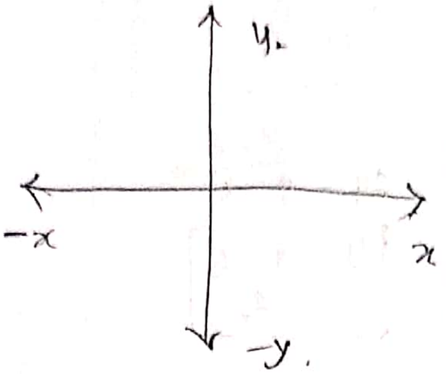
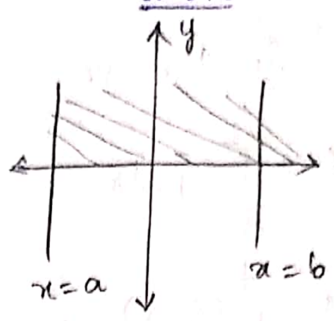
18/12/19.

Chapter - 3

Integral calculus - II.

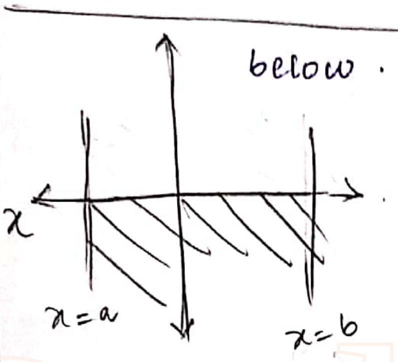


above



$$A = \int_a^b +y \cdot dx$$

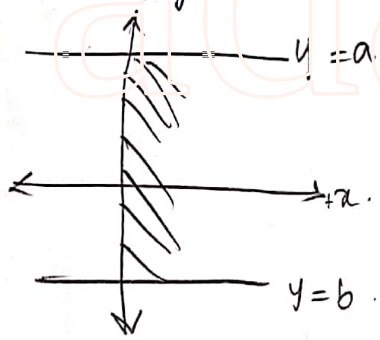
below.



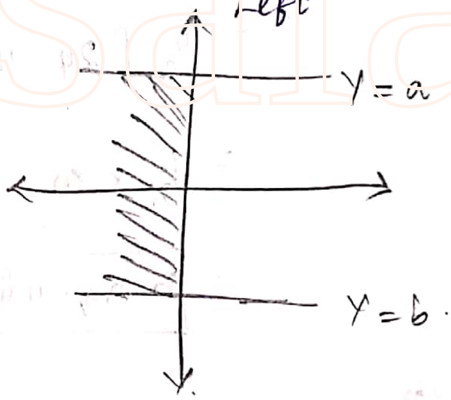
$$A = \int_a^b -y \cdot dx.$$

x-axis

Right



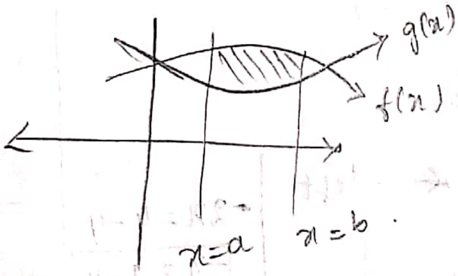
Left.



$$A = \int_a^b x \cdot dy.$$

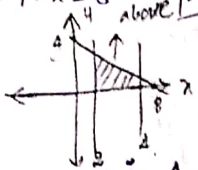
y-axis.

$$A = \int_a^b -x \cdot dy.$$



$$A = \int_a^b f(x) - g(x)\,dx$$

①. $2y+x=8$ above $\boxed{x=2}$ $\boxed{x=4}$



x-axis ↑ above.

$x=0$
$2y=8$
$\boxed{y=4}$
$y=0$ $\boxed{x=8}$
$2y=8-x$
$\boxed{y=\frac{8-x}{2}}$

$$A=\int_a^b y\,dx$$

$$A=\int_2^4 \frac{(8-x)}{2}\,dx$$

$$=\frac{1}{2}\left[8x-\frac{x^2}{2}\right]_2^4$$

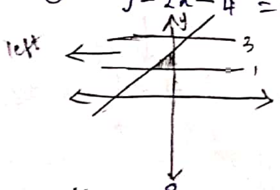
$$=\frac{1}{2}\left[32-\frac{16}{2}\right]-\left[16-\frac{4}{2}\right]$$

$$=\frac{1}{2}\left[24-14\right]$$

$$=\frac{1}{2}\times 10$$

$$=\boxed{5\text{ sq. units}}$$

② $y-2x-4=0$ $\boxed{y=1}$ $\boxed{y=3}$



y - axis ← left.

