Definitions, Results, Tips, formulae, Conditions, values

Definitions: Degree: An angle is called a right angle when its terminal side and initial sides are $\perp r$ to each other. If a right angle is divided into 90 equal parts then each part is called a degree. One degree is divided into 60 equal parts and each part is called minute. One minute is further divided into 60 equal parts and each part is called 1 second.

(or) In degrees, one complete rotation is split into 360 equal parts and each part is called 1 degree.

Note: 1) Two angles that have the exact same measure are called Congruent angles.

2) Two angles that have their measures have adding to 90° are called complimentary angle

3. Two angles that have their measures adding to 180° are Called Supplementary angle.

4) Two angles between 0° and 360° are Conjugate if their sum equal to 360°.

Coterminal angles: If the difference of two angles is $k(360)$ then they are coterminal angle.

Radian Measure. The radian measure of the angle is the ratio of the arc length it subtends to the radius of the circle in which it is the central angle. $\theta = \dfrac{\text{arc length}}{\text{radius}} = \dfrac{s}{r}$ (or) $s = r\theta$.

Relation ship between degree and radian.

$$\pi^c = 180^\circ \Rightarrow 1^\circ = \frac{\pi}{180}$$

$$1^c = \frac{180}{\pi}.$$

Note: The ratio of the circumfrance of any circle to its diameter is always constant. This constant is denoted by irrational number π

$$\text{Area of the sector} = \left(\frac{\pi r^2}{360}\right)\theta \quad \text{in degree}$$

$$= \left(\frac{\pi r^2}{2\pi}\right)\theta = \frac{r^2\theta}{2} \quad \text{in radian.}$$

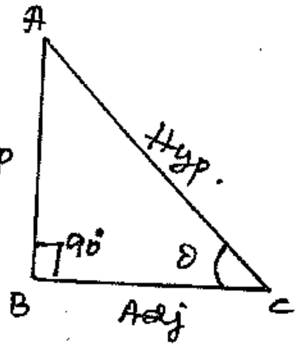
$1^c \simeq 56^\circ 17' 15''$

Radian	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	π	$3\pi/2$	2π
Degree	30	45	60	180	270	360

Basic Trigonometric ratios using right angle

$\sin\theta = \frac{Opp}{Hyp}$, $\cos\theta = \frac{Adj}{Hyp}$, $\tan\theta = \frac{Opp}{Adj}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$.



$\mathrm{cosec}\,\theta = \frac{1}{\sin\theta}$, $\mathrm{cosec}\,\theta \sin\theta = 1$, $\frac{1}{\mathrm{cosec}\,\theta} = \sin\theta$.

$\sec\theta = \frac{1}{\cos\theta}$, $\cos\theta = \frac{1}{\sec\theta}$, $\cos\theta \sec\theta = 1$

$\cot\theta = \frac{1}{\tan\theta}$, $\tan\theta = \frac{1}{\cot\theta}$, $\tan\theta \cot\theta = 1$

θ	0	30	45	60	90	180	270	360
$\sin\theta$	0	$\frac{1}{2}$	$1/\sqrt{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
$\cos\theta$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0	-1	0	1
$\tan\theta$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	∞	0	$-\infty$	0

$\sin^2\theta + \cos^2\theta = 1$, $\sin^2\theta = 1 - \cos^2\theta$ $\cos^2\theta = 1 - \sin^2\theta$.

$\sin\theta = \sqrt{1-\cos^2\theta}$ $\cos\theta = \sqrt{1-\sin^2\theta}$.

$1 + \tan^2\theta = \sec^2\theta$, $\sec^2\theta - \tan^2\theta = 1$, $1 - \sec^2\theta = -\tan^2\theta$.

$1 + \cot^2\theta = \mathrm{cosec}^2\theta$, $\mathrm{cosec}^2\theta - \cot^2\theta = 1$ $1 - \mathrm{cosec}^2\theta = -\cot^2\theta$.

Note: The Trigonometric identity is an equation that is true for all values of its domain values.

$-1 \le \cos\theta \le 1$ and $-1 \le \sin\theta \le 1$ Always.

II quadrant	I quadrant.
Silver $90+\theta$, $180-\theta$	All $90-\theta$, $360+\theta$.
Tea $108+\theta$, $270-\theta$	$270+\theta$, $360-\theta$ or $(-\theta)$
III quadrant	IV quadrant. cups.

$\left.\begin{matrix} 90 \pm \theta \\ 270 \pm \theta \end{matrix}\right\}$ Co ratio
Sin $\leftrightarrow$ cosin
Sec $\leftrightarrow$ Cosec
tan $\leftrightarrow$ cotan.

$\left.\begin{matrix} 180 \pm \theta \\ 360 \pm \theta \end{matrix}\right\}$ Same ratio
$\sin\theta \longrightarrow \sin\theta$
$\cos\theta \longrightarrow \cos\theta$

	I	II		III		IV	
	$90-\theta$	$90+\theta$	$180-\theta$	$180+\theta$	$270-\theta$	$270+\theta$	$360-\theta$ $(-\theta)$
$\sin\theta$	$\cos\theta$	$\cos\theta$	$\sin\theta$	$-\sin\theta$	$-\cos\theta$	$-\cos\theta$	$-\sin\theta$
$\cos\theta$	$\sin\theta$	$-\sin\theta$	$-\cos\theta$	$-\cos\theta$	$-\sin\theta$	$\sin\theta$	$\cos\theta$
$\tan\theta$	$\cot\theta$	$-\cot\theta$	$-\tan\theta$	$\tan\theta$	$\cot\theta$	$-\cot\theta$	$-\tan\theta$
$\operatorname{cosec}\theta$	$\sec\theta$	$\sec\theta$	$\operatorname{cosec}\theta$	$-\operatorname{cosec}\theta$	$-\sec\theta$	$-\sec\theta$	$-\operatorname{cosec}\theta$
$\sec\theta$	$\operatorname{cosec}\theta$	$-\operatorname{cosec}\theta$	$-\sec\theta$	$-\sec\theta$	$-\operatorname{cosec}\theta$	$\operatorname{cosec}\theta$	$\sec\theta$
$\cot\theta$	$\tan\theta$	$-\cot\theta$	$-\cot\theta$	$\cot\theta$	$\tan\theta$	$-\tan\theta$	$-\cot\theta$

Periodic function: A function $y = f(x)$ is said to be periodic if there exists a real number $(T>0)$ s.t $f(x+T) = f(x)\ \forall x$.

$\sin(2\pi+\theta) = \sin\theta$ $\quad \operatorname{cosec}(2\pi+\theta) = \operatorname{cosec}\theta$

$\cos(2\pi+\theta) = \cos\theta$ $\quad \sec(2\pi+\theta) = \sec\theta$.

But $\tan(\pi+\theta) = \tan\theta$ $\quad \cot(\pi+\theta) = \cot\theta$.

$\therefore$ The period of $\sin\theta$, $\cos\theta$, $\operatorname{cosec}\theta$, $\sec\theta$ are 2π

whereas the period of $\tan\theta$ and $\cot\theta = \pi$.

1) $\sin 2A = 2\sin A\cos A$

$\cos 2A = \begin{cases} \cos^2 A - \sin^2 A \\ 1 - 2\sin^2 A \\ 2\cos^2 A - 1 \end{cases}$

$\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$.

$\sin 2A = \dfrac{2\tan A}{1+\tan^2 A}$

$\cos 2A = \dfrac{1-\tan^2 A}{1+\tan^2 A}$

$\sin A = 2\sin A/2 \cos A/2$

$\cos A = \cos^2 A/2 - \sin^2 A/2$

$= 1 - 2\sin^2 A/2$

$2\cos^2 A/2 - 1$

$\tan A = \dfrac{2\tan A/2}{1-\tan^2 A/2}$

Trigonometric functions of Sum and difference of Two angles.

1) $\sin(A+B) = \sin A \cos B + \cos A \sin B$.

2) $\sin(A-B) = \sin A \cos B - \cos A \sin B$

3) $\cos(A+B) = \cos A \cos B - \sin A \sin B$

4) $\cos(A-B) = \cos A \cos B + \sin A \sin B$.

5) $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

6) $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$.

7) $\cot(A+B) = \dfrac{\cot A \cot B - 1}{\cot B + \cot A}$

8) $\cot(A-B) = \dfrac{\cot A \cot B + 1}{\cot B - \cot A}$.

9) $\tan(45+A) = \dfrac{1+\tan A}{1-\tan A}$

10) $\tan(45-A) = \dfrac{1-\tan A}{1+\tan A}$.

$\cos^2 A = \dfrac{1+\cos 2A}{2}$

$\sin^2 A = \dfrac{1-\cos 2A}{2}$

$\cos^2 A/2 = \dfrac{1+\cos A}{2}$

$\sin^2 A/2 = \dfrac{1-\cos A}{2}$

$\sin 3A = 3\sin A - 4\sin^3 A$

$\cos 3A = 4\cos^3 A - 3\cos A$

$\tan 3A = \dfrac{3\tan A - \tan^2 A}{1 - 3\tan^2 A}$.

Results: 1) $\sin(A+B) \cdot \sin(A-B) = \sin^2 A - \sin^2 B$ (or) $\cos^2 B - \cos^2 A$

2) $\cos(A+B)\cos(A-B) = \cos^2 A - \sin^2 B$ (or) $\cos^2 B - \sin^2 A$.

Product to Sum and Sum to product.

1) $\sin(A+B) + \sin(A-B) = 2\sin A \cos B$.

2) $\sin(A+B) - \sin(A-B) = 2\cos A \sin B$.

3) $\cos(A+B) + \cos(A-B) = 2\cos A \cos B$

4) $\cos(A+B) - \cos(A-B) = -2\sin A \sin B$

(or) $\cos(A-B) - \cos(A+B) = 2\sin A \sin B$.

1) $\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$

2) $\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$

3) $\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$

4) $\cos C - \cos D = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}$ (or) $\cos D - \cos C = 2\sin\frac{C+D}{2}\sin\frac{C-D}{2}$

Results: 1) $\sin(60-A)\sin A\sin(60+A) = \frac{1}{4}\sin 3A$

2) $\cos(60-A)\cos A.\cos(60+A) = \frac{1}{4}\cos 3A$

3) $\tan(60-A)\tan A\cdot\tan(60+A) = \tan 3A$.

Result: $\cos A\cos 2A\cos 2^2 A\cos 2^3 A \ldots \cos 2^{n-1} A = \dfrac{\sin 2^n A}{2^n \sin A}$.

Special Values.

$\sin 18^\circ = \cos 72^\circ = \dfrac{\sqrt{5}-1}{4}$ $\quad$ $\sin 75^\circ = \dfrac{\sqrt{3}+1}{2\sqrt{2}}$

$\sin 54 = \cos 36 = \dfrac{\sqrt{5}+1}{4}$ $\quad$ $\cos 15^\circ = \dfrac{\sqrt{3}+1}{2\sqrt{2}}$

$\cos 18 = \sin 72^\circ = \dfrac{\sqrt{10+2\sqrt{5}}}{4}$ $\quad$ $\sin 15^\circ = \dfrac{\sqrt{3}-1}{2\sqrt{2}}$

$\cos 75^\circ = \dfrac{\sqrt{3}-1}{2\sqrt{3}}$

$\cos 54 = \sin 36^\circ = \dfrac{\sqrt{10-2\sqrt{5}}}{4}$

Working Rule for solving the problems which are Conditional identities involving sines and cosines of Multiples (or) Sub multiple of the angles.

1) Take any two of the given terms and express this as a product of by using the formula occuring $\sin C$ and $\sin D$.
2) In the product so obtained express the sum of the two angles in terms of the third angle by using the given condition.
3) Expand the third term by using one of the formula $\sin 2\theta$, or $\cos 2\theta$ whichever is applicable.
4) Take out the common factor and in the other factor express the t ratios of single angle into that of the sum of angles by using the given condition
5) Use one of C, D formula to change the sum of t-ratios into a Product.

Working Rule for solving the problems which are conditional identities which involving squares of sines and cosines of Multiple (or) Sub Multiples of the angles.

$\sin^2\theta = \dfrac{1-\cos 2\theta}{2}$, $\quad \cos^2\theta = \dfrac{1+\cos 2\theta}{2}$

1. Take any two of the given terms and express in the form either $\sin^2\theta - \sin^2\varphi$ (or) $\cos^2\theta - \sin^2\varphi$

For this purpose The formula $\sin^2\alpha + \cos^2\alpha = 1$ may be used.

2) Use the formula $\sin^2\theta - \sin^2\varphi = \sin(\theta+\varphi)\sin(\theta-\varphi)$ or
$\cos^2\theta - \sin^2\varphi = \cos(\theta+\varphi)\cos(\theta-\varphi)$

3) Take out the common factor and in the other factor, express the t ratio of single angle into that the sum of angles by using the given Condition.

4) Use one of the C&D formula to change the sum of t-ratios into a product.

Working Rule to solve the problem with identities involving tangents and Cotangents of Multiples or Sub multiples of the angle.

1. Express the given condition so that LHS is the sum of two multiplies (or sub multiplies) of the angles occuring in the identity

2. Take tangents (or cotangents) on both sides.

3. Use addition formula and cross multiply

4. Arrange the terms as given in the identity.

General Solutions of Trigonometric Equations.

1) $\sin\theta = 0 \Rightarrow \theta = n\pi \quad n \in Z$

2) $\cos\theta = 0 \Rightarrow \theta = (2n+1)\frac{\pi}{2} \quad n \in Z$

3) $\tan\theta = 0 \Rightarrow \theta = n\pi \quad n \in Z.$

If α is some constant angle, Then prove That

1) $\sin\theta = \sin\alpha \Rightarrow \theta = n\pi + (-1)^n\alpha, \; n \in Z.$

2) $\cos\theta = \cos\alpha \Rightarrow \theta = 2n\pi \pm \alpha \quad n \in Z$

3) $\tan\theta = \tan\alpha \Rightarrow \theta = n\pi + \alpha \quad n \in Z.$

Working rule for solving $a\cos\theta + b\sin\theta = 0$

1. Check that $|c| \leq \sqrt{a^2+b^2}$. If $|c| > \sqrt{a^2+b^2}$ Then the equation will not have any solution

2. Put $a = r\cos\alpha$, $b = r\sin\alpha$, $r > 0$ and get $r(\cos(\theta-\alpha)) = c$
(ie) $\cos(\theta-\alpha) = \frac{c}{r}$ and a value of α is found by using the equations $\cos\alpha = a/r$ $\sin\alpha = \frac{b}{r}$ alternatively put $a = r\sin\alpha$, $b = r\cos\alpha$ and get $r\sin(\theta+\alpha) = c$ (ie) $\sin(\theta+r) = \frac{c}{r}$

Here $r = \sqrt{a^2+b^2}$ and a value of α is found by using the equations $\sin\alpha = \frac{a}{r}$ and $\cos\alpha = \frac{b}{r}$

3) write the general solution of the equation $\cos(\theta-\alpha) = \frac{c}{r}$ $\sin(\theta+\alpha) = \frac{c}{r}$ and shift the angle α on the right hand side.

Sine Formula: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$, $R = \frac{abc}{4\Delta}$

Cosine Formula: $a^2+b^2-c^2 = 2ab\cos C$, $b^2+c^2-a^2 = 2bc\cos A$, $c^2+a^2-b^2 = 2ca\cos B$

$\cos C = \frac{a^2+b^2-c^2}{2ab}$, $\cos A = \frac{b^2+c^2-a^2}{2bc}$, $\cos B = \frac{c^2+a^2-b^2}{2ca}$.

(or) $a^2 = b^2+c^2-2bc\cos A$, $b^2 = c^2+a^2-2ca\cos B$, $c^2 = a^2+b^2-2ab\cos C$.

Projection formula: $a = b\cos C + c\cos B$
$b = c\cos A + a\cos C$
$c = a\cos B + b\cos A$.

Napier's Formula: $\tan\frac{B-C}{2} = \frac{b-c}{b+c}\cot A/2$

$\tan\frac{C-A}{2} = \frac{c-a}{c+a}\cot B/2$

$\tan\frac{A-B}{2} = \frac{a-b}{a+b}\cot C/2$

Half angle Formula

$\sin\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$, $\cos A/2 = \sqrt{\frac{s(s-a)}{bc}}$, $\tan\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$

$\sin B/2 = \sqrt{\frac{(s-c)(s-a)}{ca}}$ $\cos B/2 = \sqrt{\frac{s(s-b)}{ca}}$ $s = \frac{a+b+c}{2}$

$\sin C/2 = \sqrt{\frac{(s-a)(s-b)}{ab}}$ $\cos C/2 = \sqrt{\frac{s(s-c)}{ab}}$ $\tan B/2 = \sqrt{\frac{(s-c)(s-a)}{s(s-b)}}$

$\tan C/2 = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$

$\sin A = \frac{2}{bc}\sqrt{s(s-a)(s-b)(s-c)}$ $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$ Hero's Formula

Area of the $\triangle$le $= \Delta = \frac{1}{2}ab\sin C = \frac{1}{2}bc\sin A = \frac{1}{2}ca\sin B$

(or) $\Delta = a^2\frac{\sin B\sin C}{\sin A}$, $a\cos A + b\cos B + c\cos C = 2a\sin B\sin C$

For a fixed perimeter $2s$ the area of the Δ is maximum when $a=b=c$ and the area of the $\triangle$le $= \frac{s^2}{3\sqrt{3}}$ sq. units, $\frac{2\Delta}{ab} = \sin C$, $\frac{2\Delta}{bc} = \sin A$, $\frac{2\Delta}{ca} = \sin B$

2/3/5 Marks Problems: (New)

1) TBP If $\tan^2 x = 1 - a^2$ P.T $\sec x + \tan^3 x \operatorname{cosec} x = (2-a^2)^{3/2}$. And also find the values of a for which the above result holds good.

Sol: $\sec x + \tan^3 x \operatorname{cosec} x = \sec x \left(1 + \tan^3 x \cdot \frac{\operatorname{cosec} x}{\sec x}\right)$

$$= \sqrt{1+\tan^2 x}\left(1 + \tan^3 x \cdot \frac{\cos x}{\sin x}\right)$$

$$= \sqrt{1+\tan^2 x}\,(1 + \tan^3 x \cot x)$$

$$= \sqrt{1+\tan^2 x}\,(1 + \tan^2 x).$$

$$= (1+\tan^2 x)^{3/2}$$

$$= (1+1-a^2)^{3/2} = (2-a^2)^{3/2}$$

$\because$ always $\tan^2 x \geq 0 \Rightarrow 1 - a^2 \geq 0 \Rightarrow a^2 - 1 \leq 0 \Rightarrow -1 \leq a \leq 1$

$\therefore \sec x + \tan^3 x \operatorname{cosec} x = (2-a^2)^{3/2}$

$$\Rightarrow 2 - a^2 \geq 0 \Rightarrow a^2 - 2 \leq 0 \Rightarrow -\sqrt{2} \leq a \leq \sqrt{2}$$

2) TBP $\frac{\cos^4\alpha}{\cos^2\beta} + \frac{\sin^4\alpha}{\sin^2\beta} = 1$ P.T 1) $\sin^4\alpha + \sin^4\beta = 2\sin^2\alpha \sin^2\beta$

2) $\frac{\cos^4\beta}{\cos^2\alpha} + \frac{\sin^4\beta}{\sin^2\alpha} = 1$

Sol: Given $\frac{\cos^4\alpha}{\cos^2\beta} + \frac{\sin^4\alpha}{\sin^2\beta} = 1$

$\cos^4\alpha \sin^2\beta + \sin^4\alpha \cos^2\beta = \cos^2\beta \sin^2\beta.$

$(1-\sin^2\alpha)^2 \sin^2\beta + \sin^4\alpha(1-\sin^2\beta) = (1-\sin^2\beta)\sin^2\beta.$ (change all in terms of Sin)

$(\sin^4\alpha + 1 - 2\sin^2\alpha)\sin^2\beta + \sin^4\alpha(1-\sin^2\beta) = (1-\sin^2\beta)\sin^2\beta.$

$\cancel{\sin^4\alpha \sin^2\beta} + \cancel{\sin^2\beta} - 2\sin^2\alpha \sin^2\beta + \sin^4\alpha - \cancel{\sin^4\alpha \sin^2\beta} = \cancel{\sin^2\beta} - \sin^4\beta.$

$$\sin^4\alpha + \sin^4\beta - 2\sin^2\alpha \sin^2\beta = 0$$

$$(\sin^2\alpha - \sin^2\beta)^2 = 0 \Rightarrow \sin^2\alpha = \sin^2\beta.$$

1. $\sin^4\alpha + \sin^4\beta = (\sin^2\alpha - \sin^2\beta)^2 + 2\sin^2\alpha \sin^2\beta$ $\quad a^2+b^2 = (a-b)^2 + 2ab$

$$= 2\sin^2\alpha \sin^2\beta.$$

2) $\frac{\cos^4\beta}{\cos^2\alpha} + \frac{\sin^4\beta}{\sin^2\alpha} = \frac{\cos^2\alpha \cdot \cos^2\alpha}{\cos^2\alpha} + \frac{\sin^2\beta \cdot \sin^2\alpha}{\sin^2\beta} = 1$ $\quad \because \sin^2\alpha = \sin^2\beta.$

Procedure: Just add by algebraic method and change all the terms in terms of Sin. Simplify. we get $\sin^2\alpha = \sin^2\beta$. By using this result

3) If $a = \dfrac{2\sin x}{1+\cos x+\sin x}$, then P.T $\dfrac{1-\cos x+\sin x}{1+\sin x} = a$.

TBP

Sol: $a = \dfrac{2\sin x}{1+\cos x+\sin x} = \dfrac{2\sin x}{1+\cos x+\sin x} \times \dfrac{(1-\cos x+\sin x)}{1-\cos x+\sin x}$

Procedure:
Multiplying both Nr and Dr by
$1-\cos\theta+\sin\theta$ and Simplify.

$$= \frac{2\sin x(1-\cos x+\sin x)}{[(1+\sin x)+\cos x][(1+\sin x)-\cos x]}$$

$$= \frac{2\sin x(1-\cos x+\sin x)}{(1+\sin x)^2-\cos^2 x}$$

$$= \frac{2\sin x(1-\cos x+\sin x)}{\cancel{1}+2\sin x+\sin^2 x-(\cancel{1}-\sin^2 x)}$$

$$= \frac{2\sin x(1-\cos x+\sin x)}{2\sin x+2\sin^2 x}$$

$$a = \frac{\cancel{2\sin x}(1-\cos x+\sin x)}{\cancel{2\sin x}(1+\sin x)}$$

$$a = \frac{1-\cos x+\sin x}{1+\sin x}.$$

4) If $\operatorname{cosec} x-\sin x = a^3$, $\sec x-\cos x = b^3$ P.T $a^2b^2(a^2+b^2)=1$

TBP

Given $\operatorname{cosec} x-\sin x = a^3$

$$\frac{1}{\sin x}-\sin x = a^3$$

$$\frac{1-\sin^2 x}{\sin x} = a^3$$

$$\frac{\cos^2 x}{\sin x} = a^3 \quad \text{—(1)}$$

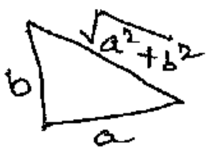
$\sec x-\cos x = b^3$

$$\frac{1}{\cos x}-\cos x = b^3$$

$$\frac{1-\cos^2 x}{\cos x} = b^3$$

$$\frac{\sin^2 x}{\cos x} = b^3 \quad \text{—(2)}$$

$\therefore \dfrac{b^3}{a^3} = \dfrac{\sin^2 x}{\cos x}\times\dfrac{\sin x}{\cos^2 x} = \tan^3 x$



$\Rightarrow \tan x = \dfrac{b}{a}$ $\quad\therefore \sin x = \dfrac{b}{\sqrt{a^2+b^2}}$

$$\cos x = \frac{a}{\sqrt{a^2+b^2}}.$$

Sub. in (1) $\dfrac{\cancel{a^2}}{(a^2+b^2)}\times\dfrac{\sqrt{a^2+b^2}}{b} = a^3 \Rightarrow a = \dfrac{1}{\sqrt{a^2+b^2}\cdot b} \Rightarrow ab = \dfrac{1}{\sqrt{a^2+b^2}}$

$a^2b^2(a^2+b^2)=1.$

5) * If $\cot x(1+\sin x) = 4m$, $\cot x(1-\sin x) = 4n$ P.T $(m^2-n^2)^2 = mn$.

TBP

Given $\cot x(1+\sin x) = 4m$

$$\frac{\cos x}{\sin x}(1+\sin x) = 4m$$

$$\cot x + \cos x = 4m \quad \text{—①}$$

$$\cot x - \cos x = 4n \quad \text{—②}$$

$①^2 - ②^2$

$$(\cot x + \cos x)^2 - (\cot x - \cos x)^2 = 16m^2 - 16n^2$$

$$\cancel{4}\cot x \cos x = \overset{4}{\cancel{16}}(m^2-n^2)$$

$$\frac{\cos^2 x}{\sin x} = 4(m^2-n^2) \quad \text{—③}$$

$① \times ②$ $(\cot x + \cos x)(\cot x - \cos x) = 16mn$

$$\cot^2 x - \cos^2 x = 16mn.$$

$$\frac{\cos^2 x}{\sin^2 x} - \cos^2 x = 16mn.$$

$$\frac{\cos^2 x - \sin^2 x \cos^2 x}{\sin^2 x} = 16mn.$$

$$\frac{\cos^2 x(1-\sin^2 x)}{\sin^2 x} = 16mn.$$

$$\Rightarrow \frac{\cos^2 x \cdot \cos^2 x}{\sin^2 x} = 16mn \Rightarrow \frac{\cos^2 x}{\sin x} = 4\sqrt{mn} \quad \text{—④}$$

From ③ and ④ $\cancel{4}(m^2-n^2) = \cancel{4}\sqrt{mn}$

$$(m^2-n^2)^2 = mn.$$

6 Eliminate θ from the equation $a\sec\theta - c\tan\theta = b$

$b\sec\theta + d\tan\theta = c$.

TBP

Sol: $a\sec\theta - c\tan\theta - b = 0$

$b\sec\theta + d\tan\theta - c = 0$ Solving for $\sec\theta$ and $\tan\theta$.

$$\begin{matrix} -c & & -b & & a & & -c \\ d & \times & -c & \times & b & \times & d \end{matrix}$$

$$\frac{\sec\theta}{c^2+bd} = \frac{\tan\theta}{ac-b^2} = \frac{1}{ad+bc}$$

$$\sec\theta = \frac{c^2+bd}{ad+bc}, \quad \tan\theta = \frac{ac-b^2}{ad+bc}$$

$$\sec^2\theta = 1+\tan^2\theta$$

$$\frac{(c^2+bd)^2}{(ad+bc)^2} = 1 + \frac{(ac-b^2)^2}{(ad+bc)^2}$$

$$(c^2+bd)^2 = (ad+bc)^2 + (ac-b^2)^2$$

7) TBP If $x = \sum_{n=0}^{\infty} \cos^{2n}\theta$, $y = \sum_{n=0}^{\infty} \sin^{2n}\theta$, $z = \sum_{n=0}^{\infty} \cos^{2n}\theta \sin^{2n}\theta$. $0 < \theta < \pi/2$ then

S.T. $xyz = x+y+z$.

Sol: $\because 1+x+x^2+x^3+\cdots = \frac{1}{1-x}$ $\quad |x|<1$.

$$x = \frac{1}{1-\cos^2 x}, \quad y = \frac{1}{1-\sin^2 x} \quad z = \frac{1}{1-\sin^2 x\cos^2 x}.$$

$$= \frac{1}{\sin^2 x} \qquad = \frac{1}{\cos^2 x} \qquad = \frac{1}{1-\sin^2 x\cos^2 x}.$$

$$xyz = \frac{1}{\sin^2 x}\cdot\frac{1}{\cos^2 x}\cdot\frac{1}{1-\sin^2 x\cos^2 x} = \frac{1}{\sin^2 x\cos^2 x(1-\sin^2 x\cos^2 x)} \quad \text{--- (1)}$$

$$x+y+z = \frac{1}{\sin^2 x} + \frac{1}{\cos^2 x} + \frac{1}{1-\sin^2 x\cos^2 x}.$$

$$= \frac{\cos^2 x(1-\sin^2 x\cos^2 x) + \sin^2 x(1-\sin^2 x\cos^2 x) + \sin^2 x\cos^2 x}{\sin^2 x\cos^2 x(1-\sin^2 x\cos^2 x)}$$

$$= \frac{\cos^2 x - \sin^2 x\cos^4 x + \sin^2 x - \sin^4 x\cos^2 x + \sin^2 x\cos^2 x}{\sin^2 x\cos^2 x(1-\sin^2 x\cos^2 x)}$$

$$= \frac{1 - \cancel{\sin^2 x\cos^2 x(\sin^2 x+\cos^2 x)} + \cancel{\sin^2 x\cos^2 x}}{\sin^2 x\cos^2 x(1-\sin^2 x\cos^2 x)}$$

$$= \frac{1}{\sin^2 x\cos^2 x(1-\sin^2 x\cos^2 x)} \quad \text{--- (2)}$$

From (1) and (2)

$$xyz = x+y+z.$$

TBP 8) If $a\cos x - b\sin x = c$ S.T $a\sin x + b\cos x = \pm\sqrt{a^2+b^2-c^2}$

Sol: Consider.

$$(a\cos x - b\sin x)^2 + (a\sin x + b\cos x)^2 = a^2\cos^2 x + b^2\sin^2 x - \cancel{2ab\sin x\cos x} + a^2\sin^2 x + b^2\cos^2 x + \cancel{2ab\sin x\cos x}$$

$$= a^2+b^2$$

$$\therefore (a\sin x + b\cos x)^2 = (a^2+b^2) - (a\cos x - b\sin x)^2$$

$$= a^2+b^2-c^2$$

$$a\sin x + b\cos x = \pm\sqrt{a^2+b^2-c^2}$$

9) TBP If $\sec x + \tan x = P$ obtain the values of $\sec x$, $\tan x$, $\sin x$ in terms of P.

Sol: we know that $\sec^2 x - \tan^2 x = 1$

$$(\sec x + \tan x)(\sec x - \tan x) = 1$$

$$p(\sec x - \tan x) = 1$$

$$\sec x - \tan x = \frac{1}{p}$$

$$(\sec x + \tan x) + (\sec x - \tan x) = p + \frac{1}{p}$$

$$(+) \quad 2\sec x = \frac{p^2+1}{p} \Rightarrow \sec x = \frac{p^2+1}{2p}.$$

$$(-) \quad 2\tan x = p - \frac{1}{p}$$

$$= \frac{p^2-1}{p} \Rightarrow \tan x = \frac{p^2-1}{2p}.$$

$$\sin x = \frac{\tan x}{\sec x} = \frac{p^2-1}{\cancel{2p}} \times \frac{\cancel{2p}}{p^2+1} = \frac{p^2-1}{p^2+1}.$$

10) If $m = a\cos^3 x + 3a\cos x \sin^2 x$, $n = a\sin^3 x + 3a\cos^2 x \sin x$. P.T $(m+n)^{2/3} + (m-n)^{2/3} = 2a^{2/3}$.

Sol: Procedure: Find $m+n$ and $m-n$. Raise the power to $2/3$ on both $m+n$ and $m-n$. Then add.

$$m+n = a\cos^3 x + 3a\cos x \sin^2 x + a\sin^3 x + 3a\cos^2 x \sin x.$$

$$= a(\cos^3 x + \sin^3 x) + 3a\sin x\cos x(\sin x + \cos x)$$

$$= a(\cos x + \sin x)^3$$

$$(m+n)^{2/3} = a^{2/3}(\cos x + \sin x)^2 \quad \text{——} ①$$

$$(m-n)^{2/3} = a^{2/3}(\cos x - \sin x)^2 \quad \text{——} ②$$

$$① + ② \quad (m+n)^{2/3} + (m-n)^{2/3} = a^{2/3}\begin{bmatrix}\cos^2 x + \sin^2 x + 2\sin x\cos x \\ + \cos^2 x + \sin^2 x - 2\sin x \cos x\end{bmatrix}$$

$$= a^{2/3}[2(\cos^2 x + \sin^2 x)]$$

$$(m+n)^{2/3} + (m-n)^{2/3} = 2a^{2/3}$$

11. If $2\tan^2\alpha\tan^2\beta\tan^2\gamma + \tan^2\alpha\tan^2\beta + \tan^2\beta\tan^2\gamma + \tan^2\alpha\tan^2\gamma = 1$. then P.T $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 1$.

Sol: Procedure: $\div$ by $\tan^2\alpha\tan^2\beta\tan^2\gamma$ and change the Trigonometric formulas and simplify $\cot^2 x = \csc^2 x - 1$

Second time $\div \csc^2\alpha\csc^2\beta\csc^2\gamma$.

$$\frac{2\tan^2\alpha\tan^2\beta\tan^2\gamma+\tan^2\alpha\tan^2\beta+\tan^2\beta\tan^2\gamma+\tan^2\alpha\tan^2\gamma}{\tan^2\alpha\tan^2\beta\tan^2\gamma}=\frac{1}{\tan^2\alpha\tan^2\beta\tan^2\gamma}$$

$$2+\frac{1}{\tan^2\gamma}+\frac{1}{\tan\alpha}+\frac{1}{\tan^2\beta}=\frac{1}{\tan^2\alpha\tan^2\beta\tan^2\gamma}.$$

$$2+\cot^2\alpha+\cot^2\beta+\cot^2\gamma=\cot^2\alpha\cot^2\gamma\cdot\cot^2\gamma.$$

$$2+(\operatorname{cosec}^2\alpha-1)+(\operatorname{cosec}^2\beta-1)+(\operatorname{cosec}^2\gamma-1)=(\operatorname{cosec}^2\alpha-1)(\operatorname{cosec}^2\beta-1)(\operatorname{cosec}^2\gamma-1)$$

$$\cancel{\operatorname{cosec}^2\alpha}+\cancel{\operatorname{cosec}^2\beta}+\cancel{\operatorname{cosec}^2\gamma}\cancel{-1}=(\operatorname{cosec}^2\alpha\operatorname{cosec}^2\beta-\operatorname{cosec}^2\alpha-\operatorname{cosec}^2\beta+1)(\operatorname{cosec}^2\gamma-1)$$

$$=\operatorname{cosec}^2\alpha\operatorname{cosec}^2\beta\operatorname{cosec}^2\gamma-\operatorname{cosec}^2\alpha\operatorname{cosec}^2\beta-\operatorname{cosec}^2\beta\operatorname{cosec}^2\gamma$$

$$-\operatorname{cosec}^2\alpha\operatorname{cosec}^2\gamma+\cancel{\operatorname{cosec}^2\alpha}+\cancel{\operatorname{cosec}^2\beta}+\cancel{\operatorname{cosec}^2\gamma}\cancel{-1}$$

$$\operatorname{cosec}^2\alpha\operatorname{cosec}^2\beta+\operatorname{cosec}^2\beta\operatorname{cosec}^2\gamma+\operatorname{cosec}^2\alpha\operatorname{cosec}^2\gamma=\operatorname{cosec}^2\alpha\operatorname{cosec}^2\beta\operatorname{cosec}^2\gamma.$$

$\div\ \operatorname{cosec}^2\alpha\operatorname{cosec}^2\beta\operatorname{cosec}^2\gamma$

$$\frac{1}{\operatorname{cosec}^2\gamma}+\frac{1}{\operatorname{cosec}^2\alpha}+\frac{1}{\operatorname{cosec}^2\beta}=1$$

$$\sin^2\alpha+\sin^2\beta+\sin^2\gamma=1.$$

12) If $m=\sin x+\cos x$, P.T $\sin^6 x+\cos^6 x=\dfrac{4-3(m^2-1)^2}{4}$.

TBP.

Sol: P.T LHS = RHS. Procedure

For LHS use $a^3+b^3=(a+b)(a^2-ab+b^2)$ and $a^2+b^2=(a+b)^2-2ab$.

RHS: Sub the m value.

$$\text{LHS}=\sin^6 x+\cos^6 x=(\sin^2 x)^3+(\cos^2 x)^3$$

$$=(\sin^2 x+\cos^2 x)\left[(\sin^2 x)^2-\sin^2 x\cos^2 x+(\cos^2 x)^2\right]$$

$$=(\sin^2 x)^2+(\cos^2 x)^2-\sin^2 x\cos^2 x.$$

$$=(\sin^2 x+\cos^2 x)^2-2\sin^2 x\cos^2 x-\sin^2 x\cos^2 x.$$

$$=1-3\sin^2 x\cos^2 x.$$

$$\text{RHS}:\ \frac{4-3\left[(\sin x+\cos x)^2-1\right]^2}{4}=\frac{4-3(\cancel{\sin^2 x+\cos^2 x}+2\sin x\cos x\cancel{-1})^2}{4}$$

$$=\frac{\cancel{4}^{1}-3\cdot\cancel{4}\sin^2 x\cos^2 x}{\cancel{4}}$$

$$=1-3\sin^2 x\cos^2 x.$$

LHS = RHS

13) If $a\cos x + b\sin x = m$ and $a\sin x - b\cos x = n$ P.T $a^2+b^2 = m^2+n^2$

Sol: Procedure: Squaring both m and n and adding.

$$m^2 = (a\cos x + b\sin x)^2 = a^2\cos^2 x + b^2\sin^2 x + 2ab\sin x\cos x.$$

$$n^2 = (a\sin x - b\cos x)^2 = a^2\sin^2 x + b^2\cos^2 x - 2ab\sin x\cos x.$$

$$m^2+n^2 = a^2(\cos^2 x + \sin^2 x) + b^2(\sin^2 x + \cos^2 x)$$

$$= a^2+b^2$$

14) If $\cos x + \sin x = \sqrt{2}\cos x$, P.T $\cos x - \sin x = \sqrt{2}\sin x$.

Sol: Procedure: Find $(\cos x + \cos x)^2 + (\cos x - \sin x)^2$ and simplify.

$$(\cos x + \sin x)^2 + (\cos x - \sin x)^2 = (\sqrt{2}\cos x)^2 + (\sqrt{2}\sin x)^2$$

$$(\sqrt{2}\cos x)^2 + (\cos x - \sin x)^2 = 2$$

$$(\cos x - \sin x)^2 = 2 - 2\cos^2 x$$

$$= 2(1-\cos^2 x)$$

$$= 2\sin^2 x.$$

$$\cos x - \sin x = \sqrt{2}\sin x.$$

15) P.T $\dfrac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \dfrac{1+\sin\theta}{\cos\theta}$

TBP.

