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10th Standard

Mathematics

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Don Mathematics for 10th standard is released with much pride. This guide is prepared based on the Tamilnadu Government's latest new syllabus.

In this book, 'KEY POINTS' and 'IMPORTANT FORMULAE' are given for headings. All the textual questions are solved. Also enormous additional creative questions are solved. Latest Govt. model question paper is given at the end of this book.

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Wishing you all the best!

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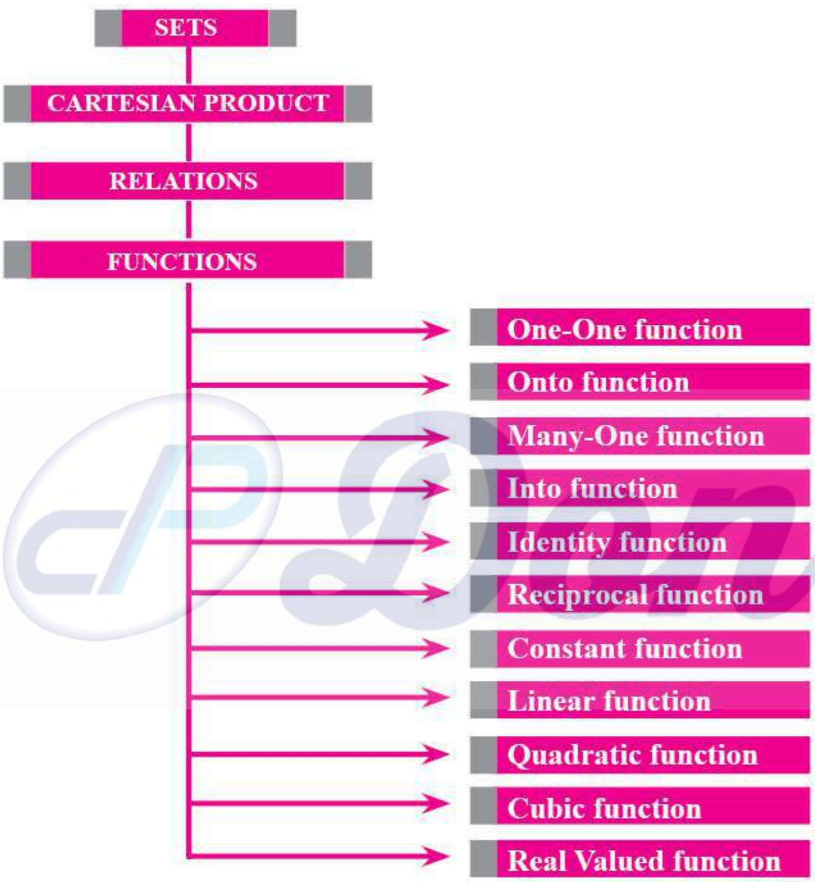
MATHEMATICAL SYMBOLS

SYMBOL	MEANING	SYMBOL	MEANING
$+$	Addition	$\sqrt{\ }$	Square root
$-$	Subtraction	$\sqrt[3]{\ }$	Cubic root
$\times$	Multiplication	$\parallel$	Parallel to
$\div$	Division	$\perp$	Perpendicular to
$\pm$	Plus or Minus	$\angle$	Angle
$=$	Equal to	$\triangle$	Triangle
$\neq$	Not equal to	$\bigcirc$	Circle
$\approx$	Equivalent to	$\square$	Square
$\simeq$	Approximately equal to	▭	Rectangle
$\equiv$ (or) $\cong$	Congruent	$\therefore$	Therefore
$\equiv$	Identically equal to	$\because$	Since (or) because
$<$	Less than	π	Pi
$\leq$	Less than or equal to	Σ	Summation
$>$	Greater than	A' (or) A^c	Complement of A
$\geq$	Greater than or equal to	$\varnothing$ (or) $\{\}$	Empty set or null set or void set
$\lvert\ \rvert$	Absolute value	$n(A)$	Number of elements in the set A
$\propto$	Proportional to	$P(A)$	Power set of A
∞	Infinity	$\lvert\lvert\lvert^{ly}$	Similarly
$\cup$	Union	Δ	Symmetric difference
$\cap$	Intersection	$\mathbb{N}$	Set of Natural numbers
$\mathbb{U}$	Universal set	$\mathbb{W}$	Set of Whole numbers
$\in$	Belongs to	$\mathbb{Z}$	Set of all Integers
$\notin$	Does not belong to	$\mathbb{R}$	Set of Real numbers
$\subset$	Proper subset of	$\mid$ (or) $:$	Such that
$\subseteq$	Subset of or is contained in	x''	x seconds
$\not\subset$	Not a proper subset of	x'	x minutes
$\nsubseteq$	Not a subset of or is not contained in	x°	x degrees
$\Longrightarrow$	Implies	$\overline{AB}$	Segment AB
$\Leftrightarrow$	Implies and is implied by	$\overrightarrow{AB}$	Ray AB
$:$	Ratio	$\overleftrightarrow{AB}$	Line AB
$.$	Decimal	x^n	x to the power n
$\%$	Percent [out of 100]		

UNIT 1 RELATIONS AND FUNCTIONS

MIND MAP



CARTESIAN PRODUCT

Key Points

- The set of all ordered pairs (a, b) such that $a \in A$ and $b \in B$ is called the Cartesian product of non-empty sets A and B and is denoted by $A \times B$.
- For three non-empty sets A, B and C, $A \times B \times C$ is the set of all ordered triplets having first element from A, second element from B and third element from C.
- If A and B are two finite sets, then the number of elements in $A \times B$, i.e., $n(A \times B) = n(A) \times n(B)$.
- If either A or B is an infinite set, then $A \times B$ is an infinite set.
- If A, B and C are finite sets, then $n(A \times B \times C) = n(A) \times n(B) \times n(C)$.
- Two ordered pairs (a, b) and (c, d) are equal if and only if $a = c$, $b = d$.
- $A \times B \neq B \times A$ unless $A = B$.

- If one of A and B or both A and B are null sets then $A \times B = \phi$.
- $A \times B$ and $B \times A$ are equivalent sets.
- For any three sets A, B and C
  (i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$
  (ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$
- $A \times (B - C) = (A \times B) - (A \times C)$
- If $A \subseteq B$, then $(A \times B) \cap (B \times A) = A^2$

Worked Examples

1.1 If A = {1, 3, 5} and B = {2, 3} then
(i) find A × B and B × A.
(ii) Is $A \times B \neq B \times A$? If not why?
(iii) Show that n (A × B) = n (B × A) = n (A) × n (B)

Sol : Given that A = {1, 3, 5} and B = {2, 3}

$A \times B = \{1, 3, 5\} \times \{2, 3\}$
$= \{(1, 2), (1, 3), (3, 2), (3, 3), (5, 2), (5, 3)\}$ ... (1)

$B \times A = \{2, 3\} \times \{1, 3, 5\}$
$= \{(2, 1), (2, 3), (2, 5), (3, 1), (3, 3), (3, 5)\}$ ... (2)

From (1) and (2) we conclude that $A \times B \neq B \times A$ as $(1, 2) \neq (2, 1)$ and $(1, 3) \neq (3, 1)$, etc.

$n(A) = 3$; $n(B) = 2$

From (1) and (2) we observe that,
$n(A \times B) = n(B \times A) = 6$
we see that, $n(A) \times n(B) = 3 \times 2 = 6$ and
$n(B) \times n(A) = 2 \times 3 = 6$

Hence,
$n(A \times B) = n(B \times A) = n(A) \times n(B) = 6$
Thus, $n(A \times B) = n(B \times A) = n(A) \times n(B)$.

1.2 If A × B = {(3, 2), (3, 4), (5, 2), (5, 4)} then find A and B.

Sol : $A \times B = \{(3, 2) (3, 4) (5, 2), (5, 4)\}$
We have A = {set of all first co-ordinates of elements of A × B}.
Therefore, A = {3, 5}
B = {set of all second co-ordinates of elements of A × B}.
Therefore, B = {2, 4}
Thus A = {3, 5} and B = {2, 4}.

1.3 Let $A = \{x \in \mathbb{N} \mid 1 < x < 4\}$, $B = \{x \in \mathbb{W} \mid 0 \leq x < 2\}$ and $C = \{x \in \mathbb{N} \mid x < 3\}$. Then verify that
(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$
(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Sol : $A = \{x \in \mathbb{N} \mid 1 < x < 4\} = \{2, 3\}$
$B = \{x \in \mathbb{W} \mid 0 \leq x < 2\} = \{0, 1\}$
$C = \{x \in \mathbb{N} \mid x < 3\} = \{1, 2\}$

(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$
$B \cup C = \{0, 1\} \cup \{1, 2\}$
$= \{0, 1, 2\}$
$A \times (B \cup C) = \{2, 3\} \times \{0, 1, 2\}$
$= \{(2, 0), (2, 1), (2, 2), (3, 0), (3, 1), (3, 2)\}$ ... (1)
$A \times B = \{2, 3\} \times \{0, 1\}$
$= \{(2, 0), (2, 1), (3, 0), (3, 1)\}$
$A \times C = \{2, 3\} \times \{1, 2\}$
$= \{(2, 1), (2, 2), (3, 1), (3, 2)\}$
$(A \times B) \cup (A \times C) = \{(2, 0), (2, 1), (3, 0), (3, 1)\} \cup \{(2, 1), (2, 2), (3, 1), (3, 2)\}$
$(A \times B) \cup (A \times C) = \{(2, 0), (2, 1), (2, 2), (3, 0), (3, 1), (3, 2)\}$ ... (2)

From (1) and (2),
$A \times (B \cup C) = (A \times B) \cup (A \times C)$ is verified.

(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$
$B \cap C = \{0, 1\} \cap \{1, 2\} = \{1\}$
$A \times (B \cap C) = \{2, 3\} \times \{1\} = \{(2, 1), (3, 1)\}$ ... (1)
$A \times B = \{2, 3\} \times \{0, 1\} = \{(2, 0), (2, 1), (3, 0), (3, 1)\}$
$A \times C = \{2 \times 3\} \times \{1, 2\}$
$= \{(2, 1), (2, 2), (3, 1), (3, 2)\}$
$(A \times B) \cap (A \times C) = \{(2, 0), (2, 1), (3, 0), (3, 1)\} \cap \{(2, 1), (2, 2), (3, 1), (3, 2)\}$
$(A \times B) \cap (A \times C) = \{(2, 1), (3, 1)\}$ ... (2)

From (1) and (2),
$A \times (B \cap C) = (A \times B) \cap (A \times C)$ is verified.

Progress Check

1. **For any two non-empty sets A and B, A × B is called as _____**

   Ans : Cartesian Product

2. **If n (A × B) = 20 and n (A) = 5 then n (B) is _____**

   Ans : n (B) = 4

3. **If A = {– 1, 1} and B = {– 1, 1} then geometrically describe the set of points of A × B**

   Ans : A × B = {(– 1, – 1), (– 1, 1), (1, –1), (1, 1)}

4. **If A, B are the line segments given by the intervals (– 4, 3) and (– 2, 3) respectively, represent the cartesian product of A and B.**

   Ans : A × B = {(– 4, – 2), (– 4, 3), (3, – 2), (3, 3)}

Thinking Corner

1. **When will A × B be equal to B × A?**

   Ans : A × B = B × A, When A = B.

Exercise 1. 1

1. **Find A × B, A × A and B × A if**

   (i) A = {2, – 2, 3} and B = {1, – 4}

   (ii) A = B = {p, q}

   (iii) A = {m, n}; $B = \phi$

   Sol :

   (i) A={2, –2, 3} and B = {1, – 4}

   A × B = {2, – 2, 3} × {1, – 4}
   = {(2, 1), (2, – 4), (– 2, 1), (– 2, – 4), (3, 1), (3, – 4)}

   A × A = {2, – 2, 3} × {2, – 2, 3}
   = {(2, 2), (2, – 2), (2, 3), (–2, 2), (– 2, – 2), (– 2, 3), (3, 2), (3, – 2), (3, 3)}

   B × A = {1, – 4} × {2, – 2, 3}
   = {(1, 2), (1, – 2), (1, 3) , (– 4, 2), (– 4, – 2), (– 4, 3)}

   (ii) A = B = {p, q}

   A × B = {p, q} × {p, q}
   = {(p, p), (p, q), (q, p), (q, q)}

   A × A = {p, q} × {p. q}
   = {(p, p), (p, q), (q, p), (q, q)}

   B × A = {p, q} × {p, q}
   = {(p, p), (p, q), (q, p), (q, q)}

   ∴ A × B = A × A = B × A

   Since A = B

   (iii) A = {m, n}, B = ϕ

   If either A or B are null sets, then A × B will also be an empty set.

   i.e., A = ϕ (or) B = ϕ

   then A × B = ϕ, B × A = ϕ

   and A × A = {m, n} × {m, n}
   = {(m, m), (m, n), (n, m), (n, n)}

2. **Let A = {1, 2, 3} and B = {x | x is a prime number less than 10}. Find A × B and B × A.**

   Sol : A = {1, 2, 3}; B = {2, 3, 5, 7}

   A × B ={(1, 2), (1, 3), (1, 5), (1, 7), (2, 2), (2, 3), (2, 5), (2, 7), (3, 2), (3, 3), (3, 5), (3, 7)}

   B × A ={(2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (5, 1), (5, 2), (5, 3), (7, 1), (7, 2), (7, 3)}

3. **If B × A= {(– 2, 3), (– 2, 4), (0, 3), (0, 4), (3, 3), (3, 4)} find A and B.**

   Sol : From B × A, All the first entries belong to the set B and all the second entries belong to A.

   ∴ A = {3, 4}

   B = {– 2, 0, 3}

4. **If A = {5, 6}, B = {4, 5, 6}, C = {5, 6, 7}, Show that A × A = (B×B) ∩ (C×C).**

   Sol : A = {5, 6}, B = {4, 5, 6}, C = {5, 6, 7}

   LHS: A × A = {5, 6} × {5, 6}
   = {(5, 5), (5, 6), (6, 5), (6, 6)}

   B × B = {4, 5, 6} × {4, 5, 6}
   = {(4, 4), (4, 5), (4, 6), (5, 4), (5, 5), (5, 6), (6, 4), (6, 5), (6, 6)}

   RHS:

   C × C = {5, 6, 7} × {5, 6, 7}
   = {(5, 5), (5, 6), (5, 7), (6, 5), (6, 6), (6, 7), (7, 5), (7, 6), (7, 7)}

   (B×B)∩(C×C) = {(5, 5), (5, 6), (6, 5), (6, 6)}

   ∴ LHS = RHS

   $A \times A = (B \times B) \cap (C \times C)$

   Hence proved.

5. Given A = {1, 2, 3}, B = {2, 3, 5}, C = {3, 4} and D = {1, 3, 5}, check if $(A\cap C)\times(B\cap D) = (A\times B)\cap(C\times D)$ is true?

Sol : A = {1, 2, 3}, B = {2, 3, 5}, C = {3, 4}, D = {1, 3, 5}

$$A\cap C = \{3\}$$
$$B\cap D = \{3, 5\}$$
$$(A\cap C)\times(B\cap D) = \{3\}\times\{3,5\} = \{(3,3),(3,5)\} \quad .....(1)$$
$$A\times B = \{(1,2),(1,3),(1,5),(2,2),(2,3),(2,5),(3,2),(3,3),(3,5)\}$$
$$C\times D = \{(3,1),(3,3),(3,5),(4,1),(4,3),(4,5)\}$$
$$(A\times B)\cap(C\times D) = \{(3,3),(3,5)\} \quad .....(2)$$

From (1) and (2), it is clear that

$\therefore \quad (A\cap C)\times(B\cap D) = (A\times B)\cap(C\times D)$

Hence it is true.

6. Let A = $\{x\in\mathbb{W}\,|\,x<2\}$, B = $\{x\in\mathbb{N}\,|\,1<x\leq 4\}$ and C = {3, 5}. Verify that

(i) $A\times(B\cup C) = (A\times B)\cup(A\times C)$

(ii) $A\times(B\cap C) = (A\times B)\cap(A\times C)$

(iii) $(A\cup B)\times C = (A\times C)\cup(B\times C)$

Sol : Given $A = \{x\in\mathbb{W}\,|\,x<2\}\Rightarrow A = \{0,1\}$

$B = \{x\in\mathbb{N}/1<x\leq 4\}\Rightarrow B = \{2,3,4\}$

C = {3, 5}

(i) $A\times(B\cup C) = (A\times B)\cup(A\times C)$

$$B\cup C = \{2,3,4,5\}$$
$$A\times(B\cup C) = \{0,1\}\times\{2,3,4,5\}$$
$$=\{(0,2),(0,3),(0,4),(0,5),(1,2),(1,3),(1,4),(1,5)\} \;...(1)$$
$$A\times B = \{0,1\}\times\{2,3,4\} = \{(0,2),(0,3),(0,4),(1,2),(1,3),(1,4)\}$$
$$A\times C = \{0,1\}\times\{3,5\} = \{(0,3),(0,5),(1,3),(1,5)\}$$
$$(A\times B)\cup(A\times C) = \{(0,2),(0,3),(0,4),(0,5),(1,2),(1,3),(1,4),(1,5)\}....(2)$$

From (1) × (2), it is clear that

$A\times(B\cup C) = (A\times B)\cup(A\times C)$

Hence verified.

(ii) $A\times(B\cap C) = (A\times B)\cap(A\times C)$

A = {0, 1}, B = {2, 3, 4}, C = {3, 5}

$$B\cap C = \{3\}$$
$$A\times(B\cap C) = \{0,1\}\times\{3\} = \{(0,3),(1,3)\} \;...(1)$$
$$A\times B = \{(0,2),(0,3),(0,4),(1,2),(1,3),(1,4)\}$$
$$A\times C = \{(0,3),(0,5),(1,3),(1,5)\}$$
$$(A\times B)\cap(A\times C) = \{(0,3),(1,3)\} \;...(2)$$

From (1) and (2), it is clear that

$A\times(B\cap C) = (A\times B)\cap(A\times C)$

Hence verified.

(iii) $(A\cup B)\times C = (A\times C)\cup(B\times C)$

A = {0, 1}, B = {2, 3, 4}, C = {3, 5}

$$(A\cup B) = \{0,1,2,3,4\}$$
$$(A\cup B)\times C = \{0,1,2,3,4\}\times\{3,5\}$$
$$= \{(0,3),(0,5),(1,3),(1,5),(2,3),(2,5),(3,3),(3,5),(4,3),(4,5)\} \;...(1)$$
$$A\times C = \{0,1\}\times\{3,5\} = \{(0,3),(0,5),(1,3),(1,5)\}$$
$$B\times C = \{2,3,4\}\times\{3,5\} = \{(2,3),(2,5),(3,3),(3,5),(4,3),(4,5)\}$$
$$(A\times C)\cup(B\times C) = \{(0,3),(0,5),(1,3),(1,5),(2,3),(2,5),(3,3),(3,5),(4,3),(4,5)\} \;...(2)$$

From (1) and (2), it is clear that

$(A\cup B)\times C = (A\times C)\cup(B\times C)$

Hence verified.

