

## Chapter 3: Indefinite Integration

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### EXERCISE 3.1 [PAGE 102]

#### Exercise 3.1 | Q 1.1 | Page 102

Integrate the following w.r.t. x :  $x^3 + x^2 - x + 1$

#### SOLUTION

$$\begin{aligned}\int (x^3 + x^2 - x + 1) dx &= \int x^3 dx + \int x^2 dx - \int x dx + \int 1 dx \\ &= \frac{x^4}{4} + \frac{x^3}{3} - \frac{x^2}{2} + x + c.\end{aligned}$$

#### SOLUTION

$$\begin{aligned}\int (x^3 + x^2 - x + 1) dx &= \int x^3 dx + \int x^2 dx - \int x dx + \int 1 dx \\ &= \frac{x^4}{4} + \frac{x^3}{3} - \frac{x^2}{2} + x + c.\end{aligned}$$

#### Exercise 3.1 | Q 1.2 | Page 102

Integrate the following w.r.t. x :  $\int x^2 \left(1 - \frac{2}{x}\right)^2 dx$

#### SOLUTION

$$\begin{aligned}&\int x^2 \left(1 - \frac{2}{x}\right)^2 dx \\ &= \int x^2 \left(1 - \frac{4}{x} + \frac{4}{x^2}\right) dx \\ &= \int (x^2 - 4x + 4) dx\end{aligned}$$

$$\begin{aligned}
&= \int x^2 dx - 4 \int x dx + 4 \int 1 dx \\
&= \frac{x^3}{3} - 4 \left( \frac{x^2}{2} \right) + 4x + c \\
&= \frac{1}{3}x^3 - 2x^2 + 4x + c.
\end{aligned}$$

**Exercise 3.1 | Q 1.3 | Page 102**

Integrate the following w.r.t.  $x$  :  $3 \sec^2 x - \frac{4}{x} + \frac{1}{x\sqrt{x}} - 7$

**SOLUTION**

$$\begin{aligned}
&\int \left( 3 \sec^2 x - \frac{4}{x} + \frac{1}{x\sqrt{x}} - 7 \right) dx \\
&= 3 \int \sec^2 x dx - 4 \int \frac{1}{x} dx + \int x^{-\frac{3}{2}} dx - 7 \int 1 dx \\
&= 3 \tan x - 4 \log|x| + \frac{x - \frac{3}{2} + 1}{-\frac{3}{2} + 1} - 7x + c \\
&= 3 \tan x - 4 \log|x| - \frac{2}{\sqrt{x}} - 7x + c
\end{aligned}$$

**Exercise 3.1 | Q 1.4 | Page 102**

Integrate the following w.r.t.  $x$  :  $2x^3 - 5x + \frac{3}{x} + \frac{4}{x^5}$

**SOLUTION**

$$\begin{aligned}
&\int \left( 2x^3 - 5x + \frac{3}{x} + \frac{4}{x^5} \right) dx \\
&= 2 \int x^3 dx - 5 \int x dx + 3 \int \frac{1}{x} dx + 4 \int x^{-5} dx \\
&= 2 \left( \frac{x^4}{4} \right) - 5 \left( \frac{x^2}{2} \right) + 3 \log|x| + 4 \left( \frac{x}{-4} \right) + c
\end{aligned}$$

$$= \frac{x^4}{2} - \frac{5}{2}x^2 + 3\log|x| - \frac{1}{x^4} + c$$

**Exercise 3.1 | Q 1.5 | Page 102**

Integrate the following w.r.t. x :  $\frac{3x^3 - 2x + 5}{x\sqrt{x}}$

**SOLUTION**

$$\begin{aligned} & \int \frac{3x^3 - 2x + 5}{x\sqrt{x}} dx \\ &= \int x^{\frac{-3}{2}} (3x^3 - 2x + 5) dx \\ &= \int \left(3x^{\frac{3}{2}} - 2x^{-\frac{1}{2}} + 5x^{-\frac{3}{2}}\right) dx \\ &= 3 \int x^{\frac{3}{2}} dx - 2 \int x^{-\frac{1}{2}} dx + 5 \int x^{-\frac{3}{2}} dx \\ &= 3 \left( \frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} \right) - 2 \left( \frac{x^{\frac{1}{2}+1}}{-\frac{1}{2}+1} \right) + 5 \left( \frac{x^{-\frac{3}{2}+1}}{-\frac{3}{2}+1} \right) + c \\ &= \frac{6}{5}x^2\sqrt{x^2} - 4\sqrt{x} - \frac{10}{\sqrt{x}} + c. \end{aligned}$$

**Exercise 3.1 | Q 2.01 | Page 102**

Evaluate the following integrals :  $\tan^2 x$

**SOLUTION**

$$\begin{aligned} \int \tan^2 x dx &= \int (\sec^2 x - 1) dx \\ &= \int \sec^2 x dx - \int 1 dx \\ &= \tan x - x + C. \end{aligned}$$

**Exercise 3.1 | Q 2.02 | Page 102**

Evaluate the following integrals :  $\int \frac{\sin 2x}{\cos x} dx$

**SOLUTION**

$$\begin{aligned}\int \frac{\sin 2x}{\cos x} dx &= \int \frac{2 \sin x \cos x}{\cos x} dx \\&= 2 \int \sin x dx \\&= -2 \cos x + c.\end{aligned}$$

**Exercise 3.1 | Q 2.03 | Page 102**

Evaluate the following integrals :  $\int \frac{\sin x}{\cos^2 x} dx$

**SOLUTION**

$$\begin{aligned}\int \frac{\sin x}{\cos^2 x} dx &= \int \left( \frac{1}{\cos x} \right) \left( \frac{\sin x}{\cos x} \right) dx \\&= \int \sec x \tan x dx \\&= \sec x + c.\end{aligned}$$

**Exercise 3.1 | Q 2.04 | Page 102**

Evaluate the following integrals :  $\int \frac{\cos 2x}{\sin^2 x} dx$

**SOLUTION**

$$\begin{aligned}\int \frac{\cos 2x}{\sin^2 x} dx &= \int \frac{(1 - 2 \sin^2 x)}{\sin^2 x} dx \\&= \int \left( \frac{1}{\sin^2 x} - \frac{2 \sin^2 x}{\sin^2 x} \right) dx\end{aligned}$$

$$= \int \operatorname{cosec}^2 x dx - 2 \int dx$$

$$= -\cot x - 2x + c.$$

**Exercise 3.1 | Q 2.05 | Page 102**

Evaluate the following integrals :  $\int \frac{\cos 2x}{\sin^2 x \cdot \cos^2 x} dx$

**SOLUTION**

$$\begin{aligned} & \int \frac{\cos 2x}{\sin^2 x \cdot \cos^2 x} dx \\ &= \int \frac{\cos^2 x - \sin^2 x}{\sin^2 x \cdot \cos^2 x} dx \\ &= \int \left( \frac{1}{\sin^2 x} - \frac{1}{\cos^2 x} \right) dx \\ &= \int \operatorname{cosec}^2 x dx - \int \sec^2 x dx \\ &= -\cot x - \tan x + c. \end{aligned}$$

**Exercise 3.1 | Q 2.06 | Page 102**

Evaluate the following integrals :  $\int \frac{\sin x}{1 + \sin x} dx$

**SOLUTION**

$$\begin{aligned} & \int \frac{\sin x}{1 + \sin x} dx \\ &= \int \frac{\sin x}{1 + \sin x} \times \frac{1 - \sin x}{1 - \sin x} dx \\ &= \int \frac{\sin x - \sin^2 x}{1 - \sin^2 x} dx \\ &= \int \frac{\sin x - \sin^2 x}{\cos^2 x} dx \end{aligned}$$

$$\begin{aligned}
&= \int \left( \frac{\sin x}{\cos^2 x} - \frac{\sin^2 x}{\cos^2 x} \right) dx \\
&= \int \left( \frac{1}{\cos x} \right) \left( \frac{\sin x}{\cos x} \right) dx - \int \tan^2 x dx \\
&= \int \sec x \tan x dx - \int (\sec^2 x - 1) dx \\
&= \int \sec x \tan x dx - \int \sec^2 x dx + \int 1 dx \\
&= \sec x - \tan x + x + C.
\end{aligned}$$

**Exercise 3.1 | Q 2.07 | Page 102**

Evaluate the following integrals :  $\int \frac{\tan x}{\sec x + \tan x} dx$

**SOLUTION**

$$\begin{aligned}
&\int \frac{\tan x}{\sec x + \tan x} dx \\
&= \int \frac{\tan x}{\sec x + \tan x} \times \frac{\sec x - \tan x}{\sec x - \tan x} dx \\
&= \int \frac{\sec x \tan x - \tan^2 x}{\sec^2 x - \tan^2 x} dx \\
&= \int \frac{\sec \tan x - (\sec^2 x - 1)}{1} dx \\
&= \int \sec x \tan x dx - \int \sec^2 x dx + \int 1 dx \\
&= \sec x - \tan x + x + C.
\end{aligned}$$

**Exercise 3.1 | Q 2.08 | Page 102**

Evaluate the following integrals :  $\int \sqrt{1 + \sin 2x} dx$

**SOLUTION**

$$\begin{aligned} & \int \sqrt{1 + \sin 2x} dx \\ &= \int \sqrt{\cos^2 x + \sin^2 x + 2 \sin x \cos x} dx \\ &= \int \sqrt{(\cos x + \sin x)^2} dx \\ &= \int (\cos x + \sin x) dx \\ &= \int \cos x dx + \int \sin x dx \\ &= \sin x - \cos x + C. \end{aligned}$$

**Exercise 3.1 | Q 2.09 | Page 102**

Evaluate the following integrals :  $\int \sqrt{1 - \cos 2x} dx$

**SOLUTION**

$$\begin{aligned} & \int \sqrt{1 - \cos 2x} dx \\ &= \int \sqrt{2 \sin^2 x} dx \\ &= \sqrt{2} \int \sin x dx \\ &= -\sqrt{2} \cos x + C. \end{aligned}$$

**Exercise 3.1 | Q 2.1 | Page 102**

Evaluate the following integrals :  $\int \sin 4x \cos 3x dx$

**SOLUTION**

$$\begin{aligned}& \int \sin 4x \cos 3x dx \\&= \frac{1}{2} \int \sin 4x \cos 3x dx \\&= \frac{1}{2} \int [\sin(4x + 3x) + \sin(4x - 3x)] dx \\&= \frac{1}{2} \int \sin 7x dx + \frac{1}{2} \int \sin x dx \\&= \frac{1}{2} \left( \frac{-\cos 7x}{7} \right) - \frac{1}{2} \cos x + c \\&= -\frac{1}{14} \cos 7x - \frac{1}{2} \cos x + c.\end{aligned}$$

**Exercise 3.1 | Q 3.01 | Page 102**

Evaluate the following integrals :  $\int \frac{x}{x+2} \cdot dx$

**SOLUTION**

$$\begin{aligned}& \int \frac{x}{x+2} \cdot dx \\&= \int \frac{(x+2)-2}{x+2} \cdot dx \\&= \int \left( \frac{x+2}{x+2} - \frac{2}{x+2} \right) \cdot dx \\&= \int 1 dx - 2 \int \frac{1}{x+2} \cdot dx \\&= x - 2 \log|x+2| + c.\end{aligned}$$

**Exercise 3.1 | Q 3.02 | Page 102**

Evaluate the following integrals :  $\int \frac{4x+3}{2x+1} \cdot dx$

**SOLUTION**

$$\begin{aligned}& \int \frac{4x+3}{2x+1} \cdot dx \\&= \int \frac{(2(2x+1)+1)}{2x+1} \cdot dx \\&= \int \left( \frac{2(2x+1)}{2x+1} + \frac{1}{2x+1} \right) \cdot dx \\&= 2 \int 1 \cdot dx + \int \frac{1}{2x+1} \cdot dx \\&= 2x + \frac{1}{2} \log|2x+1| + c.\end{aligned}$$

**Exercise 3.1 | Q 3.03 | Page 102**

Evaluate the following integrals :  $\int \frac{5x+2}{3x-4} \cdot dx$

**SOLUTION**

$$\begin{aligned}& \int \frac{5x+2}{3x-4} \cdot dx \\&= \int \frac{\frac{5}{3}(3x-4 + \frac{20}{3} + 2)}{3x-4} \cdot dx \\&= \int \frac{\frac{5}{3}(3x-4) + \frac{26}{3}}{3x-4} \cdot dx \\&= \int \left[ \frac{5}{3} + \frac{\left(\frac{26}{3}\right)}{3x-4} \right] \cdot dx \\&= \frac{5}{3} \int 1 \cdot dx + \frac{26}{3} \int \frac{1}{3x-4} \cdot dx\end{aligned}$$

$$\begin{aligned}
&= (5x)(3) + \frac{26}{3} \cdot \frac{1}{3} \log|3x - 4| + c \\
&= (5x)(3) + \frac{26}{9} \log|3x - 4| + c.
\end{aligned}$$

**Exercise 3.1 | Q 3.03 | Page 102**

Evaluate the following integrals :  $\int \frac{5x + 2}{3x - 4} \cdot dx$

**SOLUTION**

$$\begin{aligned}
&\int \frac{5x + 2}{3x - 4} \cdot dx \\
&= \int \frac{\frac{5}{3}(3x - 4) + \frac{20}{3} + 2}{3x - 4} \cdot dx \\
&= \int \frac{\frac{5}{3}(3x - 4) + \frac{26}{3}}{3x - 4} \cdot dx \\
&= \int \left[ \frac{5}{3} + \frac{\left(\frac{26}{3}\right)}{3x - 4} \right] \cdot dx \\
&= \frac{5}{3} \int 1 \cdot dx + \frac{26}{3} \int \frac{1}{3x - 4} \cdot dx \\
&= (5x)(3) + \frac{26}{3} \cdot \frac{1}{3} \log|3x - 4| + c \\
&= (5x)(3) + \frac{26}{9} \log|3x - 4| + c.
\end{aligned}$$

**Exercise 3.1 | Q 3.04 | Page 102**

Evaluate the following integrals :  $\int \frac{x - 2}{\sqrt{x + 5}} \cdot dx$

**SOLUTION**

$$\begin{aligned}& \int \frac{x-2}{\sqrt{x+5}} \cdot dx \\&= \int \frac{(x+5)-7}{\sqrt{x+5}} \cdot dx \\&= \int \left( \frac{x+5}{\sqrt{x+5}} - \frac{7}{\sqrt{x+5}} \right) \cdot dx \\&= \int (x+5)^{\frac{1}{2}} dx - 7 \int (x+5)^{-\frac{1}{2}} dx \\&= \frac{(x+5)^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} - \frac{7(x+5)^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} + c \\&= \frac{1}{3}(x+5)^{\frac{3}{2}} - 14\sqrt{x+5} + c.\end{aligned}$$

**Exercise 3.1 | Q 3.05 | Page 102**

Evaluate the following integrals :  $\int \frac{2x-7}{\sqrt{4x-1}} \cdot dx$

**SOLUTION**

$$\begin{aligned}& \int \frac{2x-7}{\sqrt{4x-1}} \cdot dx \\&= \frac{1}{2} \int \frac{2(2x-7)}{\sqrt{4x-1}} \cdot dx \\&= \frac{1}{2} \int \frac{(4x-1)-13}{\sqrt{4x-1}} \cdot dx \\&= \frac{1}{2} \int \left( \frac{4x-1}{\sqrt{4x-1}} - \frac{13}{\sqrt{4x-1}} \right) \cdot dx \\&= \frac{1}{2} \int (4x-1)^{\frac{1}{2}} \cdot dx - \frac{13}{2} \int (4x-1)^{-\frac{1}{2}} \cdot dx\end{aligned}$$

$$\begin{aligned}
&= \frac{1}{2} \int \frac{(4x-1)^{\frac{3}{2}}}{(4)(\frac{3}{2})} - \frac{13}{2} \cdot \frac{(4x-1)^{\frac{1}{2}}}{(4)(\frac{1}{2})} + c \\
&= \frac{1}{12}(4x-1)^{\frac{3}{2}} - \frac{13}{4}\sqrt{4x-1} + c.
\end{aligned}$$

**Exercise 3.1 | Q 3.06 | Page 102**

Evaluate the following integrals :  $\int \frac{\sin 4x}{\cos 2x} \cdot dx$

**SOLUTION**

$$\begin{aligned}
&\int \frac{\sin 4x}{\cos 2x} \cdot dx \\
&= \int \frac{2 \sin 2x \cos 2x}{\cos 2x} \cdot dx \\
&= 2 \int \sin 2x \cdot dx \\
&= 2 \left( -\frac{\cos 2x}{2} \right) + c \\
&= -\cos 2x + c.
\end{aligned}$$

**Exercise 3.1 | Q 3.07 | Page 102**

Evaluate the following integrals :  $\int \sqrt{1 + \sin 5x} \cdot dx$

**SOLUTION**

$$\begin{aligned}
&\int \sqrt{1 + \sin 5x} \cdot dx \\
&= \int \sqrt{\sin^2 x + \cos^2 x + 5 \sin x \cos x} \cdot dx \\
&= \int \sqrt{(\sin x + \cos x)^2} \cdot dx
\end{aligned}$$

$$\begin{aligned}
 &= \int (\sin x + \cos x) \cdot dx \\
 &= \int \sin x \, dx + \int \cos x \, dx \\
 &= \left( \frac{2}{5} \sin \frac{5x}{2} - \cos \frac{5x}{2} \right) + c.
 \end{aligned}$$

**Exercise 3.1 | Q 3.08 | Page 102**

Evaluate the following integrals :  $\int \cos^2 x \, dx$

**SOLUTION**

Recall the identity  $\cos 2x = 2 \cos^2 x - 1$ , which gives

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

Therefore,  $\int \cos^2 x \, dx$

$$\begin{aligned}
 &= \frac{1}{2} \int (1 + \cos 2x) \cdot dx \\
 &= \frac{1}{2} \int dx + \frac{1}{2} \int \cos 2x \cdot dx \\
 &= \frac{x}{2} + \frac{1}{4} \sin 2x + C.
 \end{aligned}$$

**Exercise 3.1 | Q 3.09 | Page 102**

Evaluate the following integrals :  $\int \frac{2}{\sqrt{x} - \sqrt{x+3}} \cdot dx$

**SOLUTION**

$$\begin{aligned}& \int \frac{2}{\sqrt{x} - \sqrt{x+3}} \cdot dx \\&= \int \frac{2}{\sqrt{x} - \sqrt{x+3}} \times \frac{\sqrt{x} + \sqrt{x+3}}{\sqrt{x} + \sqrt{x+3}} \cdot dx \\&= \int \frac{2(\sqrt{x} + \sqrt{x+3})}{x - (x+3)} \cdot dx \\&= -\frac{2}{3} \int (\sqrt{x} + \sqrt{x+3}) \cdot dx \\&= -\frac{2}{3} \int x^{\frac{1}{2}} dx - \frac{2}{3} \int (x+3)^{\frac{1}{2}} \cdot dx \\&= -\frac{2}{3} \cdot \frac{x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} - \frac{2}{3} \cdot \frac{(x+3)^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} + c \\&= -\frac{4}{9} \left[ x^{\frac{3}{2}} + (x+3)^{\frac{3}{2}} \right] + c.\end{aligned}$$

**Exercise 3.1 | Q 3.1 | Page 102**

Evaluate the following integrals :  $\int \frac{3}{\sqrt{7x-2} - \sqrt{7x-5}} \cdot dx$

**SOLUTION**

$$\begin{aligned}& \int \frac{3}{\sqrt{7x-2} - \sqrt{7x-5}} \cdot dx \\&= \int \frac{3}{\sqrt{7x-2} - \sqrt{7x-5}} \times \frac{\sqrt{7x-2} + \sqrt{7x-5}}{\sqrt{7x-2} + \sqrt{7x-5}} \cdot dx \\&= \int \frac{3(\sqrt{7x-2} + \sqrt{7x-5})}{(7x-2) - (7x-5)} \cdot dx \\&= \int (\sqrt{7x-2} + \sqrt{7x-5}) \cdot dx\end{aligned}$$

$$\begin{aligned}
&= \int (7x - 2)^{\frac{1}{2}} \cdot dx + \int (7x - 5)^{\frac{1}{2}} \cdot dx \\
&= \frac{(7x - 2)^{\frac{3}{2}}}{\frac{3}{2}} \times \frac{1}{7} + \frac{(7x - 5)^{\frac{3}{2}}}{\frac{3}{2}} \times \frac{1}{7} + c \\
&= \frac{2}{21}(7x - 2)^{\frac{3}{2}} + \frac{2}{21}(7x - 5)^{\frac{3}{2}} + c.
\end{aligned}$$

**Exercise 3.1 | Q 4 | Page 102**

If  $f'(x) = x - \frac{3}{x^3}$ ,  $f(1) = \frac{11}{2}$ , find  $f(x)$

**SOLUTION**

By the definition of integral,

$$\begin{aligned}
f(x) &= \int f'(x) \cdot dx \\
&= \int \left( x - \frac{3}{x^3} \right) \cdot dx \\
&= \int x \, dx - 3 \int x^{-3} \, dx \\
&= \frac{x^2}{2} - \frac{3x^{(-2)}}{(-2)} + c \\
&= \frac{x^2}{2} + \frac{3}{2x^2} + c \quad \dots(1)
\end{aligned}$$

$$f(1) = \frac{11}{2} \quad \dots(\text{Given})$$

$$\therefore \frac{1}{2} + \frac{3}{2} + c = \frac{11}{2}$$

$$\therefore c = \frac{7}{2}$$

$$\therefore f(x) = \frac{x^2}{2} + \frac{3}{2x^2} + \frac{7}{2}. \quad \dots[\text{By (1)}]$$

**EXERCISE 3.2 (A) [PAGE 110]**

Exercise 3.2 (A) | Q 1.01 | Page 110

Integrate the following functions w.r.t.  $x$  :  $\frac{(\log x)^n}{x}$ **SOLUTION**

$$\text{Let } I = \int \frac{(\log x)^n}{x} \cdot dx$$

Put  $\log x = t$ .

$$\therefore \frac{1}{x} \cdot dx = dt$$

$$\therefore I = \int t^n dt$$

$$= \frac{t^{n+1}}{n+1} + c$$

$$= \frac{1}{n+1} \cdot (\log x)^{n+1} + c.$$

Exercise 3.2 (A) | Q 1.02 | Page 110

Integrate the following functions w.r.t.  $x$  :  $\frac{(\sin^{-1} x)^{\frac{3}{2}}}{\sqrt{1-x^2}}$ **SOLUTION**

$$\text{Let } I = \int \frac{(\sin^{-1} x)^{\frac{3}{2}}}{\sqrt{1-x^2}} \cdot dx$$

Put  $\sin^{-1} x = t$ .

$$\therefore \frac{1}{\sqrt{1-x^2}} \cdot dx = dt$$

$$\begin{aligned}\therefore I &= \int t^{\frac{3}{2}} dt \\ &= \frac{t^{\frac{5}{2}}}{\frac{5}{2}} + c \\ &= \frac{2}{5} (\sin^{-1} x)^{\frac{5}{2}} + c.\end{aligned}$$

**Exercise 3.2 (A) | Q 1.03 | Page 110**

Integrate the following functions w.r.t. x :  $\frac{1+x}{x \cdot \sin(x + \log x)}$

**SOLUTION**

$$\begin{aligned}\text{Let } I &= \int \frac{1+x}{x \cdot \sin(x + \log x)} \cdot dx \\ &= \int \frac{1}{\sin(x + \log x)} \cdot \left( \frac{1+x}{x} \right) \cdot dx \\ &= \int \frac{1}{\sin(x + \log x)} \cdot \left( \frac{1}{x} + 1 \right) \cdot dx\end{aligned}$$

Put  $x + \log x = t$

$$\begin{aligned}\therefore \left( 1 + \frac{1}{x} \right) \cdot dx &= dt \\ \therefore I &= \int \frac{1}{\sin t} dt = \int \operatorname{cosec} t dt \\ &= \log |\operatorname{cosec} t - \cot t| + c \\ &= \log |\operatorname{cosec}(x + \log x) - \cot(x + \log x)| + c.\end{aligned}$$

**Exercise 3.2 (A) | Q 1.04 | Page 110**

Integrate the following functions w.r.t. x :  $\frac{x \cdot \sec^2(x^2)}{\sqrt{\tan^3(x^2)}}$

**SOLUTION**

$$\text{Let } I = \int \frac{x \cdot \sec^2(x^2)}{\sqrt{\tan^3(x^2)}} \cdot dx$$

$$\text{Put } \tan(x^2) = t$$

$$\therefore \sec^2(x^2) \times 2x \, dx = dt$$

$$\therefore x \cdot \sec^2(x^2) dx = \frac{dt}{2}$$

$$\therefore I = \int \frac{1}{\sqrt{t^3}} \cdot \frac{dt}{2}$$

$$= \frac{1}{2} \int t^{-\frac{3}{2}} dt$$

$$= \frac{1}{2} \cdot \frac{t^{-\frac{1}{2}}}{-\frac{1}{2}} + c$$

$$= \frac{-1}{\sqrt{t}} + c$$

$$= \frac{-1}{\sqrt{\tan(x^2)}} + c.$$

**Exercise 3.2 (A) | Q 1.05 | Page 110**

Integrate the following functions w.r.t. x :  $\frac{e^{3x}}{e^{3x} + 1}$

**SOLUTION**

$$\text{Let } I = \int \frac{e^{3x}}{e^{3x} + 1} \cdot dx$$

$$\text{Put } e^{3x} + 1 = t.$$

$$\therefore 3e^{3x} dx = dt$$

$$\begin{aligned}
\therefore e^{3x} dx &= \frac{dt}{3} \\
\therefore I &= \int \frac{1}{t} \cdot \frac{dt}{3} \\
&= \frac{1}{3} \int \frac{1}{t} dt \\
&= \frac{1}{3} \log|t| + c \\
&= \frac{1}{3} \log|e^{3x} + 1| + c.
\end{aligned}$$

**Exercise 3.2 (A) | Q 1.06 | Page 110**

Integrate the following functions w.r.t. x :  $\frac{x^2 + 2}{(x^2 + 1)} \cdot a^{x+\tan^{-1}x}$

**SOLUTION**

$$\begin{aligned}
\text{Let } I &= \int \frac{x^2 + 2}{(x^2 + 1)} \cdot a^{x+\tan^{-1}x} \cdot dx \\
&= \int a^{x+\tan^{-1}x} \cdot \left( \frac{x^2 + 2}{x^2 + 1} \right) \cdot dx
\end{aligned}$$

Put  $x + \tan^{-1}x = t$

$$\begin{aligned}
\therefore \left( 1 + \frac{1}{1+x^2} \right) \cdot dx &= dt \\
\therefore \left( \frac{1+x^2+1}{1+x^2} \right) \cdot dx &= dt \\
\therefore \left( \frac{x^2+2}{x^2+1} \right) \cdot dx &= dt
\end{aligned}$$

$$\therefore I = \int a^t dt = \frac{a^t}{\log a} + c$$

$$= \frac{a^{x+\tan^{-1}x}}{\log a} + c.$$

**Exercise 3.2 (A) | Q 1.07 | Page 110**

Integrate the following functions w.r.t.  $x$  :  $e^x \cdot \frac{\log(\sin e^x)}{\tan(e^x)}$

**SOLUTION**

$$\begin{aligned} \text{Let } I &= \int \frac{e^x \cdot \log(\sin e^x)}{\tan(e^x)} \cdot dx \\ &= \int \log(\sin e^x) \cdot e^x \cdot \cot(e^x) dx \end{aligned}$$

Put  $\log(\sin e^x) = t$

$$\therefore \frac{1}{\sin(e^x)} \cdot \cos(e^x) \cdot e^x dx = dt$$

$$\therefore e^x \cdot \cot(e^x) dx = dt$$

$$\therefore I = \int t dt = \frac{t^2}{2} + c$$

$$= \frac{1}{2} [\log(\sin e^x)]^2 + c.$$

**Exercise 3.2 (A) | Q 1.08 | Page 110**

Integrate the following functions w.r.t.  $x$  :  $\frac{e^{2x} + 1}{e^{2x} - 1}$

**SOLUTION**

$$\begin{aligned} \text{Let } I &= \int \frac{e^{2x} + 1}{e^{2x} - 1} \cdot dx \\ &= \int \frac{\left(\frac{e^{2x}+1}{e^x}\right)}{\left(\frac{e^{2x}-1}{e^x}\right)} \cdot dx \\ &= \int \left( \frac{e^x + e^{-x}}{e^x - e^{-x}} \right) \cdot dx \\ &= \int \frac{\frac{d}{dx}(e^x - e^{-x})}{e^x - e^{-x}} \cdot dx \\ &= \log|e^x - e^{-x}| + c. \quad \dots \left[ \because \int \frac{f'(x)}{f(x)} \cdot dx = \log|f(x)| + c \right] \end{aligned}$$

Exercise 3.2 (A) | Q 1.09 | Page 110

Integrate the following functions w.r.t. x :  $\sin^4 x \cdot \cos^3 x$

**SOLUTION**

$$\begin{aligned} \text{Let } I &= \int \sin^4 x \cdot \cos^3 x dx \\ &= \int \sin^4 x \cdot \cos^2 x \cdot \cos x dx \\ &= \int \sin^4 x (1 - \sin^2 x) \cos x dx \end{aligned}$$

Put  $\sin x = t$

$\therefore \cos x dx = dt$

$$\begin{aligned} \therefore I &= \int t^4 (1 - t^2) dt \\ &= \int (t^4 - t^6) dt \end{aligned}$$

$$\begin{aligned}
 &= \int t^4 dt - \int t^6 dt \\
 &= \frac{t^5}{5} - \frac{t^7}{7} + c \\
 &= \frac{1}{5} \sin^5 x - \frac{1}{7} \sin^7 x + c.
 \end{aligned}$$

**Exercise 3.2 (A) | Q 1.1 | Page 110**

Integrate the following functions w.r.t. x :  $\frac{1}{4x + 5x^{-11}}$

**SOLUTION**

$$\begin{aligned}
 \text{Let } I &= \int \frac{1}{4x + 5x^{-11}} \cdot dx \\
 &= \int \frac{x^{11}}{x^{11}(4x + 5x^{-11})} \cdot dx \\
 &= \int \frac{x^{11}}{4x^{12} + 5} \cdot dx \\
 &= \frac{1}{48} \int \frac{48x^{11}}{4x^{12} + 5} \cdot dx \\
 &= \frac{1}{48} \int \frac{\frac{d}{dx}(4x^{12} + 5)}{4x^{12} + 5} \cdot dx \\
 &= \frac{1}{48} \log|4x^{12} + 5| + c \quad \dots \left[ \because \int \frac{f'(x)}{f(x)} dx = \log|f(x)| + c \right]
 \end{aligned}$$

**Exercise 3.2 (A) | Q 1.11 | Page 110**

Integrate the following functions w.r.t. x :  $x^9 \cdot \sec^2(x^{10})$

**SOLUTION**

$$\text{Let } I = \int x^9 \cdot \sec^2(x^{10}) \cdot dx$$

$$\text{Put } x^{10} = t$$

$$\therefore 10x^9 dx = dt$$

$$\therefore x^9 dx = \frac{1}{10} dt$$

$$\therefore I = \int \sec^2 t \cdot \frac{dt}{10}$$

$$= \frac{1}{10} \tan t + c$$

$$= \frac{1}{10} \tan(x^{10}) + c.$$

**Exercise 3.2 (A) | Q 1.12 | Page 110**

Integrate the following functions w.r.t. x :  $e^{3\log x}(x^4 + 1)^{-1}$

**SOLUTION**

$$\text{Let } I = e^{3\log x}(x^4 + 1)^{-1} dx$$

$$= \int \frac{e^{\log x^3}}{x^4 + 1} \cdot dx$$

$$= \int \frac{x^3}{x^4 + 1} \cdot dx \quad \dots [\because e^{\log N} = N]$$

$$= \frac{1}{4} \int \frac{4x^3}{x^4 + 1} \cdot dx$$

$$= \frac{1}{4} \int \frac{\frac{d}{dx}(x^4 + 1)}{x^4 + 1} \cdot dx$$

$$= \frac{1}{4} \log|x^4 + 1| + c. \quad \dots \left[ \because \int \frac{f'(x)}{f(x)} dx = \log|f(x)| + c \right]$$

**Exercise 3.2 (A) | Q 1.13 | Page 110**

Integrate the following functions w.r.t. x :  $\frac{\sqrt{\tan x}}{\sin x \cdot \cos x}$

**SOLUTION**

$$\text{Let } I = \int \frac{\sqrt{\tan x}}{\sin x \cdot \cos x} \cdot dx$$

Dividing numerator and denominator by  $\cos^2 x$ , we get

$$\begin{aligned} I &= \int \frac{\left( \frac{\sqrt{\tan x}}{\cos^2 x} \right)}{\left( \frac{\sin x}{\cos x} \right)} \cdot dx \\ &= \int \frac{\sqrt{\tan x} \cdot \sec^2 x}{\tan x} \cdot dx \\ &= \int \frac{\sec^2 x}{\sqrt{\tan x}} \cdot dx \end{aligned}$$

Put  $\tan x = t$

$$\therefore \sec^2 x dx = dt$$

$$\therefore I = \int \frac{1}{\sqrt{t}} dt$$

$$= \int t^{-\frac{1}{2}} dt$$

$$= \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$= 2\sqrt{t} + c$$

$$= 2\sqrt{\tan x} + c.$$

**Exercise 3.2 (A) | Q 1.14 | Page 110**

Integrate the following functions w.r.t. x :  $\frac{(x - 1)^2}{(x^2 + 1)^2}$

**SOLUTION**

$$\begin{aligned}
 \text{Let } I &= \int \frac{(x - 1)^2}{(x^2 + 1)^2} \cdot dx \\
 &= \int \frac{x^2 - 2x + 1}{(x^2 + 1)^2} \cdot dx \\
 &= \int \frac{(x^2 + 1) - 2x}{(x^2 + 1)^2} \cdot dx \\
 &= \int \left[ \frac{x^2 + 1}{(x^2 + 1)^2} - \frac{2x}{(x^2 + 1)^2} \right] \cdot dx \\
 &= \int \frac{1}{x^2 + 1} dx - \int \frac{2x}{(x^2 + 1)^2} \cdot dx \\
 &= I_1 - I_2 \quad \dots(\text{Let})
 \end{aligned}$$

In  $I_2$ , Put  $x^2 + 1 = t$

$$\therefore 2x dx = dt$$

$$\begin{aligned}
 I &= \int \frac{1}{x^2 + 1} \cdot dx - \int t^{-2} dt \\
 &= \tan^{-1} x - \frac{t^{-1}}{(-1)} + c \\
 &= \tan^{-1} x + \frac{1}{x^2 + 1} + c.
 \end{aligned}$$

**Exercise 3.2 (A) | Q 1.15 | Page 110**

Integrate the following functions w.r.t. x :  $\frac{2 \sin x \cos x}{3 \cos^2 x + 4 \sin^2 x}$

**SOLUTION**

$$\text{Let } I = \int \frac{2 \sin x \cos x}{3 \cos^2 x + 4 \sin^2 x} \cdot dx$$

$$\text{Put } 3 \cos^2 x + 4 \sin^2 x = t$$

$$\therefore \left[ 3(2 \cos x) \frac{d}{dx}(\cos x) + 4(2 \sin x) \frac{d}{dx}(\sin x) \right] dx = dt$$

$$\therefore [-6 \cos x \sin x + 8 \sin x \cos x] dx = dt$$

$$\therefore 2 \sin x \cos x dx = dt$$

$$I = \int \frac{dt}{t} = \log|t| + c$$

$$= \log |3 \cos^2 x + 4 \sin^2 x| + c.$$

**Exercise 3.2 (A) | Q 1.16 | Page 110**

Integrate the following functions w.r.t. x :  $\frac{1}{\sqrt{x} + \sqrt{x^3}}$

**SOLUTION**

$$\text{Let } I = \int \frac{1}{\sqrt{x} + \sqrt{x^3}} \cdot dx$$

$$= \int \frac{1}{x^{\frac{1}{2}} + x^{\frac{3}{2}}} \cdot dx$$

$$\text{Put } x = t^2$$

$$\therefore dx = 2t dt$$

$$\text{Also } x^{\frac{1}{2}} = (t^2)^{\frac{1}{2}} = t$$

and

$$x^{\frac{3}{2}} = (t^2)^{\frac{3}{2}} = t^3$$

$$\begin{aligned}
I &= \int \frac{2tdt}{t+t^3} \\
&= 2 \int \frac{tdt}{t(1+t^2)} \\
&= 2 \int \frac{1}{1+t^2} dt \\
&= 2\tan^{-1} t + c \\
&= 2\tan^{-1}(\sqrt{x}) + c.
\end{aligned}$$

**Exercise 3.2 (A) | Q 1.17 | Page 110**

Integrate the following functions w.r.t. x :  $\frac{10x^9 10^x \cdot \log 10}{10^x + 10^{10}}$

**SOLUTION**

$$\text{Let } I = \int \frac{10x^9 10^x \cdot \log 10}{10^x + 10^{10}} \cdot dx$$

$$\text{Put } 10^x + x^{10} = t$$

$$\therefore (10^x \cdot \log 10 + 10x^9).dx = dt$$

$$\therefore I = \int \frac{1}{t} dt = \log |t| + c$$

$$= \log |10^x + x^{10}| + c.$$

**Exercise 3.2 (A) | Q 1.18 | Page 110**

Integrate the following functions w.r.t. x :  $\frac{x^n - 1}{\sqrt{1 + 4x^n}}$

**SOLUTION**

$$\text{Let } I = \int \frac{x^n - 1}{\sqrt{1 + 4x^n}} \cdot dx$$

Put  $x^n = t$

$$\therefore nx^{n-1} dx = dt$$

$$\therefore x^{n-1} dx = \frac{dt}{n}$$

$$\therefore I = \int \frac{1}{\sqrt{1 + 4t}} \cdot \frac{dt}{n}$$

$$= \frac{1}{n} \int (1 + 4t)^{-\frac{1}{2}} dt$$

$$= \frac{1}{n} \cdot \frac{(1 + 4t)^{\frac{1}{2}}}{\frac{1}{2}} \times \frac{1}{4} + c$$

$$= \frac{1}{2n} \cdot \sqrt{1 + 4x^n} + c.$$

**Exercise 3.2 (A) | Q 1.19 | Page 110**

Integrate the following functions w.r.t. x :  $(2x + 1)\sqrt{x + 2}$

**SOLUTION**

$$\text{Let } I = \int (2x + 1)\sqrt{x + 2} dx$$

Put  $x + 2 = t$

$$\therefore dx = dt$$

Also,  $x = t - 2$

$$\therefore 2x + 1 = 2(t - 2) + 1 = 2t - 3$$

$$\therefore I = \int (2t - 3)\sqrt{t} dt$$

$$\begin{aligned}
&= \int \left( 2t^{\frac{3}{2}} - 3t^{\frac{1}{2}} \right) dt \\
&= 2 \int t^{\frac{3}{2}} dt - 3 \int t^{\frac{1}{2}} dt \\
&= 2 \cdot \frac{\frac{t^5}{2}}{\left(\frac{5}{2}\right)} - 3 \cdot \frac{t^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} + c \\
&= \frac{4}{5}(x+2)^{\frac{5}{2}} - 2(x+2)^{\frac{3}{2}} + c.
\end{aligned}$$