$x=0$
$\boxed{y=4}$
$y=0$
$-2y-4=0$
$-2y=4$
$\boxed{x=-2}$

$-2x=4-y$
$\boxed{-x=\frac{4-y}{2}}$

$$A=\int_a^b -x\,dy$$

$$=\int_1^3\left(\frac{4-y}{2}\right).dy$$

$$=\frac{1}{2}\left[4y-\frac{y^2}{2}\right]_1^3$$

$$=\frac{1}{2}\left(\left(12-\frac{9}{2}\right)-\left(4-\frac{1}{2}\right)\right)$$

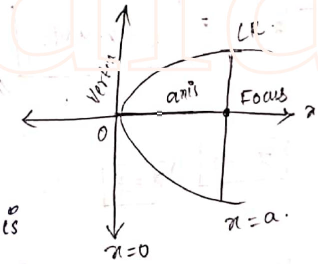
$$=\frac{1}{2}\left(12-\frac{9}{2}-4+\frac{1}{2}\right)$$

$$=\frac{1}{2}(8-4)$$

$$=\frac{1}{2}\times 4$$

$$=\boxed{2\text{ sq. units}}$$

③ $y^2=4ax$ & Latus Rectum.



x-axis

$y^2=4ax$
$y=\sqrt{4ax}$
$\boxed{y=2a^{1/2}.x^{1/2}}$

$$A=2\int_0^a y.dx$$

$$=2\int_0^a 2a^{1/2}.x^{1/2}.dx$$

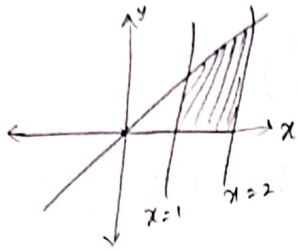
$$\frac{1}{2}+\frac{3}{2}=\frac{4}{2}=\boxed{2}$$

$$=4a^{1/2}\left(\frac{x^{3/2}}{3/2}\right)_0^a$$

$$=\frac{8}{3}a^{1/2}\times(a^{3/2}-0)$$

$$=\boxed{\frac{8a^2}{3}\text{ sq. units}}$$

(4) $y = x$ $\boxed{x=1}$, $\boxed{x=2}$



x-axis ↑ above.

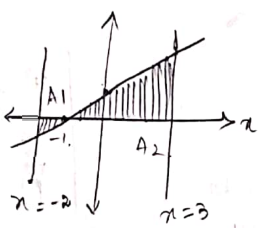
$A = \int_a^b y.dx$

$A = \int_1^2 x.dx$

$= \left(\frac{x^2}{2}\right)_1^2$

$= \left[\frac{4}{2} - \frac{1}{2}\right] = \boxed{3/2 \text{ sq. units}}$

(5) $y - 1 = x$, $\boxed{x=-2}$ $\boxed{x=3}$



x-axis

↓ ↑

$A = A_1 + A_2$

$= \int_{-2}^{-1} -y.dx + \int_{-1}^{3} y.dx$ $\boxed{y = x+1}$

$= \int_{-2}^{-1} -(x+1).dx + \int_{-1}^{3}(x+1).dx$

$= -\left[\frac{x^2}{2} + x\right]_{-2}^{-1} + \left[\frac{x^2}{2} + x\right]_{-1}^{3}$

$= -\left[\left(\frac{1}{2} - 1\right) - \left(\frac{4}{2} - 2\right)\right] + \left[\left(\frac{9}{2} + 3\right) - \left(\frac{1}{2} - 1\right)\right]$

$= \left[-\frac{1}{2} + 1 + \frac{4}{2} - 2\right] + \left[\frac{9}{2} + 3 - \frac{1}{2} + 1\right]$

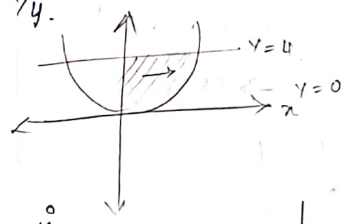
$= +\frac{1}{2} + 3$

$= \boxed{1 + \frac{1}{2} \text{ sq. units}}$

(6) $y = 4x^2$ → It shows that it is not x-axis

$\boxed{x=0}$ $y = 0$ $y = 4$.

$x^2 = 4/y$.



Y-axis →

$A = \int_0^4 x.dy$.

$x^2 = y/4$

$x = \frac{y^{1/2}}{2}$

$$= \int_0^4 \frac{y^{1/2}}{2} \cdot dy$$

$$= \frac{1}{2}\left[\frac{y^{3/2}}{3/2}\right]_0^4$$

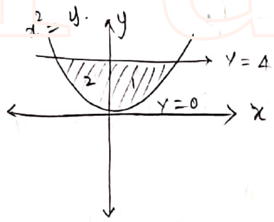
$$= \frac{1}{3}\left[4^{3/2} - 0\right)$$

$$= \frac{1}{3} \times (2^2)^{3/2}$$

$$= \frac{1 \times 8}{3}$$

$$= \boxed{8/3 \text{ sq. units}}$$

(7). $y = x^2$ & $\boxed{y = 4}$



(open upward)

y-axis → (Right)