7. Let A is the set of all natural numbers less than 8, B is the set of all prime numbers less than 8, C is the set of even prime number. Verify that

(i) $(A\cap B)\times C = (A\times C)\cap(B\times C)$

(ii) A × (B – C) = (A × B) – (A × C)

Sol : Given: 'A' is the set of all natural numbers less than 8.

$A = \{1,2,3,4,5,6,7\}$

'B' is the set of all prime numbers less than 8

$B = \{2,3,5,7\}$

'C' is the set of all even prime number

$C = \{2\}$

(i) Verify

$$(A\cap B)\times C = (A\times C)\cap(B\times C)$$
$$A\cap B = \{2,3,5,7\}$$
$$(A\cap B)\times C = \{2,3,5,7\}\times\{2\} = \{(2,2),(3,2),(5,2),(7,2)\} \;...(1)$$
$$A\times C = \{1,2,3,4,5,6,7\}\times\{2\} = \{(1,2),(2,2),(3,2),(4,2),(5,2),(6,2),(7,2)\}$$
$$B\times C = \{2,3,5,7\}\times\{2\} = \{(2,2),(3,2),(5,2),(7,2)\}$$
$$(A\times C)\cap(B\times C) = \{(2,2),(3,2),(5,2),(7,2)\} \;...(2)$$

From (1) and (2) it is clear that

$(A\cap B)\times C = (A\times C)\cap(B\times C)$

Hence verified.

(ii) Verify

$$A\times(B-C) = (A\times B)-(A\times C)$$
$$B-C = \{2,3,5,7\}-\{2\} = \{3,5,7\}$$

$$A \times (B - C) = \{1, 2, 3, 4, 5, 6, 7\} \times \{3, 5, 7\}$$
$$= \{(1, 3), (1, 5), (1, 7), (2, 3), (2, 5), (2, 7), (3, 3), (3, 5), (3, 7), (4, 3), (4, 5), (4, 7), (5, 3), (5, 5), (5, 7), (6, 3), (6, 5), (6, 7), (7, 3), (7, 5), (7, 7)\} \quad \ldots (1)$$

$$A \times B = \{1, 2, 3, 4, 5, 6, 7\} \times \{2, 3, 5, 7\}$$
$$= \{(1, 2), (1, 3), (1, 5), (1, 7), (2, 2), (2, 3), (2, 5), (2, 7), (3, 2), (3, 3), (3, 5), (3, 7), (4, 2), (4, 3), (4, 5), (4, 7), (5, 2), (5, 3), (5, 5), (5, 7), (6, 2), (6, 3), (6, 5), (6, 7), (7, 2), (7, 3), (7, 5), (7, 7)\}$$

$$A \times C = \{1, 2, 3, 4, 5, 6, 7\} \times \{2\}$$
$$= \{(1,2),(2,2),(3,2),(4,2),(5,2),(6,2),(7,2)\}$$

$$(A \times B) - (A \times C) = \{(1, 3), (1, 5), (1, 7), (2, 3), (2, 5), (2, 7), (3, 3), (3, 5), (3, 7), (4, 3), (4, 5), (4, 7), (5, 3), (5, 5), (5, 7), (6, 3), (6, 5), (6, 7), (7, 3), (7, 5), (7, 7)\} \quad \ldots (2)$$

From (1) and (2), it is clear that

$$A \times (B - C) = (A \times B) - (A \times C)$$

Hence verified.

RELATIONS

Key Points

- Let A and B be two sets. Then a relation R from A to B is a subset of A × B.
  Thus $R \subseteq A \times B$
- A and B are two non-empty sets having 'm' and 'n' number of elements respectively. Then A × B will have 'mn' elements. Total number of relations is 2^{mn}.
- A relation from set A to a set B can be represented in any one of the following forms.
  (i) Roster form (ii) Set-Builder form
  (iii) An Arrow diagram (iv) A set of ordered pairs
- If R is a relation from set A to set B, then the set of all first components or co-ordinates of the ordered pairs belonging to R is called the domain of R, while the set of all second components or co-ordinates of the ordered pairs in R is called the range of R.
- In a non-empty set A, a relation from A to itself i.e., a subset of A × A is called a relation on set A.

Worked Examples

1.4 Let A = {3, 4, 7, 8} and B = {1, 7, 10}. Which of the following sets are relations from A to B?

(i) $\mathbb{R}_1 = \{(3, 7), (4, 7), (7, 10), (8, 1)\}$

(ii) $\mathbb{R}_2 = \{(3, 1), (4, 12)\}$

(iii) $\mathbb{R}_3 = \{(3, 7), (4, 10), (7, 7), (7, 8), (8, 11), (8, 7), (8, 10)\}$

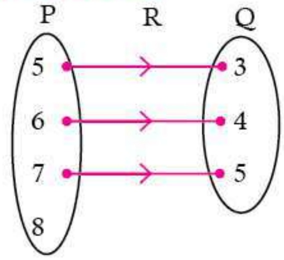
Sol : $A \times B = \{(3, 1), (3, 7), (3, 10), (4, 1), (4, 7), (4, 10), (7, 1), (7, 7), (7, 10), (8, 1), (8, 7), (8, 10)\}$

(i) We note that, $\mathbb{R}_1 \subseteq A \times B$. Thus, $\mathbb{R}_1$ is a relation from A to B.

(ii) Here, $(4, 12) \in \mathbb{R}_2$, but $(4, 12) \notin A \times B$. So, $\mathbb{R}_2$ is not a relation from A to B.

(iii) Here, $(7, 8) \in \mathbb{R}_3$, but $(7, 8) \notin A \times B$. So, $\mathbb{R}_3$ is not a relation from A to B.

1.5 The arrow diagram shows a relationship between the sets P and Q. Write the relation in (i) Set builder form (ii) Roster form (iii) What is the domain and range of $\mathbb{R}$?



Sol :

(i) Set builder form of
$\mathbb{R} = \{(x, y) \mid y = x - 2, x \in P, y \in Q\}$

(ii) Roster form $\mathbb{R} = \{(5, 3), (6, 4), (7, 5)\}$

(iii) Domain of $\mathbb{R} = \{5, 6, 7\}$;
Range of $\mathbb{R} = \{3, 4, 5\}$

Progress Check

1. Which of the following are relations from A to B?

(i) {(1, b), (1, c), (3, a), (4, b)}

(ii) {(1, a), (b, 4), (c, 3)}

(iii) {(1, a), (a, 1), (2, b), (b, 2)}

Ans : Let A = {1, 2, 3, 4} and B = {a, b, c}

Relations from A to B

(i) {(1, b), (1, c), (3, a), (4, b)}

2. Which of the following are relations from B to A?

(i) {(c, a), (c, b), (c, 1})

(ii) {(c, 1), (c, 2), (c, 3), (c, 4)}

(iii) {(a, 4), (b, 3), (c, 2})

Ans : Let A = {1, 2, 3, 4} and B = {a, b, c}

Relations from B to A

(iii) {(a, 4), (b, 3), (c, 2)}

Exercise 1. 2

1. Let A = {1, 2, 3, 7} and B = {3, 0, – 1, 7}, which of the following are relation from A to B?

(i) $\mathbb{R}_1$ = {(2, 1), (7, 1)}

(ii) $\mathbb{R}_2$ = {(– 1, 1)}

(iii) $\mathbb{R}_3$ = {(2, – 1), (7, 7), (1, 3)}

(iv) $\mathbb{R}_4$ = {(7, – 1), (0, 3), (3, 3), (0, 7)}

Sol : A = {1, 2, 3, 7}, B = {3, 0, –1, 7}

A × B = {(1, 3), (1, 0), (1, –1), (1, 7), (2, 3), (2, 0), (2, –1), (2, 7), (3, 3), (3, 0), (3, –1), (3, 7), (7, 3), (7, 0), (7, –1), (7, 7)}

(i) R_1 = {(2, 1), (7, 1)}

Since (2, 1) and (7, 1) are not the elements of A × B, R_1 is not a relation from A to B. Moreover $1 \notin B$.

(ii) R_2 = {(– 1, 1)}, $(-1,1) \notin A \times B$,

∴ R_2 is not a relation from A to B.

But $(-1,1) \in (B \times A)$ as $-1 \in B$ and $1 \in A$.

(iii) R_3 = {(2, – 1), (7, 7), (1, 3)}

It is clear that $R_3 \subseteq A \times B$

∴ R_3 is a relation from A to B.

(iv) R_4 = {(7, – 1), (0, 3), (3, 3), (0, 7)}

In this (0, 3) and (0, 7) $\in R_4$

But (0, 3) and (0, 7) are not the elements of A × B. Hence R_4 is not a relation from A to B.

2. Let A = {1, 2, 3, 4,..., 45} and R be the relation defined as "is square of" on A. Write $\mathbb{R}$ as a subset of A × A. Also, find the domain and range of $\mathbb{R}$.

Sol : A= {1, 2, 3, 4, ...,45}

Relation is "is square of" and $A \to A$ on A

A × A={(1, 1), (1, 2), (1, 3), (1, 4).....(45, 45)}

The square of '1' is $1 \in A$ and $(1,1) \in A \times A$

The square of 2 is $4 \in A$ and $(4,2) \in A \times A$

The square of 3 is $9 \in A$ and $(9,3) \in A \times A$

The square of 4 is $16 \in A$ and $(16,4) \in A \times A$

The square of 5 is $25 \in A$ and $(25,5) \in A \times A$

The square of 6 is $36 \in A$ and $(36,6) \in A \times A$

The square of 7 is $49 \notin A$.

R ={(1, 1), (4, 2), (9, 3), (16, 4), (25, 5), (36, 6)}

Domain of R = {1, 4, 9, 16, 25, 36}

Range of R = {1, 2, 3, 4, 5, 6}

3. A Relation $\mathbb{R}$ is given by the set {(x, y) / y = x + 3, $x \in$ {0, 1, 2, 3, 4, 5}}. Determine its domain and range.

Sol :

Given Set = {(x, y) / y = x + 3, $x \in$ {0, 1, 2, 3, 4, 5}}

When x = 0, y = 0 + 3 = 3

When x = 1, y = 1 + 3 = 4

When x = 2, y = 2 + 3 = 5

When x = 3, y = 3 + 3 = 6

When x = 4, y = 4 + 3 = 7

When x = 5, y = 5 + 3 = 8

∴ Relation R = {(0, 3), (1, 4), (2, 5), (3, 6), (4, 7), (5, 8)}

Domain = {0, 1, 2, 3, 4, 5}

Range = {3, 4, 5, 6, 7, 8}

4. Represent each of the given relations by (a) an arrow diagram, (b) a graph and (c) a set in roster, wherever possible.

(i) {(x, y) | x = 2y, $x \in$ {2, 3, 4, 5}, $y \in$ {1, 2, 3, 4}

(ii) {(x, y) | y = x + 3, x, y are natural numbers < 10}

Sol :

(i) Given Set-Builder form

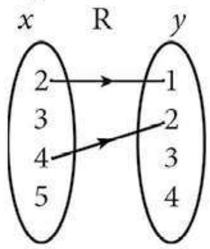
{(x, y) / x = 2y, $x \in$ {2, 3, 4, 5}, $y \in$ {1, 2, 3, 4}}

x = 2y

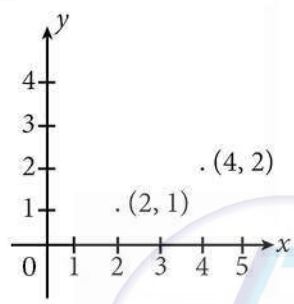
When y = 1, x = 2 (1) = 2 ∈ x

When $y = 2,\ x = 2\,(2) = 4 \in x$
When $y = 3,\ x = 2\,(3) = 6 \notin x$
When $y = 4,\ x = 2\,(4) = 8 \notin x$
$\therefore$ Relation $R = \{(2, 1), (4, 2)\}$

(a) Arrow diagram



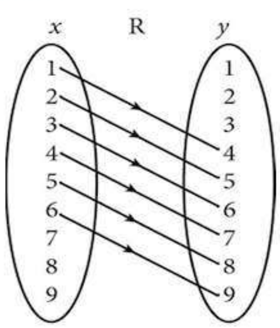
(b) Graph



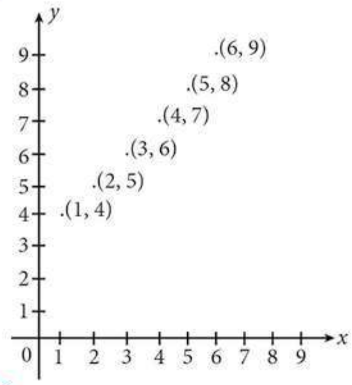
(c) Roster form $R = \{(2,1), (4,2)\}$

(ii) Given set
$\{(x, y) / y = x + 3, x, y \text{ are natural numbers} < 10\}$
When $x = 1,\ y = 1 + 3 = 4$
When $x = 2,\ y = 2 + 3 = 5$
When $x = 3,\ y = 3 + 3 = 6$
When $x = 4,\ y = 4 + 3 = 7$
When $x = 5,\ y = 5 + 3 = 8$
When $x = 6,\ y = 6 + 3 = 9$
When $x = 7,\ y = 7 + 3 = 10$ is not possible
Since x and y are less than 10.
$\therefore$ Relation $R = \{(1, 4), (2, 5), (3, 6), (4, 7), (5, 8), (6, 9)\}$

(a) Arrow diagram



(b) Graph



(c) Roster form

$R = \{(1, 4), (2, 5), (3, 6), (4, 7), (5, 8), (6, 9)\}$

5. A company has four categories of employees given by Assistants (A), Clerks (C), Managers (M) and an Executive Officer (E). The company provide ₹ 10,000, ₹ 25,000, ₹ 50,000 and ₹ 1,00,000 as salaries to the people who work in the categories A, C, M and E respectively. If A_1, A_2, A_3, A_4 and A_5 were Assistants; C_1, C_2, C_3, C_4 were Clerks; M_1, M_2, M_3 were managers and E_1, E_2 were Executive officers and if the relation $\mathbb{R}$ is defined by $x\mathbb{R}y$, where x is the salary given to person y, express the relation $\mathbb{R}$ through an ordered pair and an arrow diagram.

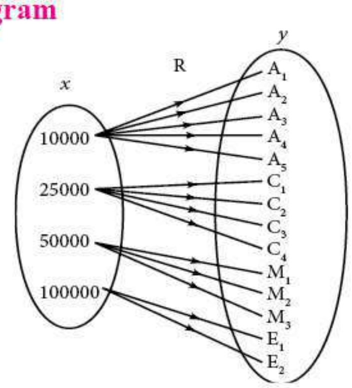
Sol :

Ordered Pair : The Domain of the relation is about the salaries given to person.

Relation is $R = \{(10000, A_1), (10000, A_2), (10000, A_3), (10000, A_4), (10000, A_5), (25000, C_1), (25000, C_2), (25000, C_3), (25000\ C_4), (50000, M_1), (50000, M_2), (50000, M_3), (100000, E_1), (100000, E_2)\}$

Relation R defined by x R y
'x' is the salary given to person 'y'.

Arrow diagram



FUNCTIONS

Key Points

- Let A and B be two non-empty sets, then a relation from A to B i.e., a subset of $A \times B$ is called a function from A to B, if
  (i) for each $a \in A$ there exists $b \in B$ such that $(a, b) \in f$
  (ii) $(a, b) \in f$ and $(a, c) \in f \Rightarrow b = c$
- A function 'f' from a set 'A' to set 'B' associates each element of set A to a unique element of set B.
- Let $f : A \to B$. Then, the set A is domain of 'f' and B is co-domain of 'f'. The set of all images of elements of A is known as the range of 'f' or image set of A under f and is denoted by f(A).

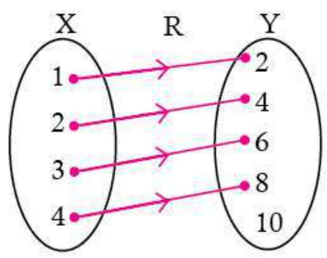
$$\text{i.e., } f(A) = \{f(x) : x \in A\} = \text{Range of } f$$
$$\therefore f(A) \subseteq B$$

- Not every curve in the cartesian plane is the graph of a function.
- Vertical line test: A set of points in the cartesian plane is the graph of a function if and only if no vertical straight line intersects the curve more than once.

Worked Examples

1.6 Let X = {1, 2, 3, 4} and Y = {2, 4, 6, 8, 10} and $\mathbb{R}$ = {(1, 2), (2, 4), (3, 6), (4, 8)}. Show that $\mathbb{R}$ is a function and find its domain, co-domain and range?

Sol : Pictorial representation of $\mathbb{R}$ is given in the figure. From the diagram, we see that for each $x \in X$, there exists only one $y \in Y$. Thus all elements in X have only image in Y. Therefore $\mathbb{R}$ is a function.



Domain X = {1, 2, 3, 4};
Co-domain Y = {2, 4, 6, 8, 10};
Range of f = {2, 4, 6, 8}.

1.7 A relation 'f' is defined by $f(x) = x^2 - 2$ where, $x \in \{-2, -1, 0, 3\}$

(i) List the elements of f (ii) Is f a function?

Sol : $f(x) = x^2 - 2$ where $x \in \{-2, -1, 0, 3\}$

(i)
$$f(-2) = (-2)^2 - 2 = 2$$
$$f(-1) = (-1)^2 - 2 = -1$$
$$f(0) = (0)^2 - 2 = -2$$
$$f(3) = (3)^2 - 2 = 7$$

Therefore, f = {(– 2, 2), (– 1, – 1), (0, – 2), (3, 7)}

(ii) We note that each element in the domain of f has a unique image. Therefore f is a function.

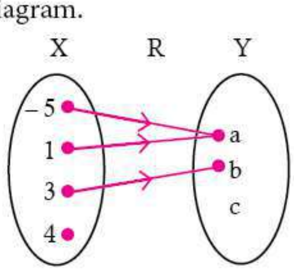
1.8 If X = {– 5, 1, 3, 4} and Y = {a, b, c}, then which of the following relations are functions from X to Y?

(i) $\mathbb{R}_1$ = {(– 5, a), (1, a), (3, b)}
(ii) $\mathbb{R}_2$ = {(– 5, b), (1, b), (3, a), (4, c)}
(iii) $\mathbb{R}_3$ = {(– 5, a), (1, a), (3, b), (4, c), (1, b)}

Sol :

(i) $\mathbb{R}_1$ = {(– 5, a), (1, a), (3, b)}

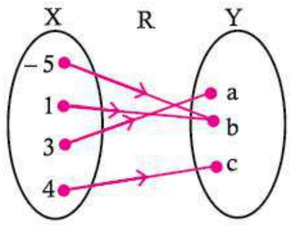
We may represent the relation $\mathbb{R}_1$ in an arrow diagram.



