EXERCISE 5.3

Question 1:

Find the sum of the following APs:
(i) 2, 7, 12,…… to 10 terms.
(ii) -37, -33, -29, …… to 12 terms.
(iii) 0.6, 1.7, 2.8, ……, to 100 terms.
(iv) 1/15, 1/12, 1/10, ……., to 11 terms.

Solution:

(i) Here, $a = 2, t_2 = 7, t_3 = 12$ and $n = 10$

$\therefore d = t_3 - t_2 = 12 - 7 = 5$

$\therefore$ The required sum,

$$S_{10} = \frac{10}{2}[2 \times 2 + (10 - 1)5]$$
$$= 5 \times 49 = \mathbf{245}.$$

(ii) Here, $a = -37$, $t_2 = -33$, $t_3 = -29$ and $n = 12$

$\therefore \quad d = t_3 - t_2 = -29 + 33 = 4$

$\therefore$ The required sum,

$$S_{12} = \frac{12}{2}[2 \times (-37) + (12 - 1)4]$$
$$= 6 \times (-30) = \mathbf{-180}.$$

(iii) Here, $a = 0.6, t_2 = 1.7, t_3 = 2.8$ and $n = 100$

$\therefore d = t_3 - t_2 = 2.8 - 1.7 = 1.1$

$\therefore$ The required sum,

$$S_{100} = \frac{100}{2}[2 \times 0.6 + (100 - 1)\ 1.1]$$
$$= 50 \times 110.1 = \mathbf{5505}.$$

(iv) Here, $a = \frac{1}{15}, t_2 = \frac{1}{12}, t_3 = \frac{1}{10}$ and $n = 11$

$$\therefore \quad d = t_3 - t_2 = \frac{1}{10} - \frac{1}{12} = \frac{6-5}{60} = \frac{1}{60}$$

$\therefore$ The required sum,

$$S_{11} = \frac{11}{2}\left[\frac{2}{15} + (11-1)\frac{1}{60}\right]$$
$$= \frac{11}{2}\left[\frac{2}{15} + \frac{1}{6}\right] = \frac{11}{2}\left[\frac{4+5}{30}\right]$$
$$= \frac{11}{2} \times \frac{3}{10} = \frac{\mathbf{33}}{\mathbf{20}}.$$

Question 2:

Find the sums given below:

(i) $7 + 10\frac{1}{2} + 14 + \ldots + 84$

(ii) 34 + 32 + 30 + … + 10

(iii) -5 + (-8) + (-11) + ….. + (-230)

Solution:

(i) $7 + 10\frac{1}{2} + 14 + \ldots\ldots + 84$

Here,

$$a = 7, d = \frac{21}{2} - \frac{7}{1} = \frac{7}{2}, a_n = 84$$

$$a_n = a + (n-1)d$$

$$\Rightarrow \quad 84 = 7 + (n-1)\frac{7}{2} \Rightarrow 84 - 7 = (n-1)\frac{7}{2}$$

$$\Rightarrow \quad 77 \times \frac{2}{7} = n - 1 \Rightarrow 22 + 1 = n \Rightarrow n = 23$$

$$S_n = \frac{n}{2}[a + l] \qquad [\because a_n = l]$$

$$S_{23} = \frac{23}{2}[7 + 84] = \frac{23}{2} \times 91 = \frac{2093}{2} = 1046.5$$

(ii) $34 + 32 + 30 + \ldots\ldots + 10$

Here,

$$a = 34, d = 32 - 34 = -2, a_n = 10$$

$$a_n = a + (n-1)d \Rightarrow 10 = 34 + (n-1)(-2)$$

$$\Rightarrow \quad 10 - 34 = (n-1)(-2) \Rightarrow \frac{-24}{-2} = n - 1 \Rightarrow n - 1 = 12$$

$$n = 12 + 1 = 13$$

$$S_{13} = \frac{13}{2}[34 + 10] = \frac{13}{2} \times 44 = 13 \times 22 = 286$$

(iii) $-5 + (-8) + (-11) + \ldots\ldots + (-230)$

Here,

$$a = -5, a_n = -230,$$

$$d = -8 - (-5) = -3$$

$$a_n = a + (n-1)d \Rightarrow -230 = -5 + (n-1)(-3)$$

$$\Rightarrow \quad \frac{-225}{-3} = n - 1 \Rightarrow 75 + 1 = n \Rightarrow n = 76$$

$$S_{76} = \frac{76}{2}[-5 + (-230)] = 38 \times (-235) = -8930$$

Question 3:

