

Chapter 1: Partition Values

EXERCISE 1.1 [PAGES 7 - 8]

Exercise 1.1 | Q 1 | Page 7

Compute all the quartiles for the following series of observations:

16, 14.9, 11.5, 11.8, 11.1, 14.5, 14, 12, 10.9, 10.7, 10.6, 10.5, 13.5, 13, 12.6.

SOLUTION

The given data can be arranged in ascending order as follows:

10.5, 10.6, 10.7, 10.9, 11.1, 11.5, 11.8, 12, 12.6, 13, 13.5, 14, 14.5, 14.9, 16

Here, $n = 15$

$$Q_1 = \text{value of } \left(\frac{n+1}{4} \right)^{\text{th}} \text{ observation}$$

$$= \text{value of } \left(\frac{15+1}{4} \right)^{\text{th}} \text{ observation}$$

$$= \text{value of } 4^{\text{th}} \text{ observation}$$

$$\therefore Q_1 = 10.9$$

$$Q_2 = \text{value of } 2 \left(\frac{n+1}{4} \right)^{\text{th}} \text{ observation}$$

$$= \text{value of } 2 \left(\frac{15+1}{4} \right)^{\text{th}} \text{ observation}$$

$$= \text{value of } (2 \times 4)^{\text{th}} \text{ observation}$$

$$= \text{value of } 8^{\text{th}} \text{ observation}$$

$$\therefore Q_2 = 12$$

$$Q_3 = \text{value of } 3 \left(\frac{n+1}{4} \right)^{\text{th}} \text{ observation}$$

$$= \text{value of } 3\left(\frac{15+1}{4}\right)^{\text{th}} \text{ observation}$$

$$= \text{value of } (3 \times 4)^{\text{th}} \text{ observation}$$

$$= \text{value of } 12^{\text{th}} \text{ observation}$$

$$\therefore Q_3 = 14$$

Exercise 1.1 | Q 2 | Page 7

The heights (in cm) of 10 students are given below: 148, 171, 158, 151, 154, 159, 152, 163, 171, 145. Calculate Q_1 and Q_3 for above data.

SOLUTION

The given data can be arranged in ascending order as follows:

145, 148, 151, 152, 154, 158, 159, 163, 171, 171.

Here, $n = 10$

$$Q_1 = \text{value of } \left(\frac{n+1}{4}\right)^{\text{th}} \text{ observation}$$

$$= \text{value of } \left(\frac{10+1}{4}\right)^{\text{th}} \text{ observation}$$

$$= \text{value of } (2.75)^{\text{th}} \text{ observation}$$

$$= \text{value of } 2^{\text{nd}} \text{ observation} + 0.75 (\text{value of } 3^{\text{rd}} \text{ observation} - \text{value of } 2^{\text{nd}} \text{ observation})$$

$$= 148 + 0.75 (151 - 148)$$

$$= 148 + 0.75 (3)$$

$$= 148 + 2.25$$

$$\therefore Q_1 = 150.25$$

$$Q_3 = \text{value of } 3\left(\frac{n+1}{4}\right)^{\text{th}} \text{ observation}$$

$$= \text{value of } 3\left(\frac{10+1}{4}\right)^{\text{th}} \text{ observation}$$

$$= \text{value of } (3 \times 2.75)^{\text{th}} \text{ observation}$$

$$= \text{value of } (8.25)^{\text{th}} \text{ observation}$$

$$= \text{value of } 8^{\text{th}} \text{ observation} + 0.25 (\text{value of } 9^{\text{th}} \text{ observation} - \text{value of } 8^{\text{th}} \text{ observation})$$

$$= 163 + 0.25 (171 - 163)$$

$$\begin{aligned}
 &= 163 + 0.25 (8) \\
 &= 163 + 2 \\
 \therefore Q_3 &= 165
 \end{aligned}$$

Exercise 1.1 | Q 3 | Page 7

Monthly consumption of electricity (in units) of families in a certain locality is given below:

205, 201, 190, 188, 195, 172, 210, 225, 215, 232, 260, 230.

Calculate electricity consumption (in units) below which 25% of families lie.

SOLUTION

To find the consumption of electricity below which 25% of the families lie, we have to find Q_1



Monthly consumption of electricity (in units) can be arranged in ascending order as follows:

172, 188, 190, 195, 201, 205, 210, 215, 225, 230, 232, 260.

Here, $n = 12$

$$Q_1 = \text{value of } \left(\frac{n+1}{4} \right)^{\text{th}} \text{ observation}$$

$$= \text{value of } \left(\frac{12+1}{4} \right)^{\text{th}} \text{ observation}$$

$$= \text{value of } (3.25)^{\text{th}} \text{ observation}$$

$$= \text{value of } 3^{\text{rd}} \text{ observation} + 0.25 (\text{value of } 4^{\text{th}} \text{ observation} - \text{value of } 3^{\text{rd}} \text{ observation})$$

$$= 190 + 0.25 (195 - 190)$$

$$= 190 + 0.25 (5)$$

$$= 190 + 1.25$$

$$= 191.25$$

\therefore the consumption of electricity below which 25% of the families lie is 191.25.

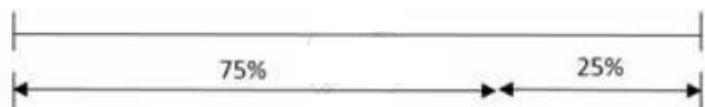
Exercise 1.1 | Q 4 | Page 7

For the following data of daily expenditure of families (in ₹), compute the expenditure below which 75% of families include their expenditure.

Daily expenditure (in ₹)	350	450	550	650	750
No. of families	16	19	24	28	13

SOLUTION

To find the expenditure below which 75% of families have their expenditure, we have to find Q_3 .



We construct the less than cumulative frequency table as given below:

Daily expenditure (in ₹)	No. of families (f)	Less than cumulative frequency (c.f.)
350	16	16
450	19	35
550	24	59
650	28	87 ← Q_3
750	13	100
Total	100	

Here, $n = 100$

$$Q_3 = \text{value of } 3 \left(\frac{n+1}{4} \right)^{\text{th}} \text{ observation}$$

$$= \text{value of } 3 \left(\frac{100+1}{4} \right)^{\text{th}} \text{ observation}$$

$$= \text{value of } (3 \times 25.25)^{\text{th}} \text{ observation}$$

$$= \text{value of } (75.75)^{\text{th}} \text{ observation}$$

Cumulative frequency which is just greater than (or equal to) 75.75 is 87.

$$\therefore Q_3 = 650$$

\therefore The expenditure below which 75% of families include their expenditure is 650.

Exercise 1.1 | Q 5 | Page 7

Calculate all the quartiles for the following frequency distribution:

No. of E-transactions per day	0	1	2	3	4	5	6	7
No. of days	10	35	45	95	64	32	10	9

SOLUTION

We construct the less than cumulative frequency table as given below:

No. of E-transaction per day	No. of days	Less than cumulative frequency (c.f.)
0	10	10
1	35	45
2	45	90 $\leftarrow Q_1$
3	95	185 $\leftarrow Q_2$
4	64	249 $\leftarrow Q_3$
5	32	281
6	10	291
7	9	300
Total	300	

Here, $n = 300$

$$Q_1 = \text{value of } \left(\frac{n+1}{4} \right)^{\text{th}} \text{ observation}$$

$$= \text{value of } \left(\frac{300+1}{4} \right)^{\text{th}} \text{ observation}$$

$$= \text{value of } (75.25)^{\text{th}} \text{ observation}$$

Cumulative frequency which is just greater than (or equal to) 75.25 is 90.

$$\therefore Q_1 = 2$$

$$Q_2 = \text{value of } 2 \left(\frac{n+1}{4} \right)^{\text{th}} \text{ observation}$$

$$= \text{value of } 2 \left(\frac{300+1}{4} \right)^{\text{th}} \text{ observation}$$

$$= \text{value of } (2 \times 75.25)^{\text{th}} \text{ observation}$$

$$= \text{value of } (150.50)^{\text{th}} \text{ observation}$$

Cumulative frequency which is just greater than (or equal to) 150.50 is 185.

$$\therefore Q_2 = 3$$

$$Q_3 = \text{value of } 3 \left(\frac{n+1}{4} \right)^{\text{th}} \text{ observation}$$

$$= \text{value of } 3 \left(\frac{300+1}{4} \right)^{\text{th}} \text{ observation}$$

$$= \text{value of } (3 \times 75.25)^{\text{th}} \text{ observation}$$

$$= \text{value of } (225.75)^{\text{th}} \text{ observation}$$

Cumulative frequency which is just greater than (or equal to) 225.75 is 249.

$$\therefore Q_3 = 4$$

Exercise 1.1 | Q 6 | Page 7

The following is the frequency distribution of heights of 200 male adults in a factory:

Height (in cm.)	No. of male adults
145 – 150	4
150 – 155	6
155 – 160	25
160 – 165	57
165 – 170	64
170 – 175	30

175 – 180	8
180 – 185	6

Find the central height.

SOLUTION

To find the central height, we have to find Q_2 .

We construct the less than cumulative frequency table as given below:

Height (in cm.)	No. of male adults (f)	Less than Cumulative frequency (c.f.)
145 – 150	4	4
150 – 155	6	10
155 – 160	25	35
160 – 165	57	92
165 – 170	64	156 ← Q_2
170 – 175	30	186
175 – 180	8	194
180 – 185	6	200
Total	200	

Here, $N = 200$

Q_2 class = class containing $\left(\frac{2N}{4}\right)^{\text{th}}$ observation

$$\therefore \frac{2N}{4} = \frac{2 \times 200}{4} = 100$$

Cumulative frequency which is just greater than (or equal to) 100 is 156.

$\therefore Q_2$ lies in the class 165 – 170

$\therefore L = 165, f = 64, \text{c.f.} = 92; h = 5$

$$Q_2 = L + \frac{h}{f} \left(\frac{2N}{4} - \text{c.f.} \right)$$

$$= 165 + \frac{5}{64} (100 - 92)$$

$$= 165 + \frac{5}{64} \times 8$$

$$= 165 + \frac{5}{8}$$

$$= 165 + 0.625$$

$$= 165.625$$

\therefore The central height is 165.625 cm.

Exercise 1.1 | Q 7 | Page 7

The following is the data of pocket expenditure per week of 50 students in a class. It is known that the median of the distribution is ₹120. Find the missing frequencies.

Expenditure per week (in ₹)	0 – 50	50 – 100	100 – 150	150 – 200	200 – 250
No. of students	7	?	15	?	3

SOLUTION

Let a and b be the missing frequencies of the class 50 – 100 and class 150 – 200 respectively.

We construct the less than cumulative frequency table as given below:

Expenditure per week (in ₹)	No. of students (f)	Less than Cumulative frequency (c.f.)
0 – 50	7	7
50 – 100	a	7 + a
100 – 150	15	22 + a ← Q ₂
150 – 200	b	22 + a + b
200 – 250	3	25 + a + b
Total	25 + a + b	

Here, $N = 25 + a + b$

Since, $N = 50$

$$\therefore 25 + a + b = 50$$

$$\therefore a + b = 25 \dots\dots\dots(i)$$

Given, Median = $Q_2 = 120$

$\therefore Q_2$ lies in the class 100 – 150.

$$\therefore L = 100, h = 50, f = 15, \frac{2N}{4} = \frac{2 \times 50}{4} = 25,$$

$$c.f. = 7 + a$$

$$Q_2 = L + \frac{h}{f} \left(\frac{2N}{4} - c.f. \right)$$

$$\therefore 120 = 100 + \frac{50}{15} [25 - (7 + a)]$$

$$\therefore 120 - 100 = \frac{10}{3} (25 - 7 - a)$$

$$\therefore 20 = \frac{10}{3} (18 - a)$$

$$\therefore \frac{60}{10} = 18 - a$$

$$\therefore 6 = 18 - a$$

$$\therefore a = 18 - 6 = 12$$

Substituting the value of a in equation (i), we get

$$12 + b = 25$$

$$\therefore b = 25 - 12 = 13$$

\therefore 12 and 13 are the missing frequencies of the class 50 – 100 and class 150 – 200 respectively.

Exercise 1.1 | Q 8 | Page 8

The following is the distribution of 160 Workers according to the wages in a certain factory:

Wages more than (in ₹)	No. of workers
8000	160
9000	155
10000	137
11000	91
12000	57
13000	23
14000	10
15000	1
16000	0

SOLUTION

The given table is a more than cumulative frequency. We transform the given table into less than cumulative frequency.

We construct the less than cumulative frequency table as given below:

Wage s (in ₹)	No. of workers (f)	Less than Cumulative frequency (c.f.)
8000 – 9000	160 – 155 = 5	5
9000 – 10000	155 – 137 = 18	23
10000 – 11000	137 – 91 = 46	69 ← Q ₁
11000 – 12000	91 – 57 = 34	103 ← Q ₂
12000 – 13000	57 – 23 = 34	137 ← Q ₃
13000 – 14000	23 – 10 = 13	150
14000 – 15000	10 – 1 = 9	159
15000 – 16000	1 – 0 = 1	160
16000 – 17000	0	160
Total	160	

Determine the values of all quartiles and interpret the results.

Here, N = 160

∴ Q₁ class = class containing $\left(\frac{N}{4}\right)^{\text{th}}$ observation

$$\therefore \frac{N}{4} = \frac{160}{4} = 40$$

Cumulative frequency which is just greater than (or equal to) 40 is 69.

∴ Q₁ lies in the class 10000 – 11000

∴ L = 10000, f = 46, c.f. = 23, h = 1000

$$\begin{aligned}
 Q_1 &= L + \frac{h}{f} \left(\frac{N}{4} - \text{c.f.} \right) \\
 &= 10000 + \frac{1000}{46} (40 - 23) \\
 &= 10000 + \frac{1000}{46} (17)
 \end{aligned}$$

$$= 10000 + \frac{17000}{46}$$

$$= 10000 + 369.57$$

$$= 10369.57$$

Q_2 class = class containing $\left(\frac{2N}{4}\right)^{\text{th}}$ observation

$$\therefore \frac{2N}{4} = \frac{2 \times 160}{4} = 80$$

Cumulative frequency which is just greater than (or equal to) 80 is 103

$\therefore Q_2$ lies in the class 11000 – 12000

$\therefore L = 11000, f = 34, \text{c.f.} = 69, h = 1000$

$$Q_2 = L + \frac{h}{f} \left(\frac{2N}{4} - \text{c.f.} \right)$$

$$= 11000 + \frac{1000}{34} (80 - 69)$$

$$= 11000 + \frac{1000}{34} (11)$$

$$= 11000 + \frac{11000}{34}$$

$$= 11000 + 323.529$$

$$= 11323.529$$

Q_3 class = class containing $\left(\frac{3N}{4}\right)^{\text{th}}$ observation

$$\frac{3N}{4} = \frac{3(160)}{4} = 120$$

Cumulative frequency which is just greater than (or equal to) 120 is 137.

