IMPERIAL

RCS Mini-Project

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1 Aim of the Project

We aim to plot the pathway of a sailboat in the sea, accounting for realistic forces and dynamic behaviour.

2 Theory

We must determine the specific effects of the various forces acting on a sailboat and plot the boat's movement.

To understand the coordinate axes, we have taken the positive x-axis along the stern, the positive y-direction along the starboard side, and the positive z-direction upwards of the sailboat.

The figure can further clarify this.

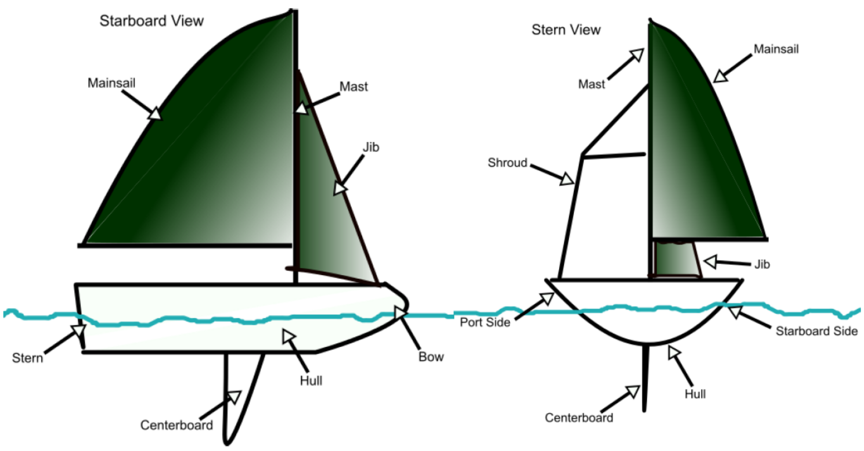

Figure 2.1: Figure 1. Parts of Sailboat [1]

The weight of the sailboat and sailors in total is given by:

$$F_{mg} = Mg = M * (-9.8)$$

The force acts downwards, so the acceleration due to gravity is taken as negative. The force of buoyancy for a stationary sailboat is defined as

$$F_b = \rho_w * V(A_w * z) * g = -F_{mg}.$$

Where A_w is the area of the boat underwater (taken to be constant), and F_b is considered equal to F_{mg}.

We also take into consideration the aerodynamic and hydrodynamic forces, which offer both drag and lift along all three axes.

We have,

$$F_{da} = 1/2 * \rho_a * Cd_a * A_{Sail} * v_{rel}^2$$

in the direction of v_{rel} (relative wind) where Cd_a is the drag coefficient due to air along all three axes, A_{sail} is the area of sail, and ρ_a is the density of air.

Also, the aerodynamic lift offered by the relative wind is calculated by

$$F_{lift} = 1/2 * \rho_a * Cl * A_{Sail} * v_{rel}^2$$

along the z-axis, where Cl is the lift coefficient along the z-direction.

We also have hydrodynamic drag forces offered by water, with different coefficients of water drag along the xy plane and z-axis. We use the following formulae for it,

$$F_{dxy} = 1/2 * \rho_w * Cd_w * A_{hull} * v_{xy}^2 \ \ (along\ v_{xy}\ direction)$$

$$F_{dz} = 1/2 * \rho_w * Cd_z * A_{hull} * v_z^2 \ \ (along\ v_z\ direction)$$

where Cd_w and Cd_z are the xy plane and z direction drag coefficients, respectively. The boat's velocity is taken as v_{xy}, and v_z in the same direction.

We also introduce an added sine wave function along the z-axis in the direction of the wind, depicting the sea waves in the sea.

$$z_w = A_w * \sin(k * (x * \cos(angle_w) + y * \sin(angle_w)) - \omega_w * t)$$

where the A_w, $angle_w$, ω_w and k are the wave amplitude, angle of the wind, angular frequency of the wave and wave number, respectively.

This is considered while calculating the forces of buoyancy along the z-axis. The total forces are calculated along the three directions as,

$$F_{total} = F_{aero} + F_{hydro} + F_{mg} + F_b$$

which further gives the acceleration along the three axes.

3 Algorithm

The algorithm can be outlined as follows:

- Set physical parameters for the boat (mass, hull area, sail area), environmental constants (air and water density and drag coefficients) and forces acting on the boat.
- Define the wind conditions, wave parameters, drag coefficients and the initial conditions (position and speed) of the boat.
- k_b is defined as the force of buoyancy divided by z. Define a time span for the simulation.
- Apply the ode45 function in MATLAB and obtain the numerical solution for the ordinary differential equations describing the motion containing the position along three axes.
- The boat function considers all forces due to wind, water, buoyancy, and gravity with separate functions.
- Extract from the solution provided by ode45 the boat's position, x, y, z.

