Physics

(Chapter – 2) (Electrostatic Potential and Capacitance)

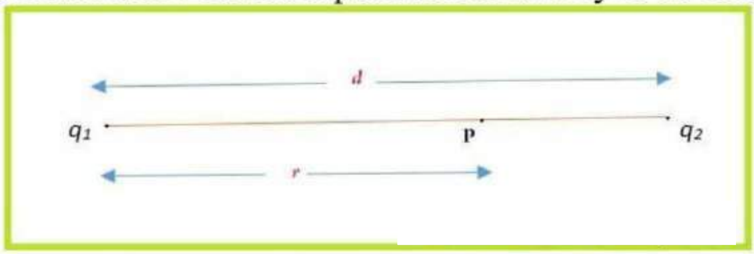
(Class - XII)

Exercises

Question 2.1:

Two charges 5×10^{-8} C and -3×10^{-8} C are located 16 cm apart. At what point(s) on the line joining the two charges is the electric potential zero? Take the potential at infinity to be zero.

Answer 2.1:



There are two charges,

$$q_1 = 5 \times 10^{-8}\ \text{C}$$
$$q_2 = -3 \times 10^{-8}\ \text{C}$$

Distance between the two charges,

d = 16 cm = 0.16 m

Consider a point P on the line joining the two charges, as shown in the given figure.

r = Distance of point P from charge q_1

Let the electric potential (V) at point P be zero.

Potential at point P is the sum of potentials caused by charges q_1 and q_2 respectively.

$$\therefore V = \frac{1}{4\pi\varepsilon_0}.\frac{q_1}{r} + \frac{1}{4\pi\varepsilon_0}.\frac{q_2}{(d-r)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Where,

ε_0 = Permittivity of free space

For V = 0, equation (i) reduces to

$$0 = \frac{1}{4\pi\varepsilon_0}.\frac{q_1}{r} + \frac{1}{4\pi\varepsilon_0}.\frac{q_2}{(d-r)} \quad \Rightarrow \frac{1}{4\pi\varepsilon_0}.\frac{q_1}{r} = -\frac{1}{4\pi\varepsilon_0}.\frac{q_2}{(d-r)}$$

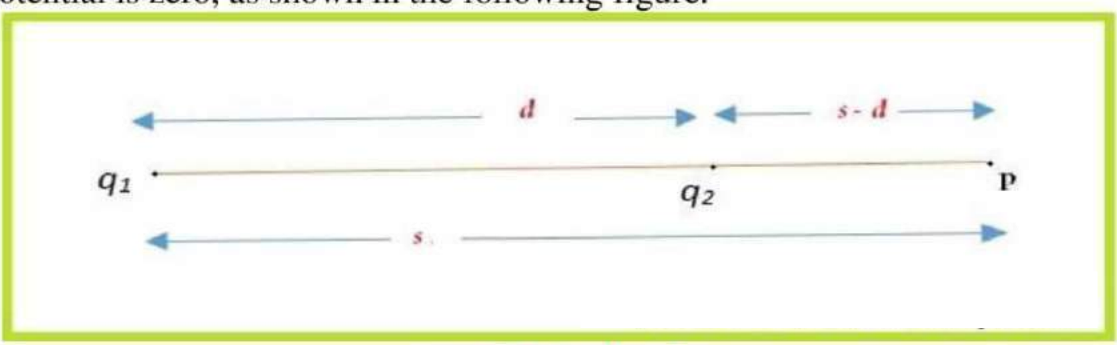
$$\Rightarrow \frac{q_1}{r} = -\frac{q_2}{(d-r)} \quad \Rightarrow \frac{5 \times 10^{-8}}{r} = -\frac{(-3 \times 10^{-8})}{(0.16 - r)}$$

$$\Rightarrow 5(0.16 - r) = 3r$$

$$\Rightarrow 0.8 = 8r \quad \Rightarrow r = 0.1\,m = 10\,cm$$

Therefore, the potential is zero at a distance of 10 cm from the positive charge between the charges.

Suppose point P is outside the system of two charges at a distance s from the negative charge, where potential is zero, as shown in the following figure.



