(x) = ${}^{12}C_x\left(\frac{1}{10}\right)^x\left(\frac{9}{10}\right)^{12-x}$, x = 1, 2, 3,...,12

P(9 defective articles) = P[X = 9]

= p(9) = ${}^{12}C_9\left(\frac{1}{10}\right)^9\left(\frac{9}{10}\right)^{12-9}$

$$= \frac{12!}{9!\,3!}\left(\frac{1}{10}\right)^9\left(\frac{9}{10}\right)^3$$

$$= \frac{12\times 11\times 10\times 9!}{9!\times 3\times 2\times 1}\times\frac{1}{10^9}\times\frac{9^3}{10^3}$$

$$= 2\times 11\times 10\cdot\frac{9^3}{10^{12}} = 22\left(\frac{9^3}{10^{11}}\right)$$

Hence, the probability of getting 9 defective articles $= 22\left(\frac{9^3}{10^{11}}\right)$.

Exercise 8.1 | Q 12.1 | Page 252

Given X ~ B(n, P) if n = 10 and p = 0.4, find E(x) and Var(X).

SOLUTION

Given: n = 10 and p = 0.4

∴ q = 1 - p = 1 - 0.4 = 0.6

∴ E(X) = np

= 10(0.4) = 4

∴ Var(X) = npq

= 10(0.4)(0.6) = 2.4

Hence, E(x) = 4, Var(X) = 2.4

Exercise 8.1 | Q 12.2 | Page 252

Given X ~ B(n, P) if p = 0.6 and E(X) = 6, find n and Var(X).

SOLUTION

Given: p = 0.6 and E(X) = 6

E(X) = np

∴ 6 = n × 0.6

n = $\frac{6}{0.6} = 10$

Now, q = 1 - p = 1 - 0.6 = 0.4

∴ Var(X) = npq

= 10 × 0.6 × 0.4 = 2.4

Hence, n = 10, Var(X) = 2.4

Exercise 8.1 | Q 12.3 | Page 252

Given X ~ B(n, P) if n = 25 and E(X) = 10, find p and SD(X).

SOLUTION

Given: n = 25 and E(X) = 10

E(X) = np

∴ 10 = 25 × p

∴ p = $\frac{10}{25} = \frac{2}{5}$

∴ q = 1 - p = $1 - \frac{2}{5} = \frac{3}{5}$

Var(X) = npq

$= 25 \times \frac{2}{5} \times \frac{3}{5} = 6$

∴ SD(X) = $\sqrt{\mathbf{Var(X)}} = \sqrt{6}$

Hence, p = $\frac{2}{5}$, S.D.(X) = $\sqrt{6}$

Exercise 8.1 | Q 12.4 | Page 252

Given X ~ B(n, P) if n = 10, E(X) = 8, find Var(X).

SOLUTION

Given: n = 10, E(X) = 8

E(X) = np

$\therefore$ 8 = 10p

$\therefore p = \frac{8}{10} = \frac{4}{5}$

$\therefore q = 1 - p = 1 - \frac{4}{5} = \frac{1}{5}$

Var(X) = npq

$= 10 \times \frac{4}{5} \times \frac{1}{5} = \frac{8}{5}$

Hence, Var(X) = $\frac{8}{5}$

MISCELLANEOUS EXERCISE 8 [PAGE 253]

Miscellaneous exercise 8 | Q 1 | Page 253

Choose the correct option from the given alternatives:
A die is thrown 100 times. If getting an even number is considered a success, then the standard deviation of the number of successes is

$\sqrt{50}$

5

25

10

SOLUTION

5

Miscellaneous exercise 8 | Q 2 | Page 253

Choose the correct option from the given alternatives:

The mean and the variance of a binomial distribution are 4 and 2 respectively. Then the probability of 2 successes is

$\frac{128}{256}$

$\frac{219}{256}$

$\frac{37}{256}$

$\frac{28}{256}$

SOLUTION

$\frac{28}{256}$

Explanation:

np = 4, npq = 2

$\therefore$ q = $\frac{1}{2}$ and p = $\frac{1}{2}$

$\therefore \mathbf{n}\left(\frac{1}{2}\right) = 4$

$\therefore$ n = 8

$\therefore$ P(X = 2) = ${}^{8}C_{2}\left(\frac{1}{2}\right)^{8} = \frac{8 \times 7}{1 \times 2} \times \frac{1}{256} = \frac{28}{256}$

Miscellaneous exercise 8 | Q 3 | Page 253

Choose the correct option from the given alternatives:

For a binomial distribution, n = 5. If P(X = 4) = P(X = 3), then p = _____

$\frac{1}{3}$

$\frac{3}{4}$

1

$\frac{2}{3}$

SOLUTION

$\frac{2}{3}$

Explanation:

P(X = 4) = P(X = 3)

$\therefore\ {}^5C_4\, p^4\, q = {}^5C_3\, p^3\, q^2$

∴ 5p = 10q

∴ 5p = 10(1 - p)

∴ p = $\frac{10(1-\text{p})}{5}$

∴ p = 2 - 2p

∴ p + 2p = 2

∴ 3p = 2

∴ p = $\frac{2}{3}$

Miscellaneous exercise 8 | Q 4 | Page 253

Choose the correct option from the given alternatives:

For a binomial distribution, n = 4. If P(X = 3) = 3P(X = 2), then p = ______

$\frac{4}{13}$

$\frac{5}{13}$

$\frac{9}{13}$

$\frac{6}{13}$

SOLUTION

9/13

Miscellaneous exercise 8 | Q 5 | Page 253

Choose the correct option from the given alternatives:

If X ~ B(4, p) and P(X = 0) = $\frac{16}{81}$, the P(X = 4) = ______

- $\frac{1}{16}$
- $\frac{1}{81}$
- $\frac{1}{27}$
- $\frac{1}{8}$

SOLUTION

$\frac{1}{81}$

Hint:

P(X = 0) = $^4C_0\ p^0\ q^4 = \frac{16}{81}$

$\therefore$ $q^4 = \left(\frac{2}{3}\right)^4$

$\therefore$ q = $\frac{2}{3}$

$\therefore$ p = 1 - q = $1 - \frac{2}{3} = \frac{1}{3}$

$\therefore$ P(X = 4) = $^4C_4 p^4 q^0 = \left(\frac{1}{3}\right)^4 = \frac{1}{81}$

Miscellaneous exercise 8 | Q 6 | Page 253

Choose the correct option from the given alternatives:

The probability of a shooter hitting a target is 3/4 How many minimum numbers of times must he fire so that the probability of hitting the target at least once is more than 0·99?

