29/11/19.

chapter - 2.

2 - Integral calculus - I.

$\int = \frac{1}{d}$

$d = \frac{1}{\int}$

[In differentiation $S \xrightarrow{(+)} C$, $C \xrightarrow{(-)} S$]

[In Integration $C \xrightarrow{(+)} S$, $S \xrightarrow{(-)} C$]

definite $\int_a^b f(x)\,dx = [f(x)]_a^b$

$= F(b) - F(a)$.

Indefinite $\int f(x)\,dx = F(x) + c$.

$\Rightarrow d(x^2+2) = 2x + 0$

$d(x^2) = 2x$.

$d(x^2-2) = 2x$.

$\int 2x\,dx = \cancel{2}\frac{x^2}{\cancel{2}} = x^2 + c$

* $\int k\,.\,dx = kx$.
* $\int x\,dx = \frac{x^2}{2}$
* $\int x^n\,dx = \frac{x^{n+1}}{(n+1).1} = \frac{x^{n+1}}{n+1}$.
* $\int e^x\,.\,dx = \frac{e^x}{1}$
* $\int e^{ax}.\,dx = \frac{e^{ax}}{a}$
* $\int \frac{1}{x}\,.\,dx = \log x$

* $\int \sin x\,.\,dx = -\cos x$
* $\int \cos x\,.\,dx = \sin x$.
* $\int \frac{f'(x)}{f(x)}\,dx = \log(f(x))$

eg. $\int \frac{2x}{(x^2+3)}\,dx$

$= \log(x^2+3)$

* $\int (f(x))^n f'(x)\,dx$

$= \frac{(f(x))^{n+1}}{n+1}$

eg. $\int (x^2+3)^2\,2x\,.\,dx$

$d(x^2+3)^2 = 2x$

$= \frac{(x^2+3)^3}{3}$

* $\int \frac{f'(x)}{\sqrt{f(x)}}\,dx = 2\sqrt{f(x)} + C$

Ex 2.4

1. $\int \sqrt{3x+5}\, dx$

$= \int (3x+5)^{1/2} \cdot dx$

$= \dfrac{(3x+5)^{3/2}}{3/2\,(3+0)} + c$

$= \boxed{2/9\,(3x+5)^{3/2} + c}$ //

(x) $\boxed{\int x^n dx = \dfrac{x^{n+1}}{n+1}}$

2. $\int \left(9x^2 - \dfrac{4}{x^2}\right)^2 dx$.

(x) $\boxed{(a-b)^2 = a^2+b^2-2ab}$

$= \int \left[81x^4 + \dfrac{16}{x^4} - 72 \cdot x^2 \cdot \dfrac{1}{x^2}\right] dx$

$= \int (81x^4 + 16x^{-4} - 72)\, dx$.

$= \left[\dfrac{81x^5}{5} + \dfrac{16x^{-3}}{-3} - 72x + c\right]$

$= \boxed{\dfrac{81x^5}{5} - \dfrac{16}{3x^3} - 72x + c}$ //

3. $\int (3+x)(2-5x)\, dx$.

$= \int (6 - 15x + 2x - 5x^2)\, dx$.

$= \boxed{6x - \dfrac{15x^2}{2} + \dfrac{2x^2}{2} - \dfrac{5x^3}{3} + c}$

(vii)

$= \boxed{6x - \dfrac{13x^2}{2} - \dfrac{5x^3}{3} + c}$ //

4. $\int \sqrt{x}\,(x^3 - 2x + 3)\, dx$.

$= \int x^{1/2} (x^3 - 2x + 3)\, dx$

$= \int (x^{7/2} - 2x^{3/2} + 3x^{1/2})\, dx$

$= \dfrac{x^{9/2}}{9/2} - \dfrac{2x^{5/2}}{5/2} + \dfrac{3x^{3/2}}{3/2} + c$.

$= \boxed{2/9\, x^{9/2} - 4/5\, x^{5/2} + 2x^{3/2} + c}$ //

5. $\int \dfrac{(8x+13)}{\sqrt{4x+7}}\, dx$.

$= \int \dfrac{(8x+14)-1}{(4x+7)^{1/2}} \cdot dx$

$= \int \dfrac{(8x+14)}{(4x+7)^{1/2}} - \dfrac{1}{(4x+7)^{1/2}} \cdot dx$

$= \int \dfrac{2\,(4x+7)^{1}}{(4x+7)^{1/2}} - (4x+7)^{-1/2} \cdot dx$.

$= 2\int (4x+7)^{1/2}\,.dx - \int (4x+7)^{-1/2}\,.dx$

$= \dfrac{2\,(4x+7)^{3/2}}{3/2\,(4)^{2}} - \dfrac{(4x+7)^{1/2}}{1/2\,(4)^{2}}$

$= \boxed{\dfrac{(4x+7)^{3/2}}{3} - \dfrac{(4x+7)^{1/2}}{2} + c}$

⑥ $\displaystyle\int \frac{1}{\sqrt{x+1}\;\sqrt{x-1}}\,dx$

$= \displaystyle\int \frac{1}{\sqrt{x+1}+\sqrt{x-1}} \times \frac{\sqrt{x+1}-\sqrt{x-1}}{\sqrt{x+1}-\sqrt{x+1}}$

$(a+b)\quad (a-b) = a^2-b^2$

$= \displaystyle\int \frac{(x+1)^{1/2}-(x-1)^{1/2}}{(x+1)-(x-1)}\qquad [(\sqrt{x})^2 = x$

$= \dfrac{1}{2}\,\dfrac{(x+1)^{3/2}}{3/2\,(1)} - \dfrac{(x-1)^{3/2}}{3/2\,(1)} + c$

$= \cancel{\tfrac{2}{2}}\;\boxed{\dfrac{(x+1)^{3/2}}{3} - \dfrac{(x-1)^{3/2}}{3} + c}$

eg 2.1

$\displaystyle\int \frac{ax^2+bx+c}{\sqrt{x}}\,dx.$

$= \displaystyle\int \frac{ax^2+bx+c}{(x)^{1/2}}$

$= \int ax^2.x^{-1/2} + bx.x^{-1/2} + cx^{-1/2}\,.dx.$

$= \int ax^{3/2} + bx^{1/2} + cx^{-1/2}\,.dx$

$= a\int x^{3/2}.dx + b\int x^{1/2}.dx + c\int x^{-1/2}\,.dx.$

$= a\left[\dfrac{x^{5/2}}{5/2}\right] + b\left[\dfrac{x^{3/2}}{3/2}\right] + c\left[\dfrac{x^{1/2}}{1/2}\right] + c.$

$= \boxed{\dfrac{2a}{5}x^{5/2} + \dfrac{2b}{3}x^{3/2} + 2c\,x^{1/2} + c}$

eg 2.2

$\int \sqrt{2x+1}\;dx.$

$\int (2x+1)^{1/2}\,.dx.$

$= \dfrac{(2x+1)^{3/2}}{2\cdot 3/2} + c$

$= \boxed{\dfrac{(2x+1)^{3/2}}{3} + c}$

eg 2.3

$$\int \frac{dx}{(2x+3)^2}$$

$$= \int (2x+3)^{-2}$$

$$= + \frac{(2x+3)^{-1}}{2(-1)}$$

$$= \frac{(2x+3)^{-1}}{-2}$$

$$= \boxed{\frac{-1\,(2x+3)}{2} + c}$$

eg 2.4

$$\int \left(x + \tfrac{1}{x}\right)^2 \cdot dx$$

$$= \int \left(x^2 + \frac{1}{x^2} - 2x \cdot \tfrac{1}{x}\right)$$

$$= \int (x^2 + x^{-2} - 2)$$

$$= \frac{x^3}{3} + \frac{x^{-1}}{-1} + 2x + c$$

$$= \boxed{\frac{x^3}{3} - \frac{1}{x} + 2x + c}$$

eg 2.5

$$\int (x^3+7)(x-4) \cdot dx$$

$$= \int (x^4 - 4x^3 + 7x - 28) \cdot dx$$

$$= \frac{x^5}{5} - \frac{\cancel{4}x^4}{\cancel{4}} + \frac{7x^2}{2} - 28x + c$$

$$= \boxed{\frac{x^5}{5} - x^4 + \frac{7x^2}{2} - 28x + c}$$ //.

eg 2.6

$$\int \frac{2x^2 - 14x + 24}{x-3}$$

48, $\frac{-6}{2}$, $\frac{-8}{2}$, -14

$$= \int \frac{\cancel{(x-3)}(2x-8)}{\cancel{(x-3)}} \cdot dx$$

$$= \int (2x+8) \cdot dx$$

$$= \frac{\cancel{2}x^2}{\cancel{2}} + 8x + c$$

$$= \boxed{x^2 + 8x + c}$$ //.

Step method

$\frac{d}{}$

$d(x) = 1$

$d(1/x) = -1/x^2$

$d(-1/x^2) = \frac{+2}{x^3}$

$d\left(\frac{2}{x^3}\right) = -\frac{6}{x^4}$.

$d = \left(\frac{8}{x^3}\right) = \frac{-24}{x^4}$.

I

$\int \frac{1}{x} \cdot dx = \log x$

$\int \frac{1}{x^2}\, dx = -\frac{1}{(1)x^1}$

$\int \frac{2}{x^3} \cdot dx = \frac{-2}{2 \cdot x^2} + c$.

$\int e^{ax} \cdot dx = \frac{e^{ax}}{a}$

$\int a^x \cdot dx = \frac{a^x}{\log a}$

$\int a^{mx+n} \cdot dx = \frac{a^{mx+n}}{m \log a}$ | $e^{\log}$

$\log x^n = n \log x$.

(8) $f'(x) = 8x^3 - 2x$ $\quad f(2) = 8$

$\int f'(x) \cdot dx = \int 8x^3 - 2x \cdot dx$

$f(x) = \frac{8x^4}{4} - \frac{2x^2}{2}$

$\boxed{f(x) = 2x^4 - x^2 + c} \rightarrow (1)$.

$f(2) = 2(2)^4 - (2)^2 + c$.

$8 = 2(16) - 4 + c$

$8 = 32 - 4 + c$

$8 = 28 + c$

$c = -28 + 8$

$\boxed{c = -20}$

$(1) \Rightarrow \boxed{2x^4 - x^2 - 20}$

(7). $f'(x) = x + b$, $f(1) = 5$ $\quad f(2) = 13$

$f'(x) = x + b$.

$\int f'(x) \cdot dx = \int (x+b) \cdot dx$.

$f(x) = \frac{x^2}{2} + bx + c \rightarrow (1)$.