$A = 2\int_a^b x \cdot dy$

$= 2\int_0^4 y^{1/2} \cdot dy$

$y = x^2$
$x = y^{1/2}$

$4^{3/2} = (2^2)^{3/2}$
$= 8$

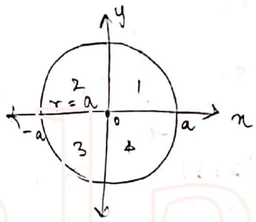
$$= 2\left(\frac{y^{3/2}}{3/2}\right)_0^4$$

$$= \frac{4}{3}\left(4^{3/2} - 0\right)$$

$$= \frac{4}{3} \times (8 - 0)$$

$$= \boxed{\frac{32}{3} \text{ sq. units}}$$

eg: 3.7



$x^2 + y^2 = a^2$

Put $y = 0 \Rightarrow x^2 = a^2$

$\boxed{x = \pm a}$

x-axis

$A = 4\int_a^b y \cdot dx$

$y^2 = a^2 - x^2$
$y = \sqrt{a^2 - x^2}$

$A = 4\int_0^a \sqrt{a^2 - x^2} \cdot dx$

$\left[\int \sqrt{a^2 - x^2} \cdot dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right.$

$$= 4\left[\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a$$

$$= 4\left[\frac{a}{2}\times 0 + \frac{a^2}{2}\sin^{-1}\frac{a}{a} - 0\right)$$

$$= 4\left[\frac{a^2}{2}\times\frac{\pi}{2}\right]$$

$$= \boxed{\pi a^2 \text{ sq. units}}$$

$\sin\pi/2 = 1$

$\pi = \sin^{-1}(1)$

eg 3.6

$y = |x+3|$. $\int_{-6}^{0} |x+3| \, dx$

$$y = \begin{cases} +(x+3) & x \geq -3 \\ -(x+3) & x < -3 \end{cases}$$

x	-3	-2	-1	0
y	0	1	2	3

x	-4	-5	-6
y	1	2	3

x-axis:

$$\int_{-6}^{0} |x+3| \, dx = \int_{-6}^{-3} -(x+3).\,dx + \int_{-3}^{0} (x+3).\,dx$$

$$= -\left(\frac{x^2}{2}+3x\right)_{-6}^{-3} + \left(\frac{x^2}{2}+3x\right)_{-3}^{0}$$

$$= -\left(\left(\frac{9}{2}-9\right) - \left(\frac{36}{2}-18\right)\right] + 0 - \left(\frac{9}{2}-9\right)$$

$$= -\frac{9}{2} + 9 - \frac{36}{2} + 18 - \frac{9}{2} + 9.$$

$$= 36 - \frac{54}{2}$$

$$= 36 - 27 = \boxed{9 \text{ sq. units}}$$

Ex : 3.2

Formula:

$\boxed{MC = C'(x)}$

1. Marginal cost $MC = \frac{dc}{dx}$, ($C \to$ cost)
2. $C = \int (MC)\,dx + k$, [$k \to$ fixed cost]
3. Average cost $AC = \frac{C}{x}$
4. Marginal Revenue function $MR = \frac{dR}{dx}$ [$R \to$ total Revenue function]
5. $R = \int (MR).\,dx + k$
6. Demand function $P = \frac{R}{x}$
7. If $P \to$ Profit, $P = \int (MR - MC).\,dx + k$.
8. Total Inventory cost $= C_1 \int_0^T I(x).\,dx$

C_1 = Holding cost , T = time period.
$I(x)$ = Inventory on Hand

(9) Amount of annuity after N payment.

$$A = \int_0^N Pe^{rt}.dt \qquad \boxed{r = \text{interest}}$$

(10) Total sale $= \int_0^r f(t).dt \qquad \boxed{r = \text{time}}$

(11) Elasticity of demand

$$\boxed{\eta_d = \frac{-P}{x}.\frac{dx}{dP}}$$

(12) $\frac{Ex}{Ey} = \frac{x\,dy}{y.dx}$

(13) Consumer surplus

$$CS = \int_0^{x_0} f(x).dx - x_0p_0 \qquad \boxed{f(x) = \text{demand}}$$

(14) Producer's surplus.

$$P_s = x_0p_0 - \int_0^{x_0} g(x)\,dx \qquad \boxed{g(x) = \text{supply}}$$

ex : 3.2

(1) Total cost $\boxed{r = 300}$

$= 10{,}000 + \int_0^r f(x).dx$

$= 10{,}000 + \int_0^{300} (2x - 240).dx$

$= 10{,}000 + \left[\frac{2x^2}{2} - 240x\right]_0^{300}$

$= 10{,}000 + (90000 - 72{,}000) - 0$

$= 10{,}000 + 18{,}000$

$= \boxed{28{,}000}$ //

(3) $\frac{dc}{dx} = 100 - 10x + 0.1x^2 = MC$

$C = \int (MC)\,dx + k$. $k = 500$

Given $\boxed{\text{fixed cost} = 500}$

$C = \int (100 - 10x + 0.1x^2) + 500$

No output $x = 0$, $C = 500$ $\therefore k = 500$

$C = 100x - \frac{\overset{5}{\cancel{10}}x^2}{\cancel{2}} + 0.1\frac{x^3}{3} + 500$