1. 2
2. 3
3. **4**
4. 5

SOLUTION

4

Hint:

$P(X \geq 1) > 0.99$

$\therefore 1 - P(X = 0) > 0.99$

$\therefore P(X = 0) < 0.01 = \dfrac{1}{100}$

$\therefore {}^nC_0 \left(\dfrac{3}{4}\right)^0 \left(\dfrac{1}{4}\right)^n < \dfrac{1}{100}$

$\therefore \left(\dfrac{1}{4}\right)^n < \dfrac{1}{100}$

$\therefore n = 4$

Miscellaneous exercise 8 | Q 7 | Page 253

Choose the correct option from the given alternatives:

If the mean and variance of a binomial distribution are 18 and 12 respectively, then n =

1. 36
2. **54**
3. 18
4. 27

SOLUTION

54

Hint:

np = 18 and npq = 12

$\therefore \dfrac{\text{npq}}{\text{np}} = \dfrac{12}{18}$

$\therefore$ q = $\dfrac{2}{3}$

$\therefore$ p = 1 - q = $1 - \dfrac{2}{3} = \dfrac{1}{3}$

$\therefore \mathbf{n}\left(\dfrac{1}{3}\right) = 18$

$\therefore$ n = 54

MISCELLANEOUS EXERCISE 8 [PAGES 253 - 524]

Miscellaneous exercise 8 | Q 1.1 | Page 253

Let X ~ B(10, 0.2). Find P(X = 1).

SOLUTION

X ~ B(10, 0.2)

$\therefore$ n = 10, p = 0.2

$\therefore$ q = 1 - p = 1 - 0.2 = 0.8

The p,m.f. of X is given by

P(X = x) = ${}^{n}C_{x}\ p^{x}\ q^{n-x}$

$\therefore$ P(X = x) = ${}^{10}C_{x}\ (0.2)^{x}\ (0.8)^{10-x}$, x = 0, 1, 2, 3,....,10

P(X = 1) = ${}^{10}C_{1}\ (0.2)^{1}\ (0.8)^{10-1}$

$$= 10(0.2)^1(0.8)^9 = 2(0.8)^9$$

Miscellaneous exercise 8 | Q 1.2 | Page 253

Let X ~ B(10, 0.2). Find P(X ≥ 1).

SOLUTION

X ~ B(10, 0.2)

∴ n = 10, p = 0.2

∴ q = 1 - p = 1 - 0.2 = 0.8

The p,m.f. of X is given by

P(X = x) = $^nC_x\ p^x\ q^{n-x}$

∴ P(X = x) = $^{10}C_x\ (0.2)^x\ (0.8)^{10-x}$, x = 0, 1, 2, 3,....,10

P(X ≥ 1) = 1 - P(X < 1)

= 1 - P(X = 0)

= 1 - $^{10}C_0(0.2)^0(0.8)^{10-0}$

$$= 1 - 1 \times 1 \times (0.8)^{10}$$

$$= 1 - (0.8)^{10}$$

Miscellaneous exercise 8 | Q 1.3 | Page 253

Let X ~ B(10, 0.2). Find P(X ≤ 8).

SOLUTION

X ~ B(10, 0.2)

∴ n = 10, p = 0.2

∴ q = 1 - p = 1 - 0.2 = 0.8

The p,m.f. of X is given by

P(X = x) = $^nC_x\ p^x\ q^{n-x}$

∴ P(X = x) = ${}^{10}C_x\ (0.2)^x\ (0.8)^{10-x}$, x = 0, 1, 2, 3,....,10

P(X ≤ 8) = 1 - P(X > 8)

= 1 - [P(X = 9) + P(X = 10)]

$$= 1 - \left[{}^{10}C_9(0.2)^9(0.8)^{10-9} + {}^{10}C_{10}(0.2)^{10}(0.8)^{10-10}\right]$$

$$= 1 - \left[10(0.2)^9(0.8)^1 + 1(0.2)^{10}(0.8)^0\right]$$

$$= 1 - (0.2)^9[10(0.8) + (0.2)]$$

$$= 1 - (0.2)^9[8 + 0.2]$$

$$= 1 - (8.2)(0.2)^9$$

Miscellaneous exercise 8 | Q 2.1 | Page 253

Let X ~ B(n, p) if n = 10, E(X) = 5, find p and Var(X).

SOLUTION

X ~ B(n, p)

Given: n = 10 and E(X) = 5

But E(X) = np   ∴ np = 5

∴ 10p = 5

∴ p = $\frac{1}{2}$

∴ q = 1 - p = $1 - \frac{1}{2} = \frac{1}{2}$

Var(X) = npq = $10\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = 2.5$

Hence, p = $\frac{1}{2}$ and Var(X) = 2.5

Miscellaneous exercise 8 | Q 2.2 | Page 253

Let X ~ B(n, p) if E(X) = 5 and Var(X) = 2.5, find n and p.

SOLUTION

X ~ B(n, p)

Given: E(X) = 5 and Var(X) = 2.5

np = 5 and npq = 2.5

$\therefore \frac{\text{npq}}{\text{np}} = \frac{2.5}{5}$

$\therefore$ q = 0.5 = $\frac{5}{10} = \frac{1}{5}$

$\therefore$ p = 1 - p = $1 - \frac{1}{2} = \frac{1}{2}$

Substituting p = $\frac{1}{2}$ in np = 5, we get

$$\text{n}\left(\frac{1}{2}\right) = 5$$

$\therefore$ n = 10

Hence, n = 10 and p = $\frac{1}{2}$

Miscellaneous exercise 8 | Q 3.1 | Page 253

If a fair coin is tossed 10 times and the probability that it shows heads 5 times.

SOLUTION

Let, X = number of heads.

p = probability that coin tossed shows a head

$\therefore$ p = $\frac{1}{2}$

q = 1 - q = $1 - \frac{1}{2} = \frac{1}{2}$

Given: n = 10

$\therefore$ X ~ B $\left(10, \frac{1}{2}\right)$

The p.m.f. of X is given by

P(X = x) = ${}^{n}C_{x}\ p^{x}\ q^{n-x}$

$\therefore$ p(x) = ${}^{10}C_{x}\left(\frac{1}{2}\right)^{x}\left(\frac{1}{2}\right)^{10-x} = {}^{10}C_{x}\left(\frac{1}{2}\right)^{10}$, x = 0, 1, 2,...,10

P(coin shows heads 5 times) = P[X = 5]

= p(5) = ${}^{10}C_{5}\cdot\left(\frac{1}{2}\right)^{10}$

$= \frac{10!}{5!\cdot 5!} \times \frac{1}{1024}$

$= \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5!}{5! \times 5 \times 4 \times 3 \times 2 \times 1} \times \frac{1}{1024} = \frac{63}{256}$

Hence, the probability that can shows heads exactly 5 times = $\frac{63}{256}$.

