

Assignment 2



①

$$x \equiv 2 \pmod{3}$$

$$x \equiv 3 \pmod{5}$$

$$x \equiv 2 \pmod{7}$$

$$x \equiv \begin{matrix} & \pmod{3} & \pmod{5} & \pmod{7} \\ 35 & + & 21 & + & 15 \end{matrix}$$

Applying mod 3

$$\begin{aligned} x &\equiv (35 + 0 + 0) \pmod{3} \\ &\equiv 2 \pmod{3} \end{aligned}$$

Applying mod 5

$$\begin{aligned} &\equiv (0 + 21 + 0) \pmod{5} \\ &\equiv 1 \pmod{5} \end{aligned}$$

So, we need to multiply 21 by 3.

Applying mod 7

$$\begin{aligned} &\equiv (0 + 0 + 15) \pmod{7} \\ &\equiv 1 \pmod{7} = 2 \pmod{7} \end{aligned}$$

(Multiply 15 by 2)

$$\equiv (35 + 21 \cdot 3 + 15 \cdot 2)$$

$$\equiv 128 \pmod{105}$$

Ans

128 mod 105 = 23

Q7)

$$30 = 2 \cdot 3 \cdot 5$$

$$g(d(2, 3, 5)) = 1$$

Using Chinese Remainder Theorem

for each $k = 0, \dots, 29 \pmod{30}$

\exists unique (a, b, c) s.t

$$k \equiv a \pmod{2} \text{ \& } b \pmod{3}$$

$$k \equiv c \pmod{5}$$

So, $N = 2! \cdot 3! \cdot 5! = 1440$ and

1440 is Ans

$$\textcircled{2} \quad x^2 \equiv 1 \pmod{3}$$

$$x \equiv 2 \pmod{4}$$

$$\Rightarrow x^2 \equiv 4 \pmod{4}$$

$$x^2 \equiv 1 + 4$$

$$\Rightarrow x^2 \equiv 5 \pmod{3}$$

$$\equiv 2 \pmod{3}$$

$$\equiv 1 \pmod{3} \text{ (Divide eqn by 2)}$$

$$x^2 \equiv 5 \pmod{4} \equiv 1 \pmod{4}$$

$$\equiv 4 \pmod{4} \text{ (Multiply 1 by 4)}$$

$$\Rightarrow x^2 \equiv \frac{1}{2} + \frac{4}{2} \equiv 1 + 2$$

$$\Rightarrow 4 + 2 \equiv 6 \pmod{12}$$

$$x \equiv 2 \pmod{12}$$

$$\& x \equiv 3 \pmod{12}$$