$\mathbb{R}_1$ is not a function as $4 \in X$ does not have an image in Y.

(ii) $\mathbb{R}_2 = \{(-5, b), (1, b), (3, a), (4, c)\}$

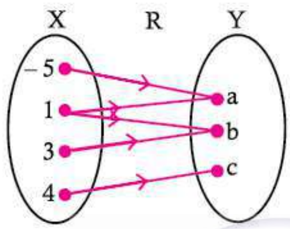
Arrow diagram of $\mathbb{R}_2$ is shown in Figure.



$\mathbb{R}_2$ is a function as each element of X has an unique image in Y.

(iii) $\mathbb{R}_3 = \{(-5, a), (1, a), (3, b), (4, c), (1, b)\}$

Representing $\mathbb{R}_3$ in an arrow diagram.



$\mathbb{R}_3$ is not a function as $1 \in X$ has two images $a \in Y$ and $b \in Y$.

Note that the image of an element should always be unique.

1.9 **Given $f(x) = 2x - x^2$, find**

(i) $f(1)$ (ii) $f(x+1)$ (iii) $f(x) + f(1)$

Sol :

(i) Replacing x with 1, we get

$$f(1) = 2(1) - (1)^2 = 2 - 1 = 1$$

(ii) Replacing x with x + 1, we get

$$\begin{aligned} f(x+1) &= 2(x+1) - (x+1)^2 \\ &= 2x + 2 - (x^2 + 2x + 1) \\ &= -x^2 + 1 \end{aligned}$$

(iii) $f(x) + f(1) = (2x - x^2) + 1 = -x^2 + 2x + 1$

[Note that $f(x) + f(1) \neq f(x+1)$.

In general $f(a+b)$ is not equal to $f(a) + f(b)$]

Progress Check

1. Relations are subsets of _____; Functions are subsets of _____.

Ans : Cartesian product; Relations.

2. True or False: All the elements of a relation should have images.

Ans : False

3. True or False: All the elements of a function should have images.

Ans : True

4. True or False: If $\mathbb{R} : A \to B$ is a relation then the domain of $\mathbb{R} = A$

Ans : True

5. If $f: \mathbb{N} \to \mathbb{N}$ is defined as $f(x) = x^2$ the pre-image(s) of 1 and 2 are _____ and _____.

Ans : 1 and None

6. The difference between relation and function is _____.

Ans : Every function is a Relation, but the relation need not be a function.

7. Let A and B be two non-empty finite sets. Then which one among the following two collection is large?

(i) The number of relations between A and B.

(ii) The number of functions between A and B.

Ans :

(i) The number of relations between A and B is large.

(ii) Number of relation is always greater than number of functions.

Thinking Corner

1. Is the relation representing the association between planets and their respective moons a function?

Ans : Yes.

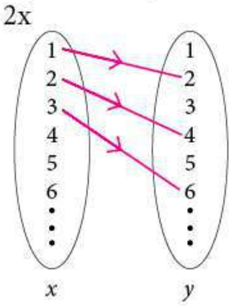
Exercise 1. 3

1. Let $f = \{(x, y) \mid x, y \in \mathbb{N} \text{ and } y = 2x\}$ be a relation on N. Find the domain, co-domain and range. Is this relation a function?

Sol :

$f = \{(x, y)/x, y \in \mathbb{N} \text{ and } y = 2x\}$

Given that y = 2x



'x' is always a natural number. Domain is the set of all first entries.

So, Domain-Set of natural numbers = N and y is always an even number as y = 2x

Range = Set of even natural numbers

Co-domain = Set of natural numbers = N

Here, the first elements (x) are having unique images. So, this relation is a function.

2. Let X = {3, 4, 6, 8}. Determine whether the relation $\mathbb{R} = \{(x, f(x)) \mid x \in X, f(x) = x^2 + 1\}$ is a function from X to $\mathbb{N}$?

Sol : Given X = {3, 4, 6, 8}

Relation $R = \{(x, f(x)) / x \in X, f(x) = x^2 + 1\}$

When x = 3, $f(3) = (3)^2 + 1 = 9 + 1 = 10 \in N$

When x = 4, $f(4) = (4)^2 + 1 = 16 + 1 = 17 \in N$

When x = 6, $f(6) = (6)^2 + 1 = 36 + 1 = 37 \in N$

When x = 8, $f(8) = (8)^2 + 1 = 64 + 1 = 65 \in N$

R = {(3, 10), (4, 17), (6, 37), (8, 65)}

Since, all the elements of X are having natural numbers as images, it is a function from X to N.

3. Given the function f : $x \to x^2 - 5x + 6$, evaluate

(i) f (– 1) **(ii) f (2a)**

(iii) f (2) **(iv) f (x – 1)**

Sol : $f(x) = x^2 - 5x + 6$

(i) $f(-1) = (-1)^2 - 5(-1) + 6 = 1 + 5 + 6 = 12$

(ii) $f(2a) = (2a)^2 - 5(2a) + 6 = 4a^2 - 10a + 6$

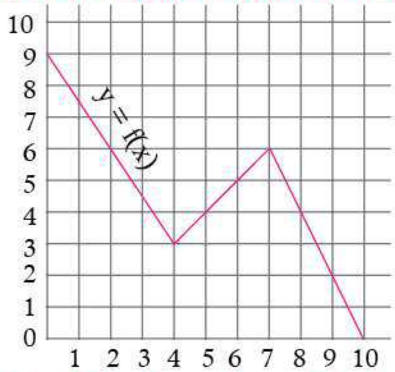
(iii) $f(2) = (2)^2 - 5(2) + 6 = 4 - 10 + 6 = 0$

(iv) $f(x-1) = (x-1)^2 - 5(x-1) + 6$

$= x^2 - 2x + 1 - 5x + 5 + 6$

$= x^2 - 7x + 12$

4. A graph representing the function f (x) is given below figure. From figure it is clear that f (9) = 2.



(i) Find the following values of the function

(a) f (0) **(b) f (7)**

(c) f (2) **(d) f (10)**

(ii) For what value of x is f (x) = 1?

(iii) Describe the following (a) Domain (b) Range.

(iv) What is the image of 6 under f?

Sol :

(i) (a) f (0) = 9

(b) f (7) = 6

(c) f (2) = 6

(d) f (10) = 0

(ii) For what value of x is f (x) = 1

From the graph, it is known that when x = 9.5, f (9.5) = 1

(iii) (a) Domain = $\{x / 0 \le x \le 10, x \in R\}$

(b) Range = $\{x / 0 \le x \le 9, x \in R\}$

(iv) Image of '6' under f is '5'.

5. Let f (x) = 2x + 5. If $x \neq 0$ then find $\frac{f(x+2) - f(2)}{x}$.

Sol :

$f(x) = 2x + 5, \quad x \neq 0$

$$\frac{f(x+2) - f(2)}{x} = \frac{[2(x+2)+5] - [2(2)+5]}{x}$$

$$= \frac{2x+4+5-9}{x} = \frac{2x+9-9}{x}$$

$$= \frac{2x}{x} = 2$$

6. A function f is defined by f (x) = 2x – 3

(i) find $\frac{f(0) + f(1)}{2}$

(ii) find x such that f (x) = 0

(iii) find x such that f (x) = x.

(iv) find x such that f (x) = f (1 – x).

Sol : f (x) = 2x – 3

(i) $$\frac{f(0) + f(1)}{2} = \frac{[2(0)-3] + [2(1)-3]}{2}$$

$$= \frac{0-3+2-3}{2} = \frac{2-6}{2}$$

$$= \frac{-4}{2} = -2$$

(ii) Given f (x) = 0

$\therefore$ 2x – 3 = 0

2x = 3 $\Rightarrow$ x = 3/2

(iii) Given f (x) = x

2x – 3 = x

2x – x = 3 $\Rightarrow$ x = 3

(iv) Given f (x) = f (1 – x)

2x – 3 = 2 (1 – x) – 3

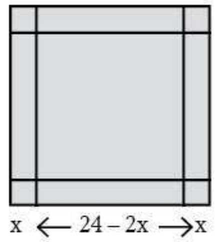
2x – 3 = 2 – 2x – 3

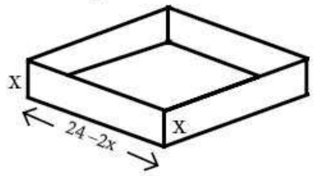
2x + 2x = 2 – 3 + 3

4x = 2

x = 2/4 = 1/2

7. An open box is to be made from a square piece of material, 24 cm on a side, by cutting equal squares from the corners and turning up the sides as shown figure. Express the volume V of the box as a function of x.





Sol : From the diagram, the solid is a cuboid.

Volume of cuboid = length × breadth × height

where $l = 24 - 2x$, $b = 24 - 2x$, $h = x$

$\therefore$ Volume $V(x) = (24 - 2x)(24 - 2x)x$

$V(x) = x(24 - 2x)^2, x > 0$

$= 4x^3 - 96x^2 + 576x, x > 0$

So, the domain is $0 < x < 12$

8. A function f is defined by $f(x) = 3 - 2x$. Find x such that $f(x^2) = (f(x))^2$.

Sol : $f(x) = 3 - 2x$

Given $f(x^2) = [f(x)]^2$

$3 - 2x^2 = (3 - 2x)^2$

$3 - 2x^2 = 9 - 12x + 4x^2$

$6x^2 - 12x + 6 = 0$

$6(x^2 - 2x + 1) = 0$

$(x - 1)^2 = 0 \Rightarrow x = 1.$

9. A plane is flying at a speed of 500 km per hour. Express the distance d travelled by the plane as function of time t in hours.

Sol : Let the distance be 'd'

Speed = 500 km/hr,

Time = 't' hours

$\therefore$ Distance = Speed × time

$d(t) = 500\,t$

10. The data in the adjacent table depicts the length of a woman's forehand and her corresponding height. Based on this data, a student finds a relationship between the height (y) and the forehand length (x) as y = ax + b, where a, b are constants.

Length *x* of forehand (in cm)	Height y (in inch)
35	56
45	65
50	69.5
55	74

(i) Check if this relation is a function.

(ii) Find a and b.

(iii) Find the height of a woman whose forehand length is 40 cm.

(iv) Find the length of forehand of a woman if her height is 53.3 inches.

Sol : $y = ax + b$; x = forehand length; y = height

x	y
35	56
45	65
50	69.5
55	74

For all the x-values, there is an image which is 'y'. Moreover, the difference between two consecutive 'y' values is constant.

$\therefore$ In $y = ax + b$,

(i) the Relation

$R = \{(35, 56), (45, 65), (50, 69.5), (55, 74)\}$ is a function.

(ii) In $y = ax + b$

when $x = 35, y = 56$

$\Rightarrow 56 = 35a + b$ ....(1)

when $x = 45, y = 65$

$\Rightarrow 65 = 45a + b$ ....(2)

Solving (1) and (2), we get $a = 0.90$ and $b = 24.5$

(iii) Given, forehand length is 40 cm

i.e., when $x = 40, y = ax + b$

So, $y = (0.90)(40) + 24.5 = 60.5$

$\therefore$ Height of woman is 60.5 inches

(iv) Given height is 53.3 inches

ie. when $y = 53.3, x = ?$

$\Rightarrow 53.3 = 0.9x + 24.5$

$53.3 - 24.5 = 0.9x$

$\Rightarrow x = \dfrac{28.8}{0.9} = 32$

length of forehand is 32 cm.

REPRESENTATION OF FUNCTIONS

Worked Examples

1. 10 **Using vertical line test, determine which of the following curves (figure a, b, c, d) represent a function?**

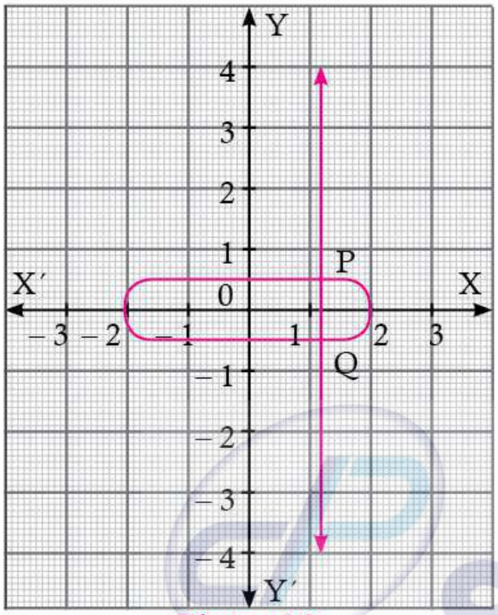


Figure (a)

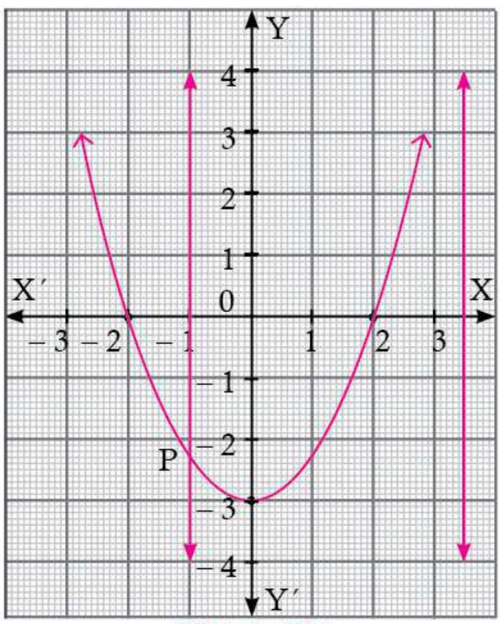


Figure (b)

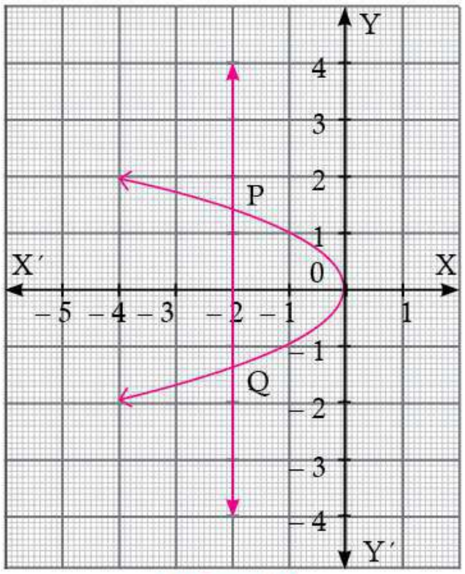


Figure (c)

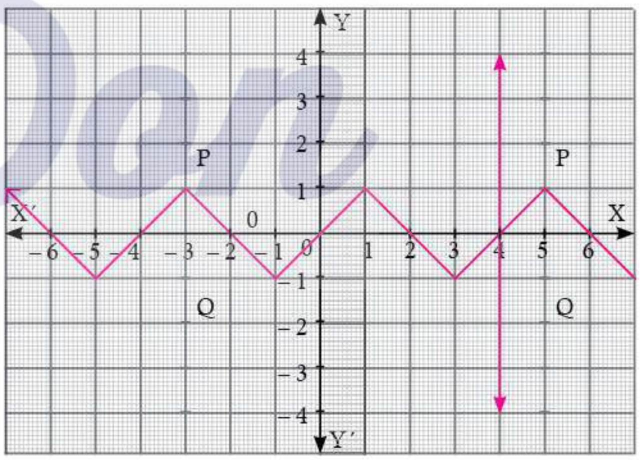


Figure (d)

Sol :

The curves in figure (a) and (c) do not represent a function as the vertical lines meet the curves in two points P and Q.

The curves in figure (b) and (d) represent a function as the vertical lines meet the curve in at most one point.

1. 11 **Let A = {1, 2, 3, 4} and B = {2, 5, 8, 11, 14} be two sets. Let f : $A \to B$ be a function given by f (x) = 3x – 1. Represent this function as**

(i) by arrow diagram

(ii) in a table form

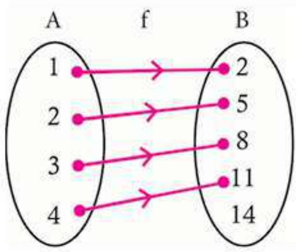
(iii) as a set of ordered pairs

(iv) in a graphical form

Sol :
$A = \{1, 2, 3, 4\}$; $B = \{2, 5, 8, 11, 14\}$;
$f(x) = 3x - 1$
$f(1) = 3(1) - 1 = 3 - 1 = 2$
$f(2) = 3(2) - 1 = 6 - 1 = 5$
$f(3) = 3(3) - 1 = 9 - 1 = 8$
$f(4) = 4(3) - 1 = 12 - 1 = 11$

(i) Arrow diagram

Let us represent the function $f : A \to B$ by an arrow diagram



(ii) Table form

The given function f can be represented in a tabular form as given below

x	1	2	3	4
f (x)	2	5	8	11

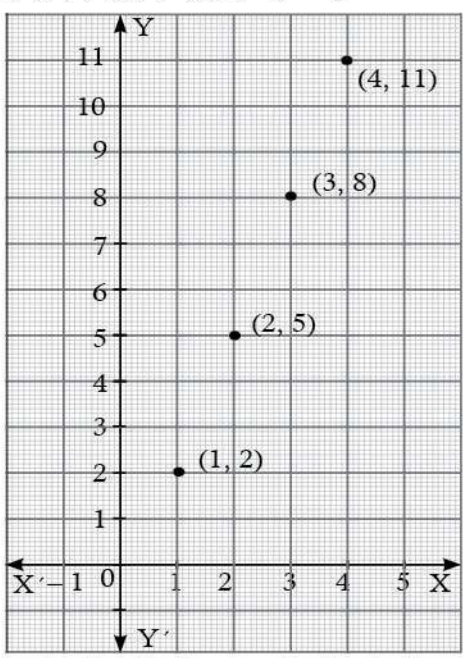
(iii) Set of ordered pairs

The function f can be represented as a set of ordered pairs as

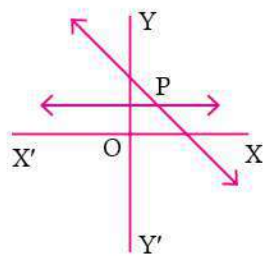
$f = \{(1, 2), (2, 5), (3, 8), (4, 11)\}$

(iv) Graphical form

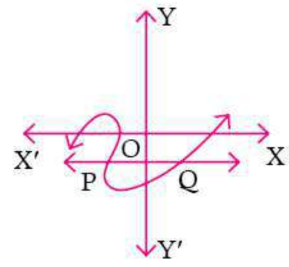
In the adjacent xy-plane the points (1, 2), (2, 5), (3, 8), (4, 11) are plotted.



