UNIT – 04 WORK, ENERGY AND POWER

TWO MARKS AND THREE MARKS:

01. Explain how the definition of work in physics is different from general perception.

1. Generally any activity can be called as work
2. But in physics, work is said to be done by the force when the force applied on a body displaces it.

02. Write the various types of potential energy. Explain the formulae.

1. The energy possessed by the body due to gravitational force gives rise to gravitational potential energy U = mgh
2. The energy due to spring force and other similar forces give rise to elastic potential energy. $U = \frac{1}{2} Kx^2$
3. The energy due to electrostatic force on charges gives rise to electrostatic potential energy. U = – E. dr

03. Write the differences between conservative and Non-conservative forces. Give two examples each.

Conservative forces	Non-conservative forces
Work done is independent of the path	Work done depends upon the path
Work done in a round trip is zero	Work done in a round trip is not zero
Total energy remains constant	Energy is dissipated as heat energy
Work done is completely recoverable	Work done is not completely recoverable
Force is the negative gradient of potential energy	No such relation exists.

04. Explain the characteristics of elastic and inelastic collision.

Elastic Collision	Inelastic Collision
Total momentum is conserved	Total momentum is conserved
Total kinetic energy is conserved	Total kinetic energy is not conserved
Forces involved are conservative forces	Forces involved are non-conservative forces
Mechanical energy is not dissipated	Mechanical energy is dissipated into heat, light, sound etc.

05. Define the following

a) Coefficient of restitution b) Power c) Law of conservation of energy
d) Loss of kinetic energy in inelastic collision.

a) Coefficient of restitution

It is defined as the ratio of velocity of separation (relative velocity) after collision to the velocity of approach (relative velocity) before collision, i.e.,

$$e = \frac{\text{Velocity of separation (after collision)}}{\text{Velocity of approach (before collision)}} ; \frac{(v_2 - v_1)}{(u_1 - u_2)}$$

b) Power

The rate of work done or energy delivered.

$$\text{Power (P)} = \frac{\text{Workdone (W)}}{\text{Time taken (t)}}$$

c) Law of conservation of energy

Energy can neither be created nor destroyed. It may be transformed from one form to another but the total energy of an isolated system remains constant.

d) Loss of kinetic energy in inelastic collision

In perfectly inelastic collision, the loss in kinetic energy during collision is transformed to another form of energy like sound, thermal, heat, light etc. Let KE_i be the total kinetic energy before collision and KE_f be the total kinetic energy after collision. Total kinetic energy before collision,

$$KE_i = \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 \text{ -----------(1)}$$

Total kinetic energy after collision, $KE_f = \frac{1}{2}(m_1 + m_2)v^2$ ------------ (2)

Then the loss of kinetic energy is Loss of KE, $\Delta Q = KE_f - KE_i$

$$= \frac{1}{2}(m_1 + m_2)v^2 - \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 \text{ -------- (3)}$$

Substituting equation $v = \frac{\mathbf{m_1 u_1 + m_2 u_2}}{\mathbf{(m_1 + m_2)}}$ in equation (3), and on simplifying (expand v by using the algebra $(a+b)^2 = a^2 + b^2 + 2ab$, we get

$$\textbf{Loss of KE, } \Delta Q = \frac{1}{2}\left(\frac{\mathbf{m_1 m_2}}{\mathbf{m_1 + m_2}}\right)(\mathbf{u_1 - u_2})^2$$

06. Define unit of power:

One watt is defined as the power when one joule of work is done in one second. $1W = 1Js^{-1}$

07. Explain Work done.

i) Work is said to be done by the force when the force applied on a body displaces it.
ii) work done is a scalar quantity. It has only magnitude and no direction.
iii) In SI system, unit of work done is N m (or) joule (J). Its dimensional formula is ML^2T^{-2}

08. When does work done becomes zero?

i) When the force is zero (F = 0). ii) When the displacement is zero (dr = 0).
iii) When the force and displacement are perpendicular ($\theta = 90^0$) to each other.