In an AP:
(i) given a = 5, d = 3, a_n = 50, find n and S_n.
(ii) given a = 7, a_{13} = 35, find d and S_{13}.
(iii) given a_{12} = 37, d = 3, find a and S_{12}.
(iv) given a_3 = -15, S_{10} = 125, find d and a_{10}.
(v) given d = 5, S_9 = 75, find a and a_9.
(vi) given a = 2, d = 8, S_n = 90, find n and a_n.
(vii) given a = 8, a_n = 62, S_n = 210, find n and d.
(viii) given a_n = 4, d = 2, S_n = -14, find n and a.
(ix) given a = 3, n = 8, S = 192, find d.
(x) given l = 28, S = 144, and there are total 9 terms. Find a.

Solution:

(i) Given: $a = 5$, and $d = 3$ and $a_n = 50$

Applying the formula, $a_n = a + (n-1)d$, we get:

$$a + (n-1)d = 50$$
$$\Rightarrow 5 + (n-1)3 = 50$$
$$\Rightarrow 3n = 48 \Rightarrow n = 16$$
$$\therefore S_n = \frac{n}{2}[2a + (n-1)d]$$
$$\Rightarrow S_{16} = \frac{16}{2}[2 \times 5 + (16-1)3]$$
$$= 8(10 + 45) = 8 \times 55 = 440.$$

Hence, n = **16** and S_n = **440**.

(ii) Given: $a = 7$ and $a_{13} = 35$

Applying the formula, $a_n = a + (n-1)d$, we get:

$$a_{13} = a + (13-1)d$$
$$\Rightarrow 35 = 7 + 12d$$
$$\Rightarrow d = \frac{28}{12} = \frac{7}{3}$$
$$\therefore S_{13} = \frac{13}{2}[2a + 12d]$$

$$= \frac{13}{2}\left[2\times 7+12\times\frac{7}{3}\right]$$

$$= \frac{13}{2}\times 42 = 273.$$

Hence, $d = \mathbf{\frac{7}{3}}$ and $S_{13} = \mathbf{273}$.

(iii) Given: $a_{12} = 37$ and $d = 3$

$\therefore \quad a_{12} = a + (12-1)d$

$\Rightarrow \quad 37 = a + 11 \times 3$

$\Rightarrow \quad a = 37 - 33 = 4$

$\therefore \quad S_{12} = \frac{12}{2}[2a + (12-1)d]$

$= 6[2 \times 4 + 11 \times 3]$

$= 6[8 + 33] = 6 \times 41 = 246.$

Hence, $a = \mathbf{4}$ and $S_{12} = \mathbf{246}$.

(iv) Given: $a_3 = 15$ and $S_{10} = 125$

$\therefore \quad a_3 = a + (3-1)d$

$\Rightarrow \quad 15 = a + 2d \quad \ldots(i)$

Now $S_{10} = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow \quad 125 = 5[2a + 9d]$

$\Rightarrow \quad 25 = 2a + 9d \quad \ldots(ii)$

Multiplying equation (i) by 2 and then subtracting equation (ii) from resulted equation, we get: $d = -1$

Putting $d = -1$ in equation (i), we get:

$a + 2d = 15$

$\Rightarrow \quad a - 2 = 15 \quad \Rightarrow \quad a = 17$

$\therefore a_{10} = a + (n-1)d$

$= 17 + 9 \times (-1) = 17 - 9 = 8.$

Hence, $d = \mathbf{-1}$ and $a_{10} = \mathbf{8}$.

(v) Given: $d = 5$ and $S_9 = 75$

$$\therefore \quad S_9 = \frac{n}{2}[2a + (9-1)d]$$

$$\Rightarrow \quad 75 = \frac{9}{2}[2a + 40]$$

$$\Rightarrow \quad -210 = 18a$$

$$\Rightarrow \quad a = \frac{-210}{18} = \frac{-35}{3}$$

Now $a_9 = a + (9-1)d$

$$= \frac{-35}{3} + 40 = \frac{85}{3}.$$

Hence, $a = \mathbf{\frac{-35}{3}}$ and $a_9 = \mathbf{\frac{85}{3}}$.

