CHAPTER 1: APPLICATION OF MATRICES AND DETERMINANTS

Formulae:

1. $(AB)^{-1} = B^{-1} A^{-1}$ if A & B are non singular matrices of same order

2. $(A^{-1})^{-1} = A$ if A is non singular, A^T is non singular

3. if A is non singular square matrix order n, Then

   i) $(adj A)^{-1} = (adj A^{-1}) = \frac{1}{|A|} A$ (ii) $|adj A| = |A|^{n-1}$

   (iii) $adj (adj A) = |A|^{n-1} A$ (iv) $adj(\lambda A) = \lambda^{n-1} adj A$, λ is nonzero scalar

   (v) $|adj(adj A)| = |A|^{(n-1)^2}$ (vi) $(adj A)^T = adj (A^T)$

4) $adj (AB) = (adj B)(adj A)$

5) A square matrix A is called orthogonal if $AA^T = A^T A = I$

6) i) $A^{-1} = \frac{1}{|A|} adj A$ iii) $(A^{-1})^{-1} = A$

7) $(\lambda A)^{-1} = \frac{1}{\lambda} A^{-1}$ iii) $A(adj A) = (adj A) A = |A| I$.

8) $A^{-1} = \pm \frac{1}{\sqrt{|adj A|}} adj A$ 9) $A = \pm \frac{1}{\sqrt{|adj A|}} adj(adj A)$

9) Methods to solve the system of linear equations $Ax = B$

  i) by matrix Inversion method $X = A^{-1} B$, $|A| \neq 0$

  ii) by cramer's rule $x = \frac{\Delta x}{\Delta}$, $y = \frac{\Delta y}{\Delta}$, $z = \frac{\Delta z}{\Delta}$, $\Delta \neq 0$.

  iii) by Gaussian elimination (Rank) method

10) i) if $\rho(A) = \rho(A|B) < n$ Then the system has infinitely many solutions

  ii) if $\rho(A) = \rho(A|B) = n$ Then The system has unique solution

  iii) if $\rho(A) \neq \rho(A|B)$ then The system is in consistent and has no solution.

11) The homogenous system of linear equation $Ax = 0$

  i) has the trival solution if $|A| \neq 0$

  ii) has a non trival solution if $|A| = 0$.

11) A matrix A is orthogonal iff A is non singular and $A^{-1} = A^T$.

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Exercise 1·1

1. Find the adjoint of the following.

(i) $\begin{bmatrix} -3 & 4 \\ 6 & 2 \end{bmatrix}$

Soln

Let $A = \begin{bmatrix} -3 & 4 \\ 6 & 2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ [∵ interchanging a_{22} & a_{11}

Sign changing a_{12} & a_{21}

$adj A = \begin{bmatrix} 2 & -4 \\ -6 & -3 \end{bmatrix}$

(ii)

$A = \begin{bmatrix} 2 & 3 & 1 \\ 3 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix}$

$adj A = \begin{bmatrix} 8-7 & 7-6 & 3-4 \\ 3-6 & 4-3 & 3-2 \\ 21-12 & 9-14 & 8-9 \end{bmatrix}$

$= \begin{bmatrix} 1 & 1 & -1 \\ -3 & 1 & 1 \\ 9 & -5 & -1 \end{bmatrix}$

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(iii) $A = \frac{1}{3}\begin{bmatrix} 2 & 2 & 1 \\ -2 & 1 & 2 \\ 1 & -2 & 2 \end{bmatrix}_{3\times3}$ $adj(KA) = K^{n-1} adj A$

Soln $n = 3$

$adj (kA) = \left(\frac{1}{3}\right)^{3-1} \begin{bmatrix} 2+4 & -2-4 & 4-1 \\ 2+4 & 4-1 & -2-4 \\ 4-1 & 2+4 & 2+4 \end{bmatrix}$

$= \frac{1}{9}\begin{bmatrix} 6 & -6 & 3 \\ 6 & 3 & -6 \\ 3 & 6 & 6 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 2 & -2 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 2 \end{bmatrix}$

2. Find the inverse (if it exists) of the following

(i) $A = \begin{pmatrix} -2 & 4 \\ 1 & -3 \end{pmatrix}$

$|A| = 6-4 = 2 \neq 0$ ∴ A^{-1} exists.

A is non singular

$adj A = \begin{pmatrix} -3 & -4 \\ -1 & -2 \end{pmatrix}$

$A^{-1} = \frac{1}{|A|} adj A = \frac{1}{2}\begin{pmatrix} -3 & -4 \\ -1 & -2 \end{pmatrix}$

(vii) $\quad A = \begin{bmatrix} 5 & 1 & 1 \\ 1 & 5 & 1 \\ 1 & 1 & 5 \end{bmatrix}$

Soln

$|A| = 5(25-1) - 1(5-1) + 1(1-5)$

$\quad = 5(24) - 1(4) + 1(-4) = 120 - 4 - 4 = 112 \neq 0$

$\qquad\qquad\qquad\qquad A^{-1}$ is exist

$adj A = \begin{pmatrix} 25-1 & 1-5 & 1-5 \\ 1-5 & 25-1 & 1-5 \\ 1-5 & 1-5 & 25-1 \end{pmatrix}$

$\quad = \begin{pmatrix} 24 & -4 & -4 \\ -4 & 24 & -4 \\ -4 & -4 & 24 \end{pmatrix}$

$A^{-1} = \dfrac{1}{|A|} adj A = \dfrac{1}{112} \begin{pmatrix} 24 & -4 & -4 \\ -4 & 24 & -4 \\ -4 & -4 & 24 \end{pmatrix}$

$\quad = \dfrac{1}{28} \begin{pmatrix} 6 & -1 & -1 \\ -1 & 6 & -1 \\ -1 & -1 & 6 \end{pmatrix}$

(iii) $\quad A = \begin{pmatrix} 2 & 3 & 1 \\ 3 & 4 & 1 \\ 3 & 7 & 2 \end{pmatrix}$

Soln

$|A| = 2(8-7) - 3(6-3) + 1(21-12) = 2 - 9 + 9 = 2 \neq 0$

$\qquad\qquad\qquad\qquad A^{-1}$ is exist

$adj A = \begin{bmatrix} 8-7 & 7-6 & 3-4 \\ 3-6 & 4-3 & 3-2 \\ 21-12 & 9-14 & 8-9 \end{bmatrix}$

$\quad = \begin{bmatrix} 1 & 1 & -1 \\ -3 & 1 & 1 \\ 9 & -5 & -1 \end{bmatrix}$

$A^{-1} = \dfrac{1}{|A|} adj A = \dfrac{1}{2} \begin{bmatrix} 1 & 1 & -1 \\ -3 & 1 & 1 \\ 9 & -5 & -1 \end{bmatrix}$

3) If $F(\alpha) = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix}$ Show that $[F(\alpha)]^{-1} = F(-\alpha)$

Soln

$|F(\alpha)| = \cos\alpha(\cos\alpha - 0) - 0 + \sin\alpha(0 + \sin\alpha) = \cos^2\alpha + \sin^2\alpha = 1$

$\qquad |F(\alpha)| = 1 \neq 0$

$adj\, F(\alpha) = \begin{bmatrix} \cos\alpha & 0 & \#\sin\alpha \\ 0 & 1 & 0 \\ +\sin\alpha & 0 & \cos\alpha \end{bmatrix}$

$$F(\alpha)^{-1} = \frac{1}{|F(\alpha)|} \text{ adj } F(\alpha) = \frac{1}{1}\begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix}$$ ④

$$= \begin{bmatrix} \cos\alpha & 0 & +\sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix} - ①$$

$$F(-\alpha) = \begin{bmatrix} \cos(-\alpha) & 0 & -\sin(-\alpha) \\ 0 & 1 & 0 \\ \sin(-\alpha) & 0 & \cos\alpha \end{bmatrix} = \begin{bmatrix} \cos\alpha & 0 & +\sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix}$$

$$F(-\alpha) = F(\alpha)^{-1}$$

$$\begin{bmatrix} \because \cos(-\theta) = \cos\theta \\ \sin(-\theta) = -\sin\theta \end{bmatrix}$$

4) If $A = \begin{pmatrix} 5 & 3 \\ -1 & -2 \end{pmatrix}$ Show that $A^2 - 3A - 7I_2 = 0$ Hence find A^{-1}

Soln

$$A^2 = A \times A = \begin{pmatrix} 5 & 3 \\ -1 & -2 \end{pmatrix}\begin{pmatrix} 5 & 3 \\ -1 & -2 \end{pmatrix} = \begin{pmatrix} 25-3 & 15+6 \\ -5+2 & -3+4 \end{pmatrix} = \begin{pmatrix} 22 & 9 \\ -3 & 1 \end{pmatrix}$$

$$-3A = -3\begin{pmatrix} 5 & 3 \\ -1 & -2 \end{pmatrix} = \begin{pmatrix} -15 & -9 \\ 3 & 6 \end{pmatrix}$$

$$-7I_2 = -7\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} -7 & 0 \\ 0 & -7 \end{pmatrix}$$

$$A^2 - 3A - 7I_2 = \begin{pmatrix} 22 & 9 \\ -3 & 1 \end{pmatrix} + \begin{pmatrix} -15 & -9 \\ 3 & 6 \end{pmatrix} + \begin{pmatrix} -7 & 0 \\ 0 & -7 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\therefore A^2 - 3A - 7I_2 = 0$$

Post multiply this eqn by A^{-1}

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$$A - 3I - 7A^{-1} = 0$$

$$A - 3I = 7A^{-1}$$

$$A^{-1} = \frac{1}{7}(A - 3I) = \frac{1}{7}\left[\begin{pmatrix} 5 & 3 \\ -1 & -2 \end{pmatrix} - 3\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right]$$

$$= \frac{1}{7}\left[\begin{pmatrix} 5 & 3 \\ -1 & -2 \end{pmatrix} + \begin{pmatrix} -3 & 0 \\ 0 & -3 \end{pmatrix}\right] = \frac{1}{7}\begin{bmatrix} 2 & 3 \\ -1 & -5 \end{bmatrix}$$

5) If $A = \frac{1}{9}\begin{bmatrix} -8 & 1 & 4 \\ 4 & 4 & 7 \\ 1 & -8 & 4 \end{bmatrix}$ Prove that $A^{-1} = A^T$

Soln

RHS $A^T = \frac{1}{9}\begin{bmatrix} -8 & 4 & 1 \\ 1 & 4 & -8 \\ 4 & 7 & 4 \end{bmatrix}$

(6)

LHS

$$n=3$$

$$|A| = \left(\frac{1}{9}\right)^3 \left[-8(16+56) -1(16-7) + 4(-32-4) \right] \qquad \because |KA| = K^n |A|$$

$$= \frac{1}{729} \left[-8(72) -1(9) + 4(-36) \right] = \frac{1}{729} \left(-576 - 23 - 144 \right)$$

$$= \frac{1}{729}(-729) = -1 \neq 0$$

$$\therefore A^{-1} \text{ exist.}$$

$$\left| adj(KA) \right| = K^{n-1} adj A$$

$$adj A = \left(\frac{1}{9}\right)^{3-1} \begin{bmatrix} 16+56 & -32-4 & 7-16 \\ 7-16 & -32-4 & 16+56 \\ -32-4 & 1-64 & -32-4 \end{bmatrix}$$

$$= \left(\frac{1}{9}\right)^2 \begin{bmatrix} 72 & -36 & -9 \\ -9 & -36 & 72 \\ -36 & -63 & -36 \end{bmatrix}$$

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$$= \frac{1}{9} \begin{bmatrix} 8 & -4 & -1 \\ -1 & -4 & 8 \\ -4 & -7 & -4 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} adj A = \frac{1}{-1} \left(\frac{1}{9} \begin{pmatrix} 8 & -4 & -1 \\ -1 & -4 & 8 \\ -4 & -7 & -4 \end{pmatrix} \right) = \frac{1}{9} \begin{pmatrix} -8 & 4 & 1 \\ 1 & +4 & 8 \\ 4 & 7 & 4 \end{pmatrix} \quad — ②$$

by ① & ② $\boxed{A^T = A^{-1}}$

6) If $A = \begin{pmatrix} 8 & -4 \\ -5 & 3 \end{pmatrix}$ verify that $A(adj A) = (adj A) A = |A| I_2$

Soln

$$|A| = 24 - 20 = 4 \neq 0 \qquad A^{-1} \text{ exist.}$$

$$adj A = \begin{pmatrix} 3 & 4 \\ 5 & 8 \end{pmatrix}$$

$$A(adj A) = \begin{pmatrix} 8 & -4 \\ -5 & 3 \end{pmatrix} \begin{pmatrix} 3 & 4 \\ 5 & 8 \end{pmatrix} = \begin{pmatrix} 24-20 & 32-32 \\ -15+15 & -20+24 \end{pmatrix} = \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix} — ①$$

$$(adj A) A = \begin{pmatrix} 3 & 4 \\ 5 & 8 \end{pmatrix} \begin{pmatrix} 8 & -4 \\ -5 & 3 \end{pmatrix} = \begin{pmatrix} 24-20 & -12+12 \\ 40-40 & -20+24 \end{pmatrix} = \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix} — ②$$

$$|A| I_2 = 4 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix} — ③$$

by ①, ② & ③ $A(adj A) = (adj A) A = |A| I_2$.

7) If $A = \begin{pmatrix} 3 & 2 \\ 7 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} -1 & -3 \\ 5 & 2 \end{pmatrix}$ verify that $(AB)^{-1} = B^{-1}A^{-1}$ ⑦

Soln

$|A| = 15 - 14 = 1 \neq 0$ A^{-1} exist.

