EXERCISE 4.3

Question 1:

Find the nature of the roots of the following quadratic equations. If the real roots exist, find them:
(i) $2x^2 - 3x + 5 = 0$
(ii) $3x^2 - 4\sqrt{3}x + 4 = 0$
(iii) $2x^2 - 6x + 3 = 0$

Solution:

(i) Given: $2x^2 - 3x + 5 = 0$

Here, $a = 2$, $b = -3$ and $c = 5$.

$\therefore$ Discriminant, $D = b^2 - 4ac$

$$= (-3)^2 - 4 \times 2 \times 5$$

$$= 9 - 40 = -31 < 0$$

Hence, the roots are **imaginary**.

(ii) Given: $3x^2 - 4\sqrt{3}x + 4 = 0$

Here, $a = 3$, $b = -4\sqrt{3}$ and $c = 4$.

$\therefore$ Discriminant, $D = b^2 - 4ac$

$$= \left(-4\sqrt{3}\right)^2 - 4 \times 3 \times 4$$

$$= 48 - 48 = 0$$

Hence, the roots are **real** and **equal**.

Now using the formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \text{ we get:}$$

$$x = \frac{-\left(-4\sqrt{3}\right) \pm \sqrt{\left(-4\sqrt{3}\right)^2 - 4 \times 3 \times 4}}{2 \times 3}$$

$$= \frac{4\sqrt{3} \pm \sqrt{48 - 48}}{6} = \frac{4\sqrt{3}}{6} = \frac{2}{\sqrt{3}}$$

Hence, the **equal roots** are $\mathbf{\frac{2}{\sqrt{3}}}$ and $\mathbf{\frac{2}{\sqrt{3}}}$.

(iii) Given: $2x^2-6x+3=0$

Here, $a = 2$, $b = -6$ and $c = 3$.

$\therefore$ Discriminant, $D = b^2 - 4ac$

$$= (-6)^2 - 4 \times 2 \times 3$$

$$= 36 - 24 = 12 > 0$$

Hence, the roots are **distinct** and **real**.

Now using the formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \text{ we get:}$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(2)(3)}}{2 \times 2}$$

$$= \frac{6 \pm \sqrt{36 - 24}}{4} = \frac{6 \pm \sqrt{12}}{4}$$

$$= \frac{6 \pm 2\sqrt{3}}{4} = \frac{3 \pm \sqrt{3}}{2}$$

i.e., $x = \mathbf{\frac{3 + \sqrt{3}}{2}}$ or $x = \mathbf{\frac{3 - \sqrt{3}}{2}}$.

Question 2:

Find the values of k for each of the following quadratic equations, so that they have two equal roots.

(1) $2x^2 + kx + 3 = 0$

(2) $kx(x - 2) + 6 = 0$

Solution:

(*i*) $2x^2 + kx + 3 = 0$

This is of the form $ax^2 + bx + c = 0$,

where, $a = 2, b = k$ and $c = 3$

Discriminant, $D = b^2 - 4ac$

$= k^2 - 4 \times 2 \times 3 = k^2 - 24$

For equal roots,

$D = 0$

$\Rightarrow k^2 - 24 = 0$

$\Rightarrow k^2 = 24$ or $k = \pm\sqrt{24}$

$\Rightarrow k = \pm\sqrt{4\times 6} = \pm 2\sqrt{6}$

(*ii*) $kx(x - 2) + 6 = 0$

$\Rightarrow kx^2 - 2kx + 6 = 0$

This is of the form $ax^2 + bx + c = 0$,

where $a = k, b = -2k$ and $c = 6$

Discriminant, $D = b^2 - 4ac$

$= (-2k)^2 - 4 \times k \times 6 = 4k^2 - 24k$

For equal roots, $D = 0$

$\Rightarrow 4k^2 - 24k = 0 \Rightarrow k(4k - 24) = 0$

$\Rightarrow k = 0$ (not possible) or $4k - 24 = 0$

$\Rightarrow 4k = 24$

$\Rightarrow k = \frac{24}{4} = 6$

Question 3:

Is it possible to design a rectangular mango grove whose length is twice its breadth, and the area is 800 m^2? If so, find its length and breadth.

Solution:

Let breadth of the rectangular be x m

Then, the length of rectangular will be 2x m.

According to question, we have

$$\begin{aligned}
\text{Length} \times \text{Breadth} &= \text{Area} \\
\Rightarrow \quad x \times 2x &= 800 \\
\Rightarrow \quad 2x^2 &= 800 \\
\Rightarrow \quad x^2 &= 400 = (20)^2 \\
\Rightarrow \quad x &= 20
\end{aligned}$$

Hence, the rectangular mango grove is **possible** to design whose breadth is **20 m** and length is **40 m**.

Question 4:

Is the following situation possible? If so, determine their present ages.
The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48.

Solution:

Let the present age of one friend be x years

Then, the present age of other friend be (20 -x) years

4 years ago, one friend's age was (x – 4) years

4 years ago, other friend's age was (20 -x – 4) = (16 -x) years

According to question,

$(x-4)(16-x) = 48$

$\Rightarrow 16x - x^2 - 64 + 4x = 48$

$\Rightarrow x^2 - 20x + 112 = 0$

This is of the form $ax^2 + bx + c = 0$, where, a = 1, b = -20 and c = 112

Discriminant, $D = b^2 - 4ac$

$= (-20)^2 - 4 \times 1 \times 112 = 400 - 448 = -48 < 0$

Since, no real roots exist.

So, the given situation is not possible.

Question 5:

Is it possible to design a rectangular park of perimeter 80 m and area 400 m^2? If so, find its length and breadth.

Solution:

Let the length of rectangular park be x.

Then, the perimeter of rectangular park

$$= 2(\text{Length} + \text{Breadth})$$

$$\Rightarrow \quad 2(x + \text{Breadth}) = 80$$

$$\Rightarrow \quad \text{Breadth} = 40 - x$$

$\therefore$ Area of rectangular park = Length × Breadth

$$\Rightarrow \quad x(40 - x) = 400$$

$$\Rightarrow \quad 40x - x^2 = 400$$

$$\Rightarrow \quad x^2 - 40x + 400 = 0$$

$$\Rightarrow \quad x^2 - 20x - 20x + 400 = 0$$

$$\Rightarrow \quad (x - 20)(x - 20) = 0$$

$$\Rightarrow \quad x = 20$$

Thus, the rectangular park is **possible** to design. So, length of park = **20 m** and its breadth = 40 – 20 = **20 m**.