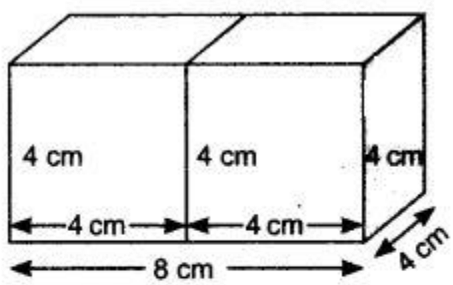
EXERCISE 12.1

Question 1:

2 cubes each of volume 64 cm^3 are joined end to end. Find the surface area of the resulting cuboid.

Solution:



Solution:

Volume of one cube = 64 cm^3

Let edge of one cube = a

Volume of the cube = $(edge)^3$

$a^3 = 64 \Rightarrow a = 4$ cm

Similarly, edge of the another cube = 4 cm.

Now, both cubes are joined together and a cuboid is formed as shown in the figure.

Now, length of the cuboid (l) = 8 cm

breadth of the cuboid (b) = 4 cm

height of the cuboid (h) = 4 cm

Surface area of the cuboid so formed = 2 (lb + bh + hl)

= 2(8 x 4 + 4 x 4 + 4 x 8)

= 2(32 + 16 + 32) = 160 cm^2

Question 2:

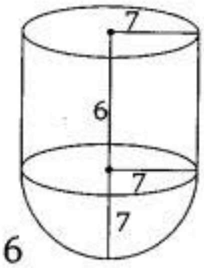
A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area of the vessel.

Solution:

Radius of hemisphere, r = $\frac{14}{2}$ = 7 cm

Height of cylinder, h = 13 – 7 = 6 cm

$\therefore$ Total inner surface area of vessel = Inner surface area of hemisphere + Inner surface area of cylinder = $2\pi r^2 + 2\pi rh$



$= 2\pi \times (7)^2 + 2\pi \times 7 \times 6$

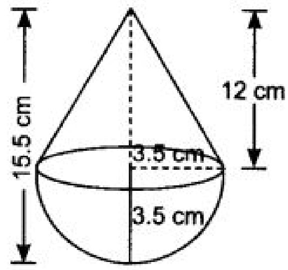
$= 98\pi + 84\pi = 182\pi$

$= 182 \times \frac{22}{7} = \mathbf{572\ cm^2}$.

Question 3:

A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of the same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy.

Solution:



Given: radius of the hemisphere = 3.5 cm

Surface area of the hemisphere = $2\pi r^2$

$= 2 \times \frac{22}{7} \times 3.5 \times 3.5\ \text{cm}^2$

$= \frac{2 \times 22 \times 35 \times 35}{7 \times 10 \times 10} = 77\ \text{cm}^2$

Height of the conical portion = 15.5 cm – 3.5 cm = 12 cm

Radius of the conical portion = 3.5 cm

Slant height of the conical portion = $\sqrt{(12)^2 + (3.5)^2}$ cm

($\because$ $l^2 = r^2 + h^2$, l = slant height, r = radius, h = height)

$= \sqrt{144 + 12.25}$ cm $= \sqrt{156.25} = 12.5$ cm

Curved surface area of the conical portion = $\pi rl = \pi \times 3.5 \times 12.5$

$= \frac{22}{7} \times \frac{35}{10} \times \frac{125}{10}\ \text{cm}^2 = \frac{11 \times 25}{2}\ \text{cm}^2 = \frac{275}{2}\ \text{cm}^2$

Total surface area of the toy = Surface area of the hemisphere + Surface area of the conical portion

$= 77\ \text{cm}^2 + \frac{275}{2}\ \text{cm}^2 = \frac{154 + 275}{2}\ \text{cm}^2$

$= \frac{429}{2}\ \text{cm}^2 = 214.5\ \text{cm}^2$

Question 4:

A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have? Find the surface area of the solid.

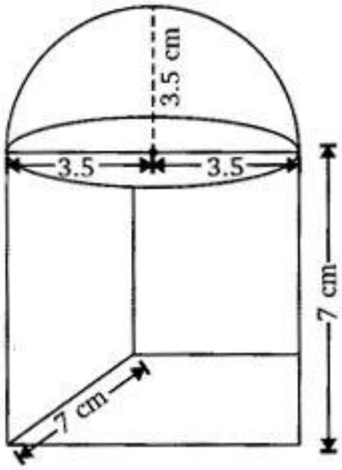
Solution:

From the figure,

the hemisphere can have greatest diameter = 7 cm

Here, radius of the hemisphere,

$r = \frac{7}{2}$ cm = 3.5 cm



The surface area of the solid formed

= Curved surface area of cube

+ Curved surface area of hemisphere

– Area of the base of hemisphere

$= 6a^2 + 2\pi r^2 - \pi r^2$

$= [6(7)^2 + 2\pi(3.5)^2 - \pi(3.5)^2]$ cm²

$= 6(7)^2 + \pi(3.5)^2$

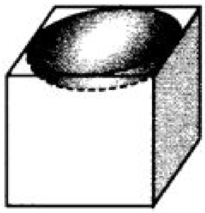
$= \left(6 \times 49 + \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\right)$ cm²

$= \left(294 + \frac{77}{7}\right)$ cm² = **332.5 cm²**.

Question 5:

A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter d of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid.