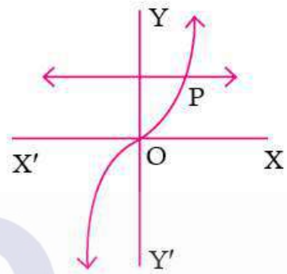
1. 12. Using horizontal line test fig (a), fig (b), fig (c), determine which of the following functions are one-one.



(a)



(b)



(c)

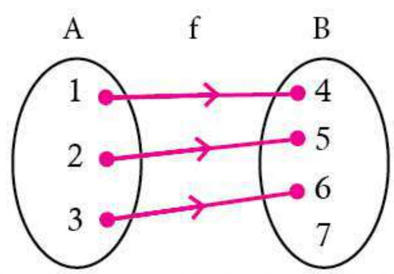
Sol : The curves in fig (a) and fig (c), represent a one-one function as the horizontal lines meet the curves in only one point P.

The curve in fig (b) does not represent a one-one function, since, the horizontal line meet the curve in two points P and Q.

1. 13. Let $A = \{1, 2, 3\}$, $B = \{4, 5, 6, 7\}$ and $f = \{(1, 4), (2, 5), (3, 6)\}$ be a function from A to B. Show that f is one-one but not onto function.

Sol : $A = \{1, 2, 3\}$, $B = \{4, 5, 6, 7\}$;
$f = \{(1, 4), (2, 5), (3, 6)\}$

Then f is a function from A to B and for different elements in A, there are different images in B. Hence f is one-one function. Note that the element 7 in the co-domain does not have any pre-image in the domain. Hence f is not onto.



Therefore f is one-one but not an onto function.

1. 14. If $A = \{-2, -1, 0, 1, 2\}$ and $f: A \to B$ is an onto function defined by $f(x) = x^2 + x + 1$ then find B.

Sol : Given $A = \{-2, -1, 0, 1, 2\}$ and

$$f(x) = x^2 + x + 1.$$
$$f(-2) = (-2)^2 + (-2) + 1 = 3$$
$$f(-1) = (-1)^2 + (-1) + 1 = 1$$
$$f(0) = 0^2 + 0 + 1 = 1$$
$$f(1) = 1^2 + 1 + 1 = 3$$
$$f(2) = 2^2 + 2 + 1 = 7$$

Since, f is an onto function, range of f = B Co-domain.

Therefore, $B = \{1, 3, 7\}$.

1. 15 Let f be a function $f: \mathbb{N} \to \mathbb{N}$ defined by $f(x) = 3x + 2,\ x \in \mathbb{N}$

(i) Find the images of 1, 2, 3

(ii) Find the pre-images of 29, 53

(iii) Identify the type of function

Sol : The function $f: \mathbb{N} \to \mathbb{N}$ defined by $f(x) = 3x + 2$

(i) If x = 1, $f(1) = 3(1) + 2 = 5$

If x = 2, $f(2) = 3(2) + 2 = 8$

If x = 3, $f(3) = 3(3) + 2 = 11$

The images of 1, 2, 3 are 5, 8, 11 respectively.

(ii) If x is the pre-image of 29, then

$$f(x) = 29.$$

Hence $3x + 2 = 29 \Rightarrow 3x = 27 \Rightarrow x = 9$.

Similarly, if x is the pre-image of 53, then $f(x) = 53$. Hence $3x + 2 = 53$

$3x = 51 \Rightarrow x = 17$.

Thus the pre-images of 29 and 53 are 9 and 17 respectively.

(iii) Since different elements of $\mathbb{N}$ have different images in the co-domain, the function f is one-one function.

The co-domain of f is $\mathbb{N}$.

But the range of f = {5, 8, 11, 14, 17 , ...} is a subset of $\mathbb{N}$.

Therefore f is not an onto function. That is, f is an into function.

Thus f is one-one and into function.

1. 16 Forensic scientists can determine the height (in cms) of a person based on the length of their thigh bone. They usually do so using the function h (b) = 2. 47 b + 54 .10 where b is the length of the thigh bone.

(i) Check if the function h is one-one

(ii) Also find the height of a person if the length of his thigh bone is 50 cms.

(iii) Find the length of the thigh bone if the height of a person is 147.96 cms.

Sol :

(i) To check if h is one - one, we assume that $h(b_1) = h(b_2)$.

Then we get,

$$2.47\, b_1 + 54.10 = 2.47\, b_2 + 54.10$$
$$2.47\, b_1 = 2.47\, b_2 \Rightarrow b_1 = b_2$$

Thus, $h(b_1) = h(b_2) \Rightarrow b_1 = b_2$.

So, the function h is one - one.

(ii) If the length of the thigh bone b = 50, then the height is

$h(50) = (2.47 \times 50) + 54.10 = 177.6$ cms.

(iii) If the height of a person is 147. 96 cms, then h (b) = 147. 96 and so the length of the thigh bone is given by 2. 47 b + 54. 10 = 147. 96.

$$b = \frac{93.86}{2.47} = 38$$

Therefore, the length of the thigh bone is 38 cms.

1. 17. Let f be a function from $\mathbb{R}$ to $\mathbb{R}$ defined by $f(x) = 3x - 5$. Find the values of a and b given that (a, 4) and (1, b) belong to f.

Sol : $f(x) = 3x - 5$ can be written as

$f = \{(x, 3x - 5) \mid x \in \mathbb{R}\}$

(a, 4) means the image of a is 4.

That is, $f(a) = 4$

$$3a - 5 = 4$$
$$\Rightarrow a = 3$$

(1, b) means the image of 1 is b.

That is, $f(1) = b$

$$3(1) - 5 = b \Rightarrow b = -2$$

1. 18. The distance S (in kms) travelled by a particle in time 't' hours is given by $S(t) = \frac{t^2 + t}{2}$. Find the distance travelled by the particle after

(i) three and half hours

(ii) eight hours and fifteen minutes.

Sol : The distance travelled by the particle is given by

$$S(t) = \frac{t^2 + t}{2}$$

(i) t = 3.5 hours. Therefore,

$$S(3.5) = \frac{(3.5)^2 + 3.5}{2} = \frac{15.75}{2} = 7.875$$

The distance travelled in 3. 5 hours is 7. 875 kms.

(ii) t = 8.25 hours. Therefore,

$$S(8.25) = \frac{(8.25)^2 + 8.25}{2} = \frac{76.3125}{3} = 38.15625$$

The distance travelled in 8.25 hours is 38.16 kms, approximately.

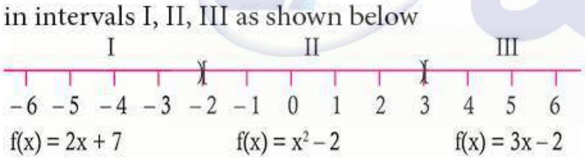
1. 19. If the function f: $R \to R$ defined by

$$f(x) = \begin{cases} 2x+7, x<-2 \\ x^2-2, -2 \le x < 3 \\ 3x-2, x \ge 3 \end{cases}$$

(i) f (4) **(ii) f (– 2)**

(iii) f (4) + 2 f (1) **(iv) $\frac{f(1)-3f(4)}{f(-3)}$**

Sol : The function f is defined by three values in intervals I, II, III as shown below



For a given value of x = a, find out the interval at which the point a is located, there after find f (a) using the particular value defined in that interval.

(i) First, we see that, x = 4 lie in the third interval. Therefore,

$f(x) = 3x - 2; f(4) = 3(4) - 2 = 10$

(ii) x = – 2 lies in the second interval. Therefore,

$f(x) = x^2 - 2; f(-2) = (-2)^2 - 2 = 2$

(iii) From **(i)**, f (4) = 10.

To find f (1), first we see that x = 1 lies in the second interval.

Therefore,

$f(x) = x^2 - 2 \Rightarrow f(1) = 1^2 - 2 = -1$

Therefore, $f(4) + 2f(1) = 10 + 2(-1) = 8$

(iv) We know that f (1) = – 1 and f (4) = 10.

For finding f (– 3), we see that x = – 3, lies in the first interval.

Therefore, $f(x) = 2x + 7$; thus,

$f(-3) = 2(-3) + 7 = 1$

Hence, $\frac{f(1)-3f(4)}{f(-3)} = \frac{-1-3(10)}{1} = -31$

Progress Check

State True (or) False

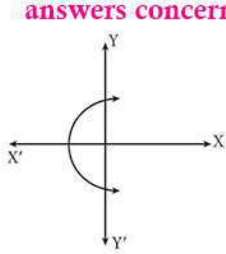
1. **All one - one functions are onto functions.**
   Ans : False
2. **There will be no one - one function from A to B when n (A) = 4, n (B) = 3.**
   Ans : True
3. **All onto functions are one - one functions.**
   Ans : False
4. **There will be no onto function from A to B when n (A) = 4, n (B) = 5.**
   Ans : True
5. **If f is a bijection from A to B, then n (A) = n (B).**
   Ans : True
6. **If n (A) = n (B), then f is a bijection from A to B.**
   Ans : False
7. **All constant functions are bijections.**
   Ans : False

Thinking Corner

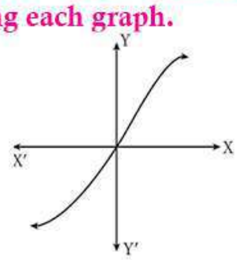
1. **Can there be a one to many function?**
   Ans : There cannot be a one to many function as the elements in Co-domain should have only one pre-image in the domain.
2. **Is an identity function one-one function?**
   Ans : Yes. It is one-to-one function.

Exercise 1. 4

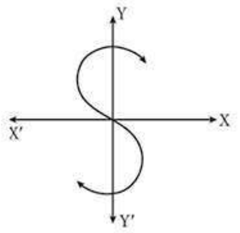
1. **Determine whether the graph given below represent functions. Give reason for your answers concerning each graph.**



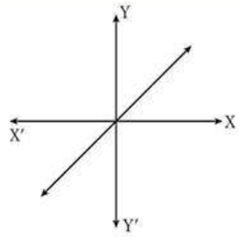
(i)



(ii)



(iii)



(iv)

Sol :

(i) It is not a function. Since, a vertical line intersects the curve in two points.

(ii) It is a function. Any vertical line drawn, will intersect the curve at only one point.

(iii) It is not a function. Vertical line intersecting the curve at two points.

(iv) It is a function. Vertical line intersects the curve at only one point.

2. Let f : $A \to B$ be a function defined by f (x) = $\frac{x}{2} - 1$, where A = {2, 4, 6, 10, 12}, B = {0, 1, 2, 4, 5, 9}. Represent f by
(i) set of ordered pairs; (ii) a table;
(iii) an arrow diagram; (iv) a graph

Sol : f : $A \to B$, $f(x) = \frac{x}{2} - 1$

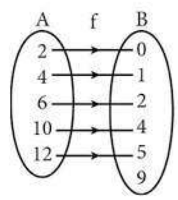
A = {2, 4, 6, 10, 12}, B = {0, 1, 2, 4, 5, 9}

(i) Set of ordered pairs
={(2, 0), (4, 1), (6, 2), (10, 4), (12, 5)}

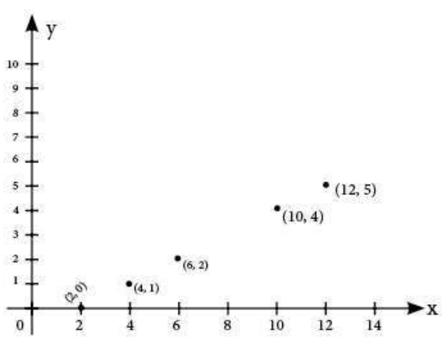
(ii) Table

x	2	4	6	10	12
f (x)	0	1	2	4	5

(iii) Arrow diagram



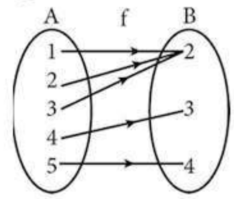
(iv) Graph



3. Represent the function f = {(1, 2), (2, 2), (3, 2), (4, 3), (5, 4)} through
(i) an arrow diagram
(ii) a table form (iii) a graph

Sol : Given function f = {(1, 2), (2, 2), (3, 2), (4, 3), (5, 4)}

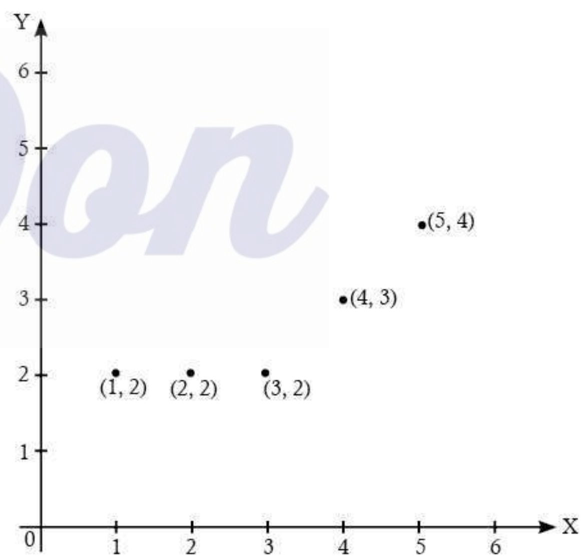
(i) Arrow diagram



(ii) Table form

x	1	2	3	4	5
f (x)	2	2	2	3	4

(iii) Graph



4. Show that the function f : $\mathbb{N} \to \mathbb{N}$ be defined by f (x) = 2x – 1 is one - one but not onto.

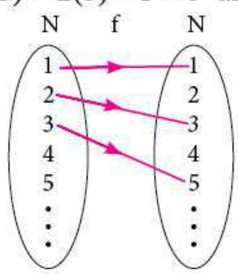
Sol : Given function f : $\mathbb{N} \to \mathbb{N}$

f (x) = 2x – 1

This function maps every element from the domain to element that is twice minus one the original. 2x – 1 is always an odd number when $x \in N$.

Clearly, each element from the domain is mapped to different element in the co-domain. So, the function is one-to-one. On the other hand, there are no elements in the domain that would map to even numbers. So, the function is not onto.

when x = 1, $f(1) = 2(1) - 1 = 1$
when x = 2, $f(2) = 2(2) - 1 = 3$
when x = 3, $f(3) = 2(3) - 1 = 5$ and so on.



5. Show that the function f : $\mathbb{N} \to \mathbb{N}$ defined by f (m) = m^2 + m + 3 is one-one function.

Sol : $f: \mathbb{N} \to \mathbb{N}$

$f(m) = m^2 + m + 3$

when m = 1, $f(1) = (1)^2 + 1 + 3 = 5$
when m = 2, $f(2) = (2)^2 + 2 + 3 = 9$
when m = 3, $f(3) = (3)^2 + 3 + 3 = 15$ and so on.

Clearly, A function for which every element of the range of the function corresponds to exactly one element of the domain.

$\therefore$ So, it is one-to-one function.

6. Let A = {1, 2, 3, 4} and B = $\mathbb{N}$. Let f : $A \to B$ be defined by f (x) = x^3

(i) Find the range of f

(ii) Identify the type of function

Sol : A = {1, 2, 3, 4} and B = N

(i) $f: A \to B$

$$f(x) = x^3$$
$$f(1) = (1)^3 = 1 \in N$$
$$f(2) = (2)^3 = 8 \in N$$
$$f(3) = (3)^3 = 27 \in N$$
$$f(4) = (4)^3 = 64 \in N$$

Range of f = {1, 8, 27, 64}

(ii) 'f' is a function from 'A' to 'B' and all the elements in 'A' having different images in 'B'.

$\therefore$ $f: A \to B$ is a function. It is one-to-one and into function and also called a cubic function.

7. In each of the following cases state whether the function is bijective or not. Justify your answer.

(i) f : $\mathbb{R} \to \mathbb{R}$ defined by f (x) = 2x + 1

(ii) f : $\mathbb{R} \to \mathbb{R}$ defined by f (x) = 3 – $4x^2$

Sol : (i) $f: R \to R$ and $f(x) = 2x + 1$

when $x = -1$, $f(-1) = 2(-1) + 1 = -1 \in R$
when $x = 0$, $f(0) = 2(0) + 1 = 1 \in R$
when $x = 1$, $f(1) = 2(1) + 1 = 3 \in R$ and so on.

For every value of $x \in R$, $f(x)$ also $\in R$.

$\therefore$ The function is well defined and it is one-to-one function (Injective)

For $f(x): R \to R$, the domain and range are also well defined. So it is an onto function. (Surjective)

Thus, the function is one -to -one onto i.e. Bijective function.

(ii) $f: R \to R$, $f(x) = 3 - 4x^2$

when $x = 0$, $f(0) = 3 - 4(0) = 3 \in R$
when $x = 1$, $f(1) = 3 - 4(1)^2 = -1 \in R$
when $x = 2$, $f(2) = 3 - 4(2)^2 = -13 \in R$
when $x = -1$, $f(-1) = 3 - 4(-1)^2 = -1 \in R$
when $x = -2$, $f(-2) = 3 - 4(-2)^2 = -13 \in R$

From this, it is clear that two or more elements having same image in the co-domain. So, it is not one-to-one and it is many-to-one function. Hence it is not Bijective.

8. Let A = {– 1, 1} and B = {0, 2}. If the function f : $A \to B$ defined by f (x) = ax + b. Is an onto function? Find a and b.

Sol : A = {– 1, 1}, B = {0, 2}

$f: A \to B$, f (x) = ax + b

when x = – 1, f (–1) = 0
when x = 1, f (1) = 2

Since, there is a constant difference between x and f (x), it is an onto function.

