EXERCISE 3.2

Question 1:

Solve the following pairs of linear equations by the substitution method:

(i) $x + y = 14, x - y = 4$

(ii) $s - t = 3, \frac{s}{3} + \frac{t}{2} = 6$

(iii) $3x - y = 3, 9x - 3y = 9$

(iv) $0.2x + 0.3y = 1.3, 0.4x + 0.5y = 2.3$

(v) $\sqrt{2}x + \sqrt{3}y = 0, \sqrt{3}x - \sqrt{8}y = 0$

(vi) $\frac{3x}{2} - \frac{5y}{3} = -2, \frac{x}{3} + \frac{y}{2} = \frac{13}{6}$

Solution:

(i) x + y = 14 .....(i)

x – y = 4 .....(ii)

From equation (ii), we have:

y = x – 4 ....(iii)

Substituting the value of y in equation (i), we get:

x + x-4 =14

⇒ 2x = 18

⇒ x = 9

Substituting x = 9 in equation (iii), we get: y = 9 – 4 = 5

Thus, the solution is x = 9 and y = 5.

(*ii*) $s - t = 3$ ... (*i*)

$\frac{s}{3} + \frac{t}{2} = 6$ ... (*ii*)

From equation (*i*), $s - t = 3 \Rightarrow s = 3 + t$

Putting the value of s in equation (*ii*), we get

$$\frac{3+t}{3} + \frac{t}{2} = 6 \Rightarrow \frac{2(3+t)+3t}{6} = 6$$

$$\Rightarrow 6 + 2t + 3t = 36 \Rightarrow 5t = 36 - 6$$

$$5t = 30 \Rightarrow t = 6$$

Putting $t = 6$ in equation (*i*), we have

$$s = 3 + 6 = 9$$

So, $\boxed{s = 9, t = 6}$

(iii) $3x - y = 3$ ... (i)

$9x - 3y = 9$ ... (ii)

From equation (i), $3x - y = 3 \Rightarrow 3x = 3 + y \Rightarrow x = \frac{3+y}{3}$

Now, putting the value of x in equation (ii), we have

$$9\left(\frac{3+y}{3}\right) - 3y = 9 \Rightarrow 3(3+y) - 3y = 9 \Rightarrow 9 + 3y - 3y = 9$$

$$9 = 9$$

∴ y can have infinite real values

∴ x can have infinite real values because $x = \frac{y+3}{3}$

(iv) $0.2x + 0.3y = 1.3$ ...(i)

$0.4x + 0.5y = 2.3$ ...(ii)

From equation (i),

$$0.2x + 0.3y = 1.3 \Rightarrow 0.2x = 1.3 - 0.3y \Rightarrow x = \frac{1.3 - 0.3y}{0.2}$$

Putting the value of x in equation (ii), we have

$$0.4\left(\frac{1.3 - 0.3y}{0.2}\right) + 0.5y = 2.3 \quad \Rightarrow 2(1.3 - 0.3y) + 0.5y = 2.3$$

$$\Rightarrow \quad 2.6 - 0.6y + 0.5y = 2.3 \quad \Rightarrow -0.1y = 2.3 - 2.6$$

$$\Rightarrow \quad -0.1y = -0.3 \quad \Rightarrow y = 3$$

Putting $y = 3$ in equation (i), we get

$$0.2x + 0.3(3) = 1.3 \Rightarrow 0.2x + 0.9 = 1.3$$

$$\Rightarrow \quad 0.2x = 1.3 - 0.9 \Rightarrow 0.2x = 0.4 \Rightarrow x = 2$$

So, $\boxed{x = 2, y = 3}$

(v) $\sqrt{2}x + \sqrt{3}y = 0$ ... (i)

$\sqrt{3}x - \sqrt{8}y = 0$ ... (ii)

From equation (i), we have

$$x = \frac{-\sqrt{3}}{\sqrt{2}}y \quad \text{... (iii)}$$

Putting this value in equation (ii), we get

$$\sqrt{3}\left(\frac{-\sqrt{3}}{\sqrt{2}}y\right) - \sqrt{8}y = 0 \quad \Rightarrow \quad \frac{-3}{\sqrt{2}}y - \sqrt{8}y = 0 \quad \Rightarrow \quad y = 0$$

Putting $y = 0$ in equation (iii), we have $x = 0$

So, $\boxed{x = 0, y = 0}$

(vi) $\frac{3}{2}x - \frac{5}{3}y = -2$ ... (i)

