EXERCISE 2.2

Question 1:

Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

(i) $x^2 - 2x - 8$

(i) Quadratic polynomial is

$$x^2 - 2x - 8 = x^2 - 4x + 2x - 8 = x(x-4) + 2(x-4) = (x+2)(x-4)$$

So, zeroes are -2 and 4 because value of $x^2 - 2x - 8$ is zero, when

$x + 2 = 0 \Rightarrow x = -2$

or $x - 4 = 0$

$\Rightarrow x = 4.$

Verification:

Zeroes are -2 and 4.

$$\alpha = -2 \text{ and } \beta = 4$$

$$\therefore \quad \alpha + \beta = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$$

$$-2 + 4 = -(-2)$$

$$2 = 2 \text{ verified.}$$

$$\alpha\beta = \frac{\text{constant term}}{\text{coefficient of } x^2}$$

$$(-2)(4) = \frac{-8}{1} = -8$$

$$-8 = -8 \text{ verified.}$$

(ii) $4s^2 - 4s + 1$

(ii) $4s^2 - 4s + 1 = 4s^2 - 2s - 2s + 1 = 2s(2s-1) - 1(2s-1) = (2s-1)(2s-1)$

So, zeroes are $\frac{1}{2}$ and $\frac{1}{2}$, because the value of $4s^2 - 4s + 1$ is zero, when

$2s - 1 = 0 \Rightarrow 2s = 1 \Rightarrow s = \frac{1}{2}$

or $2s - 1 = 0 \Rightarrow s = \frac{1}{2}$

Verification:

$$\alpha = \frac{1}{2}, \beta = \frac{1}{2}$$

$$\therefore \quad \alpha + \beta = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$$

$$\frac{1}{2} + \frac{1}{2} = -\frac{-4}{4}$$

$$1 = -(-1) = 1 \text{ verified.}$$

$$\alpha\beta = \frac{\text{constant term}}{\text{coefficient of } x^2}$$

$$\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{4}$$

(iii) $6x^2 - 3 - 7x$

(iii) $6x^2 - 3 - 7x = 6x^2 - 7x - 3 = 6x^2 - 9x + 2x - 3 = 3x(2x-3) + 1(2x-3) = (3x+1)(2x-3)$

So, zeroes are $-\frac{1}{3}$ and $\frac{3}{2}$ because the value of $6x^2 - 7x - 3$ is zero, when

$3x + 1 = 0 \Rightarrow x = \frac{-1}{3}$

$2x - 3 = 0 \Rightarrow x = \frac{3}{2}$

Verification:

$$\alpha = \frac{-1}{3}, \beta = \frac{3}{2}$$

$$\therefore \quad \alpha + \beta = -\frac{1}{3} + \frac{3}{2} = \frac{-2+9}{6} = \frac{7}{6}$$

$$= \frac{-(-7)}{6} = -\frac{\text{coefficient of } x}{\text{coeffcient of } x^2}$$

and $$\alpha\beta = \frac{-1}{3} \times \frac{3}{2} = \frac{-1}{2} = \frac{-3}{6}$$

$$= \frac{\text{constant term}}{\text{coefficient of } x^2} \text{ verified.}$$

(iv) $4u^2 + 8u$

(iv) $4u^2 + 8u = 4u(u+2)$

Zeroes are 0 and -2, so, value of $4u^2 + 8u$ is zero, when

$4u = 0 \Rightarrow u = 0$

or $u + 2 = 0 \Rightarrow u = -2$

Verification:

$$\alpha = 0, \beta = -2$$

$$\therefore \quad \alpha + \beta = 0 + (-2) = -2 = -\frac{8}{4}$$

$$= -\frac{\text{coefficient of } u}{\text{coefficient of } u^2}$$

and $$\alpha\beta = 0(-2) = \frac{0}{4}$$

$$= \frac{\text{constant term}}{\text{coefficient of } u^2} \text{ verified.}$$

(v) $t^2 - 15$

(v) Let the zeroes of the polynomial be α and β.

Then, $\alpha + \beta = -\frac{1}{4}$ and $\alpha\beta = \frac{1}{4}$

∴ Required polynomial

$= x^2 - (\alpha + \beta)x + \alpha\beta$

$= x^2 - (-\frac{1}{4}) + \frac{1}{4}$

$= 4x^2 + x + 1 = 0$

(vi) $3x^2 - x - 4$

(vi) $3x^2 - x - 4 = 3x^2 - 4x + 3x - 4$
$= x(3x - 4) + 1(3x - 4)$
$= (3x - 4)(x + 1)$

So, Zeroes are $\frac{4}{3}$ and -1.

because value of $3x^2 - x - 4$ is zero,

when $3x - 4 = 0 \Rightarrow x = \frac{4}{3}$

or $x + 1 = 0 \Rightarrow x = -1$

Verification:

$\alpha = \frac{4}{3}, \beta = -1$

$\therefore \quad \alpha + \beta = \frac{4}{3} + (-1) = \frac{1}{3} = \frac{-(-1)}{3}$

$= \frac{-\text{coefficient of } x}{\text{coefficient of } x^2}$ verified.

$\alpha\beta = \frac{4}{3}(-1) = \frac{-4}{3}$

$= \frac{\text{constant term}}{\text{coefficient of } x^2}$ verified.

Question 2:

Find a quadratic polynomial each with the given numbers as the sum and product of zeroes respectively:

(i) 1/4, - 1

(i) Let the zeroes of polynomial be α and β.

Then, $\alpha + \beta = \frac{1}{4}$ and $\alpha\beta = -1$

∴ Required polynomial is given by,

$x^2 - (\alpha + \beta)x + \alpha\beta = x^2 - \frac{1}{4}x + (-1)$

$= x^2 - \frac{1}{4}x - 1$

$= 4x^2 - x - 4$

(ii) √2, 1/3

(ii) Let the zeroes of polynomial be α and β.

Then, $\alpha + \beta = \sqrt{2}$ and $\alpha\beta = \frac{1}{3}$

∴ Required polynomial is:

$x^2 - (\alpha + \beta)x + \alpha\beta = x^2 - \sqrt{2}x + \frac{1}{3}$

$= 3x^2 - 3\sqrt{2}x + 1$

(iii) 0, √5

(iii) Let the zeroes of the polynomial be α and β.

Then, $\alpha + \beta = 0$ and $\alpha\beta = \sqrt{5}$

∴ Required polynomial

$= x^2 - (\alpha + \beta)x + \alpha\beta$

$= x^2 - 0 \times x + \sqrt{5} = x^2 + \sqrt{5}$

(iv) 1, 1

(iv) Let the zeroes of the polynomial be α and β.

Then, $\alpha + \beta = 1$ and $\alpha\beta = 1$.

∴ Required polynomial

$= x^2 - (\alpha + \beta)x + \alpha\beta$

$= x^2 - x + 1$

(v) - 1/4, 1/4

(v) Let the zeroes of the polynomial be α and β.

Then, $\alpha + \beta = -\frac{1}{4}$ and $\alpha\beta = \frac{1}{4}$

∴ Required polynomial

$= x^2 - (\alpha + \beta)x + \alpha\beta$

$= x^2 - (-\frac{1}{4}) + \frac{1}{4}$

$= 4x^2 + x + 1 = 0$

(vi) 4, 1

(vi) Let the zeroes of the polynomial be α and β.

Then, $\alpha + \beta = 4$ and $\alpha\beta = 1$.

∴ Required polynomial $= x^2 - (\alpha + \beta)x + \alpha\beta$

$= x^2 - 4x + 1$