(vi) Given: $a = 2$, $d = 8$ and $S_n = 90$.

$$\therefore S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow \quad \frac{n}{2}[4 + (n-1) \times 8] = 90$$

$$\Rightarrow \quad n[2 + (n-1)4] = 90$$

$$\Rightarrow \quad 4n^2 - 2n - 90 = 0$$

$$\Rightarrow \quad 2n^2 - n - 45 = 0$$

$$\Rightarrow \quad 2n(n-5) + 9(n-5) = 0$$

$$\Rightarrow \quad (n-5)(2n+9) = 0$$

$$\Rightarrow \quad n = 5 \text{ or } n = \frac{-9}{2}$$

Since the number of terms cannot be negative, so $n = -\frac{9}{2}$ is rejected.

$$\therefore \quad n = 5$$

Now $a_5 = 2 + 4 \times 8 = 34$

Hence, $n = \mathbf{5}$ and $a_5 = \mathbf{34}$.

(vii) Given: $a = 8$, $a_n = 62 = l$ and $S_n = 210$

$$\therefore \quad \frac{n}{2}(a + l) = 210 \qquad [\text{Given } S_n = 210]$$

$$\Rightarrow \frac{n}{2}(8 + 62) = 210 \qquad \Rightarrow \quad n = 6$$

Now, $a_6 = 62$ [Given]

$\Rightarrow \quad 8 + 5d = 62 \quad \Rightarrow \quad d = \frac{54}{5}$

Hence, $n = \mathbf{6}$ and $d = \mathbf{\frac{54}{5}}$.

(viii) Given: $a_n = 4$, $d = 2$ and $S_n = -14$

$\Rightarrow \; a + (n-1)2 = 4 \quad \Rightarrow \; a = 6 - 2n \quad ...(i)$

Now $\; S_n = -14$

$\Rightarrow \quad \frac{n}{2}[2a + (n-1)d] = -14$

$\Rightarrow \frac{n}{2}[2(6-2n) + (n-1)2] = -14 \quad$ [From (i)]

$\Rightarrow \quad \frac{n}{2}[10 - 2n] = -14$

$\Rightarrow \quad n(n-5) = 14$

$\Rightarrow \quad n^2 - 5n - 14 = 0$

$\Rightarrow \quad (n-7)(n+2) = 0$

$\Rightarrow \quad n = 7, n = -2$

Since the number of terms cannot be negative, so $n = -2$ is rejected.

$\therefore$ Putting $n = 7$ in equation (i), we get:

$a = 6 - 2 \times 7 = -8$

Hence, $n = \mathbf{7}$ and $a = \mathbf{-8}$.

(ix) Given: $a = 3$, $n = 8$ and $S = 192$

$\because \quad S_8 = 192 \quad$ [Given]

$\therefore \quad \frac{8}{2}(2 \times 3 + 7d) = 192$

$\Rightarrow \quad 4 \times (6 + 7d) = 192$

$\Rightarrow \quad 6 + 7d = 48$

$\Rightarrow \quad d = \mathbf{6}$

(x) Given: $l = 28 = t_n$ and $S = 144$

Since $n = 9$, so $t_9 = 28$ and $S_9 = 144$.

$\therefore \quad \frac{9}{2}(a + t_9) = 144$

$\Rightarrow \quad \frac{9}{2}(a + 28) = 144$

$\Rightarrow \quad a + 28 = \frac{288}{9}$

$\Rightarrow \quad a = \mathbf{4}.$

Question 4:

How many terms of AP: 9, 17, 25, … must be taken to give a sum of 636?

