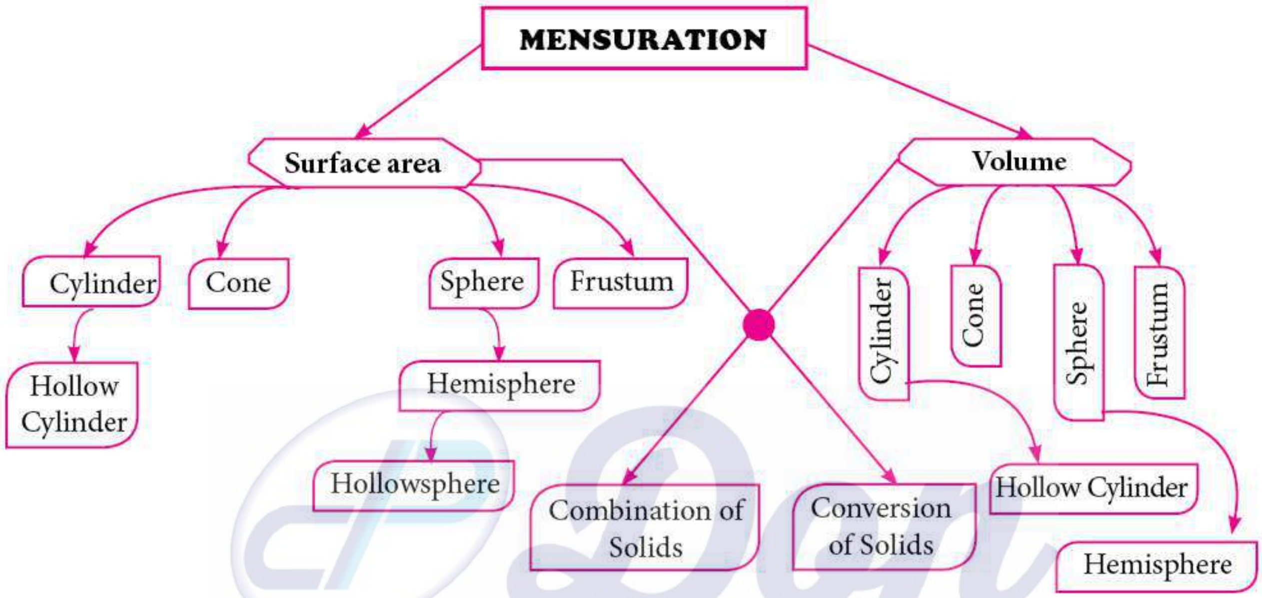
UNIT 7 MENSURATION

MIND MAP



SURFACE AREA

Key Points

- Surface area refers the term 'Total surface area'
- Use $\pi = \frac{22}{7}$ unless stated
- C.S.A of a right circular cylinder = $2\pi rh$ sq. units.
- T.S.A of a right circular cylinder = $2\pi r(h + r)$ sq. units
- C.S.A of a hollow cylinder = $2\pi(R + r)h$ sq. units
- T.S.A of a hollow cylinder = $2\pi(R + r)(R - r + h)$ sq. units
- C.S.A of a right circular cone = πrl sq. units
- T.S.A of a cone = $\pi r(l + r)$ sq. units
- C.S.A of a hemisphere = $2\pi r^2$ sq. units
- T.SA of a hemisphere = $3\pi r^2$ sq. units
- C.S.A of a hollow hemisphere = $2\pi(R^2 + r^2)$ sq. units
- T.S.A of a hollow hemisphere = $\pi(3R^2 + r^2)$ sq. units

Worked Examples

7.1 A cylindrical drum has a height of 20 cm and base radius of 14 cm. Find its curved surface area and the total surface area.

Sol : Given that,

height of the cylinder h = 20 cm;

radius r = 14 cm

Now,

C.S.A of the cylinder = $2\pi rh$ square units

C.S.A of the cylinder = $2 \times \frac{22}{7} \times 14 \times 20$

$= 2 \times 22 \times 2 \times 20$

$= 1760 \text{ cm}^2$

T.S.A. of the cylinder = $2\pi r(h + r)$ sq. units

$= 2 \times \frac{22}{7} \times 14 \times (20 + 14)$

$= 2 \times \frac{22}{7} \times 14 \times 34$

$= 2992 \text{ cm}^2$

Therefore, C.S.A. = 1760 cm^2

and T.S.A. = 2992 cm^2

7.2 The curved surface area of a right circular cylinder of height 14 cm is 88 cm². Find the diameter of the cylinder.

Sol :

Given that, C.S.A of the cylinder = 88 sq. cm

$\Rightarrow \quad 2\pi rh = 88$

$\Rightarrow 2 \times \frac{22}{7} \times r \times 14 = 88$ (given h = 14 cm)

$2r = \frac{88 \times 7}{22 \times 14} = 2$

$= 2r = 2$ cm

Therefore, diameter = 2 cm.

7.3 A garden roller whose length is 3 m long and whose diameter is 2.8 m is rolled to level a garden. How much area will it cover in 8 revolutions?

Sol :

Given that, diameter = 2r = 2.8 m and

height = 3m

radius r = 1.4 m

Area covered in one revolution = curved surface area of the cylinder

$= 2\pi rh$ square units

$= 2 \times \frac{22}{7} \times 1.4 \times 3 = 26.4$

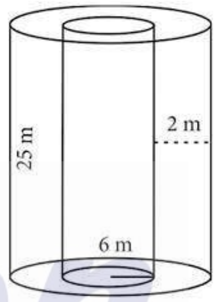
Area covered in 1 revolution = 26.4 m^2

Therefore, Area covered in 8 revolutions

$= 8 \times 26.4 = 211.2 \text{ m}^2$

7.4 If one litre of paint covers 10 m², how many litres of paint is required to paint the internal and external surface areas of a cylindrical tunnel whose thickness is 2 m, internal radius is 6 m and height is 25 m?

Sol :



Given that, height (h) = 25 m; thickness = 2 m.

internal radius (r) = 6 m

Now, external radius (R) = 6 + 2 = 8 m

C.S.A of cylindrical tunnel

= C.S.A of hollow cylinder

C.S.A of the hollow cylinder

$= 2\pi (R + r) h$ square units

$= 2 \times \frac{22}{7} (8 + 6) \times 25$

Hence, C.S.A. of the cylinder tunnel = 2200 m^2

Area covered by one litre of paint = 10 m^2

Therefore, No. of litres required to paint the

tunnel = $\frac{2200}{10} = 220$ litres.

$\therefore$ 220 litres of paint is needed to paint the tunnel.

7.5 The radius of a conical tent is 7 m and the height is 24 m. Calculate the length of the canvas used to make the tent if the width of the rectangular canvas is 4 m?

Sol:

Let r and h be the radius and height of the cone respectively.

Given that,

radius (r) = 7 m, height (h) = 24 m

Hence, $l = \sqrt{r^2 + h^2}$

$$= \sqrt{49 + 576}$$

$$l = \sqrt{625} = 25 \text{ m}$$

C.S.A of the conical tent $= \pi rl$ sq. units

Area of the canvas $= \frac{22}{7} \times 7 \times 25 = 550 \text{ m}^2$

Now, length of the canvas $= \frac{\textit{Area of canvas}}{\textit{Width}}$

$$= \frac{550}{4} = 137.5 \text{ m}$$

Therefore, the length of the canvas = 137.5 m.

7.6 If the total surface area of a cone of radius 7 cm is 704 cm² then find its slant height.

Sol :

Given that, Radius r = 7 cm

Now, total surface area of the cone $= \pi r\ (l + \text{r})$ square units

$$\text{T.S.A} = 704 \text{ cm}^2$$

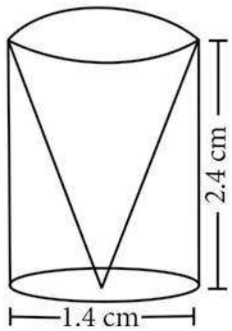
$$\Rightarrow \quad 704 = \frac{22}{7} \times 7\ (l + 7)$$

$$\Rightarrow \quad 32 = l + 7 \Rightarrow l = 25 \text{ cm}$$

Therefore, slant height of the cone = 25 cm.

7.7 From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and base is hollowed out (Fig). Find the total surface area of the remaining solid.

Sol :



Let h and r be the height and radius of the cone and cylinder respectively.

Let l be the slant height of the cone.

Given that, h = 2.4 cm and d = 1.4 cm

$$\Rightarrow \quad \text{r} = 0.7 \text{ cm}$$

Here, T.S.A of the remaining solid

= C.S.A of the cylinder + C.S.A of the cone + area of the bottom

$$= 2\pi rh + \pi rl + \pi r^2 \text{ sq. units}$$

Now, $l = \sqrt{r^2 + h^2} = \sqrt{0.49 + 5.76}$

$$= \sqrt{6.25} = 2.5 \text{ cm}$$

$$l = 2.5 \text{ cm}$$

Area of the remaining solid

$$= 2\pi rh + \pi rl + \pi r^2 \text{ sq. units}$$

$$= \pi r\ (2h + l + r)$$

$$= \frac{22}{7} \times 0.7 \times \left[(2 \times 2.4) + 2.5 + 0.7\right]$$

$$= 17.6 \text{ m}^2$$

Therefore, T.S.A of the remaining solid is 17.6 m².

7.8 Find the diameter of a sphere whose surface area is 154 m².

Sol :

Let r be the radius of the sphere.

Given that, surface area of sphere = 154 m²

$$4\pi r^2 = 154$$

$$\Rightarrow \quad 4 \times \frac{22}{7} \times r^2 = 154$$

$$\Rightarrow \quad \text{r}^2 = 154 \times \frac{1}{4} \times \frac{7}{22}$$

$$\Rightarrow \quad \text{r}^2 = \frac{49}{4} \Rightarrow \text{r} = \frac{7}{2}$$

Therefore, diameter = 2r = 7 m.

7.9 The radius of a spherical balloon increases from 12 cm to 16 cm as air being pumped into it. Find the ratio of the surface area of the balloons in the two cases.

Sol :

Let r_1 and r_2 be the radii of the balloons.

Given that, $\frac{r_1}{r_2} = \frac{12}{16} = \frac{3}{4}$

Now, ratio of C.S.A of balloons

$$= \frac{4\pi r_1^2}{4\pi r_2^2} = \frac{r_1^2}{r_2^2} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{3}{4}\right)^2 = \frac{9}{16}$$

Therefore, ratio of C.S.A of balloons is 9 : 16.

7.10 If the base area of a hemispherical solid is 1386 sq. metres then find its total surface area.

Sol :

Let r be the radius of the hemisphere.

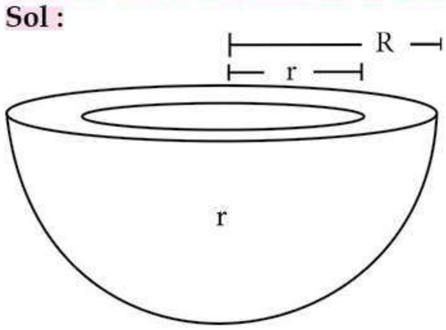
Given that, base area = πr^2 = 1386 sq. m

$$\text{T.S.A} = 3\pi r^2 \text{ sq. m}$$
$$= 3 \times 1386 = 4158 \text{ m}^2$$

Therefore, T.S.A of the hemispherical solid is 4158 m².

7.11 The internal and external radii of a hollow hemispherical shell are 3 m and 5 m respectively. Find the T.S.A and C.S.A of the shell.

Sol :



Let the internal and external radii of the hemispherical shell be r and R respectively.

Given that, R = 5 m, r = 3 m

C.S.A of the shell = $2\pi(R^2 + r^2)$ sq. units

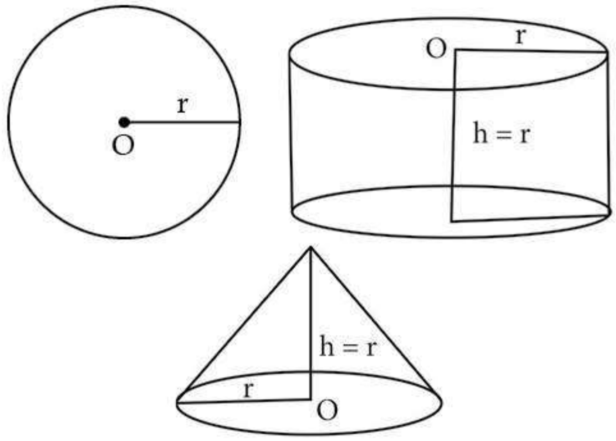
$= 2 \times \frac{22}{7} \times (25 + 9) = 213.71 \text{ m}^2$

T.S.A of the shell= $\pi(3R^2 + r^2)$ sq. units

$= \frac{22}{7}(75 + 9) = 264 \text{ m}^2$

Therefore, C.S.A= 213.71 m² and T.S.A = 264 m².

7.12 A sphere, a cylinder and a cone (Figure) are of the same radius, where as cone and cylinder are of same height. Find the ratio of their curved surface areas.



Sol :

Required Ratio = C.S.A of the sphere : C.S.A of the cylinder : C.S.A of the cone

$= 4\pi r^2 : 2\pi rh : \pi rl,$

$(l = \sqrt{r^2 + h^2} = \sqrt{2r^2} = \sqrt{2}r \text{ units})$

$= 4 : 2 : \sqrt{2} = 2\sqrt{2} : \sqrt{2} : 1$

7.13 The slant height of a frustum of a cone is 5 cm and the radii of its ends are 4 cm and 1 cm. Find its curved surface area.

Sol :

Let l, R and r be the slant height, top radius and bottom radius of the frustum.

Given that, l = 5 cm, R = 4 cm, r = 1 cm

Now, C.S,A of the frustum

$$= \pi(R + r)l \text{ sq. units}$$
$$= \frac{22}{7} \times (4 + 1) \times 5$$
$$= \frac{550}{7}$$

Therefore, C.S.A = 78.57 cm².

7.14 An industrial metallic bucket is in the shape of the frustum of a right circular cone whose top and bottom diameters are 10 m and 4 m and whose height is 4 m. Find the curved and total surface area of the bucket.

Sol :

Let h, l, R and r be the height, slant height, outer radius and inner radius of the frustum.

Given that, diameter of the top = 10 m;

Radius of the top R = 5 m.

Diameter of the bottom = 4 m;

Radius of the bottom r = 2 m, Height h = 4 m

Now, $l = \sqrt{h^2 + (R - r)^2}$

$= \sqrt{4^2 + (5 - 2)^2}$

$l = \sqrt{16 + 9} = \sqrt{25} = 5 \text{ m}$

Here, C.S.A = $\pi(R + r)l$ sq. units

$$= \frac{22}{7}(5+2)\times 5 = 110 \text{ m}^2$$

$$\text{T.S.A} = \pi(R+r)\,l + \pi R^2 + \pi r^2 \text{ sq. units}$$

$$= \frac{22}{7}\left[(5+2)5 + 25 + 4\right]$$

$$= \frac{1408}{7} = 201.14 \text{ m}^2$$

Therefore, C.S.A= 110 m² and T.S.A = 201.14 m².

Progress Check

1. **Right circular cylinder is a solid obtained by revolving _____ about _____.**

   Ans : Rectangle, one of its sides as axis.

2. **In a right circular cylinder the axis is _____ to the diameter.**

   Ans : Perpendicular.

3. **The difference between the C.S.A and T.S.A of a right circular cylinder is _____.**

   Ans : $2\pi r^2$ i.e., area of two circles.

4. **The C.S.A of a right circular cylinder of equal radius and height is _____ the area of its base.**

   Ans : 2 times.

5. **Right circular cone is a solid obtained by revolving _____ about _____.**

   Ans : right angled triangle, one of its sides containing right angle

6. **In a right circular cone the axis is _____ to the diameter.**

   Ans : Perpendicular

7. **The difference between the C.S.A and T.S.A of a cone is _____.**

   Ans : πr^2 i.e., area of base

8. **When a sector of a circle is transformed to form a cone, then match the conversions taking place between the sector and the cone.**

Sector	Cone
Radius	Circumference of the base
Area	Slant height
Arc length	Curved surface area

Ans :

Sector	Cone
Radius	Slant height
Area	Curved surface area
Arc length	Circumference of the base

9. **Every section of a sphere by a plane is a _____.**

   Ans : circle.

10. **The centre of a great circle is at the _____ of the sphere.**

    Ans : diameter.

11. **The difference between the T.S.A and C.S.A of hemisphere is _____.**

    Ans : πr^2. i.e., area of circle.

12. **The ratio of surface area of a sphere and C.S.A of hemisphere is _____.**

    Ans : 3 : 2

13. **A section of the sphere by a plane through any of its great circle is _____.**

    Ans : Largest.

14. **The portion of a right circular cone intersected between two parallel planes is _____.**

    Ans : Frustum of a cone.

15. **How many frustums can a right circular cone have?**

    Ans : Only one.

Thinking Corner

1. **When 'h' coins each of radius 'r' units and thickness 1 unit is stacked one upon the other, what would be the solid object you get? Also find its C.S.A.**

   Ans : The solid is a cylinder

   radius = r

   height = h × 1 = h

   C.S.A = $2\pi rh$ sq. units

2. **When the radius of a cylinder is double its height, find the relation between its C.S.A and base area.**

Ans :

Radius is double the height $r = 2h$

$$\text{C.S.A} = 4\pi h^2 \text{ as } 2\pi(2h)h$$

$$\text{Base area} = 4\pi h^2 \text{ as } \pi(2h)^2$$

$$\text{C.S.A} = \text{Base area}$$

3. Two circular cylinders are formed by rolling two rectangular aluminium sheets each of dimensions 12 m length and 5 m breadth, one by rolling along its length and the other along its width. Find the ratio of their curved surface areas.

Ans :

$$\left.\begin{array}{l}\textit{ratio of curved}\\ \textit{surface areas}\end{array}\right\} = \frac{2\pi r(12)}{2\pi r(5)} = \frac{12}{5} = 12:5$$

4. Give practical example of solid cone.

Ans : Ice cream Cone

5. Find surface area of a cone in terms of its radius when height is equal to radius.

Ans :

$$h = r,$$

$$l = \sqrt{h^2+r^2} = \sqrt{r^2+r^2} = \sqrt{2}r$$

$$\text{Surface Area} = \pi r\,(l+r)$$

$$= \pi r\left(\sqrt{2}r + r\right)$$

$$= \pi r^2\left(\sqrt{2}+1\right)$$

6. Compare the above surface area with the area of the base of the cone.

Ans :

Area of base of cone $= \pi r^2$

Surface area of the cone obtained in (5)

$$= \left(\sqrt{2}+1\right)\pi r^2 = \left(\sqrt{2}+1\right) \text{ times more}$$

7. Find the value of the radius of a sphere whose surface area is 36π square units.

Ans :

$$\text{Surface area} = 36\pi$$

$$4\pi r^2 = 36\pi \Rightarrow r^2 = 9 \Rightarrow r = 3$$

8. How many great circles can a sphere have?

Ans : Two circles

9. Find the surface area of the earth whose diameter is 12756 kms.

Ans :

Diameter of earth $= 12756$ kms

$$\text{Radius} \quad r = \frac{12756}{2} = 6378 \text{ kms}$$

$$\text{Surface Area} = 4\pi r^2 = 4 \times \frac{22}{7} \times (6378)^2$$

$$= \frac{3579741792}{7}$$

$$= 511391684.571 \text{ sq. km}$$

10. Shall we get a hemisphere when a sphere is cut along the small circle?

Ans : No, it is not possible to get the hemisphere, when a sphere is cut along the small circle.