$adj\, A = \begin{pmatrix} 5 & -2 \\ 7 & 3 \end{pmatrix}$

$A^{-1} = \frac{1}{|A|} adj\, A$

$= \frac{1}{1} \begin{pmatrix} 5 & -2 \\ 7 & 3 \end{pmatrix} = \begin{pmatrix} 5 & -2 \\ 7 & 3 \end{pmatrix}$

$|B| = -2 + 15 = 13 \neq 0$

$adj\, B = \begin{pmatrix} 2 & 3 \\ -5 & -1 \end{pmatrix}$

$B^{-1} = \frac{1}{|B|} adj\, B = \frac{1}{13} \begin{pmatrix} 2 & 3 \\ -5 & -1 \end{pmatrix}$

$\underline{\underline{RHS}}$

$B^{-1}A^{-1} = \frac{1}{13}\begin{pmatrix} 2 & 3 \\ -5 & -1 \end{pmatrix} \times \begin{pmatrix} 5 & -2 \\ 7 & 3 \end{pmatrix} = \frac{1}{13}\begin{pmatrix} 10-21 & -4+9 \\ -25+7 & 10-3 \end{pmatrix}$

$= \frac{1}{13}\begin{pmatrix} -11 & 5 \\ -18 & 7 \end{pmatrix}$

$\underline{\underline{LHS}}$ $AB = \begin{pmatrix} 3 & 2 \\ 7 & 5 \end{pmatrix}\begin{pmatrix} -1 & -3 \\ 5 & 2 \end{pmatrix} = \begin{pmatrix} -3+10 & -9+4 \\ -7+25 & -21+10 \end{pmatrix} = \begin{pmatrix} 7 & -5 \\ 18 & -11 \end{pmatrix}$

$|AB| = -77 + 90 = 13 \neq 0$

$adj\, AB = \begin{pmatrix} -11 & 5 \\ +18 & 7 \end{pmatrix}$

$(AB)^{-1} = \frac{1}{|AB|} adj\, AB = \frac{1}{13}\begin{pmatrix} -11 & 5 \\ 18 & 7 \end{pmatrix}$

8) If $adj\, A = \begin{pmatrix} 2 & -4 & 2 \\ -3 & 12 & -7 \\ -2 & 0 & 2 \end{pmatrix}$ find A

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Soln

$A = \pm \frac{1}{\sqrt{|adj\, A|}} adj\, (adj\, A)$

$|adj\, A| = 2(24-0) + 4(-6-14) + 2(0+24) = 48 - 80 + 48 = 16$

$adj\,(adj\, A) = \begin{pmatrix} 24 & 8 & 4 \\ 20 & 8 & 8 \\ 24 & 8 & 12 \end{pmatrix} = \frac{1}{4}\begin{pmatrix} 6 & 2 & 1 \\ 5 & 2 & 2 \\ 6 & 2 & 3 \end{pmatrix}$

$\sqrt{|adj\, A|} = \sqrt{16} = 4$

$A = \pm\, 4\left(\frac{1}{4}\right)\begin{pmatrix} 6 & 2 & 1 \\ 5 & 2 & 2 \\ 6 & 2 & 3 \end{pmatrix} = \pm \begin{pmatrix} 6 & 2 & 1 \\ 5 & 2 & 2 \\ 6 & 2 & 3 \end{pmatrix}$

9) If $adj\, A = \begin{pmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{pmatrix}$ find A^{-1}

soln

$$A^{-1} = \pm \frac{1}{\sqrt{|adj\, A|}}\ adj\, A$$

$|adj\, A| = 0 + 2(36 - 18) + 0 = 2(18) = 36.$

$$A^{-1} = \pm \frac{1}{\sqrt{36}} \begin{pmatrix} 6 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{pmatrix} = \pm \frac{1}{6} \begin{pmatrix} 6 & -2 & 0 \\ 6 & 2 & -6 \\ 3 & 0 & 6 \end{pmatrix}$$

10) Find $adj\,(adj\, A)$ if $adj\, A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ -1 & 0 & 1 \end{pmatrix}$

soln

$$adj\,(adj\, A) = \begin{pmatrix} 2-0 & 0-0 & 0-2 \\ 0-0 & 1+1 & 0-0 \\ 0+2 & 0-0 & 2-0 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 0 & -2 \\ 0 & 2 & 0 \\ 2 & 0 & 2 \end{pmatrix}$$

11) $A = \begin{pmatrix} 1 & \tan x \\ -\tan x & 1 \end{pmatrix}$ Show that $A^T A^{-1} = \begin{pmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{pmatrix}$

soln

$$|A| = 1 + \tan^2 x = \sec^2 x$$

$$adj\, A = \begin{pmatrix} 1 & -\tan x \\ \tan x & 1 \end{pmatrix}$$

$$A^{-1} = \frac{1}{|A|}\ adj\, A = \frac{1}{\sec^2 x} \begin{pmatrix} 1 & -\tan x \\ \tan x & 1 \end{pmatrix}$$

$$A^T = \begin{pmatrix} 1 & -\tan x \\ \tan x & 1 \end{pmatrix}$$

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$$A^T A^{-1} = \frac{1}{\sec^2 x} \begin{pmatrix} 1 & -\tan x \\ \tan x & 1 \end{pmatrix} \begin{pmatrix} 1 & -\tan x \\ \tan x & 1 \end{pmatrix}$$

$$= \frac{1}{\sec^2 x} \begin{pmatrix} 1-\tan^2 x & -\tan x - \tan x \\ \tan x + \tan x & -\tan^2 x + 1 \end{pmatrix}$$

$$= \frac{1}{\sec^2 x} \begin{pmatrix} 1-\tan^2 x & -2\tan x \\ 2\tan x & 1-\tan^2 x \end{pmatrix} = \cos^2 x \begin{pmatrix} 1-\dfrac{\sin^2 x}{\cos^2 x} & -2\dfrac{\sin x}{\cos x} \\ 2\dfrac{\sin x}{\cos x} & 1-\dfrac{\sin^2 x}{\cos^2 x} \end{pmatrix}$$

$$= \begin{pmatrix} \cos^2 x - \sin^2 x & -2\sin x \cos x \\ 2\sin x \cos x & \cos^2 x - \sin^2 x \end{pmatrix} = \begin{pmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{pmatrix}$$

$$= RHS.$$

$$\left[\because \cos 2A = \cos^2 A - \sin^2 A \atop \sin 2A = 2 \sin A \cos A \right]$$

Hence proved.

(12) Find the matrix A for which $A \begin{pmatrix} 5 & 3 \\ -1 & -2 \end{pmatrix} = \begin{pmatrix} 14 & 7 \\ 7 & 7 \end{pmatrix}$

soln order of A is 2×2.

let us take $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$$AB = C_{2\times 2}$$
$$m \times \boxed{n} \quad n \times p \quad m \times p$$
$$n = 2.$$
$$m = 2.$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 5 & 3 \\ -1 & -2 \end{pmatrix} = \begin{pmatrix} 14 & 7 \\ 7 & 7 \end{pmatrix}$$

$$\begin{pmatrix} 5a-b & 3a-2b \\ 5c-d & 3c-2d \end{pmatrix} = \begin{pmatrix} 14 & 7 \\ 7 & 7 \end{pmatrix}$$

$$5a - b = 14 \quad — ① \qquad 5c - d = 7 \quad — ③$$
$$3a - 2b = 7 \quad — ② \qquad 3c - 2d = 7 \quad — ④$$

② $-2 \times$ ① $\Rightarrow$
$$\begin{array}{r} 3a - 2b = 7 \\ -10a + 2b = -28 \\ \hline -7a = -21 \end{array}$$
$$\boxed{a = 3}$$

④ $-2 \times$ ③ $\Rightarrow$
$$\begin{array}{r} 3c - 2d = 7 \\ -10c + 2d = -14 \\ \hline -7c = -7 \end{array}$$
$$\boxed{c = 1}$$

① $\Rightarrow$
$$15 - b = 14$$
$$15 - 14 = b$$
$$\boxed{b = 1}$$

③ $\Rightarrow$
$$5 - d = 7$$
$$5 - 7 = d$$
$$d = -2$$

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$$\therefore A = \begin{pmatrix} 3 & 1 \\ 1 & -2 \end{pmatrix}$$

(13) Given $A = \begin{pmatrix} 1 & -1 \\ 2 & 0 \end{pmatrix}$ $B = \begin{pmatrix} 3 & -2 \\ 1 & 1 \end{pmatrix}$ $C = \begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix}$ find a matrix x such that $AXB = C$

soln

$$AXB = C$$
$$A^{-1}A (X) BB^{-1} = A^{-1} C B^{-1} \qquad (\text{Pre \& Post multiply by } A^{-1} \text{ and } B^{-1})$$
$$X = A^{-1} C B^{-1}$$

$$|A| = 0 + 2 = 2 \neq 0$$
$$adj A = \begin{pmatrix} 0 & 1 \\ -2 & 1 \end{pmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} adj A = \frac{1}{2} \begin{pmatrix} 0 & 1 \\ -2 & 1 \end{pmatrix}$$

$|B| = \qquad 3+2 = \qquad 5$

$adj\ B = \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}$ $\therefore B^{-1} = \dfrac{1}{|B|} adj\ B = \dfrac{1}{5}\begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}$

$X = A^{-1}\ C\ B^{-1}$

$= \dfrac{1}{2}\begin{pmatrix} 0 & 1 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix}\dfrac{1}{5}\begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}$

$= \dfrac{1}{10}\begin{bmatrix} 0+2 & 0+2 \\ -2+2 & -2+2 \end{bmatrix}\begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix} = \dfrac{1}{10}\begin{pmatrix} 2 & 2 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}$

$= \dfrac{1}{10}\begin{pmatrix} 2-2 & 4+6 \\ 0-0 & 0-0 \end{pmatrix} = \dfrac{1}{10}\begin{pmatrix} 0 & 10 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$

14. If $A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ show that $A^{-1} = \dfrac{1}{2}(A^2 - 3I)$

soln

$A^2 = A \times A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$

$A^2 - 3I = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix} - 3\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix} + \begin{pmatrix} -3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -3 \end{pmatrix}$

$= \begin{pmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix} \quad —①$

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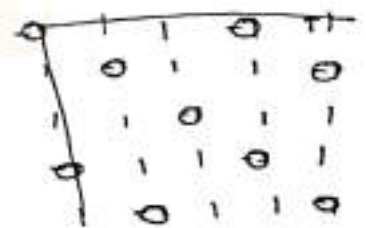முதுகலை ஆசிரியர் (கணிதம்)
அரசு மேல்நிலைப்பள்ளி
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$|A| = 0 - 1(0-1) + 1(1-0) = 1+1 = 2$

$adj\ A = \begin{pmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix}$

$A^{-1} = \dfrac{1}{|A|} adj\ A = \dfrac{1}{2}\begin{pmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix}$

$A^{-1} = \dfrac{1}{2}(A^2 - 3I)$ (using ①)

Hence proved.

15) Decrypt the received encoded message $(2 \ -3) \ (20 \ 4)$ with the encryption matrix $\begin{pmatrix} -1 & -1 \\ 2 & 1 \end{pmatrix}$ and the decryption matrix as its inverse. where the system of codes are described by the numbers 1-26 to the letters A-Z respectively and the number 0 to a blank space.

Soln

Encoding matrix $\begin{pmatrix} -1 & -1 \\ 2 & 1 \end{pmatrix}$

un coded row matrix	Encoding matrix	coded row matrix

$(2 \ -3)$ $\begin{pmatrix} -1 & -1 \\ 2 & 1 \end{pmatrix}$ $= (-2-6 \quad -2-3)$

$= (-8 \quad -5)$

$(20 \ 4)$ $\begin{pmatrix} -1 & -1 \\ 2 & 1 \end{pmatrix}$ $= (-20+8 \quad -20+4) = (-12 \ -16)$

So the encoded message is $(-8 \ -5) \ (-12 \ -16)$

The receiver will decode the message by the inverse key - post multiplying by the inverse of A.

