

Chapter: 3 Theory of Equations

(2)

Exercise 3.1

1. If the sides of a cubic box are increased by 1, 2, 3 units respectively to form a cuboid, then the volume is increased by 52 cubic units. Find the volume of the cuboid.

Soln Let the sides of cubic boxes are x, x, x .

$$\text{volume of a cube} = x^3$$

it is increased by 1, 2, 3.

$$\text{Cuboid sides be } (x+1), (x+2), (x+3)$$

Volume of a cuboid

$$(x+1)(x+2)(x+3) = x^3 + 52$$

$$(x^2 + 3x + 2)(x+3) = x^3 + 52$$

$$x^3 + 3x^2 + 2x + 3x^2 + 9x + 6 = x^3 + 52$$

$$6x^2 + 11x + 6 - 52 = 0$$

$$6x^2 + 11x - 46 = 0$$

$$6x^2 + 12x + 23x - 46 = 0$$

$$6x(x+2) + 23(x-2) = 0$$

$$x+2=0 \quad 6x+23=0$$

$$x = -2 \quad 6x = -23$$

$$x = -\frac{23}{6} \text{ (is not possible)}$$

$$\therefore x = 2$$

$$\text{Volume of a cube} = x^3 = 2^3 = 8$$

2. Construct a cubic equation with roots.

(i) 1, 2 and 3

$$\alpha = 1, \beta = 2, \gamma = 3$$

$$\alpha + \beta + \gamma = 6$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = 2 + 6 + 3 = 11$$

$$\alpha\beta\gamma = 6$$

$$x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma = 0$$

$$x^3 - 6x^2 + 11x - 6 = 0$$

(ii) 1, 1 and -2.

$$\alpha = 1, \beta = 1, \gamma = -2$$

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$$\alpha + \beta + \gamma = 1 + 1 + 0 = 2$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = 1 - 2 - 1 = -3$$

$$\alpha\beta\gamma = 1(1)(-1) = -2$$

$$x^3 - 0x^2 - 3x + 2 = 0$$

$$x^3 - 3x + 2 = 0$$

(ii)

2, -2 and 4.

$$\alpha = 2 \quad \beta = -2 \quad \gamma = 4$$

$$\alpha + \beta + \gamma = 2 - 2 + 4 = 4$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = -4 - 8 + 8 = -4$$

$$\alpha\beta\gamma = 2(-2)(4) = -16$$

$$x^3 - 4x^2 - 4x + 16 = 0$$

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If α, β and γ are the roots of the cubic equation $x^3 + 2x^2 + 3x + 4 = 0$ form a cubic equation whose roots are

(i) $2\alpha, 2\beta, 2\gamma$.

$$x^3 + 2x^2 + 3x + 4 = 0$$

$$\alpha + \beta + \gamma = -\frac{b}{a} = -\frac{2}{1} = -2$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} = \frac{3}{1} = 3$$

$$\alpha\beta\gamma = -\frac{d}{a} = -4$$

$$(i) \quad 2\alpha + 2\beta + 2\gamma = 2(\alpha + \beta + \gamma) = 2(-2) = -4$$

$$(2\alpha)(2\beta)(2\gamma) + (2\gamma)(2\alpha) = 4\alpha\beta + 4\beta\gamma + 4\gamma\alpha$$

$$= 4(\alpha\beta + \beta\gamma + \gamma\alpha) = 4(3) = 12$$

$$2\alpha(2\beta)(2\gamma) = 8(\alpha\beta\gamma) = 8(-4) = -32$$

$$x^3 - (-4)x^2 + 12x - (-32) = 0$$

$$x^3 + 4x^2 + 12x + 32 = 0$$

4. Solve the equation $3x^3 - 16x^2 + 23x - 6 = 0$ if the product of two roots is 1.

Soln: Let the roots be α, β, γ .

$$\text{Given } \alpha\beta = 1$$

$$\beta = \frac{1}{\alpha}$$

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$$\gamma = 2.$$

$$d + 1/d + 2 = \frac{16}{3} \Rightarrow d + \frac{1}{d} = \frac{16}{3} - 2 = \frac{16-6}{3} = \frac{10}{3}$$

$$(3x-1)(x-3) = 0$$

$$\begin{array}{r} -10 \\ 3 \overline{) 33} \\ \underline{3} \\ 0 \end{array}$$

(iv) if $\alpha = \frac{1}{3}$ $\beta = 3$ $\gamma = 2$.

$$2x^4 - 8x^3 + 6x^2 - 3 = 0$$

$$\alpha\beta + \beta\gamma + \gamma\alpha + \delta\alpha + \beta\delta + \gamma\delta = \frac{6}{2} = 3.$$

$$(a+b+c+d)^2 = a^2 + b^2 + c^2 + d^2 + 2(ab+ac+ad+bc+bd+cd)$$

$$\begin{aligned} (a+b+c+d)^2 &= a^2+b^2+c^2+d^2+2(ab+ac+ad+bc+bd+cd) \\ a^2+b^2+c^2+d^2 &= (a+b+c+d)^2 - 2(\alpha\beta+\beta\gamma+\gamma\delta+\delta\alpha+\beta\delta+\gamma\delta) \\ &= 4^2 - 2(13) = 16 - 6 = 10. \end{aligned}$$

soln. let the roots be $3d, 2d, p$.

sum of the roots : $3\alpha + 2\alpha + \beta = +9$
 $5\alpha + \beta = 9$ — (1)

Product of two roots: $(3\alpha)(2\alpha) + 2\alpha(\beta) + 3\alpha(\beta) = 14$
 $6\alpha^2 + 5\alpha\beta = 14$ — (2)

Product of 3 roots : $(3\alpha)(2\alpha)\beta = -24$
 $\alpha^2\beta = -4$

① $\Rightarrow \beta = 9 - 5\alpha$.

$$\textcircled{20} -) \quad 6x^2 + 5x(9-5x) = 14$$

$$6x^2 + 45x - 25x^2 = 14$$

$$\begin{aligned}
 -19x^2 + 45x - 14 &= 0 \\
 19x^2 - 45x + 14 &= 0 \\
 (x-2) \left(x - \frac{7}{19}\right) &= 0 \\
 x=2 \quad x &= \frac{7}{19}
 \end{aligned}$$

$$\begin{array}{c}
 24 \\
 \wedge \\
 \frac{235}{19} \quad -\frac{7}{19}
 \end{array}$$

(3)

