EXERCISE 8.1

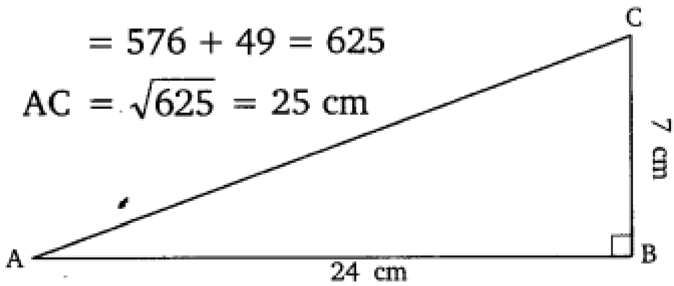
Question 1:

In ΔABC right angled at B, AB = 24 cm, BC = 7 cm. Determine:
(i) sin A, cos A
(ii) sin C, cos C

Solution:

By Pythagoras' Theorem,

$$AC^2 = AB^2 + BC^2 = (24)^2 + (7)^2$$
$$= 576 + 49 = 625$$
$$\Rightarrow \quad AC = \sqrt{625} = 25 \text{ cm}$$



$$\text{(i)}\ \sin A = \frac{BC}{AC} = \frac{7}{25}, \quad \cos A = \frac{AB}{AC} = \frac{24}{25}$$
$$\text{(ii)}\ \sin C = \frac{AB}{AC} = \frac{24}{25}, \quad \cos C = \frac{BC}{AC} = \frac{7}{25}$$

Question 2:

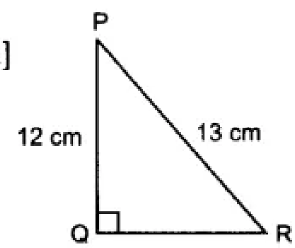
In given figure, find tan P – cot R.

Solution:

In right angled ΔPQR,

$$PR^2 = PQ^2 + QR^2 \qquad \text{[Pythagoras Theorem]}$$
$$\Rightarrow \quad (13)^2 = (12)^2 + QR^2$$
$$\Rightarrow \quad 169 - 144 = QR^2$$
$$\Rightarrow \quad 25 = QR^2 \quad \Rightarrow \quad QR = 5 \text{ cm}$$
$$\tan P = \frac{QR}{PQ} = \frac{5}{12}$$
$$\cot R = \frac{QR}{PQ} = \frac{5}{12}$$

So, $\tan P - \cot R = \frac{5}{12} - \frac{5}{12} = 0$



Question 3:

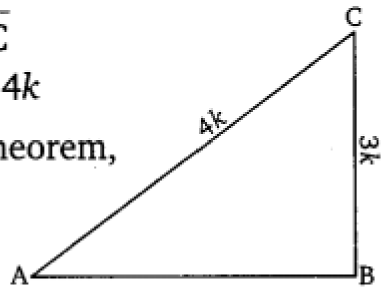
If sin A = 3/4, calculate cos A and tan A.

Solution:

Given: $\sin A = \frac{3}{4} = \frac{BC}{AC}$

Let $BC = 3k$ and $AC = 4k$

Then by Pythagoras' Theorem,



$$AB^2 = AC^2 - BC^2$$
$$= (4k)^2 - (3k)^2$$
$$= 16k^2 - 9k^2 = 7k^2$$

$$\Rightarrow AB = k\sqrt{7}$$

$$\therefore \cos A = \frac{AB}{AC} = \frac{\sqrt{7}k}{4k} = \frac{\sqrt{7}}{4}$$

$$\text{and } \tan A = \frac{BC}{AB} = \frac{3k}{\sqrt{7}k} = \frac{3}{\sqrt{7}}$$

Question 4:

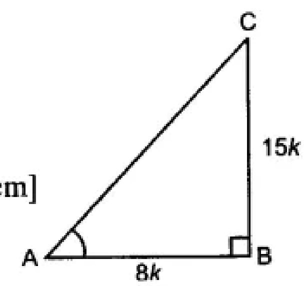
Given 15 cot A = 8, find sin A and sec A.

Solution:

$$15 \cot A = 8 \Rightarrow \cot A = \frac{8}{15} \Rightarrow \frac{AB}{BC} = \frac{8}{15}$$

Let $AB = 8k$ and $BC = 15k$

In right angled ΔABC,



$$AC^2 = AB^2 + BC^2 \quad \text{[Pythagoras theorem]}$$
$$= (8k)^2 + (15k)^2 = 64k^2 + 225k^2 = 289k^2$$

$\Rightarrow$ $$AC = \sqrt{289k^2} = 17k$$

So, $$\sin A = \frac{BC}{AC} = \frac{15k}{17k} = \frac{15}{17}$$

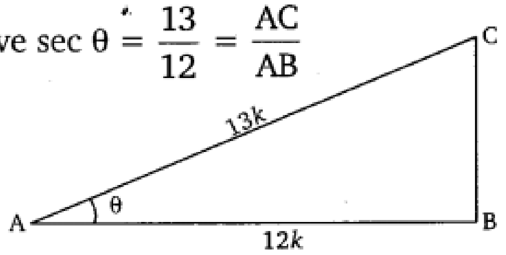
and $$\sec A = \frac{AC}{AB} = \frac{17k}{8k} = \frac{17}{8}$$

Question 5:

Given sec θ = 13/12 , calculate all other trigonometric ratios.