Sol: $$\frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \frac{(\tan\theta + \sec\theta) - (\sec^2\theta - \tan^2\theta)}{\tan\theta - \sec\theta + 1}$$

$$= \frac{(\tan\theta + \sec\theta) - (\sec\theta + \tan\theta)(\sec\theta - \tan\theta)}{\tan\theta - \sec\theta + 1}$$

$$= \frac{(\tan\theta + \sec\theta)[1 - \cancel{\sec\theta + \tan\theta}]}{\cancel{\tan\theta - \sec\theta + 1}}$$

$$= \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta} \Rightarrow = \frac{1+\sin\theta}{\cos\theta}.$$

16. P.T $(\sec A - \operatorname{cosec} A)(1 + \tan A + \cot A) = \tan A\sec A - \cot A\operatorname{cosec} A$.

Sol: LHS: $(\sec A - \operatorname{cosec} A)(1 + \tan A + \cot A)$

$$= \left(\frac{1}{\cos A} - \frac{1}{\sin A}\right)\left(1 + \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}\right)$$

$$= \left(\frac{\sin A - \cos A}{\sin A\cos A}\right)\left(\frac{\sin A\cos A + \sin^2 A + \cos^2 A}{\sin A\cos A}\right)$$

$$= \frac{\sin^3 A - \cos^3 A}{\sin^2 A\cos^2 A}.$$

RHS: $\dfrac{\sin A}{\cos A}\cdot\dfrac{1}{\cos A} - \dfrac{\cos A}{\sin A}\cdot\dfrac{1}{\sin A}$.

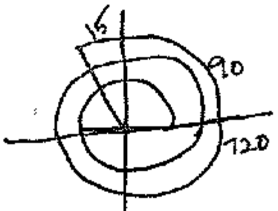
$$= \frac{\sin^3 A - \cos^3 A}{\sin^2 A\cos^2 A}$$

$\therefore$ LHS = RHS.

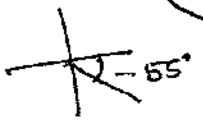
17) Identify the quadrant in which an angle of each given measure lies.

KBP. 1) 25° — I quadrant

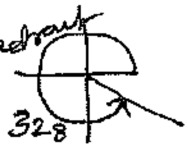
2) $\cancel{82}25$ II quadrant



3) -55° IV quadrant



4) 328° – IV quadrant



5) -230 – II quadrant.

18) For each given angle, Find the co-terminal angle with measure

KBP of θ $\quad 0 \le \theta \le 360^\circ$

1) $(395^\circ, 35^\circ)$ $\quad \therefore 35 - 395^\circ = -360$
$= -1(360)$ $\quad \because -1$ is integer

2) $(525, 885)$ $\quad \therefore 885 - 525 = 360^\circ$
$2230 - 1150$

3) $\therefore$

$(1150, 70)$ $\quad 70 - 1150 = -1080$
$= -3(360)$

4) $(-270, 90)$ $\quad 90 + 270 = 360.$

5) $(-450, 270)$ $\quad 270 + 450 = 720 = 2(360)$

19) If $a = \sec x - \tan x$, $b = \operatorname{cosec} x + \cot x$ S.T $ab + a - b + 1 = 0$

Sol: Procedure: Find ab, and $a-b$ separately and ab in $a-b$ we will get the answer.

$ab = (\operatorname{cosec} x + \cot x)(\sec x - \tan x)$

$= \sec x \operatorname{cosec} x + \sec x \cot x - \tan x \operatorname{cosec} x - \tan x \cot x$,

$= \sec x \operatorname{cosec} x + \frac{1}{\sin x} - \frac{1}{\cos x} - 1$

$= \sec x \operatorname{cosec} x + \operatorname{cosec} x - \sec x - 1$

$1 + ab = \sec x \operatorname{cosec} x + \operatorname{cosec} x - \sec x$ ——— (1)

$a - b = \sec x - \tan x - \operatorname{cosec} x - \cot x$

$= \sec x - \operatorname{cosec} x - \left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right) = \sec x - \operatorname{cosec} x - \left(\frac{\sin^2 x + \cos^2 x}{\sin x \cos x}\right)$

$= \sec x - \operatorname{cosec} x - \sec x \operatorname{cosec} x$

$a - b = -(1 + ab) \Rightarrow ab + a - b + 1 = 0$

20) If $\tan x = \frac{b}{a}$ then find the value of $\sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}}$.

Sol: Procedure. Dividing both Nr and Dr by a and sub. $\tan x = \frac{b}{a}$ and simplify.

$$\sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}} = \sqrt{\frac{1+\frac{b}{a}}{1-b/a}} + \sqrt{\frac{1-b/a}{1+b/a}}.$$

$$= \frac{\sqrt{1+\tan x}}{\sqrt{1-\tan x}} + \frac{\sqrt{1-\tan x}}{\sqrt{1+\tan x}}.$$

$$= \frac{1+\tan x + 1 - \tan x}{\sqrt{1-\tan x}\sqrt{1+\tan x}} = \frac{2}{\sqrt{1-\tan^2 x}}.$$

$$= \frac{2}{\sqrt{1-\frac{\sin x^2}{\cos^2 x}}}$$

$$= \frac{2\cos x}{\sqrt{\cos^2 x - \sin^2 x}}.$$

21) If $a\cos x + b\sin x = m$ and $a\sin x - b\cos x = n$ P.T $a^2+b^2 = m^2+n^2$

Sol: Procedure: Find m^2 and n^2, add and simplify.

$$m^2 = (a\cos x + b\sin x)^2 = a^2\cos^2 x + b^2\sin^2 x + 2ab\sin x\cos x.$$

$$n^2 = (a\sin x - b\cos x)^2 = a^2\sin^2 x + b^2\cos^2 x - 2ab\sin x\cos x.$$

$$m^2+n^2 = a^2(\sin^2 x + \cos^2 x) + b^2(\sin^2 x + \cos^2 x)$$

$$= a^2 + b^2.$$

22) If $m = \tan x + \sin x$, $n = \tan x - \sin x$ S.T $m^2 - n^2 = 4\sqrt{mn}$.

Sol: Procedure: Find both m^2-n^2 and $4\sqrt{mn}$ separately and show that they are equal.

$$m^2 - n^2 = (\tan x + \sin x)^2 - (\tan x - \sin x)^2 = 4\tan x\sin x$$

$\because (a+b)^2 - (a-b)^2 = 4ab$

$1+\tan^2 x = \sec^2 x$.

$$4\sqrt{mn} = 4\sqrt{(\tan x + \sin x)(\tan x - \sin x)}$$

$$= 4\sqrt{\tan^2 x - \sin^2 x}.$$

$$= 4\sqrt{\frac{\sin^2 x}{\cos^2 x} - \sin^2 x}. = 4\sin x\sqrt{\frac{1}{\cos^2 x} - 1}$$

$$= 4\sin x\sqrt{\sec^2 x - 1}$$

$$= 4\sin x\tan x$$

$$\therefore m^2 - n^2 = 4\sqrt{mn}.$$

P.T. 23) $\left|\sqrt{\frac{1-\sin x}{1+\sin x}}+\sqrt{\frac{1+\sin x}{1-\sin x}}\right| = -\frac{2}{\cos x}$ $\quad \pi/2 < x < \pi$

Sol: Just simplify and put the sign for II quadrant

$\because \pi/2 < x < \pi$.

$$\frac{\sqrt{1-\sin x}}{\sqrt{1+\sin x}}+\frac{\sqrt{1+\sin x}}{\sqrt{1-\sin x}} = \frac{1-\sin x+1+\sin x}{\sqrt{1+\sin x}\sqrt{1-\sin x}}$$

$$= \frac{2}{\sqrt{1-\sin^2 x}}.$$

$$= \frac{2}{\cos x} \text{ or } 2\sec x.$$

$$= -\frac{2}{\cos x}.$$

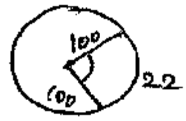
$\because \sec x = -ve$ in II quadrant

24) Find the degree measure of the angle subtended at a centre of a circle of radius 100 cm by an arc of length 22 cm as shown in fig: ($\pi = \frac{22}{7}$)

BP

Sol: $l = r\theta$

$\theta = \frac{l}{r} = \frac{22}{100} = .22^c$



$= .22 \times \frac{180}{\pi}$ deg. $\Rightarrow \cancel{.22}^{.01} \times \frac{180}{\cancel{22}} \times 7 = 12.60$

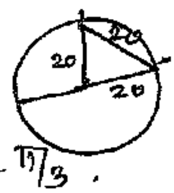
$= 12 + \frac{6}{10} \times 60$

$= 12^\circ 36'$

25) In a circular diameter 40 cm a chord of length 20 cm. Find the length of the minor arc of the chord.

BP

Sol: Since radius $= 20$
and length of the chord $= 20$.



$\therefore$ The $\triangle$le is equilateral $\triangle$le

$\therefore \theta = 60^\circ = 60 \times \frac{\pi}{180_3} = \pi/3$.

Length of the arc $= r\theta$

$= 20 \times \frac{\pi}{3} = 20 \times \frac{22}{7 \times 3} = \frac{440}{21}$ cm.

Relation between degree and Radian measure.

26 convert i) 18° ii) -108° into radians.

TBP:

Sol: $180^\circ = \pi^c$,

$1^\circ = \pi^c/180$

1) $18^\circ = \frac{\pi}{\cancel{180}_{10}} \times \cancel{18} = \frac{\pi}{10}$ radian.

2) $-108^\circ = \frac{\pi}{\cancel{180}_{5}} \times (-\cancel{108}^{3}) = -\frac{3\pi}{5}$

27 Convert 1) 30° 2) 135° 3) -225° 4) 150° 5) 330° into radian.

TBP

Sol: $180^\circ = \pi^c \Rightarrow 1^\circ = \frac{\pi^c}{180}$

1) $30^\circ = \frac{\pi}{18\cancel{0}} \times 3\cancel{0} = \frac{\pi}{6}$

2) $135^\circ = \frac{\pi}{\cancel{180}_{4}} \times \cancel{135}^{3} = \frac{3\pi}{4}$

3) $-205 = \frac{\pi}{\cancel{180}_{4}} \times (-\cancel{225}^{5}) = -\frac{5\pi}{4}$

4) $150^\circ = \frac{\pi}{\cancel{180}_{6}} \times \cancel{150}^{5} = \frac{5\pi}{6}$

5) $330^\circ = \frac{\pi}{\cancel{180}_{6}} \times \cancel{330}^{11} = \frac{11\pi}{6}$.

28) change into degree measure.

TBP: 1) $\frac{\pi}{5}^c$ 2) 6 radians.

Sol: 1) $\pi^c = 180^\circ$

$1^c = \frac{180}{\pi}$

$\frac{\pi}{5} = \frac{\cancel{180}^{36}}{\cancel{\pi}} \times \frac{\cancel{\pi}}{\cancel{5}} = 36^\circ$

2) $1^c = \frac{180}{\pi} = \frac{180 \times 7}{22}^\circ$

$6^c = \left(\frac{180 \times 7}{\cancel{22}_{11}} \times \cancel{6}^{3}\right)^\circ$

$= \left(343\frac{7}{11}\right)^\circ$

29) Change into degree measure

TBP 1) $\frac{\pi}{3}$, 2) $\frac{\pi}{9}$ 3) $\frac{2\pi}{5}$ 4) $\frac{7\pi}{3}$ 5) $\frac{10\pi}{9}$.

Sol; $1^c = \frac{\pi}{180}$, $\frac{180^\circ}{\pi}$

i) $\frac{\pi}{3}^c = \frac{180}{\cancel{\pi}} \times \frac{\cancel{\pi}}{3} = 60^\circ$

2) $\frac{\pi}{9} = \frac{180}{\cancel{\pi}} \times \frac{\cancel{\pi}}{9} = 20^\circ$

3) $\frac{2\pi}{5} = \frac{\cancel{180}^{36}}{\cancel{\pi}} \times \frac{2\cancel{\pi}}{\cancel{5}} = 72^\circ$

4) $\frac{10\pi}{9} = \frac{\cancel{180}^{20}}{\cancel{\pi}} \times \frac{10\cancel{\pi}}{\cancel{9}} = 200^\circ$.

30) What is the length of the arc intercepted by a central angle of measure 41° in a circle of radius 10 ft?
FBP

Sol: $\theta = 41^\circ = \frac{\pi}{180^\circ} \times 41^c$

$r = 41$

$\therefore l = r\theta = \frac{\pi}{180} \times 41 = \frac{22 \times 41}{7 \times 180} = \frac{902}{1260}$ ft. $\frac{\overset{82}{\overset{82}{902}}}{1260}$

31) If the arc of the same length in two circles subtend angles 60° and $\cancel{110}$ 75° at the centre. Find the ratio of their radii.
FBP

Sol: $60^\circ = \frac{\pi}{180_3} \times 60 = \pi/3$.

$75^\circ = \frac{\pi}{180} \times 75 = \frac{5\pi}{12}$

$\theta_1 = \frac{l}{r_1} = \cancel{l} = r_1\theta_1 \qquad l = r_2\theta_2$

$\therefore r_1\theta_1 = r_2\theta_2$

$\frac{r_1}{r_2} = \frac{\theta_2}{\theta_1} = \cancel{\frac{\pi}{3} \times \frac{12 4}{5\pi}} = \frac{5\pi}{12_4} \times \frac{3}{\pi}$

$r_1 : r_2 = \cancel{4:5} \quad 5:4.$

32) If the arc of the same length in two circles subtended angles 65° and 110° at the centre. Find their ratio of their radii.

33) The perimeter of a certain sector of a circle is equal to the length of the arc of a semi circle having the same radius. Express the angle of the sector in degrees and seconds.
FBP.

Sol: Perimeter of the sector $= 2r + r\theta$.

length of the semicircle $= \pi r$

Given $\quad 2r + r\theta = \pi r$

$\cancel{r}(2+\theta) = \pi \cancel{r}$

$2 + \theta = \pi$

$\theta = \pi - 2$ rad.

$= \frac{180}{\pi} \times (\pi - 2)$

$= \frac{180 \times \cancel{\pi}}{\cancel{\pi}} - \frac{180 \times 2}{\pi}$

$\theta = 180 - \frac{360^\circ}{\pi} = 180 - 114^\circ 32' 44''$

$= 65^\circ 27' 16''.$

34) What must be the radius of circular running path around.
TBP Which an athelete must run 5 times in order to describe 5 KM.

Sol: For 5 rounds = 5 Km
$= 5000$ m.
1 round $= 1000$ m.
$2\pi r = 1000$ m.
$r = \frac{\cancel{1000}^{\cancel{500}\,250} \times 7}{\cancel{2} \times \cancel{22}_{11}} = \frac{1750}{11} = 158^{\circ}18'$

35) Find the length of an arc of a circle of radius 5 cm.
TBP. Subtending a central angles 15°

Sol: $l = r\theta$. $= 5 \times \frac{\pi}{12}$ cm.	$\theta = 15^{\circ} = \cancel{15} \times \frac{\pi}{\cancel{180}\,12}$ $r = 5$ cm.

36) If the arc of the same length in two circles subtend central
TBP angles 30° and 80° find the ratio of their radii.

Sol: $\theta_1 = 30^{\circ} = \frac{\pi}{6}$
$\theta_2 = 80^{\circ} = \frac{\pi}{\cancel{180}\,9} \times \cancel{80}^{4} = \frac{4\pi}{9}$

$l = r_1\theta_1 = r_2\theta_2$
$\frac{r_1}{r_2} = \frac{\theta_2}{\theta_1}$
$= \frac{4\cancel{\pi}}{\cancel{9}\,3} \times \frac{\cancel{6}\,2}{\cancel{\pi}}$
$= \frac{8}{3}$

$\therefore r_1 : r_2 = 8 : 3$.

37) What must be the radius of a circular running path
TBP. around which an athlete must run 5 times in order to describe 1 km.

Sol: For 5 rounds = 1 KM = 1000 m.
For 1 round = 200 m.
$\cancel{2}\pi r = \cancel{2}00$ m
$r = \frac{100 \times 7}{22} = \frac{700}{22} = 31^{\circ}49'5''$.

$\frac{\cancel{18}^{9} \times 60^{\circ}}{\cancel{22}\,11}$ $\frac{540}{11}$ 49
$\frac{60}{11}$

40) A Train is moving on a circular track of 1500 m radius at the rate of 66 KM/hr. What angle will it turn in 20 seconds.
TB.P.

$r = 1500$ m.

in 20 seconds $= \frac{66000}{3600} \times 20 = \frac{3300}{9}$ ✓

~~mean~~ length $l = \frac{3300}{9}$

$\theta = \frac{l}{r} = \frac{3300}{9 \times 1500} = \frac{11}{45}$

41) If $\sec\theta + \tan\theta = P$ obtain the values of $\sec\theta$, $\tan\theta$ and $\sin\theta$ interms of P.
TBP.

Sol: Given $\sec\theta + \tan\theta = P \longrightarrow (1)$

we know $\sec^2\theta - \tan^2\theta = 1$

$(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

$P(\sec\theta - \tan\theta) = 1$

$\sec\theta - \tan\theta = 1/P \longrightarrow (2)$

(1)+(2) $2\sec\theta = P + \frac{1}{P}$

$= \frac{P^2+1}{P}$

$\sec\theta = \frac{P^2+1}{2P}$

(1)−(2) $2\tan\theta = P - \frac{1}{P}$

$= \frac{P^2-1}{P}$

$\tan\theta = \frac{P^2-1}{2P}$

$\frac{\tan\theta}{\sec\theta} = \frac{P^2-1}{P^2+1}$

$\frac{\sin\theta}{\cos\theta \cdot \cancel{\sec\theta}} = \frac{P^2-1}{P^2+1} \Rightarrow \sin\theta = \frac{P^2-1}{P^2+1}$

P.T $3(\sin x - \cos x)^4 + 6(\sin x + \cos x)^2 + 4(\sin^6 x + \cos^6 x) = 13.$

Sol: 1) $(\sin x - \cos x)^4 = \left[(\sin x - \cos x)^2\right]^2$

$= (\sin^2 x + \cos^2 x - 2\sin x\cos x)^2$

$= (1 - 2\sin x\cos x)^2$

$= 1 + 4\sin^2 x\cos^2 x - 4\sin x\cos x.$

$(\sin x + \cos x)^2 = \sin^2 x + \cos^2 x + 2\sin x\cos x$

$= 1 + 2\sin x\cos x.$

$\sin^6 x + \cos^6 x = (\sin^2 x)^3 + (\cos^2 x)^3$

$= (\sin^2 x + \cos^2 x)^3 - 3\sin^2 x\cos^2 x(\sin^2 x + \cos^2 x)$

$= 1 - 3\sin^2 x\cos^2 x.$

$\therefore$ LHS $= 3(1 + 4\sin^2 x\cos^2 x - 4\sin x\cos x) + 6(1 + 2\sin x\cos x)$
$+ 4(1 - 3\sin^2 x\cos^2 x)$

$= 3 + 12\cancel{\sin^2 x\cos^2 x} - 12\cancel{\sin x\cos x} + 6 + 12\cancel{\sin x\cos x}$
$+ 4 - 12\cancel{\sin^2 x\cos^2 x}$

$= 3 + 6 + 4$

$= 13$

2) If $\tan\theta + \sin\theta = p$, $\tan\theta - \sin\theta = q$, $p > q$ P.T $p^2 - q^2 = 4\sqrt{pq}$

Sol: $p^2 = (\tan\theta + \sin\theta)^2$

$= \tan^2\theta + \sin^2\theta + 2\tan\theta\sin\theta$

$q^2 = (\tan\theta - \sin\theta)^2$

$= \tan^2\theta + \sin^2\theta - 2\sin\theta\tan\theta.$

$p^2 - q^2 = 4\tan\theta\sin\theta$ ——— (1)

$pq = (\tan\theta + \sin\theta)(\tan\theta - \sin\theta)$

$= \tan^2\theta - \sin^2\theta$

$= \frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta = \sin^2\theta\left(\frac{1}{\cos^2\theta} - 1\right)$

$\sin^2\theta(\sec^2\theta - 1)$

$\sqrt{pq} = \sin\theta\tan\theta$

$4\sqrt{pq} = 4\sin\theta\tan\theta$ — (2)

From (1) and (2)

$p^2 - q^2 = 4\sqrt{pq}.$

$\sqrt{pq} = \sin\theta \tan\theta$

$4\sqrt{pq} = 4\tan\theta\sin\theta$ —— ②

$\therefore \quad p^2 - q^2 = 4\sqrt{pq}.$

3) P.T $\dfrac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \dfrac{1+\sin\theta}{\cos\theta}$

<u>LHS</u> $\dfrac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \dfrac{(\tan\theta + \sec\theta) - (\sec^2\theta - \tan^2\theta)}{\tan\theta - \sec\theta + 1}$

$= \dfrac{(\tan\theta + \sec\theta) - (\sec\theta + \tan\theta)(\sec\theta - \tan\theta)}{\tan\theta - \sec\theta + 1}$

$= \dfrac{(\tan\theta + \sec\theta)(1 - \cancel{\sec\theta + \tan\theta})}{(\cancel{\tan\theta - \sec\theta} + 1)}$

$= \tan\theta + \sec\theta$

$= \dfrac{\sin\theta}{\cos\theta} + \dfrac{1}{\cos\theta}$

$= \dfrac{1+\sin\theta}{\cos\theta}.$

4) S.T $\sin^2 A \tan A + \cos^2 A \cot A + 2\sin A\cos A = \tan A + \cot A.$

<u>LHS.</u> $\sin^2 A \tan A + \cos^2 A \cot A + 2\sin A \cos A.$

$= \sin^2 A \dfrac{\sin A}{\cos A} + \cos^2 A \dfrac{\cos A}{\sin A} + 2\sin A\cos A.$

$= \dfrac{\sin^4 A + \cos^4 A + 2\sin^2 A\cos^2 A}{\sin A\cos A}$

$= \dfrac{(\sin^2 A + \cos^2 A)^2}{\sin A\cos A} = \dfrac{1}{\sin A\cos A}. = \dfrac{\sin^2 A + \cos^2 A}{\sin A\cos A}$

$= \dfrac{\sin A}{\cos A} + \dfrac{\cos A}{\sin A}$

$= \tan A + \cot A$

5. If $\tan\theta + \sec\theta = x$; P.T $2\tan\theta = x - \frac{1}{x}$, P.T $2\sec\theta = x + \frac{1}{x}$

$$\sin\theta : \frac{x^2-1}{x^2+1}.$$

Sol: $x = \tan\theta + \sec\theta$

$$\frac{1}{x} = \frac{1}{\tan\theta+\sec\theta} : \frac{1}{\tan\theta+\sec\theta} \times \frac{\tan\theta-\sec\theta}{\tan\theta-\sec\theta}.$$

$$= \frac{\tan\theta - \sec\theta}{\tan^2\theta - \sec^2\theta}$$

$$= \frac{\tan\theta - \sec\theta}{-1}$$

$$= \sec\theta - \tan\theta.$$

$\therefore\ x - \frac{1}{x} = \tan\theta + \cancel{\sec\theta} - \cancel{\sec\theta} + \tan\theta$

$= 2\tan\theta.$

$x + \frac{1}{x} = \cancel{\tan\theta} + \sec\theta + \sec\theta - \cancel{\tan\theta}$

$= 2\sec\theta.$

$$\frac{\cancel{2}\tan\theta}{\cancel{2}\sec\theta} = \frac{x-\frac{1}{x}}{x+\frac{1}{x}} = \frac{\frac{x^2-1}{x}}{\frac{x^2+1}{x}}$$

$$\frac{\sin\theta}{\cancel{\cos\theta}} \times \cancel{\cos\theta} = \frac{x^2-1}{x^2+1}$$

$$\sin\theta = \frac{x^2-1}{x^2+1}$$

6) P.T $(1+\tan A + \sec A)(1+\cot A - \operatorname{cosec} A) = 2.$

LHS. $(1+\tan A + \sec A)(1+\cot A - \operatorname{cosec} A)$

$= \left(1 + \frac{\sin A}{\cos A} + \frac{1}{\cos A}\right)\left(1 + \frac{\cos A}{\sin A} - \frac{1}{\sin A}\right)$

$= (\cos A + \sin A + 1)(\sin A + \cos A - 1)$

$$\frac{(\cos A+\sin A+1)(\cos A+\sin A-1)}{\sin A\cos A}$$

$$=\frac{(\cos A+\sin A)^2-1}{\sin A\cos A.} = \frac{\cancel{\cos^2 A+\sin^2 A}+2\sin A\cos A\cancel{-1}}{\sin A\cos A}$$

$$=\frac{2\cancel{\sin A\cos A}}{\cancel{\sin A\cos A}}$$

$$=2$$

7) P.T $(1+\cot A+\tan A)(\sin A-\cos A)=\frac{\sec A}{\operatorname{cosec}^2 A}-\frac{\operatorname{cosec} A}{\sec^2 A}.$

LHS: $(1+\cot A+\tan A)(\sin A-\cos A)$

$$=\left(1+\frac{\cos A}{\sin A}+\frac{\sin A}{\cos A}\right)(\sin A-\cos A)$$

$$=\frac{(\sin A\cos A+\cos^2 A+\sin^2 A)}{\sin A\cos A}(\sin A-\cos A)$$

$$=\sin^2 A\cos A+\sin A\cos^2 A+\sin^3 A-\sin A\cos^2 A-\cos^3 A-\sin^2 A\cos A$$

$$=\cancel{\sin A}+\cancel{\cos A}+\frac{\sin^2 A}{\cos A}-\cancel{\cos A}-\frac{\cos^2 A}{\sin A}-\cancel{\sin A}$$

$$=\frac{\sin^2 A}{\cos A}-\frac{\cos^2 A}{\sin A}$$

$$=\frac{\sec A}{\operatorname{cosec}^2 A}-\frac{\operatorname{cosec} A}{\sec^2 A}.$$

8) Find the value of $\sin 18^\circ$

TBP let $A=18^\circ \Rightarrow 5A=90^\circ$

$$2A+3A=90^\circ$$

$$2A=90-3A$$

$$\sin 2A=\sin(90-3A)$$

$$=\cos 3A$$

$$2\sin A\cos A=4\cos^3 A-3\cos A$$

2

$$2\sin A=4(1-\sin^2 A)-3$$

$$=4-4\sin^2 A-3$$

$$=1-4\sin^2 A$$

$4\sin^2 A+2\sin A-1=0.$ $a=4$, $b=2$, $c=-1$

Quadratic in $\sin A$

$$\sin A=\frac{-2\pm\sqrt{4+16}}{8}$$

$$=\frac{-2\pm 2\sqrt5}{8}=\frac{-1\pm\sqrt5}{4}$$

Extras Either one mark or Two marks.

1. Change 150° into radian measure

$180^\circ = \pi^c$

$150^\circ = \frac{\pi}{\cancel{180}_6} \times \cancel{150}^5 = \frac{5\pi}{6}$.

2. Change $\frac{3\pi}{4}$ into degree measure

π radian $= 180^\circ$

$\frac{3\pi}{4} = \frac{\cancel{180}^{45}}{\cancel{\pi}} \times \frac{3\cancel{\pi}}{\cancel{4}}$

$= 135^\circ$

3. Change $\frac{1}{4}$ into degree measure.

$= \frac{1}{4} \times \frac{180}{\pi} = \frac{1}{4} \times \frac{180}{22} \times 7$

$= 14^\circ 19' 5''$

4. Express in which quadrant it appears.

1. $380 = 360 + 20$, I quadrant
2. $-140 = -(90+50)$, III quadrant
3. $1100^\circ = 3 \times 360 + 20$, I quadrant

5) 1. $\tan 735^\circ = \tan(2 \times 360 + 15) = \tan 15^\circ$

2. $\cos 980 = \cos(2 \times 360 + 260) = \cos 260$
$= \cos(270 - 10) = -\sin 10^\circ$

3. $\sin(2460) = \sin(6 \times 360 + 300)$
$= \sin 300$
$= \sin(360 - 60) = -\sin 60^\circ$
$= -\sqrt{3}/2$

4) $\cos(-870) = \cos 870$
$= \cos(2 \times 360 + 150)$
$= \cos 150$
$= \cos(180 - 30) = -\cos 30^\circ$
$= -\sqrt{3}/2$

5. $\sin(-780) = -\sin(780)$

6. $\cot(-855) = -\cot 885$

$= -\cot(2\times 360^\circ + 135)$

$= -\cot 135^\circ$

$= -\cot(180-45)$

$= \cot 45$

$= 1.$

7. $\operatorname{cosec}(2040) = \operatorname{cosec}(5\times 360 + 240)$

$= \operatorname{cosec} 240$

$= \operatorname{cosec}(180+60^\circ)$

$= -\operatorname{cosec} 60^\circ$

$= -2/\sqrt{3}$

8: $\sec(-1305) = \sec(1305)$

$= \sec(3\times 360 + 225)$

$= \sec(225)$

$= \sec(180+45)$

$= -\sec 45'$

$= -\sqrt{2}$

---

6. P.T $\dfrac{\sin 300 \tan 330 \sec 420}{\cot 135^\circ \cos 210^\circ \operatorname{cosec} 315^\circ} = -\sqrt{\dfrac{2}{3}}.$

$\sin 300 = \sin(360-60)$
$= -\sin 60$
$= -\sqrt{3}/2$

$\tan 330 = \tan(360-30)$
$= -\tan 30$
$= -1/\sqrt{3}$

$\sec 420 = \sec(360+60)$
$= \sec 60^\circ$
$= 2$

$\cot 135^\circ = \cot(180-45)$
$= -\cot 45$
$= -1$

$\cos 210 = \cos(180+30)$
$= -\cos 30'$
$= -\sqrt{3}/2$

LHS $\dfrac{\sin 300 \tan 330 \sec 420}{\cot 135 \cos 210 \operatorname{cosec} 315}$

$= \dfrac{\left(-\cancel{\frac{\sqrt{3}}{2}}\right)\left(-\frac{1}{\sqrt{3}}\right)\left(\cancel{2}\right)}{\left(\cancel{-1}\right)\left(-\cancel{\sqrt{3}/2}\right)\left(\cancel{-}\sqrt{2}\right)}$

$= -\dfrac{\sqrt{2}}{\sqrt{3}}.$

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7) Express as a function of A $\sec\left(A - \frac{3\pi}{2}\right)$

$\sec\left(-\left(\frac{3\pi}{2} - A\right)\right) = \sec\left(\frac{3\pi}{2} - A\right)$

8) TBP P.T $\dfrac{\sin(180+A)\cos(90-A)\tan(270-A)}{\sec(540-A)\cos(360+A)\operatorname{cosec}(270+A)} = -\sin A\cos^2 A.$

$\sin(180+A) = -\sin A$, $\cos(90-A) = \sin A$, $\tan(270-A) = \cot A$.

$\sec(540-A) = \sec(180-A) = -\sec A$, $\cos(360+A) = \cos A$, $\operatorname{cosec}(270+A) = -\sec A$.

LHS: $\dfrac{\sin(180+A)\cos(90-A)\tan(270-A)}{\sec(540-A)\cos(360+A)\operatorname{cosec}(270+A)}$

$= \dfrac{(-\sin A)(\sin A)(\cot A)}{(-\sec A)(\cos A)(-\sec A)}$

$= \dfrac{(-\sin A)(\cancel{\sin A})(\cos A)}{\left(-\frac{1}{\cancel{\cos A}}\right)(\cancel{\cos A})\left(-\frac{1}{\cos A}\right)(\cancel{\sin A})}$

$= -\sin A\cos^2 A$

9) Find the value of $\frac{1}{2}\sin^2 60 - \frac{1}{2}\sec 60\tan^2 30 + \frac{4}{5}\sin^2 45\tan^2 60^\circ$

Sol: $\frac{1}{2}\sin^2 60 - \frac{1}{2}\sec 60\tan^2 30 + \frac{4}{5}\sin^2 45\tan^2 60^\circ$

$= \frac{1}{2}\left(\frac{\sqrt{3}}{2}\right)^2 - \frac{1}{\cancel{2}}\cdot\cancel{2}\left(\frac{1}{\sqrt{3}}\right)^2 + \frac{4}{5}\cdot\frac{1}{2}\cdot 3$

$= \frac{3}{8} - \frac{1}{3} + \frac{12}{10} = \frac{45 - 40 + 144}{120}$

$= \frac{149}{120}$

10) P.T $\sin^4 A - \cos^4 A = 1 - 2\cos^2 A$

LHS: $\sin^4 A - \cos^4 A$

$= (\sin^2 A)^2 - (\cos^2 A)^2$

$= (\sin^2 A + \cos^2 A)(\sin^2 A - \cos^2 A)$

$= \sin^2 A - \cos^2 A$

$= 1 - \cos^2 A - \cos^2 A$

) $\sin^3 A - \cos^3 A = (\sin A - \cos A)(1 + \sin A \cos A)$

LHS $\sin^3 A - \cos^3 A = (\sin A - \cos A)(\sin^2 A + \sin A \cos A + \cos^2 A)$ $a^3 - b^3 = (a-b)(a^2+ab+b^2)$

$= (\sin A - \cos A)(1 + \sin A \cos A)$.

12) $(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2 = 2$

LHS: $(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2$

$= \sin^2\theta + \cos^2\theta + \cancel{2\sin\theta\cos\theta} + \sin^2\theta + \cos^2\theta - \cancel{2\sin\theta\cos\theta}$

$= 2(\sin^2\theta + \cos^2\theta)$

$= 2$

13) $(\tan\theta + \cot\theta)^2 = \sec^2\theta + \operatorname{cosec}^2\theta$

LHS: $(\tan\theta + \cot\theta)^2$

$= \tan^2\theta + \cot^2\theta + 2\cancel{\tan\theta}\cancel{\cot\theta}$

$= \tan^2\theta + \cot^2\theta + 2$

$= 1 + \tan^2\theta + 1 + \cot^2\theta$

$= \sec^2\theta + \operatorname{cosec}^2\theta$.

14) $\dfrac{1}{1+\sin\theta} + \dfrac{1}{1-\sin\theta} = 2\sec^2\theta$.

LHS: $\dfrac{1}{1+\sin\theta} + \dfrac{1}{1-\sin\theta}$

$= \dfrac{1 - \cancel{\sin\theta} + 1 + \cancel{\sin\theta}}{(1+\sin\theta)(1-\sin\theta)} = \dfrac{2}{1-\sin^2\theta}$

$= \dfrac{2}{\cos^2\theta} = 2\sec^2\theta$.

15) $\dfrac{\sec x + \tan x}{\sec x - \tan x} = (\sec x + \tan x)^2$

LHS: $\dfrac{\sec x + \tan x}{\sec x - \tan x} \times \dfrac{\sec x + \tan x}{\sec x + \tan x}$.

$= \dfrac{(\sec x + \tan x)^2}{\sec^2 x - \tan^2 x}$

$\because \sec^2 x - \tan^2 x = 1$

2

16) $\frac{1}{\tan\theta+\sec\theta} = \sec\theta - \tan\theta$.

LHS: $\frac{1}{\tan\theta+\sec x} \times \frac{\tan\theta-\sec\theta}{\tan\theta-\sec\theta}$

$= \frac{\tan\theta - \sec\theta}{\tan^2\theta - \sec^2\theta} = \frac{\tan\theta - \sec\theta}{-1}$

$= \sec\theta - \tan\theta$.

17) $\frac{\operatorname{cosec}\theta}{\cot\theta + \tan\theta} = \cos\theta$

LHS: $\frac{\operatorname{cosec}\theta}{\cot\theta+\tan\theta} = \frac{\operatorname{cosec}\theta}{\frac{\cos\theta}{\sin\theta}+\frac{\sin\theta}{\cos\theta}}$

$= \frac{\operatorname{cosec}\theta}{\frac{\cos^2\theta+\sin^2\theta}{\sin\theta\cos\theta}}$

$= \cancel{\operatorname{cosec}\theta} \times \cancel{\sin\theta}\cdot\cos\theta$

$= \cos\theta$.

18. $\frac{1}{\operatorname{cosec}\theta - \cot\theta} = \frac{1+\cos\theta}{\sin\theta}$.

LHS: $\frac{1}{\operatorname{cosec}\theta - \cot\theta} =$

$= \frac{1}{\operatorname{cosec}\theta - \cot\theta} \times \frac{\operatorname{cosec}\theta + \cot\theta}{\operatorname{cosec}\theta+\cot\theta}$

$= \frac{\operatorname{cosec}\theta + \cot\theta}{\operatorname{cosec}^2\theta - \cot^2\theta} = \operatorname{cosec}\theta + \cot\theta$

$= \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}$.

$= \frac{1+\cos\theta}{\sin\theta}$.

19. $(\sec\theta + \cos\theta)(\sec\theta - \cos\theta) = \tan^2\theta + \sin^2\theta$.

LHS: $(\sec\theta+\cos\theta)(\sec\theta-\cos\theta)$

$= \sec^2\theta - \cos^2\theta$

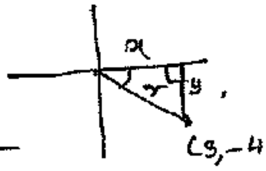
$= (\cancel{1}+\tan^2\theta) - (\cancel{1}-\sin^2\theta) = \tan^2\theta + \sin^2\theta$.

Trignometric functions and their properties.

1) The terminal side of an angle θ in standard position passes through the point $(3,-4)$. Find the trigonometric functions.

TBP

Sol: $r = \sqrt{9+16} = 5$



$\sin\theta = \frac{y}{x} = \frac{4}{5}$ $\therefore \theta$ is in IV $\quad \sin\theta = -4/5$

$\cos\theta = \frac{x}{r} = \frac{3}{5}$ $\quad$ " $\quad \cos\theta = 3/5$

$\tan\theta = \frac{y}{x} = \frac{4}{3}$ $\quad$ " $\quad \tan\theta = -4/3$

$\therefore \operatorname{cosec}\theta = -5/4$

$\sec\theta = 5/3$

$\cot\theta = -3/4.$

2) If $\sin\theta = 3/5$ and the angle θ is II quadrant find the value of other functions.

