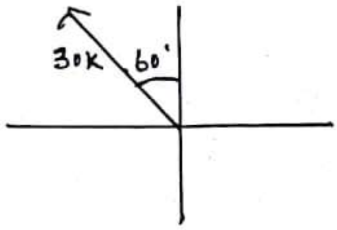
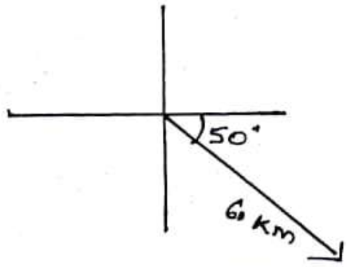
Vector Algebra - EXERCISE 8.1.

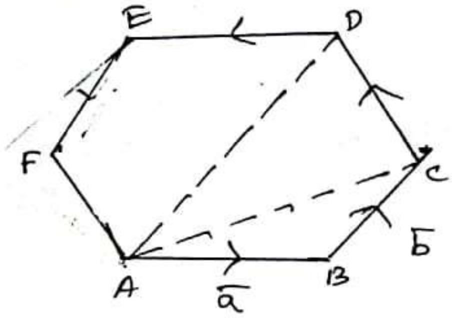
EX: 8.1. Represent Graphically the displacement of
i) 30 KM 60° west of north 2) 60KM 50° South of east.





EX: 8.2 If $\bar{a}, \bar{b}$ are vectors represented by two adjacent sides of regular hexagon, then find the vectors represented by other sides.

Sol: Let $\overline{AB} = \bar{a}$, $\overline{BC} = \bar{b}$



By Triangle law of addition

$$\overline{AC} = \bar{a} + \bar{b}$$

But $\overline{AD} = 2\overline{BC}$

$$= 2\bar{b}$$

Again by Δle law of addition.

$$\overline{AC} + \overline{CD} = \overline{AD}$$

$$\overline{CD} = \overline{AD} - \overline{AC}$$

$$= 2\bar{b} - (\bar{a} + \bar{b})$$

$$= \bar{b} - \bar{a}$$

$\overline{AB} = -\overline{DE} = -\bar{a}$

$\overline{EF} = -\overline{BC} = -\bar{b}$

$\overline{FA} = -\overline{CD} = \bar{a} - \bar{b}$.

∴ $\overline{AB} = \bar{a}$

$\overline{BC} = \bar{b}$

$\overline{CD} = \bar{b} - \bar{a}$

$\overline{DE} = -\bar{a}$

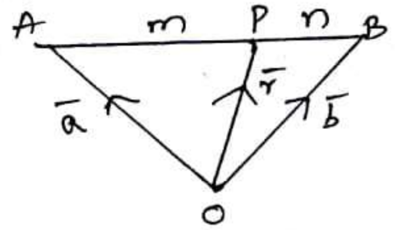
$\overline{EF} = -\bar{b}$

$\overline{FA} = \bar{a} - \bar{b}$.

Theorem: 8.1. Section Formula (Internal Division)

Let O be the origin. Let A, B be two points. Let P be the point which divides the line segment AB internally in the ratio m:n. If $\bar{a}, \bar{b}$ are the p.v of the points A and B then the p.v of the point P is given by $\overline{OP} = \dfrac{n\bar{a} + m\bar{b}}{m+n}$.

Proof: Let $\overline{OA} = \bar{a}$, $\overline{OB} = \bar{b}$ and $\overline{OP} = \bar{r}$.



Let P divides AB in the ratio $m:n$ internally

$$\frac{AP}{PB} = \frac{m}{n}$$

$$\frac{|\overline{AP}|}{|\overline{PB}|} = \frac{m}{n}.$$

(ie) $n|\overline{AP}| = m|\overline{PB}|$

But $\overline{AP} = \overline{OP} - \overline{OA}$
$= \bar{r} - \bar{a}$

$\overline{PB} = \overline{OB} - \overline{OP} = \bar{b} - \bar{r}$

$$\therefore n(\bar{r} - \bar{a}) = m(\bar{b} - \bar{r})$$

$$n\bar{r} - n\bar{a} = m\bar{b} - m\bar{r}$$

$$(n+m)\bar{r} = m\bar{b} + n\bar{a}$$

$$\bar{r} = \frac{m\bar{b} + n\bar{a}}{m+n}$$

Note: 1) If P divides AB in the ratio $m:n$ externally then

$$\bar{r} = \frac{m\bar{b} - n\bar{a}}{m-n}.$$

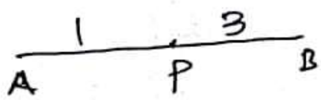
2) If P is the mid point of AB $m:n = 1:1$ then $\overline{OP}$

$$\bar{r} = \frac{\bar{a} + \bar{b}}{2}$$

Ex 8.3 Let A and B be two points with P.V. $2\bar{a} + 4\bar{b}$, $2\bar{a} - 8\bar{b}$. Find the P.V of the points which divides the line segment joining A and B in the ratio $1:3$ internally and externally.

Internally $\overline{OA} = 2\bar{a} + 4\bar{b}$

$\overline{OB} = 2\bar{a} - 8\bar{b}$



ratio $1:3$ internally.

$$\overline{OP} = \frac{1\overline{OB} + 3\overline{OA}}{1+3} = \frac{(2\bar{a} - 8\bar{b}) + 3(2\bar{a} + 4\bar{b})}{1+3}$$

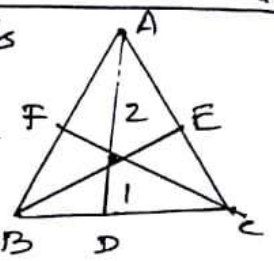
$$= \frac{8\bar{a} + 4\bar{b}}{4} = \frac{\cancel{4}(2\bar{a} + \bar{b})}{\cancel{4}}$$

Externally:
$$\overline{OP} = \frac{1\cdot\overline{OB} - 3\cdot\overline{OA}}{1-3} = \frac{(2\bar{a} - 8\bar{b}) - 3(2\bar{a} + 4\bar{b})}{-2}$$

$$= \frac{-4\bar{a} - 20\bar{b}}{-2} = \frac{\cancel{-2}(2\bar{a} + 10\bar{b})}{\cancel{-2}}$$

Theorem 8.3) The medians of a $\triangle$le are concurrent

In a $\triangle$le ABC D, E, F are the mid points of BC, CA, AB. $\therefore$ AD, BE, CF are the medians.



Let $\overline{OA} = \bar{a}$, $\overline{OB} = \bar{b}$, $\overline{OC} = \bar{c}$

$\therefore$ $\overline{OD} = \dfrac{\bar{b}+\bar{c}}{2}$, $\overline{OE} = \dfrac{\bar{c}+\bar{a}}{2}$ $\overline{OF} = \dfrac{\bar{a}+\bar{b}}{2}$

Let G_1 be the point on AD which divides AD in the ratio 2:1

$$\therefore \overline{OG_1} = \frac{2\cdot\overline{OD} + 1\cdot\overline{OA}}{2+1} = \frac{\cancel{2}\left(\frac{\bar{b}+\bar{c}}{\cancel{2}}\right)+1\cdot\bar{a}}{3} = \frac{\bar{a}+\bar{b}+\bar{c}}{3}$$

Let G_2 be the point on BE which divides in the ratio 2:1.

and G_3 be the point on CF which divides in the ratio 2:1

|||ly we can find that $\overline{OG_2} = \dfrac{\bar{a}+\bar{b}+\bar{c}}{2}$

$\overline{OG_3} = \dfrac{\bar{a}+\bar{b}+\bar{c}}{2}$

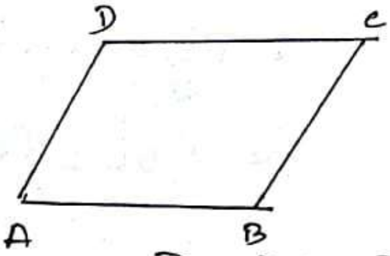
$\therefore$ G_1, G_2, G_3 are the same point. Hence medians of a $\triangle$le are concurrent.

G is the centroid of the $\triangle$le. $\therefore$ $\overline{OG} = \dfrac{\bar{a}+\bar{b}+\bar{c}}{3}$

Theorem 8.4) A quadrilateral is a parallelogram iff its diagonals bisect each other.

Proof: Let ABCD be a quadrilateral with diagonals AC and BD.

$\overline{OA} = \bar{a}$, $\overline{OB} = \bar{b}$, $\overline{OC} = \bar{c}$ $\overline{OD} = \bar{d}$



I If ABCD is a parallelogram

$\overline{AB} = \overline{DC}$

$\overline{OB} - \overline{OA} = \overline{OC} - \overline{OD}$

$\bar{b} - \bar{a} = \bar{c} - \bar{d}$

$\bar{b} + \bar{d} = \bar{a} + \bar{c}$

$\dfrac{\bar{a}+\bar{c}}{2} = \dfrac{\bar{b}+\bar{d}}{2}$

$\Rightarrow$ Mid points of AC and BD are same.

II If $\dfrac{\bar{a}+\bar{c}}{2} = \dfrac{\bar{b}+\bar{d}}{2} \Rightarrow \bar{a}+\bar{c} = \bar{b}+\bar{d}$

$\bar{c} - \bar{d} = \bar{b} - \bar{a}$

$\Rightarrow \overline{OC} - \overline{OD} = \overline{OB} - \overline{OA}$

$\Rightarrow \overline{DC} = \overline{AB}$

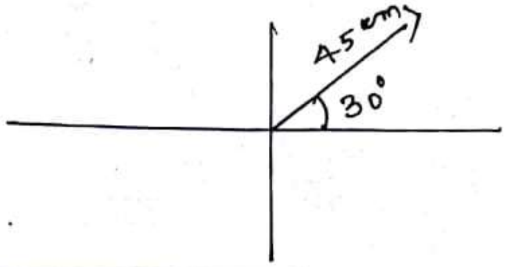
This shows that AB, CD are equal and parallel |||ly we can prove BC, AD are parallel and equal

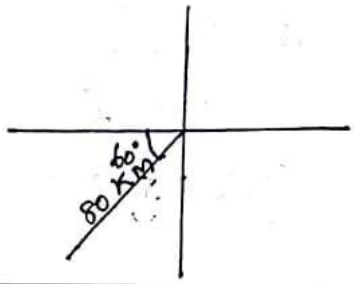
$\therefore$ ABCD is Parallelogram.

EXERCISE - 8.1

1) Represent graphically the displacement of

1) 45cm 30° north of east 2) 80 KM, 60° south of west.





3) Let $\bar{a}$ and $\bar{b}$ be the p.o.v of the points A and B. P.T the p.v of the points which trisects the line segment AB are $\frac{\bar{a}+2\bar{b}}{3}$, $\frac{\bar{b}+2\bar{a}}{3}$.

Let $\overline{OA} = \bar{a}$, $\overline{OB} = \bar{b}$

Let P be a point which divides AB in the ratio 1 : 2 internally.

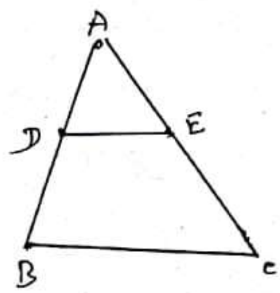
$$\overline{OP} = \frac{1\cdot\bar{b} + 2\cdot\bar{a}}{1+2} = \frac{\bar{b}+2\bar{a}}{3}$$

Let Q be the point on AB which divides AB in the ratio 2 : 1

$$\frac{2\cdot\bar{b}+\bar{a}}{2+1} = \frac{2\bar{b}+\bar{a}}{3}.$$

4) If D, E are the mid points of the sides AB and AC of a $\triangle$le ABC

P.T $\overline{BE}+\overline{DE} = \frac{3}{2}\overline{BC}$



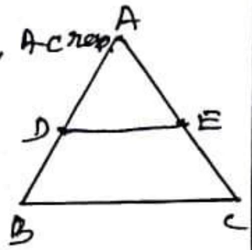
$$\overline{BE} = \overline{BC}+\overline{CE}$$
$$= \overline{BC}+\frac{1}{2}\overline{CA}$$

$$\overline{DC} = \overline{DB}+\overline{BC}$$
$$= \frac{1}{2}\overline{AB}+\overline{BC}$$

$$\therefore \overline{BE}+\overline{DC} = \overline{BC}+\frac{1}{2}\overline{CA}+\frac{1}{2}\overline{AB}+\overline{BC}$$
$$= 2\overline{BC}+\frac{1}{2}(\overline{CA}+\overline{AB})$$
$$= 2\overline{BC}+\frac{1}{2}(\overline{CB})$$
$$= 2\overline{BC}-\frac{1}{2}\overline{BC}$$
$$= 3/2\,\overline{BC}.$$

5) P.T the line segment joining the mid points of two sides of a triange is parallel to the third side whose lengths is half of the lengths of the third side.