If $x=2$, $\beta = 9 - 5(x) = 9 - 5(2) = 9 - 10 = -1$

roots are $3x, 2x, \beta$
 $3(2), 2(2), -1$ (i.e) $6, 4, -1$

If $x = \frac{7}{19}$, $\beta = 9 - 5\left(\frac{7}{19}\right) = \frac{136}{19}$

roots are $3x, 2x, \beta$ (i.e) $\frac{21}{19}, \frac{14}{19}, \frac{136}{19}$

7) If α, β and γ are the roots of the polynomial equation $ax^3 + bx^2 + cx + d = 0$ find the value of $\sum \frac{x}{\beta\gamma}$ in terms of the coefficients

Soln

$$\sum \frac{x}{\beta\gamma} = \frac{x}{\beta\gamma} + \frac{\beta}{\alpha\gamma} + \frac{\gamma}{\alpha\beta} = \frac{x^2 + \beta^2 + \gamma^2}{\alpha\beta\gamma}$$

$$\alpha + \beta + \gamma = -\frac{b}{a}, \quad \alpha\beta\gamma = -\frac{d}{a}, \quad \alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$$

$$\begin{aligned}
 x^2 + \beta^2 + \gamma^2 &= (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha) \\
 &= \left(-\frac{b}{a}\right)^2 - 2\left(\frac{c}{a}\right) = \frac{b^2}{a^2} - \frac{2c}{a}
 \end{aligned}$$

$$\begin{aligned}
 \sum \frac{x}{\beta\gamma} &= \frac{\frac{b^2}{a^2} - \frac{2c}{a}}{-\frac{d}{a}} = \frac{b^2 - 2ac}{a^2} \times \frac{a}{-d} = \frac{b^2 - 2ac}{-ad} \\
 &= \frac{2ac - b^2}{ad}
 \end{aligned}$$

8) If α, β, γ and δ are the roots of the polynomial equation $2x^4 + 5x^3 - 7x^2 + 8 = 0$. find a quadratic equation with integer coefficients whose roots are $\alpha\beta\gamma + \delta$ and $\alpha\beta\gamma\delta$.

Soln $p: \alpha + \beta + \gamma + \delta = -\frac{5}{2}$

$q: \alpha\beta\gamma\delta = -\frac{8}{2} = -4$

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$$p+q = -\frac{5}{2} - 4 = -\frac{5-8}{2} = -\frac{13}{2} \quad (6)$$

$$pq = -\frac{5}{2}(-4) = +10$$

$$x^2 - (p+q)x + pq = 0$$

$$x^2 + \frac{13}{2}x + 10 = 0$$

$$2x^2 + 13x + 20 = 0.$$

9) If p and q are the roots of the equation $lx^2 + mx + n = 0$ show that $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}} = 0$.

Soln. p & q are the roots of the equation

$$p+q = -\frac{n}{l}, \quad pq = \frac{n}{l}.$$

$$\begin{aligned} \text{L.H.S.} \quad \sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}} &= \frac{\sqrt{p}}{\sqrt{q}} + \frac{\sqrt{q}}{\sqrt{p}} + \frac{\sqrt{n}}{\sqrt{l}} = \frac{p+q}{\sqrt{p}\sqrt{q}} + \frac{\sqrt{n}}{\sqrt{l}} \\ &= \frac{-\frac{n}{l}}{\sqrt{pq}} + \frac{\sqrt{n}}{\sqrt{l}} = \frac{-\frac{n}{l}}{\frac{\sqrt{n}}{\sqrt{l}}} + \frac{\sqrt{n}}{\sqrt{l}} \\ &= \frac{-\frac{n}{l} + \frac{n}{l}}{\frac{\sqrt{n}}{\sqrt{l}}} = 0 = R.H.S. \end{aligned}$$

10)

If the equations $x^2 + px + q = 0$ and $x^2 + p'x + q' = 0$ have a common root, show that it must be equal to $\frac{pq' - p'q}{q - q'}$ or $\frac{q - q'}{p' - p}$.

Soln. Let α be common roots of $x^2 + px + q = 0$ and $x^2 + p'x + q' = 0$

$$(i) \quad x^2 + px + q = 0 \quad \text{and} \quad x^2 + p'x + q' = 0$$

$$\frac{\alpha^2}{pq' - p'q} = \frac{\alpha}{q - q'} = \frac{1}{p' - p}$$

$$\alpha^2 = \frac{pq' - p'q}{p' - p}$$

$$\alpha = \frac{q - q'}{p' - p}$$

$$\alpha = \frac{\alpha^2}{\alpha} = \frac{\frac{pq' - p'q}{p' - p}}{\frac{q - q'}{p' - p}} = \frac{pq' - p'q}{q - q'}$$

Hence proved.

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- 11) Formulate into a mathematical problem to find a number such that when its cube root is added to it, the result is 6. (7)

Soln. Let the number be x
Its cube root be $\sqrt[3]{x}$.

$$x + \sqrt[3]{x} = 6$$

$$\sqrt[3]{x} = 6 - x$$

$$x^{1/3} = 6 - x \Rightarrow x = (6 - x)^3$$

$$x = 6^3 - x^3 + 3(x^2)6 - 3(6^2)x$$

$$216 - x^3 + 18x^2 - 108x - x = 0$$

$$-x^3 + 18x^2 - 109x + 216 = 0$$

$$x^3 - 18x^2 + 109x - 216 = 0$$

- 12) A 12 metre tall tree was broken into two parts. It was found that the height of the part which was left standing was the cube root of the length of the part that was cut away. Formulate this into a mathematical problem to find the height of the part which was cut away.

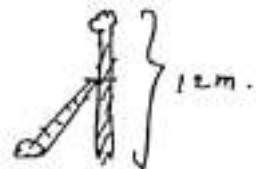
Soln Let the height of tree = 12.

$$\text{length of cut part} = x^3$$

$$\text{length of left out part} = \sqrt[3]{x^3} = x$$

$$x + x^3 = 12$$

$$x^3 + x - 12 = 0$$



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Exercise 3.2

(8)

1. If k is real, discuss the nature of the roots of the polynomial equation $2x^2 + kx + k = 0$ in terms of k

Soln

$$\Delta = b^2 - 4ac$$

$$a = 2 \quad b = k \quad c = k$$

$$\Delta = k^2 - 4 \cdot 2(k)$$

$$\Delta = k^2 - 8k$$

If $k < 0$, The polynomial has real roots ($\Delta > 0$)

If $\Delta = 0 \quad k^2 - 8k = 0$
 $k(k - 8) = 0$

$$k = 0 \text{ or } k = 8$$

$\therefore k = 0$ or $k = 8$ The roots are real & equal

If $0 < k < 8 \quad \Delta < 0$ The roots are imaginary

If $k > 8$ The roots are real and distinct.