$C(x) = 100x - 5x^2 + 0.1\frac{x^3}{3} + 500$

$$AC = \frac{C}{x} = \boxed{100 - 5x + 0.1\frac{x^2}{3} + \frac{500}{x}}$$

Fixed cost $\boxed{x=0}$ $\boxed{C=0}$

⑦. $MC = 300\,x^{2/5}$ $\therefore \boxed{K=0}$

$C = \int (MC)\,dx + k.$

$= \int 300x^{2/5} + 0$

$= \dfrac{300\,x^{7/5}}{7/5}$

$\boxed{C(x) = \dfrac{1500}{7}\,x^{7/5}}$

$AC = \dfrac{C}{x} = \dfrac{1500}{7}\,x^{7/5}$

$\boxed{AC = \dfrac{1500}{7}\,x^{2/5}}$

⑧ $MC = \dfrac{a}{\sqrt{ax+b}}$ Given $C=0,\ x=0$

$C = \int (MC)\,dx + k$

$C = \int \dfrac{a}{\sqrt{ax+b}}\,dx + k$

$= a\int (ax+b)^{-1/2}\,dx + k$

$= a\left[\dfrac{(ax+b)^{1/2}}{\frac{1}{2}\times(a+0)}\right] + k$

$\boxed{C = 2\sqrt{ax+b} + k.}$

Given $\boxed{C=0}$ $\boxed{x=0}$

$0 = 2\sqrt{0a+b} + k$

$\boxed{k = -2\sqrt{b}}$

$\therefore \boxed{C = 2\sqrt{ax+b} - 2\sqrt{b}}$

⑨ $C'(x) = \dfrac{x^2}{200} + 4 = MC.$

Integrate on both side. $\int \dfrac{dc}{dx} = \dfrac{x^2}{200} + 4$

$C = \int_a^b (MC)\cdot dx$

$C = \int_0^{200} \left(\dfrac{x^2}{200} + 4\right)\cdot dx$

$= \left[\dfrac{x^3}{600} + 4x\right]_0^{200}$

$= \dfrac{80,00,000}{600} + 800 - 0$

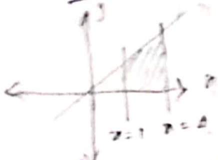
$= 13,333.3 + 800$

$= \boxed{14,133.3.}$

eg 3.1

$y = 4x + 3$

$\boxed{x=1}$ $\boxed{x=4}$



x-axis (above)

$A = \int_a^b y \cdot dx$

$= \int_1^4 4x + 3 \cdot dx$

$= \left[\frac{4x^2}{2} + 3x\right]_1^4$

$= 2(4)^2 + 3(4) - (2(1)+3)$

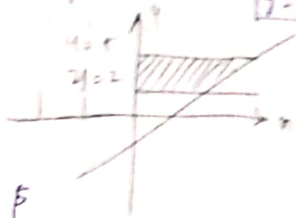
$= 2(16) + 12 - 2 - 3$

$= 32 + 12 - 5$

$= \boxed{39 \text{ sq. units}}$

eg 3.2

$x - 2y - 12 = 0$ $\boxed{y=2}$ $\boxed{y=5}$



$x - 2y - 12 = 0$

let $x = 0$

$x - 2y = 12$

$\boxed{x = +2y + 12}$

$= \int_2^5 (2y+12)\, dx$

$= \left[\frac{2y^2}{2} + 12x\right]_2^5$

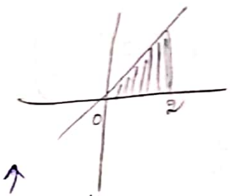
$= ((25+60) - (4+24))$

$= 85 - 28$

$= 57$ sq. units

eg 3.3

$y = 4 - x^2$ $x = 0$ $x = 2$



x-axis ↑

$A = \int_a^b y \cdot dx$

$= \int_0^2 4 - x^2 \cdot dx$

$= \left[4x - \frac{x^3}{3}\right]_0^2$

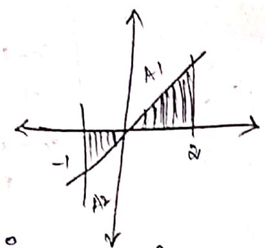
$= \left[4(2) - \frac{(2)^3}{3}\right] - 0$

$= 8 - \frac{8}{3}$

$A = 16/3$ sq. units

eg 3.4

$y = x \quad x = -1 \quad x = 2$



$-x$-axis , x-axis $\uparrow \downarrow$

$= A_1 + A_2$

$= \int_a^b -x \cdot dy + \int_a^b x \cdot dy$

$= \int_{-1}^{0} -x \cdot dy + \int_0^2 x \cdot dy$

$= -\left[\frac{x^2}{2}\right]_{-1}^{0} + \left[\frac{x^2}{2}\right]_0^2$

$= \left[0 - \left(\frac{-1}{2}\right)^2\right] + \left[\frac{(2)^2}{2} - 0\right]$