$\boxed{f(1) = \frac{1}{2} + b + c}$

① →

$5 = \frac{1}{2} + b + c$

$5 - \frac{1}{2} = b + c$

$\frac{9}{2} = b + c$

$\boxed{9 = 2b + 2c}$ → ②

① ⇒ $f(2) = \frac{4}{2} + 2b + c$

$13 = 2 + 2b + c$

$\boxed{11 = 2b + c}$ → ③

③ − ②

$11 = 2b + c$

$9 = 2b + 2c$

$2 = -c$

$\boxed{c = -2}$

3 ⇒ $11 = 2b - 2$

$2b = 13$

$\boxed{b = \frac{13}{2}}$

$\boxed{f(x) = \frac{x^2}{2} + \frac{13}{2}x - 2}$

1: 2.7

$\int \frac{x+2}{\sqrt{2x+3}} \, dx$

(×) and ÷ by 2

(8) ⑤ $\int \frac{e^{3x} + e^{5x}}{e^x + e^{-x}} \, dx$

$= \int \frac{e^{3x}(1 + e^{2x})}{e^x + \frac{1}{e^x}} \cdot dx$

$= \int \frac{e^{3x}(1 + e^{2x})}{\frac{e^{2x}+1}{e^x}} \cdot dx$

$= \boxed{\int e^{4x} \cdot dx = \frac{e^{4x}}{4} + c}$

⑥ $\int (1 - \frac{1}{x^2}) \, e^{(x + \frac{1}{x})} dx$

$= \int d \, e^{(x + \frac{1}{x})}$

$= \boxed{e^{(x + \frac{1}{x})} + c}$

(X) $a e^{ax} dx = \int d(e^{ax})$

⑦ $\int \frac{1}{x(\log x)^2} \cdot dx$

$t = \log x$

$dt = \frac{1}{x} \cdot dx$

$= \int \frac{1}{t^2} \cdot dt$

$= \frac{-1}{t} + c$

$= \boxed{\frac{-1}{\log x} + c}$

⑧ $f'(x) = e^x$

$\int f'(x) dx = \int e^x \cdot dx$

$f(x) = e^x + c$

$f(0) = e^0 + c$

$2 = 1 + c$

$\boxed{c = 1}$

$\boxed{f(x) = e^x + 1}$

$\Rightarrow \int \frac{1}{x} \cdot dx = \log|x|$

Type - I :

$\frac{ax+b}{(x+a)(x+b)} = \frac{A}{(x+a)} + \frac{B}{x+b}$

(diff)

Type - II :

$\frac{ax+b}{(x+a)^2} = \frac{A}{(x+a)} + \frac{B}{(x+a)^2}$

(same)

Type - III :

$\frac{ax+b}{ax^2+bx+c} = \frac{Ax+B}{ax^2+bx+c}$.

(no. factor)

Ex 2.2

① $\int \left(\sqrt{2x} - \frac{1}{\sqrt{2x}}\right)^2 dx$.

$= \int 2x + \frac{1}{2x} - 2$

$= \boxed{\frac{2x^2}{2} + \frac{1}{2}\log|x| - 2x + c}$ //

② $\int \frac{x^4 - x^2 + 2}{x-1} \cdot dx$.

$= \int \frac{x^2(x^2-1)+2}{x-1} \cdot dx$.

$= \int \frac{(x^2 \cancel{(x-1)}(x+1)}{\cancel{x-1}} + \frac{2}{(x-1)}$

$= \int (x^3+x^2) + \frac{2}{(x-1)} dx$.

$= \boxed{\frac{x^4}{4} + \frac{x^3}{3} + 2\log|x-1| + c}$ //

③ $\int \frac{x^3}{x+2} \cdot dx$.

$= \int \frac{x^3+2^3-2^3}{(x+2)} \cdot dx$

$= \int \frac{(x^3+2^3) - 8}{(x-2)} \cdot dx$

$= \frac{(x^3+2^3)}{(x-2)} - \frac{8}{(x-2)} \cdot dx$

$x^2 - 2x + 4$
$x+2 \,)\, x^3$
$x^3 + 2x^2$
$(-)\ (-)$
$-2x^2$
$-2x^2 - 4x$
$(+)\ (+)$
$4x$
$(-)4x+$
$-$

[∵ $a^3+b^3 = (a+b)(a^2-ab+b^2)$

$= \int \frac{(x+2)(x^2-2x+4)}{(x+2)} - \frac{8}{(x-2)} \cdot dx$.

$= \boxed{\frac{x^3}{3} + \frac{2x^2}{2} + 4x - 8\log|x-2| + c}$ //

③ $f'(x) = 1/x$ $\quad f(1) = \pi/4$

$\int f'(x)\,dx = \int 1/x\,dx.$

$f(x) = \log|x| + c$

$f(1) = \log|1| + c.$

$\pi/4 = 0 + c$

$\boxed{c = \pi/4}$

$\log 1 = 0$
$\log 0 = 1$

$\boxed{f(x) = \log|x| + \pi/4}$

⑤. $\int \frac{3x+2}{(x-2)(x-3)}\,dx.$

$\frac{3x+2}{(x-2)(x-3)} = \frac{A}{(x-2)} + \frac{B}{(x-3)}$

$\frac{3x+2}{\cancel{(x-2)(x-3)}} = \frac{A(x-3) + B(x-2)}{\cancel{(x-2)(x-3)}}.$

$(3x+2) = A(x-3) + B(x-2).$

$x=3$

$11 = 0 + B(1)$

$\boxed{B=11}$

$x=2$

$8 = A(-1) + 0$

$\boxed{A = -8}$

$\frac{3x+2}{(x-2)(x-3)} = \frac{-8}{x-2} + \frac{11}{(x-3)}$

$\int \frac{3x+2}{(x-2)(x-3)} = \int \frac{-8}{(x-2)} + \frac{11}{(x-3)}\,dx$

$= \boxed{-8\log|x-2| + 11\log|x-3| + c}$

④. $\frac{x^3+3x^2-7x+11}{x+5}$

$$\begin{array}{r|l} & x^2 - 2x + 3 \\ x+5 & x^3 + 3x^2 - 7x + 11 \\ & x^3 + 5x^2 \\ & (-)\;(-) \\ \hline & -2x^2 - 7x \\ & -2x^2 - 10x \\ & (+)\;(+) \\ \hline & +3x + 11 \\ & 3x + 15 \\ & (-)\;(-) \\ \hline & -4 \end{array}$$

$= \int x^3 - 2x + 3 - \frac{4}{x+5}$

$= \frac{x^3}{3} - \frac{\cancel{2}x^2}{\cancel{2}} + 3x - 4\log|x+5| + c.$

$= \frac{x^3}{3} - x^2 + 3x - 4\log|x+5| + c$

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(6) $\dfrac{4x^2+2x+6}{(x+1)^2(x-3)}$

$$\frac{4x^2+2x+6}{(x+1)^2(x-3)} = \frac{A}{(x-3)} + \frac{B}{(x+1)} + \frac{C}{(x+1)^2}$$

$$\frac{4x^2+2x+6}{(x+1)^2(x-3)} = \frac{A(x+1)^2 + B(x+1)(x-3) + C(x-3)}{(x+1)^2(x-3)}$$

$$4x^2+2x+6 = A(x+1)^2 + B(x+1)(x-3) + C(x-3)$$

Put $\boxed{x=-1}$

$$4(-1)^2 + 2(-1) + 6 = 0+0+(-4)$$

$$4-2+6 = -4C$$

$$8 = -4C$$

$$\boxed{C=-2}$$

Put $\boxed{x=3}$

$$4(+3)^2+2(3)+6 = A(4)^2+0+0$$

$$36+6+6 = A(16)$$

$$48 = A(16)$$

$$\boxed{A=3}$$

equating x^2.

$$4 = A+B$$

$$4 = 3+B$$

$$\boxed{B=1}$$

$$\left|\int \frac{1}{x^2}\,dx = \frac{-1}{x}\right.$$

$$\therefore \frac{4x^2+2x+6}{(x+1)^2(x-3)} = \frac{3}{x-3} + \frac{1}{(x+1)} - \frac{2}{(x+1)^2}$$

$$\int \frac{4x^2+2x+6}{(x+1)^2(x-3)}\,dx = \int \frac{3}{x-3}\,dx + \int \frac{1}{(x+1)}\,dx - \int \frac{2}{(x+1)^2}\,dx$$

$$= \boxed{3\log|x-3| + \log|x+1| + 2\frac{1}{(x+1)} + c}$$

(7) $$\frac{3x^2-2x+5}{(x-1)(x^2+5)} = \frac{A}{(x-1)} + \frac{Bx+C}{x^2+5}$$

$$\frac{3x^2-2x+5}{\cancel{(x-1)(x^2+5)}} = \frac{A(x^2+5)+(Bx+C)(x-1)}{\cancel{(x-1)(x^2+5)}}$$

$$3x^2-2x+5 = A(x^2+5) + (Bx)(x-1) + C(x-1)$$

Put $\boxed{x=1}$

$$3(1)^2-2(1)+5 = A(1^2+5)+0+0$$

$$3-2+5 = 6A$$

$$6 = 6A$$

$$\boxed{A=1}$$

Equate x.

$$-2 = -B+C$$

$$-2 = -2+C$$

$$\boxed{C=0}$$

$$\frac{3x^2-2x+5}{(x-1)(x^2+5)} = \frac{1}{(x-1)} + \frac{2x+0}{x^2+5}$$

$$\int \frac{3x^2-2x+5}{(x-1)(x^2+5)}\,dx = \int \frac{1}{(x-1)}\,dx + \int \frac{2x}{(x^2+5)}\,dx$$

$$= \boxed{\log|x-1| + \log|x^2+5| + c}$$

Formula:

(i) $\boxed{\int \sin x \, dx = -\cos x + c}$ (ii) $\boxed{\int \cos x \, dx = \sin x + c}$

(iii) $\boxed{\int \sec^2 x \, dx = \tan x + c}$ (iv) $\boxed{\int \operatorname{cosec}^2 x \, dx = -\cot x + c}$

(v) $\int \sin(ax+b)\, dx = -\frac{1}{a}\cos(ax+b) + c$

(vi) $\int \cos(ax+b)\, dx = \frac{1}{a}\sin(ax+b) + c$

(vii) $\int \sec^2(ax+b)\, dx = \frac{1}{a}\tan(ax+b) + c$

(viii) $\int \operatorname{cosec}^2(ax+b)\, dx = -\frac{1}{a}\cot(ax+b) + c$

Ex 2.11

① $\int (2\cos x - 3\sin x + 4\sec^2 x - 5\operatorname{cosec}^2 x)\, dx$

$= \boxed{2\sin x + 3\cos x + 4\tan x + 5\cot x + c}$

②. $\int \sin^3 x \, dx$

$= \int \frac{1}{4}(3\sin x - \sin 3x)\, dx$

$= \boxed{\frac{1}{4}\left[-3\cos x + \frac{\cos 3x}{3}\right] + c}$

$\sin 3A = 3\sin A - 4\sin^3 A$

$\cos 3A = 4\cos^3 A - 3\cos A$

$\sin^3 A = \frac{1}{4}[3\sin A - \sin 3A]$

③ $\int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x}\, dx$

$= \int \frac{\cos 2x + 2\left(\frac{1-\cos 2x}{2}\right)}{\cos^2 x}\, dx$

$= \int \frac{1}{\cos^2 x}\, dx$

$= \boxed{\int \sec^2 x \, dx = \tan x + c}$

$\sin^2 A = \frac{1-\cos 2A}{2}$

$\cos 2A = \cos^2 A - \sin^2 A$

$\frac{1}{\cos^2 x} = \sec^2 x$.