So the decoding matrix is $A^{-1} = \frac{1}{|A|} adj A$

$|A| = -1+2 = 1$

$adj A = \begin{pmatrix} 1 & 1 \\ -2 & -1 \end{pmatrix}$ $\therefore A^{-1} = \begin{pmatrix} 1 & 1 \\ -2 & -1 \end{pmatrix}$

The receiver decodes the coded message as follows

Coded row matrix	Decoding matrix	Decoded row matrix

$(-8 \ -5)$ $\begin{pmatrix} 1 & 1 \\ -2 & -1 \end{pmatrix}$ $= (-8+10 \quad -8+5) = (2 \ -3)$

$(-12 \ -16)$ $\begin{pmatrix} 1 & 1 \\ -2 & -1 \end{pmatrix}$ $= (-12+32 \quad -12+16) = (20 \ 4)$

So the sequence of decoded row matrices is $(2, -3) \ (20, 4)$

Thus the receiver reads the message is

$(-8 \quad -5) \ (-12 \quad -16)$

H   E   L   P

word is $\boxed{HELP}$

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⑫

Exercise 1·2

1. Find the rank of the following matrices by minor method.

(i) $A = \begin{pmatrix} 2 & -4 \\ -1 & 2 \end{pmatrix}$

$\begin{vmatrix} 2 & -4 \\ -1 & 2 \end{vmatrix} = 4 - 4 = 0.$

Rank of A is 1

$\qquad \rho(A) = 1$

(ii) $A = \begin{pmatrix} -1 & 3 \\ 4 & -7 \\ 3 & -4 \end{pmatrix}$

$\begin{vmatrix} -1 & 3 \\ 4 & -7 \end{vmatrix} = 7 - 12 = -5 \neq 0$

$\qquad \rho(A) = 2$

(iii) $A = \begin{pmatrix} 1 & -2 & -1 & 0 \\ 3 & -6 & -3 & 1 \end{pmatrix}$

$\begin{vmatrix} 1 & -2 \\ 3 & -6 \end{vmatrix} = 6 - 6 = 0$

$\begin{vmatrix} -1 & 0 \\ -3 & 1 \end{vmatrix} = -1 - 0 = -1 \neq 0$

$\qquad \rho(A) = 2$

(iv) $A = \begin{pmatrix} 1 & -2 & 3 \\ 2 & 4 & -6 \\ 5 & 1 & -1 \end{pmatrix}$

$|A| = 1(-4+6) + 2(-2+30) + 3(2-20)$

$= 1(-2) + 2(18) + 3(-18) = 2 + 56 - 54 = 4 \neq 0$

$\qquad \rho(A) = 3$

(v) $A = \begin{pmatrix} 0 & 1 & 2 & 1 \\ 0 & 2 & 4 & 3 \\ 8 & 1 & 0 & 2 \end{pmatrix}$

$\begin{vmatrix} 0 & 1 & 2 \\ 0 & 2 & 4 \\ 8 & 1 & 0 \end{vmatrix} = 8(4-4) = 8(0) = 0$

$\begin{vmatrix} 1 & 2 & 1 \\ 2 & 4 & 3 \\ 1 & 0 & 2 \end{vmatrix} = 1(8) - 2(4-3) + 1(0-4)$

$\qquad\qquad = 8 - 2(1) + 1(-4) = 8 - 2 - 4 = 2 \neq 0$

$\qquad \rho(A) = 3$

2) Find the rank of the following matrices by row reduction method.

i) $A = \begin{pmatrix} 1 & 1 & 1 & 3 \\ 2 & -1 & 3 & 4 \\ 5 & -1 & 7 & 11 \end{pmatrix}$

$\sim \begin{pmatrix} 1 & 1 & 1 & 3 \\ 0 & -6 & 2 & -4 \\ 0 & -3 & 1 & -2 \end{pmatrix}$ $\begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 5R_1 \end{array}$

$\sim \begin{pmatrix} 1 & 1 & 1 & 3 \\ 0 & -3 & 1 & -2 \\ 0 & -6 & 2 & -4 \end{pmatrix}$ $R_3 \leftrightarrow R_2$

$\sim \begin{pmatrix} 1 & 1 & 1 & 3 \\ 0 & -3 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ $R_3 \to R_3 - 2R_2$

$\qquad \rho(A) = 2$

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ii) $A = \begin{pmatrix} 1 & 2 & -1 \\ 3 & -1 & 2 \\ 1 & -2 & 3 \\ 1 & -1 & 1 \end{pmatrix}$

$\sim \begin{pmatrix} 1 & 2 & -1 \\ 0 & -7 & 5 \\ 0 & -4 & 4 \\ 0 & -3 & 2 \end{pmatrix}$ $\begin{array}{l} R_2 \to R_2 - 3R_1 \\ R_3 \to R_3 - R_1 \\ R_4 \to R_4 - R_1 \end{array}$

$\sim \begin{pmatrix} 1 & 2 & -1 \\ 0 & 7 & -5 \\ 0 & 4 & -4 \\ 0 & 3 & -2 \end{pmatrix}$ $\begin{array}{l} R_2 \to (-1)R_2 \\ R_3 \to (-1)R_3 \\ R_4 \to (-1)R_4 \end{array}$

$\sim \begin{pmatrix} 1 & 2 & -1 \\ 0 & 7 & -5 \\ 0 & 0 & -8 \\ 0 & 0 & 1 \end{pmatrix}$ $\begin{array}{l} R_3 \to 7R_3 - 4R_2 \\ R_4 \to 7R_4 - 3R_2 \end{array}$

$\sim \begin{pmatrix} 1 & 2 & -1 \\ 0 & 7 & -5 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}$ $R_3 \to \dfrac{R_3}{-8}$

$\sim \begin{pmatrix} 1 & 2 & -1 \\ 0 & 7 & -5 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ $R_4 \to R_4 - R_3$

$\qquad \rho(A) = 3.$

(14)   (14)

$$A = \begin{pmatrix} 3 & -8 & 5 & 2 \\ 2 & -5 & 1 & 4 \\ -1 & 2 & 3 & -2 \end{pmatrix}$$

$$\sim \begin{pmatrix} -1 & 2 & 3 & -2 \\ 2 & 5 & 1 & 4 \\ 3 & -8 & 5 & 2 \end{pmatrix} \quad R_1 \longleftrightarrow R_3$$

$$\sim \begin{pmatrix} 1 & -2 & -3 & 2 \\ 2 & 5 & 1 & 4 \\ 3 & -8 & 5 & 2 \end{pmatrix} \quad R_1 \to (-) R_1$$

$$\sim \begin{pmatrix} 1 & -2 & -3 & 2 \\ 0 & -1 & 7 & 0 \\ 0 & -2 & 14 & -4 \end{pmatrix} \quad \begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \end{array}$$

$$\sim \begin{pmatrix} 1 & -2 & -3 & 2 \\ 0 & -1 & 7 & 0 \\ 0 & 0 & 0 & -4 \end{pmatrix} \quad R_3 \to R_3 - 2R_1$$

$$\therefore \; \rho(A) = 3$$

3) Find the inverse of each of the following by Gauss Jordan method

(i) $A = \begin{pmatrix} 2 & -1 \\ 5 & -2 \end{pmatrix}$

soln

$$[A \mid I_2] = \begin{bmatrix} 2 & -1 & | & 1 & 0 \\ 5 & -2 & | & 0 & 1 \end{bmatrix} \xrightarrow{R_1 \to \frac{R_1}{2}} \begin{bmatrix} 1 & -\frac{1}{2} & | & \frac{1}{2} & 0 \\ 5 & -2 & | & 0 & 1 \end{bmatrix}$$

$$\xrightarrow{R_2 \to R_2 - 5R_1} \begin{bmatrix} 1 & -\frac{1}{2} & | & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & | & -\frac{5}{2} & 1 \end{bmatrix} \xrightarrow{R_2 \to 2R_2} \begin{bmatrix} 1 & -\frac{1}{2} & | & \frac{1}{2} & 0 \\ 0 & 1 & | & -5 & 2 \end{bmatrix}$$

$$\xrightarrow{R_1 \to R_1 + \frac{1}{2} R_2} \begin{bmatrix} 1 & 0 & | & -2 & 1 \\ 0 & 1 & | & -5 & 2 \end{bmatrix}$$

$\therefore$ inverse of matrix A is $\begin{pmatrix} -2 & 1 \\ -5 & 2 \end{pmatrix}$

(ii) $A = \begin{pmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 6 & -2 & -3 \end{pmatrix}$

soln

$$[A \mid I_3] = \begin{pmatrix} 1 & -1 & 0 & | & 1 & 0 & 0 \\ 1 & 0 & -1 & | & 0 & 1 & 0 \\ 6 & -2 & -3 & | & 0 & 0 & 1 \end{pmatrix}$$

$R_2 \to R_2 - R_1$
$R_3 \to R_3 - 6R_1$ $\longrightarrow$
$\begin{pmatrix} 1 & -1 & 0 & | & 1 & 0 & 0 \\ 0 & 1 & -1 & | & -1 & 1 & 0 \\ 0 & 4 & -3 & | & -6 & 0 & 1 \end{pmatrix}$

$R_3 \to R_3 - 4R_2$ $\longrightarrow$
$\begin{pmatrix} 1 & -1 & 0 & | & 1 & 0 & 0 \\ 0 & 1 & -1 & | & -1 & 1 & 0 \\ 0 & 0 & 1 & | & -2 & -4 & 1 \end{pmatrix}$

$R_2 \to R_2 + R_3$ $\longrightarrow$
$\begin{pmatrix} 1 & -1 & 0 & | & 1 & 0 & 0 \\ 0 & 1 & 0 & | & -3 & -3 & 1 \\ 0 & 0 & 1 & | & -2 & -4 & 1 \end{pmatrix}$

$R_1 \to R_1 + R_2$ $\longrightarrow$
$\begin{pmatrix} 1 & 0 & 0 & | & -2 & -3 & 1 \\ 0 & 1 & 0 & | & -3 & -3 & 1 \\ 0 & 0 & 1 & | & -2 & -4 & 1 \end{pmatrix}$

inverse of A is $\begin{pmatrix} -2 & -3 & 1 \\ -3 & -3 & 1 \\ -2 & -4 & 1 \end{pmatrix}$

(iii) $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{pmatrix}$

soln

$[A | I_3] = \begin{pmatrix} 1 & 2 & 3 & | & 1 & 0 & 0 \\ 2 & 5 & 3 & | & 0 & 1 & 0 \\ 1 & 0 & 8 & | & 0 & 0 & 1 \end{pmatrix}$

$R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - R_1$ $\longrightarrow$
$\begin{pmatrix} 1 & 2 & 3 & | & 1 & 0 & 0 \\ 0 & 1 & -3 & | & -2 & 1 & 0 \\ 0 & -2 & 5 & | & -1 & 0 & 1 \end{pmatrix}$

$R_3 \to R_3 + 2R_2$ $\longrightarrow$
$\begin{pmatrix} 1 & 2 & 3 & | & 1 & 0 & 0 \\ 0 & 1 & -3 & | & -2 & 1 & 0 \\ 0 & 0 & 1 & | & 5 & -2 & -1 \end{pmatrix}$

$R_2 \to R_2 + 3R_3$ $\longrightarrow$
$\begin{pmatrix} 1 & 2 & 3 & | & 1 & 0 & 0 \\ 0 & 1 & -3 & | & -2 & -1 & 0 \\ 0 & 0 & 1 & | & 5 & -2 & 1 \end{pmatrix}$

$R_2 \to R_2 + R_1$ $\longrightarrow$
$\begin{pmatrix} 1 & 2 & 3 & | & 1 & 0 & 0 \\ 0 & 1 & 0 & | & 13 & -5 & -3 \\ 0 & 0 & 1 & | & 5 & -2 & -1 \end{pmatrix}$

$R_1 \to R_1 - 2R_2$ $\longrightarrow$
$\begin{pmatrix} 1 & 0 & 3 & | & -25 & 10 & 6 \\ 0 & 1 & 0 & | & 13 & -5 & -3 \\ 0 & 0 & 1 & | & 5 & -2 & -1 \end{pmatrix}$

$R_1 \to R_1 - 3R_3$ $\longrightarrow$
$\begin{pmatrix} 1 & 0 & 0 & | & -40 & 16 & 9 \\ 0 & 1 & 0 & | & 13 & -5 & 3 \\ 0 & 0 & 1 & | & 5 & -2 & -1 \end{pmatrix}$

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Inverse of matrix is $\begin{pmatrix} -40 & 16 & 9 \\ 13 & -5 & 3 \\ 5 & -2 & -1 \end{pmatrix}$

Exercise 1·3

1. Solve the following system of linear equation by matrix inversion method.

(i) $2x + 5y = -2$, $x + 2y = -3$

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Soln

$\begin{pmatrix} 2 & 5 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 \\ -3 \end{pmatrix}$

$Ax = B$

$X = A^{-1} B$ $\therefore A = \begin{pmatrix} 2 & 5 \\ 1 & 2 \end{pmatrix}$

$|A| = 4 - 5 = -1$

$adj\, A = \begin{pmatrix} 2 & -5 \\ -1 & 2 \end{pmatrix}$

$A^{-1} = \dfrac{1}{|A|} adj\, A = \dfrac{1}{-1} \begin{pmatrix} 2 & -5 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} -2 & 5 \\ 1 & -2 \end{pmatrix}$

$X = A^{-1} B = \begin{pmatrix} -2 & 5 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} -2 \\ -3 \end{pmatrix} = \begin{pmatrix} 4-15 \\ -2+6 \end{pmatrix} = \begin{pmatrix} -11 \\ 4 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -11 \\ 4 \end{pmatrix}$ $\therefore \boxed{\begin{array}{l} x = -11 \\ y = 4 \end{array}}$

(ii) $2x - y = 8$, $3x - 2y = -2$

Soln

$\begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 8 \\ -2 \end{pmatrix}$ Hence $A = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix}$

$Ax = B$

$X = A^{-1} B$ $|A| = -4 + 3 = -1$

$adj\, A = \begin{pmatrix} -2 & 1 \\ -3 & 2 \end{pmatrix}$

$X = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix} \begin{pmatrix} 8 \\ -2 \end{pmatrix}$

$A^{-1} = \dfrac{1}{|A|} adj\, A = \dfrac{1}{-1} \begin{pmatrix} -2 & 1 \\ -3 & 2 \end{pmatrix}$

$= \begin{pmatrix} 16+2 \\ 24+4 \end{pmatrix} = \begin{pmatrix} 18 \\ 28 \end{pmatrix}$ $= \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 18 \\ 28 \end{pmatrix}$

$\boxed{\begin{array}{l} x = 18 \\ y = 28 \end{array}}$

(17)

(iv) $2x + 3y - z = 9$, $x + y + z = 9$, $3x - y - z = -1$

Soln

$$\begin{pmatrix} 2 & 3 & -1 \\ 1 & 1 & 1 \\ 3 & -1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 9 \\ 9 \\ -1 \end{pmatrix}$$

$A \quad x \quad = \quad B$ where $A = \begin{pmatrix} 2 & 3 & -1 \\ 1 & 1 & 1 \\ 3 & -1 & 1 \end{pmatrix}$

$x \quad = A^{-1} B$

$|A| = 2(-1+1) - 3(-1-3) - 1(-1-3) = 0 + 12 + 4 = 16$

$adj A = \begin{pmatrix} -1+1 & 1+3 & 3+1 \\ 3+1 & -2+3 & -1-2 \\ -1-3 & 9+2 & 2-3 \end{pmatrix}$

$= \begin{pmatrix} 0 & 4 & 4 \\ 4 & 1 & -3 \\ -4 & 11 & -1 \end{pmatrix}$

$A^{-1} = \dfrac{1}{|A|} adj A = \dfrac{1}{16} \begin{pmatrix} 0 & 4 & 4 \\ 4 & 1 & -3 \\ -4 & 11 & -1 \end{pmatrix}$

$X = A^{-1} B = \dfrac{1}{16} \begin{pmatrix} 0 & 4 & 4 \\ 4 & 1 & -3 \\ -4 & 11 & -1 \end{pmatrix} \begin{pmatrix} 9 \\ 9 \\ -1 \end{pmatrix} = \dfrac{1}{16} \begin{pmatrix} 36 - 4 \\ 36 + 9 + 3 \\ -36 + 99 + 1 \end{pmatrix}$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \dfrac{1}{16} \begin{pmatrix} 32 \\ 48 \\ 64 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$

	2	3	-1
	1	1	1
3	-1	-1	3 -1
	3	-1	2 3
	1	1	1 1

$\boxed{\begin{aligned} x &= 2 \\ y &= 3 \\ z &= 4 \end{aligned}}$

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(v) $x + y + z - 2 = 0$, $6x - 4y + 5z - 31 = 0$, $5x + 2y + 2z = 13$.