Substituting the values, we get

$$-a + b = 0$$
$$a + b = 2$$

Solving these equations, we get a = 1, b = 1

9. If the function f is defined by

$$f(x) = \begin{cases} x+2 & \text{if } x > 1 \\ 2 & \text{if } -1 \le x \le 1 \\ x-1 & \text{if } -3 < x < -1 \end{cases}$$ **; Find the values of**

(i) f (3) **(ii) f (0)**

(iii) f (– 1.5) **(iv) f (2) + f (– 2)**

Sol :

$$f(x) = \begin{cases} x+2 & \text{if } x > 1 \\ 2 & \text{if } -1 \le x \le 1 \\ x-1 & \text{if } -3 < x < -1 \end{cases}$$

(i) $f(3) = 3 + 2 = 5$ $[\because 3 > 1]$

(ii) $f(0) = 2$ $[\because -1 \le 0 \le 1]$

(iii) $f(-1.5) = -1.5 - 1 = -2.5$ $[\because -3 < -1.5 < -1]$

(iv) $f(2) + f(-2) = (2 + 2) + (-2 - 1)$
$= 4 - 3 = 1$ $[\because 2 > 1 \text{ and } -3 < -2 < -1]$

10. A function $f : [-5, 9] \to \mathbb{R}$ is defined as follows:

$$f(x) = \begin{cases} 6x+1 & \text{if } -5 \le x < 2 \\ 5x^2-1 & \text{if } 2 \le x < 6 \\ 3x-4 & \text{if } 6 \le x \le 9 \end{cases}$$

Find (i) $f(-3) + f(2)$ (ii) $f(7) - f(1)$

(iii) $2f(4) + f(8)$ (iv) $\dfrac{2f(-2) - f(6)}{f(4) + f(-2)}$

Sol : $f : [-5, 9] \to R$

$$f(x) = \begin{cases} 6x+1 & \text{if } -5 \le x < 2 \\ 5x^2-1 & \text{if } 2 \le x < 6 \\ 3x-4 & \text{if } 6 \le x \le 9 \end{cases}$$

(i) $f(-3) + f(2) = [6(-3) + 1] + [5(2)^2 - 1]$
$= (-18 + 1) + (20 - 1)$
$= -17 + 19 = 2$ $[\because -5 \le -3 < 2$, $2 \le 2 < 6]$

(ii) $f(7) - f(1) = [3(7) - 4] - [6(1) + 1]$
$= (21 - 4) - (6 + 1)$
$= 17 - 7 = 10$ $[\because 6 \le 7 \le 9$, $-5 \le 1 < 2]$

(iii) $2f(4) + f(8) = 2[5(4)^2 - 1] + [3(8) - 4]$
$= 2[80 - 1] + [24 - 4]$
$= 158 + 20 = 178$ $[\because 2 \le 4 < 6$, $6 \le 8 \le 9]$

(iv) $\dfrac{2f(-2) - f(6)}{f(4) + f(-2)}$

$f(-2) = 6(-2) + 1 = -12 + 1 = -11$ $[\because -5 \le -2 < 2]$

$f(6) = 3(6) - 4 = 18 - 4 = 14$ $[\because 6 \le 6 < 9]$

$f(4) = 5(4)^2 - 1 = 80 - 1 = 79$ $[\because 2 \le 4 < 6]$

$f(-2) = -11$

$$\therefore \frac{2f(-2) - f(6)}{f(4) + f(-2)} = \frac{2(-11) - 14}{79 - 11}$$
$$= \frac{-22 - 14}{68} = -\frac{36}{68} = -\frac{9}{17}$$

11. The distance S an object travels under the influence of gravity in time t seconds is given by $S(t) = \frac{1}{2}gt^2 + at + b$, where, (g is the acceleration due to gravity), a, b are constants. Check if the function S (t) is one-one.

Sol : Distance travelled by an object is given to be $S(t) = \frac{1}{2}gt^2 + at + b$

'g'- acceleration due to gravity is a constant.

'g' is the acceleration due to gravity 'a' and 'b' are constants.

't' is a variable. (t - time)

At different values of 't' , S(t) is having different values. (images in codomain) clearly $f : t \to S(t)$ is one-to-one function.

Given $S(t) = \frac{1}{2}gt^2 + at + b$ (a, b are constants)

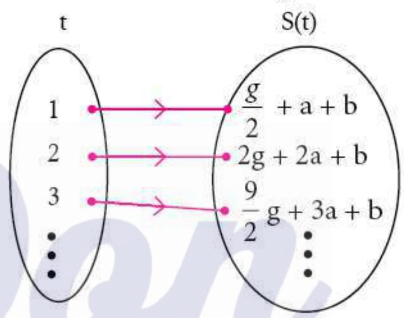
t = time in seconds

Let us take t = 1, 2, 3 ..... Seconds

When t = 1 $\Rightarrow$ $S(1) = \frac{g}{2} + a + b$

When t = 2 $\Rightarrow$ $S(2) = 2g + 2a + b$

When t = 3 $\Rightarrow$ $S(3) = \frac{9}{2}g + 3a + b$ and so on.



All the elements of t, having different images in S(t). Hence it is an One-to-one function.

12. The function 't' which maps temperature in Celsius (C) into temperature in Fahrenheit (F) is defined by t (C) = F where $F = \frac{9C}{5} + 32$. Find,

(i) t (0) (ii) t (28)

(iii) t (– 10)

(iv) the value of C when t (C) = 212

(v) the temperature when the Celsius value is equal to the Fahrenheit value.

Sol :

Given t (C) = F where $F = \frac{9C}{5} + 32$.

C - Celsius, F - Fahrenheit

$$\therefore t(C) = \frac{9C}{5} + 32$$

(i) $t(0) = \dfrac{9(0)}{5} + 32 = 0 + 32 = 32\,^\circ F$

(ii) $t(28) = \dfrac{9(28)}{5} + 32 = \dfrac{252}{5} + 32$
$= 50.4 + 32 = 82.4^\circ F$

(iii) $t(-10) = \dfrac{9(-10)}{5} + 32 = -18 + 32 = 14^\circ F$

(iv) Given t (C) = 212

$\therefore \frac{9C}{5} + 32 = 212 \Rightarrow \frac{9C}{5} = 212 - 32$

$C = 180 \times \frac{5}{9} = 100\,^{\circ}C$

(v) The temperature when the Celsius value is equal to the Fahrenheit value.

$\therefore F = C$

$\frac{9C}{5} + 32 = C$

$\frac{9C}{5} - C = -32 \Rightarrow \frac{9C - 5C}{5} = -32$

$4C = -32 \times 5 \Rightarrow C = -\frac{160}{4}$

$^{\circ}C = -40$

COMPOSITION OF FUNCTIONS

Key Points

- Let f : X→R and g : X→R be any two real functions, where X⊂R, then their sum f + g i.e., (f + g) : X→R is the function defined by (f + g)(x) = f(x) + g(x), for all x∈X.
- Their difference i.e., (f – g) : X→R is the function defined by (f – g)(x) = f(x) – g(x), for all x∈X.
- The multiplication (αf) is a function from X→R defined by a scalar by $(\alpha f)(x) = \alpha\, f(x)$, x∈X.
- The product fg : X→R is defined by (fg)(x) = f(x).g(x), for all x∈X.
- The quotient $\frac{f}{g}$ is a function defined by $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, provided g(x)≠0, x∈X.
- The composition of two functions f and g is denoted by $f \circ g$ and $(f \circ g)(x) = f[g(x)]$.
- Composition of functions is not always commutative i.e., $f \circ g \neq g \circ f$.
- Composition of functions is always associative.

Worked Examples

1.20. Find $f \circ g$ and $g \circ f$ when f (x) = 2x + 1 and g (x) = $x^2 - 2$.

Sol :

$f(x) = 2x + 1, g(x) = x^2 - 2$

$(f \circ g)(x) = f(g(x)) = f(x^2 - 2)$
$= 2(x^2 - 2) + 1 = 2x^2 - 3$

$(g \circ f)(x) = g(f(x)) = g(2x + 1)$
$= (2x + 1)^2 - 2 = 4x^2 + 4x - 1$

Thus $f \circ g = 2x^2 - 3$, $g \circ f = 4x^2 + 4x - 1$.

From the above, we see that $f \circ g \neq g \circ f$.

1.21. Represent the function f (x) = $\sqrt{2x^2 - 5x + 3}$ as a composition of two functions.

Sol :

We set $f_2(x) = 2x^2 - 5x + 3$ and $f_1(x) = \sqrt{x}$

Then, $f(x) = \sqrt{2x^2 - 5x + 3}$
$= \sqrt{f_2(x)}$
$= f_1[f_2(x)] = f_1 f_2(x)$

1.22. If f (x) = 3x – 2, g (x) = 2x + k and $f \circ g = g \circ f$, then find the value of k.

Sol :

$f(x) = 3x - 2, g(x) = 2x + k$

$(f \circ g)(x) = f(g(x)) = f(2x + k)$
$= 3(2x + k) - 2 = 6x + 3k - 2$

Thus, $(f \circ g)(x) = 6x + 3k - 2$.

$(g \circ f)(x) = g(3x - 2) = 2(3x - 2) + k$

Thus, $(g \circ f)(x) = 6x - 4 + k$.

Given that $f \circ g = g \circ f$

Therefore, $6x + 3k - 2 = 6x - 4 + k$

$6x - 6x + 3k - k = -4 + 2 \Rightarrow k = -1$

1.23. Find k if $f \circ f(k) = 5$ where f (k) = 2k – 1.

Sol :

$f \circ f(k) = f(f(k)) = f(2k - 1)$

$= 2(2k-1)-1 = 4k-3$

Thus, $f \circ f(k) = 4k - 3$

But, it is given that $f \circ f(k) = 5$

Therefore $4k - 3 = 5 \Rightarrow k = 2$.

1.24. If f (x) = 2x + 3, g (x) = 1 – 2x and h (x) = 3x. Prove that $f \circ (g \circ h) = (f \circ g) \circ h$.

Sol :

f (x) = 2x + 3, g (x) = 1 – 2x, h (x) = 3x

Now, $(f \circ g)(x) = f(g(x)) = f(1-2x)$

$= 2(1-2x)+3 = 5-4x$

Since, $(f \circ g) \circ h(x) = (f \circ g)(h(x))$

$= (f \circ g)(3x)$

$= 5 - 4(3x) = 5 - 12x$ ... (1)

$(g \circ h)(x) = g(h(x)) = g(3x)$

$= 1 - 2(3x) = 1 - 6x$

Since, $f \circ (g \circ h)(x) = f(1-6x) = 2(1-6x)+3$

$= 5 - 12x$ ... (2)

From (1) and (2), we get

$(f \circ g) \circ h = f \circ (g \circ h)$

1.25. Find x if gff (x) = fgg (x), given f (x) = 3x + 1 and g (x) = x + 3.

Sol : gff (x) = g [f {f (x)}]

(This means "g of f of f of x")

$= g[f(3x+1)] = g[3(3x+1)+1]$

$= g(9x+4) = [(9x+4)+3] = 9x+7$

fgg (x) = f [g {g (x)}]

(This means "f of g of g of x")

$= f[g(x+3)] = f[(x+3)+3]$

$f(x+6) = [3(x+6)+1] = 3x+19$

These two quantities being equal, we get 9x + 7 = 3x + 19. Solving this equation we obtain x = 2.

Progress Check

State your answer for the following questions by selecting the correct option.

1. Composition of functions is commutative

(a) Always true (b) Never true
(c) Sometimes true

Ans : (c) Sometimes true

2. Composition of functions is associative

(a) Always true (b) Never true
(c) Sometimes true

Ans : (a) Always true

3. Is a constant function a linear function?

Ans : Yes

4. Is quadratic function a one-one function?

Ans : No

5. Is cubic function a one -one function?

Ans : Yes

6. Is the reciprocal function a bijection?

Ans : Yes

7. Is f : $A \to B$ is a constant function, then the range of f will have _____ elements.

Ans : One element

Thinking Corner

1. If f (x) = x^m and g (x) = x^n does $f \circ g = g \circ f$?

Ans : $f(x) = x^m, g(x) = x^n$

$f \circ g = f[g(x)] = f(x^n) = (x^n)^m = x^{nm}$

$g \circ f = g[f(x)] = g[x^m] = (x^m)^n = x^{mn}$

$\therefore f \circ g = g \circ f$

Exercise 1. 5

1. Using the functions f and g given below, find $f \circ g$ and $g \circ f$. Check whether $f \circ g = g \circ f$.

(i) $f(x) = x - 6, g(x) = x^2$

(ii) $f(x) = \frac{2}{x}, g(x) = 2x^2 - 1$

(iii) $f(x) = \frac{x+6}{3}, g(x) = 3 - x$

(iv) $f(x) = 3 + x, g(x) = x - 4$

(v) $f(x) = 4x^2 - 1, g(x) = 1 + x$

Sol :

(i) $f(x) = x - 6, g(x) = x^2$

$f \circ g = f[g(x)] = f[x^2] = x^2 - 6$

$g \circ f = g[f(x)] = g[x-6] = (x-6)^2$

$= x^2 - 12x + 36$

$\therefore f \circ g \neq g \circ f$

(ii) $f(x) = \frac{2}{x}, g(x) = 2x^2 - 1$

$f \circ g = f[g(x)] = f[2x^2-1] = \frac{2}{2x^2-1}$

$g \circ f = g[f(x)] = g\left[\frac{2}{x}\right] = 2\left(\frac{2}{x}\right)^2 - 1$

$= 2\left(\frac{4}{x^2}\right) - 1 = \frac{8-x^2}{x^2}$

$\therefore f \circ g \neq g \circ f$

(iii) $f(x) = \dfrac{x+6}{3}$, $g(x) = 3 - x$

$f\circ g = f[g(x)] = f(3-x) = \dfrac{3-x+6}{3} = \dfrac{9-x}{3}$

$g\circ f = g[f(x)] = g\left(\dfrac{x+6}{3}\right) = 3 - \left(\dfrac{x+6}{3}\right)$

$= \dfrac{9-x-6}{3} = \dfrac{3-x}{3}$ $\therefore f\circ g \neq g\circ f$

(iv) $f(x) = 3 + x$, $g(x) = x - 4$

$f\circ g = f[g(x)] = f[x-4] = 3+x-4 = x-1$

$g\circ f = g[f(x)] = g[3+x] = 3+x-4 = x-1$

$\therefore f\circ g = g\circ f$

(v) $f(x) = 4x^2 - 1$, $g(x) = 1 + x$

$f\circ g = f[g(x)] = f(1+x) = 4(1+x)^2 - 1$

$= 4(1 + 2x + x^2) - 1$

$= 4 + 8x + 4x^2 - 1$

$= 4x^2 + 8x + 3$

$g\circ f = g[f(x)] = 1 + 4x^2 - 1$

$= 4x^2$

$\therefore f\circ g \neq g\circ f$

2. Find the value of k, such that $f\circ g = g\circ f$.

(i) $f(x) = 3x + 2$, $g(x) = 6x - k$

(ii) $f(x) = 2x - k$, $g(x) = 4x + 5$

Sol :

(i) $f(x) = 3x + 2$, $g(x) = 6x - k$

$f\circ g = f[g(x)] = f(6x-k)$

$= 3(6x-k) + 2$

$= 18x - 3k + 2$

$g\circ f = g[f(x)] = 6(3x+2) - k$

$= 18x + 12 - k$

Given $f\circ g = g\circ f$

$\therefore$ $18x - 3k + 2 = 18x + 12 - k$

$3k - k = -12 + 2$

$2k = -10$

$k = -5$

(ii) $f(x) = 2x - k$, $g(x) = 4x + 5$

$f\circ g = f[g(x)] = f[4x+5]$

$= 2(4x+5) - k$

$= 8x + 10 - k$

$g\circ f = g[f(x)] = g(2x-k) = 4(2x-k) + 5$

$= 8x - 4k + 5$

Given $f\circ g = g\circ f$

$\therefore$ $8x + 10 - k = 8x - 4k + 5$

$4k - k = 5 - 10$

$3k = -5$

$k = -5/3$

3. If $f(x) = 2x - 1$, $g(x) = \dfrac{x+1}{2}$, show that $f\circ g = g\circ f = x$.

Sol : $f(x) = 2x - 1$, $g(x) = \dfrac{x+1}{2}$

$f\circ g = f[g(x)] = f\left(\dfrac{x+1}{2}\right) = 2\left(\dfrac{x+1}{2}\right) - 1 = x + 1 - 1 = x$

$g\circ f = g[f(x)] = g(2x-1) = \dfrac{2x-1+1}{2}$

$= \dfrac{2x}{2} = x$

$\therefore$ $f\circ g = g\circ f = x$ Hence proved.

4. (i) If $f(x) = x^2 - 1$, $g(x) = x - 2$ find a, if $g\circ f(a) = 1$.

(ii) Find k, if $f(k) = 2k - 1$ and $f\circ f(k) = 5$

Sol :

(i) $f(x) = x^2 - 1$, $g(x) = x - 2$

$f(a) = a^2 - 1$

Given, $(g\circ f)(a) = 1$

$g[f(a)] = 1$

$g[a^2 - 1] = 1$

$a^2 - 1 - 2 = 1$

$a^2 - 3 = 1$

$a^2 = 1 + 3 = 4$

$a = \pm 2$

(ii) $f(k) = 2k - 1$

Given $(f\circ f)(k) = 5$

$f[f(k)] = 1$

$f[2k-1] = 1$

$\therefore$ $2(2k-1) - 1 = 5$

$4k - 2 - 1 = 5$

$4k - 3 = 5$

$4k = 5 + 3 = 8$

$k = 8/4 = 2$

5. Let $A, B, C \subseteq \mathbb{N}$ and a function $f : A \to B$ be defined by $f(x) = 2x + 1$ and $g : B \to C$ be defined by $g(x) = x^2$. Find the range of $f\circ g$ and $g\circ f$.

Sol : $A, B, C \subseteq \mathbb{N}$

$f : A \to B$ defined by $f(x) = 2x + 1$

$g : B \to C$ defined by $g(x) = x^2$

$f\circ g = f[g(x)] = f(x^2) = 2x^2 + 1$

$g\circ f = g[f(x)] = g(2x-1) = (2x+1)^2$

$= 4x^2 + 4x + 1$

'x' can take any real value and can produce any real value. Thus, the domain and range of $f\circ g$ and $g\circ f$ is R (Set of real numbers).

6. If f (x) = $x^2 - 1$. Find (a) $f \circ f$ (b) $f \circ f \circ f$

Sol : $f(x) = x^2 - 1$

(a) $f \circ f = f[f(x)] = f(x^2 - 1)$
$= (x^2 - 1)^2 - 1 = x^4 - 2x^2 + 1 - 1 = x^4 - 2x^2$

(b) $f \circ f \circ f = f[f[f(x)]] = f[f(x^2 - 1)] = f[x^4 - 2x^2]$
$= (x^4 - 2x^2)^2 - 1 = x^8 - 4x^6 + 4x^4 - 1$

7. If $f : R \to R$ and $g : R \to R$ are defined by f (x) = x^5 and g (x) = x^4 then check if f, g are one - one and $f \circ g$ is one - one.

Sol :

$f(x) = x^5, g(x) = x^4$

$f(x) = x^5$

For any value of 'x', f (x) gives us a different value (image) in co domain.

$\therefore$ f (x) is one-one function

$g(1) = 1; \quad g(-1) = 1$

Hence g(x) is not one-one function

$f \circ g = f[g(x)] = f(x^4) = (x^4)^5 = x^{20}$

$f \circ g$ is also One - One function as x is mapped with different value of $f \circ g$.

8. Consider the function f (x), g (x), h (x) as given below. Show that $(f \circ g) \circ h = f \circ (g \circ h)$ in each case.

