Chapter 9: Differentiation

EXERCISE 9.1 [PAGE 120]

Exercise 9.1 | Q 1.1 | Page 120

Find the derivative of the following function w.r.t. x.
x^{12}

SOLUTION

Let $y = x^{12}$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx} x^{12}$$

$= 12\,x^{12-1}$

$= 12\,x^{11}$

Exercise 9.1 | Q 1.2 | Page 120
Find the derivative of the following function w.r.t. x.
x^{-9}

SOLUTION

Let $y = x^{-9}$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx} x^{-9}$$

$= -9\,x^{-9-1}$

$= -9\,x^{-10}$

Exercise 9.1 | Q 1.3 | Page 120

Find the derivative of the following functions w. r. t. x.

$x^{\frac{3}{2}}$

SOLUTION

Let y = $x^{\frac{3}{2}}$

Differentiating w.r.t. x, we get

$\frac{dy}{dx} = \frac{d}{dx} x^{\frac{3}{2}}$

= $\frac{3}{2} x^{\frac{3}{2}-1}$

= $-\frac{3}{2} x^{\frac{1}{2}}$

= $\frac{3}{2}\sqrt{x}$

Exercise 9.1 | Q 1.4 | Page 120

Find the derivative of the following function w. r. t. x.

$7x\sqrt{x}$

SOLUTION

Let y = $7x\sqrt{x}$

=$7x^1 x^{\frac{1}{2}}$

$y = 7x^{\frac{3}{2}}$

Differentiating w.r.t. x, we get

$\frac{dy}{dx} = \frac{d}{dx} 7x^{\frac{3}{2}}$

= $7 \times \frac{3}{2} x^{\frac{3}{2}-1}$

$= \frac{21}{2} x^{\frac{1}{2}}$

$= \frac{21}{2} \sqrt{x}$

Exercise 9.1 | Q 1.5 | Page 120

Find the derivative of the following function w. r. t. x.
3^5

SOLUTION

Let y = 3^5

Differentiating w.r.t. x, we get

$\frac{dy}{dx} = \frac{d}{dx} 3^5 = 0$ ...[3^5 is a constant]

Exercise 9.1 | Q 2.1 | Page 120
Differentiate the following w. r. t. x.
$x^5 + 3x^4$

SOLUTION

Let y = $x^5 + 3x^4$

Differentiating w.r.t. x, we get

$\frac{dy}{dx} = \frac{d}{dx}\left(x^5 + 3x^4\right)$

$= \frac{d}{dx} x^5 + 3 \frac{d}{dx} x^4$

$= 5x^4 + 3(4x^3)$

$\frac{dy}{dx} = 5x^4 12x^3$

Exercise 9.1 | Q 2.2 | Page 120

Differentiate the following w. r. t. x.

$x\sqrt{x} + \log x - e^x$

SOLUTION

Let y = $x\sqrt{x} + \log x - e^x$

$= x\frac{3}{2} + \log x - e^x$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^{\frac{3}{2}} + \log x - e^x\right)$$

$$= \frac{d}{dx}x^{\frac{3}{2}} + \frac{d}{dx}\log x - \frac{d}{dx}e^x$$

$$= \frac{3}{2}x^{\frac{3}{2}-1} + \frac{1}{x} - e^x$$

$$= \frac{3}{2}x^{\frac{1}{2}} + \frac{1}{x} - e^x$$

$$= \frac{3}{2}\sqrt{x} + \frac{1}{x} - e^x$$

Exercise 9.1 | Q 2.3 | Page 120

Differentiate the following w. r. t. x.

$x^{\frac{5}{2}} + 5x^{\frac{7}{5}}$

SOLUTION

Let y = $x^{\frac{5}{2}} + 5x^{\frac{7}{5}}$

Differentiating w.r.t. x, we get

$$= \frac{dy}{dx} = \frac{d}{dx}\left(x^{\frac{5}{2}} + 5x^{\frac{7}{5}}\right)$$

$= \frac{d}{dx}x^{\frac{5}{2}} + 5\frac{d}{dx}x^{\frac{7}{5}}$

$= \frac{5}{2}x^{\frac{5}{2}-1} + 5\frac{7}{5}x^{\frac{7}{5}-1}$

$= \frac{5}{2}x^{\frac{3}{2}} + 7x^{\frac{2}{5}}$

Exercise 9.1 | Q 2.4 | Page 120

Differentiate the following w. r. t. x.

$\frac{2}{7}x^{\frac{7}{2}} + \frac{5}{2}x^{\frac{2}{5}}$

SOLUTION

Let y = $\frac{2}{7}x^{\frac{7}{2}} + \frac{5}{2}x^{\frac{2}{5}}$

Differentiating w.r.t. x, we get

$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{2}{7}x^{\frac{7}{2}} + \frac{5}{2}x^{\frac{2}{5}}\right)$

$= \frac{2}{7}\frac{d}{dx}x^{\frac{7}{2}} + \frac{5}{2}\frac{d}{dx}x^{\frac{2}{5}}$

$= \frac{2}{7} \times \frac{7}{2}x^{\frac{7}{2}-1} + \frac{5}{2} \times \frac{2}{5}x^{\frac{2}{5}-1}$

$= x^{\frac{5}{2}} + x^{\frac{-3}{5}}$

Exercise 9.1 | Q 2.5 | Page 120

Differentiate the following w. r. t. x.

$\sqrt{x}\left(x^2+1\right)^2$

SOLUTION

Let y = $\sqrt{x}\left(x^2+1\right)^2$

$\therefore y = x^{\frac{1}{2}}\left(x^4+2x^2+1\right)$

y = $x^{\frac{9}{2}}+2x^{\frac{5}{2}}+x^{\frac{1}{2}}$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^{\frac{9}{2}}+2x^{\frac{5}{2}}+x^{\frac{1}{2}}\right)$$

$$= \frac{d}{dx^{\frac{9}{2}}} + 2\frac{d}{dx}x^{\frac{5}{2}} + \frac{d}{dx}\sqrt{x}$$

$$= \frac{9}{2}x^{\frac{9}{2}-1} + 2\times\frac{5}{2}x^{\frac{5}{2}-1} + \frac{1}{2\sqrt{x}}$$

$$= \frac{9}{2}\frac{x^7}{2} + 5\frac{x^3}{2} + \frac{1}{2\sqrt{x}}$$

Exercise 9.1 | Q 3.1 | Page 120

Differentiate the following w. r. t. x
x^3 log x

SOLUTION

Let y = (x^3 log x)

