

UNIT – 11 WAVES

TWO MARKS AND THREE MARKS:

01. What is meant by waves?

The disturbance which carries energy and momentum from one point in space to another point in space without the transfer of the medium is known as a wave.

02. Write down the types of waves.

a) **Mechanical wave** – Waves which require a medium for propagation are known as mechanical waves.

Examples: sound waves, ripples formed on the surface of water, etc.

b) **Non mechanical wave** – Waves which do not require any medium for propagation are known as non-mechanical waves.

Example: light

Further, waves can be classified into two types

a. Transverse waves b. Longitudinal waves

03. What are transverse waves? Give one example.

In transverse wave motion, the constituents of the medium oscillate or vibrate about their mean positions in a direction perpendicular to the direction of propagation (direction of energy transfer) of waves.

Example: light (electromagnetic waves)

04. What are longitudinal waves? Give one example.

In longitudinal wave motion, the constituent of the medium oscillate or vibrate about their mean positions in a direction parallel to the direction of propagation (direction of energy transfer) of waves.

Example: Sound waves travelling in air.

05. Define wavelength.

For **transverse waves**, the distance between two neighbouring crests or troughs is known as the wavelength.

For **longitudinal waves**, the distance between two neighbouring compressions or rarefactions is known as the wavelength.

The SI unit of wavelength is meter.

06. Write down the relation between frequency, wavelength and velocity of a wave.

Dimension of wavelength is, $[\lambda] = L$;

Frequency $f = \frac{1}{\text{Time period}}$,

which implies that the dimension of frequency is, $|f| = \frac{1}{|T|} = T^{-1}$

$$\Rightarrow [\lambda f] = [\lambda] [f] = LT^{-1} = [\text{Velocity}]$$

Therefore Velocity, $\lambda f = v$

Where v is known as the wave velocity or phase velocity. This is the velocity with which the wave propagates. Wave velocity is the distance travelled by a wave in one second.

07. What is meant by interference of waves?

Interference is a phenomenon in which two waves superimpose to form a resultant wave of greater, lower or the same amplitude.

08. Explain the beat phenomenon.

When two or more waves superimpose each other with slightly different frequencies, then a sound of periodically varying amplitude at a point is observed. This phenomenon is known as beats. The number of amplitude maxima per second is called beat frequency. If we have two sources, then their difference in frequency gives the beat frequency.

$$\text{Number of beats per second } n = |f_1 - f_2| \text{ per second.}$$

09. Define intensity of sound and loudness of sound.

The **intensity of sound** is defined as “the sound power transmitted per unit area taken normal to the propagation of the sound wave”.

The **loudness of sound** is defined as “the degree of sensation of sound produced in the ear or the perception of sound by the listener”.

10. Explain Doppler Effect.

When the source and the observer are in relative motion with respect to each other and to the medium in which sound propagates, the frequency of the sound wave observed is different from the frequency of the source. This phenomenon is called Doppler Effect.

11. Explain red shift and blue shift in Doppler Effect.

The spectral lines of the star are found to shift towards red end of the spectrum (**called as red shift**) then the star is receding away from the Earth. Similarly, if the spectral lines of the star are found to shift towards the blue end of the spectrum (**called as blue shift**) then the star is approaching Earth.

12. What is meant by end correction in resonance air column apparatus?

The antinodes are not exactly formed at the open end, we have to include a correction, called end correction. $L_1 + e = \frac{\lambda}{4}$ and $L_2 + e = \frac{3\lambda}{4}$

13. Sketch the function $y = x + a$. Explain your sketch.

- i) A combination of constant and direct
- ii) A fixed amount is added at regular intervals
- iii) $y = x + a$, a suitable conclusion statement would be that,
 - 1) Y is linear with x
 - 2) Y varies linearly with x
 - 3) Y is a linear function of x, y is the intercept

14. Write down the factors affecting velocity of sound in gases.

Pressure, Temperature, Density, Humidity and wind

15. What is meant by an echo? Explain.

1) An echo is a repetition of sound produced by the reflection of sound waves from a wall, mountain or other obstructing surfaces. The speed of sound in air at 20°C is 344 m s^{-1} . If we shout at a wall which is at 344 m away, then the sound will take 1 second to reach the wall.

2) After reflection, the sound will take one more second to reach us. Therefore, we hear the echo after two seconds. Scientists have estimated that we can hear two sounds properly if the time gap or time interval between each sound is $\left(\frac{1}{10}\right)^{\text{th}}$ of a second (persistence of hearing) i.e., 0.1 s. Then,

$$\text{Velocity} = \frac{\text{Distance travelled}}{\text{Time taken}} ; = \frac{2d}{t}$$

$$2d = 344 \times 0.1 = 34.1\text{m}; \quad d = 17.2 \text{ m}$$

The minimum distance from a sound reflecting wall to hear an echo at 20°C is 17.2 meter.

16. What is reverberation?

In a closed room the sound is repeatedly reflected from the walls and it is even heard long after the sound source ceases to function. The residual sound remaining in an enclosure and the phenomenon of multiple reflections of sound is called reverberation.

17. Write characteristics of wave motion.

- 1) For the propagation of the waves, the medium must possess both inertia and elasticity, which decide the velocity of the wave in that medium.
- 2) In a given medium, the velocity of a wave is a constant whereas the constituent particles in that medium move with different velocities at different positions. Velocity is maximum at their mean position and zero at extreme positions.
- 3) Waves undergo reflections, refraction, interference, diffraction and polarization

CONCEPTUAL QUESTIONS:

- 01. Why is it that transverse waves cannot be produced in a gas? Can the transverse waves can be produced in solids and liquids?**

Transverse waves travel in the form of crests and troughs and so involve change in shape. As gas no elasticity of shape, hence transverse waves cannot be produced in it. Yes, solids and liquids have elasticity so, transverse wave can be produced.

- 02. Why is the roar of our national animal different from the sound of a mosquito?**

Roaring of a national animal and tiger produces a sound of low pitch and high intensity or loudness, whereas the buzzing of mosquito produces a sound of high pitch and low intensity or loudness.

- 03. A sound source and listener are both stationary and a strong wind is blowing. Is there a Doppler effect?**

Yes, It does not matter whether the sound source or the transmission media are in motion.

- 04. In an empty room why is it that a tone sounds louder than in the room having things like furniture etc.**

Sound is a form of energy. The furniture which act as obstacles absorbs most of energy. So the intensity of sound becomes low but in empty room, due to the absence of obstacles the intensity of sound remains mostly same but we feel it louder.

- 05. How do animals sense impending danger of hurricane?**

Some animals are believed to be sensitive to be low frequency sound waves emitted by hurricanes. They can also detect the slight drops in air and water pressure that signal a storm's approach.

- 06. Is it possible to realize whether a vessel kept under the tap is about to fill with water?**

The frequency of the note produced by an air column is inversely proportional to its length. As the level of water in the vessel rises, the length of the air column above it decreases. It produces sound of decreasing frequency. i.e. the sound becomes shorter. From the shrillness of sound, it is possible to realize whether the vessel is filled with water. $V_{\min} = 11.71 \text{ms}^{-1}$

FIVE MARKS

01. Discuss how ripples are formed in still water.

1) A stone in a trough of still water, we can see a disturbance produced at the place where the stone strikes the water surface as shown in Figure. We find that this disturbance spreads out (diverges out) in the form of concentric circles of ever increasing radii (ripples) and strike the boundary of the trough.

2) This is because some of the kinetic energy of the stone is transmitted to the water molecules on the surface. Actually the particles of the water (medium) themselves do not move outward with the disturbance.

3) This can be observed by keeping a paper strip on the water surface. The strip moves up and down when the disturbance (wave) passes on the water surface. This shows that the water molecules only undergo vibratory motion about their mean positions.

02. Briefly explain the difference between travelling waves and standing waves.

S. No.	Progressive waves	Stationary waves
1	Crests and troughs are formed in transverse progressive waves, and compression and rarefaction are formed in longitudinal progressive waves. These waves move forward or backward in a medium i.e., they will advance in a medium with a definite velocity.	Crests and troughs are formed in transverse stationary waves, and compression and rarefaction are formed in longitudinal stationary waves. These waves neither move forward nor backward in a medium i.e., they will not advance in a medium.
2	All the particles in the medium vibrate such that the amplitude of the vibration for all particles is same.	Except at nodes, all other particles of the medium vibrate such that amplitude of vibration is different for different particles. The amplitude is minimum or zero at nodes and maximum at anti-nodes.
3	These wave carry energy while propagating.	These waves do not transport energy.