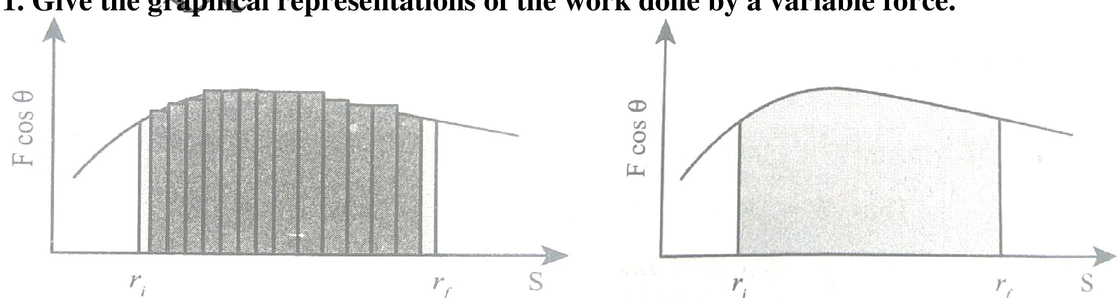
09. Define Work done by a constant force

When a constant force F acts on a body, the small work done (dW) by the force in producing a small displacement dr is given by the relation, $dW = (F \cos\theta)\, dr$

10. Define Work done by a variable force

When the component of a variable force F acts on a body, the small work done (dW) by the force in producing a small displacement dr is given by the relation $dW = (F \cos\theta)\, dr$
[F cos θ is the component of the variable force F]

11. Give the graphical representations of the work done by a variable force.



12. Define Energy, Kinetic energy and Potential Energy

Energy: The capacity to do work , Dimension : ML^2T^{-2} , SI Unit : Nm or joule .
Kinetic energy : The energy possessed by a body due to its motion.
Dimension : ML^2T^{-2} , SI Unit : Nm or joule .
Potential Energy: The energy possessed by the body by virtue of its position
Dimension : ML^2T^{-2} , SI Unit : Nm or joule .

13. Write the significance of kinetic energy in the work – kinetic energy theorem.

1. If the work done by the force on the body is **positive** then its **kinetic energy increases.**
2. If the work done by the force on the body is **negative** then its **kinetic energy decreases**.
3. If there is **no work done** by the force on the body then there is **no change** in its kinetic energy

14. Define Work – kinetic energy theorem.

The work done by the force on the body changes the kinetic energy of the body. This is called work-kinetic energy theorem.

15. Define elastic potential energy

The potential energy possessed by a spring due to a deforming force which stretches or compresses the spring is termed as elastic potential energy.

16. Define Conservative force

A force is said to be a conservative force if the work done by or against the force in moving the body depends only on the initial and final positions of the body and not on the nature of the path followed between the initial and final positions.

17. Define Non-conservative force

A force is said to be non-conservative if the work done by or against the force in moving a body depends upon the path between the initial and final positions. This means that the value of work done is different in different paths.

18. Define A*verage power*

The average power (P_{av}) is defined as the ratio of the total work done to the total time taken. $P_{av} = \frac{\text{Total work done}}{\text{Total time taken}}$

19. Define Instantaneous power

The instantaneous power (P_{inst}) is defined as the power delivered at an instant (as time interval approaches zero), $P_{inst} = \frac{dw}{dt}$

20. What is meant by collision?

Collision is a common phenomenon that happens around us every now and then. For example, carom, billiards, marbles, etc.,. Collisions can happen between two bodies with or without physical contacts.

21. What is Elastic Collision?

In a collision, the total initial kinetic energy of the bodies (before collision) is equal to the total final kinetic energy of the bodies (after collision) then, it is called as elastic collision. i.e., Total kinetic energy before collision = Total kinetic energy after collision

22. What is Inelastic Collision?

In a collision, the total initial kinetic energy of the bodies (before collision) is not equal to the total final kinetic energy of the bodies (after collision) then, it is called as inelastic collision. i.e., Total kinetic energy before collision ≠ Total kinetic energy after collision

CONCEPTUAL QUESTIONS

01. Which is conserved in inelastic collision? Total energy or Kinetic energy?

The total energy of the system is conserved in inelastic collision. The kinetic energy is not conserved because it is carried by the moving objects or it is transformed into other form of energy.

02. Is there any net work done by external forces on a car moving with a constant speed along a straight road?

At a constant speed, a car can be carrying around corners or driving in a curved path. That would cause all sorts of acceleration. But an object travelling at a constant speed, in a straight line and by Newton's first law of motion. If has no external force acting on it.

03. A charged particle moves towards another charged particle. Under what conditions the total momentum and the total energy of the system conserved?

1. If positive and negative charged particles moves towards another
2. After collision the charged particles should stick together permanent.
3. So, they should move with common velocity under this situation the total momentum and total energy of the system is conserved.

