CHAPTER I: APPLICATION OF MATRICES AND DETERMINANTS

Formulae:-

1. $(AB)^{-1} = B^{-1}A^{-1}$ if A & B are non singular matrices of same order
2. $(A^{-1})^{-1} = A$ if A is non singular, A^{-1} is non singular
3. if A is non singular square matrix order n, then

(i) $(\text{adj} A)^{-1} = (\text{adj} A^{-1}) = \frac{1}{|A|} A$ (ii) $|\text{adj} A| = |A|^{n-1}$

(iii) $\text{adj}(\text{adj} A) = |A|^{n-2} A$ (iv) $\text{adj}(\lambda A) = \lambda^{n-1} \text{adj} A$, λ is nonzero scalar

(v) $|\text{adj}(\text{adj} A)| = |A|^{(n-1)^2}$ (vi) $(\text{adj} A)^T = \text{adj}(A^T)$

4) $\text{adj}(AB) = (\text{adj} B)(\text{adj} A)$

5) A square matrix A is called orthogonal if $AA^T = A^TA = I$

6) (i) $A^{-1} = \frac{1}{|A|} \text{adj} A$ (ii) $(A^T)^{-1} = A$

7) $(\lambda A)^{-1} = \frac{1}{\lambda} A^{-1}$ (ii) $A(\text{adj} A) = (\text{adj} A) A = |A| I$.

8) $A^{-1} = \pm \frac{1}{\sqrt{|\text{adj} A|}} \text{adj} A$ 9) $A = \pm \frac{1}{\sqrt{|\text{adj} A|}} \text{adj}(\text{adj} A)$

9) Methods to solve the system of linear equations $AX = B$

(i) by matrix inversion method $X = A^{-1}B$, $|A| \neq 0$

(ii) by Cramer's rule $x = \frac{\Delta x}{\Delta}$, $y = \frac{\Delta y}{\Delta}$, $z = \frac{\Delta z}{\Delta}$, $\Delta \neq 0$.

(iii) by Gaussian elimination (Rank) method

10) (i) if $\rho(A) = \rho(A|B) < n$ Then the system has infinitely many solutions

(ii) If $\rho(A) = \rho(A|B) = n$ Then The system has unique solution

(iii) If $\rho(A) \neq \rho(A|B)$ then The system is inconsistent and has no solution.

11) The homogenous system of linear equation $AX = 0$

(i) has the trival solution if $|A| \neq 0$

(ii) has a non trival solution if $|A| = 0$.

11) A matrix A is orthogonal iff A is non singular and $A^{-1} = A^T$.

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Exercise 1.1

1. Find the adjoint of the following.

(i) $\begin{bmatrix} -3 & 4 \\ 6 & 2 \end{bmatrix}$

Soln

Let $A = \begin{bmatrix} -3 & 4 \\ 6 & 2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ [∵ interchanging a_{22} & a_{11} Sign changing a_{12} & a_{21}

$\text{adj}A = \begin{bmatrix} 2 & -4 \\ -6 & -3 \end{bmatrix}$

(ii) $A = \begin{bmatrix} 2 & 3 & 1 \\ 3 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix}$

$$\begin{array}{c|ccccc} & C_1 & C_2 & & & \\ \hline R_1 & 2 & 3 & 1 & 2 & 3 \\ R_2 & 3 & 4 & 1 & 3 & 4 \\ & 3 & 7 & 2 & 3 & 7 \\ & 2 & 3 & 1 & 2 & 3 \\ & 3 & 4 & 1 & 3 & 4 \end{array}$$

$\text{adj}A = \begin{bmatrix} 8-7 & 7-6 & 3-4 \\ 3-6 & 4-3 & 3-2 \\ 21-12 & 9-14 & 8-9 \end{bmatrix}$

$= \begin{bmatrix} 1 & 1 & -1 \\ -3 & 1 & 1 \\ 9 & -5 & -1 \end{bmatrix}$

(iii) $A = \frac{1}{3}\begin{bmatrix} 2 & 2 & 1 \\ -2 & 1 & 2 \\ 1 & -2 & 2 \end{bmatrix}_{3\times 3}$

Soln $n = 3$ $\qquad \text{adj}(KA) = K^{n-1}\,\text{adj}A$

$$\begin{array}{c|cccc} 2 & 2 & 1 & 2 & 2 \\ \hline -2 & 1 & 2 & -2 & 1 \\ 1 & -2 & 2 & 1 & -2 \\ 2 & 2 & 1 & 2 & 2 \\ -2 & 1 & 2 & -2 & 1 \end{array}$$

$\text{adj}(KA) = \left(\frac{1}{3}\right)^{3-1}\begin{bmatrix} 2+4 & -2-4 & 4-1 \\ 2+4 & 4-1 & -2-4 \\ 4-1 & 2+4 & 2+4 \end{bmatrix}$

$= \frac{1}{9}\begin{bmatrix} 6 & -6 & 3 \\ 6 & 3 & -6 \\ 3 & 6 & 6 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 2 & -2 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 2 \end{bmatrix}$

2. Find the inverse (if it exists) of the following

(i) $A = \begin{pmatrix} -2 & 4 \\ 1 & -3 \end{pmatrix}$

$|A| = 6-4 = 2 \neq 0 \quad \therefore A^{-1}$ exists.

A is non singular

$\text{adj}A = \begin{pmatrix} -3 & -4 \\ -1 & -2 \end{pmatrix}$

$A^{-1} = \frac{1}{|A|}\text{adj}A = \frac{1}{2}\begin{pmatrix} -3 & -4 \\ -1 & -2 \end{pmatrix}$.

(ii) $A = \begin{bmatrix} 5 & 1 & 1 \\ 1 & 5 & 1 \\ 1 & 1 & 5 \end{bmatrix}$

Soln

$|A| = 5(25-1) - 1(5-1) + 1(1-5) = 120 - 4 - 4 = 112 \neq 0$

$= 5(24) - 1(4) + 1(-4)$

A^{-1} is exist

$$\begin{array}{c|cccc} 5 & 1 & 1 & 5 & 1 \\ \hline 1 & 5 & 1 & 1 & 5 \\ 1 & 1 & 5 & 1 & 1 \\ 5 & 1 & 1 & 5 & 1 \\ & 5 & 1 & 1 & 5 \end{array}$$

$$\text{adj } A = \begin{pmatrix} 25-1 & 1-5 & 1-5 \\ 1-5 & 25-1 & 1-5 \\ 1-5 & 1-5 & 25-1 \end{pmatrix}$$

$$= \begin{pmatrix} 24 & -4 & -4 \\ -4 & 24 & -4 \\ -4 & -4 & 24 \end{pmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{112} \begin{pmatrix} 24 & -4 & -4 \\ -4 & 24 & -4 \\ -4 & -4 & 24 \end{pmatrix}$$

$$= \frac{1}{28} \begin{pmatrix} 6 & -1 & -1 \\ -1 & 6 & -1 \\ -1 & -1 & 6 \end{pmatrix}$$

(iii) $A = \begin{pmatrix} 2 & 3 & 1 \\ 3 & 4 & 1 \\ 3 & 7 & 2 \end{pmatrix}$

soln

$|A| = 2(8-7) - 3(6-3) + 1(21-12) = 2 - 9 + 9 = 2 \neq 0$

A^{-1} is exist

$$\begin{array}{c|cccc} 2 & 3 & 1 & 2 & 3 \\ \hline 3 & 4 & 1 & 3 & 4 \\ 3 & 7 & 2 & 3 & 7 \\ 2 & 3 & 1 & 2 & 3 \\ 3 & 4 & 1 & 3 & 4 \end{array}$$

$$\text{adj } A = \begin{bmatrix} 8-7 & 7-6 & 3-4 \\ 3-6 & 4-3 & 3-2 \\ 21-12 & 9-14 & 8-9 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & -1 \\ -3 & 1 & 1 \\ 9 & -5 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{2} \begin{bmatrix} 1 & 1 & -1 \\ -3 & 1 & 1 \\ 9 & -5 & -1 \end{bmatrix}$$

3) If $F(\alpha) = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix}$ show that $[F(\alpha)]^{-1} = F(-\alpha)$

soln

$|F(\alpha)| = \cos\alpha(\cos\alpha - 0) - 0 + \sin\alpha(0 + \sin\alpha) = \cos^2\alpha + \sin^2\alpha = 1$

$|F(\alpha)| = 1 \neq 0$

$$\text{adj } F(\alpha) = \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ +\sin\alpha & 0 & \cos\alpha \end{bmatrix}$$

$$F(\alpha)^{-1} = \frac{1}{|F(\alpha)|} \text{adj } F(\alpha) = \frac{1}{1}\begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha & 0 & +\sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix} \quad -①$$

$$F(-\alpha) = \begin{bmatrix} \cos(-\alpha) & 0 & -\sin(-\alpha) \\ 0 & 1 & 0 \\ \sin(-\alpha) & 0 & \cos\alpha \end{bmatrix} = \begin{bmatrix} \cos\alpha & 0 & +\sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix}$$

$$F(-\alpha) = F(\alpha)^{-1}$$

$[\because \cos(-\theta) = \cos\theta$
$\sin(-\theta) = -\sin\theta]$

4) If $A = \begin{pmatrix} 5 & 3 \\ -1 & -2 \end{pmatrix}$ Show that $A^2 - 3A - 7I_2 = 0$ Hence find A^{-1}

Soln

$$A^2 = A \times A = \begin{pmatrix} 5 & 3 \\ -1 & -2 \end{pmatrix}\begin{pmatrix} 5 & 3 \\ -1 & -2 \end{pmatrix} = \begin{pmatrix} 25-3 & 15+6 \\ -5+2 & -3+4 \end{pmatrix} = \begin{pmatrix} 22 & 9 \\ -3 & 1 \end{pmatrix}$$

$$-3A = -3\begin{pmatrix} 5 & 3 \\ -1 & -2 \end{pmatrix} = \begin{pmatrix} -15 & -9 \\ 3 & 6 \end{pmatrix}$$

$$-7I_2 = -7\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} -7 & 0 \\ 0 & -7 \end{pmatrix}$$

$$A^2 - 3A - 7I_2 = \begin{pmatrix} 22 & 9 \\ -3 & 1 \end{pmatrix} + \begin{pmatrix} -15 & -9 \\ 3 & 6 \end{pmatrix} + \begin{pmatrix} -7 & 0 \\ 0 & -7 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\therefore A^2 - 3A - 7I_2 = 0.$$

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Post multiply this eqn by A^{-1}

$$A - 3I - 7A^{-1} = 0$$

$$A - 3I = 7A^{-1}$$

$$A^{-1} = \frac{1}{7}(A - 3I) = \frac{1}{7}\left[\begin{pmatrix} 5 & 3 \\ -1 & -2 \end{pmatrix} - 3\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right]$$

$$= \frac{1}{7}\left[\begin{pmatrix} 5 & 3 \\ -1 & -2 \end{pmatrix} + \begin{pmatrix} -3 & 0 \\ 0 & -3 \end{pmatrix}\right] = \frac{1}{7}\begin{bmatrix} 2 & 3 \\ -1 & -5 \end{bmatrix}$$

5) If $A = \frac{1}{9}\begin{bmatrix} -8 & 1 & 4 \\ 4 & 4 & 7 \\ 1 & -8 & 4 \end{bmatrix}$ Prove that $A^{-1} = A^T$

Soln

RHS $A^T = \frac{1}{9}\begin{bmatrix} -8 & 4 & 1 \\ 1 & 4 & -8 \\ 4 & 7 & 4 \end{bmatrix}$

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LHS $n = 3$

$$|A| = \left(\frac{1}{9}\right)^3 \left[-8(16+56) - 1(16-7) + 4(-32-4)\right] \qquad \because |kA| = k^n |A|$$

$$= \frac{1}{729}\left[-8(72) - 1(9) + 4(-36)\right] = \frac{1}{729}(-576-9-144)$$

$$= \frac{1}{729}(-729) = -1 \neq 0$$

$\therefore A^{-1}$ exist.

$$|\operatorname{adj}(kA)| = k^{n-1} \operatorname{adj} A$$

$$\operatorname{adj} A = \left(\frac{1}{9}\right)^{3-1} \begin{bmatrix} 16+56 & -32-4 & 7-16 \\ 7-16 & -32-4 & 16+56 \\ -32-4 & 1-64 & -32-4 \end{bmatrix}$$

$$\begin{array}{ccccc} -8 & 1 & 4 & -8 & 1 \\ 4 & 4 & 7 & 4 & 4 \\ 1 & -8 & 4 & 1 & -8 \\ -8 & 1 & 4 & -8 & 1 \\ 4 & 4 & 7 & 4 & 4 \end{array}$$

$$= \left(\frac{1}{9}\right)^2 \begin{bmatrix} 72 & -36 & -9 \\ -9 & -36 & 72 \\ -36 & -63 & -36 \end{bmatrix}$$

$$= \frac{1}{9} \begin{bmatrix} 8 & -4 & -1 \\ -1 & -4 & 8 \\ -4 & -7 & -4 \end{bmatrix}$$

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$$A^{-1} = \frac{1}{|A|} \operatorname{adj} A = \frac{1}{-1}\left(\frac{1}{9}\begin{bmatrix} 8 & -4 & -1 \\ -1 & -4 & 8 \\ -4 & -7 & -4 \end{bmatrix}\right) = \frac{1}{9}\begin{pmatrix} -8 & 4 & 1 \\ 1 & 4 & 8 \\ 4 & 7 & 4 \end{pmatrix} \text{ — (2)}$$

by (1) & (2) $\boxed{A^T = A^{-1}}$

6) If $A = \begin{pmatrix} 8 & -4 \\ -5 & 3 \end{pmatrix}$ verify that $A(\operatorname{adj} A) = (\operatorname{adj} A)A = |A| I_2$

Soln

$|A| = 24 - 20 = 4 \neq 0$ $\quad A^{-1}$ exist.

$$\operatorname{adj} A = \begin{pmatrix} 3 & 4 \\ 5 & 8 \end{pmatrix}$$

$$A(\operatorname{adj} A) = \begin{pmatrix} 8 & -4 \\ -5 & 3 \end{pmatrix}\begin{pmatrix} 3 & 4 \\ 5 & 8 \end{pmatrix} = \begin{pmatrix} 24-20 & 32-32 \\ -15+15 & -20+24 \end{pmatrix} = \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix} \text{ — (1)}$$

$$(\operatorname{adj} A)A = \begin{pmatrix} 3 & 4 \\ 5 & 8 \end{pmatrix}\begin{pmatrix} 8 & -4 \\ -5 & 3 \end{pmatrix} = \begin{pmatrix} 24-20 & -12+12 \\ 40-40 & -20+24 \end{pmatrix} = \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix} \text{ — (2)}$$

$$|A| I_2 = 4\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix} \text{ — (3)}$$

by (1), (2) & (3) $A(\operatorname{adj} A) = (\operatorname{adj} A)A = |A| I_2$.