In a $\triangle le$ ABC D, E are the mid point of AB, AC resp.



$\therefore \overline{OD} = \frac{\overline{OA}+\overline{OB}}{2}$ $\quad \overline{OE} = \frac{\overline{OA}+\overline{OC}}{2}$

$\overline{DE} = \overline{OE} - \overline{OD} = \frac{\cancel{\overline{OA}}+\overline{OC}}{2} - \frac{\cancel{\overline{OA}}+\overline{OB}}{2}$

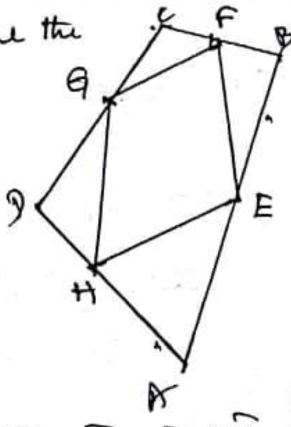
$= \frac{\overline{OC}-\overline{OB}}{2} = \frac{1}{2}(\overline{BC})$

$|\overline{DE}| = \frac{1}{2}|\overline{BC}|$

which implies that line joining of the mid points of the two sides is parallel to third side and length is equal to $1/2$ of the length of the third side.

6) P.T the line segments joining the mid points of the adjacent sides of a quadrilateral from a parallelogram.

ABCD is a quadrilateral in which E, F, G, H are the are the mid points of AB, BC, CD, DA resp.



$\overline{OE} = \frac{\overline{OA}+\overline{OB}}{2}$ $\quad \overline{OG} = \frac{\overline{OD}+\overline{OC}}{2}$

$\overline{OF} = \frac{\overline{OB}+\overline{OC}}{2}$ $\quad \overline{OH} = \frac{\overline{OD}+\overline{OA}}{2}$

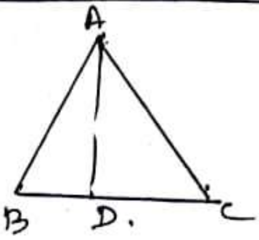
Mid point of $\overline{EG} = \dfrac{\frac{\overline{OA}+\overline{OB}}{2} + \frac{\overline{OC}+\overline{OD}}{2}}{2} = \frac{\overline{OA}+\overline{OB}+\overline{OC}+\overline{OD}}{4}.$

Mid point of $\overline{FH} = \dfrac{\frac{\overline{OB}+\overline{OC}}{2} + \frac{\overline{OD}+\overline{OA}}{2}}{2} = \frac{\overline{OA}+\overline{OB}+\overline{OC}+\overline{OD}}{4}.$

$\therefore$ The mid point of the diagonals EG and FH are same EFGH is a parallellogram.

9) If D is the mid point of the side BC of a $\triangle^{le}$ ABC P.T $\overline{AB} + \overline{AC} = 2\overline{AD}$.

Let D be the mid point of BC of the $\triangle^{le}$ ABC



$\overline{AB} = \overline{AD} + \overline{DB}$ —— ①

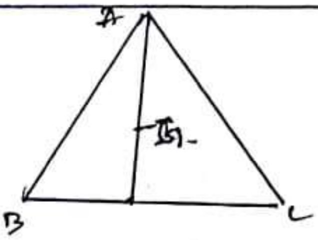
$\overline{AC} = \overline{AD} + \overline{DC}$

$= \overline{AD} - \overline{DB}$ —— ② $\because$ D is the midepoint of BC and DC and DB are opposite

$\overline{AB} + \overline{AC} = 2\overline{AD}$

10) If G is the centroid of the $\triangle^{le}$ ABC P.T $\overline{GA} + \overline{GB} + \overline{GC} = \overline{0}$

Let G be the centroid of the $\triangle^{le}$ ABC



$\overline{OG} = \dfrac{\overline{OA} + \overline{OB} + \overline{OC}}{3.}$

$3\overline{OG} = \overline{OA} + \overline{OB} + \overline{OC}$ —— ⓐ

Given $\overline{GA} + \overline{GB} + \overline{GC} = \overline{OA} - \overline{OG} + \overline{OB} - \overline{OG} + \overline{OC} - \overline{OG}$

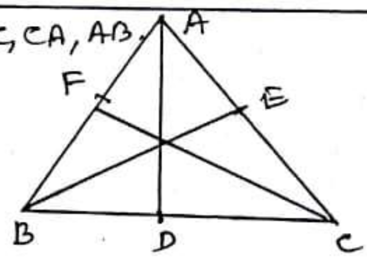
$= (\overline{OA} + \overline{OB} + \overline{OC}) - 3\overline{OG}$

$= 3\overline{OG} - 3\overline{OG}$

$= \overline{0}$

11) Let A, B, C are the vertices of a $\triangle^{le}$. Let D, E and F are the mid points of the sides BC, CA, AB resp. S.T $\overline{AD} + \overline{BE} + \overline{CF} = 0$.

Let D, E, F are the mid points of BC, CA, AB



$\overline{AD} = \overline{AB} + \overline{BD} = \overline{AB} + \frac{1}{2}\overline{BC}$

$\overline{BE} = \overline{BC} + \overline{CE} = \overline{BC} + \frac{1}{2}\overline{CA}$

$\overline{CF} = \overline{CA} + \overline{AF} = \overline{CA} + \frac{1}{2}\overline{AB}$

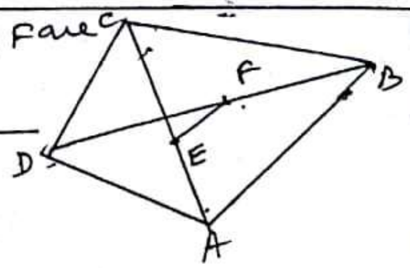
$\overline{AD} + \overline{BE} + \overline{CF} = \frac{3}{2}\overline{AB} + \frac{3}{2}\overline{BC} + \frac{3}{2}\overline{CA}$

$= \frac{3}{2}(\overline{AB} + \overline{BC} + \overline{CA})$

$= \frac{3}{2}(0) = 0$

12) If ABCD is a quadrilateral and E, F are the mid points of AC and BD resp then P.T $\overline{AB} + \overline{AD} + \overline{CB} + \overline{CD} = 4\overline{EF}$

ABCD is a quadrilateral in which, E, F are the mid points of AC, BD



$\overline{AB} = \overline{AE} + \overline{EF} + \overline{FB}$

$\overline{AD} = \overline{AE} + \overline{EF} + \overline{FD}$

$\overline{CB} = \overline{CE} + \overline{EF} + \overline{FB}$

$\overline{CD} = \overline{CE} + \overline{EF} + \overline{FD}$

$\overline{AB} + \overline{AD} + \overline{CB} + \overline{CD} = 2(\overline{AE} + \overline{CE}) + 4\overline{EF} + 2(\overline{FB} + \overline{FD})$

$= 2(0) + 4\overline{EF} + 2(0)$

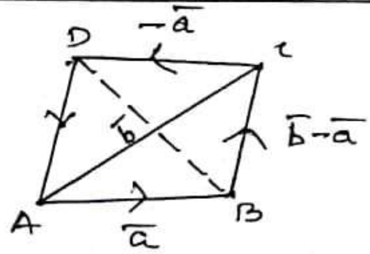
$= 4\overline{EF}$

$\because$ $\overline{AE}, \overline{CE}$ are equal but opposite

$\overline{FB}, \overline{FD}$ are equal but opposite.

7) If $\bar{a}, \bar{b}$ represent a side and a diagonal of a parallelogram find the other sides and the other diagonal.

In a parallelogram ABCD



$\overline{AB} = \bar{a}$, $\overline{AC} = \bar{b}$

By T.L.A.

$\overline{AB} + \overline{BC} = \overline{AC}$

$\overline{BC} = \overline{AC} - \overline{AB}$

$= \bar{b} - \bar{a}$

$\overline{DA}$ is equal and opposite to $\overline{BC}$

$\therefore \overline{DA} = -(\bar{a} - \bar{b})$

$\overline{CD}$ is equal and opposite to $\overline{AB}$

$\overline{CD} = -\bar{a}$.

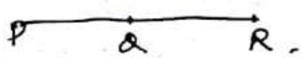
$\overline{BD} = \overline{BC} + \overline{CD}$

$= (\bar{b} - \bar{a}) + (-\bar{a}) = \bar{b} - 2\bar{a}$

8) If $\overline{PQ} + \overline{OQ} = \overline{QO} + \overline{OR}$ P.T the points P, Q, R are collinear.

Given $\overline{PQ} + \overline{OQ} = \overline{QO} + \overline{OR}$

$\overline{PQ} = \overline{QR}$ ——— (1).



Again $\overline{PQ} + \overline{QR} = \cancel{\overline{OQ}} - \overline{OP} + \overline{OR} - \cancel{\overline{OQ}}$

$= \overline{OR} - \overline{OP}$

$= \overline{PR}$ ——— (2)

From (1) and (2). P, Q, R are collinear.

EXERCISE - 8.2.

Ex 8.4. Find the unit vector along the direction of $5\hat{i} - 3\hat{j} + 4\hat{k}$

Let $\bar{a} = 5\hat{i} - 3\hat{j} + 4\hat{k}$

$|\bar{a}| = \sqrt{25+9+16} = \sqrt{50}$

$\hat{a} = \frac{\bar{a}}{|\bar{a}|} = \frac{5\hat{i} - 3\hat{j} + 4\hat{k}}{\sqrt{50}}$.

Note! unit vector parallel to $\bar{a}$ but in the opposite direction

$\hat{a} = -\frac{\bar{a}}{|\bar{a}|}$

Ex : 8.5) Find the direction ratios and direction cosines of

i) $3\hat{i} + 4\hat{j} - 6\hat{k}$ 2) $3\hat{i} - 4\hat{k}$

Let $\bar{a} = 3\hat{i} + 4\hat{j} - 6\hat{k}$

Dr's of $\bar{a}$ = 3, 4, −6. $|\bar{a}| = \sqrt{9+16+36} = \sqrt{61}$

Dc's $\left(\frac{3}{\sqrt{61}}, \frac{4}{\sqrt{61}}, \frac{-6}{\sqrt{61}}\right)$

Let $\bar{b} = 3\hat{i} - 4\hat{k}$

Dr's of $\bar{b}$ 3, 0, −4. $|\bar{b}| = \sqrt{9+16} = 5$

Dc's $\left(\frac{3}{5}, 0, \frac{-4}{5}\right)$

Ex 8.6) 1) Find the direction cosines of a vector whose dr's are 2, 3, −6.

2) Can a vector have direction angles 30°, 45°, 60°

3) Find the dc's of $\overline{AB}$ where A (2, 3, 1) B (3, −1, 2)

4) Find the dc's of the line joining (2, 3, 1) (3, −1, 2)

5) The dr's of a vector are 2, 3, 6 and its magnitude is 5 Find the vector.

1) dc's are $\frac{2}{\sqrt{4+9+36}}, \frac{3}{\sqrt{49}}, \frac{-6}{\sqrt{49}}$

(ie) $\left(\frac{2}{7}, \frac{3}{7}, \frac{-6}{7}\right)$

2) If α, β, γ are the angles with OX, OY, OZ then

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

(ie) $\cos^2 30 + \cos^2 45 + \cos^2 60 = \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2$

$= \frac{3}{4} + \frac{1}{2} + \frac{1}{4} \neq 1.$

∴ They are not the direction angles of a vector.

3) $\overline{AB} = \overline{OB} - \overline{OA} = i - 4j + \bar{k}$

$$|\overline{AB}| = \sqrt{1+16+1} = \sqrt{18}.$$

DC's $\left(\frac{1}{\sqrt{18}}, \frac{-4}{\sqrt{18}}, \frac{1}{\sqrt{18}}\right)$

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4) $\overline{AB} = \overline{OB} - \overline{OA} = \bar{i} - 4j + \bar{k}$.

The dc's of AB are $\left(\frac{1}{\sqrt{18}}, \frac{-4}{\sqrt{18}}, \frac{1}{\sqrt{18}}\right)$

Suppose if we take second point as first point

dc's are $\left(-\frac{1}{\sqrt{18}}, \frac{4}{\sqrt{18}}, \frac{1}{\sqrt{18}}\right)$

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5) dc's are $\left(\frac{2}{7}, \frac{3}{7}, \frac{6}{7}\right)$

$\therefore$ unit vector is $\frac{2}{7}\bar{i} + \frac{3}{7}j + \frac{6}{7}\bar{k}$

The required vector is $\frac{5}{7}(2\bar{i} + 3j + 6\bar{k})$

EX 8.7) S.T the points whose P.V are $2\bar{i}+3j-5\bar{k}$, $3\bar{i}+j-2\bar{k}$ and $6\bar{i}-5j+7\bar{k}$ are collinear.