FIVE MARKS QUESTIONS

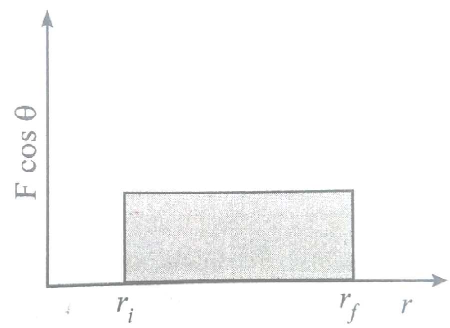
01. Explain with graphs the difference between work done by a constant force and by a variable force.

i) When a constant force F acts on a body, the small work done (dW) by the force in producing a small displacement dr is given by the relation, $dW = (F\cos\theta)\,dr$

ii) The total work done in producing a displacement from initial position $\mathbf{r_i}$ to final position $\mathbf{r_f}$ is, $W = \int_{r_i}^{r_f} dW$;

$W = \int_{r_i}^{r_f}(F\cos\theta)dr = (F\cos\theta)\int_{r_i}^{r_f}dr = (F\cos\theta)\,(r_f - r_i)$

iii) The graphical representation of the work done by a constant force .The area under the graph shows the work done by the constant force.

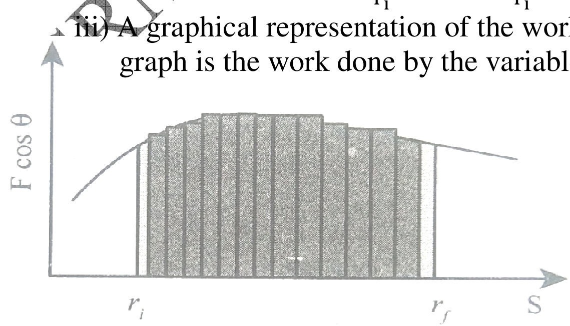


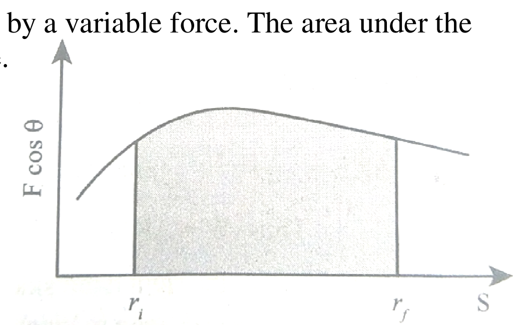
Work done by a variable force

i) When the component of a variable force F acts on a body, the small work done (dW) by the force in producing a small displacement dr is given by the relation $dW = (F\cos\theta)\,dr$ [$F\cos\theta$ is the component of the variable force F] where, F and θ are variables.

ii) The total work done for a displacement from initial position r_i to final position r_f is given by the relation, $W = \int_{r_i}^{r_f} dW$; $= \int_{r_i}^{r_f}(F\cos\theta)dr$

iii) A graphical representation of the work done by a variable force. The area under the graph is the work done by the variable force.





02. State and explain work energy principle. Mention any three examples for it.

1) It states that work done by the force acting on a body is equal to the change produced in the kinetic energy of the body.

2) Consider a body of mass m at rest on a frictionless horizontal surface.

3) The work (W) done by the constant force (F) for a displacement (s) in the same direction is, W = Fs ------------- (1)

The constant force is given by the equation, F = ma ---------- (2)

The third equation of motion can be written as, $v^2 = u^2 + 2as$

$$a = \frac{v^2 - u^2}{2s} \text{ ------------------- (3)}$$

Substituting for a in equation (2), $F = m\left(\frac{v^2-u^2}{2s}\right)$ ------------- (4)

Substituting equation (4) in (1), $W = m\left(\frac{v^2}{2s}s\right) - m\left(\frac{u^2}{2s}s\right)$

$$W = \tfrac{1}{2} mv^2 - \tfrac{1}{2} mu^2 \text{ --------------------- (5)}$$

The expression for kinetic energy:

i) The term ½ (mv^2) in the above equation is the kinetic energy of the body of mass (m) moving with velocity (v). KE = ½ mv^2 ------------------ (6)

ii) Kinetic energy of the body is always positive. From equations (5) and (6)
$\Delta KE = \tfrac{1}{2} mv^2 - \tfrac{1}{2} mu^2$ -----------------(7) thus, W = ΔKE

iii) The expression on the right hand side (RHS) of equation (7) is the change in kinetic energy (ΔKE) of the body.

iv) This implies that the work done by the force on the body changes the kinetic energy of the body. This is called work-kinetic energy theorem.

