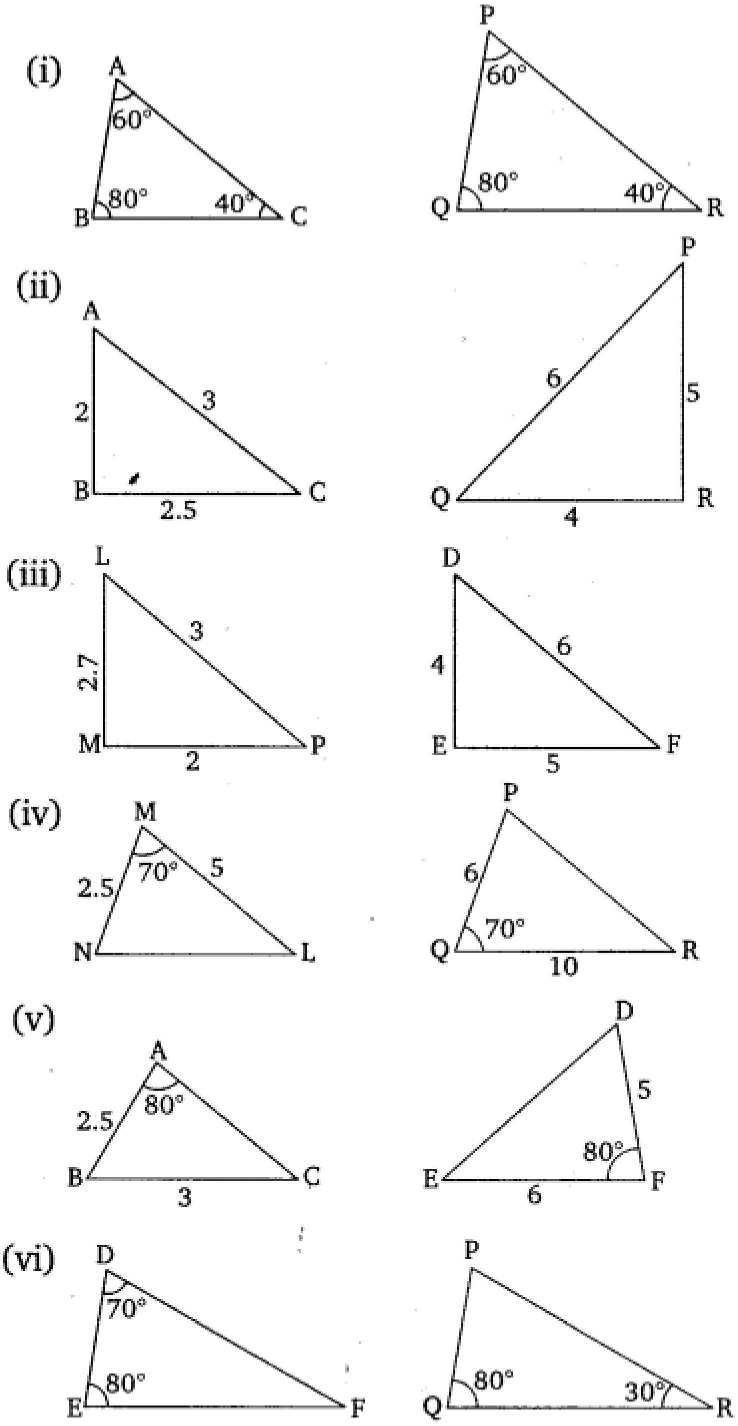
EXERCISE 6.3

Question 1:

State which pairs of triangles in the given figures are similar. Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form :



Solution:

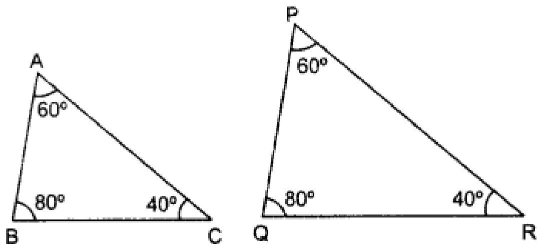
(*i*) In ΔABC and ΔPQR

$\angle A = \angle P$ [Each 60°]