**Exercise 3.2 (A) | Q 1.2 | Page 110**

Integrate the following functions w.r.t.  $x$  :  $x^5 \sqrt{a^2 + x^2}$

**SOLUTION**

$$\text{Let } I = \int x^5 \sqrt{a^2 + x^2} dx$$

$$\text{Put, } a^2 + x^2 = t$$

$$\therefore 2 dx = dt$$

$$\therefore x dx = \frac{1}{2} dt$$

$$\text{Also, } x^2 = t - a^2$$

$$\begin{aligned}
I &= \int x^2 \cdot x^2 \sqrt{a^2 + x^2} x dx \\
&= \frac{1}{2} \int (t - a^2)^2 \sqrt{t} dt \\
&= \frac{1}{2} \int (t^2 - 2a^2t + a^4) \sqrt{t} dt \\
&= \frac{1}{2} \int \left( t^{\frac{5}{2}} - 2a^2t^{\frac{3}{2}} + a^4t^{\frac{1}{2}} \right) \sqrt{t} dt \\
&= \frac{1}{2} \int t^{\frac{5}{2}} dt - a^2 \int t^{\frac{3}{2}} dt + \frac{a^4}{2} \int t^{\frac{1}{2}} dt
\end{aligned}$$

$$\begin{aligned}
&= \frac{1}{2} \cdot \frac{t^{\frac{7}{2}}}{\left(\frac{7}{2}\right)} - a^2 \cdot \frac{t^{\frac{5}{2}}}{\left(\frac{5}{2}\right)} + \frac{a^4}{2} \cdot \frac{t^{\frac{3}{2}}}{\left(\frac{3}{2}\right) + c} \\
&= \frac{1}{7} (a^2 + x^2)^{\frac{7}{2}} - \frac{2a^2}{5} (a^2 + x^2)^{\frac{5}{2}} + \frac{a^4}{3} (a^2 + x^2)^{\frac{3}{2}} + c.
\end{aligned}$$

**Exercise 3.2 (A) | Q 1.21 | Page 110**

Integrate the following functions w.r.t. x :  $(5 - 3x)(2 - 3x)^{-\frac{1}{2}}$

**SOLUTION**

$$\text{Let } I = \int (5 - 3x)(2 - 3x)^{-\frac{1}{2}} dx$$

$$\text{Put } 2 - 3x = t$$

$$\therefore -3dx = dt$$

$$\therefore dx = \frac{-dt}{3}$$

$$\text{Also, } x = \frac{2-t}{3}$$

$$\therefore I = \int \left[ 5 - 3\left(\frac{2-t}{3}\right) \right] t^{-\frac{1}{2}} \cdot \left( \frac{-dt}{3} \right)$$

$$= -\frac{1}{3} (5 - 2 + t) t^{-\frac{1}{2}} dt$$

$$= -\frac{1}{3} \int (3 + t) t^{\frac{1}{2}} dt$$

$$= -\frac{1}{3} \int \left( 3t^{-\frac{1}{2}} + t^{\frac{1}{2}} \right) dt$$

$$\begin{aligned}
&= -\frac{3}{3} \int t^{-\frac{1}{2}} dt - \frac{1}{3} \int t^{\frac{1}{2}} dt \\
&= -\frac{t^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} - \frac{1}{3} \cdot \frac{t^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} + c \\
&= -2\sqrt{2-3x} - \frac{2}{9}(2-3x)^{\frac{3}{2}} + c.
\end{aligned}$$

**Exercise 3.2 (A) | Q 1.22 | Page 110**

Integrate the following functions w.r.t. x :  $\frac{7+4+5x^2}{(2x+3)^{\frac{3}{2}}}$

**SOLUTION**

$$\begin{aligned}
\text{Let } I &= \int \frac{7+4x+5x^2}{(2x+3)^{\frac{3}{2}}} \cdot dx \\
&= \int \frac{5x^2+4x+7}{(2x+3)^{\frac{3}{2}}} \cdot dx
\end{aligned}$$

Put  $2x+3 = t$

$$\therefore 2dx = dt$$

$$\therefore dx = \frac{dt}{2}$$

$$\text{Also, } x = \frac{t-3}{2}$$

$$\begin{aligned}
\therefore I &= \int \frac{5\left(\frac{t-3}{2}\right)^2 + 4\left(\frac{t-3}{2}\right) + 7}{t^{\frac{3}{2}}} \cdot \frac{dt}{2} \\
&= \frac{1}{2} \int \frac{5\left(\frac{t^2-6t+9}{4}\right) + 2(t-3) + 7}{t^{\frac{3}{2}}} dt \\
&= \frac{1}{2} \int \frac{5t^2-30t+4+8t-24+28}{4t^{\frac{3}{2}}} dt
\end{aligned}$$

$$\begin{aligned}
&= \frac{1}{8} \int \frac{5^2 - 22t + 49}{t^{\frac{3}{2}}} dt \\
&= \frac{1}{8} \int \left( 5t^{\frac{1}{2}} - 22t^{-\frac{1}{2}} + 49t^{-\frac{3}{2}} \right) dt \\
&= \frac{5}{8} \int t^{\frac{1}{2}} dt - \frac{22}{8} \int t^{-\frac{1}{2}} dt + \frac{49}{8} \int t^{-\frac{3}{2}} dt \\
&= \frac{5}{8} \cdot \frac{t^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} - \frac{11}{4} \cdot \frac{t^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} + \frac{49}{8} \cdot \frac{t^{-\frac{1}{2}}}{\left(-\frac{1}{2}\right)} + c \\
&= \frac{5}{12} (x+3)^{\frac{3}{2}} - \frac{11}{2} \sqrt{2x+3} - \frac{49}{4} \cdot \frac{1}{\sqrt{2x+3}} + c.
\end{aligned}$$

**Exercise 3.2 (A) | Q 1.23 | Page 110**

Integrate the following functions w.r.t. x :  $\frac{x^2}{\sqrt{9-x^6}}$

**SOLUTION**

$$\text{Let } I = \int \frac{x^2}{\sqrt{9-x^6}} \cdot dx$$

$$\text{Put } x^3 = t$$

$$\therefore 3x^2 dx = dt$$

$$\therefore x^2 dx = \frac{1}{3} dt$$

$$\therefore I = \int \frac{1}{\sqrt{9-t^2}} \cdot \frac{dt}{3}$$

$$= \frac{1}{3} \int \frac{dt}{\sqrt{3^2 - t^2}}$$

$$= \frac{1}{3} \sin^{-1} \left( \frac{t}{3} \right) + c$$

$$= \frac{1}{3} \sin^{-1} \left( \frac{x^3}{3} \right) + c.$$

**Exercise 3.2 (A) | Q 1.24 | Page 110**

Integrate the following functions w.r.t. x :  $\frac{1}{x(x^3 - 1)}$

*SOLUTION*

$$\begin{aligned} \text{Let } I &= \int \frac{1}{x(x^3 - 1)} \cdot dx \\ &= \int \frac{x^{-4}}{x^{-4}x(x^3 - 1)} \cdot dx \\ &= \int \frac{x^{-4}}{1 - x^{-3}} \cdot dx \\ &= \frac{1}{3} \int \frac{3x^{-4}}{1 - x^{-3}} \cdot dx \\ &= \frac{1}{3} \int \frac{\frac{d}{dx}(1 - x^{-3})}{1 - x^{-3}} \cdot dx \\ &= \frac{1}{3} \log|1 - x^{-3}| + c \quad \dots \left[ \because \int \frac{f'(x)}{f(x)} dx = \log|f(x)| + c \right] \\ &= \frac{1}{3} \log \left| 1 - \frac{1}{x^3} \right| + c \\ &= \frac{1}{3} \log \left| \frac{x^3 - 1}{x^3} \right| + c. \end{aligned}$$

Alternative Method :

$$\text{Let } I = \int \frac{1}{x(x^3 - 1)} \cdot dx$$

$$= \int \frac{x^2}{x^3(x^3 - 1)} \cdot dx$$

Put  $x^3 = t$

$$\therefore 3x^2dx = dt$$

$$\therefore x^2dx = \frac{dt}{3}$$

$$\therefore I = \int \frac{1}{t(t-1)} \cdot \frac{dt}{3}$$

$$= \frac{1}{3} \int \frac{1}{t(t-1)} dt$$

$$= \frac{1}{3} \int \frac{t-(t-1)}{t(t-1)} dt$$

$$= \frac{1}{3} \int \left( \frac{1}{t-1} - \frac{1}{t} \right) dt$$

$$= \frac{1}{3} \left[ \int \frac{1}{t-1} dt - \int \frac{1}{t} dt \right]$$

$$= \frac{1}{3} [\log|t-1| - \log|t|] + c$$

$$= \frac{1}{3} \log \left| \frac{t-1}{t} \right| + c$$

$$= \frac{1}{3} \log \left| \frac{x^3-1}{x^3} \right| + c.$$

#### Exercise 3.2 (A) | Q 1.25 | Page 110

Integrate the following functions w.r.t. x :  $\frac{1}{x \cdot \log x \cdot \log(\log x)}$ .

**SOLUTION**

$$\begin{aligned} \text{Let } I &= \int \frac{1}{x \cdot \log x \cdot \log(\log x)} \cdot dx \\ &= \int \frac{1}{\log(\log x)} \cdot \frac{1}{x \cdot \log x} \cdot dx \end{aligned}$$

Put  $\log(\log x) = t$

$$\begin{aligned} \therefore \frac{1}{\log x} \cdot \frac{1}{x} \cdot dx &= dt \\ \therefore \frac{1}{x \cdot \log x} \cdot dx &= dt \\ \therefore I &= \int \frac{1}{t} dt = \log|t| + c \\ &= \log|\log(\log x)| + c. \end{aligned}$$

**Exercise 3.2 (A) | Q 2.01 | Page 110**

Integrate the following functions w.r.t. x :  $\frac{\cos 3x - \cos 4x}{\sin 3x + \sin 4x}$

**SOLUTION**

$$\begin{aligned} \text{Let } I &= \int \frac{\cos 3x - \cos 4x}{\sin 3x + \sin 4x} \cdot dx \\ &= \int \frac{-2 \sin\left(\frac{3x+4x}{2}\right) \sin\left(\frac{3x-4x}{2}\right)}{2 \sin\left(\frac{3x+4x}{2}\right) \cos\left(\frac{3x-4x}{2}\right)} \cdot dx \\ &= \int -\frac{\sin\left(-\frac{x}{2}\right)}{\cos\left(-\frac{x}{2}\right)} \cdot dx \\ &= \int \frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)} \cdot dx \end{aligned}$$

$$\begin{aligned}
 &= \int \tan\left(\frac{x}{2}\right) \cdot dx \\
 &= \log \frac{\left|\sec\left(\frac{x}{2}\right)\right|}{\left(\frac{1}{2}\right)} + c \\
 &= 2 \log \left|\sec\left(\frac{x}{2}\right)\right| + c.
 \end{aligned}$$

**Exercise 3.2 (A) | Q 2.02 | Page 110**

Integrate the following functions w.r.t.  $x$  :  $\frac{\cos x}{\sin(x - a)}$

**SOLUTION**

$$\begin{aligned}
 \text{Let } I &= \int \frac{\cos x}{\sin(x - a)} \cdot dx \\
 &= \int \frac{\cos[(x - a) + a]}{\sin(x - a)} \cdot dx \\
 &= \int \frac{\cos(x - a)\cos a - \sin(x - a)\sin a}{\sin(x - a)} \cdot dx \\
 &= \int \left[ \frac{\cos(x - a)\cos a}{\sin(x - a)} - \frac{\sin(x - a)\sin a}{\sin(x - a)} \right] \cdot dx \\
 &= \cos a \int \cot(x - a) dx - \sin a \int 1 dx \\
 &= \cos a \log |\sin(x - a)| - x \sin a + c.
 \end{aligned}$$

**Exercise 3.2 (A) | Q 2.03 | Page 110**

Integrate the following functions w.r.t.  $x$  :  $\frac{\sin(x - a)}{\cos(x + b)}$

**SOLUTION**

$$\text{Let } I = \int \frac{\sin(x - a)}{\cos(x + b)} \cdot dx$$

$$\begin{aligned}
&= \int \frac{\sin[(x+b)-(a+b)]}{\cos(x+b)} \cdot dx \\
&= \int \frac{\sin(x+b)\cos(a+b) - \cos(x+b)\sin(a+b)}{\cos(x+b)} \cdot dx \\
&= \int \left[ \frac{\sin(x+b)\cos(a+b)}{\cos(x+b)} - \frac{\cos(x+b)\sin(a+b)}{\cos(x+b)} \right] \cdot dx \\
&= \cos(a+b) \int \tan(x+b)dx - \sin(a+b) \int 1dx \\
&= \cos(a+b) \log |\sec(x+b)| - x \sin(a+b) + c.
\end{aligned}$$

**Exercise 3.2 (A) | Q 2.04 | Page 110**

Integrate the following functions w.r.t. x :  $\frac{1}{\sin x \cdot \cos x + 2 \cos^2 x}$

**SOLUTION**

$$\text{Let } I = \int \frac{1}{\sin x \cdot \cos x + 2 \cos^2 x} \cdot dx$$

Dividing numerator and denominator of  $\cos^2 x$ , we get

$$\begin{aligned}
I &= \int \frac{\left(\frac{1}{\cos^2 x}\right)}{\frac{\sin x}{\cos x} + 2} \cdot dx \\
&= \int \frac{\sec^2 x}{\tan x + 2} \cdot dx
\end{aligned}$$

Put  $\tan x = t$

$$\therefore \sec^2 x \cdot dx = dt$$

$$\therefore I = \int \frac{1}{t+2} dt$$

$$= \log|t+2| + c$$

$$= \log|\tan x + 2| + c.$$

**Exercise 3.2 (A) | Q 2.05 | Page 110**

Integrate the following functions w.r.t. x :  $\frac{\sin x + 2 \cos x}{3 \sin x + 4 \cos x}$

**SOLUTION**

$$\text{Let } I = \int \frac{\sin x + 2 \cos x}{3 \sin x + 4 \cos x} \cdot dx$$

Put,

$$\text{Numerator} = A \text{ (Denominator)} + B \left[ \frac{d}{dx} (\text{Denominator}) \right]$$

$$\therefore \sin x + 2 \cos x = A(3 \sin x + 4 \cos x) + B \left[ \frac{d}{dx} (3 \sin x + 4 \cos x) \right]$$

$$= A(3 \sin x + 4 \cos x) + B(3 \cos x - 4 \sin x)$$

$$\therefore \sin x + 2 \cos x = (3A - 4B)\sin x + (4A + 3B)\cos x$$

Equaliting the coefficients of  $\sin x$  and  $\cos x$  on both the sides, we get

$$3A - 4B = 1 \quad \dots(1)$$

and

$$4A + 3B = 2 \quad \dots(2)$$

Multiplying equation (1) bt 3 and equation (2) byy 4, we get

$$9A - 12B = 3$$

$$16A + 12B = 8$$

On adding, we get

$$25A = 11$$

$$\therefore A = \frac{11}{25}$$

$$\therefore \text{from (2), } 4\left(\frac{11}{25}\right) + 3B = 2$$

$$\therefore 3B = 2 - \frac{44}{25} = \frac{6}{25}$$

$$\therefore B = \frac{2}{25}$$

$$\begin{aligned}
\therefore \sin x + 2 \cos x &= \frac{11}{25}(3 \sin x + 4 \cos x) + \frac{2}{25}(3 \cos x - 4 \sin x) \\
\therefore I &= \int \left[ \frac{\frac{11}{25}(3 \sin x + 4 \cos x) + \frac{2}{25}(3 \cos x - 4 \sin x)}{3 \sin x + 4 \cos x} \right] dx \\
&= \int \left[ \frac{\frac{11}{25}}{3 \sin x + 4 \cos x} + \frac{\frac{2}{25}(3 \cos x - 4 \sin x)}{(3 \sin x + 4 \cos x)} \right] dx \\
&= \frac{11}{25} \int 1 dx + \frac{2}{25} \int \frac{3 \cos x - 4 \sin x}{3 \sin x + 4 \cos x} dx \\
&= \frac{11}{25}x + \frac{2}{25} \log|3 \sin x + 4 \cos x| + c. \quad \dots \left[ \because \int \frac{f'(x)}{f(x)} dx = \log|f(x)| + c \right]
\end{aligned}$$

**Exercise 3.2 (A) | Q 2.06 | Page 110**

Integrate the following functions w.r.t.  $x$ :  $\frac{1}{2 + 3 \tan x}$

**SOLUTION**

$$\begin{aligned}
\text{Let } I &= \int \frac{1}{2 + 3 \tan x} dx \\
&= \int \frac{1}{2 + 3 \left( \frac{\sin x}{\cos x} \right)} dx \\
&= \int \frac{\cos x}{2 \cos x + 3 \sin x} dx
\end{aligned}$$

Put,

$$\text{Numerator} = A \text{ (Denominator)} + B \left[ \frac{d}{dx} \text{ (Denominator)} \right]$$

$$\therefore \cos x = A(\cos x + 3 \sin x) + B \left[ \frac{d}{dx} (2 \cos x + 3 \sin x) \right]$$

$$= A(2 \cos x + 3 \sin x) + B(-\sin x + 3 \cos x)$$

$$\therefore \cos x = (2A + 3B)\cos x + (3A - 2B)\sin x$$

Equating the coefficients of  $\cos x$   $\sin x$  on both the sides, we get

$$2A - 3B = 1 \quad \dots(1)$$

and

$$3A - 2B = 0 \quad \dots(2)$$

Multiplying equation (1) by 22 and equation (2) by 3, we get

$$4A + 6B = 2$$

$$9A - 6B = 0$$

On adding, we get

$$13A = 2$$

$$\therefore A = \frac{2}{13}$$

$$\therefore \text{from (2), } 2B = 3A = 3\left(\frac{2}{13}\right) = \frac{6}{13}$$

$$\therefore B = \frac{3}{13}$$

$$\therefore \cos x = \frac{2}{13}(2 \cos x + 3 \sin x) + \frac{3}{13}(-2 \sin x + 3 \cos x)$$

$$\therefore I = \int \left[ \frac{\frac{2}{13}(2 \cos x + 3 \sin x) + \frac{3}{13}(-2 \sin x + 3 \cos x)}{2 \cos x + 3 \sin x} \right] dx$$

$$= \int \left[ \frac{\frac{2}{13}}{2 \cos x + 3 \sin x} + \frac{\frac{3}{13}(-2 \sin x + 3 \cos x)}{2 \cos x + 3 \sin x} \right] dx$$

$$= \frac{2}{13} \int dx + \frac{3}{13} \int \frac{-2 \sin x + 3 \cos x}{2 \cos x + 3 \sin x} dx$$

$$= \frac{2}{13}x + \frac{3}{13} \log|2 \cos x + 3 \sin x| + c. \quad \dots \left[ \because \int \frac{f'(x)}{f(x)} dx = \log|f(x)| + c \right]$$

### Exercise 3.2 (A) | Q 2.07 | Page 110

Integrate the following functions w.r.t. x :  $\frac{4e^x - 25}{2e^x - 5}$

**SOLUTION**

$$\text{Let } I = \int \frac{4e^x - 25}{2e^x - 5} \cdot dx$$

Put,

$$\text{Numerator} = A \text{ (Denominator)} + B \left[ \frac{d}{dx} (\text{Denominator}) \right]$$

$$\therefore 4e^x - 25 = A(2e^x - 5) + B \left[ \frac{d}{dx} (2e^x - 5) \right]$$

$$= A(2e^x - 5) + B(2e^x - 0)$$

$$\therefore 4e^x - 25 = (2A + 2B)e^x - 5A$$

Equating the coefficient of  $e^x$  and constant on both sides, we get

$$2A + 2B = 4 \quad \dots(1)$$

and

$$5A = 25$$

$$\therefore A = 5$$

$$\therefore \text{from (1), } 2(5) + 2B = 4$$

$$\therefore 2B = -6$$

$$\therefore B = -3$$

$$\therefore 4e^x - 25 = 5(2e^x - 5) - 3(2e^x)$$

$$\therefore I = \int \left[ \frac{5(2e^x - 5) - 3(2e^x)}{2e^x - 5} \right] \cdot dx$$

$$= \int \left[ 5 - \frac{3(2e^x)}{2e^x - 5} \right] \cdot dx$$

$$= 5 \int 1 dx - 3 \int \frac{2e^x}{2e^x - 5} \cdot dx$$

$$= 5x - 3 \log|2e^x - 5| + c \quad \dots \left[ \because \int \frac{f'(x)}{f(x)} dx = \log|f(x)| + c \right]$$

Exercise 3.2 (A) | Q 2.08 | Page 110

Integrate the following functions w.r.t. x :  $\frac{20 + 12e^x}{3e^x + 4}$

SOLUTION

$$\text{Let } I = \int \frac{20 + 12e^x}{3e^x + 4} \cdot dx$$

Put,

$$\text{Numerator} = A \text{ (Denominator)} + B \left[ \frac{d}{dx} (\text{Denominator}) \right]$$

$$\therefore 20 + 12e^x = A(3e^x + 4) + B \left[ \frac{d}{dx} (3e^x + 4) \right]$$

$$= A(3e^x + 4) + B(3e^x + 0)$$

$$\therefore 20 + 12e^x = (2A + 2B)e^x - 5A$$

Equating the coefficient of  $e^x$  and constant on both sides, we get

$$2A + 2B = 4 \quad \dots(1)$$

and

$$5A = 25$$

$$\therefore A = 5$$

$$\therefore \text{from (1), } 2(5) + 2B = 4$$

$$\therefore 2B = -6$$

$$\therefore B = -3$$

$$\therefore 20 + 12e^x = 5(3e^x + 4) - 3(3e^x)$$

$$\therefore I = \int \left[ \frac{5(3e^x + 4) - 3(3e^x)}{3e^x + 4} \right] \cdot dx$$

$$= \int \left[ 5 - \frac{3(3e^x)}{3e^x + 4} \right] \cdot dx$$

$$= 5 \int 1 dx - 3 \int \frac{3e^x}{3e^x + 4} \cdot dx$$

$$= 5x - \log|3e^x + 4| + c.$$

**Exercise 3.2 (A) | Q 2.09 | Page 110**

Integrate the following functions w.r.t. x :  $\frac{3e^{2x} + 5}{4e^{2x} - 5}$

**SOLUTION**

$$\text{Let } I = \int \frac{3e^{2x} + 5}{4e^{2x} - 5} \cdot dx$$

Put,

$$\text{Numerator} = A \text{ (Denominator)} + B \left[ \frac{d}{dx} (\text{Denominator}) \right]$$

$$\therefore 3e^{2x} + 5 = A(4e^{2x} - 5) + B \left[ \frac{d}{dx} (4e^{2x} - 5) \right]$$

$$= A(4e^{2x} - 5) + B(4e^{2x} \times 2 - 0)$$

$$\therefore 3e^{2x} + 5 = (4A + 8B)e^{2x} - 5A$$

Equating the coefficient of  $e^{2x}$  and constant on both sides, we get

$$4A + 8B = 3 \quad \dots(1)$$

and

$$-5A = 5$$

$$\therefore A = -1$$

$$\therefore \text{from (1), } 4(-1) + 8B = 3$$

$$\therefore 8B = 7$$

$$\therefore B = \frac{7}{8}$$

$$\therefore 3e^{2x} + 5 = -(4e^{2x} - 5) + \frac{7}{8}(8e^{2x})$$

$$\begin{aligned}
I &= \int \left[ \frac{- (4e^{2x} - 5) + \frac{7}{8} (8e^{2x})}{4e^{2x} - 5} \right] dx \\
&= \int \left[ -1 + \frac{\frac{7}{8} (8e^{2x})}{4e^{2x} - 5} \right] dx \\
&= \int 1 dx + \frac{7}{8} \int \frac{8e^{2x}}{4e^{2x} - 5} dx \\
&= -x + \frac{7}{8} \log|4e^{2x} - 5| + c \quad \dots \left[ \because \int \frac{f'(x)}{f(x)} dx = \log|f(x)| + c \right]
\end{aligned}$$

**Exercise 3.2 (A) | Q 2.1 | Page 110**

Integrate the following functions w.r.t. x :  $\cos^8 x \cot x$

**SOLUTION**

$$\begin{aligned}
I &= \int \cos^8 x \cot x dx \\
&= \int \cos^8 x \cdot \frac{\cos x}{\sin x} dx \\
\text{Put } \sin x &= t \\
\therefore \cos x dx &= dt \\
\cos^8 x &= (\cos^2 x)^4 = (1 - \sin^2 x)^4 \\
&= (1 - t^2)^4 = 1 - 4t^2 + 6t^4 - 4t^6 + t^8 \\
I &= \int \frac{1 - 4t^2 + 6t^4 - 4t^6 + t^8}{t} dt \\
&= \int \left[ \frac{1}{t} - 4t + 6t^3 - 4t^5 + t^7 \right] dt \\
&= \int \frac{1}{t} dt - 4 \int t dt + 6 \int t^3 dt - 4 \int t^5 dt + \int t^7 dt
\end{aligned}$$

$$\begin{aligned}
&= \log|t| - 4\left(\frac{t^2}{2}\right) + 6\left(\frac{t^4}{4}\right) - 4\left(\frac{t^6}{6}\right) + \frac{t^8}{8} + c \\
&= \log|\sin x| - 2\sin^2 x + \frac{3}{2}\sin^4 x - \frac{2}{3}\sin^6 x + \frac{\sin^8 x}{8} + c.
\end{aligned}$$

**Exercise 3.2 (A) | Q 2.11 | Page 110**

Integrate the following functions w.r.t. x :  $\tan^5 x$

**SOLUTION**

$$\begin{aligned}
\text{Let } I &= \int \tan^5 x \, dx \\
&= \int \tan^3 x \tan^2 x \, dx \\
&= \int \tan^3 x (\sec^2 x - 1) \, dx \\
&= \int (\tan^3 x \sec^2 x - \tan^3 x) \, dx \\
&= \int (\tan^3 x \sec^2 x - \tan x \cdot \tan^2 x) \, dx \\
&= \int [\tan^3 x \sec^2 x - \tan x (\sec^2 x - 1)] \, dx \\
&= \int (\tan^3 x \sec^2 x - \tan x \sec^2 x + \tan x) \, dx \\
&= \int [(\tan^3 x - \tan x) \sec^2 x + \tan x] \, dx \\
&= \int (\tan^3 x - \tan x) \sec^2 x \, dx + \int \tan x \, dx \\
&= I_1 + I_2 \\
\text{In } I_1, \text{ put } \tan x = t
\end{aligned}$$

$$\begin{aligned}
 \therefore \sec^2 x dx &= dt \\
 \therefore I &= \int (t^3 - t) dt + \int \tan x dx \\
 &= \frac{t^4}{4} - \frac{t^2}{2} + \log|\sec x| + c \\
 &= \frac{\tan^4 x}{4} - \frac{\tan^2 x}{2} + \log|\sec x| + c.
 \end{aligned}$$

**Exercise 3.2 (A) | Q 2.12 | Page 110**

Integrate the following functions w.r.t. x :  $\cos^7 x$

**SOLUTION**

$$\begin{aligned}
 \text{Let } I &= \int \cos^7 x dx \\
 &= \int \cos^6 x \cdot \cos x dx \\
 &= \int (1 - \sin^2 x)^3 \cos x dx \\
 \text{Put, } \sin x &= t \\
 \therefore \cos x dx &= dt \\
 I &= \int (1 - t^2)^3 dt \\
 &= \int (1 - 3t^2 + 3t^4 - t^6) dt \\
 &= \int 1 dt - 3 \int t^2 dt + 3 \int t^4 dt - \int t^6 dt \\
 &= t - 3\left(\frac{t^3}{3}\right) + 3\left(\frac{t^5}{5}\right) - \frac{t^7}{7} + C \\
 &= \sin x - \sin^3 x + \frac{3}{5} \sin^5 x - \frac{1}{7} \sin^7 x + C
 \end{aligned}$$

**Exercise 3.2 (A) | Q 2.13 | Page 110**

Integrate the following functions w.r.t. x :  $\tan 3x \tan 2x \tan x$

**SOLUTION**

$$\text{Let } I = \int \tan 3x \tan 2x \tan x dx$$

Consider  $\tan 3x = \tan(2x + x)$

$$= \frac{\tan 2x + \tan x}{1 - \tan 2x \tan x}$$

$$\therefore \tan 3x (1 - \tan 2x \tan x) = \tan 2x + \tan x$$

$$\therefore \tan 3x - \tan 3x \tan 2x \tan x = \tan 2x + \tan x$$

$$\therefore \tan 3x - \tan 2x - \tan x = \tan 3x \tan 2x \tan x$$

$$I = \int (\tan 3x - \tan 2x - \tan x) dx$$

$$= \int \tan 3x dx - \int \tan 2x dx - \int \tan x dx$$

$$= \frac{1}{3} \log |\sec 3x| - \frac{1}{2} \log |\sec 2x| - \log |\sec x| + c.$$

**Exercise 3.2 (A) | Q 2.14 | Page 110**

Integrate the following functions w.r.t. x :  $\sin^5 x \cos^8 x$

**SOLUTION**

$$\text{Let } I = \int \sin^5 x \cos^8 x dx$$

$$= \int \sin^4 x \cos^8 x \sin x dx$$

$$= \int (1 - \cos^2 x)^2 \cos^8 x \sin x dx$$

Put  $\cos x = t$

$$\therefore -\sin x dx = dt$$

$$\therefore \sin x dx = -dt$$

$$\begin{aligned}
I &= - \int (1-t^2)^2 t^8 dt \\
&= - \int (1-2t^2+t^4)t^8 dt \\
&= - \int (t^8 - 2t^{10} + t^{12}) dt \\
&= - \int t^8 dt + 2 \int t^{10} dt - \int t^{12} dt \\
&= - \frac{t^9}{9} + 2 \left( \frac{t^{11}}{11} \right) - \frac{t^{13}}{13} + c \\
&= - \frac{1}{9} \cos^9 x + \frac{2}{11} \cos^{11} x - \frac{1}{13} \cos^{13} x + c.
\end{aligned}$$

**Exercise 3.2 (A) | Q 2.15 | Page 110**

Integrate the following functions w.r.t.  $x$  :  $3^{\cos^2 x} \sin 2x$

**SOLUTION**

$$\text{let } I = \int 3^{\cos^2 x} \sin 2x dx$$

$$\text{Put } \cos^2 x = t$$

$$\therefore \left[ 2 \cos x \frac{d}{dx} (\cos x) \right] dx = dt$$

$$\therefore -2 \sin x \cos x dx = dt$$

$$\therefore \sin 2x dx = -dt$$

$$\begin{aligned}
I &= - \int 3^t dt \\
&= - \frac{1}{\log 3} \cdot 3^t + c \\
&= - \frac{1}{\log 3} \cdot 3^{\cos^2 x} + c.
\end{aligned}$$

**Exercise 3.2 (A) | Q 2.16 | Page 110**

Integrate the following functions w.r.t. x :  $\frac{\sin 6x}{\sin 10x \sin 4x}$

**SOLUTION**

$$\begin{aligned}
 \text{Let } I &= \int \frac{\sin 6x}{\sin 10x \sin 4x} \cdot dx \\
 &= \int \frac{\sin(10x - 4x)}{\sin 10x \sin 4x} \cdot dx \\
 &= \int \frac{\sin 10x \cos 4x - \cos 10x \sin 4x}{\sin 10x \sin 4x} \cdot dx \\
 &= \int \left[ \frac{\sin 10x \cos 4x}{\sin 10x \sin 4x} - \frac{\cos 10x \sin 4x}{\sin 10x \sin 4x} \right] \cdot dx \\
 &= \int \cot 4x dx - \int \cot 10x dx \\
 &= \frac{1}{4} \log |\sin 4x| - \frac{1}{10} \log |\sin 10x| + c.
 \end{aligned}$$

**Exercise 3.2 (A) | Q 2.17 | Page 110**

Integrate the following functions w.r.t. x :  $\frac{\sin x \cos^3 x}{1 + \cos^2 x}$

**SOLUTION**

$$\text{Let } I = \int \frac{\sin x \cos^3 x}{1 + \cos^2 x} \cdot dx$$

Put  $\cos x = t$

$$\therefore -\sin x dx = dt$$

$$\therefore \sin x dx = -dt$$

$$I = - \int \frac{t^3}{t^2 + 1} dt$$

$$\begin{aligned}
&= - \int \frac{t(t^2 + 1) - t}{t^2 + 1} dt \\
&= - \int \left[ \frac{t(t^2 + 1)}{t^2 + 1} - \frac{t}{t^2 + 1} \right] dt \\
&= - \int t dt + \int \frac{t}{t^2 + 1} dt \\
&= - \int t dt + \frac{1}{2} \int \frac{2t}{t^2 + 1} dt \\
&= \frac{t^2}{2} + \frac{1}{2} \log|t^2 + 1| + c \\
&\dots \left[ \because \frac{d}{dt}(t^2 + 1) = 2t \text{ and } \int \frac{f'(x)}{f(x)} dx = \log[f(x)] + c \right] \\
&= -\frac{1}{2} \cos^2 x + \frac{1}{2} \log|\cos^2 x + 1| + c \\
&= \frac{1}{2} [\log|\cos^2 x + 1| - \cos^2 x] + c.
\end{aligned}$$

### **EXERCISE 3.2 (B) [PAGE 123]**

**Exercise 3.2 (B) | Q 1.01 | Page 123**

Evaluate the following :  $\int \frac{1}{4x^2 - 3} \cdot dx$

**SOLUTION**

$$\begin{aligned}
I &= \int \frac{1}{4x^2 - 3} \cdot dx \\
&= \frac{1}{4} \int \frac{1}{x^2 - \frac{3}{4}} \cdot dx \\
&= \frac{1}{4} \int \frac{1}{x^2 - \left(\frac{\sqrt{3}}{2}\right)^2} \cdot dx
\end{aligned}$$

$$\begin{aligned}
&= \frac{1}{4} \frac{1}{2\left(\frac{\sqrt{3}}{2}\right)} \log \left| \frac{x - \frac{\sqrt{3}}{2}}{x + \frac{\sqrt{3}}{2}} \right| + c \\
&= \frac{1}{4\sqrt{3}} \log \left| \frac{2x - \sqrt{3}}{2x + \sqrt{3}} \right| + c.
\end{aligned}$$

**Exercise 3.2 (B) | Q 1.02 | Page 123**

Evaluate the following :  $\int \frac{1}{25 - 9x^2} \cdot dx$

**SOLUTION**

$$\begin{aligned}
I &= \int \frac{1}{25 - 9x^2} \cdot dx \\
&= \int \frac{1}{5^2 - (3x)^2} \cdot dx \\
&= \frac{1}{2(5)} \log \left| \frac{5 + 3x}{5 - 3x} \right| \cdot \frac{1}{3} + c \\
&= \frac{1}{30} \log \left| \frac{5 + 3x}{5 - 3x} \right| + c.
\end{aligned}$$

Alternative Method :

$$\begin{aligned}
&\int \frac{1}{25 - 9x^2} \cdot dx \\
&= \frac{1}{9} \int \frac{1}{\frac{25}{9}x^2} \cdot dx \\
&= \frac{1}{9} \int \frac{1}{\left(\frac{5}{3}\right)^2 - x^2} \cdot dx \\
&= \frac{1}{9} \times \frac{1}{2 \times \frac{5}{3}} \log \left| \frac{\frac{5}{3} + x}{\frac{5}{3} - x} \right| + c
\end{aligned}$$

$$= \frac{1}{30} \log \left| \frac{5+3x}{5-3x} \right| + c.$$

**Exercise 3.2 (B) | Q 1.03 | Page 123**

Evaluate the following :  $\int \frac{1}{7+2x^2} \cdot dx$

**SOLUTION**

$$\begin{aligned} I &= \int \frac{1}{7+2x^2} \cdot dx \\ &= \frac{1}{2} \int \frac{1}{\frac{7}{2} + x^2} \cdot dx \\ &= \frac{1}{2} \int \frac{1}{\left(\sqrt{\frac{7}{2}}\right)^2 + x^2} \cdot dx \\ &= \frac{1}{2} \cdot \frac{1}{\left(\sqrt{\frac{7}{2}}\right)} \tan^{-1} \left| \frac{x}{\sqrt{\frac{7}{2}}} \right| + c \\ &= \frac{1}{\sqrt{14}} \tan^{-1} \left| \frac{\sqrt{2}x}{\sqrt{7}} \right| + c. \end{aligned}$$

**Exercise 3.2 (B) | Q 1.04 | Page 123**

Evaluate the following :  $\int \frac{1}{\sqrt{3x^2 - 8}} \cdot dx$

**SOLUTION**

$$\begin{aligned} & \int \frac{1}{\sqrt{3x^2 + 8}} \cdot dx \\ &= \frac{1}{\sqrt{3}} \int \frac{1}{\sqrt{x^2 + \frac{8}{3}}} \cdot dx \\ &= \frac{1}{\sqrt{3}} \int \frac{1}{\sqrt{x^2 + \left(\sqrt{\frac{8}{3}}\right)^2}} \cdot dx \\ &= \frac{1}{\sqrt{3}} \log \left| x + \sqrt{x^2 + \left(\sqrt{\frac{8}{3}}\right)^2} \right| + c_1 \\ &= \frac{1}{\sqrt{3}} \log \left| x + \sqrt{x^2 + \frac{8}{3}} \right| + c_1 \\ &= \frac{1}{\sqrt{3}} \log \left| \frac{\sqrt{3}x + \sqrt{3x^2 + 8}}{\sqrt{3}} \right| + c_1 \\ &= \frac{1}{\sqrt{3}} \log \left| \sqrt{3}x + \sqrt{3x^2 + 8} \right| - \log \sqrt{3} + c_1 \\ &= \frac{1}{\sqrt{3}} \log \left| \sqrt{3}x + \sqrt{3x^2 + 8} \right| + c, \text{ where } c = c_1 - \log \sqrt{3} \end{aligned}$$

Alternative Method :

$$\begin{aligned} & \int \frac{1}{\sqrt{3x^2 + 8}} \cdot dx \\ &= \int \frac{1}{\sqrt{(\sqrt{3}x)^2 + (\sqrt{8})^2}} \cdot dx \end{aligned}$$

$$\begin{aligned}
&= \frac{\log |\sqrt{3}x + \sqrt{(\sqrt{3}x)^2 + \sqrt{(8)^2}}| + c}{\sqrt{3}} \\
&= \frac{1}{\sqrt{3}} \log |\sqrt{3}x + \sqrt{3x^2 + 8}| + c.
\end{aligned}$$

**Exercise 3.2 (B) | Q 1.05 | Page 123**

Evaluate the following :  $\int \frac{1}{\sqrt{11 - 4x^2}} \cdot dx$

**SOLUTION**

$$\begin{aligned}
&\int \frac{1}{\sqrt{11 - 4x^2}} \cdot dx \\
&= \int \frac{1}{\sqrt{(\sqrt{11})^2 - (2x)^2}} \cdot dx \\
&= \frac{1}{2} \sin^{-1} \left( 2 \frac{x}{\sqrt{11}} \right) + c.
\end{aligned}$$

**Exercise 3.2 (B) | Q 1.06 | Page 123**

Evaluate the following :  $\int \frac{1}{\sqrt{2x^2 - 5}} \cdot dx$

**SOLUTION**

$$\begin{aligned}
&\int \frac{1}{\sqrt{2x^2 - 5}} \cdot dx \\
&= \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{x^2 - \frac{5}{2}}} \cdot dx
\end{aligned}$$

$$= \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{x^2 - \left(\sqrt{\frac{5}{2}}\right)^2}} \cdot dx$$

$$= \frac{1}{\sqrt{2}} \log \left| x + \sqrt{x^2 - \frac{5}{2}} \right| + c.$$

**Exercise 3.2 (B) | Q 1.07 | Page 123**

Evaluate the following :  $\int \sqrt{\frac{9+x}{9-x}} \cdot dx$

**SOLUTION**

$$\begin{aligned} \text{Let } I &= \int \sqrt{\frac{9+x}{9-x}} \cdot dx \\ &= \int \sqrt{\frac{9+x}{9-x}} \times \frac{9+x}{9+x} \cdot dx \\ &= \int \frac{9+x}{\sqrt{81-x^2}} \cdot dx \\ &= \int \frac{9}{\sqrt{81-x^2}} \cdot dx + \int \frac{x}{\sqrt{81-x^2}} \cdot dx \\ &= 9 \int \frac{1}{\sqrt{9^2-x^2}} \cdot dx + \frac{1}{2} \int \frac{2x}{\sqrt{81-x^2}} \cdot dx \\ &= I_1 + I_2 \quad \dots (\text{Let}) \end{aligned}$$

$$I_1 = 9 \int \frac{1}{\sqrt{9^2-x^2}} \cdot dx$$

$$= 9 \sin^{-1}\left(\frac{x}{9}\right) + c_1$$

In  $I_2$ , put  $81-x^2 = t$

$$\therefore -2x \, dx = dt$$

$$\therefore 2x \, dx = -dt$$

$$\begin{aligned}
 I_2 &= -\frac{1}{2} \int t^{-\frac{1}{2}} dt \\
 &= -\frac{1}{2} \cdot \frac{t^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} + c_2 \\
 &= -\sqrt{81 - x^2} + c_2 \\
 I &= 9 \sin^{-1}\left(\frac{x}{9}\right) - \sqrt{81 - x^2} + c,
 \end{aligned}$$

where  $c = c_1 + c_2$ .