11. T.S.A of a hemisphere is equal to how many times the area of its base?

Ans : 3 times.

12. How many hemispheres can be obtained from a given sphere?

Ans : 2 hemispheres.

13. Give two real life examples for a frustum of a cone.

Ans : Bucket, Table lamp.

14. Can a hemisphere be considered as a frustum of a sphere?

Ans : No.

Exercise 7.1

1. The radius and height of a cylinder are in the ratio 5 : 7 and its curved surface area is 5500 sq.cm. Find its radius and height.

Sol :

Given that radius and height of a cylinder are in the ratio 5 : 7

$$\text{i.e., } \frac{r}{h} = \frac{5}{7} \Rightarrow h = \frac{7r}{5}$$

$$\text{Curved surface area} = 5500 \text{ sq. cm}$$

$$2\pi rh = 5500$$

$$\Rightarrow 2 \times \frac{22}{7} \times r \times \frac{7r}{5} = 5500$$

$$\Rightarrow \quad r^2 = \frac{5500 \times 5}{2 \times 22}$$

$$\Rightarrow \quad r^2 = 625 \Rightarrow r = 25$$

$$\therefore \quad h = \frac{7(25)}{5} = 35$$

radius = 25 cm, height = 35 cm

2. A solid iron cylinder has total surface area of 1848 sq. m. Its curved surface area is five - sixth of its total surface area. Find the radius and height of the iron cylinder.

Sol :

Given total surface area of cylinder

$$= 1848 \text{ sq. m}$$

i.e., $2\pi r\,(h + r) = 1848$

It is given that C.S.A $= \frac{5}{6}$ (T.S.A)

$$\text{C.S.A} = \frac{5}{6}(1848) = 1540$$

$$\text{C.S.A} = \frac{5}{6}\,(\text{T.S.A})$$

$$\Rightarrow \quad 2\pi rh = \frac{5}{6}(2\pi r\,(h + r))$$

$$\Rightarrow \quad h = 5r$$

we have C.S.A = 1540

$$2\pi rh = 1540$$

$$2 \times \frac{22}{7} \times r \times 5r = 1540$$

$$r^2 = \frac{1540 \times 7}{44 \times 5} = 49$$

$$r = 7$$

$$\therefore \quad h = 5r = 5(7) = 35$$

radius = 7 m, height = 35 m

3. The external radius and the length of a hallow wooden log are 16 cm and 13 cm respectively. If its thickness is 4 cm then find its T.S.A.

Sol :

External radius of hollow cylinder R = 16 cm

length h = 13 cm

Thickness R – r = 4

$\Rightarrow$ 16 – r = 4

r = 12 cm

$$\left.\begin{array}{r}\textit{Total surface area}\\ \textit{of hollow cylinder}\end{array}\right\} = 2\pi\,(R + r)(R - r + h) \text{ sq. units}$$

$$= 2 \times \frac{22}{7} \times (16 + 12)\,(4 + 13)$$

$$= 2 \times \frac{22}{7} \times 28 \times 17$$

$$= 2992 \text{ sq. cm}$$

4. A right angled triangle PQR where $\angle Q = 90^{\circ}$ is rotated about QR and PQ. If QR = 16 cm and PR = 20 cm, compare the curved surface areas of the right circular cones so formed by the triangle.

Sol :

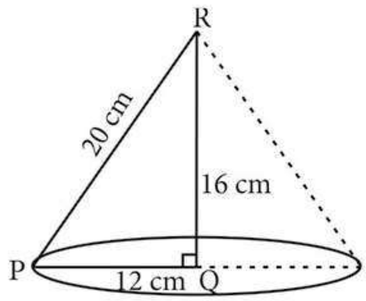
Right triangle PQR, right angled at Q and PR = 20 cm, QR = 16 cm

$$\therefore \quad PQ^2 = PR^2 - QR^2$$

$$= (20)^2 - (16)^2$$

$$= 400 - 256 = 144$$

$$\therefore \quad PQ = 12 \text{ cm}$$

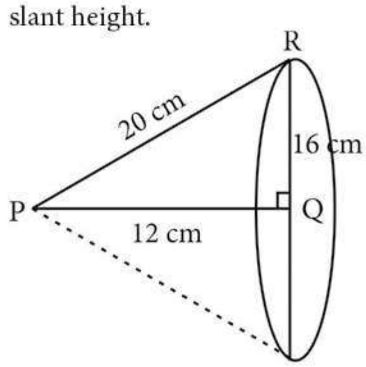


When right triangle PQR, rotates about QR, a right circular cone is formed with PQ = 12 cm as base radius and PR = 20 cm as slant height.

C.S.A of the Cone = πrl sq. units

$$= \frac{22}{7} \times 12 \times 20 = 754.29 \text{ cm}^2$$

When right triangle PQR, rotates about PQ, a right circular cone is formed with QR = 16 cm as base radius and PR = 20 cm as slant height.



C.S.A of the Cone = πrl sq.units

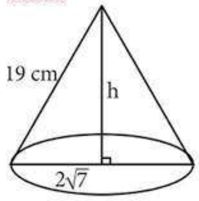
$$= \frac{22}{7} \times 16 \times 20$$

$$= 1005.71 \text{ cm}^2$$

Hence, C.S.A of the cone when rotates about PQ is larger.

5. 4 persons live in a conical tent whose slant height is 19 cm. If each person requires 22 cm² of the floor area, then find the height of the tent.

Sol :



Each person requires 22 cm² of floor area.

$\therefore$ Required base area $= 22 \times 4 = 88$ cm²

$$\Rightarrow \quad \pi r^2 = 88$$

$$r^2 = \frac{88 \times 7}{22} = 4 \times 7$$

$$r = 2\sqrt{7} \text{ cm}$$

slant height = 19 cm

$$\therefore \text{ height of the tent, } h = \sqrt{l^2 - r^2}$$

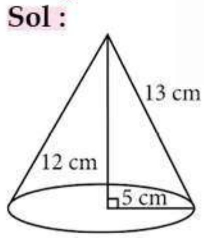
$$= \sqrt{(19)^2 - \left(2\sqrt{7}\right)^2}$$

$$= \sqrt{361 - 28} = \sqrt{330} \simeq 18.25 \text{ cm}$$

$\therefore$ Height of the tent = 18.25 cm.

6. A girl wishes to prepare birthday caps in the form of right circular cones for her birthday party, using a sheet of paper whose area is 5720 cm², how many caps can be made with radius 5 cm and height 12 cm?

Sol :



Area of the paper = 5720 cm²

Given radius of birthday cap r = 5 cm

height of birthday cap h = 12 cm

$$\therefore \text{ slant height } \quad l = \sqrt{h^2 + r^2}$$

$$= \sqrt{12^2 + 5^2} = \sqrt{144 + 25}$$

$$= \sqrt{169} = 13 \text{ cm}$$

CSA of conical cap = πrl sq. units

$$= \frac{22}{7} \times 5 \times 13 = \frac{1430}{7}$$

$\therefore$ Number of birthday caps

$$= \frac{Area\ of\ paper\ sheet}{CSA\ of\ conical\ cap}$$

$$= \frac{5720}{1430} \times 7 = 28 \text{ caps}$$

7. The ratio of the radii of two right circular cones of same height is 1 : 3. Find the ratio of their curved surface area when the height of each cone is 3 times the radius of the smaller cone.

Sol :

Let the radii of two cones be r_1 and r_2 and heights be h_1 and h_2.

$$\text{Given ratio of their radii} = \frac{r_1}{r_2} = \frac{1}{3}$$

$$\Rightarrow \quad r_1 = \frac{r_2}{3}$$

and $\quad h_1 = 3r_1, h_2 = 3r_1$

[$\because$ r_1 is the radius of smaller cone]

$$\text{Slant heights} \quad l_1 = \sqrt{h_1^2 + r_1^2}$$

$$= \sqrt{9r_1^2 + r_1^2} = \sqrt{10}\ r_1$$

$$l_2 = \sqrt{h_2^2 + r_2^2}$$

$$= \sqrt{9r_1^2 + 9r_1^2} = \sqrt{18r_1^2} = 3\sqrt{2}\ r_1$$

Ratio of curved surface areas

$$= \frac{CSA\ of\ I\ cone}{CSA\ of\ II\ cone}$$

$$= \frac{\pi r_1\ l_1}{\pi r_2\ l_2} = \frac{r_1\left(\sqrt{10}\ r_1\right)}{(3r_1)\left(3\sqrt{2}\ r_1\right)}$$

$$= \frac{\sqrt{10}}{9\sqrt{2}} = \frac{\sqrt{5}\sqrt{2}}{9\sqrt{2}} = \frac{\sqrt{5}}{9}$$

Ratio of C.S.A = $\sqrt{5} : 9$

8. The radius of a sphere increases by 25%. Find the percentage increase in its surface area.

Sol :

Let the radius of the sphere be 'r' cm

Surface area $= 4\pi r^2$

when radius is increased by 25%, then new diameter = r + 25% of r

$$= r + \frac{25\,r}{100} = \frac{5r}{4}$$

Surface area of new sphere

$$= 4\pi\left(\frac{5r}{4}\right)^2$$

$$= 4\pi\left(\frac{25r^2}{16}\right)$$

$$= \frac{25\,\pi r^2}{4}$$

Increase in surface area $= \dfrac{25\pi r^2}{4} - 4\pi r^2$

$= \dfrac{25\pi r^2 - 16\pi r^2}{4}$

$= \dfrac{9\pi r^2}{4}$

$\therefore$ Percentage increase in surface area

$= \dfrac{9\pi r^2/4}{4\pi r^2} \times 100\%$

$= \dfrac{900}{16}\% = 56.25\%$

9. The internal and external diameters of a hollow hemispherical vessel are 20 cm and 28 cm respectively. Find the cost to paint the vessel all over at ₹ 0.14 per cm².

Sol :

Internal diameter = 20 cm
External diameter = 28 cm
Internal radius = 10 cm
External radius = 14 cm

Total surface area $= \pi(3R^2 + r^2)$ sq. units

$= \dfrac{22}{7}(3(14)^2 + (10)^2)$

$= \dfrac{22}{7}[588 + 100]$

$= \dfrac{22}{7} \times 688$

$= \dfrac{15136}{7}$ cm²

Cost of painting per sq. cm = ₹ 0.14

$\therefore$ Total cost $= \dfrac{15136}{7} \times 0.14$

= ₹ 302.72

10. The frustum shaped outer portion of the table lamp has to be painted including the top part. Find the total cost of painting the lamp if the cost of painting 1 sq.cm is ₹ 2.

Sol :

From the figure r = 6 cm

R = 12 cm

h = 8 cm

$l = \sqrt{h^2 + (R - r)^2}$

$= \sqrt{8^2 + (12 - 6)^2}$

$= \sqrt{64 + 36}$

$= \sqrt{100} = 10$ cm

Area to be painted = C.S.A + area of top circular region

$= \pi(R + r)l + \pi r^2$

$= \dfrac{22}{7}(12 + 6)(10) + \dfrac{22}{7}(6)^2$

$= \dfrac{22}{7}(180) + \dfrac{22}{7}(36)$

$= \dfrac{22}{7}(180 + 36)$

$= \dfrac{22}{7}(216) = \dfrac{4752}{7} \simeq 678.86$

Cost of painting per sq. cm = ₹ 2

$\therefore$ Total cost = 678.86 × 2 = ₹ 1357.72

VOLUME

Key Points

Volume refers to the amount of space occupied by an object. The volume is measured in cubic units.

- Volume of a cylinder = $\pi r^2 h$ cu. units.
- Volume of a hollow cylinder = $\pi(R^2 - r^2)h$ cu. units
- Volume of a cone = $\frac{1}{3}\pi r^2 h$ cu. units
- Volume of a sphere = $\frac{4}{3}\pi r^3$ cu. units
- Volume of a hollow sphere = $\frac{4}{3}\pi(R^3 - r^3)$ cu. units
- Volume of a solid hemisphere = $\frac{2}{3}\pi r^3$ cu. units
- Volume of a hollow hemisphere = $\frac{2}{3}\pi(R^3 - r^3)$ cu. units
- Volume of a frustum = $\frac{\pi h}{3}(R^2 + Rr + r^2)$ cu. units

Worked Examples

7.15 Find the volume of a cylinder whose height is 2 m and whose base area is 250 m².

Sol :

Let r and h be the radius and height of the cylinder respectively.

Given that, height h = 2 m, base area = 250 m²

Now, volume of a cylinder $= \pi r^2 h$ cu. units

$= \text{base area} \times h$

$= 250 \times 2 = 500 \text{ m}^3$

Therefore, volume of the cylinder = 500 m³.

7.16 The volume of a cylindrical water tank is 1.078×10^6 litres. If the diameter of the tank is 7 m, find its height.

Sol :

Let r and h be the radius and height of the cylinder respectively.

Given that,

volume of the tank $= 1.078 \times 10^6 = 1078000$ litre

$= 1078 \text{ m}^3$

(since 1 *litre* $= \frac{1}{1000} m^3$)

Diameter $= 7 \text{ m} \Rightarrow$ Radius $= \frac{7}{2}$ m

Volume of the tank $= \pi r^2 h$ cu. units

$$1078 = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times h$$

Therefore, height of the tank h = 28 m.

7.17 Find the volume of the iron used to make a hollow cylinder of height 9 cm and whose internal and external radii are 21 cm and 28 cm respectively.

Sol :

Let r, R and h be the internal radius, external radius and height of the hollow cylinder respectively.

Given that, r = 21 cm, r = 28 cm, h = 9 cm

Now, volume of hollow cylinder

$= \pi(R^2 - r^2)h$ cu. units

$= \frac{22}{7}(28^2 - 21^2) \times 9$

$= \frac{22}{7}(784 - 441) \times 9 = 9702$

Therefore, volume of iron used = 9402 cm³.

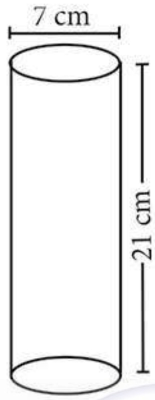
7.18 **For the cylinders A and B (Figure),**

(i) Find out the cylinder whose volume is greater?

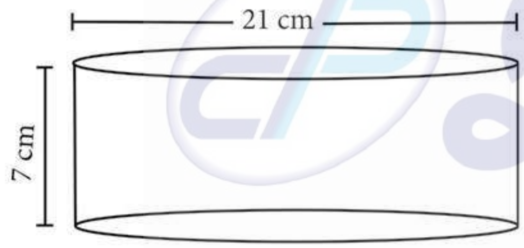
(ii) Verify whether the cylinder with greater volume has greater total surface area.

(iii) Find the ratios of the volumes of the cylinders A and B.

Sol :



Cylinder A



Cylinder B

(i) Volume of Cylinder= $\pi r^2 h$ Cu. units

Volume of Cylinder A $= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 21$

$= 808.5 \text{ cm}^3$

Volume of Cylinder B $= \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times 7$

$= 2425.5 \text{ cm}^3$

Therefore, volume of cylinder B is greater than volume of cylinder A.

(ii) T.S.A of cylinder $= 2\pi r\,(h + r)$ sq. units

T.S.A of Cylinder A $= 2 \times \frac{22}{7} \times \frac{7}{2} \times (21 + 3.5)$

$= 539 \text{ cm}^2$

T.S.A of Cylinder B $= 2 \times \frac{22}{7} \times \frac{21}{2} \times (7 + 10.5)$

$= 1155 \text{ cm}^2$

Hence verified that Cylinder B with greater volume has a greater surface area.

(iii) $\frac{\textit{Volume of cylinder A}}{\textit{Volume of cylinder B}} = \frac{808.5}{2425.5} = \frac{1}{3}$

Therefore, ratio of the volumes of cylinders A and B is 1 : 3.

7.19 **The volume of a solid right circular cone is 11088 cm^3. If its height is 24 cm then find the radius of the cone.**

Sol :

Let r and h be the radius and height of the cone respectively.

Given that, volume of the cone = 11088 cm^3

$\Rightarrow \quad \frac{1}{3}\pi r^2 h = 11088$

$\Rightarrow \quad \frac{1}{3} \times \frac{22}{7} \times r^2 \times 24 = 11088$

$\Rightarrow \quad r^2 = 441$

Therefore, radius of the cone r = 21 cm.

7.20 **The ratio of the volumes of two cones is 2 : 3. Find the ratio of their radii if the height of second cone is double the height of the first.**

Sol :

Let r_1 and h_1 be the radius and height of the cone-I and let r_2 and h_2 be the radius and height of the cone-II.

Given $h_2 = 2h_1$ and $\frac{\textit{Volume of the cone I}}{\textit{Volume of the cone II}} = \frac{2}{3}$

$$\Rightarrow \quad \frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2} = \frac{2}{3}$$

$$\Rightarrow \quad \frac{r_1^2}{r_2^2} \times \frac{h_1}{2h_1} = \frac{2}{3}$$

$$\Rightarrow \quad \frac{r_1^2}{r_2^2} = \frac{4}{3} \Rightarrow \frac{r_1}{r_2} = \frac{2}{\sqrt{3}}$$

Therefore, ratio of their radii $= 2 : \sqrt{3}$

7.21 **The volume of a solid hemisphere is 29106 cm^3. Another hemisphere whose volume is two-third of the above is carved out. Find the radius of the new hemisphere.**

Sol :

Let r be the radius of the hemisphere.

Given that,

volume of the hemisphere = 29106 cm^3

Now, volume of new hemisphere

$$= \frac{2}{3} \times (\text{Volume of original sphere})$$

$$= \frac{2}{3} \times 29106$$

Volume of new hemisphere = 19404 cm^3

$$\Rightarrow \quad \frac{2}{3}\pi r^3 = 19404$$

$$\Rightarrow \quad r^3 = \frac{19404 \times 3 \times 7}{2 \times 22} = 9261$$

$$\Rightarrow \quad r = \sqrt[3]{9261} = 21 \text{ cm}$$

Therefore, $\quad r = 21$ cm

7.22 Calculate the weight of a hollow brass sphere if the inner diameter is 14 cm and thickness is 1 mm and whose density is 17.3 g/cm^3.