④ $\int \frac{1}{\sin^2 x \cos^2 x}\, dx$

$= \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x}\, dx$

$$= \int \left(\frac{\sin^2 x}{\sin^2 x \cos^2 x} + \frac{\cos^2 x}{\sin^2 x \cos^2 x} \right) . dx$$

$$= \int (\sec^2 x + \csc^2 x)\, dx$$

$$= \boxed{\tan x - \cot x + c}$$

$\frac{1}{\cos^2 x} = \sec^2 x$

$\frac{1}{\sin^2 x} = \csc^2 x$

5. $\int \sqrt{1 - \sin 2x}\, .dx$

$$= \int \sqrt{(\cos^2 x + \sin^2 x - 2\cos x \sin x)}\,.dx$$

$$= \int \sqrt{(\cos x - \sin x)^2}\, .dx$$

$$= \int (\cos x - \sin x)\, dx$$

$$= \boxed{\sin x + \cos x + c}$$

$1 = \cos^2 x + \sin^2 x$

$\sin 2A = 2\cos A \sin A$

Integration by parts:

I LATE

- E → Exponential (eg: e^x)
- T → Trignometry (eg: $\cos x$, $\sin x$)
- A → Arithmetic (x, x^2 etc)
- L → logarithmic ($\log x$).
- I → Inverse trignometry [$(\sin^{-1} x)$ $(\tan^{-1} x)$

Formula:

$$\int u\,dv = uv - \int v\,du.$$

Selection of u by "I late".

eg: $\int x (e^x . dx)$

A. e

$\boxed{u = x}$ $\boxed{dv = e^x . dx}$

u = differentiate. [u', u'', u''']

dv = Integrate. [v, v_1, v_2, v_3]

IBP $\rightarrow \int u\,dv = uv - \int v\,du$.

BT $\rightarrow \int u\,dv = uv - u'v_1 + u''v_2 - \dots$

$\int \underset{L}{\log x}\,.dx$ $\int \underset{I}{\tan x}(\underset{I}{\sin^{-1}x}).dx$.

$\boxed{u = \log x}$ $\boxed{U = \tan x}$

$\int \underset{A}{x}\,\underset{E}{e^{\log x}}.dx$

$\boxed{u = x}$

Ex : 2.5

① $\int x e^{-x}.dx$.

$I = \int \underset{A}{x}\underset{E}{e^{-x}}.dx$.

$u = x$, $u' = 1$, $u'' = 0$

$dv = e^{-x}.dx$

$\int dv = \int e^{-x}.dx$

$v = \frac{e^{-x}}{-1} = -e^{-x}$.

$\boxed{V_1 = e^{-x}}$

$\int u\,dv = uv - u'v_1 + u''v_2 \dots$

$I = -xe^{-x} - 1e^{-x} + c$//

$\boxed{I = -e^{-x}[x+1] + c}$ //

② $\int \underset{A}{x^3}\underset{E}{e^{3x}}.dx$

$u = x^3$, $u' = 3x^2$, $u'' = 6x$, $u''' = 6$, $u'''' = 0$

$I = \int x^3 e^{3x}.dx$.

$\int u\,dv = uv - u'v_1 + u''v_2 - u'''v_3 + \dots$

$dv = e^{3x}.dx$

$\int dv = \int e^{3x}.dx$

$v = \frac{e^{3x}}{3}$

$v_1 = \frac{e^{3x}}{9}$

$v_2 = \frac{e^{3x}}{27}$

$v_3 = \frac{e^{3x}}{81}$

$I = \frac{x^3e^{3x}}{3} - \frac{3x^2e^{3x}}{9} + \frac{6xe^{3x}}{27} - \frac{6e^{3x}}{81} + c$

$= \boxed{\frac{e^{3x}}{3}\left[x^3 - x^2 + \frac{2x}{3} - \frac{2}{9}\right] + c}$ //.

③ $\int \log x.dx$.

$I = \log x.dx$.

$u = \log x$, $u' = 1/x$

$dv = dx$

$\int dv = \int dx$.

$\boxed{v = x}$

$\int u\,dv = uv - \int v\,du$

$I = \log x(x) - \int x\,\frac{1}{x}\,dx$.

$= \boxed{x\log x - x + c}$

④ $\int x\log x$

$I = \int x\log x.dx$.

$u = \log x$, $u' = 1/x$

$dv = x.dx$.

$\int dv = \int x.dx$

$\boxed{v = \frac{x^2}{2}}$

$\int u\,dv = uv - \int v\,du$

$I = \log x \times \frac{x^2}{2} - \int \frac{x^2}{2} \times \frac{1}{x}\,dx$

$= \frac{x^2}{2}\log x - \frac{1}{2}\left(\frac{x^2}{2}\right) + c$

$= \boxed{\frac{x^2}{2}\log x - \frac{x^2}{4} + c}$

⑤ $\int x^n \cdot \log x\,dx$.

$I = \int x^n \log x \cdot dx$

$u = \log x$ | $dv = x^n \cdot dx$
$u' = \frac{1}{x}$ | $\int dv = \int x^n \cdot dx$
$v = \frac{x^{n+1}}{n+1}$

$\int u\,dv = uv - \int v\,du$.

$I = \log x\left(\frac{x^{n+1}}{n+1}\right) - \int \frac{x^{n+1}}{n+1} \times \frac{1}{x} \cdot dx$

$= \left(\frac{x^{n+1}}{n+1}\right)\log x - \frac{x^{n+1}}{(n+1)(n+1)} + c$

⑥ $\int x^5 e^{x^2} \cdot dx$

$I = \int x^5 e^{x^2} \cdot dx$.

$= \int x^4\, x\, e^{x^2} dx = \int (x^2)^2\, x e^{x^2} \cdot dx$

Put $t = x^2$

$dt = 2x \cdot dx$

$\boxed{\frac{dt}{2} = x \cdot dx}$

$I = \int t^2 e^t \frac{dt}{2}$

$I = \frac{1}{2}\int t^2 e^t\,dt$.

$u = t^2$ | $dv = e^t dt$
$u' = 2t$ | $\int dv = \int e^t dt$
$u'' = 2$ | $v = e^t$
$u''' = 0$ | $v_1 = e^t$
| $v_2 = e^t$

By BT

$\int u\,dv = uv - u'v_1 + u''v_2$.

$I = \frac{1}{2}\left[t^2 e^t - 2te^t + 2e^t\right] + c$

$= \frac{e^t}{2}\left[t^2 - 2t + 2\right] + c$

$= \frac{e^{x^2}}{2}\left[x^4 - 2x^2 + 2\right] + c$

eg: 2.28

$\int (\log x)^2 \, dx$

$I = \int (\log x)^2 \cdot dx$

$u = (\log x)^2$

$u' = 2\log x \times 1/x$

$dv = dx$

$\int dv = \int dx$

$\boxed{v = x}$

By IBP:

$\int u\,dv = uv - \int v\,du$

$I = (\log x)^2 (x) - \int x \times 2\log x \times 1/x \cdot dx$

$= x(\log x)^2 - 2\int \log x \cdot dx$ — ①

$I_1 = \int \log x \cdot dx.$

$u = \log x$

$u' = 1/x$

$\int dv = \int dx$

$\boxed{v = x}$

By IBP:

$\int u\,dv = uv - \int v\,du.$

$I_1 = (\log x)(x) - \int x \times 1/x \cdot dx$

$= x\log x - x$

① ⇒

$I = x(\log x)^2 - 2(x\log x - x) + c$

$= x[(\log x)^2 - 2\log x + 2] + c$

$\boxed{\therefore n\log x = \log x^n}$

$= x((\log x)^2 - \log x^2 + 2) + c$

Formula :

Type - VI

1. $\int [f(x)]^n f'(x)\,dx = \frac{[f(x)]^{n+1}}{n+1} + c, \; n \neq -1$

2. $\int \frac{f'(x)}{f(x)}\,dx = \log|f(x)| + c$

3. $\int \frac{f'(x)}{\sqrt{f(x)}}\,dx = 2\sqrt{f(x)} + c$

4. $\int e^x [f(x) + f'(x)]\,dx = e^x f(x) + c$.

5. $\int e^{ax} [a f(x) + f'(x)]\,dx = e^{ax} f(x) + c$.

Ex. 2.6

① $\int \frac{2x+5}{x^2+5x-7}\,dx$

$d(x^2+5x-7) = 2x+5$

$= \boxed{\log|x^2+5x-7| + c}$ //

② $\int \frac{e^{3\log x}}{x^4+1}\,dx$

$= \int \frac{e^{\log x^3}}{x^4+1}\,dx$

$= \frac{1}{4}\int \frac{4x^3\,dx}{x^4+1}$ $[d(x^4+1) = 4x^3+0]$

$= \boxed{\frac{1}{4}\log|x^4+1| + c}$

③ $\int \frac{e^{2x}}{e^{2x}-2}\,dx$.

$d(e^{2x}-2) = 2e^{2x}$.

$= \frac{1}{2}\int \frac{2e^{2x}}{e^{2x}-2}\,dx$

$= \boxed{\frac{1}{2}\log|e^{2x}-2| + c}$ //

(4) $\int \frac{(\log x)^3}{x} \cdot dx.$

$= \int (\log x)^3 \cdot 1/x \cdot dx$

$\Rightarrow \left[\int f(x)^n f'(x) \cdot dx = \frac{(f(x))^{n+1}}{n+1} \right]$

$= \frac{(\log x)^4}{4} + c$

(5) $\int \frac{6x+7}{\sqrt{3x^2+7x-1}} \cdot dx.$

$d(3x^2+7x-1) = 6x+7$

$\int \frac{f'(x)}{\sqrt{f(x)}} \cdot dx = 2\sqrt{f(x)}$ ..

$= \boxed{2\sqrt{3x^2+7x-1} + c}$

6. $(4x+2)\sqrt{x^2+x+1} \cdot dx.$

$d(x^2+x+1) = (2x+1)$

$= 2\int (2x+1)(x^2+x+1)^{1/2} dx.$

$\boxed{\int f'(x) f(x)^n \cdot dx = \frac{f(x)^{n+1}}{n+1}}$

$= \frac{2(x^2+x+1)^{3/2}}{3/2}$

$= 4/3 (x^2+x+1)^{3/2}.$

(7) $x^8(1+x^9)^5.$

$d(1+x^9) = 0 + 9x^8$

$f'(x) + f(x)^n \cdot dx = \frac{f(x)^{n+1}}{n+1}$

$= 1/9 \int 9x^8 (1+x^9)^5 \cdot dx$

$= 1/9 \frac{(1+x^9)^6}{6} + c$

$= \boxed{\frac{(1+x^9)^6}{54} + c}$

(8). $\int \frac{x^{e-1} + e^{x-1}}{x^e + e^x} \cdot dx.$ $\boxed{x^n = nx^{n-1}}$

$d(x^e + e^x) = ex^{e-1} + e^x.$

$= e\left[x^{e-1} + \frac{e^x}{e}\right]$

$= e\left[x^{e-1} + e^x \cdot e^{-1}\right]$

$= e\left[x^{e-1} + e^{x-1}\right]$

$$= \frac{1}{e}\int \frac{e\,(x^{e-1}+e^{x-1})}{x^e+e^x}\,dx$$

$$= \frac{1}{e}\log|x^e+e^x| + c$$

9. $\int \frac{1}{x\log x}\,dx$

Put $t = \log x$

$$\boxed{dt = \frac{1}{x}\cdot dx}$$

$= \int \frac{1}{t}\cdot dt$ | $\int \frac{1}{x}\cdot dx = \log x$

$= \log|t| + c$

$$= \boxed{\log|\log x| + c}$$

10. $I = \int \frac{x}{2x^4-3x^2-2}\,dx = \int \frac{x\,dx}{2(x^2)^2-3(x^2)-2}$

Put $t = x^2$

$dt = 2x\cdot dx$

$$\boxed{\frac{dt}{2} = x\cdot dx}$$

$$= \int \frac{dt/2}{2t^2-3t-2} = \frac{1}{2}\int \frac{dt}{2t^2-3t-2}$$

$$= \int \frac{dt}{(t-2)(2t+1)} \rightarrow ①$$

$$\frac{1}{(t-2)(2t+1)} = \frac{A}{t-2} + \frac{B}{2t+1}$$

$$\frac{1}{\cancel{(t-2)(2t+1)}} = \frac{A(2t+1)+B(t-2)}{\cancel{(t-2)(2t+1)}}$$

$1 = A(2t+1) + B(t-2)$

Put $\boxed{t=2}$

$1 = 5A + 0$

$$\boxed{A = \frac{1}{5}}$$

$t = -\frac{1}{2}$

$1 = 0 + B(-\frac{1}{2}-2)$

$$\boxed{B = -\frac{2}{5}}$$

$$\frac{1}{(t-2)(2t+1)} = \frac{1/5}{t-2} - \frac{2/5}{2t+1}$$

$$① \Rightarrow I = \frac{1}{5}\left(\int \frac{1}{t-2} - \int \frac{2}{2t+1}\right)dt$$

