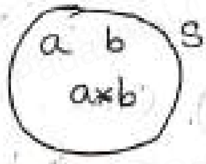
12. DISCRETE MATHEMATICS

Binary Operation (or) closure property

$\forall\, a, b \in S$, $a * b$ is unique and $a * b \in S$

where S is any nonempty set.



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Properties

1) Commutative property

$$a * b = b * a \quad \forall\, a, b \in S$$

2) Associative property

$$a * (b * c) = (a * b) * c \quad \forall\, a, b, c \in S$$

3) Existence of identity property

$e \in S$ is an identity element

$$a * e = e * a = a \quad \forall\, a \in S$$

4) Existence of Inverse property

$b \in S$ is said to be Inverse element of a

$$a * b = b * a = e \quad \forall\, a \in S \qquad b = a^{-1}$$

Exercise 12.1

1) Determine whether $*$ is a binary operation on the sets given below

(i) $a * b = a \cdot |b|$ on R

Let $a, b \in R$ $\quad |b| \in R$

$a \cdot |b| \in R$

$\Rightarrow a * b \in R \quad \forall\, a, b \in R$.

$*$ is the binary operation on R.

(ii) $a * b = \min(a, b)$ on $A = \{1, 2, 3, 4, 5\}$

Let $a, b \in A$

$\min(a, b) \in A$

$a * b \in A$

$\forall\, a, b \in A$

For example

$\min(1, 2) = 1 \in A$

$\min(1, 5) = 1 \in A$

$\min(2, 3) = 2 \in A$

$\therefore *$ is the binary operation on A.

(iii) $a*b = a\sqrt{b}$ is binary on R

Let $a,b \in R$

$\therefore a\sqrt{b} \notin R$ root of negative numbers not in R $\left[\because 2,-2 \in R,\ 2\sqrt{-2} \notin R\right.$

$\Rightarrow a*b \notin R$

$\therefore *$ is not binary on R.

2). On Z, define $\otimes$ by $(m\otimes n) = m^n + n^m \ \forall\, m,n \in Z$

Is $\otimes$ binary on Z

Let $m,n \in Z$

take $m=2$ $n=-2$

$$m^n + n^m = 2^{-2} + (-2)^2$$

$$= \frac{1}{4} + 4 = \frac{17}{4} \notin Z$$

$m^n + n^m \notin Z$

$\therefore m\otimes n \notin Z$ $\therefore \otimes$ is not binary on Z.

3) Let $*$ be defined on R by $(a*b) = a+b+ab-7$.

Is $*$ binary on R? If so find $3*\left(\frac{-7}{15}\right)$

Let $a,b \in R$

clearly $a,b,a.b \in R$

$\therefore a+b+ab-7 \in R$

$\Rightarrow a*b \in R$

$\therefore *$ is binary operator on R

$$3*\left(\frac{-7}{15}\right) = 3 - \frac{7}{15} + 3\left(\frac{-7}{15}\right) - 7$$

$$= \frac{45-7-21-105}{15} = \frac{45-133}{15} = -\frac{88}{15}$$

4) Let $A = \{a+\sqrt{5}b : a,b \in Z\}$, check whether the usual multiplication is a binary operation on A

Let $x = a+\sqrt{5}b$ $y = c+d\sqrt{5}$

$x,y \in A$, $a,b,c,d \in Z$

$$xy = (a+\sqrt{5}b)(c+\sqrt{5}d)$$

$$= ac + 5bd + \sqrt{5}ad + \sqrt{5}bc$$

$$= (ac+5bd) + \sqrt{5}(ad+bc) \in A$$

$\therefore xy \in R$

multiplication is binary on R.

5) (i). Define an operation $*$ on Q as follows

$a*b = \frac{a+b}{2}$; $a,b \in Q$. Examine the closure, commutative and associative properties satisfied by $*$ on Q.

① closure property

given $a,b \in Q \Rightarrow a+b \in Q \Rightarrow \frac{a+b}{2} \in Q \Rightarrow a*b \in Q$ $\forall a,b \in Q$.

$*$ is closure on Q.

