EXERCISE 7.1

Question 1:

Find the distance between the following pairs of points:
(i) (2, 3), (4, 1)
(ii) (-5, 7), (-1, 3)
(iii) (a, b), (-a, -b)

Solution:

(i) Let the given points be P(2, 3) and Q(4, 1).

Then $x_1 = 2, y_1 = 3, x_2 = 4$ and $y_2 = 1$

$$\therefore \text{Distance PQ} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(4-2)^2 + (1-3)^2}$$
$$= \sqrt{(2)^2 + (-2)^2} = \sqrt{4+4}$$
$$= \sqrt{8} = \mathbf{2\sqrt{2}\ units}.$$

(ii) Let the given points be P(–5, 7) and Q(–1, 3).

Then $x_1 = -5, y_1 = 7, x_2 = -1$ and $y_2 = 3$

$$\therefore \text{Distance PQ} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(-1+5)^2 + (3-7)^2}$$
$$= \sqrt{(4)^2 + (-4)^2} = \sqrt{16+16}$$
$$= \sqrt{32} = \mathbf{4\sqrt{2}\ units}.$$

(iii) Let the given points be P(a, b) and Q($-a$, $-b$).

Then $x_1 = a, y_1 = b, x_2 = -a$ and $y_2 = -b$

$$\therefore \text{Distance PQ} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(-a-a)^2 + (-b-b)^2}$$
$$= \sqrt{(-2a)^2 + (-2b)^2}$$
$$= \sqrt{4a^2 + 4b^2}$$
$$= \mathbf{2\sqrt{a^2 + b^2}\ units}.$$

Question 2:

Find the distance between the points (0, 0) and (36, 15).

Solution:

Let points be A (0, 0) and B (36, 15)

The distance between two points is

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(36 - 0)^2 + (15 - 0)^2}$$
$$= \sqrt{1296 + 225} = \sqrt{1521} = 39 \text{ units}$$

Question 3:

Determine if the points (1, 5), (2, 3) and (-2, -11) are collinear.

Solution:

Let the given points are A (1, 5), B (2, 3) and C (-2, -11). Then:

$$AB = \sqrt{(2-1)^2 + (3-5)^2} = \sqrt{(1)^2 + (-2)^2}$$
$$= \sqrt{1+4} = \sqrt{5}.$$
$$BC = \sqrt{(-2-2)^2 + (-11-3)^2} = \sqrt{(-4)^2 + (-14)^2}$$
$$= \sqrt{16+196} = \sqrt{212} = 2\sqrt{53}.$$
$$AC = \sqrt{(-2-1)^2 + (-11-5)^2} = \sqrt{(-3)^2 + (-16)^2}$$
$$= \sqrt{9+256} = \sqrt{265}.$$

Since AB + BC ≠ AC

Hence, the given points are not collinear.

Question 4:

Check whether (5, -2), (6, 4) and (7, -2) are the vertices of an isosceles triangle.

Solution:

Let points be A(5, -2), B (6, 4) and C (7, -2)

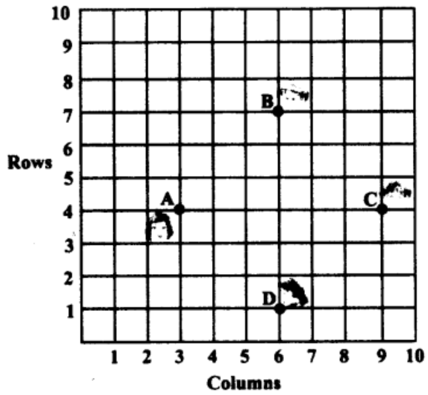
$$AB = \sqrt{(6-5)^2 + (4+2)^2} = \sqrt{1+36} = \sqrt{37}$$
$$BC = \sqrt{(7-6)^2 + (-2-4)^2} = \sqrt{1+36} = \sqrt{37}$$
$$AC = \sqrt{(7-5)^2 + (-2+2)^2} = \sqrt{4+0} = 2$$

Here, AB = BC

ΔABC is an isosceles triangle.