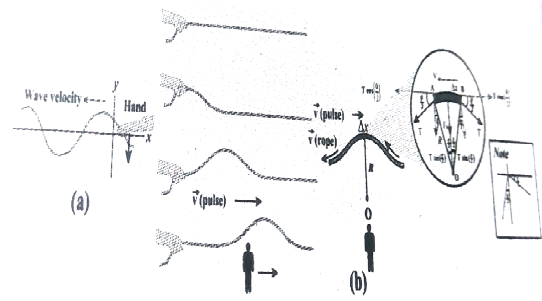
03. Show that the velocity of a travelling wave produced in a string is $v = \sqrt{\frac{T}{\mu}}$

1) Consider an elemental segment in the string as shown in the Figure. Let A and B be two points on the string at an instant of time. Let dl and dm be the length and mass of the elemental string, respectively. By definition, linear

mass density, μ is $\mu = \frac{dm}{dl}$ ----- 1

$dm = \mu dl$ ----- 2

2) The elemental string AB has a curvature which looks like an arc of a circle with centre at O, radius R and the arc subtending an angle θ at the origin O as shown in Figure. The angle θ can be written in terms of arc length and radius as $\theta = \frac{dl}{R}$. The centripetal acceleration supplied by the tension in the string is $a_{cp} = \frac{v^2}{R}$ ----- 3



3) Then, centripetal force can be obtained when mass of the string (dm) is included in equation (3)

$$F_{cp} = \frac{(dm)v^2}{R} \text{ ----- 4}$$

4) The centripetal force experienced by elemental string can be calculated by substituting equation (2) in equation (4) we get,

$$\frac{(dm)v^2}{R} = \frac{\mu v^2 dl}{R} \text{ ----- 5}$$

5) The tension T acts along the tangent of the elemental segment of the string at A and B. Since the arc length is very small, variation in the tension force can be ignored. We can resolve T into horizontal component $T \cos\left(\frac{\theta}{2}\right)$ and vertical component $T \sin\left(\frac{\theta}{2}\right)$.

6) The horizontal components at A and B are equal in magnitude but opposite in direction; therefore, they cancel each other. Since the elemental arc length AB is taken to be very small, the vertical components at A and B appears to act vertical towards the centre of the arc and hence, they add up. The net radial force F_r is $F_r = 2T \sin\left(\frac{\theta}{2}\right)$ -----6

7) Since the amplitude of the wave is very small when it is compared with the length of the string, the sine of small angle is approximated as $\sin\left(\frac{\theta}{2}\right) \approx \frac{\theta}{2}$. Hence, equation (6) can be written as $F_r = 2T \times \frac{\theta}{2} = T \theta$ -----7

8) But $\theta = \frac{dl}{R}$, therefore substituting in equation (7),

$$\text{we get } F_r = T \frac{dl}{R} \text{ -----8}$$

Applying Newton's second law to the elemental string in the radial direction, under equilibrium, the radial component of the force is equal to the centripetal force. Hence equating equation (5) and equation (8), we have

$$T \frac{dl}{R} = \mu v^2 \frac{dl}{R} \quad v = \sqrt{\frac{T}{\mu}} \text{ measured in ms}^{-1} \text{ -----9}$$

04. Describe Newton's formula for velocity of sound waves in air and also discuss the Laplace's correction.

1) Newton assumed that when sound propagates in air, the formation of compression and rarefaction takes place in a very slow manner so that the process is isothermal in nature.

2) That is, the heat produced during compression (pressure increases, volume decreases), and heat lost during rarefaction (pressure decreases, volume increases) occur over a period of time such that the temperature of the medium remains constant. Therefore, by treating the air molecules to form an ideal gas, the changes in pressure and volume obey Boyle's law,

$$PV = \text{Constant} \quad \text{----- 1}$$

3) Differentiating equation (1), we get $PdV + VdP = 0$ or

$$P = -V \frac{dP}{dV} = B_T \quad \text{----- 2}$$

where, B_T is an isothermal bulk modulus of air. Substituting equation (2) in

equation $V = \sqrt{\frac{B}{\rho}}$ the speed of sound in air is

$$V_T = \sqrt{\frac{B_T}{\rho}} = \sqrt{\frac{P}{\rho}} \quad \text{----- 3}$$

Since P is the pressure of air whose value at NTP (Normal Temperature and Pressure) is 76 cm of mercury, we have

$$P = (0.76 \times 13.6 \times 10^3 \times 9.8) \text{ N m}^{-2}$$

$\rho = 1.293 \text{ kg m}^{-3}$. here ρ is density of air

Then the speed of sound in air at Normal Temperature and Pressure (NTP) is

$$V_T = \sqrt{\frac{0.76 \times 13.6 \times 10^3 \times 9.8}{1.293}} = 279.80 \text{ ms}^{-1} \approx 280 \text{ ms}^{-1} \text{ (theoretical value)}$$

But the speed of sound in air at 0°C is experimentally observed as 332 ms^{-1} which is close upto 16% more than theoretical value

(Percentage error is $\frac{(332-280)}{332} \times 100\% = 15.6\%$) This error is not small.

Laplace's correction:

1) Laplace assumed that when the sound propagates through a medium, the particles oscillate very rapidly such that the compression and rarefaction occur very fast. Hence the exchange of heat produced due to compression and cooling effect due to rarefaction do not take place, because, air (medium) is a bad conductor of heat.

2) Since, temperature is no longer considered as a constant here, sound propagation is an adiabatic process. By adiabatic considerations, the gas obeys Poisson's law (not Boyle's law as Newton assumed), which is

$$Pv^\gamma = \text{Constant} \quad \text{----- 4}$$

Where, $\gamma = \frac{C_P}{C_V}$, which is the ratio between specific heat at constant pressure and specific heat at constant volume. Differentiating equation (4) on both the sides, we get

$$v^\gamma dP + P(\gamma V \gamma^{-1} dV) = 0 \text{ or } \gamma^P = -V \frac{dP}{dV} B_A \text{ -----5}$$

where, B_A is the adiabatic bulk modulus of air. Now, substituting equation (5)

in equation $V = \sqrt{\frac{B}{\rho}}$ the speed of sound in air is

$$V_A = \sqrt{\frac{B_T}{\rho}} = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\gamma} V_T \text{ -----6}$$

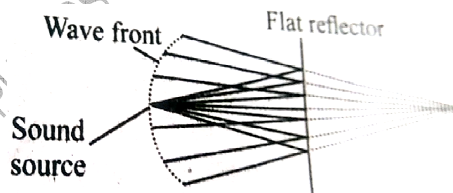
$$V_A = 331 \text{ms}^{-1}$$

05. Write short notes on reflection of sound waves from plane and curved surfaces.

1) Sound also reflects from a harder flat surface, This is called as **specular reflection**.

2) Specular reflection is observed only when the wavelength of the source is smaller than dimensions of the reflecting surface, as well as smaller than surface irregularities.

3) When the sound waves hit the plane wall, they bounce off in a manner similar to that of light. Suppose a loudspeaker is kept at an angle with respect to a wall (plane surface), then the waves coming from the source (assumed to be a point source) can be treated as spherical wave fronts (say, compressions moving like a spherical wave front).



4) Therefore, the reflected wave front on the plane surface is also spherical, such that its centre of curvature (which lies on the other side of plane surface) can be treated as the image of the sound source (virtual or imaginary loud speaker) which can be assumed to be at a position behind the plane surface.

Reflection of sound through the curved surface:

1) The behaviour of sound is different when it is reflected from different surfaces-convex or concave or plane. The sound reflected from a convex surface is spread out and so it is easily attenuated and weakened. Whereas, if it is reflected from the concave surface it will converge at a point and this can be easily amplified.

2) The parabolic reflector (curved reflector) which is used to focus the sound precisely to a point is used in designing the parabolic mics which are known as high directional microphones.

3) We know that any surface (smooth or rough) can absorb sound. For example, the sound produced in a big hall or auditorium or theatre is absorbed by the walls, ceilings, floor, seats etc.

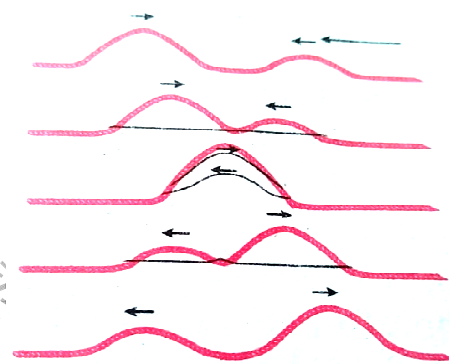
06. Briefly explain the concept of superposition principle.