$\therefore Q_3$ lies in the class 12000 – 13000

$\therefore L = 12000, f = 34, \text{c.f.} = 103; h = 1000$

$$\begin{aligned}
 Q_3 &= L + \frac{h}{f} \left(\frac{3N}{4} - \text{c.f.} \right) \\
 &= 12000 + \frac{1000}{34} (120 - 103) \\
 &= 12000 + \frac{1000}{34} (17) \\
 &= 12000 + \frac{1000}{2} \\
 &= 12000 + 500 \\
 &= 12500
 \end{aligned}$$

∴ The quartiles are

$$Q_1 = \text{Rs. } 10369.57$$

$$Q_2 = \text{Rs. } 11323.529$$

$$Q_3 = \text{Rs. } 12500$$

$$Q_1 < Q_2 < Q_3.$$

Exercise 1.1 | Q 9 | Page 8

Following is the grouped data for duration of fixed deposits of 100 senior citizens from a certain bank:

Fixed deposit (in days)	0 – 180	180 – 360	360 – 540	540 – 720	720 – 900
No. of senior citizens	15	20	25	30	10

Calculate the limits of fixed deposits of central 50% senior citizens.

SOLUTION

Fixed deposit (in days)	No. of senior citizens (f)	Less than Cumulative frequency (c.f.)
0 – 180	15	15

180 – 360	20	35 ← Q ₁
360 – 540	25	60
540 – 720	30	90 ← Q ₃
720 – 900	10	100
Total	100	

We construct the less than cumulative frequency table as given below:

To find the limits of fixed deposits of central 50% senior citizens, we have to find Q₁ and Q₃.

Here, N = 100

Q₁ class = class containing $\left(\frac{N}{4}\right)^{\text{th}}$ observation

$$\therefore \frac{N}{4} = \frac{100}{4} = 25$$

Cumulative frequency which is just greater than (or equal to) 25 is 35.

\therefore Q₁ lies in the class 180 – 360.

\therefore L = 180, f = 20, c.f. = 15, h = 180

$$\therefore Q_1 = L + \frac{h}{f} \left(\frac{N}{4} - \text{c.f.} \right)$$

$$= 180 + \frac{180}{20} (25 - 15)$$

$$= 180 + 9(10)$$

$$= 180 + 90$$

$$\therefore Q_1 = 270$$

Q₃ class = class containing $\left(\frac{3N}{4}\right)^{\text{th}}$ observation

$$\therefore \frac{3N}{4} = \frac{3(100)}{4} = 75$$

Cumulative frequency which is just greater than (or equal to) 75 is 90.

$\therefore Q_3$ lies in the class 540 – 720

$\therefore L = 540, f = 30, \text{c.f.} = 60, h = 180$

$$\therefore Q_3 = L + \frac{h}{f} \left(\frac{3N}{4} - \text{c.f.} \right)$$

$$= 540 + \frac{180}{30} (75 - 60)$$

$$= 540 + 6(15)$$

$$= 540 + 90$$

$$\therefore Q_3 = 630$$

\therefore Limits of the duration of fixed deposits of central 50% of senior citizens are from 270 to 630.

Exercise 1.1 | Q 10 | Page 8

Find the missing frequency given that the median of the distribution is 1504.

Life in hours	950 – 1150	1150 – 1350	1350 – 1550	1550 – 1750	1750 – 1950	1950 – 2150
No. of bulbs	20	43	100	–	23	13

SOLUTION

Let x be the missing frequency of the class 1550 – 1750.

Life in hours	No. of bulbs (f)	Less than Cumulative frequency (c.f.)
950 – 1150	20	20
1150 – 1350	43	63
1350 – 1550	100	163
1550 – 1750	x	163 + x
1750 – 1950	23	186 + x
1950 – 2150	13	199 + x
Total	199 + x	

Here, $N = 199 + x$

Given, Median (Q_2) = 1504

$\therefore Q_2$ lies in the class 1350 – 1550
 $\therefore L = 1350, f = 100, \text{c.f.} = 63, h = 200,$

$$\frac{2N}{4} = \frac{199 + x}{2}$$

$$Q_2 = L + \frac{h}{f} \left(\frac{2N}{4} - \text{c.f.} \right)$$

$$1540 = 1350 + \frac{200}{100} \left(\frac{199 + x}{2} - 63 \right)$$

$$1540 - 1350 = 2 \left(\frac{199 + x - 126}{2} \right)$$

$$\therefore 154 = 199 + x - 126$$

$$\therefore 154 = 73 + x$$

$$\therefore x = 81$$

EXERCISE 1.2 [PAGES 15 - 16]

Exercise 1.2 | Q 1 | Page 15

Calculate D_6 and P_{85} for the following data:
 79, 82, 36, 38, 51, 72, 68, 70, 64, 63.

SOLUTION

The given data can be arranged in ascending order as follows:
 36, 38, 51, 63, 64, 68, 70, 72, 79, 82.
 Here, $n = 10$

$$D_6 = \text{value of } 6 \left(\frac{n+1}{10} \right)^{\text{th}} \text{ observation}$$

$$= \text{value of } 6 \left(\frac{10+1}{10} \right)^{\text{th}} \text{ observation}$$

$$= \text{value of } (6 \times 1.1)^{\text{th}} \text{ observation}$$

$$= \text{value of } (6.6)^{\text{th}} \text{ observation}$$

$$= \text{value of } 6^{\text{th}} \text{ observation} + 0.6 (\text{value of } 7^{\text{th}} \text{ observation} - \text{value of } 6^{\text{th}} \text{ observation})$$

$$= 68 + 0.6 (70 - 68)$$

$$\begin{aligned}
&= 68 + 0.6 (2) \\
&= 68 + 1.2 \\
\therefore D_6 &= \mathbf{69.2}
\end{aligned}$$

$$\begin{aligned}
P_{85} &= \text{value of } 85 \left(\frac{n+1}{100} \right)^{\text{th}} \text{ observation} \\
&= \text{value of } 85 \left(\frac{10+1}{100} \right)^{\text{th}} \text{ observation}
\end{aligned}$$

$$\begin{aligned}
&= \text{value of } (85 \times 0.11)^{\text{th}} \text{ observation} \\
&= \text{value of } (9.35)^{\text{th}} \text{ observation} \\
&= \text{value of } 9^{\text{th}} \text{ observation} + 0.35 (\text{value of } 10^{\text{th}} \text{ observation} - \text{value of } 9^{\text{th}} \text{ observation}) \\
&= 79 + 0.35 (82 - 79) \\
&= 79 + 0.35 (3) \\
&= 79 + 1.05 \\
\therefore P_{85} &= \mathbf{80.05}
\end{aligned}$$

Exercise 1.2 | Q 2 | Page 15

The daily wages (in Rs.) of 15 laboures are as follows:

230, 400, 350, 200, 250, 380, 210, 225, 375, 180, 375, 450, 300, 350, 250

Calculate D_8 and P_{90} .

SOLUTION

The given data can be arranged in ascending order as follows:

180, 200, 210, 225, 230, 250, 250, 300, 350, 350, 375, 375, 380, 400, 450.

Here, $n = 15$

$$\begin{aligned}
D_8 &= \text{value of } 8 \left(\frac{n+1}{10} \right)^{\text{th}} \text{ observation} \\
&= \text{value of } 8 \left(\frac{15+1}{10} \right)^{\text{th}} \text{ observation}
\end{aligned}$$

$$\begin{aligned}
&= \text{value of } (8 \times 1.6)^{\text{th}} \text{ observation} \\
&= \text{value of } (12.8)^{\text{th}} \text{ observation} \\
&= \text{value of } 12^{\text{th}} \text{ observation} + 0.8 (\text{value of } 13^{\text{th}} \text{ observation} - \text{value of } 12^{\text{th}} \text{ observation}) \\
&= 375 + 0.8 (380 - 375) \\
&= 375 + 0.8 (5) \\
&= 375 + 4 \\
\therefore D_8 &= \mathbf{379}
\end{aligned}$$

$$P_{90} = \text{value of } 90 \left(\frac{n+1}{100} \right)^{\text{th}} \text{ observation}$$

$$= \text{value of } 90 \left(\frac{15+1}{100} \right)^{\text{th}} \text{ observation}$$

$$= \text{value of } (90 \times 0.16)^{\text{th}} \text{ observation}$$

$$= \text{value of } (14.4)^{\text{th}} \text{ observation}$$

$$= \text{value of } 14^{\text{th}} \text{ observation} + 0.4 (\text{value of } 15^{\text{th}} \text{ observation} - \text{value of } 14^{\text{th}} \text{ observation})$$

$$= 400 + 0.4 (450 - 400)$$

$$= 400 + 0.4 (50)$$

$$= 400 + 20$$

$$\therefore P_{90} = 420$$

Exercise 1.2 | Q 3 | Page 15

Calculate 2nd decile and 65th percentile for the following:

x	80	100	120	145	200	280	310	380	400	410
f	15	18	25	27	40	25	19	16	8	7

SOLUTION

We construct the less than cumulative frequency table as given below:

x	f	Less than Cumulative frequency (c.f.)
80	15	15
100	18	33
120	25	58 ← D ₂
145	27	85
200	40	125
280	25	150 ← P ₆₅
310	19	169
380	16	185
400	8	193
410	7	200
Total	200	

Here, n = 20

$$D_2 = \text{value of } 2\left(\frac{n+1}{10}\right)^{\text{th}} \text{ observation}$$

$$= \text{value of } 2\left(\frac{200+1}{10}\right)^{\text{th}} \text{ observation}$$

$$= \text{value of } (2 \times 20.1)^{\text{th}} \text{ observation}$$

$$= \text{value of } (40.2)^{\text{th}} \text{ observation}$$

Cumulative frequency which is just greater than (or equal to) 40.2 is 58.

$$\therefore D_2 = 120$$

$$P_{65} = \text{value of } 65\left(\frac{n+1}{100}\right)^{\text{th}} \text{ observation}$$

$$= \text{value of } 65\left(\frac{200+1}{100}\right)^{\text{th}} \text{ observation}$$

$$= \text{value of } (65 \times 2.01)^{\text{th}} \text{ observation}$$

$$= \text{value of } (130.65)^{\text{th}} \text{ observation}$$

Cumulative frequency which is just greater than (or equal to) 130.65 is 150.

$$\therefore P_{65} = 280$$

Exercise 1.2 | Q 4 | Page 15

From the following data calculate the rent of 15th, 65th and 92nd house.

House rent (in ₹)	11000	12000	13000	15000	14000	16000	17000	18000
No. of houses	25	17	13	14	15	8	6	2

SOLUTION

Arranging the given data in ascending order.

House Rent (in ₹)	No. of houses (f)	Less than Cumulative frequency (c.f.)
11000	25	25 ← P ₁₅
12000	17	42
13000	13	55
14000	15	70 ← P ₆₅
15000	14	84
16000	8	92
17000	6	98 ← P ₉₂
18000	2	100
Total	100	

Here, n = 100

$$P_{15} = \text{value of } 15 \left(\frac{n+1}{100} \right)^{\text{th}} \text{ observation}$$

$$= \text{value of } 15 \left(\frac{100+1}{100} \right)^{\text{th}} \text{ observation}$$

$$= \text{value of } (15 \times 1.01)^{\text{th}} \text{ observation}$$

$$= \text{value of } (15.15)^{\text{th}} \text{ observation}$$

Cumulative frequency which is just greater than (or equal to) 15.15 is 25.

$$\therefore P_{15} = 11000$$

$$P_{65} = \text{value of } 65 \left(\frac{n+1}{100} \right)^{\text{th}} \text{ observation}$$

$$= \text{value of } 65 \left(\frac{100+1}{100} \right)^{\text{th}} \text{ observation}$$

$$= \text{value of } (65 \times 1.01)^{\text{th}} \text{ observation}$$

= value of $(65.65)^{\text{th}}$ observation

Cumulative frequency which is just greater than (or equal to) 65.65 is 70.

$$\therefore P_{65} = 14000$$

$$P_{92} = \text{value of } 92 \left(\frac{n+1}{100} \right)^{\text{th}} \text{ observation}$$

$$= \text{value of } 92 \left(\frac{100+1}{100} \right)^{\text{th}} \text{ observation}$$

$$= \text{value of } (92 \times 1.01)^{\text{th}} \text{ observation}$$

$$= \text{value of } (92.92)^{\text{th}} \text{ observation}$$

Cumulative frequency which is just greater than (or equal to) 92.92 is 98.

$$\therefore P_{92} = 17000$$

Exercise 1.2 | Q 5 | Page 15

The following frequency distribution shows the weight of students in a class:

Weight (in Kg)	40	45	50	55	60	65
Number of Students	15	40	29	21	10	5

(a) Find the percentage of students whose weight is more than 50 kg.

(b) If the weight column provided is of mid values then find the percentage of students whose weight is more than 50 kg.

SOLUTION

(a)

Weight (in kg)	Number of students (f)	Less than cumulative frequency (c.f.)
40	15	15
45	40	55
50	29	84

55	21	105
60	10	115
65	5	120
Total	120	

Let the percentage of students weighing less than 50 kg be x.

$$\therefore P_x = 50$$

From the table, out of 20 students, 84 students have their weight less than 50 kg.

$$\therefore \text{Number of students weighing more than 50 kg} \\ = 120 - 84 = 36$$

$$\therefore \text{percentage of students having their weight more than 50 kg} = \frac{36}{120} \times 100 = 30\%$$

(b) The difference between any two consecutive mid values of weight is 5 kg. The class intervals must be of width 5, with 40, 45, ... as their mid values.

\therefore The class intervals will be 37.5 - 42.5, 42.5 - 47.5, etc. We construct

Weight (in kg)	Number of students (f)	Less than cumulative frequency (c.f.)
37.5 - 42.5	15	15
42.5 - 47.5	40	55
47.5 - 52.5	29	84
52.5 - 57.5	21	105
57.5 - 62.5	10	115
62.5 - 67.5	5	120
Total	120	

the less than cumulative frequency table as given below:

Here, $N = 120$

Let $P_x = 50$

The value 50 lies in the class 47.5 - 52.5.

$$\therefore L = 47.5, f = 29, c.f. = 55, h = 5$$

$$P_x = L + \frac{h}{f} \left(\frac{xN}{100} - c.f. \right)$$

$$50 = 47.5 + \frac{5}{29} \left(\frac{x \times 120}{100} - 55 \right)$$

$$\therefore 50 - 47.5 = \frac{5}{29} \left(\frac{6x}{5} - 55 \right)$$

$$\therefore 2.5 = \frac{5}{29} \left(\frac{6x}{5} - 55 \right)$$

$$\therefore \frac{6x}{5} - 55 = 14.5$$

$$\therefore \frac{6x}{5} = 55 + 14.5$$

$$\therefore \frac{6x}{5} = 69.5$$

$$\therefore x = 69.5 \times \frac{5}{6}$$

$x = 58$ (approximately)

\therefore 58% of students are having weight below 50 kg.

\therefore Percentage of students having weight above 50 kg is $100 - 58 = 42$

\therefore 42% of students are having weight above 50 kg.

Exercise 1.2 | Q 6 | Page 15

Calculate D_4 and P_{48} from the following data:

Mid value	2.5	7.5	12.5	17.5	22.55	Total
Frequency	7	18	25	30	20	100

SOLUTION

The difference between any two consecutive mid values is 5, the width of class interval = 5

\therefore Class interval with mid-value 2.5 is 0 - 5 Class interval with mid-value 7.5 is 5 - 10, etc.

