EXERCISE 11.1

Question 1:

Find the area of a sector of a circle with radius 6 cm if angle of the sector is 60°.

Solution:

Given: Radius of sector, r = 6 cm and angle of sector, θ = 60°

$$\therefore \text{Area of sector} = \pi r^2 \times \frac{\theta}{360^\circ}$$

$$= \frac{22}{7} \times 6 \times 6 \times \frac{60^\circ}{360^\circ} = \frac{22 \times 6 \times 6}{7 \times 6}$$

$$= \mathbf{18.86\ cm^2}.$$

Question 2:

Find the area of a quadrant of a circle whose circumference is 22 cm.

Solution:

Let radius of the circle = r

∴ Circumference of the circle = 2πr

According to question,

$$2\pi r = 22 \text{ cm}$$

$$\Rightarrow \quad 2 \times \frac{22}{7} \times r = 22 \Rightarrow r = \frac{22 \times 7}{2 \times 22} = \frac{7}{2} \text{cm}$$

$$\text{Area of quadrant of the circle} = \frac{\pi r^2 \theta}{360^\circ} = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{90^\circ}{360^\circ} = \frac{22 \times 7}{2 \times 2 \times 4} = \frac{77}{8} \text{ cm}^2$$

Question 3:

The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.

Solution:

Angle described by the minute hand in 60 minutes = 360°

Angle described by the minute hand in 5 minutes

$$= \frac{360^\circ \times 5}{60^\circ} = 30^\circ$$

Now, we have $\theta = 30^\circ$ and $r = 14$ cm.

∴ Required area swept by the minute hand in 5 minutes = Area of the sector with $r = 14$ cm and $\theta = 30^\circ$

$$= \left(\frac{\pi r^2 \theta}{360^\circ}\right) \text{cm}^2 = \left(\frac{22}{7} \times 14 \times 14 \times \frac{30^\circ}{360^\circ}\right) \text{cm}^2$$

$$= \mathbf{51.33\ cm^2}.$$

Question 4:

A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding:
(i) minor segment
(ii) major segment (Use π = 3.14)

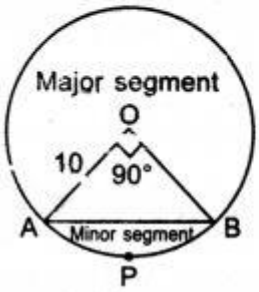
Solution:

Given: radius of the circle = 10 cm

Angle subtended by chord at centre = 90°

(i) Area of the minor segment

= Area of the sector OAPB – Area of ΔAOB formed with radius and chord



$$= \frac{\pi r^2 \theta}{360^\circ} - \frac{1}{2} r^2 \sin\theta$$

$$= 3.14 \times \frac{10 \times 10 \times 90^\circ}{360^\circ} - \frac{1}{2} \times 10 \times 10 \times \sin 90^\circ$$

$$= 3.14 \times \frac{10 \times 10}{4} - \frac{1}{2} \times 10 \times 10$$

$$= 3.14 \times 25 - 50 = 78.5 - 50 = 28.5 \text{ cm}^2$$

(*ii*) Area of the major segment = Area of the circle – Area of the minor segment

$$= \pi r^2 - 28.5 = 3.14 \times 10 \times 10 - 28.5$$

$$= 314 - 28.5 = 285.5 \text{ cm}^2$$

Question 5:

In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find:
(i) length of the arc.
(ii) area of the sector formed by the arc.
(iii) area of the segment formed by the corresponding chord.

Solution:

Given: Radius of circles, $r = 21$ cm

Angle of sector, $\theta = 60°$

(i) Length of the arc $= \frac{\theta}{360°} \times 2\pi r$

$= \frac{60°}{360°} \times 2 \times \frac{22}{7} \times 21 = \frac{1}{6} \times 2 \times 22 \times 3$

$= \mathbf{22\ cm}.$

(ii) Area of the sector formed by the arc

$= \pi r^2 \times \frac{\theta}{360°} = \frac{22}{7} \times 21 \times 21 \times \frac{60°}{360°}$

$= 11 \times 21 = \mathbf{231\ cm^2}.$

(iii) From the figure, OA = OB [Radii of same circle]

$\angle A = \angle B = \frac{1}{2}(180° - 60°) = 60°$

i.e., $\angle OAB$ is an equilateral triangle.