1) When a jerk is given to a stretched string which is tied at one end, a wave pulse is produced and the pulse travels along the string. Suppose two persons holding the stretched string on either side give a jerk simultaneously, then these two wave pulses move towards each other, meet at some point and move away from each other with their original identity.

2) Their behaviour is very different only at the crossing/meeting points; this behaviour depends on whether the two pulses have the same or different shape as figure.

3) When the pulses have the same shape, at the crossing, the total displacement is the algebraic sum of their individual displacements and hence its net amplitude is higher than the amplitudes of the individual pulses.

4) Whereas, if the two pulses have same amplitude but shapes are 180° out of phase at the crossing point, the net amplitude vanishes at that point and the pulses will recover their identities after crossing.



5) Only waves can possess such a peculiar property and it is called superposition of waves. This means that the principle of superposition explains the net behaviour of the waves when they overlap.

6) Generalizing to any number of waves i.e, if two or more waves in a medium move simultaneously, when they overlap, their total displacement is the vector sum of the individual displacements.

7) To understand mathematically, let us consider two functions which characterize the displacement of the waves, for example,

$$y_1 = A_1 \sin(kx - \omega t) \text{ and } y_2 = A_2 \cos(kx - \omega t)$$

8) Since, both y_1 and y_2 satisfy the wave equation (solutions of wave equation) then their algebraic sum $y = y_1 + y_2$ also satisfies the wave equation.

9) This means, the displacements are additive. Suppose we multiply y_1 and y_2 with some constant then their amplitude is scaled by that constant. Further, if C_1 and C_2 are used to multiply the displacements y_1 and y_2 , respectively, then, their net displacement y is $y = C_1 y_1 + C_2 y_2$

10) This can be generalized to any number of waves. In the case of n such waves in more than one dimension the displacements are written using vector notation. Here, the net displacement \vec{y} is $\vec{y} = \sum_{i=1}^n C_i \vec{y}_i$

The principle of superposition can explain the following:

- (a) Space (or spatial) Interference (also known as Interference)
- (b) Time (or Temporal) Interference (also known as Beats)
- (c) Concept of stationary waves.

11) Waves that obey principle of superposition are called linear waves (amplitude is much smaller than their wavelengths). In general, if the amplitude of the wave is not small then they are called non-linear waves. These violate the linear superposition principle, e.g. laser. In this chapter, we will focus our attention only on linear waves.

07. Explain how the interference of waves is formed.

1) Consider two harmonic waves having identical frequencies, constant phase difference ϕ and same wave form (can be treated as coherent source), but having amplitudes A_1 and A_2 , then

$$y_1 = A_1 \sin(kx - \omega t) \text{ -----1}$$

$$y_2 = A_2 \sin(kx - \omega t + \phi) \text{ ----- 2}$$

Suppose they move simultaneously in a particular direction, then interference occurs (i.e., overlap of these two waves),

$$y = y_1 + y_2 \text{ -----3}$$

2) Therefore, substituting equation (1) and equation (2) in equation (3), we get $y = A_1 \sin(kx - \omega t) + A_2 \sin(kx - \omega t + \phi)$

Using trigonometric identity $\sin(\alpha + \beta) = (\sin \alpha \cos \beta + \cos \alpha \sin \beta)$, we get

$$y = A_1 \sin(kx - \omega t) + A_2 [\sin(kx - \omega t) \cos \phi + \cos(kx - \omega t) \sin \phi]$$

$$y = \sin(kx - \omega t)(A_1 + A_2 \cos \phi) + A_2 \sin \phi \cos(kx - \omega t) \text{ -----4}$$

Let us re-define

$$A \cos \theta = (A_1 + A_2 \cos \phi) \text{ ----- 5}$$

$$\text{and } A \sin \theta = A_2 \sin \phi \text{ ----- 6}$$

then equation (4) can be rewritten as

$$y = A \sin(kx - \omega t) \cos \theta + A \cos(kx - \omega t) \sin \theta$$

$$y = A (\sin(kx - \omega t) \cos \theta + \sin \theta \cos(kx - \omega t))$$

$$y = A \sin(kx - \omega t + \theta) \text{ -----7}$$

By squaring and adding equation (5) and equation (6), we get

$$A^2 = A_1^2 + A_2^2 + 2A_1 A_2 \cos \phi \text{ -----8}$$

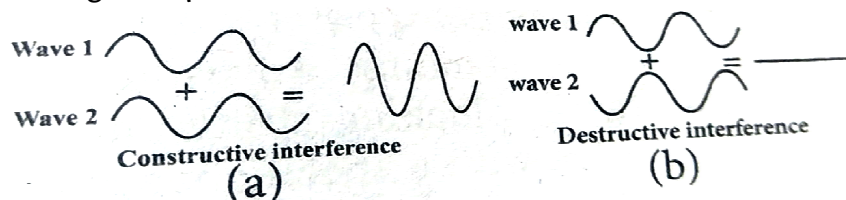
Since, intensity is square of the amplitude ($I = A^2$),

$$\text{we have } I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi \text{ -----9}$$

This means the resultant intensity at any point depends on the phase difference at that point.

a) For constructive interference:

1) When crests of one wave overlap with crests of another wave, their amplitudes will add up and we get constructive interference. The resultant wave has a larger amplitude than the individual waves as shown in Figure.



2) The constructive interference at a point occurs if there is maximum intensity at that point, which means that $\cos\phi = +1$

$$\Rightarrow \phi = 0, 2\pi, 4\pi, \dots = 2n\pi, \text{ where } n = 0, 1, 2, \dots$$

3) This is the phase difference in which two waves overlap to give constructive interference. Therefore, for this resultant wave,

$$I_{\text{maximum}} = (\sqrt{I_1} + \sqrt{I_2})^2 = (A_1 + A_2)^2$$

$$\text{Hence, the resultant amplitude } A = A_1 + A_2$$

b) For destructive interference:

1) When the trough of one wave overlaps with the crest of another wave, their amplitudes “cancel” each other and we get destructive interference as shown in Figure. The resultant amplitude is nearly zero.

2) The destructive interference occurs if there is minimum intensity at that point, which means $\cos\phi = -1 \Rightarrow \phi = \pi, 3\pi, 5\pi, \dots = (2n-1)\pi$, where $n = 0, 1, 2, \dots$ i.e. This is the phase difference in which two waves overlap to give destructive interference.

$$3) \text{ Therefore, } I_{\text{minimum}} = (\sqrt{I_1} - \sqrt{I_2})^2 = (A_1 - A_2)^2$$

$$\text{Hence, the resultant amplitude } A = A_1 - A_2$$

08. Describe the formation of beats.

Formation of beats: When two or more waves superimpose each other with slightly different frequencies, then a sound of periodically varying amplitude at a point is observed. This phenomenon is known as beats. The number of amplitude maxima per second is called beat frequency. If we have two sources, then their difference in frequency gives the beat frequency. Number of beats per second $n = |f_1 - f_2|$ per second

09. What are stationary waves? Explain the formation of stationary waves and also write down the characteristics of stationary waves.

1) When the wave hits the rigid boundary it bounces back to the original medium and can interfere with the original waves. A pattern is formed, which are known as standing waves or stationary waves.

2) Consider two harmonic progressive waves (formed by strings) that have the same amplitude and same velocity but move in opposite directions. Then the displacement of the first wave (incident wave) is

$$y_1 = A \sin(kx - \omega t) \text{ (waves move toward right) -----1}$$

and the displacement of the second wave (reflected wave) is

$$y_2 = A \sin(kx + \omega t) \text{ (waves move toward left) -----2}$$

both will interfere with each other by the principle of superposition, the net displacement is $y = y_1 + y_2$ -----3

Substituting equation (1) and equation (2) in equation (3), we get

$$y = A \sin(kx - \omega t) + A \sin(kx + \omega t) \text{ -----4}$$

Using trigonometric identity, we rewrite equation (4) as

$$y(x, t) = 2A \cos(\omega t) \sin(kx) \text{ ----- 5}$$

3) This represents a stationary wave or standing wave, which means that this wave does not move either forward or backward, whereas progressive or travelling waves will move forward or backward.