Miscellaneous exercise 8 | Q 3.2 | Page 253

If a fair coin is tossed 10 times and the probability that it shows heads in the first four tosses and tail in last six tosses.

SOLUTION

Let, X = number of heads.

p = probability that coin tossed shows a head

$\therefore$ p = $\frac{1}{2}$

q = 1 - q = $1 - \frac{1}{2} = \frac{1}{2}$

Given: n = 10

$\therefore$ X ~ B $\left(10, \frac{1}{2}\right)$

The p.m.f. of X is given by

P(X = x) = ${}^nC_x\ p^x\ q^{n-x}$

$\therefore$ p(x) = ${}^{10}C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{10-x} = {}^{10}C_x \left(\frac{1}{2}\right)^{10}$, x = 0, 1, 2,...,10

P(getting heads in first four tosses and tails in last six tosses) = P(X = 4)

= p(4) = ${}^{10}C_4 \cdot \left(\frac{1}{2}\right)^{10}$

$$= \frac{10 \times 9 \times 8 \times 7 \times 6!}{6! \times 4 \times 3 \times 2 \times 1} \times \frac{1}{1024}$$

$$= 210 \times \frac{1}{1024} = \frac{105}{512}$$

Hence, the probability that getting heads in first four tosses and tails in last six tosses = 105/512.

Miscellaneous exercise 8 | Q 4 | Page 254

The probability that a bomb will hit a target is 0.8. Find the probability that out of 10 bombs dropped, exactly 2 will miss the target.

SOLUTION

Let X = number of bombs hitting the target.

p = probability that bomb will hit the target

$\therefore$ p = 0.8 = $\frac{8}{10} = \frac{4}{5}$

$\therefore$ q = 1 - p = $1 - \frac{4}{5} = \frac{1}{5}$

Given: n = 10

$\therefore$ X ~ B $\left(10, \frac{4}{5}\right)$

The p.m.f. of X is given as :

P[X = x] = ${}^nC_x\, p^x\, q^{n-x}$

i.e. p(x) = ${}^{10}C_x \left(\frac{4}{5}\right)^x \left(\frac{1}{5}\right)^{10-x}$

P (exactly 2 bombs will miss the target)

= P (exactly 8 bombs will hit the target)

= P[X = 8] = p(8)

$= {}^{10}C_8 \left(\frac{4}{5}\right)^8 \left(\frac{1}{5}\right)^{10-8}$

$= \hat{}10C\_2\ (4/5)\hat{}8\ (1/5)\hat{}2\quad ....[\text{because}\ \ {}^nC_x = {}^nC_{n-x}]$

$= \frac{10 \times 9}{1 \times 2} \times \frac{4^8}{5^{10}} = \frac{45 \times 4^8}{5^{10}} = 45\left(\frac{2^{16}}{5^{10}}\right)$

Hence, the probability that exactly 2 bombs will miss the target = $45\left(\frac{2^{16}}{5^{10}}\right)$

Miscellaneous exercise 8 | Q 5.1 | Page 254

The probability that a mountain-bike travelling along a certain track will have a tyre burst is 0.05. Find the probability that among 17 riders: exactly one has a burst tyre.

SOLUTION

Let X = number of burst tyre.

p = probability that a mountain-bike travelling along a certain track will have a tyre burst

∴ p = 0.05

∴ q = 1 - p = 1 - 0.05 = 0.95

Given: n = 17

∴ X ~ B(17, 0.05)

The p.m.f. of X is given by

P(X = x) = $^nC_x\ p^x\ q^{n-x}$

i.e. p(x) = $^{17}C_x(0.05)^x(0.95)^{17-x}$, x = 0, 1, 2,...,17

P(exactly one has a burst tyre)

P(X = 1) = p(1) = $^{17}C_1(0.05)^1(0.95)^{17-1}$

$= 17(0.05)(0.95)^{16}$

$= 0.85(0.95)^{16}$

Hence, the probability that riders has exactly one burst tyre = $(0.85)(0.95)^{16}$

Miscellaneous exercise 8 | Q 5.2 | Page 254

The probability that a mountain-bike travelling along a certain track will have a tyre burst is 0.05. Find the probability that among 17 riders: at most three have a burst tyre

SOLUTION

Let X = number of burst tyre.

p = probability that a mountain-bike travelling along a certain track will have a tyre burst

∴ p = 0.05

∴ q = 1 - p = 1 - 0.05 = 0.95

Given: n = 17

∴ X ~ B(17, 0.05)

The p.m.f. of X is given by

P(X = x) = ${}^nC_x\, p^x\, q^{n-x}$

i.e. p(x) = ${}^{17}C_x(0.05)^x(0.95)^{17-x}$, x = 0, 1, 2,...,17

P (at most three have a burst tyre) = P(X ≤ 3)

= P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)

= p(0) + p(1) + p(2) + p(3)

$$= {}^{17}C_0(0.05)^0(0.95)^{17-0} + {}^{17}C_1(0.05)^1(0.17)^{17-1} + {}^{17}C_2(0.05)^2(0.17)^{17-2} + {}^{17}C_3(0.05)^3(0.17)^{17-3}$$

$$= 1(1)(0.95)^{17} + 17(0.05)(0.95)^{16} + \frac{17\times 16}{2\times 1}\times(0.05)^2(0.95)^{15} + \frac{17\times 16\times 15}{3\times 2\times 1}\times(0.05)^3\times(0.95)^{14}$$

$$= (0.95)^{17} + 17(0.05)\times(0.95)^{16} + 17(8)\times(0.05)^2\times(0.95)^{15} + 17(8)(5)\times(0.05)^3\times(0.95)^{14}$$

$$= (0.95)^{14}\left[(0.95)^3 + (17)(0.05)(0.95)^2 + 17(8)\times(0.05)^2\times(0.95)^1 + 17(8)(5)(0.05)^3\right]$$

$$= (0.95)^{14}[0.8574 + 0.7671 + 0.323 + 0.085]$$

$$= (2.0325)(0.95)^{14}$$

Hence, the probability that at most three riders have burst tyre $= (2.0325)(0.95)^{14}$

Miscellaneous exercise 8 | Q 5.3 | Page 254

The probability that a mountain-bike travelling along a certain track will have a tyre burst is 0.05. Find the probability that among 17 riders: two or more have burst tyre.

