Assignment · 1

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(1) For $n \in Z$ and $n > 0$
we have, $169 / 3$
$169 \mid 3^{3n+3} - 26n - 27$

Ans As for $n=1$ the result of expression is true

$\Rightarrow 3^{6} - 26 - 27 = 676 = 4 \times 169$

Let the result be true for $n = k$

$\Rightarrow 169 \mid 3^{3k+3} - 26k - 27$

We need to proove for $n = k+1$

$169 \mid 3^{3n+6} - 26(k+1) - 27$

$3^{3n+6} - 26(k+1) - 27 = 3^{3}(3^{3k+3}) - 26k - 26 - 27$

$= (3^{3k+3} - 26k - 26) + 26 \cdot [3^{3k+3} - 1]$

So, As $3^{3} = 1 \pmod{13}$
So, $3^{3k} = 1 \pmod{13}$
~~$\Rightarrow 3^{3k} = 3$~~

$\Rightarrow 3^{3k+3} = 3^{3} \,\&\, \pmod{13} \Rightarrow 3^{3k+3} = 1 \pmod{13}$

$\Rightarrow 13 \mid 3^{3k+3} - 1 \quad \& \; 169 \mid 26 \cdot (3^{3k+3} - 1)$

$\Rightarrow 169 \mid (3^{3k+3} - 26k - 27) + 26[3^{3k+3} - 1]$

Hence its true for all Positive integer

Q4)

$2 \equiv 2 \bmod 17$

$2^2 \equiv 2^2 \bmod 17$

$\equiv 4 \bmod 17$

$\Rightarrow 2^4 \equiv 16 \bmod 17 \equiv (-1) \bmod 17$

$\Rightarrow 2^8 \equiv 1 \bmod 17$

$\Rightarrow 2^{8k} \equiv 1 \bmod 17$

$2^{80} \equiv 1 \bmod 17$

$\Rightarrow 2^{81} \equiv 2 \bmod 17$

$\Rightarrow$ Remainder of $2^{81} = 2$

(b) for $n=1$

$\Rightarrow 2 + 6 \cdot 9 = 56$ so its true

let this be true for $n=k$

Now for checking $n = k+1$

$\Rightarrow 7 \mid 2^{k+1} + 6 \cdot 9^{k+1}$

$= 2^n + 6 \cdot 9^n + 1 + 48$

$= 2^n + 6 \cdot 9^n + 49$

Thus its true for $n = k+1$

Hence true for any positive integer k.

$\therefore a^2 + b^2 = 7(k+l)$

Q2 Ans

$(n+1) \equiv 1 \mod (n^2)$

$\Rightarrow (n+1)^n \equiv 1 \mod (n^2)$

$\therefore$ ~~$(n+1)^n - 1$~~ $n^2 \mid (n+1)^n - 1$

$\frac{2}{4}$

~~X~~ ~~Q~~

Q7) No is divisible by 3 or 9 by summing the digits.

Looking at one's place

0, 1, 2, 3, 4, 5, 6, 7, 8, 9

$0+1+2+\dots 9=45$ This is divisible by 9.

This excludes 19, 90, 91, 92.

$9+1+2=12$

(Unit sum is 3 (mod 9)).

10 sets of ten's digit for 2, 3, 4 ... 8

$\frac{8\cdot 9}{2}-1=35$ congruent to 8 (mod 9)

Again 19, 90, 91, 92

$1+9\cdot 3=28$

28 is congruent to 1 mod 9. So sum is congruent to 3 (mod 9) $\boxed{K=1}$