EXERCISE 1.2

Question 1:
Prove that √5 is irrational.
Solution:
Let √5 = p/q be a rational number, where p and q are co-primes and q ≠ 0.
Then, √5q = p

=> $5q^2=p^2$
⇒ $p^2 = 5q^2$ ... (i)
Since 5 divides p^2, so it will divide p also.
Let p = 5r
Then $p^2 – 25r^2$ [Squaring both sides]
⇒ $5q^2 = 25r^2$ [From(i)]
⇒ $q^2 = 5r^2$
Since 5 divides q^2, so it will divide q also. Thus, 5 is a common factor of both p and q.

This contradicts our assumption that √5 is rational.
Hence, √5 is irrational. Hence, proved.

Question 2:
Prove that 3 + 2√5 is irrational.
Solution:

Let $3 + 2\sqrt{5} = \frac{p}{q}$ be a rational number, where p and q are co-prime and q ≠ 0.
Then, $2\sqrt{5} = \frac{p}{q} - 3 = \frac{p-3q}{q}$
⇒ $\sqrt{5} = \frac{p-3q}{2q}$
since $\frac{p-3q}{2q}$ is a rational number,
therefore, √5 is a rational number. But, it is a contradiction.
Hence, 3 + √5 is irrational. Hence, proved.

Question 3:

Prove that the following are irrationals:

(i) 1/√2

Solution

(*i*) Let us assume that $\frac{1}{\sqrt{2}}$ is rational.

$\therefore$ There exists co-prime integers a and b $(b \neq 0)$ such that

$$\frac{1}{\sqrt{2}} = \frac{a}{b} \Rightarrow \sqrt{2} = \frac{b}{a}$$

Since a and b are integers, we get $\frac{b}{a}$ is rational and so $\sqrt{2}$ is rational.

But this contradicts the fact that $\sqrt{2}$ is irrational.

This contradiction has arisen because of our incorrect assumption that $\frac{1}{\sqrt{2}}$ is rational.

Hence, we conclude that $\frac{1}{\sqrt{2}}$ is irrational.

(ii) 7√5

Solution

(*ii*) Let us assume that $7\sqrt{5}$ is rational.

$\therefore$ There exists co-prime integers a and b $(b \neq 0)$ such that

$$7\sqrt{5} = \frac{a}{b} \Rightarrow \sqrt{5} = \frac{a}{7b}$$

Since a and b are integers, we get $\frac{a}{7b}$ is rational and so $\sqrt{5}$ is rational.

But this contradicts the fact that $\sqrt{5}$ is irrational.

This contradiction has arisen because of our incorrect assumption that $7\sqrt{5}$ is rational.

Hence, we conclude that $7\sqrt{5}$ is irrational.

(iii) 6 + √2

Solution

(*iii*) Let us assume that $6 + \sqrt{2}$ is rational.

$\therefore$ There exists co-prime integers a and b $(b \neq 0)$ such that

$$6 + \sqrt{2} = \frac{a}{b} \Rightarrow \sqrt{2} = \frac{a}{b} - 6$$

Since a and b are integers, we get $\frac{a}{b} - 6$ is rational and so $\sqrt{2}$ is rational.

But this contradicts the fact that $\sqrt{2}$ is irrational.

This contradiction has arisen because of our incorrect assumption that $6 + \sqrt{2}$ is rational.

Hence, we conclude that $6 + \sqrt{2}$ is irrational.