SOLUTION

Let X = number of burst tyre.

p = probability that a mountain-bike travelling along a certain track will have a tyre burst

∴ p = 0.05

∴ q = 1 - p = 1 - 0.05 = 0.95

Given: n = 17

∴ X ~ B(17, 0.05)

The p.m.f. of X is given by

P(X = x) = ${}^nC_x\, p^x\, q^{n-x}$

i.e. p(x) = ${}^{17}C_x(0.05)^x(0.95)^{17-x}$, x = 0, 1, 2,...,17

P(two or more have tyre burst)

= P(X ≥ 2) = 1 - P(X < 2)

= 1 - [P(X = 0) + P(X = 1)]

$= 1 - [p(0) + p(1)]$

$= 1 - \left[{}^{17}C_0(0.05)^0(0.95)^{17} + {}^{17}C_1(0.05)^1(0.95)^{16}\right]$

$= 1 - \left[1(1)(0.95)^{17} + 17(0.05)(0.95)^{16}\right]$

$= 1 - \left(0.95^{16}\right)[0.95 + 0.85]$

$= 1 - (1.80)(0.95)^{16}$

$= 1 - (1.8)(0.95)^{16}$

Hence, the probability that two or more riders have tyre burst = $1 - (1.8)(0.95)^{16}$.

Miscellaneous exercise 8 | Q 6 | Page 254

The probability that a lamp in a classroom will be burnt out is 0.3. Six such lamps are fitted in the class-room. If it is known that the classroom is unusable if the number of lamps burning in it is less than four, find the probability that the classroom cannot be used on a random occasion.

SOLUTION

Let X = number of lamps burnt out in the classroom.

p = probability of a lamp in a classroom will be burnt

∴ p = $0.3 = \frac{3}{10}$

∴ q = 1 - p = $1 - \frac{3}{10} = \frac{7}{10}$

Given: n = 6

∴ X ~ B$\left(6, \frac{3}{10}\right)$

The p.m.f. of X is given as:

P[X = x] = ${}^{n}C_{x}\, p^{x}\, q^{n-x}$

i.e. p(x) = ${}^{6}C_{x}\left(\frac{3}{10}\right)^{x}\left(\frac{7}{10}\right)^{6-x}$

Since, the classroom is unusable if the number of lamps burning in it is less than four, therefore

P (classroom cannot be used)

= P[X < 4] = P[X = 0] + P[X = 1] + P[X = 2] + P[X = 3]

= p(0) + p(1) + p(2) + p(3)

$$= {}^{6}C_{0}\left(\frac{3}{10}\right)^{0}\left(\frac{7}{10}\right)^{6-0} + {}^{6}C_{1}\left(\frac{3}{10}\right)^{1}\left(\frac{7}{10}\right)^{6-1} + {}^{6}C_{2}\left(\frac{3}{10}\right)^{2}\left(\frac{7}{10}\right)^{6-2} + {}^{6}C_{3}\left(\frac{3}{10}\right)^{3}\left(\frac{7}{10}\right)^{6-3}$$

$$= 1 \times 1 \times \left(\frac{7}{10}\right)^{6} + 6\left(\frac{3}{10}\right)\left(\frac{7}{10}\right)^{5} + \frac{6 \times 5}{1 \times 2} \cdot \left(\frac{3}{10}\right)^{2}\left(\frac{7}{10}\right)^{4} + \frac{6 \times 5 \times 4}{1 \times 2 \times 3} \cdot \left(\frac{3}{10}\right)^{3}\left(\frac{7}{10}\right)^{3}$$

$$= \left[7^{6} + 18 \times 7^{5} + 15 \times 9 \times 7^{4} + 20 \times 27 \times 7^{3}\right]\frac{1}{10^{6}}$$

$$= \frac{117649 + 302526 + 324135 + 185220}{10^{6}}$$

$$= \frac{929530}{10^{6}} = 0.92953$$

Hence, the probability that the classroom cannot be used on a random occasion is 0.92953.

Miscellaneous exercise 8 | Q 7 | Page 254

A lot of 100 items contain 10 defective items. Five items are selected at random from the lot and sent to the retail store. What is the probability that the store will receive at most one defective item?

SOLUTION

Let X = number of defective items.

p = probability that item is defective

$\therefore$ p = $\frac{10}{100} = \frac{1}{10}$

$\therefore$ q = $1 - p = 1 - \frac{1}{10} = \frac{9}{10}$

Given: n = 5

$\therefore X \sim B\left(5, \frac{1}{10}\right)$

The p.m.f. of X is given as:

P[X = x] = ${}^nC_x\, p^x\, q^{n-x}$

i.e. p(x) = ${}^5C_x \left(\frac{1}{10}\right)^x \left(\frac{9}{10}\right)^{5-x}$

P (store will receive at most one defective item)

= P[X ≤ 1] = P[X = 0] + P[X = 1]

= p(0) + p(1)

$$= {}^5C_0\left(\frac{1}{10}\right)^0\left(\frac{9}{10}\right)^{5-0} + {}^5C_1\left(\frac{1}{10}\right)^1\left(\frac{9}{10}\right)^{5-1}$$

$$= 1 \times 1 \times \left(\frac{9}{10}\right)^5 + 5 \times \frac{1}{10} \times \left(\frac{9}{10}\right)^4$$

$$= (0.9)^5 + (0.05)(0.9)^4$$

$$= (0.9 + 0.5)(0.9)^4$$

= $(1.4)(0.9)^4$

Hence, the probability that the store will receive at most one defective item is $(1.4)(0.9)^4$

Miscellaneous exercise 8 | Q 8 | Page 254

A large chain retailer purchases a certain kind of electronic device from a manufacturer. The manufacturer indicates that the defective rate of the device is 3%. The inspector of the retailer picks 20 items from a shipment. What is the probability that the store will receive at most one defective item?

SOLUTION

Let X = number of a defective electronic device.

p = probability that a device is defective

$\therefore$ p = 3% = $\frac{3}{100}$

$\therefore$ q = 1 - p = $1-\frac{3}{100}=\frac{97}{100}$

Given: n = 20

$\therefore$ X ~ B $\left(20, \frac{3}{100}\right)$

The p.m.f. of X is given as :

P[X = x] = ${}^nC_x\, p^x q^{n-x}$

i.e. p(x) = ${}^{20}C_x\left(\frac{3}{100}\right)^x\left(\frac{97}{100}\right)^{20-x}$

P (store will receive at most one defective item)

= P[X ≤ 1] = p[X = 0] + P[X = 1]

= p(0) + p(1)

$$={}^{20}C_0\left(\frac{3}{100}\right)^0\left(\frac{97}{100}\right)^{20-0}+{}^{20}C_1\left(\frac{3}{100}\right)^1\left(\frac{97}{100}\right)^{20-1}$$

$$=1\times1\times(0.97)^{20}+20\times(0.03)\times(0.97)^{19}$$

$$=(0.97+0.6)(0.97)^{19}$$

$$=(1.57)(0.97)^{19}$$

Hence, the probability that the store will receive at most one defective item $= (1.57)(0.97)^{19}$

[**Note:** Answer in the textbook is incorrect.]

