EXERCISE 10.2

Question 1:

From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is
(a) 7 cm
(b) 12 cm
(c) 15 cm
(d) 24.5 cm

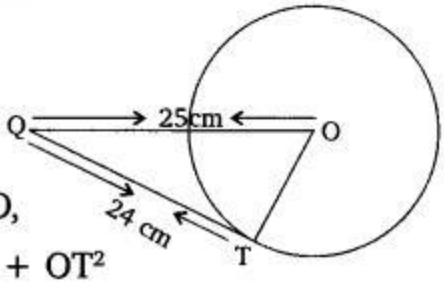
Solution:

The correct option is **(A).**

Justification:

Let OT be x cm.

Then in right ΔQTO,



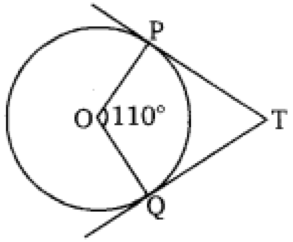
$$QO^2 = QT^2 + OT^2$$

[By Pythagoras' Theorem]

$$\Rightarrow \quad (25)^2 = (24)^2 + x^2$$
$$\Rightarrow \quad x^2 = 625 - 576 = 49$$
$$\Rightarrow \quad x = \sqrt{49} = \mathbf{7\ cm.}$$

Question 2:

In figure, if TP and TQ are the two tangents to a circle with centre O so that ∠POQ = 110°, then ∠PTQ is equal to
(a) 60°
(b) 70°
(c) 80°
(d) 90°



Solution:

$\angle OPT = 90°$
$\angle OQT = 90°$
$\angle POQ = 110°$
TPOQ is a quadrilateral,
$\therefore \angle PTQ + \angle POQ = 180°$
$\Rightarrow \angle PTQ + 110° = 180°$
$\Rightarrow \angle PTQ = 180° - 110° = 70°$
Hence, correct option is **(b)**.

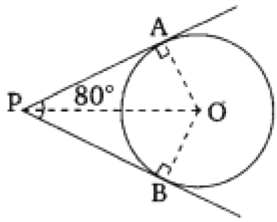
Question 3:

If tangents PA and PB from a point P to a circle with centre O are inclined to each other at angle of 80°, then ∠POA is equal to
(a) 50°
(b) 60°
(c) 70°
(d) 80°

Solution:

The correct option is **(A)**.

Justification:



In ΔPOA and ΔPOB,

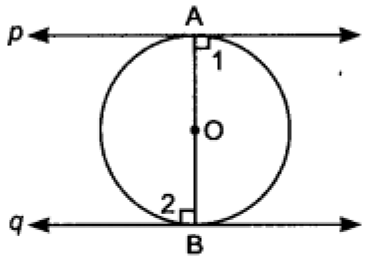
$\angle PAO = \angle PBO$ [Each of 90°]
$OA = OB$ [Radii of the circle]
$PA = PB$ [Both are tangents]
$\therefore \Delta POA \cong \Delta POB$ [By SAS congruence]
$\Rightarrow \angle APO = \angle BPO$ [CPCT]
$\Rightarrow \angle APO = \frac{1}{2}\angle APB = \frac{1}{2} \times 80° = 40°$

In ΔPAO, $\angle APO + \angle POA + \angle OAP = 180°$
$\Rightarrow 40° + \angle POA + 90° = 180°$
$\Rightarrow \angle POA = \mathbf{50°}$.

Question 4:

Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

Solution:



AB is a diameter of the circle, p and q are two tangents.

OA ⊥ p and OB ⊥ q

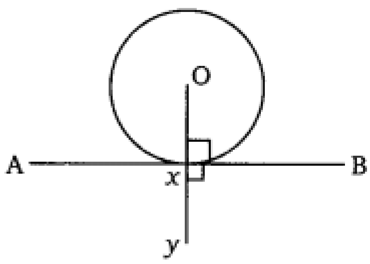
∴ ∠1 = ∠2 = 90°

⇒ p ∥q [∠1 and ∠2 are alternate angles]

Question 5:

Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre.

Solution:



In the given figure, AXB is the tangent to a circle with centre O.

Here, OX is the line perpendicular to the tangent AXB at the point of contract X.

The, we have:

∠OXB + ∠BXY = 90° + 90° = 180°

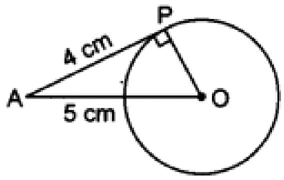
⇒ OXY is collinear, i.e., OX passes through the centre of the circle.

Hence, **Proved.**

Question 6:

The length of a tangent from a point A at distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle.