Soln

$$\begin{pmatrix} 1 & 1 & 1 \\ 6 & -4 & 5 \\ 5 & 2 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 31 \\ 13 \end{pmatrix}$$

$A x = B$ where $A = \begin{pmatrix} 1 & 1 & 1 \\ 6 & -4 & 5 \\ 5 & 2 & 2 \end{pmatrix}$

$x = A^{-1} B$.

$|A| = 1(-8-10) - 1(12-25) + 1(12+20)$

$= -18 + 13 + 32 = 27$.

$$\text{adj } A = \begin{pmatrix} -8-10 & 2-2 & 5+4 \\ 25-12 & 2-5 & 6-5 \\ 12+20 & 5-2 & -4-6 \end{pmatrix}$$

$$= \begin{pmatrix} -18 & 0 & 9 \\ 13 & -3 & 1 \\ 32 & 3 & -10 \end{pmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{ adj } A = \frac{1}{27} \begin{pmatrix} -18 & 0 & 9 \\ 13 & -3 & 1 \\ 32 & 3 & -10 \end{pmatrix}$$

$$X = A^{-1} B = \frac{1}{27} \begin{pmatrix} -18 & 0 & 9 \\ 13 & -3 & 1 \\ 32 & 3 & -10 \end{pmatrix} \begin{pmatrix} 2 \\ 31 \\ 13 \end{pmatrix} = \frac{1}{27} \begin{pmatrix} -36+0+117 \\ 26-93+13 \\ 64+93-130 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{27} \begin{pmatrix} 81 \\ -54 \\ 27 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$$

$$\boxed{\begin{matrix} x = 3 \\ y = -2 \\ z = 1 \end{matrix}}$$

2. If $A = \begin{pmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{pmatrix}$ find the products AB and BA and hence solve the system of equations $x+y+2z=1$, $3x+2y+z=7$, $2x+y+3z=2$.

Soln:

$$AB = \begin{pmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{pmatrix} = \begin{pmatrix} -5+3+6 & -5+2+3 & -10+1+9 \\ 7+3-10 & 7+2-5 & 14+1-15 \\ 1-3+2 & 1-2+1 & 2-1+3 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix} = 4 I_3$$

$$BA = \begin{pmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{pmatrix} \begin{pmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{pmatrix} = \begin{pmatrix} -5+7+2 & 1+1-2 & 3-5+2 \\ -15+14+1 & 3+2-1 & 9-10+1 \\ -10+7+3 & 2+1-3 & 6-5+3 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix} = 4 I_3.$$

$$AB = BA = 4 I_3$$

$$\left(\frac{1}{4} A\right) B = B\left(\frac{1}{4} A\right) = I \quad \Rightarrow \quad B^{-1} = \frac{1}{4} A.$$

(18)

matrix form $\begin{pmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 7 \\ 2 \end{pmatrix}$

$$A \quad x \quad = \quad B$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{4} A \begin{pmatrix} 1 \\ 7 \\ 2 \end{pmatrix} = \frac{1}{4} \begin{pmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 7 \\ 2 \end{pmatrix}$$

$$= \frac{1}{4} \begin{pmatrix} -5+7+6 \\ 7+7-10 \\ 1-7+2 \end{pmatrix} = \frac{1}{4} \begin{pmatrix} 8 \\ 4 \\ -4 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$$

$$\boxed{x = 2, \ y = 2, \ z = -1}$$

7 A man is appointed in a job with a monthy salary of certain amount and a fixed amount of annual increment. If his salary was ₹ 19800 per month at the end of the first month after 3 years of service and ₹ 23400 per month at the end of the first month after 9 years of service. find his starting salary and his annual increment.

Soln Let the starting month salary be x & increment be y.

after 3 years salary will be

$$x + 3y = 19800 \quad \text{——①}$$

after 9 years salary will be

$$x + 9y = 23400 \quad \text{——②}$$

② − ① $6y = 3600$

$$y = 600$$

① ⟹ $x + 1800 = 19800$

$$x = 19800 - 1800 = 18000$$

$$x = 18000$$

Starting month salary $= 18000$

monthly increment $= 600$.

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4) 4 men and 4 women can finish a piece of work jointly in 3 days while 2 men and 5 women can finish the same work jointly in 4 days. Find the time taken by one man alone and that of one woman alone to finish the same work by using matrix Inversion method?

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Soln.
$$4x + 4y = \tfrac{1}{3}$$
$$2x + 5y = \tfrac{1}{4}$$

matrix form $\begin{pmatrix} 4 & 4 \\ 2 & 5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1/3 \\ 1/4 \end{pmatrix}$

$AX = B$ where $A = \begin{pmatrix} 4 & 4 \\ 2 & 5 \end{pmatrix}$

$X = A^{-1}B$

$|A| = 20 - 8 = 12$, $\text{adj } A = \begin{pmatrix} 5 & -4 \\ -2 & 4 \end{pmatrix}$

$A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{12} \begin{pmatrix} 5 & -4 \\ -2 & 4 \end{pmatrix}$

$X = \frac{1}{12} \begin{pmatrix} 5 & -4 \\ -2 & 4 \end{pmatrix} \begin{pmatrix} 1/3 \\ 1/4 \end{pmatrix} = \frac{1}{12} \begin{pmatrix} 5/3 - 1 \\ -2/3 + 1 \end{pmatrix} = \frac{1}{12} \begin{pmatrix} 2/3 \\ 1/3 \end{pmatrix} = \begin{pmatrix} 1/18 \\ 1/36 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1/18 \\ 1/36 \end{pmatrix}$ ∴ one men can do 18 days
one women can do 36 days.

5) The prices of 3 commodities A, B and c are ₹ x, y & z per units respectively. A person P purchases 4 units of B and sells two units of A and 5 units of C. Person Q Purchases 2 units of C and sells 3 units of A & one unit of B. In the process. P, Q & R earn ₹ 15000, ₹ 1000 and ₹ 4000 respectively. Find the prices per unit of A, B & C.

Soln. let x, y, z are commodities of A, B & C.
$$2x + 4y + 5c = 1500 \quad ①$$
$$3x + y + 2c = 1000 \quad ②$$
$$x + 3y + c = 4000 \quad ③$$

matrix form $\begin{pmatrix} 2 & 4 & 5 \\ 3 & 1 & 2 \\ 1 & 3 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1500 \\ 1000 \\ 4000 \end{pmatrix}$

$AX = B$
$X = A^{-1}B$

(21)

$|A| = 2(1-6) - 4(3-2) + 5(9-1)$

$= 2(-5) - 4(1) + 5(8) = -10 - 4 + 40 = 26$

$adj A = \begin{pmatrix} 1-6 & 15-4 & 8-5 \\ 2-3 & 2-5 & 15-4 \\ 9-1 & 4-2 & 2-12 \end{pmatrix}$

$= \begin{pmatrix} -5 & 11 & 3 \\ -1 & -3 & 11 \\ 8 & -2 & -10 \end{pmatrix}$

$A^{-1} = \frac{1}{|A|} \, adj A = \frac{1}{26} \begin{pmatrix} -5 & 11 & 3 \\ -1 & -3 & 11 \\ 8 & -2 & 10 \end{pmatrix}$

$X = A^{-1} B = \frac{1}{26} \begin{pmatrix} -5 & 11 & 3 \\ -1 & -3 & 11 \\ 8 & -2 & 10 \end{pmatrix} \begin{pmatrix} 15000 \\ 1000 \\ 4000 \end{pmatrix}$

$= \frac{1}{26} \begin{pmatrix} -75000 + 11000 + 12000 \\ -15000 - 3000 + 44000 \\ 120000 - 2000 + 40000 \end{pmatrix} = \frac{1}{26} \begin{pmatrix} 52000 \\ 26000 \\ 78000 \end{pmatrix}$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2000 \\ 1000 \\ 3000 \end{pmatrix}$

$\boxed{\begin{array}{l} x = 2000 \\ y = 1000 \\ z = 3000 \end{array}}$

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(22)

Exercise 1·4.

1. Solve the following systems of linear equations by cramer's rule.

i) $5x - 2y + 16 = 0$ $x + 3y - 7 = 0$ $\Rightarrow$ $5x - 2y = -16$

$$x + 3y = 7$$

$$A = \begin{vmatrix} 5 & -2 \\ 1 & 3 \end{vmatrix} = 15 + 2 = 17$$

$$\Delta_x = \begin{vmatrix} -16 & -2 \\ 7 & 3 \end{vmatrix} = -48 + 14 = -34$$

$$\Delta_y = \begin{vmatrix} 5 & -16 \\ 1 & 7 \end{vmatrix} = 35 + 16 = 51$$

by cramer's rule $x = \dfrac{\Delta_x}{\Delta} = \dfrac{-34}{17} = -2$

$$y = \dfrac{\Delta_y}{\Delta} = \dfrac{51}{17} = 3$$

$$\therefore \boxed{x = -2, \; y = 3}$$

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ii) $\dfrac{3}{x} + 2y = 12$, $\dfrac{2}{x} + 3y = 13$.

$\underline{\underline{soln}}$ $\dfrac{3}{x} + 2y = 12$ —① $\dfrac{2}{x} + 3y = 13$. —②

$$2y = 12 - \dfrac{3}{x}.$$

$$\dfrac{2}{x} + 3 \left(\dfrac{1}{2}\right) \left(12 - \dfrac{3}{x}\right) = 13.$$

$$y = \dfrac{1}{2}\left(12 - \dfrac{3}{x}\right)$$

$\therefore$ $\dfrac{2}{x} + \dfrac{3}{2}(12) - \dfrac{3}{2}\left(\dfrac{3}{x}\right) = 13.$

$$\dfrac{2}{x} + 18 - \dfrac{9}{2x} = 13 \qquad \Rightarrow \qquad \dfrac{4-9}{2x} = 13 - 18$$

$$\dfrac{-5}{2x} = -5$$

$$\dfrac{1}{2x} = 1 \qquad \Rightarrow \quad 2x = 1 \Rightarrow x = \tfrac{1}{2}.$$

① $\dfrac{3}{x} + 2y = 12$

$$6 + 2y = 12$$

$$2y = 6$$

$$y = 3$$

$$\boxed{\therefore x = \tfrac{1}{2} \quad y = 3}$$

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(iii) $3x+3y-z=11$, $2x-y+2z=9$, $4x+3y+2z=25$

Sol^n

$D = \begin{vmatrix} 3 & 3 & -1 \\ 2 & -1 & 2 \\ 4 & 3 & 2 \end{vmatrix} = 3(-2-6)-3(4-8)-1(6+4)$

$\qquad = 3(-8)-3(-4)-1(10) = -24+12-10$

$\qquad = -22$

$D_x = \begin{vmatrix} 11 & 3 & -1 \\ 9 & -1 & 2 \\ 25 & 3 & 2 \end{vmatrix} = 11(-2-6)-3(18-50)-1(27+25)$

$\qquad = 11(-8)-3(-32)-1(52)$

$\qquad = -88+96-52 = -44$

$D_y = \begin{vmatrix} 3 & 11 & -1 \\ 2 & 9 & 2 \\ 4 & 25 & 2 \end{vmatrix} = 3(18-50)-11(4-8)-1(50-36)$

$\qquad = 3(-32)-11(-4)-1(14)$

$\qquad = -96+44-14 = -66$

$D_z = \begin{vmatrix} 3 & 3 & 11 \\ 2 & -1 & 9 \\ 4 & 3 & 25 \end{vmatrix} = 3(-25-27)-3(50-36)+11(6+4)$

$\qquad = 3(-52)-3(14)+11(10) = -156-42+110 = -88$

by Cramer's rule $x = \dfrac{D_x}{D} = \dfrac{-44}{-22} = 2$

$\qquad y = \dfrac{D_y}{D} = \dfrac{-66}{-22} = 3$

$\qquad z = \dfrac{D_z}{D} = \dfrac{-88}{-22} = 4$

$$\boxed{\begin{aligned} x &= 2 \\ y &= 3 \\ z &= 4 \end{aligned}}$$

(iv) $\dfrac{3}{x} - \dfrac{4}{y} - \dfrac{2}{z} - 1 = 0$, $\dfrac{1}{x}+\dfrac{2}{y}+\dfrac{1}{z}-2=0$, $\dfrac{2}{x}-\dfrac{5}{y}-\dfrac{4}{z}+1=0$.