Miscellaneous exercise 8 | Q 9 | Page 524

The probability that a certain kind of component will survive a check test is 0.5. Find the probability that exactly two of the next four components tested will survive.

SOLUTION

Let X = number of tested components survive.

p = probability that the component survives the check test

$\therefore$ p = 0.6 = $\frac{6}{10} = \frac{3}{5}$

$\therefore$ q = 1 - p = $1 - \frac{3}{5} = \frac{2}{5}$

Given: n = 4

$\therefore$ X ~ B$\left(4, \frac{3}{5}\right)$

The p.m.f. of X is given as:

P[X = x] = ${}^nC_x\ p^x\ q^{n-x}$

i.e. p(x) = ${}^4C_x\left(\frac{3}{5}\right)^x\left(\frac{2}{5}\right)^{4-x}$

P (exactly 2 components survive)

= P[X = x] = p(2)

$$= {}^4C_2\left(\frac{3}{5}\right)^2\left(\frac{2}{5}\right)^{4-2}$$

$$= \left(\frac{4 \times 3}{1 \times 2}\right) \times \left(\frac{3}{5}\right)^2\left(\frac{2}{5}\right)^2 = \frac{6 \times 9 \times 4}{625}$$

$$= \frac{216}{625} = 0.3456$$

Hence, the probability that exactly 2 of the 4 tested components survive is 0.3456.

Miscellaneous exercise 8 | Q 10 | Page 254

An examination consists of 10 multiple choice questions, in each of which a candidate has to deduce which one of five suggested answers is correct. A completely unprepared student guesses each answer completely randomly. What is the probability that this student gets 8 or more questions correct? Draw the appropriate morals.

SOLUTION

Let X = number of correct answers.

p = probability that student gets a correct answer

$\therefore$ p = $\frac{1}{5}$

$\therefore$ q = 1 - p = $1 - \frac{1}{5} = \frac{4}{5}$

Given: n = 10 (number of total questions)

$\therefore$ X ~ B $\left(10, \frac{1}{5}\right)$

The p.m.f. of X is given by

P[X = x] = ${}^nC_x\ p^x\ q^{n-x}$

i.e. p(x) = ${}^{10}C_x\ \left(\frac{1}{5}\right)^x\ \left(\frac{4}{5}\right)^{10-x}$, x = 0, 1, 2,...,10

P(student gets 8 or more questions correct)

= P(X ≥ 8) = P(X = 8) + P(X = 9) + P(X = 10)

$$= {}^{10}C_8\left(\frac{1}{5}\right)^8\left(\frac{4}{5}\right)^2 + {}^{10}C_9\left(\frac{1}{5}\right)^9\left(\frac{4}{5}\right)^1 + {}^{10}C_{10}\left(\frac{1}{5}\right)^{10}\left(\frac{4}{5}\right)^0$$

$$= \frac{10 \times 9 \times 8!}{8! \times 2 \times 1} \times \left(\frac{1}{5}\right)^8 \times \left(\frac{4}{5}\right)^2 + 10\left(\frac{1}{5}\right)^9\left(\frac{4}{5}\right)^1 + 1 \times \left(\frac{1}{5}\right)^{10}$$

$$= 45 \times \left(\frac{1}{5}\right)^8 \times \left(\frac{4}{5}\right)^2 + 10 \times \left(\frac{1}{5}\right)^9 \times \left(\frac{4}{5}\right) + \left(\frac{1}{5}\right)^{10}$$

$$= \left(\frac{1}{5}\right)^8 \left[45 \times \left(\frac{4}{5}\right)^2 + 10 \times \left(\frac{1}{5}\right) \times \left(\frac{4}{5}\right) + \left(\frac{1}{5}\right)^2\right]$$

$$= \left[45 \times \frac{16}{25} + \frac{10}{5} \times \frac{4}{5} + \frac{1}{25}\right]\left(\frac{1}{5}\right)^8$$

$$= \left[\frac{720}{25} + \frac{40}{25} + \frac{1}{25}\right]\left(\frac{1}{5^8}\right)$$

$$= \left(\frac{761}{25}\right) \times \left(\frac{1}{5^8}\right) = \frac{30.44}{5^8}$$

Hence, the probability that student gets 8 or more questions correct = $\frac{30.44}{5^8}$

Miscellaneous exercise 8 | Q 11.1 | Page 254

The probability that a machine will produce all bolts in a production run within specification is 0.998. A sample of 8 machines is taken at random. Calculate the probability that all 8 machines.

SOLUTION

Let X = number of machines that produce the bolts within specification.

p = probability that a machine produce bolts within specification

p = 0.998

and q = 1 - p = 1 - 0.998 = 0.002

Given: n = 8

∴ X ~ B (8, 0.998)

The p.m.f. of X is given by

P(X = x) = ${}^nC_x \ p^x \ q^{n-x}$

i.e. p(x) = ${}^8C_x \ (0.998)^x \ (0.002)^{8-x}$, x = 0, 1, 2,...,8

P(all 8 machines will produce all bolts within specification) = P[X = 8]

= p(8) = $^8C_8\ (0.998)^8\ (0.002)^{8-8}$

$= 1(0.998)^8 \cdot (1)$

$= (0.998)^8$

Hence, the probability that all 8 machines produce all bolts with specification $= (0.998)^8$

Miscellaneous exercise 8 | Q 11.2 | Page 524

The probability that a machine will produce all bolts in a production run within specification is 0.998. A sample of 8 machines is taken at random. Calculate the probability that 7 or 8 machines.

SOLUTION

Let X = number of machines that produce the bolts within specification.

p = probability that a machine produce bolts within specification

p = 0.998

and q = 1 - p = 1 - 0.998 = 0.002

Given: n = 8

∴ X ~ B (8, 0.998)

The p.m.f. of X is given by

P(X = x) = $^nC_x\ p^x\ q^{n-x}$

i.e. p(x) = $^8C_x\ (0.998)^x\ (0.002)^{8-x}$, x = 0, 1, 2,...,8

P(7 or 8 machines will produce all bolts within specification) = P(X = 7) + P(X = 8)

$= {}^8C_7(0.998)^7(0.002)^{8-7} + {}^8C_8(0.998)^8(0.002)^{8-8}$

$= 8 \times (0.998)^7(0.002)^1 + 1 \times (0.998)^8(0.002)^0$

$= (0.998)^7[8(0.002) + 0.998]$

$= (0.016 + 0.998)(0.998)^7$

$= (1.014) \times (0.998)^7$

Hence, the probability that 7 or 8 machines produce all bolts within specification = $(1.014) \times (0.998)^7$

Miscellaneous exercise 8 | Q 11.3 | Page 254

The probability that a machine will produce all bolts in a production run within specification is 0.998. A sample of 8 machines is taken at random. Calculate the probability that at most 6 machines will produce all bolts within specification.