$\angle B = \angle Q$ [Each 80°]

$\angle C = \angle R$ [Each 40°]

$\therefore$ $\Delta ABC \sim \Delta PQR$ [AAA criterion]



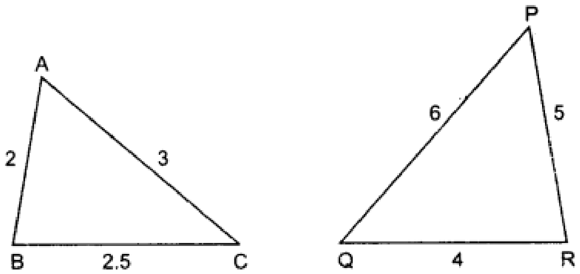
(*ii*) In ΔABC and ΔPQR

$$\frac{BC}{PR} = \frac{2.5}{5} = \frac{1}{2}$$

$$\frac{AB}{QR} = \frac{2}{4} = \frac{1}{2}$$

$$\frac{AC}{PQ} = \frac{3}{6} = \frac{1}{2}$$

Hence, $\Delta ABC \sim \Delta QRP$



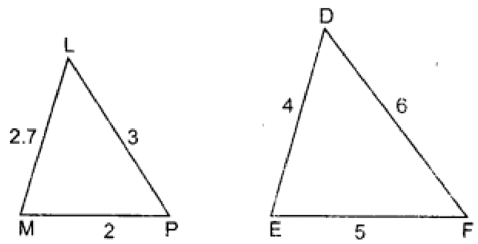
[SSS criterion]

(*iii*) In ΔLMP and ΔEFD

$$\frac{LM}{EF} = \frac{2.7}{5}$$

$$\frac{LP}{DF} = \frac{3}{6} = \frac{1}{2}$$

$$\frac{MP}{DE} = \frac{2}{4} = \frac{1}{2}$$



$\therefore$ ΔLMP is not similar to ΔEFD.

Since the three ratios are not same.

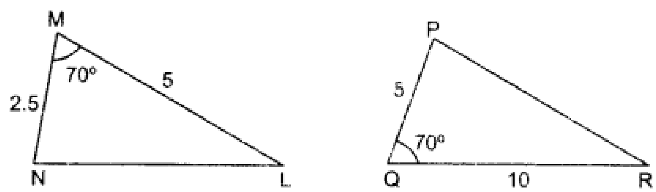
(*iv*) In ΔMNL and ΔPQR

$$\frac{MN}{PQ} = \frac{2.5}{5} = \frac{1}{2}$$

$$\frac{ML}{QR} = \frac{5}{10} = \frac{1}{2}$$

$\angle M = \angle Q = 70°$

$\Delta NML \sim \Delta PQR$ [SAS]



(*v*) In ΔABC and ΔDEF

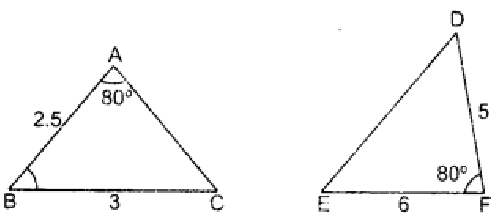
$$\frac{AB}{DF} = \frac{2.5}{5} = \frac{1}{2}$$

$$\frac{BC}{EF} = \frac{3}{6} = \frac{1}{2}$$

$\angle A = \angle F = 80°$

ΔABC is not similar to ΔDEF

$\because$ Angles between two sides are not same.



