EXERCISE 5.2

Question 1:

Fill in the blanks in the following table, given that a is the first term, d the common difference and the nth term of the AP:

	a	d	n	a_n
(i)	7	3	8	...
(ii)	–18	...	10	0
(iii)	...	–3	18	–5
(iv)	–18.9	2.5	...	3.6
(v)	3.5	0	105	...

Solution:

(i) Here, $a = 7$, $d = 3$ and $n = 8$

$$\therefore \quad a_n = a + (n - 1)d$$

$$a_8 = 7 + (8 - 1)3 = 7 + 21 = \mathbf{28}.$$

(ii) Here, $a = -18$, $n = 10$ and $a_n = 0$

$$\therefore \quad a_n = a + (n - 1)d$$

$$\Rightarrow \quad 0 = -18 + (10 - 1)d$$

$$\Rightarrow \quad 0 = -18 + 9d$$

$$\Rightarrow \quad d = \mathbf{2}.$$

(iii) Here, $d = -3$, $n = 18$ and $a_n = -5$

$$\therefore \quad a_n = a + (n - 1)d$$

$$\Rightarrow \quad -5 = a + (18 - 1) - 3$$

$$\Rightarrow \quad a = \mathbf{46}.$$

(iv) Here, $a = -18.9$, $d = 2.5$ and $a_n = 3.6$

$$\therefore \quad a_n = a + (n - 1)d$$

$$3.6 = -18.9 + (n - 1)2.5$$

$$\Rightarrow \quad 25 = 2.5n$$

$$\Rightarrow \quad n = \frac{25}{2.5} = \mathbf{10}.$$

(v) Here, $a = 3.5$, $d = 0$ and $n = 105$

$$\therefore \quad a_n = a + (n - 1)d$$

$$= 3.5 + (105 - 1)0 = \mathbf{3.5}.$$

Question 2:

Choose the correct choice in the following and justify:

i) 30th term of the AP: 10, 7, 4, …, is
(a) 97
(b) 77
(c) -77
(d) -87

ii) 11th term of the AP: -3, -12 , 2, …, is
(a) 28
(b) 22
(c) -38
(d) -48

Solution:

(i) 10, 7, 4, …,

a = 10, d = 7 − 10 = -3, n = 30

$a_n = a + (n - 1)d$

$\Rightarrow a_{30} = a + (30 - 1)\,d = a + 29\,d = 10 + 29\,(-3) = 10 - 87 = -77$

Hence, correct option is **(C).**

$(ii)\ -3, -\frac{1}{2}, 2, \ldots,$

$$a = -3,\ n = 11$$

$$d = -\frac{1}{2} - (-3) = -\frac{1}{2} + \frac{3}{1} = \frac{5}{2}$$

$$a_n = a + (n-1)d \Rightarrow a_{11} = a + (11-1)d$$

$$\Rightarrow \quad a_{11} = a + 10d = -3 + 10 \times \frac{5}{2} = -3 + 25 = 22$$

Hence, correct option is (B).

Question 3:

In the following APs, find the missing terms in the boxes:

(i) 2, $\square$, 26 (ii) $\square$, 13, $\square$,3

(iii) 5 $\square$, $\square$, $9\frac{1}{2}$ (iv) – 4, $\square$, $\square$, $\square$, $\square$, 6

(v) $\square$, 38, $\square$, $\square$, $\square$, – 22

Solution:

(i) Here, $a = 2$ and $t_3 = 26$

Then, $t_3 = a + (3-1)d$

$\Rightarrow \quad 26 = 2 + 2d \quad \Rightarrow \quad d = 12.$

$\therefore \quad t_2 = t_3 - d = 26 - 12 = 14$

Hence, the complete sequence is 2, $\boxed{14}$, 26

(ii) Here, $t_2 = 13$ and $t_4 = 3$

Then $\quad t_2 = a + (2-1)d$

$\Rightarrow \quad 13 = a + d \quad \ldots\text{(i)}$

and $\quad t_4 = a + (4-1)d$

$\Rightarrow \quad 3 = a + 3d \quad \ldots\text{(ii)}$

Subtracting equation (i) form equation (ii), we get: $d = -5$

Putting $d = -5$ in equation (i), we get:

$a = 13 + 5 = 18$

$\therefore \quad t_3 = a + (3-1)d$

$= 18 + 2 \times (-5) = 18 - 10 = 8$

Hence, the complete sequence is $\boxed{18}$, 13, $\boxed{8}$, 3

(iii) Here, $a = 5$, $t_4 = 9\frac{1}{2} = \frac{19}{2}$

Then, $\quad t_4 = a + (4-1)d$

$\Rightarrow \quad \frac{19}{2} = 5 + 3d \quad \Rightarrow \quad d = \frac{9}{6} = \frac{3}{2}$

$\therefore \quad t_3 = t_4 - d$

$= \frac{19}{2} - \frac{3}{2} = \frac{16}{2} = 8$

and $\quad t_2 = t_3 - d$

$= 8 - \frac{3}{2} = \frac{16-3}{2} = \frac{13}{2} = 6\frac{1}{2}$

Hence, the complete sequence is 5, $\boxed{6\frac{1}{2}}$, $\boxed{8}$, $9\frac{1}{2}$

(iv) Here, $a = -4$ and $t_6 = 6$

Then, $t_6 = a + (6-1)d$

$\Rightarrow \quad 6 = -4 + 5d \quad \Rightarrow \quad d = 2$

$\therefore \quad t_2 = a + d = -4 + 2 = -2$

$t_3 = a + 2d = -4 + 4 = 0$

$t_4 = a + 3d = -4 + 6 = 2$

and $\quad t_5 = a + 4d = -4 + 8 = 4$

Hence, the complete sequence is

$-4, \boxed{-2}, \boxed{0}, \boxed{2}, \boxed{4}, 6$

(v) Here, $t_2 = 38$ and $t_6 = -22$

Then $\quad t_2 = a + (2-1)d$

$\Rightarrow \quad 38 = a + d \qquad \ldots(i)$

and $\quad t_6 = a + 5d$

$\Rightarrow \quad -22 = a + 5d \qquad \ldots(ii)$

Subtracting equation (i) from equation (ii), we get: $d = -15$

Putting $d = -15$ in equation (i), we get:

$a = 38 + 15 = 53$

$\therefore \quad t_3 = a + 2d = 53 + 2 \times (-15)$

$= 53 - 30 = 23$

$t_4 = a + 3d = 53 + 3 \times (-15)$

$= 53 - 45 = 8$

$t_5 = a + 4d = 53 - 60 = -7$

Hence, the complete sequence is

$\boxed{53}, 38, \boxed{23}, \boxed{8}, \boxed{-7}, -22$

Question 4:

Which term of the AP: 3, 8, 13, 18, …, is 78?

Solution:

Given: 3, 8, 13, 18, ………,

$a = 3, d = 8 - 3 = 5$

Let nth term is 78

$a_n = 78$

$a + (n - 1) d = 78$

$\Rightarrow 3 + (n - 1) 5 = 78$

$\Rightarrow (n - 1) 5 = 78 - 3$

$\Rightarrow (n - 1) 5 = 75$

$\Rightarrow n - 1 = 15$

$\Rightarrow n = 15 + 1$

$\Rightarrow n = 16$

Hence, $a_{16} = 78$

Question 5:

Find the number of terms in each of the following APs:
(i) 7, 13, 19, …, 205
(ii) 18, $15\frac{1}{2}$, 13, …, -47

Solution:

(i) Here, $a = 7$, $d = 13 - 7 = 6$ and $l = 205$

Applying the formula $l = a + (n - 1)d$, we get:

$$205 = 7 + (n - 1) \times 6$$

$$\Rightarrow (n - 1) = \frac{198}{6} = 33$$

$$\Rightarrow \quad n = 33 + 1 = 34$$

Hence, the number of terms in this AP is **34**.