Sol :

Let r and R the inner and outer radii of the hollow sphere.

Given that, inner diameter d = 14 cm;

Inner radius r = 7 cm;

$$\text{Thickness} = 1 \text{ mm} = \frac{1}{10} \text{ cm}$$

$$\text{Outer radius} \quad R = 7 + \frac{1}{10} = \frac{71}{10} = 7.1 \text{ cm}$$

$$\text{Volume of hollow sphere} = \frac{4}{3}\pi(R^3 - r^3) \text{ cu. cm}$$

$$= \frac{4}{3} \times \frac{22}{7}(357.91 - 343) = 62.48 \text{ cm}^3$$

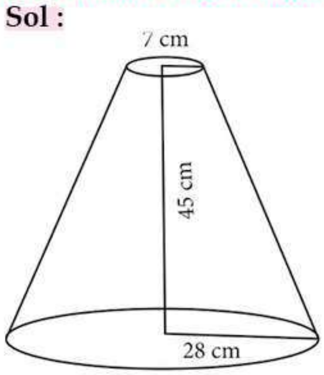
But, weight of brass in 1 cm^3 = 17.3 gm

Total weight = 17.3 × 62.48 = 1080.90 gm

Therefore, Total weight = 1080.90 grams.

7.23 If the radii of the circular ends of a frustum which is 45 cm high are 28 cm and 7 cm, find the volume of the frustum.

Sol :



Let h, r and R be the height, top and bottom radii of the frustum.

Given that, h = 45 cm, R = 28 cm, r = 7 cm

$$\text{Now, Volume} = \frac{1}{3}\pi[R^2 + Rr + r^2]h \text{ cu. units}$$

$$= \frac{1}{3} \times \frac{22}{7} \times [28^2 + (28 \times 7) + 7^2] \times 45$$

$$= \frac{1}{3} \times \frac{22}{7} \times 1029 \times 45 = 48510$$

Therefore, volume of the frustum = 48510 cm^3.

Progress Check

1. Volume of a cone is the product of its base area and _____

Ans : One third of its height.

2. If the radius of the cone is doubled, the new volume will be _____ times the original volume.

Ans : 4 times

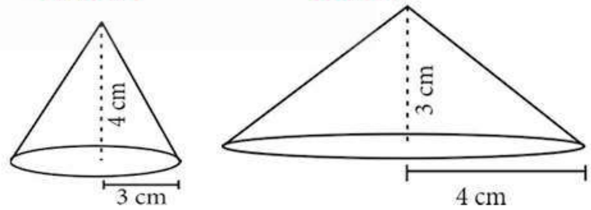
3. Consider the cones given

(i) Without doing any calculation, find out whose volume is greater?

(ii) Verify whether the cone with greater volume has greater surface area.

(iii) Volume of Cone A: Volume of Cone B = ?

Cone A **Cone B**



Ans :

(i) Volume of cone B is greater as the radius is greater

(ii) Volume of cone A

$$= \frac{1}{3}\pi(3)^2(4) = 12\pi$$

Volume of cone B

$$= \frac{1}{3}\pi(4)^2(3) = 16\pi$$

Surface area of cone A

T.S.A = $\pi r(l + r)$

$= \pi(3)(5 + 3)$

$= 24\pi$

Surface area of cone B

T.S.A = $\pi(4)(5 + 4)$

$= 36\pi$

(iii) Surface area of cone B > Surface area of cone A

$$\frac{\textit{Volume of cone A}}{\textit{Volume of cone B}} = \frac{12\pi}{16\pi} = 3:4$$

4. What is the ratio of volume to surface area of sphere?

Ans : $\dfrac{Volume\ of\ Sphere}{Volume\ of\ Surface\ area} = \dfrac{\frac{4}{3}\pi r^3}{4\pi r^2} = r : 3.$

5. The relationship between the height and radius of the hemisphere is _____.

Ans : Height = radius

6. The volume of a sphere is the product of its surface area and _____.

Ans : One third of its radius

Thinking Corner

1. If the height is inversely proportional to the square of its radius, the volume of the cylinder is _____.

Ans :

$$\text{Height } \alpha \ \frac{1}{(\text{radius})^2}$$

$$\text{Height} = k\left(\frac{1}{(\text{radius})^2}\right) \quad [\because k - \text{constant}]$$

$$\text{Volume} = \pi r^2 h$$

$$= \pi r^2\left(\frac{k}{r^2}\right) = k\pi.$$

2. What happens to the volume of the cylinder with radius r and height h, when its
(a) Radius is halved (b) Height is halved.

Ans : (a) radius is halved then radius $\frac{r}{2}$, height is h

$$\text{Volume} = \pi r^2 h = \pi\left(\frac{r}{2}\right)^2 h = \frac{\pi r^2 h}{4}$$

$$= \frac{1}{4} \text{ (Volume of original cylinder).}$$

(b) When height is halved, then height = $\frac{h}{2}$

$$\text{Volume} = \pi r^2\left(\frac{h}{2}\right) = \frac{\pi r^2 h}{2}$$

$$= \frac{1}{2} \text{ (Volume of original cylinder).}$$

3. Is it possible to find a right circular cone with equal.
(a) Height and slant height
(b) Radius and slant height
(c) Height and radius.

Ans :

(a) height = Slant height

i.e., $h = l$ = cone is not possible

(b) $r = l$ = cone is not possible

(c) $h = r$ = cone is possible.

4. There are two cones with equal volumes. What will be the ratios of their radius and heights?

Ans : Ratios of radii and heights are same.

5. A cone, a hemisphere and a cylinder have equal bases. The heights of the cone and cylinder are equal and are same as the common radius. Are they equal in volume?

Ans : No.

6. Give any two real life examples of sphere and hemisphere.

Ans : Sphere – Globe, Ball

Hemisphere – Left side of the brain

7. A plane along a great circle will split the sphere into _____ parts.

Ans : Two parts.

8. If the volume and surface area of a sphere are numerically equal, then the radius of the sphere is _____.

Ans : $\frac{4}{3}\pi r^3 = 4\pi r^2 \Rightarrow r = 3.$

9. Is it possible to obtain the volume of the full cone when the volume of the frustum is known?

Ans : Not possible.

Exercise 7. 2

1. A 14 m deep well with inner diameter 10 m is dug and the earth taken out is evenly spread all around the well to form an embankment of width 5 m. Find the height of the embankment.

Sol : Radius of well = 5m

Depth of well = 14 m

Volume of earth taken out = $\pi r^2 h$

$$= \frac{22}{7} \times (5)^2 \times 14$$

$$= 1100 \text{ m}^3$$

Now, it is spread to form an embankment, which is in the form of hollow cylinder.

Inner radius = 5 m

Width of embankment = 5 m

$\therefore$ Outer radius = 5 + 5 = 10 m

height = h

Volume of hollow cylinder = $\pi h\,(R^2 - r^2)$

$$\therefore \quad \pi h\,(R^2 - r^2) = 1100$$

$$\frac{22}{7} \times h\,(10^2 - 5^2) = 1100$$

height of the embankment

$$h = \frac{1100 \times 7}{22 \times 75} = 4.67 \text{ m}$$

2. A cylindrical glass with diameter 20 cm has water to a height of 9 cm. A small cylindrical metal of radius 5 cm and height 4 cm is immersed it completely. Calculate the raise of the water in the glass.

Sol :

Diameter of Glass = 20 cm

$\therefore$ radius = 10 cm

water upto height = 9 cm

radius of cylindrical metal = 5 cm

height of cylindrical metal = 4 cm

Volume of water displaced = Volume of cylindrical metal

$$\pi r_1^2\, h_1 = \pi r_2^2\, h_2$$

$$(10)^2\, h_1 = (5)^2\,(4)$$

$$h_1 = \frac{100}{100} = 1 \text{ cm}$$

Hence, the increase in water level is 1 cm.

3. If the circumference of a conical wooden piece is 484 cm then find its volume when its height is 105 cm.

Sol :

Given circumference = 484 cm

$$2\pi r = 484$$

$$2 \times \frac{22}{7} \times r = 484$$

$$r = \frac{484 \times 7}{44} = 77 \text{ cm}$$

height h = 105 cm

Volume of cone $= \frac{1}{3}\pi r^2 h$ cu. units

$$= \frac{1}{3} \times \frac{22}{7} \times 77 \times 77 \times 105$$

$$= 652190 \text{ cm}^3$$

4. A conical container is fully filled with petrol. The radius is 10 m and the height is 15 m. If the container can release the petrol through its bottom at the rate of 25 cu. meter per minute, in how many minutes the container will be emptied. Round off your answer to the nearest minute.

Sol :

Radius of conical container = 10 m

Height of conical container = 15 m

Volume $= \frac{1}{3}\pi r^2 h$ cu. units

$$= \frac{1}{3} \times \frac{22}{7} \times 10 \times 10 \times 15$$

$$= \frac{11000}{7}\, m^3$$

water is released at the rate of 25 m³/min

$\therefore$ Time required to empty the container

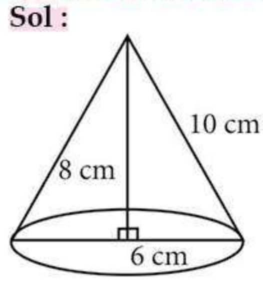
$$= \frac{\frac{11000}{7}}{25}$$

$$= \frac{11000}{7 \times 25} = 62.85$$

$\simeq$ 63 minutes (approx)

5. A right angled triangle whose sides are 6 cm, 8 cm and 10 cm is revolved about the sides containing the right angle in two ways. Find the difference in volumes of the two solids so formed.

Sol :



When right triangle is revolved about one of its sides (containing the right angle), a cone is formed

Now, Radius of the cone = 6 cm

Height of the cone = 8 cm

Slant height = 10 cm

Volume of the cone = $\frac{1}{3}\pi r^2 h$ cu. units

$$= \frac{1}{3} \times \frac{22}{7} \times (6)^2 \times 8$$

$$= \frac{22 \times 12 \times 8}{7} = 301.71 \text{ cm}^3$$

Now, Right triangle rotates about the side which is 8 cm in length

$\therefore$ Radius = 8 cm

Height = 6 cm

Slant height = 10 cm

Volume = $\frac{1}{3}\pi r^2 h$ cu. units

$$= \frac{1}{3} \times \frac{22}{7} \times (8)^2 \times 6$$

$$= \frac{2816}{7} = 402.29 \text{ cm}^3$$

Difference in volumes = 402.29 – 301.71

= 100.58 cm^3

6. The volumes of two cones of same base radius are 3600 cm^3 and 5040 cm^3. Find the ratio of heights.

Sol : Let r_1, r_2 be the radii of two cones,

Given $r_1 = r_2$ and let h_1, h_2 be the heights of two cones.

V_1 = Volumes of I cone $= \frac{1}{3}\pi r_1^2 h_1 = \frac{\pi}{3} r_1^2 h_1$

= 3600 cm^3

V_2 = Volume of II cone $= \frac{1}{3}\pi r_2^2 h_2 = 5040$ cm^3

Now, $\frac{V_1}{V_2} = \frac{3600}{5040} \Rightarrow \frac{\frac{\pi}{3} r_1^2 h_1}{\frac{\pi}{3} r_2^2 h_2} = \frac{3600}{5040}$ [$\because$ $r_1 = r_2$

$\therefore$ ratio of heights $\frac{h_1}{h_2} = \frac{5}{7} = 5 : 7$

7. If the ratio of radii of two spheres is 4 : 7, find the ratio of their volumes.

Sol :

Let r_1, r_2 be the radii of two spheres

Given $\frac{r_1}{r_2} = \frac{4}{7} \Rightarrow r_1 = \frac{4r_2}{7}$

Ratio of the volumes = $\frac{V_1}{V_2} = \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3}$

$$= \frac{\left(\frac{4r_2}{7}\right)^3}{r_2^3} = \frac{4^3}{7^3}$$

Ratio of volumes $V_1 : V_2 = 64 : 343 = \frac{64}{343}$

8. A solid sphere and a solid hemisphere have equal total surface area. Prove that the ratio of their volume is $3\sqrt{3} : 4$.

Sol :

Let r_1 and r_2 be the radii of sphere and hemisphere respectively.

Given T.S.A of sphere = T.S.A of hemisphere

$$4\pi r_1^2 = 3\pi r_2^2$$

$$\frac{r_1^2}{r_2^2} = \frac{3}{4} \Rightarrow \frac{r_1}{r_2} = \frac{\sqrt{3}}{2}$$

Ratio of their volumes : $\frac{V_1}{V_2} = \frac{\frac{4}{3}\pi r_1^3}{\frac{2}{3}\pi r_2^3} = 2\left(\frac{r_1}{r_2}\right)^3$

$$= 2\left(\frac{\sqrt{3}}{2}\right)^3$$

$$= \frac{3\sqrt{3}}{4} = 3\sqrt{3} : 4$$

Ratio of their volumes = $3\sqrt{3} : 4$

9. The outer and the inner surface areas of a spherical copper shell are 576π cm^2 and 324π cm^2 respectively. Find the volume of the material required to make the shell.

Sol :

Let R, r be the outer and inner radii respectively.

Given outer surface area = $576\pi\ cm^2$

$$4\pi R^2 = 576\pi$$

$$R^2 = 144$$

$$R = 12 \text{ cm}$$

Inner surface area = $324\pi\ cm^2$

$$4\pi r^2 = 324\pi$$

$$r^2 = 81$$

$$r = 9 \text{ cm}$$

Volume of the material required

$$= \frac{4}{3}\pi(R^3 - r^3) \text{ cu. units}$$
$$= \frac{4}{3} \times \frac{22}{7} \times (12^3 - 9^3)$$
$$= \frac{4}{3} \times \frac{22}{7} \times 999$$
$$= 4186.285 \simeq 4186.29 \text{ cm}^3$$

10. A container open at the top is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends are 8 cm and 20 cm respectively. Find the cost of milk which can completely fill a container at the rate of ₹ 40 per litre.

Sol :

Given radius of lower end $r = 8$ cm

radius of upper end $R = 20$ cm

height $h = 16$ cm

$$\text{Volume} = \frac{\pi h}{3}(R^2 + Rr + r^2) \text{ cu. units}$$
$$= \frac{22 \times 16}{7 \times 3}\left((20)^2 + (20)(8) + (8)^2\right)$$
$$= \frac{22 \times 16}{21}[400 + 160 + 64]$$
$$= \frac{22 \times 16}{21}(624) = 10459.43 \text{ cm}^3$$
$$= \frac{10459.43}{1000} \quad [\because 1000 \text{ cm}^3 = 1 \text{ litre}]$$
$$= 10.45943 \text{ litre}$$

Cost of milk per litre = ₹ 40

$\therefore$ Total cost $= 10.459 \times 40$

$= ₹\ 418.36$

VOLUME AND SURFACE AREA OF COMBINED SOLIDS.

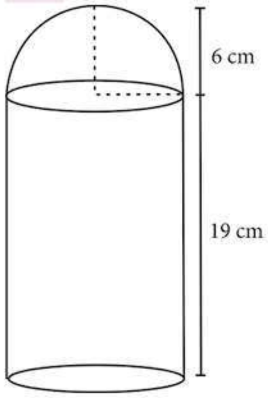
Key Points

- A combined solid is said to be a solid formed by combining two or more solids.

Worked Examples

7.24 A toy is in the shape of a cylinder surmounted by a hemisphere. The height of the toy is 25 cm. Find the total surface area of the toy if its common diameter is 12 cm.

Sol :



Let r and h be the radius and height of the cylinder respectively.

Given that, diameter $d = 12$ cm,

radius $r = 6$ cm

Total height of the toy is 25 cm

Therefore, height of the cylindrical portion

$= 25 - 6 = 19$ cm

T.S.A of the toy = C.S.A of the cylinder + C.S.A of the hemisphere + Base Area of the cylinder

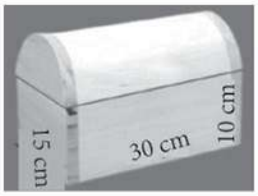
$$= 2\pi rh + 2\pi r^2 + \pi r^2$$
$$= \pi r(2h + 3r) \text{ sq. units}$$
$$= \frac{22}{7} \times 6 \times (38 + 18)$$
$$= \frac{22}{7} \times 6 \times 56 = 1056$$

Therefore, TSA of the toy = 1056 cm².

7.25 **A jewEl box (Figure) is in the shape of a cuboid of dimensions 30 cm × 15 cm × 10 cm surmounted by a half part of a cylinder as shown in the figure. Find the volume and T.S.A of the box.**

Sol :

Let l, b and h_1 be the length, breadth and height of the cuboid. Also let us take r and h_2 be the radius and height of the cylinder.



Now, volume of the box

$= \text{Volume of the cuboid} + \frac{1}{2}(\text{Volume of Cylinder})$

$= (l \times b \times h_1) + \frac{1}{2}\left(\pi r^2 h_2\right)$ cu. units

$= (30 \times 15 \times 10) + \frac{1}{2}\left(\frac{22}{7} \times \frac{15}{2} \times \frac{15}{2} \times 30\right)$

$= 4500 + 2651.79 \quad = 7151.79$

Therefore, volume of the box $= 7151.79 \text{ cm}^3$

Now, T.S.A of the box = C.S.A of the cuboid + $\frac{1}{2}$ (C.S.A of the cylinder)

$= 2(l + b)h_1 + \frac{1}{2}(2\pi r h_2)$

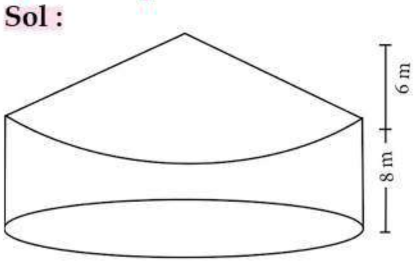
$= 2(45 \times 10) + \left(\frac{22}{7} \times \frac{15}{2} \times 30\right)$

$= 900 + 707.14 = 1607.14$

Therefore, T.S.A of the box = 1607.14 cm^2.

7.26 **Arul has to make arrangements for the accommodation of 150 persons for his family function. For this purpose, he plans to build a tent which is in the shape of cylinder surmounted by a cone. Each person occupies 4 sq.m of the space on ground and 40 cu.meter of air to breathe. What should be the height of the conical part of the tent if the height of cylindrical part is 8 m?**

Sol :



Let h_1 and h_2 be the height of cylinder and cone respectively.

Area for one person = 4 sq. m

Total No. of persons = 150

Therefore total base area = 150 × 4

$\pi r^2 = 600$

$r^2 = 600 \times \frac{7}{22} = \frac{2100}{11} \quad \text{... (1)}$

Volume of air required for 1 person = 40 m^3

Total volume of air required for 150 persons

$= 150 \times 40 = 6000 \text{ m}^3$.