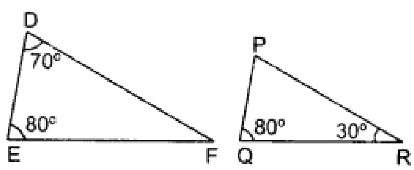
(*vi*) In ΔDEF and ΔPQR

$\angle E = \angle Q = 80°$

$\angle F = \angle R = 30°$

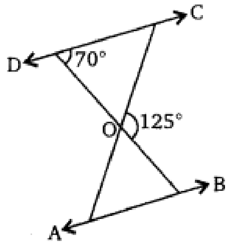
[$\therefore$ F = 180° – (80° + 70°) = 30°]

$\therefore$ $\Delta DEF \sim \Delta PQR$ [AA]



Question 2:

In the given figure, ΔODC ~ ΔOBA, ∠BOC = 125° and ∠CDO = 70°. Find ∠DOC, ∠DCO and ∠OAB.



Solution:

From the given figure,

$\angle DOC + 125^\circ = 180^\circ$ [Linear pair]

$\Rightarrow \angle DOC = 180^\circ - 125^\circ = 55^\circ$

Now, in ΔDOC,

$\angle DCO + \angle ODC + \angle DOC = 180^\circ$

[Angle sum property of a triangle]

$\Rightarrow \angle DCO + 70^\circ + 55^\circ = 180^\circ$

$\Rightarrow \angle DCO = 180^\circ - 125^\circ = 55^\circ$

Now, $\Delta ODC \sim \Delta OBA$ [Given]

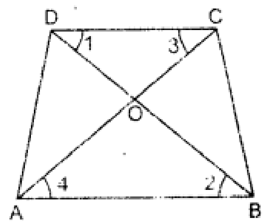
$\therefore \angle OAB = \angle OCD = 55^\circ$

Hence, ∠DOC = **55°**, ∠DCO = **55°** and ∠OAB = **55°**.

Question 3:

Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at the point O. Using a similarity criterion for two triangles, show that $OA/OC = OB/OD$.

Solution:



Given: Diagonals AC and BD intersect at O.

AB || DC

To Prove: $\frac{OA}{OC} = \frac{OB}{OD}$

Proof: In ΔAOB and ΔCOD

$\angle 1 = \angle 2$

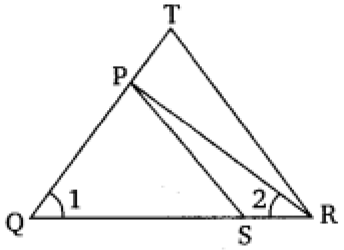
$\angle 3 = \angle 4$ [Alternate angles]

$\therefore \Delta AOB \sim \Delta COD$ [AA]

$\Rightarrow \frac{OA}{OC} = \frac{OB}{OD}$ [Corresponding sides of similar triangles]

Question 4:

In the given figure, $QR/QS = QT/PR$ and ∠1 = ∠2. show that ΔPQR ~ ΔTQR.



Solution:

From the figure,

$\angle 1 = \angle 2$

$\therefore PQ = PR$ [Sides opposite to equal angles are equal]

In ΔPQS and ΔTQR

$$\Rightarrow \frac{QR}{QS} = \frac{QT}{PR}$$

$$\Rightarrow \frac{QR}{QS} = \frac{QT}{PR} \qquad [\because PQ = PR \text{ proved above}]$$

$\angle PQS = \angle TQR = \angle 1$

$\therefore \Delta PQS \sim \Delta TQR$ [By SAS similarity]

Hence, **proved**.

Question 5:

S and T are points on sides PR and QR of ΔPQR such that ∠P = ∠RTS. Show that ΔRPQ ~ ΔRTS.

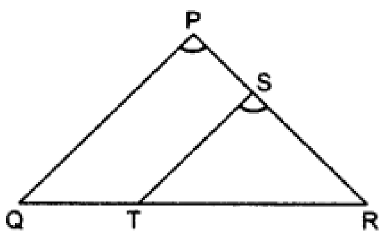
Solution:

In ΔRPQ and ΔRTS,

$\angle P = \angle RTS$ [Given]

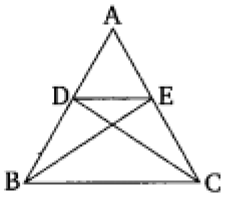
$\angle R = \angle R$ [Common]

$\therefore \quad \Delta RPQ \sim \Delta RTS$ [AA]



Question 6:

In the given figure, if ΔABE ≅ ΔACD, show that ΔADE ~ ΔABC.



