2 REAL NUMBERS

INTRODUCTION

2.1 You know that $3 \times 3 \times 3 \times 3$ is shortly written as 3^4; thus $81 = 3^4$ where 3 is known as the base and 4 is its exponent. Another name for 'exponent' is 'index'. When one writes

$$x^n = \underbrace{x \times x \times x \times ... \times x}_{n \text{ factors}} \text{ (where n is a positive integer)}$$

x is the base and n is the index (Plural for 'index' is indices'). What is x^{-n} ? It is the multiplicative inverse of x^n (with the understanding that $x^n \times x^{-n} = x^0 = 1$). Thus we write

$$x^{-n} = \frac{1}{x^n}$$

2.2 **Radical Notation :** Let n be a positive integer and r be a real number. If $r^n = x$, then r is called the n^{th} root of x and we write

$$\sqrt[n]{x} = r$$

The symbol n (read as n^{th} root) is called a radical; n is the index of the radical (hitherto we named it as exponent); and x is called the radicand.

2.2.1 **Fractional Index**

Consider again results of the form $r = \sqrt[n]{x}$.

In the adjacent notation, the index of the radical (namely n which is 3 here) tells you how many times the answer (that is 4) must be multiplied with itself to yield the radicand.

To express the powers and roots, there is one more way of representation. It involves the use of fractional indices.

We write $\sqrt[n]{x}$ as $x^{\frac{1}{n}}$

With this notation, for example

$\sqrt[3]{64}$ is $64^{\frac{1}{3}}$ and $\sqrt{25}$ is $25^{\frac{1}{2}}$.

2.2.2 Meaning of $x^{\frac{m}{n}}$, (where m and n are Positive Integers)

We interpret $x^{\frac{m}{n}}$ either as the n^{th} root of the m^{th} power of x or as the m^{th} power of the n^{th} root of x.

In symbols, $x^{\frac{m}{n}} = (x^m)^{\frac{1}{n}}$ or $(x^m)^{\frac{1}{n}} = \sqrt[n]{x^m}$ or $(\sqrt[n]{x})^m$

Exercise 2.1

1. Write the following in the form of 5n:

(i) 625 (ii) $\frac{1}{5}$ (iii) $\sqrt{5}$ (iv) $\sqrt{125}$

Sol. (i) $625 = 5^4$

$$\begin{array}{r|r} 5 & 625 \\ \hline 5 & 125 \\ \hline 5 & 25 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

(ii) $\frac{1}{5} = 5^{-1}$ (iii) $\sqrt{5} = 5^{\frac{1}{2}}$

(iv) $\sqrt{125} = \sqrt{5^3} = (5^3)^{\frac{1}{2}} = 5^{\frac{3}{2}}$

$$\begin{array}{r|r} 5 & 125 \\ \hline 5 & 25 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

2. Write the following in the form of 4^n:

(i) 16 (ii) 8 (iii) 32

Sol. (i) $16 = 4^2$

(ii) $8 = 4^1 \times 4^{\frac{1}{2}} = 4^{1+\frac{1}{2}} = 4^{\frac{3}{2}}$

(iii) $32 = 4 \times 4 \times 4^{\frac{1}{2}}$

$= 4^{2+\frac{1}{2}} = 4^{\frac{5}{2}}$

$$\begin{array}{r|r} 2 & 32 \\ \hline 2 & 16 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline 2 & 2 \\ \hline & 1 \end{array}$$

Find the value of

3. (i) $(49)^{\frac{1}{2}}$ (ii) $(243)^{\frac{2}{5}}$ (iii) $(8)^{\frac{5}{3}}$ (iv) $9^{\frac{-3}{2}}$

(v) $\left(\frac{1}{27}\right)^{\frac{-2}{3}}$ (vi) $\left(\frac{64}{125}\right)^{\frac{-2}{3}}$

Sol. (i) $(49)^{\frac{1}{2}} = (7\times 7)^{\frac{1}{2}} = 7$

(ii) $(243)^{\frac{2}{5}} = (3^5)^{\frac{2}{5}}$

$= 3^{\cancel{5}\times\frac{2}{\cancel{5}}}$

$= 3^2$

$= 9$

3	243
3	81
3	27
3	9
3	3
	1

(iii) $(8)^{\frac{5}{3}} = (2^3)^{\frac{5}{3}} = 2^{\cancel{3}\times\frac{5}{\cancel{3}}} = 2^5 = 32$

(iv) $9^{\frac{-3}{2}} = (3^2)^{\frac{-3}{2}} = 3^{\cancel{2}\times\frac{-3}{\cancel{2}}} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$

(v) $\left(\frac{1}{27}\right)^{\frac{-2}{3}} = (27)^{\frac{2}{3}} = (3^3)^{\frac{2}{3}} = 3^{\cancel{3}\times\frac{2}{\cancel{3}}} = 3^2 = 9$

(vi) $\left(\frac{64}{125}\right)^{\frac{-2}{3}} = \left(\frac{125}{64}\right)^{\frac{2}{3}} = \left(\left(\frac{5}{4}\right)^3\right)^{\frac{2}{3}} = \left(\frac{5}{4}\right)^{\cancel{3}\times\frac{2}{\cancel{3}}} = \left(\frac{5}{4}\right)^2 = \frac{25}{16}$

4. **Use a fractional index to write:**

(i) $\sqrt{5}$ (ii) $\sqrt[3]{7}$ (iii) $(\sqrt[3]{49})^5$ (iv) $\left(\frac{1}{\sqrt[3]{100}}\right)^7$

Sol. (i) $\sqrt{5} = 5^{\frac{1}{2}}$

(ii) $\sqrt[3]{7} = 7^{\frac{1}{2}}$

(iii) $(\sqrt[3]{49})^5 = 49^{\frac{5}{3}}$

(iv) $\left(\frac{1}{\sqrt[3]{100}}\right)^7 = \left(\frac{1}{100^{\frac{1}{3}}}\right)^7 = \left(\frac{1^{\frac{1}{3}}}{100^{\frac{1}{3}}}\right)^7 = \left(\frac{1}{100}\right)^{\frac{7}{3}}$

5. **Find the 5th root of**

(i) 32 (ii) 243 (iii) 100000 (iv) $\frac{1024}{2135}$

Sol. (i) $\sqrt[5]{32} = 32^{\frac{1}{5}} = (2^5)^{\frac{1}{5}} = 2^{\cancel{5}\times\frac{1}{\cancel{5}}} = 2$

(ii) $\sqrt[5]{243} = 243^{\frac{1}{5}} = (3^5)^{\frac{1}{5}} = 3^{\cancel{5}\times\frac{1}{\cancel{5}}} = 3$

(iii) $\sqrt[5]{100000} = (100000)^{\frac{1}{5}} = (10^{5})^{\frac{1}{5}} = 10$

(iv) $\sqrt[5]{\frac{1024}{3125}} = \left(\frac{1024}{3125}\right)^{\frac{1}{5}} = \left(\left(\frac{4}{5}\right)^{5}\right)^{\frac{1}{5}} = \frac{4}{5}$

2.3 Surds

A surd is an irrational root of a rational number. $\sqrt[n]{a}$ is a surd, provided $n \in N$, $n > 1$, 'a' is rational.