TBP

$\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\frac{9}{25}} = \frac{4}{5}$

$\tan\theta = 3/4.$

$\because \theta$ lies in second quadrant $\quad \sin\theta = 3/2, \operatorname{cosec}\theta = 4/3$

$\cos\theta = -4/5, \sec\theta = -5/4$

$\tan\theta = -3/4, \cot\theta = -4/3.$

3) Find the value of i) $\sin(-45)$ 2) $\cos(-45)$ 3) $\cot(-45^\circ)$

TBP

1) $\sin(-45) = -\sin 45^\circ = -1/\sqrt{2}$ $\quad \sin(-\theta) = -\sin\theta$

2) $\cos(-45) = \cos 45^\circ = 1/\sqrt{2}$ $\quad \cos(-\theta) = \cos\theta$

$\cot(-\theta) = -\cot\theta.$

3) $\cot(-45) = -\cot 45^\circ = -1.$

4) Find the value of 1) $\sin 150^\circ$ 2) $\cos 135$ 3) $\tan 120^\circ$

TBP

1) $\sin 150 = \sin(180-30) = \sin 30 = 1/2$

2) $\cos 135 = \cos(180-45) = -\cos 45 = -1/\sqrt{2}$

3) $\tan 120^\circ = \tan(180-60) = -\tan 60^\circ = -\sqrt{3}.$

5) Find the value of 1) $\sin 765^\circ$ 2) $\operatorname{cosec}(-1410)$, $\cot\left(\frac{-15\pi}{4}\right)$

TBP

1) 1. $\sin 765 = \sin(360+45) = \sin 45 = \frac{1}{\sqrt{2}}$

2) $\operatorname{cosec}(-1410) = -\operatorname{cosec}(1410)$

$= -\operatorname{cosec}$

$= -\operatorname{cosec} 330$

$= -\operatorname{cosec}(360-330)$

$= -\operatorname{cosec} 30^\circ$

$= -2$

3) $\cot\left(\frac{-15\pi}{4}\right) = -\cot\left(4\pi - \frac{\pi}{4}\right) = +\cot \pi/4$

$= +1.$

6) BP. P.T $\tan 315^\circ \cot(-405^\circ) + \cot 495 \tan(-585) = 2.$

$\tan 315^\circ = \tan(360-45) = -\tan 45 = -1$

$\cot(-405) = -\cot 405 = -\cot(360+45) = -\cot 45 = -1$

$\cot 495 = \cot(360+135) = \cot 135 = \cot(180-45)$

$= -\cot 45 = -1$

$\tan(-585) = -\tan(585) = -\tan 225$

$= -\tan(180+45)$

$= -\tan 45 = -1$

$\therefore$ $\tan 315 \cot(-405) + \cot 495 \tan(-585) = (-1)(-1) + (-1)(-1)$

$= 1+1$

$= 2$

7) Determine whether the functions are even or odd or neither.

TBP

Sol

1) $\sin^2 x - 2\cos^2 x - \cos x =$

$f(-x) = \sin^2 x - 2\cos^2 x - \cos x$

$= f(x)$ $\therefore$ even function

2) $\sin(\cos(x))$: $f(-x) = \sin(\cos(-x))$

$= \sin(\cos x) =$ even fn

$= f(x)$

3) $f(x) = \cos(\sin x) = f(-x) = \cos(\sin(-x)) = \cos(-\sin x)$

$= \cos(\sin x) = f(x)$ even.

4. $f(x) = \sin x + \cos x$

$f(-x) = -\sin x + \cos x$ $\therefore f(x) \neq f(-x)$ $\therefore$ neither even nor odd.

8) Find the values of 1) $\sin 480°$ 2) $\sin(-1110)$ 3) $\cos(300)$ 4) $\tan(1050)$ 5) $\cot 660°$ 6) $\tan \frac{19\pi}{3}$ 7) $\sin\left(-\frac{11\pi}{3}\right)$

TBP

Sol: 1. $\sin 480 = \sin(360+120) = \sin 120 = \sin(180-60)$
$= \sin 60° = \sqrt{3}/2$

2. $\sin(-1110) = -\sin(1110) = -\sin(3\times 360+30)$
$= -\sin 30 = -1/2$

3. $\cos(300) = \cos(360-60) = \cos 60° = 1/2$

4. $\tan(1050) = \tan(3\times360-30) = -\tan 30° = -\frac{1}{\sqrt{3}}$

5. $\cot(660°) = \cot(2\times360-60) = -\cot 60° = -1/\sqrt{3}$

6. $\tan\left(\frac{19\pi}{3}\right) = \tan\left(6\pi+\frac{\pi}{3}\right) = \tan\frac{\pi}{3} = \sqrt{3}.$

7. $\sin\left(-\frac{11\pi}{3}\right) = \sin\left(4\pi-\frac{\pi}{3}\right) = -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2}.$

9) PT $\dfrac{\cot(180+\theta)\sin(90-\theta)\cos(-\theta)}{\sin(270+\theta)\tan(-\theta)\operatorname{cosec}(360+\theta)} = \cos^2\theta\tan\theta.$

TBP

Sol: $\cot(180+\theta) = \cot\theta$

$\sin(90-\theta) = \cos\theta$

$\cos(-\theta) = \cos\theta$

$\sin(270+\theta) = -\cos\theta$

$\tan(-\theta) = -\tan\theta$

$\operatorname{cosec}(360+\theta) = \operatorname{cosec}\theta.$

$\therefore \dfrac{\cot(180+\theta)\sin(90-\theta)\cos(-\theta)}{\sin(270+\theta)\tan(-\theta)\operatorname{cosec}(360+\theta)}$

$= \dfrac{(\cot\theta)(\cos\theta)(\cos\theta)}{(-\cos\theta)(-\tan\theta)\cdot\operatorname{cosec}\theta}$

$= \dfrac{\cot\theta\cdot\cos^2\theta}{\cos\theta\operatorname{cosec}\theta\cdot\tan\theta} = \cos^2\theta\cdot\cot\theta.$

10) Find the values of other five trigonometric functions for the following

TBP

1) $\cos\theta = -1/2$ θ lies in the III quadrant.

$\sin\theta = \sqrt{1-\frac{1}{4}} = \sqrt{3}/2$

$\sin\theta = -\sqrt{3}/2$ $\operatorname{cosec}\theta = -2/\sqrt{3}$

$\tan\theta = \dfrac{\sqrt{3}/2}{1/2} = \sqrt{3}$ $\sec\theta = -2$

$\cot\theta = 1/\sqrt{3}$

2) $\cos\theta = 2/3$ θ lies in I quadrant.

$\sin\theta = \sqrt{1-\frac{4}{9}} = \sqrt{5}/3$

$\tan\theta = \sqrt{5}/2$, $\sec\theta = 3/2$

$\operatorname{cosec}\theta = 3/\sqrt{5}$, $\cot\theta = 2/\sqrt{5}$

3) $\sin\theta = -\frac{2}{3}$ θ lies in the $\overline{IV}$ quadrant.

$\cos\theta = \sqrt{1-\frac{4}{9}} = \sqrt{5}/3$

$\tan\theta = -2/\sqrt{5}$ $\sec\theta = 3/\sqrt{5}$

$\operatorname{cosec}\theta = -3/2$ $\cot\theta = -\sqrt{5}/2$

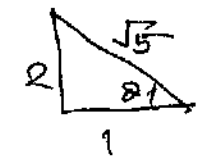
5) $\sec\theta = \frac{13}{5}$ θ is in $\overline{IV}$ quadrant

$\cos\theta = 5/13$, $\tan\theta = \frac{-12}{5}$

$\sin\theta = \sqrt{1-\frac{25}{13}} = \frac{-12}{13}$

$\operatorname{cosec}\theta = -\frac{13}{12}$

4) $\tan\theta = \frac{-2}{1}$ θ lies in $\overline{II}$ quadrant.



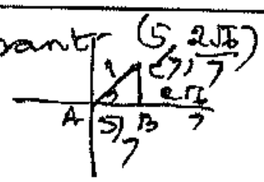
$\sin\theta = 2/\sqrt{5}$, $\operatorname{cosec}\theta = \sqrt{5}/2$

$\cos\theta = -1/\sqrt{5}$ $\sec\theta = -\sqrt{5}$

$\cot\theta = -1/2$

11) TBP $\left(\frac{7}{5}, \frac{2\sqrt{6}}{7}\right)$ is a point on the terminal side of an angle θ in standard position. Determine the trigonometric values of angle θ

Sol: $AC = \sqrt{\frac{25}{49}+\frac{24}{49}} = 1$ θ lies in I quadrant



$\sin\theta = 2\sqrt{6}/7$ $\operatorname{cosec}\theta = \frac{7}{2\sqrt{6}}$

$\cos\theta = \frac{5}{7}$ $\sec\theta = 7/5$

$\tan\theta = \frac{2}{\sqrt{6}} \times \frac{7}{5} = \frac{14}{5\sqrt{6}}$, $\cot\theta = \frac{5\sqrt{6}}{14}$.

12) Find all the angles between 0 and $360°$ which satisfies the equation $\sin^2\theta = 3/4$

$\sin^2\theta = 3/4 \Rightarrow \sin\theta = \pm\sqrt{3}/2$

If $\sin\theta = \sqrt{3}/2$ θ lies in I or $\overline{II}$ quadrant.

If θ is in I quadrant

$\cos\theta = \sqrt{1-\frac{3}{4}} = \frac{1}{2}$, $\sec\theta = 2$

$\tan\theta = \sqrt{3}$, $\cot\theta = \frac{1}{\sqrt{3}}$

If θ is in $\overline{II}$ quadrant

$\sin\theta = \sqrt{3}/2$, $\operatorname{cosec}\theta = 2/\sqrt{3}$

$\cos\theta = -1/2$, $\sec\theta = -2$

If $\sin\theta = -\sqrt{3}/2$

θ may be in $\overline{III}$ or $\overline{IV}$ quadrant

If θ is in $\overline{III}$

$\sin\theta = -\sqrt{3}/2$, $\operatorname{cosec}\theta = -2/\sqrt{3}$

$\cos\theta = -1/2$ $\sec\theta = -2$

$\tan\theta = \sqrt{3}$ $\cot\theta = 1/\sqrt{3}$

If θ is in $\overline{IV}$

$\sin\theta = -\sqrt{3}/2$ $\operatorname{cosec}\theta = -2/\sqrt{3}$

$\cos\theta = 1/2$ $\sec\theta = 2$

$\cot\theta = -1/\sqrt{3}$

13) P.T $\dfrac{\cos(90+\theta)\sec(-\theta)\tan(180-\theta)}{\sec(360-\theta)\sin(180+\theta)\cot(90-\theta)} = -1.$

Sol: $\cos(90+\theta) = -\sin\theta$ $\quad\sec(360-\theta) = \sec\theta$

$\sec(-\theta) = \sec\theta$ $\quad\sin(180+\theta) = -\sin\theta$

$\tan(180-\theta) = -\tan\theta$ $\quad\cot(90-\theta) = \tan\theta.$

$\therefore \dfrac{\cos(90+\theta)\sec(-\theta)\tan(180-\theta)}{\sec(360-\theta)\sin(180+\theta)\cot(90-\theta)} = \dfrac{(-\sin\theta)(\sec\theta)(-\tan\theta)}{(\sec\theta)(-\sin\theta)(\tan\theta)}$

$= -1.$

(14) P.T $\sin^2\frac{\pi}{18} + \sin^2\frac{\pi}{9} + \sin^2\frac{7\pi}{18} + \sin^2\frac{4\pi}{9} = 2.$

KBP

Sol: $\sin\frac{7\pi}{18} = \sin\left(\frac{9\pi-2\pi}{18}\right) = \sin\left(\frac{\pi}{2}-\frac{\pi}{9}\right) = \cos\frac{\pi}{9}.$

$\sin\frac{4\pi}{9} = \sin\frac{8\pi}{18} = \sin\left(\frac{9\pi-\pi}{18}\right) = \sin\left(\frac{\pi}{2}-\frac{\pi}{18}\right) = \cos\frac{\pi}{18}$

$\therefore$ LHS: $\sin^2\frac{\pi}{18} + \sin^2\frac{\pi}{9} + \sin^2\frac{7\pi}{18} + \sin^2\frac{4\pi}{9}$

$= \sin^2\frac{\pi}{18} + \sin^2\frac{\pi}{9} + \cos^2\frac{\pi}{18} + \cos^2\frac{\pi}{9}.$

$= \left(\sin^2\frac{\pi}{18} + \cos^2\frac{\pi}{18}\right) + \left(\sin^2\frac{\pi}{9} + \cos^2\frac{\pi}{9}\right)$

$= 1+1 = 2.$

Procedure: Change $\frac{7\pi}{18}$ as $\frac{9\pi-2\pi}{18}$

and $\frac{4\pi}{9}$ as $\frac{8\pi}{18}$ and $\frac{9\pi-\pi}{18}$

15) P.T $\sin^2\frac{\pi}{8} + \sin^2\frac{3\pi}{8} + \sin^2\frac{5\pi}{8} + \sin^2\frac{7\pi}{8} = 2.$

Sol: Procedure: $\frac{7\pi}{8} = \frac{4\pi-3\pi}{8} = \frac{\pi}{2} - \frac{3\pi}{8}$ and $\frac{5\pi}{8} = \frac{4\pi-\pi}{8} = \left(\frac{\pi}{2}-\frac{\pi}{8}\right)$

$\therefore \sin\frac{7\pi}{8} = \sin\left(\frac{\pi}{2}-\frac{3\pi}{8}\right) = \cos\left(\frac{3\pi}{8}\right.$

$\sin\left(\frac{5\pi}{8}\right) = \sin\left(\frac{4\pi-\pi}{8}\right) = \sin\left(\frac{\pi}{2}-\frac{\pi}{2}\right) = \cos\pi/2$

$\therefore$ LHS $\sin^2\pi/8 + \sin^2\frac{3\pi}{8} + \sin^2\frac{5\pi}{8} + \sin^2\frac{7\pi}{8}$

$= \sin^2\pi/8 + \sin^2\frac{3\pi}{8} + \cos^2\frac{\pi}{8} + \cos^2\frac{3\pi}{8}$

$= (\sin^2\frac{\pi}{8} + \cos^2\frac{\pi}{8}) + (\sin^2\frac{3\pi}{8} + \cos^2\frac{3\pi}{8})$

16) P.T $\cos^2\frac{\pi}{8} + \cos^2\frac{3\pi}{8} + \cos^2\frac{7\pi}{8} + \cos^2\frac{5\pi}{8} = 2$.

Same as problem (15)

17) P.T $\left[1+\cot\alpha - \sec(\alpha+\pi/2)\right]\left[1+\cot\alpha + \sec(\alpha+\pi/2)\right] = 2\cot\alpha$

Sol: Procedure. Find $\sec(\alpha+\pi/2)$ and use $(a+b)(a-b) = a^2-b^2$

and also $1+\cot^2\alpha = \operatorname{cosec}^2\alpha$.

$\sec(\alpha+\pi/2) = \operatorname{cosec}\alpha$.

$\therefore$ LHS: $\left[1+\cot\alpha - \operatorname{cosec}\alpha\right]\left[1+\cot\alpha + \operatorname{cosec}\alpha\right]$

$= (1+\cot\alpha)^2 - \operatorname{cosec}^2\alpha$.

$= 1+\cot^2\alpha + 2\cot\alpha - \operatorname{cosec}^2\alpha$.

$= \cancel{\operatorname{cosec}^2\alpha} + 2\cot\alpha - \cancel{\operatorname{cosec}^2\alpha}$

$= 2\cot\alpha$.

18) P.T $\cos\left(\frac{3\pi}{2}+\theta\right)\cos(2\pi+\theta)\left[\cot\left(\frac{3\pi}{2}-\theta\right) + \cot(2\pi+\theta)\right] = 1$

Sol: Procedure: $\frac{3\pi}{2} = 270$, Find all given trigonometric values and substitute.

$\cos(270+\theta) = +\sin\theta$ | $\cot(270-\theta) = \tan\theta$

$\cos(2\pi+\theta) = \cos\theta$ | $\cot(2\pi+\theta) = \cot\theta$.

LHS: $\cos\left(\frac{3\pi}{2}+\theta\right)\cos(2\pi+\theta)\left[\cot\left(\frac{3\pi}{2}-\theta\right) + \cot(2\pi+\theta)\right]$

$= (+\sin\theta\cdot\cos\theta)(\tan\theta + \cot\theta)$

$= \sin\theta\cdot\cos\theta\cdot\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)$

$= \cancel{\sin\theta\cos\theta}\left(\frac{\sin^2\theta+\cos^2\theta}{\cancel{\sin\theta\cos\theta}}\right) = 1$.

19) P.T $\sec\left(\frac{3\pi}{2}-\theta\right)\sec\left(\theta-\frac{5\pi}{2}\right) + \tan\left(\frac{5\pi}{2}+\theta\right)\tan\left(\theta-\frac{3\pi}{2}\right) = -1$.

Sol: Procedure: $\frac{3\pi}{2} = 270$, $\sec(-\theta) = \sec\theta$, $\tan(-\theta) = -\tan\theta$.

$\frac{5\pi}{2} = 450 = 360+90$. Use these to find the values and find LHS.

$\sec(270-\theta) = -\operatorname{cosec}\theta$. | $= \sec(450-\theta)$

$= \sec(360+90-\theta) = \sec(90-\theta)$

and $\tan(\theta - \frac{3\pi}{2}) = \tan(-(\frac{3\pi}{2} - \theta))$

$= -\tan(270 - \theta)$

$= -\cot\theta.$

$\therefore$ LHS $= (-\operatorname{cosec}\theta)(\operatorname{cosec}\theta) + (-\cot\theta)(-\cot\theta)$

$= (\cot^2\theta - \operatorname{cosec}^2\theta) = -1.$

20) Simplify $\dfrac{\cos(90+\theta)\sec(-\theta)\tan(180-\theta)}{\sec(360-\theta)\sin(180+\theta)\cot(90+\theta)} = 1$

Already workedout same model.

21) Simplify $\dfrac{\tan(90-\theta)\sec(180-\theta)\sin(-\theta)}{\sin(180+\theta)\cot(360-\theta)\operatorname{cosec}(90+\theta)}$ Ans: 1

Trigonometric identities (Sum and difference of two angles.

1) Find the values of 1) cos 15° 2) tan 165°.

*BP 1) $\cos 15° = \cos(45-30) = \cos 45 \cos 30 + \sin 45 \sin 30.$

$$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$$

$$= \frac{\sqrt{3}+1}{2\sqrt{2}}$$

Also $\sin 75° = \frac{\sqrt{3}+1}{2\sqrt{2}}$

2) $\tan(165) = \tan(120+45)$

$$= \frac{\tan 120 + \tan 45°}{1 - \tan 120 \tan 45}$$

But $\tan 120$

$= \tan(90+30)$

$= -\cot 30°$

$= -\sqrt{3}$

$$= \frac{-\sqrt{3}+1}{1+\sqrt{3}} = \frac{1-\sqrt{3}}{1+\sqrt{3}}$$

2) If $\sin x = \frac{4}{5}$ (I quadrant) and $\cos y = \frac{-12}{13}$ (second quadrant)

*BP Find 1) $\sin(x-y)$ 2) $\cos(x-y)$

Sol: $\sin x = \frac{4}{5}$ $\cos x = \sqrt{1-\frac{16}{25}} = 3/5$ $\because x$ is in I quadrant

$\cos y = \frac{-12}{13}$ $\sin y = \sqrt{1-\frac{144}{169}} = \frac{5}{13}$ $\because y$ is second quadrant

1) $\sin(x-y) = \sin x \cos y - \cos x \sin y$

$$= \frac{4}{5}\left(\frac{-12}{13}\right) - \frac{3}{5} \cdot \frac{5}{13} = \frac{-48-15}{65} = \frac{-63}{65}$$

2. $\cos(x-y) = \cos x \cos y + \sin x \sin y$

$$= \frac{3}{5}\left(\frac{-12}{13}\right) + \frac{4}{5} \cdot \frac{5}{13} = \frac{-36+20}{65} = \frac{-16}{65}$$

3) PT $\cos\left(\frac{3\pi}{4}+x\right) - \cos\left(\frac{3\pi}{4}-x\right) = -\sqrt{2} \sin x$.

*BP. LHS $\left(\cos\frac{3\pi}{4} \cos x - \sin\frac{3\pi}{4} \sin x\right) - \left(\cos\frac{3\pi}{4} \cos x + \sin\frac{3\pi}{4} \sin x\right)$

$$= -2 \sin\frac{3\pi}{4} \sin x$$

$\sin\frac{3\pi}{4} = \sin\left(\pi - \frac{\pi}{4}\right) = \sin\frac{\pi}{4}.$

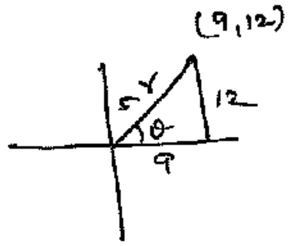
$$= -2 \cdot \frac{1}{\sqrt{2}} \sin x = -\sqrt{2} \sin x.$$

Note: $\cos(A+B) - \cos(A-B) = -2 \sin A \sin B.$

4) Point A (9,12) rotates around the origin O in the plane through 60° in the anticlock wise direction to the new position B. Find the Co-ordinates of B.

BP

Sol: Here $r\cos\theta = 9$, $r\sin\theta = 12$



$$r = \sqrt{81+144} = \sqrt{225} = 15$$

$$\therefore 15\cos\theta = 9,\ 15\sin\theta = 12$$

$$15\cos(\theta+60) = \cos\theta\cos 60 - \sin\theta\sin 60°$$

$$= 15\left(\frac{9}{15}\cdot\frac{1}{2} - \frac{12}{15}\cdot\frac{\sqrt{3}}{2}\right) = \cancel{15}\left(\frac{9-12\sqrt{3}}{\cancel{30}_2}\right)$$

$$= \frac{3}{2}(3-4\sqrt{3})$$

$$\text{lly } 15\sin(\theta+60°) = \frac{3}{2}(4+3\sqrt{3})$$

Hence $B = \left(\frac{3}{2}(3-4\sqrt{3}),\ \frac{3}{2}(4+3\sqrt{3})\right)$

5) Expand 1) $\sin(A+B+C)$, 2) $\tan(A+B+C)$

BP

Sol: 1) $\sin[A+(B+C)] = \sin A\cos(B+C) + \cos A(\sin(B+C))$

$$= \sin A(\cos B\cos C - \sin B\sin C) + \cos A(\sin B\cos C + \cos B\sin C)$$

$$= \sin A\cos B\cos C - \sin A\sin B\sin C + \cos A\sin B\cos C + \cos A\cos B\sin C.$$

2) $\tan(A+B+C) = \tan[A+(B+C)]$

$$= \frac{\tan A + \tan(B+C)}{1 - \tan A\tan(B+C)}$$

$$\tan(A+B+C) = \frac{\tan A + \dfrac{\tan B + \tan C}{1-\tan B\tan C}}{1 - \tan A\cdot\dfrac{\tan B + \tan C}{1-\tan B\tan C}}$$

$$= \frac{\tan A(1-\tan B\tan C) + \tan B + \tan C}{1-\tan B\tan C - \tan A\tan B - \tan A\tan C}$$

$$\tan(A+B+C) = \frac{\tan A + \tan B + \tan C - \tan A\tan B\tan C}{1-\tan A\tan B - \tan B\tan C - \tan A\tan C}.$$

Note: If $A+B+C = 0$ or π

From (*). $0 = \dfrac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A \tan B - \tan B \tan C - \tan C \tan A}$

$\Rightarrow \tan A \tan B \tan C = \tan A + \tan B + \tan C.$

If $A+B+C = \pi/2$ $\quad 1 = \tan A \tan B + \tan B \tan C + \tan C \tan A.$

From (*) Dr $= 0$

6) Expand $\cos(A+B+C)$ Hence P.T

TBP $\cos A \cos B \cos C = \sin A \sin B \cos C + \sin B \sin C \cos A + \sin C \sin A \cos B$

if $A+B+C = \pi/2.$

Sol: $\cos(A+B+C) = \cos[A + (B+C)]$

$= \cos A \cos(B+C) - \sin A \sin(B+C)$

$= \cos A[\cos B \cos C - \sin B \sin C] - \sin A[\sin B \cos C + \cos B \sin C]$

$= \cos A \cos B \cos C - \cos A \sin B \sin C - \sin A \sin B \cos C - \sin A \cos B \sin C.$

If $A+B+C = \pi/2$ $\cos(\pi/2) = 0$

$\therefore \cos A \cos B \cos C = \sin A \sin B \cos C + \sin B \sin C \cos A + \sin A \sin C \cos B.$

7) If $A+B+C = \pi/2$ then $\tan A \tan B + \tan B \tan C + \tan C \tan A = 1.$

TBP

Sol: $A+B+C = \pi/2$

$A+B = \frac{\pi}{2} - C \Rightarrow \tan(A+B) = \tan(\pi/2 - C)$

$\dfrac{\tan A + \tan B}{1 - \tan A \tan B} = \cot C = \dfrac{1}{\tan C}.$

$\tan A \tan C + \tan B \tan C = 1 - \tan A \tan B$

$\tan A \tan C + \tan B \tan C + \tan A \tan B = 1$

8) P.T $\sin(45+\theta) - \sin(45-\theta) = \sqrt{2}\sin\theta.$

TBP $\sin(30+\theta) + \cos(60+\theta) = \cos\theta.$

1) $\sin(A+B) - \sin(A-B) = 2\cos A \sin B.$

$\therefore \sin(45+\theta) - \sin(45-\theta) = 2\cos 45 \sin\theta = 2 \cdot \frac{1}{\sqrt{2}} \sin\theta$

$= \sqrt{2}\sin\theta.$

2) $\sin(30+\theta) + \cos(60+\theta)$

$= \sin 30 \cos\theta + \cos 30 \sin\theta + \cos 60 \cos\theta - \sin 60 \sin\theta$

$= \frac{1}{2}\cos\theta + \cancel{\frac{\sqrt{3}}{2}\sin\theta} + \frac{1}{2}\cos\theta - \cancel{\frac{\sqrt{3}}{2}\sin\theta} = \cos\theta.$

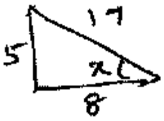
9) If $\sin x = \frac{15}{17}$ $\cos y = \frac{12}{13}$ $0 < x < \pi/2$ and $0 < y < \pi/2$

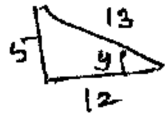
KBP

Find the value of 1) $\sin(x+y)$ 2) $\cos(x-y)$ 3) $\tan(x+y)$

Sol: x and y are in I quadrant.

$\sin x = \frac{15}{17}$ $\cos x = \frac{8}{17}$ $\tan x = 15/8$





$\sin y = \frac{5}{13}$ $\cos y = \frac{12}{13}$ $\tan y = 5/12$.

1) $\sin(x+y) = \sin x \cos y + \cos x \sin y$.

$$= \frac{15}{17} \cdot \frac{12}{13} + \frac{8}{17} \cdot \frac{5}{13} = \frac{180+40}{221} = \frac{120}{221}$$

2) $\cos(x+y) = \cos x \cos y - \sin x \sin y$

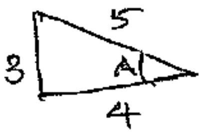
$$= \frac{8}{17} \cdot \frac{12}{13} - \frac{15}{17} \cdot \frac{5}{13} = \frac{96-25}{221} = \frac{71}{221}.$$

10) If $\sin A = 3/5$ and $\cos B = \frac{9}{41}$ $0 < A < \pi/2$ $0 < B < \pi/2$.

KBP

Find 1) $\sin(A+B)$ 2) $\cos(A-B)$

Sol: θ lies in I quadrant.



$\sin A = 3/5$ $\cos A = \frac{4}{5}$ $\tan A = 3/4$

$\sin B = \frac{40}{41}$

$\cos B = \frac{9}{41}$

$\tan B = \frac{40}{9}$.

1) $\sin(A+B) = \sin A \cos B + \cos A \sin B$.

$$= \frac{3}{5} \cdot \frac{9}{41} + \frac{4}{5} \cdot \frac{40}{41}.$$

$$= \frac{27+160}{205} = \frac{187}{205}$$

2) $\cos(A-B) = \cos A \cos B + \sin A \sin B$.

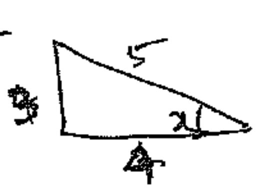
$$= \frac{4}{5} \cdot \frac{9}{41} + \frac{3}{5} \cdot \frac{40}{41} = \frac{36+120}{205} = \frac{156}{205}$$

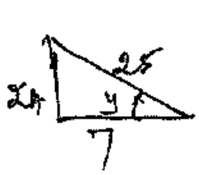
11) Find $\cos(x-y)$, given that $\cos x = -\frac{4}{5}$ $\pi < x < \frac{3\pi}{2}$, $\pi < y \leq \frac{3\pi}{2}$

KB.P

$\sin y = -\frac{24}{25}$

x, y lies in $\underline{\text{III}}$ Quadrant.





$\sin x = -\frac{3}{5}$ $\cos x = -4/5$

$\sin y = -\frac{24}{25}$ $\cos y = -\frac{7}{25}$

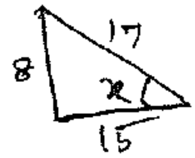
$\cos(x-y) = \cos x \cos y + \sin x \sin y$

$$= \left(-\frac{4}{5}\right)\left(-\frac{7}{25}\right) + \left(-\frac{3}{5}\right)\left(-\frac{24}{25}\right) = \frac{28+72}{125} = \frac{100}{125}\ 4/5$$

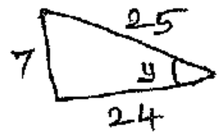
12) Find $\sin(x-y)$ given that $\sin x = \frac{8}{17}$ with $0 < x < \pi/2$ and
TBP $\cos y = -\frac{24}{25}$ with $\pi < y < \frac{3\pi}{2}$

Sol: x lies in the I quadrant, y lies in the III quadrant.

$\sin x = \frac{8}{17}$, $\cos x = 15/17$



$\cos y = \frac{-24}{25}$ $\sin y = -\frac{7}{25}$



$$\sin(x-y) = \sin x \cos y - \cos x \sin y$$

$$= \frac{8}{17}\left(-\frac{24}{25}\right) - \frac{15}{17}\left(-\frac{7}{25}\right)$$

$$= \frac{-192 + 105}{425} = \frac{87}{425}$$

13) Find the value of i) $\cos 105°$ 2) $\sin 105$ 3) $\tan \frac{7\pi}{12}$
TBP

Sol: 1) $\cos 105 = \cos(60+45)$

$$= \cos 60 \cos 45 - \sin 60 \sin 45$$

$$= \frac{1}{2} \cdot \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} \frac{1}{\sqrt{2}} = \frac{1-\sqrt{3}}{2\sqrt{2}}$$

2) $\sin 105 = \sin(60+45)$

$$= \sin 60 \cos 45 + \cos 60 \sin 45$$

$$= \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}} + \frac{1}{2} \cdot \frac{1}{\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

3) $\tan \frac{7\pi}{12} = \tan \frac{7 \times \cancel{180}^{15}}{\cancel{12}} = \tan 105$

$$= \tan(60+45) = \frac{\tan 60 + \tan 45}{1 - \tan 60 \tan 45}$$

$$= \frac{\sqrt{3}+1}{1-\sqrt{3}} = \frac{(1+\sqrt{3})(1+\sqrt{3})}{(1-\sqrt{3})(1+\sqrt{3})}$$

$$= \frac{1+3+2\sqrt{3}}{1-3}$$

$$= \frac{2(2+\sqrt{3})}{-2}$$

$$= -(2+\sqrt{3})$$

12) P.T $\sin(n+1)\theta \sin(n-1)\theta + \cos(n+1)\theta\cos(n-1)\theta = \cos 2\theta \quad n \in z.$

BP

Sol: Procedure use $\cos A\cos B + \sin A\sin B = \cos(A-B)$

LHS: $\overset{A}{\cos(n+1)\theta}\,\overset{B}{\cos(n-1)\theta} + \overset{A}{\sin(n+1)\theta}\,\overset{B}{\sin(n-1)\theta}$

$= \cos\left((n+1)-(n-1)\right)\theta$

$= \cos 2\theta$

13) If $x\cos\theta = y\cos\left(\theta+\frac{2\pi}{3}\right) = z\cos\left(\theta+\frac{4\pi}{3}\right)$ Find the value of $xy+yz+zx$

BP

Sol: Procedure: Find the values of each separately. and put They are equal to k. then find x, y, z then calculate $xy+yz+zx$.

$x\cos\theta \pm y\cos\left(\theta \mp \frac{\pi}{3}\right) = z\cos\left(\theta+\pi/3\right)$

$x\cos\theta = y\left[\cos\theta\cos\frac{\pi}{3} + \sin\theta\sin\pi/3\right] = z\left[\cos\theta\cos\frac{\pi}{3} - \sin\theta\sin\pi/3\right]$

$x\cos\theta = y\left[\cos\theta\cdot\frac{1}{2} + \frac{\sqrt{3}}{2}\sin\theta\right] = z\left[\frac{1}{2}\cos\theta - \frac{\sqrt{3}}{2}\sin\theta\right] = K.$

$x = \frac{K}{\cos\theta}, \quad y = \frac{K}{\frac{1}{2}\cos\theta + \frac{\sqrt{3}}{2}\sin\theta}, \quad z = \frac{K}{\frac{1}{2}\cos\theta - \frac{\sqrt{3}}{2}\sin\theta}.$

$\therefore xy+yz+zx = \frac{K^2}{\frac{1}{2}\cos^2\theta + \frac{\sqrt{3}}{2}\sin\theta\cos\theta} + \frac{K^2}{\frac{1}{4}\cos^2\theta - \frac{3}{4}\sin^2\theta} + \frac{K^2}{\frac{1}{2}\cos^2\theta - \frac{\sqrt{3}}{2}\sin\theta\cos\theta}$

$= \frac{2K^2}{\frac{1}{4}\cos^4\theta - \frac{3}{4}\sin^2\theta\cos^2\theta} +$

$x\cos\theta = y\cos(\pi/3-\theta) = -z\cos(\pi/3+\theta) = K.$

$\frac{K^2}{\cos\theta(\cos\frac{\pi}{3}-\theta)} + \frac{K^2}{-\cos(\frac{\pi}{3}-\theta)\cos(\pi/3+\theta)} + \frac{K^2}{-\cos\theta(\cos(\pi/3+\theta))}$

$= \frac{-K^2\left[\cos(\frac{\pi}{3}+\theta)\right] K^2\cos\theta + K^2\cos(\pi/3-\theta)]}{-\cos\theta(\cos\frac{\pi}{3}-\theta)\cdot\cos(\pi/3+\theta)}$

$= -K^2\left(\cos\frac{\pi}{3}\cos\theta - \sin\pi/3\sin\theta\right) - K^2\cos\theta + K^2\left(\cos\pi/3\cos\theta + \sin\pi/3\sin\theta\right)$

$= +K^2\frac{\sqrt{3}}{2}\sin\theta - K^2\cos\theta + \frac{\sqrt{3}}{2}K^2\sin\theta.$

13 P.T $\sin(n+1)\theta \sin(n-1)\theta + \cos(n+1)\theta \cos(n-1)\theta = \cos 2\theta \quad n \in z$.

KBP

Sol: Procedure: use $\cos A \cos B + \sin A \sin B = \cos(A-B)$

$$\cos(n+1)\theta \cos(n-1)\theta + \sin(n+1)\theta \sin(n-1)\theta$$
$$= \cos((n+1)-(n-1))\theta$$
$$= \cos 2\theta.$$

(4) If $x\cos\theta = y\cos\left(\theta + \frac{2\pi}{3}\right) = z\cos\left(\theta + \frac{4\pi}{3}\right)$. find $xy + yz + zx$.

KBP.

Sol: Procedure: simplify each one and put they are equal to K. Then find x, y, z then sub. in $xy+yz+zx$.

$$y\cos\left(\pi - \frac{\pi}{3} + \theta\right) = y\cos\left(\pi - \left(\frac{\pi}{3} - \theta\right)\right) = -y\cos\left(\frac{\pi}{3} - \theta\right)$$
$$= -y\left[\cos\pi/3 \cos\theta + \sin\pi/3 \sin\theta\right]$$
$$= -y\left[\frac{1}{2}\cos\theta + \frac{\sqrt{3}}{2}\sin\theta\right]$$

$$z\cos\left(\pi + \frac{\pi}{3} + \theta\right) = -z\cos(\pi/3 + \theta)$$
$$= -z(\cos\pi/3 \cos\theta - \sin\pi/3 \sin\theta)$$
$$= -z\left(\tfrac{1}{2}\cos\theta - \frac{\sqrt{3}}{2}\sin\theta\right)$$

Now $x\cos\theta = -y\left[\frac{1}{2}\cos\theta + \frac{\sqrt{3}}{2}\sin\theta\right] = -z\left(\frac{1}{2}\cos\theta - \frac{\sqrt{3}}{2}\sin\theta\right) = K.$

$$x = \frac{K}{\cos\theta}, \quad y = \frac{K}{-\left(\frac{1}{2}\cos\theta + \frac{\sqrt{3}}{2}\sin\theta\right)}, \quad z = \frac{K}{\frac{\sqrt{3}}{2}\sin\theta - \frac{1}{2}\cos\theta}.$$

$$\therefore xy + yz + zx = \frac{K^2}{-\cos\theta\left(\frac{1}{2}\cos\theta + \frac{\sqrt{3}}{2}\sin\theta\right)} + \frac{K^2}{-\left(\frac{1}{2}\cos\theta + \frac{\sqrt{3}}{2}\sin\theta\right)\left(\frac{\sqrt{3}}{2}\sin\theta - \frac{1}{2}\cos\theta\right)}$$
$$+ \frac{K^2}{\cos\theta\left(\frac{\sqrt{3}}{2}\sin\theta - \frac{1}{2}\cos\theta\right)}$$

$$= \frac{-K^2\left(\frac{\sqrt{3}}{2}\sin\theta - \frac{1}{2}\cos\theta\right) - K^2(\cos\theta) + K^2\left(\frac{\sqrt{3}}{2}\sin\theta + \frac{1}{2}\cos\theta\right)}{\cos\theta\left(\frac{\sqrt{3}}{2}\sin\theta + \frac{1}{2}\cos\theta\right)\left(\frac{\sqrt{3}}{2}\sin\theta - \frac{1}{2}\cos\theta\right)}$$

$$= \frac{\cancel{-K^2\frac{\sqrt{3}}{2}\sin\theta} + \frac{K^2}{2}\cos\theta - K^2\cos\theta + K^2\frac{\sqrt{3}}{2}\sin\theta + \frac{K^2}{2}\cos\theta}{\quad}$$

$$= K^2\cos\theta - K^2\cos\theta$$
$$= 0.$$

15) If $\cos(\alpha-\beta)+\cos(\beta-\gamma)+\cos(\gamma-\alpha)=-\frac{3}{2}$ P.T $\cos\alpha+\cos\beta+\cos\gamma=0$ $\sin\alpha+\sin\beta+\sin\gamma$.