SOLUTION

Let X = number of machines that produce the bolts within specification.

p = probability that a machine produce bolts within specification

p = 0.998

and q = 1 - p = 1 - 0.998 = 0.002

Given: n = 8

∴ X ~ B (8, 0.998)

The p.m.f. of X is given by

P(X = x) = ${}^nC_x\ p^x\ q^{n-x}$

i.e. p(x) = ${}^8C_x\ (0.998)^x\ (0.002)^{8-x}$, x = 0, 1, 2,...,8

P(at most 6 machines will produce all bolts with specification)

= P[X ≤ 6] = 1 - P[x > 6]

= 1 - [P (X = 7) + P(X = 8)]

= 1 - [P(7) + P(8)]

= 1 - (1.014)$(0.998)^7$

Hence, the probability that at most 6 machines will produce all bolts with specification = 1 - (1.014)$(0.998)^7$

Miscellaneous exercise 8 | Q 12 | Page 254

The probability that a machine develops a fault within the first 3 years of use is 0.003. If 40 machines are selected at random, calculate the probability that 38 or more will develop any faults within the first 3 years of use.

SOLUTION

Let X = number of machines who develops a fault.

p = probability that a machine develops a falt within the first 3 years of use

∴ p = 0.003

and q = 1 - p = 1 - 0.003 = 0.997

Given: n = 40

∴ X ~ B (40, 0.003)

The p.m.f. of X is given by

P(X = x) = ${}^nC_x\ p^x\ q^{n-x}$, x = 0, 1, 2,...,n

i.e. p(x) = ${}^{40}C_x\ (0.003)^x\ (0.997)^{40-x}$, x = 0, 1, 2, ...,40

P(38 or more machines will develop any fault)

= P(X ≥ 38) = P(X = 38) + P(X = 39) + P(X = 40)

= p(38) + p(39) + p(40)

$$= {}^{40}C_{38}(0.003)^{38}(0.997)^{40-38} + {}^{40}C_{39}(0.003)^{39}(0.997)^{40-39} + {}^{40}C_{40}(0.003)^{40}(0.997)^{40-40}$$

$$= \frac{40\times 39}{2\times 1}(0.003)^{38}(0.997)^2 + 40(0.003)^{39}(0.997)^1 + 1\cdot(0.003)^{40}(0.997)^0$$

$$= (780)(0.003)^{38}(0.997)^2 + (40)(0.003)^{39}(0.997) + 1\times(0.003)^{40}\times 1$$

$$= (0.003)^{38}\left[(780)(0.997)^2 + 40(0.003)(0.997) + (0.003)^2\right]$$

$$= (0.003)^{38}[775.327 + 0.1196 + 0.000009]$$

$$= (0.003)^{38}(775.446609)$$

$$= (775.446609)(0.003)^{38}$$

$$\approx (775.44)(0.003)^{38}$$

Hence, the probability that 38 or more machines will develop the fault within 3 years of use = $(775.44)(0.003)^{38}$.

Miscellaneous exercise 8 | Q 13.1 | Page 254

A computer installation has 10 terminals. Independently, the probability that any one terminal will require attention during a week is 0.1. Find the probabilities that 0.

SOLUTION

Let X = number of terminals which required attention during a week.

p = probability that any terminal will require attention during a week

∴ p = 0.1

and q = 1 - p = 1 - 0.1 = 0.9

Given: n = 10

∴ X ~ B (10, 0.1)

The p.m.f. of X is given by

P(X = x) = ${}^nC_x\ p^x\ q^{n-x}$

i.e. p(x) = ${}^{10}C_x\ (0.1)^x\ (0.9)^{10-x}$, x = 0, 1, 2,...,10

P(no terminal will require attention) = P(X = 0)

$$= \mathbf{p(0)} = {}^{10}C_0\ (0.1)^0\ (0.9)^{10-0}$$

$$= 1 \times 1 \times (0.9)^{10} = (0.9)^{10}$$

Hence, the probability that no terminal requires attention $(0.9)^{10}$

Miscellaneous exercise 8 | Q 13.2 | Page 254

A computer installation has 10 terminals. Independently, the probability that any one terminal will require attention during a week is 0.1. Find the probabilities that 1.

SOLUTION

Let X = number of terminals which required attention during a week.

p = probability that any terminal will require attention during a week

∴ p = 0.1

and q = 1 - p = 1 - 0.1 = 0.9

Given: n = 10

∴ X ~ B (10, 0.1)

The p.m.f. of X is given by

P(X = x) = ${}^nC_x\ p^x\ q^{n-x}$

i.e. p(x) = ${}^{10}C_x\ (0.1)^x\ (0.9)^{10-x}$, x = 0, 1, 2,...,10

P(1 terminal will require attention)

P(X = 1) = p(1) = ${}^{10}C_1\ (0.1)^1\ (0.9)^{10-1}$

$$= 10(0.1)(0.9)^9$$

$$= (1.0)(0.9)^9$$

$= (0.9)^9$

Hence, the probability that 1 terminal requires attention $= (0.9)^9$

Miscellaneous exercise 8 | Q 13.3 | Page 254

A computer installation has 10 terminals. Independently, the probability that any one terminal will require attention during a week is 0.1. Find the probabilities that 2.

SOLUTION

Let X = number of terminals which required attention during a week.

p = probability that any terminal will require attention during a week

∴ p = 0.1

and q = 1 - p = 1 - 0.1 = 0.9

Given: n = 10

∴ X ~ B (10, 0.1)

The p.m.f. of X is given by

P(X = x) = ${}^nC_x\ p^x\ q^{n-x}$

i.e. p(x) = ${}^{10}C_x\ (0.1)^x\ (0.9)^{10-x}$, x = 0, 1, 2,...,10

P(2 terminals will require attention)

P(X = 2) = p(2) = ${}^{10}C_2(0.1)^2(0.9)^{10-2}$

$$= \frac{10 \times 9}{1 \times 2}(0.1)^2(0.9)^8$$

$$= 45(0.01)(0.9)^8$$

$$= (0.45) \times (0.9)^8$$

Hence, the probability that 2 terminals require attention $= (0.45) \times (0.9)^8$

Miscellaneous exercise 8 | Q 13.4 | Page 254

A computer installation has 10 terminals. Independently, the probability that any one terminal will require attention during a week is 0.1. Find the probabilities that 3 or more, terminals will require attention during the next week.