(ii) Here, $a = 18$,

$$d = 15\frac{1}{2} - 18 = \frac{31 - 36}{2} = -\frac{5}{2}$$

and $l = -47$

Applying the formula $l = a + (n - 1)d$, we get:

$$-47 = 18 + (n - 1)\left(-\frac{5}{2}\right)$$

$$\Rightarrow (n - 1) = \frac{65 \times 2}{5} = 26$$

$$\Rightarrow \quad n = 26 + 1 = 27$$

Hence, the number of terms in this AP is **27**.

Question 6:

Check, whether -150 is a term of the AP: 11, 8, 5, 2, ….

Solution:

11, 8, 5, 2, ...

Here, $a = 11$, $d = 8 - 11 = -3$, $a_n = -150$

$a + (n - 1) d = an$

$\Rightarrow 11 + (n - 1)(-3) = -150$

$\Rightarrow (n - 1)(-3) = -150 - 11$

$\Rightarrow -3(n - 1) = -161$

$\Rightarrow n - 1 = \frac{-161}{-3}$

$\Rightarrow n = \frac{161}{3} + 1 = \frac{164}{3} = 53\frac{4}{3}$

Which is not an integral number.

Hence, -150 is not a term of the AP.

Question 7:

Find the 31st term of an AP whose 11th term is 38 and the 16th term is 73.

Solution:

Here, $t_{11} = a + 10d = 38$ ...(i)

$t_{16} = a + 15d = 73$ ...(ii)

Subtracting equation (i) from equation (ii), we get:

$5d = 35 \Rightarrow d = 7$

Substituting the value of $d = 7$ in equation (ii), we get:

$a + 10 \times 7 = 38 \Rightarrow a = 38 - 70$

$\therefore t_{31} = a + 30d = -32 + 30 \times 7$

$= -32 + 210 = 178$

Hence, the 31st term is **178**.

Question 8:

An AP consists of 50 terms of which 3rd term is 12 and the last term is 106. Find the 29th term

Solution:

Given:

$a_{50} = 106$

$a_{50} = a + (50 - 1)\,d$

$\Rightarrow a + 49d = 106$ ...(i)

and $a_3 = 12 \Rightarrow a_3 = a + (3 - 1)d \Rightarrow a + 2d = 12$ ...(ii)

Subtracting eqn. (ii) from (i), we get

$a + 49d - a - 2d = 106 - 12$

$\Rightarrow 47d = 94$

$\Rightarrow d = \frac{94}{47} = 2$

$a + 2d = 12$

$\Rightarrow a + 2 \times 2 = 12$

$\Rightarrow a + 4 = 12$

$\Rightarrow a = 12 - 4 = 8$

$a_{29} = a + (29 - 1)\,d = a + 28d = 8 + 28 \times 2 = 8 + 56 = 64$

Question 9:

If the 3rd and the 9th term of an AP are 4 and -8 respectively, which term of this AP is zero?

Solution:

Here, $t_3 = 4$

$\Rightarrow \quad a + 2d = 4$ ...(i)

and $t_9 = -8$

$\Rightarrow \quad a + 8d = -8$ ...(ii)

Subtracting equation (i) from equation (ii), we get:

$6d = -12 \Rightarrow d = -2$

From equation (i), $a + 2 \times (-2) = 4 \Rightarrow a = 8$

Now let t_n be zero.

Then $a + (n-1)d = 0$

$\Rightarrow \quad 8 + (n-1)(-2) = 0 \quad \Rightarrow \quad n = 5$

Hence, **5th term** of the given AP is zero.

Question 10:

The 17th term of an AP exceeds its 10th term by 7. Find the common difference.

Solution:

Given: $a_{17} - a_{10} = 7$

$\Rightarrow [a + (17 - 1)d] - [a + (10 - 1)d] = 7$

$\Rightarrow (a + 16d) - (a + 9d) = 7$

$\Rightarrow 7d = 7$

$\Rightarrow d = 1$

Question 11:

Which term of the AP: 3, 15, 27, 39, … will be 132 more than its 54th term?