Solution:



OA = 5 cm, AP = 4 cm

OP = Radius of the circle

$\angle OPA = 90°$ [Radius and tangent are perpendicular]

$$OA^2 = AP^2 + OP^2 \quad \text{[By Pythagoras theorem]}$$

$$5^2 = 4^2 + OP^2 \Rightarrow 25 = 16 + OP^2$$

$$\Rightarrow 25 - 16 = OP^2 \Rightarrow 9 = OP^2 \Rightarrow \sqrt{9} = OP$$

$$\Rightarrow OP = 3 \text{ cm}$$

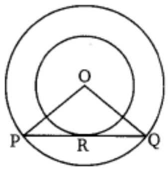
$$\therefore \text{Radius} = 3 \text{ cm}$$

Question 7:

Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

Solution:

In the given figure, PQ is the chord of the larger circle, which touches the smaller circle at R.



We have, $OP = OQ = 5$ cm [Radii of larger circle]

and $OR = 3$ cm [Radius of smaller circle]

Since PQ is tangent to the smaller circle.

$\therefore$ $OR \perp PQ$ [By theorem]

In ΔOPR and ΔOQR,

$\angle ORP = \angle ORQ$ [Each of 90°]

$OR = OR$ [Common]

$OP = OQ$ [Radii of the same circle]

$\therefore$ $\Delta OPR \cong \Delta OQR$ [By RHS congruence]

$\Rightarrow$ $PR = RQ$ [CPCT]

In ΔOPR,

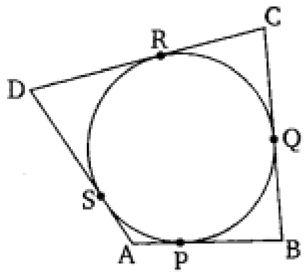
$PR^2 = OP^2 - OR^2 = (5)^2 - (3)^2 = 16$ cm

$\Rightarrow$ $PR = \sqrt{16} = 4$ cm

$\therefore$ $PQ = 2PR = 2 \times 4 = $ **8 cm**.

Question 8:

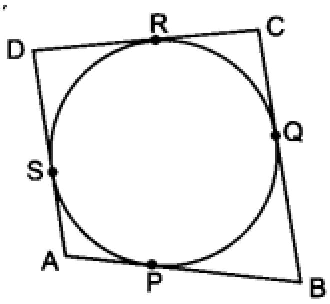
A quadrilateral ABCD is drawn to circumscribe a circle (see figure). Prove that AB + CD = AD + BC.



Solution:

AP = AS ...(i)

[Lengths of tangents from an external point are equal]



BP = BQ ...(ii)

CR = CQ ...(iii)

DR = DS ...(iv)

Adding equations (i), (ii), (iii) and (iv), we get

AP + BP + CR + DR = AS + BQ + CQ + DS

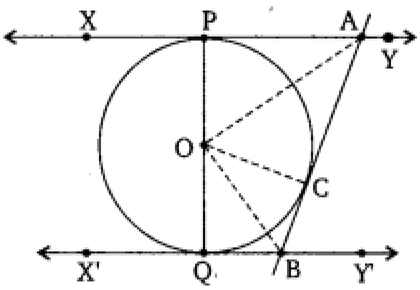
⇒ (AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)

⇒ AB + CD = AD + BC

Hence proved.

Question 9:

In figure, XY and X'Y' are two parallel tangents to a circle , x with centre O and another tangent AB with point of contact C intersecting XY at A and X'Y' at B. Prove that ∠AOB = 90°.



Solution:

In ΔAOP and ΔAOC,

$AP = AC$ [Tangents from the point]

$OP = OC$ [Radii of the same circle]

$OA = OA$ [Common]

$\therefore \Delta AOP \cong \Delta AOC$ [By SSS congruence]

$\Rightarrow \angle PAO = \angle CAO$ [CPCT]

$\Rightarrow \angle PAC = 2\angle PAO = 2\angle CAO$

$\Rightarrow \angle PAC = 2\angle OAC$ ...(i)

Similarly, $\angle QBC = 2\angle OBC$ ...(ii)

Adding equations (i) and (ii), we get:

$\angle PAC + \angle QBC = 2(\angle OAC + \angle OBC)$

$\Rightarrow 180° = 2(\angle OAC + \angle OBC)$

$\Rightarrow \angle OAC + \angle OBC = 90°$

Then, in triangle AOB, we have

$\angle AOB + \angle OAC + \angle OBC = 180°$

$\Rightarrow \angle AOB + 90° = 180°$

$\Rightarrow \angle AOB = 90°$. Hence, **proved**.

Question 10:

Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact at the centre.

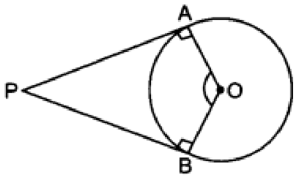
Solution:

PA and PB are two tangents, A and B are the points of contact of the tangents.

$OA \perp AP$ and $OB \perp BP$

$\angle OAP = \angle OBP = 90°$

[Radius and tangent are perpendicular to each other]



p In quadrilateral OAPB

$\angle OAP + \angle OBP + \angle APB + \angle AOB = 360°$

$\Rightarrow 90° + 90° + \angle APB + \angle AOB = 360°$

$\angle APB + \angle AOB = 360° - 180° = 180°$

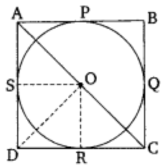
Hence, $\angle APB$ and $\angle AOB$ are supplementary angles.