4) Further, the displacement of the particle in equation (5) can be written in more compact form, $y(x, t) = A' \cos(\omega t)$ where, $A' = 2A \sin(kx)$, implying that the particular element of the string executes simple harmonic motion with amplitude equals to A' .

5) The maximum of this amplitude occurs at positions for which $\sin(kx) = 1 \Rightarrow kx = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \dots, m\pi$ where m takes half integer or half integral values. The position of maximum amplitude is known as antinodes.

Characteristics of stationary waves :

1) Stationary waves are characterized by the confinement of a wave disturbance between two rigid boundaries. This means, the wave does not move forward or backward in a medium (does not advance), it remains steady at its place. Therefore, they are called “stationary waves or standing waves”.

2) Certain points in the region in which the wave exists have maximum amplitude, called as anti-nodes and at certain points the amplitude is minimum or zero, called as nodes.

- 3) The distance between two consecutive nodes (or) anti-nodes is $\frac{\lambda}{2}$
- 4) The distance between a node and its neighbouring anti-node is $\frac{\lambda}{4}$
- 5) The transfer of energy along the standing wave is zero

10. Discuss the law of transverse vibrations in stretched strings.

i) The law of length:

For a given wire with tension T (which is fixed) and mass per unit length μ (fixed) the frequency varies inversely with the vibrating length.

Therefore, $f \propto \frac{1}{l} \Rightarrow f = \frac{C}{l} \Rightarrow l \times f = C$, where C is a constant.

ii) The law of tension:

For a given vibrating length l (fixed) and mass per unit length μ (fixed) the frequency varies directly with the square root of the tension T , $f \propto \sqrt{T}$

$\Rightarrow f = A\sqrt{T}$ where A is a constant

iii) The law of mass:

For a given vibrating length l (fixed) and tension T (fixed) the frequency varies inversely with the square root of the mass per unit length μ , $f \propto \frac{1}{\sqrt{\mu}}$

$\Rightarrow f = \frac{B}{\sqrt{\mu}}$, where B is a constant.

11. Explain the concepts of fundamental frequency, harmonics and overtones in detail.

1) Keep the rigid boundaries at $x = 0$ and $x = L$ and produce a standing waves by wiggling the string (as in plucking strings in a guitar). Standing waves with a specific wavelength are produced. Since, the amplitude must vanish at the boundaries, therefore, the displacement at the boundary $y(x = 0, t) = 0$ and $y(x = L, t) = 0$ -----1

Since the nodes formed are at a distance $\frac{\lambda_n}{2}$ apart, we have $n\left[\frac{\lambda_n}{2}\right] = L$

2) where n is an integer, L is the length between the two boundaries and λ_n is the specific wavelength that satisfy the specified boundary conditions.

Hence, $\lambda_n = \left(\frac{2L}{n}\right)$ -----2

3) Therefore, not all wavelengths are allowed. The (allowed) wavelengths should fit with the specified boundary conditions, i.e., for $n = 1$, the first mode of vibration has specific wavelength $\lambda_1 = 2L$. Similarly for $n = 2$, the second mode of vibration has specific wavelength $\lambda_2 = \left(\frac{2L}{2}\right) = L$

For $n = 3$, the third mode of vibration has specific wavelength $\lambda_3 = \left(\frac{2L}{3}\right)$

and so on. The frequency of each mode of vibration (called natural frequency) can be calculated. $f_n = \frac{v}{\lambda_n} = n \left(\frac{v}{2L} \right)$ -----3

4) The lowest natural frequency is called the fundamental frequency.

$$f_1 = \frac{v}{\lambda_1} = \left(\frac{v}{2L} \right) \text{ -----4}$$

The second natural frequency is called the first over tone.

$$f_2 = 2 \left(\frac{v}{2L} \right) = \frac{1}{L} \sqrt{\frac{T}{\mu}}$$

The third natural frequency is called the second over tone.

$$f_3 = 3 \left(\frac{v}{2L} \right) = 3 \left(\frac{1}{2L} \sqrt{\frac{T}{\mu}} \right)$$

and so on. Therefore, the nth natural frequency can be computed as integral (or integer) multiple of fundamental frequency, i.e.,

$$f_n = n f_1, \text{ where } n \text{ is an integer ----- 5}$$

5) If natural frequencies are written as integral multiple of fundamental frequencies, then the frequencies are called harmonics. Thus, the first harmonic is $f_1 = f_1$ (the fundamental frequency is called first harmonic), the second harmonic is $f_2 = 2f_1$, the third harmonic is $f_3 = 3f_1$ etc.

12. What is a sonometer? Give its construction and working. Explain how to determine the frequency of tuning fork using sonometer.

1) **Sono** means *sound* related, and sonometer implies sound-related measurements. It is a device for demonstrating the relationship between the frequency of the sound produced in the transverse standing wave in a string, and the tension, length and mass per unit length of the string.

2) Therefore, using this device, we can determine the following quantities:

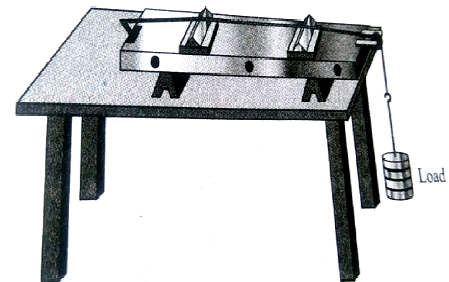
- a) the frequency of the tuning fork or frequency of alternating current
- b) the tension in the string
- c) the unknown hanging mass

Construction:

3) The sonometer is made up of a hollow box which is one meter long with a uniform metallic thin string attached to it. One end of the string is connected to a hook and the other end is connected to a weight hanger through a pulley as shown in Figure.

4) Since only one string is used, it is also known as monochord. The weights are added to the free end of the wire to increase the tension of the wire.

5) Two adjustable wooden knives are put over the board, and their positions are adjusted to change the vibrating length of the stretched wire.



Working :

6) A transverse stationary or standing wave is produced and hence, at the knife edges P and Q, nodes are formed. In between the knife edges, anti-nodes are formed.

If the length of the vibrating element is l then $l = \frac{\lambda}{2} \Rightarrow \lambda = 2l$

7) Let f be the frequency of the vibrating element, T the tension of in the string and μ the mass per unit length of the string. Then using equation $v = \sqrt{\frac{T}{\mu}}$,

we get $f = \frac{v}{\lambda} = \frac{1}{2l} \sqrt{\frac{T}{\mu}}$ in Hz -----1

8) Let ρ be the density of the material of the string and d be the diameter of the string. Then the mass per unit length μ ,

$$\mu = \text{Area} \times \text{density} = \pi r^2 \rho = \frac{\pi \rho d^2}{4}; f = \frac{v}{\lambda} = \frac{1}{2l} \sqrt{\frac{T}{\frac{\pi \rho d^2}{4}}} \quad f = \frac{1}{ld} \sqrt{\frac{T}{\pi \rho}}$$

13. Write short notes on intensity and loudness.

Intensity of sound:

1) When a sound wave is emitted by a source, the energy is carried to all possible surrounding points. The average sound energy emitted or transmitted per unit time or per second is called sound power.

2) Therefore, the intensity of sound is defined as “the sound power transmitted per unit area taken normal to the propagation of the sound wave”.

3) For a particular source (fixed source), the sound intensity is inversely proportional to the square of the distance from the source.

$$I = \frac{\text{power of the source}}{4\pi r^2} \Rightarrow I \propto \frac{1}{r^2}$$

This is known as inverse square law of sound intensity.

Loudness of sound:

1) Two sounds with same intensities need not have the same loudness. For example, the sound heard during the explosion of balloons in a silent closed room is very loud when compared to the same explosion happening in a noisy market.

2) Though the intensity of the sound is the same, the loudness is not. If the intensity of sound is increased then loudness also increases. But additionally, not only does intensity matter, the internal and subjective experience of “how loud a sound is” i.e., the sensitivity of the listener also matters here.

3) This is often called loudness. That is, loudness depends on both intensity of sound wave and sensitivity of the ear (It is purely observer dependent quantity which varies from person to person) whereas the intensity of sound does not depend on the observer.

4) The loudness of sound is defined as “the degree of sensation of sound produced in the ear or the perception of sound by the listener”.