ABP

Sol: Expand LHS and crossmultiply with 2. Take -3 to LHS as $1+1+1$. Put $1=\sin^2\alpha+\cos^2\alpha$, $1=\sin^2\beta+\cos^2\beta$, $1=\sin^2\gamma+\cos^2\gamma$. Simplify. use $(a+b+c)^2=a^2+b^2+c^2+2ab+2bc+2ca$

LHS: $(\cos\alpha\cos\beta+\sin\alpha\sin\beta)+(\cos\beta\cos\gamma+\sin\beta\sin\gamma)+(\cos\gamma\cos\alpha+\sin\gamma\sin\alpha)=-\frac{3}{2}$

$$2\cos\alpha\cos\beta+2\sin\alpha\sin\beta+2\cos\beta\cos\gamma+2\sin\beta\sin\gamma+2\cos\gamma\cos\alpha+2\sin\gamma\sin\alpha=-3.$$

$$\therefore 2\cos\alpha\cos\beta+2\sin\alpha\sin\beta+2\cos\beta\cos\gamma+2\sin\beta\sin\gamma+2\cos\gamma\cos\alpha+2\sin\gamma\sin\alpha+1+1+1=0.$$

$$2\cos\alpha\cos\beta+2\sin\alpha\sin\beta+2\cos\beta\cos\gamma+2\sin\beta\sin\gamma+2\cos\gamma\cos\alpha+2\sin\gamma\sin\alpha+\sin^2\alpha+\cos^2\alpha+\sin^2\beta+\cos^2\beta+\sin^2\gamma+\cos^2\gamma=0$$

$$\sin^2\alpha+\sin^2\beta+\sin^2\gamma+2\sin\alpha\sin\beta+2\sin\beta\sin\gamma+2\sin\gamma\sin\alpha+\cos^2\alpha+\cos^2\beta+\cos^2\gamma+2\cos\alpha\cos\beta+2\cos\beta\cos\gamma+2\cos\gamma\cos\alpha=0$$

$$(\sin\alpha+\sin\beta+\sin\gamma)^2+(\cos\alpha+\cos\beta+\cos\gamma)^2=0$$

$$\Rightarrow \sin\alpha+\sin\beta+\sin\gamma=0$$

$$\cos\alpha+\cos\beta+\cos\gamma=0.$$

16) Find the quadratic equation whose roots are $\sin 15^\circ$ and $\cos 15^\circ$.

ABP

Sol: Procedure: writedown the values of $\sin 15^\circ$ and $\cos 15^\circ$. Find the sum of the roots and product of the roots. Then form the equation $x^2-(SR)x+PR=0$.

$$\sin 15^\circ=\frac{\sqrt{3}+1}{2\sqrt{2}},\quad \cos 15=\frac{\sqrt{3}-1}{2\sqrt{2}}$$

$$SR=\frac{\sqrt{3}+1}{2\sqrt{2}}+\frac{\sqrt{3}-1}{2\sqrt{2}}=\frac{\cancel{2}\sqrt{3}}{\cancel{2}\sqrt{2}}$$

$$PR=\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)=\frac{3-1}{8}=\frac{\cancel{2}}{8}\ 1/4$$

Eqn $x^2-\frac{\sqrt{3}}{\sqrt{2}}x+\frac{1}{4}=0$

$$x^2-\frac{\sqrt{3}\sqrt{2}}{2}x+\frac{1}{4}=0\Rightarrow 4x^2-2\sqrt{6}x+1=0$$

17) P.T 1. $\sin(45+\theta) - \sin(45-\theta) = \sqrt{2}\sin\theta$

TBP. 2. $\sin(30+\theta) + \cos(60+\theta) = \cos\theta$.

Sol: Procedure: use $\sin(A+B)$, $\sin(A-B)$, $\cos(A+B)$.

1) LHS: $\sin(45+\theta) - \sin(45-\theta)$

$= (\sin\cancel{45}\cos\theta + \cos 45\sin\theta) - (\sin\cancel{45}\cos\theta - \cos 45\sin\theta)$

$= 2\cos 45\sin\theta$

$= 2\cdot\frac{1}{\sqrt{2}}\sin\theta = \sqrt{2}\sin\theta$.

2) LHS $\sin(30+\theta) + \cos(60+\theta)$

$= (\sin 30\cos\theta + \cos 30\sin\theta) + (\cos 60\cos\theta - \sin 60\sin\theta)$

$= \frac{1}{2}\cos\theta + \cancel{\frac{\sqrt{3}}{2}\sin\theta} + \frac{1}{2}\cos\theta - \cancel{\frac{\sqrt{3}}{2}\sin\theta}$

$= \cos\theta$.

18) P.T 1) $\cos(30^\circ + x) = \frac{\sqrt{3}\cos x - \sin x}{2}$

TBP. 2) $\cos(\pi+\theta) = -\cos\theta$

3) $\sin(\pi+\theta) = -\sin\theta$.

Sol: 1) LHS: $\cos(30+x) = \cos 30\cos x - \sin 30\sin x$.

$= \frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x$.

$= \frac{\sqrt{3}\cos x - \sin x}{2}$.

2. $\cos(\pi+\theta) = \cos\pi\cos\theta - \sin\pi\sin\theta$

$= -1\cdot\cos\theta - 0$

$= -\cos\theta$.

3) $\sin(\pi+\theta) = \sin\pi\cos\theta + \cos\pi\sin\theta$

$= 0 - 1\cdot\sin\theta$

$= -\sin\theta$.

19) PT $\sin 105 + \cos 105 = \cos 45^\circ$

TBP. Sol: Procedure write 105 as 60+45. and Expand.

LHS: $\sin(60+45) + \cos(60+45)$

$(\sin 60\cos 45 + \cos 60\sin 45) + (\cos 60\cos 45 - \sin 60\sin 45)$

$\cancel{\frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{2}}} + \frac{1}{2}\cdot\frac{1}{\sqrt{2}} + \frac{1}{2}\cdot\frac{1}{\sqrt{2}} - \cancel{\frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{2}}} = \frac{1}{\sqrt{2}}$

$= \cos 45^\circ$

20) P.T $\sin 75 - \sin 15' = \cos 105 + \cos 15$.

Sol: use $\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$

Procedure $\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$

LHS $= \sin 75 - \sin 15 = 2\cos\frac{90}{2}\sin\frac{60}{2}$

$= 2\cos 45 \sin 30$

$= \cancel{2}\cdot\frac{1}{\sqrt{2}}\cdot\frac{1}{\cancel{2}} = \frac{1}{\sqrt{2}}$

RHS $\cos 105 + \cos 15' = 2\cos\frac{120}{2}\cos\frac{90}{2}$

$= 2\cos 60 \cos 45$ $\quad\therefore$ LHS = RHS

$= \cancel{2}\cdot\frac{1}{\cancel{2}}\cdot\frac{1}{\sqrt{2}} = 1/\sqrt{2}$

21) P.T $\tan 75' + \cot 75^\circ = 4$.

BP. Sol: Procedure: use $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$(a+b)^2 + (a-b)^2 = 2(a^2+b^2)$ $\quad \cot(A+B) = \frac{\cot A \cos B - 1}{\cot B + \cot A}$.

LHS $\tan 75' + \cot 75^\circ = \tan(45+30) + \cot(45+30)$

$= \frac{\tan 45 + \tan 30}{1 - \tan 45 \tan 30} + \frac{\cot 45 \cot 30 - 1}{\cot 30 + \cot 45}$

$= \frac{1 + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}} + \frac{1\cdot\sqrt{3} - 1}{\sqrt{3}+1}$

$= \frac{\frac{\sqrt{3}+1}{\cancel{\sqrt{3}}}}{\frac{\sqrt{3}-1}{\cancel{\sqrt{3}}}} + \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{(\sqrt{3}+1)^2 + (\sqrt{3}-1)^2}{(\sqrt{3}-1)(\sqrt{3}+1)}$

$= \frac{2(3+1)}{3-1} = 4$

22) 1) S.T $\tan(45+A) = \frac{1+\tan A}{1-\tan A}$ $\quad$ 2) $\tan(45^\circ - A) = \frac{1-\tan A}{1+\tan A}$.

BP

1) $\tan(45+A) = \frac{\tan 45 + \tan A}{1 - \tan 45 \tan A}$

$= \frac{1+\tan A}{1-\tan A}$

2) $\tan(45-A) = \frac{\tan 45 - \tan A}{1 + \tan 45 \tan A}$

$= \frac{1-\tan A}{1+\tan A}$

23) P.T $\cot(A+B) = \dfrac{\cot A \cot B - 1}{\cot A + \cot B}$.
KBP

Sol: $\cot(A+B) = \dfrac{1}{\tan(A+B)} = \dfrac{1}{\dfrac{\tan A + \tan B}{1 - \tan A \tan B}}$

$= \dfrac{1 - \tan A \tan B}{\tan A + \tan B}$

$\div$ both Nr and Dr by $\tan A \tan B$

$= \dfrac{\dfrac{1}{\tan A \tan B} - 1}{\dfrac{\tan A}{\tan A . \tan B} + \dfrac{\tan B}{\tan A \tan B}}$

$= \dfrac{\cot A \cos B - 1}{\cot B + \tan A}$.

24 If $\tan x = \dfrac{n}{n+1}$ and $\tan y = \dfrac{1}{2n+1}$ find $\tan(x+y)$
KBP

Procedure: use $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$ and Sub. values.

Sol: $\tan(x+y) = \dfrac{\tan x + \tan y}{1 - \tan x \tan y} = \dfrac{\dfrac{n}{n+1} + \dfrac{1}{2n+1}}{1 - \left(\dfrac{n}{n+1}\right)\left(\dfrac{1}{2n+1}\right)}$

$= \dfrac{\dfrac{n(2n+1) + (n+1)}{\cancel{(n+1)(2n+1)}}}{\dfrac{(n+1)(2n+1) - n}{\cancel{(n+1)(2n+1)}}} = \dfrac{2n^2 + n + n + 1}{2n^2 + 3n + 1 - n}$

$= \dfrac{2n^2 + 2n + 1}{2n^2 + 2n + 1} = 1$ $\quad x+y = \tan^{-1}(1) = \pi/4$

25) P.T $\tan\left(\frac{\pi}{4} + \theta\right)\tan\left(\frac{3\pi}{4} + \theta\right) = -1$.

Sol: write $\frac{3\pi}{4}$ as $\pi - \frac{\pi}{4}$ and $\tan(\pi - \theta) = -\tan\theta$.

$\tan\left(\frac{3\pi}{4} + \theta\right) = \tan\left(\pi - \left(\frac{\pi}{4} - \theta\right)\right) = -\tan\left(\frac{\pi}{4} - \theta\right)$

$\therefore \left[\tan\left(\frac{\pi}{4} + \theta\right)\right]\left[-\tan\left(\frac{\pi}{4} - \theta\right)\right] = -\dfrac{2\tan 45}{1 - \tan^2 45 \tan^2\theta} = \dfrac{2}{1 - \tan^2\theta}$

$= \left(\dfrac{\tan\frac{\pi}{4} + \tan\theta}{1 - \tan\frac{\pi}{4}\tan\theta}\right)\left(\dfrac{\tan\frac{\pi}{4} - \tan\theta}{1 + \tan\frac{\pi}{4}\tan\theta}\right) =$

$$= -\left(\frac{1+\tan\theta}{1-\tan\theta}\right)\left(\frac{1-\tan\theta}{1+\tan\theta}\right).$$

$$= -1$$

26) If $\theta+\phi=\alpha$ and $\tan\theta = K\tan\phi$ P.T $\sin(\theta-\phi) = \frac{K-1}{K+1}\sin\alpha$.
4BP

Sol: by using componendo and dividendo method.

Given $\tan\theta = K\tan\phi$

$$\frac{\tan\theta}{\tan\phi} = \frac{K}{1} \Rightarrow \frac{K+1}{K-1} = \frac{\tan\theta+\tan\phi}{\tan\theta-\tan\phi}$$

$$= \frac{\frac{\sin\theta}{\cos\theta}+\frac{\sin\phi}{\cos\phi}}{\frac{\sin\theta}{\cos\theta}-\frac{\sin\phi}{\cos\phi}}$$

$$= \frac{\frac{\sin\theta\cos\phi+\cos\theta\sin\phi}{\cos\theta\cos\phi}}{\frac{\sin\theta\cos\phi-\cos\theta\sin\phi}{\cos\theta\cos\phi}}$$

$$\therefore \frac{K+1}{K-1} = \frac{\sin(\theta+\phi)}{\sin(\theta-\phi)}$$

$$\therefore \frac{\sin(\theta-\phi)}{\sin(\theta+\phi)} = \frac{K-1}{K+1}$$

$$\frac{\sin(\theta-\phi)}{\sin\alpha} = \frac{K-1}{K+1}$$

$$\sin(\theta-\phi) = \frac{K-1}{K+1}\sin\alpha.$$

27) Find the value of $\tan(\alpha+\beta)$ given that $\cot\alpha = 1/2$ $\alpha \in (\pi, \frac{3\pi}{2})$ and $\sec\beta = -\frac{5}{3}$ $\beta \in (\pi/2, \pi)$
4BP

Sol: Procedure: identify α, β belongs to which quadrant.

$$\alpha \in \left(\pi, \frac{3\pi}{2}\right) = \text{III} \quad \beta \in (\pi/2, \pi) = \text{II}$$

$$\cot\alpha = \frac{1}{2} \Rightarrow \tan\alpha = 2 \quad \therefore \alpha \in \text{III} \text{ it is +ve}$$

$$\tan^2\beta = \sec^2-1 = \frac{25}{9}-1 = \frac{16}{9}$$

$$\tan\beta = -\frac{4}{3} \quad \therefore \beta \text{ lies in II quadrant.}$$

$$\therefore \tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta} = \frac{\frac{1}{2}-\frac{4}{3}}{1-\frac{1}{2}(-4/3)} = \frac{\frac{3-8}{6}}{\frac{6+4}{6}}$$

$$= -5/10 = -1/2$$

28) P.T 1. $\sin(A+B)\sin(A-B) = \sin^2A - \sin^2B$
4BP
2. $\cos(A+B)\cos(A-B) = \cos^2A - \sin^2B$,

Procedure: Apply $\sin(A+B)$, $\sin(A-B)$, $\cos(A+B)\cos(A-B)$. and for ① change all into sin For ② change angle A to cos and B to sin.

Sol: 1) LHS $\sin(A+B)\cdot\sin(A-B)$

$= (\sin A\cos B + \cos A\sin B)(\sin A\cos B - \cos A\sin B)$

$= \sin^2 A\cos^2 B - \cos^2 A\sin^2 B.$

$= \sin^2 A(1-\sin^2 B) - (1-\sin^2 A)\sin^2 B.$

$= \sin^2 A - \cancel{\sin^2 A\sin^2 B} - \sin^2 B + \cancel{\sin^2 A\sin^2 B}$

$= \sin^2 A - \sin^2 B.$

2) $\cos(A+B)\cdot\cos(A-B)$

$= (\cos A\cos B - \sin A\sin B)(\cos A\cos B + \sin A\sin B)$

$= \cos^2 A\cos^2 B - \sin^2 A\sin^2 B.$

$= \cos^2 A(1-\sin^2 B) - (1-\cos^2 A)\sin^2 B$

$= \cos^2 A - \cancel{\cos^2 A\sin^2 B} - \sin^2 B + \cancel{\cos^2 A\sin^2 B}$

$= \cos^2 A - \sin^2 B.$

29) 1) $\sin^2(A+B) - \sin^2(A-B) = \sin 2A\sin 2B$

BP 2) $\cos 8\theta\cos 2\theta = \cos^2 5\theta - \sin^2 3\theta.$

Sol: Procedure: Use $\sin(A+B)\cdot\sin(A-B) = \sin^2 A - \sin^2 B.$
$\cos(A+B)\cdot\cos(A-B) = \cos^2 A - \sin^2 B.$

1) LHS: $\sin^2(\underset{A}{A+B}) - \sin^2(\underset{B}{A-B}) = \sin(A+B+A-B)\cdot\sin(A+B-A+B)$

$= \sin 2A\cdot\sin 2B.$

2) RHS: $\cos^2 5\theta - \sin^2 3\theta = \cos(8\theta+3\theta)\cdot\cos(8\theta-3\theta)$

$= \cos 11\theta\cdot\cos 5\theta.$

30) If $a\cos(x+y) = b(\cos(x-y))$ then P.T $(a+b)\tan x = (a-b)\tan y.$

BP by componendo dividendo method.

Sol:

$a\cos(x+y) = b\cos(x-y)$

$$\frac{a}{b} = \frac{\cos(x-y)}{\cos(x+y)}$$

Now $$\frac{a+b}{a-b} = \frac{\cos(x-y)+\cos(x+y)}{\cos(x-y)-\cos(x+y)}$$

$$= \frac{\cos x\cos y + \cancel{\sin x\sin y} + \cos x\cos y - \cancel{\sin x\sin y}}{\cancel{\cos x\cos y} + \sin x\sin y - \cancel{\cos x\cos y} + \sin x\sin y.}$$

$$\frac{a+b}{a-b} = \frac{\cancel{2}\cos x\cos y}{\cancel{2}\sin x\sin y} = \cot x\cot y$$

$$\frac{a+b}{a-b} = \frac{\cot y}{\tan x}$$

$$(a+b)\tan x = (a-b)\cot y.$$

31) P.T $\sin(n+1)x \sin(n+2)x + \cos(n+1)x\cos(n+2)x = \cos x$.

Sol: Procedure: use $\cos A\cos B + \sin A\sin B = \cos(A-B)$

LHS: $\cos(n+1)x\cos(n+2)x + \sin(n+1)x\sin(n+2)x$

$$= \cos[(\cancel{n}+1)-(\cancel{n}+2)]x.$$

$$= \cos(-x) = \cos x.$$

32) S.T $\cos\left(\frac{\pi}{4}-\theta\right)\cos\left(\frac{\pi}{4}-\varphi\right) - \sin\left(\frac{\pi}{4}-\theta\right)\sin\left(\frac{\pi}{4}-\varphi\right) = \sin(\theta+\varphi)$

Sol: LHS: Procedure; use $\cos A\cos B - \sin A\sin B = \cos(A+B)$

$$\cos\left(\frac{\pi}{4}-\theta\right)\cos\left(\frac{\pi}{4}-\varphi\right) - \sin\left(\frac{\pi}{4}-\theta\right)\sin\left(\frac{\pi}{4}-\varphi\right)$$

$$= \cos\left(\frac{\pi}{4}-\theta+\frac{\pi}{4}-\varphi\right)$$

$$= \cos(90-(\theta+\varphi))$$

$$= \cos(\theta+\varphi)$$

33) If $\cos(\theta+\varphi) = m\cos(\theta-\varphi)$ then find the value of $\frac{1-m}{1+m}\cot\varphi$.

$$\frac{1}{m} = \frac{\cos(\theta-\varphi)}{\cos(\theta+\varphi)}$$

$\cos(A-B) - \cos(A+B) = 2\sin A\sin B$

$\cos(A-B) + \cos(A+B) = 2\cos A\cos B$

Now $$\frac{1-m}{1+m} = \frac{\cos(\theta-\varphi) - \cos(\theta+\varphi)}{\cos(\theta-\varphi) + \cos(\theta+\varphi)}$$

$$= \frac{\cancel{2}\sin\theta\sin\varphi}{\cancel{2}\cos\theta\cos\varphi}$$

$$\frac{1-m}{1+m} = \tan\theta\tan\varphi.$$

$$= \frac{\tan\theta}{\cot\varphi}$$

$$\left(\frac{1-m}{1+m}\right)\cot\varphi = \tan\theta.$$

34) S.T $\sin(40+\theta)\cos(10+\theta) - \cos(40+\theta)\cdot\sin(10+\theta) = 1/2$

Procedure: use $\sin A\cos B - \cos A\sin B = \sin(A-B)$

LHS: $\sin(40+\theta)\cos(10+\theta) - \cos(40+\theta)\cdot\sin(10+\theta)$

$$= \sin(40+\cancel{\theta}-10-\cancel{\theta}) = \sin 30 = 1/2$$

35) If $\cos(\alpha+\beta) = \frac{4}{5}$, $\sin(\alpha-\beta) = \frac{5}{13}$ and $0 < \alpha, \beta < \pi/4$ then P.T $\tan(2\alpha) = \frac{56}{33}$.

Sol: Procedure: Find $\sin(\alpha+\beta)$ and $\cos(\alpha-\beta)$ then $\tan(\alpha+\beta)$ and $\tan(\alpha-\beta)$ from the given values.
$\tan 2\alpha = \tan(\alpha+\beta+\alpha-\beta) = \tan(A+B)$ type.

Since $0 < \ldots < \pi/4 \Rightarrow -\pi/4 < \alpha-\beta < \pi/4$ and $0 < \alpha+\beta < \pi/2$.

$$\cos(\alpha-\beta) = \sqrt{1-\sin^2(\alpha-\beta)} = \cancel{\sqrt{1-\frac{16}{25}} = \frac{3}{5}} = \sqrt{1-\frac{25}{169}} = \frac{12}{13}$$

$$\sin(\alpha+\beta) = \sqrt{1-\cos^2(\alpha+\beta)} = \cancel{\sqrt{1-\frac{25}{169}} = \frac{12}{13}} = \sqrt{1-\frac{16}{25}} = \frac{3}{5}$$

$$\tan(\alpha+\beta) = \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} = \frac{3/5}{4/5} = 3/4.$$

$$\tan(\alpha-\beta) = \frac{\sin(\alpha-\beta)}{\cos(\alpha-\beta)} = \frac{5/13}{12/13} = 5/12$$

$$\tan 2\alpha = \tan((\alpha+\beta)+(\alpha-\beta)) = \frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta)\cdot\tan(\alpha-\beta)}$$

$$= \frac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}\cdot\frac{5}{12}} = \frac{\frac{36+20}{48}}{\frac{48-15}{48}} = \frac{56}{33}.$$

36) If $\tan(\pi\cos\theta) = \cot(\pi\sin\theta)$ then P.T $\cos(\theta-\pi/4) = \pm\frac{1}{2\sqrt{2}}$

Sol: Procedure: Change LHS and RHS interms of Sin and cos. then cross multiply. Use $\cos A\cos B - \sin A\sin B = \cos(A+B)$ and multiply on both sides by $\frac{1}{\sqrt{2}}$ (ie) $\cos\pi/4$ and $\sin\pi/4$.

Given $\tan(\pi\cos\theta) = \cot(\pi\sin\theta)$

$$\frac{\sin(\pi\cos\theta)}{\cos(\pi\cos\theta)} = \frac{\cos(\pi\sin\theta)}{\sin(\pi\sin\theta)}.$$

$$\sin(\pi\cos\theta)\sin(\pi\sin\theta) = \cos(\pi\sin\theta)\cos(\pi\cos\theta)$$

$$\cos(\pi\sin\theta)\cdot\cos(\pi\cos\theta) - \sin(\pi\sin\theta)\sin(\pi\cos\theta) = 0$$

$$\cos(\pi\sin\theta+\pi\cos\theta) = 0$$

$$\pi\sin\theta + \pi\cos\theta = \cos^{-1}(0) = \pm\frac{\pi}{2}$$

$$\pi(\sin\theta+\cos\theta) = \pm\frac{\pi}{2}$$

36) If $\tan\alpha = \dfrac{1}{\sqrt{x(x^2+x+1)}}$, $\tan\beta = \dfrac{\sqrt{x}}{\sqrt{x^2+x+1}}$, $\tan\gamma = \sqrt{\dfrac{1}{x}+\dfrac{1}{x^2}+\dfrac{1}{x^3}}$

then P.T $\alpha+\beta=\gamma$.

Sol: Procedure: Find $\tan(\alpha+\beta)$ S.T it is equal to $\tan\gamma$.

$$\tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta} = \frac{\dfrac{1}{\sqrt{x(x^2+x+1)}} + \dfrac{\sqrt{x}}{\sqrt{x^2+x+1}}}{1-\left(\dfrac{1}{\sqrt{x(x^2+x+1)}}\right)\left(\dfrac{\sqrt{x}}{\sqrt{x^2+x+1}}\right)}$$

$$= \frac{\dfrac{1+x}{\sqrt{x}\sqrt{x^2+x+1}}}{1-\dfrac{\sqrt{x}}{\sqrt{x}(x^2+x+1)}}$$

$$= \frac{\dfrac{1+x}{\sqrt{x}\sqrt{x^2+x+1}}}{\dfrac{x^2+x+\cancel{1}-\cancel{1}}{x^2+x+1}}$$

$$= \frac{(1+x)(x^2+x+1)}{\sqrt{x}\,(x\,\cancel{(x+1)})\sqrt{x^2+x+1}}$$

$$= \frac{\sqrt{x}\sqrt{x^2+x+1}}{\sqrt{x}\cdot\sqrt{x}\cdot x}$$

$$= \sqrt{\frac{x^3+x^2+x}{x^2}}$$

$$= \sqrt{\frac{x^3+x^2+x}{x^4}} = \sqrt{\frac{1}{x}+\frac{1}{x^2}+\frac{1}{x^3}}$$

$$= \tan\gamma.$$

37) P.T $\dfrac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)} = \left(\dfrac{1+\tan x}{1-\tan x}\right)^2$ Procedure: Just expand LHS and Simplify.

Sol:
$$\frac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)} = \frac{\dfrac{\tan\frac{\pi}{4}+\tan x}{1-\tan\frac{\pi}{4}\tan x}}{\dfrac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}}$$

$$= \frac{\dfrac{1+\tan x}{1-\tan x}}{\dfrac{1-\tan x}{1+\tan x}} = \frac{(1+\tan x)^2}{(1-\tan x)^2}$$

38) If $A+B+C=\pi$ and P.T $\tan A+\tan B+\tan C$.
(BP.

If $A+B+C=\pi$

$A+B=\pi-C$

$\tan(A+B) = \tan(\pi-C)$

$\dfrac{\tan A+\tan B}{\quad} = -\tan C$.

$\tan A+\tan B = -\tan C + \tan A\tan B\tan C$

$\tan A+\tan B+\tan C = \tan A\tan B\tan C$

This result is also true in the case oblique triangle.

39) P.T) $\tan\left(\frac{\pi}{4}+\theta\right)\tan\left(\frac{\pi}{4}-\theta\right)=1$

2) $\tan\left(\frac{\pi}{4}+\theta\right)\tan\left(\frac{3\pi}{4}+\theta\right)=-1$

3) $\cot\left(\frac{\pi}{4}+\theta\right)\cot\left(\frac{\pi}{4}-\theta\right)=1$

Sol: Procedure use $\tan\left(\frac{\pi}{4}+\theta\right)=\frac{1+\tan\theta}{1-\tan\theta}$

and $\tan\left(\frac{\pi}{4}-\theta\right)=\frac{1-\tan\theta}{1+\tan\theta}$.

1) $\tan\left(\frac{\pi}{4}+\theta\right)\tan\left(\frac{\pi}{4}-\theta\right)=\frac{1+\tan\theta}{1-\tan\theta}\times\frac{1-\tan\theta}{1+\tan\theta}=1$

2) $\tan\left(\frac{3\pi}{4}+\theta\right)=\tan\left(\pi-\left(\frac{\pi}{4}-\theta\right)\right)=-\tan\left(\frac{\pi}{4}-\theta\right)$

LHS: $\tan\left(\frac{\pi}{4}+\theta\right)\left(-\tan\left(\frac{\pi}{4}-\theta\right)\right)=\left(\frac{1+\tan\theta}{1-\tan\theta}\right)\left(-\left(\frac{1-\tan\theta}{1+\tan\theta}\right)\right)$

$=-1$.

4) $\cot\left(\frac{\pi}{4}+\theta\right)\cdot\cot\left(\frac{\pi}{4}-\theta\right)=\frac{1}{\tan\left(\frac{\pi}{4}+\theta\right)\tan\left(\frac{\pi}{4}-\theta\right)}$

$=\frac{1}{\frac{1+\tan x}{1-\tan x}\times\frac{1-\tan x}{1+\tan x}}$

$=\frac{1}{1}=1.$

40) If $A+B=45^\circ$ P.T) $(1+\tan A)(1+\tan B)=2$

2) $(\cot A-1)(\cot B-1)=2$.

Sol: Procedure: use $\tan\left(\frac{\pi}{4}-B\right)=\frac{1-\tan B}{1+\tan B}$

If $A+B=45$

$B=45^\circ-A$

$\tan B=\tan\left(\frac{\pi}{4}-A\right)=\frac{1-\tan A}{1+\tan A}$.

$\therefore$ 1) L.HS $(1+\tan A)(1+\tan B)=(1+\tan A)\left(1+\frac{1-\tan A}{1+\tan A}\right)$

$=(1+\tan A)\left(\frac{1+\tan A+1-\tan A}{1+\tan A}\right)$

$=\cancel{(1+\tan A)}\cdot\frac{2}{\cancel{1+\tan A}}=2$

2) $(\cot A-1)(\cot B-1)$

$=\left(\frac{1}{\tan A}-1\right)\left(\frac{1}{\tan B}-1\right)=\frac{(1-\tan A)(1-\tan B)}{\tan A\tan B}$.

$$\frac{(1-\tan A)\left(1-\frac{1-\tan A}{1+\tan A}\right)}{\tan A \cdot \frac{1-\tan A}{1+\tan A}}$$

$$= \frac{(1-\tan A)\left(\frac{1+\tan A-1+\tan A}{1+\tan A}\right)}{\frac{\tan A\,(1-\tan A)}{(1+\tan A)}} = \frac{\cancel{(1-\tan A)}\cdot 2\cancel{\tan A}}{\cancel{\tan A}\,\cancel{(1-\tan A)}}$$

$$= 2.$$

41) If $A+B = 225^\circ$ P.T $\frac{\cot A}{1+\cot A} \cdot \frac{\cot B}{1+\cot B} = \frac{1}{2}$.

Sol: Procedure: $A+B = 225$

$B = 225 - A = (180 + \overline{45-A})$

$\tan B = \tan(180+\overline{45-A}) = \tan(45-A)$

$= \frac{1-\tan A}{1+\tan B}$.

LHS: $\frac{\frac{1}{\tan A}}{1+\frac{1}{\tan A}} \cdot \frac{\frac{1}{\tan B}}{1+\frac{1}{\tan B}} = \frac{1 \times \cancel{\tan A}}{\cancel{\tan A}\,(1+\tan A)} \times \frac{\cancel{\tan B}}{\cancel{\tan B}\,(1+\tan B)}$

$= \frac{1}{1+\tan A} \times \frac{1}{1+\tan B}$

$= \frac{1}{1+\tan A} \times \frac{1}{1+\frac{1-\tan A}{1+\tan A}}$

$= \frac{1}{\cancel{1+\tan A}} \times \frac{\cancel{1+\tan A}}{1+\tan A+1-\tan A}$

$= \frac{1}{2}$

42) P.T $\tan(A-B)+\tan(B-C)+\tan(C-A) = \tan(A-B)\cdot\tan(B-C)\tan(C-A)$

Sol: Procedure: consider $\alpha = A-B$, $\beta = B-C$, $\gamma = C-A$.
and apply $\tan(A+B)$ formula.

$\alpha+\beta+\gamma = A-B+B-C+C-A = 0$

$\alpha+\beta = -\gamma$

$\tan(\alpha+\beta) = \tan(-\gamma)$

$\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta} = -\tan\gamma$

$\Rightarrow \tan\alpha+\tan\beta = -\tan\gamma\,(1-\tan\alpha\tan\beta)$

$\tan\alpha+\tan\beta = -\tan\gamma + \tan\alpha\tan\beta\tan\gamma$

$\tan\alpha+\tan\beta+\tan\gamma = \tan\alpha\tan\beta\tan\gamma$

(e) $\tan(A-B)+\tan(B-C)+\tan(C-A) = \tan(A-B)\tan(B-C)\tan(C-A)$

Multiple angle identities and Sub Multiple angle identities

1) Find the value of $\sin 22\frac{1}{2}°$

BP Sol: Let $\theta = 45°$

$$\cos\theta = 1 - 2\sin^2\theta/2 \Rightarrow 2\sin^2\theta/2 = 1-\cos\theta$$

$$\sin^2\theta/2 = \frac{1-\cos\theta}{2}$$

$$\sin\theta/2 = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

$$\therefore \sin 22\tfrac{1}{2}° = \pm\sqrt{\frac{1-\cos 45°}{2}}$$

$$= \pm\sqrt{\frac{1-\frac{1}{\sqrt{2}}}{2}} = \pm\sqrt{\frac{\sqrt{2}-1}{2}} = \frac{\sqrt{1-\sqrt{2}}}{2}.$$ $\because 22\frac{1}{2}°$ is acute.

2) Find the value of $\sin 2\theta$ when $\sin\theta = \frac{12}{13}$, θ lies in I quadrant

BP

Sol: $\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\frac{144}{169}} = 5/13$, $\because \theta$ lies in I quadrant

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$= 2\cdot\frac{12}{13}\cdot\frac{5}{13} = \frac{120}{169}.$$

3) P.T $\sin 4A = 4\sin A\cos^3 A - 4\cos A\sin^3 A$.

BP. Sol: RHS: $4\sin A\cos^3 A - 4\cos A\sin^3 A$

$$= 4\sin A\cos A(\cos^2 A - \sin^2 A)$$

$$= 2\cdot 2\sin A\cos A\cdot\cos 2A.$$

$$= 2\sin 2A\cdot\cos 2A$$

$$= \sin 4A.$$

4) P.T $\sin x = 2^{10}\sin\left(\frac{x}{2^{10}}\right)\cos\left(\frac{x}{2}\right)\cos\left(\frac{x}{2^2}\right)\cdots\cos\left(\frac{x}{2^{10}}\right)$

BP.

Sol: LHS: $\sin x = 2\sin\frac{x}{2}\cos x/2$

$$= 2\cdot 2\sin\frac{x}{4}\cos\frac{x}{4}\cdot\cos x/2$$

$$= 4\cdot 2\cdot\sin\frac{x}{8}\cos\frac{x}{8}\cdot\cos\frac{x}{4}\cdot\cos\frac{x}{2}$$

$$= 2^4\sin\frac{x}{2^4}\cos\frac{x}{2^4}\cdot\cos\frac{x}{2^3}\cdot\cos\frac{x}{2^2}\cdot\cos\frac{x}{2}$$

we can proceed the same way

$$\sin x = 2^{10}\left(\sin\frac{x}{2^{10}}\right)\cdot\cos\frac{x}{2^{10}}\cdot\cos\frac{x}{2^9}\cdots\cos\frac{x}{2^3}\cdot\cos\frac{x}{2^2}\cdot\cos\frac{x}{2}$$

5) P.T $\frac{\sin\theta+\sin 2\theta}{1+\cos\theta+\cos 2\theta} = \tan\theta$

BP

Sol: LHS: $\frac{\sin\theta+\sin 2\theta}{1+\cos\theta+\cos 2\theta} = \frac{\sin\theta+2\sin\theta\cos\theta}{\cos\theta+1+\cos 2\theta} = \frac{\sin\theta(1+2\cos\theta)}{\cos\theta+2\cos^2\theta} = \frac{\sin\theta\,\cancel{(1+2\cos\theta)}}{\cos\theta\,\cancel{(1+2\cos\theta)}}$

$= \tan\theta.$

6) P.T $1-\frac{1}{2}\sin 2x = \frac{\sin^3 x+\cos^3 x}{\sin x+\cos x}$.
ABP

Sol: Procedure: apply $a^3+b^3=(a+b)(a^2-ab+b^2)$

RHS: $\frac{\sin^3 x+\cos^3 x}{\sin x+\cos x} = \frac{\cancel{(\sin x+\cos x)}(\sin^2 x-\sin x\cos x+\cos^2 x)}{\cancel{(\sin x+\cos x)}}$

$= 1-\sin x\cos x$

$= 1-\frac{1}{2}\cdot 2\sin x\cos x$

$= 1-\frac{1}{2}\sin 2x$.

7) Find x s.t $-\pi\le x\le \pi$ and $\cos 2x = \sin x$.
ABP

Sol: Given. $\cos 2x = \sin x$

$1-2\sin^2 x = \sin x$

$2\sin^2 x+\sin x-1=0$

$\sin x = \frac{-1\pm 3}{4} = -1$ or $\frac{1}{2}$ $\because -\pi<x<\pi$

$\sin x=\frac{1}{2} \Rightarrow x=\frac{\pi}{6}, \frac{\pi}{6},$

$=\frac{\pi}{6}, \frac{5\pi}{6}$

$\sin x=-1 \Rightarrow x=-\frac{\pi}{2}$

$\therefore x=-\frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$.

8) Find the values of 1) $\sin 18°$ 2) $\cos 18°$ 3) $\sin 72°$ 4) $\cos 36$ 5) $\sin 54°$.
ABP

Sol: Let $\theta=18°$

1) $\sin 18°$

$5\theta=90°$

$3\theta+2\theta=90$

$2\theta=90-3\theta$

$\sin 2\theta=\sin(90-3\theta)$

$=\cos 3\theta$

$2\sin\theta\cos\theta=4\cos^3\theta-3\cos\theta$

$\div\cos\theta$ $\quad 2\sin\theta=4\cos^2\theta-3$

$=4(1-\sin^2\theta)-3$

$=4-4\sin^2\theta-3$

$=1-4\sin^2\theta$

$\therefore 4\sin^2\theta+2\sin\theta-1=0$ $\quad a=4,\ b=2,\ c=1$

$\sin\theta=\frac{-2\pm\sqrt{4+16}}{8}$

$=\frac{-2\pm 2\sqrt{5}}{8}=\frac{-1\pm\sqrt{5}}{4}$ $\because \theta$ is $18°$

$\sin 18°=\frac{-1+\sqrt{5}}{4}$

2) $\cos 18°=\sqrt{1-\sin^2 18°}$

$=\sqrt{1-\left(\frac{\sqrt{5}-1}{4}\right)^2}$

$=\sqrt{\frac{16-(5+1-2\sqrt{5})}{16}}$

$=\frac{\sqrt{10+2\sqrt{5}}}{4}$.