03. Arrive at an expression for power and velocity. Give some examples for the same.

i) The work done by a force $\vec{F}$ for a displacement $d\vec{r}$ is $W = \int \vec{F}.d\vec{r}$ ------- (1)

Left hand side of the equation (1) can be written as

$$W = \int dW = \int \frac{dW}{dt}dt \quad \text{(multiplied and divided by dt)} \text{ -------------- (2)}$$

ii) Since, velocity $\vec{v} = \frac{d\vec{r}}{dt}$; $d\vec{r} = \vec{v}\,dt$. Right hand side of the equation (1) can be written as $\int \vec{F}.d\vec{r} = \int\left(\vec{F}.\frac{d\vec{r}}{dt}\right)dt = \int\left(\vec{F}.\vec{v}\right)dt \quad \left[\vec{v} = \frac{d\vec{r}}{dt}\right]$ ---------------- (30

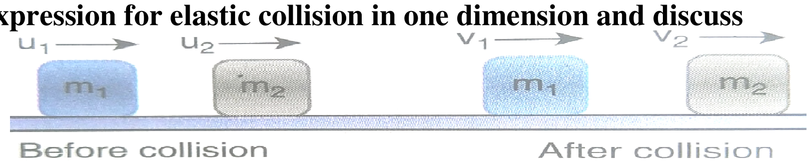
Substituting equation (2) and equation (3) in equation (1), we get

$$\int \frac{dW}{dt}dt = \int\left(\vec{F}.\vec{v}\right)dt \quad ; \int\left(\frac{dW}{dt} - \vec{F}.\vec{v}\right)dt = 0$$

iii) This relation is true for any arbitrary value of dt. This implies that the term within the bracket must be equal to zero, i.e.,

$$\frac{dW}{dt} - \vec{F}.\vec{v} = 0 \text{ (or) } \frac{dW}{dt} = \vec{F}.\vec{v}$$

04. Arrive at an expression for elastic collision in one dimension and discuss various cases.



Consider two elastic bodies of masses m_1 and m_2 moving in a straight line (along positive x direction) on a frictionless horizontal surface.

i) In order to have collision, we assume that the mass m_1 moves faster than mass m_2 i.e., $\mu_1 > \mu_2$ For elastic collision, the total linear momentum and kinetic energies of the two bodies before and after collision must remain the same.

From the law of conservation of linear momentum, Total momentum before collision (*pi*) = Total momentum after collision (*pf*)

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$ -------------- (1) (or)

$m_1(u_1 - v_1) = m_2(v_2 - u_2)$ ------------------- (2)

For elastic collision,

Total kinetic energy before collision KE_i = Total kinetic energy after collision KE_f

$\frac{1}{2} m_1u_1^2 + \frac{1}{2} m_2u_2^2 = \frac{1}{2} m_1v_1^2 + \frac{1}{2} m_2v_2^2$ ------------------ (3)

After simplifying and rearranging the terms,

$m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2)$

Using the formula, $a^2 - b^2 = (a + b)(a - b)$, we can rewrite the above equation as

$m_1(u_1 + v_1)(u_1 - v_1) = m_2(v_2 + u_2)(v_2 - u_2)$ ------------ (4)

Dividing equation (4) by (2) gives,

$$\frac{m_1(u_1+v_1)(u_1-v_1)}{m_1(u_1-v_1)} = \frac{m_2(v_2+u_2)(v_2-u_2)}{m_2(v_2-u_2)}$$

$u_1 + v_1 = v_2 + u_2$, Re-arranging $u_1 - u_2 = v_2 - v_1$ -------------- (5)

Equation (5) can be rewritten as $(u_1 - u_2 = -(v_1 - v_2)$

ii) This means that for any elastic head on collision, the relative speed of the two elastic bodies after the collision has the same magnitude as before collision but in opposite direction. Further note that this result is independent of mass.

Rewriting the above equation for v_1 and v_2 ,

$v_1 = v_2 + u_2 - u_1$ ------------- (6) **or** $v_2 = u_1 + v_1 - u_2$ ----------------(7)

To find the final velocities v_1 and v_2:

Substituting equation (7) in equation (2) gives the velocity of m_1 as

$m_1(u_1 - v_1) = m_2 (u_1 + v_1 - u_2 - u_2)$

$m_1(u_1 - v_1) = m_2 (u_1 + v_1 - 2u_2)$

$m_1u_1 - m_1v_1 = m_2u_1 + m_2v_1 - 2m_2u_2$

$m_1u_1 - m_2u_1 + 2m_2u_2 = m_1v_1 + m_2v_1$

$(m_1 - m_2) u_1 + 2m_2u_2 = (m_1 + m_2)v_1$ **(or)**

$$v_1 = \left(\frac{m_1 - m_2}{m_1+m_2}\right)u_1 + \left(\frac{2m_2}{m_1+m_2}\right)u_2 \text{ ----------(8)}$$

