

BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(MID SEMESTER EXAMINATION SP/2025)

CLASS: BTECH
BRANCH: CSE/AI/ML

SEMESTER : IV/ADD
SESSION : SP/2025

SUBJECT: CS241DESIGN AND ANALYSIS OF ALGORITHMS

TIME: 02 Hours

FULL MARKS: 25

INSTRUCTIONS:

1. The question paper contains 5 questions each of 5 marks and total 25 marks.
2. Attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Tables/Data handbook/Graph paper etc., if applicable, will be supplied to the candidates

- | | | CO | BL |
|---|-----|---------|----|
| Q.1(a) Discuss a practical scenario where asymptotic notation is more useful than considering the actual run-time of an algorithm. Also prove that if $f(n) = 2n^3 + 3n^2 + 1$ and $g(n) = 2n^2 + 3$ then $f(n) = \Omega(g(n))$. | [2] | 1 | 3 |
| Q.1(b) Consider a part of an algorithm below: <pre> int count = 0; for (int i = n; i > 0; i /= 2) { for (int j = 0; j < i; j++) { count++; } } </pre> <ol style="list-style-type: none"> 1. Find the time complexity of the above algorithm. 2. Will there be any difference in the complexity if the outer loop is substituted by for(int i=0; i<n; i*= 2) | [3] | 1 | 4 |
| Q.2(a) Solve the following recurrences using the <i>Recursion Tree</i> method: $T(n) = T(n/3) + n/2$ | [2] | 1 | 3 |
| Q.2(b) Using <i>Master's Theorem</i> solve the following recurrence relations: (1) $T(n) = 2T(n/4) + n^{0.5}$ (2) $T(n) = 2T(n/2) + \log n$ | [3] | 1 | 3 |
| Q.3(a) Suggest an algorithm which dichotomizes any randomly distributed dataset around a pivot value (say p) such that after the partition is achieved, values lesser than p are placed left to it, and the values larger than p are placed right to it. Assuming the last element as the pivot, apply this algorithm to process the following data: 8, 2, 1, 5, 6, 1, 3, 7, 4, 9, 5. | [2] | 2, 3 | 3 |
| Q.3(b) In a class, there are m boys and n girls. Their CGPAs are stored in two arrays B & G, one for the boys (B) in descending order, the other for the girls(G) in ascending order. Devise an efficient $O(m+n)$ algorithm to find out the set (A) of duplicate CGPAs that are common between both the boys and girls in ascending order. | [3] | 3 | 3 |
| Q.4(a) Finding the n^{th} order statistic in a given dataset (of size n) typically requires $(n-1)$ comparisons. Going by this we can find both the 1^{st} and the n^{th} order statistics in $(2n-2)$ comparisons. Suggest an algorithm which finds these order statistics in $kn + c$ comparisons, where k is a positive real constant and is less than 2, and c is an integer value. | [2] | 4 | 6 |
| Q.4(b) Prof Sheshadri has discovered a divide-and-conquer matrix multiplication algorithm that is based on multiplying two 70-by-70 matrices using 143,640 multiplications. Find the asymptotic efficiency of this algorithm and compare it with that of Strassen's algorithm. Give your comment on the admissible number of additions and/or subtractions required. | [3] | 5 | 3 |

PTO

Q.5(a) A data compression system needs to encode the following characters with their given frequencies: [2] 3 4
 $\{(A, 10), (B, 15), (C, 30), (D, 16), (E, 29)\}$

1. Determine the optimal prefix-free binary code for each character.
2. Critically analyse if the applied algorithm always produces an optimal encoding for lossless compression.

Q.5(b) A courier company needs to maximize the profit of a delivery truck that can carry a maximum weight of 20 kg. The available parcels (P_i) have the following weights (W_i) and values (V_i) in Rs.: [3] 3 4
 $(P_i, W_i, V_i) = \{(P_1, 4, 40), (P_2, 8, 60), (P_3, 2, 20), (P_4, 6, 50), P_5, 10, 100)\}$

Task:

1. Compute the maximum profit the company can obtain. Show all steps.
2. Critically evaluate whether the greedy strategy is guaranteed to produce the globally optimal solution.