EXERCISE 4.2

Question 1:

Find the roots of the following quadratic equations by factorisation:

(i) $x^2 - 3x - 10 = 0$
(ii) $2x^2 + x - 6 = 0$
(iii) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$
(iv) $2x^2 - x + 18 = 0$ 8
(v) $100x^2 - 20X + 1 = 0$

Solution:

(i) Given: $x^2 - 3x - 10 = 0$

$\Rightarrow x^2 - 5x + 2x - 10 = 0$

$\Rightarrow x(x - 5) + 2(x - 5) = 0$

$\Rightarrow (x - 5)(x + 2) = 0$

Either $x - 5 = 0$ or $x + 2 = 0$

$\Rightarrow x = 5$ or $x = -2$

Hence, the roots are **5** and **–2**.

(ii) Given: $2x^2 + x - 6 = 0$

$\Rightarrow 2x^2 + 4x - 3x - 6 = 0$

$\Rightarrow 2x(x + 2) - 3(x + 2) = 0$

$\Rightarrow (x + 2)(2x - 3) = 0$

Either $x + 2 = 0$ or $2x - 3 = 0$

$\Rightarrow x = -2$ or $x = \frac{3}{2}$

Hence, the roots are **–2** and $\mathbf{\frac{3}{2}}$.

(iii) Given: $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

$\Rightarrow \sqrt{2}x^2 + 5x + 2x + 5\sqrt{2} = 0$

$\Rightarrow x(\sqrt{2}x + 5) + \sqrt{2}(\sqrt{2}x + 5) = 0$

$\Rightarrow (\sqrt{2}x + 5)(x + \sqrt{2}) = 0$

Either $\sqrt{2}x + 5 = 0$ or $x + \sqrt{2} = 0$

$\Rightarrow x = -\frac{5}{\sqrt{2}}$ or $x = -\sqrt{2}$

Hence, the roots are $\mathbf{-\frac{5}{\sqrt{2}}}$ and $\mathbf{-\sqrt{2}}$.

(iv) Given: $2x^2 - x + \frac{1}{8} = 0$

$\Rightarrow \quad 16x^2 - 8x + 1 = 0$

$\Rightarrow \quad 16x^2 - 4x - 4x + 1 = 0$

$\Rightarrow \quad 4x(4x - 1) - 1(4x - 1) = 0$

$\Rightarrow \quad (4x - 1)(4x - 1) = 0$

Either $4x - 1 = 0$ or $4x - 1 = 0$

$\Rightarrow x = \frac{1}{4}$ or $x = \frac{1}{4}$

Hence, the roots are $\mathbf{\frac{1}{4}}$ and $\mathbf{\frac{1}{4}}$.

(v) Given: $100x^2 - 20x + 1 = 0$

$\Rightarrow \quad 100x^2 - 10x - 10x + 1 = 0$

$\Rightarrow \quad 10x(10x - 1) - 1(10x - 1) = 0$

$\Rightarrow \quad (10x - 1)(10x - 1) = 0$

Either $10x - 1 = 0$ or $10x - 1 = 0$

$\Rightarrow x = \frac{1}{10}$ or $x = \frac{1}{10}$

Hence, the roots are $\mathbf{\frac{1}{10}}$ and $\mathbf{\frac{1}{10}}$.

Question 2:

Solve the following situations mathematically:

(i) John and Jivanti together have 45 marbles. Both of them lost 5 marbles each and the product of the number of marbles they now have is 124. We would like to find out how many marbles they had to start with.

(ii) A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys produced in a day. On a particular day, the total cost of production was ₹750. We would like to find out the number of toys produced on that day.

Solution:

(i) Let the number of marbles John had be x

Then, the number of marbles Jivanti had = 45 -x

The number of marbles left with John, when he lost 5 marbles = x – 5

The number of marbles left with Jivanti, when she lost 5 marbles = 45 -x – 5 = 40 -x

According to question,

$$(x-5)(40-x) = 124 \Rightarrow 40x - x^2 - 200 + 5x = 124$$

$$\Rightarrow x^2 - 45x + 324 = 0 \Rightarrow x^2 - 36x - 9x + 324 = 0$$

$$\Rightarrow x(x-36) - 9(x-36) = 0 \Rightarrow (x-9)(x-36) = 0$$

$$\Rightarrow x - 9 = 0 \text{ or } x - 36 = 0$$

$$\Rightarrow x = 9 \text{ or } x = 36$$

Number of marbles they had to start with 9 and 36.

