EXERCISE 12.2

Question 1:

A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 1 cm and the height of the cone is equal to its radius. Find the volume of the solid in terms of π.

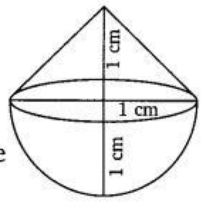
Solution:

Radius of hemisphere and cone, r = 1 cm

Height of cone, $h = 1$ cm

$\therefore$ Total volume of solid

$= $ Volume of cone + Volume of hemisphere



$$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$

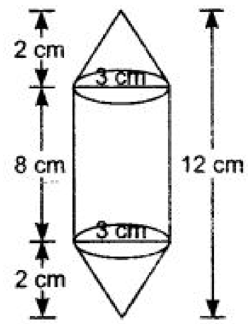
$$= \left[\frac{1}{3}\pi(1)^2 \times 1 + \frac{2}{3}\pi(1)^3\right] \text{cm}^3$$

$$= \left(\frac{1}{3}\pi + \frac{2}{3}\pi\right)\text{cm}^3 = \boldsymbol{\pi}\ \textbf{cm}^3.$$

Question 2:

Rachel, an engineering student, was asked to make a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminium sheet. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of 2 cm, find the volume of air contained in the model that Rachel made. (Assume the outer and inner dimensions of the model to be nearly the same.)

Solution:



Volume of air contained in the model = Total volume of the solid

Diameter of base of each cone = 3 cm

$\therefore$ Radius of base of each cone $= \frac{3}{2}$ cm

Height of each cone = 2 cm

$$\text{Volume of one cone} = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{3}{2}\right)^2 \times 2 \text{ cm}^3$$

$$= \frac{1}{3}\pi\left(\frac{9 \times 2}{4}\right) = \frac{3}{2}\pi \text{ cm}^3$$

$$\therefore \text{Volume of both cones} = 2 \times \frac{3}{2}\pi \text{ cm}^3 = 3\pi \text{ cm}^3$$

$$\text{Volume of the cylindrical portion} = \pi r^2 h = \pi\left(\frac{3}{2}\right)^2 \times 8 \text{ cm}^3 = \frac{\pi \times 9 \times 8}{4} \text{ cm}^3 = 18\pi \text{ cm}^3$$

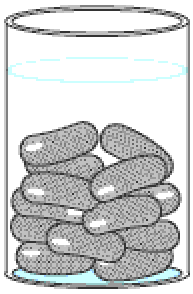
$$\text{Volume of air contained in the model} = \text{Total volume of the solid}$$

$$= 3\pi \text{ cm}^3 + 18\pi \text{ cm}^3 = 21\pi \text{ cm}^3$$

$$= \frac{21 \times 22}{7} \text{ cm}^3 = 66 \text{ cm}^3$$

Question 3:

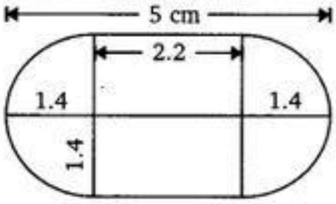
A gulab jamun, contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 gulab jamuns, each shaped like a cylinder with two hemispherical ends with length 5 cm and diameter 2.8 cm.



Solution:

Radius of hemisphere part = 1.4 cm

Length of cylindrical part = 5 – 2.8 = 2.2 cm and its radius = 1.4 cm



Volume of one *gulab jamun*

$$= 2(\text{Volume of hemispherical part}) + \text{Volume of cylindrical part}$$

$$= \left[2 \times \frac{2}{3}\pi \times (1.4)^3 + \pi(1.4)^2 \times 2.2\right] \text{cm}^3$$

$$= \pi(1.4)^2 \times \left[\frac{4}{3} \times 1.4 + 2.2\right] \text{cm}^3$$

$$= \frac{22}{7} \times 1.4 \times 1.4 \times \left[\frac{5.6}{3} + 2.2\right] \text{cm}^3$$