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}x^3\log x$$

$$= x^3\frac{d}{dx}(\log x) + (\log x)\frac{d}{dx}\left(x^3\right)$$

$$= x^3\times\frac{1}{x} + (\log x)\left(3x^2\right)$$

= x^2 + $3x^2$ log x

Exercise 9.1 | Q 3.2 | Page 120

Differentiate the following w. r. t. x

$x^{\frac{5}{2}} e^x$

SOLUTION

Let y = $x^{\frac{5}{2}} e^x$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^{\frac{5}{2}} e^x\right)$$

$$= x^{\frac{5}{2}} \frac{d}{dx}(e^x) + e^x\left(\frac{5}{2} x^{\frac{3}{2}}\right)$$

$$= x^{\frac{5}{2}}(e^x) + e^x\left(\frac{5}{2} x^{\frac{3}{2}}\right)$$

$$= e^x\left(x^{\frac{5}{2}} + \frac{5}{2} x^{\frac{3}{2}}\right)$$

Exercise 9.1 | Q 3.3 | Page 120

Differentiate the following w. r. t. x

$e^x \log x$

SOLUTION

Let y = $e^x \log x$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}(e^x \log x)$$

$$= e^x \frac{d}{dx}(\log x) + (\log x)\frac{d}{dx}(e^x)$$

$$=e^{x}\left(\frac{1}{x}\right)+(\log x)(e^{x})$$

$$= e^{x}\left(\frac{1}{x}+\log x\right)$$

Exercise 9.1 | Q 3.4 | Page 120

Differentiate the following w. r. t. x
$x^3 .3^x$

SOLUTION

Let y = $x^3\ 3^x$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx}=\frac{d}{dx}\left(x^{3}3^{x}\right)$$

$$= x^{3}\frac{d}{dx}(3^{x})+3^{x}\frac{d}{dx}\left(x^{3}\right)$$

$= (x^3)(3^x \log 3) + 3^x(3x^2)$

$= x^2\ 3^x\ (x\log 3+3)$

Exercise 9.1 | Q 4.1 | Page 120

Find the derivative of the following w. r. t.x

$$\frac{x^2+a^2}{x^2-a^2}$$

SOLUTION

Let y = $\dfrac{x^2+a^2}{x^2-a^2}$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx}=\frac{d}{dx}\left(\frac{x^2+a^2}{x^2-a^2}\right)$$

$$=\frac{\left(x^2-a^2\right)\frac{d}{dx}\left(x^2+a^2\right)-\left(x^2+a^2\right)\frac{d}{dx}\left(x^2-a^2\right)}{\left(x^2-a^2\right)^2}$$

$$=\frac{\left(x^2-a^2\right)\left(\frac{d}{dx}x^2+\frac{d}{dx}a^2\right)-\left(x^2+a^2\right)\left(\frac{d}{dx}x^2-\frac{d}{dx}a^2\right)}{\left(x^2-a^2\right)^2}$$

$$=\frac{\left(x^2-a^2\right)(2x+0)-\left(x^2+a^2\right)(2x-0)}{\left(x^2-a^2\right)^2}$$

$$=\frac{2x\left(x^2-a^2\right)-2x\left(x^2+a^2\right)}{\left(x^2-a^2\right)^2}$$

$$=\frac{2x\left(x^2-a^2-x^2-a^2\right)}{\left(x^2-a^2\right)^2}$$

$$=\frac{2x\left(-2a^2\right)}{\left(x^2-a^2\right)^2}$$

$$=\frac{-4xa^2}{\left(x^2-a^2\right)^2}$$

Exercise 9.1 | Q 4.2 | Page 120

Find the derivative of the following w. r. t.x.

$$\frac{3x^2+5}{2x^2-4}$$

SOLUTION

Let y = $\dfrac{3x^2+5}{2x^2-4}$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{3x^2+5}{2x^2-4}\right)$$

$$= \frac{\left(2x^2-4\right)\frac{d}{dx}\left(3x^2+5\right)-\left(3x^2+5\right)\frac{d}{dx}\left(2x^2-4\right)}{\left(2x^2-4\right)^2}$$

$$= \frac{\left(2x^2-4\right)(6x+0)-\left(3x^2+5\right)(4x-0)}{\left(2x^2-4\right)^2}$$

$$= \frac{6x\left(2x^2-4\right)-4x\left(3x^2+5\right)}{\left(2x^2-4\right)^2}$$

$$= \frac{2x\left[3\left(2x^2-4\right)-2\left(3x^2+5\right)\right]}{\left(2x^2-4\right)^2}$$

$$= \frac{2x\left(6x^2-12-6x^2-10\right)}{\left(2x^2-4\right)^2}$$

$$= \frac{2x(-22)}{\left(2x^2-4\right)^2}$$

$$= \frac{-44x}{\left(2x^2-4\right)^2}$$

Exercise 9.1 | Q 4.3 | Page 120

Find the derivative of the following w. r. t. x

$$\frac{\log x}{x^3-5}$$

SOLUTION

Let y = $\dfrac{\log x}{x^3 - 5}$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{\log x}{x^3 - 5}\right)$$

$$= \frac{\left(x^3 - 5\right)\frac{d}{dx}(\log x) - (\log x)\frac{d}{dx}\left(x^3 - 5\right)}{\left(x^3 - 5\right)^2}$$

$$= \frac{\left(x^3 - 5\right)\left(\frac{1}{x}\right) - \log x\left(\frac{d}{dx}\left(x^3\right) - \frac{d}{dx}(5)\right)}{\left(x^3 - 5\right)^2}$$

$$= \frac{\left(x^3 - 5\right)\frac{1}{x} - \log x\left(3x^2 - 0\right)}{\left(x^3 - 5\right)^2}$$

$$= \frac{\left(x^3 - 5\right)\frac{1}{x} - 3x^2 \log x}{\left(x^2 - 5\right)^2}$$

Exercise 9.1 | Q 4.4 | Page 120

Find the derivative of the following w. r. t.x.

$$\frac{3e^x - 2}{3e^x + 2}$$

SOLUTION

Let y = $\dfrac{3e^x - 2}{3e^x + 2}$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{3e^x - 2}{3e^x + 2}\right)$$

$$= \frac{(3e^x + 2)\frac{d}{dx}(3e^x - 2) - (3e^x - 2)\frac{d}{dx}(3e^x + 2)}{(3e^x + 2)^2}$$

$$= \frac{(3e^x)\left(\frac{d}{dx}(3e^x) - \frac{d}{dx}(2)\right) - (3e^x - 2)\left(\frac{d}{dx}(3e^x) + \frac{d}{dx}(2)\right)}{(3e^x + 2)^2}$$

$$= \frac{(3e^x + 2)(3e^x - 0) - (3e^x - 2)(3e^x + 0)}{(3e^x + 2)^2}$$

$$= \frac{3e^x(3e^x + 2) - 3e^x(3e^x - 2)}{(3e^x + 2)^2}$$

$$= \frac{3e^x(3e^x + 2 - 3e^x + 2)}{(3e^x + 2)^2}$$

$$= \frac{3e^x(4)}{(3e^x + 2)^2}$$

$$= \frac{12e^x}{(3e^x + 2)^2}$$

Exercise 9.1 | Q 4.5 | Page 120

Find the derivative of the following w. r. t.x.