(i) f (x) = x – 1, g (x) = 3x + 1 and h (x) = x^2

(ii) f (x) = x^2, g (x) = 2x and h (x) = x + 4

(iii) f (x) = x – 4, g (x) = x^2 and h (x) = 3x – 5

Sol :

(i) $f(x) = x - 1, g(x) = 3x + 1, h(x) = x^2$

$f \circ g = f[g(x)] = f[3x + 1]$
$= (3x + 1) - 1 = 3x$

$(f \circ g) \circ h = (f \circ g)[h(x)] = (f \circ g)[x^2] = 3x^2$

$g \circ h = g[h(x)] = g[x^2] = 3x^2 + 1$

$f \circ (g \circ h) = f[g(h(x))] = f[g(x^2)] = f[3x^2 + 1]$
$= (3x^2 + 1) - 1 = 3x^2$

$\therefore (f \circ g) \circ h = f \circ (g \circ h)$

Hence proved.

(ii) $f(x) = x^2, g(x) = 2x, h(x) = x + 4$

$f \circ g = f[g(x)] = f[2x]$
$= f(2x) = (2x)^2 = 4x^2$

$(f \circ g) \circ h = (f \circ g)[h(x)] = (f \circ g)[x + 4]$
$= 4(x + 4)^2$
$= 4(x^2 + 8x + 16) = 4x^2 + 32x + 64$

$g \circ h = g[h(x)] = g[x + 4] = 2(x + 4)$

$f \circ (g \circ h) = f[g(h(x))] = f[g(x + 4)]$
$= f[2(x + 4)] = [2(x + 4)]^2 = 4(x + 4)^2$
$= 4(x^2 + 8x + 16) = 4x^2 + 32x + 64$

$\therefore (f \circ g) \circ h = f \circ (g \circ h)$

(iii) $f(x) = x - 4, g(x) = x^2, h(x) = 3x - 5$

$f \circ g = f[g(x)] = f(x^2) = x^2 - 4$

$(f \circ g) \circ h = (f \circ g)[h(x)] = (f \circ g)[3x - 5]$
$= (3x - 5)^2 - 4 = 9x^2 - 30x + 25 - 4$
$= 9x^2 - 30x + 21$

$g \circ h = g[h(x)]$
$= g(3x - 5) = (3x - 5)^2 = 9x^2 - 30x + 25$

$f \circ (g \circ h) = f[g(h(x))] = f[g(3x - 5)] = f[(3x - 5)^2]$
$= 9x^2 - 30x + 25 - 4 = 9x^2 - 30x + 21$

$\therefore (f \circ g) \circ h = f \circ (g \circ h)$

9. Let f = {(– 1, 3), (0, – 1), (2, – 9)} be a linear function from $\mathbb{Z}$ into $\mathbb{Z}$. Find f (x).

Sol : f = {(– 1, 3), (0, – 1), (2, – 9)}, $f : \mathbb{Z} \to \mathbb{Z}$

Since 'f' is a linear function

$f(x) = ax + b$

in (0, – 1), when x = 0, f (0) = – 1

$\therefore a(0) + b = -1 \Rightarrow b = -1$

in (– 1, 3), when x = – 1, f (– 1) = 3

$\therefore a(-1) + b = 3 \Rightarrow -a - 1 = 3$

$-a = 4 \Rightarrow a = -4$

$\therefore \quad f(x) = -4x - 1$

10. In electrical circuit theory, a circuit C(t) is called a linear circuit if it satisfies the superposition principle given by $C(at_1 + bt_2) = aC(t_1) + bC(t_2)$, where a, b are constants. Show that the circuit C (t) = 3t is linear.

Sol : Given superposition principle is

$C(at_1 + bt_2) = aC(t_1) + bC(t_2)$ a, b are constants

If all the independent sources except for $C(t_1)$ have known fixed values, then

$C(t) = aC(t_1) + d$

where $d = bC(t_2)$

$\therefore$ C (t) is linear.

Aliter:

C(t) is linear and $t = t_1 + t_2$

Let $C(t_1) = t$ and $C(t_2) = 2t$

By Given data,

$C(at_1 + bt_2) = aC(t_1) + bC(t_2)$ ... (1)

Now $C(t) = C(t_1 + t_2)$ [$\therefore t = t_1 + t_2$]

$= C(t_1) + C(t_2)$ from (1)

$= t + 2t = 3t$

Hence the function C(t) is linear.

Exercise 1. 6

Multiple Choice Questions:

1. If n (A × B) = 6 and A = {1, 3} then n (B) is

(1) 1 (2) 2
(3) 3 (4) 6

[Ans: (3)]

Sol :

$$n(A \times B) = 6$$
$$A = \{1, 3\} \Rightarrow n(A) = 2$$
$$n(B) = \frac{n(A \times B)}{n(A)} = \frac{6}{2} = 3$$

2. A = {a, b, p}, B = {2, 3}, C = {p, q, r, s} then $n[(A \cup C) \times B]$ is

(1) 8 (2) 20
(3) 12 (4) 16

[Ans: (3)]

Sol:

$A = \{a, b, p\}$, $B = \{2, 3\}$, $C = \{p, q, r, s\}$

$$A \cup C = \{a, b, p, q, r, s\}$$
$$n(A \cup C) = 6$$
$$n(B) = 2$$
$$\therefore \ n[(A \cup C) \times B] = 6 \times 2 = 12$$

3. If A = {1, 2}, B = {1, 2, 3, 4}, C = {5, 6} and D = {5, 6, 7, 8} then state which of the following statement is true.

(1) $(A \times C) \subset (B \times D)$ (2) $(B \times D) \subset (A \times C)$
(3) $(A \times B) \subset (A \times D)$ (4) $(D \times A) \subset (B \times A)$

[Ans: (1)]

Sol : $A = \{1, 2\}$, $B = \{1, 2, 3, 4\}$,
$C = \{5, 6\}$ and $D = \{5, 6, 7, 8\}$

$$A \times C = \{(1, 5), (1, 6), (2, 5), (2, 6)\}$$
$$B \times D = \{(1, 5), (1, 6), (1, 7), (1, 8), (2, 5), (2, 6), (2, 7), (2, 8), (3, 5), (3, 6), (3, 7), (3, 8), (4, 5), (4, 6), (4, 7), (4, 8)\}$$

Hence $(A \times C) \subset (B \times D)$

4. If there are 1024 relations from a set A = {1, 2, 3, 4, 5} to a set B, then the number of elements in B is

(1) 3 (2) 2
(3) 4 (4) 8

[Ans: (2)]

Sol : $A = \{1, 2, 3, 4, 5\}$, $n(A) = 5$, $n(B) = ?$

Number of relations from A to B is 1024.

i.e., $2^{mn} = 1024$ where 'mn' is number of elements in $A \times B$.

$$2^{mn} = 2^{10} \Rightarrow mn = 10$$
$$n(B) = \frac{n(A \times B)}{n(A)} = \frac{10}{5} = 2$$

5. The range of the relation R = {(x, x^2) | x is a prime number less than 13} is

(1) {2, 3, 5, 7}
(2) {2, 3, 5, 7, 11}
(3) {4, 9, 25, 49, 121}
(4) {1, 4, 9, 25, 49, 121}

[Ans: (3)]

Sol : Set of prime numbers less than 13 is {2, 3, 5, 7, 11}

Relation $R = \{(x, x^2)/ \text{ x is a prime number less than } 13\}$

$$R = \{(2, 4), (3, 9), (5, 25), (7, 49), (11, 121)\}$$
$$\therefore \text{Range} = \{(4, 9, 25, 49, 121\}$$

6. If the ordered pairs (a + 2, 4) and (5, 2a + b) are equal then (a, b) is

(1) (2, – 2) (2) (5, 1)
(3) (2, 3) (4) (3, – 2)

[Ans: (4)]

Sol :

Given $(a + 2, 4) = (5, 2a + b)$

$a + 2 = 5$	$2a + b = 4$
$a = 5 - 2$	$2(3) + b = 4$
$= 3$	$b = 4 - 6 = -2$

7. Let n (A) = m and n (B) = n then the total number of non-empty relations that can be defined from A to B is

(1) m^n (2) n^m
(3) $2^{mn} - 1$ (4) 2^{mn}

[Ans: (4)]

8. If {(a, 8), (6, b)} represents an identity function, then the value of a and b are respectively

(1) (8, 6) (2) (8, 8)
(3) (6, 8) (4) (6, 6)

[Ans: (1)]

Sol : Given {(a, 8), (6, b)} is an Identity function

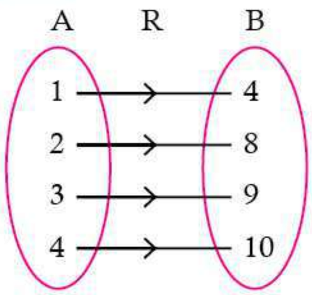
i.e., {(8, 8), (6, 6)}

$\therefore$ a = 8, b = 6

9. Let A = {1, 2, 3, 4} and B = {4, 8, 9, 10}. A function f : $A \to B$ given by f = {(1, 4), (2, 8), (3, 9), (4, 10)} is a

(1) Many-one function
(2) Identity function
(3) One-to-one function
(4) Into function **[Ans: (3)]**

Sol : A = {1, 2, 3, 4}, B = {4, 8, 9, 10}



f : A → B

and f = {(1, 4), (2, 8), (3, 9), (4, 10)}

One-to-one function

10. If f (x) = $2x^2$ and g (x) = $\frac{1}{3x}$, Then $f \circ g$ is

(1) $\frac{3}{2x^2}$ (2) $\frac{2}{3x^2}$

(3) $\frac{2}{9x^2}$ (4) $\frac{1}{6x^2}$

[Ans: (3)]

Sol :

$$f(x) = 2x^2, \; g(x) = 1/3x$$

$$f \circ g = f[g(x)] = f\left[\frac{1}{3x}\right]$$

$$= 2\left(\frac{1}{3x}\right)^2 = 2\left(\frac{1}{9x^2}\right)$$

$$= \left(\frac{2}{9x^2}\right)$$

11. If f : $A \to B$ is a bijective function and if n (B) = 7, then n (A) is equal to

(1) 7 (2) 49
(3) 1 (4) 14

[Ans: (1)]

Sol :

f : A → B is a bijective function and n(B) = 7.
f is a bijective function i.e., one-to-one onto function.
∴ A and B should have equal number of elements.

n(A) = 7

12. Let f and g be two functions given by f = {(0, 1), (2, 0), (3, – 4), (4, 2), (5, 7)} g = {(0, 2), (1, 0), (2, 4), (– 4, 2), (7, 0)} then the range of $f \circ g$ is

(1) {0, 2, 3, 4, 5} (2) {– 4, 1, 0, 2, 7}
(3) {1, 2, 3, 4, 5} (4) {0, 1, 2} **[Ans: (4)]**

Sol : f = {(0, 1), (2, 0), (3, – 4), (4, 2), (5, 7)}
g = {(0, 2), (1, 0), (2, 4), (– 4, 2), (7, 0)}

Now, f o g(0) = f[2] = 0
f o g(1) = f[0] = 1
f o g(2) = f[4] = 2
f o g(– 4) = f[2] = 0
f o g(7) = f[0] = 1

Range of 'f o g' = {0, 1, 2}

13. Let f (x) = $\sqrt{1+x^2}$ then

(1) f (xy) = f (x). f (y)
(2) $f(xy) \geq f(x).f(y)$
(3) $f(xy) \leq f(x).f(y)$
(4) None of these **[Ans: (3)]**

Sol :

$$f(x) = \sqrt{1+x^2}$$

then $f(y) = \sqrt{1+y^2}$

and $f(xy) = \sqrt{1+x^2y^2}$

Hence, it is clear that f(xy) ≤ f(x) f(y)

14. If g = {(1, 1), (2, 3), (3, 5), (4, 7)} is a function given by g (x) = $\alpha x + \beta$ then the values of α *and* β are

(1) (– 1, 2) (2) (2, – 1)
(3) (– 1, – 2) (4) (1, 2) **[Ans: (2)]**

Sol :

g = {(1, 1), (2, 3), (3, 5), (4, 7)}

$$g(x) = \alpha x + \beta$$

When α = 2 and β = – 1, g(x) = 2x – 1
Which is satisfying 'g'.

15. f (x) = $(x + 1)^3 – (x – 1)^3$ represents

(1) a linear function
(2) a cubic function
(3) a reciprocal function
(4) a quadratic function **[Ans: (4)]**

Sol :

$$f(x) = (x + 1)^3 - (x - 1)^3$$

$$= (x^3 + 3x^2 + 3x + 1) - (x^3 - 3x^2 + 3x - 1)$$

$$= 6x^2 + 2$$

UNIT EXERCISE - 1

1. If the ordered pairs $(x^2 - 3x, y^2 + 4y)$ and $(-2, 5)$ are equal, then find x and y.

Sol :

Given $(x^2 - 3x, y^2 + 4y) = (-2, 5)$

$\therefore\ x^2 - 3x = -2$	$y^2 + 4y = 5$
$x^2 - 3x + 2 = 0$	$y^2 + 4y - 5 = 0$
$(x-1)(x-2) = 0$	$(y+5)(y-1) = 0$
$x = 1, 2$	$y = -5, 1$

2. The Cartesian product A × A has 9 elements among which (– 1, 0) and (0, 1) are found. Find the set A and the remaining elements of A × A.

Sol : Since A × A has 9 elements ,

A would have 3 elements $(\because 3 \times 3 = 9)$

A × A contains (– 1, 0) and (0, 1)

$\therefore\ -1, 0 \in A$ ... (1)

Similarly (0, 1) is in A × A

So, $0, 1 \in A$ ... (2)

From (1) and (2) $\Rightarrow\ -1, 0, 1 \in A$

$$\therefore\ A = \{-1, 0, 1\}$$

$$\therefore\ A \times A = \{-1, 0, 1\} \times \{-1, 0, 1\}$$
$$= \{(-1,-1), (-1,0), (-1,1), (0,-1), (0,0), (0,1), (1,-1), (1,0), (1,1)\}$$

$\therefore$ The remaining elements of A × A are

{(– 1, – 1), (– 1, 1), (0, – 1), (0, 0), (1, – 1), (1, 0),(1,1)}

3. Given that $f(x) = \begin{cases} \sqrt{x-1} & x \ge 1 \\ 4 & x < 1 \end{cases}$. Find

(i) f (0) (ii) f (3)

(iii) f (a + 1) in terms of a. (Given that $a \ge 0$)

Sol : $f(x) = \begin{cases} \sqrt{x-1} & x \ge 1 \\ 4 & x < 1 \end{cases}$

(i) $f(0) = 4$ $[\because 0 < 1$

(ii) $f(3) = \sqrt{3-1} = \sqrt{2}$ $[\because 3 \ge 1$

(iii) $f(a+1) = \sqrt{a+1-1} = \sqrt{a}$ $[\because$ *Given* $a \ge 0$
$[\because a+1 \ge 1$

4. Let A = {9, 10, 11, 12, 13, 14, 15, 16, 17} and let $f : A \to N$ be defined by f (n) = the highest prime factor of $n \in A$. Write f as a set of ordered pairs and find the range of f.

Sol : A = {9, 10, 11, 12, 13, 14, 15, 16, 17}

$f: A \to N$, f (n)= the highest prime factor of $n \in A$

$\therefore$ f (9) = 3, f (10) = 5, f (11) = 11
f (12) = 3, f (13) = 13, f (14) = 7
f (15) = 5, f (16) = 2, f (17) = 17

Set of ordered pairs

f ={(9, 3), (10, 5), (11, 11), (12, 3), (13, 13), (14, 7), (15, 5), (16, 2), (17, 17)

Range ={2, 3, 5, 7, 11, 13, 17}

5. Find the domain of the function

$f(x) = \sqrt{1-\sqrt{1-\sqrt{1-x^2}}}$

Sol :
$$f(x) = \sqrt{1-\sqrt{1-\sqrt{1-x^2}}}$$
$$f(x) = \sqrt{1-t}$$
where $t = \sqrt{1-\sqrt{1-x^2}}$

$$1 - t \ge 0$$
$$t \le 1$$
$$\sqrt{1-\sqrt{1-x^2}} \le 1$$
Squaring $1-\sqrt{1-x^2} \le 1$
$$-\sqrt{1-x^2} \le 0$$
$$\sqrt{1-x^2} \ge 0$$
$$1 - x^2 \ge 0$$
$$x^2 \le 1$$
$\Rightarrow\ x \in [-1, 1]$ i.e., {– 1, 0, 1}

6. If $f(x) = x^2$, $g(x) = 3x$ and $h(x) = x - 2$. Prove that $(f \circ g) \circ h = f \circ (g \circ h)$.

Sol : $f(x) = x^2$, $g(x) = 3x$, $h(x) = x - 2$

$f \circ g = f[g(x)] = f(3x)$
$= (3x)^2 = 9x^2$

$(f \circ g) \circ h = (f \circ g)[h(x)] = (f \circ g)[x-2]$
$= 9(x-2)^2$

$g \circ h = g[h(x)] = g[x-2] = 3(x-2)$

$f \circ (g \circ h) = f[g(h(x))]$
$= f[3(x-2)] = [3(x-2)]^2 = 9(x-2)^2$

$\therefore\ (f \circ g) \circ h = f \circ (g \circ h)$

Hence proved.

7. A = {1, 2} and B = {1, 2, 3, 4}, C = {5, 6} and D = {5, 6, 7, 8}. Verify that whether A × C is a subset of B × D?

Sol :

A = {1, 2}, B = {1, 2, 3, 4}, C = {5, 6}, D = {5, 6, 7, 8}

$A \times C = \{1, 2\} \times \{5, 6\}$
$= \{(1,5), (1,6), (2,5), (2,6)\}$

$$B \times D = \{1, 2, 3, 4\} \times \{5, 6, 7, 8\}$$
$$= \{(1, 5), (1, 6), (1, 7), (1, 8), (2, 5), (2, 6), (2, 7), (2, 8), (3, 5), (3, 6), (3, 7), (3, 8), (4, 5), (4, 6), (4, 7), (4, 8)\}$$

$\therefore$ $(A \times C)$ is a subset of $(B \times D)$

8. If $f(x) = \frac{x-1}{x+1}, x \neq 1$ show that $f(f(x)) = -\frac{1}{x}$, provided $x \neq 0$.

Sol : $f(x) = \frac{x-1}{x+1}, x \neq -1$

$$f[f(x)] = f\left[\frac{x-1}{x+1}\right]$$

$$= \frac{\frac{x-1}{x+1}-1}{\frac{x-1}{x+1}+1} = \frac{\frac{x-1-(x+1)}{x+1}}{\frac{x-1+(x+1)}{x+1}}$$

$$= \frac{x-1-x-1}{x-1+x+1} = -\frac{2}{2x} = -\frac{1}{x}$$

Hence proved.

9. The function f and g are defined by $f(x) = 6x + 8$; $g(x) = \frac{x-2}{3}$

(i) Calculate the value of $gg\left(\frac{1}{2}\right)$

(ii) Write an expression for gf (x) in its simplest form.