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7) If $A = \begin{pmatrix} 3 & 2 \\ 7 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} -1 & -3 \\ 5 & 2 \end{pmatrix}$ verify that $(AB)^{-1} = B^{-1}A^{-1}$ ⑦

Soln

$|A| = 15 - 14 = 1 \neq 0$ $\quad A^{-1}$ exist.

$\text{adj } A = \begin{pmatrix} 5 & -2 \\ 7 & 3 \end{pmatrix}$

$A^{-1} = \frac{1}{|A|} \text{adj } A$

$= \frac{1}{1}\begin{pmatrix} 5 & -2 \\ 7 & 3 \end{pmatrix} = \begin{pmatrix} 5 & -2 \\ 7 & 3 \end{pmatrix}$

$|B| = -2 + 15 = 13 \neq 0$

$\text{adj } B = \begin{pmatrix} 2 & 3 \\ -5 & -1 \end{pmatrix}$

$B^{-1} = \frac{1}{|B|} \text{adj } B = \frac{1}{13}\begin{pmatrix} 2 & 3 \\ -5 & -1 \end{pmatrix}$

RHS

$B^{-1}A^{-1} = \frac{1}{13}\begin{pmatrix} 2 & 3 \\ -5 & -1 \end{pmatrix} \times \begin{pmatrix} 5 & -2 \\ 7 & 3 \end{pmatrix} = \frac{1}{13}\begin{pmatrix} 10-21 & -4+9 \\ -25+7 & 10-3 \end{pmatrix}$

$= \frac{1}{13}\begin{pmatrix} -11 & 5 \\ -18 & 7 \end{pmatrix}$

LHS

$AB = \begin{pmatrix} 3 & 2 \\ 7 & 5 \end{pmatrix}\begin{pmatrix} -1 & -3 \\ 5 & 2 \end{pmatrix} = \begin{pmatrix} -3+10 & -9+4 \\ -7+25 & -21+10 \end{pmatrix} = \begin{pmatrix} 7 & -5 \\ 18 & -11 \end{pmatrix}$

$|AB| = -77 + 90 = 13 \neq 0$

$\text{adj } AB = \begin{pmatrix} -11 & 5 \\ +18 & 7 \end{pmatrix}$

$(AB)^{-1} = \frac{1}{|AB|} \text{adj } AB = \frac{1}{13}\begin{pmatrix} -11 & 5 \\ 18 & 7 \end{pmatrix}$

8) If $\text{adj } A = \begin{pmatrix} 2 & -4 & 2 \\ -3 & 12 & -7 \\ -2 & 0 & 2 \end{pmatrix}$ find A

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Soln

$A = \pm \frac{1}{\sqrt{|\text{adj} A|}} \text{adj}(\text{adj} A)$

$|\text{adj} A| = 2(24-0) + 4(-6-14) + 2(0+24) = 48 - 80 + 48 = 16$

$\text{adj}(\text{adj} A) = \begin{pmatrix} 24 & 8 & 4 \\ 20 & 8 & 8 \\ 24 & 8 & 12 \end{pmatrix} = \frac{1}{4}\begin{pmatrix} 6 & 2 & 1 \\ 5 & 2 & 2 \\ 6 & 2 & 3 \end{pmatrix}$

$$\begin{matrix} -4 & 2 & 2 & -4 \\ 12 & -7 & -3 & 12 \\ 0 & 2 & -2 & 0 \\ -4 & 2 & 2 & -4 \\ 12 & -7 & -3 & 12 \end{matrix}$$

$\sqrt{|\text{adj} A|} = \sqrt{16} = 4$

$A = \pm 4\left(\frac{1}{4}\right)\begin{pmatrix} 6 & 2 & 1 \\ 5 & 2 & 2 \\ 6 & 2 & 3 \end{pmatrix} = \pm \begin{pmatrix} 6 & 2 & 1 \\ 5 & 2 & 2 \\ 6 & 2 & 3 \end{pmatrix}$

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9) If $adj\,A = \begin{pmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{pmatrix}$ find A^{-1}

Soln

$$A^{-1} = \pm \frac{1}{\sqrt{|adj\,A|}}\, adj\,A$$

$$|adj\,A| = 0 + 2(36-18) + 0 = 2(18) = 36.$$

$$A^{-1} = \pm \frac{1}{\sqrt{36}} \begin{pmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{pmatrix} = \pm \frac{1}{6} \begin{pmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ 3 & 0 & 6 \end{pmatrix}$$

10) Find $adj\,(adj\,A)$ if $adj\,A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ -1 & 0 & 1 \end{pmatrix}$

Soln

$$adj\,(adj\,A) = \begin{pmatrix} 2-0 & 0-0 & 0-2 \\ 0-0 & 1+1 & 0-0 \\ 0+2 & 0-0 & 2-0 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 0 & -2 \\ 0 & 2 & 0 \\ 2 & 0 & 2 \end{pmatrix}$$

$$\begin{array}{c|cccc} 1 & 0 & 1 & 1 & 0 \\ \hline 0 & 2 & 0 & 0 & 2 \\ -1 & 0 & 1 & -1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 2 & 0 & 0 & 2 \end{array}$$

11) $A = \begin{pmatrix} 1 & \tan x \\ -\tan x & 1 \end{pmatrix}$ Show that $A^T A^{-1} = \begin{pmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{pmatrix}$

Soln

$$|A| = 1 + \tan^2 x = \sec^2 x$$

$$adj\,A = \begin{pmatrix} 1 & -\tan x \\ \tan x & 1 \end{pmatrix}$$

$$A^{-1} = \frac{1}{|A|}\, adj\,A = \frac{1}{\sec^2 x} \begin{pmatrix} 1 & -\tan x \\ \tan x & 1 \end{pmatrix}$$

$$A^T = \begin{pmatrix} 1 & -\tan x \\ \tan x & 1 \end{pmatrix}$$

$$A^T A^{-1} = \frac{1}{\sec^2 x} \begin{pmatrix} 1 & -\tan x \\ \tan x & 1 \end{pmatrix} \begin{pmatrix} 1 & -\tan x \\ \tan x & 1 \end{pmatrix}$$

$$= \frac{1}{\sec^2 x} \begin{pmatrix} 1-\tan^2 x & -\tan x - \tan x \\ \tan x + \tan x & -\tan^2 x + 1 \end{pmatrix}$$

$$= \frac{1}{\sec^2 x} \begin{pmatrix} 1-\tan^2 x & -2\tan x \\ 2\tan x & 1-\tan^2 x \end{pmatrix} = \cos^2 x \begin{pmatrix} 1 - \frac{\sin^2 x}{\cos^2 x} & -\frac{2\sin x}{\cos x} \\ \frac{2\sin x}{\cos x} & 1 - \frac{\sin^2 x}{\cos^2 x} \end{pmatrix}$$

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$$= \begin{pmatrix} \cos^2 x - \sin^2 x & -2\sin x\cos x \\ 2\sin x\cos x & \cos^2 x - \sin^2 x \end{pmatrix} = \begin{pmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{pmatrix}$$ 9

$[\because \cos 2A = \cos^2 A - \sin^2 A$

$\sin 2A = 2\sin A\cos A]$

$=$ RHS.

Hence proved.

12) Find the matrix A for which $A\begin{pmatrix} 5 & 3 \\ -1 & -2 \end{pmatrix} = \begin{pmatrix} 14 & 7 \\ 7 & 7 \end{pmatrix}$

soln

order of A is 2×2.

Let us take $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$\underset{m\times \boxed{n \quad n}\times p}{A \quad B} = \underset{m\times p}{C_{2\times 2}}$

$n = 2.$

$m = 2.$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 5 & 3 \\ -1 & -2 \end{pmatrix} = \begin{pmatrix} 14 & 7 \\ 7 & 7 \end{pmatrix}$$

$$\begin{pmatrix} 5a-b & 3a-2b \\ 5c-d & 3c-2d \end{pmatrix} = \begin{pmatrix} 14 & 7 \\ 7 & 7 \end{pmatrix}$$

$5a - b = 14$ — ① $5c - d = 7$ — ③

$3a - 2b = 7$ — ② $3c - 2d = 7$ — ④

② $- 2\times$ ① $\Rightarrow$

$3a - 2b = 7$

$-10a + 2b = -28$

$-7a = -21$

$\boxed{a = 3}$

① $\Rightarrow$

$15 - b = 14$

$15 - 14 = b$

$\boxed{b = 1}$

④ $- 2\times$ ③ $\Rightarrow$

$3c - 2d = 7$

$-10c + 2d = -14$

$-7c = -7$

$\boxed{c = 1}$

③ $\Rightarrow$

$5 - d = 7$

$5 - 7 = d$

$d = -2$

$\therefore A = \begin{pmatrix} 3 & 1 \\ 1 & -2 \end{pmatrix}$

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13) Given $A = \begin{pmatrix} 1 & -1 \\ 2 & 0 \end{pmatrix}$ $B = \begin{pmatrix} 3 & -2 \\ 1 & 1 \end{pmatrix}$ $C = \begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix}$ find a matrix X such that $AXB = C$

soln

$AXB = C$

$A^{-1}A(X)BB^{-1} = A^{-1}CB^{-1}$

$X = A^{-1}CB^{-1}$

(Pre & Post multiply by A^{-1} and B^{-1})

$|A| = 0 + 2 = 2 \neq 0$

$\operatorname{adj} A = \begin{pmatrix} 0 & 1 \\ -2 & 1 \end{pmatrix}$

$\therefore A^{-1} = \frac{1}{|A|}\operatorname{adj} A = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ -2 & 1 \end{pmatrix}$

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$|B| = 3+2 = 5$

$\text{adj } B = \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}$ $\quad \therefore B^{-1} = \frac{1}{|B|} \text{adj } B = \frac{1}{5}\begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}$

$X = A^{-1} C B^{-1}$

$= \frac{1}{2}\begin{pmatrix} 0 & 1 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix} \frac{1}{5}\begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}$

$= \frac{1}{10}\begin{bmatrix} 0+2 & 0+2 \\ -2+2 & -2+2 \end{bmatrix}\begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix} = \frac{1}{10}\begin{pmatrix} 2 & 2 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}$

$= \frac{1}{10}\begin{pmatrix} 2-2 & 4+6 \\ 0-0 & 0-0 \end{pmatrix} = \frac{1}{10}\begin{pmatrix} 0 & 10 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$

14. If $A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ show that $A^{-1} = \frac{1}{2}(A^2 - 3I)$

soln

$A^2 = A \times A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$

$A^2 - 3I = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix} - 3\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix} + \begin{pmatrix} -3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -3 \end{pmatrix}$

$= \begin{pmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix} \quad \text{—} (1)$

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$|A| = 0 - 1(0-1) + 1(1-0) = 1+1 = 2$

$\text{adj } A = \begin{pmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix}$

$A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{2}\begin{pmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix}$

$A^{-1} = \frac{1}{2}(A^2 - 3I)$ $\quad$ using (1)

Hence proved.

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15) Decrypt the received encoded message $(2 \ -3)\ (20 \ 4)$ with the encryption matrix $\begin{pmatrix} -1 & -1 \\ 2 & 1 \end{pmatrix}$ and the decryption matrix as its inverse. where the system of codes are described by the numbers 1-26 to the letters A-Z respectively and the number 0 to a blank space.

Soln

Encoding matrix $\begin{pmatrix} -1 & -1 \\ 2 & 1 \end{pmatrix}$

un coded row matrix	Encoding matrix	coded row matrix
$(2 \ -3)$	$\begin{pmatrix} -1 & -1 \\ 2 & 1 \end{pmatrix}$	$= (-2-6 \ \ -2-3) = (-8 \ -5)$
$(20 \ 4)$	$\begin{pmatrix} -1 & -1 \\ 2 & 1 \end{pmatrix}$	$= (-20+8 \ \ -20+4) = (-12 \ -16)$

So the encoded message is $(-8 \ -5)\ (-12 \ -16)$

The receiver will decode the message by the inverse key - post multiplying by the inverse of A.