SOLUTION

Let X = number of terminals which required attention during a week.

p = probability that any terminal will require attention during a week

∴ p = 0.1

and q = 1 - p = 1 - 0.1 = 0.9

Given: n = 10

∴ X ~ B(10, 0.1)

The p.m.f. of X is given by

P(X = x) = ${}^nC_x\ p^x\ q^{n-x}$

i.e. p(x) = ${}^{10}C_x\ (0.1)^x\ (0.9)^{10-x}$ x = 0, 1, 2,...,10

P(3 or more terminals will require attention)

= P(X ≥ 3)

= 1 - P(x < 3)

= 1 - [P(X = 0) + P(X = 1) + P(X = 2)]

= 1 - [p(0) + p(1) + p(2)]

$$= 1 - \left[(0.9)^{40} + (0.9) + (0.45)(0.9)^8\right]$$

$$= 1 - \left[(0.9)^2 + (0.9)^4 + 0.45\right](0.9)^8$$

= 1 - [0.81 + 0.9 + 0.45]$(0.9)^8$

= 1 - (2.16) × $(0.9)^8$

Hence, the probability that 3 or more terminals require attention = 1 - (2.16) × $(0.9)^8$

Miscellaneous exercise 8 | Q 14.1 | Page 255

In a large school, 80% of the pupil like Mathematics. A visitor to the school asks each of 4 pupils, chosen at random, whether they like Mathematics.
Calculate the probabilities of obtaining an answer yes from 0, 1, 2, 3, 4 of the pupils.

SOLUTION

Let X = number of pupils like Mathematics.

p = probability that pupils like Mathematics

$\therefore$ p = 80% = $\frac{80}{100} = \frac{4}{5}$

and q = 1 - p = $1 - \frac{4}{5} = \frac{1}{5}$

Given: n = 4

$\therefore$ X ~ B $\left(4, \frac{4}{5}\right)$

The p.m.f. of X is given by

P(X = x) = ${}^{n}C_{x}\, p^{x}\, q^{n-x}$

i.e. p(x) = ${}^{4}C_{x} \left(\frac{4}{5}\right)^{x} \left(\frac{1}{5}\right)^{4-x}$ x = 0, 1, 2, 3, 4

The probabilities of obtaining an answer yes from 0, 1, 2, 3, 4 of pupils are P(X= 0), P(X = 1), P(X = 2), P(X = 3) and P(X = 4) respectively.

i.e. ${}^{4}C_{0} \left(\frac{4}{5}\right)^{0} \left(\frac{1}{5}\right)^{4-0}$, ${}^{4}C_{1} \left(\frac{4}{5}\right)^{1} \left(\frac{1}{5}\right)^{4-1}$, ${}^{4}C_{2} \left(\frac{4}{5}\right)^{2} \left(\frac{1}{5}\right)^{4-2}$, ${}^{4}C_{3} \left(\frac{4}{5}\right)^{3} \left(\frac{1}{5}\right)^{4-3}$ and ${}^{4}C_{4} \left(\frac{4}{5}\right)^{4} \left(\frac{1}{5}\right)^{4-4}$

i.e. $1(1)\left(\frac{1}{5}\right)^{4}, 4\left(\frac{4}{5}\right) \cdot \left(\frac{1}{5}\right)^{3}, \frac{4 \times 3}{1 \times 2}\left(\frac{16}{25}\right)\left(\frac{1}{25}\right), 4\left(\frac{64}{125}\right)\left(\frac{1}{5}\right)$ and $1 \times \left(\frac{4}{5}\right)^{4}\left(\frac{1}{5}\right)^{0}$

i.e. $\left(\frac{1}{5}\right)^{4}, \frac{16}{5}\left(\frac{1}{5}\right)^{3}, \frac{96}{5^2}\left(\frac{1}{5^2}\right), \frac{256}{5^3}\left(\frac{1}{5}\right)$ and $\frac{256}{5^4}$

i.e. $\frac{1}{5^4}, \frac{16}{5^4}, \frac{96}{5^4}, \frac{256}{5^4}, \frac{256}{5^4}$

OR $\frac{1}{625}, \frac{16}{625}, \frac{96}{625}, \frac{256}{625}$ and $\frac{256}{625}$

Miscellaneous exercise 8 | Q 14.2 | Page 255

In a large school, 80% of the pupil like Mathematics. A visitor to the school asks each of 4 pupils, chosen at random, whether they like Mathematics.
Find the probability that the visitor obtains answer yes from at least 2 pupils:
a. when the number of pupils questioned remains at 4.
b. when the number of pupils questioned is increased to 8.

SOLUTION

Let X = number of pupils like Mathematics.

p = probability that pupils like Mathematics

$\therefore$ p = 80% = $\frac{80}{100} = \frac{4}{5}$

and q = 1 - p = $1 - \frac{4}{5} = \frac{1}{5}$

Given: n = 4

$\therefore$ X ~ B $\left(4, \frac{4}{5}\right)$

The p.m.f. of X is given by

P(X = x) = ${}^nC_x\, p^x\, q^{n-x}$

i.e. p(x) = ${}^4C_x \left(\frac{4}{5}\right)^x \left(\frac{1}{5}\right)^{4-x}$ x = 0, 1, 2, 3, 4

(a) P(visitor obtains the answer yes from at least 2 pupils when the number of pupils questioned remains at 4) = P(X ≥ 2)

= P(X = 2) + P(X = 3) + P(X = 4)

$$= {}^4C_2 \left(\frac{4}{5}\right)^2 \left(\frac{1}{5}\right)^{4-2} + {}^4C_3 \left(\frac{4}{5}\right)^3 \left(\frac{1}{5}\right)^{4-3} + {}^4C_4 \left(\frac{4}{5}\right)^4 \left(\frac{1}{5}\right)^{4-4}$$

$$= \frac{4 \times 3}{1 \times 2} \times \frac{16}{5^2} \times \frac{1}{5^2} + 4 \times \frac{64}{5^3} \times \frac{1}{5} + 1 \times \frac{256}{5^4}$$

$$= \frac{96}{5^4} + \frac{256}{5^4} + \frac{256}{5^4}$$

$$= (96 + 256 + 256)\frac{1}{5^4}$$

$$= \frac{608}{5^4} = \frac{608}{625}$$

(b) P(the visitor obtains the answer yes from at least 2 pupils when number of pupils questioned is increased to 8)

= P(X ≥ 2)

= 1 - P(X < 2)

= 1 - [P(X = 0) + P(X = 1)]

$$= 1 - \left[{}^8C_0\left(\frac{4}{5}\right)^0\left(\frac{1}{5}\right)^{8-0} + {}^8C_1\left(\frac{4}{5}\right)^1\left(\frac{1}{5}\right)^{8-1}\right]$$

$$= 1 - \left[1(1)\left(\frac{1}{5}\right)^8 + (8)\left(\frac{4}{5}\right)\left(\frac{1}{5}\right)^7\right]$$

$$= 1 - \left[\frac{1}{5^8} + \frac{32}{5^8}\right]$$

$$= 1 - \frac{33}{5^8}.$$

Miscellaneous exercise 8 | Q 15.1 | Page 255

It is observed that it rains on 12 days out of 30 days. Find the probability that it rains exactly 3 days of week.