$\Rightarrow \quad \pi r^2 h_1 + \frac{1}{3}\pi r^2 h_2 = 6000$

$\Rightarrow \quad \pi r^2\left(h_1 + \frac{1}{3}h_2\right) = 6000$

$\Rightarrow \frac{22}{7} \times \frac{2100}{11}\left(8 + \frac{1}{3}h_2\right) = 6000 \quad$ [using (1)

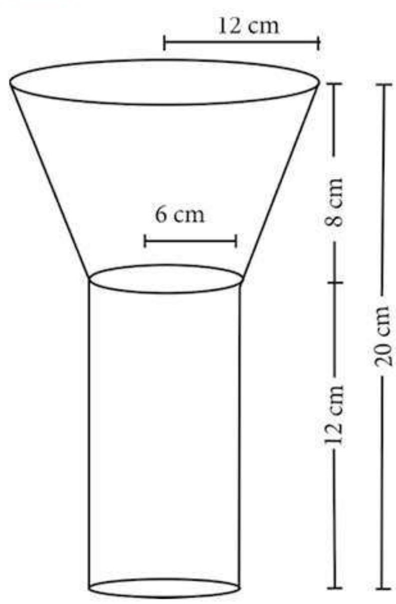
$\Rightarrow 8 + \frac{1}{3}h_2 = \frac{6000 \times 7 \times 11}{22 \times 2100}$

$\Rightarrow \quad \frac{1}{3}h_2 = 10 - 8 = 2$

Therefore, the height of the conical tent h_2 is 6 m.

7.27 **A funnel consists of a frustum of a cone attached to a cylindrical portion 12 cm long attached at the bottom. If the total height be 20 cm, diameter of the cylindrical portion be 12 cm and the diameter of the top of the funnel be 24 cm. Find the outer surface area of the funnel.**

Sol :



Let R, r be the top and bottom radii of the frustum.

Let h_1, h_2 be the heights of the frustum and cylinder respectively.

Given that, R = 12 cm, r = 6 cm, h_2 = 12 cm

Now, $h_1 = 20 - 12 = 8$ cm

Here, Slant height of the frustum

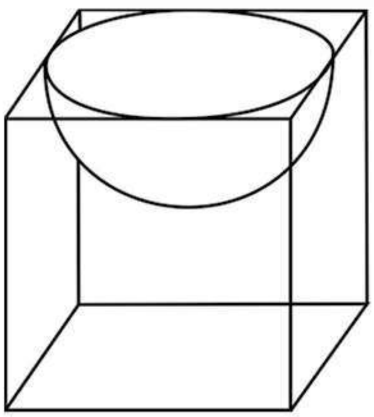
$$l = \sqrt{(R-r)^2 + h_1^2} \text{ units}$$
$$= \sqrt{36+64}$$
$$l = 10 \text{ cm}$$

Outer surface area = $2\pi r h_2 + \pi(R + r)\, l$ sq. units

$$= \pi\left[2rh_2 + (R+r)l\right]$$
$$= \pi\left[(2\times 6\times 12) + (18\times 10)\right]$$
$$= \pi\ [144 + 180]$$
$$= \frac{22}{7}\times 324 = 1018.28$$

Therefore, Outer surface area of the funnel is 1018.28 cm^2.

7.28 A hemispherical section is cut out from one face of a cubical block (figure) such that the diameter l of the hemisphere is equal to side length of the cube. Determine the surface area of the remaining solid.



Sol :

Let r be the radius of the hemisphere.

Given that, diameter of the hemisphere = side of the cube = l

Radius of the hemisphere = $\frac{l}{2}$

T.S.A of the remaining solid = Surface area of the cubical part + C.S.A of the hemispherical part – Area of the base of the hemispherical part

$$= 6\times(\text{Edge})^2 + 2\pi r^2 - \pi r^2$$
$$= 6\times(\text{Edge})^2 + \pi r^2$$
$$= 6\times(l)^2 + \pi\left(\frac{l}{2}\right)^2 = \frac{1}{4}(24+\pi)l^2$$

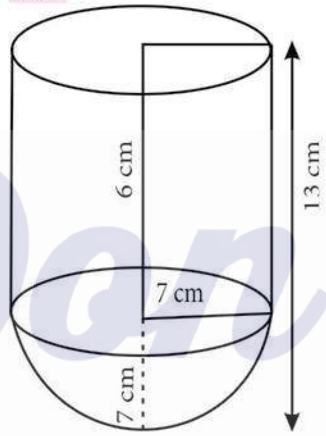
Total surface area of the remaining solid

$$= \frac{1}{4}(24+\pi)l^2 \text{ sq. units.}$$

Exercise 7. 3

1. A vessel is in form of a hemispherical bowl mounted by a hollow cylinder. The diameter is 14 cm and the height of the vessel is 13 cm. Find the capacity of the vessel.

Sol :



Diameter of the bowl = 14 cm

Radius r = 7 cm

Volume of hemisphere $= \frac{2}{3}\pi r^3$ cu. units

$$= \frac{2}{3}\times\frac{22}{7}\times 7\times 7\times 7$$
$$= \frac{2156}{3} = 718.67 \text{ cm}^3$$

Radius of cylinder 'r' = 7 cm

Height 'h' = 6 cm

Volume of cylinder = $\pi r^2 h$ cu. units

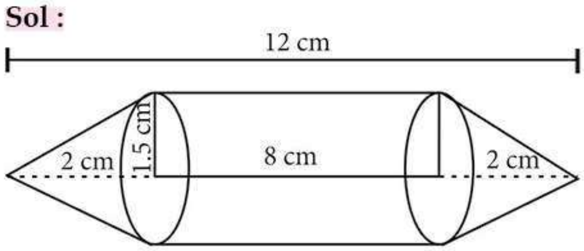
$$= \frac{22}{7}\times 7\times 7\times 6$$
$$= 924 \text{ cm}^3$$

∴ Capacity of the vessel = Volume of hemisphere + Volume of cylinder

$$= 718.67 + 924$$
$$= 1642.67 \text{ cm}^3.$$

2. Nathan, an engineering student was asked to make a model shaped like a cylinder with two cones attached at its two ends. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of 2 cm, find the volume of the model that Nathan made.

Sol :



From the figure, radius of cylinder $= \frac{3}{2} = 1.5$ cm

Height $= 8$ cm

Volume of cylinder $= \pi r^2 h$ cu. units

$= \frac{22}{7} \times 1.5 \times 1.5 \times 8$

Radius of cone $= 1.5$ cm

Height of cone $= 2$ cm

Volume of 2 cones $= 2\left(\frac{1}{3}\pi r^2 h\right)$ cu. units

$= \frac{2}{3} \times \frac{22}{7} \times 1.5 \times 1.5 \times 2$

$\therefore$ Volume of the model = Volume of cylinder + Volume of 2 cones

$= \frac{22}{7} \times (1.5)^2 \left[8 + \frac{4}{3}\right]$

$= \frac{22}{7} \times 2.25 \times \frac{28}{3}$

$= \frac{1386}{21} = 66 \text{ cm}^3$

3. From a solid cylinder whose height is 2.4 cm and the diameter 1.4 cm a cone of the same height and same diameter is carved out. Find the volume of the remaining solid to the nearest cm³.

Sol : Diameter of a solid cylinder = 1.4 cm

Radius of a solid cylinder $= \frac{1.4}{2} = 0.7$ cm

Height of a solid cylinder = 2.4 cm

Volume of the cylinder $= \pi r^2 h$ cu. units

$= \frac{22}{7} \times 0.7 \times 0.7 \times 2.4$

Radius of cone = 0.7 cm

Height of cone = 2.4 cm

Volume of cone $= \frac{1}{3}\pi r^2 h$ cu. units

$= \frac{1}{3} \times \frac{22}{7} \times 0.7 \times 0.7 \times 2.4$

$\therefore$ Volume of the remaining solid = Volume of cylinder – Volume of cone

$= \frac{22}{7} \times 0.7 \times 0.7 \times 2.4 - \frac{1}{3} \times \frac{22}{7} \times 0.7 \times 0.7 \times 2.4$

$= \frac{22}{7} \times 0.7 \times 0.7 \times 2.4 \left(1 - \frac{1}{3}\right)$

$= \frac{22}{7} \times 0.7 \times 0.7 \times 2.4 \times \frac{2}{3}$

$= 2.464 \text{ cm}^3$

Aliter: Since the height and radius of the cylinder and cone are same,

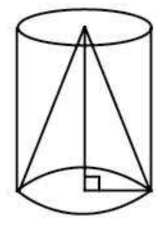
Volume of the remaining solid = Volume of cylinder – Volume of cone

$= \pi r^2 h - \frac{1}{3}\pi r^2 h$

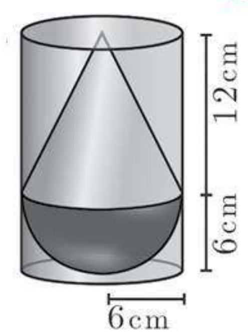
$= \frac{2}{3}\pi r^2 h$

$= \frac{2}{3} \times \frac{22}{7} \times (0.7)^2 (2.4)$

$= 2.464 \text{ cm}^3$



4. A solid consisting of a right circular cone of height 12 cm and radius 6 cm standing on a hemisphere of radius 6 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of the water displaced out of the cylinder, if the radius of the cylinder is 6 cm and height is 18 cm.



Sol :

Radius of hemisphere = 6 cm

Volume of hemisphere $= \frac{2}{3}\pi r^3$ cu. units

$= \frac{2}{3}\pi(6)^3$

$= \frac{2}{3}\pi(216)$

$= 144\pi\ cm^3$

Radius of the base of cone = 6 cm

Height of the cone = 12 cm

Volume of the cone = $\frac{1}{3}\pi r^2 h$ cu. units

$= \frac{1}{3}\pi(6)^2\ (12)$

$= 144\pi\ cm^3$

Volume of the solid = Volume of cone + Volume of hemisphere

$= 144\pi + 144\pi = 288\pi$

Volume of water displaced

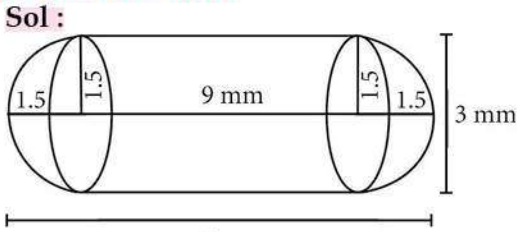
= Volume of the solid placed in the cylinder

$= 288\pi = 288 \times \frac{22}{7}$

$= 905.14\ cm^3$

5. A capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends. If the length of the entire capsule is 12 mm and the diameter of the capsule is 3 mm, how much medicine it can hold?

Sol :



From the figure,

Diameter of hemisphere = 3 mm

Radius of hemisphere = 1.5 mm

Volume of hemisphere = $\frac{2}{3}\pi r^3$ cu. units

$\therefore$ Volume of 2 hemispheres = $2\left(\frac{2}{3}\pi r^3\right)$

$= \frac{4}{3}\pi(1.5)^3$

$= 4.5\pi\ mm^3$

Radius of cylinder = 1.5 mm

Height of cylinder = 12 − 3 = 9 mm

Volume of cylinder = $\pi r^2 h$ cu. units

$= \pi(1.5)^2\ (9)$

$= 20.25\pi\ mm^3$

$\therefore$ Amount of medicine that a capsule can hold

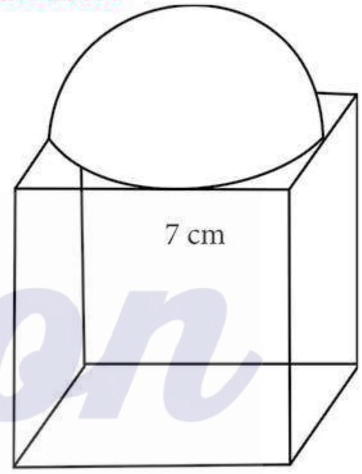
= Volume of cylinder + Volume of 2 hemispheres

$= 20.25\pi + 4.5\pi$

$= 24.75\pi\ mm^3$

$= 24.75 \times \frac{22}{7} = 77.785\ mm^3$

6. As shown in figure a cubical block of side 7 cm is surmounted by a hemisphere. Find the surface area of the solid.



Sol :

Edge of cube = 7 cm

surface area of a cube = $6a^2$ sq. units

$= 6(7)^2$

$= 294\ cm^2$

radius of hemisphere = $\frac{7}{2}$ cm

[∵ Only C.S.A is considered as the hemisphere surmounted]

C.S.A of hemisphere = $2\pi r^2$ sq. units

$= 2 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$

$= 77\ cm^2$

Surface area of the solid = T. S a of cube + C.S.A of hemisphere − area of circular region (bottom of hemisphere)

$= 294 + 77 - \left(\frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\right)$

$= 371 - 38.5$

$= 332.5\ cm^2$

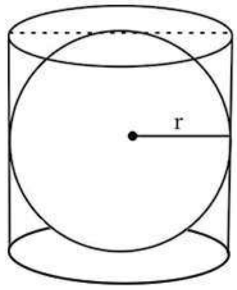
7. A right circular cylinder just enclose a sphere of radius r units. Calculate

(i) the surface area of the sphere

(ii) the curved surface area of the cylinder

(iii) the ratio of the areas obtained in (i) and (ii).

Sol :



(i) Surface area of a sphere

Radius of sphere = r

Surface area = $4\pi r^2$ sq. units

(ii) Curved surface area of cylinder

Radius of cylinder = r

Height of cylinder = r + r = 2r

Curved surface area = $2\pi rh$ sq. units

$= 2\pi r\,(2r)$

$= 4\pi r^2$ sq. units

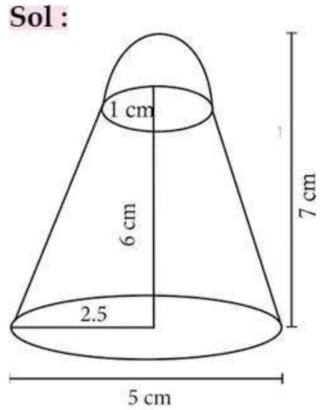
(iii) Ratio of the areas $= \dfrac{\textit{Surface area of sphere}}{\textit{CSA of cylinder}}$

$$= \frac{4\pi r^2}{4\pi r^2} = \frac{1}{1}$$

Ratio = 1 : 1.

8. A shuttlecock used for playing badminton has the shape of a frustum of a cone is mounted on a hemisphere. The diameters of the frustum are 5 cm and 2 cm. The height of the entire shuttle cock is 7 cm. Find its external surface area.

Sol :



Frustum

Radius of the base R = 2.5 cm

Radius of the top r = 1 cm

Height h = 7 – 1 = 6 cm

Slant height $l = \sqrt{h^2 + (\mathrm{R}-\mathrm{r})^2}$

$= \sqrt{(6)^2 + (2.5-1)^2}$

$= \sqrt{36 + 2.25}$

$= \sqrt{38.25} = 6.18$ cm

Curved surface area $= \pi\,(\mathrm{R}+\mathrm{r})l$ sq. units

$= \dfrac{22}{7}(2.5+1)(6.18)$

$= \dfrac{22}{7}(3.5)(6.18)$

$= 67.98$ cm^2

Radius of hemisphere = 1 cm

C.S.A of hemisphere $= 2\pi r^2$ sq. units

$= 2 \times \dfrac{22}{7} \times (1)^2$

$= 6.29$ cm^2

∴ External surface area of shuttlecock

= C.S.A Frustum + C.S.A of hemisphere

= 67.98 + 6.29 = 74.27 cm^2

CONVERSION OF SOLIDS FROM ONE SHAPE TO ANOTHER WITH NO CHANGE IN VOLUME.

Key Points

- When converting one solid to another solid, the volumes are equal but they differ in surface area.
- In melting and casting problems, Since the volumes are equal, No need to take $\pi = \dfrac{22}{7}$ as they will get cancelled when equated.

Worked Examples

7.29 A metallic sphere of radius 16 cm is melted and recast into small spheres each of radius 2 cm. How many small spheres can be obtained?

Sol :

Let the number of small spheres obtained be n.
Let r be the radius of each small sphere and R be the radius of metallic sphere.
Here, R = 16 cm, r = 2 cm
Now, n × (Volume of a small sphere) = Volume of big metallic sphere

$$n\left(\frac{4}{3}\pi r^3\right) = \frac{4}{3}\pi R^3$$

$$\Rightarrow n\left(\frac{4}{3}\pi \times 2^3\right) = \frac{4}{3}\pi \times 16^3$$

$$\Rightarrow \quad 8n = 4096 \Rightarrow n = 512$$

Therefore, there will be 512 small spheres.

7.30 A cone of height 24 cm is made up of modeling clay. A child reshapes it in the form of a cylinder of same radius as cone. Find the height of the cylinder.

Sol :

Let h_1 and h_2 be the heights of a cone and cylinder respectively.
Also, let r be the radius of the cone.

Given that, height of the cone h_1 = 24 cm; Radius of the cone and cylinder r = 6 cm

Since, Volume of cylinder = Volume of cone

$$\pi r^2 h_2 = \frac{1}{3}\pi r^2 h_1$$

$$\Rightarrow \quad h_2 = \frac{1}{3}\times h_1 \Rightarrow h_2 = \frac{1}{3}\times 24 = 8$$

Therefore, height of cylinder h_2 = 8 cm.

7.31 A right circular cylindrical container of base radius 6 cm and height 15 cm is full of ice cream. The ice cream is to be filled in cones of height 9 cm and base radius 3 cm, having a hemispherical cap. Find the number of cones needed to empty the container.

Sol :

Let h and r be the height and radius of the cylinder respectively.
Given that, h = 15 cm, r = 6 cm

Volume of the container $V = \pi r^2 h$ cubic. units

$$= \frac{22}{7}\times 6\times 6\times 15$$

Let r_1 = 3 cm, h_1 = 9 cm be the radius and height of the cone.
Also, r_1 = 3 cm is the radius of the hemispherical cap.

Volume of one ice cream cone = (Volume of the cone + Volume of the hemispherical cap)

$$= \left[\frac{1}{3}\pi r_1^2 h_1 + \frac{2}{3}\pi r_1^3\right]$$

$$= \left[\frac{1}{3}\times\frac{22}{7}\times 3\times 3\times 9 + \frac{2}{3}\times\frac{22}{7}\times 3\times 3\times 3\right]$$

$$= \left[\frac{22}{7}\times 9\,(3+2)\right] = \frac{22}{7}\times 45$$

$$\text{Number of cones} = \frac{\textit{volume of cylinder}}{\textit{volume of one ice cream cone}}$$

Number of Ice cream cones needed

$$= \frac{\frac{22}{7}\times 6\times 6\times 15}{\frac{22}{7}\times 45} = 12$$

Thus 12 ice cream cones are required to empty the cylindrical container.