Question 11:

Prove that the parallelogram circumscribing a circle is a rhombus.

Solution:

We have a parallelogram ABCD which circumscribes a circle with centre O. P, Q, R and S are the points of contact of sides AB, BC, CD and DA respectively.



In ΔORC and ΔOSA,

$\angle ORC = \angle OSA$ [Each of 90°]

$OC = OA$ [O is the midpoint of AC]

$OR = OS$ [Radii of the same circle]

$\therefore \Delta ORC \cong \Delta OSA$ [By RHS congruence]

$\Rightarrow RC = AS$ ... (i) [By CPCT]

$\Rightarrow DR = DS$ ... (ii) [Tangents from the point]

Adding equations (i) and (ii), we get:

$RC + DR = AS + DS$

$\Rightarrow DC = AD$

$\Rightarrow AB = DC, AD = BC$

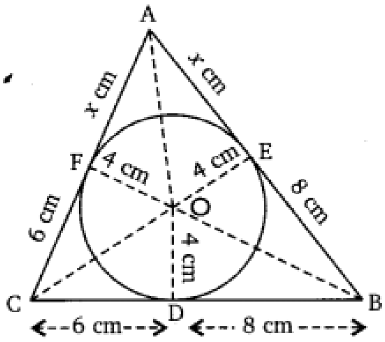
[ABCD is a parallelogram]

$\Rightarrow$ ABCD is a rhombus.

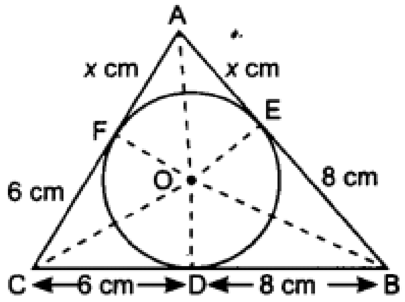
Hence, **proved**.

Question 12:

A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively (see figure). Find the sides AB and AC.



Solution:



BD = 8 cm and DC = 6 cm

BE = BD = 8 cm

CD = CF = 6 cm

Let AE = AF = x cm

In ΔABC, a = 6 + 8 = 14 cm

b = (x + 6) cm

c = (x + 8) cm

$$s = \frac{a+b+c}{2} = \frac{14+x+6+x+8}{2} = \frac{2x+28}{2} = (x+14) \text{ cm}$$

$$\text{ar}(\Delta ABC) = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{(x+14)\times x\times 8\times 6} = \sqrt{48x\times(x+14)} \text{ cm}^2 \quad ...(i)$$

Again, $\text{ar}(\Delta ABC) = \text{ar}(\Delta OBC) + \text{ar}(\Delta OCA) + \text{ar}(\Delta OAB)$

$$= \frac{1}{2}\times 4\times a + \frac{1}{2}\times 4\times b + \frac{1}{2}\times 4\times c$$

$$= 2a + 2b + 2c = 2(a+b+c) = 2\times 2(x+14) \quad ...(ii)$$

From (*i*) and (*ii*), we get

$$\sqrt{48x(x+14)} = 4(x+14) \Rightarrow 48x(x+14) = 4^2(x+14)^2$$

$$\Rightarrow 48x(x+14) = 16(x+14)^2 \Rightarrow 3x(x+14) = (x+14)^2$$

$$\Rightarrow 3x = x + 14 \Rightarrow 2x = 14 \Rightarrow x = 7$$

$$AB = x + 8 = 7 + 8 = 15 \text{ cm}$$

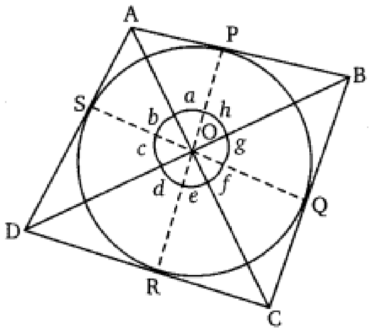
$$AC = x + 6 = 7 + 6 = 13 \text{ cm}$$

Question 13:

Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Solution:

In the figure, P, Q, R and S are the points touching the circle and sides AB, BC, CD and DA of the quadrilateral ABCD respectively.



From the figure, we observe that OA bisects $\angle SOP$.

So, $\angle a = \angle b$ ... (i)

Similarly, $\angle c = \angle d$ ... (ii)

$\angle e = \angle f$ ... (iii)

$\angle g = \angle h$ ... (iv)

$\therefore \quad 2(\angle a + \angle h + \angle e + \angle d) = 360°$

$\Rightarrow \quad (\angle a + \angle h) + (\angle e + \angle d) = 180°$

$\Rightarrow \quad \angle AOB + \angle DOC = 180°.$

Similarly, $\angle AOD + \angle BOC = 180°$

Thus, opposite sides of quadrilateral ABCD subtend supplementary angles at the centre of a circle. Hence, Proved.