$$\frac{xe^x}{x + e^x}$$

SOLUTION

Let y = $\frac{xe^x}{x + e^x}$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{xe^x}{x + e^x}\right)$$

$$= \frac{(x + e^x)\frac{d}{dx}(xe^x) - (xe^x)\frac{d}{dx}(x + e^x)}{(x + e^x)^2}$$

$$= \frac{(x + e^x)\left[x\frac{d}{dx}(e^x) + e^x\frac{d}{dx}(x)\right] - xe^x\left(\frac{d}{dx}(x) + \frac{d}{dx}(e^x)\right)}{(x + e^x)^2}$$

$$= \frac{(x+e^x)[xe^x + e^x(1)] - xe^x(1+e^x)}{(x+e^x)^2}$$

$$= \frac{(x+e^x)(xe^x + e^x) - xe^x(1+e^x)}{(x+e^x)^2}$$

$$= \frac{(x+e^x)e^x(x+1) - xe^x(1+e^x)}{(x+e^x)^2}$$

$$= \frac{e^x[(x+e^x)(x+1) - x(1+e^x)]}{(x+e^x)^2}$$

Exercise 9.1 | Q 5.1 | Page 120

Find the derivative of the following function by the first principle.
$3x^2 + 4$

SOLUTION

Let $f(x) = 3x^2 + 4$

$\therefore f(x + h) = 3(x + h)^2 + 4$

$= 3(x^2 + 2xh + h^2) + 4$

$= 3x^2 + 6xh + 3h^2 + 4$

By first principle, we get

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{(3x^2 + 6xh + 3h^2 + 4) - (3x^2 + 4)}{h}$$

$$= \lim_{h\to 0} \frac{3h^2 + 6xh}{h}$$

$$= \lim_{h\to 0} \frac{h(3h + 6x)}{h}$$

$= \lim_{h \to 0} (6x + 3h)$ ...[∵ h→0, ∴h ≠0]

= 6x + 3(0)

= 6x

Exercise 9.1 | Q 5.2 | Page 120

Find the derivative of the following function by the first principle.

$x\sqrt{x}$.

SOLUTION

Let f(x) = $x\sqrt{x} = x^{\frac{3}{2}}$

∴ f(x + h) = $(x+h)^{\frac{3}{2}}$

By first principle, we get

f ′(x) = $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{(x+h)^{\frac{3}{2}} - x^{\frac{3}{2}}}{h}$

$= \lim_{h \to 0} \frac{\left[(x+h)^{\frac{3}{2}} - x^{\frac{3}{2}}\right]\left[(x+h)^{\frac{3}{2}} + x^{\frac{3}{2}}\right]}{h\left[(x+h)^{\frac{3}{2}} + x^{\frac{3}{2}}\right]}$

$= \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h\left[(x+h)^{\frac{3}{2}} + x^{\frac{3}{2}}\right]}$

$= \lim_{h \to 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h\left[(x+h)^{\frac{3}{2}} + x^{\frac{3}{2}}\right)}$

$$= \lim_{h \to 0} \frac{h\left(3x^2 + 3xh + h^2\right)}{h\left[(x+h)^{\frac{3}{2}+x^{\frac{3}{2}}}\right]}$$

$$= \lim_{h \to 0} \frac{h\left(3x^2 + 3xh + h^2\right)}{h\left[(x+h)^{\frac{3}{2}} + x^{\frac{3}{2}}\right]}$$

$$= \lim_{h \to 0} \frac{3x^2 + 3xh + h^2}{(x+h)^{\frac{3}{2}} + x^{\frac{3}{2}}}$$ ...[∵ h →0, ∴h≠0]

$$= \frac{3x^2 + 3 \times x0 + 0^2}{(x+0)^{\frac{3}{2}} + x^{\frac{3}{2}}}$$

$$= \frac{3x^2}{2x^{\frac{3}{2}}}$$

$$= \frac{3}{2} x^{\frac{1}{2}}$$

$$= \frac{3}{2}\sqrt{x}$$

Exercise 9.1 | Q 5.3 | Page 120

Find the derivative of the following functions by the first principle.

$$\frac{1}{2x+3}$$

SOLUTION

Let f(x) = $\frac{1}{2x+3}$

∴ f(x + h) = $\frac{1}{2(x+h)+3} = \frac{1}{2x+2h+3}$

By first principle, we get

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\left(\frac{1}{2x+2h+3}\right) - \left(\frac{1}{2x+3}\right)}{h}$$

$$= \lim_{h \to 0} \frac{1}{h} \left[\frac{2x+3-2x-2h-3}{(2x+2h+3)(2x+3)} \right.$$

$$= \lim_{h \to 0} \frac{1}{h} \frac{-2h}{(2x+2h+3)(2x+3)}$$

$$= \lim_{h \to 0} \frac{-2}{(2x+2h+3)(2x+3)} \quad ...[\because h \to 0, \therefore h \neq 0]$$

$$= \frac{-2}{(2x+2 \times 0+3)(2x+3)}$$

$$= \frac{-2}{(2x+3)^2}$$

Exercise 9.1 | Q 5.4 | Page 120

Find the derivative of the following function by the first principle.

$$\frac{x-1}{2x+7}$$

SOLUTION

Let $f(x) = \frac{x-1}{2x+7}$

$\therefore f(x+h) = \frac{x+h-1}{2(x+h)+7} = \frac{x+h-1}{2x+2h+7}$

By first principle, we get

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$= \lim_{h \to 0} \dfrac{\frac{x+h+1}{2x+2h+7} - \frac{x-1}{2x+7}}{h}$

$= \lim_{h \to 0} \dfrac{1}{h}\left[\dfrac{(x+h-1)(2x+7)-(x-1)(2x+2h+7)}{(2x+2h+7)(2x+7)}\right.$

$= \lim_{h \to 0} \dfrac{1}{h}\left[\dfrac{\left(2x^2+2xh-2x+7x+7h-7-2x^2-2xh-7x+2x+2h+7\right)}{(2x+2h+7)(2x+7)}\right]$

$= \lim_{h \to 0} \dfrac{1}{h}\left[\dfrac{9h}{(2x+2h+7)(2x+7)}\right]$

$= \dfrac{9}{(2x+2\times 0+7)(2x+7)}$

$= \dfrac{9}{(2x+7)^2}$

EXERCISE 9.2 [PAGES 122 - 123]

Exercise 9.2 | Q 1.1 | Page 122

Differentiate the following function w.r.t.x.