Exercise 7. 4

1. An aluminium sphere of radius 12 cm is melted to make a cylinder of radius 8 cm. Find the height of the cylinder.

Sol :

Radius of sphere = 12 cm

Volume of sphere $= \frac{4}{3}\pi r^3$ cu. units

$$= \frac{4}{3}\pi(12)^3$$

$$= 2304\,\pi \text{ cm}^3$$

Radius of cylinder = 8 cm
height = h cm

Volume of cylinder = $\pi r^2 h$ cu . units

$$= \pi(8)^2 h$$

$= 64\pi \text{ h cm}^3$

Given that sphere is melted and cast into a cylinder

$\therefore$ Volume of cylinder = Volume of sphere

$$64\pi h = 2304\pi$$

$$h = \frac{2304\pi}{64\pi} = 36$$

$\therefore$ Height of the cylinder = 36 cm.

2. Water is flowing at the rate of 15 km per hour through a pipe of diameter 14 cm into a rectangular tank which is 50 m long and 44 m wide. Find the time in which the level of water in the tanks will rise by 21 cm.

Sol :

Diameter of cylindrical pipe = 14 cm

Radius = 7 cm

Length of the pipe = Speed of the water

= 15 km = 15000 m

Length of the water tank = 50 m

Width of the water tank = 44 m

Height of the water tank = Water level

= 21 cm

= 0.21 m

Volume of water tank = $l \times b \times h$ cu. units

$= 50 \times 44 \times 0.21 = 462 \text{ m}^3$

Volume of cylindrical pipe

= Volume of Rectangular tank

$$\pi r^2 h = 462$$

$$\frac{22}{7} \times 0.07 \times 0.07 \times h = 462$$

$$\therefore \quad h = \frac{462 \times 7}{22 \times 0.07 \times 0.07}$$

$$= \frac{3234}{0.1078} = 30000$$

$$\text{Time required} = \frac{30000}{15000} = 2 \text{ hrs.}$$

3. A conical flask is full of water. The flask has base radius r units and height h units, the water poured into a cylindrical flask of base radius xr units. Find the height of water in the cylindrical flask.

Sol :

Radius of conical flask = ‘r’ units

Height of conical flask = ‘h’ units

Volume of conical flask = Volume of water

$= \frac{1}{3}\pi r^2 h$ cu. units

Since, water is poured into the cylindrical flask

$\therefore$ Volume of cylinder = Volume of water

$$\pi(xr)^2 H = \frac{1}{3}\pi r^2 h$$

[xr – radius of cylinder, H – height]

$$x^2 r^2 H = \frac{r^2}{3} h$$

Height of the water in cylinder flask

$$H = \frac{h}{3x^2}$$

4. A solid right circular cone of diameter 14 cm and height 8 cm is melted to form a hollow sphere. If the external diameter of the sphere is 10 cm, find the internal diameter.

Sol :

Diameter of cone = 14 cm

Radius of cone = 7 cm

Height of cone = 8 cm

Volume of cone $= \frac{1}{3}\pi r^2 h$ cu. units

$$= \frac{1}{3} \times \pi \times 7 \times 7 \times 8$$

$$= \frac{392\pi}{3} \text{ cm}^3$$

External diameter of sphere = 10 cm

External radius of sphere ‘R’ = 5 cm

Internal radius = ‘r’

Given, Right Circular Cone is melted to form a hollow sphere.

i.e., Volume of hollow sphere = Volume of cone

$$\frac{4}{3}\pi(5^3 - r^3) = \frac{392\pi}{3}$$

$$5^3 - r^3 = \frac{392}{4} = 98$$

$$r^3 = 125 - 98 = 27$$

Radius r = 3 cm

$\therefore$ Internal diameter = 2r = 6 cm.

5. Seenu's house has an overhead tank in the shape of a cylinder. This is filled by pumping water from a sump (underground tank) which is in the shape of a cuboid. The sump has dimensions 2 m × 1.5 m × 1 m. The overhead tank has its radius of 60 cm and height 105 cm. Find the volume of the water left in sump after the overhead tank has been completely filled with water from the sump which has been full, initially.

Sol :

Cuboidal Tank

length = 2 m = 200 cm

width = 1.5 m = 150 cm

height = 1 m = 100 cm

Volume of cuboidal tank = $l \times b \times h$

$= 200 \times 150 \times 100$

$= 30,00,000 \text{ cm}^3$

Overhead cylindrical Tank

radius = 60 cm

height = 105 cm

volume $= \pi r^2 h$ cu. units

$= \frac{22}{7} \times 60 \times 60 \times 105$

$= 11,88,000 \text{ cm}^3$

$\therefore$ Volume of water left in the sump = Volume of Cuboidal Tank – Volume of Cylindrical Tank

$= 30,00,000 - 11,88,000$

$= 18,12,000 \text{ cm}^3$.

6. The internal and external diameter of a hollow hemispherical shell are 6 cm and 10 cm respectively. If it is melted and recast into a solid cylinder of diameter 14 cm, then find the height of the cylinder.

Sol :

Hollow Hemisphere

Internal diameter = 6 cm

Internal radius 'r' = 3 cm

External diameter = 10 cm

External radius 'R' = 5 cm

$$\left.\begin{array}{l}\text{Volume of hemisphere (or)}\\ \text{Volume of material used}\end{array}\right\} = \frac{2}{3}\pi(R^3 - r^3) \text{ cu. units}$$

$$= \frac{2}{3}\pi(5^3 - 3^3)$$

$$= \frac{2}{3}\pi(125 - 27) = \frac{196\pi}{3} \text{ cm}^3$$

Cylinder

Diameter = 14 cm

radius = 7 cm

height = h

Volume of cylinder $= \pi r^2 h$ cu. units

$= \pi(7)^2 h$

$= 49\pi h \text{ cm}^3$

Given that hollow hemisphere is melted and cast into a solid cylinder

$\therefore$ Volume of cylinder = Volume of hollow hemisphere

$$49\pi h = \frac{196\pi}{3}$$

$$h = \frac{196}{3 \times 49} = \frac{4}{3} = 1.33$$

$\therefore$ Height of the cylinder = 1.33 cm.

7. A solid sphere of radius 6 cm is melted into a hollow cylinder of uniform thickness. If the external radius of the base of the cylinder is 5 cm and its height is 32 cm, then find the thickness of the cylinder.

Sol :

Solid sphere

radius = 6 cm

volume $= \frac{4}{3}\pi r^3$ cu. units

$= \frac{4}{3}\pi(6)^3$

$= \frac{4}{3}\pi(216) = 288\pi \text{ cm}^3$

Hollow cylinder

Internal radius = 'r'

External radius = 'R' = 5 cm

Height h = 32 cm

Volume of Hollow Cylinder

$= \pi h(R^2 - r^2)$ cu. units

$= \pi(32)(25 - r^2) \text{ cm}^3$

Given that solid sphere is melted to form a hollow cylinder.

$\therefore$ Volume of Hollow Cylinder = Volume of Sphere

$$32\pi(25 - r^2) = 288\pi$$

$$25 - r^2 = \frac{288}{32} = 9$$

$$r^2 = 25 - 9 = 16$$

Internal radius r = 4 cm

$\therefore$ Thickness = External radius – Internal radius

= R – r = 5 – 4 = 1 cm.

8. A hemispherical bowl is filled to the brim with juice. The juice is poured into a cylindrical vessel whose radius is 50% more than its height. If the diameter is same for both the bowl and the cylinder then find the percentage of juice that can be transferred from the bowl into the cylindrical vessel.

Sol : Let the radius of hemispherical bowl $= r$

$\therefore$ Volume of hemispherical bowl

$$= \frac{2}{3}\pi r^3 \text{ cu. units}$$

Let the height of cylindrical vessel $= h$

Given $r = h + h\frac{50}{100} \Rightarrow r = h\left(1+\frac{50}{100}\right)$

$$h = \frac{2}{3}r$$

Now, Volume of cylindrical vessel

$$= \pi r^2\left(\frac{2r}{3}\right) = \frac{2}{3}\pi r^3$$

Hence, Volume of juice in the cylindrical vessel

$$= \frac{\frac{2}{3}\pi r^3}{\frac{2}{3}\pi r^3} \times 100\% = 100\ \%$$

Exercise 7. 5

Multiple Choice Questions:

1. The curved surface area of a right circular cone of height 15 cm and base diameter 16 cm is

(1) 60π cm^2 (2) 68π cm^2

(3) 120π cm^2 (4) 136π cm **[Ans (4)]**

Sol :

Base diameter $= 16$ cm, height $= 15$ cm

Base radius $= 8$ cm

C.S.A of cone $= \pi rl$ sq. units

$= \pi(8)\sqrt{15^2+8^2}$

$= \pi(8)\sqrt{289}$

$= 8\pi(17) = 136\pi$ cm^2

2. If two solid hemisphere of same base radius r units are joined together along their bases, then curved surface area of this new solid is

(1) $4\pi r^2$ sq. units (2) $6\pi r^2$ sq. units

(3) $3\pi r^2$ sq. units (4) $8\pi r^2$ sq. units

[Ans (1)]

Sol :

Two hemispheres of same base radius 'r' units are joined, then it is a sphere.

$\therefore$ CSA of new solid $= 4\pi r^2$ sq. units

3. The height of a right circular cone whose radius is 5 cm and slant height is 13 cm will be

(1) 12 cm (2) 10 cm

(3) 13 cm (4) 5 cm **[Ans (1)]**

Sol :

Given r = 5 cm, l = 13 cm

$\therefore$ height $h = \sqrt{l^2 - r^2} = \sqrt{13^2 - 5^2}$

$= \sqrt{169-25} = \sqrt{144} = 12$ cm

4. If the radius of the base of a right circular cylinder is halved keeping the same height, then the ratio of the volume of the cylinder thus obtained to the volume of original cylinder is

(1) 1 : 2 (2) 1 : 4

(3) 1 : 6 (4) 1 : 8 **[Ans (2)]**

Sol :

Radius of the cylinder $= r$

Height of the cylinder $= h$

Volume of the original cylinder $= \pi r^2 h$ sq. units

Radius is halved, then radius $= \frac{r}{2}$

Volume of new cylinder $= \pi\left(\frac{r}{2}\right)^2 h = \frac{\pi r^2 h}{4}$

Now, ratio $= \frac{\textit{Volume of new cylinder}}{\textit{Volume of original cylinder}}$

$$= \frac{\frac{\pi r^2 h}{4}}{\pi r^2 h} = \frac{1}{4}$$

Ratio $= 1 : 4$

5. The total surface area of a cylinder whose radius is $\frac{1}{3}$ of its height is

(1) $\frac{9\pi h^2}{8}$ sq. units (2) $24\pi h^2$ sq. units

(3) $\frac{8\pi h^2}{9}$ sq. units (4) $\frac{56\pi h^2}{9}$ sq. units

[Ans (3)]

Sol : height = h, radius $= \frac{1}{3}h$

T.S.A of cylinder $= 2\pi r$ (h + r)

$$= 2\pi \frac{h}{3}\left(h + \frac{h}{3}\right)$$

$$= 2\pi \frac{h}{3}\left(\frac{4h}{3}\right)$$

$$= \frac{8\pi h^2}{9} \text{ sq. units}$$

6. In a hollow cylinder, the sum of the external and internal radii is 14 cm and the width is 4 cm. If its height is 20 cm, the volume of the material in it is

(1) 5600π cm^3 (2) 1120π cm^3

(3) 56π cm^3 (4) 3600π cm^3

[Ans (2)]

Sol :

Internal radius = r, External radius = R

Given, sum = R + r = 14, width R – r = 4,

height h = 20 cm

Volume of material used

$$= \pi(R^2 - r^2)h \text{ cu. units}$$

$$= \pi(R + r)(R - r)h$$

$$= \pi(14)(4)\,20$$

$$= 1120\pi \text{ cm}^3$$

7. If the radius of the base of a cone is tripled and the height is doubled then the volume is

(1) made 6 times (2) made 18 times

(3) made 12 times (4) unchanged **[Ans (2)]**

Sol :

Radius is tripled, then new radius is 3r

Height is doubled, then new height is 2h

Volume of cone $= \frac{1}{3}\pi r^2 h$ cu. units

$$= \frac{1}{3}\pi(3r)^2\,(2h)$$

$$= \frac{\pi}{3}(9r)^2\,(2h)$$

$$= 18\left(\frac{1}{3}\pi r^2 h\right)$$

= 18 (Volume of cone) = 18 times

8. The total surface area of a hemi-sphere is how many times the square of its radius.

(1) π (2) 4π

(3) 3π (4) 2π **[Ans (3)]**

Sol :

TSA of hemisphere $= 3\pi r^2$

$= 3\pi$ (Square of radius)

$= 3\pi$ times

9. A solid sphere of radius x cm is melted and cast into a shape of a solid cone of same radius. The height of the cone is

(1) $3x$ cm (2) x cm

(3) $4x$ cm (4) $2x$ cm **[Ans (3)]**

Sol :

Radius of sphere = 'x' cm

Volume of sphere $= \frac{4}{3}\pi x^3$ cm^3

Radius of the cone = 'x' cm

Height = h

Volume of cone $= \frac{1}{3}\pi x^2 h$

Volume of cone = volume of sphere

[$\because$ sphere is melted and cast into a cone]

$$\frac{1}{3}\pi x^2 h = \frac{4}{3}\pi x^3$$

$\Rightarrow$ h = $4x$ cm

10. A frustum of a right circular cone is of height 16 cm with radii of its ends as 8 cm and 20 cm. Then, the volume of the frustum is

(1) 3328π cm^3 (2) 3228π cm^3

(3) 3240π cm^3 (4) 3340π cm^3 **[Ans (1)]**

Sol :

Height of frustum h = 16 cm

Radii are 8 cm and 20 cm

i.e., r = 8 cm, R = 20 cm

Volume of frustum

$$= \frac{\pi h}{3}(R^2 + Rr + r^2) \text{ cu. units}$$

$$= \frac{\pi(16)}{3}(400 + 160 + 64)$$

$$= \frac{16\pi}{3}(624) = 3328\pi \text{ cm}^3$$

11. A shuttlecock used for playing badminton has the shape of the combination of

(1) a cylinder and a sphere

(2) a hemisphere and a cone

(3) a sphere and a cone

(4) frustum of a cone and a hemisphere

[Ans (4)]

12. A spherical ball of radius r_1 units is melted to make 8 new identical balls each of radius r_2 units. Then $r_1 : r_2$ is

(1) 2 : 1 (2) 1 : 2
(3) 4 : 1 (4) 1 : 4 **[Ans (1)]**

Sol :

Volume of spherical ball of radius r_1

$$= \frac{4}{3}\pi r_1^3$$

Volume of 8 spherical balls of radius r_2

$$= 8\left(\frac{4}{3}\pi r_2^3\right)$$

Now, $\dfrac{\text{Volume of Sphere}}{\text{Volume of 8 New Spheres}}$

$$= \frac{\frac{4}{3}\pi r_1^3}{8\left(\frac{4}{3}\pi r_2^3\right)} = \frac{r_1^3}{8r_2^3} = 2:1$$

13. The volume (in cm^3) of the greatest sphere that can be cut off from a cylindrical log of wood of base radius 1 cm and height 5 cm is

(1) $\frac{4}{3}\pi$ (2) $\frac{10}{3}\pi$
(3) 5π (4) $\frac{20}{3}\pi$ **[Ans (1)]**

Sol :

Radius of cylindrical log of wood = 1 cm
Height of cylindrical log of wood = 5 cm

$\left.\begin{array}{l}\text{Diameter of Sphere of greatest volume}\\ \text{which can be cut off from cylinder}\end{array}\right\} = 2\text{ cm}$

$\therefore$ radius = 1 cm

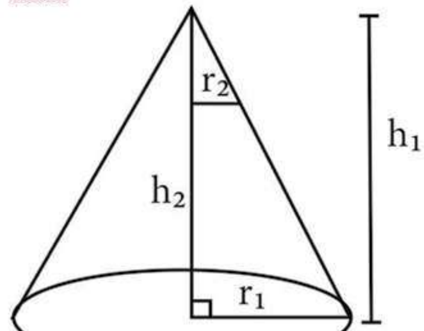
$$\text{Volume} = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3}\pi(1)^3 = \frac{4}{3}\pi\ \text{cm}^3$$

14. The height and radius of the cone of which the frustum is a part are h_1 units and r_1 units respectively. Height of the frustum is h_2 units and radius of the smaller base is r_2 units. If $h_2 : h_1 = 1 : 2$ then $r_2 : r_1$ is

(1) 1 : 3 (2) 1 : 2
(3) 2 : 1 (4) 3 : 1 **[Ans (2)]**

Sol :



Given $\dfrac{h_2}{h_1} = \dfrac{1}{2}$

$$\frac{h_2}{h_1} = \frac{r_2}{r_1} = \frac{1}{2}$$

$$r_2 : r_1 = 1 : 2$$

15. The ratio of the volumes of a cylinder, a cone and a sphere, if each has the same diameter and same height is

(1) 1 : 2 : 3 (2) 2 : 1 : 3
(3) 1 : 3 : 2 (4) 3 : 1 : 2 **[Ans (4)]**

Sol :

Diameter of cylinder = Diameter of cone = Diameter of sphere

Height of cylinder = Height of cone = Height of sphere

Volume of cylinder = $V_1 = \pi r^2 h$

Volume of cone = $V_2 = \frac{1}{3}\pi r^2 h$

Volume of sphere = $V_3 = \frac{4}{3}\pi r^3$

Radius of sphere = 'R', Diameter = 2R

Height of cylinder = Height of cone = 2R

Ratio of volumes $\Rightarrow V_1 : V_2 : V_3$

$$= \pi R^2(2R) : \frac{1}{3}\pi R^2(2R) : \frac{4}{3}\pi R^3$$

$$= 1 : \frac{1}{3} : \frac{2}{3} = 3 : 1 : 2$$

UNIT EXERCISE - 7

1. The barrel of a fountain-pen cylindrical in shape is 7 cm long and 5 mm in diameter. A full barrel of ink in the pen will be used on writing 330 words on an average. How many words can be written using a bottle of ink containing one fifth of a litre?

Sol :

Height of the barrel = h = 7 cm

Diameter = 5 mm

radius r = $\frac{5}{2}$ = 2.5 mm = 0.25 cm

Volume of cylindrical barrel $= \pi r^2 h$

$= \frac{22}{7} \times 0.25 \times 0.25 \times 7$

$= 1.375 \text{ cm}^3$

Given 1.375 cm^3 of ink is used for writing 330 words.

