10 Operations Research

Methods of finding initial Basic Feasible Solutions

For finding the initial basic feasible solution total supply must be equal to total demand. i.e $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$

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Origins \ Destinations	1	2	3	...	n	supply
1	(x_{11}) c_{11}	(x_{12}) c_{12}	(x_{13}) c_{13}	...	(x_{1n}) c_{1n}	a_1
2	(x_{21}) c_{21}	(x_{22}) c_{22}	(x_{23}) c_{23}	...	(x_{2n}) c_{2n}	a_2
⋮	⋮	⋮	⋮	⋮	⋮	⋮
m	(x_{m1}) c_{m1}	(x_{m2}) c_{m2}	(x_{m3}) c_{m3}	...	(x_{mn}) c_{mn}	a_m
demand	b_1	b_2	b_3	...	b_n	

Table 10.1

Method: 1 North west corner Rule (NWC)

Step 1: Choose the cell in the north-west corner of the transportation Table 10.1 and allocate as much as possible in this cell so that either the capacity of first row (supply) is exhausted or the destination requirement of the first column (demand) is exhausted.

i.e $x_{11} = \min(a_1, b_1)$

Step 2: * If the demand is exhausted ($b_1 < a_1$) move one cell right horizontally to the second column and allocate as much as possible (i.e) $x_{12} = \min(a_1 - x_{11}, b_2)$

* If the supply is exhausted ($b_1 > a_1$) move one cell down vertically to the second row and allocate as much as possible. (i.e) $x_{21} = \min(a_2, b_1 - x_{11})$.

* If both supply and demand are exhausted move one cell diagonally and allocate as much as possible.

Step 3: Continue the above procedure until all the allocations are made.

Method: 2 Least Cost Method (LCM)

Step 1: Find the cell with the least (minimum) cost in the transportation table.

Step 2: Allocate the maximum feasible quantity to this cell.

Step 3: Eliminate the row or column where an allocation is made

Step 4: Repeat the above steps for the reduced transportation table ~~wait~~ until all the allocations are made.

Method: 3 Vogel's Approximation Method (VAM)

Step1: Calculate the penalties for each row and each column. Here penalty means the difference between the two successive least cost in a row and in a column.

Step2: Select the row or column with the largest penalty.

Step3: In the selected row or column, allocate the maximum feasible quantity to the cell with the minimum cost.

Step4: Eliminate the row or column where all the allocations are made.

Step5: Write the reduced transportation table and repeat the steps 1 to 4.

Step6: Repeat the procedure unit all the allocations are made.

Ex 10.1

15 Find an initial basic feasible solution of the following problem using north west corner rule

	D_1	D_2	D_3	D_4	Supply
O_1	5	3	6	2	19
O_2	4	7	9	1	37
O_3	3	4	7	5	34
Demand	16	18	31	25	

Soln

Total supply = Total Demand

∴ The given problem is balanced transportation problem.

∴ there exists a feasible solution to the given problem.

Given Transportation problem is:

	D_1	D_2	D_3	D_4	supply (a_i)
O_1	5	3	6	2	19
O_2	4	7	9	1	37
O_3	3	4	7	5	34
Demand (b_j)	16	18	31	25	

First allocation

	D_1	D_2	D_3	D_4	a_i
O_1	(16) 5	3	6	2	3
O_2	4	7	9	1	37
O_3	3	4	7	5	34
b_j	0	18	31	25	

Second allocation

	D_1	D_2	D_3	D_4	a_i
O_1	(16) 5	(3) 3	6	2	0
O_2	4	7	9	1	37
O_3	3	4	7	5	34
b_j	0	15	31	25	

Third allocation

	D_1	D_2	D_3	D_4	a_i
O_1	(16) 5	(3) 3	6	2	0
O_2	4	(15) 7	9	1	22
O_3	3	4	7	5	34
b_j	0	0	31	25	

Fourth allocation

	D_1	D_2	D_3	D_4	a_i
O_1	(16) 5	(3) 3	6	2	0
O_2	4	(15) 7	(22) 9	1	0
O_3	3	4	7	5	34
b_j	0	0	9	25	

Fifth allocation

	D_1	D_2	D_3	D_4	a_i
O_1	(16) 5	(3) 3	6	2	0
O_2	4	(15) 7	(22) 9	1	0
O_3	3	4	(9) 7	5	~~34~~ 25
b_j	0	0	0	25	

Final allocation

	D_1	D_2	D_3	D_4	a_i
O_1	(16) 5	(3) 3	6	2	0
O_2	4	(15) 7	(22) 9	1	0
O_3	3	4	(9) 7	(25) 5	0
b_j	0	0	0	0	

Transportation schedule:

$O_1 \to D_1$, $O_1 \to D_2$, $O_2 \to D_2$, $O_2 \to D_3$

$O_3 \to D_3$, $O_3 \to D_4$

The total transportation cost

$$= (16\times5) + (3\times3) + (15\times7) + (22\times9) + (9\times7) + (25\times5)$$

$$= 80 + 9 + 105 + 198 + 63 + 125$$

$$= ₹\,580$$

(b) Determine an initial basic feasible solution of the following transportation problem by north west corner method.

	Bangalore	Nasik	Bhopal	Delhi	capacity
Chennai	6	8	8	5	30
Madurai	5	11	9	7	40
Trichy	8	9	7	13	50
Demand (units/day)	35	28	32	25	

Soln

Total capacity = Total demand

∴ The given problem is balanced transportation problem.

∴ there exists a feasible solution to the given problem.

Given Transportation problem is

	Bangalore	Nasik	Bhopal	Delhi	capacity (a_i)
Chennai	6	8	8	5	30
Madurai	5	11	9	7	40
Trichy	8	9	7	13	50
Demand (b_j)	35	28	32	25	

First allocation

	Bangalore	Nasik	Bhopal	Delhi	ai
Chennai	(30) 6	8	8	5	[illegible]
Madurai	5	11	9	7	40
Trichy	8	9	7	13	50
bj	35	28	32	25	

Second allocation

	Bangalore	Nasik	Bhopal	Delhi	ai
Chennai	(30) 6	8	8	5	0
Madurai	(5) 5	11	9	7	35
Trichy	8	9	7	13	50
bj	0	28	32	25	

Third allocation

	Bangalore	Nasik	Bhopal	Delhi	ai
Chennai	(30) 6	8	8	5	0
Madurai	(5) 5	(28) 11	9	7	7
Trichy	8	9	7	13	50
bj	0	0	32	25	

Fourth allocation

	Bangalore	Nasik	Bhopal	Delhi	ai
Chennai	(30) 6	8	8	5	0
Madurai	(5) 5	(28) 11	(7) 9	7	0
Trichy	8	9	7	13	50
bj	0	0	25	25	

Fifth allocation

	Bangalore	Nasik	Bhopal	Delhi	ai
Chennai	(30) 6	8	8	5	0
Madurai	(5) 5	(28) 11	(7) 9	7	0
Trichy	8	9	(25) 7	13	25
bj	0	0	0	25	

Final allocation

	Bangalore	Nasik	Bhopal	Delhi	ai
Chennai	(30) 6	8	8	5	0
Madurai	(5) 5	(28) 11	(7) 9	7	0
Trichy	8	9	(25) 7	(25) 13	0
bj	0	0	0	0	

Transportation schedule:

Chennai → Bangalore, Madurai → Bangalore
Madurai → Nasik, Madurai → Bhopal
Trichy → Bhopal, Trichy → Delhi

The total transportation cost

$= (30 \times 6) + (5 \times 5) + (28 \times 11) + (7 \times 9) + (25 \times 7) + (25 \times 13)$

$= 180 + 25 + 308 + 63 + 175 + 325$

$= ₹\ 1076$

(7) Obtain an initial basic feasible solution to the following transportation problem by using least-cost method.