We construct the less than cumulative frequency table as given below:

Class Interval	Frequency (f)	Less than cumulative frequency (c.f.)
0 - 5	7	7
5 - 10	18	25
10 - 15	25	50 ← D ₄ , P ₄₈
15 - 20	30	80
20 - 25	20	100
Total	100	

Here, N = 100

D₄ class = class containing $\left(\frac{4N}{10}\right)^{\text{th}}$ observation

$$\therefore \frac{4N}{10} = \frac{4 \times 100}{10} = 40$$

Cumulative frequency which is just greater than (or equal to) 40 is 50.

\therefore D₄ lies in the class 10 - 15

\therefore L = 10, f = 25, c.f. = 25, h = 5

$$D_4 = L + \frac{h}{f} \left(\frac{4N}{10} - \text{c.f.} \right)$$

$$= 10 + \frac{5}{25} (40 - 25)$$

$$= 10 + \frac{1}{5} (15)$$

$$= 10 + 3$$

$$\therefore D_4 = 13$$

P₄₈ class = class containing $\left(\frac{48N}{100}\right)^{\text{th}}$ observation

$$\therefore \frac{48N}{100} = \frac{48 \times 100}{100} = 48$$

Cumulative frequency which is just greater than (or equal to) 48 is 50.

$\therefore P_{48}$ lies in the class 10 - 15

$\therefore L=10, f = 25, \text{c.f.} = 25, h = 5$

$$P_{48} = L + \frac{h}{f} \left(\frac{48N}{100} - \text{c.f.} \right)$$

$$= 10 + \frac{5}{25} (48 - 25)$$

$$= 10 + \frac{1}{5} (23)$$

$$= 10 + 4.6$$

$$\therefore P_{48} = 14.6$$

Exercise 1.2 | Q 7 | Page 15

Calculate D_6 and P_{20} of the following distribution

Length (in inches)	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100	100 – 120
No. of units	1	14	35	85	90	15

SOLUTION

We construct the less than cumulative frequency table as given below:

Length (in inches)	No. of units (f)	
0 – 20	1	
20 – 40	14	
40 – 60	35	
60 – 80	85	

80 – 100	90	
100 – 120	15	
Total	240	

Here, N = 240

D_9 class = class containing $\left(\frac{9N}{10}\right)^{\text{th}}$ observation

$$\therefore \frac{9N}{10} = \frac{9 \times 240}{10} = 216$$

Cumulative frequency which is just greater than (or equal to) 216 is 225

$\therefore D_9$ lies in the class 80 – 100.

$\therefore L = 80, f = 90, \text{c.f.} = 135, h = 20$

$$\therefore D_9 = L + \frac{h}{f} \left(\frac{9N}{10} - \text{c.f.} \right)$$

$$= 80 + \frac{20}{90} (216 - 135)$$

$$= 80 + \frac{2}{9} (81)$$

$$= 80 + 18$$

$$\therefore D_9 = 98$$

P_{20} class = class containing $\left(\frac{20N}{100}\right)^{\text{th}}$ observation

$$\therefore \frac{20N}{100} = \frac{20 \times 240}{100} = 48$$

Cumulative frequency which is just greater than (or equal to) 48 is 50.

$\therefore P_{20}$ lies in the class 40 – 60

$\therefore L = 40, f = 35, \text{c.f.} = 15, h = 20$

$$P_{20} = L + \frac{h}{f} \left(\frac{20N}{100} - \text{c.f.} \right)$$

$$= 40 + \frac{20}{35} (48 - 15)$$

$$= 40 + \frac{4}{7} (33)$$

$$= 40 + \frac{132}{7}$$

$$\therefore P_{20} = 58.86$$

Exercise 1.2 | Q 8 | Page 16

Weekly Wages for group of 100 persons are given below:

Wages (in ₹)	0 – 500	500 – 1000	1000 – 1500	1500 – 2000	2000 – 2500
No. of persons	7	?	25	30	?

D_3 for this group is ₹1100 Calculate the missing frequencies.

SOLUTION

Let a and b be the missing frequencies of the class 500 – 1000 and class 2000 – 2500 respectively.

We construct the less than cumulative frequency table as given below:

Wages (in ₹)	No. of persons (f)	Less than Cumulative frequency (c.f.)
0 – 500	7	7
500 – 1000	a	7 + a
1000 – 1500	25	32 + a ← D ₃
1500 – 2000	30	62 + a
2000 – 2500	b	62 + a + b
Total	62 + a + b	

Here, $N = 62 + a + b$

Since, $N = 100$

$$\therefore 62 + a + b = 100$$

$$\therefore a + b = 38 \dots\dots\dots(i)$$

Given, $D_3 = 1100$

$\therefore D_3$ lies in the class 1000 – 1500.

$$\therefore L = 1000, h = 500, f = \frac{3N}{10} = \frac{3 \times 100}{10} = 30,$$

$$c.f. = 7 + a$$

$$\therefore D_3 = L + \frac{h}{f} \left(\frac{3N}{10} - c.f. \right)$$

$$\therefore 1100 = 1000 + \frac{500}{25} [30 - (7 + a)]$$

$$\therefore 1100 - 1000 = 20(30 - 7 - a)$$

$$\therefore 100 = 20 (23 - a)$$

$$\therefore 100 = 460 - 20a$$

$$\therefore 20a = 460 - 100$$

$$\therefore a = \frac{360}{20}$$

$$\therefore a = 18$$

Substituting the value of a in equation (i), we get

$$\therefore 18 + b = 38$$

$$\therefore b = 38 - 18$$

$$\therefore b = 20$$

\therefore 18 and 20 are the missing frequencies of the class 500 – 1000 and class 2000 – 2500 respectively.

Exercise 1.2 | Q 9 | Page 16

The weekly profit (in rupees) of 100 shops are distributed as follows:

Profit per shop	No. of shops
0 – 1000	10
1000 – 2000	16
2000 – 3000	26
3000 – 4000	20
4000 – 5000	20
5000 – 6000	5
6000 – 7000	3

Find the limits of the profit of middle 60% of the shops.

SOLUTION

To find the limits of the profit of middle 60% of the shops, we have to find P_{20} and P_{80} .

We construct the less than cumulative frequency table as given below:

Profit per shop (in rupees)	No. of shops (f)	Less than Cumulative frequency (c.f.)
0 – 1000	10	10
1000 – 2000	16	26 $\leftarrow P_{20}$
2000 – 3000	26	52
3000 – 4000	20	72
4000 – 5000	20	92 $\leftarrow P_{80}$
5000 – 6000	5	97
6000 – 7000	3	100

Total	100	
--------------	------------	--

Here, $N = 100$

P_{20} class = class containing $\left(\frac{20N}{100}\right)^{\text{th}}$ observation

$$\therefore \frac{20N}{100} = \frac{20 \times 100}{100} = 20$$

Cumulative frequency which is just greater than (or equal to) 20 is 26.

$\therefore P_{20}$ lies in the class 1000 – 2000.

$\therefore L = 1000, f = 16, \text{c.f.} = 10, h = 1000$

$$P_{20} = L + \frac{h}{f} \left(\frac{20N}{100} - \text{c.f.} \right)$$

$$= 1000 + \frac{1000}{16} (20 - 10)$$

$$= 1000 + \frac{125}{2} (10)$$

$$= 1000 + 625$$

$$\therefore P_{20} = 1625$$

P_{80} class = class containing $\left(\frac{80N}{100}\right)^{\text{th}}$ observation

$$\therefore \frac{80N}{100} = \frac{80 \times 100}{100} = 80$$

Cumulative frequency which is just greater than (or equal to) 80 is 92.

$\therefore P_{80}$ lies in the class 4000 – 5000

$\therefore L = 4000, f = 20, \text{c.f.} = 72, h = 1000$

$$P_{80} = L + \frac{h}{f} \left(\frac{80N}{100} - \text{c.f.} \right)$$

$$= 4000 + \frac{1000}{20} (80 - 72)$$

$$= 4000 + 50(8)$$

$$= 4000 + 400$$

$$\therefore P_{80} = 4400$$

\therefore the profit of middle 60% of the shops lies between the limits 1,625 to 4,400.

Exercise 1.2 | Q 10 | Page 16

In a particular factory, workers produce various types of output units. The following distribution was obtained.

Output units Produced	No. of workers
70 – 74	40
75 – 79	45
80 – 84	50
85 – 89	60
90 – 94	70
95 – 99	80
100 – 104	100

Find the percentage of workers who have produced less than 82 output units.

SOLUTION

Since the given data is not continuous, we have to convert it in the continuous form by subtracting 0.5 from the lower limit and adding 0.5 to the upper limit of every class interval.

∴ The class intervals will be 69.5 – 74.5, 74.5 – 79.5, etc.

We construct the less than cumulative frequency table as given below:

Output units produced	No. of workers (f)	Less than Cumulative frequency (c.f.)
69.5 – 74.5	40	40
74.5 – 79.5	45	85
79.5 – 84.5	50	135
84.5 – 89.5	60	195
89.5 – 94.5	70	265
94.5 – 99.5	80	345
99.5 – 104.5	100	445
Total	445	

Here, N = 445

Let $P_x = 82$

The value 82 lies in the class 79.5 – 84.5.

∴ L = 79.5, f = 50, c.f. = 85, h = 5

$$\therefore P_x = L + \frac{h}{f} \left(\frac{xN}{100} - \text{c.f.} \right)$$

$$\therefore 82 = 79.5 + \frac{5}{50} \left(\frac{x \times 445}{100} - 85 \right)$$

$$\therefore 82 - 79.5 = \frac{1}{10} \left(\frac{89x}{20} - 85 \right)$$

$$\therefore 2.5 \times 10 = \frac{89x}{20} - 20$$

$$\therefore 25 + 85 = \frac{89x}{20}$$

$$\therefore 110 = \frac{89x}{20}$$

$$\therefore x = \frac{110 \times 20}{89} = 24.72$$

∴ 24.72% of workers produced less than 82 output units.

EXERCISE 1.3 [PAGES 18 - 20]**Exercise 1.3 | Q 1 | Page 18**

The following table gives frequency distribution of marks of 100 students in an examination.

Marks	15 – 20	20 – 25	25 – 30	30 – 35	35 – 40	40 – 45	45 – 50
No. of students	9	12	23	31	10	8	7

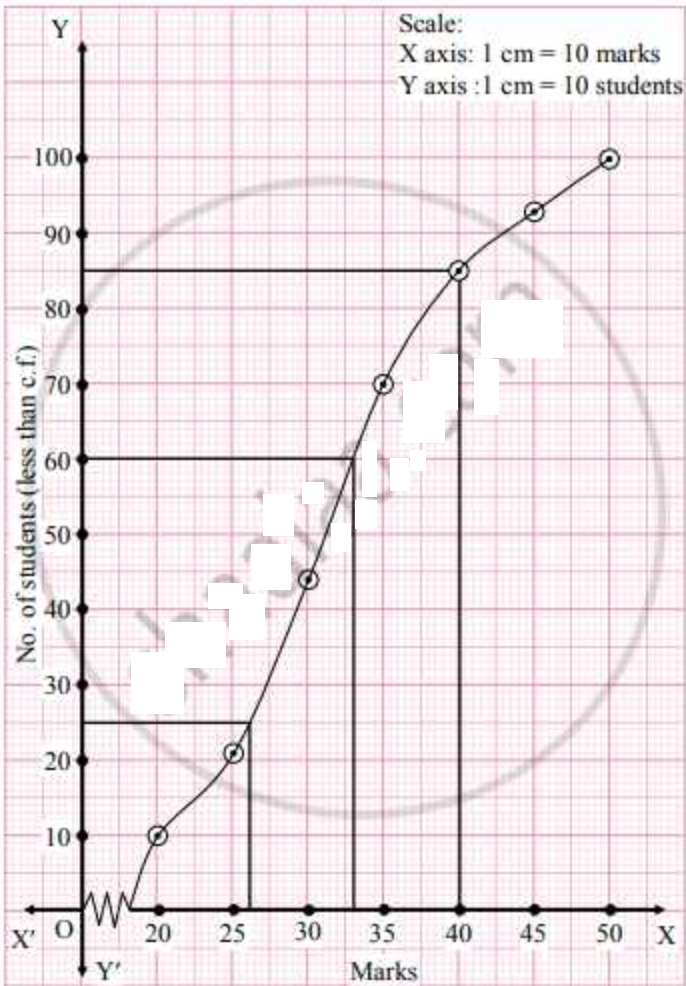
Determine D_6 , Q_1 , and P_{85} graphically.

SOLUTION

To draw an ogive curve, we construct the less than cumulative frequency table as given below:

Marks	No. of students (f)	Less than cumulative Frequency (c.f.)
15 – 20	9	9
20 – 25	12	21
25 – 30	23	44
30 – 35	31	75
35 – 40	10	85
40 – 45	8	93
45 – 50	7	100
Total	100	

The points to be plotted for less than ogive are (20, 9), (25, 21), (30, 44), (35, 75), (40, 85), (45, 93), (50, 100).



Here, $N = 100$

$$\text{For } D_6, \frac{6N}{10} = \frac{6(100)}{10} = 60$$

$$\text{For } Q_1, \frac{N}{4} = \frac{100}{4} = 25$$

$$\text{For } P_{85}, \frac{85N}{100} = \frac{85 \times 100}{100} = 85$$

\therefore We take the points having Y coordinates 60, 25, and 85 on Y-axis. From these points, we draw lines parallel to X-axis. From the points where these lines intersect the curve, we draw perpendiculars on X-axis.

X coordinates of these points give the values of D_6 , Q_1 , and P_{85} .

$$\therefore D_6 = 32.5, Q_1 = 26, P_{85} = 40$$

Exercise 1.3 | Q 2 | Page 18

The following table gives the distribution of daily wages of 500 families in a certain city.

Daily wages	No. of families
Below 100	50
100 – 200	150
200 – 300	180
300 – 400	50
400 – 500	40
500 – 600	20
600 above	10

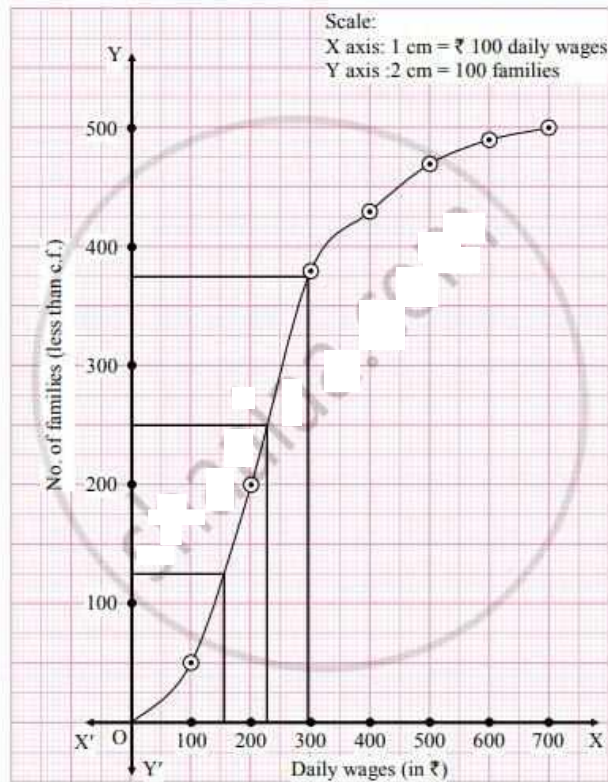
Draw a 'less than' ogive for the above data. Determine the median income and obtain the limits of income of central 50% of the families.