Question 5:

In a classroom, 4 friends are seated at the points A, B, C and D as shown in the given figure. Champa and Chameli walk into the class and after observing for a few minutes Champa asks Chameli, "Don't you think ABCD is a square?" Chameli disagrees. Using distance formula, find which of them is correct.



Solution:

From the figure, let the points along with coordinates be A (3, 4), B (6, 7), C (9, 4) and D (6, 1).
Then by distance formula, we have:

$$AB = \sqrt{(6-3)^2+(7-4)^2} = \sqrt{18} = 3\sqrt{2}.$$

$$BC = \sqrt{(9-6)^2+(4-7)^2} = \sqrt{18} = 3\sqrt{2}.$$

$$CD = \sqrt{(6-9)^2+(1-4)^2} = \sqrt{18} = 3\sqrt{2}.$$

$$DA = \sqrt{(3-6)^2+(4-1)^2} = \sqrt{18} = 3\sqrt{2}.$$

$$\text{Also, diagonal } AC = \sqrt{(9-3)^2+(4-4)^2}$$

$$= \sqrt{(6)^2+(0)^2} = \sqrt{36} = 6.$$

$$\text{and diagonal } BD = \sqrt{(6-6)^2+(1-7)^2}$$

$$= \sqrt{(0)^2+(-6)^2} = \sqrt{36} = 6.$$

$\therefore$ AB = BC = CD = DA $3\sqrt{2}$
and diagonals AC = BD = 6
Thus, ABCD is a square and Champa is correct.

Question 6:

Name the type of quadrilateral formed, if any, by the following points, and give reasons for your answer.
(i) (-1, -2), (1, 0), (-1, 2), (-3, 0)
(ii) (-3, 5), (3, 1), (0, 3), (-1, -4)
(iii) (4, 5), (7, 6), (4, 3), (1, 2)

Solution:

(i) Let points be A (-1, -2), B (1, 0), C (-1, 2) and D (-3, 0)

The distance between two points $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$AB = \sqrt{(1+1)^2 + (0+2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$
$$BC = \sqrt{(-1-1)^2 + (2-0)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$
$$CD = \sqrt{(-3+1)^2 + (0-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$
$$AD = \sqrt{(-3+1)^2 + (0+2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$
$$AC = \sqrt{(-1+1)^2 + (2+2)^2} = \sqrt{0+16} = \sqrt{16} = 4$$
$$BD = \sqrt{(-3-1)^2 + (0-0)^2} = \sqrt{16+0} = \sqrt{16} = 4$$

Here, AC = BD, AB = BC = CD = AD

Hence, the quadrilateral ABCD is a square.

(ii) Let points be A (-3, 5), B (3, 1), C (0, 3) and D (-1, -4)

$$AB = \sqrt{(3+3)^2 + (1-5)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$$
$$BC = \sqrt{(0-3)^2 + (3-1)^2} = \sqrt{9+4} = \sqrt{13}$$
$$CD = \sqrt{(-1-0)^2 + (-4-3)^2} = \sqrt{1+49} = 5\sqrt{2}$$
$$AD = \sqrt{(-3+1)^2 + (5+4)^2} = \sqrt{4+81} = \sqrt{85}$$

The given points do not form any quadrilateral.

(iii) Let points be A(4, 5), B (7, 6), C (4, 3) and D (1, 2)

$$AB = \sqrt{(7-4)^2 + (6-5)^2} = \sqrt{9+1} = \sqrt{10}$$
$$BC = \sqrt{(4-7)^2 + (3-6)^2} = \sqrt{9+9} = 3\sqrt{2}$$
$$CD = \sqrt{(1-4)^2 + (2-3)^2} = \sqrt{9+1} = \sqrt{10}$$
$$AD = \sqrt{(1-4)^2 + (2-5)^2} = \sqrt{9+9} = 3\sqrt{2}$$
$$AC = \sqrt{(4-4)^2 + (3-5)^2} = \sqrt{4} = 2$$
$$BD = \sqrt{(1-7)^2 + (2-6)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$$

Here, AB = CD, BC = AD

and AC ≠ BD

The quadrilateral ABCD is a parallelogram.