Sol^n　put $a = \dfrac{1}{x}$, $b = \dfrac{1}{y}$, $c = \dfrac{1}{z}$.

$3a-4b-2c = 1$ ————①

$a+2b+c = 2$ ————②

$2a-5b-4c = -1$. ————③

$D = \begin{vmatrix} 3 & -4 & -2 \\ 1 & 2 & 1 \\ 2 & -5 & -4 \end{vmatrix} = 3(-8+5)+4(-4-2)-2(-5-4)$

$\qquad = -9-24+18 = -15$

(24)

$$D_a = \begin{vmatrix} 1 & -4 & -2 \\ 2 & 2 & 1 \\ -1 & -5 & -4 \end{vmatrix} = 1(-8+5) + 4(-8+1) - 2(-10+2)$$
$$= -3 - 28 + 16 = -15$$

$$D_b = \begin{vmatrix} 3 & 1 & -2 \\ 1 & 2 & 1 \\ 2 & -1 & -4 \end{vmatrix} = 3(-8+1) - 1(-4-2) - 2(-1-4)$$
$$= 3(-7) - 1(-6) - 2(-5)$$
$$= -21 + 6 + 10 = -5$$

$$D_c = \begin{vmatrix} 3 & -4 & 1 \\ 1 & 2 & 2 \\ 2 & -5 & -1 \end{vmatrix} = 3(-2+10) + 4(-1-4) + 1(-5-4)$$
$$= 24 - 20 - 9 = -5$$

$$a = \frac{D_a}{D} = \frac{-15}{-15} = 1$$

$$b = \frac{D_b}{D} = \frac{-5}{-15} = \frac{1}{3}$$

$$c = \frac{D_c}{D} = \frac{-5}{-15} = \frac{1}{3}$$

$$\boxed{\begin{aligned} x &= 1 \\ y &= 3 \\ z &= 3 \end{aligned}}$$

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2) In a Competitive examination, one mark is awarded for every correct answer while $\frac{1}{4}$ mark is deducted for every wrong answer. A student answered 100 questions and got 80 marks. How many questions did he answer correctly?

soln. Total number of question 100.
Let the correct questions be x
and the wrong questions by y

$$x + y = 100 \qquad \text{—①}$$
$$x - \frac{1}{4}y = 80 \qquad \text{—②}$$

$$D = \begin{vmatrix} 1 & 1 \\ 1 & -\frac{1}{4} \end{vmatrix} = -\frac{1}{4} - 1 = -\frac{5}{4}$$

$$D_x = \begin{vmatrix} 100 & 1 \\ 80 & -\frac{1}{4} \end{vmatrix} = -25 - 80 = -105$$

$$D_y = \begin{vmatrix} 1 & 100 \\ 1 & 80 \end{vmatrix} = 80 - 100 = -20$$

$$x = \frac{D_x}{D} = \frac{+105}{+5/4} = 21 \times 4 = 84.$$

$$y = \frac{D_y}{D} = \frac{+20}{+5/4} = 4 \times 4 = 16$$

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correct ed questions = 84.
wrong questions = 16.

(25)

3) A chemist has one solution which is 50% acid and another solution which is 25% acid. How much each should be mixed to make 10 litres of a 40% acid solution?

Soln: Let two solutions be x & y.

$$x + y = 10 \quad \text{——} \quad \text{①}$$
$$0.25x + (0.50)y = (0.40)10 \quad \text{——} \quad \text{②}$$

② × 100 $25x + 50y = 400$

② ÷ 5 $5x + 10y = 80 \quad \text{——} \quad \text{③}$

$$\Delta = \begin{vmatrix} 1 & 1 \\ 5 & 10 \end{vmatrix} = 10 - 5 = 5$$

$$\Delta_x = \begin{vmatrix} 10 & 1 \\ 80 & 10 \end{vmatrix} = 100 - 80 = 20$$

$$\Delta_y = \begin{vmatrix} 1 & 10 \\ 5 & 80 \end{vmatrix} = 80 - 50 = 30$$

$$x = \frac{\Delta_x}{\Delta} = \frac{20}{5} = 4. \quad \text{(litres of 25\% solution)}$$
$$y = \frac{\Delta_y}{\Delta} = \frac{30}{5} = 6 \quad \text{(litres of 50\% solution)}$$

$$\boxed{\begin{array}{l} x = 4 \\ y = 6 \end{array}}$$

4) An amount of 65000 is invested in three bonds at the rate of 6%, 8% and 10% per annum respectively. The total annual income is ₹ 4800, The income from the third bond is ₹ 600 more than that from the second bond. determine the price of each bond.

Soln:

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4) A fish tank can be filled in 10 min using both (26) pumps A & B simultanously pump B can pump water in or out at the same rate. If pump B is inadvertently run in reverse then the tank will be filled in 30 min. How long would it take each pump to fill the tank by itself?

Soln Pump A fills $\left(\frac{1}{x}\right)^{th}$ of the tank in 1 hour

Pump B fills $\left(\frac{1}{y}\right)^{th}$ of the tank in 1 hour.

both can filled $\left(\frac{1}{10}^{th}\right)$ of the tank in 1 hour

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{10}.$$

Pump B filled in 30 min.

$$\frac{1}{x} + \frac{1}{30} = \frac{1}{10}$$

$$\frac{1}{x} = \frac{1}{10} - \frac{1}{30} = \frac{3-1}{30} = \frac{2}{30} = \frac{1}{15}. \quad (pump A)$$

$$\frac{1}{y} = \frac{1}{30}. \quad (pump B)$$

∴ pump A takes 15 min
   Pump B takes 30 min to fill the tank.

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5) A family of 3 people went out for dinner in a restaurant The cost of two dosai, three idlies & two vadas is ₹150, The cost of the two dosai, two idlies and four vadais is ₹200. Cost of five dosai, four idlies and two vadas is ₹250. The family has ₹350 in hand and ate 3 dosai and six idlies and six vadas will they be able to manage to pay the bill within amount they had?

Soln Let dosai, idlies & vadas be x, y, z (27)

$$2x + 3y + 2z = 150$$
$$2x + 2y + 4z = 200$$
$$5x + 4y + 2z = 250.$$

$$D = \begin{vmatrix} 2 & 3 & 2 \\ 2 & 2 & 4 \\ 5 & 4 & 2 \end{vmatrix} = 2(4-16) - 3(4-20) + 2(8-10)$$

$$= 2(-12) - 3(-16) + 2(-2) = -24 + 48 - 4 = 20$$

$$D_x = \begin{vmatrix} 150 & 3 & 2 \\ 200 & 2 & 4 \\ 250 & 4 & 2 \end{vmatrix} = 150(4-16) - 3(400-1000) + 2(800-500)$$
$$= 150(-12) - 3(-600) + 2(300)$$
$$= -1800 + 1800 + 600 = 600$$

$$D_y = \begin{vmatrix} 2 & 150 & 2 \\ 2 & 200 & 4 \\ 5 & 250 & 2 \end{vmatrix} = 2(400-1000) - 150(4-20) + 2(500-1000)$$
$$= 2(-600) - 150(-16) + 2(-500)$$
$$= -1200 + 2400 - 1000 = 200$$

$$D_z = \begin{vmatrix} 2 & 3 & 150 \\ 2 & 2 & 200 \\ 5 & 4 & 250 \end{vmatrix} = 2(500-800) - 3(500-1000) + 150(8-10)$$

$$= 2(-300) - 3(-500) + 150(-2)$$
$$= -600 + 1500 - 300 = 600.$$

$$x = \frac{D_x}{D} = \frac{600}{20} = 30; \quad y = \frac{D_y}{D} = \frac{200}{20} = 10; \quad z = \frac{D_z}{D} = \frac{600}{20} = 30$$

$$x = 30, \quad y = 10, \quad z = 30.$$

They ate 3 dosai, 6 idlies & 6 vadas

$$3x + 6y + 6z = 3(30) + 6(10) + 6(30)$$
$$= 90 + 60 + 180 = 330$$

They had 350 spend 330.
They can eat within the amount

Exercise 1.5   (28)

1. Solve the following systems of linear equations by Gaussian elimination method

(i) $2x - 2y + 3z = 2$, $x + 2y - z = 3$, $3x - y + 2z = 1$

Soln Augmented matrix

$$[A \mid B] = \begin{bmatrix} 2 & -2 & 3 & | & 2 \\ 1 & 2 & -1 & | & 3 \\ 3 & -1 & 2 & | & 1 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 1 & 2 & -1 & | & 3 \\ 2 & -2 & 3 & | & 2 \\ 3 & -1 & 2 & | & 1 \end{bmatrix}$$

$$\begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \end{matrix} \begin{bmatrix} 1 & 2 & -1 & | & 3 \\ 0 & -6 & 5 & | & -4 \\ 0 & -7 & 5 & | & -8 \end{bmatrix} \xrightarrow{C_1 \leftrightarrow C_3} \begin{bmatrix} 1 & -1 & 2 & | & 3 \\ 0 & 5 & -6 & | & -4 \\ 0 & 5 & -7 & | & -8 \end{bmatrix}$$

$$R_3 \to R_3 - 5R_2 \begin{bmatrix} 1 & -1 & 2 & | & 3 \\ 0 & 5 & -6 & | & -4 \\ 0 & 0 & -1 & | & -4 \end{bmatrix}$$

writing the equivalent equations from echelon form.

$-z = -4$	$5y - 6z = -4$	$x - y + 2z = 3$
$z = 4$	$5y - 24 = -4$	$x - 4 + 8 = 3$
	$5y = -4 + 24$	$x = 3 + 4 - 8$
	$5y = 20$	$x = -1$
	$y = 4$	

$$\boxed{\therefore x = -1, \; y = 4, \; z = 4}$$

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(ii) $2x + 4y + 6z = 22$, $3x + 8y + 5z = 27$, $-x + y + 2z = 2$

Soln. Augmented matrix

$$[A \mid B] = \begin{bmatrix} 2 & 4 & 6 & | & 22 \\ 3 & 8 & 5 & | & 27 \\ -1 & 1 & 2 & | & 2 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow \frac{R_1}{2}} \begin{bmatrix} 1 & 2 & 3 & | & 11 \\ 3 & 8 & 5 & | & 27 \\ -1 & 1 & 2 & | & 2 \end{bmatrix}$$

$$\begin{matrix} R_2 \to R_2 - 3R_1 \\ R_3 \to R_3 + R_1 \end{matrix} \begin{bmatrix} 1 & 2 & 3 & | & 11 \\ 0 & 2 & -4 & | & -6 \\ 0 & 3 & 5 & | & 13 \end{bmatrix} \xrightarrow{R_2 \to \frac{R_2}{2}} \begin{bmatrix} 1 & 2 & 3 & | & 11 \\ 0 & 1 & -2 & | & -3 \\ 0 & 3 & 5 & | & 13 \end{bmatrix}$$

$$R_3 \to R_3 - 3R_2 \begin{bmatrix} 1 & 2 & 3 & | & 11 \\ 0 & 1 & -2 & | & -3 \\ 0 & 0 & 11 & | & 22 \end{bmatrix}$$

writing the equivalent equations from echelon form

$$11z = 22$$
$$\boxed{z = 2}$$

$$y - 2z = -3 \qquad\qquad x + 2y + 3z = 11$$
$$y - 4 = -3 \qquad\qquad x + 2(1) + 3(2) = 11$$
$$y = -3 + 4 \qquad\qquad x + 2 + 6 = 11$$
$$\boxed{y = 1} \qquad\qquad x = 11 - 8 = 3$$
$$\boxed{x = 3}$$

$$\therefore \boxed{\begin{array}{l} x = 3 \\ y = 1 \\ z = 2 \end{array}}$$

2) If $ax^2 + bx + c$ is divided by $x+3$, $x-5$, and $x-1$. The remainders are 21, 61 & 9 respectively. Find a, b & c.

<u>Coln</u> $f(x) = ax^2 + bx + c$.

$f(-3) \Rightarrow 9a - 3b + c = 21$ — ① $(x + 3 = 0 \Rightarrow x = -3)$

$f(5) \Rightarrow 25a + 5b + c = 61$ — ② $(x - 5 = 0 \Rightarrow x = 5)$

$f(1) \Rightarrow a + b + c = 9$ — ③ $(x - 1 = 0 \Rightarrow x = 1)$

matrix form $\begin{pmatrix} 9 & -3 & 1 \\ 25 & 5 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 21 \\ 61 \\ 9 \end{pmatrix}$

$\qquad\qquad\qquad\qquad\quad A \qquad\qquad X \quad = \quad B$

$(A \mid B) = \begin{pmatrix} 9 & -3 & 1 & 21 \\ 25 & 5 & 1 & 61 \\ 1 & 1 & 1 & 9 \end{pmatrix} \xrightarrow{C_1 \leftrightarrow C_3} \begin{pmatrix} 1 & -3 & 9 & 21 \\ 1 & 5 & 25 & 61 \\ 1 & 1 & 1 & 9 \end{pmatrix}$

$\begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{array} \begin{pmatrix} 1 & -3 & 9 & 21 \\ 0 & -8 & 16 & 40 \\ 0 & 4 & -8 & -12 \end{pmatrix} \xrightarrow[R_4 \to \pm R_4]{R_2 \to \frac{R_2}{+8}} \begin{pmatrix} 1 & 1 & 1 & 9 \\ 0 & 1 & 2 & 5 \\ 0 & 1 & -2 & -3 \end{pmatrix}$

$R_3 \to R_3 - R_2 \longrightarrow \begin{pmatrix} 1 & 1 & 1 & 9 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & -4 & -8 \end{pmatrix}$

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$$-4c = -8 \quad\Big|\quad b + 2c = 5 \quad\Big|\quad a + b + c = 9$$
$$c = 2 \qquad\quad b + 4 = 5 \qquad\quad a + 1 + 2 = 9$$
$$\qquad\qquad\quad b = 5 - 4 = 1 \qquad a = 9 - 3$$
$$\qquad\qquad\qquad\qquad\qquad\qquad\qquad a = 6$$

$$\boxed{a = 6, \ b = 1, \ c = 2}$$

3) An amount of ₹65000 is invested In three bonds at the rates of 6%, 8% and 10% per annum respectively. The total annual income is ₹5000. The income from the First bond is ₹600, more than that from the Second bond. Determine the price of each bond.