$= -\left(-\frac{1}{2}\right) + \frac{4}{2}$

$= \frac{1}{2} + \frac{4}{2}$

$\boxed{A = \frac{5}{2} \text{ sq. units}}$

eg 3.5

$y^2 = 8x$ & LR

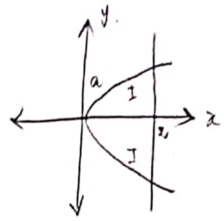
$y^2 = 4ax$

$y^2 = 8x$

$4a = 8$

$a = 8/4$

$a = 2$



x-axis $\uparrow$

$A = 2\int_a^b y \cdot dx$

$Y = \sqrt{2x} \quad x=0 \quad x=2$

$= 2\sqrt{2}\, x^{1/2}$

$= 2\int_0^2 2\sqrt{x}\; x^{1/2}$

$= 4\sqrt{2}\left[\frac{x^{3/2}}{3/2}\right]_0^2$

$= 4\sqrt{2} \times \frac{2}{3}\left[(2)^{3/2} - 0\right]$

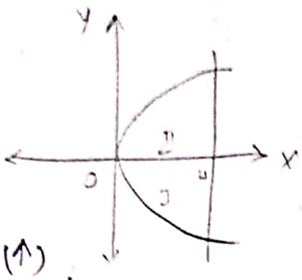
$= \frac{8}{3}(2)^{1/2} \cdot (2)^{3/2}$

$= \frac{8}{3}(2)^2$

$\boxed{A = \frac{32}{3} \text{ sq. units}}$

eg 3.8

$y^2 = 16x \quad x = 4 \quad x = 0$



x-axis (↑)

$A = 2\int_a^b y \cdot dx$

$y^2 = 16x$

$y = \sqrt{16x}$

$y = 4 \cdot x^{1/2}$.

$= 2\int_0^4 4\,(x)^{1/2} \cdot dx$

$= 8\left[\frac{x^{3/2}}{3/2}\right]_0^4$

$= 8 \times \frac{2}{3}\left[(4)^{3/2} - (0)^{3/2}\right]$

$= 16/3\,(4)^{3/2}$

$\boxed{A = \frac{128}{3} \text{ sq. units.}}$

ex. 3.2

(10) $MC = 5 + 3e^{-0.03x}$

$R = \int (MR) \cdot dx$

$R = \int_0^{100} (5 + 3e^{-0.03x}) \cdot dx$

$= \left(5x + \frac{3e^{-0.03x}}{-0.03}\right)_0^{100}$

$= 500 + \frac{e^{-3}}{-0.01} - \left(0 + \frac{e^0}{-0.01}\right)$

$= 500 + \frac{0.05}{-0.01} + \frac{1}{0.01} \times \frac{100}{100}$

$= 500 - \frac{5}{1} + \frac{100}{1}$

$= 595$

Total revenue $= 595 \times 1000$

$= \boxed{595000}$

(11) $MR = 9 - 4x^2$

$R = \int (MR)\, dx + K$.

