EXERCISE 13.3

Question 1:

The following frequency distribution gives the monthly consumption of electricity of 68 consumers of a locality. Find the median, mean and mode of the data and compare them.

Monthly consumption (in units)	65 – 85	85 – 105	105 – 125	125 – 145	145 – 165	165 – 185	185 – 205
No. of consumers	4	5	13	20	14	8	4

Solution:

Monthly consumption (in units)	Number of consumers (f_i)	Cumulative frequency (cf)	Class mark (x_i)	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
65 – 85	4	4	75	–3	–12
85 – 105	5	9	95	–2	–10
105 – 125	13	22	115	–1	–13
125 – 145	20	42	135 = a (Let)	0	0
145 – 165	14	56	155	1	14
165 – 185	8	64	175	2	16
185 – 205	4	68	195	3	12
Total	$\Sigma f_i = 68$				$\Sigma f_i u_i = 7$

We have, $\text{Mean} = a + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h = 135 + \frac{7}{68} \times 20 = 135 + \frac{35}{17}$

$= 135 + 2.06 = 137.06$ units

Here, $n = 68, \frac{n}{2} = \frac{68}{2} = 34,$

$\therefore$ Median class = 125 – 145

Here, $l = 125, n = 68, f = 20, cf = 22, h = 20$

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h = 125 + \left(\frac{34 - 22}{20}\right) \times 20$$

$= 125 + 12 = 137$ units

Maximum frequency = 20

Modal class = 125 – 145 Here, $l = 125, f_0 = 13, f_1 = 20, f_2 = 14$

$\therefore$ $$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h = 125 + \left(\frac{20 - 13}{40 - 13 - 14}\right) \times 20$$

$$= 125 + \frac{7 \times 20}{13} = 125 + \frac{140}{13} = 125 + 10.76 = 135.76 \text{ units}$$

Mean > Median > Mode

Question 2:

If the median of the distribution given below is 28.5, find the values of x and y.

Class-interval	Frequency
0 – 10	5
10 – 20	x
20 – 30	20
30 – 40	15
40 – 50	y
50 – 60	5
Total	60

Solution:

Class interval	Frequency	Cumulative frequency
0 – 10	5	5
10 – 20	x	$5 + x$(c)
20 – 30	20(f)	$25 + x$
30 – 40	15	$40 + x$
40 – 50	y	$40 + x + y$
50 – 60	5	$45 + x + y$
Total	$n = 60$	

We have $45 + x + y = 60$ ...(i) [Given]

$\because n = 60 \quad \therefore \frac{n}{2} = \frac{60}{2} = 30$

Since the median lies in the class interval (20 – 30), so the median class is (20 – 30).

Hence, $l = 20, f = 20, cf = 5 + x$ and $h = 10$.

$$\therefore \text{ Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$\Rightarrow \quad 28.5 = 20 + \left(\frac{30 - 5 - x}{20}\right) \times 10$$

$$\Rightarrow \quad 28.5 = 20 + \left(\frac{25 - x}{2}\right)$$

$$\Rightarrow \quad 57 = 40 + 25 - x \Rightarrow \quad 25 - x = 57 - 40$$

$$\Rightarrow \quad 25 - x = 17 \quad \Rightarrow \quad \mathbf{x = 25 - 17 = 8.}$$

Putting $x = 8$ in equation (i), we get:

$$\Rightarrow \quad 45 + 8 + y = 60 \quad \Rightarrow \mathbf{y = 60 - 53 = 7.}$$

Question 3:

A life insurance agent found the following data for distribution of ages of 100 policy holders. Calculate the median age, if policies are given only to persons having age 18 years onwards but less than 60 years.

Age (in years)	Number of policy holders
Below 20	2
Below 25	6
Below 30	24
Below 35	45
Below 40	78
Below 45	89
Below 50	92
Below 55	98
Below 60	100

Solution:

Age (in years)	Number of policy holders	Cumulative frequency
0 – 20	2	2
20 – 25	6 – 2 = 4	6
25 – 30	24 – 6 = 18	24
30 – 35	45 – 24 = 21	45
35 – 40	78 – 45 = 33	78
40 – 45	89 – 78 = 11	89
45 – 50	92 – 89 = 3	92
50 – 55	98 – 92 = 6	98
55 – 60	100 – 98 = 2	100
Total	100	

Here, $\frac{n}{2} = \frac{100}{2} = 50$

$\therefore$ Median class= 35 – 40, So, $l = 35, cf = 45, h = 5, f = 33$

We have, $\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h = 35 + \left(\frac{50 - 45}{33}\right) \times 5 = 35 + \frac{25}{33}$

$= 35 + 0.76 = 35.76$ years

Question 4:

The lengths of 40 leaves of a plant are measured correct to nearest millimetre, and the data obtained is represented in the following table:

Length (in mm)	Number of leaves
118 – 126	3
127 – 135	5
136 – 144	9
145 – 153	12
154 – 162	5
163 – 171	4
172 – 180	2

Solution:

Class interval	Frequency	Cumulative frequency
117.5 – 126.5	3	3
126.5 – 135.5	5	8
135.5 – 144.5	9	17 (c)
144.5 – 153.5	12 (f)	29
153.5 – 162.5	5	34
162.5 – 171.5	4	38
171.5 – 180.5	2	40
	$n = 40$	

$\because n = 40 \quad \therefore \frac{n}{2} = \frac{40}{2} = 20.$

Since 12 is the maximum frequency, so the median class is (144.5 – 153.5).