Soln: Let the amount of 3 bonds be x, y, z.

$$x + y + z = 65000 \quad —①$$

$$0.06x + 0.08y + 0.10z = 5000 \quad —② \text{ (Total income 5000)}$$

$$0.06x - 0.08y = 600 \quad —③ \text{ (1st bond is 600 more that 2nd bond)}$$

matrix form

$$[A|B] \sim \begin{pmatrix} 1 & 1 & 1 & 65000 \\ 0.06 & 0.08 & 0.10 & 5000 \\ 0.06 & -0.08 & 0 & 600 \end{pmatrix}$$

$R_2 \to R_2 - 0.06R_1$
$R_3 \to R_3 - 0.06R_1$

$$\begin{pmatrix} 1 & 1 & 1 & 65000 \\ 0 & 0.02 & 0.04 & 1100 \\ 0 & -0.14 & -0.06 & -3300 \end{pmatrix}$$

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$R_2 \to \dfrac{R_2}{5}$
$R_3 \to \dfrac{R_3}{-3}$

$$\begin{pmatrix} 1 & 1 & 1 & 65000 \\ 0 & 0.01 & 0.02 & 550 \\ 0 & 0.07 & 0.02 & 1650 \end{pmatrix}$$

$R_2 \to R_2 \times 100$
$R_3 \to R_3 \times 100$

$$\begin{pmatrix} 1 & 1 & 1 & 65000 \\ 0 & 1 & 2 & 55000 \\ 0 & 3 & 2 & 165000 \end{pmatrix}$$

$R_3 \to R_3 - 7R_2$

$$\begin{pmatrix} 1 & 1 & 1 & 65000 \\ 0 & 1 & 2 & 55000 \\ 0 & 0 & -11 & -220000 \end{pmatrix}$$

$-11z = -220000$
$z = 20000$

$y + 2z = 55000$
$y + 40000 = 55000$
$y = 15000$

$x + y + z = 65000$
$x + 20000 + 15000 = 65000$
$x = 30000$

$$\boxed{\begin{aligned} x &= 30000 \\ y &= 15000 \\ z &= 20000 \end{aligned}}$$

4) A boy is walking along the path $y = ax^2 + bx + c$ through the points $(-6, 8)$ $(-2, -12)$, and $(3, 8)$. He wants to meet his friend at $P(7, 60)$ will he meet his friend?

④

Soln $y = ax^2 + bx + c$

At $(-6, 8)$ ⇒ $8 = 36a - 6b + c$ — ①
At $(-2, -12)$ ⇒ $-12 = 4a - 2b + c$ — ②
At $(3, 8)$ ⇒ $8 = 9a + 3b + c$ — ③

Matrix form $\begin{pmatrix} 36 & -6 & 1 & 8 \\ 4 & -2 & 1 & -12 \\ 9 & 3 & 1 & 8 \end{pmatrix}$

$[A|B] = \begin{pmatrix} 36 & -6 & 1 & 8 \\ 4 & -2 & 1 & -12 \\ 9 & 3 & 1 & 8 \end{pmatrix}$
$\begin{array}{c} R_2 \to 9R_2 - R_1 \\ R_3 \to 4R_3 - 1R_1 \end{array}$
$\begin{pmatrix} 36 & -6 & 1 & 8 \\ 0 & 12 & -8 & 116 \\ 0 & -18 & -3 & -24 \end{pmatrix}$

$\begin{array}{c} R_2 \to R_2/4 \\ R_3 \to R_3/-3 \end{array}$
$\begin{pmatrix} 36 & -6 & 1 & 8 \\ 0 & 3 & -2 & 29 \\ 0 & 6 & 1 & 8 \end{pmatrix}$
$R_3 \to R_3 - 2R_2$
$\begin{pmatrix} 36 & -6 & 1 & 8 \\ 0 & +3 & -2 & 29 \\ 0 & 0 & 5 & -50 \end{pmatrix}$

$5c = -50$ $+3b - 2c = 29$ $36a - 18 - 10 = 8$
$c = -10$ $3b - 20 = 29$ $36a = 8 + 18 + 60$
                $3b = 9$ $36a = 36$
                $b = 3$ $a = 1$

At $P(7, 60)$ $y = ax^2 + bx + c$
            $60 = 1(7^2) + 3(7) - 10$
            $60 = 49 + 21 - 10$
            $60 = 60$

∴ he will meet his friend at $P(7, 60)$.

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$$\underline{Exercise\ 1\cdot6}$$ ㉛

1. Test for cosistency and if possible solve the following System of equations by rank method.

(i) $x - y + 2z = 2$, $2x + y + 4z = 7$ $4x - y + z = 4$

Soln: Augument matrix. $[A \mid B]$

$$\begin{pmatrix} 1 & -1 & 2 \\ 2 & 1 & 4 \\ 4 & -1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 7 \\ 4 \end{pmatrix}$$

$$AX = B$$

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$$[A \mid B] = \begin{pmatrix} 1 & -1 & 2 & 2 \\ 2 & 1 & 4 & 7 \\ 4 & -1 & 1 & 4 \end{pmatrix}$$

$R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - 4R_1$

$$\begin{pmatrix} 1 & -1 & 2 & 2 \\ 0 & 3 & 0 & 3 \\ 0 & 3 & -7 & -4 \end{pmatrix} \xrightarrow{R_3 \to R_3 - R_2} \begin{pmatrix} 1 & -1 & 2 & 2 \\ 0 & 3 & 0 & 3 \\ 0 & 0 & -7 & -7 \end{pmatrix}$$

$\rho(A) = 3$ $\rho(A \mid B) = 3$.

The system is consistent.

$\rho(A) = \rho(A \mid B) = 3 = n$.

it has unique solution.

$-7z = -7$ | $3y = 3$ | $x - y + 2z = 2$
$\boxed{z = 1}$ | $\boxed{y = 1}$ | $x - 1 + 2 = 2$
                                   | $\boxed{x = 1}$

∴ Solution is $x = 1$, $y = 1$, $z = 1$.

(ii) $3x + y + z = 2$, $x - 3y + 2z = 1$, $7x - y + 4z = 5$

Soln: matrix form. $\begin{pmatrix} 3 & 1 & 1 \\ 1 & -3 & 2 \\ 7 & -1 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}$

$$AX = B$$

Augumented matrix
$$[A \mid B] = \begin{pmatrix} 3 & 1 & 1 & 2 \\ 1 & -3 & 2 & 1 \\ 7 & -1 & 4 & 5 \end{pmatrix}$$

$R_1 \Leftrightarrow R_2$
$\longrightarrow$
$\begin{pmatrix} 1 & -3 & 2 & 1 \\ 3 & 1 & 1 & 2 \\ 7 & -1 & 4 & 5 \end{pmatrix}$
$\begin{array}{c} R_2 \to R_2 - 3R_1 \\ R_3 \to R_3 - 7R_1 \end{array}$
$\begin{pmatrix} 1 & -3 & 2 & 1 \\ 0 & 10 & -5 & -1 \\ 0 & 20 & -10 & -2 \end{pmatrix}$

$R_3 \to R_3 - 2R_2$
$\longrightarrow$
$\begin{pmatrix} 1 & -3 & 2 & | & 1 \\ 0 & 10 & -5 & | & -1 \\ 0 & 0 & 0 & | & 0 \end{pmatrix}$

$\rho(A) = 2 \qquad \rho(A:B) = 2$

$\rho(A) = \rho(A:B) = 2 < n.$

The system is consistent. it has infinitely many solution.   put $\boxed{z = t}$

$10y - 5z = -1$

$10y = -1 + 5z = 5t - 1$

$\boxed{y = \dfrac{5t - 1}{10}}$

$x - 3y + 2z = 1$

$x - 3\left(\dfrac{5t-1}{10}\right) + 2t = 1$

$x = 1 - 2t + 3\left(\dfrac{5t-1}{10}\right) = \dfrac{10 - 20t + 15t - 3}{10}$

$x = \dfrac{7 - 5t}{10}$

$(x, y, z) = \left(\dfrac{7-5t}{10}, \dfrac{5t-1}{10}, t\right) \quad \forall t \in R.$

(iii) $2x + 2y + z = 5, \quad x - y + z = 1, \quad 3x + y + 2z = 4.$

Soln   matrix form $\begin{pmatrix} 2 & 2 & 1 \\ 1 & -1 & 1 \\ 3 & 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \\ 4 \end{pmatrix}$

$AX = B.$

Augumented matrix $[A|B] = \begin{bmatrix} 2 & 2 & 1 & 5 \\ 1 & -1 & 1 & 1 \\ 3 & 1 & 2 & 4 \end{bmatrix}$

$R_1 \Leftrightarrow R_2$
$\longrightarrow$
$\begin{pmatrix} 1 & -1 & 1 & 1 \\ 2 & 2 & 1 & 5 \\ 3 & 1 & 2 & 4 \end{pmatrix}$
$\begin{array}{c} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \end{array}$
$\begin{pmatrix} 1 & -1 & 1 & 1 \\ 0 & 4 & -1 & 3 \\ 0 & 4 & -1 & 1 \end{pmatrix}$

$R_3 \to R_3 - R_2$
$\longrightarrow$
$\begin{pmatrix} 1 & -1 & 1 & | & 1 \\ 0 & 4 & -1 & | & 3 \\ 0 & 0 & 0 & | & -2 \end{pmatrix}$

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$\rho(A) = 2 \qquad \rho(A:B) = 3$

$\rho(A) \neq \rho(A:B)$

∴ The system is inconsistent.
it has no solution.

(iv) $2x - y + z = 2$, $6x - 3y + 3z = 6$, $4x - 2y + 2z = 4$.

Soln   Matrix form

$$[A|B] = \begin{pmatrix} 2 & -1 & 1 & 2 \\ 6 & -3 & 3 & 6 \\ 4 & -2 & 2 & 4 \end{pmatrix}$$

Augmented matrix:

$R_2 \to \dfrac{R_2}{3}$
$R_3 \to R_3/2$ $\longrightarrow$ $\begin{pmatrix} 2 & -1 & 1 & 2 \\ 2 & -1 & 1 & 2 \\ 2 & -1 & 1 & 2 \end{pmatrix}$ $\begin{array}{c} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{array} \longrightarrow$ $\begin{pmatrix} 2 & -1 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

$\rho(A) = 1$ $\rho(A:B) = 1$.

$\rho(A) = \rho(A:B) = 1 < n.$

∴ The system reduces into single equation
∴ it is consistent and has infinitely
many solution.

put $y = s$ $z = t$.

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$2x - y + z = 2$
$2x - s + t = 2$
$2x = 2 + s - t$
$x = \dfrac{2 + s - t}{2}$

$(x, y, z) = \left(\dfrac{2 + s - t}{2}, s, t \right) \; \forall \; s, t \in R.$

2) Find the value of k for which the equations
$Kx - 2y + z = 1$, $x + 2ky + z = -2$, $x - 2y + kz = 1$ have
(i) no solution (ii) unique solution (iii) infinitely many solution.

soln   Matrix form. $\begin{pmatrix} k & -2 & 1 \\ 1 & -2k & 1 \\ 1 & -2 & k \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$

$AX = B.$

Augmented matrix

$$[A|B] = \begin{pmatrix} k & -2 & 1 & 1 \\ 1 & -2k & 1 & -2 \\ 1 & -2 & k & 1 \end{pmatrix}$$

$R_1 \longleftrightarrow R_3$
$\longrightarrow$
$\begin{pmatrix} 1 & -2 & k & 1 \\ 1 & -2k & 1 & -2 \\ k & -2 & 1 & 1 \end{pmatrix}$
$\begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - kR_1 \end{matrix}$
$\longrightarrow$
$\begin{pmatrix} 1 & -2 & k & 1 \\ 0 & -2k+2 & 1-k & -3 \\ 0 & -2+2k & 1-k^2 & 1-k \end{pmatrix}$ ㉟

$R_3 \to R_3 + R_2$
$\longrightarrow$
$\begin{pmatrix} 1 & -2 & k & 1 \\ 0 & 2(1-k) & (1-k) & -3 \\ 0 & 0 & 2-k-k^2 & (k+2) \end{pmatrix}$

$\longrightarrow$
$\begin{pmatrix} 1 & -2 & k & 1 \\ 0 & 2(1-k) & (1-k) & -3 \\ 0 & 0 & (k+2)(1-k) & (k+2) \end{pmatrix}$

$2-k-k^2$
$-(k^2+k-2)$
$-(k+2)(k-1)$
$(k+2)(1-k)$

case (i) $k = -2$ $\rho(A) = 2$ $\rho(A:B) = 2$.

The system is consistent and it has infinitely many solution.

Case (ii) $k = 1$, $k \neq -2$, $\rho(A) = 2$, $\rho(A:B) = 3$.

$\rho(A) \neq \rho(A:B)$

The system is inconsistent. It has no solution.