$R = \int (9 - 4x^2) \cdot dx + k$

$R = 9x - \frac{4x^3}{3} + k$

$x = 0, \quad R = 0 \Rightarrow \boxed{k = 0}$

$$\boxed{R = 9x - \frac{4x^3}{3}}$$

demand function $P = R/x$

$$\boxed{P = 9 - \frac{4x^2}{3}}$$

(13) $MR = \frac{4}{(2x+3)^2} - 1$

$$R = \int \left(\frac{(4)}{(2x+3)^2} - 1\right) \cdot dx + k.$$

$$= \int \left(4 \frac{1}{(2x+3)^2} - 1\right) \cdot dx + k$$

$$\int \frac{1}{x^2} \cdot dx = \frac{-1}{1x}$$

$$= \left[4 \times \frac{-1}{1(2x+3) \times 2} - x\right] + k$$

$$R = \frac{-2}{2x+3} - x + k.$$

$x = 0 \quad R = 0$

$$0 = \left(\frac{-2}{(0+3)} - 0\right) + k$$

$$\boxed{k = 2/3}$$

$$R = \frac{-2}{(2x+3)} - x + 2/3$$

$$= \frac{-2}{(2x+3)} - x + 2/3.$$

$$AR = \frac{R}{x} = \frac{-2}{x(2x+3)} - 1 + \frac{2}{3x}$$

$$= \frac{-2}{x(2x+3)} + \frac{2}{3x} - 1$$

$$= \frac{-6x + 2x(2x+3)}{3x^2(2x+3)} - 1$$

$$= \frac{\cancel{-6x} + 4\cancel{x^2} + \cancel{6x}}{3\cancel{x^2}(2x+3)} - 1$$

$$= \boxed{\frac{4}{(6x+9)} - 1}$$

(12) $MR = 20e^{-x/10}\left(1 - x/10\right)$

$$R = \int (MR) \cdot dx + k$$

$$R = \int 20e^{-x/10}\left(-x/10 - 1\right) \cdot dx + k$$

$$\left[\int e^{ax}(a f(x) + f'(x)) \cdot dx = e^{ax} f(x)\right]$$

$$= 20 \int e^{-x/10}\left(-1/10(x) + 1\right) \cdot dx + k$$

$\boxed{f(x) = x}$ $\boxed{f'(x) = 1}$

$$\boxed{R = 20e^{-x/10}(x) + k}$$

$x = 0 \Rightarrow R = 0$

$\boxed{k = 0}$

$$0 = 20e^0(0) + k$$

$$R = 20xe^{-x/10}$$

Demand function $P = R/x = \boxed{20e^{-x/10}}$

(19) $MR = 20 - 5x + 3x^2$

$\int MR = \int 20 - 5x + 3x^2$

$R = 20x - \frac{5x^2}{2} + \frac{3x^3}{3} + k$

$R = 20x - \frac{5x^2}{2} + x^3 + k$

$x = 0 \quad R = 0 \quad k = 0$

$\boxed{R = 20x - \frac{5x^2}{2} + x^3}$

(20) $MR = 14 - 6x + 9x^2$

$\int MR = \int 14 - 6x + 9x^2$

$R = 14x - \frac{6x^2}{2} + \frac{9x^3}{3} + k$

$R = 14x - 3x^2 + 3x^3 + k$

When $x = 0 \quad R = 0 \quad k = 0$

$\boxed{R = 14x - 3x^2 + 3x^3}$

Demand function $P = \frac{R}{x}$

$\boxed{P = 14 - 3x + 3x^2}$

(14) $c'(x) = 5 + 0.13x$

$\int c'(x) = \int (5 + 0.13x) \cdot dx$

$\boxed{C = 5x + \frac{0.13x^2}{2} + k}$

$\boxed{x = 0}$, $\boxed{C = 120}$

$120 = 0 + 0 + k$

$k_1 = 120$

$\boxed{C = 5x + \frac{0.13x^2}{2} + 120}$

Given $R'(x) = 18$

$R = 18x + k_2$

$x = 0 \quad R = 0 \quad \boxed{k_2 = 0}$

$\boxed{R = 18x}$

Profit $f(x) = MR - MC$

$= R - C$

$= 18x - 5x - \frac{0.13x^2}{2} - 120$

$= \boxed{13x - 0.065x^2 - 120}$

(15) $R'(x) = 1500 - 4x - 3x^2$

$R = 1500x - \frac{\cancel{4}^{2}x^2}{\cancel{2}} - \frac{\cancel{3}x^3}{\cancel{3}} + k$

$R = 1500x - 2x^2 - x^3 + k$

$x = 0 \Rightarrow R = 0 \Rightarrow \boxed{k = 0}$

$\therefore R = 1500x - 2x^2 - x^3$

$AR = \frac{R}{x}$

$= \boxed{1500 - 2x - x^2}$

(16) $MR = 10 + 3x - x^2$

$\int MR = \int 10 + 3x - x^2$

$R = 10x + \frac{3x^2}{2} - \frac{x^3}{3} + k$

$R = 0,\ x = 0,\ k = 0$

$= 10x + \frac{3x^2}{2} - \frac{x^3}{3}$

Demand function

$P = \frac{R}{x}$

$= \frac{10x}{x} + \frac{3x^2}{2x} - \frac{x^3}{3x}$

$\boxed{P = 10 + \frac{3x}{2} - \frac{x^2}{3}}$

(17) $MC = \frac{14000}{\sqrt{7x+4}}$

fixed cost

$FC = 18000$

$C = \int \frac{14000}{\sqrt{7x+4}}\, dx + k$

$d(7x+4) = 7(1) + 0$
$= 7.$

$= 2000 \int \frac{7}{\sqrt{7x+4}}\, dx + k.$

$= 2000 \times 2\sqrt{7x+4} + k$

$= 4000\sqrt{7x+4} + k.$

$x = 0 \Rightarrow C = 18000$

$18000 = 4000\sqrt{4}^{\,2} + k$

$18000 = 8000 + k$

$\boxed{k = 10{,}000}$

$C = 4000 \cdot \sqrt{7x+4} + 10{,}000$

$A.C = \frac{C}{x}$

$= \frac{4000}{x}\sqrt{7x+4} + \frac{10{,}000}{x}.$

18 (A) (i)

MC $\propto x$

$$\frac{dc}{dx} = k_1 x$$

$dc = k_1 x \cdot dx$

$\int$ on both side.

$C = k_1 \frac{x^2}{2} + k_2$

$x = 0$ $\boxed{C = 5,000}$

$5000 = k_2$.

$C = k_1 \frac{x^2}{2} + 5000$

$x = 50$ $C = 5625$

$5625 = \frac{k_1 (50)^2}{2} + 5000$

$5625 - 5000 = \frac{k_1 (2500)}{2}$

$\frac{625}{1250} = k_1$

$\boxed{k_1 = 1/2}$

$$\boxed{k = \frac{x^2}{4} + 5000}$$

(5) $P = 1000$ $r = 5\% = 0.05$ $n = 5$

Amount after annuity $N = \int_0^N P e^{rt} . dt$.

$= \int_0^5 1000 e^{0.05t} . dt$

$= 1000 \left(\frac{e^{0.05t}}{0.05} \right)_0^5$

$= \frac{1000}{0.05} (e^{0.25} - e^0)$

$= \frac{1000}{0.05} (1.284 - 1)$

$= \frac{1000}{0.05} (0.284)$

$= \frac{284}{0.05}$

$= \boxed{5680.}$

(4) $f(x) = 500 - 0.03x^2$

Holding cost $C_1 = 0.3$ $t = 30$

Total inventory cost

$TIC = C_1 \int_0^t I(x) . dx$.

