Report on M/M/1 Queuing Server Simulation and Analysis

Objective

The objective of this study was to simulate an M/M/1 queue under various system loads (ρ) and compare the simulated average number of customers in the system E[N] with the theoretical expectation given by the formula $E[N]=\rho/(1-\rho)$. The analysis included:

1. **Comparison of Simulated vs. Theoretical E[N]** across different loads.
2. **Impact of Total Number of Arrival Events** on the accuracy of the simulation.
3. **Effect of System Load on Simulation Accuracy** with a fixed number of arrival events.

Model Description

The M/M/1 queue model consists of:

- **Poisson Arrivals** at rate λ.
- **Exponential Service Times** at rate μ.
- **Single Server** and **Infinite Buffer**.

The system load, ρ, is defined as $\rho=\lambda/\mu$. At steady state, the theoretical expectation for the average number of customers in the system is $E[N]=\rho/(1-\rho)$.

Methodology

1. **Simulation Setup**:
   - We defined the interarrival and service times based on exponential distributions.
2. **Load (ρ) Variation**:
   - We varied ρ from 0.1 to 0.9 in increments of 0.1 and adjusted λ accordingly, keeping $\mu=3$ fixed.
3. **Calculations**:
   - For each ρ value, we computed:
     - **Simulated E[N]** as the time-weighted average of customers in the system.
     - **Theoretical E[N]** for comparison.
4. **Repetition with Different Arrival Events**:
   - We repeated the simulations with arrival counts of 1000, 10,000, and 100,000 to analyze how increasing the number of events impacts accuracy.

Results

1. Simulated vs. Theoretical E[N] across Different Loads

The simulation results for E[N] closely followed the theoretical curve E[N]=ρ/(1−ρ) across most values of ρ, demonstrating the robustness of the simulation approach. However, the results showed minor deviations at higher loads, particularly for ρ values closer to 1 (e.g., ρ=0.8 and ρ=0.9). This divergence at high loads could be attributed to the higher variance in waiting times as ρ approaches 1.

Plot Analysis: The E[N] vs. ρ curve for simulated data showed good agreement with the theoretical curve, confirming that the simulation model aligns well with theoretical expectations.

2. Impact of Total Number of Arrival Events on Simulation Accuracy

With an increase in the number of arrival events:

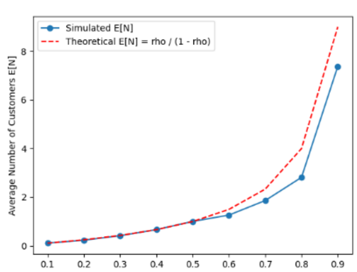
- The simulation accuracy improved, with deviations from theoretical values decreasing.
- For example, with 1000 arrivals, the simulated E[N] showed greater fluctuation around the theoretical values. However, with 10,000 and 100,000 arrivals, the results became more stable and consistent with the theoretical curve.

3. Effect of System Load on Simulation Accuracy

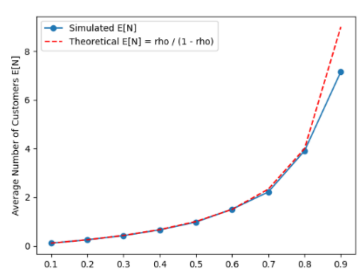
For a fixed number of arrival events, simulation accuracy decreased as the load increased:

- At lower ρ values (0.1 - 0.5), the simulated E[N] closely matched the theoretical values.
- As ρ approached 1, the system faced more congestion, with longer queues and increased waiting times, resulting in greater variance in the simulated E[N] values.

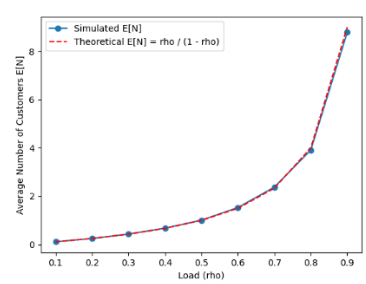
This pattern indicates that as load increases, the queue system becomes more sensitive to small variations in interarrival and service times, which amplifies the deviations from theoretical expectations.



Number of arrival events = 1000



Number of arrival events = 10,000



Number of arrival events = 100,000