$\therefore$ Number of words that can be written with one - fifth of a litre

[$\because$ 1000 cm^3 = 1 ltr; $\frac{1}{5} \times 1000$ cm^3; 200 cm^3]

$= \frac{330}{1.375} \times 200$ = 48000 words

2. A hemi-spherical tank of radius of 1.75 m is full of water. It is connected with a pipe which empties the tank at the rate of 7 litre per second. How much time will it take to empty the tank completely?

Sol :

Radius of hemispherical tank 'r' = 1.75 m

Volume of hemispherical tank $= \frac{2}{3}\pi r^3$ cu. units

$= \frac{2}{3} \times \frac{22}{7} \times (1.75)^3$

$= 11.225 \text{ m}^3$

= 11225 litre

Given that cylindrical pipe empties the tank at the rate of 7 litre per second.

$\therefore$ Time Required to empty the tank completely

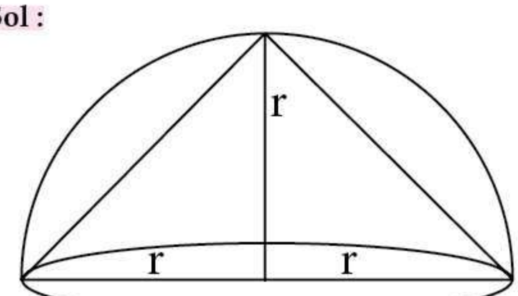
$= \frac{Volume}{Rate}$

$= \frac{11225}{7}$ = 1604 sec (app)

= 27 min (app)

3. Find the maximum volume of a cone that can be carved out of a solid hemisphere of radius r units.

Sol :



Radius of hemisphere = r units

Radius of cone = Radius of hemisphere

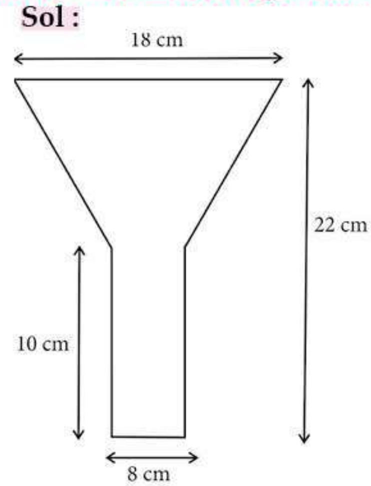
Height of cone = Radius of hemisphere

$\therefore$ Maximum volume of cone $= \frac{1}{3}\pi r^2 h$ cu. units

$= \frac{1}{3}\pi\left(r^2\right)r = \frac{1}{3}\pi r^3$ cu. units

4. An oil funnel of tin sheet consists of a cylindrical portion 10 cm long attached to a frustum of a cone. If the total height is 22 cm, the diameter of the cylindrical portion be 8 cm and the diameter of the top of the funnel be 18 cm, then find the area of the tin sheet required to make the funnel.

Sol :



Area of tin sheet required

= C.S.A of cylinder + C.S.A of frustum

Cylinder:

Radius = 4 cm

Height = 10 cm

C.S.A $= 2\pi rh$ sq. units

$$= 2 \times \frac{22}{7} \times 4 \times 10 = \frac{1760}{7} \text{ sq. units}$$

Frustum of a cone

r_1 = radius of top = 9 cm

r_2 = radius of bottom = 4 cm

Height = 22 – 10 = 12 cm

$$\text{Slant height } l = \sqrt{h^2 + (r_1 - r_2)^2}$$

$$= \sqrt{12^2 + (9-4)^2}$$

$$= \sqrt{144 + 25} = \sqrt{169} = 13 \text{ cm}$$

$$\text{C.S.A} = \pi(r_1 + r_2)l \text{ sq. units}$$

$$= \frac{22}{7}(9+4)(13)$$

$$= \frac{3718}{7} \text{ sq. units}$$

$$\therefore \text{Area of tin sheet} = \frac{1760}{7} + \frac{3718}{7}$$

$$= \frac{5478}{7} = 782.57 \text{ cm}^2.$$

5. Find the number of coins, 1.5 cm in diameter and 2 mm thickness, to be melted to form a right circular cylinder of height 10 cm and diameter 4.5 cm.

Sol :

Coin is in the form of a cylinder

Diameter of the coin = 1.5 cm

$$\text{Radius of the coin} = \frac{1.5}{2}$$

$$\text{Thickness} = \text{height} = 2 \text{ mm} = \frac{2}{10} = 0.2 \text{ cm}$$

$$\text{Volume of coin (cylinder)} = \pi r^2 h$$

$$= \pi\left(\frac{1.5}{2}\right)^2 (0.2)$$

$$= 0.1125\,\pi \text{ cm}^3$$

Diameter of cylinder = 4.5 cm

$$\text{radius} = \frac{4.5}{2} = 2.25 \text{ cm}$$

height = 10 cm

$$\text{volume} = \pi r^2 h \text{ sq. units}$$

$$= \pi(2.25)^2(10)$$

$$= 50.625\,\pi$$

$$\therefore \text{No. of coins} = \frac{\textit{Volume of cylinder}}{\textit{Volume of Coin}}$$

$$= \frac{50.625\,\pi}{0.1125\,\pi} = 450 \text{ coins.}$$

6. A hollow metallic cylinder whose external radius is 4.3 cm and internal radius is 1.1 cm and whole length is 4 cm is melted and recast into a solid cylinder of 12 cm long. Find the diameter of solid cylinder.

Sol :

Hollow metallic cylinder

External radius R = 4.3 cm

Internal radius r = 1.1 cm

Length = height = h = 4 cm

$$\text{Volume} = \pi(R^2 - r^2)h \text{ cu. units}$$

$$= \pi\left((4.3)^2 - (1.1)^2\right)(4)$$

$$= \pi\ (18.49 - 1.21)\ 4$$

$$= 69.12\,\pi \text{ cm3}$$

Solid cylinder

height h = 12 cm

radius r = ?

$$\text{volume} = \pi r^2 h \text{ sq. units}$$

$$= \pi r^2 (12)$$

Given, Hollow cylinder is melted to form solid cylinder

$\therefore$ Volume of cylinder = Volume of hollow cylinder

$$\pi r^2(12) = 69.12\,\pi$$

$$r^2 = \frac{69.12}{12} = 5.76$$

$$r = 2.4 \text{ cm}$$

Diameter of cylinder = 2r = 4.8 cm

7. The slant height of a frustum of a cone is 4 m and the perimeter of circular ends are 18 m and 16 m. Find the cost of painting its curved surface area at ₹ 100 per sq. m.

Sol :

Slant height l = 4 m

Perimeter of larger circle = $2\pi R$ = 18 cm

$$R = \frac{9}{\pi} = \frac{63}{22} \text{ m}$$

Perimeter of smaller circle = $2\pi r$ = 16 m

$$r = \frac{8}{\pi} = \frac{56}{22} \text{ m}$$

$$\text{C.S.A of Frustum of cone} = \pi l\,(R + r) \text{ sq. units}$$

$$= \frac{22}{7} \times 4 \times \left(\frac{63}{22} + \frac{56}{22}\right)$$

$$= \frac{4}{7}(119) = 68 \text{ m}^2$$

Cost of painting per sq. m = Rs. 100

$\therefore$ Cost of painting for 68 sq. m = 68×100
= Rs. 6800

8. A hemi - spherical hollow bowl has material of volume $\frac{436\pi}{3}$ cubic cm. Its external diameter is 14 cm. Find its thickness.

Sol :

External diameter of hollow hemisphere

$= 2R = 14$ cm

$\therefore$ External radius R = 7 cm

$$\text{Given volume} = \frac{436\pi}{3} \text{ cm}^3$$

$$\frac{2}{3}\pi(R^3 - r^3) = \frac{436\pi}{3}$$

$$R^3 - r^3 = 218$$

$$(7)^3 - r^3 = 218$$

$\therefore r^3 = 343 - 218 = 125 = 5$ cm

$\therefore$ Thickness = R – r = 7 – 5 = 2 cm

9. The volume of a cone is $1005\frac{5}{7}$ cu.cm. The area of its base is $201\frac{1}{7}$ sq. cm. Find the slant height of the cone.

Sol :

$$\text{Volume of a cone} = 1005\frac{5}{7} \text{ cu. cm}$$

$$\text{i.e., } \frac{1}{3}\pi r^2 h = 1005\frac{5}{7} \quad \dots (1)$$

area of base = area of circle

$= 201\frac{1}{7}$ sq. units

$$\text{i.e., } \pi r^2 = 201\frac{1}{7} \Rightarrow r^2 = 64$$

Substituting in (1), r = 8 cm

$$\frac{1}{3}\left(201\frac{1}{7}\right) h = 1005\frac{5}{7}$$

$$\frac{1}{3}\left(\frac{1408}{7}\right) h = \frac{7040}{7}$$

$$h = \frac{7040}{7} \times \frac{7}{1408} \times 3 = 15 \text{ cm}$$

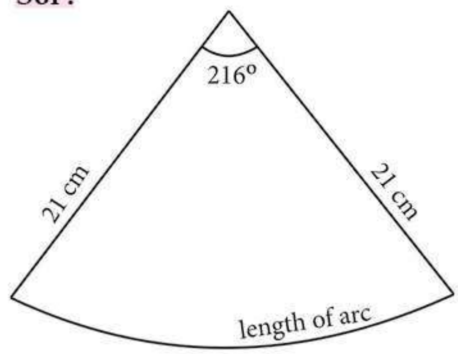
$$\therefore \text{Slant height of cone } l = \sqrt{h^2 + r^2}$$

$$= \sqrt{15^2 + 8^2} = \sqrt{225 + 64}$$

$$= \sqrt{289} = 17 \text{ cm}$$

10. A metallic sheet in the form of a sector of a circle of radius 21 cm has central angle of 216°. The sector is made into a cone by bringing the bounding radii together. Find the volume of the cone formed.

Sol :



$$\text{Length of arc of a sector} = \frac{\theta}{360^o} \times 2\pi r_1$$

$$= \frac{216^o}{360^o} \times 2\pi \times 21$$

$$= \frac{3}{5}(2\pi)(21) = \frac{126\pi}{5} \text{ cm}$$

Since, sector is made into a cone by bringing the bounding radii together.

$$\left.\begin{array}{l}\text{Circumference of base}\\ \text{of the cone}\end{array}\right\} = \left\{\begin{array}{l}\text{length of arc}\\ \text{of a sector}\end{array}\right.$$

$$\therefore 2\pi r = \frac{126\pi}{5}$$

$$\text{radius of base of a cone, r} = \frac{63}{5} \text{ cm}$$

and radius of sector = slant height of cone

$21 = l$

$$\text{Height of cone, h} = \sqrt{l^2 - r^2} = \sqrt{(21)^2 - \left(\frac{63}{5}\right)^2}$$

$$= \sqrt{441 - 158.76}$$

$$= \sqrt{282.24} = 16.8 \text{ cm}$$

$$\therefore \text{Volume of cone} = \frac{1}{3}\pi r^2 h \text{ cu. units}$$

$$= \frac{1}{3} \times \frac{22}{7} \times 12.6 \times 12.6 \times 16.8$$

$$= \frac{58677.696}{21}$$

$$= 2794.176 = 2794.18 \text{ cm}^3$$

CREATIVE QUESTIONS

I. Multiple Choice Questions

Surface area

1. The lateral surface area of a cylinder is developed into a square whose diagonal is $2\sqrt{2}$ cm. The area of the base of the cylinder (in cm²) is

(1) 3π (2) $\frac{1}{\pi}$

(3) π (4) 6π **[Ans (2)]**

Sol :

$$\text{diagonal } \sqrt{2}a = 2\sqrt{2}$$

$$\text{Side of the square (a)} = \frac{2\sqrt{2}}{\sqrt{2}} = 2 \text{ cm}$$

$$\text{Side of the square} = \text{Base perimeter} = 2 \text{ cm}$$

$$\text{i.e., } 2\pi r = 2 \Rightarrow r = \frac{1}{\pi}$$

$$\therefore \text{ Base area of cylinder} = \pi r^2$$

$$= \pi\left(\frac{1}{\pi^2}\right) = \frac{1}{\pi}$$

2. How many metres of cloth 2.5 m wide will be required to make a conical tent whose radius is 7 m and height is 24 m?

(1) 210 m (2) 220 m

(3) 230 m (4) 240 m **[Ans (2)]**

Sol :

$$\text{Slant height } l = \sqrt{r^2 + h^2}$$

$$= \sqrt{24^2 + 7^2}$$

$$= \sqrt{625} = 25 \text{ m}$$

$$\text{Area of cloth required} = \text{CSA of Cone}$$

$$= \pi r l$$

$$= \frac{22}{7} \times 7 \times 25 = 550 \text{ m}^2$$

$$\therefore \text{ Length of cloth} = \frac{550}{2.5} = 220 \text{ m}$$

3. The total surface area of a hemisphere of radius 10 cm is

(1) 942.86 cm² (2) 900 cm²

(3) 300 cm² (4) 592.86 cm² **[Ans (1)]**

Sol :

$$\text{TSA of hemisphere} = 3\pi r^2$$

$$= 3 \times \frac{22}{7} \times 10 \times 10$$

$$= \frac{6600}{7} = 942.86 \text{ cm}^2$$

3. The Curved Surface area of a right circular cone of radius 11.3 cm is 355 cm². What is its slant height?

(1) 8 cm (2) 9 cm

(3) 10 cm (4) 11 cm **[Ans (3)]**

Sol :

$$r = 11.3, \quad \text{CSA} = 355$$

$$\pi r l = 355$$

$$\frac{22}{5} \times 11.3 \times l = 355$$

$$l = \frac{355 \times 7}{22 \times 11.3} = 10 \text{ (app.)}$$

4. The Curved Surface area of a right circular cone of height 15 cm and base diameter 16 cm is

(1) 146π (2) 116π

(3) 126π (4) 136π **[Ans (4)]**

Sol :

$$h = 15, \; r = \frac{16}{2} = 8$$

$$l = \sqrt{r^2 + h^2}$$

$$= \sqrt{64 + 225} = \sqrt{289} = 17$$

$$\text{C.S.A} = \pi r l$$

$$= \pi \times 8 \times 17 = 136\pi \text{ cm}^2$$

5. The ratio of total surface area to the lateral surface area of a cylinder with base radius 80 cm and height 20 cm is

(1) 1 : 5 (2) 2 : 3

(3) 5 : 1 (4) 3 : 2 **[Ans (3)]**

Sol :

$$\text{LSA} = 2\pi r h,$$

$$\text{TSA} = 2\pi r(h + r)$$

$$\text{Ratio} = \frac{TSA}{LSA} = \frac{2\pi r(h+r)}{2\pi r h} = \frac{h+r}{h}$$

$$= \frac{20+80}{20}$$
$$= \frac{100}{20} = \frac{5}{1}$$

6. The surface area of a sphere of diameter 'r' is (sq.units)

(1) $2\pi r^2$ (2) πr^2

(3) $\frac{\pi r^2}{2}$ (4) $\frac{\pi r^2}{4}$ **[Ans (2)]**

Sol :

$$\text{Diameter} = \text{r, radius} = \frac{r}{2}$$
$$\text{Surface area} = 4\pi\left(\frac{r}{2}\right)^2$$
$$= 4\pi\left(\frac{r^2}{4}\right) = \pi r^2$$

7. The total surface area of a cone whose radius is $\frac{r}{2}$ and slant height $2l$ is (sq.units)

(1) $2\pi r(l+r)$ (2) $\pi r\left(l+\frac{r}{4}\right)$

(3) $\pi r(l+r)$ (4) $2\pi rl$ **[Ans (2)]**

Sol :

$$\text{TSA} = \pi R(L+R)$$
$$= \pi\left(\frac{r}{2}\right)\left(2l+\frac{r}{2}\right) = \pi r\left(l+\frac{r}{4}\right)$$

8. The slant height of the frustum of a cone is 4 cm and the circumference of its circular ends are 18 cm and 6 cm, then the curved surface area of the frustum is (cm²)

(1) 12 (2) 24

(3) 48 (4) 54 **[Ans (3)]**

Sol :

$$\text{Circumference } 2\pi R = 18 \Rightarrow \pi R = 9$$
$$2\pi r = 6 \Rightarrow \pi r = 3$$
$$\text{CSA of frustum} = \pi l(R+r)$$
$$= l(\pi R+\pi r)$$
$$= 4(9+3)$$
$$= 48 \text{ cm}^2$$

9. If two solid hemispheres of same base radius 'r' are joined together along their bases, then the curved surface area of this new solid is (sq.units)

(1) $4\pi r^2$ (2) $6\pi r^2$

(3) $3\pi r^2$ (4) $8\pi r^2$ **[Ans (1)]**

Sol :

When the hemispheres are joined, then the new solid is a sphere.

$$\text{CSA of a sphere} = 4\pi r^2$$

Volume

10. x and y are two cylinders of the same height. The base of x has diameter that is half the diameter of the base of y. If the height of x is doubled, the volume of x becomes

(1) equal to the volume of y

(2) double the volume of y

(3) half the volume of y

(4) greater than the volume of y **[Ans (3)]**

Sol :

Let the height of x and y be 'h' and their radii be 'r' and '2r' respectively.

$$\text{Then volume of x} = \pi r^2 h \text{ and}$$
$$\text{volume of y} = \pi(2r)^2 h = 4\pi r^2 h$$
$$\text{New volume of x} = \pi r^2(2h)$$
$$= 2\pi r^2 h$$
$$= \frac{1}{2}\text{ (volume of y)}$$

11. The Area of the base of a right circular cone is 78.5 cm² and its height is 12 cm. Find the volume

(1) 341 cm³ (2) 314 cm³

(3) 301 cm³ (4) 304 cm³ **[Ans (2)]**

Sol :

$$\text{Base Area} = 78.5$$
$$\text{i.e., } \pi r^2 = 78.5$$
$$\text{and h} = 12$$
$$\text{Volume} = \frac{1}{3}(\pi r^2)h$$
$$= \frac{1}{3}(78.5)(12)$$
$$= 314 \text{ cm}^3$$

12. The curved surface area of a cylindrical pillar is 264 m² and its volume is 924 m³, then its diameter is

(1) 12 m (2) 13 m

(3) 14 m (4) 15 m **[Ans (3)]**

Sol :

$$\text{Given CSA} = 2\pi rh = 264 \quad \ldots (1)$$

$$\text{Volume} = \pi^2 rh = 924 \quad \ldots (2)$$

$$\frac{(2)}{(1)} \Rightarrow \frac{\pi r^2 h}{2\pi rh} = \frac{924}{264} \Rightarrow r = 7$$

$$\therefore \text{ Diameter} = 2(7) = 14 \text{ m}$$

13. The volume of the sphere in 38808 cm³, then its surface area is

(1) 5544 cm² (2) 4455 cm²
(3) 4545 cm² (4) 5454 cm² **[Ans (1)]**

Sol :

$$\text{Volume} = \frac{4}{3}\pi r^3 = 38808$$

$$r^3 = 9261 = (21)^3$$

$$r = 21$$

$$\therefore \text{ Surface area} = 4\pi r^2 = 4 \times \frac{22}{7} \times 21 \times 21$$

$$= 5544 \text{ cm}^2$$

14. The volumes of two cylinders are as a : b and their heights are as c : d, then the ratio of their diameters is (cubic units)

