

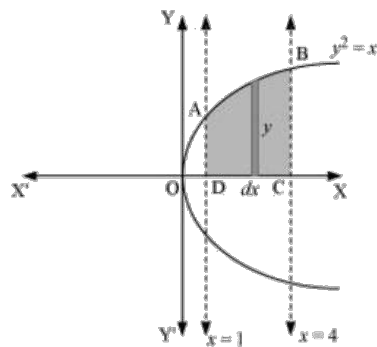
Chapter 8 Applications of Integrals

EXERCISE 8.1

Question 1:

Find the area of the region bounded by the curve $y^2 = x$ and the lines $x = 1, x = 4$ and the x -axis in the first quadrant.

Solution:

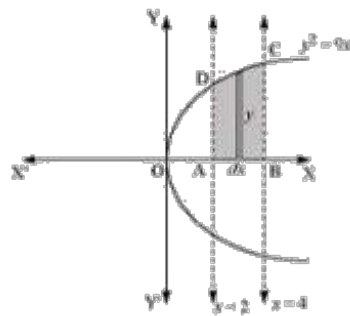


$$\begin{aligned} \text{ar}(ABCD) &= \int_1^4 y dx \\ &= \int_1^4 \sqrt{x} dx \\ &= \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_1^4 \\ &= \frac{2}{3} \left[(4)^{\frac{3}{2}} - (1)^{\frac{3}{2}} \right] \\ &= \frac{2}{3} [8 - 1] \\ &= \frac{14}{3} \end{aligned}$$

Question 2:

Find the area of the region bounded by $y^2 = 9x, x = 2, x = 4$ and the x -axis in the first quadrant.

Solution:

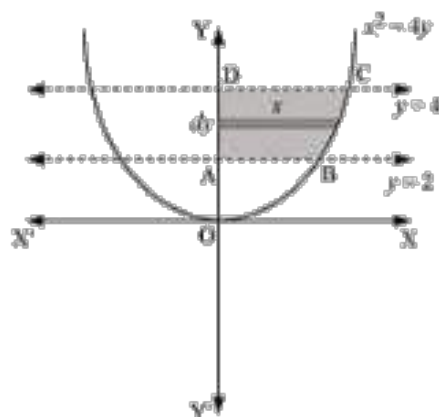


$$\begin{aligned}
 \text{ar}(ABCD) &= \int_2^4 y dx \\
 &= \int_2^4 3\sqrt{x} dx \\
 &= 3 \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_2^4 \\
 &= 2 \left[x^{\frac{3}{2}} \right]_2^4 \\
 &= 2 \left[(4)^{\frac{3}{2}} - (2)^{\frac{3}{2}} \right] \\
 &= 2 \left[8 - 2\sqrt{2} \right] \\
 &= (16 - 4\sqrt{2})
 \end{aligned}$$

Question 3:

Find the area of the region bounded by $x^2 = 4y$, $y = 2$, $y = 4$ and the y -axis in the first quadrant.

Solution:

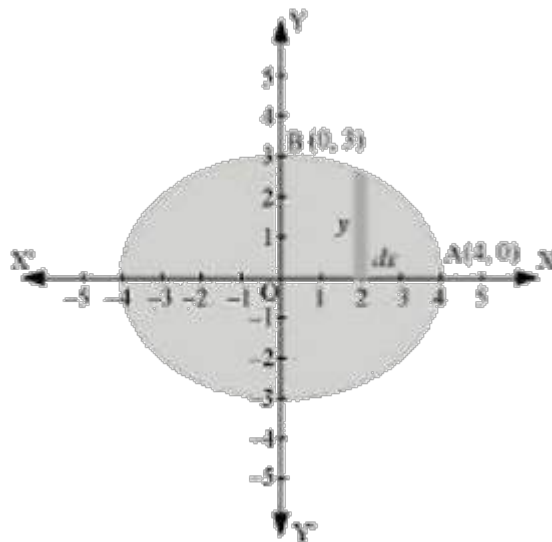


$$\begin{aligned}
 ar(ABCD) &= \int_2^4 x dy \\
 &= \int_2^4 2\sqrt{y} dy = 2 \int_2^4 \sqrt{y} dy \\
 &= 2 \left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}} \right]_2^4 \\
 &= \frac{4}{3} \left[(4)^{\frac{3}{2}} - (2)^{\frac{3}{2}} \right] = \frac{4}{3} [8 - 2\sqrt{2}] \\
 &= \left(\frac{32 - 8\sqrt{2}}{3} \right)
 \end{aligned}$$

Question 4:

Find the area of the region bounded by the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$.

Solution:



It is given that

$$\begin{aligned}
 \Rightarrow \frac{x^2}{16} + \frac{y^2}{9} &= 1 \\
 \Rightarrow \frac{y^2}{9} &= 1 - \frac{x^2}{16} \\
 \Rightarrow y &= 3\sqrt{1 - \frac{x^2}{16}}
 \end{aligned}$$

Area of ellipse = $4 \times ar(OAB)$

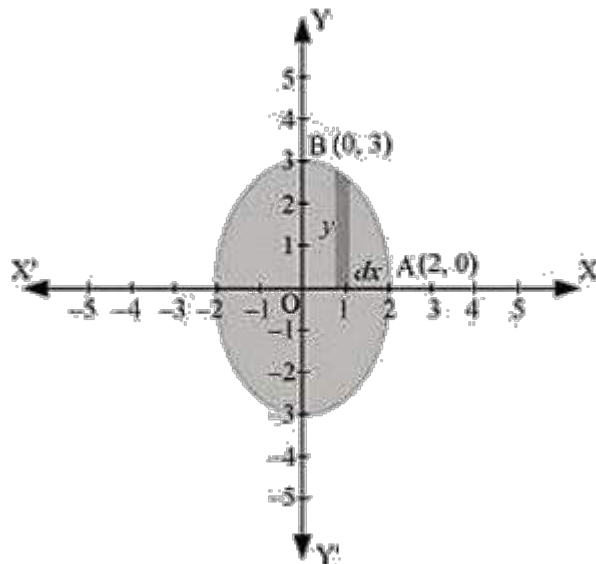
$$\begin{aligned}
 \text{ar}(OAB) &= \int_0^4 y dx \\
 &= \int_0^4 3\sqrt{1 - \frac{x^2}{16}} dx \\
 &= \frac{3}{4} \int_0^4 \sqrt{16 - x^2} dx \\
 &= \frac{3}{4} \left[\frac{x}{2} \sqrt{16 - x^2} + \frac{16}{2} \sin^{-1} \frac{x}{4} \right]_0^4 \\
 &= \frac{3}{4} \left[2\sqrt{16 - 16} + 8 \sin^{-1}(1) - 0 - 8 \sin^{-1}(0) \right] \\
 &= \frac{3}{4} \left[\frac{8\pi}{2} \right] \\
 &= \frac{3}{4} [4\pi] \\
 &= 3\pi
 \end{aligned}$$

Area of ellipse $= 4 \times 3\pi = 12\pi$ units

Question 5:

Find the area of the region bounded by the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$

Solution:



It is given that

$$\begin{aligned}
 \Rightarrow \frac{x^2}{4} + \frac{y^2}{9} &= 1 \\
 \Rightarrow y &= 3\sqrt{1 - \frac{x^2}{4}}
 \end{aligned}$$

$$\text{Area of ellipse} = 4 \times \text{ar}(OAB)$$

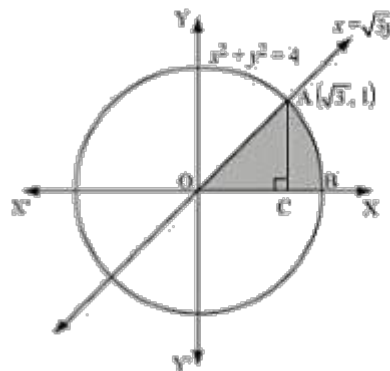
$$\begin{aligned} \text{ar}(OAB) &= \int_0^2 y dx \\ &= \int_0^2 3\sqrt{1 - \frac{x^2}{4}} dx \\ &= \frac{3}{2} \int_0^2 \sqrt{4 - x^2} dx \\ &= \frac{3}{2} \left[\frac{x}{2} \sqrt{4 - x^2} + \frac{4}{2} \sin^{-1} \frac{x}{2} \right]_0^2 \\ &= \frac{3}{2} \left[\frac{2\pi}{2} \right] \\ &= \frac{3\pi}{2} \end{aligned}$$

$$\text{Area of ellipse} = 4 \times \frac{3\pi}{2} = 6\pi \text{ units.}$$

Question 6:

Find the area of the region in the first quadrant enclosed by x -axis, line $x = \sqrt{3}y$ and the circle $x^2 + y^2 = 4$

Solution:



$$\text{ar}(OAB) = \text{ar}(\triangle OAC) + \text{ar}(ABC)$$

$$\begin{aligned} \text{ar}(\triangle OAC) &= \frac{1}{2} \times OC \times AC \\ &= \frac{1}{2} \times \sqrt{3} \times 1 \\ &= \frac{\sqrt{3}}{2} \end{aligned}$$

$$\begin{aligned}
 \text{ar}(ABC) &= \int_{\sqrt{3}}^2 y dx \\
 &= \int_{\sqrt{3}}^2 \sqrt{4-x^2} dx \\
 &= \left[\frac{x}{2} \sqrt{4-x^2} + \frac{4}{2} \sin^{-1} \left(\frac{x}{2} \right) \right]_{\sqrt{3}}^2 \\
 &= \left[2 \times \frac{\pi}{2} - \frac{\sqrt{3}}{2} \sqrt{4-3} - 2 \sin^{-1} \left(\frac{\sqrt{3}}{2} \right) \right] \\
 &= \left[\pi - \frac{\sqrt{3}}{2} - 2 \left(\frac{\pi}{3} \right) \right] \\
 &= \left[\pi - \frac{2\pi}{3} - \frac{\sqrt{3}}{2} \right] \\
 &= \left[\frac{3\pi - 2\pi}{3} - \frac{\sqrt{3}}{2} \right] \\
 &= \left[\frac{\pi}{3} - \frac{\sqrt{3}}{2} \right]
 \end{aligned}$$

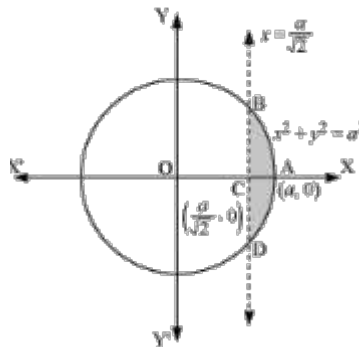
Therefore, required area enclosed $= \frac{\sqrt{3}}{2} + \frac{\pi}{3} - \frac{\sqrt{3}}{2} = \frac{\pi}{3}$ square units.