Similarly, by substituting (6) in equation (2) or substituting equation (8) in equation (7), we get the final velocity of m_2 as

$$v_2 = \left(\frac{2m_1}{m_1+m_2}\right)u_1 + \left(\frac{m_2 - m_1}{m_1+m_2}\right)u_2 \text{ ----------- (9)}$$

Case 1: When bodies has the same mass i.e., $m_1 = m_2$,

Equation (8) $\longrightarrow v_1 = (0)u_1 + \left(\frac{2m_2}{2m_2}\right)u_2$; $v_1 = u_2$ ------------- (10)

Equation (9) $\longrightarrow v_2 = \left(\frac{2m_1}{2m_1}\right)u_1 + (0)u_2$; $v_2 = u_1$ ------------- (11)

The equations (10) and (11) show that in one dimensional elastic collision, when two bodies of equal mass collide after the collision their velocities are exchanged.

Case 2: When bodies have the same mass i.e., $m_1 = m_2$, and second body (usually called target) is at rest ($u_2 = 0$),

By substituting $m_1 = m_2$ and $u_2 = 0$ in equations (8) and equations (9) we get,

From equation (8) $\longrightarrow v_1 = 0$ ---------------(12)

From equation (9) $\longrightarrow v_2 = u_1$ --------------- (13)

Equations (12) and (13) show that when the first body comes to rest the second body moves with the initial velocity of the first body.

Case 3: The first body is very much lighter than the second body

$\left(m_1 \ll m_2, \frac{m_1}{m_2} \ll 1\right)$ then the ratio $\frac{m_1}{m_2} \approx 0$. And also if the target is at rest (u_2=0)

Dividing numerator and denominator of equation (8) by m_2, we get

$$v_1 = \left(\frac{\frac{m_1}{m_2} - 1}{\frac{m_1}{m_2}+1}\right)u_1 + \left(\frac{2}{\frac{m_1}{m_2}+1}\right)(0);\ v_1 = \left(\frac{0-1}{0+1}\right)u_1;\ v_1 = -u_1 \text{ ------------(14)}$$

Similarly, Dividing numerator and denominator of equation (9) by m_2, we get

$$v_2 = \left(\frac{2\frac{m_1}{m_2}}{\frac{m_1}{m_2}+1}\right) u_1 + \left(\frac{1-\frac{m_1}{m_2}}{\frac{m_1}{m_2}+1}\right)(0)\ ;\ v_2 = (o)u_1 + \left(\frac{1-\frac{m_1}{m_2}}{\frac{m_1}{m_2}+1}\right)(0)\ ;\ v_2 = 0 \text{ ----- } (15)$$

The equation (14) implies that the first body which is lighter returns back rebounds) in the opposite direction with the same initial velocity as it has a negative sign.

The equation (15) implies that the second body which is heavier in mass continues to remain at rest even after collision. For example, if a ball is thrown at a fixed wall, the ball will bounce back from the wall with the same velocity with which it was thrown but in opposite direction.

Case 4: The second body is very much lighter than the first body

$\left(m_2 \ll m_1, \frac{m_2}{m_1} \ll 1\right)$ then the ratio $\frac{m_2}{m_1} \approx 0$. And also if the target is at rest (u_2=0)

Dividing numerator and denominator of equation (8) by m_1, we get

$$v_1 = \left(\frac{1-\frac{m_2}{m_1}}{1+\frac{m_2}{m_1}}\right)u_1 + \left(\frac{2\frac{m_2}{m_1}}{1+\frac{m_2}{m_1}}\right)(0)\ ;$$

$$v_1 = \left(\frac{0-1}{0+1}\right)u_1 + \left(\frac{0}{1+0}\right)(0)\ ;$$

$$v_1 = u_1 \text{ -------------------}(16)$$

Similarly, Dividing numerator and denominator of equation (14) by m_1, we get

$$v_1 = \left(\frac{2}{1+\frac{m_2}{m_1}}\right) u_1 + \left(\frac{\frac{m_2}{m_1}-1}{1+\frac{m_2}{m_1}}\right)(0)\ ;$$

$$v_2 = \left(\frac{2}{1+0}\right)u_1\ ;\ \ v_2 = 2u_1 \text{ ------}(17)$$

The equation (16) implies that the first body which is heavier continues to move with the same initial velocity.