(1) $\frac{ad}{bc}$ (2) $\frac{ad^2}{bc^2}$
(3) $\sqrt{\frac{ad}{bc}}$ (4) $\sqrt{\frac{a}{b} \times \frac{c}{d}}$ **[Ans (3)]**

Sol :

Let r_1 and r_2 be the radii of two cylinders respectively, and

Let the heights of two cylinders be ck and dk respectively.

$$\therefore \text{ Ratio of volumes} = \frac{\pi r_1^2 ck}{\pi r_2^2 dk} = \frac{a}{b} \text{ (given)}$$

$$\Rightarrow \frac{r_1}{r_2} = \sqrt{\frac{ad}{bc}}$$

15. A hemispherical container with radius 6 cm contains 325 ml of milk. Then the volume of milk that is needed to fill the container completely is

(1) 124.75 ml (2) 127.45 ml
(3) 217.45 ml (4) 117.45 ml **[Ans (2)]**

Sol :

$$\text{Volume of Container} = \frac{2}{3}\pi r^3 \text{ cubic units}$$

$$= \frac{2}{3} \times \frac{22}{7} \times (6)^3 \text{ cm}^3$$

$$= 452.45 \text{ ml}$$

$\therefore$ Volume of milk needed to fill the container

$$= 452.45 - 325$$

$$= 127.45 \text{ ml}$$

16. The external and internal diameters of a hemispherical bowl are 10 cm and 8 cm respectively, then the volume is (cm³)

(1) 121.87 (2) 121.78
(3) 128.71 (4) 127.81 **[Ans (4)]**

Sol :

$$\text{External radius R} = \frac{10}{2} = 5 \text{ cm}$$

$$\text{Internal radius r} = \frac{8}{2} = 4 \text{ cm}$$

$$\text{Volume} = \frac{2}{3}\pi(R^3 - r^3)$$

$$= \frac{2}{3} \times \frac{22}{7}(5^3 - 4^3)$$

$$= \frac{2}{3} \times \frac{22}{7}(61) = 127.81 \text{ cm}^3$$

17. If the volume and surface area are numerically equal then its radius is

(1) 2 units (2) 3 units
(3) 4 units (4) 5 units **[Ans (2)]**

Sol :

$$\text{Volume} = \text{Surface area}$$

$$\frac{4}{3}\pi r^3 = 4\pi r^2$$

$$r = 3$$

18. If the radius of cone is reduced to half, then the new volume would be

(1) $\frac{1}{3}\left(\frac{1}{3}\pi r^2 h\right)$ (2) $\frac{1}{3}\pi\left(\frac{r}{2}\right)^2 h$
(3) $\frac{1}{3}\pi\left(\frac{r}{9}\right)^2 h$ (4) $\frac{1}{3}\pi\left(\frac{r^2}{4}\right)\left(\frac{h}{2}\right)$ **[Ans (2)]**

Combination of Solids

19. A right circular cylinder of radius 'r' cm and height 'h' cm (h > 2r) just encloses a sphere of diameter

(1) r cm (2) 2r cm
(3) h cm (4) 2h cm **[Ans (2)]**

Sol :

Diameter of the sphere = Diameter of base of cylinder

= 2r

20. Two cones with same base radius 8 cm and height 15 cm are joined together along their bases. Then the surface area of the shape so formed is

(1) 854 cm^2 (2) 860 cm^2
(3) 864 cm^2 (4) 870 cm^2 **[Ans (1)]**

Sol :

$$\text{Slant height } l = \sqrt{r^2+h^2} = \sqrt{8^2+15^2} = \sqrt{289} = 17$$

$$\text{Surface area} = 2(\pi r l) = 2\times\frac{22}{7}\times 8\times 17 = 854.08$$

21. A cylinder circumscribes a sphere. The ratio of their volumes is

(1) 1 : 2 (2) 3 : 2
(3) 4 : 3 (4) 5 : 6 **[Ans (2)]**

Sol :

Let 'h' be the height of the cylinder then the radius of sphere $= \frac{h}{2}$ = radius of base of cylinder

$$\text{Now } \frac{V_1}{V_2} = \frac{\pi\left(\frac{h}{2}\right)^2 h}{\frac{4}{3}\pi\left(\frac{h}{2}\right)^3} = \frac{\pi\frac{h^3}{4}}{4\pi\frac{h^3}{24}} = 3:2$$

22. A sphere and a cube have the same surface area. The ratio of their volumes is

(1) $\sqrt{6}:\sqrt{\pi}$ (2) $\sqrt{3}:\sqrt{\pi}$
(3) $\pi:\sqrt{2}$ (4) None of these

[Ans (1)]

Sol :

S.A of Sphere = S.A. of cube

$$4\pi r^2 = 6a^2$$

$$r^2 = \frac{6a^2}{4\pi} = \frac{3a^2}{2\pi}$$

$$r = \frac{\sqrt{3}a}{\sqrt{2}\sqrt{\pi}}$$

$$\text{Ratio of volumes} = \frac{V_1}{V_2} = \frac{\frac{4}{3}\pi r^3}{a^3} = \frac{\frac{4}{3}\pi\left(\frac{\sqrt{3}a}{\sqrt{2}\sqrt{\pi}}\right)^3}{a^3}$$

$$= \frac{4\pi 3\sqrt{3}}{3.2\sqrt{2}\pi\sqrt{\pi}}$$

$$= \frac{2\sqrt{3}}{\sqrt{2}\sqrt{\pi}} = \frac{\sqrt{2}\sqrt{3}}{\sqrt{\pi}} = \frac{\sqrt{6}}{\sqrt{\pi}}$$

23. A sphere of radius R has volume equal to that of a cone of radius R, the height of the cone is

(1) R (2) 2R
(3) 3R (4) 4R **[Ans (4)]**

Conversion of Solids

24. A cone is 8.4 cm high and radius of its base is 2.1 cm. It is melted and recast into sphere. The radius of the sphere is

(1) 4.2 cm (2) 2.1 cm
(3) 2.4 cm (4) 1.6 cm **[Ans (2)]**

Sol :

$$\text{Volume of cone} = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(2.1)^2(8.4)$$

$$\text{Volume of sphere} = \frac{4}{3}\pi r_1^3$$

$$\text{Given } \frac{4}{3}\pi r_1^3 = \frac{1}{3}\pi(2.1)^2(8.4)$$

$$r_1^3 = (2.1)^3$$

$$r_1 = 2.1 \text{ cm}$$

25. A spherical iron ball is dropped into a vessel of base diameter 14 cm, containing water. The water level is increased by $9\frac{1}{3}$ cm. What is the radius of the ball?

(1) 3.5 cm (2) 7 cm
(3) 9 cm (4) 12 cm **[Ans (2)]**

Sol :

$$\text{Volume of spherical ball} = \frac{4}{3}\pi r^3$$

$$\text{Now } \frac{4}{3}\pi r^3 = \pi(7)^2\left(\frac{28}{3}\right)$$

$$r^3 = 7^3$$

$$\Rightarrow r = 7$$

26. Three solid spheres of gold whose radii are 1 cm, 6 cm and 8 cm respectively are melted into a single solid sphere. Then the radius of the sphere is

(1) 7 cm (2) 8 cm
(3) 9 cm (4) 10 cm **[Ans (3)]**

Sol :

$$\text{Radius of the new sphere} = \left[(1^3+6^3+8^3)\right]^{\frac{1}{3}}$$
$$= (729)^{\frac{1}{3}} = 9$$

27. A copper sphere of diameter 18 cm is drawn into a wire of diameter 4 mm. Then the length of the wire is

(1) 143 m (2) 243 m
(3) 343 m (4) 443 m **[Ans (2)]**

Sol :

$$\text{Volume of sphere} = \text{volume of wire (cylinder)}$$
$$\frac{4}{3}\pi(9\times9\times9) = \pi\times0.2\times0.2\times h$$
$$\therefore\ h = \frac{972\times5\times5}{100} = 243\text{ m}$$

28. A hemispherical bowl of internal radius 9 cm contains a liquid. This liquid is to be filled into cylindrical shaped small bottles of diameter 3 cm and height 4 cm. How many bottles will be needed to empty the bowl?

(1) 24 (2) 34
(3) 44 (4) 54 **[Ans (4)]**

Sol :

$$\text{Volume of bowl} = \frac{2}{3}\pi(9)^3 = 486\pi$$
$$\text{Volume of 1 bottle} = \pi\times\frac{3}{2}\times\frac{3}{2}\times4 = 9\pi$$
$$\therefore\ \text{No. of bottles} = \frac{486\pi}{9\pi} = 54$$

II. Very Short Answer Questions

1. The curved surface area of a right circular cylinder of height 14 cm is 88 cm². Find the diameter of the base of the cylinder.

Sol :

$$\text{Radius of the cylinder} = r\text{ cm}$$
$$\text{Height 'h'} = 14\text{ cm}$$
$$\text{C.S.A} = 2\pi rh\text{ sq. units}$$
$$\therefore\ 2\pi rh = 88$$
$$2\times\frac{22}{7}\times r\times14 = 88$$
$$r = 1\text{ cm}$$
$$\text{Diameter of the base} = 2\times1 = 2\text{ cm.}$$

2. In the hot water heating system, there is a cylindrical pipe of length 28 m and diameter 5 cm. Find the total radiating surface in the system.

Sol :

$$\text{Total radiating surface} = \text{Curved surface area of cylindrical pipe.}$$
$$= 2\pi rh$$
$$\text{Length } h = 28\text{ m}$$
$$\text{Radius } r = \frac{5}{2}\,cm$$
$$= 2.5\text{ cm} = 0.025\text{ m}$$
$$\therefore\ \text{Total radiating surface} = 2\pi rh$$
$$= 2\times\frac{22}{7}\times0.025\times28$$
$$= 4.4\text{ m}^2$$

3. Find the total surface area of a cone, if its slant height is 21 m and diameter of its base is 24 m.

Sol :

$$\text{Diameter} = 24\text{ m,}$$
$$\text{radius } r = \frac{24}{2} = 12\text{ m}$$
$$\text{Slant height } l = 21\text{ m}$$
$$\text{Total surface area} = \pi r(l+r)\text{ sq. units}$$
$$= \frac{22}{7}\times12\times(21+12)$$
$$= 1244.57\text{ m}^2\text{ (app).}$$

4. Find the radius of a sphere whose surface area is 154 cm².

Sol :

$$\text{Total surface area} = 4\pi r^2$$
$$4\pi r^2 = 154$$
$$4\times\frac{22}{7}\times r^2 = 154$$
$$r^2 = \frac{154\times7}{4\times22} = 12.25$$
$$r = \sqrt{12.25} = 3.5$$
$$\text{Radius} = 3.5\text{ cm.}$$

5. A hemispherical bowl is made of steel, 0.25 cm thick. The inner radius of the bowl is 5 cm. Find the outer curved surface area of the bowl.

Sol :

$$\text{Inner radius } r = 5\text{ cm}$$
$$\text{Thickness} = 0.25\text{ cm}$$
$$\text{outer radius } R = 5+0.25 = 5.25\text{ cm}$$

$\therefore$ Outer curved surface area $= 2\pi R^2$ Sq. units

$$= 2 \times \frac{22}{7} \times (5.25)^2$$

$$= 173.25 \text{ cm}^2$$

6. The radii of the circular ends of a bucket of height 24 cm are 15 cm and 5 cm. Find the area of its curved surface.

Sol :

Given R = 15 cm, r = 5 cm, h = 24 cm

$$l = \sqrt{h^2 + (R-r)^2}$$

$$= \sqrt{(24)^2 + (15-5)^2}$$

$$= \sqrt{676} = 26 \text{ cm}$$

$$\text{C.S.A} = \pi l (R + r)$$

$$= \frac{22}{7} \times 26 \times (15 + 5)$$

$$= \frac{11440}{7} = 1634.28 \text{ cm}^2$$

7. The circumference of the base of a cylindrical vessel is 132 cm and its height is 25 cm. How many litres of water it can hold?

Sol :

Radius = r, height = h.

Circumference $2\pi r = 132$

$\Rightarrow$ r = 21 cm

Volume $= \pi r^2 h$ cu. units

$$= \frac{22}{7} \times 21 \times 21 \times 25$$

$$= 34650 \text{ cm}^3$$

Quantity of water $= \dfrac{34650}{1000}$ litres

= 34.65 litres.

[$\because$ 1000 cm^3 = 1 ltr.]

8. The height of the cone is 15 cm. If its volume is 1570 cm^3, find the radius of the base.

Sol :

h = 15 cm, radius = r

Volume $\frac{1}{3}\pi r^2 h = 1570$

$$\frac{1}{3} \times \frac{22}{7} \times r^2 \times 15 = 1570$$

$$r = \sqrt{100}$$

radius of the base $r = \sqrt{100} = 10$ cm.

9. Find the amount of water displaced by a solid spherical ball of diameter 0.21 cm.

Sol :

Diameter = 0.21 cm;

Radius $= \dfrac{0.21}{2} = 0.105$ cm

Amount of water displaced = Volume of the ball

$$= \frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times (0.105)^3$$

$$= 0.004851 \text{ cm}^3.$$

10. A drinking glass is in the shape of a frustum of a cone of height 14 cm. The diameters of its two circular ends are 4 cm and 2 cm. Find the capacity of the glass.

Sol :

Given R = 2 cm, r = 1 cm

Capacity of glass = Volume of frustum

$$= \frac{\pi h}{3}(R^2 + Rr + r^2) \text{ cu. units.}$$

$$= \frac{22}{7} \times \frac{14}{3}((2)^2 \times (2 \times 1) + (1)^2)$$

$$= \frac{44}{3}(4 + 2 + 1)$$

$$= \frac{308}{3}$$

$$= 102.67 \text{ cm}^3$$

11. The radius of a cone is 20 cm. If its volume is 8800 cm^3, find the height of the base.

Sol :

Let Height h = h cm,

radius = 20 cm

Volume = 8800 cm^3

Volume $= \frac{1}{3}\pi r^2 h$

$$\frac{1}{3}\pi r^2 h = 8800$$

$$\frac{1}{3} \times \frac{22}{7} \times 20 \times 20 \times h = 8800$$

$$h = \frac{8800 \times 3 \times 7}{22 \times 20 \times 20}$$

height = 21 cm

III. Short Answer Questions:

1. The diameter of a roller is 84 cm and its length is 120 cm. It takes 500 complete revolutions to move once over to level a playground. Find the area of the play ground (in sq. m).

Sol :

Diameter of roller = 84 cm

$$\text{radius} = \frac{84}{2} = 42 \text{ cm} = 0.42 \text{ m}$$

$$\text{length} = h = 120 \text{ cm} = 1.2 \text{ m}$$

Area covered by roller in one revolution = C.S.A of the roller

$$= 2\pi rh$$

$$= 2 \times \frac{22}{7} \times 0.42 \times 1.2$$

$$= 3.168 \text{ m}^2$$

$\therefore$ Area covered in 500 revolutions = Area of the playground

$$= 500 \times 3.168 = 1584 \text{ sq. m}$$

2. The slant height and base diameter of a conical tomb are 25 m and 14 m respectively. Find the cost of white washing its curved surface area at the rate of ₹ 210 per 100 m²?

Sol :

diameter = 14 m; radius = 7 cm;

Slant height l = 25 m

$$\text{C.S.A} = \pi rl = \frac{22}{7} \times 7 \times 25$$

$$= 550 \text{ m}^2$$

Given, Cost of white washing per 100 m² is ₹ 210

$\therefore$ Cost of white washing the conical tomb

$$= 550 \times \frac{210}{100} = ₹\,1155$$

3. The diameter of the moon is approximately one fourth of the diameter of the Earth. Find the ratio of their surface areas.

Sol :

Let the diameter of the Earth be 'R'

$$\text{Radius of the Earth} = \frac{R}{2}$$

$$\text{Diameter of the Moon} = \frac{1}{4}R$$

$$\text{Radius of the Moon} = \frac{R}{8}$$

$\therefore$ Ratio of surface areas of moon and earth

$$= \frac{4\pi\left(\frac{R}{8}\right)^2}{4\pi\left(\frac{R}{2}\right)^2}$$

$$\therefore \text{Ratio} = \frac{1}{16} = 1 : 16$$

4. The radii of circular ends of a solid frustum of a cone are 33 cm and 27 cm and its slant height is 10 cm. Find its total surface area.

Sol :

Given R = 33 cm, r = 27 cm and l = 10 cm

$$\therefore \text{T.S.A of frustum} = \pi(R^2 + r^2 + l(R+r))$$

$$= \frac{22}{7}((33)^2 + (27)^2 + 10(33+27))$$

$$= \frac{22}{7}(1089 + 729 + 600)$$

$$= \frac{53196}{7} = 7599.43 \text{ cm}^2$$

5. The sum of the radius of the base and the height of a solid cylinder is 37 m. If the total surface area of the solid cylinder is 1628 m², find the circumference its base and volume of the cylinder.

Sol :

radius = r, height = h

T.S.A = 1628

$$2\pi r(h+r) = 1628$$

$$2\pi r(37) = 1628$$

$$\Rightarrow r = 7 \text{ m}$$

$$\text{and } 7 + h = 37$$

$$\Rightarrow h = 37 - 7 = 30 \text{ m}$$

$$\text{Circumference} = 2\pi r$$

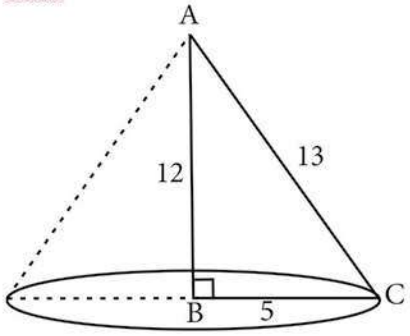
$$= 2 \times \frac{22}{7} \times 7 = 44 \text{ m}$$

$$\text{Volume} = \pi r^2 h$$

$$= \frac{22}{7} \times 7 \times 7 \times 30 = 4620 \text{ m}^3.$$

6. A right triangle ABC with sides 5 cm, 12 cm and 13 cm is revolved about the side 12 cm. Find the volume of the solid so obtained.