Question 7:

Find the area of the smaller part of the circle $x^2 + y^2 = a^2$ cut off by the line $x = \frac{a}{\sqrt{2}}$.

Solution:

The area of the smaller part of the circle, $x^2 + y^2 = a^2$ cut off by the line, $x = \frac{a}{\sqrt{2}}$, is the area ABCD.



It can be observed that the area ABCD is symmetrical about x -axis.

$$ar(ABCD) = 2 \times ar(ABC)$$

$$\begin{aligned}
 ar(ABC) &= \int_{\frac{a}{\sqrt{2}}}^a y dx \\
 &= \int_{\frac{a}{\sqrt{2}}}^a \sqrt{a^2 - x^2} dx \\
 &= \left[\frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \left(\frac{x}{a} \right) \right]_{\frac{a}{\sqrt{2}}}^a \\
 &= \left[\frac{a^2}{2} \left(\frac{\pi}{2} \right) - \frac{a}{2\sqrt{2}} \sqrt{a^2 - \frac{a^2}{2}} - \frac{a^2}{2} \sin^{-1} \left(\frac{1}{\sqrt{2}} \right) \right] \\
 &= \frac{a^2 \pi}{4} - \frac{a}{2\sqrt{2}} \cdot \frac{a}{\sqrt{2}} - \frac{a^2}{2} \left(\frac{\pi}{4} \right) \\
 &= \frac{a^2 \pi}{4} - \frac{a^2}{4} - \frac{a^2 \pi}{8} \\
 &= \frac{a^2}{4} \left[\pi - 1 - \frac{\pi}{2} \right] \\
 &= \frac{a^2}{4} \left[\frac{\pi}{2} - 1 \right]
 \end{aligned}$$

$$\begin{aligned}
 ar(ABCD) &= 2 \left[\frac{a^2}{4} \left(\frac{\pi}{2} - 1 \right) \right] \\
 &= \frac{a^2}{2} \left(\frac{\pi}{2} - 1 \right)
 \end{aligned}$$

Therefore, the required area is $\frac{a^2}{2} \left(\frac{\pi}{2} - 1 \right)$ square units.

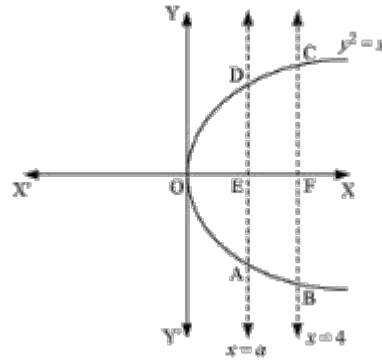
Question 8:

The area between $x = y^2$ and $x = 4$ is divided into two equal parts by the line $x = a$, find the value of a .

Solution:

The line $x = a$ divides the area bounded by the parabola and $x = 4$ into two equal parts.

Therefore, $ar(OAD) = ar(ABCD)$



It can be observed that the given area is symmetrical about x -axis.

Hence, $ar(OED) = ar(EFCD)$

$$\begin{aligned}
 ar(OED) &= \int_0^a y dx \\
 &= \int_0^a \sqrt{x} dx \\
 &= \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^a \\
 &= \frac{2}{3} a^{\frac{3}{2}} \quad \dots(1)
 \end{aligned}$$

$$\begin{aligned}
 ar(EFCD) &= \int_a^4 \sqrt{x} dx \\
 &= \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_a^4 \\
 &= \frac{2}{3} \left[8 - a^{\frac{3}{2}} \right] \quad \dots(2)
 \end{aligned}$$

From (1) and (2), we obtain

$$\Rightarrow \frac{2}{3}(a)^{\frac{3}{2}} = \frac{2}{3} \left[8 - (a)^{\frac{3}{2}} \right]$$

$$\Rightarrow 2(a)^{\frac{3}{2}} = 8$$

$$\Rightarrow (a)^{\frac{3}{2}} = 4$$

$$\Rightarrow a = (4)^{\frac{2}{3}}$$

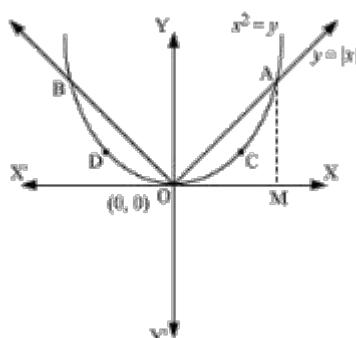
Therefore, the value of $a = (4)^{\frac{2}{3}}$.

Question 9:

Find the area of the region bounded by the parabola $y = x^2$ and the line $y = |x|$.

Solution:

The area bounded by the parabola $y = x^2$ and the line $y = |x|$, can be represented as



The given area is symmetrical about y-axis.

Therefore, $ar(OACO) = ar(ODBO)$

The point of intersection of parabola $y = x^2$ and the line $y = |x|$, is $A(1,1)$.

$ar(OACO) = ar(\Delta OAM) - ar(OMACO)$

$$ar(\Delta OAM) = \frac{1}{2} \times OM \times AM$$

$$= \frac{1}{2} \times 1 \times 1$$

$$= \frac{1}{2}$$

$$\begin{aligned}
 \text{ar}(OMACO) &= \int_0^1 y dx \\
 &= \int_0^1 x^2 dx \\
 &= \left[\frac{x^3}{3} \right]_0^1 \\
 &= \frac{1}{3}
 \end{aligned}$$

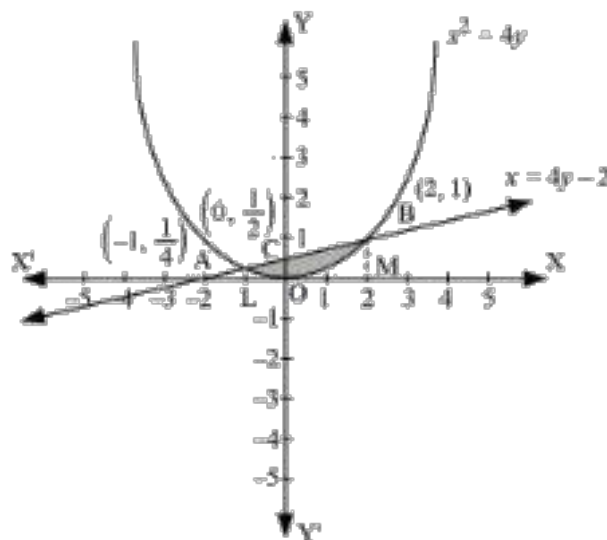
$$\begin{aligned}
 \text{ar}(OACO) &= \text{ar}(\triangle OAM) - \text{ar}(OMACO) \\
 &= \frac{1}{2} - \frac{1}{3} \\
 &= \frac{1}{6}
 \end{aligned}$$

Therefore, the required area $= 2 \left[\frac{1}{6} \right] = \frac{1}{3}$ units.

Question 10:

Find the area bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$.

Solution:



Coordinates of point $A\left(-1, \frac{1}{4}\right)$.

Coordinates of point $B(2, 1)$.

Draw AL and BM perpendicular to x -axis.

$$ar(OBAO) = ar(OBCO) + ar(OACO)$$

$$ar(OBCO) = ar(OMBC) - ar(OMBO)$$

$$\begin{aligned} &= \int_0^2 \frac{x+2}{4} dx - \int_0^2 \frac{x^2}{4} dx \\ &= \frac{1}{4} \left[\frac{x^2}{2} + 2x \right]_0^2 - \frac{1}{4} \left[\frac{x^3}{3} \right]_0^2 \\ &= \frac{1}{4} [2+4] - \frac{1}{4} \left[\frac{8}{3} \right] \\ &= \frac{3}{2} - \frac{2}{3} \\ &= \frac{5}{6} \end{aligned}$$

$$ar(OACO) = ar(OLAC) - ar(OLAO)$$

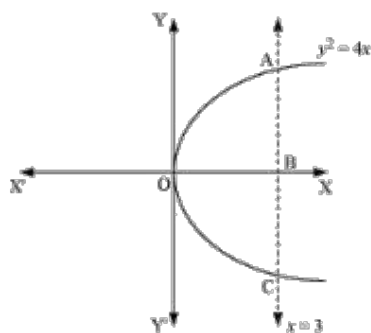
$$\begin{aligned} &= \int_{-1}^0 \frac{x+2}{4} dx - \int_{-1}^0 \frac{x^2}{4} dx \\ &= \frac{1}{4} \left[\frac{x^2}{2} + 2x \right]_{-1}^0 - \frac{1}{4} \left[\frac{x^3}{3} \right]_{-1}^0 \\ &= -\frac{1}{4} \left[\frac{(-1)^2}{2} + 2(-1) \right] - \left[-\frac{1}{4} \left(\frac{(-1)^3}{3} \right) \right] \\ &= -\frac{1}{4} \left[\frac{1}{2} - 2 \right] - \frac{1}{12} \\ &= -\frac{1}{8} + \frac{1}{2} - \frac{1}{12} \\ &= \frac{7}{24} \end{aligned}$$

$$\text{Required area} = \left(\frac{5}{6} + \frac{7}{24} \right) = \frac{9}{8} \text{ units.}$$

Question 11:

Find the area of the region bounded by the curve $y^2 = 4x$ and the line $x = 3$.

Solution:



OACO is symmetrical about x -axis.