Let $\overline{OA} = 2\bar{i}+3j-5\bar{k}$

$\overline{OB} = 3\bar{i}+j-2\bar{k}$

$\overline{OC} = 6\bar{i}-5j+7\bar{k}$.

$\overline{AB} = \overline{OB} - \overline{OA} = \bar{i} - 2j + 3\bar{k}$

$\overline{AC} = \overline{OC} - \overline{OA} = 4\bar{i} - 8j + 12\bar{k}$

$= 4(\bar{i} - 2j + 3\bar{k})$

$\overline{AC} = 4\,\overline{AB}$

$\therefore$ The three points are collinear.

Note: If $\overline{AC} = 4\overline{AB}$

$\overline{AC}$ and $\overline{AB}$ are ||el

when the three points are collinear then only AB and AC are ||el.

EX 8.8) Find the point whose P.V has magnitude 5 and ||el to $4\bar{i}-3j+10\bar{k}$

Let $\bar{a} = 4\bar{i} - 3j + 10\bar{k}$

$|\bar{a}| = \sqrt{16+9+100} = \sqrt{125} = 5\sqrt{5}$

$\hat{a} = \frac{4\bar{i} - 3j + 11\bar{k}}{5\sqrt{5}}$

$5\hat{a} = \frac{\cancel{5}(4\bar{i} - 3j + 10\bar{k})}{\cancel{5}\sqrt{5}}$ $\therefore$ Required points are $\left(\frac{4}{\sqrt{5}}, \frac{-3}{\sqrt{5}}, \frac{10}{\sqrt{5}}\right)$ $\frac{10}{\sqrt{5}} = 2\sqrt{5}$

EX 8.9) P.T the points whose pv's $2\hat{i}+4\hat{j}+3\hat{k}$, $4\hat{i}+\hat{j}+9\hat{k}$, $10\hat{i}-\hat{j}+6\hat{k}$ forms a rt triangle

let $\bar{OA} = 2i+4j+3\bar{k}$ $\quad \bar{AB} = \bar{OB}-\bar{OA} = 2i-3j+6k$

$\bar{OB} = 4i+j+9\bar{k}$ $\quad \bar{BC} = \bar{OC}-\bar{OB} = 6\hat{i}-2\hat{j}-3\bar{k}$

$\bar{OC} = 10\hat{i}-j+6k$ $\quad \vec{CA} = \bar{OA}-\bar{OC} = -8\hat{i}+5\hat{j}-3\bar{k}$

$\bar{AB}+\bar{BC}+\bar{CA} = 0$ ∴ First it is a triangle.

$|\bar{AB}| = \sqrt{4+9+36} = 7$

$|\bar{BC}| = \sqrt{36+4+9} = 7$

$|\bar{CA}| = \sqrt{64+25+9} = \sqrt{98}$.

$|AB|^2+|\bar{BC}|^2 = |\bar{CA}|^2$

$49+49 = 98$. ∴ given points forms a rt $\triangle$le

EX 8.10) S.T the vectors $5\hat{i}+6\hat{j}+7\bar{k}$, $7\hat{i}-8\hat{j}+9\bar{k}$, $3\hat{i}+20\hat{j}+5\bar{k}$ are coplanar.

Let $5\hat{i}+6\hat{j}+7\bar{k} = s(7\hat{i}-8\hat{j}+9\bar{k})+t(3\hat{i}+20\hat{j}+5\bar{k})$

$7s+3t = 5$ — ① $\quad$ ① × 5 $\quad 35s+\cancel{15t} = 25$

$-8s+20t = 6$ — ② $\quad$ ③ × 3 $\quad 27s+\cancel{15t} = 21$

$9s+5t = 7$ — ③ $\quad\quad 8s = 4$

$s = 1/2$

∴ $5\hat{i}+6\hat{j}+7\bar{k} = \frac{1}{2}(7\hat{i}-8\hat{j}+9\bar{k})+1/2(3\hat{i}+20\hat{j}+5\bar{k})$ $\quad t = 1/2$

∴ we can write one vector is a linear combination of other two vectors. Hence given vectors are coplanar.

7/8.2) S.T the vectors $2\hat{i}-\hat{j}+\bar{k}$, $3\hat{i}-4\hat{j}-4\bar{k}$, $\hat{i}-3\hat{j}-5\bar{k}$ form a right angled $\triangle$le.

$AB = 2\hat{i}-\hat{j}+\bar{k}$ $\quad |\bar{AB}| = \sqrt{4+1+1} = \sqrt{6}$

$\bar{BC} = 3\hat{i}-4\hat{j}-4\bar{k}$ $\quad |\bar{BC}| = \sqrt{9+16+16} = \sqrt{41}$

$\bar{CA} = \hat{i}-3\hat{j}-5\bar{k}$ $\quad |\bar{CA}| = \sqrt{1+9+25} = \sqrt{35}$

$\bar{CA}+\bar{AB} = \bar{BC}$ ∴ First it is a $\triangle$le.

$|\bar{CA}|^2+|\bar{AB}|^2 = |\bar{BC}|^2$

$35+6 = 41$. ∴ it is rt angled $\triangle$le.

8) Find the value of λ for which the vectors $\bar{a} = 3\hat{i} + 2\hat{j} + 9\hat{k}$
8.2
$\bar{b} = \hat{i} + \lambda\hat{j} + 3\hat{k}$ are parallel.

If $\bar{a}$ and $\bar{b}$ are ||el $\bar{b} = t\bar{a}$

$$\hat{i} + \lambda\hat{j} + 3\hat{k} = 3\left(\hat{i} + \frac{2}{3}\hat{j} + 3\hat{k}\right)$$
$$\Rightarrow \lambda = 2/3.$$

9) S.T the following vectors are coplanar.
8.2
1) $\hat{i} - 2\hat{j} + 3\hat{k}$ $-2\hat{i} + 3\hat{j} - 4\hat{k}$, $-\hat{j} + 2\hat{k}$
2) $5\hat{i} + 6\hat{j} + 7\hat{k}$, $7\hat{i} - 8\hat{j} + 9\hat{k}$, $3\hat{i} + 20\hat{j} + 5\hat{k}$.

1) $\hat{i} - 2\hat{j} + 3\hat{k} = s(-2\hat{i} + 3\hat{j} - 4\hat{k}) + t(-\hat{j} + 2\hat{k})$

$$1 = -2s \Rightarrow s = -1/2$$
$$-2 = 3s - t \Rightarrow -2 = 3(-1/2) - t$$
$$t = -\frac{3}{2} + 2 = 1/2.$$

$\therefore$ $\hat{i} - 2\hat{j} + 3\hat{k} = -\frac{1}{2}(-2\hat{i} + 3\hat{j} - 4\hat{k}) + \frac{1}{2}(-\hat{j} + 2\hat{k})$

$\therefore$ we can write one vector as a linear combination of other two vectors. Hence they are coplanar.

2) $5\hat{i} + 6\hat{j} + 7\hat{k} = s(7\hat{i} - 8\hat{j} + 9\hat{k}) + t(3\hat{i} + 20\hat{j} + 5\hat{k})$

$5 = 7s + 3t$ — ①
$6 = -8s + 20t$ — ② This is Example 8.10.
$7 = 9s + 5t$ — ③

10) S.T the points whose P.V $4\hat{i} + 5\hat{j} + \hat{k}$, $-\hat{j} - \hat{k}$, $3\hat{i} + 9\hat{j} + 4\hat{k}$,
8.2
$-4\hat{i} + 4\hat{j} + 4\hat{k}$ are coplanar.

$\overline{OA} = 4\hat{i} + 5\hat{j} + \hat{k}$
$\overline{OB} = -\hat{j} - \hat{k}$
$\overline{OC} = 3\hat{i} + 9\hat{j} + 4\hat{k}$
$\overline{OD} = -4\hat{i} + 4\hat{j} + 4\hat{k}$

$\overline{AB} = -4\hat{i} - 6\hat{j} - 2\hat{k}$ $(\overline{OB} - \overline{OA})$
$\overline{BC} = 3\hat{i} - 6\hat{j} - 2\hat{k}$
$\overline{CD} = 7\hat{i} - 5\hat{j}$.

$-4\hat{i} - 6\hat{j} - 2\hat{k} = s(3\hat{i} - 6\hat{j} - 2\hat{k}) + t(7\hat{i} - 5\hat{j})$
$$-4 = 3s + 7t$$
$$-6 = 10s - 5t$$
$$-2 = 5s \Rightarrow s = -2/5$$

$$-4 = 3(-2/5) - 7t$$

$$7t = -\frac{6}{5} + 4 = \frac{-6+20}{5}$$

$$= \frac{14}{5} \Rightarrow t = \frac{14^{2}}{5\times 7}$$

$\therefore -4i - 6j - 2k = -\frac{2}{5}(3i - 6j - 2k) + \frac{2}{5}(-7i - 5j)$

$\therefore$ one vector can be written as sum of two linear vectors Hence the given vectors are coplanar.

$\frac{11}{8.2}$) If $\bar{a} = 2i + 3j - 4k$, $\bar{b} = 3i - 4j - 5k$ c) $= -3i + 2j + 3k$ Find the magnitude, dc's of 1) $\bar{a} + \bar{b} + \bar{c}$ 2) $3\bar{a} - 2\bar{b} + 5\bar{c}$.

$\bar{a} = 2i + 3j - 4k$

$\bar{b} = 3i - 4j - 5k$

$\bar{c} = -3i + 2j + 3k$.

① $\bar{a} + \bar{b} + \bar{c} = 2i + 1j - 6k$

$|\bar{a} + \bar{b} + \bar{c}| = \sqrt{4 + 1 + 36} = \sqrt{41}$

dc's : $\frac{2}{\sqrt{41}}, \frac{1}{\sqrt{41}}, \frac{-6}{\sqrt{41}}$

$3\bar{a} = 6i + 9j - 12k$

$-2\bar{b} = -6i + 8j + 10k$.

$5\bar{c} = -15i + 10j + 15k$.

$3\bar{a} - 2\bar{b} + 5\bar{c} = -15i + 27j + 13k$

$|3\bar{a} - 2\bar{b} + 5\bar{c}| = \sqrt{1123}$

dc's : $\frac{-15}{\sqrt{1123}}, \frac{27}{\sqrt{1123}}, \frac{13}{\sqrt{1123}}$.

$\frac{12}{8.2}$) The PV of the vertices of a triangle are $i + 2j + 3k$, $3i - 4j + 5k$ and $-2i + 3j - 7k$. Find the perimeter of the $\triangle$le.

$\overline{OA} = i + 2j + 3k$

$\overline{OB} = 3i - 4j + 5k$

$\overline{OC} = -2i + 3j - 7k$.

$\overline{AB} = \overline{OB} - \overline{OA} = 2i - 6j + 2k$

$\overline{BC} = \overline{OC} - \overline{OB} = -5i + 7j - 12k$

$\overline{CA} = \overline{OA} - \overline{OC} = 3i - j + 10k$.

$|\overline{AB}| = \sqrt{4 + 36 + 4} = \sqrt{44}$

$|\overline{BC}| = \sqrt{25 + 49 + 144} = \sqrt{218}$

$|\overline{CA}| = \sqrt{9 + 1 + 100} = \sqrt{110}$

$\therefore$ Perimeter of the $\triangle$le $= \sqrt{44} + \sqrt{218} + \sqrt{110}$.

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$\frac{13}{8.2}$) Find the unit vector parallel to $3\bar{a}-2\bar{b}+4\bar{c}$ if $\bar{a}=3i-j-4\bar{k}$ $\bar{b}=-2\bar{i}+4\bar{j}-3\bar{k}$, $\bar{c}=i+2j-k$

$3\bar{a}=9i-3j-12\bar{k}$

$-2\bar{b}=4i-\cancel{8}j+6\bar{k}$

$4\bar{c}=4i+8j-4\bar{k}$

$3\bar{a}-2\bar{b}+4\bar{c}$

$=17\bar{i}+3j+10\bar{k}$

~~Let~~ $|17\bar{i}+3i+10k|=\sqrt{289+9+100}$

$=\sqrt{398}$

unit vector $\parallel^{el}$ to $3\bar{a}-2b+4\bar{c}$ $=\frac{17\bar{i}+3j+10\bar{k}}{\sqrt{398}}$

$\frac{15}{8.2}$) The PV of ~~a, b, c~~ are the points P, Q, R, S are $i+j+\bar{k}$ $2i+5j$, $3i+2j-3\bar{k}$ and $\bar{i}-6\bar{j}-\bar{k}$ resp.
P.T the lines PQ and RS are parallel.