Solution:

We have $\sec\theta = \dfrac{13}{12} = \dfrac{AC}{AB}$



Let AC = 13*k* and AB = 12*k*

Then by Pythagoras' Theorem,

$AC^2 = AB^2 + BC^2 \Rightarrow 169k^2 = 144k^2 + BC^2$

$\Rightarrow 169k^2 - 144k^2 = BC^2 \quad \Rightarrow \quad 25k^2 = BC^2$

$\Rightarrow \quad BC = \sqrt{25\,k^2} = 5k$

$$\sin\theta = \frac{BC}{AC} = \frac{5k}{13k} = \mathbf{\frac{5}{13}}$$

$$\cos\theta = \frac{AB}{AC} = \frac{12k}{13k} = \mathbf{\frac{12}{13}}$$

$$\tan\theta = \frac{BC}{AB} = \frac{5k}{12k} = \mathbf{\frac{5}{12}}$$

$$\operatorname{cosec}\theta = \frac{AC}{BC} = \frac{13k}{5k} = \mathbf{\frac{13}{5}}$$

$$\cot\theta = \frac{AB}{BC} = \frac{12k}{5k} = \mathbf{\frac{12}{5}}.$$

Question 6:

If ∠A and ∠B are acute angles such that cos A = cos B, then show that ∠A = ∠B.

Solution:

Since ∠A and ∠B are acute angles.

Then, $\angle C = 90^\circ$

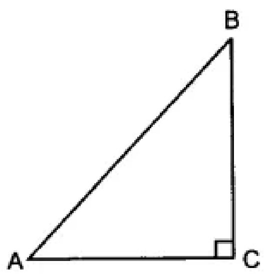
$\cos A = \cos B$ [Given]

$\Rightarrow \dfrac{AC}{AB} = \dfrac{BC}{AB}$

$\Rightarrow AC = BC$

$\therefore \angle A = \angle B$

[Angles opposite to equal sides are equal]



Question 7:

If cot θ = 78, evaluate:
(i) (1+sinθ)(1−sinθ)/(1+cosθ)(1−cosθ)
(ii) cot²θ

Solution:

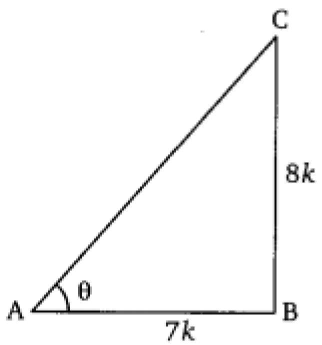
Solution:

$(1+\cos\theta)(1-\cos\theta)$

We have $\cot\theta = \frac{7}{8} = \frac{AB}{BC}$

Let $AB = 7k$ and $BC = 8k$.

Then in ΔABC,



$$AC^2 = AB^2 + BC^2$$
$$= (7k)^2 + (8k)^2 = 49k^2 + 64k^2 = 113k^2$$
$$\Rightarrow AC = k\sqrt{113}$$
$$\therefore \sin\theta = \frac{BC}{AC} = \frac{8k}{\sqrt{113}k} = \frac{8}{\sqrt{113}}$$
and
$$\cos\theta = \frac{AB}{AC} = \frac{7k}{\sqrt{113}k} = \frac{7}{\sqrt{113}}.$$

(i) $$\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{1-\sin^2\theta}{1-\cos^2\theta} = \frac{\cos^2\theta}{\sin^2\theta}$$
$$= \frac{\left(\frac{7}{\sqrt{113}}\right)^2}{\left(\frac{8}{\sqrt{113}}\right)^2} = \frac{\frac{49}{113}}{\frac{64}{113}} = \mathbf{\frac{49}{64}}.$$

(ii) $$\cot^2\theta = \frac{\cos^2\theta}{\sin^2\theta} = \mathbf{\frac{49}{64}}.$$ [From (i)]

Question 8:

If 3 cot A = 4, check whether 1−tan2A/1+tan2A = cos² A − sin² A or not.

Solution:

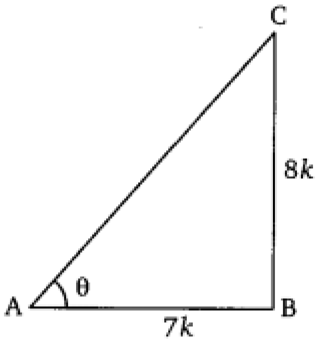
Solution:

$(1+\cos\theta)(1-\cos\theta)$

We have $\cot\theta = \dfrac{7}{8} = \dfrac{AB}{BC}$

Let $AB = 7k$ and $BC = 8k$.