- 2) Find a polynomial equation of minimum degree with rational coefficients having $2 + i\sqrt{3}$ as a root.

Soln

Let the roots be $2 + i\sqrt{3}$

another roots be $2 - i\sqrt{3}$.

$$\text{Sum of the roots} = 2 + i\sqrt{3} + 2 - i\sqrt{3} = 4$$

$$\text{Product of the roots} = (2 + i\sqrt{3})(2 - i\sqrt{3}) = 2^2 + (\sqrt{3})^2$$

$$= 4 + 3 = 7$$

$$x^2 - (SR)x + PR = 0$$

$$x^2 - 4x + 7 = 0$$

- 3) Find a polynomial equation of minimum degree with rational coefficients having $2i + 3$ as a root

Soln

Let the roots be $3 + 2i$

another root be $3 - 2i$

$$SR = 3 + 2i + 3 - 2i = 6$$

$$PR = (3 + 2i)(3 - 2i) = 3^2 + 2^2 = 9 + 4 = 13$$

$$x^2 - (SR)x + PR = 0$$

$$x^2 - 6x + 13 = 0$$

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4. Find a polynomial equation of minimum degree with rational coefficients having $\sqrt{5}-\sqrt{3}$ as a root. (9)

Soln Let the root be $\sqrt{5}-\sqrt{3}$
another root is $\sqrt{5}+\sqrt{3}$

$$SR = \sqrt{5}-\sqrt{3} + \sqrt{5}+\sqrt{3} = 2\sqrt{5}$$

$$PR = (\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3}) = \sqrt{5}^2 - \sqrt{3}^2 = 5-3=2$$

$$x^2 - (SR)x + PR = 0$$

$$x^2 - 2\sqrt{5}x + 2 = 0 \text{ which is not rational coefficient.}$$

To make rational coefficient.

$$(x^2 - 2\sqrt{5}x + 2)(x^2 + 2 + 2\sqrt{5}x) = 0$$

$$(x^2 + 2)^2 - (2\sqrt{5}x)^2 = 0$$

$$(2\sqrt{5})^2 = 4 \times 5 = 20$$

$$x^4 + 4 + 4x^2 - 20x^2 = 0$$

$$x^4 - 16x^2 + 4 = 0$$

is a rational coefficients polynomial equation.

- 5) Prove that a straight line and parabola cannot intersect at more than two points.

Soln, let the equation of st. line be $y = mx + c$

let the equation of parabola $y^2 = 4ax$

$$(mx+c)^2 = 4ax$$

$$m^2x^2 + 2mcx + c^2 - 4ax = 0$$

$$m^2x^2 + (2m - 4a)x + c^2 = 0$$

which is a quadratic equation.

\therefore This equation can't have more than two solutions.

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Exercise 3.3

(10)

1. Solve the cubic equation $2x^3 - x^2 - 18x + 9 = 0$ if sum of two of its roots vanishes

soln $2x^3 - x^2 - 18x + 9 = 0$

let the roots be α, β, γ .

Given $\alpha + \beta = 0$.

$\alpha + \beta + \gamma = \frac{1}{2}$

$\gamma = \frac{1}{2}$

$\gamma = \frac{1}{2}$

$\alpha + \beta = 0 \quad x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$x^2 - 0x + p = 0$

$(x^2 + p) = 0$

$(x - \frac{1}{2}) = 0$
 $2x - 1 = 0$

$\therefore (x^2 + p)(x - \frac{1}{2}) = 2x^3 - x^2 - 18x + 9$

$2x^3 - x^2 + 2px - p = 2x^3 - x^2 - 18x + 9$

$-p = 9$

$p = -9$

$x^2 - 9 = 0$

$x^2 = 9$

$x = \pm 3$

Roots are 3, -3, $\frac{1}{2}$.

2. Solve the equation $9x^3 - 36x^2 + 44x - 16 = 0$ if the roots form an arithmetic progression.

soln. let the roots are in A.P

let us take $a-d, a, a+d$.

Sum of the roots $= a-d + a + a+d = \frac{36}{9} = 4$

$3a = 4$

$a = \frac{4}{3}$

Product of the roots $= (a-d) a (a+d) = (a^2 - d^2) a = \frac{16}{9}$

$(\frac{4}{3}^2 - d^2) (\frac{4}{3}) = \frac{16}{9}$

$\frac{16}{9} - d^2 = \frac{4}{3}$

$\frac{16}{9} - \frac{4}{3} = d^2$

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- 4) Determine k and solve the equation $2x^3 - 6x^2 + 3x + k = 0$ if one of its roots is twice the sum of the other two roots. (12)

Soln. $2x^3 - 6x^2 + 3x + k = 0$ let the roots be α, β, γ

Given $\alpha = 2(\beta + \gamma)$

$\alpha + \beta + \gamma = 3$ $\alpha + \beta + \gamma = \frac{3}{2}$ $\alpha + \beta + \gamma = -\frac{k}{2}$ (3)

① $\therefore 2(\beta + \gamma) + (\beta + \gamma) = 3$

$3(\beta + \gamma) = 3$

$(\beta + \gamma) = 1$

$\therefore \alpha = 2(\beta + \gamma) = 2(1) = 2$

$\gamma = 1 - \beta = 1 - \left(\frac{1 + \sqrt{3}}{2}\right) = \frac{2 - 1 - \sqrt{3}}{2} = \frac{1 - \sqrt{3}}{2}$

⑤ $\Rightarrow \alpha + \beta + \gamma = -\frac{k}{2}$
 $2 + \beta + \left(\frac{1 - \sqrt{3}}{2}\right) = -\frac{k}{2}$
 $4 + 2\beta + 1 - \sqrt{3} = -k$
 $k = -4$

② $\Rightarrow 2\beta + \beta\left(\frac{1}{\beta}\right) + \frac{1}{\beta}(2) = \frac{3}{2}$
 $2\beta + 1 + \frac{2}{\beta} = \frac{3}{2}$