$\bar{OP}=i+j+\bar{k}$

$\bar{OQ}=2i+5j$

$\bar{OR}=3\bar{i}+2j-3\bar{k}$

$\bar{OS}=i-6j-\bar{k}$

$\bar{PQ}=\bar{OQ}-\bar{OP}=\bar{i}+4j-\bar{k}$

$\bar{RS}=\bar{OS}-\bar{OR}=-2\bar{i}-8j+2\bar{k}$

$\bar{RS}=-2(i+4j\cancel{+}\bar{k})$

$=-2\bar{PQ}$

$\therefore$ PQ, RS are parallel.

$\frac{16}{8.2}$) Find the value or values of m for which $m(\bar{i}+j+\bar{k})$ is a unit vector

$\hat{a}=\frac{\bar{a}}{|\bar{a}|}=\pm m(\bar{i}+j+\bar{k})$

$=\pm\frac{(\bar{i}+j+\bar{k})}{1/m}$

$\Rightarrow|\bar{a}|=\sqrt{1+1+1}=\sqrt{3}.$

$|\bar{a}|=\pm\frac{1}{m}$

$m=\pm\frac{1}{\sqrt{3}}.$

$\frac{17}{8.2}$) S.T the points A (1,1,1) B (1,2,3) C (2,-1,1) are the vertices of isosceles triangle

$\bar{OA}=i+j+\bar{k}$

$\bar{OB}=i+2j+3\bar{k}$

$\bar{OC}=2i-j+\bar{k}.$

$\bar{AB}=j+2\bar{k}$

$\bar{BC}=i-3j-2\bar{k}$

$\bar{CA}=-i+2j$

$|\bar{AB}|=\sqrt{1+4}=\sqrt{5}$

$|\bar{BC}|=\sqrt{1+9+4}=\sqrt{14}$

$|\bar{CA}|=\sqrt{1+4}=\sqrt{5}$

$|\bar{AB}|=|\bar{CA}|$ $\therefore$ it is isosceles $\triangle$le.

$\frac{1}{8.2}$) Verify whether the following ratios are dc's of some vector or not.

i) $\frac{1}{5}, \frac{3}{5}, \frac{4}{5}$ 2) $\frac{1}{\sqrt{2}}, \frac{1}{2}, \frac{1}{2}$ 3) $\frac{4}{3}, 0, \frac{3}{4}$.

W.K.T $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

1) $\frac{1}{25} + \frac{9}{25} + \frac{16}{25} \neq 1$ ∴ not dc's

2) $\frac{1}{2} + \frac{1}{4} + \frac{1}{4} = 1$ ∴ ② is dc's

3) $\frac{16}{9} + 0 + \frac{9}{16} \neq 1$. ∴ not dc's.

$\frac{2}{8.2}$) Find the direction cosines of a vector whose dr's are

1) 1, 2, 3 2) 3, −1, 3 3) 0, 0, 7.

1) let $\bar{r} = i + 2j + 3\bar{k}$

$|\bar{r}| = \sqrt{1+4+9} = \sqrt{14}$ ∴ dc's $\left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right)$.

2) let $\bar{r} = 3i - j + 3\bar{k}$

$|\bar{r}| = \sqrt{9+1+9} = \sqrt{19}$ dc's $\left(\frac{3}{\sqrt{19}}, \frac{-1}{\sqrt{19}}, \frac{3}{\sqrt{19}}\right)$.

3) Let $\bar{r} = 7\bar{k}$

$|\bar{r}| = \sqrt{49} = 7$ dc's $\left(0, 0, \frac{7}{7}\right)$

$\frac{3}{8.2}$) Find the dc's and dr's of the following vectors.

1) $3\bar{i} - 4j + 8\bar{k}$ 2) $3i + j + \bar{k}$ 3) $\hat{j}$ 4) $5i - 3\bar{j} - 48\bar{k}$

5) $3i - 3\bar{k} + 4\bar{j}$ 6) $i - \bar{k}$.

Let 1) $\bar{x} = 3i - 4j + 8\bar{k}$ $\lvert\bar{x}\rvert = \sqrt{9+16+64} = \sqrt{89}$	dr's : $(3, -4, 8)$ dc's $\left(\frac{3}{\sqrt{89}}, \frac{-4}{\sqrt{89}}, \frac{8}{\sqrt{89}}\right)$
2) $\bar{x} = 3i + j + k$ $\lvert\bar{x}\rvert = \sqrt{9+1+1} = \sqrt{10}$	dr's $(3, 1, 1)$ dc's $\left(\frac{3}{\sqrt{10}}, \frac{1}{\sqrt{10}}, \frac{1}{\sqrt{10}}\right)$
3 $\bar{x} = \bar{j}$ $\lvert\bar{x}\rvert = \sqrt{1} = 1$	dr's $(0, 1, 0)$ dc's $(0, 1, 0)$

4) $\bar{x} = 5i - 3j - 48\bar{k}$ $\lvert\bar{x}\rvert = \sqrt{2338}$	dr's $(5, -3, -48)$ dc's $\left(\frac{5}{\sqrt{2338}}, \frac{-3}{\sqrt{2338}}, \frac{-48}{\sqrt{2338}}\right)$.
5) $\bar{x} = 3\bar{i} + 4\bar{j} - 3\bar{k}$ $\lvert\bar{x}\rvert = \sqrt{9+16+9} = \sqrt{34}$	dr's $(3, 4, -3)$ dcs' $\left(\frac{3}{\sqrt{34}}, \frac{4}{\sqrt{34}}, \frac{-3}{\sqrt{34}}\right)$
6) $\bar{x} = i - k$ $\lvert\bar{x}\rvert = \sqrt{1+1} = \sqrt{2}$	dr's $(1, 0, -1)$ dcs $\left(\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}\right)$

$\frac{5)}{8.2}$ If $\frac{1}{2}, \frac{1}{\sqrt{2}}$, a are the direction cosines of some vector find a.

$\cos\alpha = \frac{1}{2}$, $\cos\beta = \frac{1}{\sqrt{2}}$, $\cos\gamma = a$

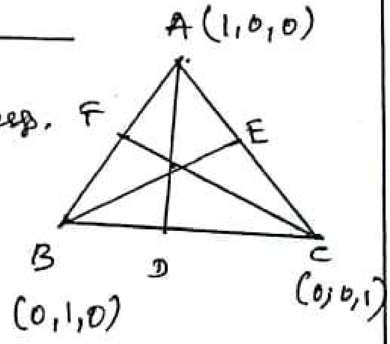
w.k.T $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$$\frac{1}{4} + \frac{1}{2} + a^2 = 1$$

$$a^2 = \frac{1}{4}$$

$$a = \pm \frac{1}{2}$$

$\frac{4)}{8.2}$ A triangle is formed by joining the points $(1,0,0)$ $(0,1,0)$ $(0,0,1)$. Find the dc's of the median.



A $(1,0,0)$ B$(0,1,0)$ C $(0,0,1)$

Let D, E, F are the mid points of BC, CA, AB resp.

AD, BE, CF are the medians.

Mid point of BC is D $= \left(0, \frac{1}{2}, \frac{1}{2}\right)$

A $= (1, 0, 0)$

D.r's of AD $= \left(-1, \frac{1}{2}, \frac{1}{2}\right)$ $\quad \sqrt{1 + \frac{1}{4} + \frac{1}{4}} = \sqrt{\frac{\cancel{6}\,3}{\cancel{4}\,2}}$

Dcs of AD $= \left(-\frac{1 \cdot \sqrt{2}}{\sqrt{3}}, \frac{\frac{1}{2} \times \sqrt{2}}{\sqrt{3}}, \frac{\frac{1}{2} \times \sqrt{2}}{\sqrt{3}}\right)$

$$= \left(-\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$$

IIIly the dc's of other medians are $\left(\frac{1}{\sqrt{6}}, \frac{-\sqrt{2}}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$, $\left(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}\right)$

4/8/2)

EXERCISE - 8.3.

Ex. 8.11) Find $\bar{a}\cdot\bar{b}$ when 1) $\bar{a} = i - j + 5k$, $\bar{b} = 3i - 2\bar{k}$
2) $\bar{a}, \bar{b}$ represents the points $(2,3,-1)$ $(-1,2,3)$

1) $\bar{a} = i - j + 5\bar{k}$
$\bar{b} = 3i - 2\bar{k}$
$\bar{a}\cdot\bar{b} = 3 + 0 - 10 = -7$

2) $\bar{a} = 2\hat{i} + 3j - \bar{k}$
$\bar{b} = -\bar{i} + 2j + 3\bar{k}$
$\bar{a}\cdot\bar{b} = -2 + 6 - 3 = 1$

Ex 8.12) Find $(\bar{a}+3\bar{b})\cdot(2\bar{a}-\bar{b})$ if $\bar{a} = i + j + 2\bar{k}$, $\bar{b} = 3i + 2j - \bar{k}$

$\bar{a} = i + j + 2\bar{k}$
$3\bar{b} = 9i + 6j - 3\bar{k}$
$(\bar{a}+3b) = 10i + 7j - \bar{k}$
$2\bar{a} = 2\bar{i} + 2j + 4\bar{k}$
$\bar{b} = 3i + 2j - \bar{k}$
$(2\bar{a}-\bar{b}) = -i + 5\bar{k}$
$(\bar{a}+3\bar{b})\cdot(2\bar{a}-\bar{b}) = -10 + 0 + 5 = -15$

Ex 8.13) If $\bar{a} = 2i + 2j + 3\bar{k}$, $\bar{b} = -i + 2j + \bar{k}$ and $\bar{c} = 3i + j$ s.t $\bar{a}+\lambda\bar{b}$ is $\perp r$ to $\bar{c}$ then find λ.

$\bar{a} = 2i + 2j + 3\bar{k}$
$\lambda\bar{b} = -\lambda\bar{i} + 2\lambda j + \lambda\bar{k}$
$\bar{a}+\lambda\bar{b} = (2-\lambda)i + (2+2\lambda)j + (3+\lambda)\bar{k}$
$\bar{c} = 3\bar{i} + j$

$\bar{a}+\lambda\bar{b}$ and $\bar{c}$ are $\perp r$ $\therefore (a+\lambda\bar{b})\cdot\bar{c} = 0$

$(\bar{a}+\lambda b)\cdot\bar{c} = (2-\lambda)3 + (2+2\lambda)\cdot 1 = 0$
$6 - 3\lambda + 2 + 2\lambda = 0$
$8 = \lambda.$

Ex 8.14) If $|\bar{a}+\bar{b}| = |\bar{a}-\bar{b}|$ P.T $\bar{a}, \bar{b}$ are $\perp r$.

$|\bar{a}+\bar{b}|^2 = |\bar{a}-b|^2$
$|\bar{a}|^2 + |\bar{b}|^2 + 2\bar{a}\cdot\bar{b} = |\bar{a}|^2 + |\bar{b}|^2 - 2\bar{a}\cdot\bar{b}$
$2\bar{a}\cdot\bar{b} + 2\bar{a}\cdot\bar{b} = 0$
$4\bar{a}\cdot\bar{b} = 0$
$\bar{a}\cdot\bar{b} = 0$
$\Rightarrow \bar{a} \perp \bar{b}.$

EX 8.15) For any vector $\bar{r}$ P.T $\bar{r} = (\bar{r}\cdot\bar{i})\bar{i} + (\bar{r}\cdot\bar{j})\bar{j} + (\bar{r}\cdot\bar{k})\bar{k}$

Let $\bar{r} = xi + yj + z\bar{k}$

$\bar{r}\cdot\bar{i} = (xi + yj + z\bar{k})\cdot\bar{i}$

$= x$

$(\bar{r}\cdot i)i = x\bar{i}$

IIIly $(\bar{r}\cdot j)j = yj$

$(\bar{r}\cdot\bar{k})\bar{k} = z\bar{k}$

$\therefore (\bar{r}\cdot i)i + (\bar{r}\cdot j)j + (\bar{r}\cdot\bar{k})\bar{k}$

$= xi + yj + z\bar{k}$

$= \bar{r}.$

EX 8.16) Find the angle between the vectors $5i + 3j + 4\bar{k}$ and $6\bar{i} - 8j - \bar{k}$.