Then in ΔABC,



$$AC^2 = AB^2 + BC^2$$
$$= (7k)^2 + (8k)^2 = 49k^2 + 64k^2 = 113k^2$$

$$\Rightarrow AC = k\sqrt{113}$$

$$\therefore \sin\theta = \frac{BC}{AC} = \frac{8k}{\sqrt{113}k} = \frac{8}{\sqrt{113}}$$

$$\text{and } \cos\theta = \frac{AB}{AC} = \frac{7k}{\sqrt{113}k} = \frac{7}{\sqrt{113}}.$$

(i) $$\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{1-\sin^2\theta}{1-\cos^2\theta} = \frac{\cos^2\theta}{\sin^2\theta}$$

$$= \frac{\left(\frac{7}{\sqrt{113}}\right)^2}{\left(\frac{8}{\sqrt{113}}\right)^2} = \frac{\frac{49}{113}}{\frac{64}{113}} = \frac{\mathbf{49}}{\mathbf{64}}.$$

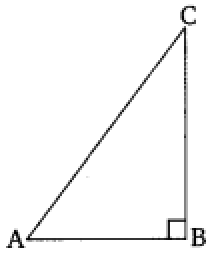
(ii) $$\cot^2\theta = \frac{\cos^2\theta}{\sin^2\theta} = \frac{\mathbf{49}}{\mathbf{64}}.$$ [From (i)]

Question 9:

In triangle ABC, right angled at B, if tan A = $1/\sqrt{3}$, find the value of:
(i) sin A cos C + cos A sin C
(ii) cos A cos C – sin A sin C

Solution:

Let ABC is a right triangle at B.



$$\because \tan A = \frac{BC}{AB} = \frac{1}{\sqrt{3}}$$

$$\therefore \frac{BC}{AB} = \frac{1}{\sqrt{3}}$$

Let $AB = \sqrt{3}k$ and $BC = k$

Then by Pythagoras' Theorem, we have:

$$AC^2 = AB^2 + BC^2 = \left(\sqrt{3}k\right)^2 + (k)^2$$

$$\Rightarrow \quad AC = \sqrt{4k^2} = 2k \qquad \text{[Hypotenuse]}$$

$$\text{Now } \sin A = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$$

$$\cos A = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$$

$$\sin C = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$$

$$\cos C = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$$

(i) sin A cos C + cos A sin C

$$= \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$$

$$= \frac{1}{4} + \frac{3}{4} = \mathbf{1}.$$

(ii) cos A cos C – sin A sin C

$$= \frac{\sqrt{3}}{2} \times \frac{1}{2} - \frac{1}{2} \times \frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = \mathbf{0}.$$

Question 10:

In ΔPQR, right-angled at Q, PR + QR = 25 cm and PQ = 5 cm. Determine the values of sin P, cos P and tan P.

Solution:

In right angled ΔPQR

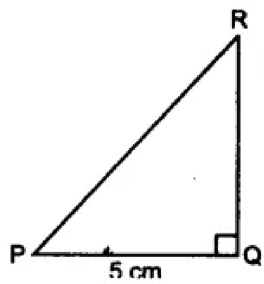
$$PR^2 = PQ^2 + QR^2 \Rightarrow PQ^2 = PR^2 - QR^2$$

$$\Rightarrow \quad (5)^2 = (PR + QR)(PR - QR)$$

$$\Rightarrow \quad 25 = 25(PR - QR) \Rightarrow \frac{25}{25} = PR - QR$$

$$\Rightarrow \quad PR - QR = 1 \quad \text{... }(i)$$

and $\quad PR + QR = 25 \quad \text{... }(ii)$



On adding equation (*i*) and (*ii*), we get

$$2PR = 26 \quad \Rightarrow \quad PR = \frac{26}{2} = 13 \text{ cm}$$

From equation (*i*),

$$PR - QR = 1 \quad \Rightarrow \quad QR = 13 - 1$$

$$QR = 12 \text{ cm}$$

$$\sin P = \frac{QR}{PR} = \frac{12}{13}$$

$$\cos P = \frac{PQ}{PR} = \frac{5}{13}$$

$$\tan P = \frac{QR}{PQ} = \frac{12}{5}$$

Question 11:

State whether the following statements are true or false. Justify your answer.
(i) The value of tan A is always less than 1.
(ii) sec A = $12/5$ for some value of angle A.
(iii) cos A is the abbreviation used for the cosecant of angle A.
(iv) cot A is the product of cot and A.
(v) sin θ = $4/3$ for some angle.

Solution:

(i) False, because tan 60° = $\sqrt{3} > 1$.

(ii) True, because sec A ≥ 1

(iii) False, because cos A abbreviation is used for cosine A.

(iv) False, because the term cot A is single not a product.

(v) False, because -1 ≤ sin θ ≤ 1.