$d(t-2) = 1$ $\qquad d(2t+1) = 2$

$$= \frac{1}{5}\left[\log|t-2| - \log|2t+1|\right] + c$$

$= \frac{1}{5}\left(\log|x^2-2| - \log|2x^2+1|\right) + c$

$= \frac{1}{5}\left(\log\left|\frac{x^2-2}{2x^2+1}\right|\right) + c.$

(11) $I = \int e^x(1+x)\log(xe^x) \cdot dx$ ex. 2.6

Put $\boxed{t = xe^x}$ uv $[d(uv) = uv' + vu']$

$dt = x\,d(e^x) + e^x(dx)$

$dt = [xe^x + e^x(1)] \cdot dx$

$= e^x(x+1)\,dx.$

$I = \int \log t \cdot dt$ $\int \log x \cdot dx = x\log x - x + c.$

$= t\log t - t + c$

$= \boxed{(xe^x)\log(xe^x) - (xe^x) + c}$

Notes:

(*) $\frac{1}{x(x^2+1)} = \frac{1}{x} - \frac{x}{x^2+1}$

$\frac{1}{x(x+1)} = \frac{1}{x} - \frac{1}{x+1}$

$\frac{x}{(x+1)^2} = \frac{1}{(x+1)} - \frac{1}{(x+1)^2}$

(12) $I = \int \frac{1}{x(x^2+1)} \cdot dx$ $\frac{1}{x(x^2+1)} = \frac{1}{x} - \frac{x}{(x^2+1)}$

$= \int\left(\frac{1}{x} - \frac{x}{x^2+1}\right) \cdot dx.$

$d(x^2+1) = 2x.$

$= \int \left(\frac{1}{x} - \frac{1}{2}\frac{2x}{x^2+1}\right).dx$

$= \boxed{\log|x| - \frac{1}{2}\log|x^2+1| + c}$ //

(13) $I = \int e^x \left[\frac{1}{x^2} - \frac{2}{x^3}\right].dx$ $\quad d\left(\frac{1}{x^2}\right) = \frac{-2}{x^3}$

$= \int e^x (f(x) + f'(x))dx$

$= e^x f(x) + c$

$= \boxed{e^x \frac{1}{x^2} + c}$ //

(14) $I = \int e^x \left[\frac{x-1}{(x+1)^3}\right]dx.$

$= \int e^x \left[\frac{x-1+1-1}{(x+1)^3}\right].dx$

$= \int e^x \left[\frac{(x+1)-2}{(x+1)^3}\right].dx$

$= \int e^x \left[\frac{\cancel{(x+1)}}{(x+1)^{\cancel{3}}} - \frac{2}{(x+1)^3}\right].dx$

$= \int e^x \left[\frac{1}{(x+1)^2} - \frac{2}{(x+1)^3}\right]$

$= \int e^x [f(x) + f'(x)].dx$ $\quad d\left(\frac{1}{(x+1)^2}\right) = \frac{-2}{(x+1)^3}$,

$= e^x f(x) + c$

$= e^x \frac{1}{(x+1)^2} + c$

(15) $I = \int e^{3x}\left[\frac{3x-1}{9x^2}\right]dx.$

$= \frac{1}{9}\int e^{3x}\left[\frac{3\cancel{x}}{\cancel{x^2}} - \frac{1}{x^2}\right].dx$

$= \frac{1}{9}\int e^{3x}\left[\frac{3}{x} - \frac{1}{x^2}\right].dx$ $\quad d\left(\frac{1}{x}\right) = -\frac{1}{x^2}$

$= \frac{1}{9}\int e^{ax}[a f(x) + f'(x)]dx$

$= \frac{1}{9}\, e^{ax} f(x) + c$

$= \frac{1}{9}\, e^{3x}\, \frac{1}{x} + c$

$= \boxed{\frac{e^{3x}}{9x} + c}$ //

(X) (X) (X) eg 2:33

$I = \int \frac{x^3}{(x^2+1)^3}.dx$

Put $t = x^2+1 \Rightarrow \boxed{x^2 = t-1}$

$dt = 2x \cdot dx \Rightarrow \frac{dt}{2} = x\,dx.$

$I = \int \frac{x^2 \cdot x\,dx}{(x^2+1)^3}$

$= \int \frac{t-1}{t^3} \cdot \frac{dt}{2}$

$= \frac{1}{2} \int \left(\frac{t}{t^3} - \frac{1}{t^3}\right) dt.$

$= \frac{1}{2} \int \left(\frac{1}{t^2} - \frac{1}{t^3}\right) . dt$

$= \frac{1}{2} \left[\frac{-1}{t} + \frac{1}{2t^2}\right] + c$

$= \frac{1}{2} \left[\frac{-1}{t} + \frac{1}{2t^2}\right] + c$

$= \frac{1}{2} \left[\frac{-1}{(x^2+1)} + \frac{1}{2(x^2+1)^2}\right] + c$

$\int \frac{1}{x} . dx = \log x$

$\int \frac{1}{x^2} . dx = -\frac{1}{x}.$

eg 2.34

$I = \int \frac{dx}{x(x^3+1)} \times \frac{x^2}{x^2}$

$= \int \frac{x^2 . dx}{x^3(x^3+1)}$

Put $t = x^3$

$dt = 3x^2 . dx$

$\frac{dt}{3} = x^2 . dx$

$= \int \frac{dt/3}{t(t+1)}$

$\frac{1}{x(x+1)} = \frac{1}{x} - \frac{1}{x+1}$

$= \frac{1}{3} \int \frac{dt}{t(t+1)}$

$= \frac{1}{3} \left[\frac{1}{t} - \frac{1}{t+1}\right] . dx$

$= \frac{1}{3} \left[\log|t| - \log|t+1|\right] + c$

$= \frac{1}{3} \left[\log|x^3| - \log|x^3+1|\right] + c$

$= \boxed{\frac{1}{3} \log\left|\frac{x^3}{x^3+1}\right| + c}$

eg : 2.37

$I = \int \frac{x e^x}{(1+x)^2} . dx$

$\frac{x}{(1+x)^2} = \frac{1}{(1+x)} - \frac{1}{(1+x)^2}$

$= \int e^x \left[\frac{1}{(1+x)} - \frac{1}{(1+x)^2}\right] . dx$

$= \int e^x (f(x) + f'(x)) . dx$

$= e^x f(x) + c.$

$= \boxed{e^x \frac{1}{1+x} + c}$

eg 2:39

$$I = \int \left(\frac{1}{(\log x)} - \frac{1}{(\log x)^2} \right) . dx$$

Put $t = \log x$.

$dt = \frac{1}{x} . dx$.

$\boxed{dx = x\,dt}$

$t = \log x$

take 'e'

$e^t = x$.

$$= \int \left(\frac{1}{t} - \frac{1}{t^2} \right) e^t . dt$$

$$= \int e^x \left(f(x) + f'(x) \right) . dx$$

$$= e^x f(x) + c.$$

$$= e^t \frac{1}{t} + c$$

$$= \frac{e^{\log x}}{\log x} + c$$

$$= \boxed{\frac{x}{\log x} + c}$$

Formula:

① $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + c$

$a^2 - b^2 = (a+b)(a-b)$

② $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log \left| \frac{a-x}{a+x} \right| + c$

③ $\int \frac{dx}{\sqrt{x^2 - a^2}} = \log \left| x + \sqrt{x^2 - a^2} \right| + c$

④ $\int \frac{dx}{\sqrt{x^2 + a^2}} = \log \left| x + \sqrt{x^2 + a^2} \right| + c$

⑤ $\int \sqrt{x^2 - a^2} . dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \log \left| x + \sqrt{x^2 - a^2} \right| + c$

⑥ $\int \sqrt{x^2 + a^2} . dx = \frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \log \left| x + \sqrt{x^2 + a^2} \right| + c$

Ex 2.7

(1) $\int \frac{1}{9-16x^2} \cdot dx$

let $\boxed{a=3}$ $\boxed{x=4x}$ $[d(4x)=4]$

$= \int \frac{1}{3^2-(4x)^2} \cdot dx$

$= \int \frac{1}{a^2-x^2} \cdot dx = \frac{1}{2a} \log \left|\frac{a+x}{a-x}\right| + c$

$= \dfrac{\frac{1}{2\times 3} \log \left|\frac{3+4x}{3-4x}\right| + c}{4}$

$= \boxed{\frac{1}{24} \log \left|\frac{3+4x}{3-4x}\right| + c}$

(3) $\int \frac{1}{2x^2-9} \cdot dx$

$= \int \frac{1}{(\sqrt{2}x)^2 - 3^2} \cdot dx$

let $\boxed{x=\sqrt{2}x}$ $\boxed{a=3}$ $[d(\sqrt{2}x)=\sqrt{2}]$

$= \int \frac{1}{x^2-a^2} dx = \frac{1}{2a} \log \left|\frac{x-a}{x+a}\right| + c$

$= \dfrac{\frac{1}{2\times 3} \log \left|\frac{\sqrt{2}x-3}{\sqrt{2}x+3}\right| + c}{\sqrt{2}}$

$= \boxed{\frac{1}{6\sqrt{2}} \log \left|\frac{\sqrt{2}x-3}{\sqrt{2}x+3}\right| + c}$

(8) $\int \frac{1}{\sqrt{9x^2-7}} \cdot dx$

let $\boxed{x=3x}$ $\boxed{a=\sqrt{7}}$ $[d(3x)=3]$

$= \int \frac{1}{\sqrt{(3x)^2-(\sqrt{7})^2}} \cdot dx$

$= \int \frac{1}{\sqrt{x^2-a^2}} \cdot dx = \log \left| x + \sqrt{x^2-a^2} \right| + c$

$= \dfrac{\log \left| 3x + \sqrt{(3x)^2-(\sqrt{7})^2} \right| + c}{3}$

$= \boxed{\frac{1}{3} \log \left| 3x + \sqrt{9x^2-7} \right| + c}$

(13) $\int \sqrt{x^2-2} \; dx$

let $\boxed{a=\sqrt{2}}$ $\boxed{x=x}$ $[d(x)=1]$

$= \int \sqrt{x^2-(\sqrt{2})^2} \cdot dx$

$= \int \sqrt{x^2-a^2} \cdot dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2} \log \left| x + \sqrt{x^2-a^2} \right|$

$= \dfrac{\frac{x}{2}\sqrt{x^2-(\sqrt{2})^2} - \frac{(\sqrt{2})^2}{2} \log \left| x + \sqrt{x^2-(\sqrt{2})^2} \right| +}{1}$

$= \boxed{\frac{x}{2}\sqrt{x^2-2} - \log \left| x + \sqrt{x^2-2} \right| + c}$

(iii) $\int \sqrt{4x^2-5}\,.dx$ $\boxed{a=\sqrt{5}}$, $\boxed{x=2x}$ $[d(2x)=2]$

$= \int \sqrt{(2x)^2-(\sqrt{5})^2}\,.dx$

$= \int \sqrt{x^2-a^2}\,.dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log|x+\sqrt{x^2-a^2}| + c$