Therefore, $ar(OACO) = 2 \times ar(AOB)$

$$\begin{aligned}
 ar(OACO) &= 2 \left[\int_0^3 y dx \right] \\
 &= 2 \left[\int_0^3 2\sqrt{x} dx \right] \\
 &= 4 \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^3 \\
 &= \frac{8}{3} \left[(3)^{\frac{3}{2}} \right] \\
 &= 8\sqrt{3}
 \end{aligned}$$

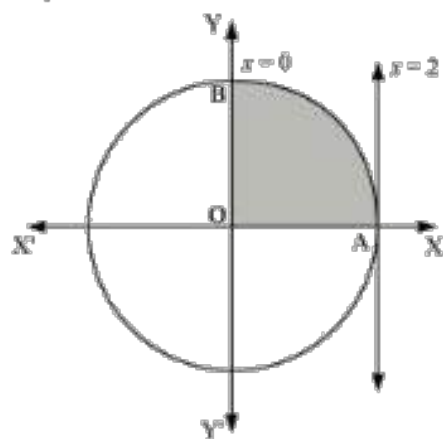
Required area is $8\sqrt{3}$ units.

Question 12:

Area lying in the first quadrant and bounded by the circle $x^2 + y^2 = 4$ and the lines $x = 0$ and $x = 2$ is

- (A) π (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{4}$

Solution:



$$\begin{aligned}ar(OAB) &= \int_0^2 y dx \\&= \int_0^2 \sqrt{4-x^2} dx \\&= \left[\frac{x}{2} \sqrt{4-x^2} + \frac{4}{2} \sin^{-1} \frac{x}{2} \right]_0^2 \\&= 2 \left(\frac{\pi}{2} \right) \\&= \pi\end{aligned}$$

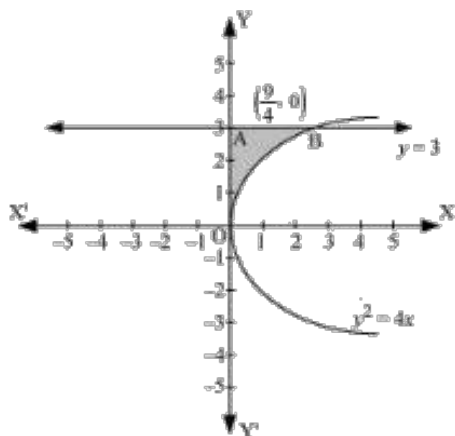
Correct answer is A.

Question 13:

Area of the region bounded by the curve $y^2 = 4x$, y -axis and the line $y = 3$ is

- (A) 2 (B) $\frac{9}{4}$ (C) $\frac{9}{3}$ (D) $\frac{9}{2}$

Solution:



$$\begin{aligned} \text{ar}(OAB) &= \int_0^3 x dy \\ &= \int_0^3 \frac{y^2}{4} dy \\ &= \frac{1}{4} \left[\frac{y^3}{3} \right]_0^3 \\ &= \frac{1}{12} (27) \\ &= \frac{9}{4} \end{aligned}$$

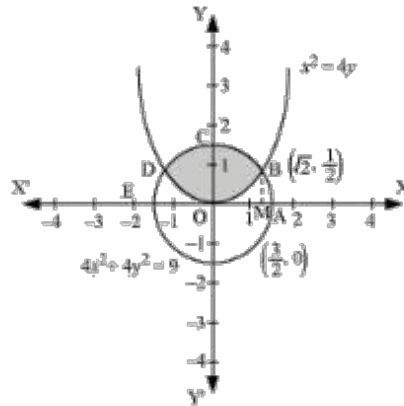
Correct answer is B.

EXERCISE 8.2

Question 1:

Find the area of the circle $4x^2 + 4y^2 = 9$ which is interior to the parabola $x^2 = 4y$.

Solution:



Solving $4x^2 + 4y^2 = 9$ and $x^2 = 4y$, point of intersection $B\left(\sqrt{2}, \frac{1}{2}\right)$ and $D\left(-\sqrt{2}, \frac{1}{2}\right)$.
Required area is symmetrical about y-axis.

$$ar(OBCDO) = 2 \times ar(OBCO)$$

Draw BM perpendicular to OA

Coordinates of M are $(\sqrt{2}, 0)$

$$ar(OBCO) = ar(OMBCO) - ar(OMBO)$$

$$\begin{aligned} &= \int_0^{\sqrt{2}} \sqrt{\frac{(9-4x^2)}{4}} dx - \int_0^{\sqrt{2}} \frac{x^2}{4} dx \\ &= \frac{1}{2} \int_0^{\sqrt{2}} \sqrt{9-4x^2} dx - \frac{1}{4} \int_0^{\sqrt{2}} x^2 dx \\ &= \frac{1}{4} \left[x\sqrt{9-4x^2} + \frac{9}{2} \sin^{-1} \frac{2x}{3} \right]_0^{\sqrt{2}} - \frac{1}{4} \left[\frac{x^3}{3} \right]_0^{\sqrt{2}} \\ &= \frac{1}{4} \left[\sqrt{2}\sqrt{9-8} + \frac{9}{2} \sin^{-1} \frac{2\sqrt{2}}{3} \right] - \frac{1}{12} (\sqrt{2})^3 \\ &= \frac{\sqrt{2}}{4} + \frac{9}{8} \sin^{-1} \frac{2\sqrt{2}}{3} - \frac{\sqrt{2}}{6} \\ &= \frac{\sqrt{2}}{12} + \frac{9}{8} \sin^{-1} \frac{2\sqrt{2}}{3} \\ &= \frac{1}{4} \left(\frac{\sqrt{2}}{3} + \frac{9}{2} \sin^{-1} \frac{2\sqrt{2}}{3} \right) \end{aligned}$$

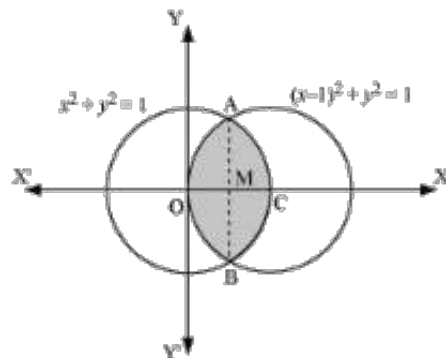
Required area OBCDO

$$= \left(2 \times \frac{1}{4} \left[\frac{\sqrt{2}}{3} + \frac{9}{2} \sin^{-1} \frac{2\sqrt{2}}{3} \right] \right) = \left[\frac{\sqrt{2}}{6} + \frac{9}{4} \sin^{-1} \frac{2\sqrt{2}}{3} \right] \text{ units.}$$

Question 2:

Find the area bounded by curves $(x-1)^2 + y^2 = 1$ and $x^2 + y^2 = 1$.

Solution:



Solving $(x-1)^2 + y^2 = 1$ and $x^2 + y^2 = 1$, point of intersection $A\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ and $B\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$
 Required area is symmetrical about x-axis.

$$ar(OBCAO) = 2 \times ar(OCAO)$$

Join AB, intersects OC at M

AM is perpendicular to OC

Coordinates of $M\left(\frac{1}{2}, 0\right)$

$$\begin{aligned}
 \text{ar}(OCAO) &= \text{ar}(OMAO) + \text{ar}(MCAM) \\
 &= \left[\int_0^{\frac{1}{2}} \sqrt{1-(x-1)^2} dx + \int_{\frac{1}{2}}^1 \sqrt{1-x^2} dx \right] \\
 &= \left[\frac{x-1}{2} \sqrt{1-(x-1)^2} + \frac{1}{2} \sin^{-1}(x-1) \right]_0^{\frac{1}{2}} + \left[\frac{x}{2} \sqrt{1-x^2} + \frac{1}{2} \sin^{-1} x \right]_{\frac{1}{2}}^1 \\
 &= \left[-\frac{1}{4} \sqrt{1-\left(\frac{1}{2}\right)^2} + \frac{1}{2} \sin^{-1}\left(\frac{1}{2}-1\right) - \frac{1}{2} \sin^{-1}(-1) \right] + \left[+\frac{1}{2} \sin^{-1}(-1) - \frac{1}{4} \sqrt{1-\left(\frac{1}{2}\right)^2} - \frac{1}{2} \sin^{-1}\left(\frac{1}{2}\right) \right] \\
 &= \left[-\frac{\sqrt{3}}{8} + \frac{1}{2} \left(-\frac{\pi}{6}\right) - \frac{1}{2} \left(-\frac{\pi}{2}\right) \right] + \left[\frac{1}{2} \left(\frac{\pi}{2}\right) - \frac{\sqrt{3}}{8} - \frac{1}{2} \left(\frac{\pi}{6}\right) \right] \\
 &= \left[-\frac{\sqrt{3}}{4} - \frac{\pi}{12} + \frac{\pi}{4} + \frac{\pi}{4} - \frac{\pi}{12} \right] \\
 &= \left[-\frac{\sqrt{3}}{4} - \frac{\pi}{6} + \frac{\pi}{2} \right] \\
 &= \left[\frac{2\pi}{6} - \frac{\sqrt{3}}{4} \right]
 \end{aligned}$$

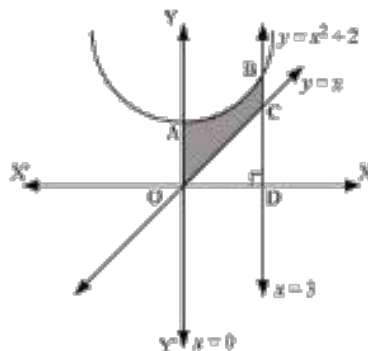
Required Area OBCAO is

$$\left(2 \times \left[\frac{2\pi}{6} - \frac{\sqrt{3}}{4} \right] \right) = \left[\frac{2\pi}{3} - \frac{\sqrt{3}}{2} \right] \text{ units.}$$

Question 3:

Find the area of the region bounded by the curves $y = x^2 + 2$, $y = x$, $x = 0$ and $x = 3$

Solution:

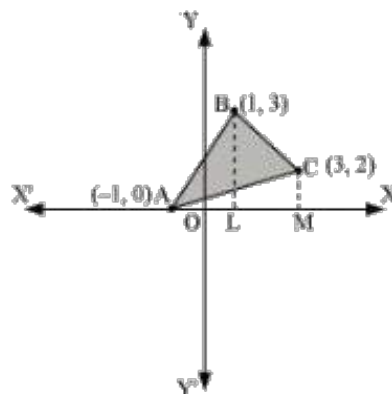


$$\begin{aligned}
 \text{ar}(\text{OCBAO}) &= \text{ar}(\text{ODBAO}) - \text{ar}(\text{ODCO}) \\
 &= \int_0^3 (x^2 + 2) dx - \int_0^3 x dx \\
 &= \left[\frac{x^3}{3} + 2x \right]_0^3 - \left[\frac{x^2}{2} \right]_0^3 \\
 &= [9 + 6] - \left[\frac{9}{2} \right] \\
 &= 15 - \frac{9}{2} \\
 &= \frac{21}{2}
 \end{aligned}$$

Question 4:

Using integration finds the area of the region bounded by the triangle whose vertices are $(-1, 0)$, $(1, 3)$ and $(3, 2)$.