Sol : $f(x) = 6x + 8, \quad g(x) = \frac{x-2}{3}$

(i) $g\left(\frac{1}{2}\right) = \frac{\frac{1}{2}-2}{3} = \frac{1-4}{2\times 3} = -\frac{3}{6} = -\frac{1}{2}$

$\therefore \quad gg\left(\frac{1}{2}\right) = g\left[g\left(\frac{1}{2}\right)\right]$

$$= g\left(-\frac{1}{2}\right) = \frac{-\frac{1}{2}-2}{3} = -\frac{1-4}{2\times 3} = -\frac{5}{6}$$

(ii) $gf(x) = g[f(x)] = g[6x + 8]$

$$= \frac{6x+8-2}{3} = \frac{6x+6}{3}$$

$$= \frac{6(x+1)}{3} = 2(x + 1)$$

10. Write the domain of the following real functions

(i) $f(x) = \frac{2x+1}{x-9}$ **(ii) $p(x) = \frac{-5}{4x^2+1}$**

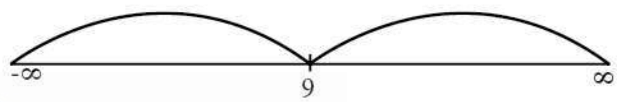
(iii) $g(x) = \sqrt{x-2}$ **(iv) $h(x) = x + 6$**

Sol :

(i) $f(x) = \frac{2x+1}{x-9}$

If $x - 9 = 0$, then

$x = 9$



$\therefore$ The domain is all values of x that make the expression defined.

i.e., $(-\infty, 9) \cup (9, \infty)$

i.e., $(x / x \neq 9) \Rightarrow R - \{9\}$

(ii) $p(x) = \frac{-5}{4x^2+1}$

Here, the expression is defined for all real values of 'x'.

i. e., $x \in R$

(iii) $g(x) = \sqrt{x-2}$

g (x) is defined real only when $x \geq 2$

(iv) $h(x) = x + 6$

h (x) is defined for all real values of 'x'.

i.e., $x \in R$.

CREATIVE QUESTIONS

I. Multiple Choice Questions

Cartesian Product

1. If A = {1, 2}, B = {0, 1}, then A × B is

(1) {(1, 0), (1, 1), (2, 0), (2, 1)}
(2) {(1, 0), (2, 1)}
(3) {(1, 1), (1, 2), (0, 1), (0, 2)}
(4) None of these **[Ans: (1)]**

Sol :

A = {1, 2}, B = {0, 1}
A × B = {(1, 0), (1, 1), (2, 0), (2, 1)}

2. If the set A has 'p' elements, B has 'q' elements, then the number of elements in A × B is

(1) p +q (2) p + q + 1
(3) pq (4) p^2 **[Ans: (3)]**

Sol :

Number of elements in A = p
Number of elements in B = q
Number of elements in A × B = pq

3. If A, B, C are any three sets, then $A\times(B\cup C)$ is equal to

(1) $(A\times B)\cup(A\times C)$ (2) $(A\cup B)\cup(A\cup C)$
(3) Both (a) and (b) (4) None of these **[Ans: (1)]**

4. Let A = {a, b, c, d}, B = {b, c, d, e}, then $n\{(A\times B)\cap(B\times A)\}$ =

(1) 3
(2) 6
(3) 9
(4) None of these **[Ans: (3)]**

Sol :

A = {a, b, c, d}, B = {b, c, d, e}

A × B = {(a,b),(a,c),(a,d),(a,e),(b,b),(b,c),(b,d),(b,e),(c,b),(c,c),(c,d),(c,e),(d,b),(d,c),(d,d),(d,e)}

B × A = {(b,a),(b,b),(b,c),(b,d),(c,a),(c,b),(c,c),(c,d),(d,a),(d,b),(d,c),(d,d),(e,a),(e,b),(e,c),(e,d)}

(A × B) ∩ (B × A) = {(b, b), (b, c), (b, d), (c, b), (c, c), (c, d), (d, b), (d, c), (d, d)}

n{(A × B) ∩ (B × A)} = 9

Relations

6. If A is the set of even numbers less than 8 and B is the set of prime numbers less than 7, then the number of relations from A to B is

(1) 2^9 (2) 9^2
(3) 3^2 (4) 2^{9-1} **[Ans: (1)]**

Sol :

A = {2, 4, 6} ⇒ n(A) = 3
B = {2, 3, 5} ⇒ n(B) = 3
∴ n(A × B) = 3 × 3 = 9
Number of relations from A to B = 2^9

7. Let N be the set of all natural numbers and let R be a relation on N defined as R = $\{(x, y) / x\in N, y\in N$ and x + 3y = 15}. Then R as set of ordered pairs is

(1) {(3, 4), (5, 3), (9, 2), (13, 2)}
(2) {(3, 5), (2, 7), (9, 2), (12, 1)}
(3) {(3, 4), (6, 3), (9, 2), (12, 1)}
(4) {(4, 5), (7, 3), (4, 5), (4, 2)} **[Ans: (3)]**

Sol :

R = {(x, y), x∈N, y∈N and x + 3y = 15}
x + 3y = 15
When x = 3, y = 4, then 3 + 3(4) = 15
When x = 6, y = 3, then 6 + 3(3) = 15
When x = 9, y = 2, then 9 + 3(2) = 15
When x = 12, y = 1, then 12 + 3(1) = 15
∴ R = {(3, 4), (6, 3), (9, 2), (12, 1)}

8. A relation R is defined from {2, 3, 4, 5} to {3, 6, 7, 10} by : x R y ⇔ x is relatively prime to y. Then, domain of R is

(1) {2, 3, 5} (2) {3, 5}
(3) {2, 3, 4} (4) {2, 3, 4, 5} **[Ans: (4)]**

Sol :

Since x is relatively prime to y,
Domain = {2, 3, 4, 5}

9. Let R be a relation from set A to a set B, then

(1) R = $A\cup B$ (2) $A\cap B$
(3) $R\subseteq A\times B$ (4) $R\subseteq B\times A$ **[Ans: (3)]**

Sol :

Relation is a subset of Cartesian Product.

Functions

10. If f (x) = $2x^2 + bx + c$ and f (0) = 3 and f (2) = 1, then f (1) is equal to

(1) – 2 (2) 0

(3) 1 (4) 2 **[Ans: (2)]**

Sol :

$$f(x) = 2x^2 + bx + c$$
$$f(0) = 3$$
$$\Rightarrow 2(0) + b(0) + c = 3 \Rightarrow c = 3$$
$$f(2) = 1$$
$$\Rightarrow 2(2)^2 + b(2) + c = 1 \Rightarrow 2b + c = -7$$
$$2b + 3 = -7$$
$$b = \frac{-10}{2} = -5$$
$$\therefore f(x) = 2x^2 - 5x + 3$$
$$f(1) = 2(1)^2 - 5(1) + 3$$
$$= 2 - 5 + 3 = 0$$

11. Let A = {x, y, z} and B = {a, b, c, d}. Which one of the following is not a function and is not a relation from A to B?

(1) {(x, a), (x, c)} (2) {(y, c), (y, d)}

(3) {(z, a), (z, d)} (4) {(z, b), (y, b), (a, d)}

[Ans: (4)]

Sol :

A = {x, y, z}, B = {a, b, c, d}

{(z, b), (y, b), (a, d)} is not a function from A to B as $a \notin A$.

12. The domain of the function 'f' given by

$$f(x) = \frac{x^2+2x+1}{x^2-x-6}$$

(1) R – {3, – 2} (2) R – {– 3, 2}

(3) R – {3, 2} (4) R – {– 3, – 2} **[Ans: (1)]**

Sol :

$$f(x) = \frac{x^2+2x+1}{x^2-x-6}$$
$$= \frac{x^2+2x+1}{(x-3)(x+2)}$$

f(x) becomes undefined when x = 3 (or) x = – 2.

Domain = R – {3, – 2}

13. Which of the following are functions?

(1) $\{(x, y) : y^2 = x,\ x, y \in R\}$

(2) $\{(x, y) : y = |x|,\ x, y \in R\}$

(3) $\{(x, y) : x^2 + y^2 = 1,\ x, y \in R\}$

(4) $\{(x, y) : x^2 - y^2 = 1,\ x, y \in R\}$ **[Ans: (2)]**

14. If $x \neq 1$ and f(x) = $\frac{x+1}{x-1}$ is a real function, then f (f(f(2))) is

(1) 1 (2) 2

(3) 3 (4) 4 **[Ans: (3)]**

Sol :

$$f(x) = \frac{x+1}{x-1}$$
$$f(2) = \frac{2+1}{2-1} = \frac{3}{1} = 3$$
$$f[f(2)] = f(3) = \frac{3+1}{3-1} = \frac{4}{2} = 2$$
$$f[f(f(2))] = f(2) = \frac{2+1}{2-1} = 3$$

15. If $2f(x) - 3f\left(\frac{1}{x}\right) = x^2, (x \neq 0)$ then f(2) = ?

(1) $\frac{-7}{4}$ (2) $\frac{5}{2}$

(3) – 1 (4) None of these **[Ans: (1)]**

Sol :

Given $2f(x) - 3f\left(\frac{1}{x}\right) = x^2$

Let x = 2

$$2f(2) - 3f\left(\frac{1}{2}\right) = 4 \quad \text{... (1)}$$

Let $x = \frac{1}{2}$

$$2f\left(\frac{1}{2}\right) - 3f(2) = \frac{1}{4} \quad \text{... (2)}$$
$$\Rightarrow f\left(\frac{1}{2}\right) = \frac{\frac{1}{4} + 3f(2)}{2}$$
$$= \frac{1+12f(2)}{8}$$

Substituting in (1), we get

$$f(2) = \frac{-7}{4}$$

Composition of Functions

16. Let $f\left(x+\frac{1}{x}\right) = x^2 + \frac{1}{x^2},\ x \neq 0,$ then f (x) is equal to

(1) $x^2 - 2$ (2) $x^2 - 1$

(3) $f\left(-\frac{a}{a+1}\right)$ (4) $f(a)$ **[Ans: (1)]**

Sol :

$$\text{Given } f\left(x+\frac{1}{x}\right) = x^2+\frac{1}{x^2}$$

$$\text{Let us find } \left(x+\frac{1}{x}\right)^2 = x^2+\frac{1}{x^2}+2x.\frac{1}{x}$$

$$= x^2+\frac{1}{x^2}+2$$

Subtracting 2 from this, we get $x^2+\frac{1}{x^2}$

$$\therefore \ f(x) = x^2 - 2$$

17. If f(x) = x – 2, g(x) = $\sqrt{x^2+1}$, then $(g\circ f)$(x) = ?

(1) $\sqrt{x^2+1}-2$ (2) $\sqrt{x^2+4x+5}$

(3) $x^2 - 1$ (4) $x^2 - 4x + 5$ **[Ans: (2)]**

Sol :

$$(g\circ f)(x) = g[f(x)]$$
$$= g[x-2]$$
$$= \sqrt{(x-2)^2+1}$$
$$= \sqrt{x^2-4x+4+1}$$
$$= \sqrt{x^2-4x+5}$$

18. Given f(2) = 3, g(3) = 2 and g(2) = 5, then $(f\circ g)$(3) =

(1) 2 (2) 3

(3) 4 (4) 5 **[Ans: (2)]**

Sol :

$$(f\circ g)(3) = f[g(3)]$$
$$= f[2] = 3$$

19. Given f = {(– 2, 1), (0, 3), (4, 5)}, g = {(1, 1), (3, 3), (4, 5)} then, Domain and range of $g\circ f$.

(1) D = {3, 0}, R = {– 2, 1}

(2) D = {3, – 2}, R = {1, 5}

(3) D = {– 2, 0}, R = {1, 3}

(4) D = {– 2, 1}, R = {0, 3} **[Ans: (3)]**

Sol :

$$(g\circ f)(-2) = g[f(-2)] = g(1) = 1$$
$$(g\circ f)(0) = g[f(0)] = g(3) = 3$$
$$(g\circ f)(4) = g[f(4)]$$
$$= g(5) = \text{undefined}$$

$$\therefore\ g\circ f = \{(-2, 1), (0, 3)\}$$

Hence the Domain = {– 2, 0},

Range = {1, 3}

II. Very Short Answer Questions

1. If $\left(\frac{x}{3}+1, y-\frac{2}{3}\right) = \left(\frac{5}{3}, \frac{1}{3}\right)$, find the values of 'x' and 'y'

Sol :

$$\frac{x}{3}+1 = \frac{5}{3}, \qquad y-\frac{2}{3} = \frac{1}{3}$$
$$\frac{x}{3} = \frac{5}{3}-1, \qquad y = \frac{1}{3}+\frac{2}{3}$$
$$\frac{x}{3} = \frac{5-3}{3}, \qquad y = \frac{3}{3}$$
$$\frac{x}{3} = \frac{2}{3}, \qquad y = 1$$
$$x = 2$$

2. If the Set 'A' has 3 elements and the Set B = {3, 4, 5}, find the number of elements in (A × B).

Sol :

'A' has 3 elements and 'B' has 3 elements, then 'A × B' will have 9 elements.

3. If G = {7, 8} and H = {5, 4, 2}, find G × H and H × G.

Sol :

G × H = {7, 8} × {5, 4, 2}

= {(7, 5), (7, 4), (7, 2), (8, 5), (8, 4), (8, 2)}

H × G = {5, 4, 2} × {7, 8}

= {(5, 7), (5, 8), (4, 7), (4, 8), (2, 7), (2, 8)}

4. If A = {– 1, 1}, find A × A × A.

Sol :

A × A = {– 1, 1} × {– 1, 1}

= {(– 1, – 1), (– 1, 1), (1, – 1), (1, 1)}

A × A × A = {(– 1, – 1), (– 1, 1), (1, – 1), (1, 1)} × {– 1, 1}

= {(– 1, – 1, – 1), (– 1, – 1, 1), (– 1, 1, – 1), (– 1, 1, 1), (1, – 1, – 1), (1, – 1, 1), (1, 1, – 1), (1, 1, 1)}

5. If A × B = {(a, x), (a, y), (b, x), (b, y)}, find 'A' and 'B'.

Sol :

A = Set of all first entries = {a, b}

B = Set of all second entries = {x, y}

6. Let A = {3, 5} and B = {7, 11}. Let R = {(a, b); a ∈ A, b ∈ B, a – b is odd}. Show that R is an empty relation from A into B.

Sol :

Since a ∈ A and b ∈ B,

$$a - b = (3-7), (3-11), (5-7), (5-11)$$
$$= -4, -8, -2, -6$$ None of them is an odd number.

Therefore, R is an empty relation.

7. What is the number of relations on A if R is a relation on a finite set A having 'n' elements?

Sol :

Set A has 'n' elements.

∴ Number of elements in $A \times A = n \times n = n^2$

The number of possible subsets of $A \times A = 2^{n^2}$

Since, each subset of A × A is a relation on A, the total number of relations on A is 2^{n^2}.

8. Let set A = {January, February, April, June, September, October, November, December} and set B = {28, 29, 30, 31}. Let R be a relation from A to B defined by R = {(a, b) ∈ A × B : 'a' month has 'b' number of days}. Write a subset of relation R connected with 'Teachers Day'.

Sol :

R = {(a, b) ∈ A × B, 'a' month has 'b' number of days}

∴ R = {(January, 31), (February, 28), (February, 29), (April, 30), (June, 30), (September, 30), (October, 31), (November, 30), December, 31)}

∴ The subset of relation R connected with Teachers Day is {(September, 30}

9. A function 'f' is defined by f (x) = 2x – 5. Write down the values of f (0), f (7) and f (– 3).

Sol :

$$f(x) = 2x - 5$$
$$f(0) = 2(0) - 5 = -5$$
$$f(7) = 2(7) - 5 = 14 - 5 = 9$$
$$f(-3) = 2(-3) - 5 = -6 - 5 = -11$$

10. If A = {1, 2, 3} and B = {a, b, c} then do

i) {(1, a), (2, b), (2, c), (3, c)} and

ii) {(2, b), (3, b)} represent a function A → B?

Sol :

(i) The two ordered pairs (2, b) and (2, c) have the same first co-ordinate. Therefore, (1, a), (2, b), (2, c), (3, c) does not represent a function A → B.

(ii) Since one element 1 of A is not associated with some element of B. i.e., 1 is not the first co-ordinate of any ordered pair. So {(2, b), (3, b)} does not represent a function from A → B.

11. Let A = {1, 2, 3} and B = {4, 5}. Let f = {(1, 4), (1, 5), (2, 4), (3, 5)}. Is 'f' a function from A into B?

Sol :

The first elements of two ordered pairs (1, 4) and (1, 5) in 'f' are same.

Therefore f = {(1, 4), (1, 5), (2, 4), (3, 5)} is not a function from A to B.

12. Find the domain of $f(x) = \dfrac{x}{x^2 - 5x + 6}$.

Sol :

$$f(x) = \frac{x}{x^2 - 5x + 6} = \frac{x}{(x-3)(x-2)}$$

When x = 2 or 3, f(x) becomes undefined.

∴ Domain of f = R – {2, 3}

13. Let f : R → R defined by $f(x) = \begin{cases} 3x - 2, & x < 0 \\ 1, & x = 0 \\ 4x + 1, & x > 0 \end{cases}$. Find f(– 3), f(0) and f(5).

Sol :

$$f(-3) = 3(-3) - 2 = -11 \text{ as } -3 < 0$$
$$f(0) = 1 \text{ as } x = 0$$
$$f(5) = 4(5) + 1 = 20 + 1 = 21 \text{ as } x > 0$$

14. Let a function f : R → A be defined as

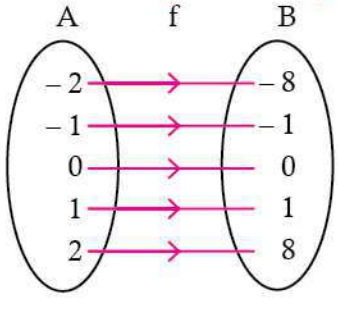
$$f(x) = \begin{cases} 1, & \text{if } x \text{ is rational} \\ -1, & \text{if } x \text{ is irrational, where } x \in R \end{cases}$$

Find $f\left(\frac{1}{3}\right)$ and $f(\sqrt{5})$.

Sol :

As $\frac{1}{3}$ is a rational number, $f\left(\frac{1}{3}\right) = 1$ and as $\sqrt{5}$ is an irrational number, $f(\sqrt{5}) = -1$.

15. Consider the following arrow diagram.



i) Is it a function? If so, write the equation form.

ii) List the elements of this function.