$:= \dfrac{\frac{\cancel{2}x}{\cancel{2}}\sqrt{(2x)^2-(\sqrt{5})^2} - \frac{(\sqrt{5})^2}{2}\log|2x+\sqrt{(2x)^2-(\sqrt{5})^2}| + c}{2}$

$= \boxed{\frac{1}{4}\left[2x\sqrt{4x^2-5} - 5\log|2x+\sqrt{4x^2-5}|\right] + c}$ //

eg 2.44

$I = \int \frac{x^2}{x^2-25}\,.dx$

$= \int \frac{(x^2-25+25)}{x^2-25}\,.dx$

$= \int \left(\frac{\cancel{x^2-25}}{\cancel{x^2-25}} + \frac{25}{x^2-25}\right).dx$

$= \int \left(1+25\frac{1}{x^2-5^2}\right).dx$

$\boxed{x=x}$, $\boxed{a=5}$ $(d(x)=1)$

$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + c$

$= x + \cancel{25}^{5} \times \frac{1}{2\times\cancel{5}}\log\left|\frac{x-5}{x+5}\right| + c$

$= \boxed{x + \frac{5}{2}\log\left|\frac{x-5}{x+5}\right| + c}$ //

(7) $I = \int \frac{e^x}{e^{2x}-9}\,.dx$

$= \int \frac{e^x}{(e^x)^2-3^2}\,.dx$

$\boxed{x=t}$ $\boxed{a=3}$ $d(t)=1$

Put $t=e^x$

$dt = e^x.dx$

$= \int \frac{dt}{t^2-3^2}$

$= \int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + c$

$= \frac{1}{2\times 3}\log\left|\frac{t-3}{t+3}\right| + c$

$= \boxed{\frac{1}{6}\log\left|\frac{e^x-3}{e^x+3}\right| + c}$ //

(11) $I = \int \frac{x^3}{\sqrt{x^8-1}} \cdot dx$

$d(x^4) = 4x^3$

$= \int \frac{x^3}{\sqrt{(x^4)^2-1^2}} \cdot dx$

$\boxed{t = x^4}$

$dt = 4x^3 \cdot dx$

$\boxed{\frac{dt}{4} = x^3 \cdot dx}$

$I = \int \frac{dt/4}{\sqrt{t^2-1^2}}$

$= \frac{1}{4}\int \frac{dt}{\sqrt{t^2-1^2}}$

$\boxed{x = t, \ a = 1 \\ d(t) = 1}$

$= \frac{1}{4}\int \frac{dx}{\sqrt{x^2-a^2}}$

$= \frac{1}{4} \log |x + \sqrt{x^2-a^2}| + c.$

$= \frac{1}{4} \log |t + \sqrt{t^2-1^2}| + c$

$= \boxed{\frac{1}{4} \log |x^4 + \sqrt{x^8-1}| + c}$

(2) $\int \frac{1}{9-8x-x^2} \cdot dx.$

$9-8x-x^2 = -[x^2 + 8x - 9]$

$= \frac{8}{2} = 4 \rightarrow \boxed{b^2 = 16}$

$= -[x^2 + 8x + 16 - 16 - 9]$

$a^2 + 2ab + b^2 = (a+b)^2$

$= -[(x+4)^2 - 5^2]$

$= 5^2 - (x+4)^2$

$I = \int \frac{dx}{5^2-(x+4)^2}.$

$\boxed{x = x+4}$ $\boxed{a = 5}$

$d(x+4) = 1$

$= \int \frac{dx}{a^2-x^2} = \frac{1}{2a} \log \left|\frac{a+x}{a-x}\right| + c$

$= \frac{1}{2 \times 5} \log \left|\frac{5+(x+4)}{5-(x+4)}\right| + c$

$= \boxed{\frac{1}{10} \log \left|\frac{5+(x+4)}{5-(x+4)}\right| + c}$

(6) $I = \int \frac{1}{2x^2+6x-8} \, dx$

$= \frac{1}{2} \int \frac{dx}{x^2+3x-4}$

$= (x^2+3x-4) = \underbrace{x^2 - 3x + \frac{9}{4}} - \frac{9}{4} - 4$

$\frac{+3}{2} = b$

$\boxed{b^2 = \frac{9}{4}}$

$= \left(x + \frac{3}{2}\right)^2 - \left(\frac{7}{4}\right)$

$= \left(x + \frac{3}{2}\right)^2 - \left(\frac{\sqrt{7}}{2}\right)^2$

$I = \frac{1}{2} \int \frac{dt}{(x+\frac{3}{2})^2 - (\frac{\sqrt{7}}{2})^2}$

$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + c$

$x = x + \frac{3}{2} \quad a = \frac{\sqrt{7}}{2}$

$= \frac{1}{2}\left[\frac{1}{2 \times \frac{\sqrt{7}}{2}} \log \left| \frac{x+\frac{3}{2} - \frac{\sqrt{7}}{2}}{x+\frac{3}{2}+\frac{\sqrt{7}}{2}} \right| \right] + c$

$= \frac{1}{2\sqrt{7}}$

$= \frac{1}{2} \int \frac{dx}{x^2+3x-4}$

$x^2 + 3x - 4 = x^2 + 3x + \frac{9}{4} - \frac{9}{4} - 4$

$+\frac{3}{2} = b$

$b^2 = \frac{9}{4}$

$= \left(x + \frac{3}{2}\right)^2 - \frac{25}{4}$

$I = \frac{1}{2} \int \frac{dx}{(x+\frac{3}{2})^2 - (\frac{5}{2})^2}$

$\boxed{x = x + \frac{3}{2}}$ $\boxed{a = \frac{5}{2}}$

$d(x) = 1$

$= \int \frac{dx}{x^2-a^2} = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + c$

$= \frac{1}{2} \times \frac{1}{2(\frac{5}{2})} \log \left| \frac{(x+\frac{3}{2}) - \frac{5}{2}}{(x+\frac{3}{2}) + \frac{5}{2}} \right| + c$

$= \frac{1}{2} \times \frac{1}{5} \log \left| \frac{\frac{2x-2}{2}}{\frac{2x+8}{2}} \right| + c$

$= \boxed{\frac{1}{10} \log \left| \frac{2x-2}{2x+8} \right| + c.}$

(4) $I = \int \frac{1}{x^2 - x - 2} \cdot dx$

$$x^2 - x - 2 = \underbrace{x^2 - x + \tfrac{1}{4}} - \tfrac{1}{4} - 2$$

$-\tfrac{1}{2} = b$

$\boxed{b^2 = \tfrac{1}{4}}$

$$= \left(x - \tfrac{1}{2}\right)^2 - \left(\tfrac{3}{2}\right)^2$$

$$I = \int \frac{1}{(x-\frac{1}{2})^2 - (\frac{3}{2})^2} \cdot dx$$

$$= \int \frac{dt}{x^2 - a^2} = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + c$$

$\boxed{x = x - \tfrac{1}{2}}$ $\boxed{a = \tfrac{3}{2}}$

$d(x - \tfrac{1}{2}) = 1$

$$= \frac{1}{\cancel{2} \times \frac{3}{\cancel{2}}} \log \left| \frac{x - \frac{1}{2} - \frac{3}{2}}{x - \frac{1}{2} + \frac{3}{2}} \right| + c$$

$$= \frac{1}{3} \log \left| \frac{\frac{2x-4}{\cancel{2}}}{\frac{2x+2}{\cancel{2}}} \right| + c$$

$$= \boxed{\frac{1}{3} \log \left| \frac{2x-4}{2x+2} \right| + c} //$$

(5) $I = \int \frac{1}{x^2 + 3x + 2} \cdot dx$

$$(x^2 + 3x + 2) = \underbrace{x^2 + 3x + \tfrac{9}{4}} - \tfrac{9}{4} + 2$$

$+\tfrac{3}{2} = b$

$\boxed{b^2 = \tfrac{9}{4}}$

$$= \left(x + \tfrac{3}{2}\right)^2 - \tfrac{1}{4}$$

$$= \left(x + \tfrac{3}{2}\right)^2 - \left(\tfrac{1}{2}\right)^2$$

$$I = \int \frac{dx}{(x+\frac{3}{2})^2 - (\frac{1}{2})^2}$$

$$= \int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + c$$

$\boxed{x = x + \tfrac{3}{2}}$ $\boxed{a = \tfrac{1}{2}}$

$\boxed{d(x + \tfrac{3}{2}) = 1}$

$$= \frac{1}{\cancel{2} \times \frac{1}{\cancel{2}}} \log \left| \frac{x + \frac{3}{2} - \frac{1}{2}}{x + \frac{3}{2} + \frac{1}{2}} \right| + c$$

$$= \log \left| \frac{\frac{2x+2}{\cancel{2}}}{\frac{2x+4}{\cancel{2}}} \right|$$

$$= \boxed{\log \left| \frac{2x+2}{2x+4} \right| + c}$$

(6) $\int \frac{1}{2x^2+6x-8} \cdot dx$

$I = 2x^2+6x-8$

$= \frac{1}{2}\int \frac{dx}{x^2+3x-4}$

(9) $\int \frac{1}{\sqrt{x^2+6x+13}} \cdot dx$

$I = \sqrt{x^2+6x+13} = \underbrace{x^2+6x+9}-9+13$

$\frac{6}{2} = 3 = b$

$\boxed{b^2 = 9}$

$= (x+3)^2+4$

$= (x+3)^2+(2)^2$

$I = \int \frac{dx}{\sqrt{(x+3)^2+(2)^2}}$

$= \int \frac{dx}{\sqrt{x^2+a^2}} = \log|x+\sqrt{x^2+a^2}|+c$

$x = x+3, \quad a = 2$

$d(x+3) = 1$

$= \boxed{\log|(x+3)+\sqrt{(x+3)^2+(2)^2}|+c}$

(12) $I = \int \sqrt{1+x+x^2} \cdot dx$

$1+x+x^2 = x^2+x+1$

$\frac{1}{2} = b$

$\boxed{b^2 = \frac{1}{4}}$

$= \underbrace{x^2+x+\frac{1}{4}}-\frac{1}{4}+1$

$= (x+\frac{1}{2})^2+\frac{3}{4}$

$= (x+\frac{1}{2})^2+\left(\frac{\sqrt{3}}{2}\right)^2$

$I = \int \sqrt{(x+\frac{1}{2})^2+\left(\frac{\sqrt{3}}{2}\right)^2} \cdot dx$

$= \int \sqrt{x^2+a^2} \cdot dx = \frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\log|x+\sqrt{x^2+a^2}|+c$

$\boxed{x = x+\frac{1}{2}}$ $\boxed{a = \frac{\sqrt{3}}{2}}$

$d(x+\frac{1}{2}) = 1$

$I = \frac{(x+\frac{1}{2})}{2}\sqrt{(x+\frac{1}{2})^2+\left(\frac{\sqrt{3}}{2}\right)^2}+\frac{\left(\frac{\sqrt{3}}{2}\right)^2}{2}\log\left|(x+\frac{1}{2})+\sqrt{(x+\frac{1}{2})^2+\left(\frac{\sqrt{3}}{2}\right)^2}\right|+c$

$= \frac{2x+1}{4}\sqrt{1+x+x^2}+\frac{3}{8}\log\left|\frac{2x+1}{2}+\sqrt{1+x+x^2}\right|+c$

