2022
MATHEMATICS

Total marks : 80   Time : 3 hours

General Instructions:
i)   *Approximately 15 minutes is allotted to read the question paper and revise the answers.*
ii)   *The question paper consists of 18 questions.*
iii) *All questions are compulsory.*
iv)   *Internal & general choice have been provided in some questions.*
v)   *Marks allocated to every question are indicated against it.*
N.B: Check to ensure that all pages of the question paper is complete as indicated on the top
   **left side.**

Section – A

1.   **Choose the correct answer from the given alternatives.**

   (a)   If α and β are the zeroes of the polynomial $f(x) = px^2 - 2x + 3p$ and $\alpha + \beta = \alpha\beta$
         then the value of p is   1
         (i) $-\dfrac{2}{3}$ (ii) $\dfrac{2}{3}$ (iii) $\dfrac{1}{3}$ (iv) $-\dfrac{1}{3}$

   (b)   The pair of equations $x + y - 40 = 0$ and $x - 2y + 14 = 0$ have   1
         (i) a unique solution.   (ii) exactly two solutions.
         (iii) infinitely many solutions.   (iv) no solution.

   (c)   If $x = 1$ is a common root of $ax^2 + ax + 2 = 0$ and $x^2 + x + b = 0$, then $ab =$ 1
         (i) 1   (ii) 2   (iii) 3   (iv) 4

   (d)   If sum of the first five terms of an AP is 30 and the 5th term is 10, then the
         sum of the first four terms of the AP is   1
         (i) 5   (ii) 10   (iii) 20   (iv) 30

   (e)   If $\tan 2\theta = \cot(\theta + 15°)$, where 2θ and $(\theta + 15°)$ are acute, the value of θ is   1
         (i) 22°   (ii) 25°   (iii) 30°   (iv) 35°

   (f)   *y*-axis divides the join of P(–4, 2) and Q(8, 3) in the ratio   1
         (i) 3 : 1   (ii) 1 : 3   (iii) 2 : 1   (iv) 1 : 2

   (g)   If tangents PA and PB from a point P to a circle with centre O are inclined to
         each other at angle of 80°, then ∠POA is equal to   1
         (i) 50°   (ii) 60°   (iii) 70°   (iv) 80°

(h)　An arc of a circle is of length 5π cm and the sector it bounds has an area of 20π cm². The radius of the circle is　　　　**1**
(i) 16 cm　　　　(ii) 12 cm　　　(iii) 8 cm　　　　(iv) 4 cm

(i)　The ratio of the total surface area to the lateral surface area of a cylinder with base radius 80 cm and height 20 cm is　　　　**1**
(i) 2 : 1　　　　(ii) 3 : 1　　　(iii) 4 : 1　　　　(iv) 5 : 1

(j)　Two dice are rolled once. The probability of getting such numbers on two dice, whose product is a perfect square, is　　　　**1**
(i) 8　　　　　　(ii) $\dfrac{1}{8}$　　　　(iii) $\dfrac{9}{2}$　　　　(iv) $\dfrac{2}{9}$

Section – B

2.　An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?　　**2**

3.　Find the value of k for the quadratic equation $kx(x-2)+6=0$ has equal roots.　　**2**

4.　Find the values of y for which the distance between the points P(2, −3) and Q(10, y) is 10 units.　　**2**

5.　In the adjoining figure, ΔABC is a right triangle in which $\angle$B = 90° and BD $\perp$ AC. If AB = x units, BC = y units and AC = z units, then find BD.　　**2**
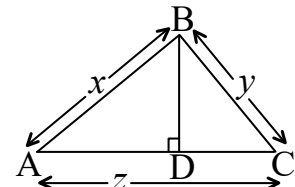
6.　The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.　　**2**

Section – C

7.　**Answer any three from the following questions (a) to (e).**　　　**3 × 3 = 9**

(a)　On dividing $x^3 - 3x^2 + x + 2$ by a polynomial g(x), the quotient and remainder were $(x - 2)$ and $(-2x + 4)$ respectively. Find g(x).