SOLUTION

To draw a ogive curve, we construct the less than cumulative frequency table as given below:

Daily wages	No. of families (f)	Less than cumulative frequency (c.f.)
Below 100	50	50
100 – 200	150	200
200 – 300	180	380
300 – 400	50	430
400 – 500	40	470
500 – 600	20	490
600 above	10	500
Total	500	

The points to be plotted for less than ogive are (100, 50), (200, 200), (300, 380), (400, 430), (500, 470), (600, 490) and (700, 500).



Here, $N = 500 = 125$

$$\text{For } Q_1, \frac{N}{4} = \frac{500}{4} = 125$$

$$\text{For } Q_2, \frac{N}{2} = \frac{500}{2} = 250$$

$$\text{For } Q_3, \frac{3N}{4} = \frac{3 \times 500}{4} = 375$$

∴ We take the points having Y coordinates 125, 250, and 375 on Y-axis. From these points, we draw lines parallel to X-axis.

From the points where these lines intersect the curve, we draw perpendiculars on X-axis. X-Co-ordinates of these points gives the values of Q_1 , Q_2 , and Q_3 .

∴ $Q_1 \approx 150$, $Q_2 \approx 228$, $Q_3 \approx 297$

∴ Median = 228

50% of families lies between Q_1 and Q_3

∴ Limits of income of central 50% of families are from ₹ 150 to ₹ 297.

Exercise 1.3 | Q 3 | Page 19

From the following distribution, determine median graphically.

Daily wages (in ₹)	No. of employees
Above 300	520
Above 400	470
Above 500	399
Above 600	210
Above 700	105
Above 800	45
Above 900	7

SOLUTION 1

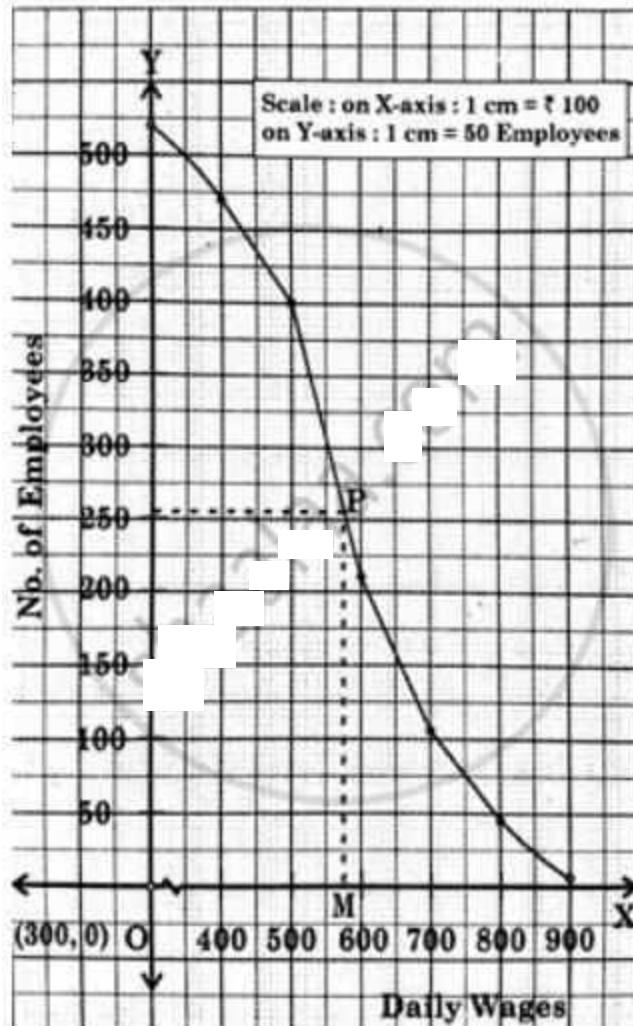
The given 'more than cumulative frequency' table is,

Daily Wages (in Rs.)	No. of employee (m.c.f)
Above 300	520
Above 400	470
Above 500	399
Above 600	210
Above 700	105
Above 800	45
Above 900	7

The 'more than ogive curve' is plotted using the given lower limit of the class interval (x) against c.f. (y)

$$\frac{N}{2} = \frac{520}{2} = 260$$

For median draw a line parallel to X-axis at Frequency



Exercise 1.3 | Q 4.1 | Page 19

The following frequency distribution shows the profit (in ₹) of shops in a particular area of city:

Profit per shop (in '000)	No. of shops
0 – 10	12
10 – 20	18
20 – 30	27
30 – 40	20
40 – 50	17
50 – 60	6

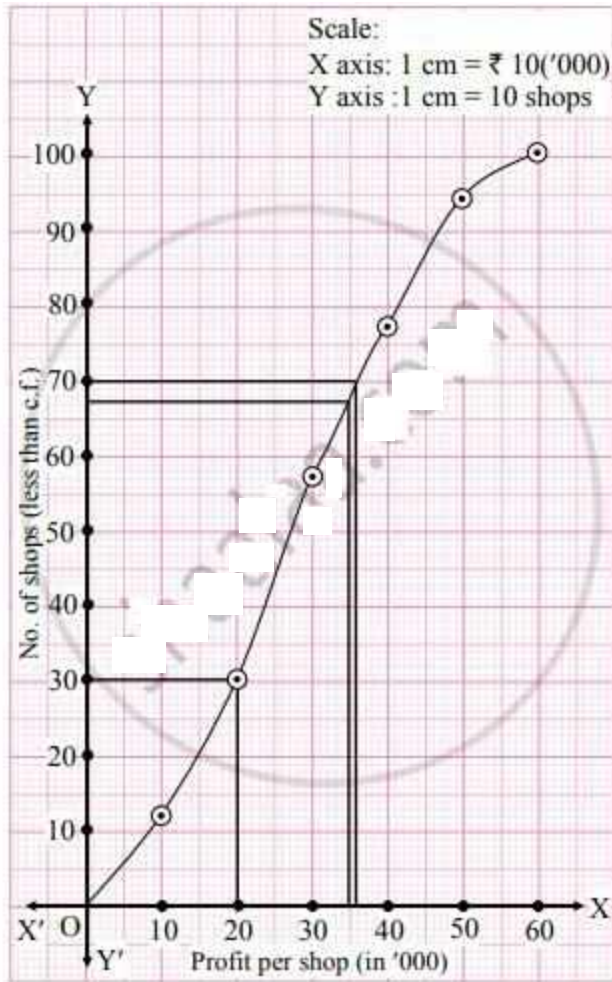
Find graphically the limits of middle 40% shops.

SOLUTION

The less than cumulative frequency table is

Profit per shop (in '000)	No. of shops (f)	Cumulative Frequency (less than type)
0 – 10	12	12
10 – 20	18	30
20 – 30	27	57
30 – 40	20	77
40 – 50	17	94
50 – 60	6	100
Total	100	

Points to be plotted are (10, 12), (20, 30), (30, 57), (40, 77), (50, 94), (60, 100).



The middle 40% shops will lie between the limits given by P_{30} and P_{70} .

$N = 100$

$$\text{For } P_{30} \quad \frac{30N}{100} = \frac{30(100)}{100} = 30$$

$$\text{For } P_{70} \quad \frac{70N}{100} = \frac{70(100)}{100} = 70$$

\therefore We take the points having Y co-ordinates 30 and 70 on Y-axis. From these points, we draw lines parallel to X-axis. From the points where these lines intersect the curve, we draw perpendiculars on X-axis.

X-Co-ordinates of these points gives the values of P_{30} and P_{70} .

$\therefore P_{30} \approx 20, P_{70} \approx 36$

Exercise 1.3 | Q 4.2 | Page 19

The following frequency distribution shows the profit (in ₹) of shops in a particular area of city:

Profit per shop (in '000)	No. of shops
0 – 10	12
10 – 20	18
20 – 30	27
30 – 40	20
40 – 50	17
50 – 60	6

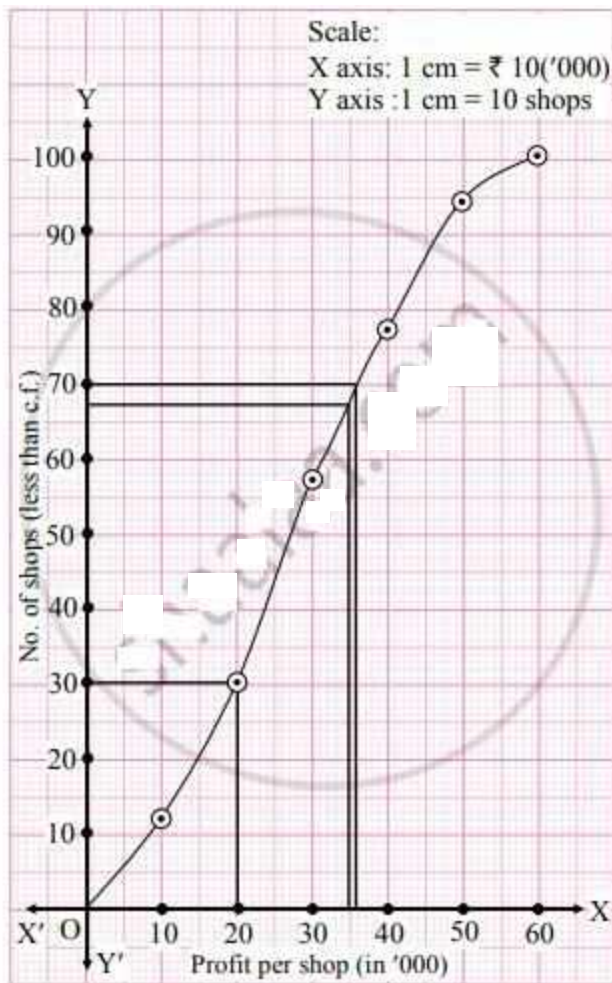
Find graphically the number of shops having profit less than 35,000 rupees.

SOLUTION

The less than cumulative frequency table is

Profit per shop (in '000)	No. of shops (f)	Less than cumulative Frequency (c.f.)
0 – 10	12	12
10 – 20	18	30
20 – 30	27	57
30 – 40	20	77
40 – 50	17	94
50 – 60	6	100
Total	100	

Points to be plotted are (10, 12), (20, 30), (30, 57), (40, 77), (50, 94), (60, 100).



Limits of middle 40% shops lie between ₹ 20,000 to ₹ 36,000

To find the number of shops having a profit less than ₹ 35,000, we take the value 35 on the X-axis. From this point, we draw a line parallel to Y-axis, and from the point where it intersects the less than o give we draw a perpendicular on Y-axis. It intersects the Y-axis at approximately 67.

∴ No. of shops having a profit less than ₹ 35,000 is 67.

Exercise 1.3 | Q 5 | Page 19

The following is the frequency distribution of overtime (per week) performed by various workers from a certain company.

Determine the values of D_2 , Q_2 , and P_{61} graphically.

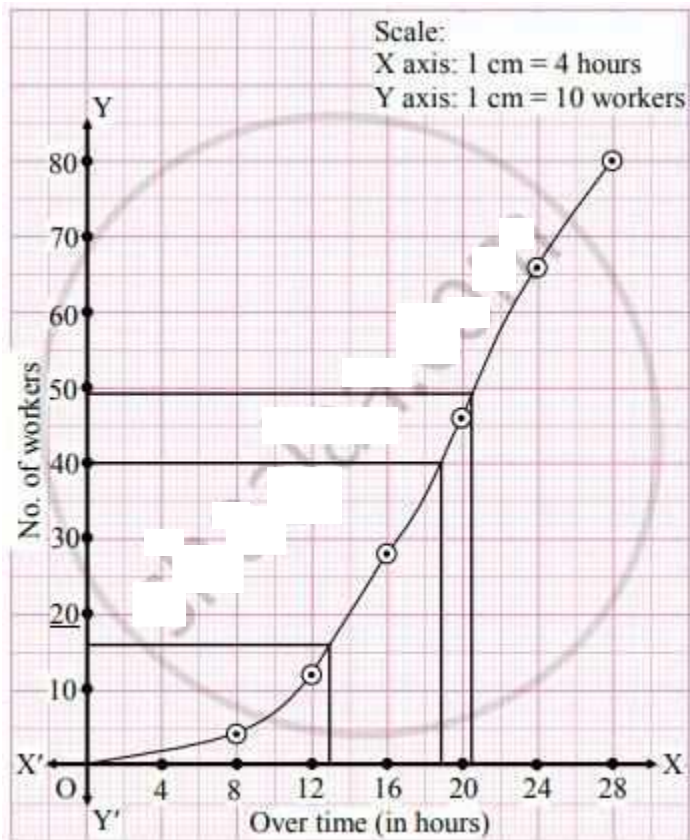
Overtime (in hours)	Below 8	8 – 12	12 – 16	16 – 20	20 – 24	24 and above
No. of workers	4	8	16	18	20	14

SOLUTION

To draw a ogive curve, we construct a less than cumulative frequency table as given below:

Overtime (in hours)	No. of workers (f)	Less than cumulative frequency (c.f.)
Below 8	4	4
8 – 12	8	12
12 – 16	16	28
16 – 20	18	46
20 – 24	20	66
24 and above	14	80
Total	80	

Points to be plotted are (8, 4), (12, 12), (16, 28), (20, 46), (24, 66) and (28, 80)
Here, $N = 80$



For D_2 , we have to consider $\frac{2N}{10} = \frac{2 \times 80}{10} = 16$

For Q_2 , we have to consider $\frac{N}{2} = \frac{80}{2} = 40$

and for P_{61} , we have to consider $\frac{61N}{100} = \frac{61 \times 80}{100} = 48.8$

\therefore We consider the values 16, 40, and 48.8 on the Y-axis. From these points, we draw the lines which are parallel to the X-axis. From the points where they intersect the less than ogive, we draw perpendiculars to X-axis. The values at the foot of perpendiculars represent the values of D_2 , Q_2 , and P_{61} respectively.

$\therefore D_2 \approx 13, Q_2 \approx 19, P_{61} \approx 20.5$

Exercise 1.3 | Q 6 | Page 19

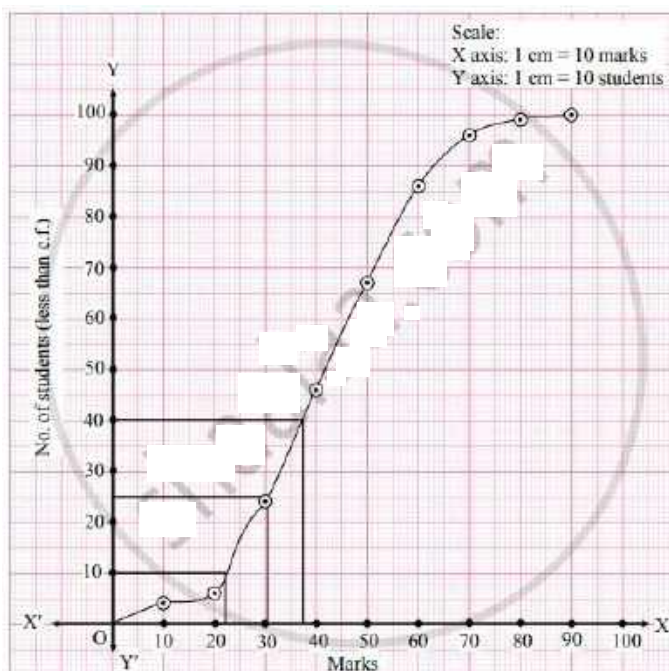
Draw ogive for the following data and hence find the values of D_1 , Q_1 , P_{40} .