Solution:



BL and CM are perpendicular to x -axis.

$$\text{ar}(\triangle ACB) = \text{ar}(\triangle ALBA) + \text{ar}(\triangle BLMCB) - \text{ar}(\triangle AMCA)$$

Equation of AB is

$$\begin{aligned}
 y - 0 &= \frac{3 - 0}{1 - (-1)}(x + 1) \\
 y &= \frac{3}{2}(x + 1)
 \end{aligned}$$

$$\begin{aligned}
 ar(ALBA) &= \int_{-1}^1 \frac{3}{2}(x+1)dx \\
 &= \frac{3}{2} \left[\frac{x^2}{2} + x \right]_{-1}^1 \\
 &= \frac{3}{2} \left[\frac{1}{2} + 1 - \frac{1}{2} + 1 \right] \\
 &= 3
 \end{aligned}$$

Equation of BC is

$$\begin{aligned}
 y-3 &= \frac{2-3}{3-1}(x-1) \\
 y &= \frac{1}{2}(-x+7)
 \end{aligned}$$

$$\begin{aligned}
 ar(BLMCB) &= \int_1^3 \frac{1}{2}(-x+7)dx \\
 &= \frac{1}{2} \left[-\frac{x^2}{2} + 7x \right]_1^3 \\
 &= \frac{1}{2} \left[-\frac{9}{2} + 21 + \frac{1}{2} - 7 \right] \\
 &= 5
 \end{aligned}$$

Equation of AC is

$$\begin{aligned}
 y-0 &= \frac{2-0}{3+1}(x+1) \\
 y &= \frac{1}{2}(x+1)
 \end{aligned}$$

$$\begin{aligned}
 ar(AMCA) &= \frac{1}{2} \int_{-1}^3 (x+1)dx \\
 &= \frac{1}{2} \left[\frac{x^2}{2} + x \right]_{-1}^3 \\
 &= \frac{1}{2} \left[\frac{9}{2} + 3 - \frac{1}{2} + 1 \right] \\
 &= 4
 \end{aligned}$$

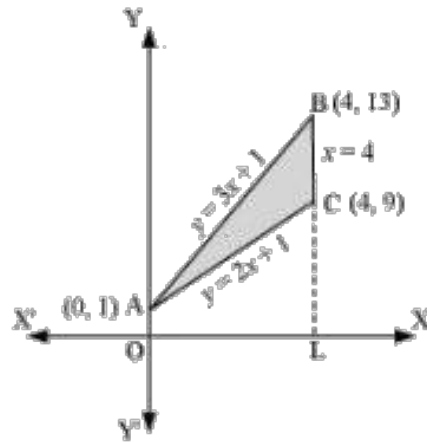
Therefore, $ar(\Delta ABC) = (3+5-4) = 4 \text{ units}$

Question 5:

Using integration find the area of the triangular region whose sides have the equations $y = 2x + 1$, $y = 3x + 1$ and $x = 4$.

Solution:

Vertices of triangle are $A(0,1)$, $B(4,13)$ and $C(4,9)$.



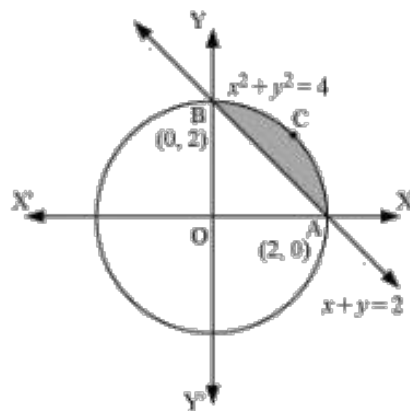
$$\begin{aligned}ar(\triangle ACB) &= ar(OLBAO) - ar(OLCAO) \\&= \int_0^4 (3x+1) dx - \int_0^4 (2x+1) dx \\&= \left[\frac{3x^2}{2} + x \right]_0^4 - \left[\frac{2x^2}{2} + x \right]_0^4 \\&= (24+4) - (16+4) \\&= 28 - 20 \\&= 8\end{aligned}$$

Question 6:

Smaller area enclosed by the circle $x^2 + y^2 = 4$ and the line $x + y = 2$ is

- (A) $2(\pi - 2)$ (B) $\pi - 2$ (C) $2\pi - 1$ (D) $2(\pi + 2)$

Solution:



$$\begin{aligned}
 \text{ar}(ACBA) &= \text{ar}(OACBO) - \text{ar}(\Delta OAB) \\
 &= \int_0^2 \sqrt{4-x^2} dx - \int_0^2 (2-x) dx \\
 &= \left[\frac{x}{2} \sqrt{4-x^2} + \frac{4}{2} \sin^{-1} \frac{x}{2} \right]_0^2 - \left[2x - \frac{x^2}{2} \right]_0^2 \\
 &= \left[2 \times \frac{\pi}{2} \right] - [4-2] \\
 &= (\pi - 2)
 \end{aligned}$$

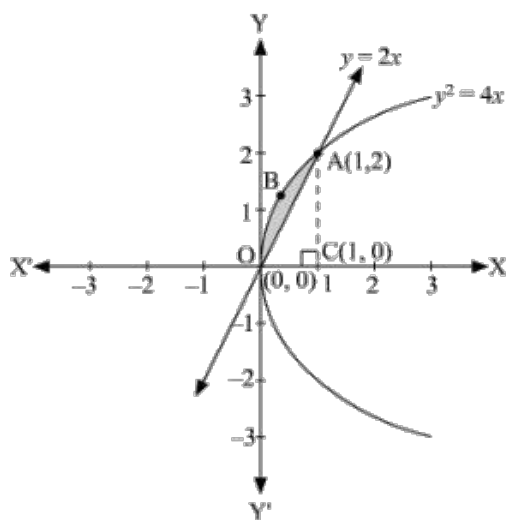
Correct answer is B.

Question 7:

Area lying between the curve $y^2 = 4x$ and $y = 2x$ is

- (A) $\frac{2}{3}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) $\frac{3}{4}$

Solution:



Points of intersection of curve $y^2 = 4x$ and $y = 2x$ are $O(0,0)$ and $A(1,2)$.

Draw AC perpendicular to x-axis.

Coordinates of C are (1,0)

$$\begin{aligned}
 ar(OBAO) &= ar(\Delta OCA) - ar(OCABO) \\
 &= \int_0^1 2x dx - \int_0^1 2\sqrt{x} dx \\
 &= 2 \left[\frac{x^2}{2} \right]_0^1 - 2 \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^1 \\
 &= \left| 1 - \frac{4}{3} \right| \\
 &= \left| -\frac{1}{3} \right| \\
 &= \frac{1}{3}
 \end{aligned}$$

Correct answer is B.

MISCELLANEOUS EXERCISE

Question 1:

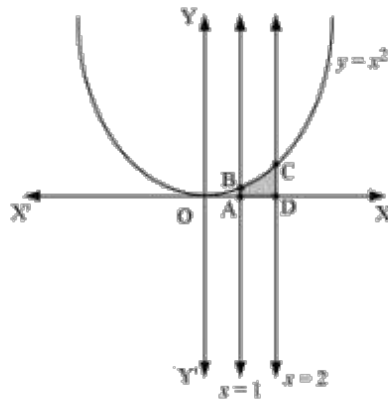
Find the area under the given curves and given lines:

(i) $y = x^2, x = 1, x = 2$ and x -axis

(ii) $y = x^4, x = 1, x = 5$ and x -axis

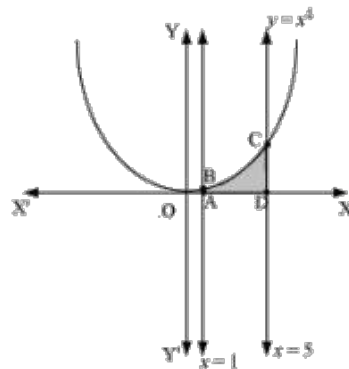
Solution:

(i) $y = x^2, x = 1, x = 2$ and x -axis



$$\begin{aligned} \text{ar}(ADCBA) &= \int_1^2 y dx \\ &= \int_1^2 x^2 dx \\ &= \left[\frac{x^3}{3} \right]_1^2 \\ &= \frac{8}{3} - \frac{1}{3} \\ &= \frac{7}{3} \end{aligned}$$

(ii) $y = x^4, x = 1, x = 5$ and x -axis

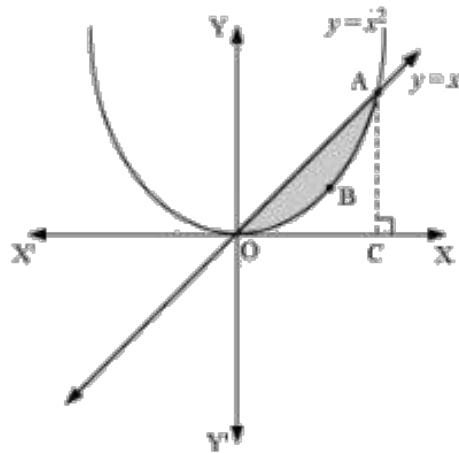


$$\begin{aligned}
 ar(ADCBA) &= \int_1^5 y dx \\
 &= \int_1^5 x^4 dx \\
 &= \left[\frac{x^5}{5} \right]_1^5 \\
 &= \frac{(5)^5}{5} - \frac{1}{5} \\
 &= (5)^4 - \frac{1}{5} \\
 &= 625 - \frac{1}{5} \\
 &= 624.8
 \end{aligned}$$

Question 2:

Find the area between the curves $y = x$ and $y = x^2$.