Solution:

Here, $a = 3$, $d = 15 - 3 = 12$

$$\therefore \quad t_n = a + (n-1)d$$
$$= 3 + (n-1)12 = 3 + 12n - 12$$
$$\Rightarrow \quad t_n = 12n - 9$$
and $$t_{54} = a + 53d = 3 + 53 \times 12$$
$$= 3 + 636 = 639$$

According to the question, we have:

$$t_n = 132 + t_{54}$$
$$\Rightarrow \quad 12n - 9 = 132 + 639$$
$$\Rightarrow \quad 12n = 780 \quad \Rightarrow \quad n = 65$$

Hence, **65th term** is the required term.

Question 12:

Two APs have the same common difference. The difference between their 100th terms is 100, what is the difference between their 1000th terms?

Solution:

Let a and A be the first term of two APs and d be the common difference.

Given:

$a_{100} - A_{100} = 100$

$\Rightarrow a + 99d - A - 99d = 100$

$\Rightarrow a - A = 100$

$\Rightarrow a_{1000} - A_{1000} = a + 999d - A - 999d$

$\Rightarrow a - A = 100$

$\Rightarrow a_{1000} - A_{1000} = 100$

Question 13:

How many three-digit numbers are divisible by 7?

Solution:

The list of three-digit numbers divisible by 7 is 105, 112, 119, ..., 994

Here, $a = 105$, $d = 112 - 105 = 7$, $t_n = 994$

$\therefore \quad t_n = a + (n-1)d$

$\Rightarrow \quad 994 = 105 + (n-1) \times 7$

$\Rightarrow \quad 889 = (n-1)\,7 \quad \Rightarrow \quad 127 = n - 1$

$\Rightarrow \quad n = 128$

Hence, there are **128** three-digit numbers divisible by 7.

Question 14:

How many multiples of 4 lie between 10 and 250?

Solution:

The multiples of 4 between 10 and 250 be 12, 16, 20, 24,...., 248

Here, $a = 12$, $d = 16 - 12 = 4$, $a_n = 248$

$a_n = a + (n - 1)\,d$

$\Rightarrow 248 = 12 + (n - 1)\,4$

$\Rightarrow 248 - 12 = (n - 1)\,4$

$\Rightarrow 236 = (n - 1)\,4$

$\Rightarrow 59 = n - 1$

$\Rightarrow n = 59 + 1 = 60$

Question 15:

For what value of n, the nth term of two APs: 63, 65, 61,… and 3, 10, 17,… are equal?

Solution:

The given APs are:

63, 65, 67, … ...(i)

and 3, 10, 17, … ...(ii)

From AP (i), we have:

The first term, $a = 63$ and the common difference $d = 2$.

$\therefore t_n = 63 + (n-1)2 = 2n + 61$

From AP (ii), we have:

The first term, $a = 3$ and the common difference $d = 7$.

$\therefore t_n = 3 + (n-1)7 = 7n - 4$

According to the question, we have:

$$7n - 4 = 2n + 61$$

$$\Rightarrow \quad 5n = 65 \Rightarrow n = 13$$

Hence, the required value of n is **13**.

Question 16:

Determine the AP whose 3rd term is 16 and 7th term exceeds the 5th term by 12.

Solution:

Given: $a_3 = 16$

$\Rightarrow a + (3 - 1)d = 16$

$\Rightarrow a + 2d = 16$

and $a_7 - a_5 = 12$

$\Rightarrow [a + (7 - 1)d] - [a + (5 - 1)d] = 12$

$\Rightarrow a + 6d - a - 4d = 12$

$\Rightarrow 2d = 12$

$\Rightarrow d = 6$

Since $a + 2d = 16$

$\Rightarrow a + 2(6) = 16$

$\Rightarrow a + 12 = 16$

$\Rightarrow a = 16 - 12 = 4$

$a_1 = a = 4$

$a_2 = a_1 + d = a + d = 4 + 6 = 10$

$a_3 = a_2 + d = 10 + 6 = 16$

$a_4 = a_3 + d = 16 + 6 = 22$

Thus, the required AP is $a_1, a_2, a_3, a_4, \ldots$, i.e. 4, 10, 16, 22

Question 17:

Find the 20th term from the last term of the AP: 3, 8, 13, …, 253.