Solution:

Given: $a = 9, d = 17 - 9 = 8, \quad S_n = 636$

$$S_n = \frac{n}{2}[2a + (n-1)d] \Rightarrow 636 = \frac{n}{2}[2 \times 9 + (n-1)8]$$

$$\Rightarrow 636 \times 2 = n[18 + 8n - 8] \Rightarrow 636 \times 2 = n(10 + 8n)$$

$$\Rightarrow 636 \times 2 = 2n(5 + 4n) \Rightarrow \frac{636 \times 2}{2} = 5n + 4n^2$$

$$\Rightarrow 4n^2 + 5n - 636 = 0 \Rightarrow 4n^2 + 53n - 48n - 636 = 0$$

$$\Rightarrow n(4n + 53) - 12(4n + 53) = 0 \Rightarrow (4n + 53)(n - 12) = 0$$

$$\Rightarrow 4n + 53 = 0 \quad \text{or} \quad n - 12 = 0$$

$$\Rightarrow n = \frac{-53}{4} \text{ (rejected)} \quad \text{or} \quad n = 12$$

Hence, $n = 12$

Question 5:

The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.

Solution:

Given: $a = 5$, $l = t_n = 45$ (last term) and $S_n = 400$

$$\therefore \quad \frac{n}{2}[a + l] = 400$$

$$\Rightarrow \frac{n}{2}[5 + 45] = 400$$

$$\Rightarrow n = \frac{400}{25} = 16$$

Now, $t_{16} = 45$

$$\Rightarrow 5 + 15d = 45$$

$$\Rightarrow d = \frac{40}{15} = \frac{8}{3}$$

Thus, $n = \mathbf{16}$ and $d = \mathbf{\frac{8}{3}}$.

Question 6:

The first and the last terms of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?

Solution:

Here, $a = 17, a_n = 350, d = 9$

$a_n = a + (n-1)d \Rightarrow 350 = 17 + (n-1)9$

$\Rightarrow \quad 350 - 17 = (n-1)9 \Rightarrow \frac{333}{9} = n - 1 \Rightarrow 37 + 1 = n \Rightarrow n = 38$

$S_{38} = \frac{38}{2}[17 + 350] = 19 \times 367 = 6973$

Question 7:

Find the sum of first 22 terms of an AP in which d = 7 and 22nd term is 149.

Solution:

Given: $d = 7$, $t_{22} = 149$, and $n = 22$

$\therefore \quad t_{22} = a + (22-1)d$

$\Rightarrow \quad 149 = a + 21 \times 7$

$\Rightarrow \quad 149 = a + 147 \quad \Rightarrow \quad a = 2$

$\therefore$ The required sum,

$S_{22} = \frac{22}{2}[2 \times 2 + (22-1)7]$

$= 11[4 + 147] = 11 \times 151 = \mathbf{1661}.$

Question 8:

Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.

Solution:

Given: $a_2 = 14$ and $a_3 = 18$

$\Rightarrow \quad a + d = 14 \quad ...(i) \quad$ and $a + 2d = 18 \quad ...(ii)$

Substracting (*i*) and (*ii*), we get

$a + 2d - a - d = 18 - 14 \Rightarrow d = 4$

Since, $a + d = 14 \Rightarrow a + 4 = 14 \Rightarrow a = 14 - 4 \Rightarrow a = 10$

So, $S_{51} = \frac{51}{2}[2 \times 10 + (51-1)4]$

$S_{51} = \frac{51}{2}[20 + 200]$

$= \frac{51}{2} \times 220 = 51 \times 110 = 5610$

Question 9:

If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.

Solution:

Given: $S_7 = 49$

$$\Rightarrow \quad \frac{7}{2}(2a + 6d) = 49$$

$$\Rightarrow \quad a + 3d = 7 \quad \text{...(i)}$$

and $S_{17} = 289$

$$\Rightarrow \quad \frac{17}{2}(2a + 16d) = 289$$

$$\Rightarrow \quad a + 8d = 17 \quad \text{... (ii)}$$

Subtracting equation (i) from equation (ii), we get: $d = 2$

Substituting the value of $d = 2$ in equation (i), we get: $a = 7 - 6 = 1$

$\therefore$ Sum of n terms,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{n}{2}[2 \times 1 + (n-1) \times 2] = n^2$$

Hence, the required sum is $\boldsymbol{n^2}$.

Question 10:

Show that $a_1, a_2, \ldots\ldots a_n, \ldots\ldots$ form an AP where an is defined as below:
(i) $a_n = 3 + 4n$
(ii) $a_n = 9 - 5n$
Also find the sum of the first 15 terms in each case.