**Exercise 3.2 (B) | Q 1.08 | Page 123**

Evaluate the following :  $\int \sqrt{\frac{2+x}{2-x}} \cdot dx$

**SOLUTION**

$$\begin{aligned}
 \text{Let } I &= \int \sqrt{\frac{2+x}{2-x}} \cdot dx \\
 &= \int \sqrt{\frac{2+x}{2-x}} \times \frac{2+x}{2+x} \cdot dx \\
 &= \int \frac{2+x}{\sqrt{4-x^2}} \cdot dx \\
 &= \int \frac{2}{\sqrt{4-x^2}} \cdot dx + \int \frac{x}{\sqrt{4-x^2}} \cdot dx \\
 &= 2 \int \frac{1}{\sqrt{2^2-x^2}} \cdot dx + \frac{1}{2} \int \frac{2x}{\sqrt{4-x^2}} \cdot dx \\
 &= I_1 + I_2 \quad \dots (\text{Let})
 \end{aligned}$$

$$\begin{aligned}
 I_1 &= 2 \int \frac{1}{\sqrt{2^2-x^2}} \cdot dx \\
 &= 2 \sin^{-1}\left(\frac{x}{2}\right) + c_1
 \end{aligned}$$

In  $I_2$ , put  $4 - x^2 = t$

$$\therefore -2x \, dx = dt$$

$$\therefore 2x \, dx = -dt$$

$$I_2 = -\frac{1}{2} \int t^{-\frac{1}{2}} dt$$

$$= -\frac{1}{2} \cdot \frac{t^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} + c_2$$

$$= -\sqrt{4 - x^2} + c_2$$

$$I = 2 \sin^{-1}\left(\frac{x}{2}\right) - \sqrt{4 - x^2} + c.$$

Exercise 3.2 (B) | Q 1.09 | Page 123

Evaluate the following :  $\int \sqrt{\frac{10+x}{10-x}} \cdot dx$

SOLUTION

$$\begin{aligned} \text{Let } I &= \int \sqrt{\frac{10+x}{10-x}} \cdot dx \\ &= \int \sqrt{\frac{10+x}{10-x}} \times \frac{10+x}{10+x} \cdot dx \\ &= \int \frac{10+x}{\sqrt{100-x^2}} \cdot dx \\ &= \int \frac{10}{\sqrt{100-x^2}} \cdot dx + \int \frac{x}{\sqrt{100-x^2}} \cdot dx \\ &= 10 \int \frac{1}{\sqrt{10^2-x^2}} \cdot dx + \frac{1}{2} \int \frac{2x}{\sqrt{100-x^2}} \cdot dx \\ &= I_1 + I_2 \quad \dots (\text{Let}) \end{aligned}$$

$$I_1 = 10 \int \frac{1}{\sqrt{10^2-x^2}} \cdot dx$$

$$= 10 \sin^{-1} \left( \frac{x}{10} \right) + c_1$$

In  $I_2$ , put  $100 - x^2 = t$

$$\therefore -2x \, dx = dt$$

$$\therefore 2x \, dx = -dt$$

$$I_2 = -\frac{1}{2} \int t^{-\frac{1}{2}} dt$$

$$= -\frac{1}{2} \cdot \frac{t^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} + c_2$$

$$= -\sqrt{100 - x^2} + c_2$$

$$I = 10 \sin^{-1} \left( \frac{x}{10} \right) - \sqrt{100 - x^2} + c.$$

**Exercise 3.2 (B) | Q 1.1 | Page 123**

Evaluate the following :  $\int \frac{1}{x^2 + 8x + 12} \cdot dx$

**SOLUTION**

$$\int \frac{1}{x^2 + 8x + 12} \cdot dx$$

$$= \int \frac{1}{(x^2 + 8x + 16) - 16 + 12} \cdot dx$$

$$= \int \frac{1}{(x+4)^2 - 2^2} \cdot dx$$

$$= \frac{1}{2(2)} \log \left| \frac{(x+4) - 2}{(x+4) + 2} \right| + c$$

$$= \frac{1}{4} \log \left| \frac{x+2}{x+6} \right| + c.$$

**Exercise 3.2 (B) | Q 1.11 | Page 123**

Evaluate the following :  $\int \frac{1}{1+x-x^2} \cdot dx$

**SOLUTION**

$$\text{Let } I = \int \frac{1}{1+x-x^2} \cdot dx$$

$$1+x-x^2 = 1-(x^2-x)$$

$$= 1 - \left( x^2 - x + \frac{1}{4} \right) + \frac{1}{4}$$

$$= \frac{5}{4} - \left( x^2 - x + \frac{1}{4} \right)$$

$$= \left( \frac{\sqrt{5}}{2} \right)^2 - \left( x - \frac{1}{2} \right)^2$$

$$\therefore I = \int \frac{1}{\left( \frac{\sqrt{5}}{2} \right)^2 - \left( x - \frac{1}{2} \right)^2} \cdot dx$$

$$= \frac{1}{2 \left( \frac{\sqrt{5}}{2} \right)} \log \left| \frac{\frac{\sqrt{5}}{2} + \left( x - \frac{1}{2} \right)}{\frac{\sqrt{5}}{2} - \left( x - \frac{1}{2} \right)} \right| + c$$

$$= \frac{1}{\sqrt{5}} \log \left| \frac{\sqrt{5} - 1 + 2x}{\sqrt{5} + 1 - 2x} \right| + c.$$

**Exercise 3.2 (B) | Q 1.12 | Page 123**

Evaluate the following :  $\frac{1}{4x^2 - 20x + 17}$

**SOLUTION**

$$\begin{aligned} & \int \frac{1}{4x^2 - 20x + 17} \cdot dx \\ &= \frac{1}{4} \int \frac{1}{x^2 - 5x + \frac{17}{4}} \cdot dx \\ &= \frac{1}{4} \int \frac{1}{\left(x^2 - 5x + \frac{25}{4}\right) - \frac{25}{4} + \frac{17}{4}} \cdot dx \\ &= \frac{1}{4} \int \frac{1}{\left(x - \frac{5}{2}\right)^2 - \left(\sqrt{2}\right)^2} \cdot dx \\ &= \frac{1}{4} \times \frac{1}{2\sqrt{2}} \log \left| \frac{x - \frac{5}{2} - \sqrt{2}}{x - \frac{5}{2} + \sqrt{2}} \right| + c \\ &= \frac{1}{8\sqrt{2}} \log \left| \frac{2x - 5 - 2\sqrt{2}}{2x - 5 + 2\sqrt{2}} \right| + c. \end{aligned}$$

**Exercise 3.2 (B) | Q 1.13 | Page 123**

Evaluate the following :  $\int \frac{1}{5 - 4x - 3x^2} \cdot dx$

**SOLUTION**

$$\begin{aligned} \text{Let } I &= \int \frac{1}{5 - 4x - 3x^2} \cdot dx \\ 5 - 4x - 3x^2 &= \left[ \frac{5}{3} - \left( x^2 + \frac{4}{3}x \right) \right] \\ &= 3 \left[ \frac{5}{3} - \left( x^2 + \frac{4}{3}x + \frac{4}{9} \right) + \frac{4}{9} \right] \\ &= 3 \left[ \frac{19}{9} - \left( x^2 + \frac{4x}{3} + \frac{4}{9} \right) \right] \end{aligned}$$

$$\begin{aligned}
&= 3 \left[ \left( \frac{\sqrt{19}}{3} \right)^2 - \left( x + \frac{2}{3} \right)^2 \right] \\
I &= \int \frac{1}{3 \left[ \left( \frac{\sqrt{19}}{3} \right)^2 - \left( x + \frac{2}{3} \right)^2 \right]} \cdot dx \\
&= \frac{1}{3} \frac{1}{2 \left( \frac{\sqrt{19}}{3} \right)} \log \left| \frac{\frac{\sqrt{19}}{3} + \left( x + \frac{2}{3} \right)}{\frac{\sqrt{19}}{3} - \left( x + \frac{2}{3} \right)} \right| + c \\
&= \frac{1}{2\sqrt{19}} \log \left| \frac{\sqrt{19} + 2 + 3x}{\sqrt{19} - 2 - 3x} \right| + c \\
&= \frac{1}{2\sqrt{19}} \log \left| \frac{3x + 2 + \sqrt{19}}{-\left( 3x + 2 - \sqrt{19} \right)} \right| + c \\
&= \frac{1}{2\sqrt{19}} \log \left| \frac{3x + 2 + \sqrt{19}}{3x + 2 - \sqrt{19}} \right| + c. \quad \dots [\because | -x | = x]
\end{aligned}$$

**Exercise 3.2 (B) | Q 1.14 | Page 123**

Evaluate the following :  $\int \frac{1}{\sqrt{3x^2 + 5x + 7}} \cdot dx$

**SOLUTION**

$$\begin{aligned}
\text{Let } I &= \int \frac{1}{\sqrt{3x^2 + 5x + 7}} \cdot dx \\
3x^2 + 5x + 7 &= 3 \left[ x^2 + \frac{5}{3}x + \frac{7}{3} \right] \\
&= 3 \left[ \left( x^2 + \frac{5x}{3} + \frac{25}{36} \right) + \left( \frac{7}{3} - \frac{25}{36} \right) \right]
\end{aligned}$$

$$\begin{aligned}
&= 3 \left[ \left( x + \frac{5}{6} \right)^2 + \left( \frac{\sqrt{59}}{6} \right)^2 \right] \\
&\therefore \sqrt{3x^2 + 5x + 7} = \sqrt{3} \sqrt{\left( x + \frac{5}{6} \right)^2 + \left( \frac{\sqrt{59}}{6} \right)^2} \\
&\therefore I = \frac{1}{\sqrt{3}} \int \frac{1}{\left( x + \frac{5}{6} \right)^2 + \left( \frac{\sqrt{59}}{6} \right)^2} \cdot dx \\
&= \frac{1}{\sqrt{3}} \log \left| x + \frac{5}{6} + \sqrt{\left( x + \frac{5}{6} \right)^2 + \left( \frac{\sqrt{59}}{6} \right)^2} \right| + c \\
&= \frac{1}{\sqrt{3}} \log \left| x + \frac{5}{6} + \sqrt{x^2 + \frac{5x}{3} + \frac{7}{3}} \right| + c.
\end{aligned}$$

**Exercise 3.2 (B) | Q 1.15 | Page 123**

Evaluate the following :  $\int \frac{1}{\sqrt{x^2 + 8x - 20}} \cdot dx$

**SOLUTION**

$$\begin{aligned}
&\int \frac{1}{\sqrt{x^2 + 8x - 20}} \cdot dx \\
&= \int \frac{1}{\sqrt{(x^2 + 8x + 16) - 16 - 20}} \cdot dx \\
&= \int \frac{1}{\sqrt{(x+4)^2 - (6)^2}} \cdot dx \\
&= \log \left| (x+4) + \sqrt{(x+4)^2 - (6)^2} \right| + c \\
&= \log \left| (x+4) + \sqrt{x^2 + 8x - 20} \right| + c.
\end{aligned}$$

**Exercise 3.2 (B) | Q 1.16 | Page 123**

Evaluate the following :  $\int \frac{1}{\sqrt{8 - 3x + 2x^2}} \cdot dx$

**SOLUTION**

$$\text{Let } I = \int \frac{1}{\sqrt{8 - 3x + 2x^2}} \cdot dx$$

$$8 - 3x + 2x^2 = 8 \left[ x^2 + \frac{3}{2}x + \frac{2}{2} \right]$$

$$= 8 \left[ \left( x^2 + \frac{3x}{2} + \frac{6}{4} \right) + \left( \frac{2}{2} - \frac{6}{4} \right) \right]$$

$$= 8 \left[ \left( x + \frac{3}{4} \right)^2 + \left( \frac{\sqrt{1}}{4} \right)^2 \right]$$

$$\therefore \sqrt{3x^2 + 5x + 7} = \sqrt{3} \sqrt{\left( x + \frac{3}{4} \right)^2 + \left( \frac{\sqrt{1}}{4} \right)^2}$$

$$\therefore I = \frac{1}{\sqrt{3}} \int \frac{1}{\left( x + \frac{3}{4} \right)^2 + \left( \frac{\sqrt{1}}{4} \right)^2} \cdot dx$$

$$= \frac{1}{\sqrt{2}} \log \left| x - \frac{3}{4} + \sqrt{\left( x + \frac{3}{4} \right)^2 + \left( \frac{\sqrt{1}}{4} \right)^2} \right| + c$$

$$= \frac{1}{\sqrt{2}} \log \left| x - \frac{3}{4} + \sqrt{x^2 - \frac{3x}{2} + 4} \right| + c.$$

**Exercise 3.2 (B) | Q 1.17 | Page 123**

Evaluate the following :  $\int \frac{1}{\sqrt{(x-3)(x+2)}} \cdot dx$

**SOLUTION**

$$\begin{aligned} \text{Let } I &= \int \frac{1}{\sqrt{(x-3)(x+2)}} \cdot dx \\ &= \int \frac{1}{\sqrt{x^2 - x - 6}} \cdot dx \\ &= \int \frac{1}{\sqrt{\left(x^2 - x + \frac{1}{4}\right) - \frac{1}{4} - 6}} \cdot dx \\ &= \int \frac{1}{\sqrt{\left(x - \frac{1}{2}\right)^2 - \left(\frac{5}{2}\right)^2}} \cdot dx \\ &= \log \left| \left(x - \frac{1}{2}\right) + \sqrt{\left(x - \frac{1}{2}\right)^2 - \left(\frac{5}{2}\right)^2} \right| + c \\ &= \log \left| \left(x - \frac{1}{2}\right) + \sqrt{x^2 - x - 6} \right| + c. \end{aligned}$$

**Exercise 3.2 (B) | Q 1.18 | Page 123**

Evaluate the following :  $\int \frac{1}{4 + 3 \cos^2 x} \cdot dx$

**SOLUTION**

$$\text{Let } I = \int \frac{1}{4 + 3 \cos^2 x} \cdot dx$$

Dividing both numerator and denominator by  $\cos^2 x$ , we get

$$\begin{aligned} I &= \int \frac{\sec^2 x}{4 \sec^2 x + 3} \cdot dx \\ &= \int \frac{\sec^2 x}{4(1 + \tan^2 x) + 3} \cdot dx \end{aligned}$$

$$= \int \frac{\sec^2 x}{4\tan^2 x + 7} \cdot dx$$

Put  $\tan x = t$

$$\therefore \sec^2 x \, dx = dt$$

$$I = \int \frac{dt}{4t^2 + 7}$$

$$= \int \frac{dt}{(2t)^2 + (\sqrt{7})^2}$$

$$= \frac{1}{\sqrt{7}} \tan^{-1} \left( \frac{2t}{\sqrt{7}} \right) \cdot \frac{1}{2} + c$$

$$= \frac{1}{2\sqrt{7}} \tan^{-1} \left( \frac{2 \tan x}{\sqrt{7}} \right) + c.$$

**Exercise 3.2 (B) | Q 1.19 | Page 123**

Evaluate the following :  $\int \frac{1}{\cos 2x + 3 \sin^2 x} \cdot dx$

**SOLUTION**

$$\text{Let } I = \int \frac{1}{\cos 2x + 3 \sin^2 x} \cdot dx$$

$$= \int \frac{1}{1 - 2 \sin^2 x + 3 \sin^2 x} \cdot dx$$

$$= \int \frac{1}{1 + \sin^2 x} \cdot dx$$

Dividing both numerator and denominator by  $\cos^2 x$ , we get

$$I = \int \frac{\sec^2 x \, dx}{\sec^2 x + \tan^2 x}$$

$$= \int \frac{\sec^2 x \, dx}{1 + \tan^2 x + \tan^2 x}$$

$$= \int \frac{\sec^2 x dx}{2\tan^2 x + 1}$$

Put  $\tan x = t$

$$\therefore \sec^2 x dx = dt$$

$$\therefore I = \int \frac{1}{2t^2 + 1} dt$$

$$= \frac{1}{2} \int \frac{1}{t^2 + \left(\frac{1}{\sqrt{2}}\right)^2} dt$$

$$= \frac{1}{2} \times \frac{1}{\left(\frac{1}{\sqrt{2}}\right)} \tan^{-1}\left(\frac{t}{\frac{1}{\sqrt{2}}}\right) + c$$

$$= \frac{1}{\sqrt{2}} \tan^{-1}\left(\sqrt{2} \tan x\right) + c.$$

**Exercise 3.2 (B) | Q 1.2 | Page 123**

Evaluate the following :  $\int \frac{\sin x}{\sin 3x} \cdot dx$

**SOLUTION**

$$\text{Let } I = \int \frac{\sin x}{\sin 3x} \cdot dx$$

$$= \int \frac{\sin x}{3 \sin x - 4 \sin^2 x} \cdot dx$$

$$= \int \frac{1}{3 - 4 \sin^2 x} \cdot dx$$

Dividing both numerator and denominator by  $\cos^2 x$ , we get

$$\begin{aligned}
 I &= \int \frac{\sec^2 x}{3 \sec^2 x - 4 \tan^2 x} \cdot dx \\
 &= \int \frac{\sec^2 x}{3(1 + \tan^2 x) - 4 \tan^2 x} \cdot dx \\
 &= \int \frac{\sec^2 x}{3 - \tan^2 x} \cdot dx
 \end{aligned}$$

Put  $\tan x = t$

$$\therefore \sec^2 x \, dx = dt$$

$$\begin{aligned}
 I &= \int \frac{dt}{(\sqrt{3})^2 - t^2} \\
 &= \frac{1}{2\sqrt{3}} \log \left| \frac{\sqrt{3} + t}{\sqrt{3} - t} \right| + c \\
 &= \frac{1}{2\sqrt{3}} \log \left| \frac{\sqrt{3} + \tan x}{\sqrt{3} - \tan x} \right| + c.
 \end{aligned}$$

**Exercise 3.2 (B) | Q 2.1 | Page 123**

Integrate the following functions w.r.t. x :  $\int \frac{1}{3 + 2 \sin x} \cdot dx$

**SOLUTION**

$$\text{Let } I = \int \frac{1}{3 + 2 \sin x} \cdot dx$$

$$\text{Put } \tan\left(\frac{x}{2}\right) = t$$

$$\therefore x = 2 \tan^{-1} t$$

$$\therefore dx = \frac{2t}{1+t^2} \text{ and } \sin x = \frac{2t}{1+t^2}$$

$$\begin{aligned}
I &= \int \frac{1}{3 + 2\left(\frac{2t}{1+t^2}\right)} \cdot \frac{2dt}{1+t^2} \\
&= \int \frac{1+t^2}{3+3t^2+4t} \cdot \frac{2dt}{1+t^2} \\
&= 2 \int \frac{1}{3t^2+4t+3} dt \\
&= \frac{2}{3} \int \frac{1}{t^2+\frac{4}{3}t+1} dt \\
&= \frac{2}{3} \int \frac{1}{\left(t^2+\frac{4}{3}t+\frac{4}{9}\right)-\frac{4}{9}+1} dt \\
&= \frac{2}{3} \int \frac{1}{\left(t+\frac{2}{3}\right)^2+\left(\frac{\sqrt{5}}{3}\right)^2} dt \\
&= \frac{2}{3} \times \frac{1}{\left(\frac{\sqrt{5}}{3}\right)} \tan^{-1} \left[ \frac{t+\frac{2}{3}}{\frac{\sqrt{5}}{3}} \right] + c \\
&= \frac{2}{\sqrt{3}} \tan^{-1} \left( \frac{3t+2}{\sqrt{5}} \right) + c \\
&= \frac{2}{\sqrt{5}} \tan^{-1} \left[ \frac{3 \tan\left(\frac{x}{2}\right) + 2}{\sqrt{5}} \right] + c.
\end{aligned}$$

**Exercise 3.2 (B) | Q 2.2 | Page 123**

Integrate the following functions w.r.t. x :  $\int \frac{1}{4 - 5 \cos x} \cdot dx$

**SOLUTION**

$$\text{Let } I = \int \frac{1}{4 - 5 \cos x} \cdot dx$$

$$\text{Put } \tan\left(\frac{x}{2}\right) = t$$

$$\therefore x = 2 \tan^{-1} t$$

$$\therefore dx = \frac{2dt}{1+t^2} \text{ and } \cos x = \frac{1-t^2}{1+t^2}$$

$$\therefore I = \int \frac{1}{4 - 5\left(\frac{1-t^2}{1+t^2}\right)} \cdot \frac{2dt}{1+t^2}$$

$$= \int \frac{1+t^2}{4(1+t^2) - 5(1-t^2)} \cdot \frac{2dt}{1+t^2}$$

$$= \int \frac{2dt}{9t^2 - 1}$$

$$= \frac{2}{9} \int \frac{1}{t^2 - \frac{1}{9}} dt$$

$$= \frac{2}{9} \int \frac{1}{t^2 - \left(\frac{1}{3}\right)^2} dt$$

$$= \frac{2}{9} \times \frac{1}{2 \times \frac{1}{3}} \log \left| \frac{t - \frac{1}{3}}{t + \frac{1}{3}} \right| + c$$

$$= \frac{1}{3} \log \left| \frac{3 \tan\left(\frac{x}{2}\right) - 1}{3 \tan\left(\frac{x}{2}\right) + 1} \right| + c.$$

**Exercise 3.2 (B) | Q 2.3 | Page 123**

Integrate the following functions w.r.t. x :  $\int \frac{1}{2 + \cos x - \sin x} \cdot dx$

**SOLUTION**

$$\text{Let } I = \int \frac{1}{2 + \cos x - \sin x} \cdot dx$$

$$\text{Put } \tan\left(\frac{x}{2}\right) = t$$

$$\therefore x = 2 \tan^{-1} t$$

$$\therefore dx = \frac{2dt}{1+t^2} \text{ and } \sin x = \frac{2t}{1+t^2}, \cos x = \frac{1-t^2}{1+t^2}$$

$$\therefore I = \int \frac{1}{2 + \left(\frac{1-t^2}{1+t^2}\right) - \left(\frac{2t}{1+t^2}\right)} \cdot \frac{2dt}{1+t^2}$$

$$= \int \frac{1+t^2}{2+2t^2+1-t^2-2t} \cdot \frac{2dt}{1+t^2}$$

$$= 2 \int \frac{1}{t^2 - 2t + 3} dt$$

$$= 2 \int \frac{1}{(t^2 - 2t + 1) + 2} dt$$

$$= 2 \int \frac{1}{(t-1)^2 + (\sqrt{2})^2} dt$$

$$= 2 \times \frac{1}{\sqrt{2}} \tan^{-1} \left( \frac{t-1}{\sqrt{2}} \right) + c$$

$$= \sqrt{2} \tan^{-1} \left[ \frac{\tan\left(\frac{x}{2}\right) - 1}{\sqrt{2}} \right] + c.$$

**Exercise 3.2 (B) | Q 2.4 | Page 123**

Integrate the following functions w.r.t. x :  $\int \frac{1}{3 + 2 \sin x - \cos x} dx$

**SOLUTION**

$$\text{Let } I = \int \frac{1}{3 + 2 \sin x - \cos x} dx$$

$$\text{Put } \tan\left(\frac{x}{2}\right) = t$$

$$\therefore x = 2 \tan^{-1} t$$

$$\therefore dx = \frac{2}{1+t^2} dt \text{ and}$$

$$\sin x = \frac{2t}{1+t^2}, \cos x = \frac{1-t^2}{1+t^2}$$

$$\begin{aligned} \therefore I &= \int \frac{1}{3 + 2\left(\frac{2t}{1+t^2}\right) - \left(\frac{1-t^2}{1+t^2}\right)} \cdot \frac{2dt}{1+t^2} \\ &= \int \frac{1+t^2}{3(1+t^2) + 4t - (1-t^2)} \cdot \frac{2dt}{1+t^2} \\ &= 2 \int \frac{dt}{4t^2 + 4t + 2} \\ &= 2 \int \frac{dt}{4t^2 + 4t + 1 + 1} \\ &= 2 \int \frac{dt}{(2t+1)^2 + 1^2} \\ &= \frac{2}{2} \tan^{-1}\left(\frac{2t+1}{1}\right) + c \\ &= \tan^{-1}\left[2 \tan^{-1}\left(\frac{x}{2}\right) + 1\right] + c. \end{aligned}$$

**Exercise 3.2 (B) | Q 2.5 | Page 123**

Integrate the following functions w.r.t. x :  $\int \frac{1}{3 - 2 \cos 2x} \cdot dx$

**SOLUTION**

$$\text{Let } I = \int \frac{1}{3 - 2 \cos 2x} \cdot dx$$

Put  $\tan x = t$

$$\therefore x = \tan^{-1} t$$

$$\therefore dx = \frac{dt}{1+t^2} \text{ and } \cos 2x = \frac{1-t^2}{1+t^2}$$

$$\therefore I = \int \frac{1}{3 - 2\left(\frac{1-t^2}{1+t^2}\right)} \cdot \frac{dt}{1+t^2}$$

$$= \int \frac{1+t^2}{3+3t^2-2+2t^2} \cdot \frac{dt}{1+t^2}$$

$$= \int \frac{1}{1+5t^2} dt$$

$$= \frac{1}{5} \int \frac{1}{\left(\frac{1}{\sqrt{5}}\right)^2 + t^2} dt$$

$$= \frac{1}{5} \times \frac{1}{\left(\frac{1}{\sqrt{5}}\right)} \tan^{-1} \left( \frac{t}{\frac{1}{\sqrt{5}}} \right) + c$$

$$= \frac{1}{\sqrt{5}} \tan^{-1} \left( \sqrt{5} \tan x \right) + c.$$

### Exercise 3.2 (B) | Q 2.6 | Page 123

Integrate the following functions w.r.t. x :  $\int \frac{1}{2 \sin 2x - 3} dx$

**SOLUTION**

$$\text{Let } I = \int \frac{1}{2 \sin 2x - 3} dx$$

Put  $\tan x = t$

$$\therefore x = \tan^{-1} t$$

$$\therefore dx = \frac{dt}{1+t^2} \text{ and } \sin 2x = \frac{2t}{1+t^2}$$

$$\therefore I = \int \frac{1}{2\left(\frac{2t}{1+t^2}\right) - 3} \cdot \frac{dt}{1+t^2}$$

$$= \int \frac{1+t^2}{4t - 3 - 3t^2} \cdot \frac{dt}{1+t^2}$$

$$= \int \frac{1}{-3t^2 + 4t - 3} dt$$

$$= \frac{1}{3} \int \frac{1}{t^2 - \frac{4}{3}t + 1} dt$$

$$= -\frac{1}{3} \int \frac{1}{(t^2 - \frac{4}{3}t + \frac{4}{9}) - \frac{4}{9} + 1} dt$$

$$= -\frac{1}{3} \int \frac{1}{\left(t - \frac{2}{3}\right)^2 + \left(\frac{\sqrt{5}}{3}\right)^2} dt$$

$$= -\frac{1}{3} \times \frac{1}{\left(\frac{\sqrt{5}}{3}\right)} \tan^{-1} \left( \frac{t - \frac{2}{3}}{\frac{\sqrt{5}}{3}} \right) + c$$

$$= -\frac{1}{\sqrt{5}} \tan^{-1} \left( \frac{3t - 2}{\sqrt{5}} \right) + c$$

$$= -\frac{1}{\sqrt{5}} \tan^{-1} \left( \frac{3 \tan x - 2}{\sqrt{5}} \right) + c.$$

**Exercise 3.2 (B) | Q 2.7 | Page 123**

Integrate the following functions w.r.t. x :  $\int \frac{1}{3 + 2 \sin 2x + 4 \cos 2x} \cdot dx$

**SOLUTION**

$$\text{Let } I = \int \frac{1}{3 + 2 \sin 2x + 4 \cos 2x} \cdot dx$$

Put  $\tan x = t$

$$\therefore x = \tan^{-1} t$$

$$\therefore dx = \frac{dt}{1+t^2} \text{ and } \sin 2x = \frac{2t}{1+t^2}, \cos 2x = \frac{1-t^2}{1+t^2}$$

$$\therefore I = \int \frac{1}{3 + 2\left(\frac{2t}{1+t^2}\right) + 4\left(\frac{1-t^2}{1+t^2}\right)} \cdot \frac{dt}{1+t^2}$$

$$= \int \frac{1+t^2}{3(1+t^2) + 4t + 4(1-t^2)} \cdot \frac{dt}{1+t^2}$$

$$= \int \frac{1}{7+4t-t^2} dt = \int \frac{1}{7-(t^2-4t+4)+4} dt$$

$$= \int \frac{1}{(\sqrt{11})^2 - (t-2)^2} dt$$

$$= \frac{1}{2\sqrt{11}} \log \left| \frac{\sqrt{11}+t-2}{\sqrt{11}-t+2} \right| + c$$

$$= \frac{1}{2\sqrt{11}} \log \left| \frac{\sqrt{11}+\tan x-2}{\sqrt{11}-\tan x+2} \right| + c.$$

**Exercise 3.2 (B) | Q 2.8 | Page 123**

Integrate the following functions w.r.t. x :  $\int \frac{1}{\cos x - \sin x} \cdot dx$

**SOLUTION**

$$\text{Let } I = \int \frac{1}{\cos x - \sin x} \cdot dx$$

Dividing each term by  $\sqrt{1^2 + (-1)^2} = \sqrt{2}$ , we get

$$\begin{aligned} I &= \frac{1}{\sqrt{2}} \int \frac{1}{\cos x \cdot \frac{1}{\sqrt{2}} - \sin x \cdot \frac{1}{\sqrt{2}}} \cdot dx \\ &= \frac{1}{\sqrt{2}} \int \frac{1}{\cos x \cdot \cos \frac{\pi}{4} - \sin x \cdot \sin \frac{\pi}{4}} \cdot dx \\ &= \frac{1}{\sqrt{2}} \int \frac{1}{\cos(x + \frac{\pi}{4})} \cdot dx \\ &= \frac{1}{\sqrt{2}} \int \sec\left(x + \frac{\pi}{4}\right) \cdot dx \\ &= \frac{1}{\sqrt{2}} \log \left| \sec\left(x + \frac{\pi}{4}\right) + \tan\left(x + \frac{\pi}{4}\right) \right| + c. \end{aligned}$$

**Exercise 3.2 (B) | Q 2.9 | Page 123**

Integrate the following functions w.r.t. x :  $\int \frac{1}{\cos x - \sqrt{3} \sin x} \cdot dx$

**SOLUTION**

$$\text{Let } I = \int \frac{1}{\cos x - \sqrt{3} \sin x} \cdot dx$$

Dividing each term by  $\sqrt{1^2 + (-\sqrt{3})^2} = \sqrt{4} = 2$ , we get

$$\begin{aligned} I &= \frac{1}{2} \int \frac{1}{\cos x \cdot \frac{1}{2} - \sin x \cdot \frac{-\sqrt{3}}{2}} \cdot dx \\ &= \frac{1}{2} \int \frac{1}{\cos x \cdot \cos \frac{\pi}{3} - \sin x \cdot \sin \frac{\pi}{3}} \cdot dx \end{aligned}$$

$$\begin{aligned}
 &= \frac{1}{2} \int \frac{1}{\cos(x + \frac{\pi}{3})} \cdot dx \\
 &= \frac{1}{2} \int \sec\left(x + \frac{\pi}{3}\right) \cdot dx \\
 &= \frac{1}{2} \log \left| \sec\left(x + \frac{\pi}{3}\right) + \tan\left(x + \frac{\pi}{3}\right) \right| + c.
 \end{aligned}$$

### EXERCISE 3.2 (C) [PAGE 128]

Exercise 3.2 (C) | Q 1.1 | Page 128

Evaluate the following integrals :  $\int \frac{3x+4}{x^2+6x+5} \cdot dx$

*SOLUTION*

$$\text{Let } I = \int \frac{3x+4}{x^2+6x+5} \cdot dx$$

$$\text{Let } 3x+4 = A \left[ \frac{d}{dx}(x^2+6x+5) \right] + B$$

$$= A(2x+6) + B$$

$$\therefore 3x+4 = 2Ax + (6A+B)$$

Comparing the coefficient of  $x$  and constant on both sides, we get

$$2A = 3 \text{ and } 6A + B = 4$$

$$\therefore A = \frac{3}{2} \text{ and } 6\left(\frac{3}{2}\right) + B = 4$$

$$\therefore B = -5$$

$$\therefore 3x+4 = \frac{3}{2}(2x+6) - 5$$

$$\therefore I = \int \frac{\frac{3}{2}(2x+6) - 5}{x^2+6x+5} \cdot dx$$

$$= \frac{3}{2} \int \frac{2x+6}{x^2+6x+5} \cdot dx - 5 \int \frac{1}{x^2+6x+5} \cdot dx \\ = \frac{3}{2} I_1 - 5I_2$$

$I_1$  is of the type  $\int \frac{f'(x)}{f(x)} \cdot dx = \log|f(x)| + c$

$$\therefore I_1 = \log|x^2 + 6x + 5| + c_1$$

$$I_2 = \int \frac{1}{x^2 + 6x + 5} \cdot dx \\ = \int \frac{1}{(x^2 + 6x + 9) - 4} \cdot dx \\ = \int \frac{1}{(x+3)^2 - 2^2} \cdot dx \\ = \frac{1}{2 \times 2} \log \left| \frac{x+3-2}{x+3+2} \right| + c_2 \\ = \frac{1}{4} \log \left| \frac{x+1}{x+5} \right| + c_2$$

$$\therefore I = \frac{3}{2} \log|x^2 + 6x + 5| - \frac{5}{4} \log \left| \frac{x+1}{x+5} \right| + c, \text{ where } c = c_1 + c_2.$$

#### Exercise 3.2 (C) | Q 1.2 | Page 128

Evaluate the following integrals :  $\int \frac{2x+1}{x^2+4x-5} \cdot dx$

**SOLUTION**

$$\text{Let } I = \int \frac{2x+1}{x^2+4x-5} \cdot dx$$

$$\text{Let } 2x+1 = A \left[ \frac{d}{dx}(x^2+4x-5) \right] + B$$

$$= A(2x+1) + B$$

$$\therefore 2x+1 = 2Ax + (4A+B)$$

Comparing the coefficient of  $x$  and constant on both sides, we get

$$4A = 2 \text{ and } 4A + B = 4$$

$$\therefore A = \frac{3}{2} \text{ and } 6\left(\frac{3}{2}\right) + B = 4$$

$$\therefore B = -5$$

$$\therefore 2x+1 = \frac{3}{2}(2x+1) - 5$$

$$\therefore I = \int \frac{\frac{3}{2}(2x+1) - 5}{x^2+6x+5} \cdot dx$$

$$= \frac{3}{2} \int \frac{2x+1}{x^2+4x-5} \cdot dx - 5 \int \frac{1}{x^2+4x+5} \cdot dx$$

$$= \frac{3}{2} I_1 - 5I_2$$

$I_1$  is of the type  $\int \frac{f'(x)}{f(x)} \cdot dx = \log|f(x)| + c$

$$\therefore I_1 = \log|x^2+4x-5| + c_1$$

$$I_2 = \int \frac{1}{x^2+4x-5} \cdot dx$$

$$= \int \frac{1}{(x^2+4x-9)-4} \cdot dx$$

$$\begin{aligned}
&= \int \frac{1}{(x+3)^2 - 2^2} \cdot dx \\
&= \log \left| \frac{x+3-2}{x+3+2} \right| + c_2 \\
&= \log \left| \frac{x-1}{x+5} \right| + c_2 \\
\therefore I &= \log|x^2 + 4x - 5| - \frac{1}{2} \log \left| \frac{x-1}{x+5} \right| + c
\end{aligned}$$

**Exercise 3.2 (C) | Q 1.3 | Page 128**

Evaluate the following integrals :  $\int \frac{2x+3}{2x^2+3x-1} \cdot dx$

**SOLUTION**

$$\begin{aligned}
\text{Let } I &= \int \frac{2x+3}{2x^2+3x-1} \cdot dx \\
\text{Let } 2x+3 &= A \left[ \frac{d}{dx} (2x^2+3x-1) \right] + B \\
&= A(4x+3) + B
\end{aligned}$$

$$\therefore 2x+3 = 4Ax + (3A+B)$$

Comparing the coefficient of x and constant on both sides, we get

$$4A = 2 \text{ and } 3A + B = 3$$

$$\therefore A = \frac{1}{2} \text{ and } 3\left(\frac{1}{2}\right) + B = 3$$

$$\therefore B = \frac{3}{2}$$

$$\therefore 2x+3 = \frac{1}{2}(4x+3) + \frac{3}{2}$$

$$\therefore I = \int \frac{\frac{1}{2}(4x+3) + \frac{3}{2}}{2x^2+3x-1} \cdot dx$$

$$\begin{aligned}
&= \frac{1}{2} \int \frac{4x+3}{2x^2+3x-1} \cdot dx + \frac{3}{2} \int \frac{1}{2x^2+3x-1} \cdot dx \\
&= \frac{1}{2} I_1 + \frac{3}{2} I_2
\end{aligned}$$

$I_1$  is of the type  $\int \frac{f'(x)}{f(x)} dx = \log|f(x)| + c$

$$\therefore I_1 = \log|2x^2 + 3x - 1| + c_1$$

$$\begin{aligned}
I_2 &= \int \frac{1}{2x^2+3x-1} \cdot dx \\
&= \frac{1}{2} \int \frac{1}{x^2 + \frac{3}{2}x - \frac{1}{2}} \cdot dx \\
&= \frac{1}{2} \int \frac{1}{\left(x^2 + \frac{3}{2}x + \frac{9}{16}\right) - \frac{9}{16} - \frac{1}{2}} \cdot dx \\
&= \frac{1}{2} \int \frac{1}{\left(x + \frac{3}{4}\right)^2 - \left(\frac{\sqrt{17}}{4}\right)^2} \cdot dx \\
&= \frac{1}{2} \times \frac{1}{2 \times \frac{\sqrt{17}}{4}} \log \left| \frac{x + \frac{3}{4} - \frac{\sqrt{17}}{4}}{x + \frac{3}{4} + \frac{\sqrt{17}}{4}} \right| + c_2 \\
&= \frac{1}{\sqrt{17}} \log \left| \frac{4x + 3 - \sqrt{17}}{4x + 3 + \sqrt{17}} \right| + c_2 \\
\therefore I &= \frac{1}{2} \log|2x^2 + 3x - 1| + \frac{3}{2\sqrt{17}} \log \left| \frac{4x + 3 - \sqrt{17}}{4x + 3 + \sqrt{17}} \right| + c, \text{ where } c = c_1 + c_2
\end{aligned}$$