$$= 0.3 \int_0^{30} (500 - 0.03x^2) \, dx$$

$$= 0.3 \left[500x - 0.03\frac{x^3}{3}\right]_0^{30}$$

$$= 0.3 \left(15000 - 0.01 \times (30)^2 - 0\right]$$

$$= 0.3 \left[15000 - 270\right]$$

$$= 0.3 \times 14730$$

$$= \boxed{4419}$$

(3) $\eta_d = \frac{4 - x}{x}$

$$\frac{-p}{x} \cdot \frac{dx}{dp} = \frac{4 - x}{x}$$

$(\times)(-1)$

$$\frac{p}{x} \cdot \frac{dx}{dp} = \frac{x - 4}{x}$$

$$\frac{dx}{x - 4} = \frac{dp}{p}$$

$d(x-4) = 1$

$\int$ on bs.

$$\int \frac{dx}{x - 4} = \int \frac{dp}{p}$$

$$\log|x - 4| = \log|p| + \log|c|$$

$$\log|x - 4| = \log|pc|$$

$$|x - 4| = |pc|$$

$$\boxed{x - 4 = pc}$$

$p = 4, \; x = 2$

$$2 - 4 = 4c$$

$$c = -2/4 = -1/2$$

$$x - 4 = p \times -1/2$$

$(\times) - 2$

$$-2x + 8 = p$$

$$\boxed{p = 8 - 2x}$$

$$\boxed{R = px = 8x - 2x^2}$$

Scanned by CamScanner

f(x)

Consumer Surplus (demand)

$$CS = \int_0^{x_0} f(x)\,dx - x_0p_0$$ [equilibrium $P_d = P_s$]

Producer's Surplus (Supply) f(x)

$$P_s = x_0p_0 - \int_0^{x_0} f(x)\,dx.$$

Exercise 3.3

① $P = 50 - 2x$, $x = 20$

$P = 50 - 40 = 10$

$\boxed{p = 10}$

$$CS = \int_0^{x_0} f(x)\,dx - x_0p_0$$

$$= \int_0^{20} (50-2x)\,dx - 200$$

$$= 150 - \frac{2x^2}{2}\Big)_0^{20} - 200$$

$$= 1000 - 400 - 200$$

$\boxed{P_d = 400}$

③ $P_d = 85 - 5x$, $P_s = 3x - 35$

Given equilibrium

$P_d = P_s$

$85 - 5x = 3x - 35$

$85 + 35 = 5x + 3x$

$120 = 8x$

$\boxed{x = 15}$

$P = 85 - 5(15) = 10$

$$CS = \int_0^{x_0} f(x)\,dx - x_0p_0$$

$$= \int_0^{15} (85-5x) - 150$$

$$= \left(85x - \frac{5x^2}{2}\right)_0^{15} - 150$$

$$= 85(15) - \frac{5\times 225}{2} - 0 - 150$$

$= 1275 - 562.5 - 0.150$

$\boxed{CS = 562.75}$

④ $P = e^{-x}$, $P = 0.5 = 1/2$

$0.5 = e^{-x}.$

$1/2 = e^{-x} = \frac{1}{e^x}.$

$2 = e^x.$

taking log

$\boxed{x = \log 2}$

$$c_s = \int_0^{x_0} f(x)dx - x_0p_0$$

$$= \int_0^{\log 2} e^{-x}.dx - \log 2 \times 1/2$$

$$= \left[\frac{e^{-x}}{-1}\right]_0^{\log 2} - \tfrac{1}{2} \log 2$$

$$= -1\left[e^{-\log 2} - e^0\right] - \tfrac{1}{2}\log 2$$

$$= -\left(\frac{1}{e^{\log 2}} - 1\right) - \tfrac{1}{2}\log 2.$$

$$= -\left(\tfrac{1}{2} - 1\right) - \tfrac{1}{2}\log 2$$

$$= \tfrac{1}{2} - \tfrac{1}{2}\log 2$$

$$= \boxed{\tfrac{1}{2}(1 - \log 2)}$$

5. $P = 7 + x \qquad x = 5$

$P = 12$

$$P_s = x_0p_0 - \int_0^{x_0} f(x)\,dx$$

$$= 60 - \int_0^5 (7+x)\,dx$$

$$= 60 - \left(7x + \frac{x^2}{2}\right)_0^5$$

$$= 60 - \left(35 + \frac{25}{2}\right) - 0$$

$$= 60 - (35 + 12.5)$$

$$= 60 - 47.5$$

$$= \boxed{12.5}$$

6. $P = \dfrac{36}{x+4}, \quad P = 6$

$$6 = \frac{36}{x+4}$$

$$x + 4 = 6$$

$$\boxed{x = 2}$$

$$\therefore c_s = \int_0^{x_0} f(x)\,dx - x_0p_0$$

$$= \int_0^2 \left(\frac{36}{x+4}\right) dx - 12$$

$$= 36\int_0^2 \left(\frac{1}{x+4}\right) dx - 12$$

$d(x+4) = 1$

$$= 36\left[\log|x+4|\right]_0^2 - 12$$

$$= 36 \log|2+4| - \log|0+4| - 12$$

$$= 36 \log 6 - \log 4 - 12$$

$$= 36 \log \frac{6}{4} - 12$$

$$= 36 \log \frac{3}{2} - 12$$

(8) $P_d = 1600 - x^2$ $P_s = 2x^2 + 400$

In equilibrium.