2.3.1 Order of a Surd

The order of a surd is the index of the root to be extracted. The order of the surd $\sqrt[n]{a}$ is n.

2.3.2 Types of Surds

Surds can be classified in different ways:

(i) **Surds of same order :** Surds of same order are surds for which the index of the root to be extracted is same. (They are also called equiradical surds).

(ii) **Simplest form of a surd :** A surd is said to be in simplest form, when it is expressed as the product of a rational factor and an irrational factor.

(iii) **Pure and Mixed Surds :** A surd is called a pure surd if its coefficient in its simplest form is 1.

(iv) **Simple and Compound Surds :** A surd with a single term is said to be a simple surd.

(v) **Binomial Surd :** A binomial surd is an algebraic sum (or difference) of 2 terms both of which could be surds or one could be a rational number and another a surd.

2.3.3 Laws of Radicals

For positive integers m, n and positive rational numbers a and b, it is worth remembering the following properties of radicals:

S.No.	Radical Notation	Index Notation
1.	$(\sqrt[n]{a})^n = a = \sqrt[n]{a^n}$	$\left(a^{\frac{1}{n}}\right)^n = a = (a^n)^{\frac{1}{n}}$
2.	$\sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{ab}$	$a^{\frac{1}{n}} \times b^{\frac{1}{n}} = (ab)^{\frac{1}{n}}$
3.	$\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} = \sqrt[n]{\sqrt[m]{a}}$	$\left(a^{\frac{1}{n}}\right)^{\frac{1}{m}} = a^{\frac{1}{nm}} = \left(a^{\frac{1}{m}}\right)^{\frac{1}{n}}$
4.	$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$	$\frac{a^{\frac{1}{n}}}{b^{\frac{1}{n}}} = \left(\frac{a}{b}\right)^{\frac{1}{n}}$

2.3.4 Four Basic Operations on Surds :

(i) Addition and subtraction of surds

(ii) Multiplication and division of surds

$\sqrt{72} = \sqrt{36 \times 2} = 6\sqrt{2}$; $\sqrt{32} = \sqrt{16 \times 2} = 4\sqrt{2}$; $\sqrt{50} = \sqrt{25 \times 2} = 5\sqrt{2}$

Exercise 2.2

1. **Simplify the following using addition and subtraction properties of surds:**

(i) $5\sqrt{3}+18\sqrt{3}-2\sqrt{3}$ (ii) $4\sqrt[3]{5}+2\sqrt[3]{5}-3\sqrt[3]{5}$

(iii) $3\sqrt{75}+5\sqrt{48}-\sqrt{243}$ (iv) $5\sqrt[3]{40}+2\sqrt[3]{625}-3\sqrt[3]{320}$

Sol. (i) $5\sqrt{3}+18\sqrt{3}-2\sqrt{3}$

$$= (5+18-2)\sqrt{3} = 21\sqrt{3}$$

(ii) $4\sqrt[3]{5}+2\sqrt[3]{5}-3\sqrt[3]{5}$

$$= (4+2-3)\sqrt[3]{5} = 3\sqrt[3]{5}$$

(iii) $3\sqrt{75}+5\sqrt{48}-\sqrt{243}$

$$= 3\sqrt{5\times5\times3}+5\sqrt{3\times2\times2\times2\times2}-\sqrt{3\times3\times3\times3\times3}$$
$$= 3\times5\sqrt{3}+5\times2\times2\sqrt{3}-3\times3\sqrt{3}$$
$$= 15\sqrt{3}+20\sqrt{3}-9\sqrt{3}$$
$$= (15+20-9)\sqrt{3} = 26\sqrt{3}$$

$$\begin{array}{r|l} 3 & 243 \\ \hline 3 & 81 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 5 & 75 \\ \hline 5 & 15 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 3 & 48 \\ \hline 2 & 16 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline 2 & 2 \\ \hline & 1 \end{array}$$

(iv) $5\sqrt[3]{40}+2\sqrt[3]{625}-3\sqrt[3]{320}$

$$= 5\sqrt[3]{2^3\times5}+2\sqrt[3]{5^3\times5}-3\sqrt[3]{2^3\times2^3\times5}$$
$$= 5\times2\times\sqrt[3]{5}+2\times5\sqrt[3]{5}-3\times2\times2\sqrt[3]{5}$$
$$= 10\sqrt[3]{5}+10\sqrt[3]{5}-12\sqrt[3]{5}$$
$$= (10+10-12)\sqrt[3]{5} = 8\sqrt[3]{5}$$

$$\begin{array}{r|l} 2 & 40 \\ \hline 2 & 20 \\ \hline 2 & 10 \\ \hline 5 & 5 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 5 & 625 \\ \hline 5 & 105 \\ \hline 5 & 25 \\ \hline 5 & 5 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 2 & 320 \\ \hline 2 & 160 \\ \hline 2 & 80 \\ \hline 2 & 40 \\ \hline 2 & 20 \\ \hline 2 & 10 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

2. **Simplify the following using multiplication and division properties of surds :**

(i) $\sqrt{3}\times\sqrt{5}\times\sqrt{2}$ (ii) $\sqrt{35}\div\sqrt{7}$

(iii) $\sqrt[3]{27}\times\sqrt[3]{8}\times\sqrt[3]{125}$ (iv) $(7\sqrt{a}-5\sqrt{b})(7\sqrt{a}+5\sqrt{b})$

(v) $\left[\sqrt{\frac{225}{729}}-\sqrt{\frac{25}{144}}\right]\div\sqrt{\frac{16}{81}}$

Sol. (i) $\sqrt{3}\times\sqrt{5}\times\sqrt{2} = \sqrt{3\times5\times2}=\sqrt{30}$

(ii) $\sqrt{35}\div\sqrt{7} = \sqrt{\frac{35}{7}}=\sqrt{5}$

(iii) $\sqrt[3]{27}\times\sqrt[3]{8}\times\sqrt[3]{125}=\sqrt[3]{27\times8\times125} = \sqrt[3]{3^3\times2^3\times5^3} = 3\times2\times5=30$

(iv) $(7\sqrt{a}-5\sqrt{b})(7\sqrt{a}+5\sqrt{b}) = (7\sqrt{a})^2-(5\sqrt{b})^2 = 49a-25b$

(v) $\left[\sqrt{\frac{225}{729}}-\sqrt{\frac{25}{144}}\right]+\sqrt{\frac{16}{81}}$

$$= \left[\sqrt{\frac{15^2}{27^2}}-\sqrt{\frac{5^2}{12^2}}\right]\times\sqrt{\frac{9^2}{4^2}}$$

$$= \left(\frac{15}{27}-\frac{5}{12}\right)\times\frac{9}{4} = \left(\frac{5}{9}-\frac{5}{12}\right)\times\frac{9}{4}$$

$$= \left(\frac{20-15}{36}\right)\times\frac{9}{4} = \frac{5}{\underset{4}{\cancel{36}}}\times\frac{\cancel{9}}{4} = \frac{5}{6}$$