For this arrangement, potential is given by,

$$V = \frac{1}{4\pi\varepsilon_0}.\frac{q_1}{s} + \frac{1}{4\pi\varepsilon_0}.\frac{q_2}{(s-d)} \dots\dots\dots\dots\dots\dots..(2)$$

Where,

ε_0 = Permittivity of free space

For V = 0, equation (2) reduces to

$$0 = \frac{1}{4\pi\varepsilon_0}.\frac{q_1}{s} + \frac{1}{4\pi\varepsilon_0}.\frac{q_2}{(s-d)} \quad \Rightarrow \frac{1}{4\pi\varepsilon_0}.\frac{q_1}{s} = -\frac{1}{4\pi\varepsilon_0}.\frac{q_2}{(s-d)}$$

$$\Rightarrow \frac{q_1}{s} = -\frac{q_2}{(s-d)}$$

$$\Rightarrow \frac{5\times 10^{-8}}{s} = -\frac{(-3\times 10^{-8})}{(s-0.16)}$$

$$\Rightarrow 5(s-0.16) = 3s$$

$$\Rightarrow 0.8 = 2s \quad \Rightarrow s = 0.4\, m \; = 40\ cm$$

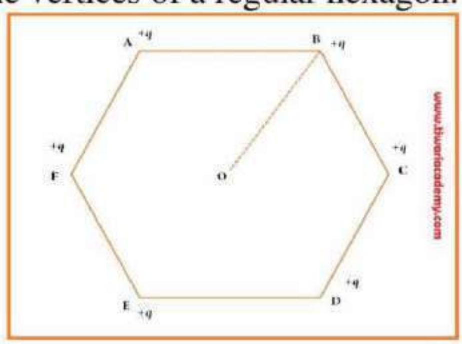
Therefore, the potential is zero at a distance of 40 cm from the positive charge outside the system of charges.

Question 2.2:

A regular hexagon of side 10 cm has a charge 5 μC at each of its vertices. Calculate the potential at the centre of the hexagon.

Answer 2.2:

The given figure shows six equal amount of charges, q, at the vertices of a regular hexagon.



Where,

Charge, q = 5 μC = 5×10^{-6} C

Side of the hexagon,

l = AB = BC = CD = DE = EF = FA = 10 cm

Distance of each vertex from centre O, d = 10 cm

Electric potential at point O,

$$V = \frac{1}{4\pi\varepsilon_0}.\frac{6\times q}{d}$$

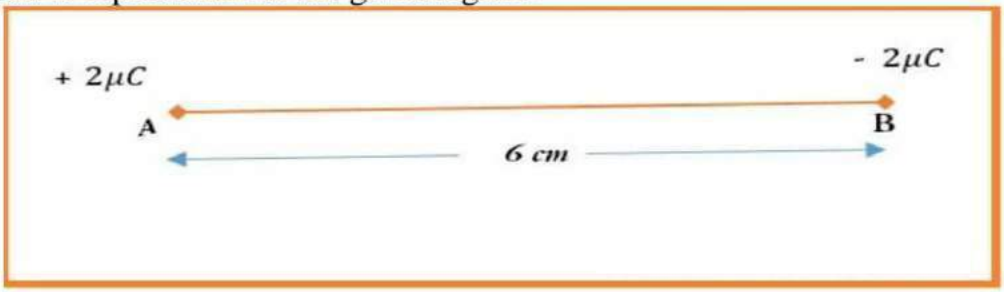
Where,

Where, ε_0 = Permittivity of free space and $\frac{1}{4\pi\varepsilon_0} = 9\times 10^9\ \mathrm{Nm^2C^{-2}}$

$$\therefore V = \frac{9\times 10^9 \times 6 \times 5 \times 10^{-6}}{0.1} = 2.7\times 10^6\ V$$

Therefore, the potential at the centre of the hexagon is 2.7×10^6 V.

Question 2.3:
Two charges 2 μC and −2 μC are placed at points A and B 6 cm apart.
(a) Identify an equipotential surface of the system.
(b) What is the direction of the electric field at every point on this surface?
Answer 2.3:
(a) The situation is represented in the given figure.



An equipotential surface is the plane on which total potential is zero everywhere. This plane is normal to line AB. The plane is located at the mid-point of line AB because the magnitude of charges is the same.
(b) The direction of the electric field at every point on this surface is normal to the plane in the direction of AB.

Question 2.4:
A spherical conductor of radius 12 cm has a charge of 1.6×10^{-7}C distributed uniformly on its surface. What is the electric field?
(a) Inside the sphere
(b) Just outside the sphere
(c) At a point 18 cm from the centre of the sphere?
Answer 2.4:
(a) Radius of the spherical conductor, r = 12 cm = 0.12 m
Charge is uniformly distributed over the conductor, $q = 1.6 \times 10^{-7}$ C
Electric field inside a spherical conductor is zero. This is because if there is field inside the conductor, then charges will move to neutralize it.
(b) Electric field E just outside the conductor is given by the relation,

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$$

Where, ε_0 = Permittivity of free space and $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ Nm}^2\text{C}^{-2}$

Therefore, $E = \frac{9\times10^9\times1.6\times10^{-7}}{(0.12)^2} = 10^5\ NC^{-1}$

Therefore, the electric field just outside the sphere is $10^5\ NC^{-1}$.