(15) $I = \int \sqrt{2x^2+4x+1} \cdot dx$

$= \int \sqrt{2(x^2+2x+1/2)} \cdot dx$

$= \sqrt{2} \int \sqrt{x^2+2x+1/2} \cdot dx$

consider:

$x^2+2x+1/2 = x^2+2x+1-1+1/2$

$\frac{x}{x} = 1 = b$, $\boxed{b^2=1}$

$= (x+1)^2 - 1/2$

$= (x+1)^2 - \left(\frac{1}{\sqrt{2}}\right)^2$

$I = \sqrt{2} \int \sqrt{(x+1)^2 - \left(\frac{1}{\sqrt{2}}\right)^2} \cdot dx$

$= \sqrt{2} \int \sqrt{x^2 - a^2} \cdot dx$

$= \sqrt{2}\ \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2} \log\left|x+\sqrt{x^2-a^2}\right| + c$

$\boxed{x = x+1}$, $\boxed{a = \frac{1}{\sqrt{2}}}$

$d(x+1) = 1$

$= \sqrt{2}\left[\frac{(x+1)}{2}\sqrt{(x+1)^2-\left(\frac{1}{\sqrt{2}}\right)^2} - \frac{\left(\frac{1}{\sqrt{2}}\right)^2}{2} \log\left|(x+1) + \sqrt{(x+1)^2-\left(\frac{1}{\sqrt{2}}\right)^2}\right| + c\right.$

$= \boxed{\sqrt{2}\left[\frac{x+1}{2}\sqrt{x^2+2x+1/2} - \frac{1}{4}\log\left|(x+1)+\sqrt{x^2+2x+1/2}\right| + c\right.}$

(16) $I = \int \frac{1}{x+\sqrt{x^2-1}} \cdot dx$

$= \int \frac{1}{\underset{(a+b)}{x+\sqrt{x^2-1}}} \times \frac{x-\sqrt{x^2-1}}{\underset{(a-b)}{x-\sqrt{x^2-1}}} \cdot dx$

$= \int \frac{x-\sqrt{x^2-1}}{\underset{a^2-b^2}{x^2-\left(\sqrt{x^2-1}\right)^2}}$

$x^2-(\sqrt{x^2-1})^2$

$\cancel{x^2} - \cancel{x^2}+1$

$= \int \left(x - \sqrt{x^2-1^2}\right) \cdot dx$ $\boxed{a=1}$ $\boxed{x=x}$

$= \boxed{\frac{x^2}{2} - \left[\frac{x}{2}\sqrt{x^2-1} - \frac{1}{2}\log\left|x+\sqrt{x^2-1}\right| + c\right.}$

Ex 2.8

I. (1). $I = \int_0^1 e^{2x} \cdot dx$

$= \left[\frac{e^{2x}}{2}\right]_0^1$

$e^0 = 1$ $\log_e e$

$e^{\log}$

$= \frac{e^2}{2} - \frac{e^0}{2}$

$= \frac{e^2}{2} - 1/2$

$\boxed{I = 1/2(e^2-1)}$

(2) $I = \int_0^{1/4} \sqrt{1-4x}\,.dx$

$= \int_0^{1/4} (1-4x)^{1/2}\,.\,dx$

$= \left[\dfrac{(1-4x)^{3/2}}{3/2 \times -4}\right]_0^{1/4}$

$= -1/6 \left((1-4\times 1/4)^{3/2} - (1-0)^{3/2}\right)$

$\boxed{I = 1/6}$

(3) $I = \int_1^2 \dfrac{x.dx}{x^2+1}$

$d(x^2+1) = 2x.$

$= 1/2 \int_1^2 \dfrac{2x.dx}{x^2+1}$

$= 1/2 \left[\log|x^2+1|\right]_1^2$

$= 1/2 \left[\log|5| - \log|2|\right]$

$\boxed{I = 1/2 \log(5/2)}$

(4) $I = \int_0^3 \dfrac{e^x.dx}{1+e^x}$

$d(1+e^x) = 0+e^x$

$e^0 = 1$

$= \left[\log|1+e^x|\right]_0^3$

$= \log|1+e^3| - \log|1+1|$

$\boxed{I = \log\left(\dfrac{1+e^3}{2}\right)}$

(5) $I = \int_0^1 x\,e^{x^2}.dx.$

Put $\boxed{t = x^2}$

x	0	1
$t=x^2$	0	1

$dt = 2x.dx.$

$\dfrac{dt}{2} = x.dx.$

$= \int_0^1 e^t.dt$

$= 1/2 (e^t)_0^1$

$= 1/2 (e^1 - e^0)$

$\boxed{I = 1/2\,[e-1]}$

(6) $I = \int_1^e \frac{dx}{x(1+\log x)^3}$

x	1	e
$t = 1+\log x$	1	2

Put $t = 1+\log x$

$dt = \frac{1}{x} \cdot dx$

$[x=1,\ t = 1+\log 1 = 1$
$(x=e\ \ t = 1+\log e = 2)]$

$= \int_1^2 \frac{dt}{t^3}$

$= \left[\frac{-1}{2t^2}\right]_1^2 = -\frac{1}{2}\left[\frac{1}{4} - \frac{1}{1}\right]$

$= -\frac{1}{2} \times -\frac{3}{4}$

$\boxed{I = 3/8}$ //

(7) $I = \int_{-1}^{1} \frac{2x+3}{x^2+3x+7}$

$[d(x^2+3x+7) = 2x+3]$

$= \left[\log|x^2+3x+7|\right]_{-1}^{1}$

$= \log|1+3+7| - \log|+1-3+7|$

$= \log|11| - \log|5|$

$= \boxed{\log\left[11/5\right]}$ //

(8) $I = \int_0^{\pi/2} \sqrt{1+\cos x}\ dx$

$= \int_0^{\pi/2} \sqrt{2\cos^2 x/2}\ .dx$

$= \sqrt{2} \int_0^{\pi/2} \cos x/2\ .dx$

$= \sqrt{2}\left[\frac{\sin x/2}{1/2}\right]_0^{\pi/2}$

$= 2\sqrt{2}\left[\sin \pi/4 - \sin 0\right]$

$= 2\sqrt{2}\left(\frac{1}{\sqrt{2}} - 0\right)$

$= \boxed{2}$

$\sin \pi/4 = \frac{1}{\sqrt{2}}$
$\sin 0 = 0$

(*) formula

$\cos^2 A = \frac{1+\cos 2A}{2}$

$1+\cos A = 2\cos^2 \frac{A}{2}$

$\sin^2 A = \frac{1-\cos 2A}{2}$

$1-\cos A = 2\sin^2 \frac{A}{2}$

(9) $I = \int_1^2 \frac{x-1}{x^2}\ .dx$

$= \int_1^2 \left(\frac{x}{x^2} - \frac{1}{x^2}\right) .dx$

$= \int_1^2 \left(\frac{1}{x} - \frac{1}{x^2}\right) .dx$

$= \left[\log|x| + \frac{1}{|x|}\right]_1^2$

$= \left(\log|2| + \frac{1}{2}\right) - \left(\log|1| + \frac{1}{1}\right)$

$= \log 2 + \frac{1}{2} - \log 1 - 1$

$= \log 2 - \frac{1}{2}$ // $= \boxed{\frac{1}{2}(2\log 2 - 1)}$

II.

① $I = \int_1^4 f(x)\cdot dx$ $\quad f(x) = \begin{cases} 4x+3, & 1 \le x \le 2 \\ 3x+5, & 2 < x \le 4 \end{cases}$

$4x+3$ $\quad$ $3x+5$
1 $\quad$ 2 $\quad$ 4

$$I = \int_1^2 (4x+3)\cdot dx + \int_2^4 (3x+5)\cdot dx$$

$$= \left[\frac{4x^2}{2} + 3x\right]_1^2 + \left[\frac{3x^2}{2} + 5x\right]_2^4$$

$$= \left(\frac{16}{2} + 6\right) - \left(\frac{4}{2} + 3\right) + \left(\frac{48}{2} + 20\right) - \left(\frac{12}{2} + 10\right)$$

$$= \frac{16}{2} + 6 - \frac{4}{2} - 3 + \frac{48}{2} + 20 - \frac{12}{2} - 10$$

$$= 58 - 21$$

$$= \boxed{37}$$

② $I = \int_0^2 f(x)\cdot dx$ $\quad = \begin{cases} 3-2x-x^2, & x \le 1 \\ x^2+2x-3, & 1 < x \le 2 \end{cases}$

$3-2x-x^2$ $\quad$ x^2+2x-3
0 $\quad$ 1 $\quad$ 2

$$I = \int_0^1 (3-2x-x^2)\cdot dx + \int_1^2 (x^2+2x-3)\cdot dx$$

$$= \left[3x - \frac{2x^2}{2} - \frac{x^3}{3}\right]_0^1 + \left[\frac{x^3}{3} + \frac{2x^2}{2} - 3x\right]_1^2$$

$$= \left(3 - 1 - \frac{1}{3}\right) - (0) + \left(\frac{8}{3} + 4 - 6\right) - \left(\frac{1}{3} + 1 - 3\right)$$

$$= 2 - \frac{1}{3} + \frac{8}{3} - \cancel{2} - \frac{1}{3} + \cancel{2}$$

$$= 2 + \frac{6}{3} = \frac{12}{3} = 4$$

③ $I = \int_{-1}^1 f'(x)$ $\quad \begin{cases} x & x \ge 0 \\ -x & x < 0 \end{cases}$

$-x$ $\quad$ x
-1 $\quad$ 0 $\quad$ 1

$$I = \int_{-1}^0 -x\cdot dx + \int_0^1 x\cdot dx$$

$$= \left(\frac{-x^2}{2}\right]_{-1}^0 + \left(\frac{x^2}{2}\right]_0^1$$

$$= \frac{1}{2}\left[(0) - (-1)\right] + (1-0)$$

$$= \frac{1}{2}\left[1+1\right]$$

$$= \boxed{1}$$

④ $f(x) = \begin{cases} cx & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$

Given $\int_0^1 f(x)\,dx = 2$

$$\int_0^1 cx\,dx = 2$$

$$\left(\frac{cx^2}{2}\right)_0^1 = 2$$

$$\left(cx^2\right)_0^1 = 4$$

$$c(1) - c(0) = 4$$

$$\boxed{c = 4}$$

eg 2.40.