② commutative property

$$a*b = \frac{a+b}{2}$$

$$= \frac{b+a}{2}$$

$$a*b = b*a \quad \forall a,b \in Q$$

commutative property is true. $\therefore *$ is commutative on Q.

③ associative property

$$a*(b*c) = a*\left(\frac{b+c}{2}\right)$$

$$= \frac{a+\left(\frac{b+c}{2}\right)}{2} = \frac{2a+b+c}{2} \times \frac{1}{2}$$

$$a*(b*c) = \frac{2a+b+c}{4} \quad \text{--- (1)}$$

$$(a*b)*c = \left(\frac{a+b}{2}\right)*c$$

$$= \frac{\left(\frac{a+b}{2}\right)+c}{2} = \frac{a+b+2c}{2} \times \frac{1}{2}$$

$$= \frac{a+b+2c}{4} \quad \text{--- (2)}$$

$\therefore$ From (1) & (2) $(a*b)*c \neq a*(b*c)$

$*$ is not associative on Q.

(ii) Define an operation $*$ on Q as follows $a*b = \frac{a+b}{2}$ $a,b \in Q$. Examine the existence of identity and the existence of inverse for the operation $*$ on Q.

(i) Existence of Identity

Let $a \in Q$, e be the identity element on Q.

By definition of $*$ $\quad a*e = \frac{a+e}{2}$

By definition of e $\quad a*e = a$.

$$\frac{a+e}{2}=a$$

$$a+e=2a$$

$$e=2a-a$$

$$e=a \quad \forall a \in Q$$

This means every element is a identity element

This is not possible.

$*$ has No Identity element.

(ii) Existence of Inverse

$*$ has no Identity element

$\therefore$ cannot defined as $a*a^{-1}=a^{-1}*a=e$.

$\therefore$ $*$ has no Inverse.

6) Fill in the following table so that the binary operation $*$ on $A=\{a,b,c\}$ is commutative.

$*$	a	b	c
a	b		
b	c	b	a
c	a		c

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(i) From table. $b*a=c$ ∴ $\boxed{* \text{ is commutative}}$

$\Rightarrow a*b=c.$

(ii) $c*a=a \Rightarrow a*c=a$

(iii) $b*c=a \Rightarrow c*b=a$

$\therefore$ given table is

$*$	a	b	c
a	b	c	a
b	c	b	a
c	a	a	c

7) consider the binary operation $*$ defined on the set $A=\{a,b,c,d\}$ by the following table.

$*$	a	b	c	d
a	a	c	b	d
b	d	a	b	c
c	c	d	a	a
d	d	b	a	c

Is it commutative and associative?

(1) <u>commutative</u> <u>property</u>

$a*b=c$ but $b*a=d \Rightarrow a*b \neq b*a$

$a*c=b$ but $c*a=c \Rightarrow a*c \neq c*a$

$\therefore *$ is not commutative.

2) <u>associative</u> <u>Property</u>

$(a*b)*c = c*c = a$

$a*(b*c) = a*b = c$

$\therefore (a*b)*c \neq a*(b*c)$

$*$ is not associative.

8) Let $A=\begin{pmatrix}1&0&1&0\\0&1&0&1\\1&0&0&1\end{pmatrix}$, $B=\begin{pmatrix}0&1&0&1\\1&0&1&0\\1&0&0&1\end{pmatrix}$, $C=\begin{pmatrix}1&1&0&1\\0&1&1&0\\1&1&1&1\end{pmatrix}$

be any three boolean matrices of the same type

Find (i) $A \vee B$ (ii) $A \wedge B$ (iii) $(A\vee B)\wedge C$ (iv) $(A\wedge B)\vee C$.