3) $\sin 72°=\sin(90-18)$

$=\cos 18°$

$=\frac{\sqrt{10+2\sqrt{5}}}{4}$

4) $\cos 36 = 1-2\sin^2 18$ $\quad (2\theta)$

$=1-2\left(\frac{\sqrt{5}-1}{4}\right)^2$

$=\frac{\sqrt{5}+1}{4}$

5) $\sin 54°=\sin(90-36)$

$=\cos 36=\frac{\sqrt{5}+1}{4}$

9) TBP If $\tan\frac{\theta}{2} = \sqrt{\frac{1-a}{1+a}}\tan\frac{\varphi}{2}$ then P.T $\cos\varphi = \frac{\cos\theta - a}{1 - a\cos\theta}$.

LHS: $\cos\varphi = \frac{1-\tan^2\frac{\varphi}{2}}{1+\tan^2\varphi/2}$ (half angle formula)

$$= \frac{1-\left(\frac{1+a}{1-a}\right)\tan^2\frac{\theta}{2}}{1+\left(\frac{1+a}{1-a}\right)\tan^2\theta/2} =$$

$$= \frac{\frac{(1-a)-(1+a)\tan^2\theta/2}{\cancel{1-a}}}{\frac{(1-a)+(1+a)\tan^2\theta/2}{\cancel{1-a}}} = \frac{(1-\tan^2\theta/2) - a(1+\tan^2\theta/2)}{(1+\tan^2\theta/2) - a(1-\tan^2\theta/2)}$$

$$= \frac{\frac{1-\tan^2\theta/2}{1+\tan^2\theta/2} - a}{1 - a\left(\frac{1-\tan^2\theta/2}{1+\tan^2\theta/2}\right)}$$

$$= \frac{\cos\theta - a}{1 - a\cos\theta}.$$

10) TBP Find the value of $\sqrt{3}\operatorname{cosec}20^\circ - \sec 20^\circ$

Sol: $\sqrt{3}\operatorname{cosec}20 - \sec 20^\circ$

$$= \frac{\sqrt{3}}{\sin 20} - \frac{1}{\cos 20}$$

$$= \frac{\sqrt{3}\cos 20 - \sin 20}{\sin 20\cos 20}$$

$$= 4\cdot\frac{\frac{\sqrt{3}}{2}\cos 20 - \frac{1}{2}\sin 20}{2\sin 20\cos 20}$$

∴ Nr by 2
Multiplying Dr by 2.

$$= 4\left[\frac{\sin 60\cos 20 - \cos 60\sin 20}{\sin 40}\right]$$

$$= 4\left[\frac{\cancel{\sin(60-20)}}{\cancel{\sin 40}}\right] = 4.$$

11) TBP P.T $\cos A\cdot\cos 2A\cos 2^2A\cos 2^3A \cdots \cos 2^nA = \frac{\sin 2^nA}{2^n\sin A}$.

12) P.T $\cos A \cos 2A \cos 2^2 A \cos 2^3 A \cdots \cos 2^{n-1} A = \dfrac{\sin 2^n A}{2^n \sin A}$
TBQ.

Sol: Procedure: to use $\sin 2A$ formula dividing and Multiplying by $2 \sin A$, similarly proceed like n step.

LHS $\cos A \cos 2A \cos 2^2 A \cos 2^3 A \cdots \cos 2^{n-1} A$.

$$= \frac{1}{2\sin A} \cdot 2\sin A \cos A \cdot \cos 2A \cos 2^2 A \cdots \cos 2^{n-1} A.$$

$$= \frac{1}{2\sin A} \cdot \sin 2A \cos 2A \cos 2^2 A \cdot \cos 2^3 A \cdots \cos 2^{n-1} A.$$

$$= \frac{1}{2^2 \sin A} \cdot 2\sin 2A \cos 2A \cdot \cos 2^2 A \cos 2^3 A \cdots \cos 2^{n-1} A.$$

$$= \frac{1}{2^2 \sin A} \cdot \sin 2^2 A \cdot \cos 2^2 A \cos 2^3 A \cdots \cos 2^{n-1} A.$$

$$= \frac{1}{2^3 \sin A} \cdot 2\sin 2^2 A \cos 2^2 A \cdot \cos 2^3 A \cdots \cos 2^{n-1} A.$$

$$= \frac{1}{2^3 \sin A} \cdot \sin 2^3 A \cos 2^3 A \cdots \cos 2^{n-1} A$$

Proceed as follows for n steps.

$$= \frac{1}{2^n \sin A} \cdot \sin 2^n A.$$

13) Find The value of $\cos 2A$, A lies in the first quadrant when,
TBP 1) $\cos A = \frac{15}{17}$ 2) $\sin A = \frac{4}{5}$ 3) $\tan A = \frac{16}{63}$.

when 1) $\cos A = \frac{15}{17}$. $\sin A = \sqrt{1 - \frac{225}{289}} = \frac{8}{17}$.

$$\therefore \cos 2A = 1 - 2\sin^2 A = 1 - 2 \cdot \frac{64}{289} = \frac{161}{289}.$$

$$\begin{array}{r} 289 \\ 128 \\ \hline 161 \end{array}$$

when 2) $\sin A = 4/5$ $\cos A = \sqrt{1 - 16/25} = 3/5$

$$\cos 2A = 2\cos^2 A - 1 = 2 \cdot \frac{9}{25} - 1$$

14) P T $\cos 5x = 16\cos^5 x - 20\cos^3 x + 5\cos x$

TBP

Sol: Procedure: take $\cos 5x = \cos(3x+2x)$. Expand. Then use $\cos 3x$, $\sin 3x$, $\cos 2x$, $\sin 2x$ formulae and simplify and change all sin into cos.

$\cos 5x = \cos(3x+2x) = \cos 3x \cos 2x - \sin 3x \sin 2x$.

$= (4\cos^3 x - 3\cos x)(2\cos^2 x - 1) - (3\sin x - 4\sin^3 x)\cdot 2\sin x \cos x$.

$= (4\cos^3 x - 3\cos x)(2\cos^2 x - 1) - (3 - 4\sin^2 x)\cdot 2\sin^2 x \cos x$

$= (4\cos^3 x - 3\cos x)(2\cos^2 x - 1) - (3 - 4(1-\cos^2 x))\cdot 2(1-\cos^2 x)\cdot \cos x$

$= (4\cos^3 x - 3\cos x)(2\cos^2 x - 1) - (\overset{(-1)}{3-4} + 4\cos^2 x)(2\cos x - 2\cos^3 x)$

$= (8\cos^5 x - 4\cos^3 x - 6\cos^3 x + 3\cos x) - (-2\cos x + 2\cos^3 x + 8\cos^3 x - 8\cos^5 x)$

$= (8\cos^5 x - 10\cos^3 x + 3\cos x) - (-2\cos x + 2\cos^3 x + 8\cos^3 x - 8\cos^5 x)$

$= 16\cos^5 x - 20\cos^3 x + 5\cos x$.

15) P.T $\cot 7\tfrac{1}{2}^\circ = \tan 82\tfrac{1}{2}^\circ = (\sqrt{3}+\sqrt{2})(\sqrt{2}+1)$ (or) $\sqrt{2}+\sqrt{3}+\sqrt{4}+\sqrt{6}$.

T.B.P

$LHS = \tan 82\tfrac{1}{2}^\circ = \tan(90 - 7\tfrac{1}{2})$

$= \cot 7\tfrac{1}{2}^\circ = \cot A$ (say)

$$\cot A = \frac{\cos A}{\sin A} = \frac{\cos A(2\cos A)}{\sin \cdot 2(\cos A)} = \frac{1+\cos 2A}{\sin 2A}$$

$$\cot 7\tfrac{1}{2}^\circ = \frac{1+\cos 15}{\sin 15^\circ} = \frac{1+\cos(45-30)}{\sin(45-30)}$$

$$= \frac{1+(\cos 45\cos 30 + \sin 45 \sin 30)}{\sin 45 \cos 30 - \cos 45 \sin 30}$$

$$= \frac{1+\frac{1}{\sqrt{2}}\cdot\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}\cdot\frac{1}{2}}{\frac{1}{\sqrt{2}}\cdot\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}\cdot\frac{1}{2}} = \frac{\frac{2\sqrt{2}+\sqrt{3}+1}{2\sqrt{2}}}{\frac{\sqrt{3}-1}{2\sqrt{2}}}$$

$$= \frac{(2\sqrt{2}+\sqrt{3}+1)(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$$

$$= \frac{2\sqrt{6}+2\sqrt{2}+3+\sqrt{3}+\sqrt{3}+1}{2}$$

$= \sqrt{6}+\sqrt{2}+\sqrt{3}+2$

$= \sqrt{6}+\sqrt{2}+\sqrt{3}+\sqrt{4}$

$= (\sqrt{3}+\sqrt{2})(\sqrt{2}+1)$

Procedure

Consider $\cot A = \frac{\cos A}{\sin A}$

make it as $\frac{1+\cos 2A}{\sin 2A}$ by Multiplying and dividing both Nr and Dr by $2\cos A$.

Apply $A = 7\tfrac{1}{2}^\circ$ (ie) $2A = 15^\circ$

Make 15° as $(45-30^\circ)$ then simplify.

16) PT $32\sqrt{3}\sin\frac{\pi}{48}\cos\frac{\pi}{48}\cos\frac{\pi}{24}\cos\frac{\pi}{12}\cos\frac{\pi}{6} = 3$

TBP

Sol: Procedure: in order to use $2\sin A\cos A = \sin 2A$ split 32 into 5, twos and join with all functions and simplify.

18) If $A+B=45$ P.T $(1+\tan A)(1+\tan B)=2$

TBP Sol: Procedure: Apply tan on both sides and add 1 on both sides to ~~simplify~~ factorise we will get answer.

Given $A+B=45°$

$$\tan(A+B)=\tan 45°$$

$$\frac{\tan A+\tan B}{1-\tan A\tan B}=1$$

$$\tan A+\tan B=1-\tan A\tan B$$

$$\tan A+\tan B+\tan A\tan B=1$$

$$(1+\tan A)+\tan B(1+\tan B)=1+1$$

$$(1+\tan A)(1+\tan B)=2$$

19) P.T $\tan\left(\frac{\pi}{4}+\theta\right)-\tan\left(\frac{\pi}{4}-\theta\right)=2\tan 2\theta$.

TBP

Sol: Procedure: Apply $\tan\left(\frac{\pi}{4}+\theta\right)=\frac{1+\tan\theta}{1-\tan\theta}$

$\tan\left(\frac{\pi}{4}-\theta\right)=\frac{1-\tan\theta}{1+\tan\theta}$.

and simplify.

LHS: $\tan\left(\frac{\pi}{4}+\theta\right)-\tan\left(\frac{\pi}{4}-\theta\right)=\frac{1+\tan\theta}{1-\tan\theta}-\frac{1-\tan\theta}{1+\tan\theta}$

$$=\frac{(1+\tan\theta)^2-(1-\tan\theta)^2}{1-\tan^2\theta}$$

$$=\frac{4\tan\theta}{1-\tan^2\theta}=\frac{2\cdot 2\tan\theta}{1-\tan^2\theta}$$

$$=2\tan 2\theta$$

20)

LHS: $32\sqrt{3}\sin\frac{\pi}{48}\cos\frac{\pi}{48}\cos\frac{\pi}{24}\cos\frac{\pi}{12}\cdot\cos\frac{\pi}{6}$. 16^{th} continuation -

$= \sqrt{3}\cdot 2\cdot \underline{2\sin\frac{\pi}{48}\cos\frac{\pi}{48}}\cdot 2\cos\frac{\pi}{24}\cdot 2\cos\frac{\pi}{12}\cdot 2\cos\frac{\pi}{6}.$

$= 2\sqrt{3}\cdot \underline{\sin\frac{2\pi}{\cancel{48}_{24}}\cdot 2\cos\frac{\pi}{24}}\cdot 2\cos\frac{\pi}{12}\cdot 2\cos\frac{\pi}{6}.$

$= 2\sqrt{3}\cdot \underline{\sin\frac{2\pi}{\cancel{24}_{12}}\cdot 2\cos\frac{\pi}{12}}\cdot 2\cos\frac{\pi}{6}$

$= 2\sqrt{3}\sin\frac{2\pi}{\cancel{12}_{6}}\cdot 2\cos\frac{\pi}{6}$

$= 2\sqrt{3}\sin\frac{2\pi}{\cancel{6}_{3}}$

$= 2\sqrt{3}\cdot\frac{\sqrt{3}}{2} = 3 = RHS.$

17) P.T $\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cdot\cos\frac{8\pi}{15}\cos\frac{16\pi}{15} = \frac{1}{16}.$

Sol: Procedure: Multiplying and dividing the LHS by $16\sin\frac{16\pi}{}$ and this 16 is also split into 4, twos and join with all f's and rearrange into decending order.

LHS $\cos\frac{2\pi}{15}\cdot\cos\frac{4\pi}{15}\cdot\cos\frac{8\pi}{15}\cos\frac{16\pi}{15}$

$= \frac{1}{16\sin\frac{2\pi}{15}}\cdot 2\sin\frac{2\pi}{15}\cos\frac{2\pi}{15}\cdot 2\cos\frac{4\pi}{15}\cdot 2\cos\frac{8\pi}{5}\cdot 2\cos\frac{16\pi}{15}$

$= \frac{1}{16\sin\frac{2\pi}{15}}\underline{2\sin\frac{2\pi}{15}\cos\frac{2\pi}{15}}\cdot 2\cos\frac{4\pi}{15}\cdot 2\cos\frac{8\pi}{16}\cdot 2\cos\frac{16\pi}{15}$

$= \frac{1}{16\sin\frac{2\pi}{15}}\cdot\sin\frac{4\pi}{15}\cdot 2\cos\frac{4\pi}{15}\cdot 2\cos\frac{8\pi}{15}\cdot 2\cos\frac{16\pi}{15}$

$= \frac{1}{16\sin\frac{2\pi}{15}}\cdot\sin\frac{8\pi}{15}\cdot 2\cos\frac{8\pi}{15}\cdot 2\cos\frac{16\pi}{15}$

$= \frac{1}{16\sin\frac{2\pi}{15}}\cdot\sin\frac{16\pi}{15}\cdot 2\cos\frac{16\pi}{15}$

$= \frac{1}{16\sin\frac{2\pi}{15}}\cdot\sin\frac{32\pi}{15}$ $\left(\sin\frac{32\pi}{15} = \sin\left(2\pi+\frac{2\pi}{15}\right) = \sin\frac{2\pi}{15}\right)$

$= \frac{1}{16\cancel{\sin\frac{2\pi}{15}}}\cdot\cancel{\sin\frac{2\pi}{15}}$

$= \frac{1}{16} = RHS.$

21) If $\tan\alpha = \frac{1}{7}$, $\sin\beta = \frac{1}{\sqrt{10}}$ P.T $\alpha + 2\beta = \frac{\pi}{4}$ $0 < \alpha < \pi/2$, $0 < \beta < \pi/2$

Sol: Procedure: α, β lies in the first quadrant. ∴ All ratios are +ve. From $\sin\beta$ find $\cos\beta$ and then find $\tan\beta$.
By using $\tan 2A = \frac{2\tan A}{1-\tan^2}$ find $\tan 2\beta$ then by using $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$ find $\tan(\alpha+2\beta)$ then we get the result.

$\cos\beta = \sqrt{1-\sin^2\beta} = \sqrt{1-\frac{1}{10}} = \frac{3}{\sqrt{10}}$ ∴ $\tan\beta = \frac{\sin\beta}{\cos\beta} = \frac{\frac{1}{\sqrt{10}}}{3/\sqrt{10}} = \frac{1}{3}$.

$$\tan 2\beta = \frac{2\tan\beta}{1-\tan^2\beta} = \frac{2\cdot\frac{1}{3}}{1-\frac{1}{9}} = \frac{2}{3} \times \frac{9}{8} = \frac{3}{4}$$

$$\tan(\alpha+2\beta) = \frac{\tan\alpha + \tan 2\beta}{1 - \tan\alpha \cdot \tan 2\beta} = \frac{\frac{1}{7}+\frac{3}{4}}{1-\frac{1}{7}\cdot\frac{3}{4}}$$

$$= \frac{\frac{4+21}{28}}{\frac{28-25}{28}} = \frac{25/28}{25/28}$$

$\tan(\alpha+2\beta) = 1$

$\alpha + 2\beta = \pi/4$.

22) If $\cos\theta = \frac{1}{2}\left(a+\frac{1}{a}\right)$ S.T $\cos 3\theta = \frac{1}{2}\left(a^3+\frac{1}{a^3}\right)$

TBP

Sol: Procedure: Take LHS as $2\cos 3\theta$ and use $\cos 3\theta$ formula. and sub. the value of $2\cos\theta$. use $a^3+b^3 = (a+b)^3 - 3ab(a+b)$

$LHS = 2\cos 3\theta = 2\left[4\cos^3\theta - 3\cos\theta\right]$

$= 8\cos^3\theta - 6\cos\theta$.

$= (2\cos\theta)^3 - 3\cdot 2\cos\theta$.

Given. sub.

$\boxed{\because 2\cos\theta = a+\frac{1}{a}}$

$= \left(a+\frac{1}{a}\right)^3 - 3\left(a+\frac{1}{a}\right)$

$= a^3 + \frac{1}{a^3} + 3a\cdot\frac{1}{a}\left(a+\frac{1}{a}\right) - 3\left(a+\frac{1}{a}\right)$

$= a^3 + \frac{1}{a^3}$

23). P.T $(1+\tan 1^\circ)(1+\tan 2^\circ)\cdots(1+\tan 44^\circ)$ is multiple of 4

TBP. Sol: we know that If $A+B = 45$ $(1+\tan A)(1+\tan B) = 2$

$1+44 = 45$ ∴ $(1+\tan 1^\circ)(1+\tan 44) = 2$

$2+43 = 45$ $(1+\tan 2)(1+\tan 43) = 2$

lll^{ly} we can get 22 sets of 2s ∴ $(1+\tan 1)(1+\tan 2^\circ)\cdots(1+\tan 44^\circ) = 2^{22} = 4^{11}$

which is the multiple of 4.

24) P.T $\sin 4\alpha = 4\tan\alpha \dfrac{(1-\tan^2\alpha)}{(1+\tan^2\alpha)^2}$
TBP

Sol: Procedure: use $\sin 2\alpha = \dfrac{2\tan\alpha}{1+\tan^2\alpha}$, $\cos 2\alpha = \dfrac{1-\tan^2\alpha}{1+\tan^2\alpha}$

$$LHS = \sin 4\alpha = \sin 2(2\alpha)$$
$$= 2\sin 2\alpha \cos 2\alpha.$$
$$= 2\cdot \frac{2\tan\alpha}{1+\tan^2\alpha} \times \frac{1-\tan^2\alpha}{1+\tan^2\alpha}$$
$$= 4\tan\alpha \frac{(1-\tan^2\alpha)}{(1+\tan^2\alpha)^2}$$

25) P.T $(1+\sec 2x)(1+\sec 4x)(1+\sec 8x)\ldots\ldots(1+\sec 2^n x) = \tan 2^n x \cot x$
TBP

Sol: Procedure: use $\sec 2x = \dfrac{1}{\cos 2x}$, $1+\cos 2x = 2\cos^2 x$
and $\cos x \cos 2x \cos 2^2 x \ldots\ldots \cos 2^{n-1} x = \dfrac{\sin 2^n x}{2^n \sin x}$.

$$LHS: (1+\sec 2x)(1+\sec 4x)(1+\sec 8x)\ldots\ldots(1+\sec 2^n x)$$
$$= \left(1+\frac{1}{\cos 2x}\right)\left(1+\frac{1}{\cos 4x}\right)\left(1+\frac{1}{\cos 8x}\right)\ldots\left(1+\frac{1}{\cos 2^n x}\right)$$
$$= \frac{(1+\cos 2x)(1+\cos 4x)(1+\cos 8x)+\ldots(1+\cos 2^n x)}{\cos 2x\cdot \cos 4x \cdot \cos 8x \ldots \cos 2^n x}.$$
$$= \frac{2\cos^2 x\cdot 2\cos^2 2x \cdot 2\cos^2 2^2 x \ldots 2\cos^2 2^{n-1} x}{\cos 2x \cos 2^2 x \cos 2^3 x \ldots \cos 2^n x}$$
$$= \frac{2^n \cos x}{\cos 2^n x}\left[\cos x\cdot \cos 2x\cdot \cos 2^2 x \ldots\ldots \cos 2^{n-1} x\right]$$
$$= \frac{\cancel{2^n}\cos x}{\cos 2^n x}\left[\frac{\sin 2^n x}{\cancel{2^n}\sin x}\right] = \tan 2^n x\cdot \cot x.$$

26) P.T $\sqrt{2+\sqrt{2+\sqrt{2+2\cos 8x}}} = 2\cos x$, $0<x<\pi/8$.

Sol: Procedure use $1+\cos x = 2\cos x/2$

$$LHS = \sqrt{2+\sqrt{2+\sqrt{2+2\cos 2(4x)}}} = \sqrt{2+\sqrt{2+\sqrt{2(1+\cos 8x)}}}$$
$$= \sqrt{2+\sqrt{2+\sqrt{4\cdot\cos^2 4x}}}$$
$$= \sqrt{2+\sqrt{2+2\cos 4x}}$$
$$= \sqrt{2+\sqrt{2(1+\cos 4x)}}$$
$$= \sqrt{2+\sqrt{2\cdot 2\cos^2 2x}}$$
$$= \sqrt{2+2\cos 2x}$$
$$= \sqrt{2(1+\cos 2x)}$$
$$= \sqrt{2\cdot 2\cos^2 x}$$
$$= 2\cos x.$$

27) P.T $\tan 1^\circ \tan 2^\circ \tan 3^\circ - - - - - - - \tan 89^\circ = 1$.

Sol: Change $\tan 89 = \tan(90-1) = \cot 1$

$\tan 88 = \tan(90-2)\ \cot 2$

$- - - - - -$

$\tan 46 = \tan(90^\circ - 44) = \cot 44$

$\therefore$ LHS: $\tan 1^\circ \tan 2^\circ \tan 3^\circ - - - - - - \tan 89^\circ$

$= (\tan 1 \cdot \tan 89)(\tan 2 \cdot \tan 88) - - - - (\tan 44)\tan(46) \cdot \tan 45$

$= (\tan 1 \cot 1)(\tan 2 \cdot \cot 2) - - - (\tan 44)(\cot 44) \cdot \tan 45$

$= 1 \cdot 1 \cdot 1 \cdot - - - \cdot 1 \cdot 1 = \underline{\underline{1}}$.

Product to sum and Sum to product identities

1) Express each of the following product as a sum (or) difference.

TBP 1. $\sin 40 \cos 30 = \frac{1}{2} 2\sin 40 \cos 30$

$= \frac{1}{2}\left[\sin 70 + \sin 10\right]$

2. $\cos 110 \sin 55^\circ = \frac{1}{2} \cdot 2\cos 110 \sin 55^\circ$

$= \frac{1}{2}\left[\sin 165 - \sin 55\right]$

3. $\sin\frac{x}{2}\cos\frac{3x}{2} = \frac{1}{2}\left[2\cos\frac{3x}{2}\sin\frac{x}{2}\right]$

$= \frac{1}{2}\left[\sin\frac{4x}{2} - \sin\frac{2x}{2}\right]$

$= \frac{1}{2}\left[\sin 2x - \sin x\right].$

2) Express each of the following sum or difference as a product.

TBP 1) $\sin 50 + \sin 20 = 2\sin\frac{70}{2}\cos\frac{30}{2}$

$= 2\sin 35^\circ \cos 15^\circ$

2) $\cos 6\theta + \cos 2\theta = 2\cos\frac{8\theta}{2}\cos\frac{4\theta}{2}$

$= 2\cos 4\theta \cos 2\theta$

3) $\cos\frac{3x}{2} - \cos\frac{9x}{2} = 2\sin\frac{\frac{12x}{2}}{2}\sin\frac{\frac{6x}{2}}{2}$

$= 2\sin\frac{\cancel{12}^{3}x}{\cancel{4}}\sin\frac{\cancel{6}^{3}x}{\cancel{4}2}$

$= 2\sin 3x \sin\frac{3x}{2}$

3) Find the value of $\sin 34^\circ + \cos 64^\circ - \cos 4^\circ$

TBP Sol: $\sin 34 + (-\cancel{2}\sin 34 \cancel{\sin 30})$

$= \sin 34 - \sin 34$

$= 0$

4) P.T $\cos 36^\circ \cos 72^\circ \cos 108 \cos 144 = \frac{1}{16}$.

TBP Sol: LHS $= \cos 36 \cos 72 \cos 108 \cos 144$

$= \cos 36 \sin(90-18)\cos(90+18)\cos(180-36)$

$= \cos 36 \sin 18(-\sin 18^\circ)(-\cos 36^\circ)$

$= \sin^2 18 \cdot \cos^2 36$

Procedure.

use $\sin 18^\circ = \frac{\sqrt{5}-1}{4}$

$\cos 36 = \frac{\sqrt{5}+1}{4}$

This type is used only when given angles are related with 90, 180, 270, 360.

5) Simplify $\dfrac{\sin 75^\circ - \sin 15^\circ}{\cos 75 + \cos 15^\circ}$ Just apply formula.
TBP

Sol: $\dfrac{\sin 75^\circ - \sin 15^\circ}{\cos 75 + \cos 15} = \dfrac{\cancel{2\cos\frac{90}{2}} \sin\frac{60}{2}}{\cancel{2\cos\frac{90}{2}} \cos\frac{60}{2}}$

$= \dfrac{\sin 30}{\cos 30} = \tan 30 = \dfrac{1}{\sqrt{3}}.$

6) S.T $\cos 10 \cos 30 \cos 50 \cos 70 = \dfrac{3}{16}.$
TBP

Sol: Procedure: use $\cos(60-A)\cos A \cos(60+A) = \frac{1}{4}\cos 3A$.
This type is used only when the angle are related with 60° (ie) write it as $60+\theta, 60-\theta, \theta$.

$\cos 10 \cos 30 \cos 50 \cos 70 = \cos 30 \cdot \cos(60-10)\cos 10 (\cos 60+10)$

$= \dfrac{\sqrt{3}}{2} \cdot \dfrac{1}{4}\cos 30^\circ$

$= \dfrac{\sqrt{3}}{8} \cdot \dfrac{\sqrt{3}}{2} = \dfrac{3}{16}.$

7) P.T $\sin 10 \sin 30 \sin 50 \sin 70 = \dfrac{1}{16}.$ Same type as problem 6

LHS $\sin 30 \sin(60-\cancel{10})\sin 10 \sin(60+10)$

$= \dfrac{1}{2} \cdot \dfrac{1}{4}\sin 30$

$= \dfrac{1}{2} \cdot \dfrac{1}{4} \cdot \dfrac{1}{2} = \dfrac{1}{16}.$

8) P.T $4\cos 12^\circ \cos 48^\circ \cos 72^\circ = \cos 36^\circ$ Same type as problem ⑥

Sol: LHS $= 4\cos 12 \cos 48 \cos 72^\circ$

$= 4\cos(60-12)\cos 12 \cos(60^\circ+12)$

$= \cancel{4} \cdot \dfrac{1}{\cancel{4}} \cos 36$

$= \cos 36^\circ.$

9) P.T $\cos\dfrac{2\pi}{15}\cos\dfrac{4\pi}{15}\cos\dfrac{8\pi}{15}\cos\dfrac{16\pi}{15} = \dfrac{1}{16}.$ already done

TBP
9) P.T $\cos\dfrac{\pi}{15}\cos\dfrac{2\pi}{15}\cos\dfrac{3\pi}{15}\cos\dfrac{4\pi}{15}\cos\dfrac{5\pi}{15}\cos\dfrac{6\pi}{15}\cos\dfrac{7\pi}{15} = \dfrac{1}{128}$

Sol: Procedure: Convert all radians into degree measure substitute the known values directly including $\sin 18$, $\cos 72^\circ$, $\sin 56$ $\cos 36$, --. then arrange the remaining to use $\sin 2A$ formula.

LHS $\cos 12 \cos 24 \cos 36 \cos 48 \cos 60 \cos 72 \cos 84$.

$$= \cos 12 \cos 24 \cos 48 \cos 84 \cdot \cos 36 \cdot \cos 72 \cdot \cos 60$$

$$= \frac{1}{8 \sin 12} \cdot 2 \sin 12 \cos 12 \cdot 2\cos 24 \cdot 2\cos 48 \cdot \cos 84 \left(\frac{\sqrt{5}-1}{4}\right)\left(\frac{\sqrt{5}+1}{4}\right) \cdot \frac{1}{2}$$

$$= \frac{1}{8 \sin 12} \sin 24 \cdot 2\cos 24 \cdot 2\cos 48 \cdot \cos 84 \quad \frac{5-1}{4 \times 4 \times 2}$$

$$= \frac{1}{8 \sin 12} \sin 48 \cdot 2\cos 48 \cos 84 \frac{1}{8}$$

$$= \frac{1}{64 \sin 12} \cdot \sin 96 \cdot \cos 84$$

$$= \frac{1}{64 \sin 12} \cdot \cos 6 \sin 6 \cdot$$

$$= \frac{1}{128 \cdot \sin 12} \cdot 2 \sin 6 \cos 6$$

$$= \frac{1}{128 \cancel{\sin 12}} \cdot \cancel{\sin 12} = \frac{1}{128}$$

10) P.T $\sin 12^\circ \sin 48^\circ \sin 54 = \frac{1}{8}$

TBP Sol: Procedure: Sub. the known value directly and use $2 \sin A \sin B = \cos(B-A) - \cos(A+B)$

LHS $\sin 12 \sin 48 \sin 54$

$$= \frac{1}{2} 2 \sin 12 \sin 48 \cdot \sin 54$$

$$= \frac{1}{2} \left[\cos(-36) - \cos 60\right] \sin 54$$

$$= \frac{1}{2} \left[\cos 36 - \cos 60\right] \sin 54$$

$$= \frac{1}{2} \left[\frac{\sqrt{5}+1}{4} - \frac{1}{2}\right] \frac{\sqrt{5}+1}{4}$$

$$= \frac{1}{2} \frac{(\sqrt{5}+1)^2}{16} - \frac{1}{4} \cdot \frac{\sqrt{5}+1}{4}$$

$$= \frac{5+1+2\sqrt{5}}{32} - \frac{2\sqrt{5}-2}{32} = \frac{4}{32} = \frac{1}{8}$$

11) S.T $\dfrac{\sin 8x \cos x - \sin 6x \cos 3x}{\cos 2x \cos x - \sin 3x \sin 4x}$

TBP.

Sol: Procedure apply $2\sin A \sin B$, $2\cos A \cos B$, $2\sin A \cos B$ formula.

LHS: $\dfrac{\sin 8x \cos x - \sin 6x \cos 3x}{\cos 2x \cos x - \sin 3x \sin 4x}$

$= \dfrac{2\sin 8x \cos x - 2\sin 6x \cos 3x}{2\cos 2x \cos x - 2\sin 3x \sin 4x}$

$= \dfrac{(\sin 9x + \sin 7x) - (\sin 9x + \sin 3x)}{(\cos(-x) + \cos 3x) - (\cos 7x + \cos x)}$

$= \dfrac{\sin 7x - \sin 3x}{\cos 7x + \cos 3x} = \dfrac{2\cos 5x \sin 2x}{2\cos 5x \cos 2x}$

$= \tan 2x.$

12. P.T $\dfrac{(\cos\theta - \cos 3\theta)(\sin 8\theta + \sin 2\theta)}{(\sin 5\theta - \sin\theta)(\cos 4\theta - \cos 6\theta)} = 1.$

TBP.

Sol: Procedure: use $\sin C + \sin D$, $\sin C - \sin D$ formula and so on.

LHS $= \dfrac{(\cos\theta - \cos 3\theta)(\sin 8\theta + \sin 2\theta)}{(\sin 5\theta - \sin\theta)(\cos 4\theta - \cos 6\theta)}$

$= \dfrac{2\sin 2\theta \sin\theta \cdot 2\sin 5\theta \cos 3\theta}{2\cos 3\theta \sin 2\theta \; 2\sin\theta \sin 5\theta}$

$= 1$

13. P.T $\dfrac{\sin 4x + \sin 2x}{\cos 4x + \cos 2x} = \tan 3x.$

TBP.

Sol: Procedure: use $\sin C + \sin D$, $\cos C + \cos D$ formula.

LHS: $\dfrac{\sin 4x + \sin 2x}{\cos 4x + \cos 2x} = \dfrac{2\sin 3x \cos x}{2\cos 3x \cos x}$

$= \tan 3x.$

14) P.T $\sin \frac{\theta}{2} \sin \frac{7\theta}{2} + \sin \frac{3\theta}{2} \sin \frac{11\theta}{2} = \sin 2\theta \sin 5\theta.$

TBP

Sol: Procedure: use $2\sin A \sin B$ formula.

LHS $= \frac{1}{2}\left[2\sin\frac{\theta}{2}\sin\frac{7\theta}{2} + 2\sin\frac{3\theta}{2}\sin\frac{11\theta}{2}\right]$

$= \frac{1}{2}\left[\left(\cancel{\cos 3\theta} - \cos 4\theta\right) + \left(\cos 4\theta - \cos 7\theta\right)\right]$

$= \frac{1}{2}\left[\cos 3\theta - \cos 7\theta\right]$

$= \frac{1}{\cancel{2}} \cancel{2} \sin 5\theta \sin 2\theta$

15 P.T $\cos(30-A)(\cos 30+A) + \cos(45-A)\cos(45+A) = \cos 2A + \frac{1}{4}$

TBP. Sol: Procedure: Apply $\cos(A+B)\cdot\cos(A-B) = \cos^2 A - \sin^2 B$

LHS: $\cos(30-A)\cos(30+A) + \cos(45-A)(\cos(45+A)$

$= \cos^2 30 - \sin^2 A + \cos^2 45 - \sin^2 A$

$= \frac{3}{4} - \sin^2 A + \frac{1}{2} - \sin^2 A$

$= \frac{1}{4} + 1 - 2\sin^2 A$

$= \frac{1}{4} + \cos 2A.$

18) P.T $\frac{\sin x + \sin 3x + \sin 5x + \sin 7x}{\cos x + \cos 3x + \cos 5x + \cos 7x} = \tan 4x.$

TBP.

Sol: Procedure: use $\sin C + \sin D$, $\cos C + \cos D$ formula two times

LHS: $\frac{\sin x + \sin 3x + \sin 5x + \sin 7x}{\cos x + \cos 3x + \cos 5x + \cos 7x}.$

$= \frac{2\sin 2x \cos x + 2\sin 6x \cos x}{2\cos 2x \cos x + 2\cos 6x \cos x}.$

$= \frac{\cancel{2\cos x}(\sin 2x + \sin 6x)}{\cancel{2\cos x}(\cos 2x) + \sin 6x)} = \frac{\cancel{2}\sin 4x \cancel{\cos 2x}}{\cancel{2}\cos 4x \cancel{\cos 2x}}$

$= \tan 4x.$

19) P.T $\frac{\sin(4A-2B) + \sin(4B-2A)}{\cos(4A-2B) + \cos(4B-2A)} = \tan(A+B)$

TBP.

Sol: Procedure: use $\sin C + \sin D$, $\cos C + \cos D$ formula.

LHS: $\frac{\sin(4A-2B) + \sin(4B-2A)}{\cos(4A-2B) + \cos(4B-2A)}$

$= \frac{2\sin\left(\frac{4A-2B+4B-2A}{2}\right)\cos\left(\frac{4A-2B-4B+2A}{2}\right)}{2\cos\left(\frac{4A-2B+4B-2A}{2}\right)\cos\left(\frac{4A-2B-4B+2A}{2}\right)}$

$= \frac{\cancel{2}\sin\left(\frac{2(A+B)}{2}\right)\cdot\cos\left(\cancel{\frac{6(A-B)}{2}}\right)}{\cancel{2}\cos\left(\frac{2(A+B)}{2}\right)\cdot\cos\left(\cancel{\frac{6(A-B)}{2}}\right)} = \tan(A+B)$

20) 18) S.T ~~$\cot(A+45)$~~ $-$ ~~$\tan(A-45)$~~ $= \frac{4\cos 2A}{1+2\sin 2A}$.
~~TBP~~

~~Sol~~:
18) P.T $\sin x + \sin 2x + \sin 3x = \sin 2x\,(1+2\cos x)$

TBP. Sol: Procedure use $\sin C + \sin D$ and arrange as $\sin 3x + \sin x + \sin 2x$ inorder to get the whole no in angle.

LHS $\sin 3x + \sin x + \sin 2x$

$= 2\sin 2x\cos x + 2\sin x\cos x$

$= 2\cos x\,(\sin 2x + \sin x)$

$= 2\cos x\,(2\sin x\cos x + \sin x)$

$= 2\sin x\cos x\,(2\cos x + 1)$

$= \sin 2x\,(1+2\cos x)$

19) P.T $1+\cos 2x + \cos 4x + \cos 6x = 4\cos x\cos 2x\cos 3x$.

TBP Sol: Same as above problem. Re arrange the LHS

LHS $= 1+\cos 4x + \cos 6x + \cos 2x$

$= 1+\cos 4x + 2\cos 4x\cos 2x$.

$= 1+\cos 2(2x) + 2\cos 4x\cos 2x$.

$= 2\cos^2 2x + 2\cos 4x\cos 2x$

$= 2\cos 2x\,(\cos 2x + \cos 4x)$

$= 2\cos 2x \cdot 2\cos 3x \cdot \cos x$

$= 4\cos x\cos 2x\cos 3x$.

Conditional Trigonometric Identities -

1) If $A+B+C=\pi$ P.T 1) $\cos A+\cos B+\cos C = 1+4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

TBP. 2) $\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2} \le \frac{1}{8}$

3) $1 < \cos A+\cos B+\cos C \le \frac{3}{2}$

Sol: Procedure: Apply $\cos C+\cos D$ formula for first $\cos A+\cos B$ and write $\cos C$ as $1-2\sin^2\frac{C}{2}$. Then $\frac{A+B}{2} = \frac{\pi}{2}-\frac{C}{2}$

and $\sin\frac{C}{2}$ as $\sin\left(\frac{\pi}{2}-\frac{A+B}{2}\right)$

Lastly Apply $\cos D-\cos C = 2\sin\frac{C+D}{2}\sin\frac{C-D}{2}$.