(c) Electric field at a point 18 m from the centre of the sphere = E_1
Distance of the point from the centre, d = 18 cm = 0.18 m

$$E_1 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{d^2} = \frac{9\times10^9\times1.6\times10^{-7}}{(1.8\times10^{-2})^2} = 4.4\times10^4\ \mathrm{N}C^{-1}$$

Therefore, the electric field at a point 18 cm from the centre of the sphere is $4.4\times10^4\ \mathrm{N}C^{-1}$.

Question 2.5:
A parallel plate capacitor with air between the plates has a capacitance of 8 pF (1pF = 10^{-12} F). What will be the capacitance if the distance between the plates is reduced by half, and the space between them is filled with a substance of dielectric constant 6?

Answer 2.5:
Capacitance between the parallel plates of the capacitor, C = 8 pF
Initially, distance between the parallel plates was d and it was filled with air.
Dielectric constant of air, k = 1
Capacitance, C, is given by the formula,

$$C = \frac{k\varepsilon_0 A}{d} = \frac{\varepsilon_0 A}{d} \ldots\ldots\ldots\ldots\ldots\ldots.(1)$$

Where,
A = Area of each plate
ε_0 = Permittivity of free space
If distance between the plates is reduced to half, then new distance, $d_1 = d/2$
Dielectric constant of the substance filled in between the plates, $k_1 = 6$
Hence, capacitance of the capacitor becomes

$$C_1 = \frac{k_1\varepsilon_0 A}{d_1} = \frac{6\varepsilon_0 A}{\frac{d}{2}} = \frac{12\varepsilon_0 A}{d} \ldots\ldots\ldots\ldots\ldots\ldots.(2)$$

Taking ratios of equations (1) and (2), we obtain
$C_1 = 2\times6\ C = 12\ C = 12\times8$ pF = 96 pF
Therefore, the capacitance between the plates is 96 pF.

Question 2.6:
Three capacitors each of capacitance 9 pF are connected in series.
(a) What is the total capacitance of the combination?
(b) What is the potential difference across each capacitor if the combination is connected to a 120 V supply?

Answer 2.6:
(a) Capacitance of each of the three capacitors, C = 9 pF

Equivalent capacitance (C_{eq}) of the combination of the capacitors is given by the relation,

$$\frac{1}{C_{eq}}=\frac{1}{C}+\frac{1}{C}+\frac{1}{C}=\frac{3}{C}=\frac{3}{9}=\frac{1}{3}$$

$$\Rightarrow \frac{1}{C_{eq}}=\frac{1}{3}\Rightarrow C_{eq}=3\ \text{pF}$$

Therefore, total capacitance of the combination is 3 pF.

(b) Supply voltage, V = 100 V

Potential difference (V_1) across each capacitor is equal to one-third of the supply voltage.

$$\therefore V_1=\frac{V}{3}=\frac{120}{3}=40\ V$$

Therefore, the potential difference across each capacitor is 40 V.

Question 2.7:

Three capacitors of capacitances 2pF, 3pF and 4pF are connected in parallel.

(a) What is the total capacitance of the combination?

(b) Determine the charge on each capacitor if the combination is connected to a 100 V supply.

Answer 2.7:

(a) Capacitances of the given capacitors: C_1 = 2 pF, C_2 = 3 pF and C_3 = 4 pF

For the parallel combination of the capacitors, equivalent capacitor is given by C_{eq} the algebraic sum,

Therefore $C_{eq} = C_1 + C_2 + C_3 = 2 + 3 + 4 = 9$ pF

Therefore, total capacitance of the combination is 9 pF.

(b) Supply voltage, V = 100 V

The voltage through all the three capacitors is same = V = 100 V

Charge on a capacitor of capacitance C and potential difference V is given by the relation,

q = VC … (i)

For C = 2 pF, charge = VC = 100 × 2 = 200 pC = 2×10^{-10} C

For C = 3 pF, charge = VC = 100 × 3 = 300 pC = 3×10^{-10} C

For C = 4 pF, charge = VC = 100 × 4 = 400 pC = 4×10^{-10} C

Question 2.8:

In a parallel plate capacitor with air between the plates, each plate has an area of 6×10^{-3} m^2 and the distance between the plates is 3 mm. Calculate the capacitance of the capacitor. If this capacitor is connected to a 100 V supply, what is the charge on each plate of the capacitor?