$$= (6.16 \times 4.07) \text{ cm}^3 = 25.07 \text{ cm}^3.$$

$\therefore$ Volume of 45 *gulab jamuns*

$$= (25.07 \times 45) \text{ cm}^3$$

$$= 1128.15 \text{ cm}^3.$$

$\therefore$ Volume of syrup $= 1128.15 \times \dfrac{30}{100}$ cm^3

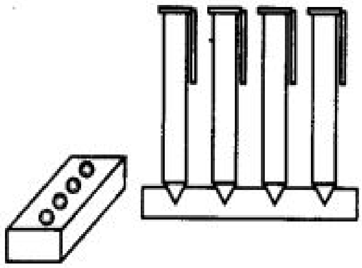
$$= 338.45 \text{ cm}^3.$$

Hence, there would be approximately 338 cm^3 syrup in 45 gulab jamuns.

Question 4:

A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth is 1.4 cm. Find the volume of wood in the entire stand.

Solution:



Solution:

Radius of one conical depression = 0.5 cm

Depth of one conical depression = 1.4 cm

Volume of one conical depression $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times (0.5)^2 \times 1.4\,\text{cm}^3$

$= \frac{1}{3} \times \frac{22}{7} \times 0.5 \times 0.5 \times 1.4\ \text{cm}^3 = 0.366\ \text{cm}^3$

$\therefore$ Volume of four conical depressions

$= 4 \times 0.366\ \text{cm}^3 = 1.464\ \text{cm}^3$

Volume of cuboidal box $= l \times b \times h$

$= 15 \times 10 \times 3.5\ \text{cm}^3 = 525\ \text{cm}^3$

Remaining volume of box = Volume of cubical box – Volume of four conical depressions

$= 525\ \text{cm}^3 - 1.464\ \text{cm}^3 = 523.5\ \text{cm}^3$

Question 5:

A vessel is in the form of an inverted cone. Its height is 8 cm and the radius of its top, which is open, is 5 cm. It is filled with water up to the brim. When lead shots, each of which is a sphere of radius 0.5 cm are dropped into the vessel, one-fourth of the water flows out. Find the number of lead shots dropped in the vessel.

Solution:

Radius of cone, r = 5 cm

Height of cone, h = 8 cm

$$\text{Volume of water in the cone} = \left[\frac{1}{3}\pi(5)^2 \times 8\right]\text{cm}^3$$

$$= \frac{200\pi}{3}\text{cm}^3$$

$$\text{Volume of water flows out} = \left(\frac{1}{4} \times \frac{200\pi}{3}\right)\text{cm}^3$$

$$= \frac{50\pi}{3}\text{ cm}^3$$

Radius of one spherical shot = 0.5 cm [Given]

$$\text{Volume of one spherical shot} = \left[\frac{4}{3}\pi(0.5)^3\right]\text{cm}^3$$

$$= \frac{\pi}{6}\text{ cm}^3$$

Number of lead shots dropped

$$= \frac{\text{Volume of water flows}}{\text{Volume of one spherical shot}}$$

$$= \frac{\frac{50}{3}\pi}{\frac{\pi}{6}} = \frac{50 \times 6}{3} = \mathbf{100}.$$

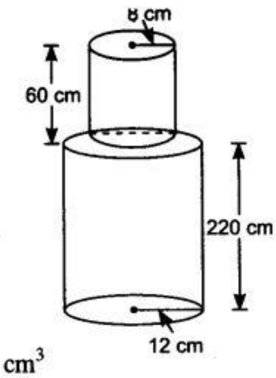
Question 6:

A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm³ of iron has approximately 8g mass.