Exercise 3.2 (C) | Q 1.4 | Page 128

Evaluate the following integrals :  $\int \frac{3x+4}{\sqrt{2x^2+2x+1}} \cdot dx$

**SOLUTION**

$$\text{Let } I = \int \frac{3x + 4}{\sqrt{2x^2 + 2x + 1}} \cdot dx$$

$$\text{Let } 3x + 4 = A \left[ \frac{d}{dx} (2x^2 + 2x + 1) \right] + B$$

$$= A(4x + 2) + B$$

$$\therefore 3x + 4 = 4Ax + (2A + B)$$

Comparing the coefficient of  $x$  and the constant on both the sides, we get

$$4A = 3 \text{ and } 2A + B = 4$$

$$\therefore A = \frac{3}{4} \text{ and } 2\left(\frac{3}{4}\right) + B = 4$$

$$\therefore B = \frac{5}{2}$$

$$\therefore 3x + 4 = (3)(4)(4x + 2) + \frac{5}{2}$$

$$\therefore I = \int \frac{\frac{3}{4}(4x + 2) + \frac{5}{2}}{\sqrt{2x^2 + 2x + 1}} \cdot dx$$

$$= \frac{3}{4} \int \frac{4x + 2}{\sqrt{2x^2 + 2x + 1}} \cdot dx + \frac{5}{2} \int \frac{1}{\sqrt{2x^2 + 2x + 1}} \cdot dx$$

$$= \frac{3}{4} I_1 + \frac{5}{2} I_2$$

In  $I_1$ , put  $2x^2 + 2x + 1 = t$

$$\therefore (4x + 2)dx = dt$$

$$\therefore I_1 = \int \frac{1}{\sqrt{t}} dt$$

$$= \int t^{-\frac{1}{2}} dt$$

$$\begin{aligned}
&= \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + c_1 \\
&= 2\sqrt{2x^2 + 2x + 1} + c \\
I_2 &= \frac{5}{\sqrt{2}} \int \frac{1}{\sqrt{x^2 + x + \frac{1}{2}}} \cdot dx \\
&= \frac{5}{\sqrt{2}} \int \frac{1}{\sqrt{(x^2 + x + \frac{1}{4}) + \frac{1}{4}}} \cdot dx \\
&= \frac{5}{\sqrt{2}} \int \frac{1}{\sqrt{(x + \frac{1}{2})^2 + (\frac{1}{2})^2}} \cdot dx \\
&= \frac{5}{\sqrt{2}} \log \left| \left( x + \frac{1}{2} \right) + \sqrt{\left( x + \frac{1}{2} \right)^2 + \left( \frac{1}{2} \right)^2} \right| + c_2 \\
&= \frac{5}{\sqrt{2}} \log \left| \left( x + \frac{1}{2} \right) + \sqrt{x^2 + x + \frac{1}{2}} \right| + c_2 \\
\therefore I &= \frac{3}{2} \sqrt{2x^2 + 2x + 1} + \frac{5}{2\sqrt{2}} \log \left| \left( x + \frac{1}{2} \right) + \sqrt{x^2 + x + \frac{1}{2}} \right| + c, \text{ where } c = c_1 + c_2.
\end{aligned}$$

**Exercise 3.2 (C) | Q 1.5 | Page 128**

x + Evaluate the following integrals :  $\int \frac{7x + 3}{\sqrt{3 + 2x - x^2}} \cdot dx$

**SOLUTION**

$$\begin{aligned}
\text{Let } I &= \int \frac{7x + 3}{\sqrt{3 + 2x - x^2}} \cdot dx \\
\text{Let } 7x + 3 &= A \left[ \frac{d}{dx} (3 + 2x - x^2) \right] + B \\
&= A(2 - 2x) + B \\
\therefore 7x + 3 &= 2Ax + (2A + B)
\end{aligned}$$

Comparing the coefficient of  $x$  and constant on both the sides, we get

$$-2A = 7 \text{ and } 2A + B = 3$$

$$\therefore A = \frac{-7}{2} \text{ and } 2\left(-\frac{7}{2}\right) + B = 3$$

$$\therefore B = 10$$

$$\therefore 7x + 3 = \frac{-7}{2}(2 - 2x) + 10$$

$$\begin{aligned}\therefore I &= \int \frac{\frac{-7}{2}(2 - 2x) + 10}{\sqrt{3 + 2x - x^2}} \cdot dx \\ &= \frac{-7}{2} \int \frac{(2 - 2x)}{\sqrt{3 + 2x - x^2}} \cdot dx + 10 \int \frac{1}{\sqrt{3 + 2x - x^2}} x \\ &= \frac{-7}{2} I_1 + 10 I_2\end{aligned}$$

$$\text{In } I_1, \text{ put } 3 + 2x - x^2 = t$$

$$\therefore (2 - 2x)dx = dt$$

$$\begin{aligned}\therefore I_1 &= \int \frac{1}{\sqrt{t}} dt \\ &= \int t^{-\frac{1}{2}} dt \\ &= \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + c_1 \\ &= 2\sqrt{3 + 2x - x^2} + c_1\end{aligned}$$

$$\begin{aligned}I_2 &= \int \frac{1}{\sqrt{3 - (x^2 - 2x + 1) + 1}} \cdot dx \\ &= \int \frac{1}{\sqrt{(2)^2 - (x - 1)^2}} \cdot dx\end{aligned}$$

$$\begin{aligned}
&= \sin^{-1} \left( \frac{x-1}{2} \right) + c_2 \\
\therefore I &= -7\sqrt{3+2x-x^2} + 10 \sin^{-1} \left( \frac{x-1}{2} \right) + c, \text{ where } c = c_1 + c_2 \\
&= \frac{5}{\sqrt{2}} \int \frac{1}{\sqrt{(x+\frac{1}{2})^2 + (\frac{1}{2})^2}} \cdot dx \\
&= \frac{5}{\sqrt{2}} \log \left| \left( x + \frac{1}{2} \right) + \sqrt{\left( x + \frac{1}{2} \right)^2 + \left( \frac{1}{2} \right)^2} \right| + c_2 \\
&= \frac{5}{\sqrt{2}} \log \left| \left( x + \frac{1}{2} \right) + \sqrt{x^2 + x + \frac{1}{2}} \right| + c_2 \\
\therefore I &= \frac{3}{2} \sqrt{2x^2 + 2x + 1} + \frac{5}{2\sqrt{2}} \log \left| \left( x + \frac{1}{2} \right) + \sqrt{x^2 + x + \frac{1}{2}} \right| + c, \text{ where } c = c_1 + c_2.
\end{aligned}$$

**Exercise 3.2 (C) | Q 1.6 | Page 128**

Evaluate the following integrals :  $\int \sqrt{\frac{x-7}{x-9}} \cdot dx$

**SOLUTION**

$$\begin{aligned}
\text{Let } I &= \int \sqrt{\frac{x-7}{x-9}} \cdot dx \\
&= \int \sqrt{\frac{x-7}{x-9} \times \frac{x-7}{x-7}} \cdot dx \\
&= \int \sqrt{\frac{(x-7)^2}{x^2 - 16x + 63}} \cdot dx \\
&= \int \frac{x-7}{\sqrt{x^2 - 16x + 63}} \cdot dx
\end{aligned}$$

$$\text{Let } x-7 = A \left[ \frac{d}{dx} (x^2 - 16x + 63) \right] + B$$

$$= A(2x - 16) + B$$

$$= 2Ax + (B - 16A)$$

Comparing the coefficient of  $x$  and constant term on both sides, we get

$$2A = 1$$

$$\therefore A = \frac{1}{2} \text{ and}$$

$$B - 16A = -7$$

$$\therefore B - 16\left(\frac{1}{2}\right) = -7$$

$$\therefore B = 1$$

$$\therefore x - 7 = \frac{1}{2}(2x - 16) + 1$$

$$\therefore I = \int \frac{\frac{1}{2}(2x - 16) + 1}{\sqrt{x^2 - 16x + 63}} \cdot dx$$

$$= \frac{1}{2} \int \frac{2x - 16}{\sqrt{x^2 - 16x + 63}} \cdot dx + \int \frac{1}{\sqrt{x^2 - 16x + 63}} \cdot dx$$

$$= \frac{1}{2}I_1 + I_2$$

In  $I_1$ , put  $x^2 - 16x + 63 = t$

$$\therefore (2x - 16)dx = dt$$

$$\therefore I_1 = \frac{1}{2} \int \frac{1}{\sqrt{t}} dt$$

$$= \frac{1}{2} \int t^{-\frac{1}{2}} dt$$

$$\begin{aligned}
&= \frac{1}{2} \frac{t^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} + c_1 \\
&= \sqrt{x^2 - 16x + 63} + c_1 \\
I_2 &= \int \frac{1}{\sqrt{x^2 - 16x + 63}} \cdot dx \\
&= \int \frac{1}{\sqrt{(x-8)^2 - 1^2}} \cdot dx \\
&= \log \left| x - 8 + \sqrt{(x-8)^2 - 1^2} \right| + c_2 \\
&= \log \left| x - 8 + \sqrt{x^2 - 16x + 63} \right| + c_2 \\
\therefore I &= \sqrt{x^2 - 16x + 63} + \log \left| x - 8 + \sqrt{x^2 - 16x + 63} \right| + c, \text{ where } c = c_1 + c_2.
\end{aligned}$$

**Exercise 3.2 (C) | Q 1.7 | Page 128**

Evaluate the following integrals :  $\int \sqrt{\frac{9-x}{x}} \cdot dx$

**SOLUTION**

$$\begin{aligned}
\text{Let } I &= \int \sqrt{\frac{9-x}{x}} \cdot dx \\
&= \int \sqrt{\frac{9-x}{x} \cdot \frac{9-x}{9-x}} \cdot dx \\
&= \int \frac{9-x}{\sqrt{9x-x^2}} \cdot dx
\end{aligned}$$

$$\text{Let } 9-x = A \left[ \frac{d}{dx} (9x-x^2) \right] + B$$

$$\begin{aligned}
&= A(9-2x) + B \\
\therefore 9-x &= (9A+B) - 2Ax
\end{aligned}$$

Comparing the coefficient of  $x$  and constant on both the sides, we get

$$-2A = -1 \text{ and } 9A + B = 9$$

$$\therefore A = \frac{1}{2} \text{ and } 9\left(\frac{1}{2}\right) + B = 9$$

$$\therefore B = \frac{9}{2}$$

$$\therefore 9 - x = \frac{1}{2}(9 - 2x) + \frac{9}{2}$$

$$\therefore I = \int \frac{\frac{1}{2}(9 - 2x) + \frac{9}{2}}{\sqrt{9x - x^2}} \cdot dx$$

$$= \frac{1}{2} \int \frac{9 - 2x}{\sqrt{9x - x^2}} \cdot dx + \frac{9}{2} \int \frac{1}{\sqrt{9x - x^2}} \cdot dx$$

$$= \frac{1}{2} I_1 + \frac{9}{2} I_2$$

In  $I_1$ , put  $9x - x^2 = t$

$$\therefore (9 - 2x)dx = dt$$

$$\therefore I_1 = \int \frac{1}{\sqrt{t}} dt$$

$$= \int t^{-\frac{1}{2}} dt$$

$$= \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + c_1$$

$$= 2\sqrt{9x - x^2} + c_1$$

$$I_2 = \int \frac{1}{\sqrt{\frac{81}{4} - (x^2 - 9x + \frac{81}{4})}} \cdot dx$$

$$\begin{aligned}
&= \int \frac{1}{\sqrt{\left(\frac{9}{2}\right)^2 - \left(x - \frac{9}{2}\right)^2}} \cdot dx \\
&= \sin^{-1} \left( \frac{x - \frac{9}{2}}{\frac{9}{2}} \right) + c_2 \\
&= \sin^{-1} \left( \frac{2x - 9}{9} \right) + c_2 \\
\therefore I &= \sqrt{9x - x^2} + \frac{9}{2} \sin^{-1} \left( \frac{2x - 9}{9} \right) + c, \text{ where } c = c_1 + c_2.
\end{aligned}$$

**Exercise 3.2 (C) | Q 1.8 | Page 128**

Evaluate the following integrals :  $\int \frac{3 \cos x}{4 \sin^2 x + 4 \sin x - 1} \cdot dx$

**SOLUTION**

$$\text{Let } I = \int \frac{3 \cos x}{4 \sin^2 x + 4 \sin x - 1} \cdot dx$$

Put  $\sin x = t$

$\therefore \cos x \, dx = dt$

$$\begin{aligned}
\therefore I &= 3 \int \frac{dt}{4t^2 + 4t - 1} \\
&= 3 \int \frac{dt}{(4t^2 + 4t + 1) - 2} \\
&= 3 \int \frac{dt}{(2t + 1)^2 - (\sqrt{2})^2} \\
&= \frac{3}{2(\sqrt{2})} \log \left| \frac{2t + 1 - \sqrt{2}}{2t + 1 + \sqrt{2}} \right| + c
\end{aligned}$$

$$\begin{aligned}
 &= \frac{3}{2(2\sqrt{2})} \log \left| \frac{2t+1-\sqrt{2}}{2t+1+\sqrt{2}} \right| + c \\
 &= \frac{3}{4\sqrt{2}} \log \left| \frac{2 \sin x + 1 - \sqrt{2}}{2 \sin x + 1 + \sqrt{2}} \right| + c.
 \end{aligned}$$

**Exercise 3.2 (C) | Q 1.9 | Page 128**

Evaluate the following integrals :  $\int \sqrt{\frac{e^{3x} - e^{2x}}{e^x + 1}} \cdot dx$

**SOLUTION**

$$\begin{aligned}
 \text{Let } I &= \int \sqrt{\frac{e^{3x} - e^{2x}}{e^x + 1}} \cdot dx \\
 &= \int \sqrt{\frac{e^{2x}(e^x - 1)}{e^x + 1}} \cdot dx \\
 &= \int e^x \sqrt{\frac{e^x - 1}{e^x + 1}} \cdot dx
 \end{aligned}$$

Put  $e^x = t$

$$\therefore e^x dx = dt$$

$$\begin{aligned}
 \therefore I &= \int \sqrt{\frac{t-1}{t+1}} dt \\
 &= \int \sqrt{\frac{t-1}{t+1} \times \frac{t-1}{t+1}} dt \\
 &= \int \sqrt{\frac{(t-1)^2}{t^2-1}} dt \\
 &= \int \frac{t-1}{\sqrt{t^2-1}} dt
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{1}{2} \int \frac{2t}{\sqrt{t^2 - 1}} dt - \int \frac{1}{\sqrt{t^2 - 1}} dt \\
 &= I_1 - I_2
 \end{aligned}$$

In  $I_1$ , put  $t^2 - 1 = \theta$

$$\therefore 2t dt = d\theta$$

$$\therefore I_1 = \frac{1}{2} \int \frac{d\theta}{\sqrt{\theta}}$$

$$= \frac{1}{2} \int \theta^{-\frac{1}{2}} d\theta$$

$$= \frac{1}{2} \frac{\theta^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} + c_1$$

$$= \sqrt{\theta} + c_1$$

$$= \sqrt{t^2 - 1} + c_1$$

$$= \sqrt{e^{2x} - 1} + c_1$$

$$I_2 = \int \frac{1}{\sqrt{t^2 - 1}} dt$$

$$= \log|t + \sqrt{t^2 - 1}| + c_2$$

$$= \log|e^x + \sqrt{e^{2x} - 1}| + c_2$$

$$\therefore I = \sqrt{e^{2x} - 1} - \log|e^x + \sqrt{e^{2x} - 1}| + c, \text{ where } c = c_1 + c_2.$$

### EXERCISE 3.3 [PAGES 137 - 138]

[Exercise 3.3 | Q 1.01 | Page 137](#)

Evaluate the following :  $\int x^2 \cdot \log x \cdot dx$

**SOLUTION**

$$\begin{aligned} \text{Let } I &= \int x^2 \cdot \log x \cdot dx \\ &= \int \log x \cdot x^2 \cdot dx \\ &= (\log x) \int x^2 \cdot dx - \int \left[ \left\{ \frac{d}{dx}(\log x) \int x^2 \cdot dx \right\} \right] \cdot dx \\ &= (\log x) \cdot \frac{x^3}{3} - \int \frac{1}{x} \cdot \frac{x^3}{3} \cdot dx \\ &= \frac{x^3}{3} \log x - \frac{1}{3} \int x^2 \cdot dx \\ &= \frac{x^3}{3} \log x - \frac{1}{3} \left( \frac{x^3}{3} \right) + c \\ &= \frac{x^3}{9} (3 \cdot \log x - 1) + c. \end{aligned}$$

**Exercise 3.3 | Q 1.02 | Page 137**

Evaluate the following :  $\int x^2 \sin 3x \, dx$

**SOLUTION**

$$\begin{aligned} \text{Let } I &= \int x^2 \sin 3x \, dx \\ &= x^2 \int \sin 3x \, dx - \int \left[ \left\{ \frac{d}{dx}(x^2) \int \sin 3x \, dx \right\} \right] \cdot dx \\ &= x^2 \left( \frac{-\cos 3x}{3} \right) - \int 2x \left( \frac{-\cos 3x}{3} \right) \cdot dx \\ &= \frac{x^2}{3} \cos 3x + \frac{2}{3} \int x \cos 3x \, dx \end{aligned}$$

$$\begin{aligned}
&= \frac{x^2}{3} \cos 3x + \frac{2}{3} \left[ x \int \cos 3x dx - \int \left\{ \frac{d}{dx}(x) \int \cos 3x dx \right\} dx \right] \\
&= \frac{x^2}{3} \cos 3x + \frac{2}{3} \left[ \frac{x \sin 3x}{3} - \int 1 \cdot \frac{\sin 3x}{3} dx \right] \\
&= -\frac{x^2}{3} \cos 3x + \frac{2}{9} x \sin 3x - \frac{2}{9} \int \sin 3x dx \\
&= -\frac{x^2}{3} \cos 3x + \frac{2}{9} x \sin 3x - \frac{2}{9} \int \left( \frac{-\cos 3x}{3} \right) + c \\
&= -\frac{x^2}{3} \cos 3x + \frac{2}{9} x \sin 3x + \frac{2}{27} \cos 3x + c.
\end{aligned}$$

**Exercise 3.3 | Q 1.03 | Page 137**

Evaluate the following :  $\int x \tan^{-1} x. dx$

**SOLUTION**

$$\begin{aligned}
\text{Let } I &= \int x \tan^{-1} x. dx \\
&= \int (\tan^{-1} x). dx \\
&= (\tan^{-1} x) \int x. dx - \int \left[ \left\{ \frac{d}{dx}(\tan^{-1} x) \int x. dx \right\} \right]. dx \\
&= (\tan^{-1} x) \left( \frac{x^2}{2} \right) - \int \left( \frac{1}{1+x^2} \right) \left( \frac{x^2}{2} \right). dx \\
&= \frac{x^2 \tan^{-1} x}{2} - \frac{1}{2} \int \frac{x^2}{x^2+1}. dx \\
&= \frac{x^2 \tan^{-1} x}{2} - \frac{1}{2} \frac{(x^2+1)-1}{x^2+1}. dx \\
&= \frac{x^2 \tan^{-1} x}{2} - \frac{1}{2} \left[ \int \left( 1 - \frac{1}{x^2+1} \right). dx \right]
\end{aligned}$$

$$= \frac{x^2}{2} \tan^{-1} x - \frac{1}{2} \left[ \int 1 \cdot dx - \int \frac{1}{x^2 + 1} \cdot dx \right]$$

$$= \frac{x^2}{2} \tan^{-1} x - \frac{1}{2} (x - \tan^{-1} x) + c.$$

**Exercise 3.3 | Q 1.04 | Page 137**

Evaluate the following :  $\int x^2 \tan^{-1} x \cdot dx$

**SOLUTION**

$$\text{Let } I = \int x^2 \tan^{-1} x \cdot dx$$

$$= \int (\tan^{-1} x) \cdot x^2 dx$$

$$= (\tan^{-1} x) \int x^2 \cdot dx - \int \left[ \left\{ \frac{d}{dx} (\tan^{-1} x) \int x^2 \cdot dx \right\} \right] \cdot dx$$

$$= (\tan^{-1} x) \left( \frac{x^3}{3} \right) - \int \left( \frac{1}{1+x^2} \right) \left( \frac{x^3}{3} \right) \cdot dx$$

$$= x \frac{3}{3} \tan^{-1} x - \frac{1}{3} \frac{x(x^2 + 1) - x}{x^2 + 1} \cdot dx$$

$$= \frac{x^3}{3} \tan^{-1} x - \frac{1}{3} \left[ \int \left\{ x - \frac{x}{x^2 + 1} \right\} \cdot dx \right]$$

$$= \frac{x^3}{3} \tan^{-1} x - \frac{1}{3} \left[ \int x \cdot dx - \frac{1}{2} \int \frac{2x}{x^2 + 1} \cdot dx \right]$$

$$= \frac{x^3}{3} \tan^{-1} x - \frac{1}{3} \left[ \frac{x^2}{2} - \frac{1}{2} \log|x^2 + 1| \right] + c$$

$$\therefore \left[ \because \frac{d}{dx} (x^2 + 1) = 2x \text{ and } \int \frac{f'(x)}{f(x)} dx = \log|f(x)| + c \right]$$

$$= \frac{x^3}{3} \tan^{-1} x - \frac{x^2}{6} + \frac{1}{6} \log|x^2 + 1| + c.$$

**Exercise 3.3 | Q 1.05 | Page 137**

Evaluate the following :  $\int x^3 \cdot \tan^{-1} x \cdot dx$

**SOLUTION**

$$\begin{aligned}
& \text{Let } I = \int x^3 \cdot \tan^{-1} x \cdot dx \\
& = \int (\tan^{-1} x) \cdot x^3 dx \\
& = (\tan^{-1} x) \int x^3 \cdot dx - \int \left[ \left\{ \frac{d}{dx} (\tan^{-1} x) \int x^3 \cdot dx \right\} \right] \cdot dx \\
& = (\tan^{-1} x) \left( \frac{x^4}{4} \right) - \int \left( \frac{1}{1+x^2} \right) \frac{x^4}{4} \cdot dx \\
& = \frac{x^4}{4} \tan^{-1} x - \frac{1}{4} \frac{(x^4 - 1) + 1}{x^2 + 1} \\
& = \frac{x^4}{4} \tan^{-1} x - \frac{1}{4} \int \frac{(x^2 - 1)(x^2 + 1) + 1}{x^2 + 1} \cdot dx \\
& = \frac{x^4}{4} \tan^{-1} x - \frac{1}{4} \int \left[ x^2 - 1 + \frac{1}{x^2 + 1} \right] \cdot dx \\
& = \frac{x^4}{4} \tan^{-1} x - \frac{1}{4} \int \left[ \int x^2 \cdot dx - \int 1 \cdot dx + \int \frac{1}{x^2 + 1} \cdot dx \right] \\
& = \frac{x^4}{4} \tan^{-1} x - \frac{1}{4} \left[ \frac{x^3}{3} - x + \tan^{-1} x \right] + c \\
& = \frac{x^4}{4} \tan^{-1} x - \tan^{-1} \frac{x}{4} - \frac{x^3}{12} - \frac{x}{4} + c \\
& = \frac{1}{4} (\tan^{-1} x) (x^4 - 1) - \frac{x}{12} (x^2 - 3) + c.
\end{aligned}$$

**Exercise 3.3 | Q 1.06 | Page 137**

Evaluate the following :  $\int (\log x)^2 \cdot dx$

**SOLUTION**

$$\text{Let } I = \int (\log x)^2 \cdot dx$$

$$\text{Put } \log x = t$$

$$\therefore x = e^t$$

$$\therefore dx = e^t dt$$

$$\therefore I = \int t^2 e^t dt$$

$$= t^2 \int e^t dt - \int \left[ \frac{d}{dx} (t^2) \int e^t - dt \right] dt$$

$$= t^2 e^t - \int 2te^t dt$$

$$= t^2 e^t - 2 \left[ t \int e^t dt - \int \left\{ \frac{d}{dt} (t) \int e^t dt \right\} dt \right]$$

$$= t^2 e^t - 2 \left[ te^t - \int 1 \cdot e^t dt \right]$$

$$= t^2 e^t - 2te^t + 2e^t + c$$

$$= e^t [t^2 - 2t + 2] + c$$

$$= x[(\log x)^2 - 2(\log x) + 2] + c.$$

Alternative Method :

$$\text{Let } I = \int (\log x)^2 \cdot dx$$

$$= \int (\log x)^2 \cdot 1 dx$$

$$= (\log x)^2 \int 1 \cdot dx - \int \left[ \frac{d}{dx} (\log x)^2 \cdot \int 1 \cdot dx \right] dx$$

$$\begin{aligned}
&= (\log x)^2 \cdot x - \int 2 \log x \cdot \frac{d}{dx}(\log x) \cdot x dx \\
&= x(\log x)^2 - \int 2 \log x \times \frac{1}{x} \times x \cdot dx \\
&= x(\log x)^2 - 2 \int (\log x) \cdot 1 dx \\
&= x(\log x)^2 - 2 \left[ (\log x) \int 1 \cdot dx - \int \left\{ \frac{d}{dx}(\log x) \int 1 \cdot dx \right\} \cdot dx \right] \\
&= x(\log x)^2 - 2 \left[ (\log x)x - \int \frac{1}{x} \times x \cdot dx \right] \\
&= x(\log x) - 2x(\log x) + 2 \int 1 \cdot dx \\
&= x(\log x)^2 - 2x(\log x) + 2x + c \\
&= x \left[ (\log x)^2 - 2(\log x) + 2 \right] + c.
\end{aligned}$$

**Exercise 3.3 | Q 1.07 | Page 137**

Evaluate the following :  $\int \sec^3 x \cdot dx$

**SOLUTION**

$$\begin{aligned}
\text{Let } I &= \int \sec^3 x \cdot dx \\
&= \int \sec x \sec^2 x \cdot dx \\
&= \sec x \int \sec^2 x \cdot dx - \int \left[ \frac{d}{dx}(\sec x) \int \sec^2 x \cdot dx \right] \cdot dx \\
&= \sec x \tan x - \int (\sec x \tan x)(\tan x) \cdot dx \\
&= \sec x \tan x - \int \sec x \tan^2 x \cdot dx
\end{aligned}$$

$$\begin{aligned}
 &= \sec x \tan x - \int \sec x (\sec^2 x - 1) \cdot dx \\
 &= \sec x \tan x - \int \sec^3 x \cdot dx + \int \sec x \cdot dx \\
 \therefore I &= \sec x \tan x - I + \log |\sec x + \tan x| \\
 \therefore 2I &= \sec x \tan x + \log |\sec x + \tan x| \\
 \therefore I &= \frac{1}{2} [\sec x \tan x + \log |\sec x + \tan x|] + c.
 \end{aligned}$$

**Exercise 3.3 | Q 1.08 | Page 137**

Evaluate the following :  $\int x \cdot \sin^2 x \cdot dx$

**SOLUTION**

$$\begin{aligned}
 &\int x \cdot \sin^2 x \cdot dx \\
 &= \int x \left( \frac{1 - \cos 2x}{2} \right) \cdot dx \\
 &= \frac{1}{2} \int (x - x \cos 2x) \cdot dx \\
 &= \frac{1}{2} \int x \cdot dx - \frac{1}{2} \int x \cos 2x \cdot dx \\
 &= \frac{1}{2} \cdot \frac{x^2}{2} - \frac{1}{2} \left[ x \int \cos 2x \cdot dx - \int \left\{ \frac{d}{dx}(x) \int \cos 2x \cdot dx \right\} \cdot dx \right] \\
 &= \frac{x^2}{4} - \frac{1}{2} \left[ x \cdot \frac{\sin 2x}{2} - \int 1 \cdot \frac{\sin 2x}{2} \cdot dx \right] \\
 &= \frac{x^2}{4} - \frac{1}{2} x \cdot \sin 2x + \frac{1}{4} \sin 2x \cdot dx
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{x^2}{4} - \frac{1}{4}x \cdot \sin 2x + \frac{1}{4} \cdot \frac{(-\cos 2x)}{2} + c \\
 &= \frac{x^2}{4} - \frac{1}{4}x \cdot \sin 2x - \frac{1}{8}\cos 2x + c
 \end{aligned}$$

**Exercise 3.3 | Q 1.09 | Page 137**

Evaluate the following :  $\int x^3 \cdot \log x \cdot dx$

**SOLUTION**

$$\begin{aligned}
 \text{Let } I &= \int x^3 \cdot \log x \cdot dx \\
 &= \int \log x \cdot x^3 \cdot dx \\
 &= (\log x) \int x^3 \cdot dx - \int \left[ \left\{ \frac{d}{dx}(\log x) \int x^3 \cdot dx \right\} \right] \cdot dx \\
 &= (\log x) \cdot \frac{x^4}{4} - \int \frac{1}{x} \cdot \frac{x^4}{4} \cdot dx \\
 &= \frac{x^4}{4} \log x - \frac{1}{4} \int x^3 \cdot dx \\
 &= \frac{x^4}{4} \log x - \frac{1}{4} \left( \frac{x^4}{4} \right) + c \\
 &= \frac{x^4}{4} \log x - \frac{x^4}{16} + c.
 \end{aligned}$$

**Exercise 3.3 | Q 1.1 | Page 137**

Evaluate the following :  $\int e^{2x} \cdot \cos 3x \cdot dx$

**SOLUTION**

$$\begin{aligned} \text{Let } I &= \int e^{2x} \cdot \cos 3x \cdot dx \\ &= e^{2x} \int \cos 3x \cdot dx - \int \left[ \frac{d}{dx}(e^{2x}) \int \cos 3x \cdot dx \right] \cdot dx \\ &= e^{2x} \cdot \frac{\sin 3x}{3} - \int e^{2x} \times 2 \times \frac{\sin 3x}{3} \cdot dx \\ &= \frac{1}{3} e^{2x} \sin 3x - \frac{2}{3} \int e^{2x} \sin 3x \cdot dx \\ &= \frac{1}{3} e^{2x} \sin 3x - \frac{2}{3} \left[ e^{2x} \int \sin 3x \cdot dx \right] \\ &= \frac{1}{3} e^{2x} \sin 3x - \frac{2}{3} \left[ e^{2x} \cdot \left( \frac{-\cos 3x}{3} \right) - \int e^{2x} \times 2 \times \left( \frac{-\cos 3x}{3} \right) \cdot dx \right] \\ &= \frac{1}{3} e^{2x} \sin 3x + \frac{2}{9} e^{2x} \cos 3x - \frac{4}{9} \int e^{2x} \cos 3x \cdot dx \\ \therefore I &= \frac{1}{3} e^{2x} \sin 3x + \frac{2}{9} e^{2x} \cos 3x - \frac{4}{9} I \\ \therefore \left( 1 + \frac{4}{9} \right) I &= \frac{1}{3} e^{2x} \sin 3x + \frac{2}{9} e^{2x} \cos 3x \\ \therefore \frac{13}{9} I &= \frac{e^{2x}}{9} (3 \sin 3x + 2 \cos 3x) \\ \therefore I &= \frac{e^{2x}}{13} (2 \cos 3x + 3 \sin 3x) + c. \end{aligned}$$

Exercise 3.3 | Q 1.11 | Page 137

Evaluate the following :  $\int x \cdot \sin^2 x \cdot dx$

**SOLUTION**

$$\begin{aligned} \text{Let } I &= \int x \cdot \sin^2 x \cdot dx \\ &= \int (\sin^{-1} x) \cdot x dx \\ &= (\sin^{-1} x) \int x \cdot dx - \int \left[ \left\{ \frac{d}{dx} (\sin^{-1} x) \int x \cdot dx \right\} \right] \cdot dx \\ &= (\sin^{-1} x) \left( \frac{x^2}{2} \right) - \int \left( \frac{1}{\sqrt{1-x^2}} \right) \left( \frac{x^2}{2} \right) \cdot dx \\ &= \frac{x^2}{2} \sin^{-1} x + \frac{1}{2} \int \frac{-x^2}{\sqrt{1-x^2}} \cdot dx \\ &= \frac{x^2}{2} \sin^{-1} x + \frac{1}{2} \int \frac{(1-x^2)-1}{\sqrt{1-x^2}} \cdot dx \\ &= \frac{x^2}{2} \sin^{-1} x + \frac{1}{2} \int \left[ \sqrt{1-x^2} - \frac{1}{\sqrt{1-x^2}} \right] \cdot dx \\ &= \frac{x^2}{2} \sin^{-1} x + \frac{1}{2} \int \sqrt{1-x^2} \cdot dx - \frac{1}{2} \int \frac{1}{\sqrt{1-x^2}} \cdot dx \\ &= \frac{x^2}{2} \sin^{-1} x + \frac{1}{2} \left[ \frac{x}{2} \sqrt{1-x^2} + \frac{1}{2} \sin^{-1} x \right] - \frac{1}{2} \sin^{-1} x + c \\ &= \frac{x^2}{2} \sin^{-1} x + \frac{1}{4} x \sqrt{1-x^2} - \frac{1}{4} \sin^{-1} x + c. \end{aligned}$$

Exercise 3.3 | Q 1.12 | Page 137

Evaluate the following :  $\int x^2 \cdot \cos^{-1} x \cdot dx$

**SOLUTION**

$$\begin{aligned}
 \text{Let } I &= \int x^2 \cdot \cos^{-1} x \cdot dx \\
 &= \int (\cos^{-1} x) \cdot x^2 dx \\
 &= (\cos^{-1} x) \int x^2 \cdot dx - \int \frac{d}{dx}(\cos^{-1} x) \int x^2 \cdot dx \Big] \cdot dx \\
 &= (\cos^{-1} x) \left( \frac{x^3}{3} \right) - \int \left( \frac{-1}{\sqrt{1-x^2}} \right) \left( \frac{x^3}{3} \right) \cdot dx \\
 &= \frac{x^3}{3} \cos^{-1} x + \frac{1}{3} \int \frac{x^2 \cdot x}{\sqrt{1-x^2}} \cdot dx \\
 &\ln \int \frac{x^3}{\sqrt{1-x^2}} \cdot dx, \text{ put } 1-x^2 = t
 \end{aligned}$$

$$\therefore -2x \cdot dx = dt$$

$$\therefore x \cdot dx = -\frac{1}{2} dt$$

$$\text{Also, } x^2 = 1-t$$

$$\begin{aligned}
 \therefore I &= \frac{x^3}{3} \cos^{-1} x + \frac{1}{3} \int \frac{(1-t)}{\sqrt{t}} \left( -\frac{1}{2} \right) \cdot dt \\
 &= \frac{x^3}{3} \cos^{-1} x - \frac{1}{6} \int \left( \frac{1}{\sqrt{t}} - \sqrt{t} \right) \cdot dt \\
 &= \frac{x^3}{3} \cos^{-1} x - \frac{1}{6} \int t^{-\frac{1}{2}} dt + \frac{1}{6} \int t^{\frac{1}{2}} \cdot dt \\
 &= \frac{x^3}{3} \cos^{-1} x - \frac{1}{6} \left( \frac{t^{\frac{1}{2}}}{\frac{1}{2}} \right) + \frac{1}{6} \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + c \\
 &= \frac{x^3}{3} \cos^{-1} x - \frac{1}{3} \sqrt{1-x^2} + \frac{1}{9} (1-x^2)^{\frac{3}{2}} + c.
 \end{aligned}$$

**Exercise 3.3 | Q 1.13 | Page 137**

Evaluate the following :  $\int \frac{\log(\log x)}{x} \cdot dx$

**SOLUTION**

$$\text{Let } I = \int \frac{\log(\log x)}{x} \cdot dx$$

$$= \int \log(\log x) \cdot \frac{1}{x} dx$$

Put  $\log x = t$

$$\therefore \frac{1}{x} \cdot dx = dt$$

$$\therefore I = \int \log t dt$$

$$= \int (\log t) \cdot 1 dt$$

$$= (\log t) \int 1 dt - \int \left[ \frac{d}{dt} (\log t) \int 1 dt \right] dt$$

$$= (\log t)t - \int \frac{1}{t} + t dt$$

$$= t \log t - \int 1 dt$$

$$= t \log t - t + c$$

$$= t(\log t - 1) + c$$

$$= (\log x)[\log(\log x) - 1] + c.$$

**Exercise 3.3 | Q 1.14 | Page 137**

Evaluate the following :  $\int \frac{t \cdot \sin^{-1} t}{\sqrt{1-t^2}} \cdot dt$

**SOLUTION**

$$\begin{aligned} \text{Let } I &= \int \frac{t \cdot \sin^{-1} t}{\sqrt{1-t^2}} \cdot dt \\ &= \int t \cdot \sin^{-1} t \cdot \frac{1}{\sqrt{1-t^2}} \cdot dt \end{aligned}$$

Put  $\sin^{-1} t = \theta$

$$\therefore \frac{1}{\sqrt{1-t^2}} \cdot dt = d\theta$$

and

$$t = \sin \theta$$

$$\begin{aligned} \therefore I &= \int (\sin \theta) \cdot \theta d\theta \\ &= \int \theta \sin \theta d\theta \\ &= \theta \int \sin \theta d\theta - \int \left[ \frac{d}{d\theta}(\theta) \int \sin \theta d\theta \right] d\theta \\ &= \theta(-\cos \theta) - \int 1 \cdot (-\cos \theta) d\theta \\ &= -\theta \cos \theta + \int \cos \theta d\theta \\ &= -\theta \cos \theta + \sin \theta + c \\ &= -\theta \cdot \sqrt{1 - \sin^2 \theta} + \sin \theta + c \\ &= -\sin^{-1} t \cdot \sqrt{1 - t^2} + t + c \\ &= -\sqrt{1 - t^2} \cdot \sin^{-1} t + t + c. \end{aligned}$$

**Exercise 3.3 | Q 1.15 | Page 137**

Evaluate the following :  $\int \cos \sqrt{x} \cdot dx$

**SOLUTION**

$$\text{Let } I = \int \cos \sqrt{x} \, dx$$

$$\text{Put } \sqrt{x} = t$$

$$\therefore x = t^2$$

$$\therefore dx = 2t \, dt$$

$$\therefore I = \int (\cos t) 2t \, dt$$

$$= \int 2t \cos t \, dt$$

$$= 2t \int \cos t \, dt - \int \left[ \frac{d}{dt}(2t) \int \cos t \, dt \right] \, dt$$

$$= 2t \sin t - \int 2 \sin t \, dt$$

$$= 2t \sin t + 2 \cos t + c$$

$$= 2[\sqrt{x} \sin \sqrt{x} + \cos \sqrt{x}] + c.$$

Exercise 3.3 | Q 1.16 | Page 137

$$\text{Evaluate the following : } \int \sin \theta \cdot \log(\cos \theta) \, d\theta$$

**SOLUTION**

$$\text{Let } I = \int \sin \theta \cdot \log(\cos \theta) \, d\theta$$

$$= \int \log(\cos \theta) \cdot \sin \theta \, d\theta$$

$$\text{Put } \cos \theta = t$$

$$\therefore -\sin \theta \, d\theta = dt$$

$$\therefore \sin \theta \, d\theta = -dt$$

$$\begin{aligned}
\therefore I &= \int \log t \cdot (-dt) \\
&= - \int (\log t) \cdot 1 dt \\
&= - \left[ (\log t) \int 1 dt - \int \left\{ \frac{d}{dt}(\log t) \int 1 dt \right\} dt \right] \\
&= - \left[ (\log t)t - \int \frac{1}{t} \cdot t dt \right] \\
&= -t \log t + \int 1 dt \\
&= -t \log t + t + c \\
&= -\cos \theta \cdot \log (\cos \theta) + \cos \theta + c \\
&= -\cos \theta [\log (\cos \theta) - 1] + c.
\end{aligned}$$