(ii) Let the number of toys produced in a day be x

Then, cost of production of each toy on that day = ₹ $(55-x)$

Total cost of production on that day = $x(55-x)$

According to question,

$$x(55-x) = 750 \Rightarrow 55x - x^2 = 750$$

$$\Rightarrow x^2 - 55x + 750 = 0 \Rightarrow x^2 - 25x - 30x + 750 = 0$$

$$\Rightarrow x(x-25) - 30(x-25) = 0 \Rightarrow (x-30)(x-25) = 0$$

$$\Rightarrow x - 30 = 0 \text{ or } x - 25 = 0$$

$$\Rightarrow x = 30 \text{ or } x = 25$$

Number of toys produced on that day was 25 or 30.

Question 3:

Find two numbers whose sum is 27 and product is 182.

Solution:

Let the first number be x, then another number will be 27 – x.

According to the questions, we have:

$$x(27 - x) = 182$$
$$\Rightarrow \quad 27x - x^2 = 182$$
$$\Rightarrow \quad x^2 - 27x + 182 = 0$$
$$\Rightarrow \quad x^2 - 14x - 13x + 182 = 0$$
$$\Rightarrow \quad (x - 14)(x - 13) = 0$$
$$\Rightarrow \quad x = 14 \text{ or } x = 13$$

Hence, the required numbers are 13 and 14.

Question 4:

Find two consecutive positive integers, the sum of whose squares is 365.

Solution:

Let the two consecutive integers be x and x + 1.

According to question,

$$x^2 + (x + 1)^2 = 365 \quad \Rightarrow \quad x^2 + x^2 + 2x + 1 = 365$$
$$\Rightarrow \quad 2x^2 + 2x - 364 = 0 \quad \Rightarrow \quad x^2 + x - 182 = 0$$
$$\Rightarrow \quad x^2 + 14x - 13x - 182 = 0 \quad \Rightarrow \quad x(x + 14) - 13(x + 14) = 0$$
$$\Rightarrow \quad (x - 13)(x + 14) = 0$$
$$\Rightarrow \quad x = 13, -14$$

(–14 is rejected because it is negative integer)

Hence, the two consecutive positive integers are 13 and 13 + 1 = 14.

Question 5:

The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides.

Solution:

Let the base of right triangle be x cm.

Then, altitude of right triangle will be (x – 7) cm

Hypotenuse = 13cm **[Given]**

By Pythagoras' theorem, we have:

$$(13)^2 = x^2 + (x-7)^2$$

$$\Rightarrow 169 = x^2 + x^2 - 14x + 49$$

$$\Rightarrow 2x^2 - 14x - 120 = 0$$

$$\Rightarrow x^2 - 7x - 60 = 0$$

$$\Rightarrow x^2 - 12x + 5x - 60 = 0$$

$$\Rightarrow (x-12)(x+5) = 0$$

$$\Rightarrow x = 12 \quad \text{or} \quad x = -5.$$

Hence, the base of the right triangle is **12 cm** and its altitude is 12 – 7 = **5 cm**.

Question 6:

A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was ₹90, find the number of articles produced and the cost of each article.

Solution:

Let total number of pottery articles produced in a day = x

Cost of production of each article = ₹ $\frac{90}{x}$

According to question,

$$2x + 3 = \frac{90}{x}$$

$$\Rightarrow x(2x+3) = 90 \quad \Rightarrow 2x^2 + 3x = 90$$

$$\Rightarrow 2x^2 + 3x - 90 = 0 \quad \Rightarrow 2x^2 + 15x - 12x - 90 = 0$$

$$\Rightarrow x(2x+15) - 6(2x+15) = 0 \quad \Rightarrow (2x+15)(x-6) = 0$$

$$\Rightarrow 2x = -15 \quad \text{or} \quad x - 6 = 0$$

$$\Rightarrow x = -\frac{15}{2} \text{ or } x = 6 \left(-\frac{15}{2} \text{ is rejected}\right)$$

$\therefore$ Number of articles produced per day = 6

Cost of production of each article = $\frac{90}{6}$ = ₹ 15