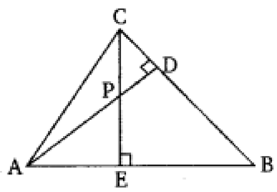
Solution:

$$\Delta ABE \cong \Delta ACD, \qquad \text{[Given]}$$
$$AB = AC \qquad \text{[By CPCT]}$$
$$\text{and} \quad AE = AD \qquad \text{[By CPCT]}$$
$$\therefore \quad \frac{AB}{AC} = \frac{AD}{AE} = \frac{1}{1}$$
$$\text{and } \angle DAE = \angle BAC \qquad \text{[Common]}$$
$$\therefore \quad \Delta ADE \sim \Delta ABC \qquad \text{[By SAS similarity]}$$

Hence, **proved**.

Question 7:

In the given figure, altitudes AD and CE of ΔABC intersect each other at the point P. Show that:
(i) ΔAEP ~ ΔCDP
(ii) ΔABD ~ ΔCBE
(iii) ΔAEP ~ ΔADB
(iv) ΔPDC ~ ΔBEC



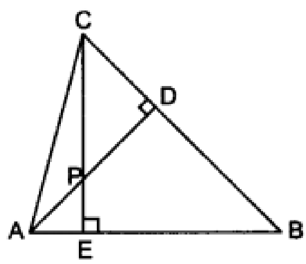
Solution:

Given: AD and CE are altitudes of the ΔABC

(*i*) **To Prove:** ΔAEP ~ ΔCDP

Proof: In ΔAEP and ΔCDP,

$$\angle AEP = \angle CDP \qquad \text{[Each 90°]}$$
$$\angle APE = \angle CPD \qquad \text{[Vertically opposite angles]}$$
$$\Delta AEP \sim \Delta CDP \qquad \text{[AA]}$$



(*ii*) In ΔABD and ΔCBE,

$$\angle ADB = \angle CEB \qquad \text{[Each 90°]}$$
$$\angle ABD = \angle CBE \qquad \text{[Common]}$$
$$\Delta ABD \sim \Delta CBE \qquad \text{[AA]}$$

(*iii*) In ΔAEP and ΔADB,

$\angle AEP = \angle ADB$ [Each 90°]

$\angle A = \angle A$ [Common]

$\therefore$ $\Delta AEP \sim \Delta ADB$ [AA]

(*iv*) In ΔPDC and ΔBEC,

$\angle PDC = \angle BEC$ [Each 90°]

$\angle PCD = \angle BCE$ [Common]

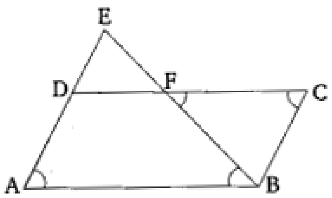
$\Delta PDC \sim \Delta BEC$ [AA]

Question 8:

E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that ΔABE ~ ΔCFB.

Solution:

In the given figure, is shown a parallelogram ABCD, in which E is a point on AD produced and BE intersects CD at F.



In parallelogram ABCD,

$\angle A = \angle C$ ...(i) [Opposite angles]

In ΔABE and ΔCFB,

$\angle EAB = \angle BCF$ [Proved above]

and $\angle ABE = \angle BFC$ [Alternate angles as DC || AB]

$\therefore$ $\Delta ABE \sim \Delta CFB$ [By AA similarity]

Hence, **proved**.

Question 9:

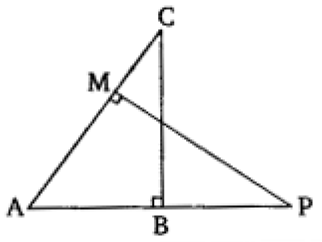
In the given figure, ABC and AMP are two right triangles, right angled at B and M respectively.