$\frac{x}{3} + \frac{y}{2} = \frac{13}{6}$ ... (ii)

From equation (i), we have

$$\frac{3}{2}x = -2 + \frac{5}{3}y \quad \Rightarrow \frac{3}{2}x = \frac{-6 + 5y}{3}$$

$$\Rightarrow \quad x = \frac{2(-6 + 5y)}{9}$$

Putting this value in equation (*ii*), we have

$$\frac{1}{3}\left[\frac{-12+10y}{9}\right]+\frac{y}{2}=\frac{13}{6} \quad \Rightarrow \frac{-12+10y}{27}+\frac{y}{2}=\frac{13}{6}$$

$$\Rightarrow \frac{-12+10y}{27}-\frac{13}{6}=\frac{-y}{2} \quad \Rightarrow \frac{-24+20y-117}{54}=\frac{-y}{2}$$

$$\Rightarrow \frac{-141+20y}{54}=\frac{-y}{2} \quad \Rightarrow -141+20y=-27y$$

$$\Rightarrow -141=-47y \quad \Rightarrow y=3$$

Now, putting $y = 3$ in equation (*i*), we have

$$\frac{3}{2}x-\frac{5}{3}(3)=-2 \Rightarrow \frac{3}{2}x-5=-2 \Rightarrow \frac{3}{2}x=3 \Rightarrow x=2$$

So, $\boxed{x=2, y=3}$

Question 2:

Solve 2x + 3y = 11 and 2x – 4y = -24 and hence find the value of'm' for which y = mx +3.

Solution:

2x + 3y = 11 ...(i)

2x – 4y = -24 ...(ii)

From equation (i), we have:

$$y=\frac{11-2x}{3} \qquad \text{...(iii)}$$

Substituting the value of y in equation (ii), we get:

$$2x-4\left(\frac{11-2x}{3}\right)=-24$$

$$\Rightarrow 2x-\frac{44-8x}{3}=-24$$

$$\Rightarrow 6x-44+8x=-72$$

$$\Rightarrow 14x=-28$$

$$\Rightarrow x=-2$$

Substituting $x = -2$ in equation (iii), we get:

$$y=\frac{11-2(-2)}{3}=\frac{11+4}{3}=5.$$

Thus, the solution is $\boldsymbol{x = -2}$ and $\boldsymbol{y = 5}$

Now, we have: $y = mx + 3$

$$\Rightarrow 5=m(-2)+3$$

$$\Rightarrow 2=-2m$$

$$\Rightarrow \boldsymbol{m=-1}.$$

Question 3:

Form the pair of linear equations for the following problems and find their solution by substitution method:

(i) The difference between two numbers is 26 and one number is three times the other. Find them.

(ii) The larger of two supplementary angles exceeds the smaller by 18 degrees. Find them.

(iii) The coach of a cricket team buys 7 bats and 6 balls for ₹3800. Later, she buys 3 bats and 5 balls for ₹1750. Find the cost of each bat and each ball.

(iv) The taxi charges in a city consist of a fixed charge together with the charge for the distance covered. For a distance of 10 km, the charge paid is ₹105 and for a journey of 15 km, the charge paid is ₹155. What are the fixed charges and the charges per km? How much does a person have to pay for travelling a distance of 25 km?

(v) A fraction becomes 911, if 2 is added to both the numerator and the denominator. If 3 is added to both the numerator and the denominator, it becomes 56. Find the fraction.

(vi) Five years hence, the age of Jacob will be three times that of his son. Five year ago, Jacob's age was seven times that of his son. What are their present ages?

Solutions:

(i) Let 1st number be x and 2nd number be y.

Let x >y

1st condition :

$x - y = 26$

2nd condition :

$x = 3y$

Putting $x = 3y$ in equation (i)

$3y - y = 26 \Rightarrow 2y = 26 \Rightarrow y = 13$

From (ii)

$x = 3 \times 13 = 39$

∴ One number is 13 and the other number is 39.

(ii) Let one angle be x and its supplementary angle = y

Let x > y

1st Condition :

$x + y = 180°$

2nd Condition :

$x - y = 18° \Rightarrow X = 18° + y$

From equation (ii), putting the value ofx in equation (i),

$18° + y + y = 180° \Rightarrow 18° + 2y = 180°$

$2y = 162° \Rightarrow y = 81°$

From (ii) $x = 18° + 81° = 99° \Rightarrow x = 99°$

∴ One angle is 81° and another angle is 99°.