(b)　Meena went to a bank to withdraw ₹2000. She asked the cashier to give her ₹50 and ₹100 notes only. Meena got 25 notes in all. Find how many notes of ₹50 and ₹100 she received.

(c)　Solve the following pair of linear equations by cross-multiplication method:

$$x - 3y - 7 = 0$$
$$3x - 3y - 15 = 0$$

(d) A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.

(e) A sum of ₹700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is ₹20 less than its preceding prize, find the value of each of the prizes.

8. **Answer any two from the following questions (a) to (d).**   **2 × 3 = 6**

(a) In triangle ABC, right angled at B, if $\tan A = \dfrac{1}{\sqrt{3}}$, find the value of $\cos A \cos C - \sin A \sin C$, with the help of a right triangle.

(b) If $\sin(A - B) = \dfrac{1}{2}$, $\cos(A + B) = \dfrac{1}{2}$, $0° < A + B \le 90°$, $A > B$, find A and B.

(c) Prove that: $\dfrac{1 + \sec A}{\sec A} = \dfrac{\sin^2 A}{1 - \cos A}$, where angle A is an acute angle.

(d) A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle 30° with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree.

9. **Case Study based question:**   **3**
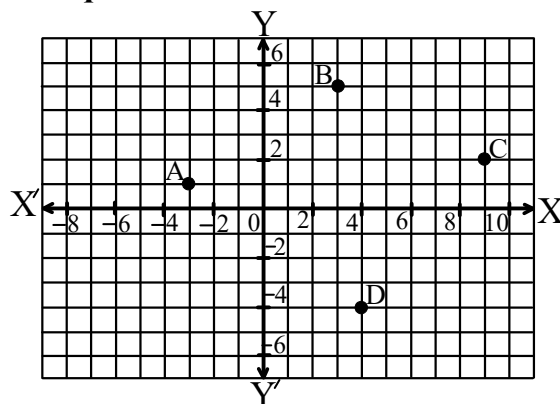
Four friends are seated at the points A, B, C and D on a lawn keeping social distancing, as shown in the figure above. One more friend wants to join them and sit exactly at the middle position E on a straight line between A and C. Based on the above information, answer the following questions (i) to (iii).

(i)   The distance between A and D is

    (a) $\sqrt{72}$ units   (b) $\sqrt{73}$ units   (c) $\sqrt{74}$ units   (d) $\sqrt{75}$ units

(ii) The coordinates of the position of E are

    (a) $\left(\dfrac{3}{2}, 3\right)$ (b) $\left(3, \dfrac{3}{2}\right)$ (c) (6, 3)   (d) (3, 6)

(iii) *x*-axis divides the distance/length between B and D in the ratio

    (a) 3 : 4   (b) 4 : 3   (c) 5 : 4   (d) 4 : 5

10. **a.** Construct an isosceles triangle whose base is 8 cm and altitude 4 cm and then another triangle whose sides are $1\dfrac{1}{2}$ times the corresponding sides of the isosceles triangle. (Traces of construction only is required.)

<div align="center">Or</div>   **3**

    b. Draw a line segment AB of length 8 cm. Taking A as centre, draw a circle of radius 4 cm and taking B as centre, draw another circle of radius 3 cm. Construct tangents to each circle from the centre of the other circle. (Traces of construction only is required.)

11. **a.** A chord of a circle of radius 15 cm subtends an angle of 60° at the centre. Find the areas of the corresponding minor and major segments of the circle. [Use $\pi = 3.14$ and $\sqrt{3} = 1.73$]

<div align="center">Or</div>   **3**

    b. How many silver coins, 1.75 cm in diameter and of thickness 2 mm, must be melted to form a cuboid of dimensions 5.5 cm × 10 cm × 3.5 cm?

12. **Answer any two from the following questions (a) to (c).**   **2 × 3 = 6**

    (a) A student noted the number of cars passing through a spot on a road for 100 periods each of 3 minutes and summarised it in the table given below. Find the mode of the data:

Number of cars	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	7	14	13	12	20	11	15	8

    (b) The lengths of 40 leaves of a plant are measured correct to the nearest millimeter, and the data obtained is represented in the following table:

Length (in mm)	118-126	127-135	136-144	145-153	154-162	163-171	172-180
Number of leaves	3	5	9	12	5	4	2

    Find the median length of the leaves.

