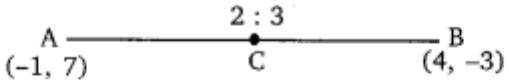
EXERCISE 7.2

Question 1:

Find the coordinates of the point which divides the join of (-1, 7) and (4, -3) in the ratio 2 : 3.

Solution:

Let the coordinates of point C be (x, y).



$$x\text{-coordinate of C} = \frac{mx_2 + nx_1}{m+n}$$

$$= \frac{2\times 4 + 3\times(-1)}{2+3} = \frac{8-3}{5} = 1.$$

$$y\text{-coordinate of C} = \frac{my_2 + ny_1}{m+n}$$

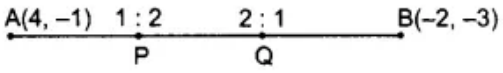
$$= \frac{2\times(-3) + 3\times(7)}{2+3} = \frac{-6+21}{5} = 3.$$

Hence, the coordinates of C are **(1, 3)**.

Question 2:

Find the coordinates of the points of trisection of the line segment joining (4, -1) and (-2, -3).

Solution:

Let points P and Q trisect the line joining the points. 

$\therefore$ AP = PQ = QB

P divides AB in the ratio 1: 2 and Q divides AB in the ratio 2:1

$$\text{P } (x\text{-coordinate}) = \frac{1\times(-2) + 2\times 4}{1+2} = \frac{-2+8}{3} = \frac{6}{3} = 2$$

$$\text{P } (y\text{-coordinate}) = \frac{1\times(-3) + 2\times(-1)}{1+2} = \frac{-3-2}{3} = -\frac{5}{3}$$

The coordinates of P are $\left(2, -\frac{5}{3}\right)$.

$$\text{Q } (x\text{-coordinate}) = \frac{2\times(-2) + 1\times(4)}{2+1} = \frac{-4+4}{3} = 0$$

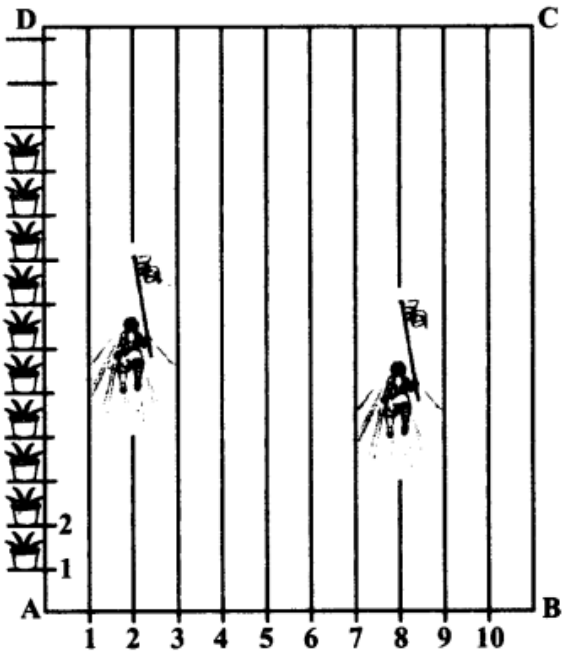
$$\text{Q } (y\text{-coordinate}) = \frac{2(-3) + 1\times(-1)}{2+1} = \frac{-6-1}{3} = -\frac{7}{3}$$

The coordinates of Q are $\left(0, -\frac{7}{3}\right)$.

Question 3:

To conduct Sports Day activities, in your rectangular shaped school ground ABCD, lines have been drawn with chalk powder at a distance of 1 m each. 100 flower pots have been placed at a distance of 1 m from each other along AD, as shown in given figure below. Niharika runs $1/4$ th the distance AD on the 2nd line and posts a green flag. Preet runs $1/5$ th distance AD on the eighth line and posts a red flag.

What is the distance between both the flags? If Rashmi has to post a blue flag exactly halfway between the line segment joining the two flags, where should she post her flag?



Solution:

From the figure, taking A as (0, 0), x- axis along AB and y- axis along AD, we will obtain the coordinates of the green flag and the red flag.