3	729
3	243
3	81
3	27
3	9
3	3
	1

3. **If $\sqrt{2}$ =1.414, $\sqrt{3}$ 1.732, $\sqrt{5}$= 2.236, $\sqrt{10}$= 3.162, then find the values of the following correct to 3 places of decimals.**

(i) $\sqrt{40}-\sqrt{20}$ **(ii)** $\sqrt{300}+\sqrt{90}-\sqrt{8}$

Sol. (i) $\sqrt{40}-\sqrt{20} = \sqrt{4\times10}-\sqrt{2\times10} = \sqrt{(4-2)10} = \sqrt{2\times10} = \sqrt{2}\times\sqrt{10}$

$$= 1.414\times3.162 = 4.471068 = 4.471$$

(ii) $\sqrt{300}+\sqrt{90}-\sqrt{8} = \sqrt{3\times100}+\sqrt{9\times10}+\sqrt{4\times2} = 10\sqrt{3}+3\sqrt{10}+2\sqrt{2}$

$$= 10\times1.732+3\times3.162+2\times1.414$$

$$= 17.32+9.486+2.828 = 29.634$$

4. **Arrange surds in descending order :**

(i) $\sqrt[3]{5},\sqrt[9]{4},\sqrt[6]{3}$ **(ii)** $\sqrt[2]{\sqrt[3]{5}},\sqrt[3]{\sqrt[4]{7}},\sqrt{\sqrt{3}}$

Sol. (i) $\sqrt[3]{5},\sqrt[9]{4},\sqrt[6]{3}$

$5^{\frac{1}{3}}$ $\therefore$ The order of the surds $\sqrt[3]{5},\sqrt[9]{4},\sqrt[6]{3}$ are 3, 9, 6.

$4^{\frac{1}{9}}$

$3^{\frac{1}{6}}$ l.c.m of 3, 9, 6 is 18

$$\therefore \frac{1}{3} = \frac{1\times6}{3\times6}=\frac{6}{18}$$

$$\frac{1}{9} = \frac{1\times2}{9\times2}=\frac{2}{18}\ ;\ \frac{1}{6} = \frac{1\times3}{6\times3}=\frac{3}{18}$$

$$\left(5^{\frac{1}{3}}\right) = 5^{\frac{6}{18}}=\left(5^6\right)^{\frac{1}{18}}=(15625)^{\frac{1}{18}}$$

$$\left(4^{\frac{1}{9}}\right) = 4^{\frac{2}{18}}=\left(4^2\right)^{\frac{1}{18}}=16^{\frac{1}{18}}$$

$$\left(3^{\frac{1}{6}}\right) = 3^{\frac{3}{18}}=\left(3^3\right)^{\frac{1}{18}}=27^{\frac{1}{18}}$$

$\therefore$ The descending order of $\sqrt[3]{5},\sqrt[9]{4},\sqrt[6]{3}$ is $(15625)^{\frac{1}{18}}>(27)^{\frac{1}{18}}>16^{\frac{1}{18}}$ i.e. $\sqrt[3]{5}>\sqrt[6]{3}>\sqrt[9]{4}$

(ii) $\sqrt[2]{\sqrt[3]{5}}, \sqrt[3]{\sqrt[4]{7}}, \sqrt{\sqrt{3}}$

The order of the surds $\sqrt[2]{\sqrt[3]{5}}, \sqrt[3]{\sqrt[4]{7}}, \sqrt[2]{\sqrt[2]{3}}$ are 6, 12, 4

l. c. m of 6, 12, 4 is 12

$$\sqrt[2]{\sqrt[3]{5}} = 5^{\frac{1}{6}} = 5^{\frac{1\times2}{6\times2}} = 5^{\frac{2}{12}} = (5^2)^{\frac{1}{12}} = 25^{\frac{1}{12}}$$

$$\sqrt[3]{\sqrt[4]{7}} = 7^{\frac{1}{12}} \; ; \; \sqrt{\sqrt{3}} = 3^{\frac{1}{4}} = 3^{\frac{1\times3}{4\times3}} = 3^{\frac{3}{12}} = (3^3)^{\frac{1}{12}} = 27^{\frac{1}{12}}$$

∴ The ascending order of the surds

$\sqrt[2]{\sqrt[3]{5}}, \sqrt[3]{\sqrt[4]{7}}, \sqrt{\sqrt{3}}$ is $7^{\frac{1}{12}} < 25^{\frac{1}{2}} < 27^{\frac{1}{2}}$, that is $\sqrt[3]{\sqrt[4]{7}} < \sqrt[2]{\sqrt[3]{5}} < \sqrt{\sqrt{3}}$

5. **Can you get a pure surd when you find**

(i) the sum of two surds **(ii) the difference of two surds**

(iii) the product of two surds **(iv) the quotient of two surds**

Justify each answer with an example.

Sol. (i) Yes $4\sqrt[3]{21} + (-3\sqrt[3]{21}) = \sqrt[3]{21}$

(ii) Yes $7\sqrt[4]{25} - 6\sqrt[4]{25} = \sqrt[4]{25}(7-6) = \sqrt[4]{25}$

(iii) Yes $\sqrt[3]{5} \times \sqrt[3]{4} = \sqrt[3]{20}$, a pure sured

(iv) Yes $= \frac{\sqrt{2\times5}}{\sqrt{2}} = \frac{\cancel{\sqrt{2}} \times \sqrt{5}}{\cancel{\sqrt{2}}} = \sqrt{5}$

6. **Can you get a rational number when you compute**

(i) the sum of two surds **(ii) the difference of two surds**

(iii) the product of two surds **(iv) the quotient of two surds**

Justify each answer with an example.

Sol. (i) Yes $(5-\sqrt{3})+(5+\sqrt{3}) = 10$, a rational number

(ii) Yes $(5+\sqrt[3]{7})-(-6+\sqrt[3]{7}) = 11$, a rational number

(iii) Yes $(5+\sqrt{3})(5-\sqrt{3}) = 25-3 = 22$, a rational number

(iv) Yes $\frac{5\sqrt{3}}{\sqrt{3}} = 5$, a rational number

2.4 Rationalisation of Surds

Rationalising factor is a term with which a term is multiplied or divided to make the whole term rational.

2.4.1 Conjugate Surds

This surd has one rational part and one radical part. In such cases, the rationalising factor has an interesting form.