	D_1	D_2	D_3	Supply
O_1	9	8	5	25
O_2	6	8	4	35
O_3	7	6	9	40
Demand	30	25	45	

Soln

Total supply = total Demand.

∴ The given problem is balanced transportation problem.

∴ there exists a feasible solution to the given problem.

Given transportation problem is

	D_1	D_2	D_3	Supply (a_i)
O_1	9	8	5	25
O_2	6	8	4	35
O_3	7	6	9	40
Demand (b_j)	30	25	45	

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First allocation

	D_1	D_2	D_3	a_i
O_1	9	8	5	25
O_2	6	8	(35) 4	0
O_3	7	6	9	40
b_j	30	25	10	

Second allocation

	D_1	D_2	D_3	a_i
O_1	9	8	(10) 5	15
O_2	6	8	(35) 4	0
O_3	7	6	9	40
b_j	30	25	0	

Third allocation

	D_1	D_2	D_3	a_i
O_1	9	8	(10) 5	15
O_2	6	8	(35) 4	0
O_3	7	(25) 6	9	15
b_j	30	0	0	

Fourth allocation

	D_1	D_2	D_3	a_i
O_1	9	8	(10) 5	15
O_2	6	8	(35) 4	0
O_3	(15) 7	(25) 6	9	0
b_j	15	0	0	

Final Allocation

	D_1	D_2	D_3	a_i
O_1	(15) 9	8	(10) 5	0
O_2	6	8	(35) 4	0
O_3	(15) 7	(25) 6	9	0
b_j	0	0	0	

Transportation schedule:

$O_1 \to D_1$, $O_1 \to D_3$, $O_2 \to D_3$

$O_3 \to D_1$, $O_3 \to D_2$

The total transportation cost

$= (15\times9) + (10\times5) + (35\times4)$

$(15\times7) + (25\times6)$

$= 135 + 50 + 140 + 105 + 150$

$= ₹\ 580$

(8) Explain vogel's approximation method by obtaining initial feasible solution of the following transportation problem.

	D_1	D_2	D_3	D_4	Supply
O_1	2	3	11	7	6
O_2	1	0	6	1	1
O_3	5	8	15	9	10
Demand	7	5	3	2	

Soln

Total supply = Total Demand

∴ The given problem is balanced transportation problem.

∴ there exists a feasible solution to the given problem.

First allocation

	D_1	D_2	D_3	D_4	ai	penalty
O_1	2	3	11	7	6	1
O_2	1	0	6	(1) 1	0	1
O_3	5	8	15	9	10	3
bj	7	5	3	1		
penalty	1	3	5	6		

Second allocation

	D_1	D_2	D_3	D_4	ai	penalty
O_1	2	(5) 3	11	7	1	1
O_3	5	8	15	9	10	3
bj	7	0	3	1		
penalty	3	5	4	2		

Third allocation

	D1	D3	D4	ai	penalty
O1	(1) 2	11	7	0	5
O3	5	15	9	10	4
bj	6	3	1		
penalty	3	4	2		

Fourth allocation

	D1	D3	D4	ai	penalty
O3	(6) 5	15	9	4	4
bj	0	3	1		
penalty	–	–	–		

Fifth and Sixth allocation

	D3	D4	ai	penalty
O3	(3) 15	(1) 9	0	6
bj	0	0		
penalty	–	–		

Thus we have the following allocations:

	D_1	D_2	D_3	D_4	a_i
O_1	(1) 2	(5) 3	11	7	6
O_2	1	0	6	(1) 1	1
O_3	(6) 5	8	(3) 15	(1) 9	10
b_j	7	5	3	2	

transportation ~~s~~ schedule:

$O_1 \to D_1$, $O_1 \to D_2$, $O_2 \to D_4$, $O_3 \to D_1$.

$O_3 \to D_3$, $O_3 \to D_4$

The total transportation cost

$= (1\times2) + (5\times3) + (1\times1) + (6\times5)$
$+ (3\times15) + (1\times9)$

$= 2 + 15 + 1 + 30 + 45 + 9$

$= ₹ 102$

(9) Consider the following transportation problem

	D_1	D_2	D_3	D_4	Availability
O_1	5	8	3	6	30
O_2	4	5	7	4	50
O_3	6	2	4	6	20
Requirement	30	40	20	10	

Determine initial basic feasible solution by VAM

Soln

Total Availability = Total Requirement

∴ The given problem is balanced transportation problem.

∴ there exists a feasible solution to the given problem.

First allocation

	D_1	D_2	D_3	D_4	a_i	penalty
O_1	5	8	3	6	30	2
O_2	4	5	7	4	50	1
O_3	6	(20) 2	4	6	0	2
b_j	30	20	20	10		
penealty	1	3	1	2		

Second allocation

	D_1	D_2	D_3	D_4	a_i	penalty
O_1	5	8	(20) 3	6	10	2
O_2	4	5	7	4	50	1
b_j	30	20	0	10		
penalty	1	3	4	2		

Third allocation

	D_1	D_2	D_4	a_i	penalty
O_1	5	8	6	10	1
O_2	4	(20) 5	4	30	1
b_j	30	0	10		
penalty	1	3	2		

Fourth allocation

	D_1	D_4	a_i	penalty
O_1	5	6	10	1
O_2	4	(10) 4	20	0
b_j	30	0		
penalty	1	2		

Fifth and sixth allocation

	D_1	a_i	penalty
O_1	(10) 5	0	—
O_2	(20) 4	0	—
b_j	0		
penalty	1		

Thus we have the following allocations:

	D_1	D_2	D_3	D_4	a_i
O_1	(10) 5	8	(20) 3	6	30
O_2	(20) 4	(20) 5	7	(10) 4	50
O_3	6	(20) 2	4	6	20
b_j	30	40	20	10	

Transportation schedule:

$O_1 \to D_1$, $O_1 \to D_3$, $O_2 \to D_1$, $O_2 \to D_2$

$O_2 \to D_4$, $O_3 \to D_2$

The total transportation cost

$= (10\times5) + (20\times3) + (20\times4) + (20\times5) + (10\times4) + (20\times2)$

$= 50 + 60 + 80 + 100 + 40 + 40$

$= ₹\,370$

(10) Determine basic feasible solution to the following transportation problem using North west corner rule.

Sinks

Origins	A	B	C	D	E	Supply
P	2	11	10	3	7	4
Q	1	4	7	2	1	8
R	3	9	4	8	12	9
Demand	3	3	4	5	6	

Soln

Total supply = Total Demand

∴ The given problem is balanced transportation problem.