(i) $A\vee B = \begin{pmatrix}1&0&1&0\\0&1&0&1\\1&0&0&1\end{pmatrix} \vee \begin{pmatrix}0&1&0&1\\1&0&1&0\\1&0&0&1\end{pmatrix} = \begin{pmatrix}1&1&1&1\\1&1&1&1\\1&0&0&1\end{pmatrix}$

(ii) $A\wedge B = \begin{pmatrix}1&0&1&0\\0&1&0&1\\1&0&0&1\end{pmatrix} \wedge \begin{pmatrix}0&1&0&1\\1&0&1&0\\1&0&0&1\end{pmatrix} = \begin{pmatrix}0&0&0&0\\0&0&0&0\\1&0&0&1\end{pmatrix}$

(iii) $(A\vee B)\wedge C = \begin{pmatrix}1&1&1&1\\1&1&1&1\\1&0&0&1\end{pmatrix} \wedge \begin{pmatrix}1&1&0&1\\0&1&1&0\\1&1&1&1\end{pmatrix}$

$= \begin{pmatrix}1&1&0&1\\0&1&1&0\\1&0&0&1\end{pmatrix}$

(iv) $(A\wedge B)\vee C = \begin{pmatrix}0&0&0&0\\0&0&0&0\\1&0&0&1\end{pmatrix} \vee \begin{pmatrix}1&1&0&1\\0&1&1&0\\1&1&1&1\end{pmatrix} =$

$= \begin{pmatrix}1&1&0&1\\0&1&1&0\\1&1&1&1\end{pmatrix}$

9 (i) Let $M = \left\{ \begin{pmatrix} x & x \\ x & x \end{pmatrix} ; x \in R - \{0\} \right\}$ and let $*$ be the matrix multiplication. Determine whether M is closed under $*$. If so examine the commutative and associative properties satisfied by $*$ on M.

Let $A = \begin{pmatrix} x & x \\ x & x \end{pmatrix}$, $B = \begin{pmatrix} y & y \\ y & y \end{pmatrix}$ where $x, y \in R - \{0\}$

$A, B \in M$

1) <u>closure property</u>

$A, B \in M$

$$\Rightarrow \quad AB = \begin{pmatrix} x & x \\ x & x \end{pmatrix} \begin{pmatrix} y & y \\ y & y \end{pmatrix}$$

$$= \begin{pmatrix} 2xy & 2xy \\ 2xy & 2xy \end{pmatrix} \in M \quad \because x, y \in R - \{0\}, \; xy \in R - \{0\}$$

$AB \in M$

$A, B \in M \Rightarrow A * B \in M$

$\therefore *$ is closed on M

2) <u>commutative Property</u>

$A, B \in M$

$$A * B = \begin{pmatrix} x & x \\ x & x \end{pmatrix} \begin{pmatrix} y & y \\ y & y \end{pmatrix}$$

$$= \begin{pmatrix} 2xy & 2xy \\ 2xy & 2xy \end{pmatrix} = \begin{pmatrix} 2yx & 2yx \\ 2yx & 2yx \end{pmatrix}$$

$$= \begin{pmatrix} y & y \\ y & y \end{pmatrix} \begin{pmatrix} x & x \\ x & x \end{pmatrix}$$

$A * B = B * A$

$*$ is commutative on M.

3) <u>associative Property</u>

Matrix multiplication is always associative.

i.e $A * (B * C) = (A * B) * C \quad \forall A, B, C \in M$

$*$ is associative on M.

(ii) Let $M = \left\{ \begin{pmatrix} x & x \\ x & x \end{pmatrix} ; x \in R - \{0\} \right\}$ and let $*$ be the matrix multiplication. Determine whether M is closed under $*$. If so examine the existence of identity, inverse property for $*$ on M.

① Closure property

$A, B \in M$ where $A = \begin{pmatrix} x & x \\ x & x \end{pmatrix}$, $B = \begin{pmatrix} y & y \\ y & y \end{pmatrix}$ $x, y \in R-\{0\}$

$$A * B = \begin{pmatrix} 2xy & 2xy \\ 2xy & 2xy \end{pmatrix} \in M$$

$$A*B \in M$$

$A, B \in M \Rightarrow A*B \in M$

$\therefore *$ is closed on M.