So the decoding matrix is $A^{-1} = \frac{1}{|A|} \text{adj } A$

$|A| = -1+2 = 1$

$\text{adj } A = \begin{pmatrix} 1 & 1 \\ -2 & -1 \end{pmatrix} \quad \therefore A^{-1} = \begin{pmatrix} 1 & 1 \\ -2 & -1 \end{pmatrix}$

The receiver decodes the coded message as follows

Coded row matrix	Decoding matrix	Decoded row matrix
$(-8 \ -5)$	$\begin{pmatrix} 1 & 1 \\ -2 & -1 \end{pmatrix}$	$= (-8+10 \ \ -8+5) = (2 \ -3)$
$(-12 \ -16)$	$\begin{pmatrix} 1 & 1 \\ -2 & -1 \end{pmatrix}$	$= (-12+32 \ \ -12+16) = (20 \ 4)$

So the sequence of decoded row matrices is $(2, -3)\ (20, 4)$

Thus the receiver reads the message is

$(-8 \ -5)\ (-12 \ -16)$

H E L P

Word is $\boxed{\text{HELP}}$

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Exercise 1.2

1. Find the rank of the following matrices by minor method.

(i) $A = \begin{pmatrix} 2 & -4 \\ -1 & 2 \end{pmatrix}$

$\begin{vmatrix} 2 & -4 \\ -1 & 2 \end{vmatrix} = 4 - 4 = 0.$

Rank of A is 1

$\rho(A) = 1$

(ii) $A = \begin{pmatrix} -1 & 3 \\ 4 & -7 \\ 3 & -4 \end{pmatrix}$

$\begin{vmatrix} -1 & 3 \\ 4 & -7 \end{vmatrix} = 7 - 12 = -5 \neq 0$

$\rho(A) = 2$

(iii) $A = \begin{pmatrix} 1 & -2 & -1 & 0 \\ 3 & -6 & -3 & 1 \end{pmatrix}$

$\begin{vmatrix} 1 & -2 \\ 3 & -6 \end{vmatrix} = 6 - 6 = 0$

$\begin{vmatrix} -1 & 0 \\ -3 & 1 \end{vmatrix} = -1 - 0 = -1 \neq 0$

$\rho(A) = 2$

(iv) $A = \begin{pmatrix} 1 & -2 & 3 \\ 2 & 4 & -6 \\ 5 & 1 & -1 \end{pmatrix}$

$|A| = 1(-4+6) + 2(-2+30) + 3(2-20)$

$= 1(-2) + 2(18) + 3(-18) = 2 + 56 - 54 = 4 \neq 0$

$\rho(A) = 3$

(v) $A = \begin{pmatrix} 0 & 1 & 2 & 1 \\ 0 & 2 & 4 & 3 \\ 8 & 1 & 0 & 2 \end{pmatrix}$

$\begin{vmatrix} 0 & 1 & 2 \\ 0 & 2 & 4 \\ 8 & 1 & 0 \end{vmatrix} = 8(4 - 4) = 8(0) = 0$

$\begin{vmatrix} 1 & 2 & 1 \\ 2 & 4 & 3 \\ 1 & 0 & 2 \end{vmatrix} = 1(8) - 2(4-3) + 1(0-4)$

$= 8 - 2(1) + 1(-4) = 8 - 2 - 4 = 2 \neq 0$

$\rho(A) = 3$

2) Find the rank of the following matrices by row reduction method.

(i) $A = \begin{pmatrix} 1 & 1 & 1 & 3 \\ 2 & -1 & 3 & 4 \\ 5 & -1 & 7 & 11 \end{pmatrix}$

$\sim \begin{pmatrix} 1 & 1 & 1 & 3 \\ 0 & -6 & 2 & -4 \\ 0 & -3 & 1 & -2 \end{pmatrix}$ $R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 5R_1$

$\sim \begin{pmatrix} 1 & 1 & 1 & 3 \\ 0 & -3 & 1 & -2 \\ 0 & -6 & 2 & -4 \end{pmatrix}$ $R_3 \leftrightarrow R_2$

$\sim \begin{pmatrix} 1 & 1 & 1 & 3 \\ 0 & -3 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ $R_3 \to R_3 - 2R_2$

$\rho(A) = 2$

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(ii) $A = \begin{pmatrix} 1 & 2 & -1 \\ 3 & -1 & 2 \\ 1 & -2 & 3 \\ 1 & -1 & 1 \end{pmatrix}$

$\sim \begin{pmatrix} 1 & 2 & -1 \\ 0 & -7 & 5 \\ 0 & -4 & 4 \\ 0 & -3 & 2 \end{pmatrix}$ $R_2 \to R_2 - 3R_1$, $R_3 \to R_3 - R_1$, $R_4 \to R_4 - R_1$

$\sim \begin{pmatrix} 1 & 2 & -1 \\ 0 & 7 & -5 \\ 0 & 4 & -4 \\ 0 & 3 & -2 \end{pmatrix}$ $R_2 \to (-1) R_2$, $R_3 \to (-1) R_3$, $R_4 \to (-1) R_4$

$\sim \begin{pmatrix} 1 & 2 & -1 \\ 0 & 7 & -5 \\ 0 & 0 & -8 \\ 0 & 0 & 1 \end{pmatrix}$ $R_3 \to 7R_3 - 4R_2$, $R_4 \to 7R_4 - 3R_2$

$\sim \begin{pmatrix} 1 & 2 & -1 \\ 0 & 7 & -5 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}$ $R_3 \to \frac{R_3}{-8}$

$\sim \begin{pmatrix} 1 & 2 & -1 \\ 0 & 7 & -5 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ $R_4 \to R_4 - R_3$

$\rho(A) = 3.$

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(iv)
$$A = \begin{pmatrix} 3 & -8 & 5 & 2 \\ 2 & -5 & 1 & 4 \\ -1 & 2 & 3 & -2 \end{pmatrix}$$

$$\sim \begin{pmatrix} -1 & 2 & 3 & -2 \\ 2 & 5 & 1 & 4 \\ 3 & -8 & 5 & 2 \end{pmatrix} \quad R_1 \leftrightarrow R_3$$

$$\sim \begin{pmatrix} 1 & -2 & -3 & 2 \\ 2 & 5 & 1 & 4 \\ 3 & -8 & 5 & 2 \end{pmatrix} \quad R_1 \to (-)R_1$$

$$\sim \begin{pmatrix} 1 & -2 & -3 & 2 \\ 0 & -1 & 7 & 0 \\ 0 & -2 & 14 & -4 \end{pmatrix} \quad \begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \end{matrix}$$

$$\sim \begin{pmatrix} 1 & -2 & -3 & 2 \\ 0 & -1 & 7 & 0 \\ 0 & 0 & 0 & -4 \end{pmatrix} \quad R_3 \to R_3 - 2R_1$$

$\therefore \rho(A) = 3$

3) Find the inverse of each of the following by Gauss Jordan method

(i) $A = \begin{pmatrix} 2 & -1 \\ 5 & -2 \end{pmatrix}$

Soln

$$[A \mid I_2] = \left[\begin{array}{cc|cc} 2 & -1 & 1 & 0 \\ 5 & -2 & 0 & 1 \end{array}\right] \xrightarrow{R_1 \to \frac{R_1}{2}} \left[\begin{array}{cc|cc} 1 & -1/2 & 1/2 & 0 \\ 5 & -2 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_2 \to R_2 - 5R_1} \left[\begin{array}{cc|cc} 1 & -1/2 & 1/2 & 0 \\ 0 & 1/2 & -5/2 & 1 \end{array}\right] \xrightarrow{R_2 \to 2R_2} \left[\begin{array}{cc|cc} 1 & -1/2 & 1/2 & 0 \\ 0 & 1 & -5 & 2 \end{array}\right]$$

$$\xrightarrow{R_1 \to R_1 + \frac{1}{2}R_2} \left[\begin{array}{cc|cc} 1 & 0 & -2 & 1 \\ 0 & 1 & -5 & 2 \end{array}\right]$$

Inverse of matrix A is $\begin{pmatrix} -2 & 1 \\ -5 & 2 \end{pmatrix}$

(ii) $A = \begin{pmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 6 & -2 & -3 \end{pmatrix}$

Soln

$$[A \mid I_3] = \left(\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 1 & 0 & -1 & 0 & 1 & 0 \\ 6 & -2 & -3 & 0 & 0 & 1 \end{array}\right)$$

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$$\xrightarrow[R_3 \to R_3 - 6R_1]{R_2 \to R_2 - R_1} \left(\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 4 & -3 & -6 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{R_3 \to R_3 - 4R_2} \left(\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & -2 & -4 & 1 \end{array}\right)$$

$$\xrightarrow{R_2 \to R_2 + R_3} \left(\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -3 & -3 & 1 \\ 0 & 0 & 1 & -2 & -4 & 1 \end{array}\right)$$

$$\xrightarrow{R_1 \to R_1 + R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & -3 & 1 \\ 0 & 1 & 0 & -3 & -3 & 1 \\ 0 & 0 & 1 & -2 & -4 & 1 \end{array}\right)$$

inverse of A is $\begin{pmatrix} -2 & -3 & 1 \\ -3 & -3 & 1 \\ -2 & -4 & 1 \end{pmatrix}$

(iii) $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{pmatrix}$

Soln

$$[A \mid I_3] = \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 5 & 3 & 0 & 1 & 0 \\ 1 & 0 & 8 & 0 & 0 & 1 \end{array}\right)$$

$$\xrightarrow[R_3 \to R_3 - R_1]{R_2 \to R_2 - 2R_1} \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & -2 & 5 & -1 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{R_3 \to R_3 + 2R_2} \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array}\right)$$

$$\xrightarrow{R_2 \to R_2 + 3R_3} \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & 0 & 1 & 5 & -2 & 1 \end{array}\right)$$

$$\xrightarrow{R_2 \to R_2 + R_1} \left(\begin{array}{cccccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 0 & 13 & -5 & -3 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array}\right)$$

$$\xrightarrow{R_1 \to R_1 - 2R_2} \left(\begin{array}{cccccc} 1 & 0 & 3 & -25 & 10 & 6 \\ 0 & 1 & 0 & 13 & -5 & -3 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array}\right)$$

$$\xrightarrow{R_1 \to R_1 - 3R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -40 & 16 & 9 \\ 0 & 1 & 0 & 13 & -5 & 3 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array}\right)$$

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Inverse of matrix is $\begin{pmatrix} -40 & 16 & 9 \\ 13 & -5 & 3 \\ 5 & -2 & -1 \end{pmatrix}$

Exercise 1.3

1. Solve the following system of linear equation by matrix inversion method.

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(i) $2x+5y=-2$, $x+2y=-3$

Soln

$$\begin{pmatrix} 2 & 5 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 \\ -3 \end{pmatrix}$$

$$AX = B$$

$$X = A^{-1}B \qquad \therefore A = \begin{pmatrix} 2 & 5 \\ 1 & 2 \end{pmatrix}$$

$$|A| = 4-5 = -1$$

$$\text{adj} A = \begin{pmatrix} 2 & -5 \\ -1 & 2 \end{pmatrix}$$

$$A^{-1} = \frac{1}{|A|}\text{adj} A = \frac{1}{-1}\begin{pmatrix} 2 & -5 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} -2 & 5 \\ 1 & -2 \end{pmatrix}$$

$$X = A^{-1}B = \begin{pmatrix} -2 & 5 \\ 1 & -2 \end{pmatrix}\begin{pmatrix} -2 \\ -3 \end{pmatrix} = \begin{pmatrix} 4-15 \\ -2+6 \end{pmatrix} = \begin{pmatrix} -11 \\ 4 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -11 \\ 4 \end{pmatrix} \qquad \therefore \boxed{\begin{matrix} x = -11 \\ y = 4 \end{matrix}}$$

(ii) $2x-y=8$, $3x-2y=-2$

Soln

$$\begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 8 \\ -2 \end{pmatrix} \qquad \text{Hence } A = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix}$$

$$AX = B$$

$$X = A^{-1}B$$

$$|A| = -4+3 = -1$$

$$\text{adj} A = \begin{pmatrix} -2 & 1 \\ -3 & 2 \end{pmatrix}$$

$$A^{-1} = \frac{1}{|A|}\text{adj} A = \frac{1}{-1}\begin{pmatrix} -2 & 1 \\ -3 & 2 \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix}$$

$$X = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix}\begin{pmatrix} 8 \\ -2 \end{pmatrix}$$

$$= \begin{pmatrix} 16+2 \\ 24+4 \end{pmatrix} = \begin{pmatrix} 18 \\ 28 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 18 \\ 28 \end{pmatrix}$$

$$\boxed{\begin{matrix} x = 18 \\ y = 28 \end{matrix}}$$

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(ii) $2x+3y-z=9,\ x+y+z=9,\ 3x-y-z=-1$