∴ there exists a feasible solution to the given problem.

Given Transportation problem is

	A	B	C	D	E	supply (a_i)
P	2	11	10	3	7	4
Q	1	4	7	2	1	8
R	3	9	4	8	12	9
Demand (b_j)	3	3	4	5	6	

First allocation

	A	B	C	D	E	a_i
P	(3) 2	11	10	3	7	1
Q	1	4	7	2	1	8
R	3	9	4	8	12	9
b_j	0	3	4	5	6	

Second allocation

	A	B	C	D	E	ai
P	(3) 2	(1) 11	10	3	7	0
Q	1	4	7	2	1	8
R	3	9	4	8	12	9
bj	0	2	4	5	6	

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Third allocation

	A	B	C	D	E	ai
P	(3) 2	(1) 11	10	3	7	0
Q	1	(2) 4	7	2	1	6
R	3	9	4	8	12	9
bj	0	0	4	5	6	

Fourth allocation

	A	B	C	D	E	ai
P	(3) 2	(1) 11	10	3	7	0
Q	1	(2) 4	(4) 7	2	1	2
R	3	9	4	8	12	9
bj	0	0	0	5	6	

Fifth allocation

	A	B	C	D	E	ai
P	(3) 2	(1) 11	10	3	7	0
Q	1	(2) 4	(4) 7	(2) 2	1	0
R	3	9	4	8	12	9
bj	0	0	0	3	6	

Sixth allocation

	A	B	C	D	E	ai
P	(3) 2	(1) 11	10	3	7	0
Q	1	(2) 4	(4) 7	(2) 2	1	0
R	3	9	4	(3) 8	12	6
bj	0	0	0	0	6	

Final allocation

	A	B	C	D	E	ai
P	(3) 2	(1) 11	10	3	7	0
Q	1	(2) 4	(4) 7	(2) 2	1	0
R	3	9	4	(3) 8	(6) 12	0
bj	0	0	0	0	0	

Transportation schedule:

P→A, P→B ; Q→B, Q→C,
Q→D, R→D, R→E

The total transportation cost
$= (3\times2)+(1\times11)+(2\times4)+(4\times7)$
$+(2\times2)+(3\times8)+(6\times12)$
$= 6+11+8+28+4+24+72$
$=₹\,153$

(11) Find the initial basic feasible solution of the following transportation problem:

	I	II	III	Demand
A	1	2	6	7
B	0	4	2	12
C	3	1	5	11
Supply	10	10	10	

Using (i) North west corner rule
(ii) Least cost method
(iii) Vogel's approximation method.

(i) North west corner rule

Total Demand = Total supply

∴ The given problem is balanced transportation problem

∴ there exists a feasible solution to the given problem.

Given Transportation problem is

	I	II	III	Demand (a_i)
A	1	2	6	7
B	0	4	2	12
C	3	1	5	11
Supply (b_j)	10	10	10	

First allocation

	I	II	III	a_i
A	(7) 1	2	6	0
B	0	4	2	12
C	3	1	5	11
b_j	3	10	10	

Second allocation

	I	II	III	ai
A	(7) 1	2	6	0
B	(3) 0	4	2	9
C	3	1	5	11
bj	0	10	10	

Third allocation

	I	II	III	ai
A	(7) 1	2	6	0
B	(3) 0	(9) 4	2	0
C	3	1	5	11
bj	0	1	10	

Fourth allocation

	I	II	III	ai
A	(7) 1	2	6	0
B	(3) 0	(9) 4	2	0
C	3	(1) 1	5	10
bj	0	0	10	

Final allocation

	I	II	III	a_i
A	(7) 1	2	6	0
B	(3) 0	(9) 4	2	0
C	3	(1) 1	(10) 5	0
b_j	0	0	0	

Transportation schedule:

A → I, B → I, B → II, C → II, C → III

The total transportation cost

$= (7 \times 1) + (3 \times 0) + (9 \times 4) + (1 \times 1) + (10 \times 5)$

$= 7 + 0 + 36 + 1 + 50$

$= ₹\,94$

(ii) Least cost Method

Soln

Total Demand = Total supply

∴ The given problem is balanced transportation problem.

∴ there exists a feasible solution to the given problem.

Given transportation problem is

	I	II	III	Demand (ai)
A	1	2	6	7
B	0	4	2	12
C	3	1	5	11
Supply (bj)	10	10	10	

First allocation

	I	II	III	ai
A	1	2	6	7
B	(10) 0	4	2	2
C	3	1	5	11
bj	0	10	10	

Second allocation

	I	II	III	ai
A	1	2	6	7
B	(10) 0	4	2	2
C	3	(10) 1	5	1
bj	0	0	10	

Third allocation

	I	II	III	a_i
A	1	2	6	7
B	(10) 0	4	(2) 2	0
C	3	(10) 1	5	1
b_j	0	0	8	

Fourth allocation

	I	II	III	a_i
A	1	2	6	7
B	(10) 0	4	(2) 2	0
C	3	(10) 1	(1) 5	0
b_j	0	0	7	

Final allocation

	I	II	III	a_i
A	1	2	(7) 6	0
B	(10) 0	4	(2) 2	0
C	3	(10) 1	(1) 5	0
b_j	0	0	0	

Transportation schedule:

A → III, B → I, B → III, C → II, C → III

The total transportation cost

$= (7\times 6) + (10\times 0) + (2\times 2) + (10\times 1)$
$+ (1\times 5)$

$= 42 + 0 + 4 + 10 + 5$

$= ₹\ 61$

(iii) Vogel's approximation method

Total Demand = Total Supply

∴ The given problem is balanced transportation problem.

∴ there exists a feasible solution to the given problem.

First allocation

	I	II	III	a_i	penalty
A	1	2	6	7	1
B	0	4	(10) 2	2	2
C	3	1	5	11	2
b_j	10	10	0		
penalty	1	1	3		

Second allocation

	I	II	a_i	penalty
A	1	2	7	1
B	(2) 0	4	0	4
C	3	1	11	2
b_j	8	10		
penalty	1	1		

Third allocation

	I	II	a_i	penalty
A	1	2	7	1
C	3	(10) 1	1	2
b_j	8	0		
penalty	2	1		

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Fourth and fifth allocation

	I	a_i	penalty
A	(7) 1	0	–
C	(1) 3	0	–
b_j	0		
penalty	2		

Thus we have the following allocations:

	I	II	III	a_i
A	(7) 1	2	6	7
B	(2) 0	4	(10) 2	12
C	(1) 3	(10) 1	5	11
b_j	10	10	10	

Transportation schedule:

A → I , B → I , B → III , C → I , C → II

The total transportation cost

$= (7 \times 1) + (2 \times 0) + (10 \times 2) + (1 \times 3) + (10 \times 1)$

$= 7 + 0 + 20 + 3 + 10$

$= ₹\ 40$

12) obtain an initial basic feasible solution to the following transportation problem by north west corner method.

	D	E	F	C	Available
A	11	13	17	14	250
B	16	18	14	10	300
C	21	24	13	10	400
Required	200	225	275	250	

Sln

Total Available = Total Required

∴ The given problem is balanced transportation problem.