② Existence of Identity property

Let $A \in M$, $E = \begin{pmatrix} e & e \\ e & e \end{pmatrix}$ be the Identity element

$$AE = A$$

$$\begin{pmatrix} x & x \\ x & x \end{pmatrix}\begin{pmatrix} e & e \\ e & e \end{pmatrix} = \begin{pmatrix} x & x \\ x & x \end{pmatrix}$$

$$\begin{pmatrix} 2xe & 2xe \\ 2xe & 2xe \end{pmatrix} = \begin{pmatrix} x & x \\ x & x \end{pmatrix}$$

$$2\cancel{x}e = \cancel{x}1$$

$$2e = 1 \qquad e = \tfrac{1}{2} \in R-\{0\}$$

Identity element $E = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \in M$

satisfies $AE = A$

Similarly $EA = A$ $\forall A \in M$

$*$ has Identity element on M.

③ Existence of Inverse property

Let $A \in M$, $A^{-1} = \begin{pmatrix} x^{-1} & x^{-1} \\ x^{-1} & x^{-1} \end{pmatrix}$ be the inverse of A

$$AA^{-1} = E$$

$$\begin{pmatrix} x & x \\ x & x \end{pmatrix}\begin{pmatrix} x^{-1} & x^{-1} \\ x^{-1} & x^{-1} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

$$\begin{pmatrix} 2xx^{-1} & 2xx^{-1} \\ 2xx^{-1} & 2xx^{-1} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

$$2xx^{-1} = \tfrac{1}{2}$$

$$x^{-1} = \frac{1}{4x} \in R-\{0\}$$

$A^{-1} = \begin{pmatrix} \frac{1}{4x} & \frac{1}{4x} \\ \frac{1}{4x} & \frac{1}{4x} \end{pmatrix} \in M$ is the inverse of A in M

Similarly we can find $A^{-1}A = E$ $\forall A \in M$

$*$ is Inverse on M.

—x—

(c) (i) Let A be $Q-\{1\}$. Define $*$ on A by $x*y=x+y-xy$. Is $*$ binary on A? If so examine the commutative and associative properties satisfied by $*$ on A.

$A=Q-\{1\}$ $x,y\in A$ $x\neq 1$ $y\neq 1$

$*$ is defined by $x*y=x+y-xy$

① <u>closure property</u>

Let $x,y\in A$ $x\neq 1, y\neq 1$

$x-1\neq 0, y-1\neq 0$

$(x-1)(y-1)\neq 0$

$xy-x-y+1\neq 0$

$1\neq x+y-xy$

$x*y\neq 1 \Rightarrow x*y\in A$

$x,y\in A\Rightarrow x*y\in A$.

$*$ is closed on A.

② <u>commutative property</u>:-

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Let $x,y\in A$ $x*y=x+y-xy$.

$=y+x-yx$

$x*y=y*x$.

$*$ is commutative on A.

③ <u>Associative property</u>:-

$(x*y)*z=(x+y-xy)*z$

$=(x+y-xy)+z-(x+y-xy)z$

$(x*y)*z=x+y+z-xy-xz-yz+xyz$ ——①

$x*(y*z)=x*(y+z-yz)$.

$=x+(y+z-yz)-x(y+z-yz)$

$x*(y*z)=x+y+z-xy-xz-yz+xyz$ ——②

From ①&② $(x*y)*z=x*(y*z)$ $\forall x,y,z\in A$

$*$ is associative on A

(ii) examine the existence of identity, inverse properties for $*$ on A.

① closure property (same as on above)

② <u>existence of Identity property</u>.

Let $x,a\in A$, e be the identity element

By definition of $*$ $\quad x*e = x+e-xe$

By definition of e $\quad x*e = x$

$$x*e - xe = x$$
$$e(1-x) = 0$$
$$e = \frac{0}{1-x} \qquad e = 0 \in A$$

Identity element $\boxed{e=0 \in A}$

$*$ has Identity element on A

③ Existence of Inverse property :-

Let $x \in A$, $\therefore$ x^{-1} be the inverse of x

By definition of $*$ $\quad x*x^{-1} = x+x^{-1}-xx^{-1}$

By definition of x^{-1} $\quad x*x^{-1} = e$

$$x+x^{-1}-xx^{-1} = 0$$
$$x^{-1}(1-x) = -x$$
$$\boxed{x^{-1} = \frac{-x}{1-x} \in A}$$

Inverse of x is $x^{-1} = \frac{-x}{1-x} \in A$ $\quad \forall x \in A$

$*$ has Inverse element for $\forall x \in A$.