$\bar{a} = 5i + 3j + 4\bar{k} \Rightarrow |\bar{a}| = \sqrt{25+9+16} = \sqrt{50} = \sqrt{25\times 2}$

$= 5\sqrt{2}$

$\bar{b} = 6\bar{i} - 8j - \bar{k} \Rightarrow |\bar{b}| = \sqrt{36+64+1} = \sqrt{101}$

$\bar{a}\cdot\bar{b} = 30 - 24 - 4 = 2$

$$\cos\theta = \frac{\bar{a}\cdot\bar{b}}{|\bar{a}||\bar{b}|} = \frac{\cancel{2}\sqrt{2}}{5\cancel{\sqrt{2}}\sqrt{101}}$$

$$\theta = \cos^{-1}\left(\frac{\sqrt{2}}{5\sqrt{101}}\right)$$

EX 8.17) Find the projection of $\bar{AB}$ on $\bar{CD}$ where A B C D are the points $(4,-3,0)$ $(7,-5,-1)$ $(-2,1,3)$ $(0,2,5)$

$\bar{AB} = \bar{OB} - \bar{OA} = 3i - 2j - \bar{k}$

$\bar{CD} = \bar{OD} - \bar{OC} = 2i + j + 2\bar{k}$

$\bar{AB}\cdot\bar{CD} = 6 - 2 - 2 = 2.$

$|\bar{CD}| = \sqrt{4+1+4} = 3.$

$$\text{Projection of } \bar{AB} \text{ on } CD = \frac{\bar{AB}\cdot\bar{CD}}{|\bar{CD}|} = 2/3.$$

EX 8.18) If $\bar{a}, \bar{b}, \bar{c}$ are the unit vectors satisfying $\bar{a} - \sqrt{3}\bar{b} + \bar{c} = 0$ then find the angle between $\bar{a}$ and $\bar{c}$.

Given $\bar{a} - \sqrt{3}\bar{b} + \bar{c} = 0$

$\bar{a} + \bar{c} = \sqrt{3}\bar{b}$

$|\bar{a}+\bar{c}| = \sqrt{3}|\bar{b}|$

$|\bar{a}+\bar{c}|^2 = 3|\bar{b}|^2$

$|\bar{a}|^2 + |\bar{b}|^2 + 2\bar{a}\cdot\bar{b} = 3|\bar{b}|^2$

$1 + 1 + 2|\bar{a}||\bar{b}|\cos\theta = 3$

$2\cos\theta = 3 - 2$

$= 1$

$\cos\theta = 1/2$

$\theta = \pi/3$

EX 8.19) S.T the points (4,−3,1) (2,−4,5) and (1,−1,0) form a rt angled $\triangle$le.

$\overline{OA} = 4i - 3j + \bar{k}$ $\quad \overline{AB} = \overline{OB} - \overline{OA} = -2i - j + 4\bar{k}$

$\overline{OB} = 2i - 4j + 5\bar{k}$ $\quad \overline{BC} = \overline{OC} - \overline{OB} = -i + 3j - 5\bar{k}$

$\overline{OC} = i - j.$ $\quad \overline{CA} = \overline{OA} - \overline{OC} = 3i - 2j + \bar{k}$

Now $\overline{AB} + \overline{BC} + \overline{CA} = 0$ ∴ the three points forms a $\triangle$le.

$\overline{AB} \cdot \overline{CA} = -6 + 2 + 4 = 0$

$\angle A = \pi/2.$

∴ it is a rt angled $\triangle$le.

1/8.3) 1) Find $\bar{a} \cdot \bar{b}$ when 1) $\bar{a} = i - 2j + \bar{k}$ $\bar{b} = 3i - 4j - 2\bar{k}$
2) $\bar{a} = 2i + 2j - \bar{k}$ $\bar{b} = 6\hat{i} - 3j + 2\bar{k}$

1) $\bar{a} = i - 2j + \bar{k}$ $\quad$ 2) $\bar{a} = 2i + 2j - \bar{k}$

$\bar{b} = 3i - 4j - 2\bar{k}$ $\quad \bar{b} = 6i - 3j + 2\bar{k}$

$\bar{a} \cdot \bar{b} = 3 + 8 - 2 = 9$ $\quad \bar{a} \cdot \bar{b} = 12 - 6 - 2 = 4.$

2/8.3) Find the value of λ for which the vectors $\bar{a}$ and $\bar{b}$ are $\perp$r.
1) $\bar{a} = 2\hat{i} + \lambda j + \bar{k}$ $\bar{b} = \hat{i} - 2j + 3\bar{k}$; 2) $\bar{a} = 2i + 4j - \bar{k}$
$\bar{b} = 3\hat{i} - 2j + \lambda\bar{k}.$

If $\bar{a}, \bar{b}$ are $\perp$r $\bar{a} \cdot \bar{b} = 0$

1) $\bar{a} \cdot \bar{b} = 2 - 2\lambda + 3 = 0$

$5 = 2\lambda \Rightarrow \lambda = 5/2$

2) $\bar{a} \cdot \bar{b} = 6 - 8 - \lambda = 0$

$-\lambda = 2$

$\lambda = -2$

3/8.3) If $\bar{a}$ and $\bar{b}$ are two vectors s.t $|\bar{a}| = 10$, $|\bar{b}| = 15$ $\bar{a} \cdot \bar{b} = 75\sqrt{2}$ find the angle between $\bar{a}$ and $\bar{b}$.

$\bar{a} \cdot \bar{b} = |\bar{a}||\bar{b}| \cos\theta$

$$\cos\theta = \frac{\bar{a} \cdot \bar{b}}{|\bar{a}||\bar{b}|} = \frac{75\sqrt{2}}{10 \times 15} = \frac{1}{\sqrt{2}}$$

$\cos\theta = \frac{1}{\sqrt{2}}$ $\quad \theta = \pi/4.$

4/8.3) Find the angle between the vectors
1) $2\hat{i} + 3j - 6\bar{k}$, $6\hat{i} - 3j + 2\bar{k}$
2) $i - j$, $j - \bar{k}$

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1) $\bar{a} = 2\bar{i}+3\bar{j}-6\bar{k}$ $|\bar{a}| = \sqrt{4+9+36} = 7.$

$\bar{b} = 6\bar{i}-3\bar{j}+2\bar{k}$ $|\bar{b}| = \sqrt{36+9+4} = 7.$

$\bar{a}\cdot\bar{b} = 12-9-12$

$= -9$ $\cos\theta = \dfrac{\bar{a}\cdot\bar{b}}{|\bar{a}||\bar{b}|} = \dfrac{-9}{7.7}.$

$\theta = \cos^{-1}\left(-\frac{9}{49}\right)$

2) $\bar{a} = \bar{i}-\bar{j} \Rightarrow |\bar{a}| = \sqrt{1+1} = \sqrt{2}$

$\bar{b} = \bar{j}-\bar{k} \Rightarrow |\bar{b}| = \sqrt{1+1} = \sqrt{2}$

$\bar{a}\cdot\bar{b} = -1$

$\cos\theta = \dfrac{\bar{a}\cdot\bar{b}}{|\bar{a}||\bar{b}|} = \dfrac{-1}{\sqrt{2}\cdot\sqrt{2}} = -\dfrac{1}{2}$ θ lies in II quadrant

$\theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3}.$

5/8.3 If $\bar{a}, \bar{b}, \bar{c}$ are the three vectors s.t $\bar{a}+2\bar{b}+\bar{c} = 0$ and $|\bar{a}| = 3$ $|\bar{b}| = 4$ $|\bar{c}| = 7$ find the angle between $\bar{a}$ and $\bar{b}$.

$\bar{a}+2\bar{b}+\bar{c} = 0$

$|\bar{a}+2\bar{b}| = |-\bar{c}|$

$|\bar{a}+2\bar{b}|^2 = |\bar{c}|^2$

$|\bar{a}|^2+4|\bar{b}|^2+4\,\bar{a}\cdot\bar{b} = |\bar{c}|^2$

$9 + 64 + 4\bar{a}\cdot\bar{b} = 49$

$4\cdot\bar{a}\cdot\bar{b} = -24$

$\bar{a}\cdot\bar{b} = -6.$

$|\bar{a}||\bar{b}|\cos\theta = -6$

$3\times4\times\cos\theta = -6$

$\Rightarrow \cos\theta = -\frac{\cancel{6}}{\cancel{12}}\frac{1}{2}$

θ lies in II quadrant

$\theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3}.$

6/8.3) S.T the vectors $\bar{a} = 2\bar{i}+3\bar{j}+6\bar{k}$, $\bar{b} = 6\bar{i}+2\bar{j}-3\bar{k}$, $\bar{c} = 3\bar{i}-6\bar{j}+2\bar{k}$ are mutually orthogonal

$\bar{a} = 2\bar{i}+3\bar{j}+6\bar{k}$

$\bar{b} = 6\bar{i}+2\bar{j}-3\bar{k}$

$\bar{c} = 3\bar{i}-6\bar{j}+2\bar{k}$

$\bar{a}\cdot\bar{b} = 12+6-18 = 0$

$\bar{b}\cdot\bar{c} = 18-12-6 = 0$

$\bar{c}\cdot\bar{a} = 6-18+12 = 0$

$\therefore \bar{a}, \bar{b}, \bar{c}$ are mutually orthogonal.

7/8.3) S.T the vectors $-\bar{i}-2\bar{j}-6\bar{k}$, $2\bar{i}-\bar{j}+\bar{k}$, $-\bar{i}+3\bar{j}+5\bar{k}$ forms a rt angled $\triangle$le.

$\bar{a} = -i - 2j - 6\bar{k}$

$\bar{b} = 2i - j + \bar{k}$

$\bar{c} = -i + 3j + 5\bar{k}$ forms

$\bar{a} + \bar{c} = \bar{b}$ $\therefore$ first it is a $\triangle$le.

$\bar{a} \cdot \bar{b} = -2 + 2 - 6 \neq 0$

$\bar{b} \cdot \bar{c} = -2 - 3 + 5 = 0$ $\therefore$ $\bar{b} \cdot \bar{c} = 0$

$\therefore$ it is a rt angled $\triangle$le.

8/8.3) If $|\bar{a}| = 5$, $|\bar{b}| = 6$, $|\bar{c}| = 7$ and $\bar{a} + \bar{b} + \bar{c} = 0$ find $\bar{a} \cdot \bar{b} + \bar{b} \cdot \bar{c} + \bar{c} \cdot \bar{a}$.

$|\bar{a} + \bar{b} + \bar{c}| = 0$

$|\bar{a} + \bar{b} + \bar{c}|^2 = 0$

$|\bar{a}|^2 + |\bar{b}|^2 + |\bar{c}|^2 + 2(\bar{a} \cdot \bar{b} + \bar{b} \cdot \bar{c} + \bar{c} \cdot \bar{a}) = 0$

$25 + 36 + 49 + 2((\bar{a} \cdot \bar{b} + \bar{b} \cdot \bar{c} + \bar{c} \cdot \bar{a}) = 0$

$2(\bar{a} \cdot \bar{b} + \bar{b} \cdot \bar{c} + \bar{c} \cdot \bar{a}) = -110$

$\bar{a} \cdot \bar{b} + \bar{b} \cdot \bar{c} + \bar{c} \cdot \bar{a} = -55$.

9/8.3) S.T the points $(2,-1,3)$ $(4,3,1)$ $(3,1,2)$ are collinear.

$\bar{OA} = 2i - j + 3\bar{k}$

$\bar{OB} = 4i + 3j + \bar{k}$

$\bar{OC} = 3i + j + 2\bar{k}$.

$\bar{AB} = \bar{OB} - \bar{OA} = 2i + 4j - 2\bar{k}$

$\bar{BC} = \bar{OC} - \bar{OB} = -i - 2j + \bar{k}$

$\bar{AB} = -2(-i - 2j + \bar{k})$

$= -2\bar{BC}$

$\therefore$ A, B, C are collinear.

10/8.3) If $\bar{a}, \bar{b}$ are the unit vectors and θ is the angle between $\bar{a}$ and $\bar{b}$ S.T 1) $\sin\theta/2 = \frac{1}{2}|\bar{a} - \bar{b}|$ 2) $\cos\theta/2 = \frac{1}{2}|\bar{a} + \bar{b}|$ 3) $\tan\theta/2 = \frac{|\bar{a} - \bar{b}|}{|\bar{a} + \bar{b}|}$

$|\bar{a} - \bar{b}|^2 = |\bar{a}|^2 + |\bar{b}|^2 - 2\bar{a} \cdot \bar{b}$

$= 1 + 1 - 2|\bar{a}||\bar{b}|\cos\theta$

$= 2(1 - \cos\theta)$

$= 2 \cdot 2\sin^2\theta/2$

$|\bar{a} - \bar{b}| = 2\sin\theta/2 \Rightarrow \frac{1}{2}|\bar{a} - \bar{b}| = \sin\theta/2$ — ①

$$|\bar{a}+\bar{b}|^2 = |\bar{a}|^2+|\bar{b}|^2+2\bar{a}\cdot\bar{b}$$
$$= 1+1+2|\bar{a}|\cdot|\bar{b}|\cos\theta$$
$$= 2+2\cos\theta$$
$$= 2(1+\cos\theta)$$
$$= 2\cdot 2\cos^2\theta/2$$
$$|\bar{a}+\bar{b}| = 2\cos\theta/2 \Rightarrow \frac{1}{2}|\bar{a}+\bar{b}| = \cos\theta/2 \longrightarrow ②$$

$$\frac{①}{②} \quad \frac{\sin\theta/2}{\cos\theta/2} = \frac{\frac{1}{2}|\bar{a}-\bar{b}|}{\frac{1}{2}|\bar{a}+\bar{b}|} = \tan\theta/2$$

$\frac{12}{8.3}$) Find the projection of the vector $i+3\hat{j}+7\hat{k}$ on the vector $2\hat{i}+6\hat{j}+3\hat{k}$.