∴ there exists a feasible solution to the given problem

Given transportation problem is

	D	E	F	C	Available (a_i)
A	11	13	17	14	250
B	16	18	14	10	300
C	21	24	13	10	400
Required (b_j)	200	225	275	250	

First allocation

	D	E	F	C	ai
A	(200) 11	13	17	14	50
B	16	18	14	10	300
C	21	24	13	10	400
bj	0	225	275	250	

Second allocation

	D	E	F	C	ai
A	(200) 11	(50) 13	17	14	0
B	16	18	14	10	300
C	21	24	13	10	400
bj	0	175	275	250	

Third allocation

	D	E	F	C	ai
A	(200) 11	(50) 13	17	14	0
B	16	(175) 18	14	10	125
C	21	24	13	10	400
bj	0	0	275	250	

Fourth allocation

	D	E	F	C	ai
A	(200) 11	(50) 13	17	14	0
B	16	(175) 18	(125) 14	10	0
C	21	24	13	10	400
bj	0	0	150	250	

Fifth allocation

	D	E	F	C	ai
A	(200) 11	(50) 13	17	14	0
B	16	(175) 18	(125) 14	10	0
C	21	24	(150) 13	10	250
bj	0	0	0	250	

Final allocation

	D	E	F	C	ai
A	(200) 11	(50) 13	17	14	0
B	16	(175) 18	(125) 14	10	0
C	21	24	(150) 13	(250) 10	0
bj	0	0	0	0	

Transportation schedule:

$A \to D$, $A \to E$, $B \to E$, $B \to F$, $C \to F$, $C \to C$

The total transportation cost

$$= (200 \times 11) + (50 \times 13) + (175 \times 18) + (125 \times 14) + (150 \times 13) + (250 \times 10)$$

$$= 2200 + 650 + 3150 + 1750 + 1950 + 2500$$

$$= ₹\ 12200$$

Assignment problem

Solution of assignment problems (Hungarian method)

First check whether the number of rows is equal to the number of columns. if it is so, the assignment problem is said to be balanced.

Step 1: Choose the least element in each row and subtract it from all the elements of that row.

Step 2: choose the least element in each column and subtract it from all the elements of that column. Step 2 has to be performed from the table obtained in step 1.

Step 3: check whether there is atleast one zero in each row and each column and make an assignment as follows.

(i) Examine the rows successively until a row with exactly one zero is found. Mark that zero by □, that means an assignment is made there, cross (x) all other zeros in its column. continue this until all the rows have been examined.

(ii) Examine the columns successively until a columns with exactly one zero is found. Mark that zero by □, that means an assignment is made there, cross (x) all other zeros in its row, continue this until all the columns have been examined.

Step 4: If each row and each column contains exactly one assignment, then the solution is optimal.

Ex 10.2

(4) Three Jobs A, B and C are to be assigned to three machines U, V and W. The processing cost for each job machine combination is shown in the matrix given below. Determine the allocation that minimizes the overall processing cost.

Machine

Job	U	V	W
A	17	25	31
B	10	25	16
C	12	14	11

(cost is in ₹ per unit)

Sln

∵ Number of rows = Number of columns

∴ The given assignment problem is balanced.

Step 1: Select a smallest element in each row and subtract this from all the elements in its row.

	U	V	W
A	0	8	14
B	0	15	6
C	1	3	0

Step 2: Select the smallest element in each column and subtract this from all the elements in its column

$$\begin{array}{c|ccc}
 & U & V & W \\
\hline
A & 0 & 5 & 14 \\
B & 0 & 12 & 6 \\
C & 1 & 0 & 0
\end{array}$$

Step 3: (from the elements without line choose least element. It should be add with double line element. It should be subtracted from elements without line. Single line element leave it)

$$\begin{array}{c|ccc}
 & U & V & W \\
\hline
A & 0 & 5 & 14 \\
B & 0 & 12 & 6 \\
C & 1 & 0 & 0
\end{array}
\qquad
\begin{array}{c|ccc}
 & U & V & W \\
\hline
A & 0 & 0 & 9 \\
B & 0 & 7 & 1 \\
C & 6 & 0 & 0
\end{array}$$

Step 4: Examine the rows with exactly one zero.

$$\begin{array}{c|ccc}
 & U & V & W \\
\hline
A & \cancel{0} & 0 & 9 \\
B & \boxed{0} & 7 & 1 \\
C & 6 & 0 & 0
\end{array}$$

Step 5: Examine columns with exactly one zero

$$\begin{array}{c|ccc|} & U & V & W \\ \hline A & \cancel{0} & 0 & 9 \\ B & \boxed{0} & 7 & 1 \\ C & 6 & \cancel{0} & \boxed{0} \\ \hline \end{array}$$

Step 6: Again Examine the rows

$$\begin{array}{c|ccc|} & U & V & W \\ \hline A & \cancel{0} & \boxed{0} & 9 \\ B & \boxed{0} & 7 & 1 \\ C & 6 & \cancel{0} & \boxed{0} \\ \hline \end{array}$$

The optimal assignment schedule and total cost is

Job	Machine	Cost
A	V	25
B	U	10
C	W	11
Total cost		46

The optimal assignment (minimum) cost = ₹ 46

(5) A computer centre has got three expert programmers. The centre needs three application programmes to be developed. The head of the computer centre, after studying carefully the programmes to be developed, estimates the computer time in minitues required by the experts to the application programme as follows.

Programmers	P	Q	R
1	120	100	80
2	80	90	110
3	110	140	120

Assign the programmers to the programme in such a way that the total computer time is least.

Soln: Number of rows = Number of columns

$\therefore$ The given assignment problem is balanced

Step 1: select a smallest element in each row and subtract this from all the elements in its row.

	P	Q	R
1	40	20	0
2	0	10	30
3	0	30	10

Step 2: Select the smallest element in each column and subtract this from all the elements in its column

$$\begin{array}{c|ccc} & P & Q & R \\ \hline 1 & 40 & 10 & 0 \\ 2 & 0 & 0 & 30 \\ 3 & 0 & 20 & 10 \end{array}$$

Step 3: Examine the rows with exactly one zero

$$\begin{array}{c|ccc} & P & Q & R \\ \hline 1 & 40 & 10 & \boxed{0} \\ 2 & \cancel{0} & 0 & 30 \\ 3 & \boxed{0} & 20 & 10 \end{array}$$

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Step 4: Examine the columns with exactly one zero.

$$\begin{array}{c|ccc} & P & Q & R \\ \hline 1 & 40 & 10 & \boxed{0} \\ 2 & \cancel{0} & \boxed{0} & 30 \\ 3 & \boxed{0} & 20 & 10 \end{array}$$

The optimal assignment schedule and total cost is

programmers	programmes	cost
1	R	80
2	Q	90
3	~~R~~ P	110
Total cost		280

The optimal assignment (minimum) cost = ₹ 280

(6) A departmental head has four subordinates and four tasks to be performed. The subordinates differ in efficiency and the tasks differ in their intrinsic difficulty. His estimates of the time each man would take to perform each task is given below

Tasks

Subordinates		1	2	3	4
	P	8	26	17	11
	Q	13	28	4	26
	R	38	19	18	15
	S	9	26	24	10

How should the tasks be allocated to subordinates so as to minimize the total man hours?