$\dfrac{x}{x+1}$

SOLUTION

Let y = $\dfrac{x}{x+1}$

Differentiating w.r.t. x, we get

$\dfrac{dy}{dx} = \dfrac{d}{dx}\left(\dfrac{x}{x+1}\right)$

$= \dfrac{(x+1)\frac{d}{dx}(x) - x\frac{d}{dx}(x+1)}{(x+1)^2}$

$$= \frac{(x+1)(1) - x(1+0)}{(x+1)^2}$$

$$= \frac{x+1-x}{(x+1)^2}$$

$$= \frac{1}{(x+1)^2}$$

Exercise 9.2 | Q 1.2 | Page 122

Differentiate the following function w.r.t.x

$$\frac{x^2+1}{x}$$

SOLUTION

Let y = $\frac{x^2+1}{x}$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{x^2+1}{x}\right)$$

$$= \frac{x\frac{d}{dx}\left(x^2+1\right) - \left(x^2+1\right)\frac{d}{dx}(x)}{x^2}$$

$$= \frac{x(2x+0) - \left(x^2+1\right)(1)}{x^2}$$

$$= \frac{2x^2 - x^2 - 1}{x^2}$$

$$\frac{dy}{dx} = \frac{x^2-1}{x^2}$$

Exercise 9.2 | Q 1.3 | Page 122

Differentiate the following function w.r.t.x.

$$\frac{1}{e^x + 1}$$

SOLUTION

Let y = $\frac{1}{e^x + 1}$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1}{e^x + 1}\right)$$

= $\frac{(e^x + 1)\frac{d}{dx}(1) - (1)\frac{d}{dx}(e^x + 1)}{(e^x + 1)2}$

= $\frac{(e^x + 1)(0) - (1)(e^x + 0)}{(e^x + 1)^2}$

= $\frac{e^x + 1 - e^x}{(e^x + 1)^2}$

= $\frac{1}{(e^x + 1)^2}$

Exercise 9.2 | Q 1.4 | Page 122

Differentiate the following function w.r.t.x

$$\frac{e^x}{e^x + 1}$$

SOLUTION

y= $\frac{e^x}{e^x+1}$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{e^x}{e^x+1}\right)$$

= $\frac{(e^x+1)\frac{d}{dx}(e^x) - \frac{d}{dx}(e^x+1)}{(e^x+1)^2}$

= $\frac{(e^x+1)e^x - e^x(e^x+0)}{(e^x+1)^2}$

= $\frac{e^x(e^x+1-e^x)}{(e^x+1)^2}$

= $\frac{e^x}{(e^x+1)^2}$

Exercise 9.2 | Q 1.5 | Page 122

Differentiate the following function w.r.t.x

$\frac{x}{\log x}$

SOLUTION

Let y = $\frac{x}{\log x}$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{x}{\log x}\right)$$

$= \dfrac{\log x \frac{d}{dx}(x) - x \frac{d}{dx}(\log x)}{(\log x)^2}$

$= \dfrac{\log x(1) - x\left(\frac{1}{x}\right)}{(\log x)^2}$

$= \dfrac{\log x - 1}{(\log x)^2}$

Exercise 9.2 | Q 1.6 | Page 122

Differentiate the following function w.r.t.x.

$\dfrac{2^x}{\log x}$

SOLUTION

Let y = $\dfrac{2^x}{\log x}$

Differentiating w.r.t. x, we get

$\dfrac{dy}{dx} = \dfrac{d}{dx}\left(\dfrac{2^x}{\log x}\right)$

$= \dfrac{\log x \frac{d}{dx}(2^x) - 2^x \frac{d}{dx}(\log x)}{(\log x)^2}$

$= \dfrac{\log x(2^x \log 2) - 2^x\left(\frac{1}{x}\right)}{(\log x)^2}$

$= \dfrac{(2^x \log x . \log 2)\left(-\frac{1}{x}\right)}{(\log x)^2}$

Exercise 9.2 | Q 1.7 | Page 122

Differentiate the following function w.r.t.x

$$\frac{(2e^x - 1)}{(2e^x + 1)}$$

SOLUTION

Let y = $\frac{2e^x - 1}{2e^x + 1}$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{2e^x - 1}{2e^x + 1}\right)$$

$$= \frac{(2e^x + 1)\frac{d}{dx}(2e^x - 1) - (2e^x - 1)\frac{d}{dx}(2e^x + 1)}{(2e^x + 1)^2}$$

$$= \frac{(2e^x + 1)(2e^x) - (2e^x - 1)(2e^x)}{(2e^x + 1)^2}$$

$$= \frac{2e^x(2e^x + 1 - 2e^x + 1)}{(2e^x + 1)^2}$$

$$= \frac{2e^x(2)}{(2e^x + 1)^2}$$

$$= \frac{4e^x}{(2e^x + 1)^2}$$

Exercise 9.2 | Q 1.8 | Page 122

Differentiate the following function w.r.t.x

$$\frac{(x + 1)(x - 1)}{(e^x + 1)}$$

SOLUTION

Let y = $\dfrac{(x+1)(x-1)}{(e^x+1)}$

∴ y = $\dfrac{x^2-1}{(e^x+1)}$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{x^2-1}{e^x+1}\right)$$

$$= \frac{(e^x+1)\frac{d}{dx}\left(x^2-1\right) - \left(x^2-1\right)\frac{d}{dx}(e^x+1)}{\left(e^x+1\right)^2}$$

$$= \frac{(x^x+1)(2x) - \left(x^2-1\right)(e^x+0)}{\left(e^x+1\right)^2}$$

$$= \frac{2xe^x+2x-x^2e^x+e^x}{\left(e^x+1\right)^2}$$

$$= \frac{2xe^x+e^x-x^2e^x+2x}{\left(e^x+1\right)^2}$$

$$= \frac{e^x\left(2x+1-x^2\right)+2x}{\left(e^x+1\right)^2}$$

Exercise 9.2 | Q 2.01 | Page 122

Solve the following example:

The demand D for a price P is given as D = 27/P, find the rate of change of demand when price is 3.

SOLUTION

Demand, D = $\dfrac{27}{P}$

Rate of change of demand = $\dfrac{dD}{dP}$

$= \dfrac{d}{dP}\left(\dfrac{27}{p}\right)$

$=27\dfrac{d}{d}P\left(\dfrac{1}{p}\right)$

$=27\dfrac{d}{dP}\left(\dfrac{1}{P}\right)$

$=27\dfrac{d}{dP}\left(P^{-1}\right)$

$=27((-1)P^{-2})$

$=27\left(\dfrac{-1}{p^2}\right) = \dfrac{-27}{p^2}$

When price P = 3,

Rate of change of demand,

$$\left(\frac{dD}{dP}\right)_{p=3} = \frac{-27}{(3)^2} = -3$$

∴ When price is 3, Rate of change of demand is -3.

Exercise 9.2 | Q 2.02 | Page 122

Solve the following example:

If for a commodity; the price-demand relation is given as D = $\dfrac{P+5}{P-1}$. Find the marginal demand when price is 2.

SOLUTION

Given, D = $\dfrac{P+5}{P-1}$

Marginal demand = $\dfrac{dD}{dP} = \dfrac{d}{dP}\left(\dfrac{P+5}{P-1}\right)$

= $\dfrac{(p-1)\frac{d}{dP}(p+5)-(p+5)\frac{d}{dP}(p-1)}{(P-1)^2}$

= $\dfrac{(p-1)(1+0)-(p+5)(1-0)}{(P-1)^2}$

= $\dfrac{P-1-P-5}{(P-1)^2}$

= $\dfrac{-6}{(P-1)^2}$

When P = 2,

Marginal demand, $\left(\dfrac{dP}{dP}\right)_{P=2}$

= $\dfrac{-6}{(2-1)^2} = -6$

∴ When price is 2, marginal demand is -6.