Sol :

Yes, it is a function and $f(x) = x^3$

Elements of $f = \{(-2,-8),(-1,-1),(0,0),(1,1),(2,8)\}$

16. If $f(x) = \dfrac{x-1}{x+1}$, then show that $f\left(\dfrac{1}{x}\right) = -f(x)$

Sol :

$$f\left(\frac{1}{x}\right) = \frac{\frac{1}{x}-1}{\frac{1}{x}+1} = \frac{\frac{1-x}{x}}{\frac{1+x}{x}} = \frac{1-x}{1+x}$$

$$= -\frac{(x-1)}{x+1} = -f(x)$$

Hence proved.

17. Given $f(x) = 3x + 2$, $g(x) = x + 5$ find $f[g(x)]$ and $g[f(x)]$.

Sol :

$$f[g(x)] = f[x+5] = 3(x+5)+2$$
$$= 3x + 15 + 2 = 3x + 17$$
$$g[f(x)] = g[3x+2] = 3x + 2 + 5 = 3x + 7$$

18. If $f(x) = 2x - 1$, then find $f \circ f$.

Sol :

$$f(x) = 2x - 1$$
$$f \circ f = f[f(x)]$$
$$= f(2x-1) = 2(2x-1) - 1$$
$$= 4x - 2 - 1 = 4x - 3$$

19. Given $f(x) = x^2 + 6$ and $g(x) = 2x + 1$, find $(g \circ f)(2)$.

Sol :

$$g \circ f = g[f(x)]$$
$$= g[x^2+6]$$
$$= 2(x^2+6)+1$$
$$= 2x^2 + 12 + 1 = 2x^2 + 13$$

Now, $(g \circ f)(2) = 2(2)^2 + 13$

$= 2(4) + 13 = 8 + 13 = 21$

III. Short Answer Questions:

1. If $A = \{1, 2, 3\}$, $B = \{3, 4\}$ and $C = \{4, 5, 6\}$, then find $(A \times B) \cup (A \times C)$.

Sol :

$$A \times B = \{1,2,3\} \times \{3,4\}$$
$$= \{(1,3),(1,4),(2,3),(2,4),(3,3),(3,4)\}$$
$$A \times C = \{1,2,3\} \times \{4,5,6\}$$
$$= \{(1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6)\}$$

$\therefore (A \times B) \cup (A \times C) = \{(1,3),(1,4),(1,5),(1,6),(2,3),(2,4),(2,5),(2,6),(3,3),(3,4),(3,5),(3,6)\}$

2. If $A = \{1, 2, 4\}$, $B = \{2, 4, 5\}$, $C = \{2, 5\}$, find $(A - B) \times (B - C)$

Sol :

$$A - B = \{1,2,4\} - \{2,4,5\}$$
$$= \{1\}$$
$$B - C = \{2,4,5\} - \{2,5\}$$
$$= \{4\}$$
$$\therefore (A - B) \times (B - C) = \{1\} \times \{4\}$$
$$= \{(1,4)\}$$

3. If $R_1 = \{(x, y) / y = 2x + 7$. where $x \in R$ and $-5 \le y \le 5\}$ is a relation, then find the domain and range of R_1.

Sol :

$R_1 = \{(x,y) / y = 2x+7, x \in R, -5 \le y \le 5\}$

$x = \{....., -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, ...\}$

$y = \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$

When $x = -6$, $y = -5$

$x = -5$, $y = -3$

$x = -4$, $y = -1$

$x = -3$, $y = 1$

$x = -2$, $y = 3$

$x = -1$, $y = 5$

$\therefore R = \{(-6,-5),(-5,-3),(-4,-1),(-3,1),(-2,3),(-1,5)\}$

Domain $= \{-6, -5, -4, -3, -2, -1\}$

Range $= \{-5, -3, -1, 1, 3, 5\}$

4. Let R be a relation in N defined by $R = \{(1 + x, 1 + x^2) / x \le 4, x \in N\}$ Find

(i) R

(ii) domain of R

(iii) range of R

Sol :

$R = \{(1+x, 1+x^2) / x \le 4, x \in N\}$

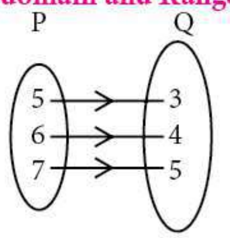
as $x \le 4$, $x \in N$, x takes the values 1, 2, 3, 4

$\therefore R = \{(2,2),(3,5),(4,10),(5,17)\}$

Domain of R $= \{2, 3, 4, 5\}$

Range of R $= \{2, 5, 10, 17\}$

5. The figure shows a relationship between the Sets P and Q. Write this relation in
(i) Set builder form
(ii) Roster form
(iii) What is its domain and Range?



Sol :

(i) Set builder form $R = \{(x, y) / y = x - 2$ for $x = 5, 6, 7\}$

(ii) Roster form $R = \{(5, 3), (6, 4), (7, 5)\}$

(iii) Domain of $R = \{5, 6, 7\}$
Range of $R = \{3, 4, 5\}$

6. Find the domain and range for the functions

(a) $f(x) = x^2 + 2$, (b) $f(t) = \dfrac{1}{t+2}$

Sol :

(a) $f(x) = x^2 + 2$ is defined for all real values of 'x'.
Hence the domain of f (x) is "all real values of x".
Since 'x^2' is always positive, $x^2 + 2$ is never less than 2.
$\therefore$ The range of f (x) is "all real numbers $f(x) \geq 2$".

(b) $f(t) = \dfrac{1}{t+2}$

Domain: The function is not defined for $t = -2$
Hence, the domain of f (t) is "all real numbers except – 2"
Range: "all real numbers except zero".

7. Let f = {(1, 1), (2, 3), (0, – 1), (– 1, – 3)} be a function from Z to Z defined by f(x) = ax + b, for some integers a, b. Determine a, b.

Sol :

$f(x) = ax + b$

for $x = 1, f(x) = 1$

$\therefore \quad a(1) + b = 1 \Rightarrow a + b = 1 \quad \dots (1)$

for $x = 2, f(x) = 3$

$\therefore \quad a(2) + b = 3 \Rightarrow 2a + b = 3 \quad \dots (2)$

Solving (1) and (2)

We get, $a = 2, b = -1$

$\therefore \quad f(x) = 2x - 1$

Also for $x = 0, f(0) = -1$ and
when $x = -1, f(-1) = -3$.

8. Find the domain and Range of the real function 'f' defined by $f(x) = \dfrac{4-x}{x-4}$.

Sol :

When $x = 4$, f(x) is not defined as $f(4) = \dfrac{0}{0}$. So, the domain is the set of all real numbers except $x = 4$ i.e., $R - \{4\}$.

and $f(x) = \dfrac{4-x}{x-4} = -\dfrac{(x-4)}{x-4} = -1, x \neq 4$

$\therefore \quad$ Range of $f(x) = \dfrac{4-x}{x-4}$ is -1.

Domain $= R - \{4\}$, Range $= \{-1\}$

9. Express the following function as set of ordered pairs and determine the range: $f : X \to R$, $f(x) = x^3 + 1$, where $X = \{-1, 0, 3, 9, 7\}$.

Sol :

$f : X \to R, f(x) = x^3 + 1$ where $X = \{-1, 0, 3, 9, 7\}$

$f(-1) = (-1)^3 + 1 = -1 + 1 = 0$

$f(0) = (0)^3 + 1 = 0 + 1 = 1$

$f(3) = (3)^3 + 1 = 27 + 1 = 28$

$f(9) = (9)^3 + 1 = 729 + 1 = 730$

$f(7) = (7)^3 + 1 = 343 + 1 = 344$

$\therefore \quad f = \{(-1, 0), (0, 1), (3, 28), (9, 730), (7, 344)\}$ and
Range of $f = \{0, 1, 28, 730, 344\}$

10. If $A = \{-1, 0, 2, 5, 1\}$ and $f : A \to Z$ is defined by $f(x) = x^2 - x - 2$. Find the range of 'f'.

Sol :

Given $A = \{-1, 0, 2, 5, 1\}$

$f(x) = x^2 - x - 2$

$f(-1) = (-1)^2 - (-1) - 2 = 0$

$f(0) = (0)^2 - 0 - 2 = -2$

$f(2) = (2)^2 - 2 - 2 = 0$

$f(5) = (5)^2 - 5 - 2 = 18$

$f(1) = (1)^2 - 1 - 2 = -2$

$\therefore$ Range of $f = \{-2, 0, 18\}$

11. If $f(x) = x^2$, find $\dfrac{f(1.1) - f(1)}{(1.1 - 1)}$

Sol :

$f(x) = x^2$

$f(1.1) = (1.1)^2 = 1.21$

$f(1) = (1)^2 = 1$

$\therefore \quad \dfrac{f(1.1) - f(1)}{(1.1 - 1)} = \dfrac{1.21 - 1}{1.1 - 1}$

$= \dfrac{0.21}{0.1} \times \dfrac{100}{100} = \dfrac{21}{10} = 2.1$

12. If $f(x) = x^2 - 2$ and $g(x) = 2x - 3$, then Evaluate

(i) $f(-3)$ (ii) $g(5)$ (iii) $f(a+1)$

Sol :

(i) $f(-3)$

$$f(x) = x^2 - 2$$
$$f(-3) = (-3)^2 - 2 = 9 - 2 = 7$$

(ii) $g(5)$

$$g(x) = 2x - 3$$
$$g(5) = 2(5) - 3 = 10 - 3 = 7$$

(iii) $f(a+1)$

$$f(x) = x^2 - 2$$
$$f(a+1) = (a+1)^2 - 2$$
$$= a^2 + 2a + 1 - 2 = a^2 + 2a - 1$$

13. Given $f(x) = 4x + 2$, Evaluate $\dfrac{f(a+h) - f(a)}{h}$.

Sol :

$$f(x) = 4x + 2$$
$$f(a+h) = 4(a+h) + 2$$
$$= 4a + 4h + 2$$
$$f(a) = 4a + 2$$
$$\therefore \frac{f(a+h) - f(a)}{h} = \frac{4a + 4h + 2 - 4a - 2}{h}$$
$$= \frac{4h}{h} = 4$$

14. Given $f(x) = 3x + 7$, $g(x) = x + 1$, $h(x) = 2x - 3$, Find $f(2) + h(1)$ and $f(0) + g(0) - h(0)$.

Sol :

$$f(2) + h(1) = 3(2) + 7 + 2(1) - 3$$
$$= 6 + 7 + 2 - 3 = 12$$
$$f(0) + g(0) - h(0) = 3(0) + 7 + 0 + 1 - (2(0) - 3)$$
$$= 0 + 7 + 0 + 1 - 0 + 3 = 11$$

15. If $f(x) = 2x^4 + x^2 + 1$ and $g(x) = \sqrt{x}$. Find $f \circ g$.

Sol :

$$f \circ g = f[g(x)] = f[\sqrt{x}]$$
$$= 2(\sqrt{x})^4 + (\sqrt{x})^2 + 1 = 2x^2 + x + 1$$

16. If $f(x) = -2x + 9$ and $g(x) = -4x^2 + 5x - 3$, find $f \circ g$ and $g \circ f$.

Sol :

$$f \circ g = f[g(x)]$$
$$= f[-4x^2 + 5x - 3]$$
$$= -2(-4x^2 + 5x - 3] + 9$$
$$= 8x^2 - 10x + 15$$
$$g \circ f = g[f(x)]$$
$$= g[-2x + 9]$$
$$= -4(-2x + 9)^2 + 5(-2x + 9) - 3$$
$$= -4(4x^2 - 36x + 81) - 10x + 45 - 3$$
$$= -16x^2 + 144x - 324 - 10x + 42$$
$$= -16x^2 + 134x - 282$$

17. If $g(x) = \sqrt[3]{x-4}$ and $h(x) = x^3 + 4$, find $(h \circ g)(-15)$.

Sol :

$$h \circ g = h[g(x)]$$
$$= h[\sqrt[3]{x-4}]$$
$$= (\sqrt[3]{x-4})^3 + 4$$
$$= (x-4)^{3 \times \frac{1}{3}} + 4$$
$$= x - 4 + 4 = x$$
$$\therefore (h \circ g)(-15) = -15$$

IV. Long Answer Questions

1. If $A = \{x / x^2 - 4x + 3 = 0\}$, $B = \{x / x^2 - x - 6 = 0\}$, $C = \{x / x^3 - 4x = 0\}$, find (i) $A \times B$ (ii) $A \times C$ (iii) $(A - B) \times C$

Sol :

$$A = \{x / x^2 - 4x + 3 = 0\}$$
$$= \{x / (x-3)(x-1) = 0\}$$
$$= \{x / x = 1, 3\}$$
$$B = \{x / x^2 - x - 6 = 0\}$$
$$= \{x / (x+2)(x-3) = 0\}$$
$$= \{x / x = -2, 3\}$$
$$C = \{x / x^3 - 4x = 0\}$$
$$= \{x / x(x^2 - 4) = 0\}$$
$$= \{x / x = 0, -2, 2\}$$
$$A = \{1, 3\},\ B = \{-2, 3\},\ C = \{-2, 0, 2\}$$

(i) $A \times B = \{1, 3\} \times \{-2, 3\}$
$$= \{(1, -2), (1, 3), (3, -2), (3, 3)\}$$

(ii) $A \times C = \{1, 3\} \times \{-2, 0, 2\}$
$$= \{(1, -2), (1, 0), (1, 2), (3, -2), (3, 0), (3, 2)\}$$

(iii) $A - B = \{1, 3\} - \{-2, 3\} = \{1\}$
$$(A - B) \times C = \{1\} \times \{-2, 0, 2\}$$
$$= \{(1, -2), (1, 0), (1, 2)\}.$$

2. If A and B be two non-empty sets, then show that $A \times B = B \times A$ if $A = B$.

Sol :

Let $A \times B = B \times A$ and $x \in A$

$$x \in A \Rightarrow (x, b) \in A \times B\ \forall\ b \in B$$
$$\Rightarrow (x, b) \in (B \times A)$$

$\because$ A × B = B × A

$\Rightarrow x \in B$

$\Rightarrow A \subseteq B$

By the definition of Cartesian Product and let y be an arbitrary element of B

$y \in B \Rightarrow (a, y) \in A \times B \;\forall\; a \in A$

$\Rightarrow (a, y) \in (B \times A)$

$\because$ A × B = B × A

By the definition of Cartesian Product

$\Rightarrow y \in A$

$B \subseteq A$

From these two results, A = B

Conversely, let A = B, then

A × B = A × A and B × A = A × A

$\therefore$ A × B = B × A

Hence A × B = B × A if A = B.

3. In the given ordered pairs (4, 6), (8, 4), (3, 3), (9, 11), (6, 3), (3, 0), (2, 3). Find the following relations. Also, find the domain and range.

(i) is two less than (ii) is less than

(iii) is greater than (iv) is equal to

Sol :

(i) R_1 is the set of all ordered pairs whose first component is two less than the second component.

$\therefore$ R_1 = {(4, 6), (9, 11)}

Domain of R_1 = Set of all first components
= {4, 9}

Range of R_1 = Set of all second components
= {6, 11}

(ii) R_2 is the set of all ordered pairs whose first component is less than the second component.

$\therefore$ R_2 = {(4, 6), (9, 11), (2, 3)}

Domain of R_2 = {4, 9, 2}

Range of R_2 = {6, 11, 3)

(iii) R_3 is the set of all ordered pairs whose first component is greater than the second component.

$\therefore$ R_3 = {(8, 4), (6, 3), (3, 0)}

Domain of R_3 = {8, 6, 3}

Range of R_3 = {4, 3, 0}

(iv) R_4 is the set of all ordered pairs whose first component is equal to the second component.

$\therefore$ R_4 = {(3, 3)}

Domain of R_4 = {3} and Range of R_4 = {3}.

4. Let A = {2, 3, 4, 5} and B = {8, 9, 10, 11}. Let R be the relation "is factor of" from A and B.

i) Write R in the roster form. Also find Domain and Range of R.

ii) Draw an arrow diagram to represent the relation.

Sol :

(i) Given, that the Relation R consists of elements (a, b) where 'a' is a factor of 'b'.

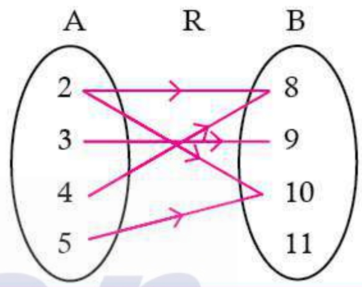
Therefore, Relation 'R' in the roster form is

R = {(2, 8), (2, 10), (3, 9), (4, 8), (5, 10)}

Domain of R = {2, 3, 4, 5}

Range of R = {8, 10, 9}

(ii) Arrow diagram representing 'R'



5. Given f (x) = 3x + 7, g (x) = – x + 8, h (x) = x² + 3x – 1. Prove that Composition of functions is associative.

Sol :

$$\begin{aligned}
f \circ g &= f[g(x)] \\
&= f[-x+8] \\
&= 3(-x+8)+7 \\
&= -3x+24+7 = -3x+31 \\
(f \circ g) \circ h &= (f \circ g)[h(x)] \\
&= (f \circ g)[x^2+3x-1] \\
&= -3(x^2+3x-1)+31 \\
&= -3x^2-9x+3+31 \\
&= -3x^2-9x+34 \qquad \ldots (1) \\
g \circ h &= g[h(x)] \\
&= g[x^2+3x-1] \\
&= -(x^2+3x-1)+8 \\
&= -x^2-3x+1+8 = -x^2-3x+9 \\
f \circ (g \circ h) &= f[g(h(x))] = f[g(x^2+3x-1)] \\
&= f[-x^2-3x+9] \\
&= 3(-x^2-3x+9)+7 \\
&= -3x^2-9x+27+7
\end{aligned}$$

$$= -3x^2 - 9x + 34 \quad \ldots (2)$$

From (1) and (2)

$$(f \circ g) \circ h = f \circ (g \circ h)$$

$\therefore$ Composition of functions is Associative.

6. If 'f' and 'g' are two real valued functions defined as $f(x) = 2x + 1$, $g(x) = x^2 + 1$, then find

(i) f + g (ii) f – g (iii) fg (iv) $\frac{f}{g}$

Sol :

(i)
$$(f+g)(x) = f(x) + g(x)$$
$$= 2x + 1 + x^2 + 1$$
$$= x^2 + 2x + 2$$

(ii)
$$(f-g)(x) = f(x) - g(x)$$
$$= (2x+1) - (x^2+1)$$
$$= 2x + 1 - x^2 - 1 = 2x - x^2$$
$$= -x^2 + 2x$$

(iii)
$$(fg)(x) = [f(x)][g(x)]$$
$$= (2x+1)(x^2+1)$$
$$= 2x^3 + x^2 + 2x + 1$$

(iv)
$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, \text{provided } g(x) \neq 0$$
$$= \frac{2x+1}{x^2+1}$$