Soln

$$\begin{pmatrix} 2 & 3 & -1 \\ 1 & 1 & 1 \\ 3 & -1 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 9 \\ 9 \\ -1 \end{pmatrix}$$

$$A \quad X = B \quad \text{where } A = \begin{pmatrix} 2 & 3 & -1 \\ 1 & 1 & 1 \\ 3 & -1 & 1 \end{pmatrix}$$

$$X = A^{-1}B$$

$$|A| = 2(-1+1) - 3(-1-3) - 1(-1-3) = 0+12+4 = 16$$

$$\text{adj}\,A = \begin{pmatrix} -1+1 & 1+3 & 3+1 \\ 3+1 & -2+3 & -1-2 \\ -1-3 & 9+2 & 2-3 \end{pmatrix}$$

$$\begin{array}{ccccc} 2 & 3 & -1 & 2 & 3 \\ 1 & 1 & 1 & 1 & 1 \\ 3 & -1 & -1 & 3 & -1 \\ 2 & 3 & -1 & 2 & 3 \\ 1 & 1 & 1 & 1 & 1 \end{array}$$

$$= \begin{pmatrix} 0 & 4 & 4 \\ 4 & 1 & -3 \\ -4 & 11 & -1 \end{pmatrix}$$

$$A^{-1} = \frac{1}{|A|}\,\text{adj}\,A = \frac{1}{16}\begin{pmatrix} 0 & 4 & 4 \\ 4 & 1 & -3 \\ -4 & 11 & -1 \end{pmatrix}$$

$$X = A^{-1}B = \frac{1}{16}\begin{pmatrix} 0 & 4 & 4 \\ 4 & 1 & -3 \\ -4 & 11 & -1 \end{pmatrix}\begin{pmatrix} 9 \\ 9 \\ -1 \end{pmatrix} = \frac{1}{16}\begin{pmatrix} 36-4 \\ 36+9+3 \\ -36+99+1 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{16}\begin{pmatrix} 32 \\ 48 \\ 64 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$$

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$$\boxed{\begin{array}{l} x = 2 \\ y = 3 \\ z = 4 \end{array}}$$

(iii) $x+y+z-2=0,\ 6x-4y+5z-31=0,\ 5x+2y+2z=13.$

Soln

$$\begin{pmatrix} 1 & 1 & 1 \\ 6 & -4 & 5 \\ 5 & 2 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 31 \\ 13 \end{pmatrix}$$

$$AX = B \quad \text{where } A = \begin{pmatrix} 1 & 1 & 1 \\ 6 & -4 & 5 \\ 5 & 2 & 2 \end{pmatrix}$$

$$X = A^{-1}B.$$

$$|A| = 1(-8-10) - 1(12-25) + 1(12+20)$$

$$= -18+13+32 = 27.$$

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$$\text{adj}\,A = \begin{pmatrix} -8-10 & 2-2 & 5+4 \\ 25-12 & 2-5 & 6-5 \\ 12+20 & 5-2 & -4-6 \end{pmatrix}$$

$$= \begin{pmatrix} -18 & 0 & 9 \\ 13 & -3 & 1 \\ 32 & 3 & -10 \end{pmatrix}$$

a	-4	5	6	-4
b	2	2	5	2
	1	1	1	1
c	-4	5	6	-4

$$A^{-1} = \frac{1}{|A|}\,\text{adj}\,A = \frac{1}{27}\begin{pmatrix} -18 & 0 & 9 \\ 13 & -3 & 1 \\ 32 & 3 & -10 \end{pmatrix}$$

$$X = A^{-1}B = \frac{1}{27}\begin{pmatrix} -18 & 0 & 9 \\ 13 & -3 & 1 \\ 32 & 3 & -10 \end{pmatrix}\begin{pmatrix} 2 \\ 31 \\ 13 \end{pmatrix} = \frac{1}{27}\begin{pmatrix} -36+0+117 \\ 26-93+13 \\ 64+93-130 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{27}\begin{pmatrix} 81 \\ -54 \\ 27 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$$

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$$\boxed{\begin{matrix} x = 3 \\ y = -2 \\ z = 1 \end{matrix}}$$

2. If $A = \begin{pmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{pmatrix}$ find the products AB and BA and hence solve the system of equations $x+y+2z=1$, $3x+2y+z=7$, $2x+y+3z=2$.

Soln

$$AB = \begin{pmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{pmatrix} = \begin{pmatrix} -5+3+6 & -5+2+3 & -10+1+9 \\ 7+3-10 & 7+2-5 & 14+1-15 \\ 1-3+2 & 1-2+1 & 2-1+3 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix} = 4I_3$$

$$BA = \begin{pmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{pmatrix}\begin{pmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{pmatrix} = \begin{pmatrix} -5+7+2 & 1+1-2 & 3-5+2 \\ -15+14+1 & 3+2-1 & 9-10+1 \\ -10+7+3 & 2+1-3 & 6-5+3 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix} = 4I_3.$$

$$AB = BA = 4I_3$$

$$\left(\frac{1}{4}A\right)B = B\left(\frac{1}{4}A\right) = I \Rightarrow B^{-1} = \frac{1}{4}A.$$

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(19)

matrix form $\begin{pmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 7 \\ 2 \end{pmatrix}$

$AX = B$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{4} A \begin{pmatrix} 1 \\ 7 \\ 2 \end{pmatrix} = \frac{1}{4}\begin{pmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 7 \\ 2 \end{pmatrix}$$

$$= \frac{1}{4}\begin{pmatrix} -5+7+6 \\ 7+7-10 \\ 1-7+2 \end{pmatrix} = \frac{1}{4}\begin{pmatrix} 8 \\ 4 \\ -4 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$$

$\boxed{x = 2,\ y = 2,\ z = -1}$

7 A man is appointed in a job with a monthly salary of certain amount and a fixed amount of annual increment. If his salary was ₹ 19800 per month at the end of the first month after 3 years of service and ₹ 23400 per month at the end of the first month after 9 years of service. find his starting salary and his annual increment.

Soln Let the starting month salary be x & increment be y.

after 3 years salary will be

$x + 3y = 19800$ — (1)

after 9 years salary will be

$x + 9y = 23400$ — (2)

(2) − (1) $6y = 3600$

$y = 600$

(1) ⇒ $x + 1800 = 19800$

$x = 19800 - 1800 = 18000$

$x = 18000$

Starting month Salary $= 18000$

monthly increment $= 6000$.

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(20)

4) 4 men and 4 women can finish a piece of work jointly in 3 days while 2 men and 5 women can finish the same work jointly in 4 days. Find the time taken by one man alone and that of one woman alone to finish the same work by using matrix inversion method?

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Soln.

$$4x+4y=\frac{1}{3}$$
$$2x+5y=\frac{1}{4}.$$

matrix form $\begin{pmatrix}4&4\\2&5\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}1/3\\1/4\end{pmatrix}$

$AX=B$ where $A=\begin{pmatrix}4&4\\2&5\end{pmatrix}$

$X=A^{-1}B.$

$|A|=20-8=12.$, $\operatorname{adj}A=\begin{pmatrix}5&-4\\-2&4\end{pmatrix}$

$A^{-1}=\frac{1}{|A|}\operatorname{adj}A=\frac{1}{12}\begin{pmatrix}5&-4\\-2&4\end{pmatrix}.$

$X=\frac{1}{12}\begin{pmatrix}5&-4\\-2&4\end{pmatrix}\begin{pmatrix}1/3\\1/4\end{pmatrix}=\frac{1}{12}\begin{pmatrix}5/3-1\\-2/3+1\end{pmatrix}=\frac{1}{12}\begin{pmatrix}2/3\\1/3\end{pmatrix}=\begin{pmatrix}\frac{1}{18}\\1/36\end{pmatrix}$

$\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}1/18\\1/36\end{pmatrix}$ ∴ one men can do 18 days
one women can do 36 days.

5) The prices of 3 commodities A, B and C are ₹ x, y + z per units respectively. A person P purchases 4 units of B and sells two units of A and 5 units of C. Person Q purchases 2 units of C and sells 3 units of A & one unit of B. In the process. P, Q & R earn ₹ 15000, ₹ 1000 and ₹ 4000 respectively. Find the prices per unit of A, B & C.

Soln

Let x, y, z are commodities of A, B & C.

$$2x+4y+5c=1500 \quad -①$$
$$3x+y+2c=1000 \quad -②$$
$$x+3y+c=4000 \quad -③$$

matrix form
$$\begin{pmatrix}2&4&5\\3&1&2\\1&3&1\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}1500\\1000\\4000\end{pmatrix}$$

$$AX=B$$
$$X=A^{-1}B$$

$$|A| = 2(1-6) - 4(3-2) + 5(9-1)$$
$$= 2(-5) - 4(1) + 5(8) = -10 - 4 + 40 = 26$$

$$\begin{array}{ccccc} 2 & 4 & 5 & 2 & 4 \\ 3 & 1 & 2 & 3 & 1 \\ 1 & 3 & 1 & 1 & 3 \\ 2 & 4 & 5 & 2 & 4 \\ 3 & 1 & 2 & 3 & 1 \end{array}$$

$$\text{adj } A = \begin{pmatrix} 1-6 & 15-4 & 8-5 \\ 2-3 & 2-5 & 15-4 \\ 9-1 & 4-6 & 2-12 \end{pmatrix}$$

$$= \begin{pmatrix} -5 & 11 & 3 \\ -1 & -3 & 11 \\ 8 & -2 & -10 \end{pmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{26}\begin{pmatrix} -5 & 11 & 3 \\ -1 & -3 & 11 \\ 8 & -2 & 10 \end{pmatrix}$$

$$X = A^{-1}B = \frac{1}{26}\begin{pmatrix} -5 & 11 & 3 \\ -1 & -3 & 11 \\ 8 & -2 & 10 \end{pmatrix}\begin{pmatrix} 15000 \\ 1000 \\ 4000 \end{pmatrix}$$

$$= \frac{1}{26}\begin{pmatrix} -75000 + 11000 + 12000 \\ -15000 - 3000 + 44000 \\ 120000 - 2000 + 40000 \end{pmatrix} = \frac{1}{26}\begin{pmatrix} 52000 \\ 26000 \\ 78000 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2000 \\ 1000 \\ 3000 \end{pmatrix}$$

$$\boxed{\begin{array}{l} x = 2000 \\ y = 1000 \\ z = 3000 \end{array}}$$

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Exercise 1.4.

1. Solve the following systems of linear equations by Cramer's rule.

i) $5x - 2y + 16 = 0 \quad x + 3y - 7 = 0 \Rightarrow 5x - 2y = -16$

$x + 3y = 7$

$\Delta = \begin{vmatrix} 5 & -2 \\ 1 & 3 \end{vmatrix} = 15 + 2 = 17$

$\Delta_x = \begin{vmatrix} -16 & -2 \\ 7 & 3 \end{vmatrix} = -48 + 14 = -34$

$\Delta_y = \begin{vmatrix} 5 & -16 \\ 1 & 7 \end{vmatrix} = 35 + 16 = 51$

by cramer's rule $x = \frac{\Delta x}{\Delta} = \frac{-34}{17} = -2$

$y = \frac{\Delta y}{\Delta} = \frac{51}{17} = 3$

$\therefore \boxed{x = -2, \ y = 3}$

(ii) $\frac{3}{x} + 2y = 12, \quad \frac{2}{x} + 3y = 13.$

soln

$\frac{3}{x} + 2y = 12$ — (1)

$2y = 12 - \frac{3}{x}.$

$y = \frac{1}{2}\left(12 - \frac{3}{x}\right).$

$\frac{2}{x} + 3y = 13.$ — (2)

$\frac{2}{x} + 3\left(\frac{1}{2}\right)\left(12 - \frac{3}{x}\right) = 13.$

$\therefore \frac{2}{x} + \frac{3}{2}(\cancel{12})^{6} - \frac{3}{2}\left(\frac{3}{x}\right) = 13.$

$\frac{2}{x} + 18 - \frac{9}{2x} = 13 \Rightarrow \frac{4-9}{2x} = 13 - 18$

$\frac{-\cancel{5}}{2x} = \cancel{-5}$

$\frac{1}{2x} = 1 \Rightarrow 2x = 1 \Rightarrow x = 1/2.$

(1) $\frac{3}{x} + 2y = 12$

$6 + 2y = 12$

$2y = 6$

$y = 3$

$\boxed{\therefore x = 1/2 \quad y = 3}$

(iii) $3x+3y-z=11,\quad 2x-y+2z=9,\quad 4x+3y+2z=25$

Soln

$$\Delta = \begin{vmatrix} 3 & 3 & -1 \\ 2 & -1 & 2 \\ 4 & 3 & 2 \end{vmatrix} = 3(-2-6)-3(4-8)-1(6+4)$$
$$= 3(-8)-3(-4)-1(10) = -24+12-10$$
$$= -22$$

$$\Delta_x = \begin{vmatrix} 11 & 3 & -1 \\ 9 & -1 & 2 \\ 25 & 3 & 2 \end{vmatrix} = 11(-2-6)-3(18-50)-1(27+25)$$
$$= 11(-8)-3(-32)-1(52)$$
$$= -88+96-52 = -44$$

$$\Delta_y = \begin{vmatrix} 3 & 11 & -1 \\ 2 & 9 & 2 \\ 4 & 25 & 2 \end{vmatrix} = 3(18-50)-11(4-8)-1(50-36)$$
$$= 3(-32)-11(-4)-1(14)$$
$$= -96+44-14 = -66.$$

$$\Delta_z = \begin{vmatrix} 3 & 3 & 11 \\ 2 & -1 & 9 \\ 4 & 3 & 25 \end{vmatrix} = 3(-25-27)-3(50-36)+11(6+4)$$
$$= 3(-52)-3(14)+11(10) = -156-42+110 = -88$$

by Cramer's rule $x = \dfrac{\Delta_x}{\Delta} = \dfrac{-44}{-22} = 2$

$y = \dfrac{\Delta_y}{\Delta} = \dfrac{-66}{-22} = 3$

$z = \dfrac{\Delta_z}{\Delta} = \dfrac{-88}{-22} = 4$

$$\boxed{\therefore\ x = 2,\ y = 3,\ z = 4}$$

(iv) $\dfrac{3}{x}-\dfrac{4}{y}-\dfrac{2}{z}-1=0,\quad \dfrac{1}{x}+\dfrac{2}{y}+\dfrac{1}{z}-2=0,\quad \dfrac{2}{x}-\dfrac{5}{y}-\dfrac{4}{z}+1=0.$

Soln Put $a=\dfrac{1}{x},\ b=\dfrac{1}{y},\ c=\dfrac{1}{z}$.