$\therefore$ Area of equilateral $\Delta OAB = \frac{\sqrt{3}}{4}(\text{Side})^2$

$= \frac{\sqrt{3}}{4}(21)^2 = \frac{441\sqrt{3}}{4}\text{cm}^2$

$\therefore$ Area of segment formed by the chord

= Area of sector – Area of equilateral triangle

$= \left(\mathbf{231 - \frac{441\sqrt{3}}{4}}\right)\mathbf{cm^2}.$

Question 6:

A chord of a circle of radius 15 cm subtends an angle of 60° at the centre. Find the areas of the corresponding minor and major segments of the circle. (Use π = 3.14 and √3 = 1.73)

Solution:

Radius of the circle = 15 cm

Angle subtended by chord at centre = 60°

$$\text{Area of the sector} = \frac{\pi r^2\theta}{360^\circ} = 3.14 \times \frac{15 \times 15 \times 60^\circ}{360^\circ} = 117.75 \text{ cm}^2$$

$$\text{Area of the triangle formed by radii and chord} = \frac{1}{2}r^2 \sin\theta$$

$$= \frac{1}{2}(15)^2 \sin 60^\circ = \frac{1}{2} \times 15 \times 15 \times \frac{\sqrt{3}}{2} = 97.31 \text{ cm}^2$$

Area of the minor segment = Area of the sector − Area of the triangle formed by radii and chord

$$= 117.75 - 97.31 = 20.44 \text{ cm}^2$$

$$\text{Area of the circle} = \pi r^2 = 3.14 \times 15 \times 15 = 706.5 \text{ cm}^2$$

Area of the major segment = Area of the circle − Area of the minor segment

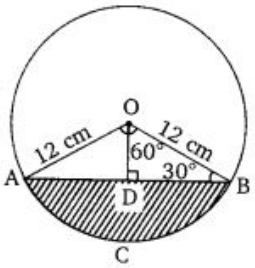
$$= 706.5 - 20.44 = 686.06 \text{ cm}^2$$

Question 7:

A chord of a circle of the radius 12 cm subtends an angle of 120° at the centre. Find the area of the corresponding segment of the circle. (Use π = 3.14 and √3 = 1.73).

Solution:

Let AB be a chord which subtends an angle 120° at the centre O of the circle.



Area of segment ACB

= Area of sector OACB − Area of ΔOAB ...(i)

$$\text{Area of sector OACB} = \frac{120^\circ}{360^\circ} \times 3.14 \times (12)^2$$

$$= \frac{1}{3} \times 3.14 \times 12 \times 12 = 150.72 \text{ cm}^2. \quad \text{...(ii)}$$

We draw OD ⊥ AB.

$\therefore \angle OBD = 180^\circ - (90^\circ + 60^\circ) = 30^\circ$

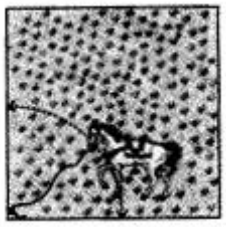
Now from ΔODB, $\sin 30^\circ = \frac{OD}{OB}$

$$\Rightarrow \quad \frac{1}{2} = \frac{OD}{12} \qquad \left[\because \sin 30^\circ = \frac{1}{2}\right]$$

$$\Rightarrow \quad OD = 6 \text{ cm}$$

Question 8:

A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope (see figure). Find:



(i) the area of that part of the field in which the horse can graze.
(ii) the increase in the grazing area if the rope were 10 m long instead of 5 m. (Use π = 3.14)

Solution:

(i) Length of the rope = Radius of the sector grazed by horse = 5 m

Here, angle of the sector = 90°

$$\text{Area of the field that horse can graze} = \frac{\pi r^2\theta}{360^\circ} = \frac{3.14 \times 5 \times 5 \times 90^\circ}{360^\circ}$$

$$= \frac{3.14 \times 5 \times 5}{4} = 19.625 \text{ cm}^2$$

(ii) **Length of the rope is increased from 5 m to 10 m**

New radius of sector grazed by horse = 10 m

$$\therefore \quad \text{Area grazed by horse} = \frac{\pi r^2\theta}{360^\circ} = \frac{3.14 \times 10 \times 10 \times 90^\circ}{360^\circ} = 78.5 \text{ cm}^2$$

$$\text{Area increased} = 78.5 - 19.625 = 58.875 \text{ cm}^2.$$

Question 9:

A brooch is made with silver wire in the form of a circle with diameter 35 mm. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors as shown in figure.
Find:

(i) the total length of the silver wire required.
(ii) the area of each sector of the brooch.

Solution:

(i) Length of silver wire required to make the perimeter of the circle, i.e.,

$$\text{Circumference} = 2\pi r = \left(2 \times \frac{22}{7} \times \frac{35}{2}\right) \text{mm}$$

$$= 110 \text{ mm}.$$

Length of wire required to make 5 diameters

$$= 5 \times 35 = 175 \text{ mm}$$

$\therefore$ Total length of silver wire required

$$= (110 + 175) \text{ mm} = \mathbf{285\ mm}.$$

(ii) Sector angle of each brooch, $\theta = \frac{360^\circ}{10} = 36^\circ$

$\therefore$ The area of each sector of the brooch

$= \frac{1}{10} \times$ Area of the circle

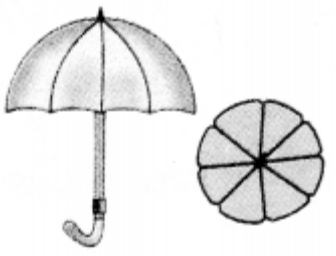
$= \frac{1}{10} \times \pi r^2 = \frac{1}{10} \times \frac{22}{7} \times \frac{35}{2} \times \frac{35}{2}$ mm^2