Marks less than	10	20	30	40	50	60	70	80	90
No. of students	4	6	24	46	67	86	96	99	100

SOLUTION

$N = 100$

To draw the less than ogive we have to plot the points (10, 4), (20, 6), (30, 24), (40, 46), (50, 67), (60, 86), (70, 96), (80, 99), (90, 100).



For D_1 , we have to consider $\frac{N}{10} = \frac{100}{10} = 10$

For Q_1 , we have to consider $\frac{N}{4} = \frac{100}{4} = 25$

For P_{40} , we have to consider $\frac{40N}{100} = \frac{40 \times 100}{100} = 40$

\therefore We consider the values 10, 25, and 40 on the Y-axis. From these points, we draw lines parallel to X-axis. From the points where they intersect the less than ogive, we draw perpendiculars on the X-axis. The values at the foot of the perpendicular represent the values of D_1 , Q_1 , and P_{40} respectively.

$\therefore D_1 \approx 22, Q_1 \approx 30.4, P_{40} \approx 37$

Exercise 1.3 | Q 7 | Page 19

The following table shows the age distribution of head of the families in a certain country. Determine the third, fifth and eighth decile of the distribution graphically.

Age of head of family (in years)	Numbers (million)
Under 35	46
35 – 45	85
45 – 55	64
55 – 65	75
65 – 75	90
75 and Above	40

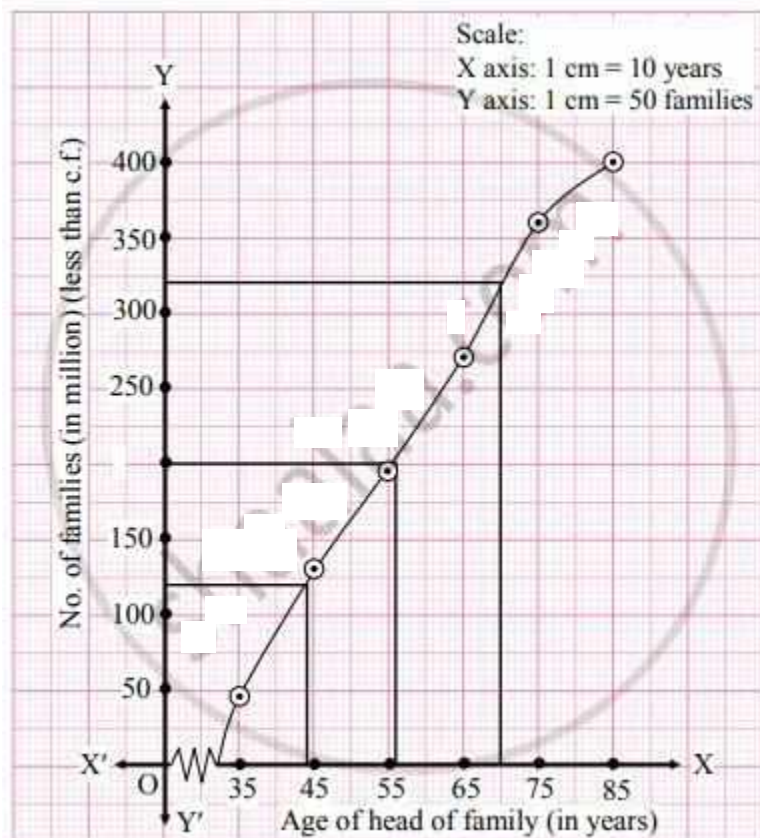
SOLUTION

To draw a ogive curve, we construct a less than cumulative frequency table as given below:

Age of head of family (in years)	Numbers (million) (f)	Less than cumulative frequency (c.f.)
Under 35	46	46
35 – 45	85	131
45 – 55	64	195

55 – 65	75	270
65 – 75	90	360
75 and Above	40	400
Total	400	

Points to be plotted are (35, 46), (45, 131), (55, 195), (65, 270), (75, 360), (85, 400).



$N = 400$

For D_3 , we have to consider $\frac{3N}{10} = \frac{3(400)}{10} = 120$,

For D_5 , we have to consider $\frac{5N}{10} = \frac{5(400)}{10} = 200$

For D_8 , we have to consider $\frac{8N}{10} = \frac{8(400)}{10} = 320$

\therefore We consider the values 120, 200, and 320 on Y-axis. From these points, we draw the lines parallel to X-axis. From the points where they intersect the less than ogive, we draw perpendiculars on the X-axis. The foot of the perpendicular represents the values of D_3 , D_5 , and D_8 .

$\therefore D_3 \approx 44$, $D_5 \approx 55.5$, and $D_8 \approx 70$

Exercise 1.3 | Q 8 | Page 19

The following table gives the distribution of females in an Indian village. Determine the median age of graphically.

Age group	No. of females (in '000)
0 – 10	175
10 – 20	100
20 – 30	68
30 – 40	48
40 – 50	25
50 – 60	50
60 – 70	23
70 – 80	8
80 – 90	2
90 – 100	1

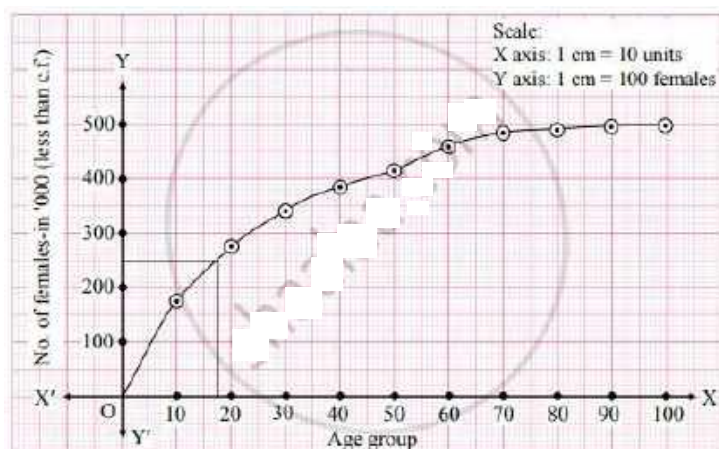
SOLUTION

To draw a ogive curve, we construct the less than cumulative frequency table as given below:

Age group	No. of females (in '000) (f)	Less than cumulative frequency (c.f.)
0 – 10	175	175
10 – 20	100	275
20 – 30	68	343
30 – 40	48	391
40 – 50	25	416
50 – 60	50	466
60 – 70	23	489
70 – 80	8	497

80 – 90	2	499
90 – 100	1	500
Total	500	

Points to be plotted are (10, 175), (20, 275), (30, 343), (40, 391), (50, 416), (60, 466), (70, 489), (80, 497), (90, 499), (100, 500)



$N = 500$

For median we have to consider $\frac{N}{2} = \frac{500}{2} = 250$

\therefore We consider the value 250 on Y-axis. From this point, we draw a line parallel to X-axis. From the point it intersects the less than ogive, we draw a perpendicular to X-axis. The foot of the perpendicular represents the value of the median.

\therefore Median ≈ 17.5

Exercise 1.3 | Q 9 | Page 19

Draw ogive for the Following distribution and hence find graphically the limits of weight of middle 50% fishes.

Weight of fishes (in gms)	800 – 890	900 – 990	1000 – 1090	1100 – 1190	1200 – 1290	1300 – 1390	1400 – 1490
No. of fishes	8	16	20	25	40	6	5

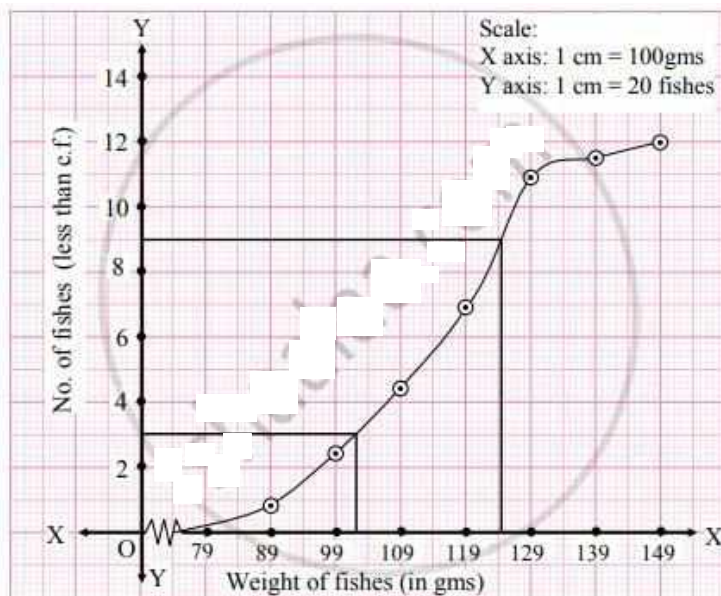
SOLUTION

Since the given data is not continuous, we have to convert it in the continuous form by subtracting 5 from the lower limit and adding 5 to the upper limit of every class interval.

To draw a ogive curve, we construct the less than cumulative frequency table as given below:

Weight of fishes (in gms)	No. of fishes (f)	Less than cumulative frequency (c.f.)
795 – 895	8	8
895 – 995	16	24
995 – 1095	20	44
1095 – 1195	25	69
1195 – 1295	40	109
1295 – 1395	6	115
1395 – 1496	5	120
Total	120	

Points to be plotted are (895, 8), (995, 24), (1095, 44), (1195, 69), (1295, 109), (1395, 115), (1495, 120).



$$N = 120$$

For Q_1 and Q_3 we have to consider $\frac{N}{4} = \frac{120}{4} = 30$, $\frac{3N}{4} = \frac{3 \times 120}{4} = 90$

For finding Q_1 and Q_3 we consider the values 30 and 90 on the Y-axis. From these points, we draw the lines which are parallel to X-axis. From the points where these lines intersect the less than ogive, we draw perpendicular on X-axis. The feet of perpendiculars represent the values of Q_1 and Q_3 .

$\therefore Q_1 \approx 1025$ and $Q_3 \approx 1248$

\therefore The limits of weight of middle 50% fishes lie between 1025 to 1248.

Exercise 1.3 | Q 10 | Page 20

Find graphically the values of D_3 and P_{65} for the data given below:

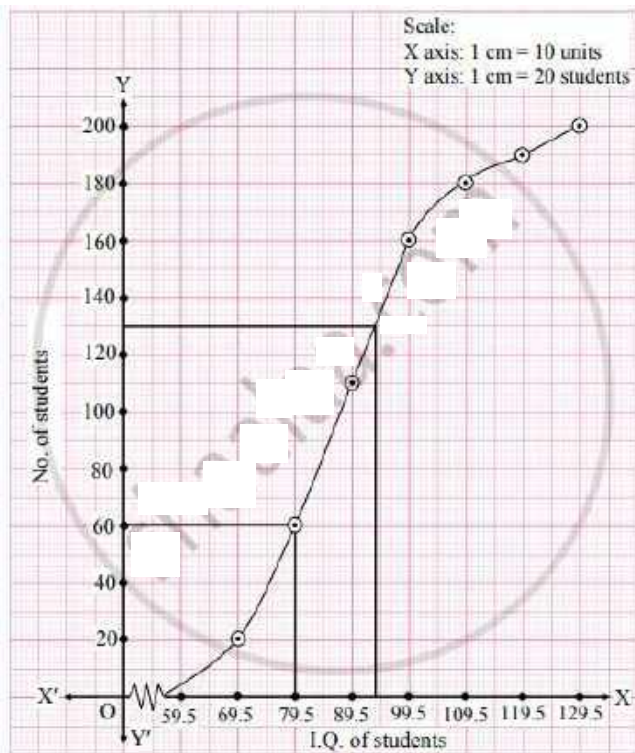
I.Q of students	60 – 69	70 – 79	80 – 89	90 – 99	100 – 109	110 – 119	120 – 129
No. of students	20	40	50	50	20	10	10

SOLUTION

Since the given data is not continuous, we have to convert it in the continuous form by subtracting 0.5 from the lower limit and adding 0.5 to the upper limit of every class interval. To draw a ogive curve, we construct the less than cumulative frequency table as given below:

I.Q. of students	No. of students (f)	Less than cumulative frequency (c.f.)
59.5 – 69.5	20	20
69.5 – 79.5	40	60
79.5 – 89.5	50	110
89.5 – 99.5	50	160
99.5 – 109.5	20	180
109.5 – 119.5	10	190
119.5 – 129.5	10	200
Total	200	

Points to be plotted are (69.5, 20), (79.5, 60), (89.5, 110), (99.5, 160), (109.5, 180), (119.5, 190), (129.5, 200).



$$N = 200$$

$$\text{For } D_3, \frac{3N}{10} = \frac{3 \times 200}{10} = 60$$

$$\text{For } P_{65}, \frac{65N}{100} = \frac{65 \times 200}{100} = 130$$

\therefore We take the values 60 and 130 on the Y-axis. From these points we draw lines parallel to X-axis and from the points where these lines intersect less than ogive, we draw perpendiculars on X-axis. The foot of perpendiculars represents the median of the values, D_3 , and P_{65} .

$$\therefore D_3 = 79.5, P_{65} = 93.5$$

MISCELLANEOUS EXERCISE 1 [PAGES 20 - 22]

Miscellaneous Exercise 1 | Q 1 | Page 20

The data gives number of accidents per day on a railway track. Compute Q_2 , P_{17} , and D_7 .

4, 2, 3, 5, 6, 3, 4, 1, 2, 3, 2, 3, 4, 3, 2.

SOLUTION

The given data can be arranged in ascending order as follows:

1, 2, 2, 2, 2, 3, 3, 3, 3, 3, 4, 4, 4, 5, 6

Here, $n = 15$

$$Q_2 = \text{value of } 2\left(\frac{n+1}{4}\right)^{\text{th}} \text{ observation}$$

$$= \text{value of } 2\left(\frac{15+1}{4}\right)^{\text{th}} \text{ observation}$$

$$= \text{value of } (2 \times 4)^{\text{th}} \text{ observation}$$

$$= \text{value of } 8^{\text{th}} \text{ observation}$$

$$\therefore Q_2 = 3$$

$$P_{17} = \text{value of } 17\left(\frac{n+1}{100}\right)^{\text{th}} \text{ observation}$$

$$= \text{value of } 17\left(\frac{15+1}{100}\right)^{\text{th}} \text{ observation}$$

$$= \text{value of } (17 \times 0.16)^{\text{th}} \text{ observation}$$

$$= \text{value of } (2.72)^{\text{th}} \text{ observation}$$

$$= \text{value of } 2^{\text{nd}} \text{ observation} + 0.72 (\text{value of } 3^{\text{rd}} \text{ observation} - \text{value of } 2^{\text{nd}} \text{ observation})$$

$$= 2 + 0.72 (2 - 2)$$

$$\therefore P_{17} = 2$$

$$\begin{aligned}
D_7 &= \text{value of } 7\left(\frac{n+1}{10}\right)^{\text{th}} \text{ observation} \\
&= \text{value of } 7\left(\frac{15+1}{10}\right)^{\text{th}} \text{ observation} \\
&= \text{value of } (7 \times 1.6)^{\text{th}} \text{ observation} \\
&= \text{value of } (11.2)^{\text{th}} \text{ observation} \\
&= \text{value of } 11^{\text{th}} \text{ observation} + 0.2 (\text{value of } 12^{\text{th}} \text{ observation} - \text{value of } 11^{\text{th}} \text{ observation}) \\
&= 4 + 0.2 (4 - 4) \\
\therefore D_7 &= 4
\end{aligned}$$

Miscellaneous Exercise 1 | Q 2 | Page 20

The distribution of daily sales of shoes (size-wise) for 100 days from a certain shop is as follows:

Size of shoes	2	4	3	5	7	6	8
No. of days	14	20	13	19	13	13	8

Compute Q_1 , D_2 , and P_{95} .