③ $\Rightarrow 2\beta + \beta(1 - \beta) + (1 - \beta)2 = \frac{3}{2}$

$2\beta + \beta - \beta^2 + 2 - 2\beta = \frac{3}{2}$

$-\beta^2 + \beta + 2 = \frac{3}{2}$

$-2\beta^2 + 2\beta + 4 - 3 = 0$

$2\beta^2 - 2\beta - 1 = 0$

$\beta = \frac{2 \pm \sqrt{4 + 8}}{4} = \frac{2 \pm \sqrt{12}}{4} = \frac{2 \pm 2\sqrt{3}}{4}$

$\beta = \frac{1 \pm \sqrt{3}}{2}$

$\therefore \gamma = \frac{1 \pm \sqrt{3}}{2}$

④ $\alpha + \beta + \gamma = -\frac{k}{2}$

$2 + \left(\frac{1 + \sqrt{3}}{2}\right) + \left(\frac{1 - \sqrt{3}}{2}\right) = -\frac{k}{2}$
 $2 + 1 = -\frac{k}{2} \Rightarrow k = -4$

- 5) Find all zeros of the polynomial $x^6 - 3x^5 - 5x^4 + 22x^3 - 39x^2 + 135x - 135$, if it is known that $1 + 2i$ and $\sqrt{3}$ are two of its zeros.

Soln Given roots $1 + 2i, \sqrt{3}$

i) Another roots be $1 - 2i, -\sqrt{3}$

SR of roots $= 1 + 2i + 1 - 2i = 2$

PR of roots $= (1 + 2i)(1 - 2i) = 1^2 + 2^2 = 1 + 4 = 5$

$x^2 - 2x + 5 = 0$

ii) SR of roots $= \sqrt{3} - \sqrt{3} = 0$

PR of roots $= (\sqrt{3})(-\sqrt{3}) = -3$

$x^2 - 0x - 3 = 0$

$x^2 - 3 = 0$

$$(x^2 - 2x + 5)(x^2 - 3) = x^4 - 2x^3 + 2x^2 + 6x - 15 \quad (13)$$

$$x^6 - 3x^5 - 5x^4 + 22x^3 - 39x^2 - 39x + 135 = 0$$

$$= (x^4 - 2x^3 + 2x^2 + 6x - 15)(x^2 + px - 9)$$

Equate coefficient of x on both sides.

$$-39 = -54 - 15p$$

$$-39 + 54 = -15p$$

$$15 = -15p$$

$$\boxed{p = -1}$$

$$\therefore x^2 - x - 9 = 0$$

$$x = \frac{1 \pm \sqrt{1 - 4 \cdot 1 \cdot (-9)}}{2 \cdot 1} = \frac{1 \pm \sqrt{1+36}}{2} = \frac{1 \pm \sqrt{37}}{2}$$

Roots are $\frac{1+\sqrt{37}}{2}, \frac{1-\sqrt{37}}{2}, 1+2i, 1-2i, \sqrt{3}, -\sqrt{3}$.

6) Solve the cubic equations

i) $2x^3 - 9x^2 + 10x = 3$

$$2x^3 - 9x^2 + 10x - 3 = 0$$

Sum of the coefficients $2 - 9 + 10 - 3 = 0$

$\therefore x = 1$ is one of the roots

$$\begin{array}{r|rrrr} 1 & 2 & -9 & 10 & -3 \\ & 0 & 2 & -7 & 3 \\ \hline & 2 & -7 & 3 & 0 \end{array}$$

$$2x^2 - 7x + 3 = 0$$

$$(x-3)(2x-1) = 0$$

$$x = 3 \text{ or } x = \frac{1}{2}$$

Roots are $3, \frac{1}{2}, 1$.

ii) Solve the equation $8x^3 - 2x^2 - 7x + 3 = 0$.

Sum of alternative coefficients are equal

$$8 - 7 = -2 + 3$$

$$1 = 1$$

$\therefore x+1$ is a root.

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$$\begin{array}{r|rrrr} -1 & 8 & -2 & -7 & 3 \\ & 0 & -8 & 10 & -3 \\ \hline & 8 & -10 & 3 & 0 \end{array}$$

$$8x^2 - 10x + 3 = 0$$

$$(4x-3)(2x-1) = 0$$

$$4x = 3$$

$$x = 3/4$$

$$2x = 1$$

$$x = 1/2$$

Roots are $3/4, 1/2, -1$.

7) Solve the equation: $x^4 - 14x^2 + 45 = 0$

Sol put $t = x^2$

$$t^2 - 14t + 45 = 0$$

$$(t-9)(t-5) = 0$$

$$t-9 = 0$$

$$t = 9$$

$$x^2 = 9$$

$$x = \pm 3$$

$$t-5 = 0$$

$$t = 5$$

$$x^2 = 5$$

$$x = \pm \sqrt{5}$$

Roots are $3, -3, \sqrt{5}, -\sqrt{5}$.

(14)

$$\begin{array}{r} 24 \\ \times 2 \\ \hline 48 \end{array}$$

$$\begin{array}{r} 45 \\ \times 5 \\ \hline 225 \end{array}$$

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பாடசாலை

Exercise 3.4

(15)

1. Solve (i) $(x-5)(x-7)(x+6)(x+4) = 504$.

Soln $(x-5)(x+4)(x-7)(x+6) - 504 = 0$

$$(x^2 - x - 20)(x^2 - x - 42) - 504 = 0$$

put $y = x^2 - x$.

$$(y-20)(y-42) - 504 = 0$$

$$y^2 - 42y - 20y + 840 - 504 = 0$$

$$y^2 - 62y + 336 = 0$$

$$(y-56)(y-6) = 0$$

$$y = 56 \quad y = 6$$

$$x^2 - x = 56 \quad \text{or} \quad x^2 - x = 6$$

$$x^2 - x - 56 = 0$$

$$(x+8)(x-7) = 0$$

$$x = 8, -7$$

$$(x+2)(x-3) = 0$$

$$x = -2, 3$$

Roots are $-2, 3, 8$ and -7 .

2) $(x-4)(x-7)(x-2)(x+1) = 16$.