$P_d = P_s$

$1600 - x^2 = 2x^2 + 400$

$1600 - 400 = 2x^2 + x^2$

$3x^2 = 1200$

$x^2 = 400$

$\boxed{x = \pm 20}$

$P = 1600 - 20^2 = 1600 - 400$

$\boxed{P = 1200}$

$P_s = x_0p_0 - \int_0^{x_0} f(x).dx$

$= 24000 - \int_0^{20} (2x^2 + 400)_0^{20}$

$= 24,000 - \left(\frac{2x^3}{3} + 400x\right)_0^{20}$

$= 24000 - \left(\frac{2(20)^3}{3} + 400+20\right) - 0$

$= 24000 - (5333.3 + 8000)$

$= 24000 - 13333.3$

$= \boxed{10666.7}$

(9) $P_d = \frac{8}{x+1} - 2$, $P_s = \frac{x+3}{2}$

equi $P_d = P_s$

$\frac{8}{x+1} - 2 = \frac{x+3}{2}$

$\frac{8 - 2x - 2}{x+1} = \frac{x+3}{2}$

$\frac{6-2x}{x+1} = \frac{x+3}{2}$

$\frac{6-2x}{x+1} = \frac{x+3}{2}$

$12 - 4x = x^2 + 3x + x + 3$

$x^2 + 8x - 9 = 0$

$\boxed{x = 1}$ $\boxed{x = -9}$

$P = \frac{1+3}{2} = \frac{4}{2} = 2$

$C_s = \int_0^{x_0} f(x)\,dx - x_0p_0$

$= \int_0^1 \left(\frac{8}{x+1} - 2\right).dx - 2$

$= \left(8 \log|x+1| - 2x\right]_0^1 - 2$

$= (8 \log 2 - 2) - (8 \log 1 - 0) - 2$

$= 8\log 2 - 2 - 2$

$C_S = 8\log 2 - 4$

$P_S = x_0p_0 - \int_0^{x_0} f(x)\,dx$

$= 2 - \int_0^1 \frac{x+3}{2}\,dx$

$= 2 - \frac{1}{2}\left[\frac{x^2}{2} + 3x\right]_0^1$

$= 2 - \frac{1}{2}\left(\frac{1}{2} + 3\right) - 0$

$= 2 - \frac{1}{2}\left(\frac{7}{2}\right)$

$= 2 - \frac{7}{4}$

$\boxed{P_S = \frac{1}{4}}$

10

$x = \sqrt{100 - p}$ | $x = \frac{p}{2} - 10$

$\frac{p}{2} - 10 = \sqrt{100 - p}$

Square:

$\left(\frac{p}{2} - 10\right)^2 = 100 - p$

$\frac{p^2}{4} + \cancel{100} - 2 \times \frac{p}{2} \times 10 - \cancel{100} + p = 0$

$\frac{p^2}{4} - 9p = 0$

$p^2 - 36p = 0$

$p(p - 36) = 0$

$\boxed{p = 0}$ $\boxed{p = 36}$ ✓

$x = \sqrt{100 - 36} = \sqrt{64}$

$x = \pm 8$

$\boxed{x = 8}$

$x = \sqrt{100 - p}$

$x^2 = 100 - p$

$\boxed{P = 100 - x^2}$

$x = \frac{P}{2} - 10$

$\frac{P}{2} = x + 10$

$\boxed{P = 2x + 20}$

$C_S = \int_0^{x_0} f(x)\,dx - x_0p_0$

$= \int_0^8 (100 - x^2)\,dx - 288$

$= \left[100x - \frac{x^3}{3}\right]_0^8 - 288$

$= 800 - \frac{512}{3} - 288$

$= 512 - \frac{512}{3}$

$= \frac{1024}{3}$

$\boxed{C_S = 341.3}$

$P_s = x_0 p_0 - \int_0^{x_0} f(x)\,dx$

$= 288 - \int_0^8 (2x+20)\,dx$

$= 288 - \left(\frac{2x^2}{2} + 20x\right)_0^8$

$= 288 - (64 + 160 - 0)$

$= 288 - 224$

$= \boxed{64}$

(11) $P_d = 25 - 3x$

$P_s = 5 + 2x$

equi $P_d = P_s$

$25 - 3x = 5 + 2x$

$25 - 5 = 2x + 3x$

$20 = 5x$

$x = \frac{20}{5}$

$\boxed{x = 4}$

$= 5 + 2(4)$

$= 5 + 8$

$\boxed{P = 13}$

$C_s = \int_0^{x_0} f(x)\,dx - x_0 p_0$

$= \int_0^4 (25 - 3x)\,dx - 52$

$= \left[25x - \frac{3x^2}{2}\right]_0^4 - 52$

$= \left[100 - \frac{48}{2}\right] - 52$

$= 76 - 52$

$\boxed{C_s = 24 \text{ units.}}$

$P_s = x_0 p_0 - \int_0^{x_0} f(x)\,dx$

$= 52 - \int_0^4 (5 + 2x)\,dx$

$= 52 - \left[5x + \frac{2x^2}{2}\right]_0^4$

$= 52 - [20 - 16]$

$= 52 - 36$

$\boxed{P_s = 16 \text{ units}}$