Exercise 2.3

1. Rationalise the denominator

(i) $\frac{1}{\sqrt{50}}$ (ii) $\frac{5}{3\sqrt{5}}$ (iii) $\frac{\sqrt{75}}{\sqrt{18}}$ (iv) $\frac{3\sqrt{5}}{\sqrt{6}}$

Sol. (i) $\frac{1}{\sqrt{50}} = \frac{1}{\sqrt{50}} \times \frac{\sqrt{50}}{\sqrt{50}} = \frac{\sqrt{50}}{50} = \frac{\sqrt{5\times5\times2}}{5\times5\times2} = \frac{\cancel{5}\sqrt{2}}{\cancel{5}\times5\times2} = \frac{\sqrt{2}}{10}$

(ii) $\frac{5}{3\sqrt{5}} = \frac{5}{3\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{\cancel{5}\sqrt{5}}{3\times\cancel{5}} = \frac{\sqrt{5}}{3}$

(iii) $\frac{\sqrt{75}}{\sqrt{18}} = \frac{\sqrt{3\times5\times5}}{\sqrt{3\times2\times3}} = \frac{5\sqrt{3}\times\sqrt{2}}{3\sqrt{2}\times\sqrt{2}} = \frac{5\sqrt{6}}{3\times2} = \frac{5\sqrt{6}}{6}$

(iv) $\frac{3\sqrt{5}}{\sqrt{6}} = \frac{3\sqrt{5}\times\sqrt{6}}{\sqrt{6}\times\sqrt{6}} = \frac{\cancel{3}\sqrt{30}}{\underset{2}{\cancel{6}}} = \frac{\sqrt{30}}{2}$

2. Rationalise the denominator and simplify

(i) $\frac{\sqrt{48}+\sqrt{32}}{\sqrt{27}-\sqrt{18}}$ (ii) $\frac{5\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ (iii) $\frac{2\sqrt{6}-\sqrt{5}}{3\sqrt{5}-2\sqrt{6}}$ (iv) $\frac{\sqrt{5}}{\sqrt{6}+2} - \frac{\sqrt{5}}{\sqrt{6}-2}$

Sol. (i) $\frac{\sqrt{48}+\sqrt{32}}{\sqrt{27}-\sqrt{18}} = \frac{\sqrt{48}+\sqrt{32}}{(\sqrt{27}-\sqrt{18})} \times \frac{\sqrt{27}+\sqrt{18}}{\sqrt{27}+\sqrt{18}}$ [Multiply the numerator and denominator by the rationalising factor $(\sqrt{27}+\sqrt{18})$.]

$$= \frac{\sqrt{48\times27}+\sqrt{32}\times\sqrt{27}+\sqrt{48\times18}+\sqrt{32\times18}}{\sqrt{27^2}-\sqrt{18^2}}$$

$$= \frac{\sqrt{3\times16\times3\times9}+\sqrt{2\times16\times3\times9}+\sqrt{3\times16\times2\times9}+\sqrt{2\times16\times2\times9}}{729-324}$$

$$= \frac{4\times3\times3+4\times3\sqrt{6}+4\times3\sqrt{6}+4\times2\times3}{405}$$

$$= \frac{36+12\sqrt{6}+12\sqrt{6}+24}{405} = \frac{60+24\sqrt{6}}{405}$$

(ii) $\frac{5\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} = \frac{(5\sqrt{3}+\sqrt{2})\times(\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})\times(\sqrt{3}-\sqrt{2})}$

$$= \frac{5\sqrt{3}\times\sqrt{3}+\sqrt{2}\times\sqrt{3}-5\sqrt{3}\times\sqrt{2}-\sqrt{2}^2}{\sqrt{3}^2-\sqrt{2}^2}$$

$$= \frac{5\times3+\sqrt{6}-5\sqrt{6}-2}{3-2} = \frac{13-4\sqrt{6}}{1} = 13-4\sqrt{6}$$

(iii) $\frac{2\sqrt{6}-\sqrt{5}}{3\sqrt{5}-2\sqrt{6}} = \frac{(2\sqrt{6}-\sqrt{5})\times3\sqrt{5}+2\sqrt{6}}{(3\sqrt{5}-2\sqrt{6})\times3\sqrt{5}+2\sqrt{6}}$

$$= \frac{2\sqrt{6}\times3\sqrt{5}-3\sqrt{5}\sqrt{5}-(2\sqrt{6})^2-2\sqrt{5}\times\sqrt{6}}{(3\sqrt{5})^2-(2\sqrt{6})^2}$$

$$= \frac{6\sqrt{30}-3\times5-4\times6-2\sqrt{30}}{9\times5-4\times6} = \frac{4\sqrt{30}-39}{45-24} = \frac{4\sqrt{30}-39}{21}$$

(iv) $$\frac{\sqrt{5}}{\sqrt{6}+2} - \frac{\sqrt{5}}{\sqrt{6}-2} = \frac{\sqrt{5}(\sqrt{6}-2)-\sqrt{5}(\sqrt{6}+2)}{(\sqrt{6}+2)(\sqrt{6}-2)}$$

$$= \frac{\cancel{\sqrt{30}}-2\sqrt{5}-\cancel{\sqrt{30}}-2\sqrt{5}}{\sqrt{6}^2-2^2} = \frac{-4\sqrt{5}}{6-4} = \frac{-\cancel{4}^{\,2}\sqrt{5}}{\cancel{2}} = -2\sqrt{5}$$

3. **Find the value of a and b if $\frac{\sqrt{7}-2}{\sqrt{7}+2} = a\sqrt{7}+b$.**

Sol. $\frac{\sqrt{7}-2}{\sqrt{7}+2} = a\sqrt{7}+b$

$$\text{L. H. S} = \frac{\sqrt{7}-2\times\sqrt{7}-2}{\sqrt{7}+2\times\sqrt{7}-2} = \frac{(\sqrt{7}-2)^2}{\sqrt{7}^2-2^2} = \frac{\sqrt{7}^2-2\sqrt{7}\times2+2^2}{7-4}$$

$$= \frac{7-4\sqrt{7}+4}{3} = \frac{11-4\sqrt{7}}{3} = \frac{11}{3} - \frac{4\sqrt{7}}{3}$$

$$\frac{-4\sqrt{7}}{3} + \frac{11}{3} = a\sqrt{7}+b$$

$$\therefore a\cancel{\sqrt{7}} = \frac{-4\sqrt{7}}{3}$$

$$\Rightarrow \quad a = \frac{-4}{3}$$

$$b = \frac{11}{3}$$

4. **If $x = \sqrt{5}+2$, then find the value of $x^2 + \frac{1}{x^2}$.**

Sol. $$\text{If } x = \sqrt{5}+2$$

$$\frac{1}{x} = \frac{1}{\sqrt{5}+2} = \frac{1}{\sqrt{5}+2}\times\frac{\sqrt{5}-2}{\sqrt{5}-2} = \frac{\sqrt{5}-2}{\sqrt{5}^2-2^2} = \frac{\sqrt{5}-2}{5-4} = \frac{\sqrt{5}-2}{1}$$

$$x+\frac{1}{x} = \sqrt{5}+\cancel{2}+\sqrt{5}-\cancel{2} = 2\sqrt{5}$$

$$\left(x+\frac{1}{x}\right)^2 = (2\sqrt{5})^2 = 4\times5 = 20$$

5. **Given $\sqrt{2} = 1.414$, find the value of $\frac{8-5\sqrt{2}}{3-2\sqrt{2}}$ (to 3 places of decimals).**

Sol. $$\frac{8-5\sqrt{2}}{3-2\sqrt{2}} = \frac{(8-5\sqrt{2})\times(3+2\sqrt{2})}{(3-2\sqrt{2})\times(3+2\sqrt{2})} = \frac{24-15\sqrt{2}+16\sqrt{2}-10\times2}{3^2-(2\sqrt{2})^2}$$

$$= \frac{24+\sqrt{2}-20}{9-4\times2} = \frac{4+\sqrt{2}}{1} = 4+1.414 = 5.414$$

2.5 Scientific Notation :

Scientific notation is a way of representing numbers that are too large or too small, to be conveniently written in decimal form. It allows the numbers to be easily recorded and handled.