Exercise 9.2 | Q 2.03 | Page 122

Solve the following example:
The demand function of a commodity is given as P = 20 + D−D^2. Find the rate at which price is changing when demand is 3.

SOLUTION

Given, P = 20 + D - D^2

Rate of change of price = $\dfrac{dP}{dD}$

= $\dfrac{d}{dD}\left(20 + D - D^2\right)$

= 0 + 1 - 2D

= 1 - 2D

Rate of change of price at D = 3 is

$\left(\dfrac{dP}{dD}\right)_{D=3}$

= 1 - 2(3) = - 5

∴ Price is changing at a rate of -5 when demand is 3.

Exercise 9.2 | Q 2.04 | Page 122

Solve the following example:

If the total cost function is given by; C = $5x^3 + 2x^2 + 7$; find the average cost and the marginal cost when x = 4.

SOLUTION

Total cost function, C = $5x^3 + 2x^2 + 7$

Average cost = $\frac{C}{x}$

$= \frac{5x^3 + 2x^2 + 7}{x}$

$= 5x^2 + 2x + \frac{7}{x}$

When x = 4,

Average cost = $5(4)^2 + 2(4) + \frac{7}{4}$

$= 80 + 8 + \frac{7}{4}$

$= \frac{320 + 32 + 7}{4}$

$= \frac{359}{4}$

Marginal cost = $\frac{dC}{dx}$

$= \frac{d}{dx}\left(5x^3 + 2x^2 + 7\right)$

$= 5\frac{d}{dx}\left(x^3\right) + 2\frac{d}{dx}\left(x^2\right) + \frac{d}{dx}(7)$

$= 5(3x^2) + 2(2x) + 0$

$= 15x^2 + 4x$

When x = 4, Marginal cost = $\left(\frac{dC}{dx}\right)_{x=4}$

$= 15(4)^2 + 4(4)$

$= 240 + 16$

$= 256$

$\therefore$ the average cost and marginal cost at x = 4 are $\frac{359}{4}$ and 256 respectively.

Exercise 9.2 | Q 2.05 | Page 122

Solve the following example:
The total cost function of producing n notebooks is given by C= 1500 − 75n + 2n^2 + n 3/5.

Find the marginal cost at n = 10.

SOLUTION

Total cost function,

$C = 1500 - 75n + 2n^2 + \frac{n^3}{5}$

Marginal Cost = $\frac{dC}{dn}$

$= \frac{d}{dn}\left(1500 - 75n + 2n^2 + \frac{n^3}{5}\right)$

$= \frac{d}{dn}(1500) - 75\frac{d}{dn}(n) + 2\frac{d}{dn}(n^2) + \frac{1}{5}\frac{d}{dn}(n^3)$

$= 0 - 75(1) + 2(2n) + \frac{1}{5}(3n^2)$

$= -75 + 4n + \frac{3n^2}{5}$

When n = 10,

Marginal cost

$$= \left(\frac{dC}{dn}\right)_{n=10} = -75 + 4(10) + \frac{3}{5}(10)^2$$

= -75 + 40 + 60

= 25

Exercise 9.2 | Q 2.06 | Page 123

Solve the following example:
The total cost of ‘t’ toy cars is given by $C=5(2^t) + 17$.
Find the marginal cost and average cost at t=3.

SOLUTION

Total cost of ‘t’ toy cars, $C = 5(2^t) + 17$

$$\text{Marginal Cost} = \frac{dC}{dt}$$

$$= \frac{d}{dt}\left[5\left(2^t\right)17\right]$$

$$=5\frac{d}{dt}\left(2^t\right) + \frac{d}{dt}(17)$$

$= 5(2^t .\log 2) + 0$

$= 5(2^t .\log 2)$

When t = 3,

$$\text{Marginal cost} = \left(\frac{dC}{dt}\right)_{t=3}$$

$= 5(2^3. \log 2) = 40 \log 2$

$$\text{Average cost} = \frac{C}{t} = \frac{5(2)^t + 17}{t}$$

$= \dfrac{40+17}{3} = 19$

$\therefore$ at t = 3, Marginal cost is 40 log 2 and Average cost is 19.

Exercise 9.2 | Q 2.07 | Page 123

Solve the following example:

If for a commodity; the demand function is given by, D = $\sqrt{75-3P}$. find the marginal demand function when P = 5

SOLUTION

Demand function, D = $\sqrt{75-3P}$

Now, Marginal demand = $\dfrac{dD}{dP}$

$= \dfrac{d}{dP}\left(\sqrt{75-3P}\right)$

$= \dfrac{1}{2\sqrt{75-3P}} \cdot \dfrac{d}{dP}(75-3P)$

$= \dfrac{1}{2\sqrt{75-3P}} \cdot (0-3\times 1)$

$= \dfrac{-3}{2\sqrt{75-3P}}$

When P = 5,

Marginal demand = $\left(\dfrac{dD}{dP}\right)_{P=5}$

$= \dfrac{-3}{2\sqrt{75-3(5)}}$

$= \dfrac{-3}{2\sqrt{60}}$

$= \dfrac{-3}{4\sqrt{15}}$

$\therefore$ Marginal demand $= \dfrac{-3}{4\sqrt{15}}$ at P = 5.

Exercise 9.2 | Q 2.08 | Page 123

Solve the following example:

The total cost of producing x units is given by $C=10e^{2x}$, find its marginal cost and average cost when x = 2

SOLUTION

Total cost, $C = 10e^{2x}$

Marginal cost $= \dfrac{dC}{dx}$

$= \dfrac{d}{dx}\left(10e^2x\right) = 10\dfrac{d}{dx}\left(e^2x\right)$

$= 10.\, e^2x.\, \dfrac{d}{dx}(2x) = 10.\, e^2x.2(1)$

$= 20e^{2x}$

When x = 2,

Marginal cost $= \left(\dfrac{dC}{dx}\right)_{x=2}$

$= 20e^4$

Average cost $= \dfrac{C}{x}$

$= \dfrac{10e^2x}{x}$

When x = 2 average cost = $\frac{10e^4}{2}$ = $5e^4$

∴ When x = 2, marginal cost is $20e^4$ and average cost is $5e^4$.

Exercise 9.2 | Q 2.09 | Page 123

Solve the following example:
The demand function is given as $P = 175 + 9D + 25D^2$. Find the revenue, average revenue, and marginal revenue when demand is 10.

