Chapter 3

Algebra

Exponents and Powers

- If 'a' is any integer, then $a \times a \times a \times$ .... a (n times) $= a^n$.
  Here 'a' is the base and n is the exponent or power.
  This method of expressing a number is called the 'Exponential form'.
  Example: $16 = 2 \times 2 \times 2 \times 2 = 2^4$
  It can be read as '2 raised to the power of 4' or '2 to the power of 4' or '2 power 4'.
  Here 2 is the base and 4 is the exponent.
  The exponent is usually written at the top right corner of the base and smaller in size when compared to the base.
- When a number is expressed as a product of factors and when the factors are repeated, then it can be expressed in the exponential form.
- The repeated factor will be the base and the number of times the factor repeats will be its exponent.

$$(-1)^n = \begin{cases} 1 & \text{if } n \text{ is even natural number} \\ -1 & \text{if } n \text{ is odd natural number} \end{cases}$$

- The exponent 2 has a special name 'squared'.
  Eg: 4^2 is read as 'four squared'.
- The exponent 3 has a special name 'cubed'.
  Eg: 4^3 is read as 'four cubed'.

Laws of Exponents

If 'a' and 'b' are non-zero numbers and 'm' and 'n' are natural numbers, then

(i) $a^m \times a^n = a^{m+n}$ (Product rule of exponents)

(ii) $a^m \div a^n = a^{m-n}$, $m > n$ (Quotient rule of exponents)

(iii) $(a^m)^n = a^{m \times n}$ (Power rule of exponents)

(iv) $(a \times b)^m = a^m \times b^n$ (Power rule of exponents)

Think

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What is Half of 2^{10}? Ragu claims the answer is 2^5. Is he correct? Discuss.

Sol : Half of 2^{10} $= \frac{1}{2} \times 2^{10} = \frac{2^{10}}{2^1}$ $\left[\text{since } \frac{a^m}{a^n} = a^{m-n}\right]$

$= 2^{10-1} = 2^9$

Half of 2^{10} is 2^9.

So Ragu's answer 2^5 is not correct.

TRY THESE

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Simply the following.

1. $23^5 \div 23^2$ **2.** $11^6 \div 11^3$ **3.** $(-5)^3 \div (-5)^2$ **4.** $7^3 \div 7^3$ **5.** $15^4 \div 15$

Sol : 1. $2^3 \div 2^5 = 2^{3-5} = 23^3$ $\left[\text{since } \frac{a^m}{a^n} = a^{m-n}\right]$

2. $11^6 \div 11^3 = 11^{6-3} = 11^3$ $\left[\text{since } \frac{a^m}{a^n} = a^{m-n}\right]$

3. $(-5)^3 \div (-5)^2 = (-5)^{3-2} = (-5)^1$ $\left[\text{since } \frac{a^m}{a^n} = a^{m-n}\right]$

4. $7^3 \div 7^3 = 7^{3-3} = 7^0 = 1$ $\left[\text{since } \frac{a^m}{a^n} = a^{m-n}; a^0 = 1\right]$

5. $15^4 \div 15 = 15^4 \div 15^1 = 15^{4-1} = 15^3$ $\left[\text{since } \frac{a^m}{a^n} = a^{m-n}\right]$

TRY THESE

(Text book Page No. 48)

Simplify and write the following in exponent form.

1. $(3^2)^3$ **2.** $[(-5)^3]^2$ **3.** $(20^6)^2$ **4.** $(10^3)^5$

Sol : 1. $(3^2)^3 = 3^{2\times3} = 3^6$ [since $(a^m)^n = a^{m\times n}$]

2. $[(-5)^3]^2 = (-5)^{3\times2} = (-5)^6$ [since $(a^m)^n = a^{m\times n}$]

3. $(20^6)^2 = 20^{6\times2} = 20^{12}$ [since $(a^m)^n = a^{m\times n}$]

4. $(10^3)^5 = 10^{3\times5} = 10^{15}$ [since $(a^m)^n = a^{m\times n}$]

Additional Questions

1. Find the degree of the following polynomials.

(i) $x^5 - x^4 + 3$ **(ii)** $2 - y^5 - y^3 + 2y^8$ **(iii)** 2

(iv) $5x^3 + 4x^2 + 7x$ **(v)** $4xy + 7x^2y + 3xy^3$

Sol :

(i) $x^5 - x^4 + 3$

The terms of the given expression are $x^5, -x^4, 3$.