Mathematical Logic

Truth tables

① Truth table for NOT

p	$\neg p$
T	F
F	T

② Truth table for AND

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	T

③ Truth table for OR

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

④ Truth table for conditional statement.

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

⑤ Truth table for Bi-conditional statement.

p	q	$p \Leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

⑥ Truth table for Exclusive OR (EOR) $\bar{\vee}$

p	q	$p \bar{\vee} q$
T	T	F
T	F	T
F	T	T
F	F	F

⑦ Tautology

A statement is said to be tautology if its truth values is always T irrespective of the truth values of its compound statements, (denoted by $\mathbb{T}$)

8) contradiction

A statement is said to be contradiction if its truth value is always F irrespective of the truth values of its compound statements. (denoted by $\mathbb{F}$)

9) contingency

A statement which is neither a tautology nor a contradiction is called contingency.

10) Duality

The dual of a statement formula is obtained by replacing $\vee$ by $\wedge$, T by F , $\mathbb{T}$ by $\mathbb{F}$
$\wedge$ by $\vee$, F by T , $\mathbb{F}$ by $\mathbb{T}$.

Exercise 12.2

1) Let p : Jupiter is a planet and q : India is an Island be any two simple statements. Give verbal sentence describing each of the following statements.

p : Jupiter is a planet
q : India is an Island.

(i) $\neg p$: Jupiter is not a planet

(ii) $p \wedge \neg q$: Jupiter is a planet and India is not an Island.

(iii) $\neg p \vee q$: Jupiter is not a planet or India is an Island.

(iv) $p \to \neg q$: If Jupiter is a planet then India is not an Island.

(v) $p \Leftrightarrow q$: Jupiter is a planet if and only if India is an Island.

2) Write each of the following sentences in symbolic form using statement variables p and q.

p: 19 is a prime number
q: All the angles of a triangle are equal.

(i) 19 is not a prime number and all the angles of a triangle are equal.

$\neg p \wedge q$

(ii) 19 is a prime number or all the angles of a triangle are not equal.

$p \vee \neg q$

(iii) 19 is a prime number and all the angles of a triangle are equal

$p \wedge q$

(iv) 19 is not a prime number $\neg p$

3) Determine the Truth value of each of the following statements.

(i) If $6+2=5$, then the milk is white.

p: $6+2=5$ F
q: The milk is white T

Symbolic form is
$p \rightarrow q$
$F \rightarrow T$ Truth value is T

(ii) China is in Europe or $\sqrt{3}$ is an integer.

p: China is in Europe F
q: $\sqrt{3}$ is an integer F

Symbolic form is $p \vee q$
$F \vee F$ Truth value is F

(iii) It is not true that $5+5=9$ or Earth is a planet.

p : It is true that $5+5=9$
q : Earth is a planet.

symbolic form is $\neg p \vee q$

Truth value T V T

Truth value is T

(iv) 11 is a prime number and all the sides of a rectangle are equal.

p : 11 is a prime number
q : all the sides of a rectangle are equal.

Symbolic form is $p \wedge q$

Truth value $T \wedge F$

Truth value is F

4) which one of the following sentences is a proposition?

(i) $4+7=12$ proposition
(ii) what are you doing? (not a proposition)
(iii) $3^n \leq 81$, $n \in N$ (proposition)
(iv) Peacock is our national bird. (proposition)
(v) How tall this mountain is! (not a proposition)

$\therefore$ (i)(iii)&(iv) are propositions.

5) write the converse, inverse, and contrapositive of each of the following implication

(i) If x and y are numbers such that $x=y$, then $x^2=y^2$

p : x and y are numbers such that $x=y$
q : $x^2=y^2$

given statement $\therefore$ symbolic form is $p \rightarrow q$

(1) converse $q \rightarrow p$

If x and y are numbers such that $x^2=y^2$ then $x=y$

② Inverse: $\neg p \to \neg q$

If x and y are numbers such that $x \neq y$ then $x^2 \neq y^2$

③ contrapositive: $\neg q \to \neg p$

If x and y are numbers such that $x^2 \neq y^2$ then $x \neq y$

(ii) If a quadrilateral is a square then it is a rectangle.

p: A quadrilateral is a square.

q: A quadrilateral is a rectangle

given statement is $p \to q$

① converse:- $q \to p$

If a quadrilateral is a rectangle then it is a square.