SOLUTION

By arranging the given data in ascending order, we construct the less than cumulative frequency table as given below:

Size of shoes	No. of days (f)	Less than cumulative frequency (c.f.)
2	14	14
3	13	27 $\leftarrow Q_1, D_2$
4	20	47
5	19	66
6	13	79
7	13	92
8	8	100 $\leftarrow P_{95}$

Total	100	
--------------	------------	--

Here, $n = 100$

$$Q_1 = \text{value of } \left(\frac{n+1}{4} \right)^{\text{th}} \text{ observation}$$

$$= \text{value of } \left(\frac{100+1}{4} \right)^{\text{th}} \text{ observation}$$

$$= \text{value of } (25.25)^{\text{th}} \text{ observation}$$

Cumulative frequency which is just greater than (or equal) to 25.25 is 27.

$$\therefore Q_1 = 3$$

$$D_2 = \text{value of } 2 \left(\frac{n+1}{10} \right)^{\text{th}} \text{ observation}$$

$$= \text{value of } 2 \left(\frac{100+1}{10} \right)^{\text{th}} \text{ observation}$$

$$= \text{value of } (2 \times 10.1)^{\text{th}} \text{ observation}$$

$$= \text{value of } (20.2)^{\text{th}} \text{ observation}$$

Cumulative frequency which is just greater than (or equal) to 20.2 is 27.

$$\therefore D_2 = 3$$

$$P_{95} = \text{value of } 95 \left(\frac{n+1}{100} \right)^{\text{th}} \text{ observation}$$

$$= \text{value of } 95 \left(\frac{100+1}{100} \right)^{\text{th}} \text{ observation}$$

$$= \text{value of } (95 \times 1.01)^{\text{th}} \text{ observation}$$

$$= \text{value of } (95.95)^{\text{th}} \text{ observation}$$

Cumulative frequency which is just greater than (or equal) to 95.95 is 100.

$$\therefore P_{95} = 8$$

Miscellaneous Exercise 1 | Q 3 | Page 20

Ten students appeared for a test in Mathematics and Statistics and they obtained the marks as follows:

Sr. No.	1	2	3	4	5	6	7	8	9	10
Marks in Mathematics	42	38	36	32	23	25	35	37	25	23
Marks in Statistics	22	26	29	34	50	45	23	28	32	36

If the median will be the criteria, in which subject, the level of knowledge of the students is higher?

SOLUTION

Marks in Mathematics can be arranged in ascending order as follows:

23, 23, 25, 25, 32, 35, 36, 37, 38, 42

Here, $n = 10$

\therefore Median = value of $\left(\frac{n+1}{2}\right)^{\text{th}}$ observation

= value of $\left(\frac{10+1}{2}\right)^{\text{th}}$ observation

= value of $(5.5)^{\text{th}}$ observation

= value of 5^{th} observation + 0.5 (value of 6^{th} observation – value of 5^{th} observation)

= $32 + 0.5 (35 - 32)$

= $32 + 0.5(3)$

= $32 + 1.5$

= 33.5

Marks in Statistics can be arranged in ascending order as follows:

22, 23, 26, 28, 29, 32, 34, 36, 45, 50

Here, $n = 10$

\therefore Median = value of $\left(\frac{n+1}{2}\right)^{\text{th}}$ observation

= value of $\left(\frac{10+1}{2}\right)^{\text{th}}$ observation

= value of $(5.5)^{\text{th}}$ observation

= value of 5^{th} observation + 0.5 (value of 6^{th} observation – value of 5^{th} observation)

= $29 + 0.5(32 - 29)$

= $29 + 0.5(3)$

= $29 + 1.5$

= 30.5

\therefore Median marks for Mathematics = 33.5

and Median marks for Statistics = 30.5

\therefore The level of knowledge in Mathematics is higher than that of Statistics.

Miscellaneous Exercise 1 | Q 4 | Page 21

In the frequency distribution of families given below, the number of families corresponding to expenditure group 2000 - 4000 is missing from the table. However value of 25^{th} percentile is 2880. Find the missing frequency.

Weekly Expenditure (₹1000)	0 – 2	2 – 4	4 – 6	6 – 8	8 – 10
No. of families	14	?	39	7	15

SOLUTION

Let x be the missing frequency of expenditure group 2000 – 4000.

We construct the less than cumulative frequency table as given below:

Weekly Expenditure	No. of families (f)	Less than cumulative frequency (c.f.)
0 – 2000	14	14
2000 – 4000	x	$14 + x \leftarrow P_{25}$
4000 – 6000	39	$53 + x$
6000 – 8000	7	$60 + x$

8000 – 10000	15	75 + x
Total	75 + x	

Here, $N = 75 + x$

Given, $P_{25} = 2880$

$\therefore P_{25}$ lies in the class 2000 – 4000.

$\therefore L = 2000, h = 2000, f = x, \text{c.f.} = 14$

$$\therefore P_{25} = L + \frac{h}{f} \left(\frac{25N}{100} - \text{c.f.} \right)$$

$$\therefore 2880 = 2000 + \frac{2000}{x} \left(\frac{75 + x}{4} - 14 \right)$$

$$\therefore 2880 - 2000 = \frac{2000}{x} \left(\frac{75 + x - 56}{4} \right)$$

$$\therefore 880x = 500(x + 19)$$

$$\therefore 880x = 500x + 9500$$

$$\therefore 880x - 500x = 9500$$

$$\therefore 380x = 9500$$

$$\therefore x = \frac{9500}{380} = 25$$

$\therefore 25$ is the missing frequency of the expenditure group 2000 – 4000.

Miscellaneous Exercise 1 | Q 5 | Page 21

Calculate Q_1 , D_6 , and P_{15} for the following data:

Mid value	25	75	125	175	225	275
Frequency	10	70	80	100	150	90

SOLUTION

Since the difference between any two consecutive mid values is 50, the width of each class interval is 50.

\therefore the class intervals will be 0 – 50, 50 – 100, etc.

We construct the less than cumulative frequency table as given below:

Class interval	Frequency (f)	Less than cumulative frequency (c.f.)
-----------------------	--------------------------	--

0 – 50	10	10
50 – 100	70	80 ← P ₁₅
100 – 150	80	160 ← Q ₁
150 – 200	100	260
200 – 250	150	410 ← D ₆
250 – 300	90	500
Total	500	

Here, N = 500

Q₁ class = class containing $\left(\frac{N}{4}\right)^{\text{th}}$ observation

$$\therefore \frac{N}{4} = \frac{500}{4} = 125$$

Cumulative frequency which is just greater than (or equal) to 125 is 160.

\therefore Q₁ lies in the class 100 – 150.

\therefore L = 100, h = 50, f = 80, c.f. = 80

$$\therefore Q_1 = L + \frac{h}{f} \left(\frac{N}{4} - \text{c.f.} \right)$$

$$= 100 + \frac{50}{80} (125 - 80)$$

$$= 100 + \frac{5}{8} (45)$$

$$= 100 + 28.125$$

$$= 128.125$$

D₆ class = class containing $\left(\frac{6N}{10}\right)^{\text{th}}$ observation

$$\therefore \frac{6N}{10} = \frac{6 \times 500}{10} = 300$$

Cumulative frequency which is just greater than (or equal) to 300 is 410.

$\therefore D_6$ lies in the class 200 – 250

$\therefore L = 200, h = 50, f = 150, \text{ c.f.} = 260$

$$\therefore D_6 = L + \frac{h}{f} \left(\frac{6N}{4} - \text{c.f.} \right)$$

$$= 200 + \frac{50}{150} (300 - 260)$$

$$= 200 + \frac{1}{3} (40)$$

$$= 200 + 13.33$$

$$= 213.33$$

$\therefore P_{15}$ class = class containing $\left(\frac{15N}{100} \right)^{\text{th}}$ observation

$$\therefore \frac{15N}{100} = \frac{15 \times 500}{100} = 75$$

Cumulative frequency which is just greater than (or equal) to 75 is 80.

$\therefore P_{15}$ lies in the class 50 – 100

$\therefore L = 50, h = 50, f = 70, \text{c.f.} = 10$

$$\therefore P_{15} = L + \frac{h}{f} \left(\frac{15N}{100} - \text{c.f.} \right)$$

$$= 50 + \frac{50}{70} (75 - 10)$$

$$= 50 + \frac{5}{7} (65)$$

$$= 50 + \frac{325}{7}$$

$$= 50 + 46.4286$$

$$= 96.4286$$

$$\therefore Q_1 = 128.125, D_6 = 213.33, P_{15} = 96.4286$$

Miscellaneous Exercise 1 | Q 6 | Page 21

Daily income for a group of 100 workers are given below:

Daily income (in ₹)	0 – 50	50 – 100	100 – 150	150 – 200	200 – 250
No. of persons	7	?	25	30	?

P_{30} for this group is ₹ 110. Calculate the missing frequencies.

SOLUTION

Let a and b be the missing frequencies of the class 50 – 100 and class 200 – 250 respectively.

We construct the less than cumulative frequency table as given below:

Daily income (in ₹)	No. of persons (f)	Less than cumulative frequency (c.f.)
0 – 50	7	7
50 – 100	a	7 + a
100 – 150	25	32 + a $\leftarrow P_{30}$

150 – 200	30	62 + a
200 – 250	b	62 + a + b
Total	62 + a + b	

Here, $N = 62 + a + b$

Since, $N = 100$

$$\therefore 62 + a + b = 100$$

$$\therefore a + b = 38 \text{(i)}$$

Given, $P_{30} = 110$

$\therefore P_{30}$ lies in the class 100 – 150.

$$\therefore L = 100, h = 50, f = 25, \frac{30N}{100} = \frac{30 \times 100}{100} = 30, \text{c.f.} = 7 + a$$

$$\therefore P_{30} = L + \frac{h}{f} \left(\frac{30N}{100} - \text{c.f.} \right)$$

$$\therefore 110 = 100 + \frac{50}{25} [30 - (7 + a)]$$

$$\therefore 110 - 100 = 2(30 - 7 - a)$$

$$\therefore 10 = 2(23 - a)$$

$$\therefore \frac{10}{2} = 23 - a$$

$$\therefore 5 = 23 - a$$

$$\therefore a = 23 - 5$$

$$\therefore a = 18$$

Substituting the value of a in equation (i), we get

$$18 + b = 38$$

$$\therefore b = 38 - 18$$

$$\therefore b = 20$$

$\therefore 18$ and 20 are the missing frequencies of the class 50 – 100 and class 200 – 250 respectively.

Miscellaneous Exercise 1 | Q 7 | Page 21

The distribution of a sample of students appearing for a C.A. examination is:

Marks	0 – 100	100 – 200	200 – 300	300 – 400	400 – 500	500 – 600
No. of students	130	150	190	220	280	130

Help C.A. institute to decide cut-off marks for qualifying an examination when 3% of students pass the examination.

SOLUTION

To decide cut off marks for qualifying an examination when 3% of students pass, we have to find P_{97} .

We construct the less than cumulative frequency table as given below:

Marks	No. of students (f)	Less than cumulative frequency (c.f.)
0 – 100	130	130
100 – 200	150	280
200 – 300	190	470
300 – 400	220	690
400 – 500	280	970
500 – 600	130	1100 ← P_{97}
Total	1100	

Here, $N = 110$

P_{97} class = class containing $\left(\frac{97N}{100}\right)^{\text{th}}$ observation

$$\therefore \frac{97N}{100} = \frac{97 \times 1100}{100} = 1067$$

Cumulative frequency which is just greater than (or equal) to 1067 is 1100.

$\therefore P_{97}$ lies in the class 500 – 600

$\therefore L = 500, h = 100, f = 130, \text{c.f.} = 970$

$$\therefore P_{97} = L + \frac{h}{f} \left(\frac{97N}{100} - \text{c.f.} \right)$$

$$= 500 + \frac{100}{130} (1067 - 970)$$

$$= 500 + \frac{10}{13} (97)$$

$$= 500 + 74.62$$

$$= 574.62 \approx 575$$

\therefore The cut-off marks for qualifying an examination is 575.

Miscellaneous Exercise 1 | Q 8 | Page 21

Determine graphically the value of median, D_3 , and P_{35} for the data given below:

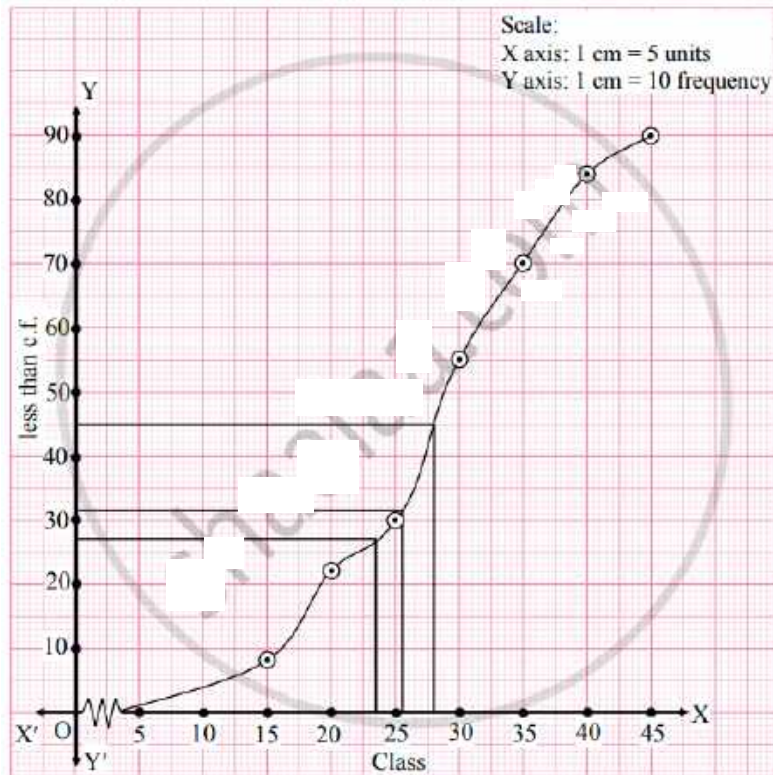
Class	10 – 15	15 – 20	20 – 25	25 – 30	30 – 35	35 – 40	40 – 45
Frequency	8	14	8	25	15	14	6

SOLUTION

To draw a ogive curve, we construct the less than cumulative frequency table as given below:

Class	Frequency (f)	Less than cumulative frequency (c.f.)
10 – 15	8	8
15 – 20	14	22
20 – 25	8	30
25 – 30	25	55
30 – 35	15	70
35 – 40	14	84
40 – 45	6	90
Total	90	

The points to be plotted for less than ogive are (15, 8), (20, 22), (25, 30), (30, 55), (35, 70), (40, 84), (45, 90).