Soln Number of rows = number of columns

∴ The given assignment problem is balanced

Step 1: Select a smallest element in each row and subtract this from all the elements in its row

	1	2	3	4
P	0	18	9	3
Q	9	24	0	22
R	23	4	3	0
S	0	17	15	1

Step 2: Select a smallest element in each column and subtract this from all the elements in its column.

	1	2	3	4
P	0	14	9	3
Q	9	20	0	22
R	23	0	3	0
S	0	13	15	1

Step 3:

	1	2	3	4
P	0	14	9	3
Q	9	20	0	22
R	23	0	3	0
S	0	13	15	1

	1	2	3	4
P	0	13	8	2
Q	10	20	0	22
R	24	0	3	0
S	0	12	14	0

Step 4: Examine the rows with exactly one zero

	1	2	3	4
P	$\boxed{0}$	13	8	2
Q	10	20	$\boxed{0}$	22
R	24	0	3	0
S	$\cancel{0}$	12	14	$\boxed{0}$

Step 5: Examine the columns with exactly one zero

$$\begin{array}{c|cccc|} & 1 & 2 & 3 & 4 \\ \hline P & \boxed{0} & 13 & 8 & 2 \\ Q & 10 & 20 & \boxed{0} & 22 \\ R & 24 & \boxed{0} & 3 & \cancel{0} \\ S & \cancel{0} & 12 & 14 & \boxed{0} \\ \hline \end{array}$$

The optimal assigment schedule and total time is

Subordinates	Tasks	Time
P	1	8
Q	3	4
R	2	19
S	4	10
Total Time		41

The optimal assignment (minimum) Time = 41 hours.

7 Find the optimal solution for the assignment problem with the following cost matrix.

Area

		1	2	3	4
Salesman	P	11	17	8	16
	Q	9	7	12	6
	R	13	16	15	12
	S	14	10	12	11

Sln

Number of rows = Number of columns

∴ The given assignment problem is balanced.

Step 1:

select a smallest element in each row and subtract this from all the elements in its row

	1	2	3	4
P	3	9	0	8
Q	3	1	6	0
R	1	4	3	0
S	4	0	2	1

Step 2: select a smallest element in each column and subtract this from all the elements in its column

$$\begin{array}{c|cccc|} & 1 & 2 & 3 & 4 \\ \hline P & 2 & 9 & 0 & 8 \\ Q & 2 & 1 & 6 & 0 \\ R & 0 & 4 & 3 & 0 \\ S & 3 & 0 & 2 & 1 \\ \hline \end{array}$$

Step 3: Examine the rows with exactly one zero

$$\begin{array}{c|cccc|} & 1 & 2 & 3 & 4 \\ \hline P & 2 & 9 & \boxed{0} & 8 \\ Q & 2 & 1 & 6 & \boxed{0} \\ R & \boxed{0} & 4 & 3 & \cancel{0} \\ S & 3 & \boxed{0} & 2 & 1 \\ \hline \end{array}$$

The optimal assignment schedule and total cost is

Salesman	Area	Cost
P	3	8
Q	4	6
R	1	13
S	2	10
Total cost		37

The optimal assignment (minimum) cost = ₹ 37

(8) Assign four trucks 1, 2, 3 and 4 to vacant spaces A, B, C, D, E and F so that distance travelled is minimized. The matrix below shows the distance.

	1	2	3	4
A	4	7	3	7
B	8	2	5	5
C	4	9	6	9
D	7	5	4	8
E	6	3	5	4
F	6	8	7	3

Soln

Number of rows ≠ Number of columns
∴ The given assignment problem is unbalanced
To balance it, introduce two dummy columns with all entries zero.
∴ The revised assignment problem is

Trucks

Vacant spaces	1	2	3	4	d_1	d_2
A	4	7	3	7	0	0
B	8	2	5	5	0	0
C	4	9	6	9	0	0
D	7	5	4	8	0	0
E	6	3	5	4	0	0
F	6	8	7	3	0	0

Step1: Since each row contains zero entry step 1 is not necessary.

Step2: Select the smallest element in each column and subtract this from all the elements in its column.

	1	2	3	4	d_1	d_2
A	0	5	0	4	0	0
B	4	0	2	2	0	0
C	0	7	3	6	0	0
D	3	3	1	5	0	0
E	2	1	2	1	0	0
F	2	6	4	0	0	0

Step3: Examine the columns with exactly one zero.

	1	2	3	4	d_1	d_2
A	~~0~~	5	$\boxed{0}$	4	~~0~~	~~0~~
B	4	$\boxed{0}$	2	2	~~0~~	~~0~~
C	0	7	3	6	0	0
D	3	3	1	5	0	0
E	2	1	2	1	0	0
F	2	6	4	$\boxed{0}$	~~0~~	~~0~~

Step 4: Examine the columns with exactly one zero

$$\begin{array}{c|cccccc|} & 1 & 2 & 3 & 4 & d_1 & d_2 \\ \hline A & \cancel{0} & 5 & \boxed{0} & 4 & \cancel{0} & \cancel{0} \\ B & 4 & \boxed{0} & 2 & 2 & \cancel{0} & \cancel{0} \\ C & \boxed{0} & 7 & 3 & 6 & \cancel{0} & \cancel{0} \\ D & 3 & 3 & 1 & 5 & \boxed{0} & \cancel{0} \\ E & 2 & 1 & 2 & 1 & 0 & \boxed{0} \\ F & 2 & 6 & 4 & \boxed{0} & \cancel{0} & \cancel{0} \\ \hline \end{array}$$

The optimal assignment schedule and total cost is

Vacant Spaces	Trucks	Cost
A	3	3
B	2	2
C	1	4
D	d_1	0
E	d_2	0
F	4	3
Total cost		12

The optimal assignment (minimum) cost = ₹ 12

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DECISION THEORY

Maximin and Minimax strategy

Maximin criteria

* Determine the ~~lowest~~ lowest outcome for each alternative.
* choose the alternative associated with the maximum of these.

Minimax criteria

* Determine the highest outcome for each alternative.
* choose the alternative associated with minimum of these.

Ex 10.3

(1) Given the following pay-off matrix (in rupees) for three strategies and two states of nature

Strategy	States-of-nature	
	E_1	E_2
S_1	40	60
S_2	10	-20
S_3	-40	150

Select a strategy using each of the following rule (i) Maximin (ii) Minimax

Soln

Strategy	States-of-nature		Minimum	Maximum
	E_1	E_2		
S_1	40	60	40	60
S_2	10	-20	-20	10
S_3	-40	150	-40	150

(i) Maximin

Max(40, -20, -40) = 40

Since maximum pay off is 40

~~S_1 is best strategy~~

∴ S_1 is the Required Strategy according to maximin principle.

(ii) Minimax

Min(60, 10, 150) = 10

Since minimum pay off is 10

∴ S_2 is the Required Strategy according to minimax principle.

(2) A farmer wants to decide which of the three crops he should plant on his 100-acre farm. The profit from each is dependent on the rainfall during the growing season. The farmer has categorized the amount of rainfall as high medium and low. His estimated profit for each is shown in the table.