The equation (17) suggests that the second body which is lighter will move with twice the initial velocity of the first body.

It means that the lighter body is thrown away from the point of collision.

05. What is inelastic collision? In which way it is different from elastic collision. Mention few examples in day to day life for inelastic collision.

1) In a collision, the total initial kinetic energy of the bodies (before collision) is not equal to the total final kinetic energy of the bodies (after collision) then, it is called as inelastic collision. i.e.,
2) Momentum is conserved. Kinetic energy is not conserved in elastic collision. Mechanical energy is dissipated into heat, light, sound etc. When a light body collides against any massive body at rest it sticks to it.
3) Total kinetic energy before collision $\neq$ Total kinetic energy after collision

$$\begin{bmatrix}\text{Total kinetic energy}\\ \text{after collision}\end{bmatrix} - \begin{bmatrix}\text{Total kinetic energy}\\ \text{before collision}\end{bmatrix} = \begin{bmatrix}\text{Loss in energy during}\\ \text{collision}\end{bmatrix}$$

4) Even though kinetic energy is not conserved but the total energy is conserved.
5) loss in kinetic energy during collision is transformed to another form of energy like sound, thermal, etc.
6) if the two colliding bodies stick together after collision such collisions are known as completely inelastic collision or perfectly inelastic collision.
7) For example when a clay putty is thrown on a moving vehicle, the clay putty (or Bubblegum) sticks to the moving vehicle and they move together with the same velocity.

06. Deduce the relation between momentum and kinetic energy.

i) Consider an object of mass m moving with a velocity $\vec{v}$. Then its linear momentum is $\vec{p}$ = m$\vec{v}$ and its kinetic energy, KE = ½ mv^2

KE = ½ mv^2 ; = ½ m($\vec{v}.\vec{v}$) --------(1)

ii) Multiplying both the numerator and denominator of equation (1) by mass, m

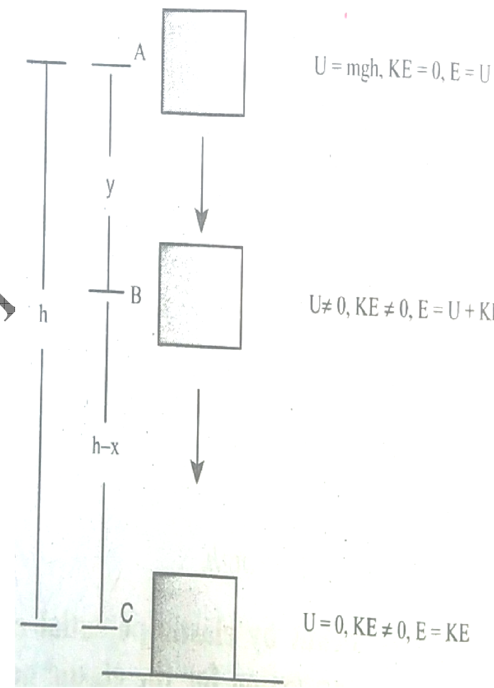
$$KE = \frac{1}{2}\frac{m^2(\vec{v}.\vec{v})}{m} \; ; \; = \frac{1}{2}\frac{(m\vec{v}).(m\vec{v})}{m} \; [\vec{p} = m\vec{v}] \; ; \; = \frac{1}{2}\frac{(\vec{p}).(\vec{p})}{m}$$

$$= \frac{\vec{p}^2}{2m} \; ; \; KE = \frac{p^2}{2m}$$

iii) Where $|\vec{p}|$is the magnitude of the momentum. The magnitude of the linear momentum can be obtained by $|\vec{p}|$ = p = $\sqrt{2m(KE)}$

iv) Note that if kinetic energy and mass are given, only the magnitude of the momentum can be calculated but not the direction of momentum. It is because the kinetic energy and mass are scalars.

07. State and prove the law of conservation of energy.



i) When an object is thrown upwards its kinetic energy goes on decreasing and consequently its potential energy keeps increasing (neglecting air resistance).

ii) When it reaches the highest point its energy is completely potential. Similarly, when the object falls back from a height its kinetic energy increases whereas its potential energy decreases.

iii) When it touches the ground its energy is completely kinetic. At the intermediate points the energy is both kinetic and potential.

iv) When the body reaches the ground the kinetic energy is completely dissipated into some other form of energy like sound, heat, light and deformation of the body etc.

v) In this example the energy transformation takes place at every point. The sum of kinetic energy and potential energy i.e., the total mechanical energy always remains constant, implying that the total energy is conserved. This is stated as the law of conservation of energy.