SOLUTION

Given, $P = 175 + 9D + 25D^2$

Total revenue, R = P.D

$= (175 + 9D + 25D^2)D$

$= 175D + 9D^2 + 25D^3$

Average revenue = P = $175 + 9D + 25D^2$

Marginal revenue = $\frac{dR}{dD}$

$= \frac{d}{dD}\left(175D + 9D^2 + 25D^3\right)$

$= 175\frac{d}{dD}(D) + 9\frac{d}{d}D\left(D^2\right) + 25\frac{d}{dD}\left(D^3\right)$

$= 175(1) + 9(2D) + 25(3D^2)$

$= 175 + 18D + 75D^2$

When D = 10,

Total revenue = $175(10) + 9(10)^2 + 25(10)^3$

= 1750 + 900 + 25000 = 27650

Average revenue = $175 + 9(10) + 25(10)^2$

= 175 + 90 + 2500 = 2765

Marginal revenue = 175 + 18(10) + 75(10)^2

= 175 + 180 + 7500 = 7855

∴ When Demand = 10,

Total revenue = 27650,

Average revenue = 2765

Marginal revenue = 7855.

Exercise 9.2 | Q 2.1 | Page 123

Solve the following example:
The supply S for a commodity at price P is given by S = $P^2 + 9P - 2$. Find the marginal supply when price is 7.

SOLUTION

Given, S = P2 + 9P - 2

Marginal supply = $\frac{dS}{dP}$

$= \frac{d}{dP}\left(p^2 + 9P - 2\right)$

$= \frac{d}{dP}\left(P^2\right) + 9\frac{d}{dP}(P) - \frac{d}{dP}(2)$

= 2P + 9(1) -0

= 2P + 9

When P = 7,

Marginal supply = $\left(\frac{dS}{dP}\right)_{P=7}$

= 2(7) + 9

= 14 + 9 = 23

∴ Marginal supply is 23, at P = 7.

Exercise 9.2 | Q 2.11 | Page 123

Solve the following example:

The cost of producing x articles is given by C = x^2 + 15x + 81. Find the average cost and marginal cost functions. Find marginal cost when x = 10. Find x for which the marginal cost equals the average cost.

SOLUTION

Given, cost C = x^2 + 15x + 81

Average cost = $\dfrac{C}{x} = \dfrac{x^2 + 15x + 81}{x}$

= x + 15 + $\dfrac{81}{x}$

and Marginal cost = $\dfrac{dC}{dx}$

= $\dfrac{d}{dx}\left(x^2 + 15x + 81\right)$

= $\dfrac{d}{dx}\left(x^2\right) + 15\dfrac{d}{dx}(x) + \dfrac{d}{dx}(81)$

= 2x + 15(1) + 0 = 2x + 15

When x = 10,

Marginal cost = $\left(\dfrac{dC}{dx}\right)_{x=10}$

= 2(10) + 15 = 35

If marginal cost = average cost, then

2x + 15 = x + 15 + $\dfrac{81}{x}$

∴ x = $\dfrac{81}{x}$

$\therefore x^2 = 81$

$\therefore x = 9$ ...[$\because x > 0$]

MISCELLANEOUS EXERCISE 9 [PAGES 123 - 124]

Miscellaneous Exercise 9 | Q 1.1 | Page 123

Differentiate the following function .w.r.t.x
x^5

SOLUTION

Let $y = x^5$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}x^5 = 5x^4$$

Miscellaneous Exercise 9 | Q 1.2 | Page 123

Differentiate the following function w.r.t.x
x^{-2}

SOLUTION

Let $y = x^{-2}$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^{-2}\right) = -2x^{-3} = \frac{-2}{x^3}$$

Miscellaneous Exercise 9 | Q 1.3 | Page 123

Differentiate the following functions w.r.t.x.
$\sqrt{x}$

SOLUTION

Let y = $\sqrt{x}$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$$

Miscellaneous Exercise 9 | Q 1.4 | Page 123

Differentiate the following function w.r.t.x

$x\sqrt{x}$

SOLUTION

Let y = $x\sqrt{x}$

$\therefore y = x^{\frac{3}{2}}$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}x^{\frac{3}{2}} = \frac{3}{2}x^{\frac{1}{2}}$$

Miscellaneous Exercise 9 | Q 1.5 | Page 123

Differentiate the followingfunctions.w.r.t.x.

$$\frac{1}{\sqrt{x}}$$

SOLUTION

Let y = $\dfrac{1}{\sqrt{x}}$

$\therefore y = x^{\frac{-1}{2}}$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{-1}{2}x^{\frac{-3}{2}} = \frac{-1}{2x^{\frac{3}{2}}}$$

Miscellaneous Exercise 9 | Q 1.6 | Page 123

Differentiate the following functions. w.r.t.x
7^x

SOLUTION

Let y = 7^x

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}7^x = 7^x \log 7$$

Miscellaneous Exercise 9 | Q 2.01 | Page 123

Find $\dfrac{dy}{dx}$ **if**

$$y = x^2 + \frac{1}{x^2}$$

SOLUTION

$$y = x^2 + \frac{1}{x^2}$$

$\therefore$ y = x^2 +x^{-2}

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^2 + x^{-2}\right)$$

= $\frac{d}{dx}\left(x^2\right) + \frac{d}{dx}\left(x^{-2}\right)$

= 2x – 2x^{-3}

= $2x - \frac{2}{x^3}$

Miscellaneous Exercise 9 | Q 2.02 | Page 123

Find $\frac{dy}{dx}$ **if**

$$y = \left(\sqrt{x} + 1\right)^2$$

SOLUTION

$$y = \left(\sqrt{x} + 1\right)^2$$

$$\therefore y = x + 2\sqrt{x} + 1$$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(x + 2\sqrt{x} + 1\right)$$

= $\frac{d}{dx}(x) + 2\frac{d}{dx}\left(\sqrt{x}\right) + \frac{d}{dx}(1)$

= $1+2\left(\frac{1}{2\sqrt{x}}\right)+0$

= $\frac{dy}{dx}=1+\frac{1}{\sqrt{x}}$

Miscellaneous Exercise 9 | Q 2.03 | Page 123

Find $\frac{dy}{dx}$ if

$y=\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2$

SOLUTION

$y=\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2$

∴ y = x + 2 + $\frac{1}{x}$

Differentiating w.r.t. x, we get

$\frac{dy}{dx}=\frac{d}{dx}\left(x+2+\frac{1}{x}\right)$

= $\frac{d}{dx}(x)+\frac{d}{dx}(2)+\frac{d}{dx}\left(\frac{1}{x}\right)$

= $1+0+\frac{d}{dx}\left(x^{-1}\right)$

= $1 + (-1)\ x^{-2}$

= $1-\ \frac{1}{x^2}$

Miscellaneous Exercise 9 | Q 2.04 | Page 123

Find $\dfrac{dy}{dx}$ if

$y = x^3 – 2x^2 + \sqrt{x} + 1$

SOLUTION

$y = x^3 – 2x^2 + \sqrt{x} + 1$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^3 - 2x^2 + \sqrt{x} + 1\right)$$

$$= \frac{d}{dx}\left(x^3\right) - 2\frac{d}{dx}\left(x^2\right) + \frac{d}{dx}\left(\sqrt{x}\right) + \frac{d}{dx}(1)$$

$$= 3x^2 - 2(2x) + \frac{d}{dx}\left(x^{\frac{1}{2}}\right) + 0$$

$$= 3x^2 - 4x + \frac{1}{2}x^{\frac{1}{2}-1}$$

$$= 3x^2 - 4x + \frac{1}{2}x^{\frac{-1}{2}}$$

$$\frac{dy}{dx} = 3x^2 - 4x + \frac{1}{2\sqrt{x}}$$

Miscellaneous Exercise 9 | Q 2.05 | Page 123

Find $\dfrac{dy}{dx}$ if

$y = x^2 + 2^x – 1$

SOLUTION

$y = x^2 + 2^x - 1$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^2 + 2^x - 1\right)$$

$$= \frac{d}{dx}\left(x^2\right) + \frac{d}{dx}\left(2^x\right) - \frac{d}{dx}(1)$$