Solution:

Given: radius of 1st cylinder = 12 cm
and height of 1st cylinder = 220 cm
∴ Volume of 1st cylinder = $\pi r^2 h$

$= \pi(12)^2 (220)\ cm^3$
$= 144 \times 220\pi\ cm^3$
$= 144 \times 220 \times 3.14\ cm^3$
$= 99475.2\ cm^3$ ...(*i*)



Given: radius of 2nd cylinder = 8 cm
and height of 2nd cylinder = 60 cm
∴ Volume of 2nd cylinder = $\pi r^2 h$

$= \pi(8)^2 (60)\ cm^3 = 64 \times 60\pi\ cm^3$
$= 64 \times 60 \times 3.14\ cm^3$
$= 12057.6\ cm^3$ ...(*ii*)

Total volume of solid = Volume of 1st cylinder + Volume of 2nd cylinder
$= 99475.2\ cm^3 + 12057.6\ cm^3 = 111532.8\ cm^3$

Given: mass of 1 cm^3 of iron = 8 g
∴ Mass of 111532.8 cm^3 of iron = 111532.8×8 g
= 892262.4 g = 892.262 kg

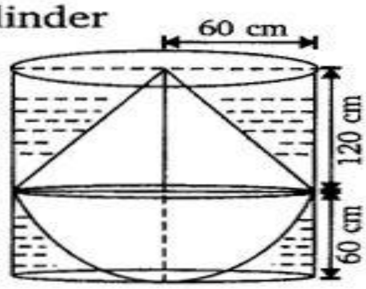
Question 7:

A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm.

Solution:

Radius of cylinder, R = 60 cm
Height of cylinder, H = 180 cm
∴ Volume of water in the cylinder

$= \pi R^2 H = \pi(60)^2 \times 180$
$= (\pi \times 3600 \times 180) cm^3$



Volume of water flows out

= Volume of conical part
+ Volume of hemispherical part

$$= \left[\frac{1}{3}\pi(60)^2 \times 120 + \frac{2}{3}\pi(60)^3\right] cm^3$$

$$= \left[\frac{4}{3}\pi(60)^3\right] cm^3$$

Volume of water left in the cylinder

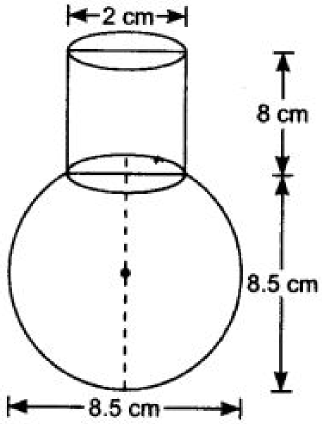
$$= \left[\pi \times 3600 \times 180 - \frac{4}{3}\pi \times 3600 \times 60\right] cm^3$$

Question 8:

A spherical glass vessel has a cylindrical neck 8 cm long, 2 cm in diameter; the diameter of the spherical part is 8.5 cm. By measuring the amount of water it holds, a child finds its volume to be 345 cm³. Check whether she is correct, taking the above as the inside measurements, and π = 3.14.

Solution:

Volume of water the glass vessel can hold = 345 cm^3 (Measured by the child)



Radius of the cylindrical part $= \frac{2}{2}$ cm $= 1$ cm

Height of the cylindrical part $= 8$ cm

$\therefore$ Volume of the cylindrical part $= \pi r^2 h$

$= 3.14 \times (1)^2 \times 8 \text{ cm}^3$

$= 25.12 \text{ cm}^3$

Diameter of the spherical part $= 8.5$ cm

$\therefore$ Radius $= \frac{8.5}{2} \text{ cm} = \frac{85}{20}$ cm

$\therefore$ Volume of the spherical part $= \frac{4}{3}\pi r^3 = \frac{4}{3} \times 3.14 \times \frac{85 \times 85 \times 85}{20 \times 20 \times 20} = 321.39 \text{ cm}^3$

Total volume of the glass vessel = Volume of the cylindrical part + Volume of the spherical part

$= 25.12 \text{ cm}^3 + 321.39 \text{ cm}^3 = 346.51 \text{ cm}^3$

Volume measured by child is 345 cm^3, which is not correct. Correct volume is 346.51 cm^3.