Soln $(x-4)(x-2)(x-7)(x+1) - 16 = 0$

$$(x^2 - 6x + 8)(x^2 - 6x - 7) - 16 = 0$$

put $y = x^2 - 6x$

$$(y+8)(y-7) - 16 = 0$$

$$y^2 - 7y + 8y - 56 - 16 = 0$$

$$y^2 + y - 72 = 0$$

$$(y+9)(y-8) = 0$$

$$y = -9 \quad y = 8$$

$$x^2 - 6x = -9$$

$$x^2 - 6x + 9 = 0$$

$$(x-3)^2 = 0$$

$$x = 3, 3$$

$$x^2 - 6x = 8$$

$$x^2 - 6x - 8 = 0$$

$$x = \frac{6 \pm \sqrt{36 - 4 \cdot 1 \cdot (-8)}}{2 \cdot 1} = \frac{6 \pm \sqrt{36 + 32}}{2}$$

$$= \frac{6 \pm \sqrt{68}}{2} = \frac{6 \pm 2\sqrt{17}}{2}$$

$$= \cancel{\frac{6}{2}} \left(\frac{3 \pm \sqrt{17}}{1} \right) = 3 \pm \sqrt{17}$$

Roots are $3, 3, 3 \pm \sqrt{17}$.

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2. solve : $(2x-1)(x+3)(x-2)(2x+3)+20=0$. (16)

Solve

$$(x+3)(x-2)(2x-1)(2x+3)+20=0$$

$$(x^2+x-6)(4x^2+6x-2x-3)+20=0$$

$$(x^2+x-6)(4x^2+4x-3)+20=0$$

$$4(x^2+x-6)(x^2+x-\frac{3}{4})+20=0$$

$$(\div 4) (x^2+x-6)(x^2+x-\frac{3}{4})+\frac{20}{4}=0$$

Put $y = x^2+x$.

$$(y-6)(y-\frac{3}{4})+\frac{20}{4}=0$$

$$y^2-\frac{3}{4}y-6y+\frac{9}{2}+\frac{20}{4}=0$$

$$y^2-y(\frac{27}{4})+\frac{38}{4}=0$$

(x4)

$$4y^2-27y+38=0$$

$$4y^2-8y-19y+38=0$$

$$(4y-19)(y-2)=0$$

$$y=2 \quad y=\frac{19}{4}$$

$$x^2+x=2 \quad x^2+x=\frac{19}{4}$$

$$x^2+x-2=0$$

$$4x^2+4x-19=0$$

$$(x+2)(x-1)=0$$

$$4x^2+4x-19=0$$

$$x=-2 \quad x=1$$

$$x = \frac{-4 \pm \sqrt{16-4 \cdot 1 \cdot (-19)}}{2 \cdot 4}$$

$$= \frac{-4 \pm \sqrt{16+76}}{16}$$

$$= \frac{-4 \pm \sqrt{92}}{8} = \frac{-4 \pm 2\sqrt{23}}{8}$$

$$= \boxed{\frac{-2 \pm \sqrt{23}}{4}}$$

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Exercise 3.5

(17)

1. Solve the following equation.

(i) $\sin^2 x - 5 \sin x + 4 = 0$

put $\sin x = t$

$t^2 - 5t + 4 = 0$

$(t-4)(t-1) = 0$

$t = 4 \quad \vee \quad t = 1$

$\sin x = 4$

(is not possible)

$\sin x = 1$

$\sin x = \sin \frac{\pi}{2}$

$x = 2n\pi + \frac{\pi}{2} \quad \forall n \in \mathbb{Z}$

2) $12x^3 + 8x = 29x^2 - 4$

$12x^3 - 29x^2 + 8x - 4 = 0$

$$\begin{array}{r|rrrr} 2 & 12 & -29 & 8 & -4 \\ & 0 & 24 & -10 & -4 \\ \hline & 12 & -5 & -2 & 0 \end{array}$$

$12x^2 - 5x - 2 = 0$

$12x^2 - 8x + 3x - 2 = 0$

$4x(3x-2) + 1(3x-2) = 0$

$(3x-2)(4x+1) = 0$

$3x = 2$

$x = \frac{2}{3}$

$4x = -1$

$x = -\frac{1}{4}$

Roots are $\frac{2}{3}, 2, -\frac{1}{4}$

3) Examine for the rational roots.

$2x^3 - x^2 - 1 = 0$

Soln

Sum of the coeffts

 $x = 1$ is a root

$2 - 1 - 1 = 0$

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$$1 \mid \begin{array}{ccc|c} 2 & -1 & 0 & -1 \\ 0 & 2 & 1 & 1 \\ \hline 2 & 1 & 1 & 0 \end{array}$$

(18)

$$2x^2 + x + 1 = 0$$

$$x = \frac{-1 \pm \sqrt{1 - 4 \cdot 2 \cdot 1}}{2 \cdot 2} = \frac{-1 \pm \sqrt{1-8}}{2} = \frac{-1 \pm i\sqrt{7}}{2}$$

which is imaginary roots

$\therefore x=1$ is rational roots.

(ii)

$$x^3 - 3x + 1 = 0.$$

$$\frac{p_0}{q} \mid n \quad a_n = 1 \quad a_0 = 1.$$

If $\frac{p}{q}$ is a root of the polynomial. (P.E) by rational root theorem.

it have no rational roots.

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3) Solve $8x^{3/2n} - 8x^{-3/2n} = 63$.

put $k = \frac{3}{2n} \therefore 8x^k - 8x^{-k} = 63$

$$8x^k - 8/x^k = 63$$

$$8x^{2k} - 8 = 63x^k$$

$$8x^{2k} - 63x^k - 8 = 0$$

$$(x^k - 8)(8x^k + 1) = 0.$$

$$x^k - 8 = 0$$

$$x^k = 8$$

$$x^{3/2n} = 8$$

$$x = 8^{2n/3}$$

$$= (2^3)^{2n/3} = (2^2)^n$$

$$x = 4^n.$$

$\therefore x = 4^n$ is a root of the equation.

个。

Ans $\frac{b^2}{49}$, $\frac{99^3}{b^2}$

$$\begin{array}{r} 1500 \\ 25 \overline{) 1250} \\ \underline{50} \\ 100 \\ \underline{50} \\ 50 \\ \underline{50} \\ 0 \end{array}$$

$$\begin{array}{r} 150 \\ - 15 \\ \hline 135 \end{array} \quad \begin{array}{r} 20 \\ - 20 \\ \hline 0 \end{array}$$

(20)

$$y = 5/2$$

$$y = \frac{10}{3}$$

$$x + \frac{1}{x} = 5/2$$

$$x + 1/x = \frac{10}{3}$$

$$\frac{x^2 + 1}{x} = 5/2$$

$$\frac{x^2 + 1}{x} = \frac{10}{3}$$

$$2x^2 + 2 - 5x = 0$$

$$3x^2 + 3 = 10x$$

$$2x^2 - 5x + 2 = 0$$

$$3x^2 - 10x + 3 = 0$$

$$x = 2, 1/2$$

$$x = 3, 1/3$$

Roots are $2, 1/2, 3, 1/3$.