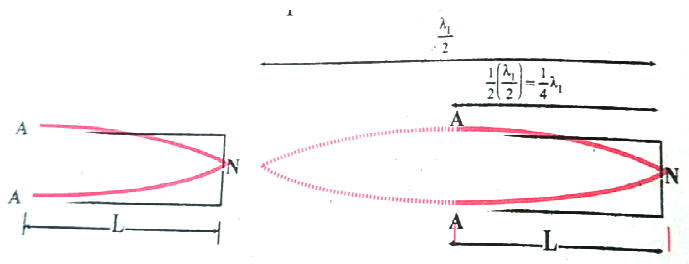
14. Explain how overtones are produced in a

(a) Closed organ pipe (b) Open organ pipe

a) Closed organ pipes:

1) It is a pipe with one end closed and the other end open. If one end of a pipe is closed, the wave reflected at this closed end is 180° out of phase with the incoming wave.

2) Thus there is no displacement of the particles at the closed end. Therefore, nodes are formed at the closed end and anti-nodes are formed at open end.



3) Consider the simplest mode of vibration of the air column called the fundamental mode. Anti-node is formed at the open end and node at closed end. From the Figure, let L be the length of the tube and the wavelength of the wave produced. For the fundamental mode of vibration, we have,

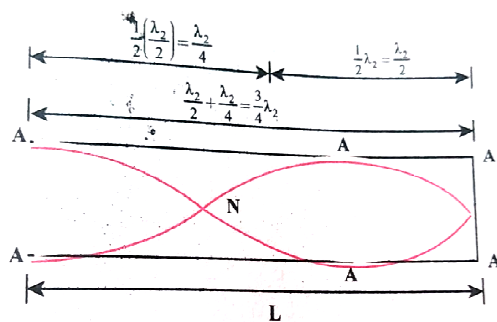
$$L = \frac{\lambda_1}{4} \text{ or } \lambda_1 = 4L ; \text{ The frequency of the note emitted is}$$

$f_1 = \frac{v}{\lambda_1} = \frac{v}{4L}$ which is called the fundamental note.

4) The frequencies higher than fundamental frequency can be produced by blowing air strongly at open end. Such frequencies are called overtones.

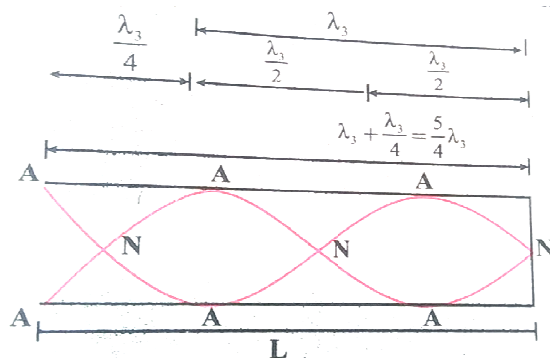
The Figure 2 shows the second mode of vibration having two nodes and two anti-nodes. $4L = 3\lambda_2$ $L = \frac{3\lambda_2}{4}$ or $\lambda_2 = \frac{4L}{3}$

The frequency of this $f_2 = \frac{v}{\lambda_2} = \frac{3v}{4L} = 3f_1$ is called first over tone, since here, the frequency is three times the fundamental frequency it is called third harmonic.



5) The Figure 3 shows third mode of vibration having three nodes and three anti-nodes. $4L = 5\lambda_3$ $L = \frac{5\lambda_3}{4}$ or $\lambda_3 = \frac{4L}{5}$

The frequency of this $f_3 = \frac{v}{\lambda_3} = \frac{5v}{4L} = 5f_1$ is called second over tone, and since $n = 5$ here, this is called fifth harmonic.



6) Hence, the closed organ pipe has only odd harmonics and frequency of the n th harmonic is $f_n = (2n+1)f_1$. Therefore, the frequencies of harmonics are in the ratio $f_1 : f_2 : f_3 : f_4 : \dots = 1 : 3 : 5 : 7 : \dots$

b) Open organ pipe :

1) It is a pipe with both the ends open. At both open ends, anti-nodes are formed. Let us consider the simplest mode of vibration of the air column called fundamental mode. Since anti-nodes are formed at the open end, a node is formed at the mid-point of the pipe.

2) From Figure, if L be the length of the tube, the wavelength of the wave produced is given by $L = \frac{\lambda_1}{2}$ or $\lambda_1 = 2L$

The frequency of the note emitted is $f_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$ which is called the fundamental note.

3) The frequencies higher than fundamental frequency can be produced by blowing air strongly at one of the open ends. Such frequencies are called overtones.

4) The Figure shows the second mode of vibration in open pipes. It has two nodes and three anti-nodes, and therefore, $L = \lambda_2$ or $\lambda_2 = L$. The frequency $f_2 = \frac{v}{\lambda_2} = \frac{v}{L}$

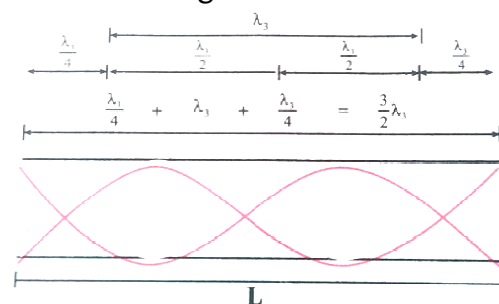
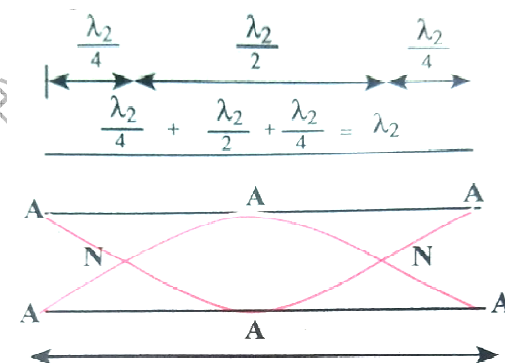
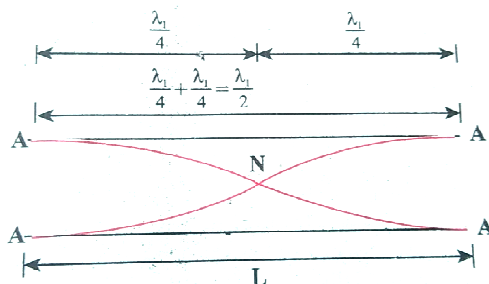
$= 2 \times \frac{v}{2L} = 2f_1$ is called **first over tone**. Since $n = 2$ here, it is called the **second harmonic**.

5) The Figure shows the third mode of vibration having three nodes and four anti-nodes $L = \frac{3}{2}\lambda_3$ or $\lambda_3 = \frac{2L}{3}$;

$$f_3 = \frac{v}{\lambda_3} = \frac{3v}{2L} = 3 \times \frac{v}{2L} = 3f_1$$

is called second over tone. Since $n = 3$ here, it is called the third harmonic.

6) Hence, the open organ pipe has all the harmonics and frequency of n^{th} harmonic is $f_n = nf_1$. Therefore, the frequencies of harmonics are in the ratio $f_1 : f_2 : f_3 : f_4 : \dots = 1 : 2 : 3 : 4 : \dots$



15. How will you determine the velocity of sound using resonance air column apparatus?

1) The resonance air column apparatus is one of the simplest techniques to measure the speed of sound in air at room temperature.

2) It consists of a cylindrical glass tube of one meter length whose one end A is open and another end B is connected to the water reservoir R through a rubber tube as shown in Figure. This cylindrical glass tube is mounted on a vertical stand with a scale attached to it.

3) The tube is partially filled with water and the water level can be adjusted by raising or lowering the water in the reservoir R. The surface of the water will act as a closed end and other as the open end.

4) Therefore, it behaves like a closed organ pipe, forming nodes at the surface of water and antinodes at the open end.

5) When a vibrating tuning fork is brought near the open end of the tube, longitudinal waves are formed inside the air column. These waves move downward as shown in Figure, and reach the surfaces of water and get reflected and produce standing waves.

6) The length of the air column is varied by changing the water level until a loud sound is produced in the air column. At this particular length the frequency of waves in the air column resonates with the frequency of the tuning fork (natural frequency of the tuning fork).