Rainfall	Estimated conditional profit (Rs.)		
	Crop A	Crop B	Crop C
High	8000	3500	5000
Medium	4500	4500	5000
Low	2000	5000	4000

If the farmer wishes to plant only crop, decide which should be his best crop using (i) maximin (ii) minimax

Soln

Rainfall	Estimated conditional profit (RS.)		
	Crop A	Crop B	Crop C
High	8000	3500	5000
Medium	4500	4500	5000
Low	200	5000	4000
Minimum	200	3500	4000
Maximum	8000	5000	5000

(i) Maximin

Max (200, 3500, 4000) = 4000

Since maximum pay off is 4000

∴ crop c is the best crop according to Maximin principle

(ii) Minimax

Min (8000, 5000, 5000) = 5000

Since minimum pay off is 5000

∴ Crop B & Crop C are the best crops according to Minimax principle.

(3) The research department of Hindustan Ltd. has recommended to pay marketing department to launch a shampoo of three different types. The marketing types of shampoo to be lauched under the following estimated pay-offs for various level of sales.

Types of shampoo	Estimated sales (in units)		
	15000	10000	5000
Egg shampoo	30	10	10
clinic shampoo	40	15	5
Deluxe shampoo	55	20	3

what will be the marketting manager's decision if (i) maximin and (ii) minimax principle applied?

Soln

Type of shampoo	Estimated sales (in units)			Minimum	Maximum
	15000	10000	5000		
Egg shampoo	30	10	10	10	30
clinic shampoo	40	15	5	5	40
Deluxe shampoo	55	20	3	3	55

(i) Maximin

$\max(10, 5, 3) = 10$

Since maximum pay off is 10

∴ marketing manager's decision is Egg shampoo according to maximin principle

(ii) Minimax

$\min(30, 40, 55) = 30$

Since minimum pay off is 30

∴ marketting manager's decision is Egg shampoo according to minimax principle.

(4) Following pay-off matrix, which is the optimal decision under each of the following rule (i) maximin (ii) minimax.

Act	States of nature			
	s_1	s_2	s_3	s_4
A_1	14	9	10	5
A_2	11	10	8	7
A_3	9	10	10	11
A_4	8	10	11	13

Soln.

Act	States of nature				Minimum	Maximum
	S_1	S_2	S_3	S_4		
A_1	14	9	10	5	5	14
A_2	11	10	8	7	7	11
A_3	9	10	10	11	9	11
A_4	8	10	11	13	8	13

(i) Maximin

$\text{Max}(5,7,9,8) = 9$

Since maximum pay off is 9

$\therefore$ Required Act is A_3 according to Maximin principle.

(ii) Minimax

$\text{Min}(14,11,11,13) = 11$

Since Minimum pay off is 11

$\therefore$ Required Acts are A_2 and A_3 according to Minimax principle.

Miscellaneous problems

(1) The following table summarizes the supply, demand and cost information for four factors S_1, S_2, S_3, S_4 shipping goods to three warehouses D_1, D_2, D_3

	D_1	D_2	D_3	supply
S_1	2	7	14	5
S_2	3	3	1	8
S_3	5	4	7	7
S_4	1	6	2	14
Demand	7	9	18	

Find an initial solution by using north west corner rule. what is the total cost for this solution?

Soln

Total supply = Total Demand

∴ The given problem is balanced transportation problem.

∴ there exists a feasible solution to the given problem.

Given Transportation problem is:

	D_1	D_2	D_3	Supply (a_i)
S_1	2	7	14	5
S_2	3	3	1	8
S_3	5	4	7	7
S_4	1	6	2	14
Demand (b_j)	7	9	18	

First allocation

	D_1	D_2	D_3	a_i
S_1	(5) 2	7	14	0
S_2	3	3	1	8
S_3	5	4	7	7
S_4	1	6	2	14
b_j	2	9	18	

Second allocation

	D_1	D_2	D_3	a_i
S_1	(5) 2	7	14	0
S_2	(2) 3	3	1	6
S_3	5	4	7	7
S_4	1	6	2	14
b_j	0	9	18	

Third allocation

	D_1	D_2	D_3	a_i
	(5) 2	7	14	0
	(2) 3	(6) 3	1	0
	5	4	7	7
	1	6	2	14
b_j	0	3	18	

Fourth allocation

	D_1	D_2	D_3	a_i
S_1	(5) 2	7	14	0
S_2	(2) 3	(6) 3	1	0
S_3	5	(3) 4	7	4
S_4	1	6	2	14
b_j	0	0	18	

Fifth allocation

	D1	D2	D3	ai
S1	(5) 2	7	14	0
S2	(2) 3	(6) 3	1	0
S3	5	(3) 4	(4) 7	0
S4	1	6	2	14
bj	0	0	14	

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Final allocation

	D1	D2	D3	ai
S1	(5) 2	7	14	0
S2	(2) 3	(6) 3	1	0
S3	5	(3) 4	(4) 7	0
S4	1	6	(14) 2	0
bj	0	0	0	

Transportation schedule:

$S_1 \to D_1$, $S_2 \to D_1$, $S_2 \to D_2$, ~~S3→D2~~
$S_3 \to D_2$, $S_3 \to D_3$, $S_4 \to D_3$

The total transportation cost

$= (5\times2) + (2\times3) + (6\times3) + (3\times4) + (4\times7) + (14\times2)$

$= 10 + 6 + 18 + 12 + 28 + 28$

$= ₹\ 102$.

(2) consider the following transportation problem

Destination

	D_1	D_2	D_3	D_4	Availability
O_1	5	8	3	6	30
O_2	4	5	7	4	50
O_3	6	2	4	6	20
Requirement	30	40	20	10	

Determine an initial basic feasible solution using a) Least cost method b) Vogel's approximation method.

Soln a) Least cost method

Total Availability = Total Requirement

$\therefore$ The given problem is balanced transportation problem.

$\therefore$ there exists a feasible solution to the given problem.

Given Transportation problem is

	D_1	D_2	D_3	D_4	Availability (a_i)
O_1	5	8	3	6	30
O_2	4	5	7	4	50
O_3	6	2	4	6	20
Requirement (b_j)	30	40	20	10	

First allocation

	D1	D2	D3	D4	ai
O1	5	8	3	6	30
O2	4	5	7	4	50
O3	6	(20) 2	4	6	0
bj	30	20	20	10	

Second allocation

	D1	D2	D3	D4	ai
O1	5	8	(20) 3	6	10
O2	4	5	7	4	50
O3	6	(20) 2	4	6	0
bj	30	20	0	10	

Third allocation

	D1	D2	D3	D4	ai
O1	5	8	(20) 3	6	10
O2	(30) 4	5	7	4	20
O3	6	(20) 2	4	6	0
bj	0	20	0	10	

Fourth allocation

	D1	D2	D3	D4	ai
O1	5	8	(20) 3	6	10
O2	(30) 4	5	7	(10) 4	10
O3	6	(20) 2	4	6	0
bj	0	20	0	0	

Fifth allocation

	D1	D2	D3	D4	ai
O1	5	8	(20) 3	6	10
O2	(30) 4	(10) 5	7	(10) 4	0
O3	6	(20) 2	4	6	0
bj	0	10	0	0	

Final allocation

	D1	D2	D3	D4	ai
O1	5	(10) 8	(20) 3	6	0
O2	(30) 4	(10) 5	7	(10) 4	0
O3	6	(20) 2	4	6	0
bj	0	0	0	0	

Transportation Schedule:

$O_1 \to D_2$, $O_1 \to D_3$, $O_2 \to D_1$, $O_2 \to D_2$

$O_2 \to D_4$, $O_3 \to D_2$

The total transportation cost

$= (10\times8) + (20\times3) + (30\times4) + (10\times5)$
$+ (10\times4) + (20\times2)$

$= 80 + 60 + 120 + 50 + 40 + 40$

$=$ ₹ 390

b) Vogel's approximation method

Total Availability = Total Requirement

∴ The given problem is balanced transportation problem.