SOLUTION

Let X = number of days it rains in a week.

p = probability that it rains

$\therefore$ p = $\frac{12}{30} = \frac{2}{5}$

and q = 1 - p = $1 - \frac{2}{5} = \frac{3}{5}$

Given: n = 7

$\therefore$ X ~ B $\left(7, \frac{2}{5}\right)$

The p.m.f. of X is given by

P(X = x) = ${}^nC_x\ p^x\ q^{n-x}$

i.e. p(x) = ${}^7C_x\ \left(\frac{2}{5}\right)^x\ \left(\frac{3}{5}\right)^{7-x}$ x = 0, 1, 2, ..., 7

P(it rains exactly 3 days of week) = P(X = 3)

= p(3) = ${}^7C_3\ \left(\frac{2}{5}\right)^3\ \left(\frac{3}{5}\right)^{7-3}$

$$= \frac{7 \times 6 \times 5}{3 \times 2 \times 1}\left(\frac{8}{125}\right)\left(\frac{81}{625}\right)$$

$$= 35\left(\frac{8}{125}\right)\left(\frac{81}{625}\right) = \frac{35 \times 8 \times 81}{5^7}$$

$$= \frac{22680}{78125} = 0.2903$$

Hence, the probability that it rains exactly 3 days of week = $35 \times 8 \times \frac{81}{5^7}$ OR 0.2903.

Miscellaneous exercise 8 | Q 15.2 | Page 255

It is observed that it rains on 12 days out of 30 days. Find the probability that it it will rain at least 2 days of given week.

SOLUTION

Let X = number of days it rains in a week.

p = probability that it rains

$\therefore$ p = $\frac{12}{30} = \frac{2}{5}$

and q = 1 - p = $1-\frac{2}{5}=\frac{3}{5}$

Given: n = 7

$\therefore$ X ~ B $\left(7, \frac{2}{5}\right)$

The p.m.f. of X is given by

P(X = x) = ${}^nC_x\, p^x\, q^{n-x}$

i.e. p(x) = ${}^7C_x \left(\frac{2}{5}\right)^x \left(\frac{3}{5}\right)^{7-x}$ x = 0, 1, 2, ..., 7

P(it will rain on at least 2 days of given week)

= P(X ≥ 2) = 1 - P(X < 2)

= 1 - [P(X = 0) + P(X = 1)]

$$= 1-\left[{}^7C_0\left(\frac{2}{5}\right)^0\left(\frac{3}{5}\right)^{7-0}+{}^7C_1\left(\frac{2}{5}\right)^1\left(\frac{3}{5}\right)^{7-1}\right]$$

$$= 1-\left[1(1)\left(\frac{3}{7}\right)^7+7\left(\frac{2}{5}\right)\left(\frac{3}{5}\right)^6\right]$$

$$= 1-\left[\frac{3}{5}+\frac{14}{5}\right]\left(\frac{3}{5}\right)^6$$

$$= 1-\left(\frac{17}{5}\right)\left(\frac{729}{(5)^6}\right)=1-\frac{12393}{5^7}$$

$$= 1-\frac{12393}{78125}=1-0.1586$$

= 0.8414

Hence, the probability that it rains at least 2 days of given week = $1-\frac{12393}{5^7}$ OR 0.8414

Miscellaneous exercise 8 | Q 16 | Page 255

If the probability of success in a single trial is 0.01. How many trials are required in order to have a probability greater than 0.5 of getting at least one success?

SOLUTION

Let X = number of successes.

p = probability of success in a single trial

∴ p = 0.01

and q = 1 - p = 1 - 0.01 = 0.99

∴ X ~ B(n, 0.01)

The p.m.f. of X is given by

P(X = x) = ${}^nC_x\ p^x q^{n-x}$

i.e. p(x) = ${}^nC_x\ (0.01)^x\ (0.99)^{n-x}$ x = 1, 2,....,n

P(at least one success)

= P(X ≥ 1) = 1 - P(X < 1)

= 1 - P(X = 0) = 1 - p(0)

$= 1 - {}^nC_0\ (0.01)^0\ (0.99)^{n-0}$

$= 1 - 1(1)(0.99)^n$

$= 1 - (0.99)^n$

Given: P(X ≥ 1) > 0.5

i.e. $1 - (0.99)^n > 0.5$

i.e. 1 - 0.5 > $(0.99)^n$

i.e. 0.5 > $(0.99)^n$

i.e. $(0.99)^n < 0.5$

i.e. log $(0.99)^n$ < log (0.5)

i.e. n log (0.99) < log 0.5

i.e. n < $\dfrac{\log 0.5}{\log 0.99}$

i.e. n < 68.96

$\therefore$ n = 68

Hence, the number of trials required in order to have probability greater than 0.5 of getting at least one success is $\dfrac{\log 0.5}{\log 0.99}$ OR 68

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In binomial distribution with five Bernoulli's trials, the probability of one and two success are 0.4096 and 0.2048 respectively. Find the probability of success.

SOLUTION

Given: X ~ B(n = 5, p)

The probability of X successes is

P(X = x) = $^nC_x\, p^x\, q^{n-x}$, x = 0, 1, 2,...,n

i.e. P(X = x) = $^5C_x\, p^x\, q^{5-x}$, x = 0, 1, 2, 3, 4, 5

Probabilities of one and two successes are

P(X = 1) = $^5C_1\, p^1\, q^{5-1}$

and P(X = 2) = $^5C_2\, p^2\, q^{5-2}$ respectively

Given: P(X = 1) = 0.4096 and P(X = 2) = 0.2048

$\therefore \dfrac{\mathrm{P(X = 2)}}{\mathrm{P(X = 1)}} = \dfrac{0.2048}{0.4096}$

i.e. $\dfrac{^5C_2\, p^2\, q^{5-2}}{^5C_1\, p^2\, q^{5-1}} = \dfrac{1}{2}$

i.e. $2 \times {}^5C_2\, p^2\, q^3 = 1 \times {}^5C_1\, \mathrm{pq}^4$

i.e. $\mathbf{2 \times \frac{5 \times 4}{1 \times 2} \times p^2q^3 = 1 \times 5 \times pq^4}$

i.e. $\mathbf{20p^2q^3 = 5pq^4}$

i.e. 4p = q

i.e. 4p = 1 - p

i.e. 5p = 1

∴ p = $\mathbf{\frac{1}{5}}$

Hence, the probability of success is $\mathbf{\frac{1}{5}}$.