Case (iii) $k \neq 1$ $k \neq -2$

$\rho(A) = \rho(A:B) = 3 = n$.

The system is consistent. it has unique solution.

3) Investigate the values of λ and μ the system of linear equations $2x+3y+5z=9$, $7x+3y-5z=8$, $2x+3y+\lambda z=\mu$ have (i) no solution (ii) a unique solution (iii) an infinite number of solutions.

soln.   matrix form $\begin{pmatrix} 2 & 3 & 5 \\ 7 & 3 & -5 \\ 2 & 3 & \lambda \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 9 \\ 8 \\ \mu \end{pmatrix}$

$A \quad X = B$

Augmented matrix
$[A|B] = \begin{pmatrix} 2 & 3 & 5 & 9 \\ 7 & 3 & -5 & 8 \\ 2 & 3 & \lambda & \mu \end{pmatrix}$

$\begin{matrix} R_2 \to 2R_2 - 7R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$
$\longrightarrow$
$\begin{pmatrix} 2 & 3 & 5 & 9 \\ 0 & -15 & -45 & -47 \\ 0 & 0 & \lambda-5 & \mu-9 \end{pmatrix}$

Case (i)   If $\lambda = 5$ $\mu = 9$ (36)
$\rho(A) = \rho(A:B) = 2 < n$

The system is consistent it has infinitely many solutions

Case (ii)   if $\lambda \neq 5$ $\mu = 9$

$\rho(A) = 3$ $\rho(A:B) = 3$

The system is consistent it has unique solutions

Case (iii)   if $\lambda = 5$ $\mu \neq 9$

$\rho(A) = 2$ $\rho(A:B) = 3$

The system is inconsistent it has no solution.

Exercise 1.7   (37)

1. Solve the following system of homogeneous equations

(i) $3x + 2y + 7z = 0$, $4x - 3y - 2z = 0$, $5x + 9y + 23z = 0$

Soln Matrix form $\begin{pmatrix} 3 & 2 & 7 \\ 4 & -3 & -2 \\ 5 & 9 & 23 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$$AX = B$$

Augmented matrix

$$[A|B] = \begin{pmatrix} 3 & 2 & 7 & 0 \\ 4 & -3 & -2 & 0 \\ 5 & 9 & 23 & 0 \end{pmatrix}$$

$R_2 \to 3R_2 - 4R_1$
$R_3 \to 3R_3 - 5R_1$ $\begin{pmatrix} 3 & 2 & 7 & 0 \\ 0 & -17 & -34 & 0 \\ 0 & 17 & 34 & 0 \end{pmatrix}$ $R_3 \to R_3 + R_2$ $\begin{pmatrix} 3 & 2 & 7 & 0 \\ 0 & -17 & -34 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

writing into equation form.

$-17y - 34z = 0$ — ①
$3x + 2y + 7z = 0$. — ②

put $z = t$ $-17y = 34z$

$y = \dfrac{34t}{-17} = -2t$

① ⟹ $3x + 2(-2t) + 7t = 0$
$3x - 4t + 7t = 0$
$3x + 3t = 0$
$3x = -3t$
$x = -t$

$(x, y, z) = (-t, -2t, t) \quad \forall \, t \in R.$

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(ii) $2x + 3y - z = 0$, $x - y - 2z = 0$, $3x + y + 3z = 0$

Soln matrix form. $\begin{pmatrix} 2 & 3 & -1 \\ 1 & -1 & -2 \\ 3 & 1 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$$AX = B$$

Augmented matrix

$$(A|B) = \begin{pmatrix} 2 & 3 & -1 & 0 \\ 1 & -1 & -2 & 0 \\ 3 & 1 & 3 & 0 \end{pmatrix}$$

$R_1 \leftrightarrow R_2$
$\longrightarrow$ $\begin{pmatrix} 1 & -1 & -2 & 0 \\ 2 & 3 & -1 & 0 \\ 3 & 1 & 3 & 0 \end{pmatrix}$ $\begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \end{matrix}$ $\longrightarrow$ $\begin{pmatrix} 1 & -1 & -2 & 0 \\ 0 & 5 & 3 & 0 \\ 0 & 4 & 7 & 0 \end{pmatrix}$

$R_3 \to 5R_3 - 4R_2$ $\longrightarrow$ $\begin{pmatrix} 1 & -1 & -2 & 0 \\ 0 & 5 & 3 & 0 \\ 0 & 0 & 33 & 0 \end{pmatrix}$

Convert into equation form.

$33z = 0$. $\quad$ $5y + 3z = 0$ $\quad$ $x - y - 2z = 0$

$z = 0$ $\qquad$ $5y = 0$ $\qquad\qquad$ $x - 0 - 0 = 0$

$\qquad\qquad\qquad$ $y = 0$ $\qquad\qquad\qquad$ $x = 0$

$\therefore x = 0 \quad y = 0 \quad z = 0$

$\therefore$ It has only trivial solution.

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2) Determine the values of λ for which the following system of equations

$x + y + 3z = 0$, $4x + 3y + \lambda z = 0$, $2x + y + 2z = 0$ has

i) unique solution ii) a non trivial solution.

Soln Matrix form $\begin{pmatrix} 1 & 1 & 3 \\ 4 & 3 & 1 \\ 2 & 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$\qquad\qquad\qquad\qquad\qquad\qquad\quad$ $AX = B$

$[A|B] = \begin{pmatrix} 1 & 1 & 3 & 0 \\ 4 & 3 & 1 & 0 \\ 2 & 1 & 2 & 0 \end{pmatrix}$ $\begin{matrix} R_2 \to R_2 - 4R_1 \\ R_3 \to R_3 - 2R_2 \end{matrix}$ $\begin{pmatrix} 1 & 1 & 3 & 0 \\ 0 & -1 & \lambda - 12 & 0 \\ 0 & -1 & -4 & 0 \end{pmatrix}$

$R_2 \leftrightarrow R_3$ $\longrightarrow$ $\begin{pmatrix} 1 & 1 & 3 & 0 \\ 0 & -1 & -4 & 0 \\ 0 & -1 & \lambda - 12 & 0 \end{pmatrix}$ $R_3 \to R_3 - R_2$ $\longrightarrow$ $\begin{pmatrix} 1 & 1 & 3 & 0 \\ 0 & -1 & -4 & 0 \\ 0 & 0 & \lambda - 8 & 0 \end{pmatrix}$

Case i) if $\lambda = 8$

$\qquad\qquad\qquad$ $\rho(A) = \rho(A|B) = 2$

The system is consistent It has Infinitely many solutions
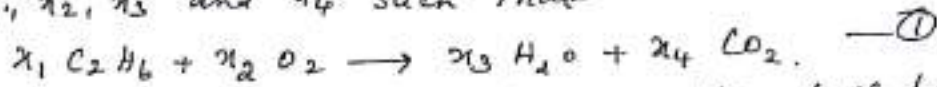
case ii) if $\lambda \neq 8$.

$\qquad\qquad$ $\rho(A) = 3$ $\qquad$ $\rho(A|B) = 3$

The system is consistent it has unique solution.

it has homogenous eqn $\therefore$ it has solution $\boxed{\begin{matrix} x = 0 \\ y = 0 \\ z = 0 \end{matrix}}$

3) By using Gaussian elimination method, balance the chemical reaction equation

$$C_2 H_6 + O_2 \longrightarrow H_2O + CO_2.$$

Soln We are searching for positive integers x_1, x_2, x_3 and x_4 such that

$$x_1 C_2 H_6 + x_2 O_2 \longrightarrow x_3 H_2 O + x_4 CO_2. \quad —①$$

The number of carbon atoms on the left hand side of ① should be equal to the number of carbon atoms on the RHS of (1) so we get a linear homogenous equation

$$2x_1 = x_4 \implies 2x_1 - x_4 = 0 \quad —②$$
$$6x_1 = 2x_3 \implies 6x_1 - 2x_3 = 0 \implies 3x_1 - x_3 = 0 \quad —③$$
$$2x_2 = x_3 + 2x_4 \implies 2x_2 - x_3 - 2x_4 = 0 \quad —④$$

eqn ②, ③ & ④ constitute a homogenous system of linear equations in four unknowns.

$$[A|B] = \begin{bmatrix} 2 & 0 & 0 & -1 & 0 \\ 3 & 0 & -1 & 0 & 0 \\ 0 & 2 & -1 & -2 & 0 \end{bmatrix}$$

By Gaussion elimination method, we get

$$[A|B] \xrightarrow{R_2 \to 2R_2 - 3R_1} \begin{pmatrix} 2 & 0 & 0 & -1 & 0 \\ 0 & 0 & -2 & 3 & 0 \\ 0 & 2 & -1 & -2 & 0 \end{pmatrix}$$

$$\xrightarrow{R_2 \leftrightarrow R_3} \begin{pmatrix} 2 & 0 & 0 & -1 & 0 \\ 0 & 2 & -1 & -2 & 0 \\ 0 & 0 & -2 & 3 & 0 \end{pmatrix}$$

$$\rho(A) = \rho(A|B) = 3 < 4$$

The system is consistent and has infinitely number of solutions.

writing the equations using the echelon form we get $-2x_3 + 3x_4 = 0$,

$$2x_2 - x_3 - 2x_4 = 0$$
$$2x_1 - x_4 = 0.$$

So one of the unknowns should be chosen arbitrarily as on non zero real number

④

put $x_4 = t$

$2x_1 = x_4$ $-2x_3 + 3x_4 = 0$ $2x_2 - x_3 - 2x_4 = 0$

$\boxed{x_1 = \dfrac{t}{2}}$ $-2x_3 = -3t$ $2x_2 - \dfrac{3}{2}t - 2t = 0$

$\boxed{x_3 = \dfrac{3}{2}t}$ $2x_2 = \dfrac{3}{2}t + 2t = \dfrac{7t}{2}$

$\boxed{x_2 = \dfrac{7t}{4}}$

$(x_1, x_2, x_3, x_4) = \left(\dfrac{t}{2}, \dfrac{7t}{4}, \dfrac{3}{2}t, t\right) \ \forall \ t \in R$

let us choose $t = 4$.

we get $x_1 = \dfrac{4}{2} = 2$ $x_2 = \dfrac{7t}{4} = \dfrac{7(4)}{4} = 7$

$\boxed{x_1 = 2}$ $\boxed{x_2 = 7}$

$x_3 = \dfrac{3}{2}(4)$ $x_4 = t$

$\boxed{x_3 = 6}$ $x_4 = 4$.

$\{x_1 = 2, \ x_2 = 7, \ x_3 = 6, \ x_4 = 4.$

So the balanced equation is

$$2C_2H_6 + 7O_2 \rightarrow 6H_2O + 4CO_2.$$

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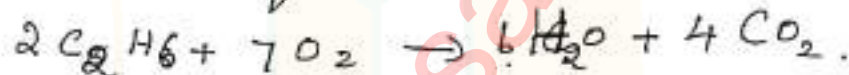
Exercise 1.8   41

1. Choose the correct answer

(i) if $|adj(adj A)| = |A|^9$, then the order of the square matrix A is

Soln $|adj(adj A)| = |A|^{(n-1)^2}$

$(n-1)^2 = 9 = 3^2$

$n - 1 = 3$

$\boxed{n = 4}$ Ans : (2) 4.

2) If A is a 3×3 non singular matrix such that $B = A^{-1} A^T$
$AA^T = A^T A$ and $BB^T = ?$

Soln $AA^T = A^T A$.   & $B = A^{-1} A^T$,

$B^T = (A^{-1})^T (A^T)^T = (A^{-1})^T A$

$BB^{-1} = (A^{-1} A^T)(A^{-1})^T A = A^{-1}(A A^{-1})^T A$

$= A^{-1}(I)^T A = A^{-1} A = I$

Ans (2) I

3) If $A = \begin{pmatrix} 3 & 5 \\ 1 & 2 \end{pmatrix}_{2\times 2}$, $B = adj A$ and $C = 3A$ then $\dfrac{|adj B|}{|C|} = ?$

Soln

$\dfrac{|adj B|}{|C|} = \dfrac{|adj(adj A)|}{|3A|} = \dfrac{|A|^{n-1}}{3^n |A|} = \dfrac{|A|^1}{3^2 |A|} = \dfrac{1}{9}$

Here $n = 2$

Ans (2) $\dfrac{1}{9}$.

4) If $A \times \begin{pmatrix} 1 & -2 \\ 1 & 4 \end{pmatrix} = \begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix}$ then $A = ?$

Soln order of A is 2×2 let A be $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1 & -2 \\ 1 & 4 \end{pmatrix} = \begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix}$

$a + b = 6$ $c + d = 0$
$-2a + 4b = 0$ $-2c + 4d = 6$.