$N = 90$

For median, consider $\frac{N}{2} = \frac{90}{2} = 45$

For D_3 , consider $\frac{3N}{10} = \frac{3 \times 90}{10} = 27$

For P_{35} , consider $\frac{35N}{100} = \frac{35 \times 90}{100} = 31.5$

\therefore We take the values 45, 27, and 31.5 on the Y-axis and draw lines from these points parallel to X-axis. From the points where they intersect the less than ogive, we draw perpendicular on the X-axis. Foot of the perpendicular represents the values of median, D_3 , and P_{35} respectively.

\therefore Median ≈ 29 , $D_3 \approx 23.5$, $P_{35} \approx 26$.

Miscellaneous Exercise 1 | Q 9 | Page 21

The I.Q. test of 500 students of a college is as follows:

I.Q.	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80	80 – 90	90 – 100
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Number of students	41	52	64	180	67	45	40	11
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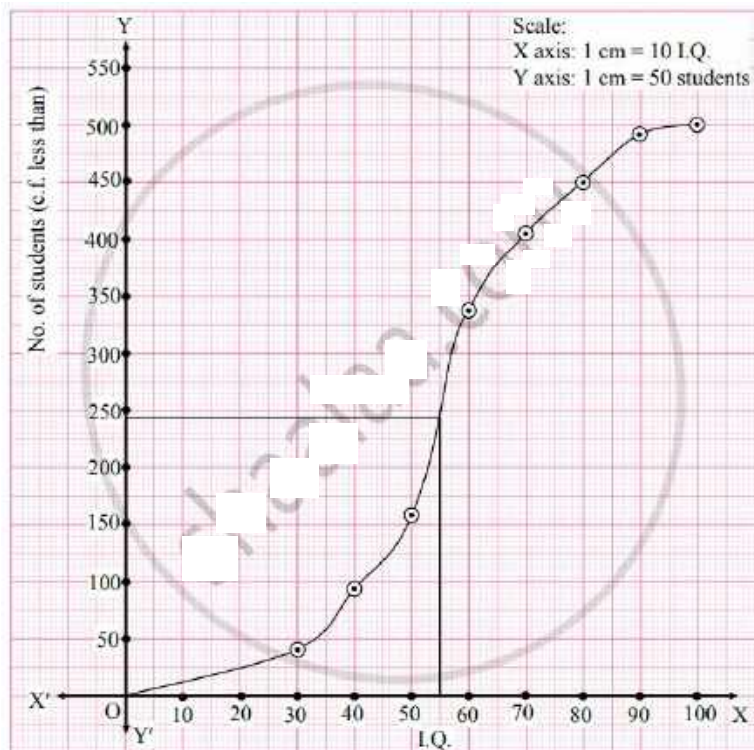
Find graphically the number of students whose I.Q. is more than 55 graphically.

SOLUTION

To draw a ogive curve, we construct the less than cumulative frequency table as given below:

I.Q.	Number of students (f)	Less than cumulative frequency (c.f.)
20 – 30	41	41
30 – 40	52	93
40 – 50	64	157
50 – 60	180	337
60 – 70	67	404
70 – 80	45	449
80 – 90	40	489
90 – 100	11	500
Total	500	

The points to be plotted for less than ogive are (30, 41), (40, 93), (50, 157), (60, 337), (70, 404), (80, 449), (90, 489), (100, 500)



To find the number of students whose I.Q. is more than 55, we consider the value 55 on the X-axis. From this point, we draw a line that is parallel to Y-axis. From the point this line intersects the less than ogive, we draw a perpendicular on the Y-axis. Foot of the perpendicular gives the number of students whose I.Q. is less than 55.

Foot of perpendicular ≈ 244

\therefore No. of students whose I.Q. is less than 55 ≈ 244

\therefore No. of Students whose I.Q. is more than 55 = $500 - 244 = 256$

Miscellaneous Exercise 1 | Q 10 | Page 21

Draw an ogive for the following distribution. Determine the median graphically and verify your result by mathematical formula.

Height (in cms.)	No. of students
145 – 150	2
150 – 155	5
155 – 160	9
160 – 165	15
165 – 170	16

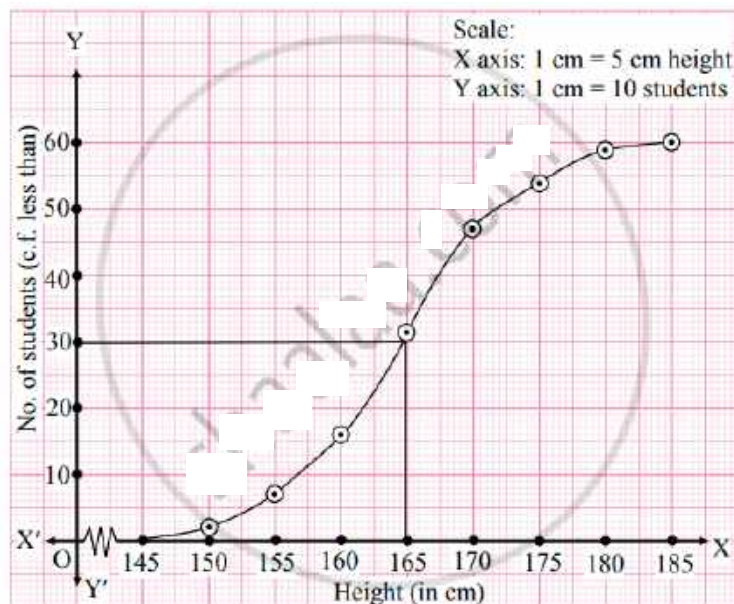
170 – 175	7
175 – 180	5
180 – 185	1

SOLUTION

To draw a ogive curve, we construct the less than cumulative frequency table as given below:

Height (in cms)	No. of students (f)	Less than cumulative frequency (c.f.)
145 – 150	2	2
150 – 155	5	7
155 – 160	9	16
160 – 165	15	31
165 – 170	16	47
170 – 175	7	54
175 – 180	5	59
180 – 185	1	60
Total	60	

The points to be plotted for less than ogive are (150, 2), (155, 7), (160, 16), (165, 31), (170, 47), (175, 54), (180, 59) and (185, 60).



$$N = 60$$

$$\therefore \frac{N}{2} = \frac{60}{2} = 30$$

\therefore We take the value 30 on the Y-axis and from this point, we draw a line parallel to X-axis. From the point where this line intersects the less than ogive, we draw a perpendicular on X-axis. Foot of the perpendicular gives the value of median.

\therefore Median ≈ 164.67

Now, let us calculate the median from the mathematical formula.

$$\therefore \frac{N}{2} = 30$$

The median lies in the class interval of 160 – 165.

$$\therefore L = 160, h = 5, f = 15, \text{c.f.} = 16$$

$$\text{Median} = L + \frac{h}{f} \left(\frac{N}{2} - \text{c.f.} \right)$$

$$= 160 + \frac{5}{15} (30 - 16)$$

$$= 160 + \frac{1}{3} \times 14$$

$$= 160 + 4.67$$

$$= 164.67$$

Miscellaneous Exercise 1 | Q 11 | Page 21

In a group of 25 students, 7 students failed and 6 students got distinction and the marks of the remaining 12 students are 61, 36, 44, 59, 52, 56, 41, 37, 39, 38, 41, 64. Find the median marks of the whole group.

SOLUTION

$$n = 25$$

$$\text{Median} = \frac{n + 1}{2}$$

$$= \frac{25 + 1}{2}$$

$$= 13^{\text{th}} \text{ observation}$$

$$= 13^{\text{th}} \text{ observation}$$

We have been stated that 7 students failed (assuming passing marks on 35) and 6 students got distinction (assuming distinction as 70+), and the marks of the remaining 12 students (who will be situated between the two groups mentioned above, if arranged in ascending order), we have,

F, F, F, F, F, F, F, 36, 37, 38, 39, 41, 41, 44, 52, 56, 59, 61, 64, D, D, D, D, D, D

∴ median = 13th observation = 41.

Miscellaneous Exercise 1 | Q 12 | Page 21

The median weight of a group of 79 students is found to be 55 kg. 6 more students are added to this group whose weights are 50, 51, 52, 59.5, 60, 61 kg. What will be the value of median of the combined group if the lowest and the highest weights were 53 kg and 59 kg respectively?

SOLUTION

$$n = 79$$

$$\text{Median} = 55 \text{ kg}$$

$$\text{Lowest observation} = 53 \text{ kg}$$

$$\text{Highest observation} = 59 \text{ kg}$$

6 new students are added to the group having weights in Kg as follows:

50, 51, 52, 59.5, 60, 61

From the above, we see that of the 6 new students, 3 have weights which are below the lowest weight of the earlier group and 3 have weights which are above the highest weight of the earlier group.

∴ the median remains the same

$$\therefore \text{median} = 55 \text{ kg}$$

Miscellaneous Exercise 1 | Q 13 | Page 22

The median of the following incomplete table is 92. Find the missing frequencies:

C.I.	30 – 50	50 – 70	70 – 90	90 – 110	110 – 130	130 – 150	Total
f	6	?	18	20	?	10	80

SOLUTION

Let a and b be the missing frequencies of the class 50 – 70 and class 110 – 130 respectively.

We construct the less than cumulative frequency table as given below:

C.I.	f	Less than cumulative frequency (c.f.)
30 – 50	6	6
50 – 70	a	6 + a
70 – 90	18	24 + a
90 – 110	20	44 + a ← Q ₂
110 – 130	B	44 + a + b
130 – 150	10	54 + a + b
Total	N = 80	

Here, $N = 54 + a + b$

Since, $N = 80$

$$\therefore 54 + a + b = 80$$

$$\therefore a + b = 26 \text{(i)}$$

Given, Median = $Q_2 = 92$

$\therefore Q_2$ lies in the class 90 – 110.

$$\therefore L = 90, h = 20, f = 20, \frac{2N}{4} = \frac{2 \times 80}{4} = 40, \text{c.f.} = 24 + a$$

$$\therefore Q_2 = L + \frac{h}{f} \left(\frac{2N}{4} - \text{c.f.} \right)$$

$$\therefore 92 = 90 + \frac{20}{20} [40 - (24 + a)]$$

$$\therefore 92 - 90 = 40 - 24 - a$$

$$\therefore 2 = 16 - a$$

$$\therefore a = 14$$

Substituting the value of a in equation (i), we get

$$14 + b = 26$$

$$\therefore b = 26 - 14 = 12$$

\therefore 14 and 12 are the missing frequencies of the class 50 – 70 and class 110 – 130 respectively.

Miscellaneous Exercise 1 | Q 14 | Page 22

A company produces tubes which are packed in batches of 100. An analysis of the defective tubes in different batches has received the following information:

No. of defective tubes	Less than 5	5 – 9	10 – 14	15 – 9	
No. of tubes	45	51	84	39	

Estimate the number of defective tubes in the central batch.

SOLUTION

To find the number of defective tubes in the central batch, we have to find Q_2 .

Since, the given data is not continuous, we have to convert it in the continuous form by subtracting 0.5 from the lower limit and adding 0.5 to the upper limit of every class interval.

\therefore the class intervals will be

Less than 4.5, 4.5 – 9.5, etc.

We construct the less than cumulative frequency table as given below:

No. of defective tubes	No. of tubes (f)	Less than cumulative frequency (c.f.)
Less than 4.5	45	45
4.5 – 9.5	51	96
9.5 – 14.5	84	180 $\leftarrow Q_2$

14.5 – 19.5	39	219
19.5 – 24.5	20	239
24.5 – 29.5	8	247
29.5 and above	4	251
Total	251	

Here, N = 251

Q_2 class = class containing $\left(\frac{2N}{4}\right)^{\text{th}}$ observation

$$\therefore \frac{2N}{4} = \frac{2 \times 251}{4} = 125.5$$

Cumulative frequency which is just greater than (or equal to) 125.5 is 180.

$\therefore Q_2$ lies in the class 9.5 – 14.5

$\therefore L = 9.5, h = 5, f = 84, \text{c.f.} = 96$

$$\therefore Q_2 = L + \frac{h}{f} \left(\frac{2N}{4} - \text{c.f.} \right)$$

$$= 9.5 + \frac{5}{84} (125.5 - 96)$$

$$= 9.5 + \frac{5}{84} \times 29.5$$

$$= 9.5 + \frac{147.5}{84}$$

$$= 9.5 + 1.76$$

$$= 11.26$$

Miscellaneous Exercise 1 | Q 15 | Page 22

In a college, there are 500 students in junior college, 5% score less than 25 marks, 68 score from 26 to 30 marks, 30% score from 31 to 35 marks, 70 score from 36 to 40 marks, 20% score from 41 to 45 marks and the rest score 46 and above marks. What is the median marks?

SOLUTION

Given data can be written in tabulated form as follows:

Marks	No. of students
Less than 25	$5\% \text{ of } 500 = \frac{5}{100} \times 500 = 25$
26 – 30	68
31 – 35	$30\% \text{ of } 500 = \frac{30}{100} \times 500 = 150$
36 – 40	70
41 – 45	$20\% \text{ of } 500 = \frac{20}{100} \times 500 = 100$
46 and above	$500 - (25 + 68 + 15 + 70 + 100) = 87$

Since, the given data is not continuous, we have to convert it in the continuous form by subtracting 0.5 from the lower limit and adding 0.5 to the upper limit of every class interval.

∴ the class intervals will be

Less than 25.5, 25.5 – 30.5 etc.

We construct the less than cumulative frequency table as given below:

Marks	No. of students (f)	Less than cumulative frequency (c.f.)
Less than 25.5	25	25
25.5 – 30.5	68	93
30.5 – 35.5	150	243
35.5 – 40.5	70	313 ← Q_2
40.5 – 45.5	100	413
45.5 and above	87	500
Total	500	

Here, $N = 500$

Q_2 class = class containing $\left(\frac{2N}{4}\right)^{\text{th}}$ observation

$$\therefore \frac{2N}{4} = \frac{2 \times 500}{4} = 250$$

Cumulative frequency which is just greater than (or equal to) 250 is 313.

$\therefore Q_2$ lies in the class 35.5 – 40.5.