Solution:

On reversing the given AP, the new AP will be
253, 248, ..., 13, 8, 3

Now, $a = 253$ and $d = 248 - 253 = -5$

$$\therefore t_{20} = a + (n-1)d = 253 + (20-1)(-5)$$
$$= 253 + 19 \times (-5) = 158.$$

Hence, the **20th term** from the last term is 158.

Question 18:

The sum of the 4th and 8th terms of an AP is 24 and the sum of the 6th and 10th terms is 44. Find the first three terms of the AP.

Solution:

Given: $a_4 + a_8 = 24$ and $a_6 + a_{10} = 44$

$\Rightarrow a + (4-1)d + a + (8-1)d = 24$

$\Rightarrow a + 3d + a + 7d = 24$ and $a + (6-1)d + a + (10-1)d = 44$

$\Rightarrow a + 5d + a + 9d = 44$

$\Rightarrow 2a + 10d = 24$ ...(i) and $2a + 14d = 44$ ...(ii)

Subtracting eqn (i) from (ii), we get

$$2a + 14d - 2a - 10d = 44 - 24 \Rightarrow 4d = 20 \Rightarrow d = \frac{20}{4} = 5$$
$$2a + 10d = 24 \Rightarrow 2a + 10 \times 5 = 24$$
$$2a = 24 - 50$$
$$2a = -26$$
$$a = \frac{-26}{2} = -13$$
$$a_1 = a = -13$$
$$a_2 = a_1 + d = -13 + 5 = -8$$
$$a_3 = a_2 + d = -8 + 5 = -3$$

Hence, the first three terms are, –13, –8, –3

Question 19:

Subba Rao started work in 1995 at an annual salary of ₹ 5000 and received an increment of ₹ 200 each year. In which year did his income reach ₹ 7000 ?

Solution:

Salary for the year 1995 = ₹ 5000

Salary for the year 1996 = ₹ 5000 + 200 = ₹ 5200

Salary for the year 1997 = ₹ 5200 + 200 = ₹ 5400

Thus, in the form of $t_1, t_2, t_3, \ldots, t_n$, we have:

5000, 5200, 5400, ..., 7000

It is an AP in which $a = 5000$ and $d = 200$

Let after n years his salary will be ₹ 7000.

Then $t_n = 7000$

$$\Rightarrow \quad a + (n-1)d = 7000$$

$$\Rightarrow \quad 5000 + (n-1)200 = 7000$$

$$\Rightarrow \quad (n-1) = \frac{2000}{200} = 10$$

$$\Rightarrow \quad n = \mathbf{11}$$

Now to find year we consider AP: 1995, 1996, 1997....

Here $a = 1995$ and $d = 1$

$$\therefore \quad t_{11} = 1995 + (11-1) \times 1$$

$$= 1995 + 10 = 2005$$

Hence, in the year **2005**, his salary was ₹ **7000**.

Question 20:

Ramkali saved ₹ 5 in the first week of a year and then increased her weekly saving by ₹ 1.75. If in the nth week, her weekly saving become ₹ 20.75, find n.

Solution:

Given: a = ₹ 5, d = ₹ 1.75

a_n = ₹ 20.75

$a + (n - 1)\,d - 20.75$

$\Rightarrow 5 + (n - 1)\,1.75 = 20.75$

$\Rightarrow (n - 1) \times 1.75 = 20.75 - 5$

$\Rightarrow (n - 1)\,1.75 = 15.75$

$\Rightarrow n - 1 = 9$

$\Rightarrow n = 9 + 1$

$\Rightarrow n = 10$

Hence, in 10th week Ramkali's saving will be ₹ 20.75.