**Exercise 3.3 | Q 1.17 | Page 137**

Evaluate the following :  $\int x \cdot \cos^3 x \cdot dx$

**SOLUTION**

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$$\begin{aligned}
\cos 3x &= 4 \cos^3 x - 3 \cos x \\
\therefore \cos 3x + 3 \cos x &= 4 \cos^3 x \\
\therefore \int \cos^3 x \cdot dx &= \frac{1}{4} \cos 3x + \frac{3}{4} \cos x \\
\therefore \int \cos^3 x \cdot dx &= \frac{1}{4} \int \cos 3x \cdot dx + \frac{3}{4} \int \cos x \cdot dx \\
&= \frac{1}{4} \left( \frac{\sin 3x}{3} \right) + \frac{3}{4} \sin x \\
&= \frac{\sin 3x}{12} + \frac{3 \sin x}{4} \quad \dots(1)
\end{aligned}$$

Let  $I = \int x \cos^3 x \cdot dx$

$$\begin{aligned}
\text{Let } I &= \int x \cos^3 x \cdot dx \\
&= x \int \cos^3 x \cdot dx - \int \left[ \left\{ \frac{d}{dx}(x) \int \cos^3 x \cdot dx \right\} \right] \cdot dx \\
&= x \left[ \frac{\sin 3x}{12} + \frac{3 \sin x}{4} \right] - \int 1 \cdot \left( \frac{\sin 3x}{12} + \frac{3 \sin x}{4} \right) \cdot dx \quad \dots[\text{By (1)}] \\
&= \frac{x \sin 3x}{12} + \frac{3x \sin x}{4} - \frac{1}{12} \int \sin 3x \cdot dx - \frac{3}{4} \int \sin x \cdot dx \\
&= \frac{x \sin 3x}{12} + \frac{3x \sin x}{4} - \frac{1}{12} \left( \frac{-\cos 3x}{3} \right) - \frac{3}{4}(-\cos x) + c
\end{aligned}$$

Exercise 3.3 | Q 1.18 | Page 137

Evaluate the following :  $\int \frac{\sin(\log x)^2}{x} \cdot \log x \cdot dx$

**SOLUTION**

$$\text{Let } I = \int \frac{\sin(\log x)^2}{x} \cdot \log x \cdot dx$$

$$\text{Put } (\log x)^2 = t$$

$$\therefore 2 \log x \cdot \frac{1}{x} \cdot dx = dt$$

$$\therefore \frac{1}{x} \log x \cdot dx = \frac{1}{2} dt$$

$$\therefore I = \frac{1}{2} \int \sin t \cdot dt$$

$$= -\frac{1}{2} \cos t + c$$

$$= -\frac{1}{2} \cos [(\log x)^2] + c.$$

**Exercise 3.3 | Q 1.19 | Page 137**

Evaluate the following :  $\int \frac{\log x}{x} \cdot dx$

**SOLUTION**

$$\text{Let } I = \int \frac{\log x}{x} \cdot dx$$

$$\text{Put } \log x = t \quad \therefore \frac{1}{x} \cdot dx = dt$$

$$\therefore I = \int t \cdot dt$$

$$= \frac{1}{2}t^2 + c$$

$$= \frac{1}{2}(\log x)^2 + c$$

**Exercise 3.3 | Q 1.2 | Page 137**

Evaluate the following :  $\int x \cdot \sin 2x \cdot \cos 5x \cdot dx$

**SOLUTION**

$$\text{Let } I = \int x \cdot \sin 2x \cdot \cos 5x \cdot dx$$

$$\sin 2x \cos 5x = \frac{1}{2}[2 \sin 2x \cos 5x]$$

$$= \frac{1}{2}[\sin(2x + 5x) + \sin(2x - 5x)]$$

$$= \frac{1}{2}[\sin 7x - \sin 3x]$$

$$\therefore \int \sin 2x \cos 5x \cdot dx = \frac{1}{2} \left[ \int \sin 7x \cdot dx - \int \sin 3x \cdot dx \right]$$

$$= \frac{1}{2} \left( \frac{-\cos 7x}{7} \right) - \frac{1}{2} \left( \frac{-\cos 3x}{3} \right)$$

$$= -\frac{1}{14} \cos 7x + \frac{1}{6} \cos 3x \quad \dots(1)$$

$$\begin{aligned} I &= \int x \sin 2x \cos 5x. dx \\ &= x \int \sin 2x \cos 5x. dx - \int \left[ \frac{d}{dx}(x) \int \sin 2x \cos 5x. dx \right]. dx \\ &= x \left[ -\frac{1}{14} \cos 7x + \frac{1}{6} \cos 3x \right] - \int 1. \left( -\frac{1}{14} \cos 7x + \frac{1}{6} \cos 3x \right). dx \quad \dots[\text{By (1)}] \\ &= -\frac{x}{14} \cos 7x + \frac{x}{6} \cos 3x + \frac{1}{14} \int \cos 7x. dx - \frac{1}{6} \int \cos 3x. dx \\ &= -\frac{x}{14} \cos 7x + \frac{x}{6} \cos 3x + \frac{1}{14} \left( \frac{\sin 7x}{7} \right) - \frac{1}{6} \left( \frac{\sin 3x}{3} \right) + c \\ &= -\frac{x}{14} \cos 7x + \frac{x}{6} \cos 3x + \frac{\sin 7x}{98} - \frac{\sin 3x}{18} + c. \end{aligned}$$

**Exercise 3.3 | Q 1.21 | Page 137**

Evaluate the following :  $\int \cos(\sqrt[3]{x}). dx$

**SOLUTION**

$$\text{Let } I = \int \cos(\sqrt[3]{x}). dx$$

$$\text{Put } \sqrt[3]{x} = t$$

$$\therefore x = t^3$$

$$\therefore dx = 3t^2 dt$$

$$\therefore I = \int 3t^2 \cos t. dt$$

$$= 3t^2 \int \cos t. dt - \int \left[ \frac{d}{dt}(3t)^2 \int \cos t. dt \right]. dt$$

$$= 3t^2 \sin t - \int 6t \sin t. dt$$

$$\begin{aligned}
&= 3t^2 \sin t - \left[ 6t \sin t \cdot dt - \int \left\{ \frac{d}{dt}(6t) \int \sin t \cdot dt \right\} \cdot dt \right] \\
&= 3t^2 \sin t - \left[ 6t(-\cos t) - \int 6(-\cos t) \cdot dt \right] \\
&= 3t^2 \sin t + 6t \cos t - 6 \sin t + c \\
&= 3(t^2 - 2) \sin t + 6t \cos t + c \\
&= 3\left(x^{\frac{2}{3}} - 2\right) \sin(\sqrt[3]{x}) + 6\sqrt[3]{x} \cos(\sqrt[3]{x}) + c.
\end{aligned}$$

**Exercise 3.3 | Q 2.01 | Page 138**

Integrate the following functions w.r.t. x :  $e^{2x} \cdot \sin 3x$

**SOLUTION**

$$\begin{aligned}
\text{Let } I &= \int e^{2x} \cdot \sin 3x \cdot dx \\
&= e^{2x} \cdot \sin 3x - \int \left[ \frac{d}{dx}(e^{2x}) \int \sin 3x \cdot dx \right] \cdot dx \\
&= e^{2x} \cdot \frac{\sin 3x}{3} - \int e^{2x} \times 2 \times \frac{\sin 3x}{3} \cdot dx \\
&= \frac{1}{3} e^{2x} \sin 3x - \frac{2}{3} \int e^{2x} \sin 3x \cdot dx \\
&= \frac{1}{3} e^{2x} \sin 3x - \frac{2}{3} \left[ e^{2x} \int \sin 3x \cdot dx \right] \\
&= \frac{1}{3} e^{2x} \sin 3x - \frac{2}{3} \left[ e^{2x} \cdot \left( \frac{-\cos 3x}{3} \right) - \int e^{2x} \times 2 \times \left( \frac{-\sin 3x}{3} \right) \cdot dx \right] \\
&= \frac{1}{3} e^{2x} \sin 3x - \frac{2}{13} e^{2x} \cos 3x - \frac{4}{13} \int e^{2x} \cos 3x \cdot dx \\
\therefore I &= \frac{1}{3} e^{2x} \sin 3x - \frac{2}{13} e^{2x} \cos 3x - \frac{4}{13} I
\end{aligned}$$

$$\begin{aligned}\therefore \left(1 + \frac{4}{13}\right)I &= \frac{1}{3}e^{2x} \sin 3x - \frac{2}{13}e^{2x} \cos 3x \\ \therefore \frac{13}{13}I &= \frac{e^{2x}}{13}(2 \sin 3x - 3 \cos 3x) \\ \therefore I &= \frac{e^{2x}}{13}(2 \sin 3x - 3 \cos 3x) + c.\end{aligned}$$

**Exercise 3.3 | Q 2.02 | Page 138**

Integrate the following functions w.r.t.  $x$  :  $e^{-x} \cos 2x$

**SOLUTION**

$$\begin{aligned}\text{Let } I &= \int e^{-x} \cos 2x \, dx \\ &= e^{-x} \int \cos 2x \, dx - \int \left[ \frac{d}{dx}(e^{2x}) \int \sin 2x \, dx \right] \, dx \\ &= e^{-x} \cdot \frac{\cos 2x}{3} - \int e^{-x} \times 2 \times \frac{\sin 2x}{3} \, dx \\ &= \frac{1}{3}e^{-x} \cos 2x - \frac{2}{3} \int e^{-x} \sin 2x \, dx \\ &= \frac{1}{3}e^{-x} \cos 2x - \frac{2}{3} \left[ e^{-x} \int \sin 2x \, dx \right] \\ &= \frac{1}{3}e^{-x} \cos 2x - \frac{2}{3} \left[ e^{-x} \cdot \left( \frac{-\cos 2x}{3} \right) - \int e^{-x} \times 2 \times \left( \frac{-\cos 2x}{3} \right) \, dx \right] \\ &= \frac{1}{3}e^{-x} \cos 2x + \frac{2}{5}e^{-x} \cos 2x - \frac{2}{5} \int e^{-x} \sin 2x \, dx \\ \therefore I &= \frac{1}{3}e^{-x} \cos 2x + \frac{2}{5}e^{-x} \cos 2x - \frac{3}{5}I \\ \therefore \left(1 + \frac{4}{5}\right)I &= \frac{1}{3}e^{-x} \cos 2x + \frac{2}{5}e^{-x} \sin 2x \\ \therefore \frac{e^{-x}}{5}I &= \frac{e^{-x}}{5}(2 \cos 2x + 2 \sin 2x)\end{aligned}$$

$$\therefore I = \frac{e^{-x}}{5} (2 \cos 2x + 2 \sin 2x) + c.$$

Exercise 3.3 | Q 2.03 | Page 138

Integrate the following functions w.r.t. x :  $\sin(\log x)$

**SOLUTION**

$$\text{Let } I = \int \sin(\log x)x. dx$$

Put  $\log x = t$

$$\therefore x = e^t$$

$$\therefore dx = e^t dt$$

$$\therefore I = \int \sin t \times e^t \cdot t$$

$$= \int e^t \sin t. dt$$

$$= e^t \int \sin t. dt - \int \left[ \frac{d}{dt}(e^t) \int \sin t. dt \right]. dt$$

$$= e^t(-\cos t) - \int e^t(-\cot). dt$$

$$= -e^t \cos t + \int e^t \cos t. dt$$

$$= -e^t \cos t + e^t \int \cos. dt - \int \left[ \frac{d}{dt}(e^t) \int \cos. dt \right]. dt$$

$$= -e^t \cos t + e^t \sin t - \int e^t \sin t. dt$$

$$\therefore I = -e^t \cos t + e^t \sin t - I$$

$$\therefore 2I = e^t (\sin t - \cos t)$$

$$\therefore I = \frac{e^t}{2} (\sin t - \cos t) + c$$

$$= \frac{x}{2} [\sin(\log x) - \cos(\log x)] + c.$$

Exercise 3.3 | Q 2.04 | Page 138

Integrate the following functions w.r.t. x :  $\sqrt{5x^2 + 3}$

**SOLUTION**

$$\text{Let } I = \int \sqrt{5x^2 + 3} dx$$

$$= \sqrt{5} \int \sqrt{x^2 + \frac{3}{5}} dx$$

$$= \sqrt{5} \left[ \frac{x}{2} \sqrt{x^2 + \frac{3}{5}} + \frac{\left(\frac{3}{5}\right)}{2} \log \left| x + \sqrt{x^2 + \frac{3}{5}} \right| \right] + c$$

$$= \frac{\sqrt{5}}{2} \left[ x \sqrt{x^2 + \frac{3}{5}} + \frac{3}{5} \log \left| x + \sqrt{x^2 + \frac{3}{5}} \right| \right] + c.$$

Exercise 3.3 | Q 2.05 | Page 138

Integrate the following functions w.r.t. x :  $x^2 \cdot \sqrt{a^2 - x^6}$

**SOLUTION**

$$\text{Let } I = \int x^2 \cdot \sqrt{a^2 - x^6} \cdot dx$$

$$\text{Put } x^3 = t$$

$$\therefore 3x^2 \cdot dx = dt$$

$$\therefore x^2 \cdot dx = \frac{1}{3} \cdot dt$$

$$\therefore I = \int \sqrt{a^2 - t^2} \cdot \frac{dt}{3} = \frac{1}{3} \int \sqrt{a^2 - t^2} \cdot dt$$

$$= \frac{1}{3} \left[ \frac{t}{2} \sqrt{a^2 - t^2} + \frac{a^2}{2} \sin^{-1} \left( \frac{t}{a} \right) \right] + c$$

$$= \frac{1}{6} \left[ x^3 \sqrt{a^2 - x^6} + a^2 \sin^{-1} \left( \frac{x^3}{a} \right) \right] + c.$$

**Exercise 3.3 | Q 2.06 | Page 138**

Integrate the following functions w.r.t. x :  $\sqrt{(x-3)(7-x)}$

**SOLUTION**

$$\text{Let } I = \int \sqrt{(x-3)(7-x)} \cdot dx$$

$$= \int \sqrt{-x^2 + 10x - 21} \cdot dx$$

$$= \int \sqrt{-(x^2 - 10x + 21)} \cdot dx$$

$$= \int \sqrt{4 - (x^2 - 10x + 25)} \cdot dx$$

$$= \int \sqrt{2^2 - (x-5)^2}$$

$$\begin{aligned}
 &= \left( \frac{x-5}{2} \right) \sqrt{2^2 - (x-5)^2} + \frac{2^2}{2} \sin^{-1} \left( \frac{x-5}{2} \right) + c \\
 &= \left( \frac{x-5}{2} \right) \sqrt{(x-3)(7-x)} + 2 \sin^{-1} \left( \frac{x-5}{2} \right) + c.
 \end{aligned}$$

**Exercise 3.3 | Q 2.07 | Page 138**

Integrate the following functions w.r.t. x :  $\sqrt{4^x(4^x + 4)}$

**SOLUTION**

$$\begin{aligned}
 \text{Let } I &= \int \sqrt{4^x(4^x + 4)} \cdot dx \\
 &= \int 2^x \sqrt{(2^x)^2 + 2^2} \cdot dx
 \end{aligned}$$

Put  $2^x = t$

$$\therefore 2^x \log 2 \, dx = dt$$

$$\therefore 2^x \, dx = \frac{1}{\log 2} \cdot dt$$

$$\begin{aligned}
 \therefore I &= \int \sqrt{t^2 + 2^2} \cdot \frac{dt}{\log 2} \\
 &= \frac{1}{\log 2} \int \sqrt{t^2 + 2^2} \cdot dt \\
 &= \frac{1}{\log 2} \left[ \frac{t}{2} \sqrt{t^2 + 2^2} + \frac{2^2}{2} \log \left| t + \sqrt{t^2 + 2^2} \right| \right] + c \\
 &= \frac{1}{\log 2} \left[ \frac{2^x}{2} \sqrt{4^x + 4} + 2 \log \left| 2^x + \sqrt{4^x + 4} \right| \right] + c
 \end{aligned}$$

**Exercise 3.3 | Q 2.08 | Page 138**

Integrate the following functions w.r.t. x :  $(x+1)\sqrt{2x^2 + 3}$

**SOLUTION**

$$\text{Let } I = \int (x+1) \sqrt{2x^2 + 3} \, dx$$

$$\text{Let } x+1 = A \left[ \frac{d}{dx} (2x^2 + 3) \right] + B$$

$$= A(4x) + B$$

$$= 4Ax + B$$

Comparing the coefficients of and constant on both sides, we get

$$4A = 1, B = 1$$

$$\therefore A = \frac{1}{4}, B = 1$$

$$\therefore x+1 = \frac{1}{4}(4x)+1$$

$$\therefore I = \int \left[ \frac{1}{4}(4x) + 1 \right] \sqrt{2x^2 + 3} \, dx$$

$$= \frac{1}{4} \int 4x \sqrt{2x^2 + 3} \, dx + \int \sqrt{2x^2 + 3} \, dx.$$

$$= I_1 + I_2$$

$$\ln I_1 = \text{put } 2x^2 + 3 = t$$

$$\therefore 4x \, dx = dt$$

$$\therefore I_1 = \frac{1}{4} \int t^{1/2} \, dt$$

$$= \frac{1}{4} \left( \frac{t^{3/2}}{\frac{3}{2}} \right) + c_1$$

$$= \frac{1}{6} (2x^2 + 3)^{3/2} + c_1$$

$$I_2 = \int \sqrt{2x^2 + 3} \, dx$$

$$\begin{aligned}
I_2 &= \int \sqrt{2x^2 + 3} \cdot dx \\
&= \sqrt{2} \int \sqrt{x^2 + \frac{3}{2}} \cdot dx \\
&= \sqrt{2} \left[ \frac{x}{2} \sqrt{x^2 + \frac{3}{2}} + \frac{\left(\frac{3}{2}\right)}{2} \log \left| x + \sqrt{x^2 + \frac{3}{2}} \right| \right] + c_2 \\
&= \sqrt{2} \left[ \frac{x}{2} \sqrt{x^2 + \frac{3}{2}} + \frac{3}{4} \log \left| x + \sqrt{x^2 + \frac{3}{2}} \right| \right] + c_2 \\
\therefore I &= \frac{1}{6} (2x^2 + 3)^{\frac{3}{2}} + \sqrt{2} \left[ \frac{x}{2} \sqrt{x^2 + \frac{3}{2}} + \frac{3}{4} \log \left| x + \sqrt{x^2 + \frac{3}{2}} \right| \right] + c, \text{ where } c = c_1 + c_2.
\end{aligned}$$

**Exercise 3.3 | Q 2.09 | Page 138**

Integrate the following functions w.r.t.  $x$  :  $x \sqrt{5 - 4x - x^2}$

**SOLUTION**

$$\text{Let } I = \int x \sqrt{5 - 4x - x^2} \cdot dx$$

$$\text{Let } x = A \left[ \frac{d}{dx} (5 - 4x - x^2) \right] + B$$

$$= A [-4 - 2x] + B$$

$$= -2Ax + (B - 4A)$$

Comparing the coefficients of  $x$  and the constant term on both the sides, we get

$$-2A = 1, B - 4A = 0$$

$$\therefore A = -\frac{1}{2}, B = 4A = 4 \left( -\frac{1}{2} \right) = -2$$

$$\therefore x = -\frac{1}{2}(-4 - 2x) - 2$$

$$\begin{aligned}
I &= \int \left[ -\frac{1}{2}(-4 - 2x) - 2 \right] \sqrt{5 - 4x - x^2} \, dx \\
&= -\frac{1}{2} \int (-4 - 2x) \sqrt{5 - 4x - x^2} \, dx - 2 \int \sqrt{5 - 4x - x^2} \, dx \\
&= I_1 - I_2
\end{aligned}$$

In  $I_1$ , put  $5 - 4x - x^2 = t$

$$\therefore (-4 - 2x)dx = dt$$

$$\begin{aligned}
I_1 &= \frac{1}{2} \int t^{\frac{1}{2}} \, dt \\
&= -\frac{1}{2} \left( \frac{t^{\frac{3}{2}}}{\frac{3}{2}} \right) + c_1 \\
&= -\frac{1}{3} (5 - 4x - x^2)^{\frac{3}{2}} + c_1
\end{aligned}$$

$$\begin{aligned}
I_2 &= 2 \int \sqrt{5 - 4x - x^2} \, dx \\
&= 2 \int \sqrt{5 - (x^2 + 4x)} \, dx \\
&= 2 \int \sqrt{9 - (x^2 + 4x + 4)} \, dx \\
&= 2 \int \sqrt{3^2 - (x + 2)^2} \, dx \\
&= 2 \left[ \left( \frac{x+2}{2} \right) \sqrt{3^2 - (x+2)^2} + \frac{3^2}{2} \sin^{-1} \left( \frac{x+2}{3} \right) \right] + c_2 \\
&= (x+2) \sqrt{5 - 4x - x^2} + 9 \sin^{-1} \left( \frac{x+2}{3} \right) + c_2
\end{aligned}$$

$$\therefore I = -\frac{1}{3} (5 - 4x - x^2)^{\frac{3}{2}} - (x+2) \sqrt{5 - 4x - x^2} - 9 \sin^{-1} \left( \frac{x+2}{3} \right) + c, \text{ where } c = c_1 + c_2.$$

**Exercise 3.3 | Q 2.1 | Page 138**

Integrate the following functions w.r.t.  $x$  :  $\sec^2 x \cdot \sqrt{\tan^2 x + \tan x - 7}$

**SOLUTION**

$$\text{Let } I = \int \sec^2 x \cdot \sqrt{\tan^2 x + \tan x - 7}$$

Put  $\tan x = t$

$$\therefore \sec^2 x \cdot dx = dt$$

$$\therefore I = \int \sqrt{t^2 + t - 7} \cdot dt$$

$$= \int \sqrt{t^2 + t + \frac{1}{4} - \frac{29}{4}} \cdot dt$$

$$= \int \sqrt{\left(t + \frac{1}{2}\right)^2 - \left(\frac{\sqrt{29}}{2}\right)^2} \cdot dt$$

$$= \left(\frac{t + \frac{1}{2}}{2}\right) \sqrt{\left(t + \frac{1}{2}\right)^2 - \frac{29}{4}} - \frac{\left(\frac{29}{4}\right)}{2} \log \left| \left(t + \frac{1}{2}\right) + \sqrt{\left(t + \frac{1}{2}\right)^2 - \frac{29}{4}} \right| + c$$

$$= \frac{(2t+1)}{4} \sqrt{t^2 + t - 7} - \frac{29}{8} \log \left| \left(t + \frac{1}{2}\right) + \sqrt{t^2 + t - 7} \right| + c$$

$$= \left(\frac{2 \tan x + 1}{4}\right) \sqrt{\tan^2 x + \tan x - 7} - \frac{29}{8} \log \left| \left(\tan x + \frac{1}{2}\right) + \sqrt{\tan^2 x + \tan x - 7} \right| + c.$$

**Exercise 3.3 | Q 2.11 | Page 138**

Integrate the following functions w.r.t.  $x$  :  $\sqrt{x^2 + 2x + 5}$

**SOLUTION**

$$\begin{aligned}
\text{Let } I &= \int \sqrt{x^2 + 2x + 5} \, dx \\
&= \int \sqrt{x^2 + 2x + 1 + 4} \, dx \\
&= \int \sqrt{(x+1)^2 + 2^2} \, dx \\
&= \left( \frac{x+1}{2} \right) \int \sqrt{(x+1)^2 + 2^2} + \frac{2^2}{2} \log \left| (x+1) + \sqrt{(x+1)^2 + 2^2} \right| + C \\
&= \left( \frac{x+1}{2} \right) \sqrt{x^2 + 2x + 5} + 2 \log \left| (x+1) + \sqrt{x^2 + 2x + 5} \right| + c.
\end{aligned}$$

**Exercise 3.3 | Q 2.12 | Page 138**

Integrate the following functions w.r.t.  $x$  :  $\sqrt{2x^2 + 3x + 4}$

**SOLUTION**

$$\begin{aligned}
\text{Let } I &= \int \sqrt{2x^2 + 3x + 4} \, dx \\
&= \sqrt{2} \int \sqrt{x^2 + \frac{3}{2}x + 2} \, dx \\
&= \sqrt{2} \int \sqrt{\left(x^2 + \frac{3}{2}x + \frac{9}{16}\right) - \frac{9}{16} + 2} \, dx \\
&= \sqrt{2} \int \sqrt{\left(x + \frac{3}{4}\right)^2 + \left(\frac{\sqrt{23}}{4}\right)^2} \, dx \\
&= \sqrt{2} \left[ \frac{\left(x + \frac{3}{4}\right)}{2} \sqrt{\left(x + \frac{3}{4}\right)^2 + \left(\frac{\sqrt{23}}{4}\right)^2} + \frac{\left(\frac{23}{16}\right)}{2} \log \left| \left(x + \frac{3}{4}\right) + \sqrt{\left(x + \frac{3}{4}\right)^2 + \left(\frac{\sqrt{23}}{4}\right)^2} \right| \right] + C \\
&= s\sqrt{2} \left[ \left(\frac{4x+3}{8}\right) \sqrt{x^2 + \frac{3}{2}x + 2} + \frac{23}{32} \log \left| \left(x + \frac{3}{4}\right) + \sqrt{x^2 + \frac{3}{2}x + 2} \right| \right] + c.
\end{aligned}$$

**Exercise 3.3 | Q 3.1 | Page 138**

Integrate the following functions w.r.t.  $x$  :  $[2 + \cot x - \operatorname{cosec}^2 x]e^x$

**SOLUTION**

$$\text{Let } I = \int e^x [2 + \cot x - \operatorname{cosec}^2 x]. dx$$

Put  $f(x) = 2 + \cot x$

$$\therefore f'(x) = \frac{d}{dx}(2 + \cot x)$$

$$= \frac{d}{dx}(2) + \frac{d}{dx}(\cot x)$$

$$= 0 - \operatorname{cosec}^2 x$$

$$= -\operatorname{cosec}^2 x$$

$$\therefore I = \int e^x [f(x) + f'(x)]. dx$$

$$= e^x f(x) + c$$

$$= e^x (2 + \cot x) + c.$$

**Exercise 3.3 | Q 3.2 | Page 138**

Integrate the following functions w.r.t.  $x$  :  $\left( \frac{1 + \sin x}{1 + \cos x} \right). e^x$

**SOLUTION**

$$\text{Let } I = \int e^x \left( \frac{1 + \sin x}{1 + \cos x} \right). dx$$

$$= \int e^x \left[ \frac{1 + 2 \sin \frac{x}{2} \cos \frac{x}{2}}{2 \cos^2 \frac{x}{2}} \right]. dx$$

$$= \int e^x \left[ \frac{1}{2 \cos^2 \frac{x}{2}} + \frac{2 \sin \frac{x}{2} \cos \frac{x}{2}}{2 \cos^2 \frac{x}{2}} \right]. dx$$

$$= \int e^x \left[ \frac{1}{2} \sec^2 \frac{x}{2} + \tan \left( \frac{x}{2} \right) \right] dx$$

$$\text{Put } f(x) = \tan \left( \frac{x}{2} \right)$$

$$\therefore f'(x) = \frac{d}{dx} \left[ \tan \frac{x}{2} \right]$$

$$= \sec^2 \frac{x}{2} \cdot \frac{1}{2}$$

$$= \frac{1}{2} \sec^2 \frac{x}{2}$$

$$\therefore I = \int e^x [f(x) + f'(x)] dx$$

$$= e^x f(x) + c$$

$$= e^x \cdot \tan \left( \frac{x}{2} \right) + c.$$

**Exercise 3.3 | Q 3.3 | Page 138**

Integrate the following functions w.r.t.  $x$  :  $e^x \cdot \left( \frac{1}{x} - \frac{1}{x^2} \right)$

**SOLUTION**

$$\text{Let } I = \int e^x \cdot \left( \frac{1}{x} - \frac{1}{x^2} \right) dx$$

$$\text{Let } f(x) = \frac{1}{x}$$

$$\therefore f'(x) = -\frac{1}{x^2}$$

$$\therefore I = \int e^x [f(x) + f'(x)] dx$$

$$= e^x f(x) + c$$

$$= e^x \cdot \frac{1}{x} + c.$$

**Exercise 3.3 | Q 3.4 | Page 138**

Integrate the following functions w.r.t. x :  $\left[ \frac{x}{(x+1)^2} \right] \cdot e^x$

**SOLUTION**

$$\begin{aligned} \text{Let } I &= \int e^x \left[ \frac{x}{(x+1)^2} \right] \cdot dx \\ &= \int e^x \left[ \frac{(x+1)-1}{(x+1)^2} \right] \cdot dx \\ &= \int e^x \left[ \frac{1}{x+1} - \frac{1}{(x+1)^2} \right] \cdot dx \end{aligned}$$

$$\begin{aligned} \text{Let } f(x) &= \frac{1}{x+1} \\ &= (x+1)^{-1} \\ \therefore f'(x) &= \frac{d}{dx} (x+1)^{-1} \\ &= -(x+1)^{-2} \frac{d}{dx} (x+1) \end{aligned}$$

$$\begin{aligned} &= \frac{-1}{(x+1)^2} \times 1 \\ &= \frac{-1}{(x+1)^2} \end{aligned}$$

$$\therefore I = \int e^x [f(x) + f'(x)] \cdot dx$$

$$= e^x \cdot f(x) + C$$

$$= \frac{e^x}{x+1} + C.$$

**Exercise 3.3 | Q 3.5 | Page 138**

Integrate the following functions w.r.t.  $x$  :  $\frac{e^x}{x} \left[ x(\log x)^2 + 2(\log x) \right]$

**SOLUTION**

$$\begin{aligned} \text{Let } I &= \int \frac{e^x}{x} \left[ x(\log x)^2 + 2\log x \right] dx \\ &= \int e^x \left[ (\log x)^2 + \frac{2\log x}{x} \right] dx \end{aligned}$$

$$\text{Put } f(x) = (\log x)^2$$

$$\begin{aligned} \therefore f'(x) &= \frac{d}{dx} (\log x)^2 \\ &= 2(\log x) \cdot \frac{d}{dx} (\log x) \\ &= \frac{2\log x}{x} \end{aligned}$$

$$\therefore I = \int e^x [f(x) + f'(x)] dx$$

$$= e^x \cdot f(x) + c$$

$$= e^x \cdot (\log x)^2 + c.$$

**Exercise 3.3 | Q 3.6 | Page 138**

Integrate the following functions w.r.t.  $x$  :  $e^{5x} \cdot \left[ \frac{5x \cdot \log x + 1}{x} \right]$

**SOLUTION**

$$\begin{aligned} \text{Let } I &= \int e^{5x} \left[ \frac{5x \cdot \log x + 1}{x} \right] dx \\ &= \int e^{5x} \left[ 5\log x + \frac{1}{x} \right] dx \end{aligned}$$

Put  $5x = t$

$$\therefore 5dx = dt$$

$$\therefore dx = \frac{1}{5}dt$$

$$\text{Also, } x = \frac{t}{5}$$

$$\therefore I = \frac{1}{5} \int e^t \left[ 5 \log\left(\frac{t}{5}\right) + \frac{5}{t} \right] dt$$

$$\text{Let } f(t) = 5 \log\left(\frac{t}{5}\right)$$

$$= 5 \log t - 5 \log 5$$

$$\therefore f'(t) = \frac{d}{dt} [5 \log t - 5 \log 5]$$

$$= \frac{5}{t} - 0$$

$$= \frac{5}{t}$$

$$\therefore I = \frac{1}{5} \int e^t [f(t) + f'(t)] dt$$

$$= \frac{1}{5} e^t f(t) + c$$

$$= \frac{1}{5} e^t \cdot 5 \log\left(\frac{t}{5}\right) + c$$

$$= e^{5x} \log x + c.$$

Exercise 3.3 | Q 3.7 | Page 138

Integrate the following functions w.r.t. x :  $e^{\sin^{-1} x} \cdot \left[ \frac{x + \sqrt{1 - x^2}}{\sqrt{1 - x^2}} \right]$

**SOLUTION**

$$\begin{aligned} \text{Let } I &= \int e^{\sin^{-1} x} \left[ \frac{x + \sqrt{1 - x^2}}{\sqrt{1 - x^2}} \right] dx \\ &= \int e^{\sin^{-1} x} \left[ x + \sqrt{1 - x^2} \right] \cdot \frac{1}{\sqrt{1 - x^2}} dx \end{aligned}$$

Put  $\sin^{-1} x = t$

$$\therefore \frac{1}{\sqrt{1 - x^2}} dx = dt$$

and  $x = \sin t$

$$\begin{aligned} \therefore I &= \int e^t \left[ \sin t + \sqrt{1 - \sin^2 t} \right] dt \\ &= \int e^t \left[ \sin t + \sqrt{\cos^2 t} \right] dt \\ &= \int e^t (\sin t + \cos t) dt \end{aligned}$$

Let  $f(t) = \sin t$

$\therefore f'(t) = \cos t$

$$\begin{aligned} \therefore I &= \int e^t [f(t) + f'(t)] dt \\ &= e^t \cdot f(t) + c \\ &= e^t \cdot \sin t + c \\ &= e^{\sin^{-1} x} \cdot x + c. \end{aligned}$$

**Exercise 3.3 | Q 3.8 | Page 138**

Integrate the following functions w.r.t.  $x$  :  $\log(1 + x)^{(1+x)}$

**SOLUTION**

$$\begin{aligned} \text{Let } I &= \int \log(1+x)^{(1+x)} \cdot dx \\ &= \int (1+x) \log(1+x) \cdot dx \\ &= \int [\log(1+x)](1+x) \cdot dx \\ &= \left[ \log(1+x) \int (1+x) \cdot dx - \int \left[ \frac{d}{dt} \{\log(1+x)\} \int (1+x) \cdot dx \right] \cdot dx \right] \\ &= [\log(1+x)] \left[ \frac{(1+x)^2}{2} \right] - \int \frac{1}{x+1} \cdot \frac{(x+1)^2}{2} \cdot dx \\ &= \frac{(x+1)^2}{2} \cdot \log(1+x) - \frac{1}{2} \int (x+1) \cdot dx \\ &= \frac{(x+1)^2}{2} \cdot \log(1+x) - \frac{1}{2} \cdot \frac{(x+1)^2}{2} + c \\ &= \frac{(x+1)^2}{2} \left[ \log(1+x) - \frac{1}{2} \right] + c. \end{aligned}$$

**Exercise 3.3 | Q 3.9 | Page 138**

Integrate the following functions w.r.t. x : cosec ( $\log x$ ) $[1 - \cot (\log x)]$

**SOLUTION**

$$\text{Let } I = \int \operatorname{cosec}(\log x)[1 - \cot(\log x)]. dx$$

Put  $\log x = t$

$$\therefore e^t$$

$$\therefore dx = e^t \cdot dt$$

$$\begin{aligned}
\therefore I &= \int \operatorname{cosec} t (1 - \cot t) \cdot e^t dt \\
&= \int e^t [\operatorname{cosec} t - \operatorname{cosec} t \cot t] dt \\
&= \int e^t \left[ \operatorname{cosec} t + \frac{d}{dt} (\operatorname{cosec} t) \right] dt \\
&= e^t \operatorname{cosec} t + c \quad \dots \left[ \because \int e^t [f(t) + f'(t)] dt = e^t f(t) + c \right] \\
&= x \cdot \operatorname{cosec}(\log x) + c.
\end{aligned}$$

### EXERCISE 3.4 [PAGES 144 - 145]

**Exercise 3.4 | Q 1.01 | Page 144**

Integrate the following w.r.t. x :  $\frac{x^2 + 2}{(x-1)(x+2)(x+3)}$

**SOLUTION**

$$\text{Let } I = \int \frac{x^2 + 2}{(x-1)(x+2)(x+3)} \cdot dx$$

$$\text{Let } \frac{x^2 + 2}{(x-1)(x+2)(x+3)}$$

$$= \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{x+3}$$

$$\therefore x^2 + 2 = A(x+2)(x+3) + B(x-1)(x+3) + C(x-1)(x+2)$$

Put  $x - 1 = 0$ , i.e.  $x = 1$ , we get

$$1 + 2 = A(3)(4) + B(0)(4) + C(0)(3)$$

$$\therefore 3 = 12A$$

$$\therefore A = \frac{1}{4}$$

Put  $x + 2 = 0$ , i.e.  $x = -2$ , we get

$$4 + 2 = A(0)(1) + B(-3)(1) + C(-3)(0)$$

$$\therefore 6 = -3B$$

$$\therefore B = -2$$

Put  $x + 3 = 0$ , i.e.  $x = -3$  we get

$$9 + 2 = A(-1)(0) + B(-4)(0) + C(-4)(-1)$$

$$\therefore 11 = 4C$$

$$\therefore C = \frac{11}{4}$$

$$\therefore \frac{x^2 + 2}{(x-1)(x+2)(x+3)} = \frac{\left(\frac{1}{4}\right)}{x-1} + \frac{-2}{x+2} + \frac{\left(\frac{11}{4}\right)}{x+3}$$

$$\therefore I = \int \left[ \frac{\left(\frac{1}{4}\right)}{x-1} + \frac{-2}{x+2} + \frac{\left(\frac{11}{4}\right)}{x+3} \right] dx$$

$$= \frac{1}{4} \int \frac{1}{x-1} dx - 2 \int \frac{1}{x+2} dx + \frac{11}{4} \int \frac{1}{x+3} dx$$

$$= \frac{1}{4} \log|x-1| - 2 \log|x+2| + \frac{11}{4} \log|x+3| + c.$$

#### Exercise 3.4 | Q 1.02 | Page 144

Integrate the following w.r.t.  $x$  :  $\frac{x^2}{(x^2 + 1)(x^2 - 2)(x^2 + 3)}$

**SOLUTION**

$$\text{Let } I = \int \frac{x^2}{(x^2 + 1)(x^2 - 2)(x^2 + 3)} \cdot dx$$

$$\text{Consider, } \frac{x^2}{(x^2 + 1)(x^2 - 2)(x^2 + 3)}$$

For finding partial fractions only, put  $x^2 = t$ .

$$\begin{aligned} \therefore \frac{x^2}{(x^2 + 1)(x^2 - 2)(x^2 + 3)} &= \frac{t}{(t - 1)(t - 2)(t + 3)} \\ &= \frac{A}{t + 1} + \frac{B}{t - 2} + \frac{C}{t + 3} \quad \dots(\text{Say}) \end{aligned}$$

$$\therefore t = A(t - 2)(t + 3) + B(t + 1)(t + 3) + C(t + 1)(t - 2)$$

Put  $t + 1 = 0$ , i.e.  $t = -1$ , we get

$$-1 = A(-3)(2) + B(0)(2) + C(0)(-3)$$

$$\therefore -1 = -6A$$

$$\therefore A = \frac{1}{6}$$

Put  $t - 2 = 0$ , i.e.  $t = 2$ , we get

$$2 = A(0)(5) + B(3)(5) + C(3)(0)$$

$$\therefore 2 = 15B$$

$$\therefore B = \frac{2}{15}$$

Put  $t + 3 = 0$ , i.e.  $t = -3$ , we get

$$-3 = A(-5)(0) + B(-2)(0) + C(-2)(-5)$$

$$-3 = 10C$$

$$\therefore C = -\frac{3}{10}$$

$$\begin{aligned}
& \therefore \frac{t}{(t+1)(t-2)(t+3)} = \frac{\left(\frac{1}{6}\right)}{t+1} + \frac{\left(\frac{2}{15}\right)}{x^2-2} + \frac{\left(\frac{-3}{10}\right)}{x^2+3} \\
& \therefore \frac{x^2}{(x^2+1)(x^2-2)(x^2+3)} = \frac{\left(\frac{1}{6}\right)}{x^2+1} + \frac{\left(\frac{2}{15}\right)}{x^2-2} + \frac{\left(\frac{-3}{10}\right)}{x^2+3} \\
& \therefore I = \int \left[ \frac{\left(\frac{1}{6}\right)}{x^2+1} + \frac{\left(\frac{2}{15}\right)}{x^2-2} + \frac{\left(\frac{-3}{10}\right)}{x^2+3} \right] dx \\
& = \frac{1}{6} \int \frac{1}{1+x^2} dx + \frac{2}{15} \int \frac{1}{x^2 - (\sqrt{2})^2} dx - \frac{3}{10} \int \frac{1}{x^2 + (\sqrt{3})^2} dx \\
& = \frac{1}{6} \tan^{-1} x + \frac{2}{15} \times \frac{1}{2\sqrt{2}} \log \left| \frac{x-\sqrt{2}}{x+\sqrt{2}} \right| - \frac{3}{10} \times \frac{1}{\sqrt{3}} \tan^{-1} \left( \frac{x}{\sqrt{3}} \right) + c \\
& = \frac{1}{6} \tan^{-1} x + \frac{1}{15\sqrt{2}} \log \left| \frac{x-\sqrt{2}}{x+\sqrt{2}} \right| - \frac{\sqrt{3}}{10} \tan^{-1} \left( \frac{x}{\sqrt{3}} \right) + c.
\end{aligned}$$