$\therefore L = 35.5, h = 5, f = 70, \text{c.f.} = 243$

$$\therefore \text{Median} = Q_2 = L + \frac{h}{f} \left(\frac{2N}{4} - \text{c.f.} \right)$$

$$= 35.5 + \frac{5}{70} (250 - 243)$$

$$= 35.5 + \frac{1}{14} (7)$$

$$= 35.5 + 0.5$$

$$= 36$$

Miscellaneous Exercise 1 | Q 16 | Page 22

Draw a cumulative frequency curve more than type for the following data and hence locate Q_1 and Q_3 . Also, find the number of workers with daily wages

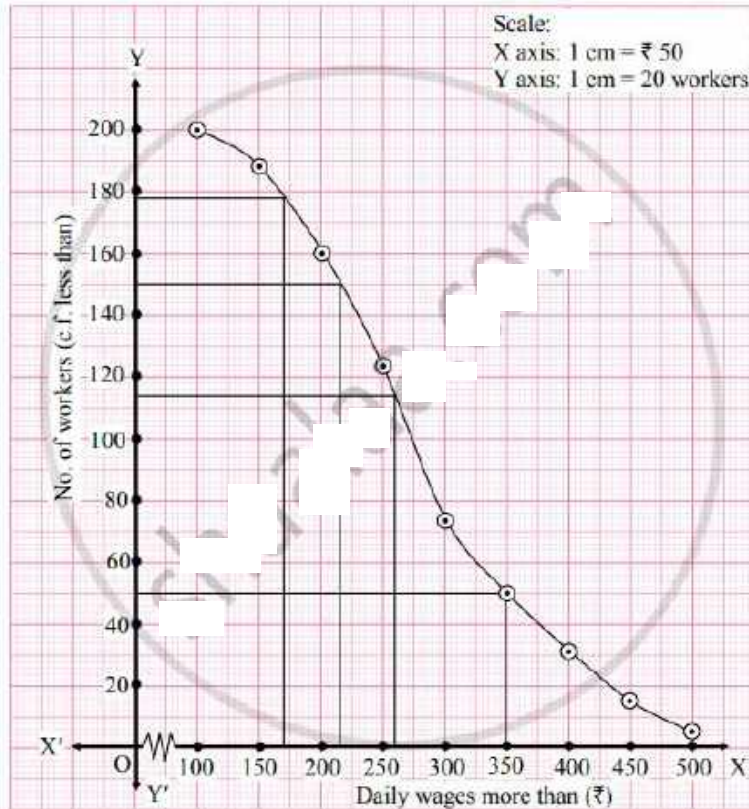
(i) Between ₹ 170 and ₹ 260

(ii) less than ₹ 260

Daily wages more than (₹)	100	150	200	250	300	350	400	450	500
No. of workers	200	188	160	124	74	49	31	15	5

SOLUTION

For more than ogive points to be plotted are (100, 200), (150, 188), (200, 160), (250, 124), (300, 74), (350, 49), (400, 31), (450, 15), (500, 5)



Here, $N = 200$

$$\text{For } Q_1, \frac{N}{4} = \frac{200}{4} = 50,$$

$$\text{For } Q_3, \frac{3N}{4} - \frac{3 \times 200}{4} = 150$$

We take the points having Y co-ordinates 50 and 150 on Y-axis. From these points, we draw lines that are parallel to X-axis. From the points of intersection of these lines with the curve, we draw perpendicular on X-axis. X-Co-ordinates of these points gives the values of Q_1 and Q_3 .

Since X-axis has daily wages more than and not less than the given amounts.

$$\therefore Q_1 = Q_3 \text{ and } Q_3 = Q_1$$

$$\therefore Q_1 \approx 215, Q_3 \approx 348$$

(i) To find the number of workers with daily wages between ₹ 170 and ₹ 260,

Take the values 170 and 260 on X-axis. From these points, we draw lines parallel to Y-axis. From the point where they intersect the more than ogive, we draw perpendiculars on Y-axis.

The points where they intersect the Y-axis gives the values 178 and 114.

$$\therefore \text{Number of workers having daily wages between ₹ 170 and ₹ 260} = 178 - 114 = 64$$

(ii) To find the number of workers having daily wages less than ₹ 260, we consider the value 260 on the X-axis. From this point, we draw a line that is parallel to Y-axis. From the point where the line intersects the more than ogive, we draw a perpendicular on the Y-axis. Foot of perpendicular gives the number of workers having daily wages more than 260.

Foot of perpendicular ≈ 114

\therefore No. of worker whose daily wages more than ₹ 260 ≈ 114

\therefore No. of workers whose daily wages less than ₹ 260 = $200 - 114 = 86$

Miscellaneous Exercise 1 | Q 17 | Page 22

Draw ogive of both the types for the following frequency distribution and hence find median.

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80	80 – 90	90 – 100
No. of students	5	5	8	12	16	15	10	8	5	2

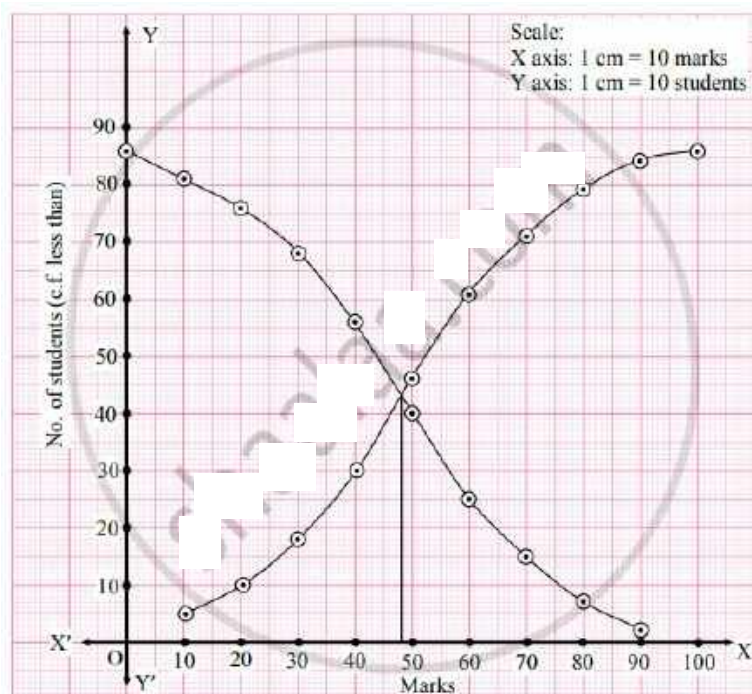
SOLUTION

Marks	No. of students	Less than cumulative frequency (c.f.)	More than cumulative frequency (c.f.)
0 – 10	5	5	86
10 – 20	5	10	81
20 – 30	8	18	76
30 – 40	12	30	68
40 – 50	16	46	56
50 – 60	15	61	40
60 – 70	10	71	25
70 – 80	8	79	15
80 – 90	5	84	7

90 – 100	2	86	2
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For less than given points to be plotted are (10, 5), (20, 10), (30, 18), (40, 30), (50, 46), (60, 61), (70, 71), (80, 79), (90, 84), (100, 86)

For more than given points to be plotted are (0, 86), (10, 81), (20, 76), (30, 68), (40, 56), (50, 40), (60, 25), (70, 15), (80, 7), (90, 2)



From the point of intersection of two ogives. We draw a perpendicular on X-axis. The point where it meets the X-axis gives the value of the median.
 \therefore Median \approx 48

Miscellaneous Exercise 1 | Q 18 | Page 22

Find Q_1 , D_6 , and P_{78} for the following data:

C.I.	8 – 8.95	9 – 9.95	10 – 10.95	11 – 11.95	12 – 12.95
f	5	10	20	10	5

SOLUTION

Since the given data is not in the form of a continuous frequency distribution, we have to convert it into that form by subtracting 0.025 from the lower limit and adding 0.025 to the upper limit of each class interval.

\therefore the class intervals will be 7.975 – 8.975, 8.975 – 9.975, etc.

We construct the less than cumulative frequency table as given below:

C.I.	f	Less than cumulative frequency (c.f.)
7.975 – 8.975	5	5
8.975 – 9.975	10	15 ← Q ₁
9.975 – 10.975	20	35 ← D ₆
10.975 – 11.975	10	45 ← P ₇₈
11.975 – 12.975	5	50
Total	50	

Here, N = 50

Q₁ class = class containing $\left(\frac{N}{4}\right)^{\text{th}}$ observation

$$\therefore \frac{N}{4} = \frac{50}{4} = 12.5$$

Cumulative frequency which is just greater than (or equal) to 12.5 is 15.

∴ Q₁ lies in the class 8.975 – 9.975

∴ L = 8.975, h = 1, f = 10, c.f. = 5

$$Q_1 = L + \frac{h}{f} \left(\frac{N}{4} - \text{c.f.} \right)$$

$$= 8.975 + \frac{1}{10} (12.5 - 5)$$

$$= 8.975 + 0.1(7.5)$$

$$= 8.975 + 0.75$$

$$= 9.725$$

D₆ class = class containing $\left(\frac{6N}{10}\right)^{\text{th}}$ observation

$$\therefore \frac{6N}{10} = \frac{6 \times 50}{10} = 30$$

Cumulative frequency which is just greater than (or equal) to 30 is 35.

$\therefore D_6$ lies in the class 9.975 – 10.975

$\therefore L = 9.975, h = 1, f = 20, \text{c.f.} = 15$

$$D_6 = L + \frac{h}{f} \left(\frac{6N}{10} - \text{c.f.} \right)$$

$$= 9.975 + \frac{1}{20} (30 - 15)$$

$$= 9.975 + 0.05(15)$$

$$= 9.975 + 0.75$$

$$= 10.725$$

P_{78} class = class containing $\left(\frac{78N}{100} \right)^{\text{th}}$ observation

$$\frac{78N}{100} = \frac{78 \times 50}{100} = 39$$

Cumulative frequency which is just greater than (or equal) to 39 is 45.

$\therefore P_{78}$ lies in the class 10.975 – 11.975

$\therefore L = 10.975, h = 1, f = 10, \text{c.f.} = 35$

$$P_{78} = L + \frac{h}{f} \left(\frac{78N}{100} - \text{c.f.} \right)$$

$$= 10.975 + \frac{1}{10} (39 - 35)$$

$$= 10.975 + 0.1(4)$$

$$= 10.975 + 0.4$$

$$= 11.375$$

Weight (kg)	40 – 45	45 – 50	50 – 55	55 – 60	60 – 65	65 – 70	70 – 75	75 – 80
No. of person	4	15	20	30	20	10	8	4

For above data, find all quartiles and number of persons weighing between 57 kg and 72.

SOLUTION

We construct the less than cumulative frequency table as given below:

Weight (kg)	No. of persons (f)	Less than cumulative frequency (c.f.)
40 – 45	4	4
45 – 50	15	19
50 – 55	20	39 $\leftarrow Q_1$
55 – 60	30	69 $\leftarrow Q_2, P_x$
60 – 65	20	89 $\leftarrow Q_3$
65 – 70	10	99
70 – 75	8	107 $\leftarrow P_y$
75 – 80	4	111
Total	N = 111	

Here, N = 111

Q_1 class = class containing $\left(\frac{N}{4}\right)^{\text{th}}$ observation

$$\therefore \frac{N}{4} = \frac{111}{4} = 27.75$$

Cumulative frequency which is just greater than (or equal) to 27.75 is 39.

$\therefore Q_1$ lies in the class 50 – 55.

$\therefore L = 50, h = 5, f = 20, \text{c.f.} = 19$

$$\therefore Q_1 = L + \frac{h}{f} \left(\frac{N}{4} - \text{c.f.} \right)$$

$$= 50 + \frac{5}{20} (27.75 - 19)$$

$$= 50 + \frac{5}{20} \times 8.75$$

$$= 50 + \frac{43.75}{20}$$

$$= 50 + 2.1875$$

$$= 52.1875$$

Q_2 class = class containing $\left(\frac{2N}{4}\right)^{\text{th}}$ observation

$$\therefore \frac{2N}{4} = \frac{2 \times 111}{4} = 55.5$$

Cumulative frequency which is just greater than (or equal) to 55.5 is 69.

$\therefore Q_2$ lies in the class 55 – 60.

$\therefore L = 55, h = 5, f = 30, \text{c.f.} = 39$

$$\therefore Q_2 = L + \frac{h}{f} \left(\frac{2N}{4} - \text{c.f.} \right)$$

$$= 55 + \frac{5}{30}(55.5 - 39)$$

$$= 55 + \frac{1}{6} \times 16.5$$

$$= 55 + 2.75$$

$$= 57.75$$

Q_3 class = class containing $\left(\frac{3N}{4}\right)^{\text{th}}$ observation

$$\therefore \frac{3N}{4} = \frac{3 \times 111}{4} = 83.25$$

Cumulative frequency which is just greater than (or equal) to 83.25 is 89.

$\therefore Q_3$ lies in the class 60 – 65.

$\therefore L = 60, h = 5, f = 20, \text{c.f.} = 69$

$$\therefore Q_3 = L + \frac{h}{f} \left(\frac{3N}{4} - \text{c.f.} \right)$$

$$= 60 + \frac{5}{20}(83.25 - 69)$$

$$= 60 + \frac{1}{4} \times 14.25$$

$$= 60 + 3.5625$$

$$= 63.5625$$

In order to find the number of persons between 57 kg and 72 kg,

We need to find x in P_x , where $P_x = 57$ kg and y in P_y , where $P_y = 72$ kg

Then $(y - x)$ would be the % of persons weighing between 57 kg and 72 kg

$$P_x = 57$$

$$\therefore L + \frac{h}{f} \left(\frac{x \times N}{100} - \text{c.f.} \right) = 57$$

$$\therefore 55 + \frac{5}{30}(1.11x - 39) = 57$$

$$\therefore \frac{1}{6}(1.11x - 39) = 57$$

$$\therefore 1.11x - 39 = 12$$

$$\therefore 1.11x = 51$$

$$\therefore x = 45.95$$

$$\therefore P_x = 72$$

$$\therefore L + \frac{h}{f} \left(\frac{y \times N}{100} - \text{c.f.} \right) = 72$$

$$\therefore 70 + \frac{5}{8}(1.11y - 99) = 72$$

$$\therefore 0.625(1.11y - 99) = 72$$

$$\therefore 1.11y - 99 = 3.2$$

$$\therefore 1.11y = 102.2$$

$$\therefore y = 92.07$$

$$\therefore \% \text{ of people weighing between 57 kg and 72 kg} = 92.07 - 45.95 = 46.12 \%$$

$$\therefore \text{No. of people weighing between 57 kg and 72 kg} = 111 \times 46.12\% = 51.1932 \approx 51$$

Miscellaneous Exercise 1 | Q 20 | Page 22

For the following data showing weights of 100 employees, find the maximum weight of the lightest 25% of employees.

Weight (kg)	45 – 50	50 – 55	55 – 60	60 – 65	65 – 70	70 – 75	75 – 80
No. of employees	6	8	15	26	20	14	11

SOLUTION

We construct the less than cumulative frequency table as given below:

Weight (kg)	No. of employees (f)	Less than cumulative frequency (c.f.)
45 – 50	6	6
50 – 55	8	14
55 – 60	15	29 ← Q ₁
60 – 65	26	55
65 – 70	20	75
70 – 75	14	89
75 – 80	11	100
Total	N = 100	

Here, N = 100

Q₁ class = class containing $\left(\frac{N}{4}\right)^{\text{th}}$ observation

$$\therefore \frac{N}{4} = \frac{100}{4} = 25$$

Cumulative frequency which is just greater than (or equal) to 25 is 29.

\therefore Q₁ lies in the class 55 – 60.

\therefore L = 55, h = 5, f = 15, c.f. = 14

$$\therefore Q_1 = L + \frac{h}{f} \left(\frac{N}{4} - \text{c.f.} \right)$$

$$= 55 + \frac{5}{15} (25 - 14)$$

$$= 55 + \frac{1}{3} \times 11$$

$$= 55 + 3.67$$

$$= 58.67$$

\therefore Maximum weight of the lightest 25% of employees is 58.67 kg.