② Inverse:- $\neg p \to \neg q$

If a quadrilateral is not a square then it is not a rectangle.

③ Contrapositive:

If a quadrilateral is not a rectangle then it is not a square.

6) construct the truth table for the following

(i) $\neg p \wedge \neg q$

p	q	$\neg p$	$\neg q$	$\neg p \wedge \neg q$
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

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(ii) $\neg(p \wedge \neg q)$

p	q	$\neg q$	$p \wedge \neg q$	$\neg(p \wedge \neg q)$
T	T	F	F	T
T	F	T	T	F
F	T	F	F	T
F	F	T	F	T

(iii) $(p \vee q) \vee \neg q$

p	q	$p \vee q$	$\neg q$	$(p \vee q) \vee (\neg q)$
T	T	T	F	T
T	F	T	T	T
F	T	T	F	T
F	F	F	T	T

(iv) $(\neg p \rightarrow r) \wedge (p \leftrightarrow q)$

p	q	r	$\neg p$	$\neg p \rightarrow r$	$p \leftrightarrow q$	$(\neg p \rightarrow r) \wedge (p \leftrightarrow q)$
T	T	T	F	T	T	T
T	T	F	F	T	T	T
T	F	T	F	T	F	F
T	F	F	F	T	F	F
F	T	T	T	T	F	F
F	T	F	T	F	F	F
F	F	T	T	T	T	T
F	F	F	T	F	T	F

7) Verify whether the following compound propositions are tautologies or contradictions or contingency.

(i) $(p \wedge q) \wedge \neg (p \vee q)$

p	q	$p \wedge q$	$p \vee q$	$\neg (p \vee q)$	$(p \wedge q) \wedge \neg (p \vee q)$
T	T	T	T	F	F
T	F	F	T	F	F
F	T	F	T	F	F
F	F	F	F	T	F

Last column contains only F

This is a contradiction

(ii) $((p \vee q) \wedge \neg p) \rightarrow q$

p	q	$p \vee q$	$\neg p$	$(p \vee q) \wedge \neg p$	$((p \vee q) \wedge \neg p) \rightarrow q$
T	T	T	F	F	T
T	F	T	F	F	T
F	T	T	T	T	T
F	F	F	T	F	T

Last column contains only T

This is a Tautology.

(iii) $(p\to q) \leftrightarrow (\neg p \to q)$

p	q	$p\to q$	$\neg p$	$\neg p \to q$	$(p\to q)\leftrightarrow(\neg p\to q)$
T	T	T	F	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	F	T	T	F	F

Last contains T and F

$\therefore$ This is a contingency.

(iv) $((p\to q)\wedge(q\to r)) \to (p\to r)$

p	q	r	$p\to q$	$q\to r$	$p\to r$	$(p\to q)\wedge(q\to r)$	$((p\to q)\wedge(q\to r))\to(p\to r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	T	F	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

Last contains only T

$\therefore$ This is a tautology.

8) Show that (i) $\neg(p\wedge q) \equiv \neg p \vee \neg q$

TABLE①

p	q	$p\wedge q$	$\neg(p\wedge q)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

TABLE②

p	q	$\neg p$	$\neg q$	$\neg p \vee \neg q$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

FROM Table ①&② Last columns are Identical.

$\therefore \neg(p\wedge q) \equiv \neg p \vee \neg q$.