Let $\bar{a} = \hat{i}+3\hat{j}+7\hat{k}$
$\bar{b} = 2\hat{i}+6\hat{j}+3\hat{k}$

$\bar{a}\cdot\bar{b} = 2+18+21 = 41$
$|\bar{b}| = \sqrt{4+36+9} = 7.$

Projection of $\bar{a}$ on $\bar{b}$ is $\frac{\bar{a}\cdot\bar{b}}{|\bar{b}|} = \frac{41}{7}.$

$\frac{13}{8.3}$) Find λ when the projection of $\bar{a}=\lambda\hat{i}+\hat{j}+4\hat{k}$ on $\bar{b}=2\hat{i}+6\hat{j}+3\hat{k}$ is 4 units.

$\bar{a} = \lambda\hat{i}+\hat{j}+4\hat{k}$
$\bar{b} = 2\hat{i}+6\hat{j}+3\hat{k}$

$\bar{a}\cdot\bar{b} = 2\lambda+6+12$
$= 2\lambda+18$

$|\bar{b}| = \sqrt{4+36+9} = 7.$

Projection of $\bar{a}$ on $\bar{b}$ = $\frac{\bar{a}\cdot\bar{b}}{|\bar{b}|} = 4 \Rightarrow \frac{2\lambda+18}{7} = 4.$

$\therefore 2\lambda+18 = 28$
$2\lambda = 10$
$\lambda = 5$

$\frac{14}{8.3}$) Three vectors $\bar{a},\bar{b},\bar{c}$ are s.t $|\bar{a}|=2$ $|\bar{b}|=3$ $|\bar{c}|=4$ and $\bar{a}+\bar{b}+\bar{c}=0$ Then find $4\bar{a}\cdot\bar{b}+3\bar{b}\cdot\bar{c}+3\bar{c}\cdot\bar{a}$.

$\bar{a}+\bar{b}+\bar{c}=0$ $\lvert\bar{a}+\bar{b}\rvert = \lvert-\bar{c}\rvert$ $\lvert\bar{a}+\bar{b}\rvert^2 = \lvert\bar{c}\rvert^2$ $\lvert\bar{a}\rvert^2+\lvert\bar{b}\rvert^2+2\bar{a}\cdot\bar{b} = \lvert\bar{c}\rvert^2$ $4+9+2\bar{a}\cdot\bar{b} = 16$ $\bar{a}\cdot\bar{b} = 3/2$	$\lvert\bar{a}+\bar{c}\rvert^2 = \lvert\bar{b}\rvert^2$ $\lvert\bar{a}\rvert^2+\lvert\bar{c}\rvert^2+2\bar{a}\cdot\bar{c} = \lvert\bar{b}\rvert^2$ $4+16+2\bar{a}\cdot\bar{c} = 9$ $\bar{a}\cdot\bar{c} = \frac{-11}{2}$	$\lvert\bar{b}+\bar{c}\rvert^2 = \lvert\bar{a}\rvert^2$ $\lvert\bar{b}\rvert^2+\lvert\bar{c}\rvert^2+2\bar{b}\cdot\bar{c} = \lvert\bar{a}\rvert^2$ $9+16+2\bar{b}\cdot\bar{c} = 4$ $\bar{b}\cdot\bar{c} = -\frac{21}{2}$

$\therefore 4\bar{a}\cdot\bar{b}+3\bar{b}\cdot\bar{c}+3\bar{c}\cdot\bar{a} = 4\cdot\frac{3}{2} - \frac{3\cdot 21}{2} - \frac{3\cdot 11}{2} = \frac{12-63-33}{2}$
$= -42$

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EXERCISE - 8.4.

EX 8.20) Find $|\bar{a}\times\bar{b}|$ if $\bar{a} = 3i+4j$ and $\bar{b} = i+j+k$

$$\bar{a}\times\bar{b} = \begin{vmatrix} i & j & k \\ 3 & 4 & 0 \\ 1 & 1 & 1 \end{vmatrix} = i(4-0) - j(3-0) + k(3-4)$$
$$= 4i - 3j - k$$

$$|\bar{a}\times\bar{b}| = \sqrt{16+9+1} = \sqrt{26}$$

EX 8.21) If $\bar{a} = -3i+4j-7k$ and $\bar{b} = 6i+2j-3k$

verify 1) $\bar{a}$ and $\bar{a}\times\bar{b}$ are $\perp r$

2) $\bar{b}$ and $\bar{a}\times\bar{b}$ are $\perp r$.

$$\bar{a}\times\bar{b} = \begin{vmatrix} i & j & k \\ -3 & 4 & -7 \\ 6 & 2 & -3 \end{vmatrix} = i(-12+14) - j(9+42) + k(-6-24)$$
$$= 2i - 51j - 30k.$$

$$\bar{a}\cdot(\bar{a}\times\bar{b}) = (-3i+4j-7k)\cdot(2i-51j-30k)$$
$$= -6 + 204 + 210 = 0$$

$\therefore$ $\bar{a}$ and $\bar{a}\times\bar{b}$ are $\perp r$.

$$\bar{b}\cdot(\bar{a}\times\bar{b}) = (6i+2j-3k)\cdot(2i-51j-30k)$$
$$= 12 - 102 + 90 = 0.$$

$\therefore$ $\bar{b}$, $\bar{a}\times\bar{b}$ are $\perp r$.

EX.8.22) Find the vectors of magnitude 6 which are $\perp r$ to both vectors $\bar{a} = 4i - j + 3k$, $\bar{b} = -2i + j - 2k$.

$$\bar{a}\times\bar{b} = \begin{vmatrix} i & j & k \\ 4 & -1 & 3 \\ -2 & 1 & -2 \end{vmatrix} = i(2-3) - j(-8+6) + k(4-2)$$
$$= -i + 2j + 2k$$

$$|\bar{a}\times\bar{b}| = \sqrt{1+4+4} = 3.$$

$$\hat{n} = \pm\frac{\bar{a}\times\bar{b}}{|\bar{a}\times\bar{b}|} = \pm\frac{(-i+2j+2k)}{3}$$

$$6\hat{n} = \pm\frac{\cancel{6}^{2}(-i+2j+2k)}{\cancel{3}}$$

EX: 8.23) Find the cosine and sine angle between the vectors $\bar{a} = 2i + j + 3k$ $\bar{b} = 4i - 2j + 2k$.

$\bar{a} = 2\bar{i} + \hat{j} + 3\bar{k} \Rightarrow |\bar{a}| = \sqrt{4+1+9} = \sqrt{14}$

$\bar{b} = 4\bar{i} - 2\hat{j} + 2\bar{k} \Rightarrow |\bar{b}| = \sqrt{16+4+4} = \sqrt{24}$

$\bar{a}\cdot\bar{b} = 8 - 2 + 6 = 12.$

$$\cos\theta = \frac{\bar{a}\cdot\bar{b}}{|\bar{a}||\bar{b}|} = \frac{12}{\sqrt{14}\sqrt{24}}$$

$$\bar{a}\times\bar{b} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 2 & 1 & 3 \\ 4 & -2 & 2 \end{vmatrix} = i(2+6) - j(4-12) + k(-4-4)$$

$= 8i + 8j - 8\bar{k}$

$= 8(i + j - \bar{k})$

$|\bar{a}\times\bar{b}| = 8\sqrt{1+1+1} = 8\sqrt{3}.$

$$\sin\theta = \frac{|\bar{a}\times\bar{b}|}{|\bar{a}||\bar{b}|} = \frac{8\sqrt{3}}{\sqrt{14}\sqrt{24}} = \frac{\overset{4}{\cancel{8}} \times \sqrt{3}}{\cancel{\sqrt{2}}\sqrt{7}\cancel{\sqrt{2}}\sqrt{12}} = \frac{\overset{\sqrt{4}}{\cancel{4}} \times \cancel{\sqrt{3}}}{\sqrt{7}\times\cancel{\sqrt{4}}\times\cancel{\sqrt{3}}}$$

$$= \frac{2}{\sqrt{7}}$$

Ex 8.24) Find the area of the parallelogram whose adjacent sides are $\bar{a} = 3\bar{i} + \hat{j} + 4\bar{k}$ $\bar{b} = \bar{i} - \hat{j} + \bar{k}$

$$\bar{a}\times\bar{b} = \begin{vmatrix} \bar{i} & j & \bar{k} \\ 3 & 1 & 4 \\ 1 & -1 & 1 \end{vmatrix} = i(1+4) - j(3-4) + \bar{k}(-3-1)$$

$= 5i + \hat{j} - 4\bar{k}$

$|\bar{a}\times\bar{b}| = \sqrt{25+1+16} = \sqrt{42}.$

Area of the Parallelogram $|\bar{a}\times\bar{b}| = \sqrt{42}$ units.

Ex 8.25) For any two vectors $\bar{a}$ and $\bar{b}$ P.T $|\bar{a}\times\bar{b}|^2 + |\bar{a}\cdot\bar{b}|^2 = |\bar{a}|^2|\bar{b}|^2$

$|\bar{a}\times\bar{b}| = |\bar{a}||\bar{b}|\sin\theta$

$|\bar{a}\times\bar{b}|^2 = |\bar{a}|^2|\bar{b}|^2\sin^2\theta$ —— (1)

$\bar{a}\cdot\bar{b} = |\bar{a}||\bar{b}|\cos\theta$

$(\bar{a}\cdot\bar{b})^2 = |\bar{a}|^2|\bar{b}|^2\cos^2\theta$ —— (2)

(1)+(2) $|\bar{a}\times\bar{b}|^2 + (\bar{a}\cdot\bar{b})^2 = |\bar{a}|^2|\bar{b}|^2(\sin^2\theta + \cos^2\theta)$

$= |\bar{a}|^2|\bar{b}|^2$

Ex 8.26) Find the area of the $\triangle$le having the points A (1,0,0) B (0,1,0) C (0,0,1)

$\bar{OA} = \hat{i}$ $\quad \bar{AB} = \bar{OB} - \bar{OA} = -i + j$

$\bar{OB} = \bar{j}$ $\quad \bar{AC} = \bar{OC} - OA = -i + k$

$\bar{OC} = \bar{k}$

Area of the $\triangle le = \frac{1}{2}|\vec{AB} \times \vec{AC}|$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} i & j & k \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix} = i(1-0) - j(-1-0) + \vec{k}(0+1)$$
$$= \vec{i} + \vec{j} + \vec{k}.$$

$$|\vec{AB} \times \vec{AC}| = \sqrt{1+1+1} = \sqrt{3}.$$

Area of the $\triangle le$ ABC $= \frac{1}{2}|\vec{AB} \times \vec{AC}| = \frac{1}{2} \cdot \sqrt{3} : \sqrt{3}/_2$

$\frac{1}{8.4}$) Find the magnitude of $\vec{a} \times \vec{b}$ if $\vec{a} = 2i + j + 3\vec{k}$ $\vec{b} = 3i + 5j - 2\vec{k}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} i & j & \vec{k} \\ 2 & 1 & 3 \\ 3 & 5 & -2 \end{vmatrix} = i(-2-15) - j(-4-9) + \vec{k}(10-3)$$
$$= -17\vec{i} + 13\vec{j} + 7\vec{k}.$$

$$|\vec{a} \times \vec{b}| = \sqrt{289+169+49} = \sqrt{507}$$

$\frac{2}{8.4}$) P.T $\vec{a} \times (\vec{b} + \vec{c}) + \vec{b} \times (\vec{c} + \vec{a}) + \vec{c} \times (\vec{a} + \vec{b}) = \vec{0}$.