7) At resonance, the frequency of sound waves produced is equal to the frequency of the tuning fork. This will occur only when the length of air column is proportional to $\left(\frac{1}{4}\right)^{th}$ of the wavelength of the sound waves produced. Let the first resonance occur at length L_1 , then $\frac{1}{4}\lambda = L_1$

8) But since the antinodes are not exactly formed at the open end, we have to include a correction, called end correction e , by assuming that the antinode is formed at some small distance above the open end. Including this end correction, the first resonance is $\frac{1}{4}\lambda = L_1 + e$

9) Now the length of the air column is increased to get the second resonance. Let L_2 be the length at which the second resonance occurs. Again taking end correction into account, $\frac{3}{4}\lambda = L_2 + e$

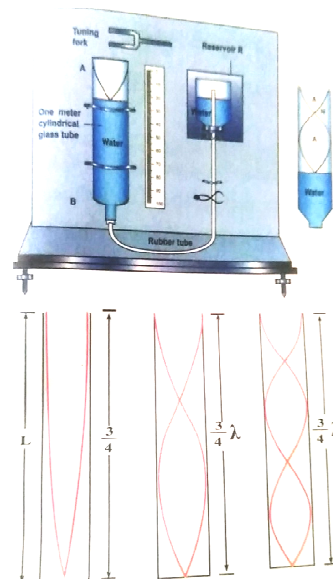
In order to avoid end correction,

let us take the difference of equation $\frac{1}{4}\lambda = L_1 + e$,

and equation $f_1 : f_2 : f_3 : f_4 : \dots = 1 : 2 : 3 : 4 : \dots$

$$\frac{3}{4}\lambda - \frac{1}{4}\lambda = (L_2 + e - L_1 + e)$$

$$\Rightarrow \frac{1}{2}\lambda = L_2 - L_1 = \Delta L \Rightarrow \lambda = 2 \Delta L$$



16. What is meant by Doppler effect? Discuss the following cases

(1) Source in motion and Observer at rest

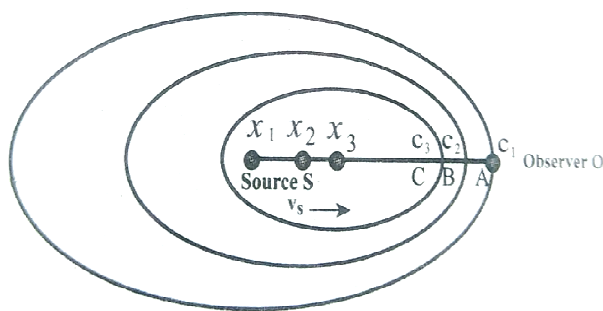
(a) Source moves towards observer (b) Source moves away from the observer

Doppler effect :

When the source and the observer are in relative motion with respect to each other and to the medium in which sound propagates, the frequency of the sound wave observed is different from the frequency of the source. This phenomenon is called Doppler Effect.

a) Source moves towards the observer:

1) Suppose a source S moves to the right as shown in Figure, with a velocity v_s and let the frequency of the sound waves produced by the source be f_s . We assume the velocity of sound in a medium is v .



2) The compression (sound wave front) produced by the source S at three successive instants of time are shown in the Figure. When S is at position x_1 the compression is at C_1 .

3) When S is at position x_2 , the compression is at C_2 and similarly for x_3 and C_3 . Assume that if C_1 reaches the observer's position A then at that instant C_2 reaches the point B and C_3 reaches the point C as shown in the Figure.

4) It is obvious to see that the distance between compressions C_2 and C_3 is shorter than distance between C_1 and C_2 .

5) This means the wavelength decreases when the source S moves towards the observer O (since sound travels longitudinally and wavelength is the distance between two consecutive compressions). But frequency is inversely related to wavelength and therefore, frequency increases.

6) Let λ be the wavelength of the source S as measured by the observer when S is at position x_1 and λ' be wavelength of the source observed by the observer when S moves to position x_2 . Then the change in wavelength is $\Delta\lambda = \lambda - \lambda' = v_s t$, where t is the time taken by the source to travel between x_1 and x_2 . Therefore, $\lambda' = \lambda - v_s t$ But $t = \frac{\lambda}{v}$

7) On substituting equation $t = \frac{\lambda}{v}$ in equation $\lambda' = \lambda - v_s t$, we get $\lambda' = \lambda \left(1 - \frac{v_s}{v}\right)$

Since frequency is inversely proportional to wavelength, we have $f' = \frac{v'}{\lambda'}$ and

$f = \frac{v_s}{\lambda}$ Hence, $f' = \frac{f}{\left(1 - \frac{v_s}{v}\right)}$, Since, $\frac{v_s}{v} \ll 1$ we use the binomial expansion and

retaining only first order in $\frac{v_s}{v}$ we get $f' = f \left(1 + \frac{v_s}{v}\right)$

b) Source moves away from the observer:

Since the velocity here of the source is opposite in direction when compared to case (a), therefore, changing the sign of the velocity of the source in the above case i.e, by substituting ($v_s \rightarrow -v_s$) in equation $\lambda' = \lambda - v_s t$,

$$\text{we get } f' = \frac{f}{\left(1 - \frac{v_s}{v}\right)}$$

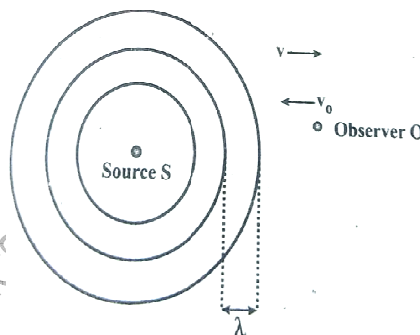
Using binomial expansion again, we get, $f' = f \left(1 - \frac{v_s}{v}\right)$

2) Observer in motion and Source at rest.

a) Observer moves towards Source b) Observer resides away from the Source

a) Observer moves towards Source:

1) Let us assume that the observer O moves towards the source S with velocity v_o . The source S is at rest and the velocity of sound waves (with respect to the medium) produced by the source is v . From the Figure, we observe that both v_o and v are in opposite direction. Then, their relative velocity is $v_r = v + v_o$. The wavelength of the sound $\lambda = \frac{v}{f}$, which means the frequency



observed by the observer O is $f' = \frac{v_r}{\lambda}$. Then $f' = \frac{v_r}{\lambda} = \left(\frac{v+v_o}{v}\right)f = f \left(1 + \frac{v_o}{v}\right)$

b) Observer recedes away from the Source:

2) If the observer O is moving away (receding away) from the source S, then velocity v_o and v moves in the same direction. Therefore, their relative velocity is $v_r = v - v_o$. Hence, the frequency observed by the observer O is $f' = \frac{v_r}{\lambda}$. Then $f' = \frac{v_r}{\lambda} = \left(\frac{v-v_o}{v}\right)f = f \left(1 - \frac{v_o}{v}\right)$

3) Both are in motion

- a) Source and Observer approach each other
- b) Source and Observer resides from each other
- c) Source chases Observer
- d) Observer chases Source

a) Source and Observer approach each other

1) Let v_s and v_o be the respective velocities of source and observer approaching each other as shown in Figure.

In order to calculate the apparent frequency observed by the observer, as a simple calculation,



2) let us have a dummy (behaving as observer or source) in between the source and observer. Since the dummy is at rest, the dummy (observer)

observes the apparent frequency due to approaching source as given in equation $f' = \frac{f}{(1 - \frac{v_s}{v})}$ as $f_d = \frac{f}{(1 - \frac{v_s}{v})}$ ----- 1

3) At that instant of time, the true observer approaches the dummy from the other side. Since the source (true source) comes in a direction opposite to true observer, the dummy (source) is treated as stationary source for the true observer at that instant.

4) Hence, apparent frequency when the true observer approaches the stationary source (dummy source), from equation

$$f' = \frac{v_r}{\lambda} = \left(\frac{v + v_0}{v} \right) f = f \left(1 + \frac{v_0}{v} \right) \text{ is}$$

$$f' = f_d \left(1 + \frac{v_0}{v} \right) \Rightarrow f_d = \frac{f'}{\left(1 + \frac{v_0}{v} \right)} \text{ ----- 2}$$

Since this is true for any arbitrary time, therefore, comparing equation (1) and equation (2), we get $\frac{f}{(1 - \frac{v_s}{v})} = \frac{f'}{(1 + \frac{v_0}{v})}$; $\Rightarrow \frac{vf'}{(v + v_0)} = \frac{vf}{(v - v_s)}$

Hence, the apparent frequency as seen by the observer is $f' = \left(\frac{v + v_0}{v - v_s} \right) f$ ----- 3

b) Source and Observer recedes from each other:

1) Here, we can derive the result as in the previous case. Instead of a detailed calculation, by inspection from Figure, we notice that the velocity of the source and the observer each point in opposite directions with respect to the case in (a) and hence, we substitute ($v_s \rightarrow -v_s$) and ($v_0 \rightarrow -v_0$) in equation (3), and therefore, the apparent frequency observed by the observer when the source and observer recede from each



other is $f' = \left(\frac{v - v_0}{v + v_s} \right) f$ ----- 4

c) Source chases Observer:

Only the observer's velocity is oppositely directed when compared to case (a). Therefore, substituting ($v_0 \rightarrow -v_0$) in equation (3),

we get $f' = \left(\frac{v - v_0}{v - v_s} \right) f$ ----- 5

d) Observer chases Source:

Only the source velocity is oppositely directed when compared to case (a). Therefore, substituting $v_s \rightarrow -v_s$ in equation (3), we get $f' = \left(\frac{v + v_0}{v + v_s} \right) f$

17. Write the expression for the velocity of longitudinal waves in an elastic medium.

1) Consider an elastic medium (here we assume air) having a fixed mass contained in a long tube (cylinder) whose cross sectional area is A and maintained under a pressure P. One can generate longitudinal waves in the fluid either by displacing the fluid using a piston or by keeping a vibrating tuning fork at one end of the tube.