Prove that:

Prove that:

(i) $\Delta ABC \sim \Delta AMP$

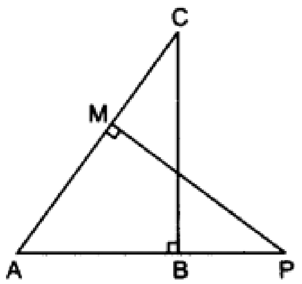
(ii) $\dfrac{CA}{PA} = \dfrac{BC}{MP}$



Solution:

(*i*) $\Delta ABC \sim \Delta AMP$

(*ii*) $\dfrac{CA}{PA} = \dfrac{BC}{MP}$



Solution:

(*i*) In ΔABC and ΔAMP,

$\angle B = \angle AMP$ [Each 90°]

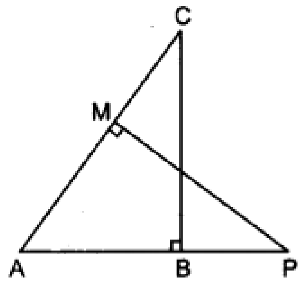
$\angle A = \angle A$ [Common]

$\Rightarrow \Delta ABC \sim \Delta AMP$ [AA]

(*ii*) $\Delta ABC \sim \Delta AMP$ [proved above]

$\Rightarrow \dfrac{CA}{PA} = \dfrac{CB}{PM}$

[Ratio of the corresponding sides of similar triangles]



Question 10:

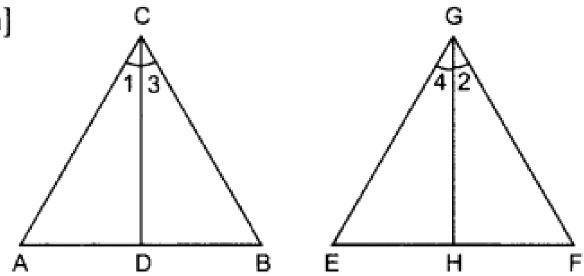
CD and GH are respectively the bisectors of ∠ACB and ∠EGF such that D and H lie on sides AB and FE of ΔABC and ΔEFG respectively. If ΔABC ~ ΔFEG, show that

(i) $\frac{CD}{GH} = \frac{AC}{FG}$ (ii) $\Delta DCB \sim \Delta HGE$

(iii) $\Delta DCA \sim \Delta HGF$

Solution:

$\Delta ABC \sim \Delta FEG$ [Given]



$\Rightarrow \angle A = \angle F$

$\angle B = \angle E$

$\angle C = \angle G$

and $\frac{AB}{FE} = \frac{BC}{EG} = \frac{AC}{FG}$

(*i*) In ΔACD and ΔFGH,

$\angle A = \angle F$ [Given]

$\angle 1 = \angle 2$ $[\frac{1}{2}\angle C = \frac{1}{2}\angle G]$

$\therefore \Delta ACD \sim \Delta FGH$ [AA]

$\Rightarrow \frac{CD}{GH} = \frac{AC}{FG}$ [Corresponding sides of similar triangles]

(*ii*) $\frac{CD}{GH} = \frac{AC}{FG}$ [Proved above]

But $\frac{AC}{FG} = \frac{BC}{EG}$

$\therefore \frac{CD}{GH} = \frac{BC}{EG}$

In ΔDCB and ΔHGE,

$\angle 3 = \angle 4$ $[\frac{1}{2}\angle C = \frac{1}{2}\angle G]$

$\frac{CD}{GH} = \frac{BC}{EG}$ [Proved above]

$\therefore \Delta DCB \sim \Delta HGE$ [SAS]