#### Exercise 3.4 | Q 1.03 | Page 144

Integrate the following w.r.t. x :  $\frac{12x+3}{6x^2+13x-63}$

**SOLUTION**

$$\text{Let } I = \int \frac{12x+3}{6x^2+13x-63} dx$$

$$\text{Let } \frac{12x+3}{6x^2+13x-63}$$

$$= \frac{12x+3}{(2x+9)(3x-7)}$$

$$= \frac{A}{2x+9} + \frac{B}{3x-7}$$

$$\therefore 12x+3 = A(3x-7) + B(2x+9)$$

Put  $2x + 9 = 0$ , i.e.  $x = \frac{-9}{2}$ , we get

$$12\left(\frac{-9}{2}\right) + 3 = A\left(\frac{-27}{2} - 7\right) + B(0)$$

$$\therefore -51 = \frac{-41}{2}A$$

$$\therefore A = \frac{102}{41}$$

Put  $3x - 7 = 0$ , i.e.  $x = \frac{7}{3}$ , we get

$$12\left(\frac{7}{3}\right) + 3 = A(0) + B\left(\frac{14}{3} + 9\right)$$

$$\therefore 31 = \frac{41}{3}B$$

$$\therefore B = \frac{93}{41}$$

$$\therefore \frac{12x + 3}{6x^2 + 13x - 63} = \frac{\left(\frac{102}{41}\right)}{2x + 9} + \frac{\left(\frac{93}{41}\right)}{3x - 7}$$

$$\therefore I = \int \left[ \frac{\left(\frac{102}{41}\right)}{2x + 9} + \frac{\left(\frac{93}{41}\right)}{3x - 7} \right] dx$$

$$= \frac{102}{41} \int \frac{1}{2x + 9} dx + \frac{93}{41} \int \frac{1}{3x - 7} dx$$

$$= \frac{102}{41} \cdot \frac{\log|2x + 9|}{2} + \frac{93}{41} \cdot \frac{\log|3x - 7|}{3} + c$$

$$= \frac{51}{41} \log|2x + 9| + \frac{31}{41} \log|3x - 7| + c.$$

#### Exercise 3.4 | Q 1.04 | Page 145

Integrate the following w.r.t. x :  $\frac{2x}{4 - 3x - x^2}$

**SOLUTION**

$$\text{Let } I = \int \frac{2x}{4 - 3x - x^2} \cdot dx$$

$$\text{Let } \frac{2x}{4 - 3x - x^2}$$

$$= \frac{2x}{(4 + x)(1 - x)}$$

$$= \frac{A}{4 + x} + \frac{B}{1 - x}$$

$$\therefore 2x = A(1 - x) + B(4 + x)$$

Put  $4 + x = 0$ , i.e.  $x = -4$ , we get

$$-8 = A(5) + B(0)$$

$$\therefore A = -\frac{8}{5}$$

Put  $1 - x = 0$ , i.e  $x = 1$ , we

$$2 = A(0) + B(5)$$

$$\therefore B = \frac{2}{5}$$

$$\therefore \frac{2x}{4 - 3x - x^2} = \frac{\left(-\frac{8}{5}\right)}{4 + x} + \frac{\left(\frac{2}{5}\right)}{1 - x}$$

$$\therefore I = \int \left[ \frac{\frac{-8}{5}}{4 + x} + \frac{\left(\frac{2}{5}\right)}{1 - x} \right] \cdot dx$$

$$= -\frac{8}{5} \int \frac{1}{4 + x} \cdot dx + \frac{2}{5} \int \frac{1}{1 - x} \cdot dx$$

$$= -\frac{8}{5} \log|4 + x| + \frac{2}{5} \cdot \frac{\log|1 - x|}{-1} + c$$

$$= -\frac{8}{5} \log|4 + x| - \frac{2}{5} \log|1 - x| + c.$$

**Exercise 3.4 | Q 1.05 | Page 145**

Integrate the following w.r.t. x :  $\frac{x^2 + x - 1}{x^2 + x - 6}$

**SOLUTION**

$$\begin{aligned} \text{Let } I &= \int \frac{x^2 + x - 1}{x^2 + x - 6} \cdot dx \\ &= \int \frac{(x^2 + x - 6) + 5}{x^2 + x - 6} \cdot dx \\ &= \int \left[ 1 + \frac{5}{x^2 + x - 6} \right] \cdot dx \\ &= \int 1 dx + 5 \int \frac{1}{x^2 + x - 6} \cdot dx \end{aligned}$$

$$\begin{aligned} \text{Let } \frac{1}{x^2 + x - 6} &= \frac{1}{(x+3)(x-2)} \\ &= \frac{A}{x+3} + \frac{B}{x-2} \end{aligned}$$

$$\therefore 1 = A(x-2) + B(x+3)$$

Put  $x = 3$ , i.e.  $x = -3$ , we get

$$1 = A(-5) + B(0)$$

$$\therefore A = \frac{-1}{5}$$

Put  $x = 2$ , i.e.  $x = 2$ , we get

$$1 = A(0) + B(5)$$

$$\therefore B = \frac{1}{5}$$

$$\begin{aligned}
\therefore \frac{1}{x^2 + x - 6} &= \frac{\left(-\frac{1}{5}\right)}{x+3} + \frac{\left(\frac{1}{5}\right)}{x-2} \\
\therefore I &= \int 1 dx + 5 \int \left[ \frac{\left(-\frac{1}{5}\right)}{x+3} + \frac{\left(\frac{1}{5}\right)}{x-2} \right] dx \\
&= \int 1 dx - \int \frac{1}{x+3} dx + \int \frac{1}{x-2} dx \\
&= x - \log|x+3| + \log|x-2| + C \\
&= x + \log \left| \frac{x-2}{x+3} \right| + c.
\end{aligned}$$

**Exercise 3.4 | Q 1.06 | Page 145**

Integrate the following w.r.t. x :  $\frac{6x^3 + 5x^2 - 7}{3x^2 - 2x - 1}$

**SOLUTION**

$$\begin{aligned}
\text{Let } I &= \int \frac{6x^3 + 5x^2 - 7}{3x^2 - 2x - 1} dx \\
3x^2 - 2x - 1) \overline{6x^3 + 5x^2 - 7} &(2x + 3 \\
&\quad 6x^3 - 4x^2 - 2x \\
&\quad \underline{- \quad + \quad +} \\
&\quad 9x^2 + 2x - 7 \\
&\quad 9x^2 - 6x - 3 \\
&\quad \underline{- \quad + \quad +} \\
&\quad 8x - 4 \\
\therefore I &= \int \left[ (2x + 3) + \frac{8x - 4}{3x^2 - 2x - 1} \right] dx \\
&= \int 2x + 3 + \int \frac{8x - 4}{(x-1)(3x+1)} dx
\end{aligned}$$

$$\text{Let } \frac{8x - 4}{(x - 1)(3x + 1)}$$

$$= \frac{A}{x - 1} + \frac{B}{3x + 1}$$

$$\therefore 8x - 4 = A(3x + 1) + B(x - 1)$$

Put  $x - 1 = 0$ , i.e.  $x = 1$ , we get

$$8 - 4 = A(4) + B(0)$$

$$\therefore A = 1$$

Put  $3x + 1 = 0$ , i.e.  $x = -\frac{1}{3}$ , we get

$$8\left(-\frac{1}{3}\right) - 4 = A(0) + B\left(-\frac{4}{3}\right)$$

$$\therefore \frac{-8 - 12}{3} = -\frac{4B}{3}$$

$$\therefore B = 5$$

$$\therefore \frac{8x - 4}{(x - 1)(3x + 1)} = \frac{1}{x - 1} + \frac{5}{3x + 1}$$

$$\therefore I = 2 \int x dx + 3 \int 1 dx + \int \left[ \frac{1}{x - 1} + \frac{5}{3x + 1} \right] dx$$

$$= 2\left(\frac{x^2}{2}\right) + 3x + \int \frac{1}{x - 1} dx + 5 \int \frac{1}{3x + 1} dx$$

$$= x^2 + 3x + \log|x - 1| + \frac{5}{3} \log|3x + 1| + C.$$

#### Exercise 3.4 | Q 1.07 | Page 145

Integrate the following w.r.t.  $x$  :  $\frac{12x^2 - 2x - 9}{(4x^2 - 1)(x + 3)}$

**SOLUTION**

$$\text{Let } I = \int \frac{12x^2 - 2x - 9}{(4x^2 - 1)(x + 3)} \cdot dx$$

$$\text{Let } \frac{12x^2 - 2x - 9}{(4x^2 - 1)(x + 3)} = \frac{A}{4x^2 - 1} + \frac{B}{x + 3}$$

$$\therefore 12x^2 - 2x - 9 = A(x + 3) + B(4x^2 - 1)$$

Put  $4x^2 - 1 = 0$ , i.e.  $x^2 = \frac{1}{4}$ , i.e  $x = \frac{1}{2}$  we get

$$12 \times \left(\frac{1}{2}\right)^2 - 2 \times \left(\frac{1}{2}\right) - 9 = A\left(\frac{7}{2}\right) + B(0)$$

$$\therefore -7 = \frac{7A}{2}$$

$$\therefore A = -2$$

Put  $x + 3 = 0$ , i.e.  $x = -3$ , we get

$$12(-3)^2 - 2(-3) - 9 = A(0) + B(4(3^2) - 1)$$

$$\therefore 105 = 35B$$

$$\therefore B = 3$$

$$\frac{12x^2 - 2x - 9}{(4x^2 - 1)(x + 3)} = \frac{-2}{4x^2 - 1} + \frac{3}{x + 3}$$

$$\therefore I = \int \left[ \frac{-2}{4x^2 - 1} + \frac{3}{x + 3} \right] \cdot dx$$

$$= (-2) \int \frac{1}{(2x)^2 - 1} \cdot dx + 3 \int \frac{1}{x + 3} \cdot dx$$

$$= \frac{1}{2} \log \left| \frac{2x + 1}{2x - 1} \right| + 3 \log|x + 3| + c.$$

**Exercise 3.4 | Q 1.08 | Page 145**

Integrate the following w.r.t. x :  $\frac{1}{x(x^5 + 1)}$

**SOLUTION**

$$\text{Let } I = \int \frac{1}{x(x^5 + 1)} \cdot dx$$

$$= \int \frac{x^4}{x^5(x^5 + 1)} \cdot dx$$

Put  $x^5 = t$ .

Then  $5x^4 dx = dt$

$$\therefore x^4 dx = \frac{dt}{5}$$

$$\therefore I = \int \frac{1}{t(t+1)} \cdot \frac{dt}{5}$$

$$= \frac{1}{5} \int \frac{(t+1) - t}{t(t+1)} \cdot dt$$

$$= \frac{1}{5} \int \left( \frac{1}{t} - \frac{1}{t+1} \right) \cdot dt$$

$$= \frac{1}{5} \left[ \int \frac{1}{t} dt - \int \frac{1}{t+1} dt \right]$$

$$= \frac{1}{5} [\log|t| - \log|t+1|] + c$$

$$= \frac{1}{5} \log \left| \frac{t}{t+1} \right| + c$$

$$= \frac{1}{5} \log \left| \frac{x^5}{x^5 + 1} \right| + c.$$

Exercise 3.4 | Q 1.09 | Page 145

Integrate the following w.r.t. x:  $\frac{2x^2 - 1}{x^4 + 9x^2 + 20}$

**SOLUTION**

$$\text{Let } I = \int \frac{2x^2 - 1}{x^4 + 9x^2 + 20} \cdot dx$$

$$\text{Consider, } \frac{2x^2 - 1}{x^4 + 9x^2 + 20}$$

For finding partial fractions only, put  $x^2 = t$ .

$$\therefore \frac{2x^2 - 1}{x^4 + 9x^2 + 20} = \frac{t}{(t-1)(t-2)(t+3)}$$

$$= \frac{A}{t+1} + \frac{B}{t-2} + \frac{C}{t+3} \quad \dots(\text{Say})$$

$$\therefore t = A(t-2)(t+3) + B(t+1)(t+3) + C(t+1)(t-2)$$

Put  $t + 1 = 0$ , i.e.  $t = -1$ , we get

$$-1 = A(-3)(2) + B(0)(2) + C(0)(-3)$$

$$\therefore -1 = -6A$$

$$\therefore A = \frac{1}{6}$$

Put  $t - 2 = 0$ , i.e.  $t = 2$ , we get

$$2 = A(0)(5) + B(3)(5) + C(3)(0)$$

$$\therefore 2 = 15B$$

$$\therefore 2 = 15B$$

$$\therefore B = \frac{2}{15}$$

Put  $t + 3 = 0$ , i.e.  $t = -3$ , we get

$$-3 = A(-5)(0) + B(-2)(0) + C(-2)(-5)$$

$$-3 = 10C$$

$$\therefore C = -\frac{3}{10}$$

$$\begin{aligned}
& \therefore \frac{t}{(t+1)(t-2)(t+3)} = \frac{\left(\frac{1}{6}\right)}{t+1} + \frac{\left(\frac{2}{15}\right)}{x^2-2} + \frac{\left(\frac{-3}{10}\right)}{x^2+3} \\
& \therefore \frac{x^2}{(x^2+1)(x^2-2)(x^2+3)} = \frac{\left(\frac{1}{6}\right)}{x^2+1} + \frac{\left(\frac{2}{15}\right)}{x^2-2} + \frac{\left(\frac{-3}{10}\right)}{x^2+3} \\
& \therefore I = \int \left[ \frac{\left(\frac{1}{6}\right)}{x^2+1} + \frac{\left(\frac{2}{15}\right)}{x^2-2} + \frac{\left(\frac{-3}{10}\right)}{x^2+3} \right] dx \\
& = \frac{1}{6} \int \frac{1}{1+x^2} dx + \frac{2}{15} \int \frac{1}{x^2 - (\sqrt{2})^2} dx - \frac{3}{10} \int \frac{1}{x^2 + (\sqrt{3})^2} dx \\
& = \frac{1}{6} \tan^{-1} x + \frac{2}{15} \times \frac{1}{2\sqrt{2}} \log \left| \frac{x-\sqrt{2}}{x+\sqrt{2}} \right| - \frac{3}{10} \times \frac{1}{\sqrt{3}} \tan^{-1} \left( \frac{x}{\sqrt{3}} \right) + c \\
& = \frac{11}{\sqrt{5}} \tan^{-1} \left( \frac{x}{\sqrt{5}} \right) - \frac{9}{2} \tan^{-1} \left( \frac{x}{2} \right) + c.
\end{aligned}$$

**Exercise 3.4 | Q 1.1 | Page 145**

Integrate the following w.r.t. x:  $\frac{x^2+3}{(x^2-1)(x^2-2)}$

**SOLUTION**

$$\text{Let } I = \int \frac{x^2 + 3}{(x^2 - 1)(x^2 - 2)} \cdot dx$$

$$\text{Consider, } \frac{x^2 + 3}{(x^2 - 1)(x^2 - 2)}$$

For finding partial fractions only, put  $x^2 = t$ .

$$\therefore \frac{x^2 + 3}{(x^2 - 1)(x^2 - 2)} = \frac{t}{(t + 1)(t - 2)}$$

$$= \frac{A}{t + 1} + \frac{B}{t - 2} \quad \dots(\text{Say})$$

$$\therefore t = A(t - 2)(t + 3) + B(t + 1)(t + 3) + C(t + 1)(t - 2)$$

Put  $t + 1 = 0$ , i.e.  $t = -1$ , we get

$$-1 = A(-3)(2) + B(0)(2) + C(0)(-3)$$

$$\therefore -1 = -6A$$

$$\therefore A = \frac{1}{6}$$

Put  $t - 2 = 0$ , i.e.  $t = 2$ , we get

$$2 = A(0)(5) + B(3)(5) + C(3)(0)$$

$$\therefore 2 = 15B$$

$$\therefore B = \frac{2}{15}$$

Put  $t + 3 = 0$ , i.e.  $t = -3$ , we get

$$-3 = A(-5)(0) + B(-2)(0) + C(-2)(-5)$$

$$-3 = 10C$$

$$\therefore C = -\frac{3}{10}$$

$$\begin{aligned}
& \therefore \frac{t}{(t+1)(t-2)(t+3)} = \frac{\left(\frac{1}{6}\right)}{t+1} + \frac{\left(\frac{2}{15}\right)}{x^2-2} + \frac{\left(\frac{-3}{10}\right)}{x^2+3} \\
& \therefore \frac{x^2}{(x^2+1)(x^2-2)(x^2+3)} = \frac{\left(\frac{1}{6}\right)}{x^2+1} + \frac{\left(\frac{2}{15}\right)}{x^2-2} + \frac{\left(\frac{-3}{10}\right)}{x^2+3} \\
& \therefore I = \int \left[ \frac{\left(\frac{1}{6}\right)}{x^2+1} + \frac{\left(\frac{2}{15}\right)}{x^2-2} + \frac{\left(\frac{-3}{10}\right)}{x^2+3} \right] dx \\
& = \frac{1}{6} \int \frac{1}{1+x^2} \cdot dx + \frac{2}{15} \int \frac{1}{x^2 - (\sqrt{2})^2} \cdot dx - \frac{3}{10} \int \frac{1}{x^2 + (\sqrt{3})^2} \cdot dx \\
& = \frac{1}{6} \tan^{-1} x + \frac{2}{15} \times \frac{1}{2\sqrt{2}} \log \left| \frac{x-\sqrt{2}}{x+\sqrt{2}} \right| - \frac{3}{10} \times \frac{1}{\sqrt{3}} \tan^{-1} \left( \frac{x}{\sqrt{3}} \right) + c \\
& = 2 \log \left| \frac{x+1}{x-1} \right| + \frac{5}{2\sqrt{2}} \log \left| \frac{x-\sqrt{2}}{x+\sqrt{2}} \right| + c.
\end{aligned}$$

**Exercise 3.4 | Q 1.11 | Page 145**

Integrate the following w.r.t. x :  $\frac{2x}{(2+x^2)(3+x^2)}$

**SOLUTION**

$$\text{Let } I = \int \frac{2x}{(2+x^2)(3+x^2)} \cdot dx$$

$$\text{Put } x^2 = t$$

$$\therefore 2x \, dx = dt$$

$$\therefore I = \int \frac{1}{(2+t)(3+t)} \cdot dt$$

$$= \int \frac{(3+t)-(2+t)}{(2+t)(3+t)} \cdot dt$$

$$= \int \left[ \frac{1}{2+t} - \frac{1}{3+t} \right] \cdot dt$$

$$= \log|2+t| - \log|3+t| + c$$

$$= \log\left|\frac{2+t}{3+t}\right| + c$$

$$= \log\left|\frac{2+x^2}{3+x^2}\right| + c.$$

**Exercise 3.4 | Q 1.12 | Page 145**

Integrate the following w.r.t. x :  $\frac{2^x}{4^x - 3 \cdot 2^x - 4}$

**SOLUTION**

$$\begin{aligned} \text{Let } I &= \int \frac{2^x}{4^x - 3 \cdot 2^x - 4} \cdot dx \\ &= \int \frac{2^x}{(2^x)^2 - 3 \cdot 2^x - 4} \end{aligned}$$

Put  $2^x = t$

$$\therefore 2^x \log 2 \, dx = dt$$

$$\therefore 2^x \, dx = \frac{1}{\log 2} \cdot dt$$

$$\begin{aligned} \therefore I &= \frac{1}{\log 2} \int \frac{dt}{t^2 - 3t - 4} \\ &= \frac{1}{\log 2} \int \frac{1}{(t+1)(t-4)} \cdot dt \\ &= \frac{1}{5 \log 2} \int \frac{(t+1) - (t-4)}{(t-4)(t+1)} \cdot dt \quad \dots[\text{Note this step.}] \\ &= \frac{1}{5 \log 2} \int \left[ \frac{1}{t-4} - \frac{1}{t+1} \right] \cdot dt \\ &= \frac{1}{5 \log 2} \left[ \int \frac{1}{t-4} \cdot dt - \int \frac{1}{t+1} \cdot dt \right] \end{aligned}$$

$$= \frac{1}{5 \log 2} [\log|t - 4| - \log|t + 1|] + c$$

$$= \frac{1}{5 \log 2} \log \left| \frac{2^x - 4}{2^x + 1} \right| + c.$$

**Exercise 3.4 | Q 1.13 | Page 145**

Integrate the following w.r.t. x :  $\frac{3x - 2}{(x + 1)^2(x + 3)}$

**SOLUTION**

$$\text{Let } I = \int \frac{3x - 2}{(x + 1)^2(x + 3)} \cdot dx$$

$$\text{Let } \frac{3x - 2}{(x + 1)^2(x + 3)} = \frac{A}{x + 1} + \frac{B}{(x + 1)^2} + \frac{C}{x + 3}$$

$$\therefore 3x - 2 = A(x + 1)(x + 3) + B(x + 3) + C(x + 1)^2$$

Put  $x + 1 = 0$ , i.e.  $x = -1$ , we get

$$-3 - 2 = A(0)(2) + B(2) + C(0)$$

$$\therefore -5 = 2B$$

$$\therefore B = -\frac{5}{2}$$

Put  $x + 3 = 0$ , i.e.  $x = -3$ , we get

$$-9 - 2 = A(-2)(0) + B(0) + C(-2)2$$

$$\therefore -11 = 4C$$

$$\therefore C = -\frac{11}{4}$$

Put  $x = 0$ , we get

$$-2 = A(1)(3) + B(3) + C(1)$$

$$\therefore -2 = 3A + 3B + C$$

$$\begin{aligned}
\therefore -2 &= 3A - \frac{15}{2} - \frac{11}{4} \\
\therefore 3A &= -2 + \frac{15}{2} + \frac{11}{4} \\
&= \frac{-8 + 30 + 11}{4} \\
\therefore A &= \frac{11}{4} \\
\therefore \frac{3x - 2}{(x+1)^2(x+3)} &= \frac{\left(\frac{11}{4}\right)}{x+1} + \frac{\left(-\frac{5}{4}\right)}{(x+1)^2} + \frac{\left(-\frac{11}{4}\right)}{x+3} \\
\therefore I &= \int \left[ \frac{\left(\frac{11}{4}\right)}{x+1} + \frac{\left(-\frac{5}{2}\right)}{(x+1)^2} + \frac{\left(-\frac{11}{4}\right)}{x+3} \right] \\
&= \frac{11}{4} \int \frac{1}{x+1} \cdot dx - \frac{5}{2} \int (x+1)^{-2} \cdot dx - \frac{11}{4} \int \frac{1}{x+3} \cdot dx \\
&= \frac{11}{4} \log|x+1| - \frac{5}{2} \cdot \frac{(x+1)^{-1}}{-1} \cdot \frac{1}{1} - \frac{11}{4} \log|x+3| + c \\
&= \frac{11}{4} \log \left| \frac{x+1}{x+3} \right| + \frac{5}{2(x+1)} + c. \\
&= 2 \log \left| \frac{x+1}{x-1} \right| + \frac{5}{2\sqrt{2}} \log \left| \frac{x-\sqrt{2}}{x+\sqrt{2}} \right| + c.
\end{aligned}$$

Exercise 3.4 | Q 1.14 | Page 145

Integrate the following w.r.t. x :  $\frac{5x^2 + 20x + 6}{x^3 + 2x^2 + x}$

**SOLUTION**

$$\begin{aligned} \text{Let } I &= \int \frac{5x^2 + 20x + 6}{x^3 + 2x^2 + x} \cdot dx \\ &= \int \frac{5x^2 + 20x + 6}{x(x^2 + 2x + 1)} \cdot dx \\ &= \int \frac{5x^2 + 20x + 6}{x(x+1)^2} \cdot dx \\ \text{Let } \frac{5x^2 + 20x + 6}{x(x+1)^2} &= \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2} \end{aligned}$$

$$\therefore 5x^2 + 20x + 6 = A(x+1)^2 + Bx(x+1) + Cx$$

Put  $x = 0$ , we get

$$0 + 0 + 6 = A(1) + B(0)(1) + C(0)$$

$$\therefore A = 6$$

Put  $x + 1 = 0$ , i.e.  $x = -1$ , we get

$$5(1) + 20(-1) + 6 = A(0) + B(-1)(0) + C(-1)$$

$$\therefore -9 = -C$$

$$\therefore C = 9$$

Put  $x = 1$ , we get

$$5(1) + 20(1) + 6 = A(4) + B(1)(2) + C(1)$$

But  $A = 6$  and  $C = 9$

$$\therefore 31 = 24 + 2B + 9$$

$$\therefore B = -1$$

$$\begin{aligned} \therefore \frac{5x^2 + 20x + 6}{x(x+1)^2} &= \frac{6}{x} - \frac{1}{x+1} + \frac{9}{(x+1)^2} \\ \therefore I &= \int \left[ \frac{6}{x} - \frac{1}{x+1} + \frac{9}{(x+1)^2} \right] \cdot dx \end{aligned}$$

$$\begin{aligned}
&= 6 \int \frac{1}{x} \cdot dx - \int \frac{1}{x+1} \cdot dx + 9 \int (x+1)^{-2} \cdot dx \\
&= 6 \log|x| - \log|x+1| + 9 \cdot \frac{(x+1)^{-1}}{-1} + c \\
&= \log|x^6| - \log|x+1| - \frac{9}{(x+1)} + c \\
&= \log \left| \frac{x^6}{x+1} \right| - \frac{9}{(x+1)} + c.
\end{aligned}$$

**Exercise 3.4 | Q 1.15 | Page 145**

Integrate the following w.r.t. x :  $\frac{1}{x(1+4x^3+3x^6)}$

**SOLUTION**

$$\begin{aligned}
\text{Let } I &= \int \frac{1}{x(1+4x^3+3x^6)} \cdot dx \\
&= \int \frac{x^2}{x^3(1+4x^3+3x^6)} \cdot dx
\end{aligned}$$

Put  $x^3 = t$

$$\therefore 3x^2 dx = dt$$

$$\therefore x^2 dx (1)(3) \cdot dt$$

$$\begin{aligned}
\therefore I &= \frac{1}{3} \int \frac{1}{t(1+4t+3t^2)} \cdot dt \\
&= \frac{1}{3} \int \frac{1}{t(t+1)(3t+1)} \cdot dt
\end{aligned}$$

$$\text{Let } \frac{1}{t(t+1)(3t+1)} = \frac{A}{t} + \frac{B}{t+1} + \frac{C}{3t+1}$$

$$\therefore 1 = A(t+1)(3t+1) + Bt(t+1) + Ct(t+1)$$

Put  $t = 0$ , we get

$$1 = A(1) + B(0) + C(0)$$

$$\therefore A = 1$$

Put  $t + 1 = 0$ , i.e.  $t = -1$  we get

$$1 = A(0) + B(-1)(-2) + C(0)$$

$$\therefore B = \frac{1}{2}$$

Put  $3t + 1 = 0$ , i.e.  $t = -\frac{1}{3}$ , we get

$$1 = A(0) + B(0) + C\left(-\frac{1}{3}\right)\left(\frac{2}{3}\right)$$

$$\therefore C = -\frac{9}{2}$$

$$\therefore \frac{1}{t(t+1)(3t+1)} = \frac{1}{t} + \frac{\left(\frac{1}{2}\right)}{t+1} + \frac{\left(-\frac{9}{2}\right)}{3t+1}$$

$$\therefore I = \frac{1}{3} \int \left[ \frac{1}{t} + \frac{\left(\frac{1}{2}\right)}{t+1} + \frac{\left(-\frac{9}{2}\right)}{3t+1} \right] \cdot dt$$

$$= \frac{1}{3} \left[ \int \frac{1}{t} \cdot dt + \frac{1}{2} \int \frac{1}{t+1} \cdot dt - \frac{9}{2} \int \frac{1}{3t+1} \cdot dt \right]$$

$$= (1)(3) \left[ \log \left| t \right| + \frac{1}{2} \log \left| t+1 \right| - \frac{9}{2} \cdot \frac{1}{3} \log \left| 3t+1 \right| \right] + c$$

$$= \frac{1}{3} \log |x^3| + \frac{1}{2} \log |x^3 + 1| - \frac{3}{2} \log |3x^3 + 1| + c$$

$$= \log |x| + \frac{1}{2} \log |x^3 + 1| - \frac{3}{2} \log |3x^3 + 1| + c.$$

Exercise 3.4 | Q 1.16 | Page 145

Integrate the following w.r.t. x :  $\frac{1}{x^3 - 1}$

**SOLUTION**

$$\begin{aligned} \text{Let } I &= \int \frac{1}{x^3 - 1} \cdot dx \\ &= \int \frac{1}{(x-1)(x^2+x+1)} \cdot dx \\ \text{Let } \frac{1}{(x-1)(x^2+x+1)} &= \frac{A}{x-1} + \frac{Bx+C}{x^2+x+1} \end{aligned}$$

$$\therefore 1 = A(x^2 + x + 1) + (Bx + C)(x - 1)$$

Put  $x - 1 = 0$  i.e  $x = 1$ , we get

$$1 = A(3) + (B + C)(0)$$

$$\therefore A = \frac{1}{3}$$

Put  $x = 0$ , we get

$$1 = A(1) + C(-1)$$

$$\therefore C = A - 1 = -\frac{2}{3}$$

Comparing the coefficients of  $x^2$  on both the sides, we get

$$0 = A + B$$

$$\therefore B = -A = -\frac{1}{3}$$

$$\begin{aligned} \therefore \frac{1}{(x-1)(x^2+x+1)} &= \frac{\left(\frac{1}{3}\right)}{x-1} + \frac{\left(-\frac{1}{3}x - \frac{2}{3}\right)}{x^2+x+1} \\ &= \frac{1}{3} \left[ \frac{1}{x-1} - \frac{x+2}{x^2+x+1} \right] \end{aligned}$$

$$\text{Let } x+2 = p \left[ \frac{d}{dx} (x^2 + x + 1) \right] + q$$

Comparing coefficient of  $x$  and the constant term on both the sides, we get

$$2p = 1 \text{ i.e. } p = \frac{1}{2} \text{ and } p + q = 2$$

$$\therefore q = 2 - p = 2 - \frac{1}{2} = \frac{3}{2}$$

$$\therefore x + 2 = \frac{1}{2}(2x + 1) + \frac{3}{2}$$

$$\begin{aligned}\therefore \frac{1}{(x+1)(x^2+x+1)} &= \frac{1}{3} \left[ \frac{1}{x-1} - \frac{\frac{1}{2}(2x+1) + \frac{3}{2}}{(x^2+x+1)} \right] \\ &= \frac{1}{3} \left[ \frac{1}{x-1} - \frac{1}{2} \left( \frac{2x+1}{x^2+x+1} \right) - \frac{\left(\frac{3}{2}\right)}{x^2+x+1} \right] \\ \therefore I &= \frac{1}{3} \int \left[ \frac{1}{x-1} - \frac{1}{2} \left( \frac{2x+1}{x^2+x+1} \right) - \frac{\left(\frac{3}{2}\right)}{x^2+x+1} \right] \cdot dx \\ &= \frac{1}{3} \int \frac{1}{x-1} \cdot dx - \frac{1}{6} \int \frac{2x+1}{x^2+x+1} \cdot dx - \frac{1}{2} \int \frac{1}{x^2+x+\frac{1}{4}+\frac{3}{4}} \cdot dx \\ &= \frac{1}{3} \log|x-1| - \frac{1}{6} \int \frac{\frac{d}{dx}(x^2+x+1)}{x^2+x+1} \cdot dx - \frac{1}{2} \int \frac{1}{\left(x+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} \cdot dx \\ &= \frac{1}{3} \log|x-1| - \frac{1}{6} \log|x^2+x+1| - \frac{1}{2} \frac{1}{\left(\frac{\sqrt{3}}{2}\right)} \tan^{-1} \left[ \frac{\left(x+\frac{1}{2}\right)}{\left(\frac{\sqrt{3}}{2}\right)} \right] + c \\ &= \frac{1}{3} \log|x-1| - \frac{1}{6} \log|x^2+x+1| - \frac{1}{\sqrt{3}} \tan^{-1} \left( \frac{2x+1}{\sqrt{3}} \right) + c.\end{aligned}$$

**Exercise 3.4 | Q 1.17 | Page 145**

Integrate the following w.r.t. x :  $\frac{(3\sin-2) \cdot \cos x}{5 - 4 \sin x - \cos^2 x}$

**SOLUTION**

$$\begin{aligned} \text{Let } I &= \int \frac{(3\sin x - 2) \cdot \cos x}{5 - 4 \sin x - \cos^2 x} \cdot dx \\ &= \int \frac{(3 \sin x - 2) \cdot \cos x}{5 - (1 - \sin^2 x) - 4 \sin x} \cdot dx \\ &= \int \frac{(3 \sin x - 2) \cdot \cos x}{5 - 1 + \sin^2 x - 4 \sin x} \cdot dx \\ &= \int \frac{(3 \sin x - 2) \cdot \cos x}{\sin^2 x - 4 \sin x + 4} \cdot dx \end{aligned}$$

Put  $\sin x = t$

$$\therefore \cos x \, dx = dt$$

$$\begin{aligned} \therefore I &= \int \frac{3t - 2}{t^2 - 4t + 4} \cdot dt \\ &= \int \frac{3t - 2}{(t - 2)^2} \cdot dt \end{aligned}$$

$$\text{Let } \frac{3t - 2}{(t - 2)^2} = \frac{A}{t - 2} + \frac{B}{(t - 2)^2}$$

$$\therefore 3t - 2 = A(t - 2) + B$$

Put  $t - 2 = 0$ , i.e.  $t = 2$ , we get

$$4 = A(0) + B$$

$$\therefore B = 4$$

Put  $t = 0$ , we get

$$-2 = A(-2) + B$$

$$\therefore -2 = -2A + 4$$

$$\therefore 2A = 6$$

$$\therefore A = 3$$

$$\therefore \frac{3t - 2}{(t - 2)^2} = \frac{3}{t - 2} + \frac{4}{(t - 2)^2}$$

$$\begin{aligned}
\therefore I &= \int \left[ \frac{3}{t-2} + \frac{4}{(t-2)^2} \right] \cdot dt \\
&= 3 \int \frac{1}{t-2} \cdot dt + 4 \int (t-2)^{-2} \cdot dt \\
&= 3 \log|t-2| + 4 \cdot \frac{(t-2)^{-1}}{-1} \cdot \frac{1}{1} + c \\
&= 3 \log|t-2| - \frac{4}{(t-2)} + c \\
&= 3 \log|\sin x - 2| - \frac{4}{(\sin x - 2)} + c.
\end{aligned}$$

Exercise 3.4 | Q 1.18 | Page 145

Integrate the following w.r.t. x :  $\frac{1}{\sin x + \sin 2x}$

**SOLUTION**

$$\begin{aligned}
\text{Let } I &= \int \frac{1}{\sin x + \sin 2x} \cdot dx \\
&= \int \frac{1}{\sin x + 2 \sin x \cos x} \cdot dx \\
&= \int \frac{dx}{\sin x(1 + 2 \cos x)} \\
&= \int \frac{\sin x \cdot dx}{\sin^2 x(1 + 2 \cos x)} \\
&= \int \frac{\sin x \cdot dx}{(1 - \cos^2 x)(1 + 2 \cos x)} \\
&= \int \frac{\sin x \cdot dx}{(1 - \cos x)(1 + \cos x)(1 + 2 \cos x)}
\end{aligned}$$

Put  $\cos x = t$

$$\therefore -\sin x \cdot dx = dt$$

$$\therefore \sin x \cdot dx = -dt$$

$$\therefore 1 = \int \frac{-dt}{(1-t)(1+t)(1+2t)}$$

$$= - \int \frac{dt}{(1-t)(1+t)(1+2t)}$$

$$\text{Let } \frac{1}{(1-t)(1+t)(1+2t)} = \frac{A}{1-t} + \frac{B}{1+t} + \frac{C}{1+2t}$$

$$\therefore 1 = A(1+t)(1+2t) + B(1-t)(1+2t) + C(1-t)(1+t)$$

Putting  $1-t=0$ , i.e.  $t=1$ , we get

$$1 = A(2)(3) + B(0)(3) + C(0)(2)$$

$$\therefore A = \frac{1}{6}$$

Putting  $1-t=0$ , i.e.  $t=-1$ , we get

$$1 = A(0)(-1) + B(2)(-1) + C(2)(0)$$

$$\therefore B = -\frac{1}{2}$$

Putting  $1+2t=0$ , i.e.  $t=-\frac{1}{2}$ , we get

$$1 = A(0) + B(0) + C\left(\frac{3}{2}\right)\left(\frac{1}{2}\right)$$

$$\therefore C = \frac{4}{3}$$

$$\therefore \frac{1}{(1-t)(1+t)(1+2t)} = \frac{\left(\frac{1}{6}\right)}{1-t} + \frac{\left(-\frac{1}{2}\right)}{1+t} + \frac{\left(\frac{4}{3}\right)}{1+2t}$$

$$\begin{aligned}
I &= \int \left[ \frac{\left(\frac{1}{6}\right)}{1-t} + \frac{\left(\frac{-1}{2}\right)}{1+t} + \frac{\left(\frac{4}{3}\right)}{1+2t} \right] \cdot dt \\
&= -\frac{1}{6} \int \frac{1}{1-t} \cdot dt + \frac{1}{2} \int \frac{1}{1+t} \cdot dt - \frac{4}{3} \int \frac{1}{1+2t} \cdot dt \\
&= -\frac{1}{6} \cdot \frac{\log|1-t|}{-1} + \frac{1}{2} \log|1+t| - \frac{4}{3} \cdot \frac{\log|1+2t|}{2} + c \\
&= -\frac{1}{6} \log|1-\cos x| + \frac{1}{2} \log|1+\cos x| - \frac{2}{3} \log|1+2\cos x| + c \\
&= \frac{1}{2} \log|\cos x + 1| + \frac{1}{6} \log|\cos x - 1| - \frac{2}{3} \log|2\cos x + 1| + c.
\end{aligned}$$

**Exercise 3.4 | Q 1.19 | Page 145**

Integrate the following w.r.t.  $x$  :  $\frac{1}{2 \sin x + \sin 2x}$

**SOLUTION**

$$\begin{aligned}
\text{Let } I &= \int \frac{1}{2 \sin x + \sin 2x} \cdot dx \\
&= \int \frac{1}{2 \sin x + 2 \sin x \cos x} \cdot dx \\
&= \int \frac{1}{2 \sin x(1 + \cos x)} \cdot dx \\
&= \int \frac{1}{2 \sin^2 x(1 + \cos x)} \cdot dx \\
&= \int \frac{\sin x}{2(1 - \cos^2 x)(1 + \cos x)} \\
&= \int \frac{\sin \cdot dx}{2(1 - \cos x)(1 + \cos x)(1 + \cos x)} \\
&= \int \frac{\sin \cdot dx}{2(1 - \cos x)(1 + \cos x)^2}
\end{aligned}$$