Solution:



Given: edge of the cube $= l$

Surface area of the cube $= 6(\text{edge})^2 = 6l^2$

Diameter of the hemisphere $= l$

$\therefore$ Radius of the hemisphere $= \frac{l}{2}$

Inner surface area of the hemisphere $= 2\pi r^2 = 2\pi\left(\frac{l}{2}\right)^2 = \frac{2\pi l^2}{4} = \frac{\pi l^2}{2}$

Area of circular portion of the hemisphere $= \pi r^2 = \pi\left(\frac{l}{2}\right)^2 = \frac{\pi l^2}{4}$

Remaining surface area of the cubical box = (Surface area of the cubical box + Inner surface area of the hemisphere – Area of the circular region)

$$= 6l^2 + \frac{\pi l^2}{2} - \frac{\pi l^2}{4} = 6l^2 + \frac{\pi l^2}{4} = \frac{l^2}{4}(24 + \pi)$$

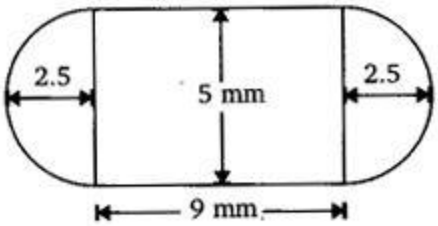
Question 6:

A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends. The length of the entire capsule is 14 mm and the diameter of the capsule is 5 mm. Find its surface area.

Solution:

Radius of cylinder, r = 2.5 mm

Length of cylinder, h = 9 mm



Total surface area of capsule

$= $ Surface area of cylindrical part + 2 × Curved surface area of hemisphere

$= 2\pi rh + 2(2\pi r^2) = 2\pi r(h + 2r)$

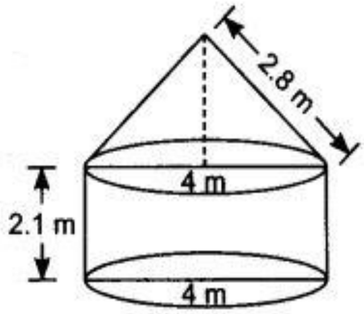
$= 2 \times \frac{22}{7} \times 2.5(9 + 2 \times 2.5)\ \text{mm}^2$

$= 5 \times \frac{22}{7}(14)\ \text{mm}^2 = \mathbf{220\ mm^2}.$

Question 7:

A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m respectively, and the slant height of the top is 2.8 m, find the area of the canvas used for making the tent. Also, find the cost of the canvas of the tent at the rate of Rs 500 per m^2. (Note that the base of the tent will not be covered with canvas.)

Solution:



Radius of the cylindrical part $= \frac{4}{2} = 2$ m

Height of the cylindrical part = 2.1 m

Curved surface area of the cylindrical part $= 2\pi rh$

$$= 2 \times \frac{22}{7} \times 2 \times 2.1 \text{ m}^2$$
$$= 4 \times 22 \times 0.3 \text{ m}^2$$
$$= 22 \times 1.2 = 26.4 \text{ m}^2$$

Curved surface area of the top $= \pi rl$

$$= \frac{22}{7} \times 2 \times 2.8 \text{ m}^2 = 44 \times 0.4 \text{ m}^2 = 17.6 \text{ m}^2$$

Total area of the canvas $= 26.4 + 17.6 = 44$ m^2

Cost of the canvas = ₹ 500/m^2

Total cost = Cost of canvas per m^2 × Total surface area of the canvas

Total cost $= 44 \times 500 =$ ₹ 22,000

Question 8:

From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm^2.

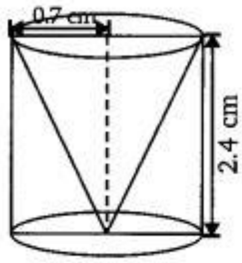
Solution:

Radius of cylinder, $r = \dfrac{1.4}{2} = 0.7$ cm

Height of cylinder, $h = 2.4$ cm

Radius of cone, $r = 0.7$ cm

Height of cone, $h = 2.4$ cm



Total surface area of the remaining solid

= Outer curved surface area of cylinder

+ Area of bottom of cylinder

+ Inner curved surface area of conical cavity

$= 2\pi rh + \pi r^2 + \pi rl$

$= \pi r(2h + r + l)$

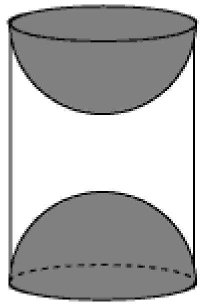
$= \dfrac{22}{7} \times 0.7\left[2 \times 2.4 + 0.7 + \sqrt{(0.7)^2 + (2.4)^2}\right]$

$= 22 \times 0.1\ [4.8 + 0.7 + 2.5]$

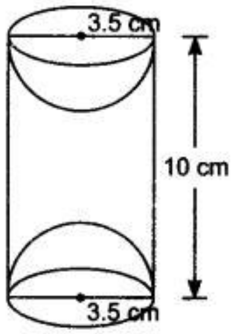
$= 22 \times 0.1 \times 8 = 17.6\ \text{cm}^2 \approx \mathbf{18\ cm^2}$.

Question 9:

A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in figure. If the height of the cylinder is 10 cm, and its base is of radius 3.5 cm, find the total surface area of the article.



Solution:



Solution:

Given: height of the cylinder = 10 cm,

base radius = 3.5 cm

Curved surface area of the cylinder $= 2\pi rh$

$$= \frac{2 \times 22 \times 35 \times 10}{7 \times 10} \text{ cm}^2 = 220 \text{ cm}^2$$

Inner surface area of a hemispherical cavity

$$= 2\pi r^2 = 2 \times \frac{22}{7} \times \frac{35 \times 35}{10 \times 10} \text{ cm}^2 = 77 \text{ cm}^2$$

Inner surface area of both hemispherical cavity $= 77 \text{ cm}^2 + 77 \text{ cm}^2 = 154 \text{ cm}^2$

Total surface area of the solid = Curved surface area of the solid

+ Inner surface area of both hemispherical ends

$$= 220 \text{ cm}^2 + 154 \text{ cm}^2 = 374 \text{ cm}^2$$