2.5.1 Writing a Number in Scientific Notation

Expressing a number N in the form of $N = a \times 10^n$ where, $1 \leq a < 10$ and 'n' is an integer is called as Scientific Notation.

2.5.2 Converting Scientific Notation to Decimal Form

The reverse process of converting a number in scientific notation to the decimal form is easily done when the following steps are followed:

(i) Write the decimal number.

(ii) Move the decimal point by the number of places specified by the power of 10, to the right if positive, or to the left if negative. Add zeros if necessary.

(iii) Rewrite the number in decimal form.

2.5.3 Arithmetic of Numbers in Scientific Notation

(i) If the indices in the scientific notation of two numbers are the same, addition (or subtraction) is easily performed.

(ii) The product or quotient of numbers in scientific notation can be easily done if we make use of the laws of radicals appropriately.

Exercise 2.4

1. Represent the following numbers in the scientific notation:

(i) 569430000000 **(ii) 2000.57**

(iii) 0.0000006000 **(iv) 0.0009000002**

Sol. (i) $569430000000 = 5.6943 \times 10^{11}$

(ii) $2000.57 = 2.00057 \times 10^3$

(iii) $0.0000006\cancel{000} = 6.0 \times 10^{-7}$

(iv) $0.0009000002 = 9.000002 \times 10^{-4}$

2. Write the following numbers in decimal form:

(i) 3.459×10^6 **(ii) 5.678×10^4**

(iii) 1.00005×10^{-5} **(iv) 2.530009×10^{-7}**

Sol. (i) $3.459 \times 10^6 = 3459000$

(ii) $5.678 \times 10^4 = 56780$

(iii) $1.00005 \times 10^{-5} = 0.0000100005$

(iv) $2.530009 \times 10^{-7} = 0.0000002530009$

3. Write the following numbers in decimal notation:

(i) $(300000)^2 \times (20000)^4$ **(ii) $(0.000001)^{11} \div (0.005)^3$**

(iii) $\{(0.00003)^6 \times (0.00005)^4\} \div \{(0.009)^3 \times (0.05)^2\}$

Sol. (i) $(300000)^2 \times (20000)^4 = (3.0 \times 10^5)^2 \times (2.0 \times 10^4)^4$

$= 3^2 \times 10^{10} \times 2^4 \times 10^{16} = 9 \times 16 \times 10^{10+16}$

$= 144 \times 10^{26} = 1.44 \times 10^{28}$

(ii) $(0.000001)^{11} \div (0.005)^3$

$$\frac{(0.000001)^{11}}{(0.005)^3} = \frac{(1.0\times10^{-6})^{11}}{(5.0\times10^{-3})^3} = \frac{(1.0)^{11}\times10^{-66}}{(5.0)^3\times10^{-9}} = 1\times10^{-66+9}\times5^3 = 125\times10^{-57}$$

$$= 1.25\times10^2\times10^{-57} = 1.25\times10^{(-57+2)} = 1.25\times10^{-55}$$

(iii) $\{(0.00003)^6\times(0.00005)^4\} \div \{(0.009)^3\times(0.05)^2\}$

$$= \frac{(3.0\times10^{-5})^6\times(5.0\times10^{-5})^4}{(9.0\times10^{-3})^3\times(5.0\times10^{-2})^2}$$

$$= \frac{3^6\times10^{-30}\times5^4\times10^{-20}}{9^3\times10^{-9}\times5^2\times10^{-4}} = \frac{3^6\times5^4\times10^{-30-20}}{(3^2)^3\times10^{-9-4}\times5^2} = \frac{\cancel{3^6}\times5^4\times10^{-50}}{\cancel{3^6}\times5^2\times10^{-13}}$$

$$= 5^{4-2}\times10^{-50+13}$$

$$= 5^2\times10^{-37} = 25\times10^{-37} = 2.5\times10^1\times10^{-37} = 2.5\times10^{-36}$$

4. **Represent the following information in scientific notation:**

(i) The world population is nearly 7000,000,000.

(ii) One light year means the distance 9460528400000000 km.

(iii) Mass of an electron is 0.000 000 000 000 000 000 000 000 000 00091093822 kg.

Sol. (i) The world population is nearly

7000,000,000 $= 7.0\times10^9$

(ii) One light year means the distance

9460528400000000 km $= 9.4605284\times10^{15}$ km

(iii) Mass of an electron is

0.000 000 000 000 000 000 000 000 000 00091093822 kg. $= 9.1093822\times10^{-31}$ kg

5. **Simplify :**

(i) $(2.75\times10^7)+(1.23\times10^8)$ (ii) $(1.598\times10^{17})-(4.58\times10^{15})$

(iii) $(1.02\times10^{10})\times(1.20\times10^{-3})$ (iv) $(8.41\times10^4)\div(4.3\times10^5)$

Sol. (i)

$$2.75\times10^7 = 27500000$$

$$1.23\times10^8 = 123000000$$

$$(2.75\times10^7)+(1.23\times10^8) = 27500000 + 123000000 = 150500000 = 1.505\times10^8$$

(ii) $(1.598\times10^{17})-(4.58\times10^{15})$

$$1.598\times10^{17} = 159800000000000000$$

$$4.58\times10^{15} = 4580000000000000$$

$$(1.598\times10^{17}) - (4.58\times10^{15})$$

$$= 159800000000000000 - 4580000000000000 = 155220000000000000 = 1.5522\times10^{1}$$

(iii) $(1.02 \times 10^{10}) \times (1.20 \times 10^{-3})$

$$= 1.02 \times 10^{10} \times 1.20 \times 10^{-3}$$
$$= 1.02 \times 1.2\cancel{0} \times 10^{10-3} = 1.224 \times 10^{7}$$

(iv) $(8.41 \times 10^4) \div (4.3 \times 10^5) = \dfrac{8.41 \times 10^4}{4.3 \times 10^5} = \dfrac{841 \times 10^{-2} \times 10^4}{43 \times 10^{-1} \times 10^5}$

$$= \frac{841 \times 10^{-2+4}}{43 \times 10^{-1+5}} = \frac{841}{43} \times \frac{10^2}{10^4} = 19.5581395 \times 10^{-2}$$
$$= 1.95581395 \times 10^1 \times 10^{-2}$$
$$= 1.95581395 \times 10^{-1} = 0.195581395$$

Exercise 2.5

Multiple Choice Questions :