Question 7:

Find the point on the x-axis which is equidistant from (2, -5) and (-2, 9).

Solution:

Let A (2, -5) and B (-2, 9) be the given points.

Also let P (x, 0) be the point on x-axis such that

$$PA = PB$$

Then $PA^2 = PB^2$

$$\Rightarrow (x-2)^2 + (0+5)^2 = (x+2)^2 + (0-9)^2$$

$$\Rightarrow (x-2)^2 - (x+2)^2 = 81 - 25$$

$$\Rightarrow (x-2+x+2)(x-2-x-2) = 56$$

$$\Rightarrow (2x)(-4) = 56$$

$$\Rightarrow -8x = 56$$

$$\Rightarrow x = -7$$

Hence, the required point is **(–7, 0)**.

Question 8:

Find the values of y for which the distance between the points P (2, -3) and Q (10, y) is 10 units.

Solution:

Points P (2, -3), Q (10, y) and PQ = 10 units

The distance between two points is

$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = PQ \Rightarrow \sqrt{(10-2)^2+(y+3)^2} = 10$$

$$\Rightarrow 64 + y^2 + 9 + 6y = 100 \Rightarrow y^2 + 6y + 73 - 100 = 0$$

$$\Rightarrow y^2 + 6y - 27 = 0 \Rightarrow y^2 + 9y - 3y - 27 = 0$$

$$\Rightarrow y(y+9) - 3(y+9) = 0 \Rightarrow (y-3)(y+9) = 0$$

$$\Rightarrow y - 3 = 0 \text{ or } y + 9 = 0$$

$$\Rightarrow y = 3 \text{ or } -9$$

Question 9:

If Q (0, 1) is equidistant from P (5, -3), and R (x, 6), find the values of x. Also, find the distances QR and PR.

Solution:

Given that Q(0, 1) is equidistant from P(5, -3) and R(x, 6)

$$\therefore \quad QP = QR \Rightarrow QP^2 = QR^2$$

$$\Rightarrow (5-0)^2 + (-3-1)^2 = (x-0)^2 + (6-1)^2$$

$$\Rightarrow 25 + 16 = x^2 + 25$$

$$\Rightarrow x^2 = 16 \Rightarrow x = \pm \mathbf{4}$$

$$QR = \sqrt{(x-0)^2 + (6-1)^2} = \sqrt{x^2 + 5^2}$$

$$= \sqrt{(4)^2 + 5^2} = \sqrt{16+25} = \sqrt{\mathbf{41}}$$

$$PR = \sqrt{(x-5)^2 + (6+3)^2}$$

$$= \sqrt{(4-5)^2 + (6+3)^2}$$

$$= \sqrt{(-1)^2 + (9)^2} = \sqrt{1+81} = \sqrt{\mathbf{82}}$$

$$\text{Also, } PR = \sqrt{(-4-5)^2 + (6+3)^2} \quad [\text{Taking } x = -4]$$

$$= \sqrt{(-9)^2 + (9)^2} = \sqrt{162} = \mathbf{9\sqrt{2}}$$

Hence, $QR = \sqrt{41}$ and $PR = \sqrt{82}$, $9\sqrt{2}$.

Question 10:

Find a relation between x and y such that the point (x, y) is equidistant from the points (3, 6) and (-3, 4).

Solution:

Points A(3, 6) and B(-3, 4) are equidistant from point P(x, y)

$$AP = BP \Rightarrow \sqrt{(x-3)^2 + (y-6)^2} = \sqrt{(x+3)^2 + (y-4)^2}$$

$$\Rightarrow (x-3)^2 + (y-6)^2 = (x+3)^2 + (y-4)^2$$

$$\Rightarrow x^2 + 9 - 6x + y^2 + 36 - 12y = x^2 + 9 + 6x + y^2 + 16 - 8y$$

$$\Rightarrow -6x - 6x - 12y + 8y + 45 - 25 = 0 \Rightarrow -12x - 4y + 20 = 0$$

Dividing by -4, we get $3x + y - 5 = 0$