Solve the equations we get $b = 2, a = 4$ $d = 1$ $c = -1$

$A = \begin{pmatrix} 4 & 2 \\ 1 & -1 \end{pmatrix}$ Ans (3) $\begin{pmatrix} 4 & 2 \\ -1 & 1 \end{pmatrix}$

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5) If $A = \begin{pmatrix} 7 & 3 \\ 4 & 2 \end{pmatrix}$ then $9I - A = ?$ ㊷

Sol^n

$|A| = 14 - 12 = 2$, $adj\, A = \begin{pmatrix} 2 & -3 \\ -4 & 7 \end{pmatrix}$, $A^{-1} = \frac{1}{|A|} adj\, A$

$A^{-1} = \frac{1}{2} \begin{pmatrix} 2 & -3 \\ -4 & 7 \end{pmatrix} \Rightarrow 2A^{-1} = \begin{pmatrix} 2 & -3 \\ -4 & 7 \end{pmatrix}$ —①

$9I - A = 9 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 7 & 3 \\ 4 & 2 \end{pmatrix} = \begin{pmatrix} 9 & 0 \\ 0 & 9 \end{pmatrix} - \begin{pmatrix} 7 & 3 \\ 4 & 2 \end{pmatrix} = \begin{pmatrix} 2 & -3 \\ -4 & 7 \end{pmatrix}$ —②

by ① & ② $9I - A = 2A^{-1}$ Ans: (4) $2A^{-1}$

6) If $A = \begin{pmatrix} 2 & 0 \\ 1 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 4 \\ 2 & 0 \end{pmatrix}$ then $|adj\,(AB)| = ?$

Sol^n

$AB = \begin{pmatrix} 2 & 0 \\ 1 & 5 \end{pmatrix} \begin{pmatrix} 1 & 4 \\ 2 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 8 \\ 11 & 4 \end{pmatrix}$

$adj\, AB = \begin{pmatrix} 4 & -8 \\ -11 & 2 \end{pmatrix}$ $\therefore |adj\, AB| = 8 - 88 = -80$

Ans (2) -80.

7) If $P = \begin{pmatrix} 1 & x & 0 \\ 1 & 3 & 0 \\ 2 & 4 & -2 \end{pmatrix}$ is the adjoint of 3×3 matrix A

and $|A| = 4$ then x is

$|adj\, A| = |A|^{n-1}$ $n = 3$

$1(-6+0) - x(-2) = 4^2$.

$-6 + 2x = 16$

$2x = 16 + 6 = 22$

$\boxed{x = 11}$ Ans (4) 11

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8) If $A = \begin{pmatrix} 3 & 1 & -1 \\ 2 & -2 & 0 \\ 1 & 2 & -1 \end{pmatrix}$ and $A^{-1} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ then

the value of a_{23} is

a_{23} in A^{-1} is $- \begin{vmatrix} 3 & 1 \\ 2 & 0 \end{vmatrix} = -(0+2) = -2$

$|A| = 3(2) - 1(-2) -1(4+2)$

$= 6 + 2 - 6 = 2$.

$A^{-1} = \frac{1}{|A|} adj\, A$. a_{23} is $\frac{1}{2}(-2) = -1$

Ans (4) -1

9) If A, B and c are invertible matrices of some order, ㊸ then which one of the following is not true.

Ans (2)

10. If $(AB)^{-1} = \begin{pmatrix} 12 & -17 \\ -19 & 27 \end{pmatrix}$ and $A^{-1} = \begin{pmatrix} 1 & -1 \\ -2 & 3 \end{pmatrix}$ then $B^{-1} = ?$

Soln

$(AB)^{-1} = B^{-1} A^{-1}$ $(A^{-1})^{-1} = A$.

$(AB)^{-1} A = B^{-1} (A^{-1}) A$ $|A^{-1}| = 3-2 = 1$

$(AB)^{-1} A = B^{-1}$ $adj\ A^{-1} = \begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix}$

     $A = \frac{1}{|A^{-1}|} adj\ A^{-1} = \begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix}$

$B^{-1} = \begin{pmatrix} 12 & -17 \\ -19 & 27 \end{pmatrix} \begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix}$

$= \begin{pmatrix} 36-34 & 12*17 \\ -57+54 & -19+27 \end{pmatrix} = \begin{pmatrix} 2 & -5 \\ -3 & 8 \end{pmatrix}$ Ans (1) $\begin{pmatrix} 2 & -5 \\ -3 & 8 \end{pmatrix}$

11) If $A^T A^{-1}$ is symmetric then $A^2 = ?$

Soln

$(A^T A^{-1})^T = A^T A^{-1}$

$(A^{-1})^T (A^T)^T = A^T A^{-1}$

$(A^{-1})^T A = A^T A^{-1}$

$A^T (A^{-1})^T A A = (A^T) A^T A^{-1} A$.   [Pre & post multiply by A^T and A

$A^2 = (A^T)^2$ Ans (2) $(A^T)^2$

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12) If A is non singular matrix such that $A^{-1} \begin{pmatrix} 5 & 3 \\ -2 & -1 \end{pmatrix}$ then $(A^T)^{-1} = ?$

Soln

$A^{-1} = \begin{pmatrix} 5 & 3 \\ -2 & -1 \end{pmatrix}$ $|A^{-1}| = -5+6 = 1$, $adj\ A^{-1} = \begin{pmatrix} -1 & -3 \\ 2 & 5 \end{pmatrix}$

$A = (A^{-1})^{-1} = \frac{1}{1} \begin{pmatrix} -1 & -3 \\ 2 & 5 \end{pmatrix} = \begin{pmatrix} -1 & -3 \\ 2 & 5 \end{pmatrix}$

$A^T = \begin{pmatrix} 5 & 3 \\ -2 & -1 \end{pmatrix}$

$(A^T)^{-1} = \frac{1}{-5+6} \begin{pmatrix} -1 & -3 \\ 2 & 5 \end{pmatrix} = \frac{1}{1} \begin{pmatrix} -1 & -3 \\ 2 & 5 \end{pmatrix} = \begin{pmatrix} -1 & -3 \\ 2 & 5 \end{pmatrix} = A$

Ans (3) $\begin{pmatrix} -1 & -3 \\ 2 & 5 \end{pmatrix}$

13) If $A = \begin{pmatrix} 3/5 & 4/5 \\ x & 3/5 \end{pmatrix}$ and $A^T = A^{-1}$ then the value of x is

Soln

$$A^T = A^{-1}$$

$$AA^T = I$$

$$\begin{pmatrix} 3/5 & 4/5 \\ x & 3/5 \end{pmatrix} \begin{pmatrix} 3/5 & x \\ 4/5 & 3/5 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

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$$\tfrac{3}{5}x + \tfrac{12}{25} = 0$$

$$\tfrac{3}{5}x = -\tfrac{12}{25}$$

Ans ① $\tfrac{4}{5}$.

$$x = -\tfrac{4}{5}$$

14) If $A = \begin{pmatrix} 1 & \tan \theta/2 \\ -\tan \theta/2 & 1 \end{pmatrix}$ and $AB = I$ then $B = ?$

Soln

$$AB = I \quad \Rightarrow \quad B = A^{-1} = \frac{1}{1+\tan^2 \theta/2} \begin{pmatrix} 1 & -\tan \theta/2 \\ \tan \theta/2 & 1 \end{pmatrix}$$

$$= \frac{1}{\sec^2 \theta/2} \begin{pmatrix} 1 & -\tan \theta/2 \\ \tan \theta/2 & 1 \end{pmatrix} = \cos^2 \theta/2 \; A^T$$

Ans (2) $\cos^2 \theta/2 \; A^T$

15) If $A = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}$ and $A(adj A) = \begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$ then $k = ?$

Soln

$$adj A = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$$

$$A(adj A) = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} = \begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$$

$$\cos^2\alpha + \sin^2\alpha = k$$

$$k = 1.$$

Ans (4) $k = 1$

16) If $A = \begin{pmatrix} 2 & 3 \\ 5 & -2 \end{pmatrix}$ be such that $\lambda A^{-1} = A$ the $\lambda = ?$

Soln

$$\lambda A^{-1} = A$$

$$\lambda I = A^2 = \begin{pmatrix} 2 & 3 \\ 5 & -2 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ 5 & -2 \end{pmatrix} = \begin{pmatrix} 4+15 & 6-6 \\ 10-10 & 15+4 \end{pmatrix}$$

$$\lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 19 & 0 \\ 0 & 19 \end{pmatrix}$$

$$\Rightarrow \lambda = 19.$$

Ans (3) 19

17) If $adj\,A = \begin{pmatrix} 2 & 3 \\ 4 & -1 \end{pmatrix}$ and $adj\,B = \begin{pmatrix} 1 & -2 \\ -3 & 1 \end{pmatrix}$ then $adj\,AB$ is $\textcircled{45}$

Soln $adj(AB) = (adj\,A)(adj\,B) = \begin{pmatrix} 2 & 3 \\ 4 & -1 \end{pmatrix}\begin{pmatrix} 1 & -2 \\ -3 & 1 \end{pmatrix}$

$= \begin{pmatrix} 2-9 & -4+3 \\ 4+3 & -8-1 \end{pmatrix} = \begin{pmatrix} -7 & -1 \\ 7 & -9 \end{pmatrix}$ Ans ① $\begin{pmatrix} -7 & -1 \\ 7 & -9 \end{pmatrix}$

18) The rank of the matrix $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ -1 & -2 & -3 & -4 \end{pmatrix}$ is

Soln

$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ -1 & -2 & -3 & -4 \end{pmatrix} \xrightarrow[R_3 \to -R_3]{R_2 \to \frac{R_2}{2}} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix} \xrightarrow[R_3 \to R_3 - R_1]{R_2 \to R_2 - R_1} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & \cdot & \cdot \\ 0 & \cdot & \cdot & \cdot \end{pmatrix}$

$\rho(A) = 1.$ Ans ① 1

19) If $x^a y^b = e^m$, $x^c y^d = e^n$, $\Delta_1 = \begin{vmatrix} m & b \\ n & d \end{vmatrix}$ $\Delta_2 = \begin{vmatrix} a & m \\ c & n \end{vmatrix}$,

$\Delta_3 = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$ Then the values of x and y are respectively

Soln $x^a y^b = e^m$ $\Delta_1 = (ad-bc)\log x$ —① $\Delta_3 = (ad-bc)$.

$a\log x + b\log y = m$ —① $\Delta_2 = \begin{vmatrix} a & m \\ c & n \end{vmatrix}$ ① $\Rightarrow$ $\Delta_1 = \Delta_3 \log x$

$x^c y^d = e^n$ $= an - cm$ $\dfrac{\Delta_1}{\Delta_3} = \log x$

$c\log x + d\log y = n$ —② $= ac\log n + ad\log y$ $\boxed{e^{\Delta_1/\Delta_3} = x}$

$\Delta_1 = \begin{vmatrix} m & b \\ n & d \end{vmatrix}$ $-ac\log x - bc\log y$ ② $\Rightarrow \dfrac{\Delta_2}{\Delta_3} = \log y$

$= md - nb$ $= (ad-bc)\log y$ $\boxed{e^{\Delta_2/\Delta_3} = y}$

$= ad\log x + bd\log y - bc\log x$ —②   Ans (4) $\left(e^{\Delta_1/\Delta_3}, e^{\Delta_2/\Delta_3} \right)$

$-bd\log y$

20) Which of the following is / are correct?

Soln (i) adjoint of diagonal matrix is also a diagonal matrix

(iv) $A(adj\,A) = (adj\,A)A = |A| I.$

                                      Ans (4)

21) If $\rho(A) = \rho(A|B)$, then the system $AX = B$ of linear equation is

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Soln consistent.   Ans (2)

22) If $0 \le \theta \le \pi$ and the system of equations

$x + (\sin\theta)y - (\cos\theta)z = 0$, $\cos\theta\,x - y + z = 0$

$(\sin\theta)x + y - z = 0$ has a non trivial solution then θ is

Soln: If $AX = 0$ has a non trivial solution if $|A| = 0$.   (46)

$\begin{vmatrix} 1 & \sin\theta & -\cos\theta \\ \cos\theta & -1 & 1 \\ \sin\theta & 1 & -1 \end{vmatrix} = 0$ $\Rightarrow$ $1(1-1) - \sin\theta(-\cos\theta - \sin\theta)$
$\qquad\qquad -\cos\theta(\cos\theta + \sin\theta) = 0$

$\Rightarrow + \sin\theta\cos\theta + \sin^2\theta - \cos^2\theta - \cos\theta\sin\theta = 0$

$\qquad\qquad \sin^2\theta = \cos^2\theta$

$\qquad\qquad \sin\theta = \cos\theta$

$\qquad\qquad \theta = \frac{\pi}{4}$.   Ans (H) $\theta = \frac{\pi}{4}$.

23) The augmented matrix of a system of linear equation
is $\begin{pmatrix} 1 & 2 & 7 & 3 \\ 0 & 1 & 4 & 6 \\ 0 & 0 & \lambda-7 & \mu+5 \end{pmatrix}$. The system has infinitely

many solution if

Soln: $\rho(A) = \rho(A:B) = 2$

$\lambda - 7 = 0$ & $\mu + 5 = 0$ Ans (A) $\lambda \leq 7$ $\mu: -5$
$\lambda = 7$ $\qquad$ $\mu = -5$

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24) Let $A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$ & $AB = \begin{pmatrix} 3 & 1 & -1 \\ 1 & 3 & x \\ -1 & 1 & 3 \end{pmatrix}$ If B is the

inverse of A. Then the value of x is

Soln: $B = A^{-1}$

$4A^{-1} = \begin{pmatrix} 3 & 1 & -1 \\ 1 & 3 & x \\ -1 & 1 & 3 \end{pmatrix}$ $\Rightarrow$ $A^{-1} = \frac{1}{4}\begin{pmatrix} 3 & 1 & -1 \\ 1 & 3 & x \\ -1 & 1 & 3 \end{pmatrix}$

x is a_{32} minor matrix of A.

$x = -\begin{vmatrix} 2 & 1 \\ -1 & -1 \end{vmatrix} = -(-2+1) = -(-1) = 1$ Ans (H) 1

25) If $A = \begin{pmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix}$ then $adj(adj A)$ is

Soln: $adj(adj A) = |A|^{n-1} \cdot A = |A|^2 |A|$

$|A| = 3(-3+4) + 3(2) + 4(-2) = 3+6-8 = 1$.

$adj(adjA) = 1^2\begin{pmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix} = \begin{pmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix} = A$

Ans (1) $\begin{pmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix}$

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