The green flag is at $\frac{1}{4}$th of the total distance

$$= \frac{1}{4} \times 100 = 25 \text{ m in 2nd line.}$$

$\therefore$ The coordinates of green flag are (2, 25).

Similarly, coordinates of red flag are (8, 20).

Distance between two flags,

$$D = \sqrt{(8-2)^2 + (20-25)^2}$$

$$= \sqrt{(6)^2 + (-5)^2} = \sqrt{36+25} = \sqrt{\mathbf{61}} \text{ m.}$$

Now, blue flag is posted at the midpoint of the distance between two flags

$$\therefore \text{ Coordinates of blue flag} = \left(\frac{2+8}{2}, \frac{25+20}{2}\right)$$

$$= (5, 22.5)$$

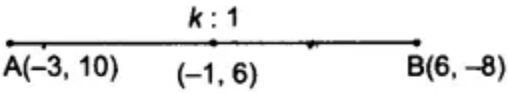
Hence, the blue flag will be posted in 5th line at a distance of **22.5 m**.

Question 4:

Find the ratio in which the line segment joining the points (-3, 10) and (6, -8) is divided by (-1, 6).

Solution:

Let the required ratio be k : 1



$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2}$$

$$-1 = \frac{k \times 6 + 1 \times (-3)}{k + 1}$$

$$-k - 1 = 6k - 3 \Rightarrow 7k = 2 \Rightarrow k = \frac{2}{7}$$

$$y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2} \Rightarrow 6 = \frac{k \times (-8) + 1 \times (10)}{k + 1}$$

$$\Rightarrow \quad 6k + 6 = -8k + 10 \Rightarrow 14k = 4 \Rightarrow k = \frac{4}{14} = \frac{2}{7}$$

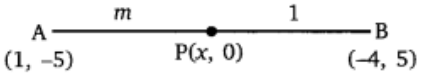
The required ratio is 2 : 7.

Question 5:

Find the ratio in which line segment joining A (1, -5) and B (-4, 5) is divided by the x-axis. Also, find the coordinates of the point of division.

Solution:

Let p(x, 0) be the point which divides the line segmwnt joining A(1,-5) and B(-4, 5) in the ratio m : 1.



Then using section formula, we get:

$$(x, 0) = \left(\frac{m \times -4 + 1 \times 1}{m + 1}, \frac{m \times 5 + 1 \times -5}{m + 1}\right)$$

$$\Rightarrow \quad 0 = \frac{m \times 5 + 1 \times (-5)}{m + 1} \quad \text{[Taking } y\text{-coordinates]}$$

$$\Rightarrow \quad 5m - 5 = 0 \quad \Rightarrow \quad m = 1$$

$$\Rightarrow \quad m : 1 = 1 : 1$$

Hence, the required ratio is **1 : 1**.

Since the ratio is 1 : 1, so P is the mid-point.

$$\therefore x = \frac{1 - 4}{2} = \frac{-3}{2}.$$

Hence, $\left(-\frac{3}{2}, 0\right)$ is the required point.

Question 6:

If (1, 2), (4, y), (x, 6) and (3, 5) are the vertices of a parallelogram taken in order, find x and y.

Solution:

Mid-point of AC = Mid-point of BD

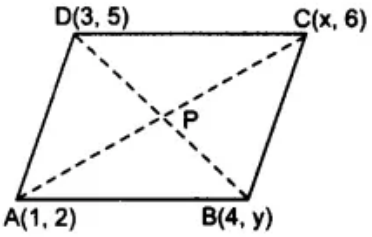
$$\Rightarrow \quad \frac{x+1}{2}, \frac{6+2}{2} = \frac{4+3}{2}, \frac{y+5}{2}$$

$$\Rightarrow \quad \frac{x+1}{2} = \frac{7}{2} \text{ and } \frac{6+2}{2} = \frac{y+5}{2}$$

$$\Rightarrow \quad x + 1 = 7 \text{ and } 8 = y + 5$$

$$\Rightarrow \quad x = 7 - 1 \text{ and } y = 8 - 5 = 3$$

$$\Rightarrow \quad x = 6 \quad \text{and } y = 3$$



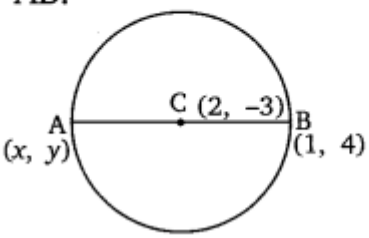
Question 7:

Find the coordinates of a point A, where AB is the diameter of a circle whose centre is (2, -3) and B is (1, 4).