Solution:



Point of intersection of $y = x$ and $y = x^2$ is A (1,1).

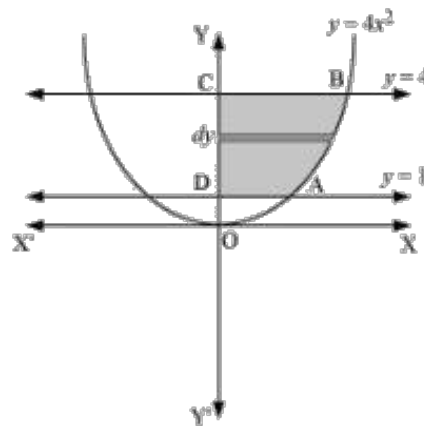
Draw AC perpendicular to x-axis.

$$\begin{aligned}
 \text{ar}(OBAO) &= \text{ar}(\triangle OCA) - \text{ar}(OCABO) \\
 &= \int_0^1 x dx - \int_0^1 x^2 dx \\
 &= \left[\frac{x^2}{2} \right]_0^1 - \left[\frac{x^3}{3} \right]_0^1 \\
 &= \frac{1}{2} - \frac{1}{3} \\
 &= \frac{1}{6}
 \end{aligned}$$

Question 3:

Find the area of the region lying in the first quadrant and bounded by $y = 4x^2$, $x = 0$, $y = 1$ and $y = 4$.

Solution:



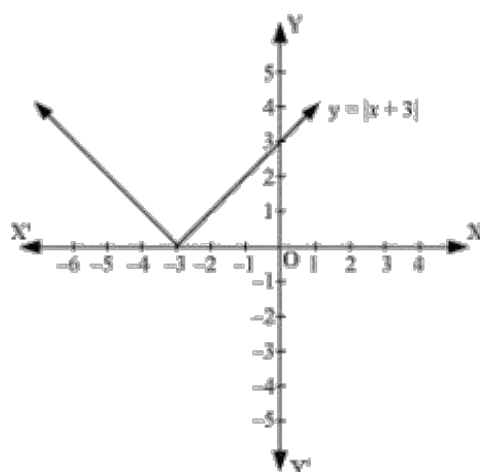
$$\begin{aligned}
 \text{ar}(ABCD) &= \int_1^4 x dy \\
 &= \int_1^4 \frac{\sqrt{y}}{2} dy \\
 &= \frac{1}{2} \left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}} \right]_1^4 \\
 &= \frac{1}{3} \left[(4)^{\frac{3}{2}} - 1 \right] \\
 &= \frac{1}{3} [8 - 1] \\
 &= \frac{7}{3}
 \end{aligned}$$

Question 4:

Sketch the graph of $y = |x+3|$ and evaluate $\int_{-6}^0 |x+3| dx$.

Solution:

X	-6	-5	-4	-3	-2	-1	0
Y	3	2	1	0	1	2	3



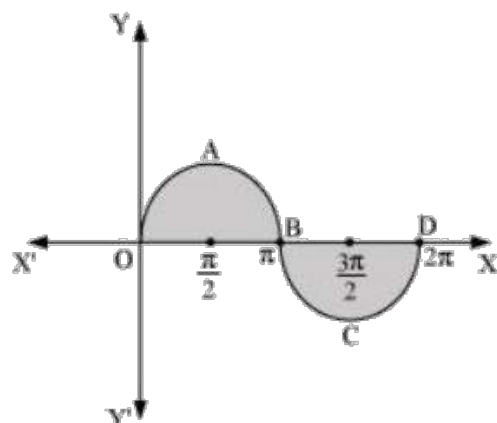
$(x+3) \leq 0$ for $-6 \leq x \leq -3$ and $(x+3) \geq 0$ for $-3 \leq x \leq 0$

$$\begin{aligned}\int_{-6}^0 |(x+3)| dx &= -\int_{-6}^{-3} (x+3) dx + \int_{-3}^0 (x+3) dx \\&= -\left[\frac{x^2}{2} + 3x \right]_{-6}^{-3} + \left[\frac{x^2}{2} + 3x \right]_{-3}^0 \\&= -\left[\left(\frac{(-3)^2}{2} + 3(-3) \right) - \left(\frac{(-6)^2}{2} + 3(-6) \right) \right] + \left[0 - \left(\frac{(-3)^2}{2} + 3(-3) \right) \right] \\&= -\left[-\frac{9}{2} \right] - \left[-\frac{9}{2} \right] \\&= 9\end{aligned}$$

Question 5:

Find the area bounded by the curve $y = \sin x$ between $x = 0$ and $x = 2\pi$.

Solution:



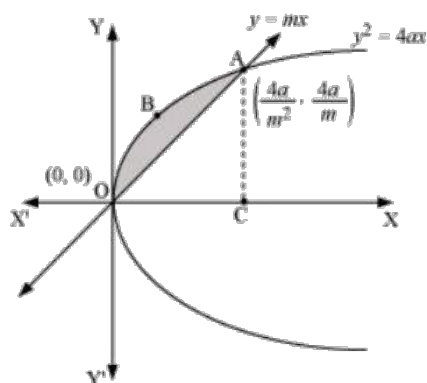
Area bounded by the curve = Area OABO + Area BCDB

$$\begin{aligned}
 ar(OABO) + ar(BCDB) &= \int_0^{\pi} \sin x dx + \left| \int_{\pi}^{2\pi} \sin x dx \right| \\
 &= [-\cos x]_0^{\pi} + \left| [-\cos x]_{\pi}^{2\pi} \right| \\
 &= [-\cos \pi + \cos 0] + |-\cos 2\pi + \cos \pi| \\
 &= 1 + 1 + |(-1 - 1)| \\
 &= 2 + |-2| \\
 &= 2 + 2 \\
 &= 4
 \end{aligned}$$

Question 6:

Find the area enclosed between the parabola $y^2 = 4ax$ and the line $y = mx$.

Solution:



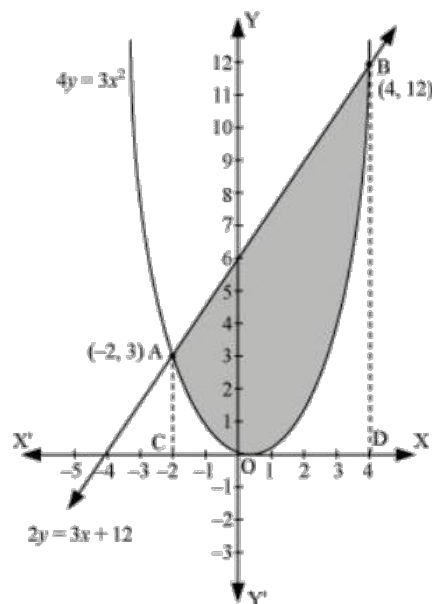
Points of intersection of curves are $(0,0)$ and $\left(\frac{4a}{m^2}, \frac{4a}{m}\right)$
 Draw AC perpendicular to x-axis.

$$\begin{aligned}
 \text{ar}(OABO) &= \text{ar}(OCABO) - \text{ar}(\Delta OCA) \\
 &= \int_0^{\frac{4a}{m^2}} 2\sqrt{ax} dx - \int_0^{\frac{4a}{m^2}} mx dx \\
 &= 2\sqrt{a} \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^{\frac{4a}{m^2}} - m \left[\frac{x^2}{2} \right]_0^{\frac{4a}{m^2}} \\
 &= \frac{4}{3} \sqrt{a} \left(\frac{4a}{m^2} \right)^{\frac{3}{2}} - \frac{m}{2} \left[\left(\frac{4a}{m^2} \right)^2 \right] \\
 &= \frac{32a^2}{3m^3} - \frac{m}{2} \left(\frac{16a^2}{m^4} \right) \\
 &= \frac{32a^2}{3m^3} - \frac{8a^2}{m^3} \\
 &= \frac{8a^2}{3m^3}
 \end{aligned}$$

Question 7:

Find the area enclosed by the parabola $4y = 3x^2$ and the line $2y = 3x + 12$.

Solution:



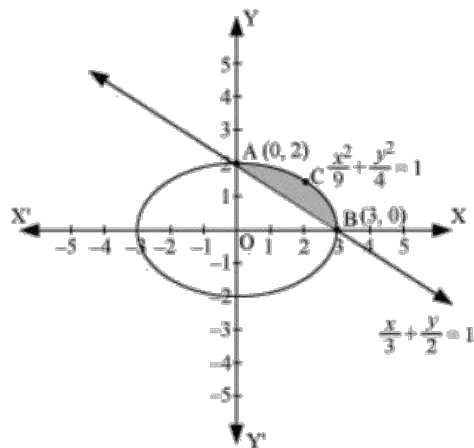
Points of intersection of curves are $A(-2, 3)$ and $B(4, 12)$.
Draw AC and BD perpendicular to x -axis.

$$\begin{aligned}
 \text{ar}(OBAO) &= \text{ar}(CDBA) - \text{ar}(ODBO + OACO) \\
 &= \int_{-2}^4 \frac{1}{2}(3x+12)dx - \int_{-2}^4 \frac{3x^2}{4}dx \\
 &= \frac{1}{2} \left[\frac{3x^2}{2} + 12x \right]_{-2}^4 - \frac{3}{4} \left[\frac{x^3}{3} \right]_{-2}^4 \\
 &= \frac{1}{2} [24 + 48 - 6 + 24] - \frac{1}{4} [64 + 8] \\
 &= \frac{1}{2} [90] - \frac{1}{4} [72] \\
 &= 45 - 18 \\
 &= 27
 \end{aligned}$$

Question 8:

Find the area of the smaller region bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the line $\frac{x}{3} + \frac{y}{2} = 1$.