(*iii*) In ΔDCA and ΔHGF,

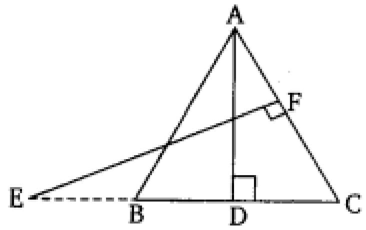
$\angle 1 = \angle 2$ [Bisectors]

$\frac{CD}{GH} = \frac{AC}{FG}$ [As proved]

$\Rightarrow DCA \sim \Delta HGF$ [SAS]

Question 11:

n the given figure, E is a point on side CB produced of an isosceles triangle ABC with AB = AC. If AD ⊥ BC and EF ⊥ AC, prove that ΔABD ~ ΔECF.



Solution:

Given: ΔABC is an isosceles triangle.

So, AB = AC [Given]

$\therefore \angle ABC = \angle ACB$ ...(i)

[Angles opposite to equal sides are equal]

In ΔABD and ΔECF,

$\angle ABD = \angle ECF$ [Proved above]

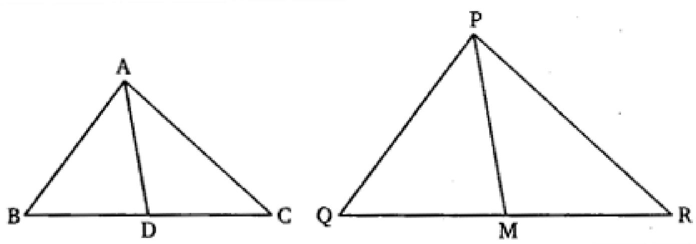
$\angle ADB = \angle EFC$ [Each of 90°]

$\Delta ABD \sim \Delta ECF$ [By AA similarity]

Hence, **proved**.

Question 12:

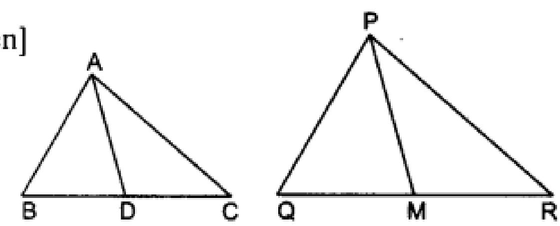
Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of ΔPQR (see in given figure). Show that ΔABC ~ ΔbPQR.



Solution:

In ΔABC and ΔPQR,

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$$ [Given]



or $$\frac{AB}{PQ} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR} = \frac{AD}{PM}$$

$$\Rightarrow \quad \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$$

$\Rightarrow \quad \Delta ABD \sim \Delta PQM$ [SAS]

$\therefore \quad \angle B = \angle Q$ [Corresponding angles of similar triangles]

In ΔABC and ΔPQR,

$$\frac{AB}{PQ} = \frac{BC}{QR}$$ [Given]

$\angle B = \angle Q$ [As proved]

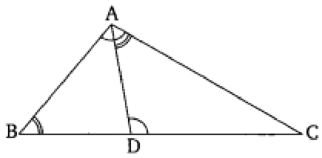
$\therefore \quad \Delta ABC \sim \Delta PQR$ [SAS]

Question 13:

D is a point on the side BC of a triangle ABC, such that ∠ADC = ∠BAC. Show that CA² = CB.CD.

Solution:

In the figure given below is shown a triangle ABC in which $\angle ADC = \angle BAC$.



In ΔABC and ΔDAC,

$\angle C = \angle C$ [Common]

$\angle BAC = \angle ADC$ [Given]

$\therefore \ \Delta ABC \sim \Delta DAC$ [By AA similarity]

Thus, their corresponding sides are proportional.

$$\therefore \quad \frac{CA}{CD} = \frac{CB}{CA}$$

$\Rightarrow \ CA^2 = CB \times CD$ Hence, **proved**.

Question 14:

Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that ΔABC ~ ΔPQR.

Solution:

Construction: Draw DE||AC and MS||PR

Proof: In ΔABC, D is mid-point of BC [given DE||AC (const.)]