(iii) Let cost of 1 bat = ₹x and cost of 1 ball = ₹y

1st Condition:

7x + 6y = 3800

2nd Condition:

3x + 5y = 1750

From equation (ii), we get

Putting $x = \frac{1750 - 5y}{3}$ in equation (i), we get

$$7\left[\frac{1750 - 5y}{3}\right] + 6y = 3800 \quad \Rightarrow \frac{12250 - 35y}{3} + 6y = 3800$$

$$\Rightarrow \quad 12250 - 35y + 18y = 11400 \quad \Rightarrow 12250 - 17y = 11400$$

$$\Rightarrow \quad -17y = 11400 - 12250 \quad \Rightarrow -17y = -850$$

$$\Rightarrow \quad \boxed{y = 50}$$

Putting the value of y in equation (i), we have

$$7x + 6 \times 50 = 3800 \Rightarrow 7x = 3800 - 300$$

$$\Rightarrow \quad 7x = 3500 \Rightarrow \quad \boxed{x = 500}$$

$\therefore$ Cost of one bat = ₹ 500 and cost of one ball = ₹ 50.

putting x = 1750-5y/3 in equation (i), we get

Cost of one bat = ₹ 500 and cost of one ball = ₹ 50.

(iv) Let fixed charges be ₹.v and charge for per km be ₹y.

A.T.Q.

1st Condition :

x + IOy = 105

2nd Condition :

x + 15y = 155

From equation (i), we get

x= 105 – 10y

Putting this value in equation (ii), we have

105 – 10y + 15y = 155 ⇒ 105 + 5y = 155

⇒ 5y = 155 – 105 ⇒ 5y = 50 ⇒ y = 10

Now, puttingy = 10 in equation (i), we have

x + 10(10) = 105 ⇒ x + 100 = 105 ⇒ x = 5

Fixed charges is ₹ 5 and charges per km is ₹ 10.

3rd Condition :

For distance of 25 km

x + 25y = 5 + 25(10) = 5 + 250 = 255

Amount paid for travelling 25 km is ₹ 255.

(v) Let numerator be x and denominator be y.

$\therefore$ Fraction is x/y

A.T.Q.

1st condition :

$$\frac{x+2}{y+2} = \frac{9}{11} \quad \Rightarrow 11x + 22 = 9y + 18$$

$$11x - 9y = 18 - 22 \quad \Rightarrow 11x - 9y = -4 \quad \ldots (i)$$

2nd condition :

$$\frac{x+3}{y+3} = \frac{5}{6} \quad \Rightarrow 6x + 18 = 5y + 15 \Rightarrow 6x - 5y = 15 - 18$$

$$\Rightarrow \quad 6x - 5y = -3 \quad \ldots (ii)$$

From equation (i), we get

$$11x = 9y - 4 \quad \Rightarrow x = \frac{9y-4}{11}$$

Putting this value in equation (ii), we have

$$6\left[\frac{9y-4}{11}\right] - 5y = -3 \quad \Rightarrow \frac{54y-24}{11} - 5y = -3$$

$$\Rightarrow \quad 54y - 24 - 55y = -33 \quad \Rightarrow -y = -33 + 24$$

$$\Rightarrow \quad -y = -9 \quad \Rightarrow \boxed{y = 9}$$

Putting the value of y in equation (i), we get

$$\Rightarrow \quad 11x - 9(9) = -4 \quad \Rightarrow 11x - 81 = -4$$

$$\Rightarrow \quad 11x = -4 + 81 \quad \Rightarrow 11x = 77 \quad \Rightarrow \quad \boxed{x = 7}$$

$\therefore$ Fraction is $\frac{x}{y} = \frac{7}{9}$.

(vi) Let present age of Jacob be x years and that of his son bey years.

A.T.Q.

1st Condition :

$x + 5 = 3(y + 5) \Rightarrow x + 5 = 3y + 15 \Rightarrow x - 3y = 15 - 5 \Rightarrow x - 3y = 10$

2nd Condition:

$x - 5 = 7(y - 5) \Rightarrow x - 5 = 7y - 35 \Rightarrow x = 7y - 35 + 5$

$\Rightarrow x = 7y - 30$

Putting the value of 'x' in equation (i), we get

$7y - 30 - 3y = 10$

$4y - 30 = 10$

$4y = 40$ $y = 10 \Rightarrow y = 10$

putting the value of y in equation(ii), we get

$x = 7(10) - 30 = 70 - 30 \Rightarrow x = 40$

Hence, the present age of Jacob is 40 years and that of his son is 10 years.