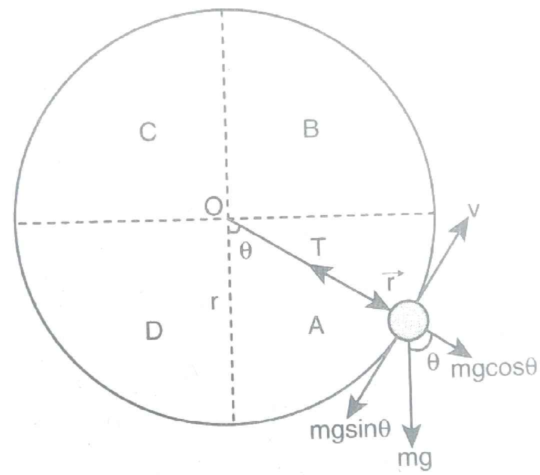
vi) The law of conservation of energy states that energy can neither be created nor destroyed. It may be transformed from one form to another but the total energy of an isolated system remains constant.

vii) The figure illustrates that, if an object starts from rest at height *h*, the total energy is purely potential energy (U=mgh) and the kinetic energy (KE) is zero at *h*. When the object falls at some distance *y*, the potential energy and the kinetic energy are not zero whereas, the total energy remains same as measured at height *h*. When the object is about to touch the ground, the potential energy is zero and total energy is purely kinetic.

08. Derive an expression for the velocity of the body moving in a vertical circle and also find a tension at the bottom and the top of the circle.

1) A body of mass (m) attached to one end of a mass less and inextensible string executes circular motion in a vertical plane with the other end of the string fixed. The length of the string becomes the radius ($\vec{r}$) of the circular path.

2) The motion of the body by taking the free body diagram (FBD) at a position where the position vector ($\vec{r}$) makes an angle θ with the vertically downward direction and the instantaneous velocity .



3) There are two forces acting on the mass.
1. Gravitational force which acts downward
2. Tension along the string.

Applying Newton's second law on the mass,
In the tangential direction,

mg sin θ = ma$_t$;

mg sin θ = $-$ m $\left(\frac{dv}{dt}\right)$

where , a$_t$ = $-\left(\frac{dv}{dt}\right)$ is tangential retardation in the radial direction,

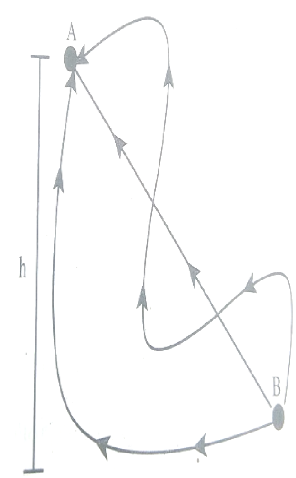
T – mg cos θ = m a$_r$;

T – mg cos θ = $\frac{mv^2}{r}$

where , a$_r$ = $\frac{v^2}{r}$ is the centripetal acceleration.

09. What is conservative force? State how it is determined from potential energy?

i) A force is said to be a conservative force if the work done by or against the force in moving the body depends only on the initial and final positions of the body and not on the nature of the path followed between the initial and final positions.



ii) Consider an object at point A on the Earth. It can be taken to another point B at a height h above the surface of the Earth by three paths.

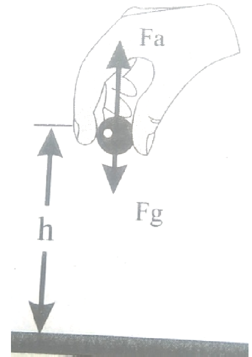
iii) Whatever may be the path, the work done against the gravitational force is the same as long as the initial and final positions are the same.

iv) This is the reason why gravitational force is a conservative force.

v) Conservative force is equal to the negative gradient of the potential energy. In one dimensional case, F$_x$ = $-\frac{dU}{dx}$

vi) Examples for conservative forces are elastic spring force, electrostatic force, magnetic force, gravitational force, etc.