Solution:

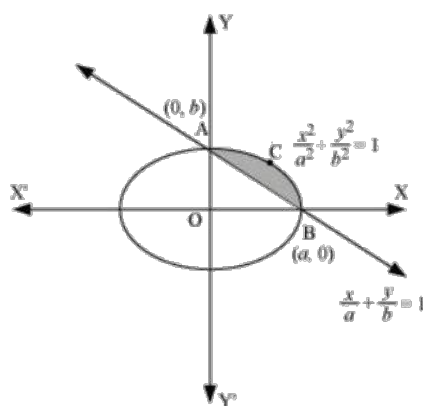


$$\begin{aligned}
 \text{ar}(BCAB) &= \text{ar}(OBCAO) - \text{ar}(OBAO) \\
 &= \int_0^3 2\sqrt{1 - \frac{x^2}{9}} dx - \int_0^3 2\left(1 - \frac{x}{3}\right) dx \\
 &= \frac{2}{3} \left[\int_0^3 \sqrt{9 - x^2} dx \right] - \frac{2}{3} \int_0^3 (3 - x) dx \\
 &= \frac{2}{3} \left[\frac{x}{2} \sqrt{9 - x^2} + \frac{9}{2} \sin^{-1} \frac{x}{3} \right]_0^3 - \frac{2}{3} \left[3x - \frac{x^2}{2} \right]_0^3 \\
 &= \frac{2}{3} \left[\frac{9}{2} \left(\frac{\pi}{2} \right) \right] - \frac{2}{3} \left[9 - \frac{9}{2} \right] \\
 &= \frac{2}{3} \left[\frac{9\pi}{4} - \frac{9}{2} \right] \\
 &= \frac{2}{3} \times \frac{9}{4} (\pi - 2) \\
 &= \frac{3}{2} (\pi - 2)
 \end{aligned}$$

Question 9:

Find the area of the smaller region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the line $\frac{x}{a} + \frac{y}{b} = 1$.

Solution:



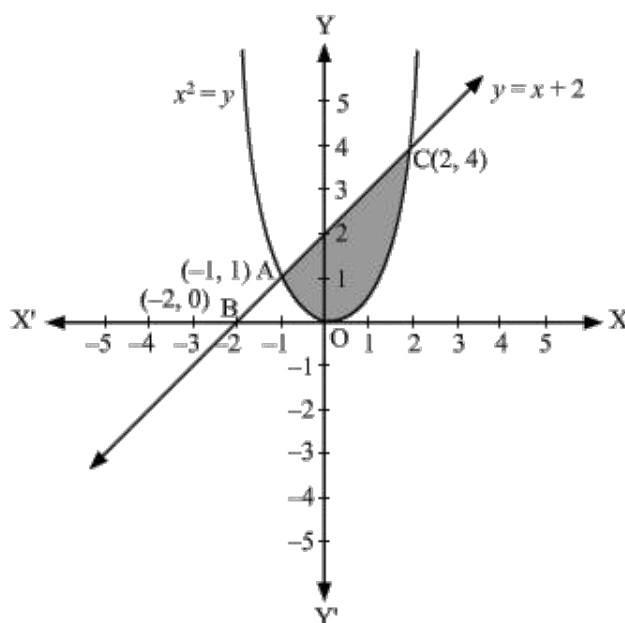
$$\begin{aligned}
 \text{ar}(CBA) &= \text{ar}(OBCAO) - \text{ar}(OBAO) \\
 &= \int_0^a b \sqrt{1 - \frac{x^2}{a^2}} dx - \int_0^a b \left(1 - \frac{x}{a}\right) dx \\
 &= \frac{b}{a} \left[\left\{ \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} \right\}_0^a - \left\{ ax - \frac{x^2}{2} \right\}_0^a \right] \\
 &= \frac{b}{a} \left[\left\{ \frac{a^2}{2} \left(\frac{\pi}{2} \right) \right\} - \left\{ a^2 - \frac{a^2}{2} \right\} \right] \\
 &= \frac{b}{a} \left[\frac{a^2 \pi}{4} - \frac{a^2}{2} \right] \\
 &= \frac{ba^2}{2a} \left[\frac{\pi}{2} - 1 \right] \\
 &= \frac{ab}{2} \left[\frac{\pi}{2} - 1 \right] \\
 &= \frac{ab}{4} (\pi - 2)
 \end{aligned}$$

Question 10:

Find the area of the region enclosed by the parabola $x^2 = y$, the line $y = x + 2$ and x-axis.

Solution:

Point of intersection of $x^2 = y$ and $y = x + 2$, is $A(-1, 1)$ and $C(2, 4)$.



Now required Area = Area of trapezium ALMB- Area of ALODBM

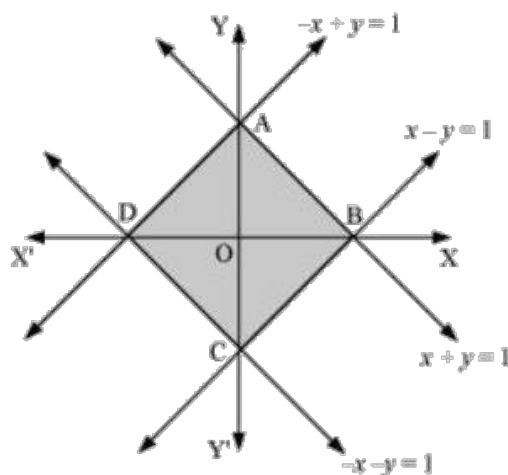
$$\begin{aligned}
 ar(trap.ALMB) - ar(ALODBM) &= \int_{-1}^2 (x+2) dx - \int_{-1}^2 x^2 dx \\
 &= \left[\frac{x^2}{2} + 2x \right]_{-1}^2 - \left[\frac{x^3}{3} \right]_{-1}^2 \\
 &= \left[2 + 4 - \frac{1}{2} + 2 \right] - \left[\frac{8}{3} + \frac{1}{3} \right] \\
 &= \frac{15}{2} - 3 \\
 &= \frac{9}{2}
 \end{aligned}$$

Question 11:

Using the method of integration find the area bounded by the curve $|x| + |y| = 1$

[Hint: The required region is bounded by lines $x + y = 1$, $x - y = 1$, $-x + y = 1$ and $-x - y = 1$]

Solution:



Curve intersects axis at points $A(0,1)$, $B(1,0)$, $C(0,-1)$ and $D(-1,0)$.

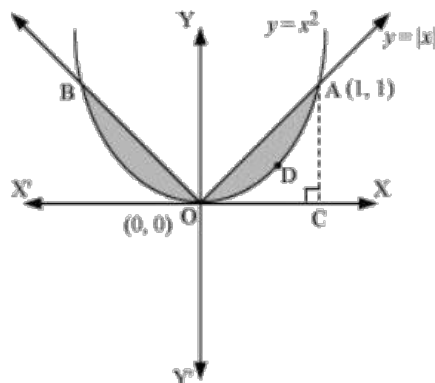
Curve is symmetrical about x -axis and y -axis.

$$ar(ADCB) = 4 \times ar(OBAO)$$

$$\begin{aligned}
 &= 4 \int_0^1 (1-x) dx \\
 &= 4 \left(x - \frac{x^2}{2} \right)_0^1 \\
 &= 4 \left[1 - \frac{1}{2} \right] \\
 &= 2
 \end{aligned}$$

Question 12:

Find the area bounded by curves $\{(x, y) : y \geq x^2 \text{ and } y = |x|\}$

Solution:

Required area is symmetrical about y-axis.

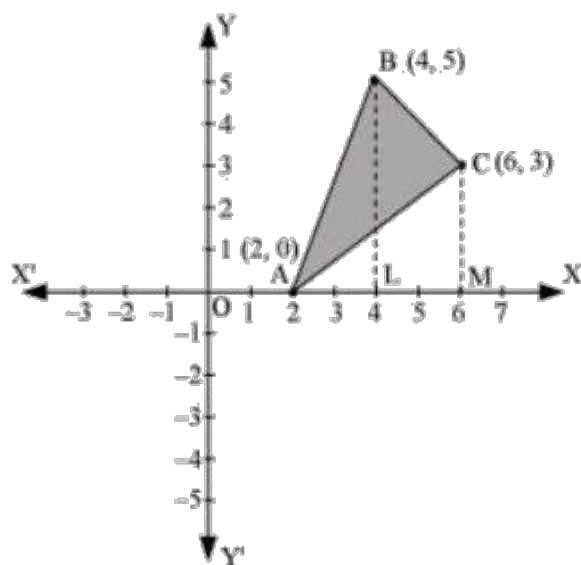
Required area = $2[\text{Area (OCAO)} - \text{Area (OCADO)}]$

$$\begin{aligned}
 2[\text{ar}(\text{OCAO}) - \text{ar}(\text{OCADO})] &= 2\left[\int_0^1 x dx - \int_0^1 x^2 dx\right] \\
 &= 2\left[\left[\frac{x^2}{2}\right]_0^1 - \left[\frac{x^3}{3}\right]_0^1\right] \\
 &= 2\left[\frac{1}{2} - \frac{1}{3}\right] \\
 &= 2\left[\frac{1}{6}\right] \\
 &= \frac{1}{3}
 \end{aligned}$$

Question 13:

Using the method of integration find the area of the triangle ABC, coordinates of whose vertices are $A(2, 0)$, $B(4, 5)$ and $C(6, 3)$.