2) Let us assume that the direction of propagation of waves coincides with the axis of the cylinder. Let ρ be the density of the fluid which is initially at rest.

At $t = 0$, the piston at left end of the tube is set in motion toward the right with a speed u .

3) Let u be the velocity of the piston and v be the velocity of the elastic wave. In time interval Δt , the distance moved by the piston $\Delta d = u \Delta t$. Now, the distance moved by the elastic disturbance is $\Delta x = v \Delta t$. Let Δm be the mass of the air that has attained a velocity v in a time Δt .

Therefore, $\Delta m = \rho A \Delta x = \rho A (v \Delta t)$

4) Then, the momentum imparted due to motion of piston with velocity u is $\Delta p = [\rho A (v \Delta t)]u$

But the change in momentum is impulse.

The net impulse is $I = (\Delta P A) \Delta t$ Or $(\Delta P A) \Delta t = [\rho A (v \Delta t)]u$

$$\Delta P = \rho v u \text{ -----1}$$

5) When the sound wave passes through air, the small volume element (ΔV) of the air undergoes regular compressions and rarefactions. So, the change in pressure can also be written as $\Delta P = B \frac{\Delta V}{V}$ where, V is original volume and B is known as bulk modulus of the elastic medium.

But $V = A \Delta x = A v \Delta t$ and

$$\Delta V = A \Delta d = A u \Delta t$$

$$\text{Therefore, } \Delta P = B \frac{A u \Delta t}{A v \Delta t} = B \frac{u}{v} \text{ ----- 2}$$

Comparing equation (1) and equation (2),

$$\text{we get } \rho v u = B \frac{u}{v} \text{ or } v^2 = \frac{B}{\rho} \Rightarrow v = \sqrt{\frac{B}{\rho}} \text{ -----3}$$

In general, the velocity of a longitudinal wave in elastic medium is $v = \sqrt{\frac{E}{\rho}}$, where E is the modulus of elasticity of the medium.

Cases: For a solid:

i) one dimension rod (1D)

$v = \sqrt{\frac{Y}{\rho}}$, -----4 where Y is the Young's modulus of the material of the rod and ρ is the density of the rod. The 1D rod will have only Young's modulus.

ii) Three dimension rod (3D)

$$\text{The speed of longitudinal wave in a solid is } v = \sqrt{\frac{4 + \frac{3}{2}\eta}{\rho}} \text{ -----5}$$

where η is the modulus of rigidity, K is the bulk modulus and ρ is the density of the rod.

Cases: For liquids:

$$v = \sqrt{\frac{K}{\rho}}, \text{ ----- 6 where, K is the bulk modulus and } \rho \text{ is the density of the rod.}$$

18. Discuss the effect of pressure, temperature, density , humidity and wind.

a) Effect of pressure:

1) For a fixed temperature, when the pressure varies, correspondingly density also varies such that the ratio $\left(\frac{P}{\rho}\right)$ becomes constant. This means that the speed of sound is independent of pressure for a fixed temperature.

2) If the temperature remains same at the top and the bottom of a mountain then the speed of sound will remain same at these two points. But, in practice, the temperatures are not same at top and bottom of a mountain; hence, the speed of sound is different at different points.

b) Effect of temperature:

Since $v \propto T$,

1) The speed of sound varies directly to the square root of temperature in kelvin. Let v_0 be the speed of sound at temperature at 0°C or 273 K and v be the speed of sound at any arbitrary temperature T (in kelvin),

$$\text{then } \frac{v}{v_0} = \sqrt{\frac{T}{273}} = \sqrt{\frac{273+t}{273}}$$

$$v = v_0 \sqrt{1 + \frac{t}{273}} \cong v_0 \left(1 + \frac{t}{546}\right) \text{ (using binomial expansion)}$$

Since $v_0 = 331\text{ m s}^{-1}$ at 0°C , v at any temperature in $t^\circ\text{C}$ is

$$v = (331 + 0.60t) \text{ ms}^{-1}$$

2) Thus the speed of sound in air increases by 0.61 ms^{-1} per degree celcius rise in temperature. Note that when the temperature is increased, the molecules will vibrate faster due to gain in thermal energy and hence, speed of sound increases.

c) Effect of density:

1) Let us consider two gases with different densities having same temperature and pressure. Then the speed of sound in the two gases are

$$v_1 = \sqrt{\frac{\gamma_1 P}{\rho_1}} \text{ -----1 and } v_2 = \sqrt{\frac{\gamma_2 P}{\rho_2}} \text{ -----2}$$

$$\text{Taking ratio of equation (1) and equation (2), we get } \frac{v_1}{v_2} = \frac{\sqrt{\frac{\gamma_1 P}{\rho_1}}}{\sqrt{\frac{\gamma_2 P}{\rho_2}}} = \sqrt{\frac{\gamma_1 \rho_2}{\gamma_2 \rho_1}}$$

$$\text{For gases having same value of } \gamma, \frac{v_1}{v_2} = \sqrt{\frac{\rho_2}{\rho_1}} \text{ ----- 3}$$

Thus the velocity of sound in a gas is inversely proportional to the square root of the density of the gas.

d) Effect of moisture (humidity):

1) We know that density of moist air is 0.625 of that of dry air, which means the presence of moisture in air (increase in humidity) decreases its density. Therefore, speed of sound increases with rise in humidity.

$$\text{From equation } v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\gamma^c T}$$

$$v = \sqrt{\frac{\gamma P}{\rho}} \text{ Let } \rho_1, v_1 \text{ and } \rho_2, v_2 \text{ be the density and speeds of sound in dry air}$$

$$\text{and moist air, respectively. Then } \frac{v_1}{v_2} = \frac{\sqrt{\frac{\gamma_1 P}{\rho_1}}}{\sqrt{\frac{\gamma_2 P}{\rho_2}}} = \sqrt{\frac{\rho_2}{\rho_1}} \text{ if } \gamma_1 = \gamma_2$$

Since P is the total atmospheric pressure, it can be shown that $\frac{\rho_2}{\rho_1} = \frac{P}{P_1 + 0.625 P_2}$

e) Effect of wind:

The speed of sound is also affected by blowing of wind. In the direction along the wind blowing, the speed of sound increases whereas in the direction opposite to wind blowing, the speed of sound decreases.

19. Write the applications of reflection of sound waves:

a) Stethoscope: It works on the principle of multiple reflections.

It consists of three main parts:

i) Chest piece (ii) Ear piece (iii) Rubber tube

i) Chest piece: It consists of a small disc-shaped resonator (diaphragm) which is very sensitive to sound and amplifies the sound it detects.

ii) Ear piece: It is made up of metal tubes which are used to hear sounds detected by the chest piece.

iii) Rubber tube: This tube connects both chest piece and ear piece. It is used to transmit the sound signal detected by the diaphragm, to the ear piece. The sound of heart beats (or lungs) or any sound produced by internal organs can be detected, and it reaches the ear piece through this tube by multiple reflections.

b) Echo:

1) An echo is a repetition of sound produced by the reflection of sound waves from a wall, mountain or other obstructing surfaces. The speed of sound in air at 20°C is 344 m s⁻¹. If we shout at a wall which is at 344 m away, then the sound will take 1 second to reach the wall.