Solution:

(i)

$$a_n = 3 + 4n$$

Putting

$$n = 1, 2, 3, \ldots\ldots$$

$$a_1 = 3 + 4 \times 1 = 3 + 4 = 7$$

$$a_2 = 3 + 4 \times 2 = 3 + 8 = 11$$

$$d = a_2 - a_1 = 11 - 7 = 4$$

$$a_3 = 3 + 4 \times 3 = 3 + 12 = 15$$

$$d = a_3 - a_2 = 15 - 11 = 4$$

Thus, the sequence 7, 11, 15, ... is an AP.

$$S_{15} = \frac{15}{2}[2 \times 7 + (15 - 1) \times 4] = \frac{15}{2}[14 + 56] = \frac{15}{2} \times 70 = 525$$

(ii)

$$a_n = 9 - 5n$$

Putting $n = 1, 2, 3, \ldots$

$$a_1 = 9 - 5 \times 1 = 9 - 5 = 4$$

$$a_2 = 9 - 5 \times 2 = 9 - 10 = -1$$

$$d = a_2 - a_1 = -1 - 4 = -5$$

$$a_3 = 9 - 5 \times 3 = 9 - 15 = -6$$

$$d = a_3 - a_2 = -6 - (-1) = -6 + 1 = -5$$

Thus, the sequence 4, –1, –6, ..., is an A.P.

$$a = 4, d = -1 - 4 = -5$$

$$S_{15} = \frac{15}{2}[2 \times 4 + (15 - 1)(-5)] = \frac{15}{2}[8 - 70]$$

$$= \frac{15}{2} \times (-62) = -465$$

Question 11:

If the sum of the first n terms of an AP is $4n - n^2$, what is the first term (that is S_1)? What is the sum of first two terms? What is the second term? Similarly, find the 3rd, the 10th and the nth terms.

Solution:

We have $S_n = 4n - n^2$ ...(i)

Putting $n = 1$ in equation (i), we get:

$S_1 = 4 - 1 = \mathbf{3}$.

In an AP, $S_1 = t_1 = a$. So $a = 3$.

Putting $n = 2$ in equation (i), we get:

$S_2 = 4 \times 2 - (2)^2 = \mathbf{4}$.

$t_2 = S_2 - S_1 = 4 - 3 = \mathbf{1}$

Putting $n = 3$ in equation (i), we get:

$S_3 = 4 \times 3 - 3^2 = 12 - 9$

$t_3 = S_3 - S_2 = 3 - 4 = -1$

$\therefore$ Third term is $\mathbf{-1}$.

Putting $n = 9$ in equation (i), we get:

$S_9 = 4 \times 9 - 9^2 = 36 - 81 = -45$

Similarly, $S_{10} = 4 \times 10 - 10^2 = 40 - 100 = -60$

$t_{10} = S_{10} - S_9 = -60 - (-45) = -60 + 45 = \mathbf{-15}$

$\therefore$ Tenth term is -15.

Here, common difference, $d = t_2 - t_1$

$$= 1 - 3 = -2$$

$\therefore$ nth term, $t_n = a + (n - 1)d$

$$= 3 + (n - 1)(-2)$$

$$= 3 - 2n + 2 = \mathbf{5 - 2n}.$$

Question 12:

Find the sum of the first 40 positive integers divisible by 6.

Solution:

Let 6, 12, 18, ..., 240 be divisible by 6

$$a = 6, d = 12 - 6 = 6$$

$$a_{40} = 6 + (40 - 1) \times 6 = 240$$

$$S_{40} = \frac{40}{2}[a + a_{40}] = 20[6 + 240] = 20 \times 246 = 4920$$

Question 13:

Find the sum of the first 15 multiples of 8.

Solution:

The list of multiples of 8 is

8, 16, 24, 32, 40, ...

It is an AP in which $a = 8$ and $d = 8$.

Applying the formula $S_n = \frac{n}{2}[2a + (n - 1)d]$,

we get:

$$S_{15} = \frac{15}{2}[2 \times 8 + (15 - 1)8]$$

$$= \frac{15}{2}[16 + 112] = \frac{15}{2} \times 128 = 960.$$

Hence, the sum of the first 15 multiples of 8 is **960**.