Sol :



Triangle ABC is revolved about the side AB

$$= 12 \text{ cm}$$

$$\text{Volume of cone} = \frac{1}{3}\pi r^2 h \text{ cu. units}$$

$$= \frac{1}{3}\pi (5)^2 (12)$$

$$= 100 \times \frac{22}{7}$$

$$= 314 \text{ cm}^3 \text{ (app)}$$

7. The diameter of a metallic ball is 4.2 cm. What is the mass of ball if the density of the metal is 8.9 g per cm³?

Sol :

$$\text{Diameter} = 4.2 \text{ cm};$$

$$\text{Radius} = 2.1 \text{ cm}$$

$$\text{Volume} = \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times (2.1)^3$$

$$= 38.808 \text{ cm}^3$$

$$\text{Mass of the ball} = \text{Volume} \times \text{Density}$$

$$= 38.808 \times 8.9 = 345.3912 \text{ g.}$$

8. A bucket is in the form of a cone. Its depth is 24 cm and the diameters of the top and bottom ends are 30 cm and 10 cm respectively. Find the capacity of the bucket.

Sol :

Given: $R = \frac{30}{2} = 15$ cm, $r = \frac{10}{2} = 5$ cm and $h = 24$ cm

$$\text{Volume} = \frac{1}{3}\pi h(R^2 + Rr + r^2) \text{ cubic units.}$$

$$= \frac{1}{3} \times \frac{22}{7} \times 24 \times (15^2 + 15 \times 5 + 5^2)$$

$$= \frac{22}{7} \times 8 \times (225 + 75 + 25)$$

$$= \frac{57200}{7} = 8171.42 \text{ cm}^3$$

IV. Long Answer Questions

1. A factory manufactures 1,20,000 pencils daily. The pencils are cylindric in shape, each of length 25 cm and circumference 1.5 cm. Determine the cost of colouring the curved surface of the pencils manufactured in one day at ₹ 0.05 per dm².

Sol :

Let the radius of the base be 'r' cm.

$$\text{Circumference} = 2\pi r$$

$$2\pi r = 1.5 \text{ cm}$$

$$r = \frac{1.5}{2\pi} = \frac{10.5}{44} cm$$

Curved surface area of pencil

$$= 2\pi rh$$

$$= 2 \times \frac{22}{7} \times \frac{10.5}{44} \times 25$$

Cost of colouring

$$100 \text{ cm}^2 = ₹. 0.05$$

[$\because$ 1. sq. dm = 100 sq. cm]

$$\text{Cost of colouring} = \frac{0.05}{100} \times 2 \times \frac{22}{7} \times \frac{10.5}{44} \times 25$$

$$= ₹ \frac{3}{160}$$

Hence, cost of colouring 1,20,000 pencils

$$= \frac{3}{160} \times 120000 = ₹ 2250.$$

2. There are two cones. The curved surface area of one is twice that of the other. The slant height of the later is twice that of the former. Find the ratio of their radii.

Sol :

Let r_1 be the radius and l_1 be the slant height of first cone and r_2 be the radius and l_2 be the slant height of the second cone.

$$\text{C.S.A of I cone} = \pi r_1 l_1$$

$$\text{C.S.A of II cone} = \pi r_2 l_2$$

$$\text{Given } \pi r_1 l_1 = 2\pi r_2 l_2$$

$$\Rightarrow r_1 l_1 = 2r_2 l_2$$

$$\text{and } r_1 l_1 = 2r_2 (2l_1)$$

$$r_1 l_1 = 4r_2 l_1$$

$$r_1 = 4r_2$$

$$\frac{r_1}{r_2} = \frac{4}{1}$$

Hence, the ratio of their radii is 4 : 1.

3. The diameter of a sphere is decreased by 25%. By what percent does its curved surface area decrease?

Sol :

Let the diameter be 'x' units

$$\text{radius} = \frac{x}{2} \text{ units}$$

$$\therefore \quad \text{C.S.A of sphere} = 4\pi r^2 \text{ sq. units.}$$

$$= 4\pi\left(\frac{x}{2}\right)^2$$

$$= 4\pi\left(\frac{x^2}{4}\right) = \pi x^2$$

Given that diameter is decreased by 25%.

$$\text{New diameter} = x - 25\% \text{ of } x$$

$$= x\left(1 - \frac{25}{100}\right) = \frac{3}{4}x$$

$$\text{New Radius} = \frac{3x}{8}$$

$$\text{C.S.A of new sphere} = 4\pi\left(\frac{3x}{8}\right)^2 = \frac{9\pi x^2}{16}$$

$$\text{Decrease in C.S.A} = \pi x^2 - \frac{9\pi x^2}{16} = \frac{7\pi x^2}{16}$$

Hence, percentage in decrease

$$= \frac{\frac{7\pi x^2}{16}}{\pi x^2} \times 100\%$$

$$= 43.75\%.$$

4. A solid cylinder has total surface area of 462 sq. cm. Its curved surface area is one-third its total surface area. Find the volume of cylinder.

Sol :

$$\text{radius} = r, \text{ height} = h$$

$$\text{T.S.A} = 2\pi r\,(h+r) \text{ sq. units}$$

$$\text{Given, } 2\pi r\,(h+r) = 462$$

$$r\,(h+r) = \frac{462 \times 7}{2 \times 22} = \frac{147}{2} \qquad \dots (1)$$

$$\text{C.S.A} = 2\pi rh \text{ sq. units}$$

$$\text{Given, } 2\pi rh = \frac{1}{3}(2\pi r\,(h+r))$$

$$\Rightarrow \quad h = \frac{h+r}{3} \Rightarrow r = 2h \qquad \dots (2)$$

Substituting (2) in (1)

$$2h\,(h + 2h) = \frac{147}{2}$$

$$6h^2 = \frac{147}{2}$$

$$h^2 = \frac{49}{4} \Rightarrow h = \frac{7}{2}$$

$$\therefore \quad r = 2h = 2\left(\frac{7}{2}\right) = 7 \text{ cm}$$

$$\therefore \quad \text{Volume} = \pi r^2 h = \frac{22}{7} \times (7)^2 \left(\frac{7}{2}\right)$$

$$= 539 \text{ cm}^3.$$

5. A cone of height 24 cm has a curved surface area 550 cm². Find its volume.

Sol :

$$\text{radius} = r,$$

$$\text{slant height} = l$$

$$\text{and } l^2 = r^2 + h^2$$

$$= r^2 + 24^2 = r^2 + 576$$

$$\text{C.S.A} = \pi r l$$

$$= \frac{22}{7} \times r \times \sqrt{r^2 + 576} \text{ cm}^2$$

$$\frac{22}{7} \times r \times \sqrt{r^2 + 576} = 550$$

$$r\sqrt{r^2 + 576} = 175$$

Squaring both sides

$$r^2\,(r^2 + 576) = (175)^2$$

$$\Rightarrow \quad r^4 + 576\,r^2 - (175)^2 = 0$$

$$\Rightarrow \quad (r^2 - 49)\,(r^2 + 625) = 0$$

$$r^2 + 625 \neq 0, \quad \therefore \quad r^2 - 49 = 0$$

$$r^2 = 49$$

$$r = 7 \text{ cm.}$$

$$\therefore \quad \text{Volume} = \frac{1}{3}\pi r^2 h \text{ cubic units.}$$

$$= \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 24$$

$$= 1232 \text{ cm}^3.$$

6. A hemispherical tank is made up of an iron sheet 1 cm thick. If the inner radius is 1 m, then find the volume of the iron used to make the tank.

Sol :

Internal radius = 'r' m = 1 m

External radius = 'R' m = 1 + 0.01 = 1.01 m

Volume of iron used = External volume – Internal volume

$$= \frac{2}{3}\pi\,(R^3 - r^3) \text{ cubic units}$$

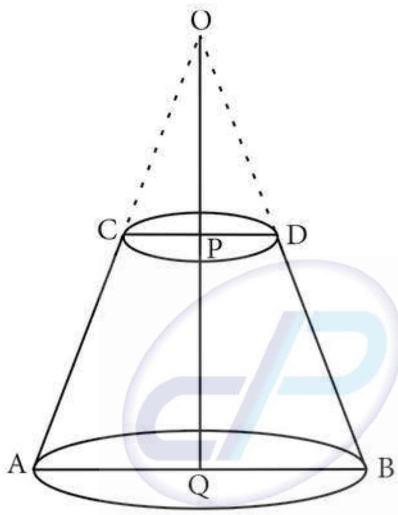
$$= \frac{2}{3}\pi((1.01)^3 - 1^3)$$

$$= \frac{2}{3} \times \frac{22}{7} \times 0.030301$$

$$= 0.06348 \text{ m}^3 \text{ (app).}$$

7. The height of a cone is 30 cm. A small cone is cut off at the top by a plane parallel to the base. If its volume is $\frac{1}{27}$ of the volume of the given cone, at what height above the base is the section made?

Sol :



Volume of the original cone OAB

$$= \frac{1}{3}\pi R^2 H$$

$$= \frac{1}{3}\pi(R^2)(30) = 10\pi R^2 \text{ cm}^3$$

Volume of small cone OCD

$$= \frac{1}{3}\pi r^2 h \text{ cubic units.}$$

Given Volume of cone OCD

$$= \frac{1}{27} \text{ (Volume of cone OAB)}$$

$$\frac{1}{3}\pi r^2 h = \frac{1}{27}(10\pi R^2)$$

$$h = \frac{10\pi R^2}{27}\left(\frac{3}{\pi r^2}\right)$$

$$= \frac{10}{9}\left(\frac{R}{r}\right)^2$$

From similar triangles OQB and OPD.

$$\text{We get } \frac{QB}{PD} = \frac{OQ}{OP} = \frac{30}{h}$$

$$\Rightarrow \frac{R}{r} = \frac{30}{h} \quad \ldots (2)$$

Substituting (2) in (1),

$$h = \frac{10}{9}\left(\frac{30}{h}\right)^2$$

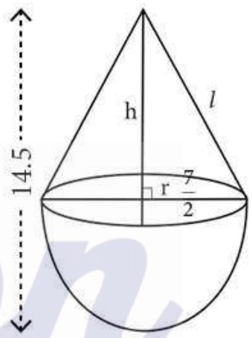
$$h^3 = 1000$$

$$h = 10 \text{ cm}$$

Hence, at (30 – 10) = 20 cm above the base, the section is made.

8. A toy is in the form of a cone on a hemisphere of diameter 7 cm. The total height of the toy is 14.5 cm. Find the volume and the total surface area of the toy.

Sol :



Radius of hemisphere 'r' $= \frac{7}{2} = 3.5$ cm

Radius of base of cone = 3.5 cm

Total height of toy = 14.5 cm

Height of conical part = 14.5 – 3.5 = 11 cm

$$\text{Slant height of cone } l = \sqrt{h^2 + r^2}$$

$$= \sqrt{11^2 + 3.5^2}$$

$$= \sqrt{121 + 12.25}$$

$$= \sqrt{133.25} = 11.54 \text{ cm}$$

Volume of toy = Volume of cone + Volume of hemisphere

$$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$

$$= \frac{\pi r^2}{3}(h + 2r)$$

$$= \frac{22}{3 \times 7} \times 12.25 \times 18$$

$$= 231 \text{ cm}^3$$

Total surface area of toy = C.S.A of cone + C.S.A of hemisphere

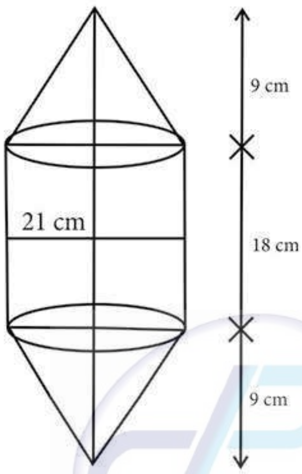
$= \pi rl + 2\pi r^2 = \pi r(l + 2r)$

$= \frac{22}{7} \times 3.5\ (11.54 + 2 \times 3.5)$

$= 22 \times 0.5 \times 18.54 = 203.94\ \text{cm}^2$

9. A petrol tank is a cylinder of base diameter 21 cm and length 18 cm fitted with conical ends each of axis length 9 cm. Determine the capacity of the tank.

Sol :



Volume of Cylindrical Portion

$= \pi r^2 h$

$= \frac{22}{7} \times \left(\frac{21}{2}\right)^2 \times 18$

$= 6237\ \text{cm}^3$

Volume of two Conical ends

$= 2\left(\frac{1}{3}\pi r^2 h\right)$

$= \frac{2}{3} \times \frac{22}{7} \times \left(\frac{21}{2}\right)^2 \times 9\ cm^3$

$= \frac{174636}{84}$

$= 2079\ \text{cm}^3$

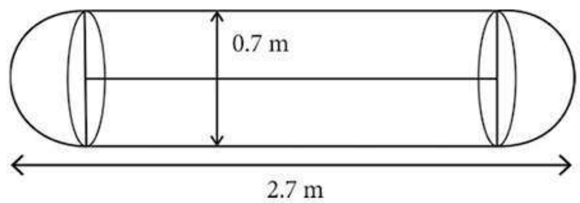
$\therefore$ Capacity of the tank = Volume of Cylinder + Volume of 2 Cones

$= 6237 + 2079$

$= 8316\ \text{cm}^3$

10. Find the volume of a solid in the form of a right circular cylinder with hemispherical ends whose total length is 2.7 m and the diameter of each hemispherical end is 0.7 m.

Sol :



radius of hemispherical ends

$= \frac{1}{2} \times 0.7$

$= \frac{0.7}{2} m = \frac{7}{20} m$

Total length of Solid = 2.7 m

Volume of two hemispheres

$= 2\left(\frac{2}{3}\pi r^3\right)$ cu. units

$= \frac{4}{3} \times \frac{22}{7} \times \left(\frac{7}{20}\right)^3$

$= 0.1797\ \text{m}^3$

Volume of cylinder $= \pi r^2 h$

$= \frac{22}{7} \times \left(\frac{7}{20}\right)^2 \times 2$

$= 0.77\ \text{m}^3$

$\therefore$ Volume of solid $= 0.1797 + 0.77$

$= 0.95\ \text{m}^3$ (app)

11. A hemispherical bowl of internal diameter 36 cm contains a liquid. This liquid is to be filled into cylindrical bottles of radius 3 cm and height 6 cm. How many such bottles are required to empty the bowl?

Sol :

Radius of hemispherical bowl $= \frac{36}{2} = 18$ cm

Volume of hemispherical bowl $= \frac{2}{3}\pi r^3$ cubic units

$= \frac{2}{3}\pi (18)^3\ cm^3$

Height of cylindrical bottle = 6 cm

Radius of cylindrical bottle = 3 cm

Volume of cylindrical bottle $= \pi r^2 h$ cu. units

$= \pi (3)^2 (6)$

$\therefore$ Number of bottles required $= \dfrac{\textit{Volume of hemispherical bowl}}{\textit{Volume of a bottle}}$

$$\therefore \text{Number of bottles required} = \frac{\frac{2}{3}\pi(18)^3}{\pi(3)^2(6)} = 72.$$

12. A vessel in the form of an inverted cone. Its height is 8 cm and the radius is 5 cm. It is filled with water upto the brim. When lead shots each of which is a sphere of radius 0.5 cm are dropped into the vessel, one fourth of the water flows out. Find the number of lead shots dropped into the vessel.

Sol :

radius of cone r = 5 cm, height h = 8 cm

$$\text{volume of cone} = \frac{1}{3}\pi r^2 h \text{ cubic units}$$

$$= \frac{1}{3}\pi(5)^2(8) = \frac{200\pi}{3} cm^3$$

Given that the cone is filled to the brim. When lead shots are dropped, one fourth of the water flown out. The volume of water flown out

$$= \frac{1}{4}\times\frac{200\pi}{3} = \frac{50\pi}{3} cm^3$$

$$\text{Volume of lead shot} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{1}{2}\right)^3$$

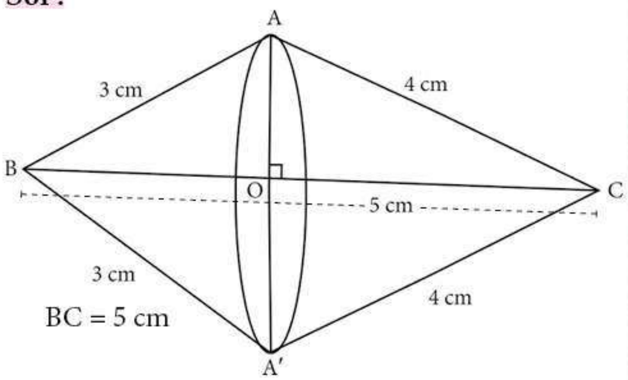
$$= \frac{\pi}{6} cm^3$$

$\therefore$ Number of lead shots dropped into the vessel

$$= \frac{50\pi/3}{\pi/6} = 100.$$

13. A right triangle with sides 3 cm and 4 cm is revolved around its hypotenuse. Find the volume of the double cone thus formed.

Sol :



Let ΔABC be the right triangle, right angled at A whose sides are AB and AC measure 3 cm and 4 cm respectively.

The length of the side BC (hypotenuse)

$$BC = \sqrt{3^2+4^2}$$

$$= \sqrt{25}$$

$$= 5 \text{ cm}$$

By revolving, ΔABC around its hypotenuse BC, the double Cone is formed.

This solid consists two cones namely BAA' and CAA'

AO or $A'O$ is the common radius.

Height of the Cone CAA' is CO and slant height is 4 cm.

Height of the Cone BAA' is BO and slant height is 3 cm.

Now ΔAOB is similar to ΔCAB

$\therefore$ Corresponding sides are proportional

$$\text{i.e., } \frac{AO}{AC} = \frac{AB}{BC} = \frac{BO}{AB}$$

$$\Rightarrow \frac{AO}{4} = \frac{3}{5} \Rightarrow AO = \frac{12}{5} cm$$

$$\text{Similarly, } \frac{BO}{3} = \frac{3}{5} \Rightarrow BO = \frac{9}{5} cm$$

$$CO = BC - BO$$

$$= 5 - \frac{9}{5} = \frac{16}{5} cm$$

$\therefore$ Volume of double Cone

$$v = \frac{12}{5}, h_1 = BO = \frac{9}{5}, h_2 = CO = \frac{16}{5}$$

$$= \left(\frac{1}{3}\pi r^2 \times BO\right) + \left(\frac{1}{3}\pi r^2 \times CO\right)$$

$$= \frac{1}{3}\pi r^2 (BO + CO)$$

$$= \frac{22}{7\times3}\left(\frac{12}{5}\right)^2\left(\frac{9}{5}+\frac{16}{5}\right)$$

$$= \frac{22}{7\times3}\times\frac{12}{5}\times\frac{12}{5}\times 5$$

$$= 30.17 \text{ cm}^3$$