1. Which of the following statement is false?

(1) The square root of 25 is 5 or –5 (2) $\sqrt{25} = 5$

(3) $-\sqrt{25} = -5$ (4) $\sqrt{25} = \pm 5$

[Ans. (4) $\sqrt{25} = \pm 5$]

2. Which one of the following is not a rational number?

(1) $\sqrt{\frac{8}{18}}$ (2) $\frac{7}{3}$ (3) $\sqrt{0.01}$ (4) $\sqrt{13}$

Hint : (1) $\sqrt{\frac{8}{18}} = \sqrt{\frac{4}{9}} = \frac{2}{3}$ is a rational number

(2) $\frac{7}{3}$ is a rational number

(3) $\sqrt{0.01} = \sqrt{\frac{1}{100}} = \frac{1}{10}$ is a rational number

(4) $\sqrt{13}$ is not a rational number

[Ans. (4) $\sqrt{13}$]

3. In simplest form, $\sqrt{640}$ is

(1) $8\sqrt{10}$ (2) $10\sqrt{8}$ (3) $2\sqrt{20}$ (4) $4\sqrt{5}$

Hint : $\sqrt{640}$ is $\sqrt{64 \times 10} = 8\sqrt{10}$

[Ans. (1) $8\sqrt{10}$]

4. $\sqrt{27} + \sqrt{12} =$

(1) $\sqrt{39}$ (2) $5\sqrt{6}$ (3) $5\sqrt{3}$ (4) $3\sqrt{5}$

Hint : $\sqrt{27} + \sqrt{12} = \sqrt{9 \times 3} + \sqrt{4 \times 3} = 3\sqrt{3} + 2\sqrt{3} = 5\sqrt{3}$ [Ans. (3) $5\sqrt{3}$]

5. If $\sqrt{80} = k\sqrt{5}$, then $k =$

(1) 2 (2) 4 (3) 8 (4) 16

Hint : $\sqrt{80} = \sqrt{16 \times 5} = 4\sqrt{5} = k\sqrt{5} \Rightarrow k = 4$

[Ans. (2) 4]

6. $4\sqrt{7} \times 2\sqrt{3} =$

(1) $6\sqrt{10}$ (2) $8\sqrt{21}$ (3) $8\sqrt{10}$ (4) $6\sqrt{21}$

Hint : $4\sqrt{7} \times 2\sqrt{3} = 8 \times \sqrt{7\times3} = 8\sqrt{21}$ **[Ans. (2) $8\sqrt{21}$]**

7. When written with a rational denominator, the expression $\frac{2\sqrt{3}}{3\sqrt{2}}$ can be simplified as

(1) $\frac{\sqrt{2}}{3}$ (2) $\frac{\sqrt{3}}{2}$ (3) $\frac{\sqrt{6}}{3}$ (4) $\frac{2}{3}$

Hint : $\frac{2\sqrt{3}}{3\sqrt{2}} = \frac{2\sqrt{3}}{3\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{6}}{3\times2} = \frac{\cancel{2}\sqrt{6}}{\cancel{6}3}$ **[Ans. (3) $\frac{\sqrt{6}}{3}$]**

8. When $(2\sqrt{5}-\sqrt{2})^2$ is simplified, we get

(1) $4\sqrt{5}+2\sqrt{2}$ (2) $22-4\sqrt{10}$

(3) $8-4\sqrt{10}$ (4) $2\sqrt{10}-2$

Hint : $(2\sqrt{5}-\sqrt{2})^2 = (2\sqrt{5})^2 - 2\times2\sqrt{5}\times\sqrt{2} + \sqrt{2}^2$

$= 4\times5 - 4\sqrt{10} + 2 = 22 - 4\sqrt{10}$ **[Ans. (2) $22-4\sqrt{10}$]**

9. $\frac{\sqrt[3]{18}}{\sqrt[3]{2}}$ is same as

(1) 3 (2) $\sqrt[3]{9}$ (3) 9 (4) $\sqrt[3]{3}$

Hint : $\frac{\sqrt[3]{18}}{\sqrt[3]{2}} = \frac{\sqrt[3]{2\times3\times3}}{\sqrt[3]{2}} = \frac{\cancel{\sqrt[3]{2}}\times\sqrt[3]{9}}{\cancel{\sqrt[3]{2}}} = \sqrt[3]{9}$ **[Ans. (2) $\sqrt[3]{9}$]**

10. $(0.000729)^{\frac{-3}{4}} \times (0.09)^{\frac{-3}{4}} =$ ________.

(1) $\frac{10^3}{3^3}$ (2) $\frac{10^5}{3^5}$ (3) $\frac{10^2}{3^2}$ (4) $\frac{10^6}{3^6}$

Hint : $(0.000729)^{\frac{-3}{4}} \times (0.09)^{\frac{-3}{4}}$

$$= (7.29\times10^{-4})^{\frac{-3}{4}} \times (9\times10^{-2})^{\frac{-3}{4}} = (7.29)^{\frac{-3}{4}} \times 10^{-\cancel{4}\times\frac{-3}{\cancel{4}}} \times 9^{\frac{-3}{4}} \times 10^{\cancel{-2}\times\frac{3}{\cancel{4}2}}$$

$$= (7.29)^{\frac{-3}{4}} \times 10^{+3} \times 9^{\frac{-3}{4}} \times 10^{\frac{3}{2}} = (729\times10^{-2})^{\frac{-3}{4}} \times 10^{3+\frac{3}{2}} \times 9^{\frac{-3}{4}}$$

$$= (9^3\times10^2)^{\frac{-3}{4}} \times 10^{\frac{9}{2}} \times 9^{\frac{-3}{4}} = (9^3)^{\frac{-3}{4}} \times 10^{\cancel{2}\times\frac{-3}{\cancel{4}2}} \times 10^{\frac{9}{2}} \times 9^{\frac{-3}{4}}$$

$$= 9^{\frac{-9}{4}} \times 10^{\frac{3}{2}+\frac{9}{2}} \times 9^{\frac{-3}{4}} = 9^{\frac{-9}{4}-\frac{3}{4}} \times 10^{\frac{\cancel{12}6}{\cancel{2}}}$$

$$= 9^{\frac{-12}{4}} \times 10^6 = 9^{-3}\times10^6 = \frac{10^6}{9^3} = \frac{10^6}{(3^2)^3} = \frac{10^6}{3^6}$$

[Ans. (4) $\frac{10^6}{(3^6)}$]

11. If $\sqrt{9^x} = \sqrt[3]{9^2}$, then $x =$ ______

(1) $\frac{2}{3}$ (2) $\frac{4}{3}$ (3) $\frac{1}{3}$ (4) $\frac{5}{3}$

Hint :

$$(9^x)^{\frac{1}{2}} = (9^2)^{\frac{1}{3}} ; \; 9^{\frac{x}{2}} = 9^{\frac{2}{3}}$$

$$\frac{x}{2} = \frac{2}{3} ; \; 3x = 4 ;$$

$$x = \frac{4}{3}$$

[Ans. (2) $\frac{4}{3}$]