LHS: 1) $\cos A+\cos B+\cos C = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}+\cos C$

$= 2\cos\left(\frac{\pi}{2}-\frac{C}{2}\right)\cdot\cos\frac{A-B}{2}+1-2\sin^2\frac{C}{2}$

$= 1+2\sin\frac{C}{2}\cdot\cos\frac{A-B}{2}-2\sin^2\frac{C}{2}$

$= 1+2\sin\frac{C}{2}\left[\cos\frac{A-B}{2}-\sin\left(\frac{\pi}{2}-\frac{A+B}{2}\right)\right]$

$= 1+2\sin\frac{C}{2}\left[\cos\frac{A-B}{2}-\cos\frac{A+B}{2}\right]$

$= 1+2\sin\frac{C}{2}\left[2\sin\frac{\cancel{A}+B+A\cancel{+B}}{2\times 2}\sin\frac{A+B-A+B}{2\times 2}\right]$

$= 1+4\sin\frac{C}{2}\sin\frac{2A}{4}\sin\frac{2B}{4}$

$= 1+4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$.

2) let $u = \sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

$= -\frac{1}{2}\left[\cos\frac{A+B}{2}-\cos\frac{A-B}{2}\right]\sin\frac{C}{2}$

$= -\frac{1}{2}\left[\cos\frac{A+B}{2}-\cos\frac{A-B}{2}\right]\sin\left(\frac{\pi}{2}-\frac{A+B}{2}\right)$

$= -\frac{1}{2}\left[\cos\frac{A+B}{2}-\cos\frac{A-B}{2}\right]\cos\frac{A+B}{2}$

$2u = -\cos^2\frac{A+B}{2}+\cos\frac{A+B}{2}\cos\frac{A-B}{2}$

$\cos^2\frac{A+B}{2}-\cos\frac{A+B}{2}\cos\frac{A-B}{2}+2u=0$ This is quadratic Eqn in $\cos\frac{A+B}{2}$

$a=1$

$b=-\cos\frac{A-B}{2}$

$c=2u$.

$b^2-4ac \ge 0$

$\cos^2\frac{A-B}{2}-8u \ge 0$

$\cos^2\frac{A-B}{2} \ge 8u$

$\Rightarrow 8u \le \cos^2\frac{A-B}{2} \le 1$

$u \le \frac{1}{8}\cos^2\frac{A-B}{2} \le \frac{1}{8}$

$\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2} \le 18$ (?) Since $\cos\frac{A+B}{2}$ real

3) $\cos A + \cos B + \cos C > 1$ and

$\cos A + \cos B + \cos C \le 1 + 4 \cdot \frac{1}{8}$

$1 < \cos A + \cos B + \cos C \le 3/8$

2) P.T $\sin A/2 + \sin B/2 + \sin C/2 = 1 + 4 \sin\left(\frac{\pi - A}{4}\right) \sin\left(\frac{\pi - B}{4}\right) \sin\left(\frac{\pi - C}{4}\right)$

TBP if $A+B+C = \pi$

Sol: Procedure: change sin terms into cos term first as

Express the sum of the first two terms as a product (by using C and D) and simplify the first factor from the relation connecting the angles.

Express the third term in terms of simplified factor

Take out common factor and express the sum with in brackets, first in terms of the first two angles and express it as a product (C, D formula)

LHS: $\sin A/2 + \sin \frac{B}{2} + \sin C/2 = \cos\left(\frac{\pi}{2} - A/2\right) + \cos\left(\frac{\pi}{2} - \frac{B}{2}\right) + \cos\left(\pi/2 - C/2\right)$

$$= 2\cos\left(\frac{\pi}{2} - \frac{A+B}{4}\right)\cos\left(\frac{B-A}{4}\right) + \left[1 - 2\sin^2\left(\frac{\pi}{4} - \frac{C}{4}\right)\right]$$

$$= 1 + 2\sin\frac{A+B}{4}\cos\frac{B-A}{4} - 2\sin^2\left(\frac{\pi}{4} - C/4\right)$$

$$= 1 + 2\sin\left(\frac{\pi}{4} - \frac{C}{4}\right)\cos\frac{B-A}{4} - 2\sin^2\left(\pi/4 - C/4\right)$$

$$= 1 - 2\sin\left(\frac{\pi}{4} - \frac{C}{4}\right)\left[\cos\frac{B-A}{4} - \sin\left(\frac{\pi}{4} - \frac{C}{4}\right)\right]$$

$$= 1 - 2\sin\left(\frac{\pi}{4} - \frac{C}{4}\right)\left[\cos\frac{B-A}{4} - \sin\frac{\pi}{4} - \left(\frac{A+B}{4} + \frac{\pi}{4}\right)\right]$$

$$= 1 - 2\sin\left(\frac{\pi}{4} - \frac{C}{4}\right)\left[\cos\frac{B-A}{4} - \sin\frac{A+B}{4}\right]$$

$$= 1 - 2\sin\left(\frac{\pi}{4} - \frac{C}{4}\right)\left[\cos\frac{B-A}{4} - \cos\left(\frac{\pi}{2} - \frac{A+B}{4}\right)\right]$$

$$= 1 - 2\sin\left(\frac{\pi}{4} - \frac{C}{4}\right)\left[2\sin\left(\frac{\frac{\pi}{2} - \frac{A+B}{2} + \frac{B-A}{4}}{2}\right)\sin\left[\left(\frac{\pi}{2} - \frac{A+B}{2}\right) - \frac{B-A}{2}\right]\right]$$

$$= 1 - 2\sin\left(\frac{\pi}{4} - \frac{C}{4}\right)\left[2\sin\frac{\frac{\pi}{2} - \frac{A}{2}}{2}\sin\frac{\frac{\pi}{2} - \frac{B}{2}}{2}\right]$$

$$= 1 - 4\sin\left(\frac{\pi}{4} - \frac{C}{4}\right)\sin\left(\frac{\pi}{4} - A/2\right)\sin\left(\frac{\pi}{4} - \frac{B}{2}\right)$$

3) If $A+B+C = 180$ P.T $\sin 2A + \sin 2B + \sin 2C = 4\sin A \sin B \sin C$.

TBP: Procedure: Same as above.

LHS: $\sin 2A + \sin 2B + \sin 2C = 2\sin(A+B)\cos(A-B) + \sin 2C$

$$= 2\sin(\pi - C)\cos(A-B) + \sin 2C$$

$$= 2\sin C\cos(A-B) + 2\sin C\cos C$$

$$= 2\sin C[\cos(A-B) + \cos C]$$

$$= 2\sin C[\cos(A-B) + \cos(\pi - \overline{A+B})]$$

$= 2\sin C\left[\cos(A-B) - \cos(A+B)\right]$

$= 2\sin C \cdot 2\sin A \sin B$

$= 4\sin A \sin B \sin C.$

4) If $A+B+C=0$ P.T $\cos A + \cos B - \cos C = -1 + 4\cos A/2 \cos B/2 \sin C/2$ TBP.

Sol: LHS: $\cos A + \cos B - \cos C = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2} - \cos C$

TBP.

Procedure: Proceed as above Problem.

$= 2\cos\left(\frac{\pi}{2} - \frac{C}{2}\frac{(A+B)}{2}\right)\cos\left(\frac{A-B}{2}\right) - \cos C$

$= 2\sin\frac{C}{2}\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) - \left(\cancel{\cos C}\ 1 - 2\sin^2 C/2\right)$

$= 2\sin\frac{C}{2}\left[\cos\frac{A-B}{2} + \sin\frac{C}{2}\right] - 1$

$= 2\sin\frac{C}{2}\left[\cos\frac{A-B}{2} + \sin\left(\frac{\pi}{2} - \frac{A+B}{2}\right)\right] - 1$

$= 2\sin C/2\left[\cos\frac{A-B}{2} + \cos\frac{A+B}{2}\right]$

$= 2\cdot\sin C/2 \; 2\cos A/2 \cos B/2 - 1$

$= -1 + 4\cancel{\cos}\ A/2 \cos B/2 \sin C/2$

5) If $A+B+C=180^\circ$ P.T $\sin A + \sin B + \sin C = 4\cos A/2 \cos B/2 \cos C/2$

Sol: Procedure: Proceed as above problem.

LHS: $\sin A + \sin B + \sin C = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2} + \sin C$

$= 2\sin\left(\frac{\pi}{2} - \frac{C}{2}\right)\cos\left(\frac{A-B}{2}\right) + \sin C.$

$= 2\cos(C/2)\cos\left(\frac{A-B}{2}\right) + 2\sin C/2 \cos C/2$

$= 2\cos C/2\left[\cos\frac{A-B}{2} + \sin\left(\frac{C}{2}\right)\right]$

$= 2\cos C/2\left[\cos\frac{A-B}{2} + \sin\left(\frac{\pi}{2} - \frac{A+B}{2}\right)\right]$

$= 2\cos C/2\left[\cos\frac{A-B}{2} + \cos\frac{A+B}{2}\right]$

$= 2\cos C/2 \cdot 2\cos A/2 \cos B/2$

$= 4\cos A/2 \cos B/2 \cos C/2$

6) If $A+B+C=180^\circ$ P.T $\sin 2A + \sin 2B - \sin 2C = 4\cos A \cos B \sin C.$

Sol: Procedure: Same as above problem.

LHS: $\sin 2A + \sin 2B - \sin 2C = 2\sin(A+B)\cos(A-B) - \sin 2C.$

$= 2\sin(\pi - C)\cos(A-B) - \sin 2C.$

$= 2\sin C\cos(A-B) - 2\sin C\cos C$

$= 2\sin C(\cos(A-B) - \cos C)$

$= 2\sin C(\cos(A-B) - \cos(\pi - \overline{A+B}))$

$= 2\sin C(\cos(A-B) + \cos(A+B))$

$= 2\sin C\, 2\cos A \cos B$

$= 4\cos A \cos B \sin C.$

7) P.T If $A+B+C=\pi$ P.T $\cos^2 A+\cos^2 B+\cos^2 C = 1-2\cos A\cos B\cos C$.

TBP. Sol: Procedure: change the squares of Sines or cosines into cosines of double the angles by using $\cos^2 A = \frac{1+\cos 2A}{2}$, $\sin^2 A = \frac{1-\cos 2A}{2}$ and then prove it as in the previous type.

LHS: $\cos^2 A+\cos^2 B+\cos^2 C = \frac{1+\cos 2A}{2}+\frac{1+\cos 2B}{2}+\frac{1+\cos 2C}{2}$

$= \frac{3}{2}+\frac{1}{2}\left[\cos 2A+\cos 2B+\cos 2C\right]$ —— (A)

Now let $\cos 2A+\cos 2B+\cos 2C = 2\cos(A+B)\cos(A-B)+\cos 2C$.

$= 2\cos(\pi-C)\cos(A-B)+\cos 2C$

$= -2\cos C\cos(A-B)+2\cos^2 C-1$

$= -1 \cancel{+} 2\cos C\,(\cos(A-B) \cancel{+} \cos C)$

$= -1 \cancel{+} 2\cos C\left[\cos(A-B) \cancel{+} \cos(\pi-\overline{A+B})\right]$

$= -1 \cancel{+} 2\cos C\left[\cos(A-B) \cdot 1 \cos(A+B)\right]$

$= -1 \cancel{+} 4\cos C\cos A\cos B$.

Sub in (A)

$= \frac{3}{2}+\frac{1}{2}(-1-4\cos A\cos B\cos C)$

$= 1 \cancel{+} 2\cos A\cos B\cos C$.

8) P.T $\sin^2 A+\sin^2 B+\sin^2 C = 2+2\cos A\cos B\cos C$.

TBP Sol: Procedure Same as previous problem.

LHS: $\sin^2 A+\sin^2 B+\sin^2 C = \frac{1-\cos 2A}{2}+\frac{1-\cos 2B}{2}+\frac{1-\cos 2C}{2}$

$= \frac{3}{2}-\frac{1}{2}(\cos 2A+\cos 2B+\cos 2C)$ —— (A)

Let $\cos 2A+\cos 2B+\cos 2C = 2\cos(A+B)\cos(A-B)+\cos 2C$.

$= 2\cos(\pi-C)\cos(A-B)+\cos 2C$

$= -2\cos C\cos(A-B)+2\cos^2 C-1$

$= -1-2\cos C(\cos(A-B)-\cos C)$

$= -1-2\cos C(\cos(A-B)-\cos(\pi-\overline{A+B}))$

$= -1-2\cos C\left[\cos(A-B)+\cos(A+B)\right]$

$= -1-2\cos C\cdot 2\,\overset{\cos A\cos B}{\cancel{\sin A\sin B}}$

$= -1-4\cos A\cos B\cos C$.

Sub in (A)

$\frac{3}{2}-\frac{1}{2}(-1-4\cos A\cos B\cos C)$

$= 2+2\cos A\cos B\cos C$.

9) P.T $\sin^2 A + \sin^2 B - \sin^2 C = 2\sin A \sin B \cos C$.

TBP Sol: Procedure: Same as above problem.

LHS: $\sin^2 A + \sin^2 B - \sin^2 C = \frac{1-\cos 2A}{2} + \frac{1-\cos 2B}{2} - \frac{1-\cos 2C}{2}$

$$= \frac{1}{2} - \frac{1}{2}(\cos 2A + \cos 2B - \cos 2C) \quad \text{— } (*)$$

Let $\cos 2A + \cos 2B - \cos 2C = 2\cos(A+B)\cos(A-B) - \cos 2C$

$$= 2\cos(\pi - C)\cos(A-B) - \cos 2C$$

$$= -2\cos C\cos(A-B) - (2\cos^2 C - 1)$$

$$= 1 - 2\cos C(\cos(A-B) + \cos C)$$

$$= 1 - 2\cos C(\cos(A-B) + \cos(\pi - \overline{A+B}))$$

$$= 1 - 2\cos C[\cos(A-B) - \cos(A+B)]$$

$$= 1 - 4\sin A\sin B\cos C.$$

Sub in $(*)$

$$\sin^2 A + \sin^2 B + \sin^2 C = \frac{1}{2} - \frac{1}{2}(1 - 4\sin A\sin B\cos C)$$

$$= \cancel{\tfrac{1}{2}} - \cancel{\tfrac{1}{2}} + \frac{1}{2}\cdot 4\sin A\sin B\cos C$$

$$= 2\sin A\sin B\cos C.$$

10 P.T $\tan A/2 \tan B/2 + \tan B/2 \tan C/2 + \tan A/2 \tan C/2 = 1$.

TBP. Sol: Procedure: Apply $\tan\left(\frac{A+B}{2}\right) = \tan\left(\frac{\pi}{2} - C/2\right)$

$A + B + C = \pi$

$\therefore \frac{A}{2} + \frac{B}{2} = \frac{\pi}{2} - \frac{C}{2}$

$$\tan\left(\frac{A}{2} + \frac{B}{2}\right) = \tan\left(\frac{\pi}{2} - \frac{C}{2}\right)$$

$$\frac{\tan A/2 + \tan B/2}{1 - \tan A/2 \tan B/2} = +\cot C/2$$

$$\frac{\tan A/2 + \tan B/2}{1 - \tan A/2 \tan B/2} = \frac{1}{\tan C/2}$$

$$\tan A/2 \tan C/2 + \tan B/2 \tan C/2 = 1 - \tan A/2 \tan B/2$$

$$\tan A/2 \tan B/2 + \tan B/2 \tan C/2 + \tan A/2 \tan C/2 = 1$$

10) P.T $\sin A + \sin B + \sin C = 4 \cos A/2 \cos B/2 \cos C/2$

TBP: Sol: Already done.

11) If $A+B+C = \frac{\pi}{2}$ P.T $\sin 2A + \sin 2B + \sin 2C = 4 \cos A \cos B \cos C$.

TBP Sol: Procedure: Refer problem 3. $A+B = \frac{\pi}{2} - C$

LHS: $\sin 2A + \sin 2B + \sin 2C = 2\sin(A+B)\cos(A-B) + \sin 2C$

$= 2\sin\left(\frac{\pi}{2} - C\right)\cos(A-B) + \sin 2C$

$= 2\cos C \cos(A-B) + 2\sin C \cos C$

$= 2\cos C\left[\cos(A-B) + \sin C\right]$

$= 2\cos C\left[\cos(A-B) + \sin\left(\frac{\pi}{2} - \overline{A+B}\right)\right]$

$= 2\cos C\left[\cos(A-B) + \cos(A+B)\right]$

$= 2 \cdot \cos C \cdot 2\cos A \cos B$.

$= 4\cos A \cos B \cos C$.

12) If $A+B+C = \frac{\pi}{2}$ P.T $\cos 2A + \cos 2B + \cos 2C = 1 + 4\sin A \sin B \sin C$.

TBP Sol: Procedure: Same as above problem.

LHS: $\cos 2A + \cos 2B + \cos 2C = 2\cos(A+B) \cdot \cos(A-B) + \cos 2C$.

$= 2\cos\left(\frac{\pi}{2} - C\right)\cos(A-B) + \cos 2C$

$= 2\sin C \cos(A-C) + (1 - 2\sin^2 C)$

$= 1 + 2\sin C\left[\cos(A-B) - \sin C\right]$

$= 1 + 2\sin C\left[\cos(A-B) - \sin\left(\frac{\pi}{2} - \overline{A+B}\right)\right]$

$= 1 + 2\sin C\left[\cos(A-B) - \cos(A+B)\right]$

$= 1 + 2\sin C\left[\cos(A-B)\right.$

$= 1 + 4\sin C \cdot \sin A \cdot \sin B$

13) If $A+B+C = \pi$ P.T $\sin(B+C-A) + \sin(C+A-B) + \sin(A+B-C)$
$= 4\sin A \sin B \sin C$.

TBP

Sol: Procedure: First reduce the sum to the form $\sin 2A + \sin 2B + \sin 2C$ by using $A+B+C = \pi$

LHS $\sin(B+C-A) + \sin(C+A-B) + \sin(A+B-C)$

$= \sin(\pi - A - A) + \sin(\pi - B - B) + \sin(\pi - C - C)$

$= \sin 2A + \sin 2B + \sin 2C$.

then Proceed as problem (3).

14) If $A+B+C=2S$ P.T $\sin(S-A)+\sin(S-B)+\sin(S-C)-\sin S$
TBP. $= 4\sin A/2 \sin B/2 \sin C/2$

Sol: Procedure: Apply $\sin C+\sin D$ and $\sin C-\sin D$ formulas.

LHS: $\sin(S-A)+\sin(S-B)+\sin(S-C)-\sin S$.

$$= 2\sin\frac{S-A+S-B}{2}\cos\frac{S-A-S+B}{2} + 2\cos\frac{S-C+S}{2}\sin\frac{S-C-S}{2}$$

$$= 2\sin\frac{2S-A-B}{2}\cos\frac{B-A}{2} + 2\cos\frac{2S-C}{2}\sin(-C/2)$$

$$= 2\sin\frac{C}{2}\cos\frac{B-A}{2} + 2\cos\frac{A+B}{2}\sin(-C/2)$$

$$= 2\sin C/2 \cos\frac{B-A}{2} - 2\cos\frac{A+B}{2}\sin C/2$$

$$= 2\sin C/2\left[\cos\frac{A-B}{2} - \cos\frac{A+B}{2}\right]$$

$$= 2\sin C/2 \cdot 2\sin A/2 \sin B/2$$

$$= 4\sin A/2 \sin B/2 \sin C/2$$

$\cos\left(\frac{B-A}{2}\right) = \cos\left(-\frac{(A-B)}{2}\right) = \cos\frac{A-B}{2}$

15) If ΔABC is a right triangle and if $\angle A = \pi/2$ P.T

TBP

1) $\cos^2 B + \cos^2 C = 1$

2) $\sin^2 B + \sin^2 C = 1$

3) $\cos B - \cos C = -1 + 2\sqrt{2}\cos B/2 \sin C/2$.

Sol: $\because \angle A = \pi/2$ $\quad B+C = \frac{\pi}{2}$

$B = \frac{\pi}{2} - C$.

1) LHS: $\cos^2 B + \cos^2 C$

$= (\cos(\frac{\pi}{2}-C))^2 + \cos^2 C$

$= (\sin C)^2 + \cos^2 C$

$= \sin^2 C + \cos^2 C = 1$

2) LHS $\sin^2 B + \sin^2 C$

$(\sin(\pi/2 - C))^2 + \sin^2 C$

$\cos^2 C + \sin^2 C = 1$

3) LHS: $\cos B - \cos C = -2\sin\frac{B+C}{2}\sin\frac{B-C}{2}$

$= -2\sin\frac{\pi}{4}$

Trigonometric Equations -

1) Find the principal solution of i) $\sin\theta = 1/2$ 2) $\sin\theta = -\sqrt{3}/2$ 3) $\operatorname{cosec}\theta = -2$ 4) $\cos\theta = 1/2$

TBP. Sol: 1) Since $\sin\theta = 1/2 > 0$ θ lies in the first quadrant

$\sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$.

$\therefore$ $\theta = \frac{\pi}{6}$ is the pricipal solution.

2) $\sin\theta = -\sqrt{3}/2 < 0$ Principal value lies in IV quadrant

$\sin\theta = -\frac{\sqrt{3}}{2} = -\sin(\sqrt{3}/2) = -\frac{\pi}{3}$

Hence $\theta = -\pi/3$ is principal solution.

3) $\operatorname{cosec}\theta = -2 \Rightarrow \sin\theta = -1/2 < 0$

θ lies in IV quadrant.

$\sin\theta = -1/2 = -\sin(1/2)$

$\theta = -\frac{\pi}{6}$

$\therefore$ Principal Solution $\theta = -\pi/6$.

4) $\cos\theta = 1/2 > 0$, θ lies in the I quadrant

$\cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$ is the principal solution.

2) Find the general solution of $\sin\theta = -\sqrt{3}/2$

TBP.

Sol: The general solution of $\sin\theta = \sin\alpha$. $\alpha \in [-\pi/2, \pi/2]$ is

$\theta = n\pi + (-1)^n \alpha$ $n \in Z$.

$\sin\theta = -\frac{\sqrt{3}}{2} = \sin\left(\frac{-\pi}{3}\right) \Rightarrow \alpha = -\pi/3$.

$\therefore$ The General Solution is $\theta = n\pi + (-1)^n(-\pi/3)$ $n \in Z$.

3) Find the general solution of i) $\sec\theta = -2$ 2) $\tan\theta = \sqrt{3}$

TBP. Solu: 1) $\sec\theta = -2 \Rightarrow \cos\theta = -\frac{1}{2}$

$\Rightarrow \cos\left(\pi - \frac{\pi}{3}\right) = -1/2 \Rightarrow \alpha = \frac{2\pi}{3}$.

General solution $\theta = 2n\pi \pm \frac{2\pi}{3}$ $n \in Z$.

2) $\tan\theta = \sqrt{3}$, θ lies in I quadrant.

$\tan\theta = \tan\left(\frac{\pi}{3}\right) \Rightarrow \alpha = \frac{\pi}{3}$.

General solution $\theta = n\pi + \frac{\pi}{3}$ $n \in Z$.

4) Solve $3\cos^2\theta = \sin^2\theta$

T.B.P

Sol: $3\cos^2\theta = \sin^2\theta$

$3\cos^2\theta = 1 - \cos^2\theta$

$4\cos^2\theta = 1 \Rightarrow \cos^2\theta = \frac{1}{4}$

$\cos\theta = \pm \frac{1}{2}$.

when $\cos\theta = \frac{1}{2} = \cos(\pi/3)$

$\therefore$ The general Solution is $\theta = n\pi \pm \pi/3 \quad n \in \mathbb{Z}$.

when $\cos\theta = (-1/2) = \cos(\pi - \pi/3) = \cos\left(\frac{2\pi}{3}\right)$

Then the general solution is $\theta = n\pi \pm \frac{2\pi}{3}$.

4) Solve $\sin x + \sin 5x = \sin 3x$.

TBP. Sol: LHS: $\sin 5x + \sin x = 2\sin 3x \cos 2x = \sin 3x$

$\Rightarrow \sin 3x\,(2\cos 2x - 1) = 0$

$\sin 3x = 0$ (or) $2\cos 2x = 1$

$\cos 2x = 1/2$

when $\sin 3x = 0 = \sin(0)$

$\therefore \quad 3x = n\pi \Rightarrow x = \frac{n\pi}{3} \quad n \in \mathbb{Z}$.

when $\cos 2x = \frac{1}{2} = \cos(\pi/3)$

$2x = \pi/3$.

$\therefore 2x = 2n\pi \pm \frac{\pi}{3}$

$x = n\pi \pm \pi/6 \quad n \in \mathbb{Z}$.

5) Solve $\cos x + \sin x = \cos 2x + \sin 2x$.

T.B.P. Sol $\cos x + \sin x = \cos 2x + \sin 2x$.

$\cos x - \cos 2x = \sin 2x - \sin x$.

$2\sin\left(\frac{x+2x}{2}\right)\sin\left(\frac{2x-x}{2}\right) = 2\cos\left(\frac{2x+x}{2}\right)\sin\left(\frac{2x-x}{2}\right)$

$\cancel{2}\sin\frac{3x}{2}\sin\frac{x}{2} = \cancel{2}\cos\frac{3x}{2}\sin\frac{x}{2}$

$\sin\frac{3x}{2}\sin x/2 - \cos\frac{3x}{2}\sin x/2 = 0$

$\sin x/2\left(\sin\frac{3x}{2} - \cos\frac{3x}{2}\right) = 0$

$\Rightarrow \sin\frac{x}{2} = 0$ (or) $\sin\frac{3x}{2} = \cos 3x/2$

If $\sin\frac{x}{2} = 0 \Rightarrow \frac{x}{2} = 0 = \alpha$

$\therefore \frac{x}{2} = n\pi$

$x = 2n\pi \quad n \in \mathbb{Z}$

when $\sin\frac{3x}{2} = \cos\frac{3x}{2}$

$\tan\frac{3x}{2} = 1 = \tan\frac{\pi}{4}$

$\frac{3x}{2} = n\pi + \frac{\pi}{4} \Rightarrow 3x = 2n\pi + \frac{\pi}{2}$

$x = \frac{2n\pi}{3} + \pi/6$

6) Solve the equation $\sin 9\theta = \sin\theta$.

TBP: Sol: $\sin 9\theta = \sin\theta$

$\Rightarrow \sin 9\theta - \sin\theta = 0 \quad = 2\cos 5\theta \sin 4\theta = 0$

$\Rightarrow \cos 5\theta = 0$ (or) $\sin 4\theta = 0$

When $\sin 4\theta = 0 = \cancel{R}=0$

$\therefore 4\theta = n\pi$

$\theta = \frac{n\pi}{4} \quad n \in Z$

When $\cos 5\theta = 0$

$5\theta = \pi/2$

$= (2n+1)\pi/2$

$\theta = (2n+1)\frac{\pi}{10} \quad n \in z.$

7) Solve: $\tan 2x = -\cot(x + \pi/3)$

TBP Sol: $\tan 2x = -\cot(x+\pi/3)$

$= \tan\left(\frac{\pi}{2} + x + \pi/3\right)$

$\tan 2x = \tan\left(\frac{5\pi}{6} + x\right)$

$\therefore 2x = n\pi + \frac{5x}{6} + x.$

$x = n\pi + \frac{5x}{6} \quad n \in Z.$

8) Solve $\sin x - 3\sin 2x + \sin 3x = \cos x - 3\cos 2x + \cos 3x.$

TBP: Sol: $\sin x + \sin 3x - 3\sin 2x = \cos x + \cos 3x - 3\cos 2x.$

$2\sin 2x \cos x - 3\sin 2x = 2\cos 2x \cos x - 3\cos 2x.$

$2\sin 2x \cos x - 2\cos 2x \cos x = 3(\sin 2x - \cos 2x)$

$2\cos x(\sin 2x - \cos 2x) = 3(\sin 2x - \cos 2x)$

$(\sin 2x - \cos 2x)[2\cos x - 3] = 0$

$\sin 2x - \cos 2x = 0 \quad \because 2\cos x - 3 \neq 0$

$\tan 2x = 1 = \tan\frac{\pi}{4}$

$2x = n\pi + \frac{\pi}{4}$

$x = \frac{n\pi}{2} + \frac{\pi}{8}$

9) Solve $\sin x + \cos x = 1 + \sin x \cos x$

TBP. Sol: Let $t = \sin x + \cos x.$

$1 + 2\sin x \cos x = (\sin^2 x + \cos^2 x) + 2\sin x \cos x$

$= (\sin x + \cos x)^2 = t^2$

$\Rightarrow \sin x \cos x = \frac{t^2-1}{2}$

$\therefore$ ~~[illegible]~~ $1 + \frac{t^2-1}{2} - t = 0$

$2 + t^2 - 1 - 2t = 0$

$t^2 - 2t + 1 = 0 \Rightarrow (t-1)^2 = 0 \therefore t = 1$

$r = \sqrt{1+1} = \sqrt{2}$

Hence $\sin x + \cos x = 1$

$\sqrt{2}\left(\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x\right) = 1$

$\sqrt{2}\left(\cos\frac{\pi}{4}\sin x + \sin\frac{\pi}{4}\cos x\right) = 1$

$\sin\left(\frac{\pi}{4} + x\right) = \frac{1}{\sqrt{2}} = \sin\frac{\pi}{4}$

$\frac{\pi}{4} + x = n\pi (-1)^n\frac{\pi}{4}$

$x = n\pi (-1)^n\frac{\pi}{4} - \frac{\pi}{4}.$

(or)

$\sqrt{2}\left(\sin\frac{\pi}{4}\sin x + \cos\frac{\pi}{4}\cos x\right)$

$= \sqrt{2}\cos\left(\frac{\pi}{4} - x\right) = 1$

$\cos\left(\frac{\pi}{4} - x\right) = \frac{1}{\sqrt{2}} = \cos\frac{\pi}{4}$

$\frac{\pi}{4} - x = 2n\pi \pm \frac{\pi}{4}$

(or) $x - \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{4}$

$x = 2n\pi + \frac{2\pi}{4}$ (or) $x = 2n\pi$

10) Solve: $2\sin^2 x + \sin^2 2x = 2$.

TBP. Sol: LHS: $2\sin^2 x + \sin^2 2x$.

$= 2\sin^2 x + (2\sin x\cos x)^2 = 2$

$= \cancel{2}\sin^2 x + \cancel{4}^{2}\sin^2 x\cos^2 x = \cancel{2}^{1}$

$\cancel{1} - \cos^2 x + 2\sin^2 x\cos^2 x = \cancel{1}$

$\cos^2 x(2\sin^2 x - 1) = 0$

$\cos^2 x = 0$

$\cos x = 0$

$x = (2n+1)\pi/2 \quad n \in Z$

$2\sin^2 x = 1$

$\sin^2 x = \frac{1}{2}$

$\sin^2 x = \frac{1}{2} = \sin^2(\pi/4)$

$x = n\pi \pm \frac{\pi}{4} \quad n \in Z$.

11) Solve: P.T for any a, b $-\sqrt{a^2+b^2} \le a\sin\theta + b\cos\theta \le \sqrt{a^2+b^2}$

TBP: Sol; $a\sin\theta + b\cos\theta = \sqrt{a^2+b^2}\left[\frac{1}{\sqrt{a^2+b^2}}\sin\theta + \frac{1}{\sqrt{a^2+b^2}}\cos\theta\right]$

$= \sqrt{a^2+b^2}\left[\cos\alpha\sin\theta + \sin\alpha\cos\theta\right]$

where $\cos\alpha = \frac{a}{\sqrt{a^2+b^2}}$

$\sin\alpha = \frac{b}{\sqrt{a^2+b^2}}$

$= \sqrt{a^2+b^2}\left[\sin(\theta+\alpha)\right]$

$|a\sin\theta + b\cos\theta| \le \sqrt{a^2+b^2} \Rightarrow -\sqrt{a^2-b^2} \le (a\sin\theta + b\cos\theta) \le \sqrt{a^2+b^2}$

12) Solve $\sqrt{3}\sin\theta - \cos\theta = \sqrt{2}$

TBP. Sol: $\sqrt{3}\sin\theta - \cos\theta = \sqrt{2}$ $\quad r = \sqrt{3+1} = 2 > c = \sqrt{2}$

∴ The Eqn. may be written as

$$\frac{\sqrt{3}}{2}\sin\theta - \frac{1}{\sqrt{2}}\cos\theta = 1$$

$$\sin\theta\cos\frac{\pi}{6} - \cos\theta\sin\frac{\pi}{6} = 1$$

$$\sin(\theta - \pi/6) = \sin\frac{\pi}{4}$$

$$\therefore \theta - \frac{\pi}{6} = n\pi \pm (-1)^n \cdot \frac{\pi}{4}.$$

$$\theta = n\pi + \frac{\pi}{6} \pm (-1)^n \pi/4 \quad n \in \mathbb{Z}.$$

13) Solve $\sqrt{3}\tan^2\theta + (\sqrt{3}-1)\tan\theta - 1 = 0$

TBP Sol: $\sqrt{3}\tan^2\theta + (\sqrt{3}-1)\tan\theta - 1 = 0$

$$\sqrt{3}\tan^2\theta + \sqrt{3}\tan\theta - \tan\theta - 1 = 0$$

$$\sqrt{3}\tan\theta(\tan\theta + 1) - 1(\tan + 1) = 0$$

$$(\tan\theta + 1)(\sqrt{3}\tan\theta - 1) = 0$$

$\sqrt{3}\tan\theta - 1 = 0$ $\qquad$ $\tan\theta + 1 = 0$

$\sqrt{3}\tan\theta = 1$

$\tan\theta = \frac{1}{\sqrt{3}} = \tan\frac{\pi}{6}$

$\theta = n\pi + \frac{\pi}{6}\ n \in \mathbb{Z}.$

$\tan\theta = -1$

$= \tan(-\pi/4)$

$\theta = n\pi - \frac{\pi}{4}\quad n \in \mathbb{Z}.$

14) Solve: $\sin\theta + \sin 3\theta + \sin 5\theta = 0$

TBP: Sol: Procedure: use $\sin C + \sin D$

$$(\sin 5\theta + \sin\theta) + \sin 3\theta = 0$$

$$2\sin 3\theta\cos 2\theta + \sin 3\theta = 0$$

$$\sin 3\theta(2\cos 2\theta + 1) = 0$$

$\sin 3\theta = 0$ $\qquad$ $2\cos 2\theta + 1 = 0$

$3\theta = n\pi$

$\theta = \frac{n\pi}{3}\ n \in \mathbb{Z}$

$2\cos 2\theta = -1$

$\cos 2\theta = -1/2 = \cos(\pi - \pi/3)$

$2\theta = 2n\pi \pm \frac{2\pi}{3},\ n \in \mathbb{Z}$

$\theta = n\pi \pm \frac{\pi}{3}$

$\theta = (3n \pm 1)\pi/3$

15) Find the general solution of $\tan\theta + \tan(\theta+\pi/3) + \tan(\theta+\frac{2\pi}{3}) = \sqrt{3}$

TBP: Sol: Procedure: use $\tan(A+B)$ and $\pi/3$, $2\pi/3$ values.

$$\tan\theta + \tan\left(\theta+\frac{\pi}{3}\right) + \tan\left(\theta+\frac{2\pi}{3}\right) = \sqrt{3}.$$

$\tan\frac{2\pi}{3} = \tan(\pi - \pi/3) = -\tan\pi/3 = -\sqrt{3}$

$$\tan\theta + \frac{\tan\theta + \tan\frac{\pi}{3}}{1-\tan\theta\tan\pi/3} + \frac{\tan\theta + \tan\frac{2\pi}{3}}{1-\tan\theta\tan\frac{2\pi}{3}} = \sqrt{3}$$

$$\tan\theta + \frac{\tan\theta+\sqrt{3}}{1-\sqrt{3}\tan\theta} + \frac{\tan\theta-\sqrt{3}}{1+\sqrt{3}\tan\theta} = \sqrt{3}$$

$$\frac{\tan\theta(1-3\tan^2\theta) + \tan\theta + \cancel{\sqrt{3}\tan^2\theta} + \cancel{\sqrt{3}} + 3\tan\theta + \tan\theta - \cancel{\sqrt{3}\tan\theta} - \cancel{\sqrt{3}} + 3\tan\theta}{1-3\tan^2\theta} = \sqrt{3}$$

$$\frac{9\tan\theta - 3\tan^3\theta}{1-3\tan^2\theta} = \sqrt{3} \Rightarrow \frac{3(3\tan\theta - \tan^3\theta)}{1-3\tan^2\theta} = \sqrt{3}$$

$$= \sqrt{3}$$

$$3\tan 3\theta = \sqrt{3}$$

$$\tan 3\theta = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$$

$$3\theta = n\pi + \frac{\pi}{6}$$

$$\theta = \frac{n\pi}{3} + \frac{\pi}{18} \quad n \in \mathbb{Z}.$$

16) Solve: $2\cos^2\theta + 3\sin\theta - 3 = 0$

TBP Sol: Procedure: change $\cos^2\theta$ into $1-\sin^2\theta$ and then solve.

$$2\cos^2\theta + 3\sin\theta - 3 = 0$$

$$2(1-\sin^2\theta) + 3\sin\theta - 3 = 0$$

$$2 - 2\sin^2\theta + 3\sin\theta - 3 = 0 \qquad a=2,\ b=-3,\ c=1$$

$$2\sin^2\theta - 3\sin\theta + 1 = 0$$

$$\sin\theta = \frac{3\pm\sqrt{9-8}}{4} = \frac{3\pm1}{4} \text{ (or) } \cancel{\tfrac{4}{4}}1,\ \cancel{\tfrac{2}{4}}\tfrac{1}{2}$$

when $\sin\theta = 1 \Rightarrow (\pi/2)$

Solution: $(-1)^n \cdot \pi/2$

when $\sin\theta = \frac{1}{2} = \sin\left(\frac{\pi}{6}\right)$

Solution $\theta = n\pi + (-1)^n \cdot \frac{\pi}{6} \quad n \in \mathbb{Z}.$

17) solve $\cot\theta + \operatorname{cosec}\theta = \sqrt{3}$

TBP Sol: $\frac{\cos\theta}{\sin\theta} + \frac{1}{\sin\theta} = \sqrt{3}$

$\cos\theta + 1 = \sqrt{3}\sin\theta$

$\sqrt{3}\sin\theta - \cos\theta = 1$

$r = \sqrt{3+1} = 2$

$\therefore \frac{\sqrt{3}}{2}\sin\theta - \frac{1}{2}\cos\theta = \frac{1}{2}$

$\sin\frac{\pi}{3}\sin\theta - \cos\frac{\pi}{3}\cos\theta = \frac{1}{2}$

$\cos\frac{\pi}{3}\cos\theta - \sin\frac{\pi}{3}\sin\theta = -\frac{1}{2}$

$\cos\left(\frac{\pi}{3}+\theta\right) = \cos\left(\pi - \frac{\pi}{3}\right)$

$= \cos\frac{2\pi}{3}$

$\therefore \theta + \pi/3 = 2n\pi \pm \frac{2\pi}{3}$

(ie) $\theta = 2n\pi + \frac{\pi}{3}$ (or) $\theta = 2n\pi - \pi$

18) solve : $\sin\theta + \sqrt{3}\cos\theta = 1$

TBP Sol: $r = \sqrt{1+3} = 2$

$\frac{1}{2}\sin\theta + \frac{\sqrt{3}}{2}\cos\theta = 1/2$

$\sin\frac{\pi}{6}\sin\theta + \cos\frac{\pi}{6}\cos\theta = \cos\left(\frac{\pi}{3}\right)$

$= \cos\pi/3$

$\cos\left(\theta - \frac{\pi}{6}\right)$

$\theta - \frac{\pi}{6} = 2n\pi \pm \frac{\pi}{3}.$

$\theta = 2n\pi + \frac{\pi}{2}$ (or) $\theta = 2n\pi - \frac{\pi}{6}.$

19 Solve : $\cos\theta + \sin\theta = \sqrt{2}$

TBP. Sol: $r = \sqrt{1+1} = \sqrt{2}$

$\cos\theta + \sin\theta = \sqrt{2}$

$\frac{1}{\sqrt{2}}\cos\theta + \frac{1}{\sqrt{2}}\sin\theta = 1$

$\cos\frac{\pi}{4}\cos\theta + \sin\frac{\pi}{4}\sin\theta = 1$

$\cos(\theta - \pi/4) = \cos 0$

$\theta - \pi/4 = 2n\pi$

$\theta = 2n\pi + \pi/4.$

20) Solve: $2\cos^2 x - 7\cos x + 3 = 0$

TBP Sol: $2\cos^2 x - 7x + 3 = 0$ $\quad a = 2,\ b = -7,\ c = 3$

$$\cos x = \frac{7 \pm \sqrt{49-24}}{4}$$

$$= \frac{7 \pm 5}{4} = 3, \tfrac{1}{2}$$

$\because$ $\cos\theta$ is inbetween -1 and 1, 3 is not possible.