$$\int \frac{dx}{16-x^2} \cdot dx$$

$$= \int \frac{1}{4^2 - x^2} \cdot dx \qquad \text{let } \boxed{a = 4} \quad \boxed{x = x} \quad d(x) = 1$$

$$= \int \frac{1}{a^2 - x^2} \cdot dx = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + c$$

$$= \frac{1}{2 \times 4} \log \left| \frac{4+x}{4-x} \right| + c.$$

$$= \boxed{\frac{1}{8} \log \left| \frac{4+x}{4-x} \right| + c} //$$

eg 2.41

$$\int \frac{dx}{1-25x^2}$$

$$= \int \frac{1}{1^2 - (5x)^2} \qquad \text{let } \boxed{a = 1} \quad \boxed{x = 5x} \quad [d(5x) = 5]$$

$$= \int \frac{1}{a^2 - x^2} dx = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + c$$

$$= \underbrace{\frac{1}{2 \times 1} \log \left| \frac{1+5x}{1-5x} \right|}_{5} + c$$

$$= \boxed{\frac{1}{10} \log \left| \frac{1+5x}{1-5x} \right| + c} /$$

eg 2.42

$$\int \frac{dx}{2+x-x^2}$$

$$= (2+x-x^2) = -(x^2-x-2)$$

$$= -\frac{1}{2} = b$$

$$\boxed{b^2 = \frac{1}{4}}$$

$$= -(\underbrace{x^2 - x + \frac{1}{4}} - \frac{1}{4} - 2)$$

$$= (x - \frac{1}{2})^2 - (\frac{3}{2})^2$$

$$I = \int \frac{1}{(x-\frac{1}{2})^2 - (\frac{3}{2})^2} \cdot dx$$

$$= \int \frac{dt}{x^2 - a^2} = \frac{1}{2a} \log |x \cdots$$

$\therefore \boxed{\sqrt[3]{x} = x^{1/3}}$

eg 2.58

$$\int_0^1 (e^x - 4a^x + 2 + \sqrt[3]{x}) \cdot dx$$

$$= \left[e^x - \frac{4a^x}{\log a} + 2x - \frac{x^{4/3}}{4/3}\right]_0^1$$

$$= \left[\left(e^1 - \frac{4a}{\log a} + 2 + \frac{1}{4/3}\right) - \left(e^0 - \frac{4(a)^0}{\log a} + 0 + 0\right)\right]$$

$$= \left[e - \frac{4a}{\log a} + 2 + 3/4 - 1 + \frac{4}{\log a}\right]$$

$$= e + \frac{4}{\log a}(1-a) + 7/4$$

eg 2.65

$$I = \int_{-1}^{1} x\sqrt{x+1} \cdot dx$$

Put $t = x+1 \Rightarrow x = t-1$

$dt = dx$.

x	-1	1
$t = x+1$	0	2

$$I = \int_0^2 (t-1)\sqrt{t} \cdot dt$$

$$= \int_0^2 (t^1 - 1)t^{1/2} \cdot dt$$

$$= \int_0^2 (t^{3/2} - t^{1/2}) \cdot dt$$

$$= \left[\frac{t^{5/2}}{5/2} - \frac{t^{3/2}}{3/2}\right]_0^2$$

$$= \left[2/5\, t^{5/2} - 2/3\, t^{3/2}\right]_0^2$$

$$= 2/5\,(2)^{5/2} - 2/3\,(2)^{3/2} - 0$$

$$= 2(2)^{3/2}\left[\frac{2^1}{5} - 1/3\right]$$

$$= 2(2\sqrt{2})\,\frac{6-5}{15}$$

$$= \boxed{\frac{4\sqrt{2}}{15}}$$

eg 2.66

$$\int_0^\infty e^{-x/2} \cdot dx$$

$$= \left(\frac{e^{-x/2}}{-1/2}\right)_0^\infty$$

$$= -2\left(\frac{1}{e^{x/2}}\right)_0^\infty$$

$$= -2\left(\frac{1}{e^\infty} - \frac{1}{e^0}\right)$$

$$= -2\left(\frac{1}{\infty} - \frac{1}{1}\right)$$

$$= \boxed{2}$$

$\infty = \frac{1}{0}$

$\frac{1}{\infty} = \frac{1}{\frac{1}{0}} = 0$

eg 2.67

$$I = \int_0^\infty x^2 e^{-x^3} \cdot dx$$

Put $t = x^3$

$dt = 3x^2 \cdot dx$

$$\boxed{\frac{dt}{3} = x^2 \cdot dx}$$

x	0	∞
$t = x^3$	0	∞

$$I = \int_0^{\infty} e^{-t} \cdot \frac{dt}{3} = \frac{1}{3}\left[\frac{e^{-t}}{-1}\right]_0^{\infty}$$

$$= -\frac{1}{3}\left[\frac{1}{e^{t}}\right]_0^{\infty}$$

$$= -\frac{1}{3}\left(\frac{1}{e^{\infty}} - \frac{1}{e^{0}}\right)$$

$$= -\frac{1}{3}(0-1)$$

$$= \boxed{+\frac{1}{3}}$$

eg: 2.70

$$I = \int_0^{\pi/2} x \cdot \sin x \cdot dx$$

A T

$u = x$ | $dv = \sin x \cdot dx$

$u' = 1$ | $\int dv = \int \sin x \cdot dx$

$u'' = 0$ | $\boxed{V = -\cos x}$

$V_1 = -\sin x$

$$\int u\,dv = (uv - u'v_1 + u''v_2 - \dots)$$

$\sin \pi/2 = 1$

$\cos \pi/2 = 0$

$$I = [-x\cos x + 1\sin x]_0^{\pi/2}$$

$$= \left(-\frac{\pi}{2}\cos\frac{\pi}{2} + \sin\frac{\pi}{2}\right) - (0 + \sin 0)$$

$$= \boxed{1}$$

eg: 2.71

$$\int_1^a 3x^2 \cdot dx = -1$$

$$\Rightarrow \left[\frac{\cancel{3}x^3}{\cancel{3}}\right]_1^a = -1$$

$$\Rightarrow a^3 - 1 = -1$$

$$a^3 = 0$$

$$\boxed{a = 0}$$

eg 2.72

$$\int_a^b dx = 1, \quad \int_a^b x \cdot dx = 1$$

$$[x]_a^b = 1$$

$$\boxed{b - a = 1}$$

$$\left[\frac{x^2}{2}\right]_a^b = 1$$

$$[x^2]_a^b = 2$$

$$b^2 - a^2 = 2$$

$$(b-a)(b+a) = 2$$

$$1(b+a) = 2$$

$$\boxed{b+a = 2}$$

$b - a = 1$

$b + a = 2$

$2b = 3$

$$\boxed{b = 3/2}$$

$$b + a = 2,$$

$$3/2 + a = 2$$

$$a = 2 - 3/2$$

$$\boxed{a = 1/2}$$

The value of $\boxed{a = 1/2}$ and $\boxed{b = 3/2}$

eg 2.56

$$\int_0^1 (x^3+7x^2-5x)\cdot dx$$

$$= \left[\frac{x^4}{4} + \frac{7x^3}{3} - \frac{5x^2}{2}\right]_0^1$$

$$= \left[\frac{1\times 6}{4\times 6} + \frac{7\times 8}{3\times 8} - \frac{5\times 12}{2\times 12}\right]$$

$$= \frac{6+56-60}{24}$$

$$= \frac{\cancel{+2}}{\cancel{24}}$$

$$= \boxed{1/12}$$

C.Q

(1) $\int_0^{\pi/2} \cos^9 x\, dx$

(2) $\int_0^{\pi/2} \cos^6 x\, dx$

(3) $\int_0^{\pi/2} \sin^5 x\, dx$

(4) $\int_0^{\pi/2} \sin^8 x\, dx$

Formula:

$$\int_a^b f(x)\cdot dx = -\int_b^a f(x)\cdot dx$$

(*) $\int_0^a f(x)\cdot dx = \int_0^a f(a-x)\cdot dx$

(*) $\int_{-a}^a f(x)\cdot dx = 2\int_0^a f(x)\cdot dx$, f is even $f(-x)=f(x)$,

$\hookrightarrow 0$, f is odd. $f(-x) = -f(x)$.

$f(x)=x \Rightarrow f(-x) = -x = -f(x)$ f is odd.

$f(x) = \cos x \Rightarrow f(-x) = \cos(-x) = \cos = f(x)$

*. $\int_a^b f(x)\cdot dx = \int_a^b f(a+b-x)\cdot dx$.

ex 2.9

① $I = \int_{-\pi/4}^{\pi/4} x^3 \cdot \cos^3 x \cdot dx$

$f(x) = x^3 \cos^3 x$

$f(-x) = -x^3 \cos^3 x$

$= -f(x)$

f is odd

$\boxed{I = 0}$

② $I = \int_{-\pi/2}^{\pi/2} \sin^2\theta \, d\theta$

$f(\theta) = \sin^2\theta$

$f(-\theta) = f(\theta)$

f is even

$I = 2\int_0^{\pi/2} \sin^2\theta \, d\theta$

$= \cancel{2} \times \frac{1}{\cancel{2}} \times \pi/2$

$I = \boxed{\pi/2}$

③ $I = \int_{-1}^{1} \log\left(\frac{2-x}{2+x}\right) dx$

$I = \int_{-1}^{1} \log(2-x) - \log(2+x) \cdot dx$

$f(x) = \log(2-x) - \log(2+x)$

$f(-x) = \log(2+x) - \log(2-x)$

$= -[\log(2-x) - \log(2+x)]$

$f(-x) = -f(x)$

f is odd

$\boxed{I = 0}$

④ $I = \int_0^{\pi/2} \frac{\sin^7 x}{\sin^7 x + \cos^7 x} \cdot dx \rightarrow ①$

$\sin \pi/2 - \theta = \cos\theta$

$\cos \pi/2 - \theta = \sin\theta$

$x = a - x = \pi/2 - x$

$I = \int_0^{\pi/2} \frac{\cos^7 x}{\cos^7 x + \sin^7 x} \cdot dx \rightarrow ②$

① + ②

$2I = \int_0^{\pi/2} \frac{\cancel{(\sin^7 x + \cos^7 x)}}{\cancel{(\sin^7 x + \cos^7 x)}} \cdot dx$

$2I = [x]_0^{\pi/2} = \pi/2 - 0$

$\boxed{I = \pi/4}$

⑤ $\int_0^1 \log\left(\frac{1}{x} - 1\right) \cdot dx$

$I = \int_0^1 \log\left(\frac{1-x}{x}\right) \cdot dx \rightarrow ①$

$x = a - x = 1 - x$

$$I = \int_0^1 \log\left(\frac{x}{1-x}\right).dx \quad -② $$

①+②

$$2I = \int_0^1 \left(\log \frac{1-x}{x} + \log \frac{x}{1-x}\right).dx$$

$$= \int_0^1 \log \frac{1-\cancel{x}}{\cancel{x}} \times \frac{\cancel{x}}{1-\cancel{x}} .dx$$

$$= \int_0^1 0.dx = 0$$

$$2I = 0 \quad \boxed{I = 0}$$

⑥ $\int_0^1 \frac{x}{(1-x)^{3/4}}.dx$

$$I = \int_0^1 \frac{x}{(1-x)^{3/4}}.dx$$

$x = a-x = 1-x$

$$I = \int_0^1 \frac{1-x}{(x)^{3/4}}$$

$$= \int_0^1 (1-x)\,x^{-3/4}.dx$$

$$= \int_0^1 (x^{-3/4} - x^{1/4}).dx$$

$$= \left(\frac{x^{1/4}}{1/4} - \frac{x^{5/4}}{5/4}\right)_0^1$$

$$= \frac{1}{1/4} - \frac{1}{5/4} - 0$$

$$= 4/1 - 4/5$$

$$= \frac{20-4}{5} = \boxed{16/5}$$

eg. 2.79

$$I = \int_2^5 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{7-x}}.dx \rightarrow ①$$

$x = a+b-x = 2+5-x$

$\boxed{x = 7-x}$

$$I = \int_2^5 \frac{\sqrt{7-x}}{\sqrt{7-x}+\sqrt{x}}.dx \rightarrow ②$$

①+②

$$2I = \int_2^5 \frac{(\sqrt{x}+\sqrt{7-x})}{\sqrt{7-x}+\sqrt{x}} dx.$$

$$= [x]_2^5$$

$$2I = 5-2$$

$$\boxed{I = 3/2}$$

C.a ① $\int_0^{\pi/2} \frac{f(\sin x)}{f(\sin x)+f(\cos x)}$ ⑤ $\int_0^3 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{3-x}}.dx$

Gamma Integral : (Γ)

(+ve) $\rightarrow$ (0 to ∞)

$$\Gamma(n) = \int_0^\infty x^{n-1} e^{-x} . dx$$

$$\Gamma(n+1) = n!$$

$$\Gamma(n) = (n-1)\Gamma(n-1)$$

$$\Gamma(n+1) = n\Gamma(n)$$

$$\Gamma(1/2) = \sqrt{\pi}$$

$$\int_0^\infty x^n e^{-ax} . dx = \frac{n!}{a^{n+1}}$$

ex : 2.10

① (i) $\Gamma(4) = \Gamma(3+1) = 3!$

$= 3 \times 2 = \boxed{6}$ //

(ii) $\Gamma(9/2) = 7/2 \times 5/2 \times 3/2 \times 1/2 \times \sqrt{\pi}$

$= \boxed{\frac{105}{16}\sqrt{\pi}}$ //

(iii) $I = \int_0^\infty e^{-mx} x^6 . dx$

By G.I,

$$\int_0^\infty x^n e^{-ax} . dx = \frac{n!}{a^{n+1}}.$$

$\boxed{a=m}$ $\boxed{n=6}$

$\boxed{I = \frac{6!}{m^7}}$ //

(iv) $I = \int_0^\infty e^{-x/2} x^5 . dx$.