$3a-4b-2c=1$ —(1)

$a+2b+c=2$ —(2)

$2a-5b-4c=-1.$ —(3)

$$\Delta = \begin{vmatrix} 3 & -4 & -2 \\ 1 & 2 & 1 \\ 2 & -5 & -4 \end{vmatrix} = 3(-8+5)+4(-4-2)-2(-5-4)$$
$$= -9-24+18 = -15$$

$$\Delta_a = \begin{vmatrix} 1 & -4 & -2 \\ 2 & 2 & 1 \\ -1 & -5 & -4 \end{vmatrix} = 1(-8+5)+4(-8+1)-2(-10+2)$$
$$= -3-28+16 = -15$$

$$\Delta_b = \begin{vmatrix} 3 & 1 & -2 \\ 1 & 2 & 1 \\ 2 & -1 & -4 \end{vmatrix} = 3(-8+1) - 1(-4-2) - 2(-1-4)$$
$$= 3(-7) - 1(-6) - 2(-5)$$
$$= -21+6+10 = -5$$

$$\Delta_c = \begin{vmatrix} 3 & -4 & 1 \\ 1 & 2 & 2 \\ 2 & -5 & -1 \end{vmatrix} = 3(-2+10)+4(-1-4)+1(-5-4)$$
$$= 24-20-9 = -5$$

$$a = \frac{\Delta_a}{\Delta} = \frac{-15}{-15} = 1$$

$$b = \frac{\Delta_b}{\Delta} = \frac{-5}{-15} = \frac{1}{3}$$

$$c = \frac{\Delta_c}{\Delta} = \frac{-5}{-15} = \frac{1}{3}$$

$$\boxed{x = 1 \quad y = 3 \quad z = 3}$$

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2) In a competitive examination, one mark is awarded for every correct answer while 1/4 mark is deducted for every wrong answer. A student answered 100 questions and got 80 marks. How many questions did he answer correctly?

Soln. Total number of question 100.

Let the correct questions be x and the wrong questions by y

$$x + y = 100 \quad — (1)$$

$$x - \tfrac{1}{4}y = 80. \quad — (2).$$

$$\Delta = \begin{vmatrix} 1 & 1 \\ 1 & -\frac{1}{4} \end{vmatrix} = -\tfrac{1}{4} - 1 = -\tfrac{5}{4}.$$

$$\Delta_x = \begin{vmatrix} 100 & 1 \\ 80 & -\frac{1}{4} \end{vmatrix} = -25 - 80 = -105$$

$$\Delta_y = \begin{vmatrix} 1 & 100 \\ 1 & 80 \end{vmatrix} = 80 - 100 = -20$$

$$x = \frac{\Delta_x}{\Delta} = \frac{+105}{+5/4} = 21 \times 4 = 84.$$

$$y = \Delta_y/\Delta = \frac{+20}{+5/4} = 4 \times 4 = 16$$

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correct ed questions = 84.

wrong Questions = 16.

3) A chemist has one solution which is 50% acid and another solution which is 25% acid. How much each should be mixed to make 10 litres of a 40% acid solution?

soln Let two solutions be x & y.

$$x+y = 10 \quad \text{--- } ①$$

$$0.25x + (0.50)y = (0.40)10 \quad \text{--- } ②$$

② × 100 $\qquad 25x + 50y = 400$

② ÷ 5 $\qquad 5x + 10y = 80 \quad \text{--- } ③$

$$\Delta = \begin{vmatrix} 1 & 1 \\ 5 & 10 \end{vmatrix} = 10 - 5 = 5$$

$$\Delta_x = \begin{vmatrix} 10 & 1 \\ 80 & 10 \end{vmatrix} = 100 - 80 = 20$$

$$\Delta_y = \begin{vmatrix} 1 & 10 \\ 5 & 80 \end{vmatrix} = 80 - 50 = 30$$

$x = \frac{\Delta x}{\Delta} = \frac{20}{5} = 4$. (litres of 25% solution)

$y = \frac{\Delta y}{\Delta} = \frac{30}{5} = 6$ (litres of 50% solution)

$$\boxed{\begin{matrix} x=4 \\ y=6 \end{matrix}}$$

4) ~~An amount of 65000 is invested in three bonds at the rate of 6%, 8% and 10% per annum respectively. The total annual income is ₹ 4800, The income from the third bond is ₹ 600 more than that from the second bond. determine the price of each bond.~~

soln

(26)

4) A fish tank can be filled in 10 min using both pumps A & B simultanously pump B can pump water in or out at the same rate. If pump B is inadvertently run in reverse then the tank will be filled in 30 min. How long would it take each pump to fill the tank by itself?

Soln Pump A fills $(\frac{1}{x})^{th}$ of the tank in 1 hour

Pump B fills $(\frac{1}{y})^{th}$ of the tank in 1 hour.

both can filled $(\frac{1}{10}^{th})$ of the tank in 1 hour

$$\frac{1}{x}+\frac{1}{y} = \frac{1}{10}.$$

Pump B filled in 30 min.

$$\frac{1}{x}+\frac{1}{30} = \frac{1}{10}$$

$$\frac{1}{x} = \frac{1}{10}-\frac{1}{30} = \frac{3-1}{30} = \frac{2}{30} = \frac{1}{15}. \text{ (Pump A)}$$

$$\frac{1}{y} = \frac{1}{30}. \text{ (Pump B)}$$

∴ Pump A takes 15 min to fill the tank.

Pump B takes 30 min

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5) A family of 3 people went out for dinner in a restaurant. The cost of two dosai, three idlies & two vadas is ₹150, The cost of the two dosai, two idlies and four vadais is ₹200. Cost of five dosai, four idlies and two vadas is ₹250. The family has ₹350 in hand and ate 3 dosai and six idlies and six vadas will they be able to manage to pay the bill within amount they had?

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Soln Let dosai, idlies & vadas be x, y, z

$$2x + 3y + 2z = 150$$
$$2x + 2y + 4z = 200$$
$$5x + 4y + 2z = 250.$$

$$\Delta = \begin{vmatrix} 2 & 3 & 2 \\ 2 & 2 & 4 \\ 5 & 4 & 2 \end{vmatrix} = 2(4-16) - 3(4-20) + 2(8-10)$$

$$= 2(-12) - 3(-16) + 2(-2) = -24 + 48 - 4 = 20$$

$$\Delta x = \begin{vmatrix} 150 & 3 & 2 \\ 200 & 2 & 4 \\ 250 & 4 & 2 \end{vmatrix} = 150(4-16) - 3(400-1000) + 2(800-500)$$
$$= 150(-12) - 3(-600) + 2(300)$$
$$= -1800 + 1800 + 600 = 600$$

$$\Delta y = \begin{vmatrix} 2 & 150 & 2 \\ 2 & 200 & 4 \\ 5 & 250 & 2 \end{vmatrix} = 2(400-1000) - 150(4-20) + 2(500-1000)$$
$$= 2(-600) - 150(-16) + 2(-500)$$
$$= -1200 + 2400 - 1000 = 200$$

$$\Delta z = \begin{vmatrix} 2 & 3 & 150 \\ 2 & 2 & 200 \\ 5 & 4 & 250 \end{vmatrix} = 2(500-800) - 3(500-1000) + 150(8-10)$$

$$= 2(-300) - 3(-500) + 150(-2)$$
$$= -600 + 1500 - 300 = 600.$$

$$x = \frac{\Delta x}{\Delta} = \frac{600}{20} = 30; \quad y = \frac{\Delta y}{\Delta} = \frac{200}{20} = 10; \quad z = \frac{\Delta z}{\Delta} = \frac{600}{20} = 30$$

$$x = 30, \quad y = 10, \quad z = 30.$$

They ate 3 dosai, 6 idlies & 6 vadas

$$3x + 6y + 6z = 3(30) + 6(10) + 6(30)$$
$$= 90 + 60 + 180 = 330$$

They had 350 spend 330. They can eat with-in the amount

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Exercise 1.5

1. Solve the following systems of linear equations by Gaussian elimination method

(i) $2x-2y+3z=2, \quad x+2y-z=3, \quad 3x-y+2z=1$

Soln Augmented matrix

$$[A|B] = \left[\begin{array}{ccc|c} 2 & -2 & 3 & 2 \\ 1 & 2 & -1 & 3 \\ 3 & -1 & 2 & 1 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 2 & -2 & 3 & 2 \\ 3 & -1 & 2 & 1 \end{array}\right]$$

$$\xrightarrow[R_3 \to R_3 - 3R_1]{R_2 \to R_2 - 2R_1} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & -6 & 5 & -4 \\ 0 & -7 & 5 & -8 \end{array}\right] \xrightarrow{C_1 \leftrightarrow C_3} \left[\begin{array}{ccc|c} 1 & -1 & 2 & 3 \\ 0 & 5 & -6 & -4 \\ 0 & 5 & -7 & -8 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 - 5R_2} \left[\begin{array}{ccc|c} 1 & -1 & 2 & 3 \\ 0 & 5 & -6 & -4 \\ 0 & 0 & -1 & -4 \end{array}\right]$$

writing the equivalent equations from echelon form.

$-z = -4$
$z = 4$

$5y - 6z = -4$
$5y - 24 = -4$
$5y = -4 + 24$
$5y = 20$
$y = 4$

$x - y + 2z = 3$
$x - 4 + 8 = 3$
$x = 3 + 4 - 8$
$x = -1$

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$$\boxed{\therefore x = -1, \ y = 4, \ z = 4}$$

(ii) $2x+4y+6z=22, \quad 3x+8y+5z=27, \quad -x+y+2z=2$

Soln. Augmented matrix

$$[A|B] = \left[\begin{array}{ccc|c} 2 & 4 & 6 & 22 \\ 3 & 8 & 5 & 27 \\ -1 & 1 & 2 & 2 \end{array}\right] \xrightarrow{R_1 \to \frac{R_1}{2}} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 11 \\ 3 & 8 & 5 & 27 \\ -1 & 1 & 2 & 2 \end{array}\right]$$

$$\xrightarrow[R_3 \to R_3 + R_1]{R_2 \to R_2 - 3R_1} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 11 \\ 0 & 2 & -4 & -6 \\ 0 & 3 & 5 & 13 \end{array}\right] \xrightarrow{R_2 \to \frac{R_2}{2}} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 11 \\ 0 & 1 & -2 & -3 \\ 0 & 3 & 5 & 13 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 - 3R_2} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 11 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 11 & 22 \end{array}\right]$$

writing the equivalent equations from echelon form

$11z = 22$

$\boxed{z = 2}$

$y - 2z = -3$

$y - 4 = -3$

$y = -3 + 4$

$\boxed{y = 1}$

$x + 2y + 3z = 11$

$x + 2(1) + 3(2) = 11$

$x + 2 + 6 = 11$

$x = 11 - 8 = 3$

$\boxed{x = 3}$

$\therefore \boxed{x = 3,\ y = 1,\ z = 2}$

2) If $ax^2 + bx + c$ is divided by $x+3$, $x-5$, and $x-1$. The remainders are $21, 61$ & 9 respectively. Find a, b & c.

