# EXERCISE 5.1

**Question 1:**

**In which of the following situations, does the list of numbers involved make an arithmetic progression and why?**

**(i)** The taxi fare after each km when the fare is ₹ 15 for the first km and ₹ 8 for each additional km.

(i) Let $t_n$ be the taxi fare for first $n$ km.
Then $t_1 = a = 15$, $t_2 = 15 + 8 = 23$,
$t_3 = 23 + 8 = 31$
So, the list will be as follows: 15, 23, 31, ...
Here $t_2 - t_1 = t_3 - t_2 = ... = 8$
Thus, this situation forms **an AP**.

**(ii)** The amount of air present in a cylinder when a vacuum pump removes 14 of the air remaining in the cylinder at a time.

(ii) Let the first term be $x$ units.
Then, $t_1 = a = x$

$$t_2 = x - \frac{1}{4}x = \frac{3}{4}x \text{ units}$$

$$t_3 = \frac{3}{4}x - \frac{1}{4}\left(\frac{3}{4}x\right) = \frac{9}{16}x \text{ units}$$

$$t_4 = \frac{9}{16}x - \frac{1}{4}\left(\frac{9}{16}x\right) = \frac{27}{64}x \text{ units .....}$$

The list of numbers is $x, \frac{3}{4}x, \frac{9}{16}x, \frac{27}{64}x, ...$
Since $t_2 - t_1 \neq t_3 - t_2$, therefore, it is **not an AP**.

**(iii)** The cost of digging a well after every meter of digging, when it costs ₹ 150 for the first meter and rises by ₹ 50 for each subsequent meter.

(iii) First term $a$ = ₹ 150.
Common difference for every subsequent metre is ₹ 50.

$t_1 = a = 150$
$t_2 = a + d = 150 + 50 = 200$
$t_3 = a + 2d = 150 + 2 \times 50 = 250$
$t_4 = a + 3d = 150 + 150 = 300$
Since $t_2 - t_1 = t_3 - t_2 = 50$, therefore, it is **an AP**.

**(iv)** The amount of money in the account every year, when ₹ 10000 is deposited at compound interest at 8% per annum.

(iv) Let $t_n$ be the amount of money in the $n$th year.

Then, $t_1 = a = 10{,}000$.

$$t_2 = 10{,}000 + 10{,}000 \times \frac{8}{100}$$

$$= 10{,}000 + 800 = 10{,}800$$

$$t_3 = 10{,}800 + 10{,}800 \times \frac{8}{100}$$

$$= 10{,}800 + 864 = 11{,}664$$

$$t_4 = 11{,}664 + 11{,}664 \times \frac{8}{100}$$

$$= 11{,}664 + 933.12 = 12597.12$$

The list is 10000, 10800, 11664, 12597.12, ...

Here, $t_2 - t_1 \neq t_3 - t_2$, therefore, it is **not an AP**.

## Question 2:

**Write first four terms of the AP, when the first term a and the common difference d are given as follows:**

**(i) a = 10, d = 10**

**Solution**:

(i) Given: a = 10, d = 10

$a_1 = 10$,

$a_2 = 10 + 10 = 20$

$a_3 = 20 + 10 = 30$

$a_4 = 30 + 10 = 40$

Thus, the first four terms of the AP are 10, 20, 30, 40.

**(ii) a = -2, d = 0**

**Solution**:

(ii) Given: a = − 2, d = 0

The first four terms of the AP are -2, -2, -2, -2.

**(iii) a = 4, d = -3**

**Solution**:

(iii) $a_1 = 4, d = -3$

$a_2 = a_1 + d = 4 - 3 = 1$

$a_3 = a_2 + d = 1 - 3 = -2$

$a_4 = a_3 + d = -2 - 3 = -5$

Thus, the first four terms of the AP are 4, 1, -2, -5.