12. **Which is the best example of a number written in scientific notation?**

(1) 0.5×10^5 (2) 0.1254 (3) 5.367×10^{-3} (4) 12.5×10^2

[Ans. (3) 5.367×10^{-3}]

13. **What is 5.92×10^{-3} written in decimal form?**

(1) 0.000592 (2) 0.00592 (3) 0.0592 (4) 0.592

Hint : $5.92 \times 10^{-3} = .00592$

[Ans. (2) 0.00592]

14. **The length and breadth of a rectangular plot are 5×10^5 and 4×10^4 metre respectively. Its area is ______.**

(1) $9 \times 10^1\ m^2$ (2) $9 \times 10^9\ m^2$ (3) $2 \times 10^{10}\ m^2$ (4) $20 \times 10^{20}\ m^2$

Hint :

$$l = 5 \times 10^5 \text{ metres} ; \; b = 4 \times 10^4 \text{ metres}$$

$$\therefore \text{Area} = l \times b = 5 \times 10^5 \times 4 \times 10^4 = 20 \times 10^{5+4}$$

$$= 20 \times 10^9 = 2.0 \times 10^1 \times 10^9 = 2 \times 10^{10}\ m^2$$

[Ans. (3) $2 \times 10^{10}\ m^2$]

Additional Questions and Answers

EXERCISE 2.1

1. **Evaluate : (i) 10^{-4} (ii) $\left(\frac{1}{9}\right)^{-3}$ (iii) $(0.01)^{-2}$**

Sol. (i)

$$10^{-4} = \frac{1}{10^4} = \frac{1}{10000} = 0.0001$$

(ii)

$$\left(\frac{1}{9}\right)^{-3} = \frac{1}{\left(\frac{1}{9}\right)^3} = \frac{1}{\left(\frac{1}{729}\right)} 729$$

(iii)

$$(0.01)^{-2} = \left(\frac{1}{100}\right)^2 = \frac{1}{\left(\frac{1}{100}\right)^{-2}} = \frac{1}{\frac{1}{10000}} = 10000$$

2. Express the following in the form 3^n :

(i) 27 (ii) 243 (iii) $\frac{1}{9}$ (iv) $\sqrt{3}$ (v) $\sqrt{27}$

Sol (i) $27 = 3 \times 3 \times 3$; therefore $27 = 3^3$

(ii) $243 = 3 \times 3 \times 3 \times 3 \times 3 = 3^5$

(iii) $\frac{1}{9} = \frac{1}{3\times 3} = \frac{1}{3^2} = 3^{-2}$

(iv) $\sqrt{3} = 3^{\frac{1}{2}}$

(v) $\sqrt{27} = \sqrt{3}\times\sqrt{3}\times\sqrt{3} = \left(3^{\frac{1}{2}}\right)^3 = (3)^{\left(\frac{3}{2}\right)}$

3. Find the value of $625^{\frac{3}{4}}$:

Sol $625^{\frac{3}{4}} = \left(\sqrt[4]{625}\right)^3 = \left(\sqrt[4]{5^4}\right)^3 = 5^3 = 5 \times 5 \times 5 = 125$

4. Find the value of $729^{-\frac{5}{6}}$:

Sol $729^{-\frac{5}{6}} = \frac{1}{729^{\frac{5}{6}}} = \frac{1}{\left(\sqrt[6]{729}\right)^5} = \frac{1}{\left(\sqrt[6]{3^6}\right)^5} = \frac{1}{3^5} = \frac{1}{243}$

5. Use a fractional index to write :

(i) $\left(5\sqrt{125}\right)^7$ (ii) $\sqrt[3]{7}$

Sol. (i) $\left(5\sqrt{125}\right)^7 = 125^{\frac{7}{5}}$

(ii) $\sqrt[3]{7} = 7^{\frac{1}{3}}$

EXERCISE 2.2

1. Can you reduce the following numbers to surds of same order.

(i) $\sqrt{5}$ (ii) $\sqrt[3]{5}$ (iii) $\sqrt[4]{5}$

Sol. (i) $\sqrt{5} = 5^{\frac{1}{2}} = 5^{\frac{6}{12}} = \sqrt[12]{5^6} = \sqrt[12]{15625}$

(ii) $\sqrt[3]{5} = 5^{\frac{1}{3}} = 5^{\frac{4}{12}} = \sqrt[12]{5^4} = \sqrt[12]{625}$

(iii) $\sqrt[4]{5} = 5^{\frac{1}{4}} = 5^{\frac{3}{12}} = \sqrt[12]{5^3} = \sqrt[12]{125}$

Now the surds have same order

2. Express the surds in the simplest form

(i) $\sqrt{27}$ (ii) $\sqrt[3]{128}$

Sol. (i) $\sqrt{27} = \sqrt{3\times3\times3} = 3\sqrt{3}$

(ii) $\sqrt[3]{128} = \sqrt[3]{2\times2\times2\times2\times2\times2\times2} = 4\sqrt[3]{2}$

3. Show that $\sqrt[3]{2} > \sqrt[5]{3}$.

Sol.

$$\sqrt[3]{2} = \sqrt[15]{2^5} = \sqrt[15]{32}$$
$$\sqrt[5]{3} = \sqrt[15]{3^3} = \sqrt[15]{27}$$
$$\sqrt[15]{32} > \sqrt[15]{27}$$

Therefore, $$\sqrt[3]{2} > \sqrt[5]{3}$$

4. Arrange in ascending order : $\sqrt[3]{5}, \sqrt[4]{7}, \sqrt[2]{6}$.

Sol. The order of the surds $\sqrt[3]{5}, \sqrt[4]{7}$ and $\sqrt[2]{6}$ are, 3, 4, 2. L.C.M. of 3, 4, 2 = 12.

$$\sqrt[3]{5} = 5^{\frac{1}{3}} = 5^{\frac{4}{12}} = (625)^{\frac{1}{12}}$$
$$\sqrt[4]{7} = 7^{\frac{1}{4}} = 7^{\frac{3}{12}} = (343)^{\frac{1}{12}}$$
$$\sqrt[2]{6} = 6^{\frac{1}{2}} = 6^{\frac{6}{12}} = (46656)^{\frac{1}{12}}$$

The ascending order of the surds $\sqrt[3]{5}, \sqrt[4]{7}, \sqrt[2]{6}$ is $(343)^{\frac{1}{12}} < (625)^{\frac{1}{12}} < (46656)^{\frac{1}{12}}$ the
$\sqrt[4]{7} < \sqrt[3]{5} < \sqrt[2]{6}$

5. Express the following surds in its simplest form $\sqrt[4]{324}$.

Sol.

$$\sqrt[4]{324} = \sqrt[4]{81 \times 4} = \sqrt[4]{3^4 \times 4} = \sqrt[4]{3^4} \times \sqrt[4]{4} \quad \left[\because \sqrt[n]{a^n} \times \sqrt[n]{b} = \sqrt[n]{a\ldots}\right.$$
$$= 3 \times \sqrt[4]{4} \quad \left[\because \sqrt[n]{a^n} = \ldots\right.$$

order = 4 ; radicand = 4; Coefficient = 3

6. Add $\sqrt[5]{11}$ and $\sqrt[7]{11}$. Check whether the sum is rational or irrational.

Sol.
$$\sqrt[5]{11} + \sqrt[7]{11} = (5+7)\sqrt{11} = 12\sqrt{11}$$

EXERCISE 2.3

1. Subtract $6\sqrt{7}$ from $9\sqrt{7}$. Is the answer rational or irrational?

Sol. $9\sqrt{7} - 6\sqrt{7} = (9-6)\sqrt{7} = 3\sqrt{7}$ The answer is irrational.