$= 2x + 2^x \log 2 - 0$

$= 2x + 2^x \log 2$

Miscellaneous Exercise 9 | Q 2.06 | Page 123

Find $\frac{dy}{dx}$ if

$y = (1 - x)(2 - x)$

SOLUTION

$y = (1 - x)(2 - x)$

$= 2 - 3x + x^2$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(2 - 3x + x^2\right)$$

$$= \frac{d}{dx}(2) - 3\frac{d}{dx}(x) + \frac{d}{dx}\left(x^2\right)$$

$= 0 - 3(1) + 2x$

$= -3 + 2x$

Miscellaneous Exercise 9 | Q 2.07 | Page 123

Find $\dfrac{dy}{dx}$ **if**

$$y = \frac{1+x}{2+x}$$

SOLUTION

$$y = \frac{1+x}{2+x}$$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1+x}{2+x}\right)$$

$$= \frac{(2+x)\frac{d}{dx}(1+x) - (1+x)\frac{d}{dx}(2+x)}{(2+x)^2}$$

$$= \frac{(2+x)(0+1) - (1+x)(0+1)}{(2+x)^2}$$

$$\frac{dy}{dx} = \frac{(2+x) - (1+x)}{(2+x)^2}$$

$$= \frac{2+x-1-x}{(2+x)^2}$$

$$= \frac{1}{(2+x)^2}$$

Miscellaneous Exercise 9 | Q 2.08 | Page 123

Find $\dfrac{dy}{dx}$ **if**

$$y = \frac{(\log x + 1)}{x}$$

SOLUTION

$$y = \frac{(\log x + 1)}{x}$$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left[\frac{\log x + 1}{x}\right]$$

$$\frac{x \frac{d}{dx}(\log x + 1) - (\log x + 1)\frac{d}{dx}(x)}{x^2}$$

$$\frac{x\left(\frac{1}{x} + 0\right) - (\log x + 1)(1)}{x^2}$$

$$= \frac{1 - \log x - 1}{x^2}$$

$$= \frac{-\log x}{x^2}$$

Miscellaneous Exercise 9 | Q 2.09 | Page 123

Find $\frac{dy}{dx}$ if

$$y = \frac{e^x}{\log x}$$

SOLUTION

$$y = \frac{e^x}{\log x}$$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{e^x}{\log x}\right)$$

$$= \frac{(\log x)\frac{d}{dx}(e^x) - (e^x)\frac{d}{dx}(\log x)}{(\log x)^2}$$

$$= \frac{(\log x)e^x - e^x\left(\frac{1}{x}\right)}{(\log x)^2}$$

$$= \frac{e^x\left(\log x - \frac{1}{x}\right)}{(\log x)^2}$$

Miscellaneous Exercise 9 | Q 2.1 | Page 123

Find $\frac{dy}{dx}$ if

$y = x \log x (x^2 + 1)$

SOLUTION

$y = x \log x (x^2 + 1)$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}(x)(\log x)\left(x^2+1\right)$$

$$(x)(\log x)\frac{d}{dx}\left(x^2+1\right) - \left(x^2+1\right)\frac{d}{dx}((x)(\log x))$$

$$= (x\log x)(2x+0) + \left(x^2+1\right)\left[x\frac{d}{dx}(\log x) + (\log x)\frac{d}{dx}(x)\right]$$

$$=2x^2\log x + \left(x^2+1\right)\left[x \times \frac{1}{x} + (\log x)(1)\right]$$

$= 2x^2 \log x + (x^2 + 1) (1 + \log x)$

$=2x^2 \log x + (x^2 + 1) + (x^2 + 1) \log x$

Miscellaneous Exercise 9 | Q 3.01 | Page 124

Solve the following.

The relation between price (P) and demand (D) of a cup of Tea is given by D = 12/P. Find the rate at which the demand changes when the price is Rs. 2/- Interpret the result.

SOLUTION

Demand, D = $\dfrac{12}{P}$

Rate of change of demand = $\dfrac{dD}{dP}$

$= \dfrac{d}{dP}\left(\dfrac{12}{P}\right)$

$= 12\dfrac{d}{dP}\left(P^{-1}\right) - 12\left((-1)P^{-2}\right)$

$= 12\left(\dfrac{-1}{P^2}\right) = \dfrac{-12}{P^2}$

When price P = 2,

Rate of change of demand, $\left(\dfrac{dD}{dP}\right)_{P=2} = \dfrac{-12}{(2)^2} = -3$

∴ When price is 2, Rate of change of demand is -3

Here, rate of change of demand is negative

∴ demand would fall when the price becomes ₹ 2.

Miscellaneous Exercise 9 | Q 3.02 | Page 124

Solve the following.

The demand (D) of biscuits at price P is given by D = 64/P^3, find the marginal demand when price is Rs. 4/-.

SOLUTION

Given demand D = $\frac{64}{P^3}$

Now, marginal demand = $\frac{dD}{dP}$

$= \frac{d}{dP}\left(\frac{64}{p^3}\right)$

$= 64\frac{d}{dP}\left(p^{-3}\right)$

= 64 (– 3) P^{-4}

$= \frac{-192}{p^4}$

When P = 4

Marginal demand = $\left(\frac{dD}{dP}\right)_{p=4}$

$= \frac{-192}{(4)^4}$

$= \frac{-192}{256}$

$= \frac{-3}{4}$

Miscellaneous Exercise 9 | Q 3.03 | Page 124

Solve the following:
The supply S of electric bulbs at price P is given by $S = 2P^3 + 5$. Find the marginal supply when the price is Rs. 5/- Interpret the result.

SOLUTION

Given, supply S = $2p^3 + 5$

Now, marginal supply = $\frac{dS}{dp}$

$= \frac{d}{dp}\left(2p^3 + 5\right)$

$= 2\frac{d}{dp}\left(p^3\right) + \frac{d}{dp}(5)$

= $2(3p^2)+0$

= $6p^2$

∴ When p = 5

Marginal supply = $\left(\frac{dS}{dp}\right)_{p=5}$

$= 6(5)^2 = 150$

Here, the rate of change of supply with respect to the price is positive which indicates that the supply increases.