(10)

$$x^4 + 3x^3 - 3x - 1 = 0$$

Sol This equation is Type I even degree reciprocal equation. Hence it can be rewritten as

$$\left(x^2 + \frac{1}{x^2}\right) + 3\left(x + \frac{1}{x}\right) + 0 = 0. \quad \text{--- (6)}$$

$$y = x + \frac{1}{x}$$

$$\Rightarrow y^2 = x^2 + \frac{1}{x^2} + 2$$

$$y^2 + 2 = x^2 + \frac{1}{x^2}$$

(7)

$$(y^2 + 2) + 3y = 0$$

$$y^2 + 3y + 2 = 0$$

$$(y + 2)(y + 1) = 0$$

$$y = -2, y = -1.$$

$$x + \frac{1}{x} = -2$$

$$x + \frac{1}{x} = -1$$

$$\frac{x^2 - 1}{x} = -2$$

$$\frac{x^2 - 1}{x} = -1$$

$$x^2 - 1 = -2x$$

$$x^2 - 1 = -x$$

$$x^2 + 2x + 1 = 0$$

$$x^2 + x + 1 = 0$$

$$(x + 1)^2 = 0$$

$$x = \frac{-1 \pm \sqrt{1 - 4 \cdot 1 \cdot (-1)}}{2 \cdot 1} = \frac{-1 \pm \sqrt{5}}{2}$$

$$x = -1, -1$$

$$= \frac{-1 \pm \sqrt{5}}{2}$$

Roots are $-1, -1, \frac{-1 \pm \sqrt{5}}{2}$.

7)

Solve the equation $6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0$ if it is known that $\frac{1}{3}$ is a solution. (21)

Soln $6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0$

$$6\left(x^2 + \frac{1}{x^2}\right) - 5\left(x + \frac{1}{x}\right) - 38 = 0$$

$$x + \frac{1}{x} = y ; y^2 - 2 = x^2 + \frac{1}{x^2}$$

$$6(y^2 - 2) - 5y - 38 = 0$$

$$6y^2 - 12 - 5y - 38 = 0$$

$$6y^2 - 5y - 50 = 0$$

$$6y^2 - 20y + 15y - 50 = 0$$

$$2y(3y - 10) + 5(3y - 10) = 0$$

$$(3y - 10)(2y + 5) = 0$$

$$3y = 10$$

$$2y = -5$$

$$y = \frac{10}{3}$$

$$y = -\frac{5}{2}$$

$$x + \frac{1}{x} = \frac{10}{3}$$

$$\frac{x^2 + 1}{x} = \frac{10}{3}$$

$$3x^2 + 3 = 10x$$

$$3x^2 - 10x + 3 = 0$$

$$(x - 3)(3x - 1) = 0$$

$$x = 3$$

$$x = \frac{1}{3}$$

$$x + \frac{1}{x} = -\frac{5}{2}$$

$$\frac{x^2 + 1}{x} = -\frac{5}{2}$$

$$2x^2 + 2 = -5x$$

$$2x^2 - 5x + 2 = 0$$

$$(x - 2)\left(x - \frac{1}{2}\right) = 0$$

$$x = 2, x = \frac{1}{2}$$

Roots are $3, \frac{1}{3}, 2, \frac{1}{2}$

b) Find all real numbers satisfying

$$4^x - 3(2^{x+2}) + 2^5 = 0$$

$$(2^2)^x - 3(2^x \cdot 2^2) + 2^5 = 0$$

$$(2^x)^2 - 12(2^x) + 2^5 = 0$$

Put $2^x = t$ $t^2 - 12t + 32 = 0$

$$(t - 8)(t - 4) = 0$$

$$t = 8$$

$$t = 4$$

$$2^x = 8 = 2^3$$

$$x = 3$$

$$2^x = 4 = (2^2)^2$$

$$x = \pm 2$$

Exercise 3.6

1. Discuss the maximum possible number of positive and negative roots of the polynomial equation
 $9x^9 - 4x^8 + 4x^7 - 3x^6 + 2x^5 + x^3 + 7x^2 + 7x + 2 = 0$.

Soln. clearly there are 4 sign changes for the given polynomial $P(x)$. and hence number of positive roots of $P(x)$ can't be more than four.
 $P(-x) = -9x^9 - 4x^8 - 4x^7 - 3x^6 - 2x^5 - x^3 + 7x^2 - 7x + 2 = 0$
 There is two sign changes. hence the number of negative roots can't be more than two.
 \therefore it has at most 4 positive roots and at most two negative roots.

2. Discuss the maximum possible number of positive and negative roots of the polynomial equations $x^2 - 5x + 6$ and $x^2 - 5x + 16$. Also draw rough sketch of the graphs.

$$P(x) = (x^2 - 5x + 6)(x^2 - 5x + 16)$$

$$= x^4 - 5x^3 + 16x^2 - 5x^3 + 25x^2 - 80x + 6x^2 - 30x + 96 = 0$$

$$x^4 - 10x^3 + 57x^2 - 110x + 96 = 0$$

~~It~~ it has two sign change
 \therefore it has two +ve real roots

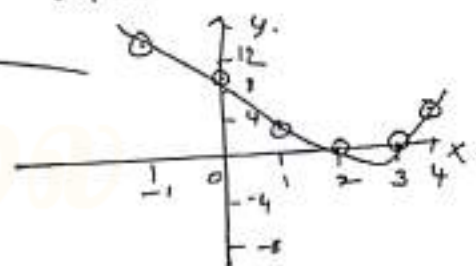
$$P(-x) = x^4 + 10x^3 + 57x^2 + 110x + 96$$

it has no sign change.
 no -ve real roots.

$$y = x^2 - 5x + 6$$

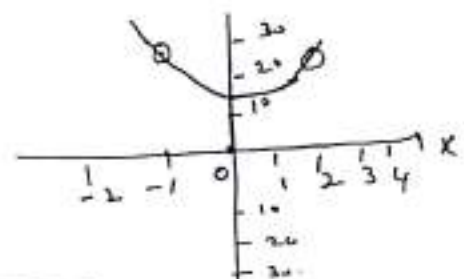
$x=1$	$y = 1 - 5 + 6 = 2$
$x=2$	$y = 4 - 10 + 6 = 0$
$x=0$	$y = 6$
$x=3$	$y = 9 - 15 + 6 = 0$
$x=-1$	$y = 1 + 5 + 6 = 12$
$x=4$	$y = 16 - 20 + 6 = 2$

$(1, 2), (0, 6), (-1, 12)$



$$y = x^2 - 5x + 16$$

x	0	1	-1	2	4
y	16	12	23	16	.