- Use plot3 to plot the path of the boat in three-dimensional space and indicate the motion of the boat with red markers.
- Draw four-wave lines parallel to the boat's course in the direction of the wind. Loop through with four different offsets in the y direction to generate parallel wave lines.
- Using plot 3, graph the four-wave lines in blue.
- Determine the minimum and maximum values of the x, y, and z positions and define these limits of the plot.
- Label the x, y, and z axes.
- Set the title and configure the legend for the boat's path and the waves.
- Modify the grid, the axes and the font size. Configure the figure's size and layout.
- Export the plot to a PDF file

The idea behind this algorithm involves simulating and visualising a sailboat's movement in a windy, wavy environment, considering realistic forces and dynamic behaviour.

4 Output

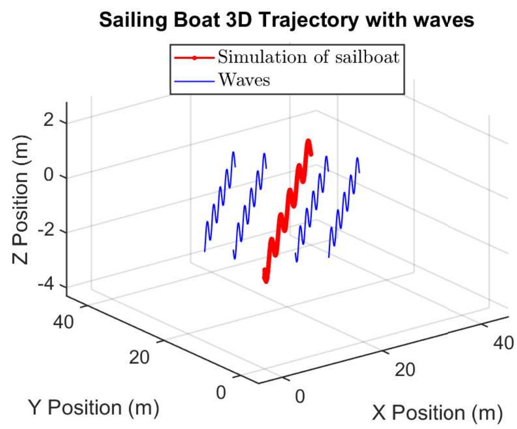

Figure 4.1: Output at wave amplitude of 0.5m along wind direction (output pdf)

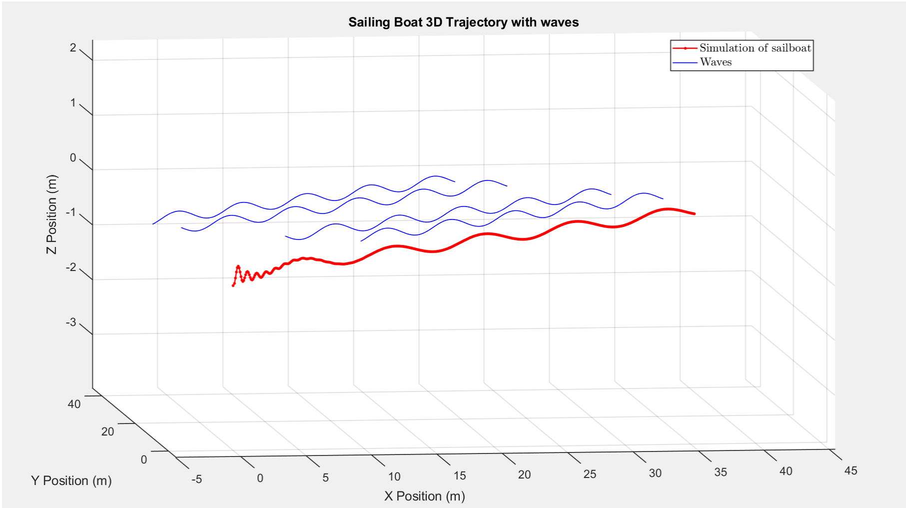

Figure 4.2: Output at wave amplitude of 0.1m along wind direction

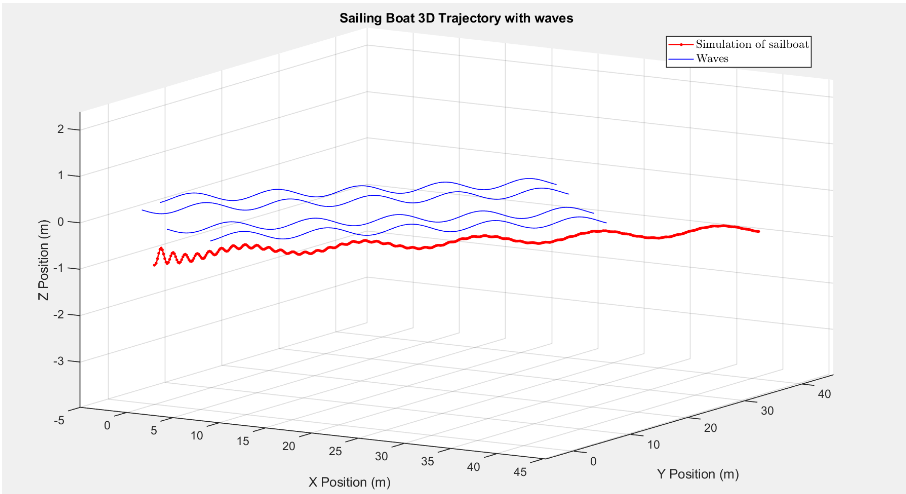

Figure 4.3: Output at wave amplitude of 0.1m along wind direction without considering the aerodynamic drag along z direction

5 Discussion

The following simulation shows the boat's motion in the sea. In the three plots, we have varied the wave amplitude in the first two from $A_w = 0.5$m to $A_w = 0.1$m. In the third plot, we have removed the aerodynamic drag along the z-axis to study the wavy motion more effectively and to investigate how drag along the z-axis reduces the boat's oscillations.

Bibliography

[1] K. Stukbauer, "The forces affecting a sailboat," *Undergraduate Journal of Mathematical Modeling: One+ Two*, vol. 8, no. 1, p. 2, 2017.