Put  $\cos x = t$

$$\therefore -\sin x \cdot dx = dt$$

$$\therefore \sin x \cdot dx = -dt$$

$$\therefore I = -\frac{1}{2} \int \frac{1}{(1-t)(1+t)^2} \cdot dt$$

$$= \frac{1}{2} \int \frac{1}{(t-1)(t+1)^2} \cdot dt$$

$$\text{Let } \frac{1}{(t-1)(t+1)^2} = \frac{A}{t-1} + \frac{B}{t+1} + \frac{C}{(t+1)^2}$$

$$\therefore 1 = A(t+1)^2 + B(t-1)(t+1) + C(t-1)$$

Put  $t+1 = 0$ , i.e.,  $t = 1$ , we get

$$\therefore 1 = A(0) + B(0) + C(-2)$$

$$\therefore C = -\frac{1}{2}$$

Put  $t-1 = 0$ , i.e.,  $t = 1$ , we get

$$\therefore 1 = A(4) + B(0) + C(0)$$

$$\therefore A = \frac{1}{4}$$

Comparing coefficients of  $t^2$  on both the sides, we get

$$0 = A + B$$

$$\therefore B = -A = -\frac{1}{4}$$

$$\therefore \frac{1}{(t-1)(t+1)^2} = \frac{\left(\frac{1}{4}\right)}{t-1} + \frac{\left(-\frac{1}{4}\right)}{t+1} + \frac{\left(-\frac{1}{2}\right)}{(t+1)^2}$$

$$\therefore I = \frac{1}{2} \int \left[ \frac{\left(\frac{1}{4}\right)}{t-1} + \frac{\left(-\frac{1}{4}\right)}{t+1} + \frac{\left(-\frac{1}{2}\right)}{(t+1)^2} \right] \cdot dt$$

$$\begin{aligned}
&= \frac{1}{8} \int \frac{1}{t-1} \cdot dt - \frac{1}{8} \int 91 \frac{)}{t+1} \cdot dt - \frac{1}{4} \int \frac{1}{(t-1)^2} \cdot dt \\
&= \frac{1}{8} \log|t-1| - \frac{1}{8} \log|t+1| - \frac{1}{4} \frac{(t+1)^{-1}}{(-1)} + c \\
&= \frac{1}{8} \log \left| \frac{t-1}{t+1} \right| + \frac{1}{4} \cdot \frac{1}{t+1} + c \\
&= \frac{1}{8} \log \left| \frac{\cos x - 1}{\cos x + 1} \right| + \frac{1}{4(\cos x + 1)} + c.
\end{aligned}$$

Exercise 3.4 | Q 1.2 | Page 145

Integrate the following w.r.t. x :  $\frac{1}{\sin 2x + \cos x}$

**SOLUTION**

$$\begin{aligned}
\text{Let } I &= \int \frac{1}{\sin 2x + \cos x} \cdot dx \\
&= \int \frac{1}{\sin x + \sin 2x \cos x} \cdot dx \\
&= \int \frac{dx}{\sin x(1 + 2 \cos x)} \\
&= \int \frac{\sin x \cdot dx}{\sin^2 x(1 + 2 \cos x)} \\
&= \int \frac{\sin \cdot dx}{(1 - \cos^2 x)(1 + 2 \cos x)} \\
&= \int \frac{\sin \cdot dx}{(1 - \cos x)(1 + \sin 2x)(1 + \cos x)}
\end{aligned}$$

Put  $\cos x = t$

$$\therefore -\sin x \cdot dx = dt$$

$$\therefore \sin x \cdot dx = - dt$$

$$\therefore I = \int \frac{-dt}{(1-t)(1+t)(1+2t)}$$

$$= - \int \frac{dt}{(1-t)(1+t)(1+2t)}$$

$$\text{Let } \frac{1}{(1-t)(1+t)(1+2t)} = \frac{A}{1-t} + \frac{B}{1+t} + \frac{C}{1+2t}$$

$$\therefore 1 = A(1+t)(1+2t) + B(1-t)(1+2t) + C(1-t)(1+t)$$

Putting  $1-t=0$ , i.e.  $t=1$ , we get

$$1 = A(2)(3) + B(0)(3) + C(0)(2)$$

$$\therefore A = \frac{1}{6}$$

Putting  $1-t=0$ , i.e.  $t=-1$ , we get

$$1 = A(0)(-1) + B(2)(-1) + C(2)(0)$$

$$\therefore B = -\frac{1}{2}$$

Putting  $1+2t=0$ , i.e.  $t=-\frac{1}{2}$ , we get

$$1 = A(0) + B(0) + C\left(\frac{3}{2}\right)\left(\frac{1}{2}\right)$$

$$\therefore C = \frac{4}{3}$$

$$\begin{aligned}
& \therefore \frac{1}{(1-t)(1+t)(1+2t)} = \frac{\left(\frac{1}{6}\right)}{1-t} + \frac{\left(-\frac{1}{2}\right)}{1+t} + \frac{\left(\frac{4}{3}\right)}{1+2t} \\
& \therefore I = \int \left[ \frac{\left(\frac{1}{6}\right)}{1-t} + \frac{\left(-\frac{1}{2}\right)}{1+t} + \frac{\left(\frac{4}{3}\right)}{1+2t} \right] \cdot dt \\
& = -\frac{1}{6} \int \frac{1}{1-t} \cdot dt + \frac{1}{2} \int \frac{1}{1+t} \cdot dt - \frac{4}{3} \int \frac{1}{1+2t} \cdot dt \\
& = -\frac{1}{6} \cdot \frac{\log|1-t|}{-1} + \frac{1}{2} \log|1+t| - \frac{4}{3} \cdot \frac{\log|1+2t|}{2} + c \\
& = -\frac{1}{6} \log|\sin x + 1| + \frac{1}{2} \log|\sin x - 1| - \frac{2}{3} \log|\sin x + 2| + c \\
& = -\frac{1}{6} \log|1 - \sin x| - \frac{1}{2} \log|1 + \sin x| + \frac{2}{3} \log|1 + 2 \sin x| + c.
\end{aligned}$$

**Exercise 3.4 | Q 1.21 | Page 145**

Integrate the following w.r.t. x :  $\frac{1}{\sin x \cdot (3 + 2 \cos x)}$

**SOLUTION**

$$\begin{aligned}
& \text{Let } I = \frac{1}{\sin x \cdot (3 + 2 \cos x)} \cdot dx \\
& = \int \frac{\sin x}{\sin^2 x \cdot (3 + 2 \cos x)} \cdot dx \\
& = \int \frac{\sin x}{(1 - \cos^2 x)(3 + 2 \cos x)} \cdot dx \\
& = \int \frac{\sin x}{(1 - \cos x)(1 + \cos x)(3 + 2 \cos x)} \cdot dx
\end{aligned}$$

Put  $\cos x = t$

$$\therefore -\sin x \cdot dx = dt$$

$$\therefore \sin x \cdot dx = -dt$$

$$\therefore 1 = \int \frac{1}{(1-t)(1+t)(3+2t)} \cdot (-dt)$$

$$= \int \frac{-1}{(1-t)(1+t)(3+2t)} \cdot dt$$

$$\text{Let } \frac{-1}{(1-t)(1+t)(3+2t)} = \frac{A}{1-t} + \frac{B}{1+t} + \frac{C}{3+2t}$$

$$\therefore -1 = A(1+t)(3+2t) + B(1-t)(3+2t) + C(1-t)(1+t)$$

Put  $1-t=0$ , i.e.  $t=1$ , we get

$$-1 = A(2)(5) + B(0)(5) + C(0)(2)$$

$$\therefore -1 = 10A$$

$$\therefore A = \frac{-1}{10}$$

Put  $1+t=0$ , i.e.  $t=-1$ , we get

$$-1 = A(0)(1) + B(2)(1) + C(2)(0)$$

$$\therefore -1 = 2B$$

$$\therefore B = -\frac{1}{2}$$

Put  $3+2t=0$ , i.e.  $t=-\frac{3}{2}$ , we get

$$-1 = A\left(-\frac{1}{2}\right)(0) + B\left(\frac{5}{2}\right)(0) + C\left(\frac{5}{2}\right)\left(-\frac{1}{2}\right)$$

$$\therefore -1 = -\frac{5}{4}C$$

$$\therefore C = \frac{4}{5}$$

$$\therefore \frac{-1}{(1-t)(1+t)(3+2t)} = \frac{\left(\frac{-1}{10}\right)}{1-t} + \frac{\left(-\frac{1}{2}\right)}{1+t} + \frac{\left(\frac{4}{5}\right)}{3+2t}$$

$$\begin{aligned}
I &= \int \left[ \frac{\left(\frac{-1}{10}\right)}{1-t} + \frac{\left(-\frac{1}{2}\right)}{1+t} + \frac{\left(\frac{4}{5}\right)}{3+2t} \right] \cdot dt \\
&= -\frac{1}{10} \int \frac{1}{1-t} \cdot dt - \frac{1}{2} \int \frac{1}{1+t} \cdot dt + \frac{4}{5} \int \frac{1}{3+2t} \cdot dt \\
&= -\frac{1}{10} \frac{\log|1-t|}{-1} - \frac{1}{2} \log|1+t| + \frac{4}{5} \frac{\log|3+2t|}{2} + c \\
&= \frac{1}{10} \log|1-\cos x| - \frac{1}{2} \log|1+\cos x| + \frac{2}{5} \log|3+2\cos x| + c.
\end{aligned}$$

**Exercise 3.4 | Q 1.22**

Integrate the following w.r.t.  $x$  :  $\frac{5 \cdot e^x}{(e^x + 1)(e^{2x} + 9)}$

**SOLUTION**

$$\text{Let } I = \int \frac{5 \cdot e^x}{(e^x + 1)(e^{2x} + 9)} \cdot dx$$

Put  $e^x = t$

$$\therefore e^x \cdot dx = dt$$

$$\therefore I = 5 \int \frac{1}{(t+1)(t^2+9)} \cdot dt$$

$$\text{Let } \frac{1}{(t+1)(t^2+9)} = \frac{A}{t+1} + \frac{Bt+C}{t^2+9}$$

$$\therefore 1 = A(t^2 + 9) + (Bt + C)(t + 1)$$

Put  $t + 1 = 0$ , i.e.  $t = -1$ , we get

$$1 = A(1 + 9) + C(0)$$

$$\therefore A = \frac{1}{10}$$

Put  $t = 0$ , we get

$$1 = A(9) + C(1)$$

$$\therefore C = 1 - 9A = 1 - \frac{9}{10} = \frac{1}{10}$$

Comparing coefficients of  $t^2$  on both the sides, we get

$$0 = A + B$$

$$\therefore B = -A = -\frac{1}{10}$$

$$\therefore \frac{1}{(t+1)(t^2+9)} = \frac{\left(\frac{1}{10}\right)}{t+1} + \frac{\left(-\frac{1}{10}t + \frac{1}{10}\right)}{t^2+9}$$

$$\begin{aligned} \therefore I &= 5 \int \left[ \frac{\left(\frac{1}{10}\right)}{t+1} + \frac{\left(-\frac{1}{10}t + \frac{1}{10}\right)}{t^2+9} \right] \cdot dt \\ &= \frac{1}{2} \int \frac{1}{t+1} \cdot dt - \frac{1}{2} \int \frac{t}{t^2+9} \cdot dt + \frac{1}{2} \int \frac{t}{t^2+9} \cdot dt \\ &= \frac{1}{2} \log|t+1| - \frac{1}{4} \int \frac{2t}{t^2+9} \cdot dt + \frac{1}{2} \cdot (1) \cdot (3) \tan^{-1}\left(\frac{t}{3}\right) \\ &= \frac{1}{2} \log|t+1| - \frac{1}{4} \int \frac{\frac{d}{dt}(t^2+9)}{t^2+9} \cdot dt + \frac{1}{6} \tan^{-1}\left(\frac{t}{3}\right) \\ &= \frac{1}{2} \log|t+1| - \frac{1}{4} \log|t^2+9| + \frac{1}{6} \tan^{-1}\left(\frac{t}{3}\right) + c \\ &= \frac{1}{2} \log|e^x+1| - \frac{1}{4} \log|e^{2x}+9| + \frac{1}{6} \tan^{-1}\left(\frac{e^x}{3}\right) + c. \end{aligned}$$

Exercise 3.4 | Q 1.23 | Page 145

Integrate the following w.r.t.  $x$  :

$$\frac{2 \log x + 3}{x(3 \log x + 2) \left[ (\log x)^2 + 1 \right]}$$

**SOLUTION**

$$\text{Let } I = \int \frac{2 \log x + 3}{x(3 \log x + 2) \left[ (\log x)^2 + 1 \right]} \cdot dx$$

Put  $\log x = t$

$$\therefore \frac{1}{x} \cdot dx = dt$$

$$\therefore I = \int \frac{2t + 3}{(3t + 2)(t^2 + 1)} \cdot dt$$

$$\text{Let } \frac{2t + 3}{(3t + 2)(t^2 + 1)} = \frac{A}{3t + 2} + \frac{Bt + C}{t^2 + 1}$$

$$\therefore 2t + 3 = A(t^2 + 1) + (Bt + C)(3t + 2)$$

Put  $3t + 2 = 0$  i.e,  $t = -\frac{2}{3}$ , we get

$$2\left(\frac{-2}{3}\right) + 3 = A\left(\frac{4}{9} + 1\right) + \left(\frac{-2}{3}B + C\right)(0)$$

$$\therefore \frac{5}{3} = A\left(\frac{13}{9}\right)$$

$$\therefore A = \frac{15}{13}$$

Put  $t = 0$ , we get

$$3 = A(1) + C(2) = \frac{15}{13} + 2C$$

$$\therefore 2C = 3 - \frac{15}{13} = \frac{24}{13}$$

$$\therefore C = \frac{12}{13}$$

Comparing coefficient of  $t^2$  on both the sides, we get

$$0 = A + 3B$$

$$\begin{aligned}
\therefore B &= -\frac{A}{3} = -\frac{5}{13} \\
\therefore \frac{2t+3}{(3t+2)(t^2+1)} &= \frac{\left(\frac{15}{13}\right)}{3t+2} + \frac{\left(-\frac{5}{13}t + \frac{2}{13}\right)}{t^2+1} \\
\therefore I &= \int \left[ \frac{\left(\frac{15}{13}\right)}{3t+2} + \frac{\left(-\frac{5}{13}t + \frac{12}{13}\right)}{t^2+1} \right] \cdot dt \\
&= \frac{15}{13} \int \frac{1}{3t+2} \cdot dt - \frac{5}{26} \int \frac{2t}{t^2+1} \cdot dt + \frac{12}{13} \int \frac{1}{t^2+1} \cdot dt \\
&= \frac{15}{13} \cdot \frac{1}{3} \log|3t+2| - \frac{5}{26} \log|t^2+1| + \frac{12}{13} \tan^{-1}(t) + c \\
\therefore \left[ \because \frac{d}{dt}(t^2+1) = 2t \text{ and } \int \frac{f'(x)}{f(x)} dx = \log|f(t)| + c \right] \\
&= \frac{5}{13} \log|3 \log x + 2| - \frac{5}{26} \log|(\log x)^2 + 1| + \frac{12}{13} \tan^{-1}(\log x) + c.
\end{aligned}$$

### MISCELLANEOUS EXERCISE 3 [PAGES 148 - 150]

Miscellaneous Exercise 3 | Q 1.01 | Page 148

**Choose the correct option from the given alternatives :**

$$\int \frac{1+x+\sqrt{x+x^2}}{\sqrt{x}+\sqrt{1+x}} \cdot dx =$$

$\frac{1}{2}\sqrt{x+1} + c$

$\frac{2}{3}(x+1)^{\frac{3}{2}} + c$

$\sqrt{x+1} + c$

$2(x-1)^{\frac{3}{2}} + c$

**SOLUTION**

$$\frac{2}{3}(x+1)^{\frac{3}{2}} + c$$

Miscellaneous Exercise 3 | Q 1.02 | Page 148

**Choose the correct options from the given alternatives :**

$$\int \frac{1}{x+x^5} \cdot dx = f(x) + c, \text{ then } \int \frac{x^4}{x+x^5} \cdot dx =$$

**log x - f(x) + c**

$f(x) + \log x + c$

$f(x) - \log x + c$

$\frac{1}{5}x^5 f(x) + c$

**SOLUTION**

$\log x - f(x) + c$

[Hint:  $\int \frac{x^4}{x+x^5} \cdot dx = \int \frac{(x^4+1)-1}{x(x^4+1)} \cdot dx$

$$= \int \left( \frac{1}{x} - \frac{1}{x+x^5} \right) \cdot dx$$

$$= \log x - f(x) + c].$$

Miscellaneous Exercise 3 | Q 1.03 | Page 148

**Choose the correct options from the given alternatives :**

$$\int \frac{\log(3x)}{x \log(9x)} \cdot dx =$$

$\log(3x) - \log(9x) + c$

**log (x) - (log 3) · log (log 9x) + c**

$\log 9 - (\log x) \cdot \log(\log 3x) + c$

$\log(x) + (\log 3) \cdot \log(\log 9x) + c$

**SOLUTION**

$$\log(x) - (\log 3) \cdot \log(\log 9x) + c$$

[ Hint :  $\int \frac{\log 3x}{x \log(x)} \cdot dx = \int \frac{\log\left(\frac{9x}{3}\right)}{x \log(9x)} \cdot dx$

$$= \int \frac{\log(9x) - \log 3}{x \log(9x)} \cdot dx$$
$$= \int \left[ \frac{1}{x} - \frac{\log 3}{x \log(9x)} \right] \cdot dx$$
$$= \int \frac{1}{x} \cdot dx - (\log 3) \int \frac{\left(\frac{1}{x}\right)}{\log(9x)} \cdot dx$$
$$= \log(x) - (\log 3) \cdot \log(\log 9x) + c].$$

Miscellaneous Exercise 3 | Q 1.04 | Page 148

**Choose the correct options from the given alternatives :**

$$\int \frac{\sin^m x}{\cos^{m+2} x} \cdot dx =$$
$$\frac{\tan^{m+1} x}{m+1} + c$$

$$(m+2)\tan^{m+1} x + c$$

$$\frac{\tan^m x}{m} + c$$

$$(m+1)\tan^{m+1} x + c$$

**SOLUTION**

$$\frac{\tan^{m+1} x}{m+1} + c$$

**Choose the correct options from the given alternatives :**

$$\int \tan(\sin^{-1} x) \cdot dx =$$

$$(1 - x^2)^{-\frac{1}{2}} + c$$

$$(1 - x^2)^{\frac{1}{2}} + c$$

$$\frac{\tan^m x}{\sqrt{1 - x^2}} + c$$

$$-\sqrt{1 - x^2} + c$$

**SOLUTION**

$$-\sqrt{1 - x^2} + c$$

$$\left[ \text{Hint : } \sin^{-1} x = \tan^{-1} \left( \frac{x}{\sqrt{1 - x^2}} \right) \right].$$

**Choose the correct options from the given alternatives :**

$$\int \frac{x - \sin x}{1 - \cos x} \cdot dx =$$

$$x \cot\left(\frac{x}{2}\right) + c$$

$$-x \cot\left(\frac{x}{2}\right) + c$$

$$\cot\left(\frac{x}{2}\right) + c$$

$$x \tan\left(\frac{x}{2}\right) + c$$

**SOLUTION**

$$\begin{aligned}
 & -x \cot\left(\frac{x}{2}\right) + c \\
 [\text{Hint:}] \quad & \int \frac{x - \sin x}{1 - \cos x} \cdot dx = \int \frac{x - 2 \sin\left(\frac{x}{2}\right) \cos\left(\frac{x}{2}\right)}{2 \sin^2\left(\frac{x}{2}\right)} \cdot dx \\
 & = \frac{1}{2} \int x \operatorname{cosec}^2\left(\frac{x}{2}\right) \cdot dx - \int \cot\left(\frac{x}{2}\right) \cdot dx \\
 & = \frac{1}{2} \left[ x \int \operatorname{cosec}^2\left(\frac{x}{2}\right) \cdot dx - \int \left[ \frac{d}{dx}(x) \int \operatorname{cosec}^2\left(\frac{x}{2}\right)^{dx} \right] \cdot dx \right] - \int \cot\left(\frac{x}{2}\right) \cdot dx \\
 & = \frac{1}{2} \left[ x \left\{ \frac{-\cot\left(\frac{x}{2}\right)}{\left(\frac{1}{2}\right)} \right\} - \int 1 \cdot \frac{-\cot\left(\frac{x}{2}\right)}{\left(\frac{1}{2}\right)} \cdot dx - \int \cot\left(\frac{x}{2}\right) \cdot dx \right] \\
 & = x \cot\left(\frac{x}{2}\right) + \int \cot\left(\frac{x}{2}\right) \cdot dx - \int \cot\left(\frac{x}{2}\right) \cdot dx \\
 & = -x \cot\left(\frac{x}{2}\right) + c].
 \end{aligned}$$

Miscellaneous Exercise 3 | Q 1.07 | Page 148

**Choose the correct options from the given alternatives :**

If  $f(x) = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$ ,  $g(x) = e^{\sin^{-1} x}$ , then  $\int f(x) \cdot g(x) \cdot dx =$

$e^{\sin^{-1} x} \cdot (\sin^{-1} x - 1) + c$   
 $e^{\sin^{-1} x} \cdot (1 - \sin^{-1} x) + c$   
 $e^{\sin^{-1} x} \cdot (\sin^{-1} x + 1) + c$   
 $-e^{\sin^{-1} x} \cdot (\sin^{-1} x + 1) + c$

**SOLUTION**

$$e^{\sin^{-1} x} \cdot (\sin^{-1} x - 1) + c$$

**Choose the correct options from the given alternatives :**

If  $\int \tan^3 x \cdot \sec^3 x \cdot dx = \left(\frac{1}{m}\right) \sec^m x - \left(\frac{1}{n}\right) \sec^n x + c$ , then  $(m, n) =$

(5, 3)

(3, 5)

$\left(\frac{1}{5}, \frac{1}{3}\right)$

(4, 4)

**SOLUTION**

(5, 3)

$$\begin{aligned} [\text{Hint: } & \int \tan^3 x \cdot \sec^3 x \cdot dx \\ &= \int \sec^2 x \cdot \tan^2 x \cdot \sec x \tan x \cdot dx \\ &= \int \sec^2 x (\sec^2 x - 1) \sec x \tan x \cdot dx \end{aligned}$$

Put  $\sec x = t$ .

**Choose the correct options from the given alternatives :**

$$\begin{aligned} \int \frac{1}{\cos x - \cos^2 x} \cdot dx &= \\ &\log(\operatorname{cosec} x - \cot x) + \tan\left(\frac{x}{2}\right) + c \end{aligned}$$

$\sin 2x - \cos x + c$

$\log(\sec x + \tan x) - \cot\left(\frac{x}{2}\right) + c$

$\cos 2x - \sin x + c$

**SOLUTION**

$$\log(\sec x + \tan x) - \cot\left(\frac{x}{2}\right) + c$$

[ Hint :  $\int \frac{1}{\cos x - \cos^2 x} \cdot dx$

$$= \int \frac{1}{\cos x(1 - \cos x)} \cdot dx$$

$$= \int \frac{(1 - \cos x) + \cos x}{\cos x(1 - \cos x)} \cdot dx$$

$$= \int \left( \frac{1}{\cos x} + \frac{1}{1 - \cos x} \right) \cdot dx$$

$$= \int \left[ \sec x + \frac{1}{2} \operatorname{cosec}^2\left(\frac{x}{2}\right) \right] \cdot dx$$

$$= \log|\sec x + \tan x| \frac{1}{2} \frac{\left(-\frac{\cot x}{2}\right)}{\frac{1}{2}} + c$$

$$= \log|\sec x + \tan x| - \cot\left(\frac{x}{2}\right) + c ] .$$

Miscellaneous Exercise 3 | Q 1.1 | Page 149

**Choose the correct options from the given alternatives :**

$$\int \frac{\sqrt{\cot x}}{\sin x \cdot \cos x} \cdot dx =$$

2 $\sqrt{\cot x} + c$   
 $-2\sqrt{\cot x} + c$   
 $\frac{1}{2}\sqrt{\cot x} + c$   
 $\sqrt{\cot x} + c$

**SOLUTION**

$$-2\sqrt{\cot x} + c$$

**Choose the correct options from the given alternatives :**

$$\int \frac{e^x(x-1)}{x^2} \cdot dx =$$

$$\frac{e^x}{x} + c$$

$$\frac{x}{e^x} + c$$

$$\left(x - \frac{1}{x}\right)e^x + c$$

$$xe^{-x} + c$$

*SOLUTION*

$$\frac{e^x}{x} + c$$

**Choose the correct options from the given alternatives :**

$$\int \sin(\log x) \cdot dx =$$

$$\frac{x}{2} [\sin(\log x) - \cos(\log x)] + c$$

$$\frac{x}{2} [\sin(\log x) + \cos(\log x)] + c$$

$$\frac{x}{2} [\cos(\log x) - \sin(\log x)] + c$$

$$\frac{x}{4} [\cos(\log x) - \sin(\log x)] + c$$

*SOLUTION*

$$\frac{x}{2} [\sin(\log x) - \cos(\log x)] + c$$

**Choose the correct options from the given alternatives :**

$$\int f x^x (1 + \log x) \cdot dx$$

$$\frac{1}{2} (1 + \log x)^2 + c$$

$$x^{2x} + c$$

$$x^x \log x + c$$

$$x^x + c$$

**SOLUTION**

$$x^x + c$$

$$[\text{ Hint : } \frac{d}{dx}(x^x) = x^x(1 + \log x)].$$

**Choose the correct options from the given alternatives :**

$$\int \cos^{-\frac{3}{7}} x \cdot \sin^{-\frac{11}{7}} x \cdot dx =$$

$$\log(\sin^{-\frac{4}{7}} x) + c$$

$$\frac{4}{7} \tan^{\frac{4}{7}} x + c$$

$$-\frac{7}{4} \tan^{-\frac{4}{7}} x + c$$

$$\log(\cos^{\frac{3}{7}} x) + c$$

**SOLUTION**

$$-\frac{7}{4} \tan^{-\frac{4}{7}} x + c$$

$$[\text{ Hint : } \int \cos^{-\frac{3}{7}} x \sin^{-\frac{11}{7}} x \cdot dx]$$

$$\begin{aligned}
 &= \int \frac{\sin^{-\frac{11}{7}} x}{\cos^{-\frac{11}{7}} x \cdot \cos^2 x} \cdot dx \\
 &= \int \tan^{-\frac{11}{7}} x \sec^2 x \cdot dx
 \end{aligned}$$

Put  $\tan x = t$ .

Miscellaneous Exercise 3 | Q 1.15 | Page 149

**Choose the correct options from the given alternatives :**

$$2 \int \frac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin^2 x} \cdot dx =$$

**sin 2x + c**

cos 2x + c

tan 2x + c

2 sin 2x + c

**SOLUTION**

sin 2x + c

Miscellaneous Exercise 3 | Q 1.16 | Page 149

**Choose the correct options from the given alternatives :**

$$\begin{aligned}
 &\int \frac{dx}{\cos x \sqrt{\sin^2 x - \cos^2 x}} \cdot dx = \\
 &\log(\tan x - \sqrt{\tan^2 x - 1}) + c
 \end{aligned}$$

**$\sin^{-1} (\tan x) + c$**

$1 + \sin^{-1} (\cot x) + c$

**$\log(\tan x + \sqrt{\tan^2 x - 1}) + c$**

**SOLUTION**

$$\log\left(\tan x + \sqrt{\tan^2 x - 1}\right) + c$$

[ Hint :  $\int \frac{dx}{\cos x \sqrt{\sin^2 x - \cos^2 x}}$

$$= \int \frac{\sec 2x \cdot dx}{\sqrt{\tan 2x - 1}} \quad \dots[\text{Dividing by } \cos^2 x]$$

Put  $\tan x = t$ .]

Miscellaneous Exercise 3 | Q 1.17 | Page 150

**Choose the correct options from the given alternatives :**

$$\int \frac{\log x}{(\log ex)^2} \cdot dx =$$
$$\frac{x}{1 + \log x} + c$$
$$x(1 + \log x) + c$$
$$\frac{x}{1 + \log x} + c$$
$$\frac{x}{1 - \log x} + c$$

**SOLUTION**

$$\frac{x}{1 + \log x} + c$$

Miscellaneous Exercise 3 | Q 1.18 | Page 150

**Choose the correct options from the given alternatives :**

$$\int [\sin(\log x) + \cos(\log x)] \cdot dx =$$

$$x \cos(\log x) + c$$

$$\sin(\log x) + c$$

$$\cos(\log x) + c$$

$$\mathbf{x \sin(\log x) + c}$$

SOLUTION

$$x \sin(\log x) + c$$

Miscellaneous Exercise 3 | Q 1.19 | Page 150

Choose the correct options from the given alternatives :

$$\int \frac{\cos 2x - 1}{\cos 2x + 1} \cdot dx =$$

$$\tan x - x + c$$

$$x + \tan x + c$$

$$\mathbf{x - \tan x + c}$$

$$-x - \cot x + c$$

SOLUTION

$$x - \tan x + c$$

$$[\text{Hint : } \int \frac{\cos 2x - 1}{\cos 2x + 1} \cdot dx]$$

$$= \int \frac{-(1 - \cos 2x)}{1 + \cos^2 x} \cdot dx$$

$$= \int \frac{-2 \sin^2 x}{2 \cos^2 x} \cdot dx$$

$$= \int (\sec^2 x - 1) \cdot dx$$

$$= -\tan x + x + c.$$

**Choose the correct options from the given alternatives :**

$$\int \frac{e^{2x} + e^{-2x}}{e^x} \cdot dx =$$

$$e^x - \frac{1}{3e^{3x}} + c$$

$$e^x + \frac{1}{3e^{3x}} + c$$

$$e^{-x} + \frac{1}{3e^{3x}} + c$$

$$e^{-x} - \frac{1}{3e^{3x}} + c$$

**SOLUTION**

$$e^x - \frac{1}{3e^{3x}} + c$$

$$[\text{Hint : } \int \frac{e^{2x} + e^{-2x}}{e^x} \cdot dx$$

$$= \int e^x \cdot dx + \int e^{-3x} \cdot dx$$

$$= e^x + \frac{e^{-3x}}{(-3)} + c$$

$$= e^x - \frac{1}{3e^{3x}} + c].$$

Integrate the following with respect to the respective variable :  $(x - 2)^2 \sqrt{x}$

**SOLUTION**

$$\begin{aligned} \text{Let } I &= \int (x-2)^2 \sqrt{x} \cdot dx \\ &= \int (x^2 - 4x + 4) \sqrt{x} \cdot dx \\ &= \int \left( x^{\frac{5}{2}} - 4x^{\frac{3}{2}} + 4x^{\frac{1}{2}} \right) \cdot dx \\ &= \int x^{\frac{5}{2}} \cdot dx - 4 \int x^{\frac{3}{2}} \cdot dx + 4 \int x^{\frac{1}{2}} \cdot dx \\ &= \frac{x^{\frac{7}{2}}}{\left(\frac{7}{2}\right)} - 4 \frac{x^{\frac{5}{2}}}{\left(\frac{5}{2}\right)} + 4 \frac{x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} \\ &= \frac{2}{7}x^{\frac{7}{2}} - 8x^{\frac{5}{2}} + \frac{8}{3}x^{\frac{3}{2}} + c. \end{aligned}$$

**Miscellaneous Exercise 3 | Q 2.2 | Page 150**

integrate the following with respect to the respective variable :  $\frac{x^2}{x+1}$

**SOLUTION**

$$\begin{aligned} \text{Let } I &= \int \frac{x^7}{x+1} \cdot dx \\ &= \int \frac{(x^7+1)-1}{x+1} \cdot dx \\ &= \int \frac{(x+1)(x^6-x^5+x^4-x^3+x^2-x+1)-1}{x+1} \cdot dx \\ &= \int \left[ x^6 - x^5 + x^4 - x^3 + x^2 - x + 1 - \frac{1}{x+1} \right] \cdot dx \\ &= \int x^6 \cdot dx - \int x^5 \cdot dx + \int x^4 \cdot dx - \int x^3 \cdot dx + \int x^2 \cdot dx - \int x \cdot dx + \int 1 \cdot dx - \int \frac{1}{x+1} \cdot dx \\ &= \frac{x^7}{7} - \frac{x^6}{6} + \frac{x^5}{5} - \frac{x^4}{4} + \frac{x^3}{3} - \frac{x^2}{2} + x - \log|x+1| + c. \end{aligned}$$

Miscellaneous Exercise 3 | Q 2.3 | Page 150

Integrate the following with respect to the respective variable :  $(6x + 5)^{\frac{3}{2}}$

**SOLUTION**

$$\begin{aligned} & \int (6x + 5)^{\frac{3}{2}} \cdot dx \\ &= \frac{(6x + 5)^{\frac{5}{2}}}{6 \times \frac{5}{2}} + c \\ &= \frac{1}{15} (6x + 5)^{\frac{5}{2}} + c. \end{aligned}$$

Miscellaneous Exercise 3 | Q 2.4 | Page 150

Integrate the following with respect to the respective variable :  $\frac{t^3}{(t + 1)^2}$

**SOLUTION**

$$\begin{aligned} \text{Let } I &= \int \frac{t^2}{(t + 1)^2} \cdot dt \\ &= \int \frac{(t^3 + 1) - 1}{(t + 1)^2} \cdot dt \\ &= \int \frac{(t + 1)(t^2 - t + 1) - 1}{(t + 1)^2} \cdot dt \\ &= \int \left[ \frac{t^2 - t + 1}{t + 1} - \frac{1}{(t + 1)^2} \right] \cdot dt \\ &= \int \left[ \frac{(t + 1)(t - 2) + 3}{t + 1} - \frac{1}{(t + 1)^2} \right] \cdot dt \end{aligned}$$

$$\begin{aligned}
&= \int \left[ t - 2 + \frac{3}{t+1} - \frac{1}{(t+1)^2} \right] \cdot dt \\
&= \int t \cdot dt - 2 \int 1 \cdot dt + 3 \int \frac{1}{t+1} \cdot dt - \int \frac{1}{(t+1)^2} \cdot dt \\
&= \frac{t^2}{2} - 2t + 3|\log|t+1|| - \frac{(t+1)-1}{(-1)} + c \\
&= \frac{t^2}{2} - 2t + 3\log|t+1| + \frac{1}{t+1} + c.
\end{aligned}$$

Miscellaneous Exercise 3 | Q 2.5 | Page 150

Integrate the following with respect to the respective variable :  $\frac{3 - 2 \sin x}{\cos^2 x}$

**SOLUTION**

$$\begin{aligned}
\text{Let } I &= \int \frac{3 - 2 \sin x}{\cos^2 x} \cdot dx \\
&= \int \left( \frac{3}{\cos^2 x} - \frac{2 \sin x}{\cos^2 x} \right) \cdot dx \\
&= 3 \int \sec^2 x \cdot dx - 2 \int \sec x \tan x \cdot dx \\
&= 3 \tan x - 2 \sec x + c.
\end{aligned}$$

Miscellaneous Exercise 3 | Q 2.6 | Page 150

Integrate the following with respect to the respective variable :  $\frac{\sin^6 \theta + \cos^6 \theta}{\sin^2 \theta \cdot \cos^2 \theta}$

**SOLUTION**

$$\begin{aligned}
& \int \frac{\sin^6 \theta + \cos^6 \theta}{\sin^2 \theta \cdot \cos^2 \theta} \\
&= \int \left[ \frac{(\sin^2 \theta + \cos^2 \theta)^3 - 3 \sin^2 \theta \cdot \cos^2 \theta (\sin^2 \theta + \cos^2 \theta)}{\sin^2 \theta \cdot \cos^2 \theta} \right] \cdot d\theta \quad \dots [\because a^3 + b^3 = (a+b)^3 - 3ab(a+b)] \\
&= \int \left[ \frac{(1)^3 - 3 \sin^2 \theta \cdot \cos^2 \theta (1)}{\sin^2 \theta \cdot \cos^2 \theta} \right] \cdot d\theta \\
&= \int \left[ \frac{1}{\sin^2 \theta \cdot \cos^2 \theta} - 3 \right] \cdot d\theta \\
&= \int \left[ \frac{\sin^2 \theta + \cos^2 \theta}{\sin^2 \theta \cdot \cos^2 \theta} - 3 \right] \cdot d\theta \\
&= \int \left( \frac{1}{\cos^2 \theta} + \frac{1}{\sin^2 \theta} - 3 \right) \cdot d\theta \\
&= \int (\sec^2 \theta + \operatorname{cosec}^2 \theta - 3) \cdot d\theta \\
&= \int \sec^2 \theta \cdot d\theta + \int \operatorname{cosec}^2 \theta \cdot d\theta - 3 \int 1 \cdot d\theta \\
&= \tan \theta - \cot \theta - 3\theta + c.
\end{aligned}$$

#### Miscellaneous Exercise 3 | Q 2.7 | Page 150

Integrate the following with respect to the respective variable :  $\cos 3x \cos 2x \cos x$

**SOLUTION**

$$\text{Let } I = \int \cos 3x \cos 2x \cos x \cdot dx$$

$$\text{Consider } \cos 3x \cos 2x \cos x = \frac{1}{2} \cos 3x [2 \cos 2x \cos x]$$

$$\begin{aligned}
&= \frac{1}{2} \cos 3x [\cos(2x+x) + \cos(2x-x)] \\
&= \frac{1}{2} [\cos^2 3x + \cos 3x \cos x] \\
&= \frac{1}{4} [2 \cos^2 3x + 2 \cos 3x \cos x]
\end{aligned}$$

$$\begin{aligned}
&= \frac{1}{4} [1 + \cos 6x + \cos(3x + x) + \cos(3x - x)] \\
&= \frac{1}{4} [1 + \cos 6x + \cos 4x + \cos 2x] \\
\therefore I &= \frac{1}{4} \int [1 + \cos 6x + \cos 4x + \cos 2x] \cdot dx \\
&= \frac{1}{4} \int 1 \cdot dx + \frac{1}{4} \int \cos 6x \cdot dx + \frac{1}{4} \int \cos 4x \cdot dx + \frac{1}{4} \int \cos 2x \cdot dx \\
&= \frac{x}{4} + \frac{1}{4} \left( \frac{\sin 6x}{6} \right) + \frac{1}{4} \left( \frac{\sin 4x}{4} \right) + \frac{1}{4} \left( \frac{\sin 2x}{2} \right) + c \\
&= \frac{1}{48} [12x + 2 \sin 6x + 3 \sin 4x + 6 \sin 2x] + c.
\end{aligned}$$

Miscellaneous Exercise 3 | Q 2.8 | Page 150

Integrate the following with respect to the respective variable :  $\frac{\cos 7x - \cos 8x}{1 + 2 \cos 5x}$