2. Simplify : $\sqrt{44} + \sqrt{99} - \sqrt{275}$.

Sol.
$$\sqrt{44} + \sqrt{99} - \sqrt{275} = \sqrt{4 \times 11} + \sqrt{9 \times 11} - \sqrt{11 \times 25}$$
$$= (2\sqrt{11} + 3\sqrt{11}) - 5\sqrt{11} = 5\sqrt{11} - 5\sqrt{11} = 0$$

3. Simplify : $-2\sqrt[4]{768} + 5\sqrt[4]{1875} + 3\sqrt[4]{48}$.

Sol.
$$-2\sqrt[4]{768} + 5\sqrt[4]{1875} + 3\sqrt[4]{48} = -2\sqrt[4]{256 \times 3} + 5\sqrt[4]{625 \times 3} + 3\sqrt[4]{16 \times 3}$$
$$= -2\sqrt[4]{4^4 \times 3} + 5\sqrt[4]{5^4 \times 3} + 3\sqrt[4]{2^4 \times 3}$$
$$= -2 \times 4\sqrt[4]{3} - 5 \times 5\sqrt[4]{3} + 3 \times 2\sqrt[4]{3}$$
$$= -8\sqrt[4]{3} + 25\sqrt[4]{3} + 6\sqrt[4]{3} = 23\sqrt[4]{3}$$

Multiply $\sqrt[4]{400}$ and $\sqrt[4]{567}$.

4\. **Sol.**

$$\sqrt[4]{400}\times\sqrt[4]{567} = \left(\sqrt[4]{2\times2\times2\times2\times5\times5}\right)\left(\sqrt[4]{3\times3\times3\times3\times7}\right)$$
$$= \left(2\times\sqrt[4]{25}\right)\times\left(3\times\sqrt[4]{7}\right) = 6\times\left(\sqrt[4]{25}\times\sqrt[4]{7}\right)$$
$$= 6\times\left(\sqrt[4]{25}\times\sqrt[4]{7}\right) = 6\times\sqrt[4]{25\times7} = 6\sqrt[4]{175}$$

Compute and give the answer in the simplest form : $3\sqrt{162}\times7\sqrt{50}\times6\sqrt{98}$

5\. **Sol.**

$$3\sqrt{162}\times7\sqrt{50}\times6\sqrt{98} = \left(3\times9\sqrt{2}\times7\times5\sqrt{2}\times6\times7\sqrt{2}\right)$$
$$= 3\times7\times6\times9\times5\times7\times\sqrt{2}\times\sqrt{2}\times\sqrt{2} = 79380\sqrt{2}$$

EXERCISE 2.4

1\. **Write in scientific notation : $(60000000)^3$**

Sol.
$$(60000000)^3 = (6.0\times10^7)^4 = (6.0)^4\times(10^7)^4$$
$$= 1296\times10^{28} = 1.296\times10^3\times10^{28} = 1.296\times10^{31}$$

2\. **Write in scientific notation : $(0.00000004)^3$**

Sol.
$$(0.00000004)^3 = (4.0\times10^{-8})^3 = (4.0)^3\times(10^{-8})^3$$
$$= 64\times10^{-24} = 6.4\times10\times10^{-24} = 6.4\times10^{-23}$$

3\. **Write in scientific notation : $(500000)^5\times(3000)^3$**

Sol.
$$(500000)^5\times(3000)^3 = (5.0\times10^5)^3\times(3.0\times10^3)^3$$
$$= (5.0)^2\times(10^5)^2\times(3.0)^3\times(10^3)^3$$
$$= 25\times10^{10}\times27\times10^9 = 675\times10^{19}$$
$$= 675.0\times10^{19} = 6.75\times10^2\times10^{19} = 6.75\times10^{21}$$

EXERCISE 2.5

MULTIPLE CHOICE QUESTIONS :

1\. **Which of the following is not an irrational number?**

(1) $\sqrt{2}$ (2) $\sqrt{5}$ (3) $\sqrt{3}$ (4) $\sqrt{25}$ **[Ans. (4) $\sqrt{25}$]**

2\. **In simple form, $\sqrt[3]{54}$ is?**

(1) $3\sqrt[3]{2}$ (2) $\sqrt[3]{27}$ (3) $3\sqrt{2}$ (4) $\sqrt{3}$ **[Ans. (1) $3\sqrt[3]{2}$]**

3\. **$\sqrt[3]{192}+\sqrt[3]{24}$**

(1) $3\sqrt[3]{6}$ (2) $6\sqrt[3]{3}$ (3) $\sqrt[3]{216}$ (4) $\sqrt[6]{216}$ **[Ans. (2) $6\sqrt[3]{3}$]**

4\. **$\sqrt[4]{405} = h\sqrt[4]{5}$, then $h =$**

(1) 5 (2) 4 (3) 2 (4) 3 **[Ans. (4) 3]**

5. $5\sqrt{21} \times 6\sqrt{10}$

(1) $30\sqrt{210}$ (2) 30 (3) $\sqrt{210}$ (4) $210\sqrt{30}$

[Ans. (1) $30\sqrt{21}$

6. Rationalising the denominator $\frac{1}{\sqrt[3]{3}} =$

(1) 3 (2) $\frac{3^{\frac{2}{3}}}{3}$ (3) $\sqrt{3}$ (4) $\sqrt[3]{3}$ [Ans. (2) $\frac{3^{\frac{2}{3}}}{3}$

7. When $(2\sqrt{3} - \sqrt{5})^2$ is simplified, we get

(1) 17 (2) $\sqrt{15}$ (3) $17-4\sqrt{15}$ (4) $4\sqrt{15}$

[Ans. (3) $17-4\sqrt{15}$

8. $\frac{\sqrt[3]{27}}{\sqrt[5]{3}}$ is equal to ____________.

(1) $\sqrt[5]{9}$ (2) $\sqrt[5]{6}$ (3) $\sqrt[5]{24}$ (4) $\sqrt[5]{30}$ [Ans. (1) $\sqrt[5]{9}$

9. Which is the best example of a number written in scientific notation?

(1) 2.71×10^5 (2) 0.6×10^4 (3) 0.9871 (4) 125.4×10^4

[Ans. (1) 2.71×10^5

10. The length of a square is 1.2×10^3 m. Its area is ________.

(1) 14.4×10^6 (2) 1.44×10^6 (3) 0.144×10 (4) 1440 [Ans. (2) 1.44×10^6

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