Degree of each of the terms : 5, 4, 0

Term with highest degree: x^5

Therefore degree of the expression in 5.

(ii) $2 - y^5 - y^3 + 2y^8$

The terms of the given expression are $2, -y^5, -y^3, 2y^8$.

Degree of each of the terms : 0, 2, 3, 8.

Term with highest degree: $2y^8$

Therefore degree of the expression in 8.

(iii) 2

Degree of the constant term is 0.

$\therefore$ Degree of 2 is 0.

(iv) $5x^3 + 4x^2 + 7x$

The terms of the given expression are $5x^3, 4x^2, 7x$

Degree of each of the terms : 3, 2, 1.

Term with highest degree: $5x^3$

Therefore degree of the expression in 3.

(v) $4xy + 7x^2y + 3xy^3$

The terms of the given expression are $4xy, 7x^2y, 3xy^3$

Degree of each of the terms : 2, 3, 4

Term with highest degree: $3xy^3$

Therefore degree of the expression in 4.

2. State whether a given pair of terms in like or unlike terms.

(i) 1, 100 **(ii)** $-7x, \frac{5}{2}x$

(iii) $4m^2p, 4mp^2$ **(iv)** $12xz, 12x^2z^2$

Sol :

(i) 1, 100 is a pair of like terms.

[$\because 1 = x^o$ and $100 = 100x^o$]

Exercise 3.4

Miscellaneous Practice Problems

1. $6^2 \times 6^m = 6^5$, find the value of 'm'.

Sol : $6^2 \times 6^m = 6^5$

$6^{2+m} = 6^5$ [Since $a^m \times a^n = a^{m+n}$]

Equating the powers, we get

$$2 + m = 5$$
$$m = 5 - 2 = 3$$

2. Find the unit digit of $124^{128} \times 126^{124}$.

Sol : In 124^{128}, the unit digit of base 124 is 4 and the power is 128 (even power).

Therefore, unit digit of 124^{128} is 4.

Also in 126^{124}, the unit digit of base 126 is 6 and the power is 124 (even power).

Therefore, unit digit of 126^{124} is 6.

Product of the unit digits = 6 × 6 = 36

∴ Unit digit of the $124^{128} \times 126^{124}$ is 6.

3. Find the unit digit of the numeric expression: $16^{23} + 71^{48} + 59^{61}$

Sol : In 16^{23}, the unit digit of base 16 is 6 and the power is 23 (odd power).

Therefore, unit digit of 16^{23} is 6.

In 71^{48}, the unit digit of base 71 is 1 and the power is 48 (even power).

Therefore, unit digit of 71^{48} is 1.

Also in 59^{61}, the unit digit of base 59 is 9 and the power is 61 (odd power).

Therefore, unit digit of 59^{61} is 9.

Sum of the unit digits = 6 + 1 + 9 = 16

∴ Unit digit of the given expression is 6.

4. Find the value of $\dfrac{(-1)^6 \times (-1)^7 \times (-1)^8}{(-1)^3 \times (-1)^5}$

Sol : $$\frac{(-1)^6 \times (-1)^7 \times (-1)^8}{(-1)^3 \times (-1)^5} = \frac{(-1)^{6+7+8}}{(-1)^{3+5}} = \frac{(-1)^{21}}{(-1)^8} = (-1)^{21-8} \text{ [By Quotient rule]}$$

$$= (-1)^{13}$$
$$= -1$$

[Since the power 13 is odd positive number]

$$\therefore \frac{(-1)^6 \times (-1)^7 \times (-1)^8}{(-1)^3 \times (-1)^5} = 1$$