$\therefore$ $\cos x = \tfrac{1}{2} = \cos\frac{\pi}{3}$.

$$x = 2n\pi \pm \pi/3.$$

21) Solve: $\cos 2\theta = \dfrac{\sqrt{5}+1}{4}$

TBP Solve: $\cos 2\theta = \dfrac{\sqrt{5}+1}{4}$

$$\cos 2\theta = \cos\frac{\pi}{5}$$

$$\therefore 2\theta = 2n\pi \pm \frac{\pi}{5}$$

$$\theta = n\pi \pm \frac{\pi}{10}.$$

22) Solve $\sin 2\theta - \cos 2\theta - \sin\theta + \cos\theta = 0$

TBP Sol: $(\sin 2\theta - \sin\theta) - (\cos 2\theta - \cos\theta) = 0$

$$2\cos\frac{3\theta}{2}\sin\theta/2 + 2\sin\frac{3\theta}{2}\sin\theta/2 = 0$$

$$2\sin\theta/2\left(\cos\frac{3\theta}{2} + \sin\frac{3\theta}{2}\right) = 0$$

$\sin\theta/2 = 0$

$\theta/2 = n\pi$

$\theta = 2n\pi$

$\cos\frac{3\theta}{2} + \sin\frac{3\theta}{2} = 0$

$$\frac{\cos 3\theta/2}{\cos 3\theta/2} + \frac{\sin 3\theta/2}{\cos 3\theta/2} = 0$$

$\tan 3\theta/2 = -1$

$= \tan\left(\pi - \frac{\pi}{4}\right)$

$\tan 3\theta/2 = \tan\frac{3\pi}{4}$

$\frac{3\theta}{2} = n\pi + \frac{3\pi}{4}$

$= (4n+3)\frac{\pi}{4}$

$3\theta = (4n+3)\pi$

$\theta = (4n+3)\frac{\pi}{3}$

23) solve $\cos x + \cos 3x - 2\cos 2x = 0$

Sol: $\cos x + \cos 3x - 2\cos 2x = 0$

TBP

$2\cos 2x \cos x - 2\cos 2x = 0$

$2\cos 2x(\cos x - 1) = 0$

$2\cos 2x = 0$

$\cos 2x = 0$

$2x = (2n+1)\frac{\pi}{2}$

$x = (2n+1)\frac{\pi}{4}$

$\cos x - 1 = 0$

$\cos x = 1$

$\cos x = \cos 0$

$x = 2n\pi \pm 0$

$x = 2n\pi$.

24) solve $\sin 5x - \sin x = \cos 3x$.

TBP Sol: $\sin 5x - \sin x = \cos 3x$

$2\cos 3x \sin 2x - \cos 3x = 0$

$2\cos 3x(\sin 2x - 1) = 0$

$2\cos 3x = 0$

$\cos 3x = 0$

$3x = (2n\pi + 1)\frac{\pi}{2}$

$x = (2n+1)\frac{\pi}{6}$

$\sin 2x - 1 = 0$

$\sin 2x = 1$

$= \sin\frac{\pi}{2}$

$2x = 2n\pi + \frac{\pi}{2}$

$= (2n+1)\pi/2$

$x = (2n+1)\frac{\pi}{4}$.

Properties of triangle

Theorem: In any triangle, the lengths of the sides are proportional to the sines of the opposite angle:

(ie) $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Proof:

Case i) $\angle A$ is acute.

Produce BO to meet the circle at D

$\angle BDC = \angle BAC = A$

$\angle BCD = 90^\circ$

$\sin \angle BCD = \dfrac{BC}{BD}$ (or) $\sin A = \dfrac{a}{2R}$ (or) $\dfrac{a}{\sin A} = 2R$.

Case (ii) $\angle A$ is right angle

In this case O must be on the side BC of the ΔABC

$\therefore \dfrac{a}{\sin A} = \dfrac{BC}{\sin 90} = \dfrac{2R}{1} \Rightarrow \dfrac{a}{\sin A} = 2R$

Case iii) $\angle A$ obtuse

Produce BO to meet the circle at D

$\angle BDC + \angle BAC = 180^\circ$

$\therefore \angle BDC = 180 - \angle BAC = 180 - A$

$\therefore \angle BCD = 90^\circ$

$\sin \angle BDC = \dfrac{BC}{BD}$ (or) $\sin(180 - A) = \sin A = \dfrac{a}{2R}$

$\Rightarrow \dfrac{a}{\sin A} = 2R$.

|||ly we can prove that $\dfrac{b}{\sin B} = 2R$ and $\dfrac{c}{\sin C} = 2R$.

$\therefore \dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C} = 2R$.

Note: $\dfrac{a}{b} = \dfrac{\sin A}{\sin B}$, $\dfrac{b}{c} = \dfrac{\sin B}{\sin C}$; $\dfrac{a}{c} = \dfrac{\sin A}{\sin C}$.

Napier's Formula

In a $\triangle$le ABC where

1) $\tan\frac{A-B}{2} = \frac{a-b}{a+b}\cot C/2$

2) $\tan\frac{B-C}{2} = \frac{b-c}{b+c}\cot A/2$

3) $\tan\frac{C-A}{2} = \frac{c-a}{c+a}\cot B/2$

Proof:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$$

Let $\frac{b-c}{b+c}\cot$ $\frac{a-b}{a+b}\cot C/2 = \frac{2R\sin A - 2R\sin B}{2R\sin A + 2R\sin C}\cdot\cot C/2$

$$= \frac{\cancel{2R}(\sin A - \sin B)}{\cancel{2R}(\sin A + \sin B)}\cdot\cot C/2$$

$$= \frac{\cancel{2}\cos\frac{A+B}{2}\sin\cancel{\frac{A-B}{2}}}{\cancel{2}\sin\frac{A+B}{2}\cdot\frac{A-B}{2}}\cot C/2$$

$$= \cot\frac{A+B}{2}\tan\frac{A-B}{2}\cot C/2$$

$$= \cot\left(\frac{\pi}{2}-\frac{C}{2}\right)\cdot\tan\frac{A-B}{2}\cot C/2$$

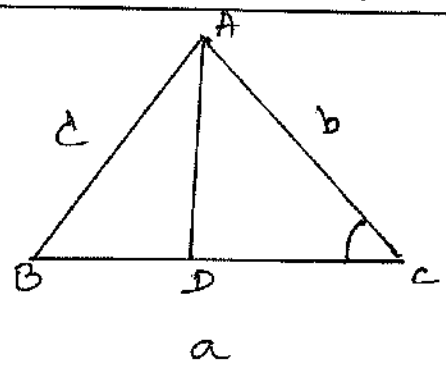
$$= \cancel{\tan C/2}\cdot\tan\frac{A-B}{2}\cancel{\cot C/2}$$

$$\frac{a-b}{a+b}\cot C/2 = \tan\frac{A-B}{2}$$

Cosine formula. $a^2 = b^2+c^2-2bc\cos A$, $b^2 = c^2+a^2-2ca\cos B$, $c^2 = a^2+b^2-2ab\cos C$

(or) $\cos A = \frac{b^2+c^2-a^2}{2bc}$, $\cos B = \frac{c^2+a^2-b^2}{2ca}$, $\cos C = \frac{a^2+b^2-c^2}{2ab}$.

Proof In $\triangle$le ABC Draw $AD \perp BC$.



$AB^2 = AD^2+BD^2$

$c^2 = AD^2+BD^2$ ——— ①

$\sin C = \frac{AD}{AC} \Rightarrow AD = AC\sin C = b\sin C$ ——— ②

$BD = BC - DC = a - b\cos C$ ——— ③

$\therefore c^2 = b^2\sin^2 C + (a-b\cos C)^2$

$c^2 = b^2\sin^2 C + (a^2+b^2\cos^2 C - 2ab\cos C) = a^2+b^2-2ab\cos C.$

Note: 1) $a^2 = b^2 + c^2 - 2bc\cos A$ says that the square of the sides is sum of the squares of the other two sides diminished by twice the product of those two sides and the cosine of the included angle.

2) The law of cosides can be viewed as generalaisation of Pythagorean theorem.

3) $c^2 = a^2 + b^2 - 2bc\cos C$. Since $-\cos C < 1$

$c^2 < a^2 + b^2 + 2ab$

$\Rightarrow c < a+b$ ||ly $a < b+c$, $b < c+a$.

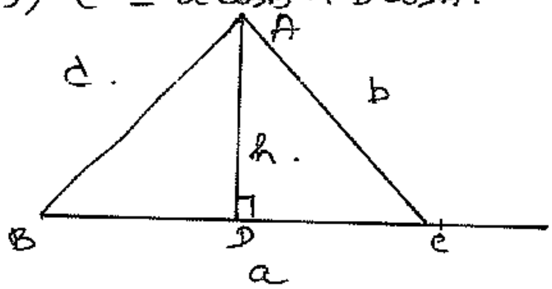
(ie) In any Δle sum of the two sides is greater than third side

<u>Projection Forormula:</u> In a Δle ABC 1) $a = b\cos C + c\cos B$

2) $b = c\cos A + a\cos C$

3) $c = a\cos B + b\cos A$.

<u>Proof:</u> In Δle ABC



$a = BC$, Draw $AD \perp BC$

$BC = BD + DC$

$= \frac{BD}{AB}.AB + \frac{DC.AC}{AC}$

$a = (\cos B)c + (\cos C)b$.

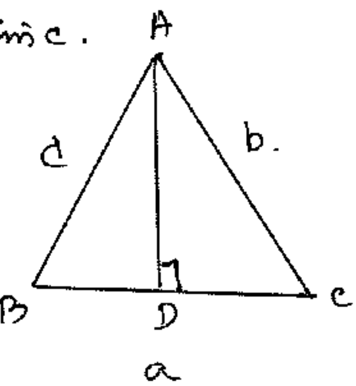
$a = c\cos B + b\cos C$

<u>Area of the Δle ABC</u>

In Δle ABC, area of the Δle is

$\Delta = \frac{1}{2}bc\sin A = \frac{1}{2}ca\sin B = \frac{1}{2}ab\sin C$.

Proof: In Δ ABC Draw $AD \perp BC$.



$\frac{AD}{AC} = \sin C$, $\Rightarrow AD = b\sin C$

$\Delta = \frac{1}{2}bh$ [b = base, h = height]

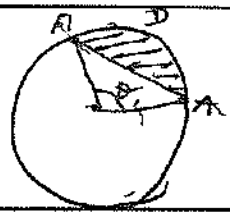
$\Delta = \frac{1}{2}a.b\sin C$

Note: 1) $\Delta = \frac{1}{2}ab\sin C$ says that the area is equal to one half of the product of two sides and the sine of their included angle.

2) Area of the segment ABD = Area of the sector − Area of the Δle OAB

$$= \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$$

$$= \frac{1}{2}r^2(\theta - \sin\theta)$$



Half angle formula: In ΔABC

1) $\sin A/2 = \sqrt{\frac{(s-b)(s-c)}{bc}}$ 2) $\cos A/2 = \sqrt{\frac{s(s-a)}{bc}}$ 3) $\tan A/2 = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$

where s is the semiperimeter

Proof: $\sin A/2 = \sqrt{\sin^2 A/2} = \sqrt{\frac{1-\cos A}{2}} = \sqrt{\frac{1}{2}\left(1 - \frac{b^2+c^2-a^2}{2bc}\right)}$

$$= \sqrt{\frac{2bc-b^2-c^2+a^2}{4bc}} = \sqrt{\frac{a^2-(b-c)^2}{4bc}}$$

$$= \sqrt{\frac{(a+b-c)(a-b+c)}{4bc}}$$

$$= \sqrt{\frac{(a+b+c-2c)(a+b+c-2b)}{4bc}}$$

$$= \sqrt{\frac{2(s-c)\,2(s-b)}{4bc}}$$

$$\sin A/2 = \sqrt{\frac{(s-c)(s-b)}{bc}}.$$

Note $\sin A = \frac{2}{bc}\sqrt{s(s-a)(s-b)(s-c)}$

Area of the Δle (Heron's Formula)

In ΔABC $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$ where s is the semiperimeter of ΔABC

Proof: $\Delta = \frac{1}{2}ab\sin C = \frac{1}{2}ab\left(2\sin\frac{C}{2}\cos\frac{C}{2}\right)$

$$= ab\sqrt{\frac{(s-a)(s-b)}{ab}}\sqrt{\frac{s(s-c)}{ab}}$$

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

1) The government plans to have a circular zoological park of diameter 8 KM. A separate area in the form of a segment formed by chord of length 4 KM is to be allotted exclusively for a veterinary hospital in the park. Find the area of the segment to be allotted for the veterinary hospital.

TBP

Area of the sector $= \frac{1}{2} r^2 (\theta - \sin\theta)$

$\because$ Δ is equilateral $\theta = \pi/3$

Area of the sector $= \frac{1}{2} 16 (\pi/3 - \sin \pi/3)$

$= 8(\pi/3 - \frac{\sqrt{3}}{2})$

$= \frac{8}{6}(2\pi - 3\sqrt{3})\, m^2$

2) In a ΔABC P.T $b^2 \sin 2C + c^2 \sin 2B = 2bc \sin A$.

TBP: Sol: We know $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$.

$\therefore a = 2R\sin A \quad b = 2R\sin B, \quad c = 2R\sin C$.

LHS $b^2 \sin 2C + c^2 \sin 2B = 4R^2 \sin^2 B \sin 2C + 4R^2 \sin^2 C \sin 2B$

$= 4R^2 \cdot \sin^2 B \cdot 2\sin C \cos C + 4R^2 \sin^2 C\, 2\sin B \cos B$

$= 8R^2 \sin B \sin C (\sin B \cos C + \cos B \sin C)$

$= 8R^2 \sin B \sin C \cdot \sin(B+C)$.

$= 8R^2 \sin B \sin C \sin(\pi - A)$

$= 8R^2 \sin C \sin B \sin A$

$= 8R^2 \left(\frac{c}{2R} \cdot \frac{b}{2R}\right) \sin A$

$= 2bc \sin A$.

3) Using sine formula prove projection formula (or)

TBP P.T $a = b\cos C + c\cos B$ by using sine formula.

Sol: Procedure Sine formula $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$.

RHS: $b\cos C + c\cos B$

$= 2R\sin B\cos C + 2R\sin C\cos B$

$= 2R(\sin B\cos C + \sin C\cos B)$.

$= 2R(\sin(B+C))$

$= 2R(\sin(180 - A)) = 2R\sin A = a$.

(ie) $b\cos C + c\cos B = a$

4) using cosine formula derive projection formula.

TBP (or) P.T $a = b\cos C + c\cos B$ by using cosine formula.

Sol: Procedure: $\cos C = \frac{a^2+b^2-c^2}{2ab}$, $\cos B = \frac{c^2+a^2-b^2}{2ca}$

LHS: $b\cos C + c\cos B$

$$= \cancel{b}\left(\frac{a^2+b^2-c^2}{2a\cancel{b}}\right) + \cancel{c}\left(\frac{c^2+a^2-b^2}{2\cancel{c}a}\right)$$

$$= \frac{a^2+\cancel{b^2}-\cancel{c^2}+\cancel{c^2}+a^2-\cancel{b^2}}{2a}$$

$$= \frac{\cancel{2}a^{\cancel{2}}}{\cancel{2}\cancel{a}} = a.$$

5) using projection formula prove cosine formula.

(or) P.T $\cos C = \frac{a^2+b^2-c^2}{2ab}$ By using projection formula.

Sol: Projection formula

$$a = b\cos C + c\cos B \quad \text{--- (1)}$$
$$b = c\cos A + a\cos C \quad \text{--- (2)}$$
$$c = a\cos B + b\cos A \quad \text{--- (3)}$$

(1) × a + (2) × b + (3) (−c)

$$a^2 = ab\cos C + a\cancel{c\cos B}.$$
$$b^2 = b\cancel{c\cos A} + ba\cos C$$
$$-c^2 = -\cancel{ca\cos B} - c\cancel{b\cos A}$$

$$a^2+b^2-c^2 = 2ab\cos C$$

$$\frac{a^2+b^2-c^2}{2ab} = \cos C.$$

6) (TBP) PT $a\sin\left(\frac{A}{2}+B\right) = (b+c)\sin\frac{A}{2}$

Sol: Procedure: $\frac{b}{a} = \frac{\sin B}{\sin A}$, $\frac{c}{a} = \frac{\sin C}{\sin a}$ and use C, D formula.

$\sin A = 2\sin\frac{A}{2}\cos\frac{A}{2}$

$\sin B + \sin C = 2\sin\frac{B+C}{2}\cos\frac{B-C}{2}$

$\sin A = 2\sin\frac{A}{2}\cos\frac{A}{2}$

$\sin\frac{B+C}{2} = \sin\left(\frac{180-A}{2}\right) = \cos\frac{A}{2}$

RHS: $(b+c)\sin\frac{A}{2} = a\left(\frac{b}{a}+\frac{c}{a}\right)\sin\frac{A}{2}$

$$= a\left(\frac{\sin B}{\sin A}+\frac{\sin C}{\sin A}\right)\sin\frac{A}{2}$$

$$= a\left(\frac{2\sin\left(\frac{B+C}{2}\right)\cos\frac{B-C}{2}}{2\cancel{\sin\frac{A}{2}}\cos\frac{A}{2}}\right)\cancel{\sin\frac{A}{2}}$$

$$= \frac{a \cdot \cancel{2\cos^2 A/2} \cos \frac{B-C}{2}}{2\cancel{\cos A/2}}$$ $\qquad C = 180 - (A+B)$

$$= a \cos \frac{B-C}{2} = a \cos \left(\frac{B-(180-\overline{A+B})}{2} \right)$$

$$= a \cos (B - 180 + A + B)$$

$$= a \cos \left(\frac{A+2B-180}{2} \right)$$

$$= a \cos \left(A/2 + B - 90 \right)$$

$$= a \cos \left(90 - (A/2 + B) \right)$$

$$= a \sin (A/2 + B)$$

8) P.T $a(\cos B + \cos C) = 2(b+c)\sin^2 A/2$

TBP

Sol: LHS: $a(\cos B + \cos C)$

$$= 2R \sin A \left(2\cos \frac{B+C}{2} \cos \frac{B-C}{2} \right)$$

$$= 4R \cdot 2\sin A/2 \cos A/2 \cdot \cos \frac{B+C}{2} \cos \frac{B-C}{2}$$

$$= 8R \sin A/2 \cos A/2 \cos \left(\frac{\pi}{2} - A/2 \right) \cos \frac{B-C}{2}$$

$$= 8R \sin A/2 \cos A/2 \sin A/2 \cos \frac{B-C}{2}$$

$$= \cancel{8}4R \sin^2 A/2 \left(2\cos A/2 \cos \frac{B-C}{2} \right)$$

$$= 4R \sin^2 A/2 \left(2\cos \left(\frac{\pi}{2} - \frac{B+C}{2} \right) \cos \frac{B-C}{2} \right)$$

$$= 4R \sin^2 A/2 \left(2\sin \frac{B+C}{2} \cos \frac{B-C}{2} \right)$$

$$= 4R \sin^2 A/2 \left[\sin B + \sin C \right]$$

$$= 2\sin^2 A/2 \left(2R\sin B + 2R\sin C \right)$$

$$= 2\sin^2 A/2 (b+c)$$

9) P.T $a\cos A + b\cos B + c\cos C = 2a \sin B \sin C$.

TBP

Sol: LHS: $a\cos A + b\cos B + c\cos C$ $\qquad$ use $\sin 2A + \sin 2B + \sin 2C = 4\sin A \sin B \sin C$

$$= 2R\sin A \cos A + 2R\sin B \cos B + 2R\sin C \cos C.$$

$$= R\left[\sin 2A + \sin 2B + \sin 2C \right]$$

$$= R\left[4\sin A \sin B \sin C \right] \quad (S.F)$$

$$= 2\left[2R\sin A \sin B \sin C \right]$$

$$= 2a \sin B \sin C.$$

10) In a $\triangle$le ABC if $\cos C = \frac{\sin A}{2\sin B}$ S.T the triangle is isosceles.
TBQ

Sol: Use $\frac{a}{b} = \frac{\sin A}{\sin B}$ and cosine formula.

Given $\cos C = \frac{\sin A}{2\sin B} = \frac{a}{2b}$.

$$\frac{a^2+b^2-c^2}{\cancel{2}a\cancel{b}} = \frac{a}{\cancel{2b}}$$

$$\cancel{a^2}+b^2-c^2 = \cancel{a^2}$$

$b^2 = c^2 \Rightarrow b = c$ $\therefore$ the triangle is isosceles.

11) In a $\triangle ABC$ P.T $\frac{\sin B}{\sin C} = \frac{c - a\cos B}{b - a\cos C}$.
TBP

Sol: apply projection formula $c = a\cos B + b\cos A$ $b = 2R\sin B$

$b = c\cos A + a\cos C$ $c = 2R\sin C$

RHS: $\frac{c - a\cos B}{b - a\cos C} = \frac{a\cancel{\cos B} + b\cos A - a\cancel{\cos B}}{c\cos A + a\cancel{\cos C} - a\cancel{\cos C}}$

$$= \frac{b\cancel{\cos A}}{c\cancel{\cos A}} = \frac{b}{c} = \frac{\cancel{2R}\sin B}{\cancel{2R}\sin C}$$

$$= \frac{\sin B}{\sin C}.$$

12) In a $\triangle ABC$ $\frac{a+b}{a-b} = \tan\frac{A+B}{2}\cot\frac{A-B}{2}$
TBP

Sol: Procedure: use sine formula and C, D formula.

LHS: $\frac{a+b}{a-b} = \frac{2R\sin A + 2R\sin B}{2R\sin A - 2R\sin B}$

$$= \frac{\cancel{2R}(\sin A + \sin B)}{\cancel{2R}(\sin A - \sin B)} = \frac{\cancel{2}\sin\frac{A+B}{2}\cos\frac{A-B}{2}}{\cancel{2}\cos\frac{A+B}{2}\sin\frac{A-B}{2}}$$

$$= \tan\frac{A+B}{2}\cot\frac{A-B}{2}$$

13) P.T $\frac{a\sin(B-C)}{b^2-c^2} = \frac{b\sin(C-A)}{c^2-a^2} = \frac{c\sin(A-B)}{a^2-b^2}$
In any $\triangle$.

Sol: Use sine formula

Consider $\frac{a\sin(B-C)}{b^2-c^2} = \frac{2R\sin A\sin(B-C)}{4R^2\sin^2 B - 4R^2\sin^2 C}$

$$= \frac{\cancel{2}R \sin(\pi - \overline{B+C}) \sin(B-C)}{\cancel{4}^{2}R^2 (\sin^2 B - \sin^2 C)}$$

$$= \frac{1}{2R} \frac{\sin(B+C)\sin(B-C)}{\sin^2 B - \sin^2 C}.$$

$$= \frac{1}{2R} \frac{(\sin^2 B - \sin^2 C)}{(\sin^2 B - \sin^2 C)} = \frac{1}{2R}.$$

Illy we can prove that $\dfrac{b \sin(C-A)}{c^2 - a^2} = \dfrac{c \sin(A-B)}{a^2 - b^2} = \dfrac{1}{2R}$

$$\therefore \frac{a \sin(B-C)}{b^2 - c^2} = \frac{b \sin(C-A)}{c^2 - a^2} = \frac{c \sin(A-B)}{a^2 - b^2} = \frac{1}{2R}.$$

14) In any ΔABC P.T $\dfrac{a^2 - c^2}{b^2} = \dfrac{\sin(A-C)}{\sin(A+C)}$

TBP.

Sol: Procedure: Sine Formula.

LHS: $$\frac{a^2 - c^2}{b^2} = \frac{\cancel{2}4R^2 \sin^2 A - 4R^2 \sin^2 C}{\cancel{2}4R^2 \sin^2 B}.$$

$$= \frac{\cancel{4R^2}(\sin^2 A - \sin^2 C)}{\cancel{4R^2} \sin^2 B}$$

$$= \frac{\sin(A+C)\sin(A-C)}{\sin^2 B}.$$

$$= \frac{\sin(\pi - B)\sin(A-C)}{\sin^2 B}$$

$$= \frac{\cancel{\sin B} \cdot \sin(A-C)}{\sin^{\cancel{2}} B}$$

$$= \frac{\sin(A-C)}{\sin(\pi - \overline{A+B})} = \frac{\sin(A-C)}{\sin(A+C)}$$

15) If in a ΔABC $\dfrac{\sin A}{\sin C} = \dfrac{\sin(A-B)}{\sin(B-C)}$ P.T a^2, b^2, c^2 are in AP.

TBP

Sol: Procedure use $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c} = k$

$$\because \frac{\sin A}{\sin C} = \frac{\sin(A-B)}{\sin(B-C)}$$

$$\frac{\sin(\pi - \overline{B+C})}{\sin(\pi - \overline{A+B})} = \frac{\sin(A-B)}{\sin(B-C)}$$

$$\frac{\sin(B+C)}{\sin(A+B)} = \frac{\sin(A-B)}{\sin(B-C)}$$

$$\sin(B+C)\sin(B-C) = \sin(A+B)\sin(A-B)$$

$$\sin^2 B - \sin^2 C = \sin^2 A - \sin^2 B$$

$k^2b^2 - k^2c^2 = k^2a^2 - k^2b^2$

$b^2 - c^2 = a^2 - b^2$

$2b^2 = a^2 + c^2 \Rightarrow a^2, b^2, c^2$ are in AP.

16) The angles of a triangle ABC, are in A.P and if $b:c = \sqrt{3}:\sqrt{2}$ find A.

TBP) Sol: $\because$ A, B, C are in AP $2B = A + C$ and Sine formula.

In any $\triangle$le $A + B + C = 180°$

$2B + B = 180$

$3B = 180°$

$B = 60°$

Given $b:c = \sqrt{3}:\sqrt{2}$

But $\dfrac{b}{\sin B} = \dfrac{c}{\sin C}$.

$\dfrac{\sqrt{3}}{\sin 60} = \dfrac{\sqrt{2}}{\sin C} \Rightarrow \dfrac{\sqrt{3}}{\sqrt{3}/2} = \dfrac{\sqrt{2}}{\sin C}$.

$2 = \dfrac{\sqrt{2}}{\sin C}$

$\sin C = \dfrac{\sqrt{2}}{2} = 1/\sqrt{2}$

$\Rightarrow C = 45°$

$A + B + C = 180$

$A + 60 + 45 = 180 \quad \therefore A = 75°$.

17) In any $\triangle ABC$, P.T $b^2 \sin 2C + c^2 \sin 2B = 2bc \sin A$.
TBP

Sol: Procedure: use Sine formula

LHS: $b^2 \sin 2C + c^2 \sin 2B = 4R^2 \sin^2 B \sin 2C + 4R^2 \sin^2 C \sin 2B$

$= 4R^2 (2 \sin^2 B \sin C \cos C + 2 \sin^2 C \sin B \cos B)$

$= 8R^2 \sin B \sin C (\sin B \cos C + \cos B \sin C)$

$= 8R^2 \sin B \sin C \cdot \sin(B+C)$

$= 8R^2 \sin B \sin C \sin(\pi - A)$

$= 8R^2 \sin B \sin C \sin A$.

$= 2 \cdot 2R \sin B \cdot 2R \sin C \cdot \sin A$

$= 2bc \sin A$

18) In a $\triangle ABC$ P.T $\sin\left(\dfrac{B-C}{2}\right) = \dfrac{b-c}{a} \cos A/2$
TBP

Sol: Procedure: use sine formula and C, D formula, 2A formula

RHS: $\dfrac{b-c}{a} \cos A/2 = \dfrac{2R \sin B - 2R \sin C}{2R \sin A} \cdot \cos A/2$

$= \dfrac{2R(\sin B - \sin C)}{2R \sin A} \cdot \cos A/2$

$$= \frac{\cancel{2}\cos\frac{B+C}{2}\sin\frac{B-C}{2}\cdot\cancel{\cos A/2}}{\cancel{2}\sin A/2\,\cancel{\cos A/2}}$$

$$= \frac{\cos\left(\frac{\pi}{2}-\frac{A}{2}\right)\cdot\sin\frac{B-C}{2}}{\sin A/2}$$

$$= \frac{\cancel{\sin A/2}\,\sin\frac{B-C}{2}}{\cancel{\sin A/2}} = \sin\frac{B-C}{2}$$

19) If the three angles in a triangle are in the ratio 1:2:3 then P.T the corresponding sides are in the ratio $1:\sqrt{3}:2$.
TBP

Sol: Procedure use Sine formula.

If angles are $\theta, 2\theta, 3\theta$

$\theta+2\theta+3\theta = 180°$

$\theta = 30°$

We know $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$\frac{a}{\sin 30} = \frac{b}{\sin 60} = \frac{c}{\sin 90°}$

$\Rightarrow a:b:c = \sin 30° : \sin 60° : \sin 90°$

$= \frac{1}{2} : \frac{\sqrt{3}}{2} : 1$

$= 1 : \sqrt{3} : 2$

20) In a ΔABC P.T $(b+c)\cos A + (c+a)\cos B + (a+b)\cos C = a+b+c$
TBP

Sol: Procedure: use cosine formula, arrange last steps as a+b+c.

LHS $= (b+c)\cos A + (c+a)\cos B + (a+b)\cos C$

$= b\cos A + c\cos A + c\cos B + a\cos B + a\cos C + b\cos C$

$= (b\cos C + c\cos B) + (c\cos A + a\cos C) + (a\cos B + b\cos A)$

$= a+b+c.$

21) In a ΔABC P.T $\frac{a^2+b^2}{a^2+c^2} = \frac{1+\cos(A-B)\cos C}{1+\cos(A-C)\cos B}$.
TBP

Sol: Procedure use Sine formula.

LHS: $\frac{a^2+b^2}{a^2+c^2} = \frac{4R^2\sin^2 A + 4R^2\sin^2 B}{4R^2\sin^2 A + 4R^2\sin^2 C} = \frac{\cancel{4R^2}(\sin^2 A + \sin^2 B)}{\cancel{4R^2}(\sin^2 A + \sin^2 C)}$

$= \frac{1-\cos^2 A + \sin^2 B}{1-\cos^2 A + \sin^2 C}.$

$$= \frac{1-(\cos^2 A - \sin^2 B)}{1-(\cos^2 A - \sin^2 C)}$$

$$= \frac{1-\cos(A+B)\cdot\cos(A-B)}{1-\cos(A+C)\cos(A-C)}$$

$$= \frac{1-\cos(\pi-C)\cos(A-B)}{1-\cos(\pi-B)\cos(A-C)}$$

$$= \frac{1+\cos(A-B)\cos C}{1+\cos(A-C)\cos B}.$$

22) Derive cosine formula using sine formula.

TBP

Sol: we know $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R.$

$$\frac{b^2+c^2-a^2}{2bc} = \frac{4R^2\sin^2 B + 4R^2\sin^2 C - 4R^2\sin^2 A}{2\cdot 2R\sin B\cdot 2R\sin C}$$

$$= \frac{\cancel{4}R^2(\sin^2 B+\sin^2 C-\sin^2 A)}{2\cdot\cancel{4}R^2\cdot\sin B\sin C}$$

$$= \frac{\sin^2 B+\sin(C+A)\sin(C-A)}{2\sin B\sin C}$$

$$= \frac{\sin^2 B+\sin(\pi-B)\sin(C-A)}{2\sin B\sin C}$$

$$= \frac{\sin^2 B+\sin B\cdot\sin(C-A)}{2\sin B\sin C}$$

$$= \frac{\cancel{\sin B}(\sin B+\sin(C-A))}{2\cancel{\sin B}\cdot\sin C}$$

$$= \frac{\sin(\pi-\overline{C+A})+\sin(C-A)}{2\sin C}$$

$$= \frac{\sin(C+A)+\sin(C-A)}{2\sin C}$$

$$= \frac{2\cancel{\sin C}\cos A}{2\cancel{\sin C}} = \cos A$$

23) Using Heron's formula S.T the equilateral △le has the maximum area for any fixed perimeter

TBP.

Sol: Procedure : use If $xyz \le k$ maximum occurs when $x = y = z$.

Let ΔABC be a triangle with constant perimeter $2s$. $\therefore s$ is const

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

Δ is maximum when $(s-a)(s-b)(s-c)$ is maximum.

(ie) $(s-a)(s-b)(s-c) \le \left(\frac{(s-a)+(s-b)+(s-c)}{3}\right)^3 = \frac{s^3}{27}$

$(s-a)(s-b)(s-c) \le \frac{s^3}{27}$ $\quad \because GM \le AM$.

(ie) Equality occurs when $s-a = s-b = s-c$

(ie) when $a = b = c$ max of $(s-a)(s-b)(s-c) = \frac{s^3}{27}$

$\therefore$ For fixed perimeter $2s$, the area of the Δle is maxi when $a = b = c$.

$\therefore$ Max area $= \sqrt{\frac{s \cdot s^3}{27}} = \frac{s^2}{3\sqrt{3}}$ sq. units.

24) An Engineer has to develop a triangular shaped park with a perimeter 120m in a village. The park to be developed must be maximum area. Find out the dimension of the park. TBP

Sol: Perimeter of the Δle $= 120$ m.

(X) For a fixed perimeter $2s$, the area of the triangle is maximum when $a = b = c$

$\therefore$ Side of the Δle $= \frac{120}{3} = 40$m.

$a = b = c = 40$m.

25) A rop of length 12m is given. Find the largest area of the triangle formed by this rope and find the dimensions of the triangle so formed. TBP

Sol: Perimeter of the Δle $= 12$ m.

(X) For a fixed perimeter $2s$ the area of the triangle is maximum when $a = b = c$.

$\therefore$ Side of the Δle $= \frac{12}{3} = 4$

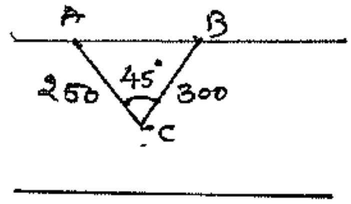
$\therefore a = b = c = 4$.

Area of the Δle $= \frac{s^2}{3\sqrt{3}}$ $\quad \boxed{\begin{array}{l} 2s = 12 \\ s = 6 \end{array}}$

$= \frac{\cancel{36}^{12}}{\cancel{3}\sqrt{3}} = \frac{12\sqrt{3}}{3} = 4\sqrt{3}$ sq. mts

26) Two trees A and B are on the same side of a river. From a point C in the river the distance of ~~the~~ trees A and B are 250m and 300m respectively of the angle C is 45° Find the distance between the trees. $\sqrt{2} : 1.44$.

Sol: Procedure: Use cosine formula.



$$AB^2 = AC^2 + BC^2 - 2AB \cdot AC \cdot \cos 45^\circ$$

$$= 250^2 + 300^2 - 2 \times 250 \times 300 \times \frac{1}{\sqrt{2}}$$

$$= 62500 + 90000 - 75000\sqrt{2}$$

$$= 62500 + 90000 - 75000 \times 1.44$$

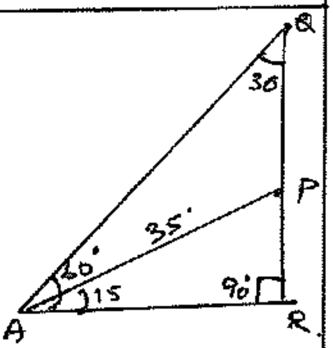
$$= 152500 - 108000$$

$$= 44500$$

$$AB = \sqrt{44500} = 210.95 \text{ m}.$$

27) A tree stand vertically on a hill side which makes an angle 15° with the horizontal. From a point on the ground 35 m down the hill from the base of the tree. The angle of elevation of the top of the tree is 60° Find the height of the tree.