10. Derive an expression for the potential energy of a body near the surface of the Earth.



1) The gravitational potential energy (U) at some height is equal to the amount of work required to take the object from ground to that height with constant velocity.
2) Consider a body of mass being moved from ground to the height h against the gravitational force.
3) The gravitational force $\vec{F}_g$ acting on the body is, $\vec{F}_g = - mg\hat{j}$ (as the force is in y direction, unit vector is used). Here, negative sign implies that the force is acting vertically downwards. In order to move the body without acceleration (or with constant velocity), an external applied force $\vec{F}_a$ equal in magnitude but opposite to that of gravitational force $\vec{F}_g$ has to be applied on the body i.e., $\vec{F}_a - \vec{F}_g$ This implies that $\vec{F}_a = - mg\hat{j}$
4) The positive sign implies that the applied force is in vertically upward direction. Hence, when the body is lifted up its velocity remains unchanged and thus its kinetic energy also remains constant.
5) The gravitational potential energy (U) at some height h is equal to the amount of work required to take the object from the ground to that height h.

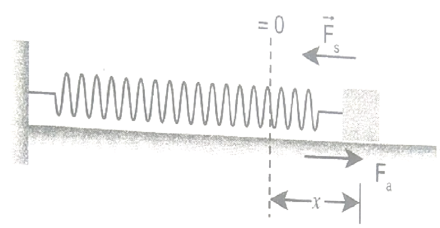
$$U = \int \vec{F}_a . d\vec{r}$$
$$= \int_0^h |\vec{F}_a| \, |d\vec{r}| \cos\theta$$

6) Since the displacement and the applied force are in the same upward direction, the angle between them, $\theta = 0$.

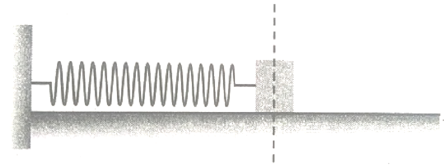
Hence Cos 0 = 1 and $|\vec{F}_a|$ = mg and $|d\vec{r}|$ = dr

$$U = mg \int_0^h dr \; ;$$
$$U = mg \, [r]_0^h \; ; \; \mathbf{U = mgh}$$

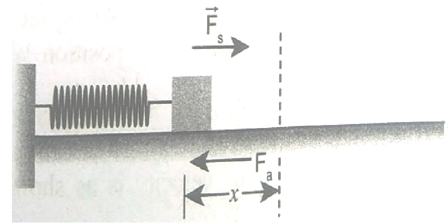
11. What is meant by elastic potential energy? Derive an expression for the elastic potential energy of the spring?



1) The potential energy possessed by a spring due to a deforming force which stretches or compresses the spring is termed as elastic potential energy. The work done by the applied force against the restoring force of the spring is stored as the elastic potential energy in the spring.



2) Consider a spring-mass system. Let us assume a mass, lying on a smooth m horizontal table. Here, x = 0 is the equilibrium position. One end of the spring is attached to a rigid wall and the other end to the mass.



3) As long as the spring remains in equilibrium position, its potential energy is zero. Now an external force $\vec{F}_a$ is applied so that it is stretched by a distance (x) in the direction of the force.

4) There is a restoring force called spring force $\vec{F}_s$ developed in the spring which tries to bring the mass back to its original position. This applied force and the spring force are equal in magnitude but opposite in direction i.e., $\vec{F}_a = -\vec{F}_s$. According Hooke's law, the restoring force developed in the spring is $\vec{F}_s = -k\vec{x}$

5) The negative sign in the above expression implies that the spring force is always opposite to that of displacement $\vec{x}$ and k is the force constant. Therefore applied force. is $\vec{F}_a = +k\vec{x}$. The positive sign implies that the applied force is in the direction of displacement $\vec{x}$. The spring force is an example of variable force as it depends on the displacement $\vec{x}$. Let the spring be stretched to a small distance $d\vec{x}$. The work done by the applied force on the spring to stretch it by a displacement $\vec{x}$ is stored as elastic potential energy

$$U = \int \vec{F}_a . d\vec{r}$$

$$= \int_0^x |\vec{F}_a| \, |d\vec{r}| \cos\theta \; ;$$

$$= \int_0^x F_a dx \cos\theta$$

6) The applied force $\vec{F}_a$ and the displacement $d\vec{r}$(i.e., here dx) are in the same direction. As, the initial position is taken as the equilibrium position or mean position, x=0 is the lower limit of integration.

$$U = \int_0^x kx dx \; ;$$

$$U = k \left[\frac{x^2}{2}\right]_0^x \; ;$$

$$U = \tfrac{1}{2} kx^2 \text{ ---------------(1)}$$

7) If the initial position is not zero, and if the mass is changed from position x_i to x_f , then the elastic potential energy is

$$U = \tfrac{1}{2} k (x_f^2 - x_i^2) \text{ --------------(2)}$$

From equations (1) and (2), we observe that the potential energy of the stretched spring depends on the force constant k and elongation or compression x.

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