Answer 2.8:

Area of each plate of the parallel plate capacitor, A = 6×10^{-3} m^2

Distance between the plates, d = 3 mm = 3×10^{-3} m

Supply voltage, V = 100 V

Capacitance C of a parallel plate capacitor is given by, $C = \frac{\varepsilon_0 A}{d}$

Where,

ε_0 = Permittivity of free space = $8.854 \times 10^{-12}\ N^{-1}\ m^{-2}\ C^{-2}$

$$\therefore C = \frac{8.854 \times 10^{-12} \times 6 \times 10^{-3}}{3 \times 10^{-3}} = 17.71 \times 10^{-12}\ F = 17.71\ pF$$

So, charge on each plate of the capacitor

$$q = VC = 100 \times 17.71 \times 10^{-12}\ C = 1.771 \times 10^{-9}\ C$$

Therefore, capacitance of the capacitor is 17.71 pF and charge on each plate is 1.771×10^{-9} C.

Question 2.9:

Explain what would happen if in the capacitor given in Exercise 2.8, a 3 mm thick mica sheet (of dielectric constant = 6) were inserted between the plates,

(a) While the voltage supply remained connected.

(b) After the supply was disconnected.

Answer 2.9:

(a) Dielectric constant of the mica sheet, $k = 6$

If voltage supply remained connected, voltage between two plates will be constant.

Supply voltage, V = 100 V

Initial capacitance, $C = 1.771 \times 10^{-11}$ F

New capacitance, $C_1 = kC = 6 \times 1.771 \times 10^{-11}$ F = 106 pF

New charge, $q_1 = C_1V = 106 \times 100$ pC = 1.06×10^{-8} C

Potential across the plates remains 100 V.

(b) Dielectric constant, $k = 6$

Initial capacitance, $C = 1.771 \times 10^{-11}$ F

New capacitance, $C_1 = kC = 6 \times 1.771 \times 10^{-11}$ F = 106 pF

If supply voltage is removed, then there will be constant amount of charge in the plates.

Charge = 1.771×10^{-9} C

Potential across the plates is given by,

$$V_1 = \frac{q}{C_1} = \frac{1.771 \times 10^{-9}}{106 \times 10^{-12}} = 16.7\ V$$

Question 2.10:

A 12 pF capacitor is connected to a 50V battery. How much electrostatic energy is stored in the capacitor?

Answer 2.10:

Capacitor of the capacitance, C = 12 pF = 12×10^{-12} F

Potential difference, V = 50 V

Electrostatic energy stored in the capacitor is given by the relation,

$$E = \frac{1}{2}CV^2 = \frac{1}{2} \times 12 \times 10^{-12} \times (50)^2 \text{ J} = 1.5 \times 10^{-8} \text{ J}$$

Therefore, the electrostatic energy stored in the capacitor is 1.5×10^{-8} J.

Question 2.11:

A 600 pF capacitor is charged by a 200 V supply. It is then disconnected from the supply and is connected to another uncharged 600 pF capacitor. How much electrostatic energy is lost in the process?

Answer 2.11:

Capacitance of the capacitor, C = 600 pF
Potential difference, V = 200 V
Electrostatic energy stored in the capacitor is given by,

$$E_1 = \frac{1}{2}CV^2$$

$= \frac{1}{2} \times (600 \times 10^{-12}) \times (200)^2 \text{ J}$
$= 1.2 \times 10^{-5} \text{ J}$

If supply is disconnected from the capacitor and another capacitor of capacitance C = 600 pF is connected to it, then equivalent capacitance (C_{eq}) of the combination is given by,

$$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{1}{C}$$

$\Rightarrow \frac{1}{C_{eq}} = \frac{1}{600} + \frac{1}{600} = \frac{2}{600} = \frac{1}{300}$
$\Rightarrow C_{eq} = 300 \text{ pF}$

New electrostatic energy can be calculated as

$$E_2 = \frac{1}{2}C_{eq}V^2$$

$= \frac{1}{2} \times 300 \times (200)^2 \text{ J}$
$= 0.6 \times 10^{-5} \text{ J}$

Loss in electrostatic energy = $E_1 - E_2$
$= 1.2 \times 10^{-5} - 0.6 \times 10^{-5} \text{ J}$
$= 0.6 \times 10^{-5} \text{ J}$
$= 6 \times 10^{-6} \text{ J}$

Therefore, the electrostatic energy lost in the process is 6×10^{-6} J .