EXERCISE 8.2

Question 1:

Evaluate the following:

(i) $\sin 60° \cos 30° + \sin 30° \cos 60°$

(ii) $2\tan^2 45° + \cos^2 30° - \sin^2 60°$

(iii) $\dfrac{\cos 45°}{\sec 30° + \text{cosec } 30°}$

(iv) $\dfrac{\sin 30° + \tan 45° - \text{cosec } 60°}{\sec 30° + \cos 60° + \cot 45°}$

(v) $\dfrac{5\cos^2 60° + 4\sec^2 30° - \tan^2 45°}{\sin^2 30° + \cos^2 30°}$

Solution:

(i) $\sin 60° \cos 30° + \sin 30° \cos 60°$

$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$$

$$= \frac{3}{4} + \frac{1}{4} = \frac{3+1}{4} = \frac{4}{4} = \mathbf{1}.$$

(ii) $2\tan^2 45° + \cos^2 30° - \sin^2 60°$

$$= 2(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$$

$$= 2 + \frac{3}{4} - \frac{3}{4} = \frac{8+3-3}{4} = \frac{8}{4} = \mathbf{2}.$$

(iii) $$\frac{\cos 45°}{\sec 30° + \text{cosec } 30°} = \frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}} + 2} = \frac{\frac{1}{\sqrt{2}}}{\frac{2+2\sqrt{3}}{\sqrt{3}}}$$

$$= \frac{\sqrt{3}}{\sqrt{2}\left(2+2\sqrt{3}\right)} = \frac{\sqrt{3}}{2\sqrt{2}+2\sqrt{6}}$$

$$= \frac{\sqrt{3}}{2\sqrt{2}+2\sqrt{6}} \times \frac{2\sqrt{2}-2\sqrt{6}}{2\sqrt{2}-2\sqrt{6}}$$

$$= \frac{\sqrt{3}\left(2\sqrt{2}-2\sqrt{6}\right)}{\left(2\sqrt{2}\right)^2 - \left(2\sqrt{6}\right)^2} = \frac{2\sqrt{6}-2\sqrt{18}}{-16}$$

$$= \frac{2\sqrt{6}-6\sqrt{2}}{-16} = \frac{2\left(\sqrt{6}-3\sqrt{2}\right)}{-16} = \mathbf{\frac{3\sqrt{2}-\sqrt{6}}{8}}.$$

(iv) $\dfrac{\sin 30° + \tan 45° - \operatorname{cosec} 60°}{\sec 30° + \cos 60° + \cot 45°}$

$$= \frac{\frac{1}{2}+1-\frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}}+\frac{1}{2}+1} = \frac{\frac{\sqrt{3}+2\sqrt{3}-4}{2\sqrt{3}}}{\frac{4+\sqrt{3}+2\sqrt{3}}{2\sqrt{3}}}$$

$$= \frac{\sqrt{3}+2\sqrt{3}-4}{4+\sqrt{3}+2\sqrt{3}} = \frac{3\sqrt{3}-4}{4+3\sqrt{3}} \times \frac{4-3\sqrt{3}}{4-3\sqrt{3}}$$

$$= \frac{12\sqrt{3}-27-16+12\sqrt{3}}{-11} = \frac{24\sqrt{3}-43}{-11}$$

$$= \mathbf{\frac{43-24\sqrt{3}}{11}}.$$

(v) $\dfrac{5\cos^2 60° + 4\sec^2 30° - \tan^2 45°}{\sin^2 30° + \cos^2 30°}$

$$= \frac{5\times\left(\frac{1}{2}\right)^2 + 4\times\left(\frac{2}{\sqrt{3}}\right)^2 - 1}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{\frac{5}{4}+\frac{16}{3}-1}{\frac{1}{4}+\frac{3}{4}} = \mathbf{\frac{67}{12}}.$$

Question 2:

Choose the correct option and justify your choice:

(i) $\dfrac{2\tan 30^\circ}{1+\tan^2 30^\circ} =$

(A) $\sin 60^\circ$ (B) $\cos 60^\circ$ (C) $\tan 60^\circ$ (D) $\sin 30^\circ$

(ii) $\dfrac{1-\tan^2 45^\circ}{1+\tan^2 45^\circ} =$

(A) $\tan 90^\circ$ (B) 1 (C) $\sin 45^\circ$ (D) 0

(iii) $\sin 2A = 2\sin A$ is true when $A =$

(A) 0° (B) 30° (C) 45° (D) 60°

(iv) $\dfrac{2\tan 30^\circ}{1-\tan^2 30^\circ} =$

(A) $\cos 60^\circ$ (B) $\sin 60^\circ$ (C) $\tan 60^\circ$ (D) $\sin 30^\circ$

Solution:

(i) $$\frac{2\tan 30^\circ}{1+\tan^2 30^\circ} = \frac{2\left(\frac{1}{\sqrt{3}}\right)}{1+\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{\frac{1}{1}+\frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{3+1}{3}} = \frac{2}{\sqrt{3}} \times \frac{3}{4} = \frac{3}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{3\sqrt{3}}{2\times 3} = \frac{\sqrt{3}}{2} = \sin 60^\circ$$

Correct option is (A)

(ii) $$\frac{1-\tan^2 45^\circ}{1+\tan^2 45^\circ} = \frac{1-(1)^2}{1+(1)^2} = \frac{1-1}{1+1} = 0$$

Correct option is (D)

(iii) $\sin 2A = 2\sin A$, for $A = 0^\circ$

LHS $= \sin 2A = \sin 2\times 0 = \sin 0^\circ = 0$

RHS $= 2\sin A = 2\sin 0^\circ = 2\times 0 = 0$

Correct option is (A)

(iv) $$\frac{2\tan 30^\circ}{1-\tan^2 30^\circ} = \frac{2\left(\frac{1}{\sqrt{3}}\right)}{1-\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{\frac{1}{1}-\frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{3-1}{3}} = \frac{2}{\sqrt{3}} \times \frac{3}{2} = \frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{3\sqrt{3}}{3} = \sqrt{3} = \tan 60^\circ$$

Correct option is (C)

Question 3:

If tan (A + B) = √3 and tan (A – B) = $1/\sqrt{3}$; 0° < A + B ≤ 90°; A > B, find A and B.

Solution:

tan (A + B) = √3 = tan 60°

⇒ A + B = 60° …(i)

tan (A – B) = $\frac{1}{\sqrt{3}}$ = tan 30°

⇒ A – B = 30° …(ii)

On Adding equation (i) and (ii), we get

2A = 90° ⇒ A = 45°

From (i), we get:

45° + B = 60° ⇒ B = 15°

Thus, A = 45°, B = 15°

Question 4:

State whether the following statements are true or false. Justify your answer.
(i) sin (A + B) = sin A + sin B.
(ii) The value of sin θ increases as θ increases.
(iii) The value of cos θ increases as θ increases.
(iv) sin θ = cos θ for all values of θ.
(v) cot A is not defined for A = 0°.

Solution:

(*i*) Let, A = 60° and B = 30°

Then, LHS = sin(60° + 30°) = sin 90° = 1

and RHS = sin A + sin B = sin 60° + sin 30° = $\frac{\sqrt{3}}{2} + \frac{1}{2} = \frac{\sqrt{3}+1}{2}$

∴ LHS ≠ RHS **(False)**

(*ii*) sin 0° = 0, sin 30° = $\frac{1}{2}$, sin 45° = $\frac{1}{\sqrt{2}}$, sin 60° = $\frac{\sqrt{3}}{2}$, sin 90° = 1

∴ Value of sin θ increases as θ increases. **(True)**

(*iii*) cos 0° = 1, cos 30° = $\frac{\sqrt{3}}{2}$ = 0.87, cos 45° = $\frac{1}{\sqrt{2}}$, cos 60° = $\frac{1}{2}$,

cos 90° = 0

∴ Value of cos θ decreases as θ increases. **(False)**

(*iv*) sin 30° = $\frac{1}{2}$, cos 30° = $\frac{\sqrt{3}}{2}$

sin 30° ≠ cos 30° **(False)**

(*v*) cot 0° = $\frac{\cos 0°}{\sin 0°} = \frac{1}{0}$(not defined) **(True)**