**(iv) a = -1, d = ½**

**Solution**:

(iv)

$$a_1 = -1, d = \frac{1}{2}$$

$$a_2 = a_1 + d = \frac{-1}{1} + \frac{1}{2} = \frac{-1}{2}$$

$$a_3 = a_2 + d = \frac{-1}{2} + \frac{1}{2} = 0$$

$$a_4 = a_3 + d = 0 + \frac{1}{2} = \frac{1}{2}$$

Thus, the first four terms of the AP are $-1, -\frac{1}{2}, 0, \frac{1}{2}$.

**(v) a = -1.25, d = -0.25**

**Solution**:

(v) $a_1 = -1.25, d = -0.25$

$a_2 = a_1 + d = -1.25 - 0.25 = -1.50$

$a_3 = a_2 + d = -1.50 - 0.25 = -1.75$

$a_4 = a_3 + d = -1.75 - 0.25 = -2$

Thus, the first four terms of the AP are -1.25, -1.50, -1.75, -2.

## Question 3:

**For the following APs, write the first term and the common difference:**
**(i) 3, 1, -1, -3, .....**
**(ii) -5, -1, 3, 7, .....**
**(iii) $1/3$ , $5/3$ , $9/3$, $13/3$ , .....**
**(iv) 0.6, 1.7, 2.8, 3.9, .....**

## Solution:

let a be the first item and d the common difference, then:

(i) $a = \mathbf{3}$ and $d = t_2 - t_1 = 1 - 3 = \mathbf{-2}$.

(ii) $a = \mathbf{-5}$ and $d = t_2 - t_1 = -1 - (-5) = \mathbf{4}$

(iii) $a = \mathbf{\frac{1}{3}}$ and $d = t_2 - t_1 = \frac{5}{3} - \frac{1}{3} = \mathbf{\frac{4}{3}}$

(iv) $a = \mathbf{0.6}$ and $d = t_2 - t_1 = 1.7 - 0.6 = \mathbf{1.1}$.

## Question 4:

**Which of the following are APs ? If they form an AP, find the common difference d and write three more terms.**

(i) 2, 4, 8, 16, .....
(ii) 2, $\frac{5}{2}$ , 3, $\frac{7}{2}$ , .....
(iii) -1.2, -3.2, -5.2, -7.2, .....
(iv) -10, -6, -2,2, .....
(v) 3, $3 + \sqrt{2}$, $3 + 2\sqrt{2}$, $3 + 3\sqrt{2}$, .....
(vi) 0.2, 0.22, 0.222, 0.2222, .....
(vii) 0, -4, -8, -12, .....
(viii) $\frac{-1}{2}$ , $\frac{-1}{2}$ , $\frac{-1}{2}$ , $\frac{-1}{2}$ , .....
(ix) 1, 3, 9, 27, .....
(x) a, 2a, 3a, 4a, .....
(xi) a, a2, a3, a4, .....
(xii) $\sqrt{2}$, $\sqrt{8}$, $\sqrt{18}$, $\sqrt{32}$, .....
(xiii) $\sqrt{3}$, $\sqrt{6}$, $\sqrt{9}$, $\sqrt{12}$, .....
(xiv) 12, 32, 52, 72, .....
(xv) 12, 52, 72, 73, .....

## Solution:

(i) 2, 4, 8, 16, .....
$a_2 - a_1 = 4 - 2 = 2$
$a_3 - a_2 = 8 - 4 = 4$
$a_2 - a_1 \neq a_3 - a_2$
Thus, the given sequence is not an AP.

(*ii*) $2, \frac{5}{2}, 3, \frac{7}{2}, \ldots$

$$a_2 - a_1 = \frac{5}{2} - \frac{2}{1} = \frac{1}{2}$$
$$a_3 - a_2 = \frac{3}{1} - \frac{5}{2} = \frac{1}{2}$$
$$a_2 - a_1 = a_3 - a_2$$

Thus, the given sequence is an AP.

$$a_1 = 2, d = \frac{1}{2}$$

Next three terms are $a_5 = a_4 + d = \frac{7}{2} + \frac{1}{2} = 4,$

$$a_6 = a_5 + d = 4 + \frac{1}{2} = \frac{9}{2}, a_7 = a_6 + d = \frac{9}{2} + \frac{1}{2} = 5$$

