Ques 4:

A complete undirected graph is a graph where the total no of edges is n(n-1)/2 and there must be a path from each node to every other node

For 1 vertex, we have: (1)

For 2 vertices, we have : (1) -- (2)

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For 3 vertices, we have : (1) -- (2)
                            |     |
                            |     |
                           (3)---->(this is not an edge, just showing connection between
3 and 2)
```

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For 4 vertices, we hvae : (1) -- (2)
                            |     |
                            |     |
                          (3)----(4)
```
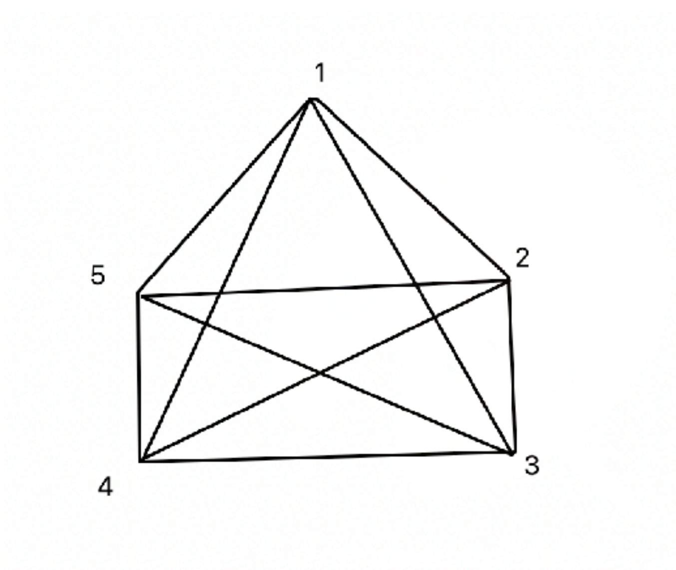

For 5 vertices, it will have a shape of a pentagon with a star inside



In order to prove that an n vertex complete graph is n(n-1) / 2. We can use mathematical induction to prove this.

1st case: n = 0. For 0 nodes, there is exists no eddges

2nd case: $n = 1$. For 1 node, there is no edge because it is an isolated vertex.

3rd case: $n = k$. For k nodes, suppose that $Ek = K(K-1)/2$ and we take a complete graph with $k + 1$ vertices. Therefore if we take one vertex and remove it together wit the edges from it, we get a graph with k vertices

Therefore,

$Ek+ 1 = Ek + n$

Thus, by our assumption:

$Ek + 1 = k(k-1)/2 + k = k^2 + k/2 = k(k+1)/2$.

Ques 5:

The graph is strongly connected as there is a path from each vertex to each vertex for all the total vertices

0 to 1 : 0 -> 1
1 to 0 : 1 -> 2 -> 0
1 to 2:   1 -> 2
2 to 1:   2-> 0 -> 1
3 to 2:   3-> 2
2 to 3:   2 -> 0 -> 3
3 to 0:   3 -> 2 -> 0
0 to 3:   0 -> 3
2 to 0:   2 -> 0
0 to 2:   0 -> 1-> 2 or 0 -> 3 -> 2
1 to 3: 1-> 2 -> 3
3 to 1: 3-> 2->0-> 1

Ques 6:

Adjacency Matrix:

	0	**1**	**2**	**3**
0	0	1	1	1
1	1	0	1	1
2	1	1	0	1
3	1	1	1	0

Adjacent list:

0 —> 1, 2, 3
1 —> 0, 2, 3
2 —> 0, 1, 3
3 —> 0, 1, 2

Multi Adjaceny list:

	0	1	2	3
0				
1				
2				
3				

Question 7: We know that an edge connects two nodes. That means when an edge is introduced, then then degree of both nodes increase by 1. Thus sum of both nodes will be two times always.

Question 8 :

a) We know that in order for a graph to be considered connected, all nodes must have an edge connected to them. Whenever an edge is introduced, it always connects two vertices. There fore if there 1 edge, there are at most two edges connected to them. That means whenever there are n vertices, the number of edges must be n -1 because at most 1 edge connects two edges, therefore it takes one less edge than total vertices to be connected to all of them.

To prove that graphs with n-1 edges are trees, a simple proof would be a graph without any cycles. Trees are acyclic in nature. Since there are n-1 edges in a tree that means that at most there is one edge less than the total number of vertices. If that is true, that means it is not possible to form a cycle because in order to form a cycle, there must be at least n edges for n vertices (also making the graph complete). Thus proving that the graph will also be a tree

b) Minimum no of edges for a strongly connected graph with n vertices is n. These have a form of a circular arrangement.