LHS: $\vec{a} \times (\vec{b} + \vec{c}) + \vec{b} \times (\vec{c} + \vec{a}) + \vec{c} \times (\vec{a} + \vec{b}) + \vec{c} \times \vec{a} + \vec{c} \times \vec{b}$

$$= \vec{a} \times \vec{b} + \vec{a} \times \vec{c} + \vec{b} \times \vec{c} + \vec{b} \times \vec{a} + \vec{c} \times \vec{a} + \vec{c} \times \vec{b}$$
$$= \vec{a} \times \vec{b} + \vec{a} \times \vec{c} + \vec{b} \times \vec{c} - \vec{a} \times \vec{b} - \vec{a} \times \vec{c} - \vec{b} \times \vec{c}$$
$$= \cancel{\vec{a} \times \vec{b}} + \cancel{\vec{a} \times \vec{c}} + \cancel{\vec{b} \times \vec{c}} - \cancel{\vec{a} \times \vec{b}}$$
$$= \vec{0}$$

$\frac{3}{8.4}$) Find the vectors of magnitude $10\sqrt{3}$ that are $\perp r$ to the plane which contains $i + 2j + \vec{k}$ and $i + 3j + 4\vec{k}$.

$$\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 1 & 2 & 1 \\ 1 & 3 & 4 \end{vmatrix} = i(8-3) - j(4-1) + \vec{k}(3-2)$$
$$= 5\vec{i} - 3\vec{j} + \vec{k}$$

$$|\vec{a} \times \vec{b}| = \sqrt{25+9+1} = \sqrt{35}$$

$$\hat{n} = \pm \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|} = \pm \frac{(5i - 3j + \vec{k})}{\sqrt{35}}$$

$$10\sqrt{3}\,\hat{n} = \pm 10\sqrt{3} \frac{(5i - 3j + \vec{k})}{\sqrt{35}}$$

$\frac{4}{8.4}$) Find the unit vectors $\perp r$ to each of the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ where $\vec{a} = i + j + \vec{k}$ $\vec{b} = i + 2j + 3\vec{k}$.

Let $\vec{c} = \vec{a} + \vec{b} = 2i + 3j + 4\vec{k}$
$\vec{d} = \vec{a} - \vec{b} = -j - 2k$.

$$\bar{c}\times\bar{d} = \begin{vmatrix} i & j & k \\ 2 & 3 & 4 \\ 0 & -1 & -2 \end{vmatrix} = i(-6+4) - j(-4-0) + k(-2-0)$$

$$= -2\bar{i} + 4\bar{j} - 2\bar{k} = -2(i+2j+k)$$

$$|\bar{c}\times\bar{d}| = \sqrt{4+16+4} = \sqrt{24}.$$

$$\hat{n} = \pm \frac{\bar{c}\times\bar{d}}{|\bar{c}\times\bar{d}|} = \pm \frac{(-2i+4j-2k)}{\sqrt{24}}.$$

$$= \pm \frac{\cancel{2}(i-2j+k)}{\cancel{2}\sqrt{6}}$$

$\frac{5}{8.4}$) Find the area of the parallelogram whose adjacent sides are determined by the vectors $i+2j+3k$, $3i-2j+k$.

$$\bar{a}\times\bar{b} = \begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ 3 & -2 & 1 \end{vmatrix} = i(2+6) - j(1-9) + \bar{k}(-2-6)$$

$$= 8\bar{i} + 8\bar{j} - 8\bar{k}.$$

$$= 8(i+j-k)$$

$$|\bar{a}\times\bar{b}| = 8\sqrt{1+1+1} = 8\sqrt{3}.$$

Area of the parallelogram $|\bar{a}\times\bar{b}| = 8\sqrt{3}$ sq. units.

$\frac{6}{8.4}$) Find the area of the tle whose vertices are $A(3,-1,2)$ $B(1,-1,-3)$ $C(4,-3,1)$

$$\bar{AB} = \bar{OB} - \bar{OA} = -2i - 5k$$

$$\bar{AC} = \bar{OC} - \bar{OA} = i - 2j - k.$$

$$\bar{AB}\times\bar{AC} = \begin{vmatrix} i & j & k \\ -2 & 0 & -5 \\ 1 & -2 & -1 \end{vmatrix} = i(0-10) - j(2+5) + \bar{k}(4-0)$$

$$= -10i - 7j + 4\bar{k}.$$

$$|\bar{AB}\times\bar{AC}| = \sqrt{100+49+16} = \sqrt{165}.$$

Area of the Δle: $\frac{1}{2}|\bar{AB}\times\bar{AC}| = \frac{1}{2}\sqrt{165}.$

$\frac{7}{8.4}$) If $\bar{a}, \bar{b}, \bar{c}$ are the P.V of A, B, C of a Δle ABC. S.T the area of the $ABC = \frac{1}{2}|\bar{a}\times\bar{b} + \bar{b}\times\bar{c} + \bar{c}\times\bar{a}|$ also deduce the condition for collinearity of the three points A, B, C.

Let $\overline{OA} = \bar{a}$ $\qquad \overline{AB} = \overline{OB} - \overline{OA} = \bar{b} - \bar{a}$

$\overline{OB} = \bar{b}$

$\overline{OC} = \bar{c}$ $\qquad \overline{AC} = \overline{OC} - \overline{OA} = \bar{c} - \bar{a}$

$$\overline{AB} \times \overline{AC} = (\bar{b} - \bar{a}) \times (\bar{c} - \bar{a})$$

$$= \bar{b} \times \bar{c} - \bar{b} \times \bar{a} - \bar{a} \times \bar{c} + \cancel{\bar{a} \times \bar{a}}^{\,0}$$

$$= \bar{b} \times \bar{c} + \bar{a} \times \bar{b} + \bar{c} \times \bar{a}$$

$$|\overline{AB} \times \overline{AC}| = |\bar{a} \times \bar{b} + \bar{b} \times \bar{c} + \bar{c} \times \bar{a}|$$

Area of the $\triangle$le $= \frac{1}{2} |\overline{AB} \times \overline{AC}|$

$$= \frac{1}{2} |\bar{a} \times \bar{b} + \bar{b} \times \bar{c} + \bar{c} \times \bar{a}|$$

If the three points are collinear Area of the $\triangle$le is 0.

$$\frac{1}{2} |\bar{a} \times \bar{b} + \bar{b} \times \bar{c} + \bar{c} \times \bar{a}| = 0$$

$$|\bar{a} \times \bar{b} + \bar{b} \times \bar{c} + \bar{c} \times \bar{a}| = 0.$$

$\frac{10}{8.4}$) Find the angle between the vectors $2\hat{i} + \hat{j} - \hat{k}$ and $\hat{i} + 2\hat{j} + \hat{k}$ using vector product.

$\bar{a} = 2\hat{i} + \hat{j} - \hat{k} \Rightarrow |\bar{a}| = \sqrt{4+1+1} = \sqrt{6}$

$\bar{b} = \hat{i} + 2\hat{j} + \hat{k} \qquad |\bar{b}| = \sqrt{1+4+1} = \sqrt{6}.$

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 1 & 2 & 1 \end{vmatrix} = \hat{i}(1+2) - \hat{j}(2+1) + \hat{k}(4-1)$$

$$= 3\hat{i} - 3\hat{j} + 3\hat{k}$$

$$= 3(\hat{i} - \hat{j} + \hat{k})$$

$$|\bar{a} \times \bar{b}| = 3\sqrt{1+1+1} = 3\sqrt{3}$$

$$\sin\theta = \frac{|\bar{a} \times \bar{b}|}{|\bar{a}||\bar{b}|} = \frac{3\sqrt{3}}{\sqrt{6}\sqrt{6}} = \frac{\cancel{3}\sqrt{3}}{\cancel{6}2}$$

$$\theta = \sin^{-1}(\sqrt{3}/2)$$

$$= \frac{\pi}{3}.$$

EXERCISE - 8.5 (one Mark)

1. The value of $\overline{AB}+\overline{BC}+\overline{CD}+\overline{DA}$ is

1) $\overline{AD}$ 2) $\overline{CA}$ 3) $\overline{0}$ 4) $-\overline{AD}$

By polygon law $\overline{AB}+\overline{BC}+\overline{CD}+\overline{DA}=0$.

∵ The starting point and end point are same at the same time the vectors continiously going on.

2) If $\bar{a}+2\bar{b}$ and $3\bar{a}+m\bar{b}$ are parallel then the value of m is

1) 3 2) $\frac{1}{3}$ 3) 6 4) $\frac{1}{6}$.

$$\bar{a}+2\bar{b}=3\left(\bar{a}+\frac{m}{3}\bar{b}\right)$$

$$\Rightarrow \frac{m}{3}=2 \Rightarrow m=6.$$

3) The unit vector parallel to the resultant of the vectors $\hat{i}+\hat{j}-\hat{k}$ and $\hat{i}-2\hat{j}+\hat{k}$ is

1) $\frac{\hat{i}-\hat{j}+\hat{k}}{\sqrt{5}}$ 2) $\frac{2\hat{i}+\hat{j}}{\sqrt{5}}$ 3) $\frac{2\hat{i}-\hat{j}+\hat{k}}{\sqrt{5}}$ 4) $\frac{2\hat{i}-\hat{j}}{\sqrt{5}}$

Let the Resultant Vector $\bar{a}=2\hat{i}-\hat{j}$

$$|\bar{a}|=\sqrt{4+1}=\sqrt{5}$$

$$\hat{a}=\frac{\bar{a}}{|\bar{a}|}=\frac{2\hat{i}-\hat{j}}{\sqrt{5}}$$

4) A vector $\overline{op}$ makes 60° and 45° with the positive direction of the x axis and y axis. Then the angle between $\overline{op}$ and the z axis is.

1) 45° 2) 60° 3) 90° 4) 30

$$\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$$

$$(\cos 60)^2+(\cos 45)^2+\cos^2\gamma=1$$

$$\frac{1}{4}+\frac{1}{2}+\cos^2\gamma=1$$

$$\cos^2\gamma=\frac{1}{4}$$

$$\cos\gamma=\frac{1}{2}$$

$$\gamma=60°$$

5) If $\overline{BA}=3\hat{i}+2\hat{j}+\hat{k}$ and the p.v of B is $\hat{i}+3\hat{j}-\hat{k}$ then the PV of A is

1) $4\hat{i}+2\hat{j}+\hat{k}$ 2) $4\hat{i}+5\hat{j}$ 3) $4\hat{i}$ 4) $-4\hat{i}$

$\overline{BA} = \overline{OA} - \overline{OB} = 3\bar{i} + 2\bar{j} + \bar{k}$

$\overline{OA} = (3\bar{i} + 2\bar{j} + \bar{k}) + \overline{OB}$

$= (3\bar{i} + 2\bar{j} + \bar{k}) + (\bar{i} + 3\bar{j} - \bar{k})$

$= 4\bar{i} + 5\bar{j}$

6) A vector makes equal angle with the positive direction of the co-ordinate axes. Then each angle is equal to

1) $\cos^{-1}(1/3)$ 2) $\cos^{-1}(2/3)$ 3) ✓ $\cos^{-1}(1/\sqrt{3})$ 4) $\cos^{-1}(2/\sqrt{3})$

Given $\alpha = \beta = \gamma$.

$\cos^2\alpha + \cos^2\alpha + \cos^2\alpha = 1$

$3\cos^2\alpha = 1$

$\cos^2\alpha = \frac{1}{3} \Rightarrow \cos\alpha = \frac{1}{\sqrt{3}}$

$\alpha = \cos^{-1}(1/\sqrt{3})$

7) The vectors $\bar{a}-\bar{b}$, $\bar{b}-\bar{c}$, $\bar{c}-\bar{a}$ are

1) Parallel to each other 2) unit vector.

3) mutually ⊥r vectors 4) ✓ coplanar vectors.

$$\begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{vmatrix} = 1(1) - 1(+1)$$

$= 0$ ∴ They are coplanar.

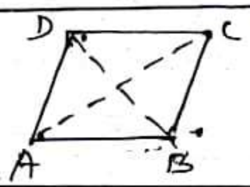
8) If ABCD is a parallelogram then $\overline{AB} + \overline{AD} + \overline{CB} + \overline{CD}$ is equal to

1) $2(\overline{AB} + \overline{AD}$ 2) $4\overline{AC}$ 3) $4\overline{BD}$ 4) ✓ $\bar{0}$

$\overline{AB} + \overline{AD} = \overline{AB} + \overline{BC}$

$\overline{CB} + \overline{CD} = -\overline{BC} - \overline{AB}$

∴ $\overline{AB} + \overline{AD} + \overline{CB} + \overline{CD} = \bar{0}$.