8)(ii) $\neg(p\to q) \equiv p \wedge \neg q$

Table ①

p	q	$p\to q$	$\neg(p\to q)$
T	T	T	F
T	F	F	T
F	T	T	F
F	F	T	F

Table②

p	q	$\neg q$	$p\wedge\neg q$
T	T	F	F
T	F	T	T
F	T	F	F
F	F	T	F

From table ① & ②
Last columns are identical $\therefore \neg(p\to\neg q) \equiv p\wedge(\neg\neg q)$

9) prove that $q\to p \equiv \neg p \to \neg q$

TABLE ①

p	q	$q\to p$
T	T	T
T	F	T
F	T	F
F	F	T

TABLE ②

p	q	$\neg p$	$\neg q$	$\neg p\to\neg q$
T	T	F	F	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

From table ① & ② The last columns are identical.

$\therefore q\to p \equiv \neg p\to\neg q$

10) show that $p\to q$ and $q\to p$ are not equivalent.

TABLE ①

p	q	$p\to q$
T	T	T
T	F	F
F	T	T
F	F	T

TABLE ②

p	q	$q\to p$
T	T	T
T	F	T
F	T	F
F	F	T

From table ① & ② Last column & are not identical $\therefore p\to q \not\equiv q\to p$.

11). Show that $\neg(p\leftrightarrow q) \equiv p\leftrightarrow\neg q$

TABLE ①

p	q	$p\leftrightarrow q$	$\neg(p\leftrightarrow q)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	T	F

TABLE ②

p	q	$\neg q$	$p\leftrightarrow\neg q$
T	T	F	F
T	F	T	T
F	T	F	T
F	F	T	F

From table ① & ② Last columns are identical $\therefore \neg(p\leftrightarrow q) \equiv p\leftrightarrow\neg q$

12) check whether the statement $p\to(q\to p)$ is a tautology or a contradiction without using the truth table.

$$p\to(q\to p) \equiv p\to(\neg q\vee p)$$
$$\equiv \neg p\vee(\neg q\vee p)$$
$$\equiv \neg p\vee(p\vee\neg q) \quad (\because \text{commutative Law})$$
$$\equiv (\neg p\vee p)\vee\neg q \quad [\because \text{associative Law}]$$

$$\equiv T \vee \neg q$$

$$\equiv T$$

$\therefore\ p \to (q \to p)$ is a Tautology.

13) Using truth table check whether the statements $\neg(p \vee q) \vee (\neg p \wedge q)$ and $\neg p$ are logically equivalent

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p$	$(\neg p \wedge q)$	$\neg(p \vee q) \vee (\neg p \wedge q)$
T	T	T	F	F	F	F
T	F	T	F	F	F	F
F	T	T	F	T	T	T
F	F	F	T	T	F	T

From the table Last column and the Fifth columns are Identical.

$$\therefore\ \neg(p \vee q) \vee (\neg p \wedge q) \equiv \neg p$$

14) Prove $p \to (q \to r) \equiv (p \wedge q) \to r$ without using truth table.

$$p \to (q \to r) \equiv p \to (\neg q \vee r)$$

$$\equiv \neg p \vee (\neg q \vee r)$$

$$\equiv (\neg p \vee \neg q) \vee r \quad [\because \text{associative Law}]$$

$$\equiv \neg(p \wedge q) \vee r$$

$$\equiv (p \wedge q) \to r$$

Hence proved.

15) Prove that $p \to (\neg q \vee r) \equiv \neg p \vee (\neg q \vee r)$ using Truth table.

TABLE ① $p \to (\neg q \vee r)$

p	q	r	¬q	(¬q ∨ r)	p → (¬q ∨ r)
T	T	T	F	T	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	F	T	T
F	T	F	F	F	T
F	F	T	T	T	T
F	F	F	T	T	T

TABLE ②

p	q	r	¬p	¬q	(¬q ∨ r)	¬p ∨ (¬q ∨ r)
T	T	T	F	F	T	T
T	T	F	F	F	F	F
T	F	T	F	T	T	T
T	F	F	F	T	T	T
F	T	T	T	F	T	T
F	T	F	T	F	F	T
F	F	T	T	T	T	T
F	F	F	T	T	T	T

From Table ①&② Last colums are Identical.

$$\therefore p \to (\neg q \vee r) \equiv \neg p \vee (\neg q \vee r)$$

(Using one table is also good)

—x—

Need suggestions
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