$\therefore$ E is mid-point of AB (converse of mid-point theorem)

$$\Rightarrow \quad DE = \frac{1}{2}AC$$

$$\text{Similarly} \quad SM = \frac{1}{2}PR$$

$$\text{Now,} \quad \frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM} \quad \text{[given]}$$

$$\Rightarrow \quad \frac{2AE}{2PS} = \frac{2DE}{2SM} = \frac{AD}{PM} \Rightarrow \frac{AE}{PS} = \frac{DE}{SM} = \frac{AD}{PM}$$

$\therefore \quad \Delta ADE \sim \Delta PMS$ [SSS similarly]

$\Rightarrow \quad \angle 1 = \angle 3$

Similarly $\quad \angle 2 = \angle 4$

$$\angle 1 + \angle 2 = \angle 3 + \angle 4 \Rightarrow \angle A = \angle P$$

Now, in ΔABC and PQR,

$$\frac{AB}{PQ} = \frac{AC}{PR} \quad \text{[Given]}$$

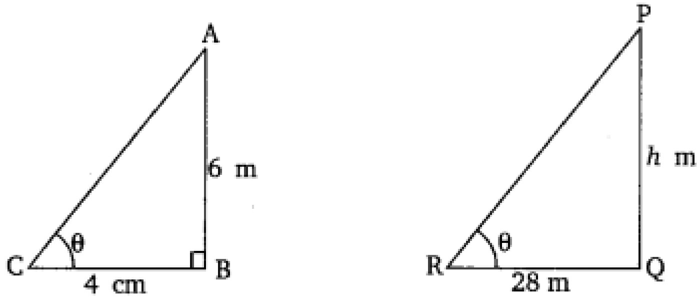
$\angle A = \angle P$ [Proved above]

$\therefore \quad \Delta ABC \sim \Delta PQR$ (SAS)

Question 15:

A vertical pole of length 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

Solution:



Let in ΔABC, AB be the pole and BC its shadow. Also let in ΔPQR, PQ be the tower of height h m and QR be its shadow.

Then when Q is the altitude of the sun.

$\Delta ABC \sim \Delta PQR$ [By AA similarity]

$$\Rightarrow \quad \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$

$$\Rightarrow \quad \frac{BC}{QR} = \frac{AC}{PR}$$

$$\Rightarrow \quad \frac{4}{28} = \frac{6}{h}$$

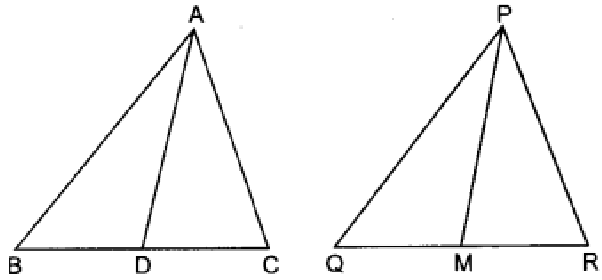
$$\Rightarrow \quad h = \frac{6 \times 28}{4} = 42 \text{ m.}$$

Hence, the height of the tower is 42 m.

Question 16:

If AD and PM are medians of triangles ABC and PQR respectively, where $\Delta ABC \sim \Delta PQR$. Prove that $AB/PQ = AD/PM$.

Solution:



When $\Delta ABC \sim \Delta PQR$

$\Rightarrow \angle ABC = \angle PQR$

$$\frac{AB}{PQ} = \frac{BC}{QR}$$

$$\frac{AB}{PQ} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR}$$

$$\frac{AB}{PQ} = \frac{BD}{QM}$$

In ΔABD and ΔPQM,

$$\frac{AB}{PQ} = \frac{BD}{QM} \quad \text{[As proved]}$$

$$\angle B = \angle Q$$

$\therefore \Delta ABD \sim \Delta PQM$

$$\frac{AB}{PQ} = \frac{AD}{PM} \quad \text{[Corresponding sides of similar triangles]}$$