2) After reflection, the sound will take one more second to reach us. Therefore, we hear the echo after two seconds. Scientists have estimated that we can hear two sounds properly if the time gap or time interval between each sound is $\left(\frac{1}{10}\right)^{\text{th}}$ of a second (persistence of hearing) i.e., 0.1 s. Then,

$$\text{Velocity} = \frac{\text{Distance travelled}}{\text{Time taken}} ; = \frac{2d}{t}$$

$$2d = 344 \times 0.1 = 34.1\text{m}; \quad d = 17.2 \text{ m}$$

The minimum distance from a sound reflecting wall to hear an echo at 20°C is 17.2 meter.

c) SONAR: SOund NAavigation and Ranging. Sonar systems make use of reflections of sound waves in water to locate the position or motion of an object. Similarly, dolphins and bats use the sonar principle to find their way in the darkness.

d) Reverberation: In a closed room the sound is repeatedly reflected from the walls and it is even heard long after the sound source ceases to function.

The residual sound remaining in an enclosure and the phenomenon of multiple reflections of sound is called reverberation.

The duration for which the sound persists is called reverberation time. It should be noted that the reverberation time greatly affects the quality of sound heard in a hall. Therefore, halls are constructed with some optimum reverberation time.

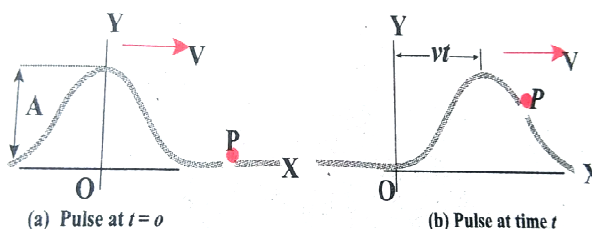
19. Write characteristics of progressive waves:

- 1) Particles in the medium vibrate about their mean positions with the same amplitude.
- 2) The phase of every particle ranges from 0 to 2π .
- 3) No particle remains at rest permanently. During wave propagation, particles come to the rest position only twice at the extreme points.
- 4) Transverse progressive waves are characterized by crests and troughs whereas longitudinal progressive waves are characterized by compressions and rarefactions.
- 5) When the particles pass through the mean position they always move with the same maximum velocity.
- 6) The displacement, velocity and acceleration of particles separated from each other by $n\lambda$ are the same, where n is an integer, and λ is the wavelength.

20. Derive the equation of a plane progressive wave.

1) A jerk on a stretched string at time $t = 0$ s. Let us assume that the wave pulse created during this disturbance moves along positive x direction with constant speed v as shown in Figure .

2) We can represent the shape of the wave pulse, mathematically as $y = y(x, 0) = f(x)$ at time $t = 0$ s. Assume that the shape of the wave pulse remains the same during the propagation. After some time t , the pulse moving towards the right and any point on it can be represented by x' (read it as x prime) as shown in Figure. Then, $y(x, t) = f(x') = f(x - vt)$



3) Similarly, if the wave pulse moves towards left with constant speed v , then $y = f(x + vt)$. Both waves $y = f(x + vt)$ and $y = f(x - vt)$ will satisfy the following one dimensional differential equation known as the

$$\text{wave equation } \frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2}.$$

4) where the symbol ∂ represent partial derivative. Not all the solutions satisfying this differential equation can represent waves, because any physical acceptable wave must take finite values for all values of x and t .

5) But if the function represents a wave then it must satisfy the differential equation. Since, in one dimension (one independent variable), the partial derivative with respect to x is the same as total derivative in coordinate x , we write $\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2}$.

21. Explain the Graphical representation of the wave.

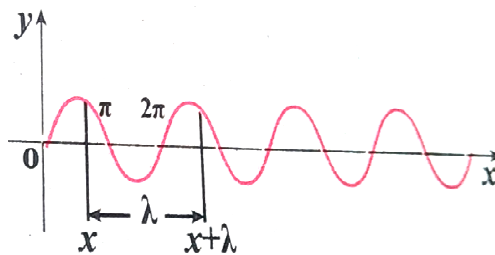
Let us graphically represent the two forms of the wave variation

a) Space (or Spatial) variation graph

b) Time (or Temporal) variation graph

a) Space variation graph

1) By keeping the time fixed, the change in displacement with respect to x is plotted. Let us consider a sinusoidal graph, $y = A \sin(kx)$ as shown in the Figure, where k is a constant. Since the wavelength λ denotes the distance between any two points in the same state of motion, the displacement y is the same at both the ends.



$$y = x \text{ and } y = x + \lambda, \text{ i.e.,}$$

$$y = A \sin(kx) = A \sin(k(x + \lambda))$$

$$= A \sin(kx + k\lambda) \text{ ----- 1}$$

The sine function is a periodic function with period 2π . Hence,

$$y = A \sin(kx + 2\pi) = A \sin(kx) \text{ ----- 2}$$

Comparing equation (1) and equation (2), we get

$$kx + k\lambda = kx + 2\pi, \text{ This implies } k = \frac{2\pi}{\lambda} \text{ rad m}^{-1} \text{ ----- 3}$$

where k is called wave number. This measures how many wavelengths are present in 2π radians.

The spatial periodicity of the wave is $\lambda = \frac{2\pi}{k}$ in m

Then, At $t = 0$ $y(x, 0) = y(x + \lambda, 0)$ and At any time t , $y(x, t) = y(x + \lambda, t)$

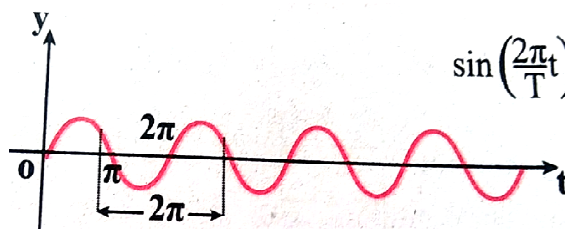
b) Time variation graph

1) By keeping the position fixed, the change in displacement with respect to time is plotted. Let us consider a sinusoidal graph, $y = A \sin(\omega t)$ as shown in the Figure, where ω is angular frequency of the wave which measures how quickly wave oscillates in time or number of cycles per second

2) The temporal periodicity or time period is $T = \frac{2\pi}{\omega} \Rightarrow \omega = \frac{2\pi}{T}$

The angular frequency is related to frequency f by the expression $\omega = 2\pi f$, where the frequency f is defined as the number of oscillations made by the medium particle per second.

3) Since inverse of frequency is time period, we have, $T = \frac{1}{f}$ in seconds



This is the time taken by a medium particle to complete one oscillation. Hence, we can define the speed of a wave (wave speed, v) as the distance traversed by the wave per second $v = \frac{\lambda}{T} = \lambda f$ in ms^{-1}

22. Derive the relation between intensity and loudness.

1) According to Weber-Fechner's law, "loudness (L) is proportional to the logarithm of the actual intensity (I) measured with an accurate non-human instrument". This means that $L \propto \ln I$, $L = k \ln I$ where k is a constant, which depends on the unit of measurement.

2) The difference between two loudness, L_1 and L_0 measures the relative loudness between two precisely measured intensities and is called as sound intensity level.

3) Sound intensity level is $\Delta L = L_1 - L_0 = k \ln I_1 - k \ln I_0 = k \ln \left[\frac{I_1}{I_0} \right]$
if $k = 1$, then sound intensity level is measured in bel, in honour of Alexander Graham Bell. Therefore, $\Delta L = \ln \left[\frac{I_1}{I_0} \right]$ bel

4) However, this is practically a bigger unit, so we use a convenient smaller unit, called decibel. Thus, decibel $= \frac{1}{10}$ bel ,

5) Therefore, by multiplying and dividing by 10,
we get $\Delta L = 10 \left(\ln \left[\frac{I_1}{I_0} \right] \right) \frac{1}{10} \text{ bel} ; \Delta L = 10 \ln \left[\frac{I_1}{I_0} \right] \text{ decibel}$ with $k = 10$

For practical purposes, we use logarithm to base 10 instead of natural logarithm, $\Delta L = 10 \log_{10} \left[\frac{I_1}{I_0} \right] \text{ decibel}$.

PREPARED BY

RAJENDRAN M, M.Sc., B.Ed., C.C.A.,
P. G. ASSISTANT IN PHYSICS,
DEPARTMENT OF PHYSICS,
SRM HIGHER SECONDARY SCHOOL,
KAVERIYAMPOONDI,
THIRUVANNAMALAI DISTRICT.

For your Feedback & Suggestion: mrrkphysics@gmail.com, murasabiphysics@gmail.com