Let us use a step size as $\beta_n = \dfrac{\alpha}{\bar{O}_n}$ where

$\cancel{Q_{n+1} = \beta_n R_n + (1-\beta_n) Q_n}$ $\bar{O}_n = \bar{O}_{n-1} + \alpha(1 - \bar{O}_{n-1})$

also $\bar{O}_0 = 0$

Q3 $Q_{n+1} = \beta_n R_n + (1-\beta_n) Q_n$

$$= \beta_n R_n + (1-\beta_n)[\beta_{n-1} R_{n-1} + (1-\beta_{n-1}) Q_{n-1}]$$

$$= \beta_n R_n + (1-\beta_n)(\beta_{n-1}) R_{n-1} + (1-\beta_n)(1-\beta_{n-1}) Q_{n-1}$$

Following the pattern, we can see that

$$\text{coefficient of } Q_i = \prod_{k=i}^{k=n} (1-\beta_k)$$

$$\text{coefficient of } Q_1 = (1-\beta_n)(1-\beta_{n-1}) \ldots (1-\beta_1)$$

Further $1-\beta_j = 1 - \dfrac{\alpha}{\bar{O}_j} = \dfrac{\bar{O}_j - \alpha}{\bar{O}_j} = \dfrac{\bar{O}_{j-1}(1-\alpha)}{\bar{O}_j}$

Hence coeff of Q_1 =

$$\left\{\frac{(1-\alpha)\bar{O}_{n-1}}{\bar{O}_n}\right\} \cdot \left\{\frac{(1-\alpha)\bar{O}_{n-2}}{\bar{O}_{n-1}}\right\} \cdot \left\{\frac{(1-\alpha)\bar{O}_{n-3}}{\bar{O}_{n-2}}\right\} \cdots \left\{\frac{(1-\alpha)\bar{O}_0}{\bar{O}_1}\right\}$$

Since $\bar{O}_0 = 0$

coeff. of $Q_1 = 0$ (no bias)

Since coeff of $Q_i = \prod_{k=i}^{k=n}(1-\beta_k)$, this is an exponential recency weighted average.

since $\cancel{\sum_{i=1}^{n} \left\{ \prod_{k=i}^{k=n} (1-\beta_k) \right\} = 1}$

sum of coeffs of $R_i = 1$, i.e,

$$\beta_n + (1-\beta_n)(\beta_{n-1}) + (1-\beta_n)(1-\beta_{n-1})(\beta_{n-2}) + \ldots = 1$$