Here, $l = 144.5, f = 12, cf = 17$ and $h = 9$

$$\therefore \text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$= 144.5 + \left(\frac{20 - 17}{12}\right) \times 9$$

$$= 144.5 + \frac{9}{4}$$

$$= 144.5 + 2.25 = 146.75 \text{ mm}.$$

Hence, the median length of leaves is **146.75 mm**.

Question 5:

The following table gives the distribution of the lifetime of 400 neon lamps:

Life time (in hours)	Number of lamps
1500 – 2000	14
2000 – 2500	56
2500 – 3000	60
3000 – 3500	86
3500 – 4000	74
4000 – 4500	62
4500 – 5000	48

Solution:

Lifetime (in hours)	Number of lamps	*cf*
1500 – 2000	14	14
2000 – 2500	56	70
2500 – 3000	60	130
3000 – 3500	86	216
3500 – 4000	74	290
4000 – 4500	62	352
4500 – 5000	48	400
Total	400	

Here, $\frac{n}{2} = \frac{400}{2} = 200$

$\therefore$ Median class $= 3000 - 3500$

So, $f = 86, cf = 130, h = 500$

We have, $\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$

$= 3000 + \left(\frac{200 - 130}{86}\right) \times 500$

$= 3000 + \frac{35000}{86} = 3000 + 406.98 = 3406.98$ hours

Question 6:

100 surnames were randomly picked up from a local telephone directory and the frequency distribution of the number of letters in the English alphabet in the surnames was obtained as follows:

Number of letters	Number of surnames
1 – 4	6
4 – 7	30
7 – 10	40
10 – 13	16
13 – 16	4
16 – 19	4

Solution:

Here, $h = 3$.

Class interval	Frequency (f_i)	Cumulative frequency (cf)	Class marks (x_i)	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
1 – 4	6	6	2.5	–2	–12
4 – 7	30(f_1)	36(c)	5.5	–1	–30
7 – 10	40(f_m)	76	8.5 = a	0	0
10 – 13	16(f_2)	92	11.5	1	16
13 – 16	4	96	14.5	2	8
16 – 19	4	100	17.5	3	12
			$n = 100$		$\Sigma f_i u_i = -6$

$\because \quad n = 100$

$\therefore \quad \frac{n}{2} = \frac{100}{2} = 50$

Since 40 is the maximum frequency, so the median class is (7 – 10).

Here, $l = 7, f_m = 40, cf = 36$ and $h = 3$.

$$\therefore \textbf{Median} = l + \left(\frac{\frac{n}{2} - cf}{f_m}\right) \times h$$

$$= 7 + \left(\frac{50 - 36}{40}\right) \times 3 = 7 + \frac{14}{40} \times 3$$

$$= 7 + \frac{21}{20} = 7 + \frac{10.5}{10}$$

$$= 7 + 1.05 = \textbf{8.05}$$

$$\textbf{Mean} = a + \frac{\sum f_i u_i}{\sum f_i} \times h = 8.5 + \frac{(-6)}{100} \times 3$$

$$= 8.5 + \frac{(-18)}{100} = 8.50 - 0.18 = \textbf{8.32}.$$

Now since the maximum number of letters in surnames = 40

$\therefore$ Modal class = 7 – 10

$$\therefore \textbf{Mode} = l + \left(\frac{f_m - f_1}{2f_m - f_1 - f_2}\right) \times h$$

$$= 7 + \left(\frac{40 - 30}{80 - 30 - 16}\right) \times 3$$

$$= 7 + \frac{10}{34} \times 3 = 7 + \frac{30}{34} = 7 + 0.88$$

$$= \textbf{7.88}.$$

Question 7:

The distribution below gives the weight of 30 students of a class. Find the median weight of the students.

Weight (in kg)	Number of students
40 – 45	2
45 – 50	3
50 – 55	8
55 – 60	6
60 – 65	6
65 – 70	3
70 – 75	2

Solution:

Weight (in kg)	Number of students (f_i)	cf
40 – 45	2	2
45 – 50	3	5
50 – 55	8	13
55 – 60	6	19
60 – 65	6	25
65 – 70	3	28
70 – 75	2	30
Total	30	

Here, $\frac{n}{2} = \frac{30}{2} = 15,$

$\therefore$ Median class $= 55 - 60,$

So, $l = 55, f = 6, cf = 13, h = 5$

$$\text{Median weight} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$= 55 + \left(\frac{15 - 13}{6}\right) \times 5 = 55 + \frac{5}{3}$$

$$= 55 + 1.67 = 56.67 \text{ kg}$$