Question 14:

Find the sum of the odd numbers between 0 and 50.

Solution:

Let odd numbers between 0 and 50 be 1, 3, 5, 7,........, 49.

Here, $a = 1, d = 3 - 1 = 2,$

$a_n = 49$

We have, $a_n = a + (n - 1)d \Rightarrow a + (n - 1)d = 49$

$\Rightarrow 1 + (n - 1)\,2 = 49 \Rightarrow (n - 1)\,2 = 49 - 1 \Rightarrow (n - 1) = \frac{48}{2}$

$\Rightarrow n - 1 = 24 \Rightarrow n = 24 + 1 = 25$

$\therefore S_{25} = \frac{25}{2}[a + a_{25}] = \frac{25}{2}[1 + 49]$

$= \frac{25}{2} \times 50 = 25 \times 25 = 625$

Question 15:

A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows:
₹ 200 for the first day, ₹ 250 for the second day, ₹ 300 for the third day, etc. the penalty for each succeeding day being ₹ 50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days?

Solution:

Penalty for the first day = ₹ 200

Penalty for the second day = ₹ 250

Penalty for the third day = ₹ 300

The list is ₹ 200, ₹ 250, ₹ 300, ...

It is an AP in which $a = 200$, $d = 50$ and $n = 30$.

$$\therefore \text{Penalty for 30 days} = \frac{30}{2}(2a + 29d)$$

$$= 15(2 \times 200 + 29 \times 50)$$

$$= 15(400 + 1450) = ₹\ 27{,}750.$$

Hence, the penalty for 30 days is ₹ **27,750**.

Question 16:

A sum of ₹ 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is ₹ 20 less than its preceding prize, find the value of each of the prizes.

Solution:

Let 1st prize be of ₹ a

2nd prize be ₹ (a – 20) and

3rd prize be ₹ (a – 20 – 20) = ₹ (a – 40)

Then seven prizes are

₹ a, ₹ $(a-20)$, ₹ $(a-40)$, ..., ₹ $(a-120)$ and $S_7 = 700$

$$a_1 = a, d = ₹\ (a - 20 - a) = -₹\ 20$$

We have, $$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_7 = \frac{7}{2}[2a + (7-1)d] \Rightarrow 700 = \frac{7}{2}[2a + 6d]$$

$$\Rightarrow \quad 700 = \frac{7}{2}[2a + 6(-20)] \Rightarrow 700 \times \frac{2}{7} = 2a - 120$$

$$\Rightarrow \quad 200 + 120 = 2a \Rightarrow 320 = 2a \Rightarrow a = \frac{320}{2} = ₹\ 160$$

So, the seven prizes are ₹ 160, ₹ 140, ₹ 120, ₹ 100, ₹ 80, ₹ 60 and ₹ 40.

Question 17:

In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, eg. a section of Class I will plant 1 tree, a section of Class II will plant 2 trees and so on till Class XII. There are three sections of each class. How many trees will be planted by the students?

Solution:

Trees planted by 3 sections of class I = $3 \times 1 = 3$

Trees planted by 3 sections of class II = $3 \times 2 = 6$

Trees planted by 3 sections of class III = $3 \times 3 = 9$

$\vdots \quad \vdots \quad \vdots$

Trees planted by 3 sections of class XII

$= 3 \times 12 = 36$

Thus, we have the list of trees planted 3, 6, 9, ... 36.

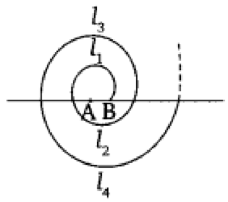
It is an AP in which $a = 3$, $d = 3$, $n = 12$ and $l = 36$.

$\therefore$ Total number of trees planted

$$= \frac{12}{2}(3 + 36) = 6 \times 39 = \mathbf{234}.$$

Question 18:

A spiral is made up of successive semicircles, with centres alternately at A and B, starting with centre at A, of radii 0.5 cm, 1.0 cm, 1.5 cm, 2.0 cm,... as shown in figure. What is the total length of such a spiral made up of thirteen consecutive semicircles?
(Take π = 22/7)



[Hint: Length of successive semicircles is l_1, l_2, l_3, l_4, ... with centres at A, B, respectively.]