**SOLUTION**

$$\begin{aligned}
&\int \frac{\cos 7x - \cos 8x}{1 + 2 \cos 5x} \cdot dx \\
&= \int \frac{\sin 5x(\cos 7x - \cos 8x)}{\sin 5x(1 + 2 \cos 5x)} \cdot dx \\
&= \int \frac{\sin 5x(\cos 7x - \cos 8x)}{\sin 5x + 2 \sin 5x \cos 5x} \cdot dx \\
&= \int \frac{\sin 5x(\cos 7x - \cos 8x)}{\sin 5x + \sin 10x} \cdot dx \\
&= \int \frac{2 \sin\left(\frac{5x}{2}\right) \cdot \cos\left(\frac{5x}{2}\right) \times 2 \sin\left(\frac{7x+8x}{2}\right) \cdot \sin\left(\frac{8x-7x}{2}\right)}{2 \sin\left(\frac{10x+5x}{2}\right) \cdot \cos\left(\frac{10x-5x}{2}\right)} \cdot dx \\
&= \int \frac{2 \sin\left(\frac{5x}{2}\right) \cdot \cos\left(\frac{5x}{2}\right) \times 2 \sin\left(\frac{15x}{2}\right) \cdot \sin\left(\frac{x}{2}\right)}{2 \sin\left(\frac{15x}{2}\right) \cdot \cos\left(\frac{5x}{2}\right)} \cdot dx
\end{aligned}$$

$$\begin{aligned}
&= \int 2 \sin\left(\frac{5x}{2}\right) \cdot \sin\left(\frac{x}{2}\right) \cdot dx \\
&= \int \left[ \cos\left(\frac{5x}{2} - \frac{x}{2}\right) - \cos\left(\frac{5x}{2} + \frac{x}{2}\right) \right] \cdot dx \\
&= \int (\cos 2x - \cos 3x) \cdot dx \\
&= \int \cos 2x \cdot dx - \int \cos 3x \cdot dx \\
&= \frac{\sin 2x}{2} - \frac{\sin 3x}{3} + c.
\end{aligned}$$

Miscellaneous Exercise 3 | Q 2.9 | Page 150

Integrate the following with respect to the respective variable :  $\cot^{-1}\left(\frac{1 + \sin x}{\cos x}\right)$

**SOLUTION**

$$\begin{aligned}
\text{Let } I &= \int \cot^{-1}\left(\frac{1 + \sin x}{\cos x}\right) \cdot dx \\
\frac{1 + \sin x}{\cos x} &= \frac{1 + \cos\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right)} \\
&= \frac{2 \cos^2\left(\frac{\pi}{4} - \frac{x}{2}\right)}{2 \sin\left(\frac{\pi}{4} - \frac{x}{2}\right) \cdot \cos\left(\frac{\pi}{4} - \frac{x}{2}\right)} \\
&= \cot\left(\frac{\pi}{6} - \frac{x}{2}\right) \\
\therefore I &= \int \cot^{-1}\left[\cot\left(\frac{\pi}{4} - \frac{x}{2}\right)\right] \cdot dx \\
&= \int \left(\frac{\pi}{4} - \frac{x}{2}\right) \cdot dx \\
&= \frac{\pi}{4} \int 1 \cdot dx - \frac{1}{2} \int x \cdot dx
\end{aligned}$$

$$\begin{aligned}
 &= \frac{\pi}{4} \cdot x - \frac{1}{2} \cdot \frac{x^2}{2} + c \\
 &= \frac{\pi}{4}x - \frac{1}{4}x^2 + c.
 \end{aligned}$$

Miscellaneous Exercise 3 | Q 3.01 | Page 150

Integrate the following w.r.t. x:  $\frac{(1 + \log x)^2}{x}$

**SOLUTION**

$$\text{Let } I = \int \frac{(1 + \log x)^2}{x} \cdot dx$$

$$\text{Put } 1 + \log x = t$$

$$\therefore \frac{1}{x} \cdot dx = dt$$

$$\therefore I = \int t^3 \cdot dt = \frac{1}{4}t^4 + c$$

$$= \frac{1}{4}(1 + \log x)^4 + c.$$

Miscellaneous Exercise 3 | Q 3.02 | Page 150

Integrate the following w.r.t.x :  $\cot^{-1}(1 - x + x^2)$

**SOLUTION**

$$\begin{aligned}
 \text{Let } I &= \int \cot^{-1}(1-x+x^2) \cdot dx \\
 &= \int \tan^{-1}\left(\frac{1}{1-x+x^2}\right) \cdot dx \\
 &= \int \tan^{-1}\left[\frac{x+(1-x)}{1-x(1-x)}\right] \\
 &= \int [\tan^{-1}x + \tan^{-1}(1-x)] \cdot dx \\
 &= \int \tan^{-1}x \cdot dx + \int \tan^{-1}(1-x) \cdot dx \\
 \therefore I &= I_1 + I_2 \quad \dots(1)
 \end{aligned}$$

$$\begin{aligned}
 I_1 &= \int \tan^{-1}x \cdot dx = \int (\tan^{-1}x) 1 \cdot dx \\
 &= (\tan^{-1}x) \cdot \int 1 dx - \left[ \frac{d}{dx}(\tan^{-1}x) \cdot \int 1 dx \right] \cdot dx \\
 &= (\tan^{-1}x)x - \int \frac{1}{1+x^2} \cdot x \cdot dx \\
 &= x \tan^{-1}x - \frac{1}{2} \int \frac{2x}{1+x^2} \cdot dx \\
 \therefore I_1 &= x \tan^{-1}x - \frac{1}{2} \log|1+x^2| + c_1 \\
 &\quad \dots \left[ \because \frac{d}{dx}(1+x^2) = 2x \text{ and } \int \frac{f'(x)}{f(x)} dx = \log|f(x)| + c \right]
 \end{aligned}$$

$$\begin{aligned}
 I_2 &= \int \tan^{-1}(1-x) \cdot dx \\
 &= \int \tan^{-1}(1-x) \cdot 1 dx
 \end{aligned}$$

$$\begin{aligned}
&= [\tan^{-1}(1-x)] \cdot \int 1 dx - \int \left\{ \frac{d}{dx} [\tan^{-1}(1-x)] \cdot \int 1 dx \right\} \cdot dx \\
&= [\tan^{-1}(1-x)] \cdot x - \int \frac{1}{1+(1-x)^2} \cdot (-1) \cdot x dx \\
&= x \tan^{-1}(1-x) + \int \frac{x}{1+1-2x+x^2} \cdot dx \\
&= x \tan^{-1}(1-x) + \int \frac{x}{2-2x+x^2} \cdot dx \\
&\text{Let } x = A \left[ \frac{d}{dx} (2-2x+x^2) \right] + B
\end{aligned}$$

$$\therefore x = A(-2+2x) + B = 2Ax + (-2A+B)$$

Comparing the coefficient of x and constant on both the sides, we get

$$1 = 2A \text{ and } 0 = -2A + B$$

$$\therefore A = \frac{1}{2} \text{ and } 0 = -2\left(\frac{1}{2}\right) + B$$

$$\therefore B = 1$$

$$\therefore x = \frac{1}{2}(-2+2x) + 1$$

$$\begin{aligned}
&\therefore I_2 = x \tan^{-1}(1-x) + \int \frac{\frac{1}{2}(-2+2x)+1}{2-2x+x^2} \cdot dx \\
&= x \tan^{-1}(1-x) + \frac{1}{2} \frac{-2+2x}{2-2x+x^2} \cdot dx + \int \frac{1}{2-2x+x^2} \cdot dx \\
&= x \tan^{-1}(1-x) + \frac{1}{2} \log|2-2x+x^2| + \int \frac{1}{1+(1-2x+x^2)} \cdot dx \\
&= x \tan^{-1}(1-x) + \frac{1}{2} \log|x^2-2x+2| + \int \frac{1}{1+(1-x^2)} \cdot dx
\end{aligned}$$

$$\begin{aligned}
&= x \tan^{-1}(1-x) + \frac{1}{2} \log|x^2 - 2x + 2| + \frac{1}{1} \frac{\tan^{-1}(1-x)}{-1} + c_2 \\
&= x \tan^{-1}(1-x) + \frac{1}{2} \log|x^2 - 2x + 2| - \tan^{-1}(1-x) + c_2 \\
&= (x-1) \tan^{-1}(1-x) + \frac{1}{2} \log|x^2 - 2x + 2| + c_2 \\
\therefore I &= -(1-x) \tan^{-1}(1-x) + \frac{1}{2} \log|x^2 - 2x + 2| + c_2 \quad \dots(3)
\end{aligned}$$

From (1), (2) and (3), we get

$$\begin{aligned}
I &= x \tan^{-1} x - \frac{1}{2} \log|1+x^2| + c_1 - (1-x) \tan^{-1}(1-x) + \frac{1}{2} \log|x^2 - 2x + 2| + c_2 \\
&= x \tan^{-1} x - \frac{1}{2} \log|1+x^2| - (1-x) \tan^{-1}(1-x) + \frac{1}{2} |x^2 - 2x + 2| + c, \text{ where } c = c_1 + c_2.
\end{aligned}$$

### Miscellaneous Exercise 3 | Q 3.03 | Page 150

Integrate the following w.r.t.x :  $\frac{1}{x \sin^2(\log x)}$

**SOLUTION**

$$\text{Let } I = \int \frac{1}{x \sin^2(\log x)} \cdot dx$$

Put  $\log x = t$

$$\therefore \frac{1}{x} \cdot dx = dt$$

$$\therefore I = \int \frac{1}{\sin^2 t} \cdot dt$$

$$= \int \operatorname{cosec}^2 dt$$

$$= -\cot t + c$$

$$= \cot(\log x) + c.$$

Miscellaneous Exercise 3 | Q 3.04 | Page 150

Integrate the following w.r.t.x :  $\sqrt{x} \sec\left(x^{\frac{3}{2}}\right) \cdot \tan\left(x^{\frac{3}{2}}\right)$

**SOLUTION**

$$\text{Let } I = \int \sqrt{x} \sec\left(x^{\frac{3}{2}}\right) \cdot \tan\left(x^{\frac{3}{2}}\right) dx$$

$$\text{Put } x^{\frac{3}{2}} = t$$

$$\therefore \frac{3}{2} \sqrt{x} \cdot dx = dt$$

$$\therefore \sqrt{x} \cdot dx = \frac{2}{3} \cdot dt$$

$$\therefore I = \frac{2}{3} \int \sec t \tan t \cdot dt$$

$$= \frac{2}{3} \sec t + c$$

$$= \frac{2}{3} \sec\left(x^{\frac{3}{2}}\right) + c.$$

Miscellaneous Exercise 3 | Q 3.05 | Page 150

Integrate the following w.r.t.x :  $\log(1 + \cos x) - x \tan\left(\frac{x}{2}\right)$

**SOLUTION**

$$\begin{aligned}
 \text{Let } I &= \int \left[ \log(1 + \cos x) - x \tan\left(\frac{x}{2}\right) \right] \cdot dx \\
 &= \int \left[ \log(1 + \cos x) \cdot 1 dx - \int x \tan\left(\frac{x}{2}\right) \cdot dx \right] \\
 &= [\log(1 + \cos x)] \cdot \int 1 dx - \int \left\{ \frac{d}{dx} [\log(1 + \cos x)] \cdot \int 1 dx \right\} \cdot dx - x \tan\left(\frac{x}{2}\right) \cdot dx \\
 &= [\log(1 + \cos x)] \cdot (x) - \int \frac{1}{1 + \cos x} \cdot (0 - \sin x) \cdot x dx - \int x \tan\left(\frac{x}{2}\right) \cdot dx \\
 &= x \cdot \log(1 + \cos x) + \int x \cdot \frac{\sin x}{1 + \cos x} \cdot dx - \int x \tan\left(\frac{x}{2}\right) \cdot dx + c \\
 &= x \cdot \log(1 + \cos x) + \int x \cdot \frac{2 \sin\left(\frac{x}{2}\right) \cdot \cos\left(\frac{x}{2}\right)}{2 \cos^2\left(\frac{x}{2}\right)} \cdot dx - \int x \tan\left(\frac{x}{2}\right) \cdot dx + c \\
 &= x \log(1 + \cos x) + \int x \cdot \tan\left(\frac{x}{2}\right) \cdot dx - \int x \tan\left(\frac{x}{2}\right) \cdot dx + c \\
 &= x \log(1 + \cos x) + c.
 \end{aligned}$$

**Miscellaneous Exercise 3 | Q 3.06 | Page 150**

Integrate the following w.r.t.x :  $\frac{x^2}{\sqrt{1-x^6}}$

**SOLUTION**

$$\text{Let } I = \int \frac{x^2}{\sqrt{1-x^6}} \cdot dx$$

Put  $x^3 = t$

$$\therefore 3x^2 dx = dt$$

$$\therefore x^2 dx = \frac{1}{3} \cdot dt$$

$$\begin{aligned}\therefore I &= \frac{1}{3} \int \frac{1}{\sqrt{1-t^2}} \cdot dt \\ &= \frac{1}{3} \sin^{-1}(t) + c \\ &= \frac{1}{3} \sin^{-1}(x^3) + c.\end{aligned}$$

Miscellaneous Exercise 3 | Q 3.07 | Page 150

Integrate the following w.r.t.x :  $\frac{1}{(1-\cos 4x)(3-\cot 2x)}$

**SOLUTION**

$$\begin{aligned}\text{Let } I &= \int \frac{1}{(1-\cos 4x)(3-\cot 2x)} \cdot dx \\ &= \int \frac{1}{2 \sin^2 2x(3-\cot 2x)} \cdot dx \\ &= \frac{1}{2} \int \frac{\operatorname{cosec}^2 x}{3-\cot 2x} \cdot dx\end{aligned}$$

Put  $3 - \cot 2x = t$

$$\therefore 2 \operatorname{cosec}^2 2x \cdot dx = dt$$

$$\therefore \operatorname{cosec}^2 2x \cdot dx = \frac{1}{2} \cdot dt$$

$$\begin{aligned}\therefore I &= \frac{1}{4} \int \frac{1}{t} \cdot dt \\ &= \frac{1}{4} \log|t| + c \\ &= \frac{1}{4} \log|3 - \cot 2x| + c.\end{aligned}$$

Miscellaneous Exercise 3 | Q 3.08 | Page 150

Integrate the following w.r.t.x :  $\log(\log x) + (\log x)^{-2}$

**SOLUTION**

$$\text{Let } I = \int [\log(\log x) + (\log x)^{-2}] \cdot dx$$

$$= \int \left[ \log(\log x) + \frac{1}{(\log x)^2} \right] \cdot dx$$

$$\text{Put } \log x = t$$

$$\therefore x = e^t$$

$$\therefore x = e^t \cdot dt$$

$$\therefore I = \int \left( \log t + \frac{1}{t^2} \right) e^t \cdot dt$$

$$= \int e^t \left( \log t + \frac{1}{t} - \frac{1}{t} + \frac{1}{t^2} \right) \cdot dt$$

$$= \int \left[ e^t \left( \log t \frac{1}{t} \right) + e^t \left( -\frac{1}{t} + \frac{1}{t^2} \right) \right] \cdot dt$$

$$= \int e^t \left( \log t + \frac{1}{t} \right) \cdot dt - \int e^t \left( \frac{1}{t} - \frac{1}{t^2} \right) \cdot dt$$

$$= I_1 - I_2$$

$$\text{In } I_1, \text{ Put } f(t) = \log t. \text{ Then } f'(t) = \left( \frac{1}{t} \right)$$

$$\therefore I_1 = \int e^t [f(t) + f'(t)] \cdot dt$$

$$= e^t f(t)$$

$$= e^t \log t$$

$$\text{In } I_2, \text{ Put } g(t) = \left( \frac{1}{t} \right). \text{ Then } g'(t) = -\left( \frac{1}{t^2} \right)$$

$$\therefore I_2 = \int e^t [g(t) + g'(t)] \cdot dt$$

$$= e^t g(t)$$

$$= e^t \cdot \left( \frac{1}{t} \right)$$

$$\therefore I = e^t \log t - \frac{e^t}{t} + c$$

$$= x \log(\log x) - \frac{x}{\log x} + c.$$

Miscellaneous Exercise 3 | Q 3.09 | Page 150

Integrate the following w.r.t.x :  $\frac{1}{2 \cos x + 3 \sin x}$

**SOLUTION**

$$\text{Let } I = \int \frac{1}{2 \cos x + 3 \sin x} \cdot dx$$

$$= \int \frac{1}{3 \sin x + 2 \cos x} \cdot dx$$

Dividing numerator and denominator by

$\sqrt{3^2 + 2^2} = \sqrt{13}$ , we get

$$I = \int \frac{\left( \frac{1}{\sqrt{13}} \right)}{\frac{3}{\sqrt{13}} \sin x + \frac{2}{\sqrt{13}} \cos x} \cdot dx$$

$$\text{Since, } \left( \frac{3}{\sqrt{13}} \right)^2 + \left( \frac{2}{\sqrt{13}} \right)^2 = \frac{9}{13} + \frac{4}{13} = 1,$$

$$\text{we take } \frac{3}{\sqrt{13}} = \cos \infty, \frac{2}{\sqrt{13}} = \sin \infty$$

$$\text{so that } \infty = \frac{2}{3} \text{ and } \infty = \tan^{-1} \left( \frac{2}{3} \right)$$

$$\begin{aligned}
I &= \frac{1}{\sqrt{13}} \int \frac{1}{\sin x + \cos x + \cos x \sin x} \cdot dx \\
&= \frac{1}{\sqrt{13}} \int \frac{1}{\sin(x + \infty)} \cdot dx \\
&= \frac{1}{\sqrt{13}} \int \csc(x + \infty) \cdot dx \\
&= \frac{1}{\sqrt{13}} \log|\tan|\tan\left(\frac{x + \infty}{2}\right)| + c \\
&= \frac{1}{\sqrt{13}} \log\left|\tan\left(\frac{x + \tan^{-1} \frac{2}{3}}{2}\right)\right| + c.
\end{aligned}$$

Alternative Method

$$\text{Let } I = \int \frac{1}{2 \cos x + 3 \sin x} \cdot dx$$

$$\text{Put } \tan\left(\frac{x}{2}\right) = t$$

$$\therefore \frac{x}{2} = \tan^{-1} t$$

$$\therefore x = 2\tan^{-1} t$$

$$\therefore dx = \frac{2}{1+t^2} \cdot dt$$

and

$$\sin x = \frac{2t}{1+t^2}$$

and

$$\cos x = \frac{1-t^2}{1+t^2}$$

$$\begin{aligned}\therefore I &= \int \frac{1}{2\left(\frac{1-t^2}{1+t^2}\right) + 3\left(\frac{2t}{1+t^2}\right)} \cdot \frac{2dt}{1+t^2} \\&= \int \frac{1+t^2}{2-2t^2+6t} \cdot \frac{2dt}{1+t^2} \\&= \int \frac{1}{1-t^2+3t} \cdot dt \\&= \int \frac{1}{1-\left(t^2-3t+\frac{9}{4}\right)+\frac{9}{4}} \cdot dt \\&= \int \frac{1}{\left(\frac{\sqrt{13}}{2}\right)^2 - \left(t-\frac{3}{2}\right)^2} \cdot dt \\&= \frac{1}{2 \times \frac{\sqrt{13}}{2}} \log \left| \frac{\frac{\sqrt{13}}{2} + t - \frac{3}{2}}{\frac{\sqrt{13}}{2} - t + \frac{3}{2}} \right| + c \\&= \frac{1}{\sqrt{13}} \log \left| \frac{\sqrt{13} + 2t - 3}{\sqrt{13} - 2t + 3} \right| + c \\&= \frac{1}{\sqrt{13}} \log \left| \frac{\sqrt{13} + 2 \tan\left(\frac{x}{2}\right) - 3}{\sqrt{13} - 2 \tan\left(\frac{x}{2}\right) - 3} \right| + c.\end{aligned}$$

Miscellaneous Exercise 3 | Q 3.1 | Page 150

Integrate the following w.r.t.x :  $\frac{1}{x^3 \sqrt{x^2 - 1}}$

**SOLUTION**

$$\text{Let } I = \int \frac{1}{x^3 \sqrt{x^2 - 1}} \cdot dx$$

Put  $x = \sec\theta$

$$\therefore dx = \sec\theta \tan\theta d\theta$$

$$\therefore I = \int \frac{\sec\theta \tan\theta d\theta}{\sec 3\theta \sqrt{\sec^2\theta - 1}}$$

$$= \int \frac{\sec\theta \tan\theta d\theta}{\sec^3\theta \sqrt{\tan^2\theta}} \cdot d\theta$$

$$\therefore I = \int \cos^2\theta \cdot d\theta$$

$$= \frac{1}{2} \int (1 + \cos 2\theta) \cdot d\theta$$

$$= \frac{1}{2} \int d\theta + \frac{1}{2} \int \cos 2\theta \cdot d\theta$$

$$= \frac{\theta}{2} + \frac{1}{2} \left( \frac{\sin 2\theta}{2} \right) + c \quad \dots(1)$$

$$\therefore x = \sec\theta$$

$$\therefore \theta = \sec^{-1}x$$

$$\sin 2\theta = 2 \sin\theta \cos\theta$$

$$= 2\sqrt{1 - \cos^2\theta} \cdot \cos\theta$$

$$= 2\sqrt{1 - \frac{1}{x^2}} \left( \frac{1}{x} \right) \dots \left[ \because \sec\theta = x \Rightarrow \cos\theta = \frac{1}{x} \right]$$

$$= \frac{2\sqrt{x^2 - 1}}{x^2}$$

∴ from (1), we have

$$I = \frac{1}{2} \sec^{-1} x + \frac{1}{2} \frac{\sqrt{x^2 - 1}}{x^2} + c.$$

Miscellaneous Exercise 3 | Q 3.11 | Page 150

Integrate the following w.r.t.x :  $\frac{3x + 1}{\sqrt{-2x^2 + x + 3}}$

**SOLUTION**

$$\text{Let } I = \int \frac{3x + 1}{\sqrt{-2x^2 + x + 3}} \cdot dx$$

$$\text{Let } 3x + 1 = A \left[ \frac{d}{dx} (-2x^2 + x + 3) \right] + B$$

$$= A(2 - 2x) + B$$

$$\therefore 3x + 1 = 2Ax + (2A + B)$$

Comparing the coefficient of x and constant on both the sides, we get

$$-2A = 7 \text{ and } 2A + B = 3$$

$$\therefore A = \frac{-7}{2} \text{ and } 2\left(\frac{-7}{2}\right) + B = 3$$

$$\therefore B = 10$$

$$\therefore 7x + 3 = \frac{-7}{2}(2 - 2x) + 10$$

$$\therefore I = \int \frac{\frac{-7}{2}(2 - 2x) + 10}{\sqrt{3 + 2x - x^2}} \cdot dx$$

$$= \frac{-7}{2} \int \frac{(2 - 2x)}{\sqrt{3 + 2x - x^2}} \cdot dx + 10 \int \frac{1}{\sqrt{3 + 2x - x^2}} x$$

$$= \frac{-7}{2} I_1 + 10 I_2$$

In I<sub>1</sub>, put  $3 + 2x - x^2 = t$

$$\therefore (2 - 2x)dx = dt$$

$$\therefore I_1 = \int \frac{1}{\sqrt{t}} dt$$

$$= \int t^{-\frac{1}{2}} dt$$

$$= \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + c_1$$

$$= 2\sqrt{3 + 2x - x^2} + c_1$$

$$I_2 = \int \frac{1}{\sqrt{3 - (x^2 - 2x + 1) + 1}} \cdot dx$$

$$= \int \frac{1}{\sqrt{(2)^2 - (x - 1)^2}} \cdot dx$$

$$= \sin^{-1}\left(\frac{x - 1}{2}\right) + c_2$$

$$= \frac{3}{2}\sqrt{-2x^2 + x + 3} + \frac{7}{4\sqrt{2}}\sin^{-1}\left(\frac{4x - 1}{5}\right) + c.$$

Miscellaneous Exercise 3 | Q 3.12 | Page 150

Integrate the following w.r.t.x :  $\log(x^2 + 1)$

**SOLUTION**

$$\begin{aligned} \text{Let } I &= \int \log(x^2 + 1) \cdot dx \\ &= \int [\log(x^2 + 1)] \cdot 1 dx \\ &= [\log(x^2 + 1)] \int 1 dx - \int \left[ \frac{d}{dx} \{\log(x^2 + 1)\} \int 1 dx \right] \cdot dx \\ &= [\log(x^2 + 1)] \cdot x - \int \frac{1}{x^2 + 1} \cdot dx (x^2 + 1) \cdot x dx \\ &= x \log(x^2 + 1) - \int \frac{2x^2}{x^2 + 1} \cdot dx \\ &= x \log(x^2 + 1) - \int \frac{2x^2 + 2 - 2}{x^2 + 1} \cdot dx \\ &= x \log(x^2 + 1) - \int \left[ \frac{2(x^2 + 1)}{x^2 + 1} - \frac{2}{x^2 + 1} \right] \cdot dx \\ &= x \log(x^2 + 1) - \int \left[ 2 \int 1 dx - 2 \int \frac{1}{x^2 + 1} \cdot dx \right] \\ &= x \log(x^2 + 1) - 2x + 2 \tan^{-1} x + C. \end{aligned}$$

Miscellaneous Exercise 3 | Q 3.13 | Page 150

Integrate the following w.r.t.x :  $e^{2x} \sin x \cos x$

**SOLUTION**

$$\begin{aligned}
 \text{Let } I &= \int e^{2x} \cdot \sin x \cos x \cdot dx \\
 &= \frac{1}{2} \int e(2x) \cdot 2 \sin x \cos x dx \\
 &= \frac{1}{2} \int e^{2x} \cdot \sin 2x \cdot dx \quad \dots(1) \\
 &= \frac{1}{2} \left[ e^{2x} \int \sin 2x \cdot dx - \int \left\{ \frac{d}{dx}(e^{2x}) \int \sin 2x \cdot dx \right\} \cdot dx \right] \\
 &= \frac{1}{2} \left[ e(2x) \left( \frac{-\cos 2x}{2} \right) - \int e^{2x} \times 2 \times \left( \frac{-\cos 2x}{2} \right) \cdot dx \right] \\
 &= -\frac{1}{4} e^{2x} \cos 2x + \frac{1}{2} \int e^{2x} \cos 2x \cdot dx \\
 &= -\frac{1}{4} e^{2x} \cos 2x + \frac{1}{2} \left[ e^{2x} \int \cos 2x \cdot dx - \int \left\{ \frac{d}{dx}(e^{2x}) \int \cos 2x \cdot dx \right\} \cdot dx \right] \\
 &= \frac{1}{4} e^{2x} \cos 2x + \frac{1}{2} \left[ e^{2x} \cdot \frac{\sin 2x}{2} - \int e^{2x} \times 2 \times \frac{\sin 2x}{2} \cdot dx \right] \\
 &= -\frac{1}{4} e^{2x} \cos 2x + \frac{1}{4} e^{2x} \sin 2x - \frac{1}{2} \int e^{2x} \sin 2x \cdot dx \\
 \therefore I &= -\frac{1}{4} e^{2x} \cos 2x + \frac{1}{4} e^{2x} \sin 2x - I \quad \dots[\text{By (1)}] \\
 \therefore 2I &= -\frac{1}{4} e^{2x} \cos 2x + \frac{1}{4} e^{2x} \sin 2x \\
 \therefore I &= \frac{e^{2x}}{8} (\sin 2x - \cos 2x) + c.
 \end{aligned}$$

Miscellaneous Exercise 3 | Q 3.14 | Page 150

Integrate the following w.r.t.x :  $\frac{x^2}{(x-1)(3x-1)(3x-2)}$

**SOLUTION**

$$\text{Let } I = \int \frac{x^2}{(x-1)(3x-1)(3x-2)} \cdot dx$$

$$\begin{aligned}\text{Let } & \frac{x^2}{(x-1)(3x-1)(3x-2)} \\ &= \frac{A}{x-1} + \frac{B}{3x-1} + \frac{C}{3x-2}\end{aligned}$$

$$\therefore x^2 = A(3x-1)(3x-2) + B(x-1)(3x-2) + C(x-1)(3x-1)$$

Put  $x-1=0$ , i.e.  $x=1$ , we get

$$\therefore x^2 = A(2)(1) + B(0)(1) + C(0)(2)$$

$$\therefore 2 = 4A$$

$$\therefore A = \frac{1}{2}$$

Put  $x+2=0$ , i.e.  $x=-2$ , we get

$$2+2 = A(0)(1) + B(-3)(1) + C(-3)(0)$$

$$\therefore 6 = -3B$$

$$\therefore B = -2$$

Put  $x+3=0$ , i.e.  $x=-3$  we get

$$9+2 = A(-1)(0) + B(-4)(0) + C(-4)(-1)$$

$$\therefore 11 = 4C$$

$$\therefore C = \frac{11}{4}$$

$$\therefore \frac{x^2+2}{(3x-1)(x-1)(3x-2)} = \frac{\left(\frac{1}{4}\right)}{3x-1} + \frac{-2}{x-1} + \frac{\left(\frac{11}{4}\right)}{3x-2}$$

$$\therefore I = \int \left[ \frac{\left(\frac{1}{4}\right)}{3x-1} + \frac{-2}{x-1} + \frac{\left(\frac{11}{4}\right)}{3x-2} \right] \cdot dx$$

$$\begin{aligned}
&= \frac{1}{18} \int \frac{1}{3x-1} \cdot dx - 2 \int \frac{1}{x-1} \cdot dx + \frac{4}{9} \int \frac{1}{3x-2} \cdot dx \\
&= \frac{1}{18} \log|3x-1| + \frac{1}{2} \log|x-1| - \frac{4}{9} \log|3x-2| + c.
\end{aligned}$$

Miscellaneous Exercise 3 | Q 3.15 | Page 150

Integrate the following w.r.t.x :  $\frac{1}{\sin x + \sin 2x}$

**SOLUTION**

$$\begin{aligned}
\text{Let } I &= \int \frac{1}{\sin x + \sin 2x} \cdot dx \\
&= \int \frac{1}{\sin x + 2 \sin x \cos x} \cdot dx \\
&= \int \frac{dx}{\sin x(1 + 2 \cos x)} \\
&= \int \frac{\sin x \cdot dx}{\sin^2 x(1 + 2 \cos x)} \\
&= \int \frac{\sin \cdot dx}{(1 - \cos^2 x)(1 + 2 \cos x)} \\
&= \int \frac{\sin \cdot dx}{(1 - \cos x)(1 + \cos x)(1 + 2 \cos x)}
\end{aligned}$$

Put  $\cos x = t$

$$\therefore -\sin x \cdot dx = dt$$

$$\therefore \sin x \cdot dx = -dt$$

$$\begin{aligned}
\therefore I &= \int \frac{-dt}{(1-t)(1+t)(1+2t)} \\
&= - \int \frac{dt}{(1-t)(1+t)(1+2t)}
\end{aligned}$$

$$\text{Let } \frac{1}{(1-t)(1+t)(1+2t)} = \frac{A}{1-t} + \frac{B}{1+t} + \frac{C}{1+2t}$$

$$\therefore 1 = A(1+t)(1+2t) + B(1-t)(1+2t) + C(1-t)(1+t)$$

Putting  $1-t=0$ , i.e.  $t=1$ , we get

$$1 = A(2)(3) + B(0)(3) + C(0)(2)$$

$$\therefore A = \frac{1}{6}$$

Putting  $1-t=0$ , i.e.  $t=-1$ , we get

$$1 = A(0)(-1) + B(2)(-1) + C(2)(0)$$

$$\therefore B = -\frac{1}{2}$$

Putting  $1+2t=0$ , i.e.  $t=-\frac{1}{2}$ , we get

$$1 = A(0) + B(0) + C\left(\frac{3}{2}\right)\left(\frac{1}{2}\right)$$

$$\therefore C = \frac{4}{3}$$

$$\therefore \frac{1}{(1-t)(1+t)(1+2t)} = \frac{\left(\frac{1}{6}\right)}{1-t} + \frac{\left(\frac{-1}{2}\right)}{1+t} + \frac{\left(\frac{4}{3}\right)}{1+2t}$$

$$\therefore I = \int \left[ \frac{\left(\frac{1}{6}\right)}{1-t} + \frac{\left(\frac{-1}{2}\right)}{1+t} + \frac{\left(\frac{4}{3}\right)}{1+2t} \right] \cdot dt$$

$$= \frac{1}{6} \int \frac{1}{1-t} \cdot dt + \frac{1}{2} \int \frac{1}{1+t} \cdot dt - \frac{4}{3} \int \frac{1}{1+2t} \cdot dt$$

$$= \frac{1}{6} \cdot \frac{\log|1-t|}{-1} + \frac{1}{2} \log|1+t| - \frac{4}{3} \cdot \frac{\log|1+2t|}{2} + c$$

$$= \frac{1}{6} \log|1-\cos x| + \frac{1}{2} \log|1+\cos x| - \frac{2}{3} \log|1+2\cos x| + c.$$

#### Miscellaneous Exercise 3 | Q 3.16 | Page 150

Integrate the following w.r.t.x :  $\sec^2 x \sqrt{7 + 2\tan x - \tan^2 x}$

**SOLUTION**

$$\text{Let } I = \int \sec^2 x \sqrt{7 + 2 \tan x - \tan^2 x} \cdot dx$$

Put  $\tan x = t$

$$\therefore \sec^2 x \cdot dx = dt$$

$$\begin{aligned} \therefore I &= \int \sqrt{7 + 2t - t^2} \cdot dt \\ &= \int \sqrt{7 - (t^2 - 2t)} \cdot dt \\ &= \int \sqrt{8 - (t^2 - 2t + 1)} \cdot dt \\ &= \int \sqrt{(2\sqrt{2})^2 - (t-1)^2} \cdot dt \\ &= \left( \frac{t-1}{2} \right) \sqrt{(2\sqrt{2})^2 - (t-1)^2} + \frac{(2\sqrt{2})^2}{2} \sin^{-1} \left( \frac{t-1}{2\sqrt{2}} \right) + c \\ &= \left( \frac{t-1}{2} \right) \sqrt{7 + 2t - t^2} + 4 \sin^{-1} \left( \frac{t-1}{2\sqrt{2}} \right) + c \\ &= \left( \frac{\tan x - 1}{2} \right) \sqrt{7 + 2 \tan x - \tan^2 x} + 4 \sin^{-1} \left( \frac{\tan x - 1}{2\sqrt{2}} \right) + c. \end{aligned}$$

**Miscellaneous Exercise 3 | Q 3.17 | Page 150**

Integrate the following w.r.t.x :  $\frac{x+5}{x^3+3x^2-x-3}$

**SOLUTION**

$$\begin{aligned} \text{Let } I &= \int \frac{x+5}{x^3 + 3x^2 - x - 3} \cdot dx \\ &= \int \frac{x+5}{x^2(x+3) - (x+3)} \cdot dx \\ &= \int \frac{x+5}{(x+3)(x^2-1)} \\ &= \int \frac{x+5}{(x+3)(x-1)(x+1)} \cdot dx \end{aligned}$$

$$\therefore x^2 + 2 = A(x+2)(x+3) + B(x-1)(x+3) + C(x-1)(x+2)$$

Put  $x - 1 = 0$ , i.e.  $x = 1$ , we get

$$1 + 2 = A(3)(4) + B(0)(4) + C(0)(3)$$

$$\therefore 3 = 12A$$

$$\therefore A = \frac{1}{4}$$

Put  $x + 2 = 0$ , i.e.  $x = -2$ , we get

$$4 + 2 = A(0)(1) + B(-3)(1) + C(-3)(0)$$

$$\therefore 6 = -3B$$

$$\therefore B = -2$$

Put  $x + 3 = 0$ , i.e.  $x = -3$  we get

$$9 + 2 = A(-1)(0) + B(-4)(0) + C(-4)(-1)$$

$$\therefore 11 = 4C$$

$$\therefore C = \frac{11}{4}$$

$$\begin{aligned}\therefore \frac{x^2 + 2}{(x-1)(x+2)(x+3)} &= \frac{\left(\frac{1}{4}\right)}{x-1} + \frac{-2}{x+1} + \frac{\left(\frac{11}{4}\right)}{x+3} \\ \therefore I &= \int \left[ \frac{\left(\frac{1}{4}\right)}{x-1} + \frac{-2}{x+1} + \frac{\left(\frac{11}{4}\right)}{x+3} \right] dx \\ &= \frac{1}{4} \int \frac{1}{x-1} dx - 2 \int \frac{1}{x+1} dx + \frac{11}{4} \int \frac{1}{x+3} dx \\ &\quad \frac{3}{4} \log|x-1| - \log|x+1| + \frac{1}{4} \log|x+3| + c.\end{aligned}$$

**Miscellaneous Exercise 3 | Q 3.18 | Page 150**

Integrate the following w.r.t. x :  $\frac{1}{x(x^5 + 1)}$

*SOLUTION*

$$\begin{aligned}\text{Let } I &= \int \frac{1}{x(x^5 + 1)} dx \\ &= \int \frac{x^4}{x^5(x^5 + 1)} dx\end{aligned}$$

Put  $x^5 = t$ .

Then  $5x^4 dx = dt$

$$\begin{aligned}\therefore x^4 dx &= \frac{dt}{5} \\ \therefore I &= \int \frac{1}{t(t+1)} \cdot \frac{dt}{5} \\ &= \frac{1}{5} \int \frac{(t+1) - t}{t(t+1)} dt \\ &= \frac{1}{5} \int \left( \frac{1}{t} - \frac{1}{t+1} \right) dt\end{aligned}$$

$$\begin{aligned}
&= \frac{1}{5} \left[ \int \frac{1}{t} dt - \int \frac{1}{t+1} dt \right] \\
&= \frac{1}{5} [\log|t| - \log|t+1|] + c \\
&= \frac{1}{5} \log \left| \frac{t}{t+1} \right| + c \\
&= \frac{1}{5} \log \left| \frac{x^5}{x^5 + 1} \right| + c.
\end{aligned}$$

Miscellaneous Exercise 3 | Q 3.19 | Page 150

Integrate the following w.r.t.x :  $\frac{\sqrt{\tan x}}{\sin x \cdot \cos x}$

**SOLUTION**

$$\text{Let } I = \int \frac{\sqrt{\tan x}}{\sin x \cdot \cos x} \cdot dx$$

Dividing numerator and denominator by  $\cos^2 x$ , we get

$$\begin{aligned}
I &= \int \frac{\left( \frac{\sqrt{\tan x}}{\cos^2} \right)}{\left( \frac{\sin x}{\cos x} \right)} \cdot dx \\
&= \int \frac{\sqrt{\tan x} \cdot \sec^2 x}{\tan x} \cdot dx \\
&= \int \frac{\sec^2 x}{\sqrt{\tan x}} \cdot dx
\end{aligned}$$

Put  $\tan x = t$

$$\therefore \sec^2 x \cdot dx = dt$$

$$\therefore I = \int \frac{1}{\sqrt{t}} \cdot dt$$

$$= \int t^{-\frac{1}{2}} \cdot dt$$

$$= \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$= 2\sqrt{t} + c$$

$$= 2\sqrt{\tan x} + c.$$

**Miscellaneous Exercise 3 | Q 3.2 | Page 150**

Integrate the following w.r.t.x :  $\sec^4 x \cosec^2 x$

**SOLUTION**

$$\text{Let } I = \int \sec^4 x \cosec^2 x \cdot dx$$

$$= \int \sec^4 x \cosec^2 x \cdot \sec^2 x \cdot \sec^2 x \cdot dx$$

Put  $\tan x = t$

$$\therefore \sec^2 x \cdot dx = dt$$

$$\text{Also, } \sec^2 x \cosec^2 x = (1 + \tan^2 x)(1 + \cot^2 x)$$

$$= (1 + t^2) \left( 1 + \frac{1}{t^2} \right)$$

$$= (1 + t^2) \left( \frac{t^2 + 1}{t^2} \right)$$

$$= \frac{t^4 + 2t^2 + 1}{t^2}$$

$$= t^2 + 2 + \frac{1}{t^2}$$

$$\therefore 1 = \int \left( t^2 + 2 + \frac{1}{t^2} \right) \cdot dt$$

$$= \int t^2 \cdot dt + 2 \int dt + \int \frac{1}{t^2} \cdot dt$$

$$= \frac{t^3}{3} + 2t + \frac{t^{-1}}{(-1)} + c$$

$$= \frac{1}{3} \tan^3 x + 2 \tan x - \frac{1}{\tan x} + c$$

$$= \frac{1}{3 \cot^3 x} + \frac{2}{\cot x} - \cot x + c.$$