Soln

$f(x) = ax^2 + bx + c.$

$f(-3) \Rightarrow 9a - 3b + c = 21$ — ① $(x+3=0 \Rightarrow x=-3)$

$f(5) \Rightarrow 25a + 5b + c = 61$ — ② $(x-5=0 \Rightarrow x=5)$

$f(1) \Rightarrow a + b + c = 9$ — ③ $(x-1=0 \Rightarrow x=1)$

matrix form
$$\begin{pmatrix} 9 & -3 & 1 \\ 25 & 5 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 21 \\ 61 \\ 9 \end{pmatrix}$$

$A \quad X \quad = B$

$$(A|B) = \begin{pmatrix} 9 & -3 & 1 & 21 \\ 25 & 5 & 1 & 61 \\ 1 & 1 & 1 & 9 \end{pmatrix} \xrightarrow{C_1 \leftrightarrow C_3} \begin{pmatrix} 1 & -3 & 9 & 21 \\ 1 & 5 & 25 & 61 \\ 1 & 1 & 1 & 9 \end{pmatrix}$$

$$\xrightarrow[R_3 \to R_3 - R_1]{R_2 \to R_2 - R_1} \begin{pmatrix} 1 & -3 & 9 & 21 \\ 0 & 8 & 16 & 40 \\ 0 & 4 & -8 & -12 \end{pmatrix} \xrightarrow[R_4 \to R_4/4]{R_2 \to R_2/8} \begin{pmatrix} 1 & 1 & 1 & 9 \\ 0 & 1 & 2 & 5 \\ 0 & 1 & -2 & -3 \end{pmatrix}$$

$$\xrightarrow{R_3 \to R_3 - R_2} \begin{pmatrix} 1 & 1 & 1 & 9 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & -4 & -8 \end{pmatrix}$$

$-4c = -8$

$c = 2$

$b + 2c = 5$

$b + 4 = 5$

$b = 5 - 4 = 1$

$a + b + c = 9$

$a + 1 + 2 = 9$

$a = 9 - 3$

$a = 6$

$\boxed{a = 6,\ b = 1,\ c = 2}$

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3) An amount of ₹65000 is invested in three bonds at the rates of 6%, 8% and 10% per annum respectively. The total annual income is ₹~~4800~~ 5000. The income from the ~~third~~ first bond is ₹600, more than that from the second bond. Determine the price of each bond. (5M)

Soln: Let the amount of 3 bonds be x, y, z.

$$x+y+z = 65000 \quad —①$$

$$0.06x+0.08y+0.10z = 5000 \quad —② \text{ (Total income 5000)}$$

$$0.06x - 0.08y = 600 \quad —③ \text{ (}1^{st}\text{ bond is 600 more that } 2^{nd}\text{ bond)}$$

matrix form

$$[A|B] \sim \begin{pmatrix} 1 & 1 & 1 & 65000 \\ 0.06 & 0.08 & 0.10 & 5000 \\ 0.06 & -0.08 & 0 & 600 \end{pmatrix}$$

$$\xrightarrow[R_3 \to R_3 - 0.06R_1]{R_2 \to R_2 - 0.06R_1} \begin{pmatrix} 1 & 1 & 1 & 65000 \\ 0 & 0.02 & 0.04 & 1100 \\ 0 & -0.14 & -0.06 & -3300 \end{pmatrix}$$

$$\xrightarrow[R_3 \to R_3/-3]{R_2 \to R_2/2} \begin{pmatrix} 1 & 1 & 1 & 65000 \\ 0 & 0.01 & 0.02 & 550 \\ 0 & 0.07 & 0.03 & 1650 \end{pmatrix}$$

$$\xrightarrow[R_3 \to R_3 \times 100]{R_2 \to R_2 \times 100} \begin{pmatrix} 1 & 1 & 1 & 65000 \\ 0 & 1 & 2 & 55000 \\ 0 & 7 & 3 & 165000 \end{pmatrix}$$

$$\xrightarrow{R_3 \to R_3 - 7R_2} \begin{pmatrix} 1 & 1 & 1 & 65000 \\ 0 & 1 & 2 & 55000 \\ 0 & 0 & -11 & -220000 \end{pmatrix}$$

$$-11z = -220000 \qquad z = 20000$$

$$y + 2z = 55000 \qquad y + 40000 = 55000 \qquad y = 15000$$

$$x+y+z = 65000 \qquad x + 20000 + 15000 = 65000 \qquad x = 30000$$

$$\boxed{\begin{aligned} x &= 30000 \\ y &= 15000 \\ z &= 20000 \end{aligned}}$$

4) A boy is walking along the path $y = ax^2+bx+c$ through the points $(-6, 8)$, $(-2, -12)$, and $(3, 8)$. He wants to meet his friend at $P(7, 60)$. will he meet his friend?

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(3)

Soln $y = ax^2 + bx + c$

At $(-6, 8) \Rightarrow 8 = 36a - 6b + c$ — (1)

At $(-2, -12) \Rightarrow -12 = 4a - 2b + c$ — (2)

At $(3, 8) \Rightarrow 8 = 9a + 3b + c$ — (3)

Matrix form $\begin{pmatrix} 36 & -6 & 1 & 8 \\ 4 & -2 & 1 & -12 \\ 9 & 3 & 1 & 8 \end{pmatrix}$

$$[A|B] = \begin{pmatrix} 36 & -6 & 1 & 8 \\ 4 & -2 & 1 & -12 \\ 9 & 3 & 1 & 8 \end{pmatrix} \xrightarrow[R_3 \to 4R_3 - 1R_1]{R_2 \to 9R_2 - R_1} \begin{pmatrix} 36 & -6 & 1 & 8 \\ 0 & 12 & -8 & 116 \\ 0 & -18 & -3 & -24 \end{pmatrix}$$

$$\xrightarrow[R_3 \to R_3/-3]{R_2 \to R_2/4} \begin{pmatrix} 36 & -6 & 1 & 8 \\ 0 & 3 & -2 & 29 \\ 0 & 6 & 1 & 8 \end{pmatrix} \xrightarrow{R_3 \to R_3 - 2R_2} \begin{pmatrix} 36 & -6 & 1 & 8 \\ 0 & +3 & -2 & 29 \\ 0 & 0 & 5 & -50 \end{pmatrix}$$

$5c = -50$	$+3b - 2c = 29$	$36a - 18 - 10 = 8$
$c = -10$	$3b - 20 = 29$	$36a = 8 + 18 + 10$
	$3b = 9$	$36a = 36$
	$b = 3$	$a = 1$

At $P(7, 60)$ $\quad y = ax^2 + bx + c$

$60 = 1(7^2) + 3(7) - 10$

$60 = 49 + 21 - 10$

$60 = 60$

$\therefore$ he will meet his friend at $P(7, 60)$.

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Exercise 1.6

1. Test for cosistency and if possible solve the following system of equations by rank method.

(i) $x-y+2z=2, \quad 2x+y+4z=7 \quad 4x-y+z=4.$

Soln Augument matrix. $[A \mid B]$

$$\begin{pmatrix} 1 & -1 & 2 \\ 2 & 1 & 4 \\ 4 & -1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 7 \\ 4 \end{pmatrix}$$

$$AX = B$$

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$$[A \mid B] = \left(\begin{array}{ccc|c} 1 & -1 & 2 & 2 \\ 2 & 1 & 4 & 7 \\ 4 & -1 & 1 & 4 \end{array}\right)$$

$$\xrightarrow[R_3 \to R_3 - 4R_1]{R_2 \to R_2 - 2R_1} \left(\begin{array}{ccc|c} 1 & -1 & 2 & 2 \\ 0 & 3 & 0 & 3 \\ 0 & 3 & -7 & -4 \end{array}\right) \xrightarrow{R_3 \to R_3 - R_2} \left(\begin{array}{ccc|c} 1 & -1 & 2 & 2 \\ 0 & 3 & 0 & 3 \\ 0 & 0 & -7 & -7 \end{array}\right)$$

$$\rho(A) = 3 \qquad \rho(A \mid B) = 3.$$

The system is consistent.

$$\rho(A) = \rho(A \mid B) = 3 = n.$$

it has unique solution.

$-7z = -7$	$3y = 3$	$x - y + 2z = 2$
$\boxed{z = 1}$	$\boxed{y = 1}$	$x - 1 + \cancel{2} = \cancel{2}$
		$\boxed{x = 1}$

$\therefore$ Solution is $x = 1, \; y = 1, \; z = 1.$

(ii) $3x+y+z=2, \quad x-3y+2z=1, \quad 7x-y+4z=5$

Soln Matrix form.

$$\begin{pmatrix} 3 & 1 & 1 \\ 1 & -3 & 2 \\ 7 & -1 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}$$

$$AX = B$$

Augumented matrix

$$[A \mid B] = \begin{pmatrix} 3 & 1 & 1 & 2 \\ 1 & -3 & 2 & 1 \\ 7 & -1 & 4 & 5 \end{pmatrix}$$

$$\xrightarrow{R_1 \leftrightarrow R_2} \begin{pmatrix} 1 & -3 & 2 & 1 \\ 3 & 1 & 1 & 2 \\ 7 & -1 & 4 & 5 \end{pmatrix} \xrightarrow[R_3 \to R_3 - 7R_1]{R_2 \to R_2 - 3R_1} \begin{pmatrix} 1 & -3 & 2 & 1 \\ 0 & 10 & -5 & -1 \\ 0 & 20 & -10 & -2 \end{pmatrix}$$

$$\xrightarrow{R_3 \to R_3 - 2R_2} \left(\begin{array}{ccc|c} 1 & -3 & 2 & 1 \\ 0 & 10 & -5 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$\rho(A) = 2 \quad \rho(A:B) = 2$

$\rho(A) = \rho(A:B) = 2 < n.$

The system is consistent. it has infinitely many solution. put $\boxed{z = t}$

$10y - 5z = -1$

$10y = -1 + 5z = 5t - 1$

$\boxed{y = \frac{5t-1}{10}}$

$x - 3y + 2z = 1$

$x - 3\left(\frac{5t-1}{10}\right) + 2t = 1$

$x = 1 - 2t + 3\left(\frac{5t-1}{10}\right) = \frac{10 - 20t + 15t - 3}{10}$

$x = \frac{7-5t}{10}$

$(x, y, z) = \left(\frac{7-5t}{10}, \frac{5t-1}{10}, t\right) \quad \forall\ t \in R.$

(iii) $2x + 2y + z = 5, \quad x - y + z = 1, \quad 3x + y + 2z = 4.$

Soln matrix form $\begin{pmatrix} 2 & 2 & 1 \\ 1 & -1 & 1 \\ 3 & 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \\ 4 \end{pmatrix}$

$AX = B.$

Augumented matrix $[A|B] = \begin{bmatrix} 2 & 2 & 1 & 5 \\ 1 & -1 & 1 & 1 \\ 3 & 1 & 2 & 4 \end{bmatrix}$

$$\xrightarrow{R_1 \leftrightarrow R_2} \begin{pmatrix} 1 & -1 & 1 & 1 \\ 2 & 2 & 1 & 5 \\ 3 & 1 & 2 & 4 \end{pmatrix} \xrightarrow[R_3 \to R_3 - 3R_1]{R_2 \to R_2 - 2R_1} \begin{pmatrix} 1 & -1 & 1 & 1 \\ 0 & 4 & -1 & 3 \\ 0 & 4 & -1 & 1 \end{pmatrix}$$

$$\xrightarrow{R_3 \to R_3 - R_2} \left(\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 0 & 4 & -1 & 3 \\ 0 & 0 & 0 & -2 \end{array}\right)$$

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$\rho(A) = 2 \quad \rho(A:B) = 3$

$\rho(A) \neq \rho(A:B)$

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$\therefore$ The system is in consistent.
it has no solution.

(iv) $2x-y+z=2$, $6x-3y+3z=6$, $4x-2y+2z=4$.

soln Matrix form

$$[A|B] = \begin{pmatrix} 2 & -1 & 1 & 2 \\ 6 & -3 & 3 & 6 \\ 4 & -2 & 2 & 4 \end{pmatrix}$$

Augumented matrix.

$$\xrightarrow[R_3 \to R_3/2]{R_2 \to \frac{R_2}{3}} \begin{pmatrix} 2 & -1 & 1 & 2 \\ 2 & -1 & 1 & 2 \\ 2 & -1 & 1 & 2 \end{pmatrix} \xrightarrow[R_3 \to R_3 - R_1]{R_2 \to R_2 - R_1} \begin{pmatrix} 2 & -1 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$\rho(A) = 1 \quad \rho(A:B) = 1$.

$\rho(A) = \rho(A:B) = 1 < n$.

$\therefore$ The system reduces into single equation
$\therefore$ it is consistent and has infinitely many solution.

put $y = s \quad z = t$.

$2x - y + z = 2$

$2x - s + t = 2$

$2x = 2 + s - t$

$x = \dfrac{2+s-t}{2}$

$(x, y, z) = \left(\dfrac{2+s-t}{2}, s, t\right) \quad \forall\, s, t \in R$.

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2) Find the value of k for which the equations $kx-2y+z=1$, $x-2ky+z=-2$, $x-2y+kz=1$ have (i) no solution (ii) unique solution (iii) infinitely many solution.

soln Matrix form.

$$\begin{pmatrix} k & -2 & 1 \\ 1 & -2k & 1 \\ 1 & -2 & k \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

$A \quad X \quad = B$.

Augumented matrix

$$[A|B] = \begin{pmatrix} k & -2 & 1 & 1 \\ 1 & -2k & 1 & -2 \\ 1 & -2 & k & 1 \end{pmatrix}$$

$$\xrightarrow{R_1 \leftrightarrow R_3} \begin{pmatrix} 1 & -2 & k & 1 \\ 1 & -2k & 1 & -2 \\ k & -2 & 1 & 1 \end{pmatrix} \xrightarrow[R_3 \to R_3 - kR_1]{R_2 \to R_2 - R_1} \begin{pmatrix} 1 & -2 & k & 1 \\ 0 & -2k+2 & 1-k & -3 \\ 0 & -2+2k & 1-k^2 & 1-k \end{pmatrix}$$

$$\xrightarrow{R_3 \to R_3 + R_2} \begin{pmatrix} 1 & -2 & k & 1 \\ 0 & 2(1-k) & (1-k) & -3 \\ 0 & 0 & 2-k-k^2 & (k+2) \end{pmatrix}$$

$$2-k-k^2$$
$$-(k^2+k-2)$$
$$-(k+2)(k-1)$$
$$(k+2)(1-k)$$

$$\longrightarrow \begin{pmatrix} 1 & -2 & k & 1 \\ 0 & 2(1-k) & (1-k) & -3 \\ 0 & 0 & (k+2)(1-k) & (k+2) \end{pmatrix}$$

Case (i) $k = -2$ $\rho(A) = 2$ $\rho(A:B) = 2$.