Solution:



Equation of AB is

$$y - 0 = \frac{5 - 0}{4 - 2}(x - 2)$$

$$2y = 5x - 10$$

$$y = \frac{5}{2}(x - 2)$$

Equation of BC is

$$y - 5 = \frac{3 - 5}{6 - 4}(x - 4)$$

$$2y - 10 = -2x + 8$$

$$2y = -2x + 18$$

$$y = -x + 9$$

Equation of CA is

$$y - 3 = \frac{0 - 3}{2 - 6}(x - 6)$$

$$-4y + 12 = -3x + 18$$

$$4y = 3x - 6$$

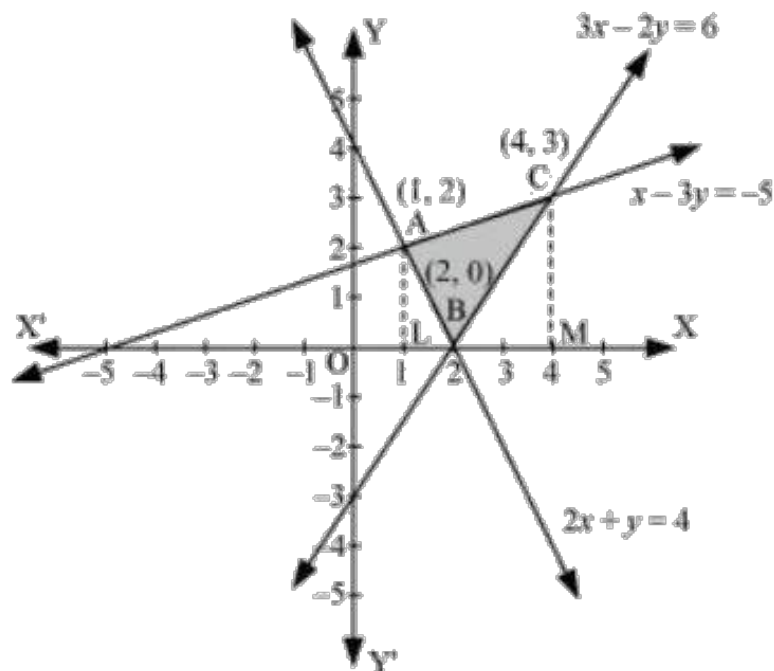
$$y = \frac{3}{4}(x - 2)$$

$$\begin{aligned}
 \text{ar}(\Delta ABC) &= \text{ar}(ABLA) + \text{ar}(BLMCB) - \text{ar}(ACMA) \\
 &= \int_2^4 \frac{5}{2}(x-2) dx + \int_4^6 (-x+9) dx - \int_2^6 \frac{3}{4}(x-2) dx \\
 &= \frac{5}{2} \left[\frac{x^2}{2} - 2x \right]_2^4 + \left[-\frac{x^2}{2} + 9x \right]_4^6 - \frac{3}{4} \left[\frac{x^2}{2} - 2x \right]_2^6 \\
 &= \frac{5}{2} [8 - 8 - 2 + 4] + [-18 + 54 + 8 - 36] - \frac{3}{4} [18 - 12 - 2 + 4] \\
 &= 5 + 8 - \frac{3}{4}(8) \\
 &= 13 - 6 \\
 &= 7 \text{ units.}
 \end{aligned}$$

Question 14:

Using the method of integration find the area of the region bounded by lines:
 $2x + y = 4$, $3x - 2y = 6$ and $x - 3y + 5 = 0$.

Solution:



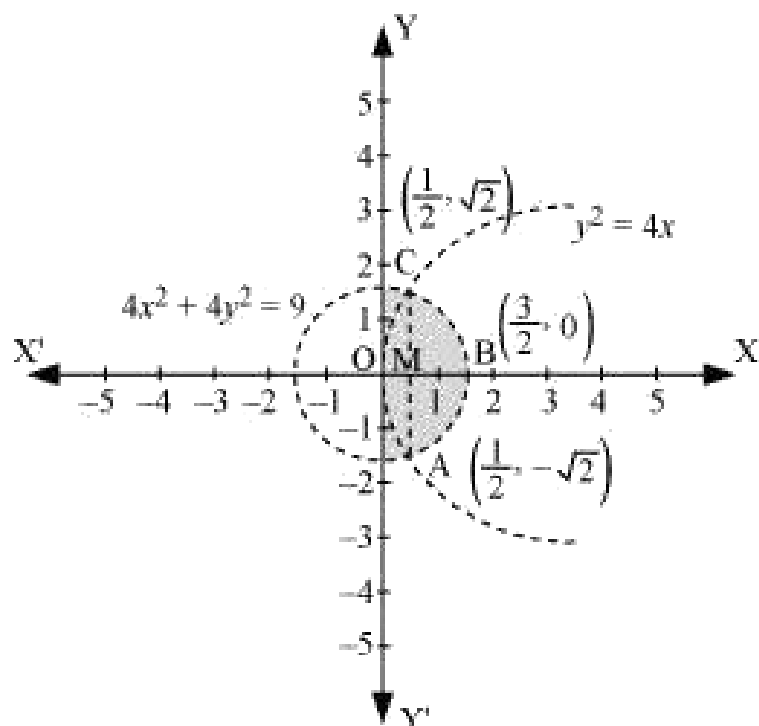
AL and CM are perpendicular on x-axis.

$$\begin{aligned}
ar(\Delta ABC) &= ar(ALMCA) - ar(ALB) - ar(CMB) \\
&= \int_1^4 \left(\frac{x+5}{3} \right) dx - \int_1^2 (4-2x) dx - \int_2^4 \left(\frac{3x-6}{2} \right) dx \\
&= \frac{1}{3} \left[\frac{x^2}{2} + 5x \right]_1^4 - \left[4x - x^2 \right]_1^2 - \frac{1}{2} \left[\frac{3x^2}{2} - 6x \right]_2^4 \\
&= \frac{1}{3} \left[8 + 20 - \frac{1}{2} - 5 \right] - [8 - 4 - 4 + 1] - \frac{1}{2} [24 - 24 - 6 + 12] \\
&= \left(\frac{1}{3} \times \frac{45}{2} \right) - (1) - \frac{1}{2}(6) \\
&= \frac{15}{2} - 1 - 3 \\
&= \frac{15}{2} - 4 \\
&= \frac{15-8}{2} \\
&= \frac{7}{2}
\end{aligned}$$

Question 15:

Find the area of the region $\{(x, y) : y^2 \leq 4x, 4x^2 + 4y^2 \leq 9\}$

Solution:



Points of intersection of curves are $\left(\frac{1}{2}, \sqrt{2}\right)$ and $\left(\frac{1}{2}, -\sqrt{2}\right)$.

Required area is OABCO.

Area OABCO is symmetrical about x -axis.

$$\text{Area OABCO} = 2 \times \text{Area OBC}$$

$$\text{ar}(OBCO) = \text{ar}(OMC) + \text{ar}(MBC)$$

$$= \int_0^{\frac{1}{2}} 2\sqrt{x} dx + \int_{\frac{1}{2}}^{\frac{3}{2}} \frac{1}{2} \sqrt{9 - 4x^2} dx$$

$$= \int_0^{\frac{1}{2}} 2\sqrt{x} dx + \int_{\frac{1}{2}}^{\frac{3}{2}} \frac{1}{2} \sqrt{(3)^2 - (2x)^2} dx$$

$$\text{put } 2x = t \Rightarrow dx = \frac{dt}{2}$$

$$\text{When } x = \frac{3}{2}, t = 3 \text{ and when } x = \frac{1}{2}, t = 1$$

$$\begin{aligned} \text{ar}(OBCO) &= \int_0^{\frac{1}{2}} 2\sqrt{x} dx + \frac{1}{4} \int_1^3 \sqrt{(3)^2 - (t)^2} dt \\ &= 2 \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^{\frac{1}{2}} + \frac{1}{4} \left[\frac{t}{2} \sqrt{9-t^2} + \frac{9}{2} \sin^{-1} \left(\frac{t}{3} \right) \right]_1^3 \\ &= 2 \left[\frac{2}{3} \left(\frac{1}{2} \right)^{\frac{3}{2}} \right] + \frac{1}{4} \left[\left\{ \frac{3}{2} \sqrt{9-(3)^2} + \frac{9}{2} \sin^{-1} \left(\frac{3}{3} \right) \right\} - \left\{ \frac{1}{2} \sqrt{9-(1)^2} + \frac{9}{2} \sin^{-1} \left(\frac{1}{3} \right) \right\} \right] \\ &= \frac{2}{3\sqrt{2}} + \frac{1}{4} \left[\left\{ 0 + \frac{9}{2} \sin^{-1}(1) \right\} - \left\{ \frac{1}{2} \sqrt{8} + \frac{9}{2} \sin^{-1} \left(\frac{1}{3} \right) \right\} \right] \\ &= \frac{\sqrt{2}}{3} + \frac{1}{4} \left[\frac{9\pi}{4} - \sqrt{2} - \frac{9}{2} \sin^{-1} \left(\frac{1}{3} \right) \right] \\ &= \frac{\sqrt{2}}{3} + \frac{9\pi}{16} - \sqrt{2} - \frac{9}{8} \sin^{-1} \left(\frac{1}{3} \right) \\ &= \frac{9\pi}{16} - \frac{9}{8} \sin^{-1} \left(\frac{1}{3} \right) + \frac{\sqrt{2}}{12} \end{aligned}$$

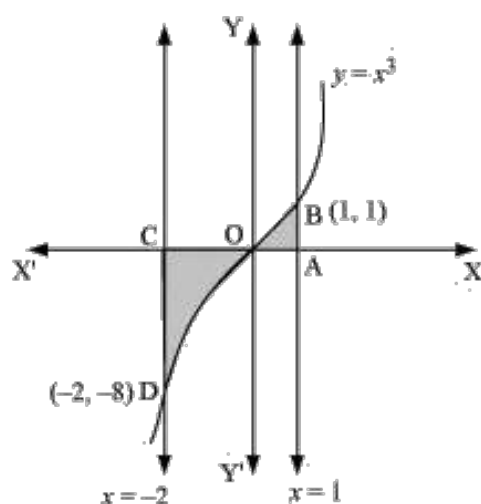
$$\begin{aligned} \text{ar}(OABCO) &= 2 \times \text{ar}(OBC) \\ &= 2 \times \frac{9\pi}{16} - \frac{9}{8} \sin^{-1} \left(\frac{1}{3} \right) + \frac{\sqrt{2}}{12} \\ &= \frac{9\pi}{8} - \frac{9}{4} \sin^{-1} \left(\frac{1}{3} \right) + \frac{\sqrt{2}}{6} \\ &= \frac{9\pi}{8} - \frac{9}{4} \sin^{-1} \left(\frac{1}{3} \right) + \frac{1}{3\sqrt{2}} \end{aligned}$$