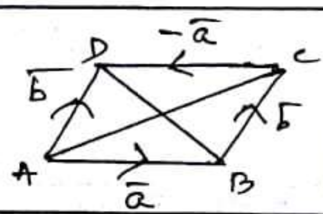


9) One of the diagonals of the parallelogram ABCD with adjacent sides $\bar{a}$ and $\bar{b}$ is $\bar{a}+\bar{b}$ then the other diagonal is

1) $\bar{a}-\bar{b}$ 2) ✓ $\bar{b}-\bar{a}$ 3) $\bar{a}+\bar{b}$ 4) $\frac{\bar{a}+\bar{b}}{2}$

$\overline{BD} = \overline{BC} + \overline{CD}$

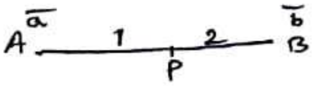
$= \bar{b} - \bar{a}$



10) If $\bar{a}, \bar{b}$ are the P.V. of A and B. Then which of the following points whose P V lies on AB.

1) $\bar{a}+\bar{b}$ 2) $\frac{2\bar{a}-\bar{b}}{2}$, 3) $\frac{2\bar{a}+\bar{b}}{3}$, 4) $\frac{\bar{a}-\bar{b}}{3}$

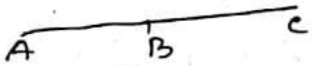
$$\bar{OP} = \frac{1\cdot\bar{b}+2\bar{a}}{1+2} = \frac{2\bar{a}+\bar{b}}{3}$$



11) If $\bar{a}, \bar{b}, \bar{c}$ are the P.V of three collinear points then which of the following is True.

1) $\bar{a} = \bar{b}+\bar{c}$ 2) $2\bar{a} = \bar{b}+\bar{c}$ 3) $\bar{b} = \bar{c}+\bar{a}$ 4) $4\bar{a}+\bar{b}+\bar{c}=0$

$\bar{OA} = \bar{a}$, $\bar{OB} = \bar{b}$, $\bar{OC} = \bar{c}$



X

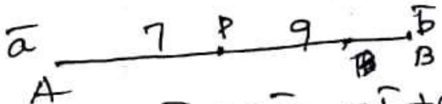
If the three points are collinear. $\bar{AB}+\bar{BC} = \bar{AC}$

$$\bar{OB}-\bar{OA}+\cancel{\bar{OC}}-\bar{OB} = \cancel{\bar{OC}}-\bar{OA}$$

12) If $\bar{r} = \frac{9\bar{a}+7\bar{b}}{16}$ then the point P whose p.v. $\bar{r}$ divides the line Joining the points with PV $\bar{a}, \bar{b}$ in the ratio

1) 7 : 9 internally 2) 9 : 7 internally

3) 9 : 7 externally 4) 7 : 9 externally.



$$\bar{r} = \frac{7\bar{b}+9\bar{a}}{7+9} = \frac{7\bar{b}+9\bar{a}}{16}.$$

13) If $\lambda\bar{i}+2\lambda\bar{j}+2\lambda\bar{k}$ is a unit vector then the value of λ is

1) 1/3 2) 1/4 3) 1/9 4) 1/2

$$\hat{a} = \frac{\bar{a}}{|\bar{a}|} = \frac{\lambda\bar{i}+2\lambda\bar{j}+2\lambda\bar{k}}{1}.$$

$$|\bar{a}| = \sqrt{\lambda^2+4\lambda^2+4\lambda^2} = 1 \Rightarrow 3\lambda = 1$$

$$\lambda = 1/3.$$

14) Two vertices of a triangle have P.V $3\bar{i}+4\bar{j}-4\bar{k}$ and $2\bar{i}+3\bar{j}+4\bar{k}$. If the P.V of centroid is $\bar{i}+2\bar{j}+3\bar{k}$ then the PV of the third vertex is

1) $-2\bar{i}-\bar{j}+9\bar{k}$ 2) $-2\bar{i}-\bar{j}-6\bar{k}$ 3) $2\bar{i}-\bar{j}+6\bar{k}$ 4) $-2\bar{i}+\bar{j}+6\bar{k}$

$\bar{OA} = 3\bar{i}+4\bar{j}-4\bar{k}$ $\bar{OB} = 2\bar{i}+3\bar{j}+4\bar{k}$ $\bar{OA}+\bar{OB} = 5\bar{i}+7\bar{j}$	$\bar{OA}+\bar{OB}+\bar{OC} = 3(\bar{i}+2\bar{j}+3\bar{k})$ $\bar{OC} = (3\bar{i}+6\bar{j}+9\bar{k})-(5\bar{i}+7\bar{j})$ $= -2\bar{i}-\bar{i}+9\bar{k}$

15) If $|\bar{a}+\bar{b}| = 60$, $|\bar{a}-\bar{b}| = 40$ $|\bar{b}| = 46$ then $|\bar{a}|$ is

1) 42 2) 12 3) 22 4) 32

$$|\bar{a}+\bar{b}|^2 + |\bar{a}-\bar{b}|^2 = 60^2 + 40^2$$
$$2\left[|\bar{a}|^2 + |\bar{b}|^2\right] = 3600 + 1600$$
$$= 5200$$
$$|\bar{a}|^2 + 46^2 = 2600$$
$$|\bar{a}|^2 = 2600 - 2116$$
$$= 484 \Rightarrow |\bar{a}| = 22$$

46×46
276
184
2116

22×22
44
44

16) If $\bar{a}$ and $\bar{b}$ having same magnitude and angle between them is 60° and their scalar product is $\frac{1}{2}$ then $|\bar{a}|$ is

1) 2 2) 3 3) 7 4) 1

$$\bar{a}\cdot\bar{b} = |\bar{a}||\bar{b}|\cos\theta = \frac{1}{2}$$
$$|\bar{a}||\bar{a}|\cos 60^\circ = \frac{1}{2}$$
$$|\bar{a}|^2 = \frac{1/2}{1/2} \Rightarrow |\bar{a}| = 1$$

17) The value of $\theta \in (0, \pi/2)$ for which the vectors $\bar{a} = (\sin\theta)\hat{i} + (\cos\theta)\hat{j}$ and $\bar{b} = \hat{i} - \sqrt{3}\hat{j} + 2\hat{k}$ are $\perp r$ is equal to

1) $\pi/3$ 2) $\pi/6$ 3) $\pi/4$ 4) $\pi/2$

$$\bar{a}\cdot\bar{b} = \sin\theta - \sqrt{3}\cos\theta = 0$$
$$\sin\theta = \sqrt{3}\cos\theta$$
$$\frac{\sin\theta}{\cos\theta} = \sqrt{3} \Rightarrow \tan\theta = \sqrt{3}$$
$$\theta = \pi/3$$

18) If $|\bar{a}| = 13$ $|\bar{b}| = 5$ and $\bar{a}\cdot\bar{b} = 60$ then $|\bar{a}\times\bar{b}|$ is

1) 15 2) 35 3) 45 4) 25.

$$|\bar{a}\times\bar{b}|^2 + (\bar{a}\cdot\bar{b})^2 = |\bar{a}|^2|\bar{b}|^2$$
$$= 169 \times 25$$
$$|\bar{a}\times\bar{b}|^2 = 4225 - 3600$$
$$= 625$$
$$|\bar{a}\times\bar{b}| = 25$$

19) Vectors $\bar{a}$ and $\bar{b}$ inclined at 120°. If $|\bar{a}| = 1$, $|\bar{b}| = 2$ find $(\bar{a}+3\bar{b})\times(3\bar{a}-\bar{b})$ is

1) 225 2) 275 3) 325 4) 300

$(\bar{a}+3\bar{b})\times(3\bar{a}-\bar{b}) = \cancel{3\bar{a}\times\bar{a}} - \bar{a}\times\bar{b} + 9\,\bar{b}\times\bar{a} - \cancel{3\,\bar{b}\times\bar{b}}$

$= -10\,\bar{a}\times\bar{b}$

$= -10\cdot|\bar{a}||\bar{b}|\sin 120^\circ$

$\therefore [(a+3\bar{b})\times(3\bar{a}-\bar{b})]^2 = 100\times 3 = 300$.

$= +10\cdot 1\cdot 2\cdot \sin 60^\circ$

$= 10\times 1\times \cancel{2}\times\frac{\sqrt{3}}{\cancel{2}} = 10\sqrt{3}$

20) If $\bar{a}$ and $\bar{b}$ are two vectors of magnitude 2 and inclined at an angle 60° then angle between $\bar{a}$ and $\bar{a}+\bar{b}$

1) 30° 2) 60° 3) 45° 4) 90°.

$\bar{a}\cdot(\bar{a}+\bar{b}) = |\bar{a}|^2 + \bar{a}\cdot\bar{b}$

$= |\bar{a}|^2 + |\bar{a}||\bar{b}|\cos\theta$.

21) If the projection of $5\bar{i}-\bar{j}-3\bar{k}$ on $\bar{i}+3\bar{j}+\lambda\bar{k}$ is same as the projection of $\bar{i}+3\bar{j}+\lambda k$ on $5\bar{i}-\bar{j}-3k$ then λ is

1) ± 4 2) ± 3 3) ± 5 4) ± 1.

$$\frac{(5\bar{i}-\bar{j}-3\bar{k})(\bar{i}+3\bar{j}+\lambda\bar{k})}{\sqrt{1+9+\lambda^2}} = \frac{(\bar{i}+3\bar{j}+\lambda\bar{k})\cdot(5\bar{i}-\bar{j}-3\bar{k})}{\sqrt{25+1+9}}$$

$$\frac{5-3-3\lambda}{\sqrt{10+\lambda^2}} = \frac{5-3-3\lambda}{\sqrt{35}} \Rightarrow \frac{\cancel{2-3\lambda}}{\sqrt{10+\lambda^2}} = \frac{\cancel{2-3\lambda}}{\sqrt{35}}$$

$\sqrt{35} = \sqrt{10+\lambda^2}$

sq. on both sides.

$35 = 10+\lambda^2 \Rightarrow \lambda^2 = 25$

$\lambda = \pm 5$

22) If $\overset{OA}{(1,2,4)}$ and $\overset{OB}{(2,-3\lambda,-3)}$ are the initial and terminal points of the vectors $\bar{i}+5\bar{j}-7K$ then the value of λ is equal to

1) $7/3$ 2) $-7/3$ 3) $-5/3$ 4) $5/3$.

$\overline{OB}-\overline{OA} = \bar{i}+(-3\lambda-2)\bar{j}-7k = \bar{i}+5\bar{j}-7\bar{k}$

$\Rightarrow -3\lambda-2 = 5$

$-3\lambda = 7 \quad \lambda = -7/3$

23) If the points whose P.V $10\bar{i}+3\bar{j}$, $12\bar{i}-5\bar{j}$, $a\bar{i}+11\bar{j}$ are collinear then a is equal to 1) 6 2) 3 3) 5 4) 8

$\overline{OA} = 10\hat{i} + 3\hat{j}$ $\quad \overline{AB} = \overline{OB} - \overline{OA} = 2\hat{i} - 8\hat{j}$

$\overline{OB} = 12\hat{i} - 5\hat{j}$ $\quad \overline{AC} = \overline{OC} - \overline{OA} = (a-10)\hat{i} + 8\hat{j}$ $\quad \Rightarrow 2 = 10 - a$

$\overline{OC} = a\hat{i} + 11\hat{j}.$ $\quad = -[(10-a)\hat{i} - 8\hat{j}]$ $\quad a = 10 - 2 = 8.$

24) If $\bar{a} = \hat{i} + \hat{j} + \bar{k}$, $\bar{b} = 2\hat{i} + x\hat{j} + \bar{k}$, $\bar{c} = \hat{i} - \hat{j} + 4\bar{k}$ and $\bar{a} \cdot (\bar{b} \times \bar{c}) = 70$ then x is equal to

1) 5 2) 7 3) 26 4) 10.

$$\begin{vmatrix} 1 & 1 & 1 \\ 2 & x & 1 \\ 1 & -1 & 4 \end{vmatrix} = 70$$

$1(4x+1) - 1(8-1) + 1(-2-x) = 70$

$4x + 1 - 7 - 2 - x = 70$

$3x = 78$

$x = 26.$

25) $\bar{a} = \hat{i} + 2\hat{j} + 2\hat{k}$ $\quad |\bar{b}| = 5$ and the angle between $\bar{a}$ and $\bar{b}$ is $\frac{\pi}{6}$. Then the area of the $\triangle$le formed by these two vectors as two sides is 1) $7/4$ 2) $15/4$ 3) $\frac{3}{4}$

Area of the $\triangle$le $= \frac{1}{2}|\bar{a} \times \bar{b}|$ $\qquad |\bar{a}| = \sqrt{1+4+4} = 3$

$= \frac{1}{2}|\bar{a}|\,|\bar{b}|\sin\theta$

$= \frac{1}{2}\, 3 \times 5 \times \sin 30^\circ$

$= \frac{1}{2} \times 3 \times 5 \times \frac{1}{2} = \frac{15}{4}$