(c) A box contains 90 discs which are numbered from 1 to 90. If one disc is drawn at random from the box, find the probability that it bears: (i) a two-digit number, (ii) a perfect square number, (iii) a number divisible by 5.

Section – D

13. **a.** Draw the graphs of the equations $x - y + 1 = 0$ and $3x + 2y - 12 = 0$. Determine the coordinates of the vertices of the triangle formed by these lines and the *x*-axis and shade the triangular region.

<div align="center">Or</div> 5

b. Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speeds of the two cars?

14. **a.** A statue, 1.6 m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45°. Find the height of the pedestal.

<div align="center">Or</div> 5

b. A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60°. After sometime, the angle of elevation reduces to 30°. Find the distance travelled by the balloon during the interval.
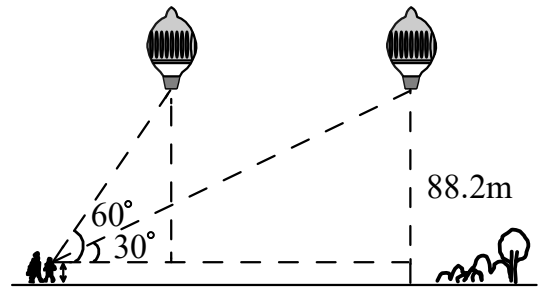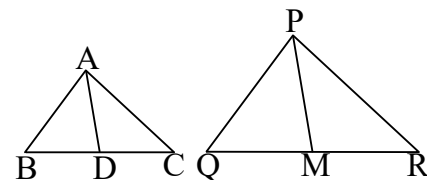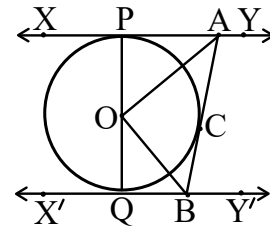

15. **a.** State and Prove Pythagoras Theorem.

<div align="center">Or</div> 5

b. Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of ΔPQR. Show that ΔABC ~ ΔPQR.
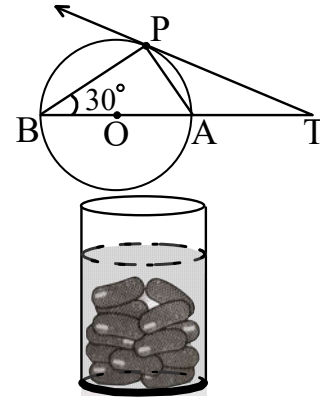

16. **a.** In the adjoining figure, XY and X′Y′ are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X′Y′ at B. Prove that ∠AOB = 90°

<div align="center">Or</div> 5

b. In the adjoining figure, O is the centre of the circle and TP is the tangent to the circle from an external point T. If ∠PBT = 30°, prove that BA : AT = 2 : 1

17. a. A *gulab jamun*, contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 *gulab jamuns*, each shaped like a cylinder with two hemispherical ends with length 5 cm and diameter 2.8 cm.

Or   5

b. A container, opened from the top and made up of a metal sheet, is in the form of a frustrum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm, respectively. Find the cost of the milk which can completely fill the container @ of ₹20 per litre. Also find the cost of metal sheet used to make the container, if it costs ₹8 per 100 cm². [Take π = 3.14]

18. a. The following distribution shows the daily pocket allowance of children of a locality. Find the mean pocket allowance by step-deviation method.

Daily pocket allowance (in ₹)	More than 11	More than 13	More than 15	More than 17	More than 19	More than 21	More than 23
Number of children	64	57	51	42	29	9	4

Or   5

b. The distribution of heights (in cm) of 96 children is given below:

Height (in cm)	Number of children
124-128	5
128-132	8
132-136	17
136-140	24
140-144	16
144-148	12
148-152	6
152-156	4
156-160	3
160-164	1

Draw a less than type cumulative frequency curve for this data and use it to compute median height of the children.

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