By G.I,

$$\int_0^\infty x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$$

$\boxed{n=5}$ $\boxed{a = 1/2}$

$$I = \frac{5!}{(1/2)^6} = 2^6 (5!)$$

eg 2.80

(iv) $\int_0^\infty e^{-x^2} . dx$

$$\Gamma(n) = \int_0^\infty t^{n-1} . e^{-t} . dt$$

$\boxed{t = x^2}$, $dt = 2x \cdot dx$

$$\Gamma(n) = \int_0^\infty (x^2)^{n-1} e^{-x^2} (2x \cdot dx)$$

$$= 2\int_0^\infty x^{2n-2} \cdot e^{-x^2} \cdot x^1 \cdot dx$$

$$= 2\int_0^\infty x^{2n-1} \cdot e^{-x^2} \cdot dx$$

Put $\boxed{n = 1/2}$ $\left(\begin{array}{c} 2n-1=0 \\ 2n=1 \\ \boxed{n = 1/2} \end{array} \right)$

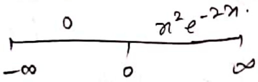
$$\Gamma(1/2) = 2\int_0^\infty x^0 e^{-x^2} \cdot dx \qquad (x^0 = 1)$$

$$\sqrt{\pi} = 2\int_0^\infty e^{-x^2} \cdot dx$$

$$\int_0^\infty e^{-x^2} \cdot dx = \boxed{\frac{\sqrt{\pi}}{2}} \;//$$

(2) $f(x) = \begin{cases} x^2 e^{-2x}, & x \geq 0 \\ 0, & \text{otherwise} \end{cases}$

$$\int_0^\infty f(x) \cdot dx$$



$$\int_0^\infty x^2 e^{-2x} \cdot dx = \frac{n!}{a^{n+1}}$$

$\boxed{a = 2}$ $\boxed{n = 2}$ $= \frac{2!}{2^3} = \frac{2\times 1}{2^3}$

$$= \frac{2}{2^3} = \boxed{1/4} \;//$$

eg: 2.80

(i) $\Gamma(6) = \Gamma(5+1)$

$= \boxed{5!} = 5\times4\times3\times2\times1 = \boxed{120} \;//$

(ii) $\Gamma(7/2) = 5/2 \times 3/2 \times 1/2 \times \sqrt{\pi}$

$= \boxed{15/8 \sqrt{\pi}} \;//$

(iii) $\int_0^\infty e^{-2x} x^5 \cdot dx$.

$\Rightarrow \int_0^\infty e^{-ax} x^n \cdot dx = \frac{n!}{a^{n+1}}$ $\boxed{n = 5}$ $\boxed{a = 2}$

$$= \frac{5!}{2^{5+1}}$$

$$= \boxed{\frac{5!}{2^6}}$$

Formula

$$\int_a^b f(x).dx = \lim_{n\to\infty} h\left(f(a)+f(a+h)+\dots\right)$$

$$= \lim_{n\to\infty} \sum_{r=1}^{n} h f(a+rh).$$

$$\boxed{h = \frac{b-a}{n}}$$

$$1+2+\dots+n = \frac{n(n+1)}{2} = \sum_{r=1}^{n} r.$$

$$1^2+2^2+\dots+n^2 = \frac{n(n+1)(2n+1)}{6} = \sum_{r=1}^{n} r^2$$

$$1^3+2^3+\dots+n^3 = \left(\frac{n(n+1)}{2}\right)^2 = \sum_{r=1}^{n} r^3.$$

eg: ex: 2.11

$$\int_1^3 x.dx.$$

$\boxed{a=1}$ $\boxed{b=3}$ $\boxed{f(x)=x}$

$$h = \frac{b-a}{n} = \frac{3-1}{n} = \boxed{\frac{2}{n}}$$

$$f(a+rh) = f\left(1+r\left(\frac{2}{n}\right)\right)$$

$$= f\left(1+\frac{2r}{n}\right) = f(x)$$

$$f\left(1+\frac{2r}{n}\right) = \left(1+\frac{2r}{n}\right)$$

$$\int_1^3 x.dx = \lim_{n\to\infty} \sum_{r=1}^{n} h f(a+rh)$$

$$= \lim_{n\to\infty} \sum_{r=1}^{n} \frac{2}{n} f\left(1+\frac{2r}{n}\right)$$

$$= \lim_{n\to\infty} \sum_{r=1}^{n} \frac{2}{n} + \sum_{r=1}^{n} \cancel{\left(1+\frac{2r}{n}\right)} \frac{4r}{n^2}$$

$$= \lim_{n\to\infty} \frac{2}{n}\sum_{r=1}^{n} 1 + \frac{4}{n^2}\sum_{r=1}^{n} r$$

$$\sum_{r=1}^{n} 1 = n$$

$$= \lim_{n\to\infty} \frac{2}{\cancel{n}} \times \cancel{n} + \frac{4}{n^2} \frac{n(n+1)}{2}$$

$$= \lim_{n\to\infty} 2 + \frac{4}{\cancel{n^2}} \left(\frac{\cancel{n^2}\left(1+\frac{1}{n}\right)}{2}\right)$$

$$= \lim_{n\to\infty} 2 + \frac{\cancel{4}^{2}\left(1+\frac{1}{\infty}\right)}{\cancel{2}}$$

$$= 2 + 2(1+0)$$

$$= 4$$

$$\int_1^3 x\,dx$$

$$= \left(\frac{x^2}{2}\right)_1^3$$

$$= \frac{9}{2} - \frac{1}{2}$$

$$= \frac{8}{2} = 4$$

① $\int_0^1 (x+4)\cdot dx$

$\boxed{a=0}$ $\boxed{b=1}$ $f(x) = x+4.$

$h = \frac{b-a}{n} = \frac{1-0}{n} = \boxed{1/n} \to h$

$f(a+rh) = f(0+r\,1/n)$

$= f\left(\frac{r}{n}\right)$

$= \boxed{\frac{r}{n}+4}$

$\int_0^1 (x+4)\cdot dx = \lim_{n\to\infty} \sum_{r=1}^{n} h f(a+rh)$

$= \lim_{n\to\infty} \sum_{r=1}^{n} 1/n \left(\frac{r}{n}+4\right)$

$\left(\frac{r}{n^2} + \frac{4}{n}\right)$

$= \lim_{n\to\infty} \sum_{r=1}^{n} \frac{r}{n^2} + \sum_{r=1}^{n} \frac{4}{n}$

$= \lim_{n\to\infty} \frac{1}{n^2} \sum_{r=1}^{n} r + \frac{4}{n} \sum_{r=1}^{n} 1$

$= \lim_{n\to\infty} \frac{1}{n^2} \frac{n(n+1)}{2} + \frac{4}{\cancel{n}} \times \cancel{n}$

$= \lim_{n\to\infty} \frac{1}{\cancel{n^2}} \frac{\cancel{n^2}(1+1/n)}{2} + 4$

$= \frac{1+1/\infty}{2} + 4$

$= 1/2 + 4$

$\int_0^1 (x+4)\cdot dx = \boxed{9/2}$

Checking

$\int_0^1 (x+4)\cdot dx$

$= \left[\frac{x^2}{2} + 4x\right]_0^1$

$= 1/2 + 4 = 9/2$

② $\int_0^1 x^2 \cdot dx$

$\boxed{a=0}$ $\boxed{b=1}$ $h = \frac{b-a}{n} = \boxed{1/n}$

$f(x) = x^2$

$\boxed{f(a+rh) = \frac{r^2}{n^2}}$

$\int_0^1 x^2\cdot dx = \lim_{n\to\infty} \sum_{r=1}^{n} h f(a+rh)$

$= \lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{n} \frac{r^2}{n^2}$

$= \lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{n^3} r^2$

$= \lim_{n\to\infty} \frac{1}{n^3} \sum_{r=1}^{n} r^2$

$= \lim_{n\to\infty} \frac{1}{n^3} \frac{n(n+1)(2n+1)}{6}$

$= \lim_{n\to\infty} \frac{1}{\cancel{n^3}} \frac{\cancel{n^3}(1+1/n)(2+1/n)}{6}$

$= \frac{(1+1/\infty)(2+1/\infty)}{6}$

$= \frac{1\times 2}{6} = 2/6 = \boxed{1/3}$

checking

$\int_0^1 x^2\cdot dx$

$= \frac{x^3}{3} = \boxed{\frac{1}{3}}$

③. $\int_1^3 (2x+3)\,dx$

$\boxed{a=1}$ $\boxed{b=3}$ $h = \frac{b-a}{n} = \frac{3-1}{n}$

$f(a+rh)$ $\boxed{h = 2/n}$

$= f\left(1+r\left(\frac{2}{n}\right)\right)$

$= f\left(1+\frac{2r}{n}\right)$

$= 2\left(1+\frac{2r}{n}\right)+3$

$= \boxed{\left(2+\frac{4r}{n}\right)+3}$

$\int_0^2 (2x+3)\,dx = \lim_{n\to\infty} \sum_{r=1}^{n} hf(a+rh)$

$= \lim_{n\to\infty} \sum_{r=1}^{n} \frac{2}{n}\left(2+\frac{4r}{n}\right)+3$

$= \lim_{n\to\infty} \sum_{r=1}^{n} \left(\frac{4}{n}+\frac{8r}{n^2}\right)+3$

$= \lim_{n\to\infty} \frac{4}{n}\sum_{r=1}^{n} 1 + \frac{8}{n^2}\sum_{r=1}^{n} r + 3$

$= \lim_{n\to\infty} \frac{4}{\cancel{n}} \times \cancel{n} + \frac{8}{n^2}\frac{n(n+1)}{2} + 3$

$= \lim_{n\to\infty} 4 + \frac{8}{\cancel{n^2}}\frac{\cancel{n^2}(1+1/n)}{2} + 3$

$= \lim_{n\to\infty} 4 + 8\frac{(1+1/n)}{2} + 3$

$= 4 + 8\frac{(1+0)}{2} + 3$

$= 4 + \frac{\cancel{8}^4}{\cancel{2}} + 3$

$=$

Checking:

$\left[\frac{\cancel{2}x^2}{\cancel{2}} + 3x\right]_1^3$

$= \left[(3)^2 + 3(3)\right] - \left[1 + 3\right]$

$= 9 + 9 - 4$

$= \boxed{14}$