(*iii*) $-1.2, -3.2, -5.2, -7.2, \ldots$

$$a_2 - a_1 = -3.2 - (-1.2) = -3.2 + 1.2 = -2$$
$$a_3 - a_2 = -5.2 - (-3.2) = -5.2 + 3.2 = -2$$
$$a_3 - a_2 = a_2 - a_1$$

Thus, the given sequence is an AP.

$$a_1 = -1.2, d = -2$$

Next three terms are $a_5 = a_4 + d = -7.2 + (-2) = -9.2,$

$$a_6 = a_5 + d = (-9.2) + (-2) = -11.2$$
$$a_7 = a_6 + d = (-11.2) + (-2) = -13.2$$

(*iv*) $-10, -6, -2, 2, \ldots$

$$a_2 - a_1 = -6 - (-10) = 4$$
$$a_3 - a_2 = -2 - (-6) = 4$$

Thus, the given sequence is an AP.

$$a_1 = -10, d = 4$$

Next three terms are $a_5 = a_4 + d = 2 + 4 = 6, a_6 = a_5 + d = 6 + 4 = 10$

$$a_7 = a_6 + d = 10 + 4 = 14$$

(*v*) $3, 3 + \sqrt{2}, 3 + 2\sqrt{2}, 3 + 3\sqrt{2}, \ldots$

$$a_2 - a_1 = 3 + \sqrt{2} - 3 = \sqrt{2}$$
$$a_3 - a_2 = 3 + 2\sqrt{2} - 3 - \sqrt{2} = \sqrt{2}$$

Thus, the given sequence is an AP.

$$a_1 = 3, d = \sqrt{2}$$

Next three terms are $a_5 = a_4 + d = 3 + 3\sqrt{2} + \sqrt{2} = 3 + 4\sqrt{2}$

$$a_6 = a_5 + d = 3 + 4\sqrt{2} + \sqrt{2} = 3 + 5\sqrt{2}$$
$$a_7 = a_6 + d = 3 + 5\sqrt{2} + \sqrt{2} = 3 + 6\sqrt{2}$$

(*vi*) 0.2, 0.22, 0.222, 0.2222, ...

$$a_2 - a_1 = 0.22 - 0.2 = 0.02$$
$$a_3 - a_2 = 0.222 - 0.22 = 0.002$$
$$a_3 - a_2 \neq a_2 - a_1$$

Thus, the given sequence is not an AP.

(*vii*) $0, -4, -8, -12, \ldots\ldots$

$$a_2 - a_1 = -4 - 0 = -4$$
$$a_3 - a_2 = -8 - (-4) = -4$$
$$a_3 - a_2 = a_2 - a_1$$

Thus, the given sequence is an AP.

$$a_1 = 0, d = -4$$

Next three terms are $a_5 = a_4 + d = -12 + (-4) = -16$

$$a_6 = a_5 + d = -16 - 4 = -20$$
$$a_7 = a_6 + d = -20 - 4 = -24$$

(*viii*) $\frac{-1}{2}, \frac{-1}{2}, \frac{-1}{2}, \frac{-1}{2}, \ldots$

$$a_2 - a_1 = \frac{-1}{2} - \left(\frac{-1}{2}\right) = \frac{-1}{2} + \frac{1}{2} = 0$$
$$a_3 - a_2 = \frac{-1}{2} - \left(\frac{-1}{2}\right) = 0$$
$$a_3 - a_2 = a_2 - a_1$$

Thus, the given sequence is an AP.

$$a_1 = \frac{-1}{2}, d = 0$$

Next three terms are $a_5 = a_4 + d = \frac{-1}{2}, a_6 = a_5 + d = \frac{-1}{2}, a_7 = a_6 + d = \frac{-1}{2}.$

(*ix*) $1, 3, 9, 27, \ldots.$

$$a_2 - a_1 = 3 - 1 = 2$$
$$a_3 - a_2 = 9 - 3 = 6$$
$$a_3 - a_2 \neq a_2 - a_1$$

Thus, the given sequence is not an AP.