Solution:

Let the coordinates of the point A be (x, y).

Then as C(2, –3) is the mid-point of diameter AB.



$$\therefore \text{ Coordinates of C} = \left(\frac{x+1}{2}, \frac{y+4}{2}\right)$$

$$\Rightarrow \quad 2 = \frac{x+1}{2} \quad \Rightarrow \quad x = 3$$

$$\text{Also,} \quad -3 = \frac{y+4}{2} \quad \Rightarrow \quad y = -10$$

Hence, the coordinates of A are **(3, –10)**.

Question 8:

If A and B are (-2, -2) and (2, -4), respectively, find the coordinates of P such that AP = 3/7 AB and P lies on the line segment AB.

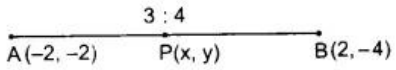
Solution:

$$AP = \frac{3}{7}AB$$

$$BP = AB - AP$$

$$= \frac{AB}{1} - \frac{3}{7}AB = \frac{7AB - 3AB}{7} = \frac{4AB}{7}$$

$$\frac{AP}{BP} = \frac{\frac{3}{7}AB}{\frac{4}{7}AB} = 3:4$$



$$x = \frac{3(2) + 4(-2)}{3+4} = \frac{6-8}{7} = -\frac{2}{7}$$

$$y = \frac{3(-4) + 4(-2)}{3+4} = \frac{-12-8}{7} = -\frac{20}{7}$$

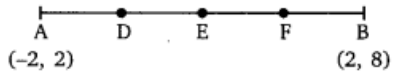
Hence, the coordinates of P are $\left(-\frac{2}{7}, -\frac{20}{7}\right)$.

Question 9:

Find the coordinates of the points which divide the line segment joining A (-2, 2) and B (2, 8) into four equal parts.

Solution:

let points D, E and F divide AB into four equal parts such that AD = DE = EF = FB



From the above figure, E is the mid-point of AB.

$$\therefore \text{Coordinates of E} = \left(\frac{-2+2}{2}, \frac{2+8}{2}\right) = (0, 5)$$

D is the mid-point of AE.

$$\therefore \text{Coordinates of D} = \left(\frac{-2+0}{2}, \frac{2+5}{2}\right)$$

$$= \left(-1, \frac{7}{2}\right)$$

F is the mid-point of EB.

$$\therefore \text{Coordinates of F} = \left(\frac{0+2}{2}, \frac{5+8}{2}\right) = \left(1, \frac{13}{2}\right)$$

Hence, the required points are $\left(\mathbf{-1, \frac{7}{2}}\right)$, **(0, 5)** and $\left(\mathbf{1, \frac{13}{2}}\right)$.

Question 10:

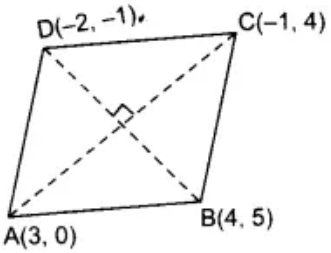
Find the area of a rhombus if its vertices are (3, 0), (4, 5), (-1, 4) and (-2, -1) taken in order. [Hint: Area of a rhombus = ½ (product of its diagonals)]

Solution:

Let points be A (3, 0), B(4, 5), C(–1, 4) and D(–2, –1)

$$AC = \sqrt{(-1-3)^2+(4-0)^2} = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$$

$$BD = \sqrt{(4+2)^2+(5+1)^2} = \sqrt{36+36} = 6\sqrt{2}$$



$$\text{Area of a rhombus} = \frac{1}{2} \times AC \times BD$$

$$= \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2}$$

$$= \frac{1}{2} \times 4 \times 6 \times 2 = 24 \text{ square units}$$