Sol: Let PQ be the tree AR be the point on Ground



In a $\triangle$ ARQ

$\angle RAQ = 60^\circ$ $\angle ARQ = 90$ $\angle AQP$

In $\triangle APQ$ $\angle PAQ = 60 - 15 = 45^\circ$

By Sine Rule: $\frac{35}{\sin 30} = \frac{PQ}{\sin 45}$

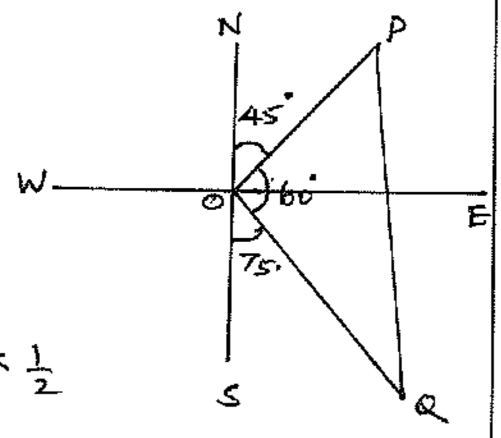
$$= \frac{35}{1/2} = \frac{PQ}{1/\sqrt{2}}$$

$$\frac{70}{\sqrt{2}} = PQ$$

$$\therefore PQ = \frac{70 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = 35\sqrt{2} \text{ m}.$$

28) Two ships leave a port at the same time. One goes 24 km/hr in the direction N 45° E and the other travels 32 km/hr in the direction S 75° E. Find the distance between the ships at the end of 3 hrs.

Sol: Procedure: use cosine formula.



$OP = 24 \times 3 = 72$ km

$OQ = 32 \times 3 = 96$ km.

By cosine formula

$PQ^2 = OP^2 + OQ^2 - 2\,OP \cdot OQ \cos 60°$

$= 72^2 + 96^2 - 2 \times 72 \times 96 \times \frac{1}{2}$

$= 5184 + 9216 - 6912$

$= 7488$

$PQ = \sqrt{7488} = 86.53$ km.

Solutions of $\triangle$le

1. In a $\triangle$le ABC $a = 3$, $b = 5$, $c = 7$. Find the values of $\cos A$, $\cos B$, $\cos C$

TBP

Sol: $a = 3$, $b = 5$, $c = 7$

$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc} = \dfrac{25 + 49 - 9}{70} = \dfrac{65}{70}\ \dfrac{13}{14}$

$\cos B = \dfrac{c^2 + a^2 - b^2}{2ca} = \dfrac{49 + 9 - 25}{2 \times 7 \times 3} = \dfrac{33}{2 \times 7 \times 3} = 11/14$

lly $\cos C = -1/2$

2. In $\triangle$le ABC $A = 30°$ $B = 60°$ and $c = 10$ Find a and b.

TBP

Sol: $A = 30$, $B = 60$ $c = 10$

$C = 180 - (30 + 60)$
$= 90°$

$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$.

$\dfrac{a}{\sin 30} = \dfrac{b}{\sin 60} = \dfrac{10}{\sin 90°}$

$\dfrac{a}{1/2} = \dfrac{b}{\sqrt{3}/2} = 10 \Rightarrow a = 5$

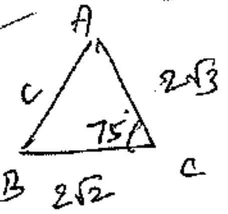
$b = 10 \times \dfrac{\sqrt{3}}{2} = 5\sqrt{3}$

3) In $\triangle$ ABC, $a = 2\sqrt{2}$ $b = 2\sqrt{3}$, $C = 75°$ find the other sides and angles.

TBP

Sol: $a = 2\sqrt{2}$, $b = 2\sqrt{3}$, $c = 75^\circ$

$$\cos C = \frac{a^2+b^2-c^2}{2ab} \Rightarrow \cos 75^\circ = \frac{8+12-c^2}{8\sqrt{6}}$$



$$= \frac{\sqrt{3}-1}{\cancel{2}\sqrt{2}} = \frac{20-c^2}{4\cancel{8}\sqrt{6}\sqrt{3}}$$

$$\frac{\sqrt{3}-1}{\cancel{2}\sqrt{2}} = \frac{20-c^2}{\cancel{8}_4\sqrt{2}\sqrt{3}}$$

c

$c^2 = a^2+b^2-2ab\cos C$

$$4\sqrt{3}(\sqrt{3}-1) = 20-c^2$$

$$4\sqrt{3}(\sqrt{3}-1) = 20-c^2$$

$$c^2 = 20-(12-4\sqrt{3})$$

c

$$c^2 = 20-12+4\sqrt{3}$$

$$= 8+4\sqrt{3} = 4($$

$A+B = $

$$= \cancel{8}+4\sqrt{3} =$$

$$6+2+4\sqrt{3}$$

$$= 4(2+\sqrt{3}) = 2($$

$$(\sqrt{6}+\sqrt{2})^2 = 2\times 2\sqrt{3}$$

$(\sqrt{6}+\sqrt{2})$

$2+6+2\sqrt{12}$

$8+4\sqrt{3}$

$$\cos A = \frac{b^2+c^2-a^2}{2bc} = \frac{12+8+4\sqrt{3}-8}{2\cdot 2\sqrt{3}}$$

---

4) Find the area of the $\triangle$le whose sides are 13 cm, 14 cm, 15 cm

TBP: Sol: $2s = a+b+c \Rightarrow s = \frac{13+14+15}{2} = 21$

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{21\times 8\times 7\times 6}$$

$$= \sqrt{7\times 3\times 4\times 2\times 7\times 3\times 2}$$

$$= 7\times 3\times 2\times 2 = 84 \text{ sq.cm.}$$

---

5) In any $\triangle$le ABC P.T $a\cos A + b\cos B + c\cos C = \frac{8\Delta^2}{abc}$.

TBP Sol: we know $a\cos A + b\cos B + c\cos C = 2a\sin B\sin C$.

$$\text{LHS}: a\cos A + b\cos B + c\cos C = 2a\sin B\sin C$$

$$= 2\cancel{a}\cdot\left(\frac{2\Delta}{c\cancel{a}}\right)\left(\frac{2\Delta}{ab}\right)$$

$$= \frac{8\Delta^2}{abc}.$$

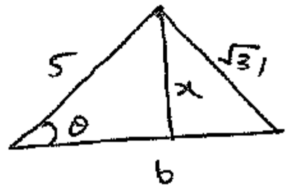
---

6) Suppose that there are two cell phone towers within range of cell phone. Two towers are located at 6 km apart along a straight highway running east to west and the cell phone is north of the highway. The signal is 5 km from the first tower and $\sqrt{31}$ km from the second tower. Determine the cell phone north and east of the first tower and how far it is from the highway.

TBP:

Sol: Let θ be the position of the cell phone form north to east of the first tower.

By cosine formula.



$$(\sqrt{31})^2 = 25+36-2\times5\times6\cos\theta$$

$$60\cos\theta = 61-31$$

$$= 30$$

$$\cos\theta = \frac{30}{60} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$$

Let x be the distance of the cell phone's position from the highway

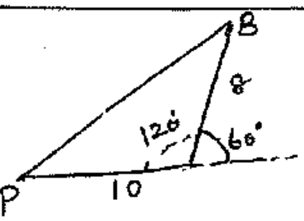
$$\sin\theta = \frac{x}{5} \Rightarrow x = 5\sin 60$$

$$= 5\cdot\frac{\sqrt{3}}{2} \text{ KM.}$$

7) Suppose that a boat travels 10 KM from the port towards north and then turns 60° to its left. If the boat travels further 8 KM, how far from the port is the boat.

TBP

Sol: By using cosine formula



$$BP^2 = 10^2+8^2-2\cdot10\cdot8\cos120$$

$$= 100+64-160\left(-\cos\frac{\pi}{3}\right)$$

$$= 164+\cancel{160}^{80}\times\frac{1}{\cancel{2}}$$

$$= 244$$

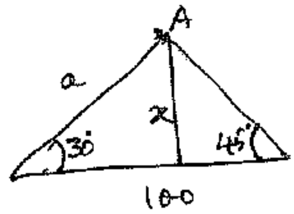
$$= 4\times61$$

$$BP = 2\sqrt{61} \text{ KM.}$$

8) Suppose two radar stations located 100 km apart, each detect a fighter aircraft between them. The angle of elevation measured by the first station is 30° whereas the angle of elevation measured by the second station is 45°. Find the altitude of the aircraft at the instant.

TBP

Sol: $\angle A = 180-(30+45) = 105^\circ$



$$\frac{a}{\sin A} = \frac{100}{\sin 105} \Rightarrow \frac{100}{\frac{\sqrt{3}+1}{2\sqrt{2}}}\times\frac{1}{\sqrt{2}}$$

$$= \frac{200(\sqrt{3}-1)}{2} = 100(\sqrt{3}-1)$$

$$\sin 30^\circ = \frac{x}{a} \Rightarrow x = \cancel{100}^{50}(\sqrt{3}-1)\cdot\frac{1}{\cancel{2}} = 50(\sqrt{3}-1) \text{ KM.}$$

9) If the sides of ΔABC are $a=4$, $b=6$, $c=8$ then S.T $4\cos B+3\cos C=2$

TBP

Sol: $\cos B = \frac{c^2+a^2-b^2}{2ca} = \frac{64+16-36}{2\cdot 8\cdot 4}$

$= \cancel{\frac{44}{64}} \frac{11}{16}$

$\cos C = \frac{a^2+b^2-c^2}{2ab} = \frac{16+36-64}{2\times 4\times 6} = \frac{-12}{2\times 4\times 6} = -\frac{3}{4}$

LHS $= 4\cos B + 3\cos C = 4\times\frac{11}{16} \quad \frac{}{4} = \frac{8}{4} = 2$

10) In a $\triangle$le ABC $a=\sqrt{3}-1$, $b=\sqrt{3}+1$, $C=60^\circ$ Find the other side and other two angles.
TBP

Sol: $a=\sqrt{3}-1 \quad b=\sqrt{3}+1 \quad c=60^\circ$

$c^2 = a^2+b^2-2ab\cos c$

$= (\sqrt{3}-1)^2+(\sqrt{3}+1)^2-2(\sqrt{3}+1)(\sqrt{3}-1)\cos 60^\circ$

$= 2(3+1) - \cancel{2}(3-1)\cdot\frac{1}{\cancel{2}}$

$= 8-2$

$= 6.$

$c=\sqrt{6}.$

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{\sqrt{6}}{\sin 60^\circ}$

$\frac{\sqrt{3}+1}{\sin B} = \frac{\sqrt{6}\sqrt{3}\sqrt{2}}{\sqrt{3}/2} = \sqrt{2}\cdot 2$

$\sin B = \frac{\sqrt{3}+1}{2\sqrt{2}} \Rightarrow B = 105^\circ$

$\frac{\sqrt{3}-1}{\sin A} = \frac{\sqrt{6}}{\sqrt{3}/2} = \frac{\cancel{\sqrt{3}}\times\sqrt{2}\times 2}{\cancel{\sqrt{3}}}$

$\sin A = \frac{\sqrt{3}-1}{2\sqrt{2}} = \Rightarrow A = 15^\circ$

11) In a $\triangle$le ABC P.T if $a=12$ $b=8$ $C=30^\circ$ area is 24 sq.cm.
TBP

Sol: $\Delta = \frac{1}{2}ab\sin C$

$= \frac{1}{2}\cdot 12\cdot 8\sin 30$

$= \frac{1}{\cancel{2}}\cdot\cancel{12}^{3}\cdot 8\cdot\frac{1}{\cancel{2}} = 24$ sq.cm.

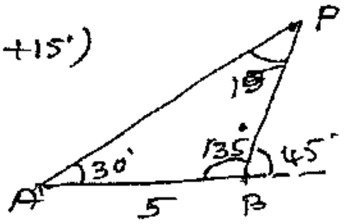
12) In a $\triangle$le ABC, $a=18$, $b=24$ cm, $c=30$ area of the $\triangle$le is 216 sq.cm.
TBP

Sol: $s = \frac{18+24+30}{2} = 36$

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{36 \times 18 \times 12 \times 6}$$
$$= \sqrt{36 \times 9 \times 2 \times 2 \times 6 \times 6}$$
$$= 6 \times 3 \times 2 \times 6$$
$$= 216 \text{ sq.cm.}$$

13) Two soldiers A and B in two different underground bunkers on a straight road, spot an intruder at the top of a hill. The angle of elevation of the intruder from A and B to the ground level in the esterndirection are 30° and 45° Resp. If A and B stand 5 KM apart Find the distance of the intruder from B.
TBC

Sol: From the diag. $\angle P = \theta = 180 - (135 + 15')$
$= 15$



$$\frac{5}{\sin\theta} = \frac{x}{\sin 30}$$

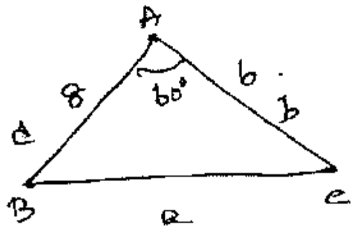
$$\frac{5}{\sin 15} = \frac{x}{1/2} \Rightarrow 2x = \frac{5}{\sin 15}$$

$$\cancel{2}x = \frac{5 \times \cancel{2}\sqrt{2}}{\sqrt{3}-1}$$

$$x = \frac{5\sqrt{2}}{\sqrt{3}-1}$$

14) A researcher wants to determine the width of a pond from east to west which cannot be done by actual measurement from a point P. He finds the distance to the eastern most point of the pond to be 8 KM while the distance to western most point from P to be 6 KM if the angle between the two. sight is 60° Find the width of the pond
TBP

Sol: $\angle A = 60°$ $b = 6$, $c = 8$

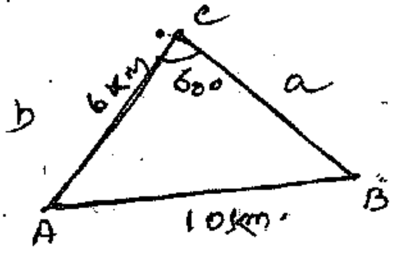


$$a^2 = b^2 + c^2 - 2bc\cos A.$$
$$= 36 + 64 - 2 \times 6 \times 8 \times \cos 60°$$
$$= 100 - \cancel{96}^{48} \cdot \frac{1}{\cancel{2}}$$
$$= 52$$
$$= 4 \times 13.$$
$$a = 2\sqrt{13} \text{ km}.$$

15) Two Navy helicopters A and B are flying over the Bay of Bengal at same altitude from the sea level to search a missing boat

TBP. - Pilots of Both the helicopters sight the boat at the same time while they are apart 10 KM from each other. If the distance of the boat from A is 6 KM if the line segment AB subtends $60°$ at the boat Find the distance of the boat from B.

Sol: $\angle C = 60°$ $b = 6$, $c = 10$



$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{6}{\sin B} = \frac{10}{\sin 60} \Rightarrow \sin B = \frac{\cancel{6}^{3} \times \sqrt{3}}{10\,\cancel{2}} = \frac{3\sqrt{3}}{10}$$

c

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \quad a = b\cos C + c\cos B$$

$$a = 6\cos 60 + c\sqrt{1-\sin^2 B}$$

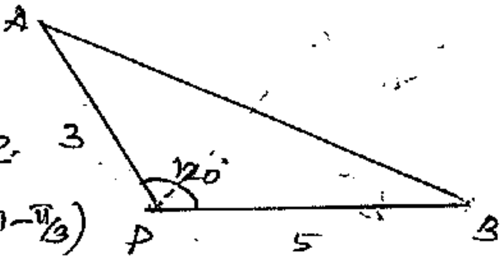
$$= \cancel{6}^{3} \times \frac{1}{\cancel{2}} + 10\sqrt{\frac{100-27}{100}}$$

$$= 3 + \frac{\cancel{10}(\sqrt{73})}{\cancel{10}} = 3+\sqrt{73} \text{ KM.}$$

16) A straight tunnel is to be made through a mountain. A surveyor observes the Two extremities A and B of the

TBP tunnel to be built from a point P in front of the mountain If $AP = 3$ KM $BP = 5$ KM $\angle APB = 120°$ then find the lengths of The tunnel to be built.

Sol: $AP = 3$, $BP = 5$ $\angle APB = 120$



$$AB^2 = AP^2 + BP^2 - 2\,AP \cdot BP \cos \angle P$$

$$= 9 + 25 - 2 \cdot 3 \cdot 5 \cos(\pi - \pi/3)$$

$$= 9 + 25 + 2 \cdot 3 \cdot 5 \cdot \frac{1}{2}$$

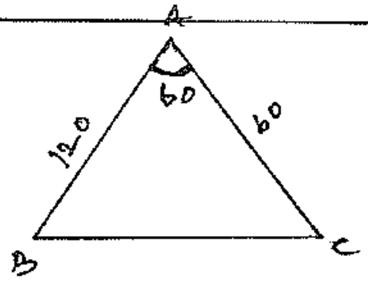
$$= 49$$

$$AB = 7$$

17) A farmer wants to purchase triangular shaped land with sides 120 feet and 60 feet and angle included between these two sides

TBP is $60°$ If the land costs Rs 500 per sq. feet find the amount need to purchase the land Also find the perimeter of the land.

Sol: $AB = 120$ $AC = 60$ $\angle A = 60°$



$\Delta = \frac{1}{2} AB \cdot AC \cdot \sin 60$

$= \frac{1}{\cancel{2}} \times 120 \times \overset{15}{60} \times \frac{\sqrt{3}}{\cancel{2}}$

$= 1800 \times 1.732$ sq feet

Total cost $= 500 \times 1800 \times 1.732$

$= 900000 \times 1.732$

$= 1558800000$ Rs.

$= 15,58,800$

$a^2 = b^2 + c^2 - 2bc\cos 60°$

$= 14400 + 3600 - \cancel{2} \times 120 \times 60 \cdot \frac{1}{\cancel{2}}$

$= 18000 - 7200$

$a^2 = 108,00 = 400 \times 27$

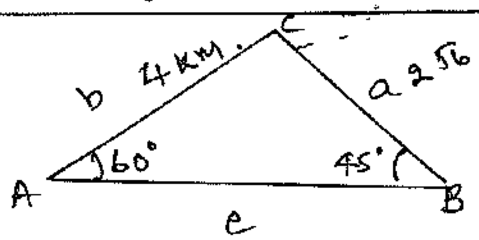
$a = 20\sqrt{27}$

$\therefore$ Perimeter $= 120 + 60 + 20\sqrt{27}$

$= 180 + 20\sqrt{27}$ feet.

18) A man starts his morning walk at a point A reaches to points B and C and finially back to A s.t $\angle A = 60°$ $\angle B = 45°$ $AC = 4$ KM
TBP Find the total distance he covered during his morning walk.

Sol: $b = 4$ km $\angle A = 60$ $\angle B = 45°$



$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.

$\frac{a}{\sin 60} = \frac{4}{\sin 45}$

$\frac{\cancel{2}a}{\sqrt{3}} = \overset{2}{\cancel{4}}\sqrt{2}$

$a = 2\sqrt{6}$.

$c = a\cos B + b\cos A$

$= 2\sqrt{6} \cdot \cos 45 + 4 \times \cos 60$

$= 2\sqrt{6} \times \frac{1}{\sqrt{2}} + \cancel{4}^{2} \frac{1}{\cancel{2}}$

$= 2\sqrt{3} + 2$

$= 2\sqrt{3} + 2$

$= 2(\sqrt{3}+1)$

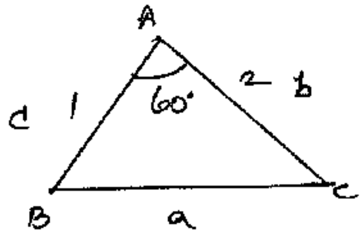
Total distance

$= 4 + 2\sqrt{6} + 2\sqrt{3} + 2$

$= 2\sqrt{6} + 2\sqrt{3} + 6$

19) A plane is 1 KM from one landmark and 2 KM from another. From the planes point of view the land between them subtends an angle 60°. How far apart are landmarks.

Sol: $\angle A = 60^\circ$ $b = 2$, $c = 1$



$$a^2 = b^2 + c^2 - 2bc\cos 60^\circ$$

$$= 4 + 1 - 2 \cdot \cancel{2} \cdot 1 \cdot \frac{1}{\cancel{2}}$$

$$= 5 - 2$$

$$= 3$$

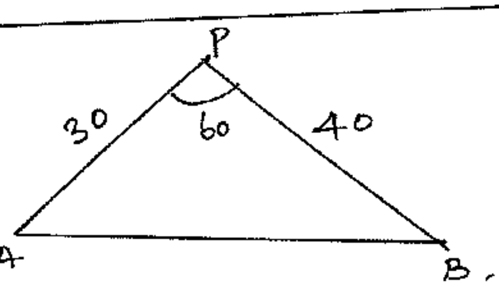
$$a = \sqrt{3}.$$

(if $\theta = 45^\circ$ the book answer is correct).

20) Two vehicles leave the same place P at the same time moving along two different roads. One vehicle moves at an average speed of 60/KM per hr. and the other vehicle moves at an average speed of 80/KM p.hr. After half an hour the vehicle reach the destinations A and B. If AB subtends 60° at the initial point P then find AB.

TBP

Sol: $AP = 30$, $PB = 40$ $\angle P = 60^\circ$



By cosine formula

$$AB^2 = AP^2 + PB^2 - 2\,AP \cdot PB\cos 60^\circ$$

$$= 30^2 + 40^2 - \cancel{2} \cdot 30 \cdot 40 \cdot \frac{1}{\cancel{2}}$$

$$= 900 + 1600 - 1200$$

$$= 1300$$

$$= 100 \times 13$$

$$AB = 10\sqrt{13}\ \text{KM}.$$

(Note: The Book answer is correct only when after one hour)

21) In any $\triangle$le ABC P.T that the area $\Delta = \dfrac{b^2 + c^2 - a^2}{4\cot A}$.

TBP.

Sol: RHS : $$\frac{b^2 + c^2 - a^2}{4\cot A} = \frac{\cancel{2}bc\cos A}{2\,\cancel{4}\,\frac{\cos A}{\sin A}}$$

$$= \frac{1}{2}bc\sin A$$

$$= \Delta.$$

Exercise - 3.2

4) In a circle of diameter 40 cm a chord of length 20 cm find the length of the minor arc of the chord.

Sol: diameter of the circle $r = 20$ cm.

Let $AB = 20$ cm

$\therefore$ The $\triangle$le is equilateral $\triangle$le.

TBP

$\therefore \angle AOB = 60^\circ$

$$l = r\theta = 20 \times \frac{\pi}{3}$$

$$= 20 \times \frac{1}{3} \times \frac{22}{7}$$

$$= 20.95 \text{ cm}$$

5) Find the degree measure of the angle subtended at the centre of circle of radius 100 cm by an arc of length 22 cm.

Sol: $r = 100$ cm

Length of the arc $= 11$ cm.

$$\theta = \frac{l}{r} = \frac{22}{100}$$

$\pi = 180^\circ$

TBP

$$\theta = 18\cancel{0} \times \frac{7}{\cancel{22}} \times \frac{\cancel{22}}{10\cancel{0}}$$

$$= \frac{18 \times 7}{10} = 12^\circ \tfrac{3}{5}$$

$$\simeq 12^\circ 36'$$

9) An airplane propeller rotates 1000 times per minute. Find the number of degrees that a point on the edge of the propeller will rotate in one second.

Sol: One complete rotation : 360°

(ie) number of degrees taken in $1' = 1000 \times 360^\circ$

TBP

" $60'' = 1000 \times 360$

" $1'' = \frac{1000 \times \cancel{360}^{6}}{\cancel{60}}$

$= 6000^\circ$

$\therefore\ x = 0, \pi, \frac{3\pi}{2}, \pi.$

2) $2\cos^2 x + 1 = -3\cos x.$

$2\cos^2 x + 3\cos x + 1 = 0$

$(\cos x + 1)(2\cos x + 1) = 0$

$\cos x + 1 = 0 \qquad 2\cos x + 1 = 0$

$\cos x = -1 \qquad 2\cos x = -1$

$\cos x = -1/2$

TBP

When $\cos x = -1$

$\cos x = -\cos(\cancel{0})$

$x = \pi$

When $2\cos x + 1 = 0$

$\cos x = -1/2$

$x = \pi + \pi/3$ ~~or~~ $\pi - \pi/3$

$= 4\pi/3$ (or) $2\pi/3.$

$\therefore$ The values are $x = \pi, 2\pi/3, 4\pi/3$

3) $2\sin^2 x + 1 = 3\sin x.$

$2\sin^2 x + 1 = 3\sin x$

$2\sin^2 x - 3\sin x + 1 = 0$

$(\sin x - 1)(2\sin x - 1) = 0$

TBP

$\sin x - 1 = 0 \qquad 2\sin x - 1 = 0$

$\sin x = 1 \qquad 2\sin x = 1$

$x = \pi/2 \qquad \sin x = 1/2$

$x = \pi/6$

$\therefore$ The values are $x = \pi/6, \pi/2$

4) $\cos 2x = 1 - 3\sin x$

$\cos 2x = 1 - 3\sin x$

$\cancel{1} - 2\sin^2 x = \cancel{1} - 3\sin x.$

$-2\sin^2 x + 3\sin x = 0$

$2\sin^2 x = 3\sin x$

TBP

$\sin x(2\sin x - 3) = 0$

$\sin x = 0 \qquad 2\sin x = 3$

$\sin x = 3/2$ not possible.

$x = 0, \pi$

$\therefore$ The values are $x = 0, \pi.$

Exercise - 3.7

3) If $x+y+z = xyz$, P.T $\frac{2x}{1-x^2}+\frac{2y}{1-y^2}+\frac{2z}{1-z^2} = \frac{2x}{1-x^2}\cdot\frac{2y}{1-y^2}\cdot\frac{2z}{1-z^2}$

Sol: Given $x+y+z = xyz$

Let $x = \tan A$, $y = \tan B$, $z = \tan C$

TBP

$\tan A + \tan B + \tan C = \tan A \tan B \tan C$.

Let $\tan(A+B+C) = 0 \Rightarrow \tan[(A+B)+C]$

$$\frac{\tan(A+B)+\tan C}{1-\tan(A+B)\tan C} = 0$$

$$\Rightarrow \tan(A+B)+\tan C = 0$$

$$\frac{\tan A+\tan B}{1-\tan A\tan B} + \tan C = 0$$

$$\Rightarrow \tan A+\tan B+\tan C = \tan A + \tan B + \tan C.$$

$\therefore \tan(A+B+C) = 0$

$\Rightarrow \tan(2A+2B+2C) = 0$

$\Rightarrow \tan 2A + \tan 2B + \tan 2C = \tan 2A \cdot 2\tan B \tan 2C$ — ①

$\because x = \tan A$, $\tan 2A = \frac{2\tan A}{1-\tan^2 A} = \frac{2x}{1-x^2}$

$|||^{ly}$ $\tan 2B = \frac{2y}{1-y^2}$

$\tan 2C = \frac{2z}{1-z^2}$

Sub. in ①

$$\frac{2x}{1-x^2}+\frac{2y}{1-y^2}+\frac{2z}{1-z^2} = \frac{2x}{1-x^2}\cdot\frac{2y}{1-y^2}\cdot\frac{2z}{1-z^2}.$$

Exercise 3.9

6) In a ΔABC $\angle A = 60$ P.T $b+c = 2a\cos\left(\frac{B-C}{2}\right)$

Sol: $A+B+C = 180$

$B+C = 180-60 = 120$

TBP

$\frac{B+C}{2} = 60°$

We know $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$.

$\Rightarrow a = 2R\sin A$, $b = 2R\sin B$, $c = 2R\sin C$.

RHS: $2a\cos\left(\frac{B-C}{2}\right) = 2\cdot 2R\sin A\cos\left(\frac{B-C}{2}\right)$

$= 2\cdot 2R\sin 60°\cos\left(\frac{B-C}{2}\right)$

$= 4R \cdot \sin\left(\frac{B+C}{2}\right) \cdot \cos\left(\frac{B-C}{2}\right)$

$= 2R \cdot 2\sin\left(\frac{B+C}{2} + \frac{B-C}{2}\right) + \sin\left(\frac{B+C}{2} - \frac{B-C}{2}\right)$

$= 2R\left[\sin B + \sin C\right]$

$= 2R\left[\frac{b}{2R} + \frac{c}{2R}\right]. = \cancel{2R}\left[\frac{b+c}{\cancel{2R}}\right]$

$= b+c$

$=$ LHS.

Exercise – 3.8

1) Find the principle solution and general solution of the following.

1) $\sin\theta = -\frac{1}{2}$ 2) $\cot\theta = \sqrt{3}$ 3) $\tan\theta = \frac{1}{\sqrt{3}}$ ~~4)~~

1) $\sin\theta = -\frac{1}{\sqrt{2}}$ θ lies in $\overline{\text{IV}}$ quadrant $\because$ Range is $[-\pi/2, \pi/2]$

TBP

$\sin\alpha = \frac{\pi}{4}$

$\therefore \theta = -\pi/4$

2. $\cot\theta = \sqrt{3}$ θ lies in the I quadrant.

$\tan\theta = \frac{1}{\sqrt{3}}$

$\theta = \frac{\pi}{6}$

3. $\tan\theta = -\frac{1}{\sqrt{3}}$ θ lies in the $\overline{\text{IV}}$ quadrant.

$= \pi/6$

$\theta = -\pi/6$.

2) Solve the following equations for which solutions lies in the $0 < \theta \leq 2\pi$

1) $\sin^4 x = \sin^2 x$ 2) $2\cos^2 x + 1 = -3\cos x$ 3) $2\sin^2 x + 1 = 3\sin x$

4) $\cos 2x = 1 - 3\sin x$.

Sol: 1) $\sin^4 x = \sin^2 x \Rightarrow \sin^2 x(\sin^2 x - 1) = 0$

$\sin^2 x = 0$ $\quad$ $\sin^2 x - 1 = 0$

$\sin x = 0$ $\quad$ $\sin x = \pm 1$

TBP

If $\sin x = 0$ $\quad$ $\sin x = 1$

$x = 0, \pi$ $\quad$ $x = \pi/2$

$\sin x = -1$

$x = \pi + \pi/2 = \frac{3\pi}{2}$

Exercise : 3.9

8) In a Δle ABC PT $(a^2-b^2+c^2)\tan B=(a^2+b^2-c^2)\tan C$.

Sol: $(a^2-b^2+c^2)\tan B=(a^2+b^2-c^2)\tan C$.

TBP

$$\tan B\cdot\cot C=\frac{a^2+b^2-c^2}{a^2-b^2+c^2}$$

$$=\frac{k^2\sin^2A+k^2\sin^2B-k^2\sin^2C}{k^2\sin^2A-k^2\sin^2B+k^2\sin^2C}$$

$$=\frac{\sin^2A+\sin^2B-\sin^2C}{\sin^2A-\sin^2B+\sin^2C}$$

$$=\frac{\sin(A+C)\cdot\sin(A-C)+\sin^2B}{\sin(A+B)\sin(A-B)+\sin^2C}$$

$$=\frac{\sin B\cdot\sin(A-C)+\sin^2B}{\sin C\,\sin(A-B)+\sin^2C}$$

$$=\frac{\sin B\left[\sin(A-C)+\sin B\right]}{\sin C\left[\sin(A-B)+\sin C\right]}$$

$$=\frac{\sin B\left[\sin(A-C)+\sin(A+C)\right]}{\sin C\left[\sin(A-B)\ \sin(A+B)\right]}$$

$$=\frac{\sin B}{\sin C}\cdot\frac{\cancel{2\sin}A\cos C}{\cancel{2\sin}A\cos B}$$

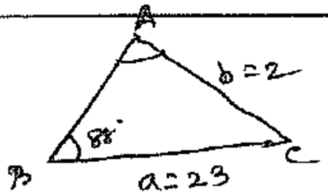
$$=\tan B\cdot\cot C.$$

Exercise - 3.10

1) Determine whether the following measurements produce one Δle, two Δs, no triangle $\angle B=88^\circ$, $a=23$, $b=2$ Solve if solution exists.

Sine formula $\dfrac{a}{\sin A}=\dfrac{b}{\sin B}$

TBP



$$\frac{\sin A}{a}=\frac{\sin B}{b}$$

$$\sin A=\frac{a(\sin B)}{a}=\frac{23\sin 88^\circ}{2}$$

$$=23\times 0.99$$

$\sin A=22.99$. which is not possible
Solution does not exist

16) Suppose that a satellite in space, an earth station and the centre of earth all lie in the same plane. Let r be the radius of earth and R be the radius of distance from the centre of earth to the satellite. Let d be the distance from the earth station to the satellite. Let 30 be the angle of elevation from the earth station to the satellite. If the line segment connecting earth station and satellite subtends angle at the centre of earth then P.T $d = \sqrt{1+\left(\frac{r}{R}\right)^2 - 2\frac{r}{R}\cos\alpha}$

TBP

Sol: Let S be the satellite, $CE = r$, $SE = d$, $SC = R$.

By cosine formula $d^2 = r^2 + R^2 - 2rR\cos\alpha$

$$\frac{d^2}{R^2} = \frac{r^2}{R^2} + 1 - \frac{2rR\cos\alpha}{R^2}$$

$$= 1 + \frac{r^2}{R^2} - \frac{2r\cos\alpha}{R}$$

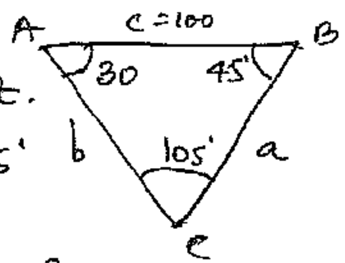
$$d^2 = R^2\left[1+\left(\frac{r}{R}\right)^2 - 2\frac{r}{R}\cos\alpha\right]$$

$$d = R\sqrt{1+\left(\frac{r}{R}\right)^2 - 2\frac{r}{R}\cos\alpha}$$

12) A fighter jet has to hit a small target by flying a horizontal distance when the target is sighted, the pilot measures the angle of depression to be 30°. If after 100 KM, the target has an angle of depression of 45° how far is the target from the fighter jet at that instant

TBP

Sol: Let C be the position of the target.



A, B are the position of the fighter jet.

$\angle A = 30$ $\angle B = 45°$ $\therefore \angle C = 105°$

$AB = 100$ KM.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \Rightarrow \frac{a}{\sin 30} = \frac{b}{\sin 45} = \frac{c}{\sin 105}$$

we know $\sin 105° = \frac{\sqrt{3}+1}{2\sqrt{2}}$

$$\therefore \frac{a}{1/2} = \frac{100 \times 2\sqrt{2}}{\sqrt{3}+1} = \frac{100\sqrt{2}}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$= \frac{\cancel{100}^{50}(\sqrt{6}-\sqrt{2})}{\cancel{3-1}}$$

$$\therefore a = 50(\sqrt{6}-\sqrt{2}) \text{ KM}.$$

Inverse Trigonometric function:

1. Find the principle value of 1) $\sin^{-1}(1/\sqrt{2})$ 2) $\cos^{-1}(\sqrt{3}/2)$ 3) $\operatorname{cosec}^{-1}(-1)$

TBP 4) $\sec^{-1}(-\sqrt{2})$ 5) $\tan^{-1}(\sqrt{3})$

Sol: 1) $\sin^{-1}(1/\sqrt{2})$

Let $y = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ where $-\pi/2 \le y \le \pi/2$

$\sin y = \frac{1}{\sqrt{2}} \Rightarrow y = \pi/4$

$\therefore$ The principle value of $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \pi/4$.

2) $\cos^{-1}(\sqrt{3}/2)$

Let $y = \cos^{-1}(\sqrt{3}/2)$ where $0 \le y \le \pi$

$\cos y = \frac{\sqrt{3}}{2}$

$y = \frac{\pi}{6}$

$\therefore$ The principle value of $\cos^{-1}(\sqrt{3}/2) = \pi/6$.

3) $\operatorname{cosec}^{-1}(-1)$

Let $y = \operatorname{cosec}^{-1}(-1)$ where $-\pi/2 \le y \le \pi/2$

$\operatorname{cosec} y = -1$ $[\operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta]$

$= -\operatorname{cosec}(\pi/2)$

$= \operatorname{cosec}(-\pi/2)$

$\therefore$ The principle value of $\operatorname{cosec}^{-1}(-1) = -\pi/2$

4) $\sec^{-1}(-\sqrt{2})$

Let $y = \sec^{-1}(-\sqrt{2})$ where $0 \le y \le \pi$

$\sec(y) = -\sqrt{2}$

$= \sec(-\sqrt{2})$ (θ lies in II quadrant)

$= \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

$\therefore$ The principle value of $\sec^{-1}(-\sqrt{2})$ is $\frac{3\pi}{4}$

5) $\tan^{-1}(\sqrt{3})$

Let $y = \tan^{-1}(\sqrt{3})$ where $-\pi/2 \le \theta \le \pi/2$

$\tan y = \sqrt{3}$

$y = \pi/3$ $\therefore$ The principle value of $\tan^{-1}(\sqrt{3}) = \pi/3$.

2) A man standing directly opposite to one side of a road of width x meter views a circular shaped traffic green signal of diameter a meter on the other side of the road. The bottom of the green signal is b meter height from the horizontal level of viewer's eye. If α denotes the angle subtend by the diameter of the green signal at the viewer's eye, then P.T $\alpha = \tan^{-1}\left(\frac{a+b}{x}\right) - \tan^{-1}\left(\frac{b}{x}\right)$

TBP-

Sol: Diameter of the signal $= a$

$DA = b$

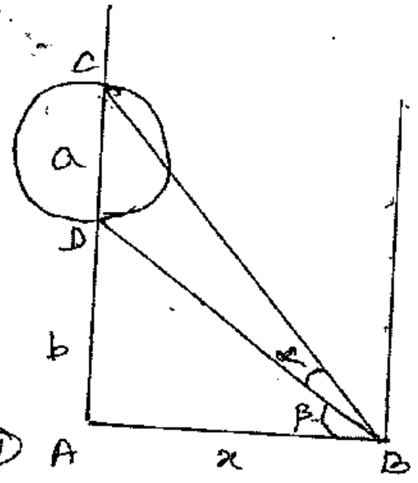
$\angle CBD = \alpha$

$AB = x$ mts.

let $\angle DBA = \beta$.

From $\triangle ABD$ $\tan\beta = \frac{b}{x}$.

$\beta = \tan^{-1}\left(\frac{b}{x}\right)$ —— (1)



From $\triangle$le ABC $\tan(\alpha+\beta) = \frac{a+b}{x}$

$\alpha+\beta = \tan^{-1}\left(\frac{a+b}{x}\right)$ —— (2)

(2) - (1)

$\alpha+\beta-\beta = \tan^{-1}\left(\frac{a+b}{x}\right) - \tan^{-1}\left(\frac{b}{x}\right)$

$\alpha = \tan^{-1}\left(\frac{a+b}{x}\right) - \tan^{-1}\left(\frac{b}{x}\right)$

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