[$\because$ Area of circle $= \pi r^2$, $r = \frac{35}{2}$]

$= \frac{385}{4}$ **mm^2.**

Question 10:

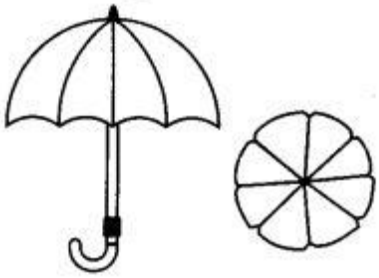
An umbrella has 8 ribs which are equally spaced (see figure). Assuming umbrella to be a flat circle of radius / 45 cm, find the area between the two consecutive ribs of the umbrella.



Solution:

Radius of the circle = 45 cm Number of ribs = 8

central angle of the circle



Angle between two consecutive ribs $= \frac{\text{central angle of the circle}}{\text{number of the sectors (ribs)}} = \frac{360^\circ}{8} = 45^\circ$

Area between two consecutive ribs = Area of one sector of the circle

$= \frac{\pi r^2 \theta}{360^\circ} = \frac{22}{7} \times \frac{45 \times 45 \times 45^\circ}{360^\circ}$

$= \frac{11 \times 45 \times 9 \times 5}{7 \times 4} = \frac{22275}{28}$ cm^2

Question 11:

A car has two wipers which do not overlap.
Each wiper has a blade of length 25 cm sweeping through an angle of 115°. Find the total area cleaned at each sweep of the blades.

Solution:

The blade of each wiper sweeps in the form of a sector of radius 25 cm.

The sector angle, $\theta = 115°$ [Given]

$\therefore$ The area covered by one blade

$= \frac{115°}{360°} \times \pi \times (25)^2$

$\therefore$ The area covered by both blades

$$= \left(2 \times \frac{115°}{360°} \times \frac{22}{7} \times 625\right) \text{cm}^2$$

$$= \left(\frac{23}{18} \times \frac{11}{7} \times 625\right) = \mathbf{\frac{158125}{126}}\ \mathbf{cm^2}.$$

Question 12:

To warn ships for underwater rocks, a lighthouse spreads a red colored light over a sector of angle 80° to a distance of 16.5 km. Find the area of the sea over which the ships are warned. (Use π = 3.14)

Solution:

Angle of the sector = 80°

Distance covered = 16.5 km

$\therefore$ **Radius of the sector formed = 16.5 km**

Area of the sea over which ships are warned = Area of the sector $= \frac{\pi r^2 \theta}{360°}$

$$= \frac{3.14 \times 16.5 \times 16.5 \times 80°}{360°} = 189.97 \text{ km}^2$$

Question 13:

A round table cover has six equal designs as shown in the figure. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of ₹0.35 per cm². (Use √3= 1.7)

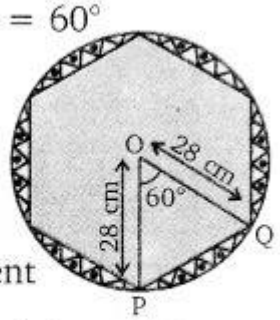
Solution:

Here, radius of the cover $(r) = 28$ cm

and central angle $(\theta) = \frac{360^\circ}{6} = 60^\circ$

We join OP and OQ

Then ΔOPQ is an equilateral triangle with side 28 cm.



∴ Area of one designed segment

= Area of sector OPQ – Area of the ΔOPQ

$$= \frac{60^\circ}{360^\circ} \times \pi \times (28)^2 - \frac{\sqrt{3}}{4} \times (28)^2$$

$$= (28)^2 \times \left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right) = (28)^2 \times \left(\frac{22}{7 \times 6} - \frac{1.7}{4}\right)$$

$$= 28^2 \times \left(\frac{11}{21} - \frac{1.7}{4}\right) = 28 \times 28 \times \left(\frac{44 - 35.7}{84}\right)$$

$$= \frac{28 \times 28 \times 8.3}{12} = 77.47 \text{ cm}^2.$$

∴ Total area of 6 designed segments

$$= 6 \times 77.47 = 464.82 \text{ cm}^2.$$

∴ Cost of making the designs

$$= ₹\ 464.82 \times 0.35 = ₹\ 162.69.$$

Question 14:

Tick the correct answer in the following: Area of a sector of angle p (in degrees) of a circle with radius R is

(a) $\frac{p}{180} \times 2\pi R$ (b) $\frac{p}{180} \times \pi R^2$

(c) $\frac{p}{360} \times 2\pi R$ (d) $\frac{p}{720} \times 2\pi R^2$

Solution:

Sector angle is p in degrees

Radius of the circle = R

Area of the sector $= \frac{\pi R^2 p}{360^\circ} = \frac{(\pi R^2 p)2}{720^\circ} = \frac{p}{720^\circ} \times 2\pi R^2$

Ans: (*d*)