Miscellaneous Exercise 9 | Q 3.04 | Page 124
Solve the following:
The marginal cost of producing x items is given by C = $x^2 + 4x + 4$. Find the average cost and the marginal cost. What is the marginal cost when x = 7.

SOLUTION

Total cost, C = $x^2 + 4x + 4$

Now, Average cost = $\frac{c}{x} = \frac{x^2 + 4x + 4}{x}$

$= x + 4 + \frac{4}{x}$

and Marginal cost = $\frac{dc}{dx} \frac{d}{dx}\left(x^2 + 4x + 4\right)$

$= \frac{d}{dx}\left(x^2\right) + 4\frac{d}{dx}(x) + \frac{d}{dx}(4)$

= 2x + 4(1) + 0

= 2x + 4

∴ When x = 7,

Marginal cost = $\left(\frac{dC}{dx}\right)_{x=7}$

= 2(7) + 4

= 14 + 4

= 18

Miscellaneous Exercise 9 | Q 3.05 | Page 124

Solve the following:
The Demand D for a price P is given as D = 27/P, Find the rate of change of demand when the price is Rs. 3/-.

SOLUTION

Demand, D = $\frac{27}{P}$

Rate of change of demand = $\frac{dD}{dP}$

$= \frac{d}{dP}\left(\frac{27}{P}\right)$

$= 27\frac{d}{dP}\left(\frac{1}{P}\right)$

$= 27\frac{d}{dP}\left(p^{-1}\right)$

= $27((-1)p^{-2})$

$$=27\left(\frac{-1}{p^2}\right)=\frac{-27}{p^2}$$

When price P = 3,

Rate of change of demand,

$$\left(\frac{dD}{dP}\right)_{p=3}=\frac{-27}{(3)^2}=-3$$

∴ When price is 3, Rate of change of demand is -3.

Miscellaneous Exercise 9 | Q 3.06 | Page 124

Solve the following.

If for a commodity; the price demand relation is given be D = $\left(\frac{P+5}{P-1}\right)$. Find the marginal demand when price is Rs. 2/-.

SOLUTION

Given, D = $\left(\frac{P+5}{P-1}\right)$

Marginal demand = $\left(\frac{dD}{dP}\right)=\frac{d}{dP}\left(\frac{P+5}{P-1}\right)$

$$=\frac{(P-1)\frac{d}{dP}(P+5)-(P+5)\frac{d}{dP}(P-1)}{(P-1)^2}$$

$$=\frac{(P-1)(1+0)-(P+5)(1-0)}{(P-1)^2}$$

$$=\frac{P-1-P-5}{(P-1)^2}$$

$$=\frac{-6}{(P-1)^2}$$

When P = 2,

Marginal demand, $\left(\frac{dP}{dP}\right)_{P=2}$

$= \frac{-6}{(2-1)^2}$

= -6

∴ When price is 2, marginal demand is -6.

Miscellaneous Exercise 9 | Q 3.07 | Page 124

Solve the following.
The price function P of a commodity is given as P = 20 + D − D^2 where D is demand. Find the rate at which price (P) is changing when demand D = 3.

SOLUTION

Given, P = 20 + D - D^2

Rate of change of price = $\frac{dP}{dP}$

$= \frac{d}{dD}\left(20 + D - D^2\right)$

= 0 + 1 - 2D

= 1 -2D

Rate of change of price at D = 3 is

$\left(\frac{dP}{dD}\right)_{D=3}$ = 1 -2(3) = -5

∴ Price is changing at a rate of -5 when demand is 3.

Miscellaneous Exercise 9 | Q 3.08 | Page 124

Solve the following.
If the total cost function is given by C = 5x^3 + 2x^2 + 1; Find the average cost and the marginal cost when x = 4.

SOLUTION

Total cost function C = $5x^3 + 2x^2 + 1$

Average cost = $\frac{C}{x}$

$= \frac{5x^3 + 2x^2 + 1}{x}$

$= 5x^2 + 2x + \frac{1}{x}$

When x = 4, Average cost = $5(4)^2 + 2(4) + \frac{1}{4}$

$= 80 + 8 + \frac{1}{4}$

$= \frac{320 + 32 + 1}{4}$

$= \frac{353}{4}$

Marginal cost = $\frac{dC}{dx}$

$= \frac{d}{dx}\left(5x^3 + 2x^2 + 1\right)$

$=5\frac{d}{dx}\left(x^3\right) + 2\frac{d}{dx}\left(x^2\right) + \frac{d}{dx}(1)$

$=5(3x^2) + 2(2x) + 0$

$= 15x^2 + 4x$

When x = 4, marginal cost = $\left(\frac{dC}{dx}\right)_{x=4}$

$= 15(4)^2 + 4(4)$

$= 240 + 16$

$= 256$

$\therefore$ The average cost and marginal cost at x = 4 are $\frac{353}{4}$ and 256 respectively.

Miscellaneous Exercise 9 | Q 3.09 | Page 124

Solve the following.
The supply S for a commodity at price P is given by S= $P^2 + 9P - 2$. Find the marginal supply when price Rs. 7/-.

SOLUTION

Given, $S = P^2 + 9P - 2$

Marginal supply = $\frac{dS}{dP}$

$= \frac{d}{dP}\left(P^2 + 9P - 2\right)$

$= \frac{d}{dP}\left(p^2\right) + 9\frac{d}{dP}(P) - \frac{d}{dP}(2)$

$= 2P + 9(1) - 0$

$= 2P + 9$

When P = 7,

Marginal supply = $\left(\frac{dS}{dP}\right)_{P=7}$

$= 2(7) + 9$

$= 14 + 9 = 23$

$\therefore$ Marginal supply is 23, at P = 7.

Miscellaneous Exercise 9 | Q 3.1 | Page 124

Solve the following.
The cost of producing x articles is given by C = $x^2 + 15x + 81$. Find the average cost and marginal cost functions. Find the marginal cost when x = 10. Find x for which the marginal cost equals the average cost.

SOLUTION

Given, cost C = $x^2 + 15x + 81$

Average cost = $\frac{C}{x} = \frac{x^2 + 15x + 81}{x}$

= $x + 15 + \frac{81}{x}$

and Marginal cost = $\frac{dC}{dx}$

= $\frac{d}{dx}\left(x^2 + 15x + 81\right)$

= $\frac{d}{dx}\left(x^2\right) + 15\frac{d}{dx}(x) + \frac{d}{dx}(81)$

= 2x + 15(1) + 0 = 2x + 15

When x = 10,

Marginal cost = $\left(\frac{dC}{dx}\right)_{x=10}$

= 2(10) + 15 = 35

If marginal cost = average cost, then

2x + 15 = x + 15 + $\frac{81}{x}$

∴ x = $\frac{81}{x}$

∴ $x^2 = 81$

∴ x = 9 ...[∵ x > 0]