Question 16:

Area bounded by the curve $y = x^3$, the x -axis and the coordinates $x = -2$ and $x = 1$ is

- (A) -9 (B) $-\frac{15}{4}$ (C) $\frac{15}{4}$ (D) $\frac{17}{4}$

Solution:



$$\begin{aligned}
 \text{required area} &= \int_{-2}^0 y dx + \int_0^1 y dx \\
 &= \int_{-2}^0 x^3 dx + \int_0^1 x^3 dx \\
 &= \left[\frac{x^4}{4} \right]_{-2}^0 + \left[\frac{x^4}{4} \right]_0^1 \\
 &= \left[\frac{(-2)^4}{4} + \frac{1}{4} \right] \\
 &= \left(4 + \frac{1}{4} \right) = \frac{17}{4}
 \end{aligned}$$

Correct answer is D

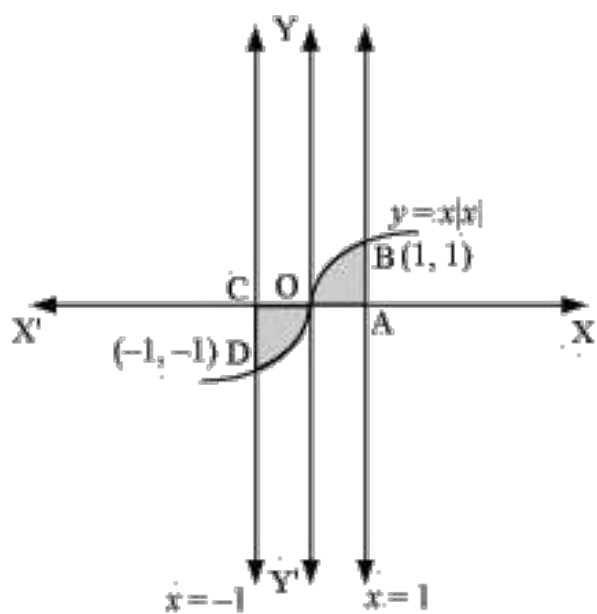
Question17:

The area bounded by the curve $y = x|x|$, x -axis and the coordinates $x = -1$ and $x = 1$ is given by

[Hint: $y = x^2$ if $x > 0$ and $y = -x^2$ if $x < 0$]

- (A) 0 (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{4}{3}$

Solution:



$$\begin{aligned}
 \text{required area} &= \int_{-1}^1 y dx \\
 &= \int_{-1}^1 x|x| dx \\
 &= \int_{-1}^0 x^2 dx + \int_0^1 x^2 dx \\
 &= \left[\frac{x^3}{3} \right]_{-1}^0 + \left[\frac{x^3}{3} \right]_0^1 \\
 &= -\left(-\frac{1}{3} \right) + \frac{1}{3} \\
 &= \frac{2}{3}
 \end{aligned}$$

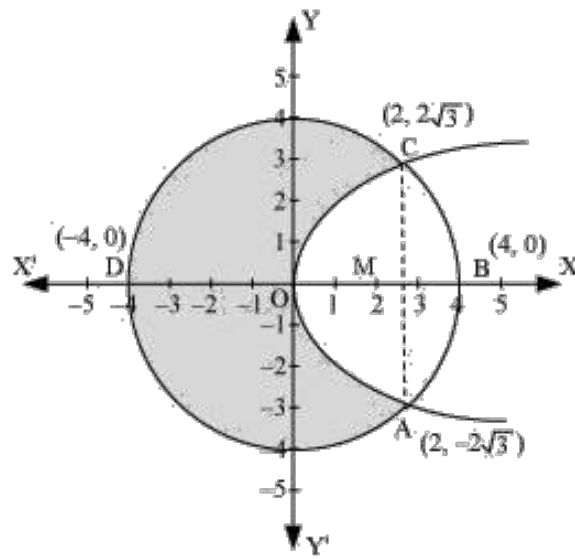
Correct answer is C.

Question 18:

The area of the circle $x^2 + y^2 = 16$ exterior to the parabola $y^2 = 6x$.

- (A) $\frac{4}{3}(4\pi - \sqrt{3})$ (B) $\frac{4}{3}(4\pi + \sqrt{3})$ (C) $\frac{4}{3}(8\pi - \sqrt{3})$ (D) $\frac{4}{3}(8\pi + \sqrt{3})$

Solution:



Required area = 2[Area (OADO) + Area (ADBA)]

$$\begin{aligned}
 2[ar(OADO) + ar(ADBA)] &= 2\left[\int_0^2 \sqrt{6x} dx + \int_2^4 \sqrt{16-x^2} dx\right] \\
 &= 2\int_0^2 \sqrt{6x} dx + 2\int_2^4 \sqrt{16-x^2} dx \\
 &= 2\sqrt{6}\int_0^2 \sqrt{x} dx + 2\int_2^4 \sqrt{16-x^2} dx \\
 &= 2\sqrt{6} \times \frac{2}{3} \left[x^{\frac{3}{2}}\right]_0^2 + 2\left[\frac{x}{2}\sqrt{16-x^2} + \frac{16}{2}\sin^{-1}\left(\frac{x}{4}\right)\right]_2^4 \\
 &= \frac{4\sqrt{6}}{3}(2\sqrt{2}-0) + 2\left[\left\{0+8\sin^{-1}(1)\right\}-\left\{2\sqrt{3}+8\sin^{-1}\left(\frac{1}{2}\right)\right\}\right] \\
 &= \frac{16\sqrt{3}}{3} + 2\left[8 \times \frac{\pi}{2} - 2\sqrt{3} - 8 \times \frac{\pi}{6}\right] \\
 &= \frac{16\sqrt{3}}{3} + 8\pi - 4\sqrt{3} - \frac{8\pi}{3} \\
 &= \frac{16\sqrt{3} + 24\pi - 12\sqrt{3} - 8\pi}{3} \\
 &= \frac{4\sqrt{3} + 16\pi}{3} \\
 &= \frac{4}{3}[4\pi + \sqrt{3}] \text{ units}
 \end{aligned}$$

$$\text{Area of circle} = \pi (r)^2$$

$$= \pi (4)^2$$

$$= 16\pi$$

$$\text{required area} = 16\pi - \frac{4}{3} [4\pi + \sqrt{3}]$$

$$= \frac{4}{3} [4 \times 3\pi - 4\pi - \sqrt{3}]$$

$$= \frac{4}{3} (8\pi - \sqrt{3})$$

Correct answer is C.

Question 19:

The area bounded by the y -axis, $y = \cos x$ and $y = \sin x$ when $0 \leq x \leq \frac{\pi}{2}$.

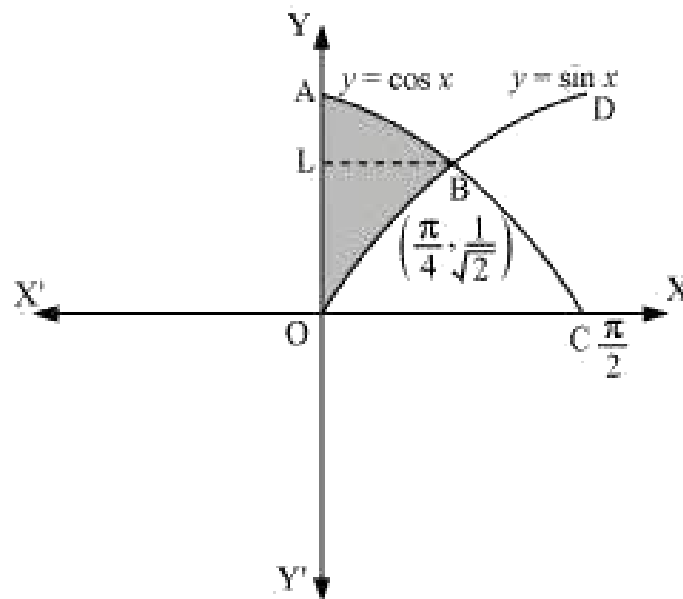
(A) $2(\sqrt{2}-1)$

(B) $\sqrt{2}-1$

(C) $\sqrt{2}+1$

(D) $\sqrt{2}$

Solution:



Required area = Area (ABLA) + Area (OBLO)

$$\begin{aligned}
ar(ABLA) + ar(OBLO) &= \int_{\frac{1}{\sqrt{2}}}^1 x dy + \int_0^{\frac{1}{\sqrt{2}}} x dy \\
&= \int_{\frac{1}{\sqrt{2}}}^1 \cos^{-1} y dy + \int_0^{\frac{1}{\sqrt{2}}} \sin^{-1} x dy \\
&= \left[y \cos^{-1} y - \sqrt{1-y^2} \right]_{\frac{1}{\sqrt{2}}}^1 + \left[x \sin^{-1} x + \sqrt{1-x^2} \right]_0^{\frac{1}{\sqrt{2}}} \\
&= \left[\cos^{-1}(1) - \frac{1}{\sqrt{2}} \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) + \sqrt{1-\frac{1}{2}} \right] + \left[\frac{1}{\sqrt{2}} \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) + \sqrt{1-\frac{1}{2}} - 1 \right] \\
&= \frac{-\pi}{4\sqrt{2}} + \frac{1}{\sqrt{2}} + \frac{\pi}{4\sqrt{2}} + \frac{1}{\sqrt{2}} - 1 \\
&= \frac{2}{\sqrt{2}} - 1 \\
&= \sqrt{2} - 1
\end{aligned}$$

Correct answer is B.