∴ there exists a feasible solution to the given problem.

First allocation

	D1	D2	D3	D4	ai	penalty
O1	5	8	3	6	30	2
O2	4	5	7	4	50	1
O3	6	(20) 2	4	6	0	2
bj	30	20	20	10		
penalty	1	3	1	2		

Second allocation

	D_1	D_2	D_3	D_4	a_i	penalty
O_1	5	8	(20) 3	6	10	2
O_2	4	5	7	4	50	1
b_j	30	20	0	10		
penalty	1	3	4	2		

Third allocation

	D_1	D_2 ~~D_3~~	D_4	a_i	penalty
O_1	5	8	6	10	1
O_2	4	(20) 5	4	30	1
b_j	30	0	10		
penalty	1	3	2		

Fourth allocation

	D_1	D_4	a_i	penalty
O_1	5	6	10	1
O_2	4	(10) 4	20	0
b_j	30	0		
penalty	1	2		

~~Fifth allocation~~, Fifth and sixth allocation

	D1	ai	Penalty
O1	(10) 5	0	—
O2	(20) 4	0	—
bj	0		
penalty	1		

Thus we have the following allocations:

	D1	D2	D3	D4	ai
O1	(10) 5	8	(20) 3	6	30
O2	(20) 4	(20) 5	7	(10) 4	50
O3	6	(20) 2	4	6	20
bj	30	40	20	10	

~~The~~ transportation schedule:

$O_1 \to D_1$, $O_1 \to D_3$, $O_2 \to D_1$, $O_2 \to D_2$

$O_2 \to D_4$, $O_3 \to D_2$

The total transportation cost

$= (10\times5) + (20\times3) + (20\times4)$

$+ (20\times5) + (10\times4) + (20\times2)$

$= 50 + 60 + 80 + 100 + 40 + 40$

$=$ ₹ 370

(3) Determine an initial basic feasible solution to the following transportion problem by using (i) North west corner rule (ii) least cost method.

Destination

	D_1	D_2	D_3	supply
S_1	9	8	5	25
Source S_2	6	8	4	35
S_3	7	6	9	40
Requirement	30	25	45	

Soln (i) North west corner rule

Total supply = Total Requirement

∴ The given problem is balanced transportation problem.

∴ there exists a feasible solution to the given problem.

Given Transportation problem:

	D_1	D_2	D_3	Supply (a_i)
S_1	9	8	5	25
S_2	6	8	4	35
S_3	7	6	9	40
Requirement (b_j)	30	25	45	

First allocation

	D_1	D_2	D_3	a_i
S_1	(25) 9	8	5	0
S_2	6	8	4	35
S_3	7	6	9	40
b_j	5	25	45	

Second allocation

	D_1	D_2	D_3	a_i
S_1	(25) 9	8	5	0
S_2	(5) 6	8	4	30
S_3	7	6	9	40
b_j	0	25	45	

Third allocation

	D1	D2	D3	ai
S1	(25) 9	8	5	0
S2	(5) 6	(25) 8	4	5
S3	7	6	9	40
bj	0	0	45	

Fourth allocation

	D1	D2	D3	ai
S1	(25) 9	8	5	0
S2	(5) 6	(25) 8	(5) 4	0
S3	7	6	9	40
bj	0	0	40	

Final allocation

	D1	D2	D3	ai
S1	(25) 9	8	5	0
S2	(5) 6	(25) 8	(5) 4	0
S3	7	6	(40) 9	0
bj	0	0	0	

Transportation ~~cost~~ schedule:

$S_1 \to D_1$, $S_2 \to D_1$, $S_2 \to D_2$, $S_2 \to D_3$

$S_3 \to D_3$

The total transportation cost

$= (25\times9) + (5\times6) + (25\times8) + (5\times4)$

$+ (40\times9)$

$= 225 + 30 + 200 + 20 + 360$

$= ₹\,835$

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(ii) least cost method

Total supply = Total Requirement

∴ The given problem is balanced transportation problem.

∴ there exists a feasible solution to the given problem.

Given Transportation problem:

	D_1	D_2	D_3	supply (a_i)
S_1	9	8	5	25
S_2	6	8	4	35
S_3	7	6	9	40
Requirement (b_j)	30	25	45	

First allocation

	D_1	D_2	D_3	a_i
S_1	9	8	5	25
S_2	6	8	(35) 4	0
S_3	7	6	9	40
b_j	30	25	10	

Second allocation

	D_1	D_2	D_3	a_i
S_1	9	8	(10) 5	15
S_2	6	8	(35) 4	0
S_3	7	6	9	40
b_j	30	25	0	

Third allocation

	D_1	D_2	D_3	~~D4~~	a_i
S_1	9	8	(10) 5		15
S_2	6	8	(35) 4		0
S_3	7	(25) 6	9		15
b_j	30	0	0		

Fourth allocation

	D_1	D_2	D_3	a_i
S_1	9	8	(10) 5	15
S_2	6	8	(35) 4	0
S_3	(15) 7	(25) 6	9	0
b_j	15	0	0	

Final allocation

	D_1	D_2	D_3	a_i
S_1	(15) 9	8	(10) 5	0
S_2	6	8	(35) 4	0
S_3	(15) 7	(25) 6	9	0
b_j	0	0	0	

Transportation schedule:

$S_1 \to D_1$, $S_1 \to D_3$, $S_2 \to D_3$, $S_3 \to D_1$

$S_3 \to D_2$

The total transportation cost

$$= (15\times9) + (10\times5) + (35\times4) + (15\times7) + (25\times6)$$

$$= 135 + 50 + 140 + 105 + 150$$

$$= ₹.580$$

(4) Explain Vogel's approximation method by obtaining initial basic feasible solution of the following transportation problem.

Destination

		D1	D2	D3	D4	supply
	O1	2	3	11	7	6
Origin	O2	1	0	6	1	1
	O3	5	8	15	9	10
	Demand	7	5	3	2	

Soln

Total supply = Total Demand

∴ The given problem is balanced transportation problem

∴ there exists a feasible solution to the given problem.