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3. Show that the equation $x^9 - 5x^5 + 4x^4 + 2x^2 + 1 = 0$ (23)
has at least 6 imaginary solutions.

Soln $P(x)$ has only one sign change

it has at most one +ve roots

$$P(-x) = -x^9 + 5x^5 + 4x^4 + 2x^2 + 1 = 0$$

it has only one sign changes

it has at most one -ve roots

Clearly 0 is not a roots.

So maximum number of real roots is 3

and hence there are at least six imaginary solutions

4. Determine the number of positive & negative roots of
the equation $x^9 - 5x^8 - 14x^7 = 0$

Soln $P(x) = x^9 - 5x^8 - 14x^7$. it has only one sign change

\therefore it has one +ve roots.

$$P(-x) = -x^9 - 5x^8 + 14x^7$$

it has one -ve roots.

\therefore It has one +ve & one -ve roots.

5. Find the exact number of real roots & imaginary
of the equation $x^9 + 9x^7 + 7x^5 + 5x^3 + 3x$.

Soln $P(x) = x^9 + 9x^7 + 7x^5 + 5x^3 + 3x$.

it has no sign change. So it has no +ve
real roots.

$$P(-x) = -x^9 - 9x^7 - 7x^5 - 5x^3 - 3x$$

it has no sign changes.

So it has no -ve real roots.

it has no positive real roots and no
negative real roots.

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Exercise 3.7

(24)

1. Choose the most suitable answer

1. A zero of $x^3 + 64$ is

$$x^3 = -64$$

$$x^3 = -4 \times -4 \times -4 = (-4)^3$$

$$x = -4$$

Ans (4) -4

2. If f and g are polynomials of degree m and n respectively and if $h(x) = (f \circ g)(x)$ then the degree of h issoln

For example $f(x) = x^m$ $g(x) = x^n$
 degree = m degree = n

$$(f \circ g)(x) = f(g(x)) = f(x^n) = (x^n)^m = x^{mn}$$

$$\text{degree} = mn$$

Ans (1) mn .3. A polynomial equation in x of degree n always has
Ans (3) n imaginary roots.4) If α, β and γ are the roots $x^3 + px^2 + qx + r$
then $\sum \frac{1}{\alpha}$ is

$$\alpha + \beta + \gamma + 8 = -p, \quad \alpha\beta + \beta\gamma + \gamma\alpha = q, \quad \alpha\beta\gamma = -r$$

$$\alpha\beta\gamma = -r$$

$$\sum \frac{1}{\alpha} = \frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\gamma} = \frac{q}{-r}$$

Ans (1) $-\frac{q}{r}$.

(5)

According to the rational root theorem which number is not possible rational root of $4x^7 + 2x^4 - 10x^3 - 5$?

By rational root theorem.

 $\frac{p}{q}$ is a root of a polynomial $a_0 = -5$ $a_n = 4$ $(4, -5)$ p must divide 5 & q must divide 4.Possible values of p are $+1, -1, +5, -5$.Possible values of q are $+1, -1, -5/4, 4/5$ $5/4$ is not possible rational rootsAns (2) $5/4$.

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- 4) The polynomial $x^3 - kx^2 + 9x$ has three real roots
iff k satisfies

$$D \geq 0 \quad k^2 - 4 \cdot 1 \cdot 9 \geq 0$$

$$k^2 \geq 36$$

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$$|k| \geq 6$$

Ans. (4) $|k| \geq 6$

- 7) The number of real number of $[0, 2\pi]$ satisfying
 $\sin^4 x - 2\sin^2 x + 1 = 0$
put $\sin^2 x = t$

$$t^2 - 2t + 1 = 0 \Rightarrow (t-1)^2 = 0$$

$$t = 1, \quad t = 1$$

$$\sin^2 x = 1$$

$$\sin x = \pm 1$$

$$\therefore \sin x = 1$$

$$\sin x = \sin \frac{\pi}{2}$$

$$x = \frac{\pi}{2}$$

$$\sin x = -1$$

$$\sin x = \sin(\pi + \frac{\pi}{2})$$

$$x = \frac{3\pi}{2}$$

Number of real numbers in $[0, 2\pi]$ is 2.

- 9) If $x^3 + 12x^2 + 10ax + 1999$ definitely has a +ve roots if
if it is a +ve root need to minimum one sign change
 $\therefore a \leq 0$ Then only we can get one sign change

Ans (3) $a \leq 0$

- 10) The polynomial $x^3 + 2x + 3$ has.
 $P(x) = x^3 + 2x + 3$, no sign changes \Rightarrow no +ve real roots
 $P(-x) = -x^3 - 2x + 3$, one sign change \Rightarrow one -ve real root
 \therefore it has one negative & two imaginary roots

- 11) The number of +ve roots of the polynomial
 $\sum_{j=0}^n n C_j (-1)^j x^j$ is
 $\sum_{j=0}^n n C_j (-1)^j x^j = x^n - n C_1 x^{n-1} + n C_2 x^{n-2} - \dots + (-1)^n$
 $P(x)$ has 'n' changes \therefore it has n +ve roots
 $P(-x)$ has no changes
 \therefore No -ve roots.