The system is consistent and it has infinitely many solution.

Case (ii) $k = 1$, $k \neq -2$, $\rho(A) = 2$, $\rho(A:B) = 3$.

$\rho(A) \neq \rho(A:B)$

The system is inconsistent. It has no solution.

Case (iii) $k \neq 1$ $k \neq -2$

$\rho(A) = \rho(A:B) = 3 = n$.

The system is consistent. it has unique solution.

3) Investigate the values of λ and μ the system of linear equations $2x+3y+5z=9$, $7x+3y-5z=8$, $2x+3y+\lambda z=\mu$ have (i) no solution (ii) a unique solution (iii) an infinite number of solutions.

Soln. matrix form
$$\begin{pmatrix} 2 & 3 & 5 \\ 7 & 3 & -5 \\ 2 & 3 & \lambda \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 9 \\ 8 \\ \mu \end{pmatrix}$$

$$A \quad X = B$$

Augmented matrix
$$[A|B] = \begin{pmatrix} 2 & 3 & 5 & 9 \\ 7 & 3 & -5 & 8 \\ 2 & 3 & \lambda & \mu \end{pmatrix}$$

$$\xrightarrow[R_3 \to R_3 - R_1]{R_2 \to 2R_2 - 7R_1} \begin{pmatrix} 2 & 3 & 5 & 9 \\ 0 & -15 & -45 & -47 \\ 0 & 0 & \lambda-5 & \mu-9 \end{pmatrix}$$

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Case (i) If $\lambda = 5 \quad \mu = 9$

$\rho(A) = \rho(A:B) = 2 < n$

The system is consistent it has infinitely many solution

Case (ii) if $\lambda \neq 5 \quad \mu = 9$

$\rho(A) = 3 \quad \rho(A:B) = 3$

The system is consistent it has unique solution

Case (iii) if $\lambda = 5 \quad \mu \neq 9$

$\rho(A) = 2 \quad \rho(A:B) = 3$

The system is inconsistent it has no solution.

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Exercise 1.7

1. Solve the following system of homogeneous equations

(i) $3x+2y+7z=0$, $4x-3y-2z=0$, $5x+9y+23z=0$

Soln Matrix form $\begin{pmatrix} 3 & 2 & 7 \\ 4 & -3 & -2 \\ 5 & 9 & 23 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$AX = B$

Augmented matrix

$[A|B] = \begin{pmatrix} 3 & 2 & 7 & 0 \\ 4 & -3 & -2 & 0 \\ 5 & 9 & 23 & 0 \end{pmatrix}$

$\xrightarrow[R_3 \to 3R_3 - 5R_1]{R_2 \to 3R_2 - 4R_1} \begin{pmatrix} 3 & 2 & 7 & 0 \\ 0 & -17 & -34 & 0 \\ 0 & 17 & 34 & 0 \end{pmatrix} \xrightarrow{R_3 \to R_3 + R_2} \left(\begin{array}{ccc|c} 3 & 2 & 7 & 0 \\ 0 & -17 & -34 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$

writing into equation form.

$-17y - 34z = 0$ — (1)

$3x+2y+7z=0.$ — (2)

put $z = t$ $\quad -17y = 34z$

$y = \frac{34t}{-17} = -2t.$

(1) $\Rightarrow$ $3x + 2(-2t) + 7t = 0$

$3x - 4t + 7t = 0$

$3x + 3t = 0$

$3x = -3t$

$x = -t$

$(x, y, z) = (-t, -2t, t) \quad \forall\ t \in R.$

(ii) $2x+3y-z=0$, $x-y-2z=0$, $3x+y+3z=0$

Soln matrix form. $\begin{pmatrix} 2 & 3 & -1 \\ 1 & -1 & -2 \\ 3 & 1 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$AX = B$

Augmented matrix

$(A|B) = \begin{pmatrix} 2 & 3 & -1 & 0 \\ 1 & -1 & -2 & 0 \\ 3 & 1 & 3 & 0 \end{pmatrix}$

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$$\xrightarrow{R_1 \leftrightarrow R_2} \begin{pmatrix} 1 & -1 & -2 & 0 \\ 2 & 3 & -1 & 0 \\ 3 & 1 & 3 & 0 \end{pmatrix} \xrightarrow[R_3 \to R_3 - 3R_1]{R_2 \to R_2 - 2R_1} \begin{pmatrix} 1 & -1 & -2 & 0 \\ 0 & 5 & 3 & 0 \\ 0 & 4 & 9 & 0 \end{pmatrix}$$

$$\xrightarrow{R_3 \to 5R_3 - 4R_2} \begin{pmatrix} 1 & -1 & -2 & 0 \\ 0 & 5 & 3 & 0 \\ 0 & 0 & 33 & 0 \end{pmatrix}$$

Convert into equation form.

$33z = 0.$ $z = 0$ | $5y + 3z = 0$, $5y = 0$, $y = 0$ | $x - y - 2z = 0$, $x - 0 - 0 = 0$, $x = 0$

$\therefore x = 0 \quad y = 0 \quad z = 0$

$\therefore$ It has only trival solution.

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2) Determine the values of λ for which the following system of equations $x + y + 3z = 0$, $4x + 3y + \lambda z = 0$, $2x + y + 2z = 0$ has (i) unique solution (ii) a non trival solution.

Soln Matrix form $\begin{pmatrix} 1 & 1 & 3 \\ 4 & 3 & \lambda \\ 2 & 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$AX = B$

$$[A|B] = \begin{pmatrix} 1 & 1 & 3 & 0 \\ 4 & 3 & \lambda & 0 \\ 2 & 1 & 2 & 0 \end{pmatrix} \xrightarrow[R_3 \to R_3 - 2R_1]{R_2 \to R_2 - 4R_1} \begin{pmatrix} 1 & 1 & 3 & 0 \\ 0 & -1 & \lambda - 12 & 0 \\ 0 & -1 & -4 & 0 \end{pmatrix}$$

$$\xrightarrow{R_2 \leftrightarrow R_3} \begin{pmatrix} 1 & 1 & 3 & 0 \\ 0 & -1 & -4 & 0 \\ 0 & -1 & \lambda - 12 & 0 \end{pmatrix} \xrightarrow{R_3 \to R_3 - R_2} \begin{pmatrix} 1 & 1 & 3 & 0 \\ 0 & -1 & -4 & 0 \\ 0 & 0 & \lambda - 8 & 0 \end{pmatrix}$$

Case (i) if $\lambda = 8$

$\rho(A) = \rho(A:B) = 2$

The system is consistent It has Infinitely many solution

case (ii) If $\lambda \neq 8$.

$\rho(A) = 3 \quad \rho(A:B) = 3$

The system is consistent It has unique solution.

it has homogeneous eqn $\therefore$ it has solution $\boxed{x = 0,\ y = 0,\ z = 0}$

3) By using Gaussian elimation method, balance the chemical reaction equation

$C_2H_6 + O_2 \longrightarrow H_2O + CO_2$.

Soln We are searching for positive integers x_1, x_2, x_3 and x_4 such that

$x_1 C_2H_6 + x_2 O_2 \longrightarrow x_3 H_2O + x_4 CO_2$. — ①

The number of carbon atoms on the left hand side of ① should be equal to the number of carbon atoms on the RHS of (1) So we get a linear homogenous equation

$2x_1 = x_4 \Rightarrow 2x_1 - x_4 = 0$ — ②

$6x_1 = 2x_3 \Rightarrow 6x_1 - 2x_3 = 0 \Rightarrow 3x_1 - x_3 = 0$ — ③

$2x_2 = x_3 + 2x_4 \Rightarrow 2x_2 - x_3 - 2x_4 = 0$ — ④

eqn ②, ③ & ④ constitute a homogenous system of linear equations in four unknowns.

$$[A|B] = \left[\begin{array}{cccc|c} 2 & 0 & 0 & -1 & 0 \\ 3 & 0 & -1 & 0 & 0 \\ 0 & 2 & -1 & -2 & 0 \end{array}\right]$$

By Gaussian elimination method, we get

$$[A|B] \xrightarrow{R_2 \to 2R_2 - 3R_1} \left(\begin{array}{ccccc} 2 & 0 & 0 & -1 & 0 \\ 0 & 0 & -2 & 3 & 0 \\ 0 & 2 & -1 & -2 & 0 \end{array}\right)$$

$$\xrightarrow{R_2 \leftrightarrow R_3} \left(\begin{array}{cccc|c} 2 & 0 & 0 & -1 & 0 \\ 0 & 2 & -1 & -2 & 0 \\ 0 & 0 & -2 & 3 & 0 \end{array}\right)$$

$\rho(A) = \rho(A|B) = 3 < 4$.

The system is consistent and has infinitely number of solutions.

writting the equations using the echelon form we get
$-2x_3 + 3x_4 = 0$,
$2x_2 - x_3 - 2x_4 = 0$
$2x_1 - x_4 = 0$.

So one of the unknowns should be chosen arbitrarily as on non zero real number put $x_4 = t$

$2x_1 = x_4$

$\boxed{x_1 = \frac{t}{2}}$

$-2x_3 + 3x_4 = 0$

$-2x_3 = -3t$

$\boxed{x_3 = \frac{3}{2}t}$

$2x_2 - x_3 - 2x_4 = 0$

$2x_2 - \frac{3}{2}t - 2t = 0$

$2x_2 = \frac{3}{2}t + 2t = \frac{7t}{2}$

$\boxed{x_2 = \frac{7t}{4}}$.

$(x_1, x_2, x_3, x_4) = (t/_2, 7t/_4, 3/_2 t, t) \ \forall t \in R$

Let us choose $t = 4$.

We get $x_1 = \frac{4}{2} = 2$

$\boxed{x_1 = 2}$

$x_2 = \frac{7t}{4} = \frac{7(4)}{4} = 7$

$\boxed{x_2 = 7}$

$x_3 = \frac{3}{2}(4)$

$\boxed{x_3 = 6}$

$x_4 = t$

$x_4 = 4$.

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$\therefore x_1 = 2, \ x_2 = 7, \ x_3 = 6, \ x_4 = 4$.

So the balanced equation is

$$2C_2H_6 + 7O_2 \rightarrow 6H_2O + 4CO_2.$$

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Exercise 1.8

1. Choose the correct answer

(i) If $|adj(adj A)| = |A|^9$, then the order of the square matrix A is 2.

Soln $|adj(adj A)| = |A|^{(n-1)^2}$

$(n-1)^2 = 9 = 3^2$

$n-1 = 3$

$\boxed{n = 4}$

Ans : (2) 4.

2) If A is a 3×3 non singular matrix such that $AA^T = A^TA$ and $B = A^{-1}A^T$, $|BB^T| = ?$

Soln $AA^T = A^TA$ & $B = A^{-1}A^T$,

$B^T = (A^{-1})^T (A^T)^T = (A^{-1})^T A$

$BB^{-1} = (A^{-1}A^T)(A^{-1})^T A = A^{-1}(AA^{-1})^T A$

$= A^{-1}(I)^T A = A^{-1}A = I$

Ans (3) I

3) If $A = \begin{pmatrix} 3 & 5 \\ 1 & 2 \end{pmatrix}_{2\times 2}$, $B = adj A$ and $C = 3A$ then $\frac{|adj B|}{|C|} = ?$

Soln

$$\frac{|adj B|}{|C|} = \frac{|adj(adj A)|}{|3A|} = \frac{|A|^{n-1}}{3^n |A|} = \frac{\cancel{|A|}^1}{3^2 \cancel{|A|}} = \frac{1}{9}$$

Here n = 2

Ans (2) $\frac{1}{9}$.

4) If $A \times \begin{pmatrix} 1 & -2 \\ 1 & 4 \end{pmatrix} = \begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix}$ then A = ?

Soln order of A is 2×2 Let A be $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 & -2 \\ 1 & 4 \end{pmatrix} = \begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix}$$

$a+b=6$	$c+d=0$
$-2a+4b=0$	$-2c+4d=6.$

Solve the equations we get $b=2$, $a=4$ $d=1$ $c=-1$

$A = \begin{pmatrix} 4 & 2 \\ 1 & -1 \end{pmatrix}$ Ans (3) $\begin{pmatrix} 4 & 2 \\ -1 & 1 \end{pmatrix}$

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5) If $A=\begin{pmatrix}7&3\\4&2\end{pmatrix}$ then $9I-A=?$ (42)

Soln

$|A| = 14-12 = 2$, $\text{adj}\,A = \begin{pmatrix}2&-3\\-4&7\end{pmatrix}$, $A^{-1} = \frac{1}{|A|}\text{adj}\,A$

$A^{-1} = \frac{1}{2}\begin{pmatrix}2&-3\\-4&7\end{pmatrix} \Rightarrow 2A^{-1} = \begin{pmatrix}2&-3\\-4&7\end{pmatrix}$ —(1)

$9I - A = 9\begin{pmatrix}1&0\\0&1\end{pmatrix} - \begin{pmatrix}7&3\\4&2\end{pmatrix} = \begin{pmatrix}9&0\\0&9\end{pmatrix} - \begin{pmatrix}7&3\\4&2\end{pmatrix} = \begin{pmatrix}2&-3\\-4&7\end{pmatrix}$ —(2)

by (1) & (2) $9I - A = 2A^{-1}$

Ans : (4) $2A^{-1}$

6) If $A=\begin{pmatrix}2&0\\1&5\end{pmatrix}$ and $B=\begin{pmatrix}1&4\\2&0\end{pmatrix}$ then $|\text{adj}\,(AB)| = ?$

Soln

$AB = \begin{pmatrix}2&0\\1&5\end{pmatrix}\begin{pmatrix}1&4\\2&0\end{pmatrix} = \begin{pmatrix}2&8\\11&4\end{pmatrix}$

$\text{adj}\,AB = \begin{pmatrix}4&-8\\-11&2\end{pmatrix}$ $\therefore |\text{adj}\,AB| = 8-88 = -80$

Ans (2) -80.