First allocation

	D1	D2	D3	D4	ai	Penalty
O1	2	3	11	7	6	1
O2	1	0	6	(1) 1	0	1
O3	5	8	15	9	10	3
bj	7	5	3	1		
Penalty	1	3	5	6		

Second allocation

	D1	D2	D3	D4	ai	penalty
O1	2	(5) 3	11	7	1	1
O3	5	8	15	9	10	3
bj	7	0	3	1		
penalty	3	5	4	2		

Third allocation

	D1	D3	D4	ai	penalty
O1	(1) 2	11	7	0	5
O3	5	15	9	10	4
bj	6	3	1		
penalty	3	4	2		

Fourth, fifth and sixth allocation

	D1	D3	D4	bj	penalty
O3	(6) 5	(3) 15	(1) 9	0	4
ai	0	0	0		
penalty	—	—	—		

Thus we have the following allocations:

	D_1	D_2	D_3	D_4	a_i
O_1	(1) 2	(5) 3	11	7	6
O_2	1	0	6	(1) 1	1
O_3	(6) 5	8	(3) 15	(1) 9	10
b_j	7	5	3	2	

Transportation schedule:

$O_1 \to D_1$, $O_1 \to D_2$, $O_2 \to D_4$

$O_3 \to D_1$, $O_3 \to D_3$, $O_3 \to D_4$

The total transportation cost

$= (1\times2) + (5\times3) + (1\times1) + (6\times5)$
$+ (3\times15) + (1\times9)$

$= 2 + 15 + 1 + 30 + 45 + 9$

$= ₹\,102$

5) A car hire company has one car at each of five depots a, b, c, d and e. A customer in each of the five towers A, B, C, D and E requires a car. The distance (in miles) between the depots (origins) and the towers (destinations) where the customers are given in the following distance matrix.

	a	b	c	d	e
A	160	130	175	190	200
B	135	120	130	160	175
C	140	110	155	170	185
D	50	50	80	80	110
E	55	35	70	80	105

How should the cars be assigned to the customers so as to minimize the distance travelled?

Soln:

Number of rows = Number of columns

∴ The given assignment problem is balanced.

Step1: Select a ~~number~~ smallest element in each row and subtract this from all the elements in its row

	a	b	c	d	e
A	30	0	45	60	70
B	15	0	10	40	55
C	30	0	45	60	75
D	0	0	30	30	60
E	20	0	35	45	70

Step2: Select a smallest element in each column and subtract this from all the elements in its column

	a	b	c	d	e
A	30	0	35	30	15
B	15	0	0	10	0
C	30	0	35	30	20
D	0	0	20	0	5
E	20	0	25	15	15

Step3:

	a	b	c	d	e
A	30	0	35	30	15
B	15	0	0	10	0
C	30	0	35	30	20
D	0	0	20	0	5
E	20	0	25	15	15

	a	b	c	d	e
A	15	0	20	15	0
B	15	15	0	10	0
C	15	0	20	15	5
D	0	15	20	0	5
E	5	0	10	0	0

Step4: Examine the rows with exactly one zero

	a	b	c	d	e
A	15	~~0~~	20	15	0
B	15	15	0	10	0
C	15	$\boxed{0}$	20	15	5
D	0	15	20	0	5
E	5	~~0~~	10	0	0

Step 5: Examine the columns with exactly one zero

	a	b	c	d	e
A	15	~~0~~	20	15	$\boxed{0}$
B	15	15	$\boxed{0}$	10	~~0~~
C	15	$\boxed{0}$	20	15	5
D	$\boxed{0}$	15	20	~~0~~	5
E	5	~~0~~	10	$\boxed{0}$	~~0~~

The optimal assignment schedule and total distance is

Towers	Depots	Distance
A	e	200
B	c	130
C	b	110
D	a	50
E	d	80
Total Distance		570

The optimal assignment (minimum) Distance = 570 miles

(b) A natural truck-rental service has a surplus of one truck in each of the cities 1, 2, 3, 4, 5 and 6 and a deficit of one truck in each of the cities 7, 8, 9, 10, 11 and 12. The distance (in kilometers) between the cities with a surplus and the cities with a deficit are displayed below:

To

From	7	8	9	10	11	12
1	31	62	29	42	15	41
2	12	19	39	55	71	40
3	17	29	50	41	22	22
4	35	40	38	42	27	33
5	19	30	29	16	20	33
6	72	30	30	50	41	20

How should the truck be dispersed so as to minimize the total distance travelled?

Soln

Number of rows = Number of columns

∴ The given assignment problem is balanced.

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Step 1: Select a smallest element in each row and subtract this from all the elements in its row.

	7	8	9	10	11	12
1	16	47	14	27	0	26
2	0	7	27	43	59	28
3	0	12	33	24	5	5
4	8	13	11	15	0	6
5	2	14	13	0	4	17
6	52	10	10	30	21	0

Step 2: Select a smallest element in each column and subtract this from all the elements in its column.

	7	8	9	10	11	12
1	16	40	4	27	0	26
2	0	0	17	43	59	28
3	0	5	23	24	5	5
4	8	6	1	15	0	6
5	2	7	3	0	4	17
6	52	3	0	30	21	0

Step 3:

	7	8	9	10	11	12
1	16	40	4	27	0	26
2	0	0	17	43	59	28
3	0	5	23	24	5	5
4	8	6	1	15	0	6
5	2	7	3	0	4	17
6	52	3	0	30	21	0

	7	8	9	10	11	12
1	16	39	3	27	0	25
2	1	0	17	44	60	28
3	0	4	22	24	5	4
4	8	5	0	15	0	5
5	2	6	2	0	4	16
6	53	3	0	31	22	0

Step 4: Examine the rows with exactly one zero.

	7	8	9	10	11	12
1	16	39	3	27	$\boxed{0}$	25
2	1	$\boxed{0}$	17	44	60	28
3	$\boxed{0}$	4	22	24	5	4
4	8	5	$\boxed{0}$	15	$\cancel{0}$	5
5	2	6	2	$\boxed{0}$	4	16
6	53	3	$\cancel{0}$	31	22	$\boxed{0}$

The optimal assignment schedule and total distance is

From	To	Distance
1	11	15
2	8	19
3	7	17
4	9	38
5	10	16
6	12	20
Total distance		125

The optimal assignment (minimum) Distance = 125 kms

7) A person wants to invest in one of three alternative investment plans: stock, Bonds and Debentures. It is assumed that the person wishes to invest all of the funds in a plan. The pay-off matrix based on three potential economic conditions is given in the following table:

Alternative	Economic Conditions		
	High growth (Rs)	Normal growth (Rs)	Slow growth (Rs)
Stocks	10000	7000	3000
Bonds	8000	6000	1000
Debentures	6000	6000	6000

Determine the best investment plan using each of following criteria (i) Maxmin (ii) Minimax.

Soln

Alternative	Economic conditions			minimum	maximum
	High growth (Rs)	Normal growth (Rs)	Slow growth (Rs)		
Stocks	10000	7000	3000	3000	10000
Bonds	8000	6000	1000	1000	8000
Debentures	6000	6000	6000	6000	6000

(i) Maxmin

Max(3000, 1000, 6000) = 6000

Since maximum pay off is 6000

∴ Debentures is the best investment plan according to maxmin principle.

(ii) Minimax

Min(10000, 8000, 6000) = 6000

Since minimum pay off is 6000

∴ Debentures is the best investment plan according to minimax principle.

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