Solution:

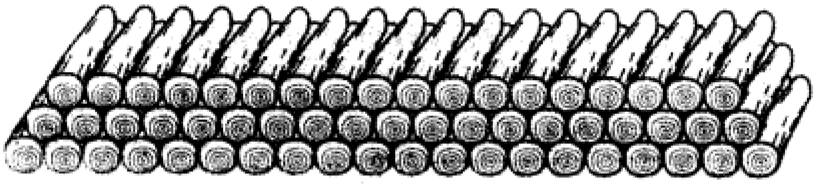
$R_1 = 0.5$ cm, $R_2 = 1.0$ cm, $R_3 = 1.5$ cm

$a = 0.5$ cm, $d = 1.0$ cm $- 0.5$ cm $= 0.5$ cm

$$\begin{aligned}
\text{Length of spiral} &= 13 \text{ consecutive semicircles} = \pi R_1 + \pi R_2 + \pi R_3 + \ldots\ldots + \pi R_{13} \\
&= \pi\,[R_1 + R_2 + R_3 + \ldots\ldots R_{13}] = \pi\,[0.5 + 1.0 + 1.5 + \ldots. + 6.5] \\
&= \pi\left\{\frac{13}{2}[2 \times 0.5 + (13 - 1)(0.5)]\right\} = \pi\left[\frac{13}{2}(1 + 12 \times 0.5)\right] \\
&= \pi \times \left(\frac{13}{2} \times 7\right) = \frac{22}{7} \times \frac{13}{2} \times 7 = 143 \text{ cm}
\end{aligned}$$

Question 19:

200 logs are stacked in the following manner 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on (see Figure). In how many rows are the 200 logs placed and how many logs are in the top row?



Solution:

Number of logs row-wise forms an AP as:

20, 19, 18, 17, ...

Here, $a = 20$ and $d = -1$

Let the number of rows be n.

Then, $S_n = 200$

$$\Rightarrow \frac{n}{2}[2 \times 20 + (n-1) \times (-1)] = 200$$

$$\Rightarrow \frac{n}{2}[41 - n] = 200$$

$$\Rightarrow 41n - n^2 = 400$$

$$\Rightarrow n^2 - 41n + 400 = 0$$

$$\therefore \quad n = \frac{41 \pm \sqrt{(-41)^2 - 4 \times 1 \times 400}}{2}$$

$$= \frac{41 \pm \sqrt{1681 - 1600}}{2} = \frac{41 \pm 9}{2}$$

$$\Rightarrow \quad n = 25 \quad \text{or} \quad n = 16$$

Rejecting the value $n = 25$, we get:

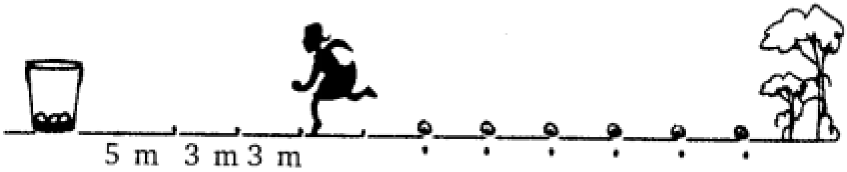
Number of logs in the 16th row

$$= 20 + 15(-1) = 5.$$

Thus, the number of rows is **16** and the number of logs in the top row is **5**.

Question 20:

In a potato race, a bucket is placed at the starting point, which is 5 m from the first potato, and the other potatoes are placed 3 m apart in a straight line. There are ten potatoes in the line



A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?

Solution:

Distance between the first potato and the bucket = 5 m

Distance between next 2 potatoes = 3 m each

So, series is 5 m, 8 m, 11 m,

Here, a = 5 m, d = (8 – 5) m = 3 m

Total distance travelled for 10 potatoes = 2 [5 + 8 + 11 + ... + 10 terms]

$= 2[\frac{10}{2}\{2 \times 5 + (10 - 1)\,3\}]$

$= 2[5\{10 + 27\}] = 2(37 \times 5) = 37 \times 10 = 370$ m.