Ans (2) n

CHAPTER: 4 INVERSE TRIGONOMETRIC FUNCTIONS

Inverse function	$\sin^{-1}x$	$\cos^{-1}x$	$\tan^{-1}x$	$\operatorname{cosec}^{-1}x$	$\sec^{-1}x$	$\cot^{-1}x$
Domain	$[-1, 1]$	$[-1, 1]$	\mathbb{R}	$[-1, -1] \cup [1, \infty)$	$(-\infty, -1] \cup [1, \infty)$	\mathbb{R}
Range	$[-\frac{\pi}{2}, \frac{\pi}{2}]$	$[0, \pi]$	$(-\frac{\pi}{2}, \frac{\pi}{2})$	$[-\frac{\pi}{2}, \frac{\pi}{2}] - \{0\}$	$(0, \pi) - \{\frac{\pi}{2}\}$	$(0, \pi)$

2. Property I
- (i) $\sin^{-1}(\sin \theta) = \theta$ if $\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ (ii) $\cos^{-1}(\cos \theta) = \theta$ if $\theta \in [0, \pi]$
 (iii) $\tan^{-1}(\tan \theta) = \theta$ if $\theta \in (-\frac{\pi}{2}, \frac{\pi}{2})$ (iv) $\operatorname{cosec}^{-1}(\operatorname{cosec} \theta) = \theta$ if $\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}] - \{0\}$
 (v) $\sec^{-1}(\sec \theta) = \theta$ if $\theta \in [0, \pi] - \{\frac{\pi}{2}\}$
 (vi) $\cot^{-1}(\cot \theta) = \theta$ if $\theta \in (0, \pi)$

3. Property II
- (i) $\sin(\sin^{-1}x) = x$ if $x \in [-1, 1]$ (ii) $\cos(\cos^{-1}x) = x$ if $x \in [-1, 1]$
 (iii) $\tan(\tan^{-1}x) = x$ if $x \in \mathbb{R}$ (iv) $\operatorname{cosec}(\operatorname{cosec}^{-1}x) = x$ if $x \in \mathbb{R} - \{0\}$
 (v) $\sec(\sec^{-1}x) = x$ if $x \in \mathbb{R} - \{0\}$ (vi) $\cot(\cot^{-1}x) = x$ if $x \in \mathbb{R}$

4. Property III
- (i) $\sin^{-1}(\frac{1}{x}) = \operatorname{cosec}^{-1}x$ if $x \in \mathbb{R} - \{0\}$ (ii) $\cos^{-1}(\frac{1}{x}) = \sec^{-1}x$ if $x \in \mathbb{R} - \{0\}$

5. Property IV (Reflection identities)
- (i) $\sin^{-1}(-x) = -\sin^{-1}x$ if $x \in [-1, 1]$
 (ii) $\tan^{-1}(-x) = -\tan^{-1}x$ if $x \in \mathbb{R}$
 (iii) $\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1}x$ if $|x| \geq 1$ or $x \in \mathbb{R} - \{0\}$
 (iv) $\cos^{-1}(-x) = \pi - \cos^{-1}x$ if $x \in [-1, 1]$
 (v) $\sec^{-1}(-x) = \pi - \sec^{-1}x$ if $|x| \geq 1$ or $x \in \mathbb{R} - \{0\}$
 (vi) $\cot^{-1}(-x) = \pi - \cot^{-1}x$ if $x \in \mathbb{R}$

6. Property V (Complementary inverse identities)
- (i) $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$, $x \in [-1, 1]$ (ii) $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$, $x \in \mathbb{R}$
 (iii) $\operatorname{cosec}^{-1}x + \sec^{-1}x = \frac{\pi}{2}$, $x \in \mathbb{R} - \{0\}$ or $|x| \geq 1$

7. Property VI

$$\sin^{-1}x + \sin^{-1}y = \sin^{-1}(x\sqrt{1-y^2} + y\sqrt{1-x^2})$$

where either $x^2 + y^2 \leq 1$ or $xy < 0$.

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$$i) \sin^{-1}x + \sin^{-1}y = \sin^{-1}[x\sqrt{1-y^2} + y\sqrt{1-x^2}] \text{ where } x^2 + y^2 \leq 1 \text{ or } xy > 0$$

$$ii) \cos^{-1}x + \cos^{-1}y = \cos^{-1}[xy - \sqrt{1-x^2}\sqrt{1-y^2}] \text{ if } x+y \geq 0$$

$$iv) \cos^{-1}x - \cos^{-1}y = \cos^{-1}[xy + \sqrt{1-x^2}\sqrt{1-y^2}] \text{ if } x > y$$

$$v) \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right) \text{ if } xy < 1$$

$$vi) \tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right) \text{ if } xy > -1$$

Property VII

$$i) 2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right), |x| < 1$$

$$ii) 2\tan^{-1}x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), x \geq 0$$

$$iii) 2\tan^{-1}x = \sin^{-1}\left(\frac{2x}{1+x^2}\right), |x| \leq 1$$

Property VIII

$$i) \sin^{-1}(2x\sqrt{1-x^2}) = 2\sin^{-1}x \text{ if } |x| \leq \frac{1}{\sqrt{2}} \text{ or } -\frac{1}{\sqrt{2}} \leq x \leq \frac{1}{\sqrt{2}}$$

$$ii) \sin^{-1}(2x\sqrt{1-x^2}) = 2\cos^{-1}x \text{ if } \frac{1}{\sqrt{2}} \leq x \leq 1$$

Property IX

$$i) \sin^{-1}x = \cos^{-1}\sqrt{1-x^2} \text{ if } 0 \leq x \leq 1 \quad ii) \sin^{-1}x = -\cos^{-1}\sqrt{1-x^2} \text{ if } -1 \leq x \leq 0$$

$$iii) \sin^{-1}x = \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right) \text{ if } -1 < x < 1 \quad iv) \cos^{-1}x = \sin^{-1}\sqrt{1-x^2} \text{ if } 0 \leq x \leq 1$$

$$v) \cos^{-1}x = \pi - \sin^{-1}\sqrt{1-x^2} \text{ if } -1 \leq x < 0 \quad vi) \tan^{-1}x = \sin^{-1}\frac{x}{\sqrt{1+x^2}} = \cos^{-1}\frac{1}{\sqrt{1+x^2}} \text{ if } x > 0$$

Property X

$$i) 3\sin^{-1}x = \sin^{-1}(3x - 4x^3), x \in [-\frac{1}{2}, \frac{1}{2}]$$

$$ii) 3\cos^{-1}x = \cos^{-1}(4x^3 - 3x), x \in [\frac{1}{2}, 1]$$

Trigonometric function	Sine	Cosine	tangent	Cosecant	secant	Cotangent
Domain	$[-\frac{\pi}{2}, \frac{\pi}{2}]$	$[0, \pi]$	$(-\frac{\pi}{2}, \frac{\pi}{2})$	$[-\frac{\pi}{2}, \frac{\pi}{2}] \setminus \{0\}$	$[0, \pi] \setminus \{\frac{\pi}{2}\}$	$(0, \pi)$
Range	$[-1, 1]$	$[-1, 1]$	\mathbb{R}	$\mathbb{R} \setminus (-1, 1)$	$\mathbb{R} \setminus (-1, 1)$	\mathbb{R}

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