7. If $P = \begin{pmatrix}1&x&0\\1&3&0\\2&4&-2\end{pmatrix}$ is the adjoint of 3×3 matrix A and $|A| = 4$ then x is

$n=3$

$|\text{adj}\,A| = |A|^{n-1}$
$\qquad\qquad\quad = 4^2.$

$1(-6+0) - x(-2) = 16$

$-6+2x = 16$

$2x = 16+6 = 22$

$\boxed{x=11}$

Ans (4) 11

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8) If $A = \begin{pmatrix}3&1&-1\\2&-2&0\\1&2&-1\end{pmatrix}$ and $A^{-1} = \begin{pmatrix}a_{11}&a_{12}&a_{13}\\a_{21}&a_{22}&a_{23}\\a_{31}&a_{32}&a_{33}\end{pmatrix}$ then the value of a_{23} is

a_{23} in A^{-1} is $-\begin{vmatrix}3&-1\\2&0\end{vmatrix} = -(0+2) = -2$

$|A| = 3(2) - 1(-2) - 1(4+2)$
$\quad = 6+2-6 = 2.$

$A^{-1} = \frac{1}{|A|}\text{adj}\,A.$ $\quad a_{23}$ is $\frac{1}{2}(-2) = -1$

Ans (4) -1

9) If A, B and C are invertible matrices of some order, then which one of the following is not true.

Ans (2)

10. If $(AB)^{-1} = \begin{pmatrix} 12 & -17 \\ -19 & 27 \end{pmatrix}$ and $A^{-1} = \begin{pmatrix} 1 & -1 \\ -2 & 3 \end{pmatrix}$ then $B^{-1} = ?$

Soln

$(AB)^{-1} = B^{-1}A^{-1}$

$(AB)^{-1}A = B^{-1}(A^{-1})A$

$(AB)^{-1}A = B^{-1}$

$(A^{-1})^{-1} = A$.

$|A^{-1}| = 3-2 = 1$

$\operatorname{adj} A^{-1} = \begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix}$

$A = \frac{1}{|A^{-1}|} \operatorname{adj} A^{-1} = \begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix}$

$B^{-1} = \begin{pmatrix} 12 & -17 \\ -19 & 27 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix}$

$= \begin{pmatrix} 36-34 & 12-17 \\ -57+54 & -19+27 \end{pmatrix} = \begin{pmatrix} 2 & -5 \\ -3 & 8 \end{pmatrix}$

Ans (1) $\begin{pmatrix} 2 & -5 \\ -3 & 8 \end{pmatrix}$

11) If $A^T A^{-1}$ is symmetric then $A^2 = ?$

Soln

$(A^T A^{-1})^T = A^T A^{-1}$

$(A^{-1})^T (A^T)^T = A^T A^{-1}$

$(A^{-1})^T A = A^T A^{-1}$

$A^T (A^{-1})^T A A = (A^T) A^T A^{-1} A$. (Pre & post multiply by A^T and A)

$A^2 = (A^T)^2$

Ans (2) $(A^T)^2$

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12) If A is non singular matrix such that $A^{-1} \begin{pmatrix} 5 & 3 \\ -2 & -1 \end{pmatrix}$ then $(A^T)^{-1} = ?$

Soln

$A^{-1} = \begin{pmatrix} 5 & 3 \\ -2 & -1 \end{pmatrix}$ $|A^{-1}| = -5+6 = 1$, $\operatorname{adj} A^{-1} = \begin{pmatrix} -1 & -3 \\ 2 & 5 \end{pmatrix}$

$A = (A^{-1})^{-1} = \frac{1}{1}\begin{pmatrix} -1 & -3 \\ 2 & 5 \end{pmatrix} = \begin{pmatrix} -1 & -3 \\ 2 & 5 \end{pmatrix}$

$A^T = \begin{pmatrix} 5 & 3 \\ -2 & -1 \end{pmatrix}$

$(A^{-1})^{-1} = \frac{1}{-5+6}\begin{pmatrix} -1 & -3 \\ 2 & 5 \end{pmatrix} = \frac{1}{1}\begin{pmatrix} -1 & -3 \\ 2 & 5 \end{pmatrix} = \begin{pmatrix} -1 & -3 \\ 2 & 5 \end{pmatrix} = A$

Ans (3) $\begin{pmatrix} -1 & -3 \\ 2 & 5 \end{pmatrix}$

13) If $A = \begin{pmatrix} 3/5 & 4/5 \\ x & 3/5 \end{pmatrix}$ and $A^T = A^{-1}$ then the value of x is

Soln

$A^T = A^{-1}$

$AA^T = I$

$\begin{pmatrix} 3/5 & 4/5 \\ x & 3/5 \end{pmatrix}\begin{pmatrix} 3/5 & x \\ 4/5 & 3/5 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

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$\frac{3}{5}x + \frac{12}{25} = 0$

$\frac{3}{5}x = -\frac{12}{25}$ Ans ① $\frac{4}{5}$.

$x = -\frac{4}{5}$

14) If $A = \begin{pmatrix} 1 & \tan\theta/2 \\ -\tan\theta/2 & 1 \end{pmatrix}$ and $AB = I$ then $B = ?$

Soln

$AB = I \Rightarrow B = A^{-1} = \frac{1}{1+\tan^2\theta/2}\begin{pmatrix} 1 & -\tan\theta/2 \\ \tan\theta/2 & 1 \end{pmatrix}$

$= \frac{1}{\sec^2\theta/2}\begin{pmatrix} 1 & -\tan\theta/2 \\ \tan\theta/2 & 1 \end{pmatrix} = \cos^2\theta/2 \; A^T$

Ans (2) $\cos^2\theta/2 \; A^T$

15) If $A = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ and $A(\operatorname{adj} A) = \begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$ then $k = ?$

Soln

$\operatorname{adj} A = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

$A(\operatorname{adj} A) = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} = \begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$

$\cos^2\theta + \sin^2\theta = k$

$k = 1.$ Ans (4) $k = 1$

16) If $A = \begin{pmatrix} 2 & 3 \\ 5 & -2 \end{pmatrix}$ be such that $\lambda A^{-1} = A$ the $\lambda = ?$

Soln

$\lambda A^{-1} = A$

$\lambda I = A^2 = \begin{pmatrix} 2 & 3 \\ 5 & -2 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 5 & -2 \end{pmatrix} = \begin{pmatrix} 4+15 & 6-6 \\ 10-10 & 15+4 \end{pmatrix}$

$\lambda\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 19 & 0 \\ 0 & 19 \end{pmatrix}$

$\Rightarrow \lambda = 19.$ Ans ③ 19

17) If adj $A = \begin{pmatrix} 2 & 3 \\ 4 & -1 \end{pmatrix}$ and adj $B = \begin{pmatrix} 1 & -2 \\ -3 & 1 \end{pmatrix}$ then adj AB is

Soln adj $(AB) = (\text{adj} A)(\text{adj} B) = \begin{pmatrix} 2 & 3 \\ 4 & -1 \end{pmatrix}\begin{pmatrix} 1 & -2 \\ -3 & 1 \end{pmatrix}$

$= \begin{pmatrix} 2-9 & -4+3 \\ 4+3 & -8-1 \end{pmatrix} = \begin{pmatrix} -7 & -1 \\ 7 & -9 \end{pmatrix}$ Ans (1) $\begin{pmatrix} -7 & -1 \\ 7 & -9 \end{pmatrix}$

18) The rank of the matrix $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ -1 & -2 & -3 & -4 \end{pmatrix}$ is

soln

$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ -1 & -2 & -3 & -4 \end{pmatrix} \xrightarrow[R_3 \to -R_3]{R_2 \to \frac{R_2}{2}} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix} \xrightarrow[R_3 \to R_3 - R_1]{R_2 \to R_2 - R_1} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

$\rho(A) = 1$. Ans (1) 1

19) If $x^a y^b = e^m$, $x^c y^d = e^n$, $\Delta_1 = \begin{vmatrix} m & b \\ n & d \end{vmatrix}$ $\Delta_2 = \begin{vmatrix} a & m \\ c & n \end{vmatrix}$, $\Delta_3 = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$ Then the values of x and y are respectively

Soln $x^a y^b = e^m$

$a\log x + b\log y = m$ — (1)

$x^c y^d = e^n$

$c\log x + d\log y = n$ — (2)

$\Delta_1 = \begin{vmatrix} m & b \\ n & d \end{vmatrix}$

$= md - nb$

$= ad\log x + bd\log y - bc\log x - bd\log y$

$\Delta_1 = (ad-bc)\log x$ — (1)

||ly $\Delta_2 = \begin{vmatrix} a & m \\ c & n \end{vmatrix}$

$= an - cm$

$= ac\log x + ad\log y - ac\log x - bc\log y$

$= (ad-bc)\log y$ — (2)

$\Delta_3 = (ad-bc)$.

(1) $\Rightarrow$ $\Delta_1 = \Delta_3 \log x$

$\frac{\Delta_1}{\Delta_3} = \log x$

$\boxed{e^{\Delta_1/\Delta_3} = x}$

(2) $\Rightarrow$ $\frac{\Delta_2}{\Delta_3} = \log y$

$\boxed{e^{\Delta_2/\Delta_3} = y}$

Ans (4) $\left(e^{\Delta_1/\Delta_3}, e^{\Delta_2/\Delta_3}\right)$

20) Which of the following is / are correct?

soln (i) adjoint of diagonal matrix is also a diagonal matrix

(iv) $A(\text{adj} A) = (\text{adj} A) A = |A| I$.

Ans (4)

21) If $\rho(A) = \rho(A|B)$, then the system $AX = B$ of linear equation is

soln consistent.

Ans (2)

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22) If $0 \le \theta \le \pi$ and the system of equations $x + (\sin\theta)y - (\cos\theta)z = 0$, $\cos\theta\, x - y + z = 0$, $(\sin\theta)x + y - z = 0$ has a non trivial solution then θ is

Soln If $Ax = 0$ has a non trival solution if $|A| = 0$.

$$\begin{vmatrix} 1 & \sin\alpha & -\cos\alpha \\ \cos\alpha & -1 & 1 \\ \sin\alpha & 1 & -1 \end{vmatrix} = 0 \Rightarrow 1(1-1) - \sin\alpha(-\cos\alpha - \sin\alpha) - \cos\alpha(\cos\alpha + \sin\alpha) = 0$$

$$\Rightarrow + \sin\alpha\cos\alpha + \sin^2\alpha - \cos^2\alpha - \cos\alpha\sin\alpha = 0$$

$$\sin^2\alpha = \cos^2\alpha$$

$$\sin\alpha = \cos\alpha$$

$$\alpha = \pi/4.$$

Ans (4) $\alpha = \pi/4$.

23) The augumented matrix of a system of linear equation is $\begin{pmatrix} 1 & 2 & 7 & 3 \\ 0 & 1 & 4 & 6 \\ 0 & 0 & \lambda-7 & \mu+5 \end{pmatrix}$. The system has infinitely many solution if

Soln $\rho(A) = \rho(A:B) = 2$

$\lambda - 7 = 0$ & $\mu + 5 = 0$

$\lambda = 7$ $\mu = -5$.

Ans (5) $\lambda = 7$ $\mu = -5$

24) Let $A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$ & $4B = \begin{pmatrix} 3 & 1 & -1 \\ 1 & 3 & x \\ -1 & 1 & 3 \end{pmatrix}$ If B is the inverse of A. Then the value of x is

Soln $B = A^{-1}$

$$4A^{-1} = \begin{pmatrix} 3 & 1 & -1 \\ 1 & 3 & x \\ -1 & 1 & 3 \end{pmatrix} \Rightarrow A^{-1} = \frac{1}{4}\begin{pmatrix} 3 & 1 & -1 \\ 1 & 3 & x \\ -1 & 1 & 3 \end{pmatrix}$$

x is a_{32} minor matrix. of A.

$$x = -\begin{vmatrix} 2 & 1 \\ -1 & -1 \end{vmatrix} = -(-2+1) = -(-1) = 1$$

Ans (4) 1

25) If $A = \begin{pmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix}$ then $\text{adj}(\text{adj} A)$ is

Soln
$$\text{adj}(\text{adj} A) = |A|^{n-2} A = |A|^{1} |A|$$

$$|A| = 3(-3+4) + 3(2) + 4(-2) = 3+6-8 = 1.$$

$$\text{adj}(\text{adj}A) = 1^2 \begin{pmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix} = \begin{pmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix} = A$$

Ans (1) $\begin{pmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix}$